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WIGGIN

AMERICAN

CIVIL ENGINEERS'

HANDBOOK

AMERICAN CIVIL ENGINEERS' HANDBOOK

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THADDEUS MERRIMAN

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FIFTH EDITION, THOROUGHLY REVISED AND ENLARGED

TOTAL ISSUE, FIFTY-FIVE THOUSAND

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The Publishers and the Editor-in-Chief will be grateful to readers of this volume who will kindly call attention to any errors of omission or of commission therein. It is intended to make our publications standard works of study and reference, and, to that end, the greatest accuracy is sought. It rarely happens that the early editions of books are free from errors; but it is the endeavor of the Publishers to have them removed, and it is therefore desired that the Editor-in-Chief may be aided in his task of revision, from time to time, by the kindly criticism of readers.

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MANSFIELD MERRIMAN

MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

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EDWARD R. MAURER

PROFESSOR OF MECHANICS IN UNIVERSITY OF WISCONSIN

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CHEMISTRY, PHYSICS, METEOROLOGY, WEIGHTS AND MEASURES

ORIGINALLY WRITTEN BY

LOUIS A. FISCHER

LATE CHIEF OF WEIGHTS AND MEASURES DIVISION OF U. S. BUREAU OF STANDARDS

Revised under direction of Wilmer Souder, Physicist, National Bureau of Standards

For Titles of Articles in Section 3 see page 241

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FORMERLY CHIEF OF THE BUREAU OF YARDS AND DOCKS OF THE NAVY
MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

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JOHN S. CRANDELL

PROFESSOR OF HIGHWAY ENGINEERING IN UNIVERSITY OF ILLINOIS

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FRED. ASA BARNES

DIRECTOR OF THE SCHOOL OF CIVIL ENGINEERS AND PROFESSOR RAILROAD ENGINEERING IN CORNELL UNIVERSITY
MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

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WILLIAM A. DELMAR

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AND

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26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975
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75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957
	0	1	2	3	4	5	6	7	8	9

2. Logarithms of Trigonometric Functions

Angle	Log Arc	Log Sin	Log Tan	Log Sec	Log Csc	Log Cot	Log Cos		
1°	$\bar{2}.2419$	$\bar{2}.2419$	$\bar{2}.2419$	0.0001	1.7581	1.7581	$\bar{1}.9999$	0.1913	89
2	$\bar{2}.5429$	$\bar{2}.5428$	$\bar{2}.5431$	0.0003	1.4572	1.4569	$\bar{1}.9997$	0.1864	88
3	$\bar{2}.7190$	$\bar{2}.7188$	$\bar{2}.7194$	0.0006	1.2812	1.2806	$\bar{1}.9994$	0.1814	87
4	$\bar{2}.8439$	$\bar{2}.8436$	$\bar{2}.8446$	0.0011	1.1564	1.1554	$\bar{1}.9980$	0.1764	86
5	$\bar{2}.9408$	$\bar{2}.9403$	$\bar{2}.9420$	0.0017	1.0597	1.0580	$\bar{1}.9983$	0.1713	85°
6°	$\bar{1}.0200$	$\bar{1}.0192$	$\bar{1}.0216$	0.0024	0.9808	0.9784	$\bar{1}.9976$	0.1662	84
7	$\bar{1}.0870$	$\bar{1}.0859$	$\bar{1}.0891$	0.0032	0.9141	0.9109	$\bar{1}.9968$	0.1610	83
8	$\bar{1}.1450$	$\bar{1}.1436$	$\bar{1}.1478$	0.0042	0.8564	0.8522	$\bar{1}.9958$	0.1557	82
9	$\bar{1}.1961$	$\bar{1}.1943$	$\bar{1}.1997$	0.0054	0.8057	0.8003	$\bar{1}.9946$	0.1504	81
10	$\bar{1}.2419$	$\bar{1}.2397$	$\bar{1}.2463$	0.0066	0.7603	0.7537	$\bar{1}.9934$	0.1450	80°
11°	$\bar{1}.2833$	$\bar{1}.2806$	$\bar{1}.2887$	0.0081	0.7194	0.7113	$\bar{1}.9919$	0.1395	79
12	$\bar{1}.3211$	$\bar{1}.3179$	$\bar{1}.3275$	0.0096	0.6821	0.6725	$\bar{1}.9904$	0.1340	78
13	$\bar{1}.3558$	$\bar{1}.3521$	$\bar{1}.3634$	0.0113	0.6479	0.6366	$\bar{1}.9887$	0.1284	77
14	$\bar{1}.3880$	$\bar{1}.3837$	$\bar{1}.3968$	0.0131	0.6163	0.6032	$\bar{1}.9869$	0.1227	76
15	$\bar{1}.4180$	$\bar{1}.4130$	$\bar{1}.4281$	0.0151	0.5870	0.5719	$\bar{1}.9849$	0.1169	75°
16°	$\bar{1}.4460$	$\bar{1}.4403$	$\bar{1}.4575$	0.0172	0.5597	0.5425	$\bar{1}.9828$	0.1111	74
17	$\bar{1}.4723$	$\bar{1}.4659$	$\bar{1}.4853$	0.0194	0.5341	0.5147	$\bar{1}.9806$	0.1052	73
18	$\bar{1}.4971$	$\bar{1}.4900$	$\bar{1}.5118$	0.0218	0.5100	0.4882	$\bar{1}.9782$	0.0992	72
19	$\bar{1}.5206$	$\bar{1}.5126$	$\bar{1}.5370$	0.0243	0.4874	0.4630	$\bar{1}.9757$	0.0931	71
20	$\bar{1}.5429$	$\bar{1}.5341$	$\bar{1}.5611$	0.0270	0.4659	0.4389	$\bar{1}.9730$	0.0870	70°
21°	$\bar{1}.5641$	$\bar{1}.5543$	$\bar{1}.5842$	0.0298	0.4457	0.4158	$\bar{1}.9702$	0.0807	69
22	$\bar{1}.5843$	$\bar{1}.5736$	$\bar{1}.6064$	0.0328	0.4264	0.3936	$\bar{1}.9672$	0.0744	68
23	$\bar{1}.6036$	$\bar{1}.5919$	$\bar{1}.6279$	0.0360	0.4081	0.3721	$\bar{1}.9640$	0.0680	67
24	$\bar{1}.6221$	$\bar{1}.6093$	$\bar{1}.6486$	0.0393	0.3907	0.3514	$\bar{1}.9607$	0.0614	66
25	$\bar{1}.6398$	$\bar{1}.6259$	$\bar{1}.6687$	0.0427	0.3741	0.3313	$\bar{1}.9573$	0.0548	65°
26°	$\bar{1}.6569$	$\bar{1}.6418$	$\bar{1}.6882$	0.0463	0.3582	0.3118	$\bar{1}.9537$	0.0481	64
27	$\bar{1}.6732$	$\bar{1}.6570$	$\bar{1}.7072$	0.0501	0.3430	0.2928	$\bar{1}.9499$	0.0412	63
28	$\bar{1}.6890$	$\bar{1}.6716$	$\bar{1}.7257$	0.0541	0.3284	0.2743	$\bar{1}.9459$	0.0343	62
29	$\bar{1}.7042$	$\bar{1}.6856$	$\bar{1}.7438$	0.0582	0.3144	0.2562	$\bar{1}.9418$	0.0272	61
30	$\bar{1}.7190$	$\bar{1}.6990$	$\bar{1}.7614$	0.0625	0.3010	0.2386	$\bar{1}.9375$	0.0200	60°
31°	$\bar{1}.7332$	$\bar{1}.7118$	$\bar{1}.7788$	0.0669	0.2882	0.2212	$\bar{1}.9331$	0.0127	59
32	$\bar{1}.7470$	$\bar{1}.7242$	$\bar{1}.7958$	0.0716	0.2758	0.2042	$\bar{1}.9284$	0.0053	58
33	$\bar{1}.7604$	$\bar{1}.7361$	$\bar{1}.8125$	0.0764	0.2639	0.1875	$\bar{1}.9236$	$\bar{1}.9978$	57
34	$\bar{1}.7734$	$\bar{1}.7476$	$\bar{1}.8290$	0.0814	0.2524	0.1710	$\bar{1}.9186$	$\bar{1}.9901$	56
35	$\bar{1}.7859$	$\bar{1}.7586$	$\bar{1}.8452$	0.0866	0.2414	0.1548	$\bar{1}.9134$	$\bar{1}.9822$	55°
36°	$\bar{1}.7982$	$\bar{1}.7692$	$\bar{1}.8613$	0.0920	0.2308	0.1387	$\bar{1}.9080$	$\bar{1}.9743$	54
37	$\bar{1}.8101$	$\bar{1}.7795$	$\bar{1}.8771$	0.0977	0.2205	0.1229	$\bar{1}.9023$	$\bar{1}.9662$	53
38	$\bar{1}.8217$	$\bar{1}.7893$	$\bar{1}.8928$	0.1035	0.2107	0.1072	$\bar{1}.8965$	$\bar{1}.9579$	52
39	$\bar{1}.8329$	$\bar{1}.7989$	$\bar{1}.9084$	0.1095	0.2011	0.0916	$\bar{1}.8905$	$\bar{1}.9494$	51
40	$\bar{1}.8439$	$\bar{1}.8081$	$\bar{1}.9238$	0.1157	0.1919	0.0762	$\bar{1}.8843$	$\bar{1}.9408$	50°
41°	$\bar{1}.8547$	$\bar{1}.8169$	$\bar{1}.9392$	0.1222	0.1831	0.0608	$\bar{1}.8778$	$\bar{1}.9321$	49
42	$\bar{1}.8651$	$\bar{1}.8255$	$\bar{1}.9544$	0.1289	0.1745	0.0456	$\bar{1}.8711$	$\bar{1}.9231$	48
43	$\bar{1}.8753$	$\bar{1}.8338$	$\bar{1}.9697$	0.1359	0.1662	0.0303	$\bar{1}.8641$	$\bar{1}.9140$	47
44	$\bar{1}.8853$	$\bar{1}.8418$	$\bar{1}.9848$	0.1431	0.1582	0.0152	$\bar{1}.8569$	$\bar{1}.9046$	46
45	$\bar{1}.8951$	$\bar{1}.8495$	0.0000	0.1505	0.1505	0.0000	$\bar{1}.8495$	$\bar{1}.8951$	45°
		Log Cos	Log Cot	Log Csc	Log Sec	Log Tan	Log Sin	Log Arc	Angle

Explanation on p. 37.

3. Natural Trigonometric Functions

Angle	Arc	Sin	Tan	Sec	Cosec	Cot	Cos		
1°	0.0175	0.0175	0.0175	1.0002	57.299	57.290	0.9998	1.5533	89
2	0.0349	0.0349	0.0349	1.0006	28.654	28.636	0.9994	1.5359	88
3	0.0524	0.0523	0.0524	1.0014	19.107	19.081	0.9986	1.5184	87
4	0.0698	0.0698	0.0699	1.0024	14.336	14.301	0.9976	1.5010	86
5	0.0873	0.0872	0.0875	1.0038	11.474	11.430	0.9962	1.4835	85°
6°	0.1047	0.1045	0.1051	1.0055	9.5668	9.5144	0.9945	1.4661	84
7	0.1222	0.1219	0.1228	1.0075	8.2055	8.1443	0.9925	1.4486	83
8	0.1396	0.1392	0.1405	1.0098	7.1853	7.1154	0.9903	1.4312	82
9	0.1571	0.1564	0.1584	1.0125	6.3925	6.3138	0.9877	1.4137	81
10	0.1745	0.1736	0.1763	1.0154	5.7588	5.6713	0.9848	1.3963	80°
11°	0.1920	0.1908	0.1944	1.0187	5.2408	5.1446	0.9816	1.3788	79
12	0.2094	0.2079	0.2126	1.0223	4.8097	4.7046	0.9781	1.3614	78
13	0 2269	0.2250	0.2309	1.0263	4.4454	4.3315	0.9744	1.3439	77
14	0.2443	0.2419	0.2493	1.0306	4.1336	4.0108	0.9703	1.3265	76
15	0.2618	0.2588	0.2679	1.0353	3.8637	3.7321	0.9659	1.3090	75°
16°	0.2793	0.2756	0.2867	1.0403	3.6280	3.4874	0.9613	1.2915	74
17	0.2967	0.2924	0.3057	1.0457	3.4203	3.2709	0.9563	1.2741	73
18	0.3142	0.3090	0.3249	1.0515	3.2361	3.0777	0.9511	1.2566	72
19	0.3316	0.3256	0.3443	1.0576	3.0716	2.9042	0.9455	1.2392	71
20	0.3491	0.3420	0.3640	1.0642	2.9238	2.7475	0.9397	1.2217	70°
21°	0.3665	0.3584	0.3839	1.0711	2.7904	2.6051	0.9336	1.2043	69
22	0.3840	0.3746	0.4040	1.0785	2.6695	2.4751	0.9272	1.1868	68
23	0.4014	0.3907	0.4245	1.0864	2.5593	2.3559	0.9205	1.1694	67
24	0.4189	0.4067	0.4452	1.0946	2.4586	2.2460	0.9135	1.1519	66
25	0.4363	0.4226	0.4663	1.1034	2.3662	2.1445	0.9063	1.1345	65°
26°	0.4538	0.4384	0.4877	1.1126	2.2812	2.0503	0.8988	1.1170	64
27	0.4712	0.4540	0.5095	1.1223	2.2027	1.9626	0.8910	1.0996	63
28	0.4887	0.4695	0.5317	1.1326	2.1301	1.8807	0.8829	1.0821	62
29	0.5061	0.4848	0.5543	1.1434	2.0627	1.8040	0.8746	1.0647	61
30	0.5236	0.5000	0.5774	1.1547	2.0000	1.7321	0.8660	1.0472	60°
31°	0.5411	0.5150	0.6009	1.1666	1.9416	1.6643	0.8572	1.0297	59
32	0.5585	0.5299	0.6249	1.1792	1.8871	1.6003	0.8480	1.0123	58
33	0.5760	0.5446	0.6494	1.1924	1.8361	1.5399	0.8387	0.9948	57
34	0.5934	0.5592	0.6745	1.2062	1.7883	1.4826	0.8290	0.9774	56
35	0.6109	0.5736	0.7002	1.2208	1.7434	1.4281	0.8192	0.9599	55°
36°	0.6283	0.5878	0.7265	1.2361	1.7013	1.3764	0.8090	0.9425	54
37	0.6458	0.6018	0.7536	1.2521	1.6616	1.3270	0.7986	0.9250	53
38	0.6632	0.6157	0.7813	1.2690	1.6243	1.2799	0.7880	0.9076	52
39	0.6807	0.6293	0.8098	1.2868	1.5890	1.2349	0.7771	0.8901	51
40	0.6981	0.6428	0.8391	1.3054	1.5557	1.1918	0.7660	0.8727	50°
41°	0.7156	0.6561	0.8693	1.3250	1.5243	1.1504	0.7547	0.8552	49
42	0.7330	0.6691	0.9004	1.3456	1.4945	1.1106	0.7431	0.8378	48
43	0.7505	0.6820	0.9325	1.3673	1.4663	1.0724	0.7314	0.8203	47
44	0.7679	0.6947	0.9657	1.3902	1.4396	1.0355	0.7193	0.8029	46
45	0.7854	0.7071	1.0000	1.4142	1.4142	1.0000	0.7071	0.7854	45°
		Cos	Cot	Cosec	Sec	Tan	Sin	Arc	Angle

Explanation on p. 38

4. Reciprocals

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
0.10	10.000	9.901	9.804	9.709	9.615	9.524	9.434	9.346	9.259	9.174
0.11	9.091	9.009	8.929	8.850	8.772	8.696	8.621	8.547	8.475	8.403
0.12	8.333	8.264	8.197	8.130	8.065	8.000	7.937	7.874	7.813	7.752
0.13	7.692	7.634	7.576	7.519	7.463	7.407	7.353	7.299	7.246	7.194
0.14	7.143	7.092	7.042	6.993	6.944	6.897	6.849	6.803	6.757	6.711
0.15	6.667	6.623	6.579	6.536	6.494	6.452	6.410	6.369	6.329	6.289
0.16	6.250	6.211	6.173	6.135	6.098	6.061	6.024	5.988	5.952	5.917
0.17	5.882	5.848	5.814	5.780	5.747	5.714	5.682	5.650	5.618	5.587
0.18	5.556	5.525	5.495	5.464	5.435	5.405	5.376	5.348	5.319	5.291
0.19	5.263	5.236	5.208	5.181	5.155	5.128	5.102	5.076	5.051	5.025
0.20	5.000	4.975	4.950	4.926	4.902	4.878	4.854	4.831	4.808	4.785
0.21	4.762	4.739	4.717	4.695	4.673	4.651	4.630	4.608	4.587	4.566
0.22	4.545	4.525	4.505	4.484	4.464	4.444	4.425	4.405	4.386	4.367
0.23	4.348	4.329	4.310	4.292	4.274	4.255	4.237	4.219	4.202	4.184
0.24	4.167	4.149	4.132	4.115	4.098	4.082	4.065	4.049	4.032	4.016
0.25	4.000	3.984	3.968	3.953	3.937	3.922	3.906	3.891	3.876	3.861
0.26	3.846	3.831	3.817	3.802	3.788	3.774	3.759	3.745	3.731	3.717
0.27	3.704	3.690	3.676	3.663	3.650	3.636	3.623	3.610	3.597	3.584
0.28	3.571	3.559	3.546	3.534	3.521	3.509	3.497	3.484	3.472	3.460
0.29	3.448	3.436	3.425	3.413	3.401	3.390	3.378	3.367	3.356	3.344
0.30	3.333	3.322	3.311	3.300	3.289	3.279	3.268	3.257	3.247	3.236
0.31	3.226	3.215	3.205	3.195	3.185	3.175	3.165	3.155	3.145	3.135
0.32	3.125	3.115	3.106	3.096	3.086	3.077	3.067	3.058	3.049	3.040
0.33	3.030	3.021	3.012	3.003	2.994	2.985	2.976	2.967	2.959	2.950
0.34	2.941	2.933	2.924	2.915	2.907	2.899	2.890	2.882	2.874	2.865
0.35	2.857	2.849	2.841	2.833	2.825	2.817	2.809	2.801	2.793	2.786
0.36	2.778	2.770	2.762	2.755	2.747	2.740	2.732	2.725	2.717	2.710
0.37	2.703	2.695	2.688	2.681	2.674	2.667	2.660	2.653	2.646	2.639
0.38	2.632	2.625	2.618	2.611	2.604	2.597	2.591	2.584	2.577	2.571
0.39	2.564	2.558	2.551	2.545	2.538	2.532	2.525	2.519	2.513	2.506
0.40	2.500	2.494	2.488	2.481	2.475	2.469	2.463	2.457	2.451	2.445
0.41	2.439	2.433	2.427	2.421	2.415	2.410	2.404	2.398	2.392	2.387
0.42	2.381	2.375	2.370	2.364	2.358	2.353	2.347	2.342	2.336	2.331
0.43	2.326	2.320	2.315	2.309	2.304	2.299	2.294	2.288	2.283	2.278
0.44	2.273	2.268	2.262	2.257	2.252	2.247	2.242	2.237	2.232	2.227
0.45	2.222	2.217	2.212	2.208	2.203	2.198	2.193	2.188	2.183	2.179
0.46	2.174	2.169	2.165	2.160	2.155	2.151	2.146	2.141	2.137	2.132
0.47	2.128	2.123	2.119	2.114	2.110	2.105	2.101	2.096	2.092	2.088
0.48	2.083	2.079	2.075	2.070	2.066	2.062	2.058	2.053	2.049	2.045
0.49	2.041	2.037	2.033	2.028	2.024	2.020	2.016	2.012	2.008	2.004
0.50	2.000	1.996	1.992	1.988	1.984	1.980	1.976	1.972	1.969	1.965
0.51	1.961	1.957	1.953	1.949	1.946	1.942	1.938	1.934	1.931	1.927
0.52	1.923	1.919	1.916	1.912	1.908	1.905	1.901	1.898	1.894	1.890
0.53	1.887	1.883	1.880	1.876	1.873	1.869	1.866	1.862	1.859	1.855
0.54	1.852	1.848	1.845	1.842	1.838	1.835	1.832	1.828	1.825	1.821
n	0	1	2	3	4	5	6	7	8	9

of Numbers

n	0	1	2	3	4	5	6	7	8	9
0.55	1.818	1.815	1.812	1.808	1.805	1.802	1.799	1.795	1.792	1.789
0.56	1.786	1.783	1.779	1.776	1.773	1.770	1.767	1.764	1.761	1.757
0.57	1.754	1.751	1.748	1.745	1.742	1.739	1.736	1.733	1.730	1.727
0.58	1.724	1.721	1.718	1.715	1.712	1.709	1.706	1.704	1.701	1.698
0.59	1.695	1.692	1.689	1.686	1.684	1.681	1.678	1.675	1.672	1.669
0.60	1.667	1.664	1.661	1.658	1.656	1.653	1.650	1.647	1.645	1.642
0.61	1.639	1.637	1.634	1.631	1.629	1.626	1.623	1.621	1.618	1.616
0.62	1.613	1.610	1.608	1.605	1.603	1.600	1.597	1.595	1.592	1.590
0.63	1.587	1.585	1.582	1.580	1.577	1.575	1.572	1.570	1.567	1.565
0.64	1.562	1.560	1.558	1.555	1.553	1.550	1.548	1.546	1.543	1.541
0.65	1.538	1.536	1.534	1.531	1.529	1.527	1.524	1.522	1.520	1.517
0.66	1.515	1.513	1.511	1.508	1.506	1.504	1.502	1.499	1.497	1.495
0.67	1.493	1.490	1.488	1.486	1.484	1.481	1.479	1.477	1.475	1.473
0.68	1.471	1.468	1.466	1.464	1.462	1.460	1.458	1.456	1.453	1.451
0.69	1.449	1.447	1.445	1.443	1.441	1.439	1.437	1.435	1.433	1.431
0.70	1.429	1.427	1.425	1.422	1.420	1.418	1.416	1.414	1.412	1.410
0.71	1.408	1.406	1.404	1.403	1.401	1.399	1.397	1.395	1.393	1.391
0.72	1.389	1.387	1.385	1.383	1.381	1.379	1.377	1.376	1.374	1.372
0.73	1.370	1.368	1.366	1.364	1.362	1.361	1.359	1.357	1.355	1.353
0.74	1.351	1.350	1.348	1.346	1.344	1.342	1.340	1.339	1.337	1.335
0.75	1.333	1.332	1.330	1.328	1.326	1.325	1.323	1.321	1.319	1.318
0.76	1.316	1.314	1.312	1.311	1.309	1.307	1.305	1.304	1.302	1.300
0.77	1.299	1.297	1.295	1.294	1.292	1.290	1.289	1.287	1.285	1.284
0.78	1.282	1.280	1.279	1.277	1.276	1.274	1.272	1.271	1.269	1.267
0.79	1.266	1.264	1.263	1.261	1.259	1.258	1.256	1.255	1.253	1.252
0.80	1.250	1.248	1.247	1.245	1.244	1.242	1.241	1.239	1.238	1.236
0.81	1.235	1.233	1.232	1.230	1.229	1.227	1.225	1.224	1.222	1.221
0.82	1.220	1.218	1.217	1.215	1.214	1.212	1.211	1.209	1.208	1.206
0.83	1.205	1.203	1.202	1.200	1.199	1.198	1.196	1.195	1.193	1.192
0.84	1.190	1.189	1.188	1.186	1.185	1.183	1.182	1.181	1.179	1.178
0.85	1.176	1.175	1.174	1.172	1.171	1.170	1.168	1.167	1.166	1.164
0.86	1.163	1.161	1.160	1.159	1.157	1.156	1.155	1.153	1.152	1.151
0.87	1.149	1.148	1.147	1.145	1.144	1.143	1.142	1.140	1.139	1.138
0.88	1.136	1.135	1.134	1.133	1.131	1.130	1.129	1.127	1.126	1.125
0.89	1.124	1.122	1.121	1.120	1.119	1.117	1.116	1.115	1.114	1.112
0.90	1.111	1.110	1.109	1.107	1.106	1.105	1.104	1.103	1.101	1.100
0.91	1.099	1.098	1.096	1.095	1.094	1.093	1.092	1.091	1.089	1.088
0.92	1.087	1.086	1.085	1.083	1.082	1.081	1.080	1.079	1.078	1.076
0.93	1.075	1.074	1.073	1.072	1.071	1.070	1.068	1.067	1.066	1.065
0.94	1.064	1.063	1.062	1.060	1.059	1.058	1.057	1.056	1.055	1.054
0.95	1.053	1.052	1.050	1.049	1.048	1.047	1.046	1.045	1.044	1.043
0.96	1.042	1.041	1.040	1.038	1.037	1.036	1.035	1.034	1.033	1.032
0.97	1.031	1.030	1.029	1.028	1.027	1.026	1.025	1.024	1.022	1.021
0.98	1.020	1.019	1.018	1.017	1.016	1.015	1.014	1.013	1.012	1.011
0.99	1.010	1.009	1.008	1.007	1.006	1.005	1.004	1.003	1.002	1.001
n	0	1	2	3	4	5	6	7	8	9

5. Squares of Num-

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.020	1.040	1.061	1.082	1.103	1.124	1.145	1.166	1.188
1.1	1.210	1.232	1.254	1.277	1.300	1.323	1.346	1.369	1.392	1.416
1.2	1.440	1.464	1.488	1.513	1.538	1.563	1.588	1.613	1.638	1.664
1.3	1.690	1.716	1.742	1.769	1.796	1.823	1.850	1.877	1.904	1.932
1.4	1.960	1.988	2.016	2.045	2.074	2.103	2.132	2.161	2.190	2.220
1.5	2.250	2.280	2.310	2.341	2.372	2.403	2.434	2.465	2.496	2.528
1.6	2.560	2.592	2.624	2.657	2.690	2.723	2.756	2.789	2.822	2.856
1.7	2.890	2.924	2.958	2.993	3.028	3.063	3.098	3.133	3.168	3.204
1.8	3.240	3.276	3.312	3.349	3.386	3.423	3.460	3.497	3.534	3.572
1.9	3.610	3.648	3.686	3.725	3.764	3.803	3.842	3.881	3.920	3.960
2.0	4.000	4.040	4.080	4.121	4.162	4.203	4.244	4.285	4.326	4.368
2.1	4.410	4.452	4.494	4.537	4.580	4.623	4.666	4.709	4.752	4.796
2.2	4.840	4.884	4.928	4.973	5.018	5.063	5.108	5.153	5.198	5.244
2.3	5.290	5.336	5.382	5.429	5.476	5.523	5.570	5.617	5.664	5.712
2.4	5.760	5.808	5.856	5.905	5.954	6.003	6.052	6.101	6.150	6.200
2.5	6.250	6.300	6.350	6.401	6.452	6.503	6.554	6.605	6.656	6.708
2.6	6.760	6.812	6.864	6.917	6.970	7.023	7.076	7.129	7.182	7.236
2.7	7.290	7.344	7.398	7.453	7.508	7.563	7.618	7.673	7.728	7.784
2.8	7.840	7.896	7.952	8.009	8.066	8.123	8.180	8.237	8.294	8.352
2.9	8.410	8.468	8.526	8.585	8.644	8.703	8.762	8.821	8.880	8.940
3.0	9.000	9.060	9.120	9.181	9.242	9.303	9.364	9.425	9.486	9.548
3.1	9.610	9.672	9.734	9.797	9.860	9.923	9.986	10.05	10.11	10.18
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27
4.4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94
4.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91
4.9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98
5.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14
n	0	1	2	3	4	5	6	7	8	9

bers from 1.00 to 9.99

n	0	1	2	3	4	5	6	7	8	9
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38
5.7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72
8.3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39
8.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90
9.7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80
n	0	1	2	3	4	5	6	7	8	9

6. Square Roots of

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091
1.2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136
1.3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261
1.6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338
1.8	1.342	1.345	1.349	1.353	1.355	1.360	1.364	1.367	1.371	1.375
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609
2.6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700
2.9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758
3.1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814
3.3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841
3.4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895
3.6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921
3.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947
3.8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972
3.9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022
4.1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095
4.4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142
4.6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166
4.7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211
4.9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256
5.1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278
5.2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322
5.4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343
n	0	1	2	3	4	5	6	7	8	9

Numbers from 1.00 to 99.9

Continued on p. 20

n	0	1	2	3	4	5	6	7	8	9
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364
5.6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385
5.7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406
5.8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427
5.9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468
6.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488
6.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508
6.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	5.528
6.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567
6.6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587
6.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606
6.8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625
6.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663
7.1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681
7.2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700
7.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718
7.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755
7.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773
7.7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791
7.8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809
7.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844
8.1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862
8.2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879
8.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897
8.4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931
8.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948
8.7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965
8.8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982
8.9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015
9.1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032
9.2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048
9.3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064
9.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097
9.6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113
9.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129
9.8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145
9.9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161
n	0	1	2	3	4	5	6	7	8	9

Square Roots of

Continued from p. 19

n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
10	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302
11	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450
12	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592
13	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728
14	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987
16	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111
17	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231
18	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347
19	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572
21	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680
22	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785
23	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889
24	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990
25	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089
26	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187
27	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282
28	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376
29	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468
30	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559
31	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648
32	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736
33	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822
34	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908
35	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992
36	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075
37	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156
38	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237
39	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317
40	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395
41	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473
42	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550
43	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626
44	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701
45	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775
46	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848
47	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921
48	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993
49	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064
50	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134
51	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204
52	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273
53	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342
54	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409
n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9

Numbers from 1.00 to 99.9

n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
55	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477
56	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543
57	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609
58	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675
59	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740
60	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804
61	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868
62	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931
63	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994
64	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056
65	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118
66	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179
67	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240
68	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301
69	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361
70	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420
71	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479
72	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538
73	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597
74	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654
75	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712
76	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769
77	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826
78	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883
79	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939
80	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994
81	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050
82	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105
83	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160
84	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214
85	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268
86	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322
87	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375
88	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429
89	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482
90	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534
91	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586
92	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638
93	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690
94	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742
95	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793
96	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844
97	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894
98	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945
99	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995
n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9

7. Cubes of Num-

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685
1.2	1.728	1.772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	4.020
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735
1.8	5.832	5.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	8.870	8.999	9.129
2.1	9.261	9.394	9.528	9.664	9.800	9.938	10.08	10.22	10.36	10.50
2.2	10.65	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01
2.3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	13.48	13.65
2.4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44
2.5	15.62	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37
2.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47
2.7	19.68	19.90	20.12	20.35	20.57	20.80	21.02	21.25	21.48	21.72
2.8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14
2.9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	26.46	26.73
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50
3.1	29.79	30.08	30.37	30.66	30.96	31.26	31.55	31.86	32.16	32.46
3.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61
3.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96
3.4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51
3.5	42.88	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42
4.1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52
4.5	91.12	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70
4.6	97.34	97.97	98.61	99.25	99.90	100.5	101.2	101.8	102.5	103.2
4.7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9
4.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9
4.9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9
5.1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8
5.2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0
5.3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6
5.4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5
n	0	1	2	3	4	5	6	7	8	9

bers from 1.00 to 9.99

n	0	1	2	3	4	5	6	7	8	9
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7
5.6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2
5.7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1
5.8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3
5.9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9
6.1	227.0	228.1	229.2	230.3	231.5	232.6	233.7	234.9	236.0	237.2
6.2	238.3	239.5	240.6	241.8	243.0	244.1	245.3	246.5	247.7	248.9
6.3	250.0	251.2	252.4	253.6	254.8	256.0	257.3	258.5	259.7	260.9
6.4	262.1	263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4
6.5	274.6	275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2
6.6	287.5	288.8	290.1	291.4	292.8	294.1	295.4	296.7	298.1	299.4
6.7	300.8	302.1	303.5	304.8	306.2	307.5	308.9	310.3	311.7	313.0
6.8	314.4	315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1
6.9	328.5	329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5
7.0	343.0	344.5	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4
7.1	357.9	359.4	360.9	362.5	364.0	365.5	367.1	368.6	370.1	371.7
7.2	373.2	374.8	376.4	377.9	379.5	381.1	382.7	384.2	385.8	387.4
7.3	389.0	390.6	392.2	393.8	395.4	397.1	398.7	400.3	401.9	403.6
7.4	405.2	406.9	408.5	410.2	411.8	413.5	415.2	416.8	418.5	420.2
7.5	421.9	423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5	437.2
7.6	439.0	440.7	442.5	444.2	445.9	447.7	449.5	451.2	453.0	454.8
7.7	456.5	458.3	460.1	461.9	463.7	465.5	467.3	469.1	470.9	472.7
7.8	474.6	476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2
7.9	493.0	494.9	496.8	498.7	500.6	502.5	504.4	506.3	508.2	510.1
8.0	512.0	513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5	529.5
8.1	531.4	533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4
8.2	551.4	553.4	555.4	557.4	559.5	561.5	563.6	565.6	567.7	569.7
8.3	571.8	573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5	590.6
8.4	592.7	594.8	596.9	599.1	601.2	603.4	605.5	607.6	609.8	612.0
8.5	614.1	616.3	618.5	620.7	622.8	625.0	627.2	629.4	631.6	633.8
8.6	636.1	638.3	640.5	642.7	645.0	647.2	649.5	651.7	654.0	656.2
8.7	658.5	660.8	663.1	665.3	667.6	669.9	672.2	674.5	676.8	679.2
8.8	681.5	683.8	686.1	688.5	690.8	693.2	695.5	697.9	700.2	702.6
8.9	705.0	707.3	709.7	712.1	714.5	716.9	719.3	721.7	724.2	726.6
9.0	729.0	731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1
9.1	753.6	756.1	758.6	761.0	763.6	766.1	768.6	771.1	773.6	776.2
9.2	778.7	781.2	783.8	786.3	788.9	791.5	794.0	796.6	799.2	801.8
9.3	804.4	807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9
9.4	830.6	833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7
9.5	857.4	860.1	862.8	865.5	868.3	871.0	873.7	876.5	879.2	882.0
9.6	884.7	887.5	890.3	893.1	895.8	898.6	901.4	904.2	907.0	909.9
9.7	912.7	915.5	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3
9.8	941.2	944.1	947.0	949.9	952.8	955.7	958.6	961.5	964.4	967.4
9.9	970.3	973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0
n	0	1	2	3	4	5	6	7	8	9

8. Cube Roots of Numbers

n	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	n	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
10	2.1544	4.6416	10.000	55	3.8030	8.1932	17.652
11	2.2240	4.7914	10.323	56	3.8259	8.2426	17.758
12	2.2894	4.9324	10.627	57	3.8485	8.2913	17.863
13	2.3513	5.0658	10.914	58	3.8709	8.3396	17.967
14	2.4101	5.1925	11.187	59	3.8930	8.3872	18.070
15	2.4662	5.3133	11.447	60	3.9149	8.4343	18.171
16	2.5198	5.4288	11.696	61	3.9365	8.4809	18.272
17	2.5713	5.5397	11.935	62	3.9579	8.5270	18.371
18	2.6207	5.6462	12.164	63	3.9791	8.5726	18.469
19	2.6684	5.7489	12.386	64	4.0000	8.6177	18.566
20	2.7144	5.8480	12.599	65	4.0207	8.6624	18.663
21	2.7589	5.9439	12.806	66	4.0412	8.7066	18.758
22	2.8020	6.0368	13.006	67	4.0615	8.7503	18.852
23	2.8439	6.1269	13.200	68	4.0817	8.7937	18.945
24	2.8845	6.2145	13.389	69	4.1016	8.8366	19.038
25	2.9240	6.2996	13.572	70	4.1213	8.8790	19.129
26	2.9625	6.3825	13.751	71	4.1408	8.9211	19.220
27	3.0000	6.4633	13.925	72	4.1602	8.9628	19.310
28	3.0366	6.5421	14.095	73	4.1793	9.0041	19.399
29	3.0723	6.6191	14.260	74	4.1983	9.0450	19.487
30	3.1072	6.6943	14.422	75	4.2172	9.0856	19.574
31	3.1414	6.7679	14.581	76	4.2358	9.1258	19.661
32	3.1748	6.8399	14.736	77	4.2543	9.1657	19.747
33	3.2075	6.9104	14.888	78	4.2727	9.2052	19.832
34	3.2396	6.9795	15.037	79	4.2908	9.2443	19.916
35	3.2711	7.0473	15.183	80	4.3089	9.2832	20.000
36	3.3019	7.1138	15.326	81	4.3267	9.3217	20.083
37	3.3322	7.1791	15.467	82	4.3445	9.3599	20.165
38	3.3620	7.2432	15.605	83	4.3621	9.3978	20.247
39	3.3912	7.3061	15.741	84	4.3795	9.4354	20.328
40	3.4200	7.3681	15.874	85	4.3968	9.4727	20.408
41	3.4482	7.4290	16.005	86	4.4140	9.5097	20.488
42	3.4760	7.4889	16.134	87	4.4310	9.5464	20.567
43	3.5034	7.5478	16.261	88	4.4480	9.5828	20.646
44	3.5303	7.6059	16.386	89	4.4647	9.6190	20.724
45	3.5569	7.6631	16.510	90	4.4814	9.6549	20.801
46	3.5830	7.7194	16.631	91	4.4979	9.6905	20.878
47	3.6088	7.7750	16.751	92	4.5144	9.7259	20.954
48	3.6342	7.8297	16.869	93	4.5307	9.7610	21.029
49	3.6593	7.8837	16.985	94	4.5468	9.7959	21.105
50	3.6840	7.9370	17.100	95	4.5629	9.8305	21.179
51	3.7084	7.9896	17.213	96	4.5789	9.8648	21.253
52	3.7325	8.0415	17.325	97	4.5947	9.8990	21.327
53	3.7563	8.0927	17.435	98	4.6104	9 9329	21.400
54	3.7798	8.1433	17.544	99	4.6261	9.9666	21.472

Explanation on p. 38

9. Three-Halves Powers of Numbers

n	0	1	2	3	4	5	6	7	8	9
0.0	0.000	0.001	0.003	0.005	0.008	0.011	0.015	0.019	0.023	0.027
0.1	0.032	0.036	0.042	0.047	0.052	0.058	0.064	0.070	0.076	0.083
0.2	0.089	0.096	0.103	0.110	0.118	0.125	0.133	0.140	0.148	0.156
0.3	0.164	0.173	0.181	0.190	0.198	0.207	0.216	0.225	0.234	0.244
0.4	0.253	0.263	0.272	0.282	0.292	0.302	0.312	0.322	0.333	0.343
0.5	0.354	0.364	0.375	0.386	0.397	0.408	0.419	0.430	0.442	0.453
0.6	0.465	0.476	0.488	0.500	0.512	0.524	0.536	0.548	0.561	0.573
0.7	0.586	0.598	0.611	0.624	0.637	0.650	0.663	0.676	0.689	0.702
0.8	0.716	0.729	0.743	0.756	0.770	0.784	0.798	0.811	0.826	0.840
0.9	0.854	0.868	0.882	0.897	0.911	0.926	0.941	0.955	0.970	0.985
1.0	1.000	1.015	1.030	1.045	1.061	1.076	1.091	1.107	1.122	1.138
1.1	1.154	1.170	1.185	1.201	1.217	1.233	1.249	1.266	1.282	1.298
1.2	1.315	1.331	1.348	1.364	1.381	1.398	1.414	1.431	1.448	1.465
1.3	1.482	1.499	1.517	1.534	1.551	1.569	1.586	1.604	1.621	1.639
1.4	1.657	1.674	1.692	1.710	1.728	1.746	1.764	1.782	1.800	1.819
1.5	1.837	1.856	1.874	1.893	1.911	1.930	1.948	1.967	1.986	2.005
1.6	2.024	2.043	2.062	2.081	2.100	2.119	2.139	2.158	2.178	2.197
1.7	2.217	2.236	2.256	2.275	2.295	2.315	2.335	2.355	2.375	2.395
1.8	2.415	2.435	2.455	2.476	2.496	2.516	2.537	2.557	2.578	2.598
1.9	2.619	2.640	2.660	2.681	2.702	2.723	2.744	2.765	2.786	2.807
2.0	2.828	2.850	2.871	2.892	2.914	2.935	2.957	2.978	3.000	3.021
2.1	3.043	3.065	3.087	3.109	3.131	3.153	3.175	3.197	3.219	3.241
2.2	3.263	3.285	3.308	3.330	3.353	3.375	3.398	3.420	3.443	3.465
2.3	3.488	3.511	3.534	3.557	3.580	3.602	3.626	3.649	3.672	3.695
2.4	3.718	3.741	3.765	3.788	3.811	3.835	3.858	3.882	3.906	3.929
2.5	3.953	3.977	4.000	4.024	4.048	4.072	4.096	4.120	4.144	4.168
2.6	4.192	4.217	4.241	4.265	4.289	4.314	4.338	4.363	4.387	4.412
2.7	4.437	4.461	4.486	4.511	4.536	4.560	4.585	4.610	4.635	4.660
2.8	4.685	4.710	4.736	4.761	4.786	4.811	4.837	4.862	4.888	4.913
2.9	4.939	4.964	4.990	5.015	5.041	5.067	5.093	5.118	5.144	5.170
3.0	5.196	5.222	5.248	5.274	5.300	5.327	5.353	5.379	5.405	5.432
3.1	5.458	5.485	5.511	5.538	5.564	5.591	5.617	5.644	5.671	5.698
3.2	5.724	5.751	5.778	5.805	5.832	5.859	5.886	5.913	5.940	5.968
3.3	5.995	6.022	6.049	6.077	6.104	6.132	6.159	6.186	6.214	6.242
3.4	6.269	6.297	6.325	6.352	6.380	6.408	6.436	6.464	6.492	6.520
3.5	6.548	6.576	6.604	6.632	6.660	6.689	6.717	6.745	6.774	6.802
3.6	6.831	6.859	6.888	6.916	6.945	6.973	7.002	7.031	7.059	7.088
3.7	7.117	7.146	7.175	7.204	7.233	7.262	7.291	7.320	7.349	7.378
3.8	7.408	7.437	7.466	7.495	7.525	7.554	7.584	7.613	7.643	7.672
3.9	7.702	7.732	7.761	7.791	7.821	7.850	7.880	7.910	7.940	7.970
4.0	8.000	8.030	8.060	8.090	8.120	8.150	8.181	8.211	8.241	8.272
4.1	8.302	8.332	8.363	8.393	8.424	8.454	8.485	8.515	8.546	8.577
4.2	8.607	8.638	8.669	8.700	8.731	8.762	8.793	8.824	8.855	8.886
4.3	8.917	8.948	8.979	9.010	9.041	9.073	9.104	9.135	9.167	9.198
4.4	9.230	9.261	9.293	9.324	9.356	9.387	9.419	9.451	9.482	9.514
n	0	1	2	3	4	5	6	7	8	9

Explanation on p. 38

10. Fifth Powers and Roots; Five-Halves Powers and Roots

n	n^5	$n^{1/5}$	$n^{5/2}$	$n^{2/5}$	n	n^5	$n^{1/5}$	$n^{5/2}$	$n^{2/5}$
0.1	0.0000	0.6310	0.0032	0.3981	**4.6**	2059.6	1.3569	45.383	1.8412
0.2	0.0003	0.7248	0.0179	0.5253	**4.7**	2293.5	1.3628	47.890	1.8571
0.3	0.0024	0.7860	0.0493	0.6178	**4.8**	2548.0	1.3685	50.478	1.8728
0.4	0.0102	0.8326	0.1012	0.6931	**4.9**	2824.8	1.3742	53.148	1.8883
0.5	0.0312	0.8706	0.1768	0.7579	**5.0**	3125.0	1.3797	55.902	1.9037
0.6	0.0778	0.9029	0.2789	0.8152	**5.1**	3450.3	1.3852	58.739	1.9188
0.7	0.1681	0.9311	0.4100	0.8670	**5.2**	3802.0	1.3906	61.661	1.9338
0.8	0.3277	0.9564	0.5724	0.9146	**5.3**	4182.0	1.3959	64.668	1.9485
0.9	0.5905	0.9791	0.7684	0.9587	**5.4**	4591.7	1.4011	67.762	1.9632
1.0	1.0000	1.0000	1.0000	1.0000	**5.5**	5032.8	1.4063	70.943	1.9776
1.1	1.6105	1.0192	1.2691	1.0389	**5.6**	5507.3	1.4114	74.211	1.9919
1.2	2.4883	1.0371	1.5774	1.0757	**5.7**	6016.9	1.4164	77.569	2.0061
1.3	3.7129	1.0539	1.9269	1.1107	**5.8**	6563.6	1.4213	81.016	2.0201
1.4	5.3782	1.0696	2.3191	1.1441	**5.9**	7149.2	1.4262	84.553	2.0340
1.5	7.5938	1.0845	2.7557	1.1761	**6.0**	7776.0	1.4310	88.182	2.0477
1.6	10.486	1.0986	3.2382	1.2068	**6.1**	8446.0	1.4357	91.902	2.0613
1.7	14.199	1.1120	3.7681	1.2365	**6.2**	9161.3	1.4404	95.715	2.0747
1.8	18.896	1.1247	4.3469	1.2651	**6.3**	9924.4	1.4450	99.621	2.0880
1.9	24.761	1.1370	4.9760	1.2927	**6.4**	10737.	1.4496	103.62	2.1012
2.0	32.000	1.1487	5.6569	1.3195	**6.5**	11603.	1.4541	107.72	2.1143
2.1	40 841	1.1600	6.3907	1.3455	**6.6**	12523.	1.4585	111.91	2.1272
2.2	51.536	1.1708	7.1789	1.3708	**6.7**	13501.	1.4629	116.19	2.1401
2.3	64.363	1.1813	8.0227	1.3954	**6.8**	14539.	1.4672	120.58	2.1528
2.4	79.626	1.1914	8.9234	1.4193	**6.9**	15640.	1.4715	125.06	2.1654
2.5	97.656	1.2011	9.8821	1.4427	**7.0**	16807.	1.4758	129.64	2.1779
2.6	118.81	1.2106	10.900	1.4655	**7.1**	18042.	1.4800	134.32	2.1903
2.7	143.49	1.2198	11.979	1.4878	**7.2**	19349.	1.4841	139.10	2.2026
2.8	172.10	1.2287	13.119	1.5096	**7.3**	20731.	1.4882	143.98	2.2148
2.9	205.11	1.2373	14.322	1.5309	**7.4**	22190.	1.4923	148.96	2.2269
3.0	243.00	1.2457	15.588	1.5518	**7.5**	23730.	1.4963	154.05	2.2388
3.1	286.29	1.2539	16.920	1.5723	**7.6**	25355.	1.5002	159.23	2.2507
3.2	335.54	1.2619	18.318	1.5924	**7.7**	27068.	1.5042	164.52	2.2625
3.3	391.35	1.2697	19.783	1.6122	**7.8**	28872.	1.5081	169.92	2.2742
3.4	454.35	1.2773	21.316	1.6315	**7.9**	30771.	1.5119	175.42	2.2859
3.5	525.22	1.2847	22.918	1.6505	**8.0**	32768.	1.5157	181.02	2.2974
3.6	604.66	1.2920	24.590	1.6692	**8.2**	37074.	1.5232	192.55	2.3202
3.7	693.44	1.2991	26.333	1.6876	**8.4**	41821.	1.5306	204.50	2.3427
3.8	792.35	1.3060	28.149	1.7057	**8.6**	47043.	1.5738	216.89	2.3648
3.9	902.24	1.3128	30.037	1.7236	**8.8**	52773.	1.5449	229.72	2.3867
4.0	1024.0	1.3195	32.000	1.7411	**9.0**	59049.	1.5518	243.00	2.4082
4.1	1158.6	1.3260	34.038	1.7584	**9.2**	65908.	1.5587	256.73	2.4295
4.2	1306.9	1.3324	36.151	1.7754	**9.4**	73390.	1.5654	270.91	2.4505
4.3	1470.1	1.3387	38.342	1.7922	**9.6**	81537.	1.5720	285.55	2.4712
4.4	1649.2	1.3449	40.610	1.8088	**9.8**	90392.	1.5785	300.65	2.4917
4.5	1845.3	1.3510	42.957	1.8251	**10**	100000.	1.5849	316.23	2.5119

Explanation on p. 39

11. Circumferences of Circles

Diameters in Units and Tenths

d	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.314	0.628	0.942	1.257	1.571	1.885	2.199	2.513	2.827
1	3.142	3.456	3.770	4.084	4.398	4.712	5.027	5.341	5.655	5.969
2	6.283	6.597	6.912	7.226	7.540	7.854	8.168	8.482	8.796	9.111
3	9.425	9.739	10.05	10.37	10.68	11.00	11.31	11.62	11.94	12.25
4	12.57	12.88	13.19	13.51	13.82	14.14	14.45	14.77	15.08	15.39
5	15.71	16.02	16.34	16.65	16.96	17.28	17.59	17.91	18.22	18.54
6	18.85	19.16	19.48	19.79	20.11	20.42	20.73	21.05	21.36	21.68
7	21.99	22.31	22.62	22.93	23.25	23.56	23.88	24.19	24.50	24.82
8	25.13	25.45	25.76	26.08	26.39	26.70	27.02	27.33	27.65	27.96
9	28.27	28.59	28.90	29.22	29.53	29.85	30.16	30.47	30.79	31.10
10	31.42	31.73	32.04	32.36	32.67	32.99	33.30	33.62	33.93	34.24
11	34.56	34.87	35.19	35.50	35.81	36.13	36.44	36.76	37.07	37.38
12	37.70	38.01	38.33	38.64	38.96	39.27	39.58	39.90	40.21	40.53
13	40.84	41.15	41.47	41.78	42.10	42.41	42.73	43.04	43.35	43.67
14	43.98	44.30	44.61	44.92	45.24	45.55	45.87	46.18	46.50	46.81
15	47.12	47.44	47.75	48.07	48.38	48.69	49.01	49.32	49.64	49.95
16	50.27	50.58	50.89	51.21	51.52	51.84	52.15	52.46	52.78	53.09
17	53.41	53.72	54.04	54.35	54.66	54.98	55.29	55.61	55.92	56.23
18	56.55	56.86	57.18	57.49	57.81	58.12	58.43	58.75	59.06	59.38
19	59.69	60.00	60.32	60.63	60.95	61.26	61.58	61.89	62.20	62.52

Explanation on p. 39

12. Circumferences of Circles

Diameters in Units and Eighths

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.3927	0.7854	1.1781	1.5708	1.9635	2.3562	2.7489
1	3.1416	3.5343	3.9270	4.3197	4.7124	5.1051	5.4978	5.8905
2	6.2832	6.6759	7.0686	7.4613	7.8540	8.2467	8.6394	9.0321
3	9.4248	9.8175	10.210	10.603	10.996	11.388	11.781	12.174
4	12.566	12.959	13.352	13.744	14.137	14.530	14.923	15.315
5	15.708	16.101	16.493	16.886	17.279	17.671	18.064	18.457
6	18.850	19.242	19.635	20.028	20.420	20.813	21.206	21.598
7	21.991	22.384	22.777	23.169	23.562	23.955	24.347	24.740
8	25.133	25.525	25.918	26.311	26.704	27.096	27.489	27.882
9	28.274	28.667	29.060	29.452	29.845	30.238	30.631	31.023
10	31.416	31.809	32.201	32.594	32.987	33.379	33.772	34.165
11	34.558	34.950	35.343	35.736	36.128	36.521	36.914	37.306
12	37.699	38.092	38.485	38.877	39.270	39.663	40.055	40.448
13	40.841	41.233	41.626	42.019	42.412	42.804	43.197	43.590
14	43.982	44.375	44.768	45.160	45.553	45.946	46.338	46.731
15	47.124	47.517	47.909	48.302	48.695	49.087	49.480	49.873
16	50.265	50.658	51.051	51.444	51.836	52.229	52.622	53.014
17	53.407	53.800	54.192	54.585	54.978	55.371	55.763	56.156
18	56.549	56.941	57.334	57.727	58.119	58.512	58.905	59.298
19	59.690	60.083	60.476	60.868	61.261	61.654	62.046	62.439

Explanation on p. 39

13. Circular

Explanation on p. 39

Central angle, degrees	Length of chord	Rise of arc	Area of segment	Central angle, degrees	Length of chord	Rise of arc	Area of segment
1	0.0175	0.0000	0.00000	46	0.7815	0.0795	0.04176
2	0.0349	0.0002	0.00000	47	0.7975	0.0829	0.04448
3	0.0524	0.0003	0.00001	48	0.8135	0.0865	0.04731
4	0.0698	0.0006	0.00003	49	0.8294	0.0900	0.05025
5	0.0872	0.0010	0.00006	50	0.8452	0.0937	0.05331
6	0.1047	0.0014	0.00010	51	0.8610	0.0974	0.05649
7	0.1221	0.0019	0.00015	52	0.8767	0.1012	0.05978
8	0.1395	0.0024	0.00023	53	0.8924	0.1051	0.06319
9	0.1569	0.0031	0.00032	54	0.9080	0.1090	0.06673
10	0.1743	0.0038	0.00044	55	0.9235	0.1130	0.07039
11	0.1917	0.0046	0.00059	56	0.9389	0.1171	0.07417
12	0.2091	0.0055	0.00076	57	0.9543	0.1212	0.07808
13	0.2264	0.0064	0.00097	58	0.9696	0.1254	0.08212
14	0.2437	0.0075	0.00121	59	0.9848	0.1296	0.08629
15	0.2611	0.0086	0.00149	60	1.0000	0.1340	0.09059
16	0.2783	0.0097	0.00181	61	1.0151	0.1384	0.09502
17	0.2956	0.0110	0.00217	62	1.0301	0.1428	0.09958
18	0.3129	0.0123	0.00257	63	1.0450	0.1474	0.10428
19	0.3301	0.0137	0.00302	64	1.0598	0.1520	0.10911
20	0.3473	0.0152	0.00352	65	1.0746	0.1566	0.11408
21	0.3645	0.0167	0.00408	66	1.0893	0.1613	0.11919
22	0.3816	0.0184	0.00468	67	1.1039	0.1661	0.12443
23	0.3987	0.0201	0.00535	68	1.1184	0.1710	0.12982
24	0.4158	0.0219	0.00607	69	1.1328	0.1759	0.13535
25	0.4329	0.0237	0.00686	70	1.1472	0.1808	0.14102
26	0.4499	0.0256	0.00771	71	1.1614	0.1859	0.14683
27	0.4669	0.0276	0.00862	72	1.1756	0.1910	0.15279
28	0.4838	0.0297	0.00961	73	1.1896	0.1961	0.15889
29	0.5008	0.0319	0.01067	74	1.2036	0.2014	0.16514
30	0.5176	0.0341	0.01180	75	1.2175	0.2066	0.17154
31	0.5345	0.0364	0.01301	76	1.2313	0.2120	0.17808
32	0.5512	0.0387	0.01429	77	1.2450	0.2174	0.18477
33	0.5680	0.0412	0.01566	78	1.2586	0.2229	0.19160
34	0.5847	0.0437	0.01711	79	1.2722	0.2284	0.19859
35	0.6014	0.0463	0.01864	80	1.2856	0.2340	0.20573
36	0.6180	0.0489	0.02027	81	1.2989	0.2396	0.21301
37	0.6346	0.0517	0.02198	82	1.3121	0.2453	0.22045
38	0.6511	0.0545	0.02378	83	1.3252	0.2510	0.22804
39	0.6676	0.0574	0.02568	84	1.3383	0.2569	0.23578
40	0.6840	0.0603	0.02767	85	1.3512	0.2627	0.24367
41	0.7004	0.0633	0.02976	86	1.3640	0.2686	0.25171
42	0.7167	0.0664	0.03195	87	1.3767	0.2746	0.25990
43	0.7330	0.0696	0.03425	88	1.3893	0.2807	0.26825
44	0.7492	0.0728	0.03664	89	1.4018	0.2867	0.27675
45	0.7654	0.0761	0.03915	90	1.4142	0.2929	0.28540

Segments

Central angle, degrees	Length of chord	Rise of arc	Area of segment	Central angle, degrees	Length of chord	Rise of arc	Area of segment
91	1.4265	0.2991	0.29420	136	1.8544	0.6254	0.83949
92	1.4387	0.3053	0.30316	137	1.8608	0.6335	0.85455
93	1.4507	0.3116	0.31226	138	1.8672	0.6416	0.86971
94	1.4627	0.3180	0.32152	139	1.8733	0.6498	0.88497
95	1.4746	0.3244	0.33093	140	1.8794	0.6580	0.90034
96	1.4863	0.3309	0.34050	141	1.8853	0.6662	0.91580
97	1.4979	0.3374	0.35021	142	1.8910	0.6744	0.93135
98	1.5094	0.3439	0.36008	143	1.8966	0.6827	0.94700
99	1.5208	0.3506	0.37009	144	1.9021	0.6910	0.96274
100	1.5321	0.3572	0.38026	145	1.9074	0.6993	0.97858
101	1.5432	0.3639	0.39058	146	1.9126	0.7076	0.99449
102	1.5543	0.3707	0.40104	147	1.9176	0.7160	1.01050
103	1.5652	0.3775	0.41166	148	1.9225	0.7244	1.02658
104	1.5760	0.3843	0.42242	149	1.9273	0.7328	1.04275
105	1.5867	0.3912	0.43333	150	1.9319	0.7412	1.05900
106	1.5973	0.3982	0.44439	151	1.9363	0.7496	1.07532
107	1.6077	0.4052	0.45560	152	0.9406	1.7581	1.09171
108	1.6180	0.4122	0.46695	153	1.9447	0.7666	1.10818
109	1.6282	0.4193	0.47844	154	1.9487	0.7750	1.12472
110	1.6383	0.4264	0.49008	155	1.9526	0.7836	1.14132
111	1.6483	0.4336	0.50187	156	1.9563	0.7921	1.15799
112	1.6581	0.4408	0.51379	157	1.9598	0.8006	1.17472
113	1.6678	0.4481	0.52586	158	1.9633	0.8092	1.19151
114	1.6773	0.4554	0.53807	159	1.9665	0.8178	1.20835
115	1.6868	0.4627	0.55041	160	1.9696	0.8264	1.22525
116	1.6961	0.4701	0.56289	161	1.9726	0.8350	1.24221
117	1.7053	0.4775	0.57551	162	1.9754	0.8436	1.25921
118	1.7143	0.4850	0.58827	163	1.9780	0.8522	1.27626
119	1.7233	0.4925	0.60116	164	1.9805	0.8608	1.29335
120	1.7321	0.5000	0.61418	165	1.9829	0.8695	1.31049
121	1.7407	0.5076	0.62734	166	1.9851	0.8781	1.32766
122	1.7492	0.5152	0.64063	167	1.9871	0.8868	1.34487
123	1.7576	0.5228	0.65404	168	1.9890	1.8955	1.36212
124	1.7659	0.5305	0.66759	169	1.9908	0.9042	1.37940
125	1.7740	0.5383	0.68125	170	1.9924	0.9128	1.39671
126	1.7820	0.5460	0.69505	171	1.9938	0.9215	1.41404
127	1.7899	0.5538	0.70897	172	1.9951	0.9302	1.43140
128	1.7976	0.5616	0.72301	173	1.9963	0.9390	1.44878
129	1.8052	0.5695	0.73716	174	1.9973	0.9477	1.46617
130	1.8126	0.5774	0.75144	175	1.9981	0.9564	1.48359
131	1.8199	0.5853	0.76584	176	1.9988	0.9651	1.50101
132	1.8271	0.5933	0.78034	177	1.9993	0.9738	1.51845
133	1.8341	0.6013	0.79497	178	1.9997	0.9825	1.53589
134	1.8410	0.6093	0.80970	179	1.9999	0.9913	1.55334
135	1.8478	0.6173	0.82454	180	2.0000	1.0000	1.57080

14. Areas of Circles for Diam-

Explanation on p. 39

d	0	1	2	3	4	5	6	7	8	9
1.0	0.785	0.801	0.817	0.833	0.849	0.866	0.882	0.899	0.916	0.933
1.1	0.950	0.968	0.985	1.003	1.021	1.039	1.057	1.075	1.094	1.112
1.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307
1.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517
1.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986
1.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243
1.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516
1.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806
1.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431
2.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767
2.2	3.801	3.836	3.871	3.906	3.941	3.976	4.012	4.047	4.083	4.119
2.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486
2.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269
2.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683
2.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114
2.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560
2.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499
3.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992
3.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501
3.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026
3.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.01	10.07	10.12
3.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69
3.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28
3.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88
3.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14
4.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79
4.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45
4.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14
4.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55
4.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28
4.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02
4.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78
4.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56
5.0	19.63	19.71	19.79	19.87	19.95	20.03	20.11	20.19	20.27	20.35
5.1	20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16
5.2	21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98
5.3	22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82
5.4	22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67
d	0	1	2	3	4	5	6	7	8	9

eters in Units and Hundredths

d	0	1	2	3	4	5	6	7	8	9
5.5	23.76	23.84	23.93	24.02	24.11	24.19	24 28	24.37	24.45	24.54
5.6	24.63	24.72	24.81	24.89	24.98	25.07	25.16	25.25	25.34	25.43
5.7	25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33
5.8	26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25
5.9	27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.09	28.18
6.0	28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13
6.1	29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09
6.2	30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07
6.3	31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.87	31.97	32.07
6.4	32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08
6.5	33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11
6.6	34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15
6.7	35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21
6.8	36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28
6.9	37.39	37.50	37.61	37.72	37.83	37.94	38.05	38.16	38.26	38.37
7.0	38.48	38.59	38.70	38.82	38.93	39.04	39.15	39.26	39.37	39.48
7.1	39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60
7.2	40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74
7.3	41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89
7.4	43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06
7.5	44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25
7.6	45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45
7.7	46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66
7.8	47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89
7.9	49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14
8.0	50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40
8.1	51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68
8.2	52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98
8.3	54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29
8.4	55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61
8.5	56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95
8.6	58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31
8.7	59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.68
8.8	60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07
8.9	62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48
9.0	63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90
9.1	65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33
9.2	66.48	66.62	66.77	66.91	67.06	67.20	67.35	67.49	67.64	67.78
9.3	67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25
9.4	69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73
9.5	70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23
9.6	72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75
9.7	73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28
9.8	75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82
9.9	76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38
d	0	1	2	3	4	5	6	7	8	9

15. Areas of Circles

Diameters in Units and Eighths

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.0123	0.0491	0.1104	0.1963	0.3068	0.4418	0.6013
1	0.7854	0.9940	1.2272	1.4849	1.7671	2.0739	2.4053	2.7612
2	3.1416	3.5466	3.9761	4.4301	4.9087	5.4119	5.9396	6.4918
3	7.0686	7.6699	8.2958	8.9462	9.6211	10.321	11.045	11.793
4	12.566	13.364	14.186	15.033	15.904	16.800	17.721	18.665
5	19.635	20.629	21.648	22.691	23.758	24.850	25.967	27.109
6	28.274	29.465	30.680	31.919	33.183	34.472	35.785	37.122
7	38.485	39.871	41.282	42.718	44.179	45.664	47.173	48.707
8	50.265	51.849	53.456	55.088	56.745	58.426	60.132	61.862
9	63.617	65.397	67.201	69.029	70.882	72.760	74.662	76.589
10	78.540	80.516	82.516	84.541	86.590	88.664	90.763	92.886
11	95.033	97.205	99.402	101.62	103.87	106.14	108.43	110.75
12	113.10	115.47	117.86	120.28	122.72	125.19	127.68	130.19
13	132.73	135.30	137.89	140.50	143.14	145.80	148.49	151.20
14	153.94	156.70	159.48	162.30	165.13	167.99	170.87	173.78
15	176.71	179.67	182.65	185.66	188.69	191.75	194.83	197.93
16	201.06	204.22	207.39	210.60	213.82	217.08	220.35	223.65
17	226.98	230.33	233.71	237.10	240.53	243.98	247.45	250.95
18	254.47	258.02	261.59	265.18	268.80	272.45	276.12	279.81
19	283.53	287.27	291.04	294.83	298.65	302.49	306.35	310.24
20	314.16	318.10	322.06	326.05	330.06	334.10	338.16	342.25
21	346.36	350.50	354.66	358.84	363.05	367.28	371.54	375.83
22	380.13	384.46	388.82	393.20	397.61	402.04	406.49	410.97
23	415.48	420.00	424.56	429.13	433.74	438.36	443.01	447.69
24	452.39	457.11	461.86	466.64	471.44	476.26	481.11	485.98
25	490.87	495.79	500.74	505.71	510.71	515.72	520.77	525.84
26	530.93	536.05	541.19	546.35	551.55	556.76	562.00	567.27
27	572.56	577.87	583.21	588.57	593.96	599.37	604.81	610.27
28	615.75	621.26	626.80	632.36	637.94	643.55	649.18	654.84
29	660.52	666.23	671.96	677.71	683.49	689.30	695.13	700.98
30	706.86	712.76	718.69	724.64	730.62	736.62	742.64	748.69
31	754.77	760.87	766.99	773.14	779.31	785.51	791.73	797.98
32	804.25	810.54	816.86	823.21	829.58	835.97	842.39	848.83
33	855.30	861.79	868.31	874.85	881.41	888.00	894.62	901.26
34	907.92	914.61	921.32	928.06	934.82	941.61	948.42	955.25
35	962.11	969.00	975.91	982.84	989.80	996.78	1003.8	1010.8
36	1017.9	1025.0	1032.1	1039.2	1046.3	1053.5	1060.7	1068.0
37	1075.2	1082.5	1089.8	1097.1	1104.5	1111.8	1119.2	1126.7
38	1134.1	1141.6	1149.1	1156.6	1164.2	1171.7	1179.3	1186.9
39	1194.6	1202.3	1210.0	1217.7	1225.4	1233.2	1241.0	1248.8
40	1256.6	1264.5	1272.4	1280.3	1288.2	1296.2	1304.2	1312.2
41	1320.3	1328.3	1336.4	1344.5	1352.7	1360.8	1369.0	1377.2
42	1385.4	1393.7	1402.0	1410.3	1418.6	1427.0	1435.4	1443.8
43	1452.2	1460.7	1469.1	1477.6	1486.2	1494.7	1503.3	1511.9
44	1520.5	1529.2	1537.9	1546.6	1555.3	1564.0	1572.8	1581.6
d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8

Explanation on p. 39

16. Volumes of Spheres

Diameters in Units and Tenths

d	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.001	0.004	0.014	0.034	0.065	0.113	0.180	0.268	0.382
1	0.524	0.697	0.905	1.150	1.437	1.767	2.145	2.572	3.054	3.591
2	4.189	4.849	5.575	6.371	7.238	8.181	9.203	10.31	11.49	12.77
3	14.14	15.60	17.16	18.82	20.58	22.45	24.43	26.52	28.73	31.06
4	33.51	36.09	38.79	41.63	44.60	47.71	50.97	54.36	57.91	61.60
5	65.45	69.46	73.62	77.95	82.45	87.11	91.95	96.97	102.2	107.5
6	113.1	118.8	124.8	130.9	137.3	143.8	150.5	157.5	164.6	172.0
7	179.6	187.4	195.4	203.7	212.2	220.9	229.8	239.0	248.5	258.2
8	268.1	278.3	288.7	299.4	310.3	321.6	333.0	344.8	356.8	369.1
9	381.7	394.6	407.7	421.2	434.9	448.9	463.2	477.9	492.8	508.0
10	523.6	539.5	555.6	572.2	589.0	606.1	623.6	641.4	659.6	678.1
11	696.9	716.1	735.6	755.5	775.7	796.3	817.3	838.6	860.3	882.3
12	904.8	927.6	950.8	974.3	998.3	1023	1047	1073	1098	1124
13	1150	1177	1204	1232	1260	1288	1317	1346	1376	1406
14	1437	1468	1499	1531	1563	1596	1630	1663	1697	1732
15	1767	1803	1839	1875	1912	1950	1988	2026	2065	2105
16	2145	2185	2226	2268	2310	2352	2395	2439	2483	2527
17	2572	2618	2664	2711	2758	2806	2855	2903	2953	3003
18	3054	3105	3157	3209	3262	3315	3369	3424	3479	3535
19	3591	3648	3706	3764	3823	3882	3942	4003	4064	4126

Explanation on p. 39

17. Volumes of Spheres

Diameters in Units and Eighths

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.0010	0.0082	0.0276	0.0654	0.1278	0.2209	0.3508
1	0.5236	0.7455	1.0227	1.3612	1.7671	2.2468	2.8062	3.4515
2	4.1888	5.0243	5.9641	7.0144	8.1812	9.4708	10.889	12.443
3	14.137	15.979	17.974	20.129	22.449	24.942	27.612	30.466
4	33.510	36.751	40.194	43.846	47.713	51.800	56.115	60.663
5	65.450	70.482	75.766	81.308	87.114	93.189	99.541	106.17
6	113.10	120.31	127.83	135.66	143.79	152.25	161.03	170.14
7	179.59	189.39	199.53	210.03	220.89	232.12	243.73	255.71
8	268.08	280.85	294.01	307.58	321.56	335.95	350.77	366.02
9	381.70	397.83	414.40	431.43	448.92	466.88	485.30	504.21
10	523.60	543.48	563.86	584.74	606.13	628.04	650.47	673.42
11	696.91	720.94	745.51	770.64	796.33	822.58	849.40	876.80
12	904.78	933.35	962.51	992.28	1022.7	1053.6	1085.2	1117.5
13	1150.3	1183.8	1218.0	1252.8	1288.2	1324.4	1361.2	1398.6
14	1436.8	1475.6	1515.1	1555.3	1596.3	1637.9	1680.3	1723.3
15	1767.1	1811.7	1857.0	1903.0	1949.8	1997.4	2045.7	2094.8
16	2144.7	2195.3	2246.8	2299.0	2352.1	2405.9	2460.6	2516.1
17	2572.4	2629.6	2687.6	2746.5	2806.2	2866.7	2928.2	2990.5
18	3053.6	3117.7	3182.6	3248.5	3315.2	3382.9	3451.5	3520.9
19	3591.4	3662.7	3735.0	3808.2	3882.4	3957.6	4033.7	4110.7

Explanation on p. 39

18. Natural Hyperbolic Functions

u	Sinh u	Cosh u	Tanh u	u	Sinh u	Cosh u	Tanh u
0.00	0.0000	1.0000	0.0000	2.25	4.6912	4.7966	0.9780
0.05	0.0500	1.0013	0.0500	2.30	4.9370	5.0372	0.9801
0.10	0.1002	1.0050	0.0997	2.35	5.1951	5.2905	0.9820
0.15	0.1506	1.0113	0.1489	2.40	5.4662	5.5569	0.9837
0.20	0.2013	1.0201	0.1974	2.45	5.7510	5.8373	0.9853
0.25	0.2526	1.0314	0.2449	2.50	6.0502	6.1323	0.9866
0.30	0.3045	1.0453	0.2913	2.55	6.3645	6.4426	0.9879
0.35	0.3572	1.0619	0.3364	2.60	6.6947	6.7690	0.9890
0.40	0.4108	1.0811	0.3800	2.65	7.0417	7.1123	0.9900
0.45	0.4653	1.1030	0.4219	2.70	7.4063	7.4735	0.9910
0.50	0.5211	1.1276	0.4621	2.75	7.7894	7.8533	0.9918
0.55	0.5782	1.1551	0.5005	2.80	8.1919	8.2527	0.9926
0.60	0.6367	1.1855	0.5370	2.85	8.6150	8.6728	0.9933
0.65	0.6967	1.2188	0.5717	2.90	9.0596	9.1146	0.9940
0.70	0.7586	1.2552	0.6044	2.95	9.5268	9.5791	0.9945
0.75	0.8223	1.2947	0.6352	3.00	10.018	10.068	0.9950
0.80	0.8881	1.3374	0.6640	3.05	10.534	10.581	0.9955
0.85	0.9561	1.3835	0.6911	3.10	11.076	11.122	0.9959
0.90	1.0265	1.4331	0.7163	3.15	11.647	11.690	0.9963
0.95	1.0995	1.4862	0.7398	3.20	12.246	12.287	0.9967
1.00	1.1752	1.5431	0.7616	3.25	12.876	12.915	0.9970
1.05	1.2539	1.6038	0.7818	3.30	13.538	13.575	0.9973
1.10	1.3356	1.6685	0.8005	3.35	14.234	14.269	0.9976
1.15	1.4208	1.7374	0.8178	3.40	14.965	14.999	0.9978
1.20	1.5095	1.8107	0.8337	3.45	15.734	15.766	0.9980
1.25	1.6019	1.8884	0.8483	3.50	16.543	16.573	0.9982
1.30	1.6984	1.9709	0.8617	3.55	17.392	17.421	0.9984
1.35	1.7991	2.0583	0.8741	3.60	18.285	18.313	0.9985
1.40	1.9043	2.1509	0.8854	3.65	19.224	19.250	0.9987
1.45	2.0143	2.2488	0.8957	3.70	20.211	20.236	0.9988
1.50	2.1293	2.3524	0.9052	3.75	21.249	21.272	0.9989
1.55	2.2496	2.4619	0.9138	3.80	22.339	22.362	0.9990
1.60	2.3756	2.5775	0.9217	3.85	23.486	23.507	0.9991
1.65	2.5075	2.6995	0.9289	3.90	24.691	24.711	0.9992
1.70	2.6456	2.8283	0.9354	3.95	25.958	25.977	0.9993
1.75	2.7904	2.9642	0.9414	4.0	27.290	27.308	0.9993
1.80	2.9422	3.1075	0.9468	4.1	30.162	30.178	0.9995
1.85	3.1013	3.2585	0.9518	4.2	33.336	33.351	0.9996
1.90	3.2682	3.4177	0.9562	4.3	36.843	36.857	0.9996
1.95	3.4432	3.5855	0.9603	4.4	40.719	40.732	0.9997
2.00	3.6269	3.7622	0.9640	4.5	45.003	45.014	0.9998
2.05	3.8196	3.9483	0.9674	4.6	49.737	49.747	0.9998
2.10	4.0219	4.1443	0.9705	4.7	54.969	54.978	0.9999
2.15	4.2342	4.3507	0.9732	4.8	60.751	60.759	0.9999
2.20	4.4571	4.5679	0.9757	4.9	67.141	67.149	0.9999

Explanation on p. 39

19. Napierian Logarithms of Numbers from 1 to 119

n	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	−∞	0.0000	0.6931	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794	2.1972
1	2.3026	2.3979	2.4849	2.5649	2.6391	2.7081	2.7726	2.8332	2.8904	2.9444
2	2.9957	3.0445	3.0910	3.1355	3.1781	3.2189	3.2581	3.2958	3.3322	3.3673
3	3.4012	3.4340	3.4657	3.4965	3.5264	3.5553	3.5835	3.6109	3.6376	3.6636
4	3.6889	3.7136	3.7377	3.7612	3.7842	3.8067	3.8286	3.8501	3.8712	3.8918
5	3.9120	3.9318	3.9512	3.9703	3.9890	4.0073	4.0254	4.0430	4.0604	4.0775
6	4.0943	4.1109	4.1271	4.1431	4.1589	4.1744	4.1897	4.2047	4.2195	4.2341
7	4.2485	4.2627	4.2767	4.2905	4.3041	4.3175	4.3307	4.3438	4.3567	4.3694
8	4.3820	4.3944	4.4067	4.4188	4.4308	4.4427	4.4543	4.4659	4.4773	4.4886
9	4.4998	4.5109	4.5218	4.5326	4.5433	4.5539	4.5643	4.5747	4.5850	4.5951
10	4.6052	4.6151	4.6250	4.6347	4.6444	4.6540	4.6634	4.6728	4.6821	4.6913
11	4.7005	4.7095	4.7185	4.7274	4.7362	4.7449	4.7536	4.7622	4.7707	4.7791

Explanation on p. 39

20. Multipliers for Transferring Logarithms

	Common to Napierian	Example	
1	2.302585093	Example.	
2	4.605170186	Find Nap log of 105	
3	6.907755279		
4	9.210340372	Com log 105 =	2.02119
5	11.512925465	2	4.605170
6	13.815510558	.02	46052
		1	2303
7	16.118095651	1	230
8	18.420680744	9	207
9	20.723265837	Nap log 105 =	4.65396

	Napierian to Common	Example	
1	0.434294482	Example.	
2	0.868588964	Find number corresponding to Nap log $\bar{1}.6078$	
3	1.302883446		
		.6	0.26058
4	1.737177928	07	304
5	2.171472410	8	35
6	2.605766891		+0.26397
7	3.040061373	$\bar{1}$	−0.43429
8	3.474355855	Com log =	$\bar{1}.8297$
9	3.908650337	Number =	0.6756

21. Multipliers for Finding Lengths of Circular Arcs

	Degrees	Minutes	Seconds
1	0.017453293	0.000290888	0.000004848
2	0.034906585	0.000581776	0.000009696
3	0.052359878	0.000872665	0.000014544
4	0.069813170	0.001163553	0.000019393
5	0.087266463	0.001454441	0.000024241
6	0.104719755	0.001745329	0.000029089
7	0.122173048	0.002036217	0.000033937
8	0.139626340	0.002327106	0.000038785
9	0.157079633	0.002617994	0.000043633

Example.

Find length of arc for a central angle of 48° 45′ in circle of 12 ft. radius.

40°	0.698132
8°	.139626
40′	.011636
5′	.001454
	0.85085
	12
Length =	10.210 ft.

22. Mathematical Constants

Symbol	Number	Logarithm
π	3.1415927	0.4971499
2π	6.2831853	0.7981799
3π	9.4247780	0.9742711
4π	12.5663706	1.0992099
5π	15.7079633	1.1961200
6π	18.8495559	1.2753011
7π	21.9911486	1.3422479
8π	25.1327412	0.4002399
9π	28.2743339	1.4513924
$4\pi/3$	4.1887902	0.6220886
$\pi/2$	1.5707963	0.1961199
$\pi/4$	0.7853982	$\bar{1}.8950899$
$\pi/6$	0.5235988	$\bar{1}.7189986$
$\pi/30$	0.1047198	$\bar{1}.0200286$
$\pi/180$	0.0174533	$\bar{2}.2418774$
$1/\pi$	0.3183099	$\bar{1}.5028501$
$2/\pi$	0.6366198	$\bar{1}.8038801$
$180/\pi$	57.2957795	1.7581226
$10800/\pi$	3437.74677	3.5362739
$648000/\pi$	206264.806	5.3144251
π^2	9.8696044	0.9942997
$1/\pi^2$	0.1013212	$\bar{1}.0057003$
π^3	31.0062767	1.4914496
$1/\pi^3$	0.0322516	$\bar{2}.5085504$

Symbol	Number	Logarithm
$\sqrt{\pi}$	1.7724539	0.2485749
$1/\sqrt{\pi}$	0.5641896	$\bar{1}.7514251$
$\pi\sqrt{2}$	4.4428829	0.6476649
$\sqrt{2\pi}$	2.5066282	0.3990899
$\sqrt{\pi/2}$	1.2533141	0.0980599
$\sqrt{2/\pi}$	0.7978844	$\bar{1}.9019401$
ϵ	2.7182818	0.4342945
ϵ^2	7.3890568	0.8685890
$1/\epsilon$	0.3678794	$\bar{1}.5657055$
$1/\epsilon^2$	0.1353353	$\bar{1}.1314110$
μ	0.4342945	$\bar{1}.6377843$
$1/\mu$	2.3025851	0.3622157
sin 1°	0.0174524	$\bar{2}.2418553$
sin 1′	0.0002909	$\bar{4}.4637261$
sin 1″	0.0000048	$\bar{6}.6855749$
2	2.	0.3010300
$\sqrt{2}$	1.4142136	0.1505150
$\sqrt{1/2}$	0.7071068	$\bar{1}.8494850$
3	3.	0.4771213
$\sqrt{3}$	1.7320508	0.2385606
$\sqrt{1/3}$	0.5773503	$\bar{1}.7614394$

Explanation on p. 39

23. Decimal Equivalents of Common Fractions

Fraction	Decimal	Logarithm	Fraction	Decimal	Logarithm	Fraction	Decimal	Logarithm
1/2	0.5	$\bar{1}.69897$	1/8	0.125	$\bar{1}.09691$	1/32	0.03125	$\bar{2}.49485$
1/3	0.33333	$\bar{1}.52288$	3/8	0.375	$\bar{1}.57403$	3/32	0.09375	$\bar{2}.97197$
2/3	0.66667	$\bar{1}.82391$	5/8	0.625	$\bar{1}.79588$	5/32	0.15625	$\bar{1}.19382$
1/4	0.25	$\bar{1}.39794$	7/8	0.875	$\bar{1}.94201$	7/32	0.21875	$\bar{1}.33995$
3/4	0.75	$\bar{1}.87506$	1/12	0.08333	$\bar{2}.92082$	9/32	0.28125	$\bar{1}.44909$
1/5	0.2	$\bar{1}.30103$	5/12	0.41667	$\bar{1}.61979$	11/32	0.34375	$\bar{1}.53624$
2/5	0.4	$\bar{1}.60206$	7/12	0.58333	$\bar{1}.76592$	13/32	0.40625	$\bar{1}.60879$
3/5	0.6	$\bar{1}.77815$	11/12	0.91667	$\bar{1}.96211$	15/32	0.46875	$\bar{1}.67094$
4/5	0.8	$\bar{1}.90309$	1/16	0.0625	$\bar{2}.79588$	17/32	0.53125	$\bar{1}.72530$
1/6	0.16667	$\bar{1}.22185$	3/16	0.1875	$\bar{1}.27300$	19/32	0.59375	$\bar{1}.77360$
5/6	0.83333	$\bar{1}.92082$	5/16	0.3125	$\bar{1}.49485$	21/32	0.65625	$\bar{1}.81707$
1/7	0.14286	$\bar{1}.15490$	7/16	0.4375	$\bar{1}.64098$	23/32	0.71875	$\bar{1}.85658$
2/7	0.28571	$\bar{1}.45593$	9/16	0.5625	$\bar{1}.75012$	25/32	0.78125	$\bar{1}.89279$
3/7	0.42857	$\bar{1}.63202$	11/16	0.6875	$\bar{1}.83727$	27/32	0.84375	$\bar{1}.92621$
4/7	0.57143	$\bar{1}.75696$	13/16	0.8125	$\bar{1}.90982$	29/32	0.90625	$\bar{1}.95725$
5/7	0.71429	$\bar{1}.85387$	15/16	0.9375	$\bar{1}.97197$	31/32	0.96875	$\bar{1}.98621$
6/7	0.85714	$\bar{1}.93305$						

24. Explanation and Use of the Mathematical Tables

Arguments and Functions are the two kinds of numbers that appear in a table, the former being the numbers for which the values of the functions have been computed; thus, in sin θ, values of θ are arguments and those of sin θ are functions. In this book arguments are generally in bold-face type and functions in common type. An argument is placed at the side of the table and sometimes part of it is placed at the side and part at the top or foot; thus, when the reciprocal of 0.192 is sought, 0.19 is found at left-hand side and .002 at top, then 5.208 is found at the intersection of the horizontal row and the vertical column. **Interpolation** is the process of finding the function for an argument that falls between two tabular arguments. This is generally done by regarding the function as varying uniformly between the two adjacent tabular values. Hence if the given argument a is half-way between two tabular arguments a_1 and a_2, the required function f is the mean of the two corresponding tabular functions f_1 and f_2. In general when $a_2 > a_1$ and $f_2 > f_1$, the value of f for an argument a which lies between a_1 and a_2 is

$$\mathbf{f} = \mathbf{f}_1 + \frac{\mathbf{a} - \mathbf{a}_1}{\mathbf{a}_2 - \mathbf{a}_1}(\mathbf{f}_2 - \mathbf{f}_1) \qquad \text{or} \qquad \mathbf{f} = \mathbf{f}_2 - \frac{\mathbf{a}_2 - \mathbf{a}}{\mathbf{a}_2 - \mathbf{a}_1}(\mathbf{f}_2 - \mathbf{f}_1)$$

the first formula being preferable when $a < 1/2\,(a_1 + a_2)$, although both are applicable for any a. Thus by Table 14 the area of a circle for diameter 2.533 is found to be 5.039. When the functions decrease as the arguments increase, $f_2 - f_1$ is negative; thus log cot 42° 17′ is found from Table 2 to be 0.0413.

When a function is given and it is required to find the argument, the same process applies if a_1, a, a_2 are regarded as the functions and f_1, f, f_2 as the arguments. For example, when a cotangent is given as 1.8501 and the angle is required, the function is seen from Table 3 to fall between 1.8807 and 1.8040 which correspond to the arguments 28° and 29°; here the difference of the tabular functions is .0767 for a difference of 60′ in the arguments, while the difference 1.8807 − 1.8501 is 0.0306; then 306/767 = .4 and .4 × 60 = 24, whence the required angle is 28° 24′, closely.

An error of one or two units usually occurs in the last figure of a function obtained by interpolation, since the tabular values do not really vary in the manner supposed. An interpolated argument is liable to a similar error, hence care should be taken not to extend the figures too far; in the last numerical example, however, it happens that the last figures should be 231.

Logarithms of Numbers (Tables 1 and 26). The word logarithm and its abbreviation log, when used without qualification, refer to a common logarithm which is defined by the equation $10^{\log n} = n$. Table 1 gives the decimal part, or mantissa of a logarithm, while the integral part or characteristic is to be supplied according to the following rules. When the number is greater that 1, the characteristic of its log is positive and is one less than the number of figures preceding the decimal point; thus,

$$\log 6.54 = 0.81558 \quad \log 65.4 = 1.81558 \quad \log 654 = 2.81558 \quad \log 6540 = 3.81558$$

When the number is less than 1, the characteristic of its log is negative and is numerically one greater than the number of ciphers immediately following the decimal point, thus the four-place log of 6 is 0.7782, and

$$\log 0.6 = \bar{1}.7782 \qquad \log 0.06 = \bar{2}.7782 \qquad \log 0.006 = \bar{3}.7782 \qquad \log 0.0006 = \bar{4}.7782$$

Here the mantissa is positive, so that $\bar{2}.7782$ is the same as $-2 + 0.7782$. When the given number is an integral power of 10, the mantissa is zero, so that log 1000 = 3, log 100 = 2, log 10 = 1, log 1 = 0, log 0.1 = −1, log 0.01 = − 2, log 0.001 = − 3, or in general $\log 10^m = m$.

Multiplication and Division of numbers may be performed by the help of logarithms and the use of the following rules:

To multiply *a* by *b*,	$\log a + \log b = \log ab$
To divide *a* by *b*,	$\log a - \log b = \log a/b$

Here log *a* and log *b* are obtained from Table 1 or 26 and the above rules for the characteristic; then the numbers corresponding to log *ab* and log *a/b* are found from the Table. For example, to multiply 68.31 by 0.2754, the sum of the logs is 1.27444 and its corresponding number is 18.812, the last decimal being one unit in error.

Powers and Roots of numbers are most conveniently computed by logarithms and the use of the following rules:

To raise *a* to the *n*th power,	$n \log a = \log a^n$
To extract the *n*th root of *a*,	$\frac{1}{n} \log a = \log a^{1/n}$

For example to raise 0.6831 to the 1.53 power: $1.53 \times \bar{1}.83448 = -1.53 + 1.27675 = \bar{2}.47 + 1.27675 = \bar{1}.74675$ which is log of 0.55815. To find the fifth root of 0.6831: one-fifth of $\bar{1}.83448$ is $1/5\,(-5 + 4.83448) = \bar{1}.96690$ which is log of 0.9262; or it is perhaps better to multiply by 0.2 instead of dividing by 5, thus $0.2\,(\bar{1}.83448) = 0.2\,(-1 + 0.83448) = -0.2 + 0.16690 = -1 + 0.8 + 0.16690 = \bar{1}.96690$.

Table 2 contains logarithms of trigonometric functions to four decimal places at intervals of 1°, the characteristics being given. When the angle is less than 45° look for it on the left side and for the name of the function at the top; when greater than 45° look for it on the right side and for the function at the foot. In many books these functions are called logarithmic sines, logarithmic tangents, etc., while the characteristics are written 8 and 9 instead of $\bar{2}$ and $\bar{1}$, thus requiring some multiple of 10 to be subtracted later. Here the final logarithm of a computation is correct without such subtraction; thus to compute $(\cos 18°)^3$ write $\log (\cos 18°)^3 = 3 \log \cos 18° = \bar{1}.9346$, which is log of 0.8601. If any one wishes to use the old but illogical method, he can, in taking a log from a table, write 9. instead of $\bar{1}$. and 8. instead of $\bar{2}$. as in Table 27.

Table 2 gives four-place logs of trigonometric functions. The columns for log arc refer to the angles expressed in radians or to the arc of the circle expressed in terms of radius unity. Thus for 10°, the arc is 0.1745 and log arc to 10° is $\bar{1}.2419$; the arc of 57°.3 is 1.0000 and log arc 57°.3 is 0.0000. The above remarks regarding characteristics should be noted.

Natural Trigonometric Functions (Arts. 3 and 28–29). Table 3 gives four-place functions for every degree and Tables 28–29 give five-place for functions for intervals of 1′ in angle. For explanation of the term arc, see preceding paragraph. For rules regarding the signs of functions greater than 90° see Sect. 2. The secant of an angle is the reciprocal of its cosine and cosecant is the reciprocal of sine. The logarithms of the functions in Table 3 are given in Table 2, and those of the functions in Tables 28 and 29 are in Table 27. Log sec = − log cos and log cosec = − log sin.

Reciprocals, Powers, and Roots (Arts. 4–10, pp. 14–26). The arrangement of most of these is like that of a logarithmic table, the last figure of the argument being given at the top and foot. By properly moving the decimal point in both argument and function, Tables 4–8 may be used for other numbers than those given. For example, reciprocals of 0.0504, 0.504, 5.04, 50.4 are 19.84, 1.984, 0.1984, 0.01984 to four significant figures; square roots of 3.04 and 304 are 1.744 and 17.44, while those of 30.4 and 3040 are 5.514 and 55.14. When the decimal point is moved one place in a number it is moved two places in the square and three places in the cube.

The Three-halves Powers in Table 9 have been taken from Horton's Weir Experiments, Coefficients and Formulas (Water Supply and Irrigation Paper 200 of U. S. Geological Survey, 1907) and are here given to four decimal places only. All those which end in 5 in Horton's table have been recomputed.

Table 10 has been computed for this Handbook, the work being done in duplicate. To find $(10\,n)^5$, $(10\,n)^{1/5}$, $(10\,n)^{5/2}$, and $(10\,n)^{2/5}$ multiply the tabular values by 100000, 1.5849, 316.23, and 2.5119. To find $(0.1\,n)^5$, $(0.1\,n)^{1/5}$, $(0.1\,n)^{5/2}$ and $(0.1\,n)^{2/5}$ multiply the tabular values by 0.00001, 0.63096, 0.0031623, and 0.39811.

Circles and Spheres (Arts. 11–17, pp. 27–33). Tables 11, 14, 16 give circumferences of circles, areas of circles, and volumes of spheres for diameters in units and tenths. When the decimal point is moved one place in a diameter, it is to be moved one place in a circumference, two places in an area, and three places in a volume. Thus, for diameters 1.53 and 15.3 ft., the areas of the circles are 1.839 and 183.9 sq. ft., and the volumes of the spheres are 1.875 and 1875 cu. ft When the diameters are given in units and eighths, Tables 12, 15, 17 are to be used.

Table 13, which gives properties of circular segments, has been taken from Taschenbuch Verein Hütte, 1905. Lengths of the arc of the segment are given in Table 3 up to 90° and for larger angles may be found from Table 21. Example: for central angles of 35° and 105° and a radius r, the arcs are $0.6109\,r$ and $1.8326\,r$, the chords are $0.6014\,r$ and $1.5867\,r$, the rises are $0.0463 =$ and $0.3912\,r$, and the areas are $0.01864\,r^2$ and $0.4\,3333\,r^2$.

Areas of Spheres may be found by multiplying the circular areas in Tables 14 and 15 by 4, since the area of a sphere is equal to that of four great circles. Thus, area of a sphere for diameter 1.53 is 7.356, the last figure being one unit in error.

Miscellaneous Tables (Arts. 18–23, pp. 34–36). Table 18 gives hyperbolic sines, cosines and tangents for arguments up to 4.9. The hyperbolic cotangent, coth u, may be found by taking the reciprocal of tanh u. Powers of the Napierian base, $\epsilon = 2.71828$, may be obtained by taking the sum of cosh u and sinh u when u is positive and their difference when u is negative; thus, $\epsilon^{2.5} = 12.1825$ and $\epsilon^{-2.5} = 0.0821$.

Napierian Logarithms, often called hyperbolic logs or natural logs, are given in Table 19 for numbers from 1 to 119. The abbreviation Nap log designates these, and they are defined by the equation $\epsilon^{\text{Nap log } n} = n$, where ϵ is the Napierian base 2.71828. Extended tables of Napierian logs are unnecessary since they may be easily computed from common logs as shown by the example in Table 20.

Table 21 gives multiples for computing lengths of circular arcs for any given central angle. In Table 22, the symbol π represents the ratio of the circumference of a circle to its diameter, ϵ the base of the Napierian system of logarithms, and μ the modulus of the common system of logarithms; $1/\mu$ is the Nap log of 10.

Circumferences and areas of circles may be computed with accuracy to six or seven significant figures by the help of the multiples of π given in Table 22 using circumference $= \pi d$ and area $= \pi r^2$ where d is the diameter and r the radius.

25. Precision of Results Computed from Tables

Numbers in Tables are not generally precise in the last figure, and hence a lack of precision occurs in the final result of a computation. It is important that a computor should use the tables so as to obtain the most precise result and also that he should not attribute to the result a precision that does not exist. In general no more than four reliable significant figures can be obtained from a four-place table; indeed the fourth significant figure is liable to an error of one-quarter of one unit. Hence the Tables 28 29 cannot be used for cases where six or more significant figures are required to be accurately determined, but their use is limited to cases where only four or five are needed. The greater part of the computations in engineering require only three or four significant figures to be determined with accuracy.

The values given in tables may have a maximum error of one-half a unit in the last figure. Thus in Table 3 the value of sin 18° is given as 0.3090, this meaning that the true value lies between 0.30895 and 0.30905; in Table 28 it is given as 0.30902, this meaning that the true value lies between 0.309015 and 0.309025. When a quantity, like 576.48 sin 18° is to be computed by the four-place natural sine, the multiplication gives 178.13232, but the last four figures have no precision, since all that can be concluded by the use of the number 0.3090 is that the result lies between 178.103498 and 178.161142. Hence the result of this multiplication should be written 178.1, and it should be recognized that the last figure may have an error of one-half a unit. When a five-place sine is used the numerical value of $a \sin \theta$ is liable to an error of half a unit in the fifth significant figure when $\sin \theta$ is taken from Table 28 without interpolation.

An interpolated value may have double the error of a tabular value, since it is obtained by using the difference of two tabular values one of which may have a positive error while the other may have a negative error. Thus sin 8° 18′.5 is found from Table 28 to be 0.14450, this meaning that the true value lies between 0.14449 and 0.14451. Hence the quantity $a \sin \theta$ when obtained by the multiplication of a by a sine interpolated from a five-place table may have an error of one unit in the fifth significant figure. All figures beyond the fifth have no precision whatever and are entirely misleading.

The Probable Error of a number taken from a table is that error which is as likely as not to be exceeded. When the number is directly taken from the table its probable error is one-fourth of a unit in the last figure; when it is obtained by interpolation its probable error is one-half of a unit in the last place. An interpolated value has really a probable error slightly greater than that just stated, since the function does not vary uniformly between two tabular values as the method of interpolation implies.

When the product of two quantities f_1 and f_2 is to be obtained by using tabular values, the product $f_1 f_2$ may have a maximum error $(f_1 + f_2)\, r$ and its probable error is $1/2\ (f_1 + f_2)\, r$ where r is half a unit in the last place when the numbers are directly taken from the table and one unit when interpolation is made. Thus, the four-place product of 4.12^2 and 7.43^2, when obtained by multiplying the four-place squares given in Table 5, is 936.7, the probable error of which is 0.18; the last figure is here really three units in error but an error so large will occur only about one time in a hundred. For cases of this kind logarithms give more precise results; thus by four-place logs the product of 4.12^2 and 7.43^2 is 937.2 where the last figure is only one unit in error.

The quotient f_1/f_2 also has the probable error $1/2\ (f_1 + f_2)\, r$. When several quantities taken from tables are combined either by direct multiplication or division, the probable error of the result is $1/2\ (f_1 + f_2 + f_3 + \ldots)\, r$ and its maximum error may be twice as great. When logarithms are used, the probable and maximum errors are much smaller.

Inexperienced computors sometimes, in making interpolations, use all the figures obtained in the multiplication of differences, and thus carry the work several places further than the tabular values warrant. This procedure not only entails additional work and gives extra figures which are wholly inaccurate, but it leads the computor to suppose that his results have a far higher degree of precision than is actually the case, hence vitiating his judgment and perhaps leading to the deceit of others as well as of himself. The above indicates that no more significant figures should appear in intermediate work or final results than are given in the tables which are used.

In this discussion the data of the computation are regarded as free from error. When the data are obtained by measurement, these are affected by the errors and uncertainties of the measurements and hence the probable errors of the computed results are greater than those noted above. See Sect. 2, Art. 14.

SECTION 2

MATHEMATICS AND MECHANICS

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ELEMENTARY MATHEMATICS

1. Algebra

Notation. Known or constant quantities are usually represented by the first letters of the English alphabet and unknown or variable quantities by the last letters. The Greek letter π is universally used for 3.14159 . . . In this book ϵ is used for 2.71828 . . .. and other Greek letters are rarely used except for angles. Names of Greek letters are

α Alpha	η Eta	ν Nu	τ Tau
β Beta	θ Theta	ξ Xi	υ Upsilon
γ Gamma	ι Iota	o Omicron	ϕ Phi
Δ, δ Delta	κ Kappa	π Pi	χ Chi
ϵ Epsilon	λ Lambda	ρ Rho	ψ Psi
ζ Zeta	μ Mu	Σ, σ Sigma	ω Omega

In this volume the sign / is often used to indicate division; thus, 3/8 is the same as $\frac{3}{8}$ and a/b is the same as $a \div b$. For the product of the first n natural numbers $n!$ is used.

Powers and Roots. The notation a^n denotes the nth power of a, even when n is negative or fractional. Rules for powers are: $(+a)^n = +a^n$, $(-a)^n = +a^n$ if n is an even integer and $-a^n$ if n is odd; also $(a^m)^n = a^{mn}$, $a^{-n} = 1/a^n$, $a^0 = 1$. For multiplication and division of powers, $a^m \times a^n = a^{m+n}$ and $a^m/a^n = a^{m-n}$.

The notation $a^{1/n}$ or $\sqrt[n]{a}$ denotes the nth root of a. Replacing m and n in the last paragraph by $1/m$ and $1/n$, the rules there given for powers apply also to roots. An even root of a positive number is positive or negative and an odd root is positive; an odd root of a negative number is negative and an even root is impossible or imaginary.

The symbol i denotes $\sqrt{-1}$, which is the simplest imaginary quantity. If a is positive and n even, then $(-a)^{1/n} = a^{1/n}i$. Powers of i are $i^2 = -1$, $i^3 = -\sqrt{-1}$, $i^4 = +1$. A complex quantity is one partly real and partly imaginary, as $a + bi$; The complex numbers $-1/2\,(1 + 3^{1/2}i)$ and $-1/2\,(1 - 3^{1/2}i)$ are two of the cube roots of unity.

Factors of some algebraic expressions are

$$a^2 - b^2 = (a - b)(a + b) \qquad a^2 + b^2 = (a - bi)(a + bi)$$
$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \qquad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + b^{n-1})$$
$$a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots - b^{n-1}) \text{ if } n \text{ is even}$$
$$a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots + b^{n-1}) \text{ if } n \text{ is odd}$$

When a and b are small compared to 1, then approximately,

$$(1 + a)(1 + b) = 1 + a + b \qquad (1 + a)^m(1 + b)^n = 1 + ma + nb$$
$$1/(1 + a) = 1 - a \qquad 1/(1 - a) = 1 + a \qquad \sqrt{ab} = 1/2(a + b)$$

Logarithms. When $y = b^x$ and b is a fixed number, x is the logarithm of y to the base b, or $\log_b y = x$. When $b = 10$ the logarithms are in the common system; thus since $100 = 10^2$, and $0.1 = 10^{-1}$, the common logs of 100 and 0.1 are $+2$ and -1. When $x = 0$, $y = 1$, and hence for all systems the log of 1 is 0. Numbers greater than 1 have positive logs and those less than 1 negative logs. When $b = 2.71828$ . . . the logs are in the Napierian system.

Proportion. All rules are included in this, where m, n, p, and q have any values:

$$\text{if} \quad \frac{a}{b} = \frac{c}{d}, \qquad \text{then} \quad \frac{ma + nb}{pa + qb} = \frac{mc + nd}{pc + qd}.$$

A Permutation is any arrangement that can be made of several things; thus, for three letters there are six arrangements: abc, acb, bac, bca, cab, cba. The number of permutations of n things taken r at a time is $n(n - 1)$ . . .

$(n - r + 1)$. For example the number of permutations of seven things taken three at a time is $7 \times 6 \times 5 = 210$. A **combination** is a group made by taking things without reference to their order; thus for three letters there is only one combination abc. The number of combinations of n things taken in groups of r is $[n(n-1)\ldots(n-r+1)]/r!$. For example, when seven lines radiate from a point the number of combinations of these taken two at a time is $7 \times 6/2 = 21$, and hence there are 42 angles that can be measured.

The Binomial Theorem or formula is a means for writing out the power of a binomial in a series. It is

$$(a \pm b)^n = a^n \pm na^{n-1}b + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 \pm \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \ldots$$

When n is a positive integer it holds in all cases; when n is a fraction or negative it holds only if a is greater than b. In the first case the expansion (right-hand member) has $n + 1$ terms; in the second the number of terms is infinite. The coefficients of the powers of a and b in the expansion are called binomial coefficients; the general formula for the rth coefficient is $Cr = [n(n-1)(n-2)\ldots(n-r+1)] \div [1 \times 2 \times 3 \times \ldots r]$. Thus when $n = 5$, the coefficients are 1, 5, 10, 10, 5, 1.

If x stands for b/a, then $(a \pm b)^n = a^n(1 \pm x)^n$, and the expansion of a binomial of the type $(a \pm b)$ can be expressed by that of a binomial of the type $(1 \pm x)$. When n is a positive integer the following hold in all cases, but when n is a fraction or negative they hold only if x is between 0 and 1.

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{1 \cdot 2}x^2 \pm \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^3 + \ldots$$

$$\frac{1}{1 \pm x} = (1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + x^4 \mp x^5 + \ldots$$

$$\sqrt{1 \pm x} = (1 \pm x)^{1/2} = 1 \pm \frac{1}{2}x - \frac{1 \cdot 1}{2 \cdot 4}x^2 \pm \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \ldots$$

$$\frac{1}{\sqrt{1 \pm x}} = (1 \pm x)^{-1/2} = 1 \mp \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 \mp \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \ldots$$

When x is small compared to 1, the first two or three terms in the four preceding expansions give generally a sufficient approximation in ordinary computations.

A Series is a succession of numbers which proceed according to some fixed law. A converging series is one whose sum, as the number of its terms is indefinitely increased, approaches some finite value as a limit; a diverging series is one whose sum, so taken, increases indefinitely. An **arithmetic series**, or arithmetic progression, is one in which the differences between successive terms are equal, as in a, $a + d$, $a + 2d$, $a + 3d$, . . . ; the nth term is $a + (n-1)d$, and the sum of the n terms is $1/2\, n\, [2a + (n-1)d]$. A **geometric series**, or geometric progression, is one in which the ratios of successive terms are equal, as in a, ar, ar^2, ar^3, ... ; the nth term is ar^{n-1}, and the sum of the n terms is $a(r^n - 1)/(r - 1)$.

Some special series (see also preceding paragraph for Binomial series):

$$1 + 2 + 3 + 4 + \ldots + (n-1) + n = 1/2\, n(n+1)$$
$$2 + 4 + 6 + 8 + \ldots + (2n-2) + 2n = n(n+1)$$
$$1 + 3 + 5 + 7 + \ldots + (2n-3) + (2n-1) = n^2$$
$$1 + 2^2 + 3^2 + 4^2 + \ldots + (n-1)^2 + n^2 = 1/6\, n(n+1)(2n+1)$$
$$1 + 2^3 + 3^3 + 4^3 + \ldots + (n-1)^3 + n^3 = 1/4\, n^2(n+1)^2$$

$$a^x = 1 + \frac{x}{1}\log_\epsilon a + \frac{x^2}{2!}\log_\epsilon^2 a + \frac{x^3}{3!}\log_\epsilon^3 a + \ldots \quad (1)$$

$$\epsilon^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \text{ where } \epsilon = 2.71828 \ldots \quad (2)$$

$$1/2 \log_\epsilon x = \frac{x-1}{x+1} + 1/3 \frac{(x-1)^3}{(x+1)^3} + 1/5 \frac{(x-1)^5}{(x+1)^5} + \ldots$$

If $x^2 < 1$, $\log_\epsilon (1 \pm x) = \pm x - 1/2\, x^2 \pm 1/3\, x^3 - 1/4\, x^4 \pm \ldots$ (3)

$$1/2 \log_\epsilon \frac{1+x}{1-x} = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots \quad (4)$$

In right-hand members of the following, x is the angles in radians:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots \quad (5) \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots \quad (6)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2\,x^5}{3 \cdot 5} + \frac{17\,x^7}{3^2 \cdot 5 \cdot 7} \ldots \quad (7)$$

Approximations when x is Small. In series (1) the first two terms, or at most three, are generally sufficient when x is small compared to a, and in (2), (3), (4) when x is small compared to 1. In (5), (6), (7), two terms, or even one, may be sufficient when x is less than the arc of 10°; thus, for 8° the value of x is $(8/180)\pi = 0.13963$, and $\sin x = x - 1/6\, x^3 = 0.13918$, which is one unit in error in the fifth decimal place. When the angle is less than 5°, the values of x and $\sin x$ do not differ more than one unit in the fourth decimal place. (See Sect. 1, Art. 5.)

2. Solution of Equations

Algebraic Equations, rational and integral, and of the first, second, third, or fourth degrees have been solved algebraically; that is, formulas have been found for the values of the unknown, the roots of the equation.

A Linear Equation, one of the first degree, as $ax + b = 0$, has only one root, namely $x = -b/a$.

A Quadratic Equation, one of the second degree, as $x^2 + 2\,ax + b = 0$ has two roots, namely $-a \pm (a^2 - b)^{1/2}$. If $a = 0$, the roots are equal and opposite in sign; if $b > a^2$ both roots are imaginary.

A Cubic Equation, or one of the third degree, can be written in the form $x^3 + 3\,ax^2 + 3\,bx + 2\,c = 0$; it has three roots, here called x_1, x_2, and x_3. To determine these compute B and C from $B = -a^2 + b$ and $C = a^3 - 3/2\,ab + c$, and s_1 and s_2 from $s_1 = \left(-C + \sqrt{B^3 + C^2}\right)^{1/3}$ and $s_2 = \left(-C - \sqrt{B^3 + C^2}\right)^{1/3}$

then
$$x_1 = -a + (s_1 + s_2)$$
$$x_2 = -a - 1/2\,(s_1 + s_2) + 1/2\sqrt{-3}\,(s^1 - s_2)$$
$$x^3 = -a - 1/2\,(s_1 + s_2) - 1/2\sqrt{-3}\,(s^1 - s_2)$$

When $B^3 + C^2$ is negative the numerical solution leads to irreducible imaginary forms although the three roots are real.

Equations of Degree Higher than the Second can generally be solved best by factoring, by trial, or graphically. In the following $f(x)$ is used to denote any expression containing x, and is read "a function of x," and so $f(x) = 0$ may denote any equation containing x; the value of $f(x)$ when $x = a$ is denoted by $f(a)$.

(1) Factoring Method: Factor $f(x)$ and equate each factor to zero; solve each of these new equations for x; these roots are also roots of $f(x) = 0$. Thus, to find the roots of $x^3 - 19\,x - 30 = 0$, note that $x^3 - 19\,x - 30 = (x + 3)(x^2 - 3\,x - 10)$; from $x + 3 = 0$, $x = -3$ and from $x^2 - 3\,x - 10 = 0$, $x = +5$ and -2; hence -3, $+5$, and -2 are the roots sought.

(2) Trial Method: Make guesses at the roots until a value is found which nearly satisfies the equation; such a value is an approximate root. In finding it, note that if $f(x) = 0$ designates the equation, and if x_1 and x_2 are numbers such that $f(x_1)$ and $f(x_2)$ are of opposite sign, then there is an odd number of roots of $f(x) = 0$ between x_1 and x_2. Thus, to find a root of $f(x) = x^3 + 4x^2 - 5x - 15 = 0$: trying $x = 1$ and $x = 3$ one gets $f(1) = -15$ and $f(3) = +33$; 1 and 3 are not close values of the root but there is a root between them; trying 2 one gets $f(2) = -1$, a closer value. Trying 2.1, one gets $f(2.1) = +1.4$; the root is between 2 and 2.1.

(3) Plotting Method: The equation to be solved being $f(x) = 0$, plot the graph of $y = f(x)$ (see Art. 3); the abscissas of the intersections of the graph with the x axis are roots of $f(x) = 0$. Or, break up $f(x)$ into the difference of two functions of x as $f_1(x)$ and $f_2(x)$ and plot the graphs of $y = f_1(x)$ and $y = f_2(x)$; then the abscissas of the intersections of the two graphs are roots of $f(x) = 0$. Thus, to solve $x^3 + 4x^2 - 5x - 15 = 0$: the graph of $y = x^3 + 4x^2 - 5x - 15$ is shown by the solid curve of Fig. 1, and the abscissas of its intersections with the x axis are about 2.1, -1.6, and -4.3, which are also the roots sought. Or, again, $x^3 + 4x^2 - 5x - 15$ can be broken up into $(x^3 + 4x^2) - (5x + 15)$; the dashed curve is the graph of $y = x^3 + 4x^2$ and the straight line is the graph of $y = 5x + 15$; the abscissas of the intersections of these two graphs are about 2.1, -1.6 and -4.3, and they are the roots sought.

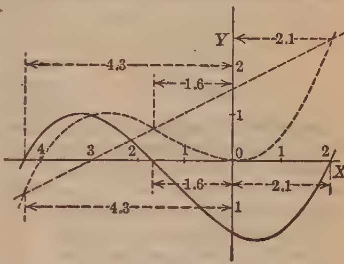

Fig. 1

Transcendental Equations are those involving trigonometry or logarithms; such equations with one unknown can be solved by one of the foregoing methods. For example, to find the roots of $\tan x - 2x = 0$ (x being in radians): plot the graphs of $y = \tan x$ and $y = 2x$, and determine the abscissas of the intersections of the graphs; the abscissas are the roots, one being $x = 1.16$.

To find the horizontal tension t in a catenary cable whose length is 22 ft., span 20 ft., and weight 10 lb. per lin. ft. The equation to be solved is $e^{10/t} - e^{-10/t} - 22 = 0$. By trial t is found to lie between 12 and 14; trying $t = 13$, the equation reduces to $+0.0298 = 0$; trying $t = 13.1$ it reduces to $-0.0012 = 0$, hence the root is a little less than $= 13.1$.

An equation which arises in the theory of a column round at one end and fixed at the other is $x - \tan x = 0$, where x is to be in radians. This equation has many roots, the smallest one being $x = 4.49341$.

3. Graphic Representations

Rectangular Coordinates. XX' and YY' (Fig. 2) are two rectangular coordinate axes or x and y axes; and XOY, YOX', $X'O'Y$, and $Y'OX$ are the first, second, third, and fourth quadrants respectively; O is the origin of coordinates. The position of any point in the drawing, as P, relative to the axes may be specified by two coordinates, the x coordinate, or abscissa of P, and the y coordinate, or ordinate of P. The abscissa, denoted by x, is the distance of P from the y axis, and the ordinate, denoted by y, is the distance of P from the x axis; the coordinates have signs as follows:

x is positive when P is to the right of the y axis,
x is negative when P is to the left of the y axis,
y is positive when P is above the x axis,
y is negative when P is below the x axis.

When P is actually plotted by means of its coordinates, the coordinates are also said to have been plotted. If the coordinates of P are x and y, say, P is referred to as the point (x, y).

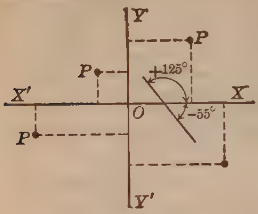

Fig. 2

Graphs. An equation between any two variables as x and y is sometimes written $y = f(x)$, for brevity; it is read "y equals a function of x." The graph of any equation as $y = f(x)$ is a graphical representation of the relation between y and x; to determine the graph compute values of y corresponding to assumed values of x, plot points whose coordinates are the corresponding values of x and y computed, and then draw a smooth curve through the points.

Logarithmic Plotting. The equation $y = f(x)$ may be represented by a graph obtained by plotting corresponding values of $\log x$ and $\log y$, instead of x and y, values of y having been obtained from $y = f(x)$ for assumed values of x; such a graph is a logarithmic graph. If values of x and $\log y$, or y and $\log x$ are plotted, the graphs are semi-logarithmic. These graphs are more advantageous than the ordinary one in some cases, and can be got as readily by using logarithmic or semi-logarithmic rulings, as in Figs. 4 and 6 in which the unequal divisions are like those on an ordinary slide rule, based on the logarithms of numbers, and the figures on the rulings are not logarithms but the corresponding numbers, also as on a slide rule. If $y = f(x)$ is graphed on such ruling by plotting corresponding values of x and y, the graphs are logarithmic or semi-logarithmic, exactly like those first described.

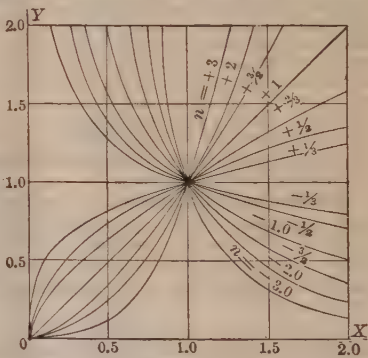

Fig. 3

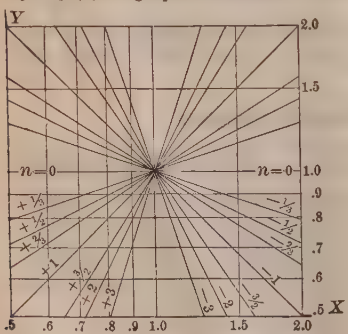

Fig. 4

The intercept of a graph on the x (or y) axis is the x (or y) coordinate of the intersection of the graph with x (or y) axis. The **slope angle** of a straight line is the angle between the line and the positive x axis; the slope angles of all lines may be expressed by angles between 0 and 180°, or $-90°$ and $+90°$ (see Fig. 2). The slope, or gradient, of a graph at any point is the tangent of the slope angle of its tangent line at that point.

The graph of $y = mx + c$, into the form of which any equation between two variables of the first degree can be put, is straight; m = the slope of the graph and c = its intercept on the y axis. Thus $y = -2x + 3$ has a graph sloping upwards to the left at an angle of 63-1/2° and cutting the y axis 3 units above the origin.

The graph of $y - b = m(x - a)^n + c$ is curved, its shape depending upon the numerical values of m and n; a, b, and c affect only the position of the graph with respect to the coordinate axes. Fig. 3 represents graphs of $y = x^n$ for the different values of n noted. The corresponding graphs of $y = mx^n$ are enlargements in the y direction if $m > 1$ and reductions if $1 > m > 0$ of those shown; if m is negative, the effect is enlargement or reduction as explained and a rotation of the graphs through 180° about the x axis. The logarithmic graph of $y = x^n$ is straight; Fig. 4 shows these graphs corresponding to those of Fig. 3. The logarithmic graphs of $y = mx^n$ are also straight, but they do not pass through the point (1, 1).

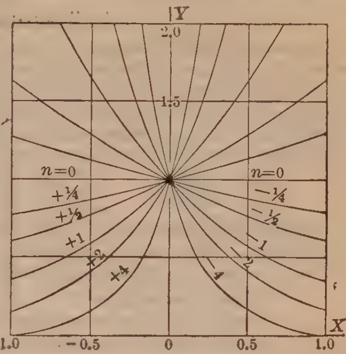

Fig. 5

The graph of $(y - a) = m\epsilon^{nx-b} + c$, a "logarithmic or compound interest" equation, is curved; its shape depends on the numerical values of m and n; a, b, and c affect only the position of the curve relative to the coordinate axes. Fig. 5 represents graphs of $y = \epsilon^{nx}$ for the several values of n noted. In $y = m\epsilon^{nx}$, m affects the graphs shown as explained in the preceding paragraph. The semi-logarithmic graph of $y = \epsilon^{nx}$ is straight; Fig. 6 shows these graphs corresponding to those of Fig. 5. The semi-logarithmic graph of $y = m\epsilon^{nx}$ is also straight, but it does not pass through the point (0, 1).

To Find an Equation for a Given Graph. If the graph, plotted on axes x and y, is straight, the equation is of the first degree; let (x_1, y_1) and (x_2, y_2) be the coordinates of any two points of the graph; the equation is $(y - y_1)(x_2 - x_1) = (x = x_1)(y_2 = y_1)$. If the intersection of the graph with either axis is available, that intersection may well be taken as one of the points; if it passes through the origin, that may be advantageously chosen. If the graph is curved, there may be no corresponding equation but an equation can usually be found which will approximately fit at least a limited portion of the graph.

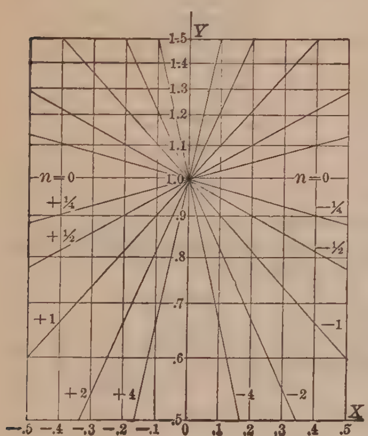

Fig. 6

(1) If the graph resembles a portion of one of the family represented in Fig. 3, plot its logarithmic graph; if this is practically straight, the original graph can be closely represented by $y = mx^n$. To determine m and n: $m = y$ when $x = 1$, and this value of y can be got from either graph, probably more accurately from the logarithmic; then in $\log (y/m) \doteq n \log x$ substitute the coordinates x and y of any point and solve for n.

(2) If the graph resembles a portion of one of the family represented in Fig. 5, plot semi-logarithmically (Fig. 6); if the resulting graph is practically straight, then the

original graph can be closely represented by $y = m\epsilon^{nx}$. To determine m and n, the value of m is found from $m = y$ when $x = 0$, and this value of y can be got from either graph, probably most accurately from the semi-logarithmic graph; then in $\log_\epsilon y = \log_\epsilon m + nx$ substitute the x and y coordinates of any point on either graph and solve for n.

4. Trigonometry

Functions of Acute Angles. If from a point in either line bounding an angle (Fig. 7) a perpendicular is drawn to the other line, then in the right triangle thus formed the leg adjacent to α is the base, and the leg opposite, the perpendicular; they are denoted by b and p respectively, and the hypotenuse by h. The sine, cosine, tangent, cotangent, secant, and cosecant of the angle are defined (and abbreviated) thus:

$$\sin\alpha = p/h \qquad \cos\alpha = b/h \qquad \tan\alpha = p/b$$
$$\csc\alpha = h/p \qquad \sec\alpha = h/b \qquad \cot\alpha = b/p$$

The versed sine, abbreviated vers, is unity minus the cosine; thus $\text{vers}\,\alpha = 1 - \cos\alpha$. The exsecant, abbreviated exsec, is secant minus unity, thus $\text{exsec}\,\alpha = \sec\alpha - 1$. From the foregoing it follows that

$$\sin\alpha/\cos\alpha = \tan\alpha \qquad \cos\alpha/\sin\alpha = \cot\alpha$$
$$\sin\alpha\csc\alpha = \cos\alpha\sec\alpha = \tan\alpha\cot\alpha = 1$$
$$\sin^2\alpha + \cos^2\alpha = \sec^2\alpha - \tan^2\alpha = \csc^2\alpha - \cot^2\alpha = 1$$

If $\alpha + \beta = 90°$, $\sin\alpha = \cos\beta$, $\tan\alpha = \cot\beta$, $\sec\alpha = \csc\beta$. If the arc $A''B'$ (Fig. 7) be struck from O with a radius unity, then in the triangle $OA'B'$ h is unity and in the triangle $OA''B''$ b is unity. Then also

$$\sin\alpha = A'B' \qquad \cos\alpha = OA' \qquad \tan\alpha = A''B''$$
$$\sec\alpha = OB'' \qquad \text{vers}\,\alpha = A'A'' \qquad \text{exsec}\,\alpha = B'B''$$

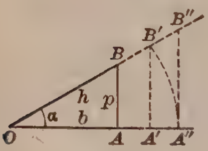

Fig. 7

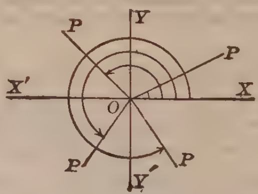

Fig. 8

Functions of any Angle. Let $\alpha = XOP$ (Fig. 8), x and y = the coordinates of P, and $h = OP$. Then whether α is in the first, second, third, or fourth quadrant, the functions of α are defined as follows:

$$\sin\alpha = y/h \qquad \cos\alpha = x/h \qquad \tan\alpha = y/x$$
$$\csc\alpha = h/y \qquad \sec\alpha = h/x \qquad \cot\alpha = x/y$$

Since x and y may be positive or negative, the functions may also be; the sign of a function depends on the quadrant of α, thus:

quad	sin	cos	tan	cot	sec	csc
1	+	+	+	+	+	+
2	+	−	−	−	−	+
3	−	−	+	+	−	−
4	−	+	−	−	+	−

The values of the functions for the four cardinal angles are:

angle	sin	cos	tan	cot	sec	csc
0°	0	+ 1	0	∞	+ 1	∞
90	+ 1	0	∞	0	∞	+ 1
180	0	− 1	0	∞	− 1	∞
270	− 1	0	∞	0	∞	− 1
360	0	+ 1	0	∞	+ 1	∞

Functions of angles greater than 90° are not given in trigonometric tables generally. These can be obtained from the functions of acute angles by means of the following in which a_2, a_3, and a_4 denote angles in the second, third, and fourth quadrants respectively.

a_2	$180° - a_2$	$a_2 - 90°$	a_3	$a_3 - 180°$	$270° - a_3$	a_4	$360° - a_4$	$a_4 - 270°$
sin =	+ sin =	+ cos	sin =	− sin =	− cos	sin =	− sin =	− cos
cos =	− cos =	− sin	cos =	− cos =	− sin	cos =	+ cos =	+ sin
tan =	− tan =	− cot	tan =	+ tan =	+ cot	tan =	− tan =	− cot

Negative Angles. Angles in the preceding figures and those referred to in the foregoing were regarded as measured from OX in the counterclockwise direction and considered positive; an angle regarded as measured clockwise is considered negative. OP (Fig. 8) may be specified by a positive or a negative angle, the arithmetic sum of the two being 360°. If α denotes merely the numerical value of any angle, then

$$\sin(-\alpha) = -\sin\alpha \qquad \cos(-\alpha) = \cos\alpha \qquad \tan(-\alpha) = -\tan\alpha$$
$$\csc(-\alpha) = -\csc\alpha \qquad \sec(-\alpha) = \sec\alpha \qquad \cot(-\alpha) = -\cot\alpha$$

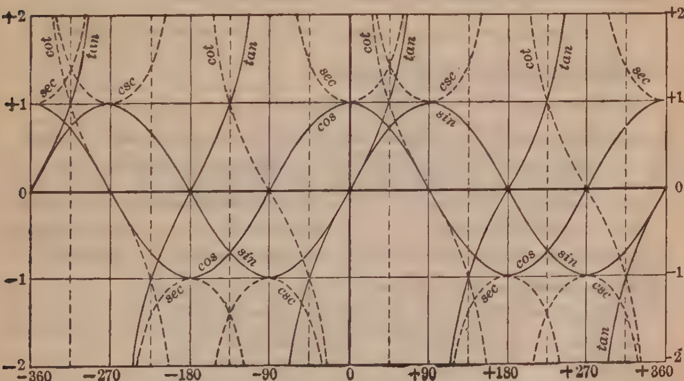

Fig. 9. Graphs of Trigonometric Functions

Graphs of sine, cosine, tangent, cotangent, secant, cosecant are given in Fig. 9 for values of angle from − 360° to + 360°. The graphs of the last four functions have infinite branches, asymptotic to the solid vertical lines adjacent at the upper and lower margins. The figure shows the sign and periodicity of any function of any angle between the limits named and roughly the relative values of the functions of any particular angle.

Inverse, or Anti-Functions. The symbol $\sin^{-1} x$ means the angle whose sine is x, and is read inverse sine of x and anti-sine of x (also arc sine x). Similarly $\cos^{-1} x$, $\tan^{-1} x$, $\cot^{-1} x$, $\sec^{-1} x$, $\csc^{-1} x$, $\text{vers}^{-1} x$, the last meaning an angle α such that $(1 - \cos\alpha) = x$. While the direct functions (sine, etc.) are single valued, the indirect are many valued; thus $\sin 30° = 0.5$, but $\sin^{-1} 0.5 = 30°$ or $150°$ . . .

Trigonometric Relations. For functions of an angle expressed in terms of each of the others, the following abbreviations are used:

$$A = \pm\sqrt{1 - \sin^2\alpha} \qquad B = \pm\sqrt{1 - \cos^2\alpha} \qquad C = \pm\sqrt{1 + \tan^2\alpha}$$

$$F = \pm\sqrt{\csc^2\alpha - 1} \qquad E = \pm\sqrt{\sec^2\alpha - 1} \qquad D = \pm\sqrt{1 + \cot^2\alpha}$$

$$\sin\alpha = B = \tan\alpha/C = 1/D = E/\sec\alpha = 1/\csc\alpha$$

$$\cos\alpha = A = 1/C = \cot\alpha/D = 1/\sec\alpha = F/\csc\alpha$$

$$\tan\alpha = \sin\alpha/A = B/\cos\alpha = 1/\cot\alpha = E = 1/F$$

$$\cot\alpha = A/\sin\alpha = \cos\alpha/B = 1/\tan\alpha = 1/E = F$$

$$\sec\alpha = 1/A = 1/\cos\alpha = C = D/\cot\alpha = \csc\alpha/F$$

$$\csc\alpha = 1/\sin\alpha = 1/B = C\tan\alpha = D = \sec\alpha/E$$

Functions of the sum and difference of two angles:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = (\tan\alpha \pm \tan\beta)/(1 \mp \tan\alpha\tan\beta)$$

$$\cot(\alpha \pm \beta) = (\cot\beta\cot\alpha \mp 1)/(\cot\beta \pm \cot\alpha).$$

If x is small, say 3 or 4°, then the following are close approximations, in which the coefficients x must be expressed in radians (1° = 0.01745 radian).

$$\sin(\alpha \pm x) = \sin\alpha \pm x\cos\alpha, \qquad \cos(\alpha \pm x) = \cos\alpha \mp x\sin\alpha$$

Functions of half and double angles:

$$\sin 1/2\,\alpha = \sqrt{1/2\,(1 - \cos\alpha)} = 1/2\sqrt{1 + \sin\alpha} - 1/2\sqrt{1 - \sin\alpha}$$

$$\cos 1/2\,\alpha = \sqrt{1/2\,(1 + \cos\alpha)} = 1/2\sqrt{1 + \sin\alpha} + 1/2\sqrt{1 - \sin\alpha}$$

$$\tan 1/2\,\alpha = \sqrt{(1-\cos\alpha)/(1+\cos\alpha)} = (1-\cos\alpha)/\sin\alpha = \sin\alpha/(1+\cos\alpha)$$

$$\cot 1/2\,\alpha = \sqrt{(1+\cos\alpha)/(1-\cos\alpha)} = (1+\cos\alpha)/\sin\alpha = \sin\alpha/(1-\cos\alpha)$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha, \qquad \cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha,$$

$$\tan 2\alpha = 2\tan\alpha/(1 - \tan^2\alpha) \qquad \cot 2\alpha = (\cot^2\alpha - 1)/2\cot\alpha.$$

Sums and products of functions:

$$\sin\alpha\cos\beta = 1/2\sin(\alpha + \beta) + 1/2\sin(\alpha - \beta)$$

$$\cos\alpha\sin\beta = 1/2\sin(\alpha + \beta) - 1/2\sin(\alpha - \beta)$$

$$\sin\alpha\sin\beta = 1/2\cos(\alpha - \beta) - 1/2\cos(\alpha + \beta)$$

$$\cos\alpha\cos\beta = 1/2\cos(\alpha - \beta) + 1/2\cos(\alpha + \beta)$$

$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$

$$\sin\alpha + \sin\beta = 2\sin 1/2\,(\alpha + \beta)\cos 1/2\,(\alpha - \beta)$$

$$\sin\alpha - \sin\beta = 2\cos 1/2\,(\alpha + \beta)\sin 1/2\,(\alpha - \beta)$$

$$\cos\alpha + \cos\beta = 2\cos 1/2\,(\alpha + \beta)\cos 1/2\,(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin 1/2\,(\alpha + \beta)\sin 1/2\,(\alpha - \beta)$$

5. Plane and Spherical Triangles

Plane Triangles. The three angles are denoted by α, β, and γ, and the respective opposite sides by a, b, and c; also s denotes $1/2\ (a + b + c)$, r radius of inscribed circle, R radius of circumscribed circle, and A area of the triangle. Then, $\alpha + \beta + \gamma = 180°$

$$a/\sin\alpha = b/\sin\beta = c/\sin\gamma \text{ ("law of sines")}$$
$$c^2 = a^2 + b^2 - 2\,ab\cos\gamma, \qquad r = \sqrt{(s-a)(s-b)(s-c)/s}$$
$$A = sr = 1/2\ ab\sin\gamma, \qquad R = 1/2\ a\csc\alpha.$$

Solution of Right Triangles. If an acute angle and one side or if two sides of a right triangle are given the other elements can be determined. Let h = hypotenuse, α and β acute angles, and a and b the legs opposite them, respectively. The acute angles are complementary, that is, $\alpha + \beta = 90°$; the area is $1/2\ ab$ always. Five cases may be distinguished:

Given h and α; $\quad a = h\sin\alpha, \quad b = h\cos\alpha$
Given a and α; $\quad b = a\cot\alpha, \quad h = a\csc\alpha$
Given b and α; $\quad a = b\tan\alpha, \quad h = b\sec\alpha$
Given a and h; $\quad \alpha = \sin^{-1} a/h,\ b = \sqrt{(h+a)(h-a)}$
Given a and b; $\quad \alpha = \tan^{-1} a/b,\ h = \sqrt{a^2 + b^2}$

Solution of Oblique Triangles. If any three of the six elements (three angles and three sides) of a triangle are known, the remaining three can be determined provided one of the given three is a side. Any problem will fall under one of four cases.

Case 1. Given one side and two angles. Then the third angle equals 180° minus the sum of the two given. If the given side is a, then

$$b = a\sin\beta/\sin\alpha \quad \text{and} \quad c = a\sin\gamma/\sin\alpha.$$

Case 2. Given two sides and the included angle (a, b and γ). Then

$$1/2(\alpha + \beta) = 90° - 1/2\gamma,\ 1/2(\alpha - \beta) = \tan^{-1}[\tan 1/2(\alpha+\beta)\cdot(a-b)/(a+b)]$$
$$\alpha = 1/2\ (\alpha + \beta) + 1/2\ (\alpha - \beta) \qquad \beta = 1/2\ (\alpha + \beta) - 1/2\ (\alpha - \beta)$$
$$c = a\sin\gamma/\sin\alpha.$$

Case 3. Given two sides a and b and the angle α opposite one of them. Then $\sin\beta = (b/a)\sin\alpha$, giving two values of β, one acute and one obtuse unless $\sin\beta > 1$, in which case the data are impossible. Calling these two angles β_1 and β_2 respectively, then

corresponding to β_1, $\gamma_1 = 180 - (\alpha + \beta_1)$ and $c_1 = a\sin\gamma_1/\sin\alpha$
corresponding to β_2, $\gamma_2 = 180 - (\alpha + \beta_2)$ and $c_2 = a\sin\gamma_2/\sin\alpha$

That is, there are two solutions unless $\gamma_2 < 0$, when only the first holds. (The meanings of these exceptions, $\sin\beta > 1$ and $\gamma_2 < 0$, will become evident if a geometrical construction of the triangle is attempted.)

Case 4. Given the three sides. Let s denote $1/2\ (a + b + c)$. Then

$$\cos 1/2\,\alpha = \sqrt{s(s-a)/bc} \quad \cos 1/2\,\beta = \sqrt{s(s-b)/ca} \quad \cos 1/2\,\gamma = \sqrt{s(s-c)/ab}$$

Spherical Triangles. The intersection of the surface of a sphere with a plane through its center is a great circle of the sphere; an arc of the circle is measured by the angle between the radiuses of the sphere drawn to the ends of the arc. A spherical angle is the angle between two intersecting arcs of great circles; it is measured by the angle between their tangents at the intersection. A spherical triangle is a portion of the surface of a sphere bounded by arcs of three great circles. Spherical triangles are classified in the same way as plane triangles: isosceles, equilateral, right etc. The spherical excess of a triangle is the excess of the sum of its angles over 180°. Any spherical triangle can be solved if any three of its six elements (three sides and three angles) are given; from the following formulas a solution may be made in any given case. The notation is as follows: α, β, and γ represent the three angles, while a, b, and c designate the opposite sides respectively, and e = the spherical excess.

$$\sin a/\sin\alpha = \sin b/\sin\beta = \sin c/\sin\gamma$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos\gamma$$

$$\cos\gamma = -\cos\alpha\cos\beta + \sin\alpha\sin\beta\cos c$$

$$\cot 1/2\, e = (\cot 1/2\, a \cot 1/2\, b + \cos\gamma)/\sin\gamma$$

A spherical right triangle can be solved if any two of its elements, not including the right angle, are given. In the following h is the hypotenuse, a and b the other two sides, and α and β the angles opposite a and b respectively.

$$\sin a = \sin h \sin\alpha = \tan b \cot\beta \qquad \cos\alpha = \cos a \sin\beta = \cot h \tan b$$

$$\cos h = \cos a \cos b = \cot\alpha\cot\beta$$

6. Geometry and Mensuration

Angles. The common unit of angle is the degree (one-ninetieth of a right angle); the degree is divided into 60 minutes and the minute into 60 seconds. Another unit is the *radian*, called also the unit in circular measure of angles; it is an angle equal to that between two radiuses of a circle which embrace an arc equal to the radius. Then

$$1 \text{ radian} = 180 \div \pi \text{ degrees} = 57.296°, \qquad 1 \text{ degree} = 0.0175 \text{ radian}$$

If an angle α subtends a circular arc whose length and radius are s and r respectively, then $\alpha = s/r$ or $s = r\alpha$ provided that s and r are expressed in the same unit and α in radians.

Triangles. Let a, b, and c = lengths of sides; α, β, and γ the opposite angles, h the altitude to the side b, s denote $1/2\,(a + b + c)$, and A = area.

$$A = 1/2\, bh = 1/2\, ba \sin\gamma = 1/2\, b^2 \sin\gamma \sin\alpha/\sin\beta = \sqrt{s(s-a)(s-b)(s-c)}$$

Quadrilaterals. Let D_1 and D_2 = lengths of the two diagonals and a the angle between them, then $A = 1/2 D_1 D_2 \sin a$. Parallelogram: let a = one side, b = base, a = angle between a and b, h = altitude; then $A = bh = ab \sin a$. Trapezoid: let b and B = parallel sides, and h = altitude; then $A = 1/2\,(b + B)h$.

Polygons. The area A equals the sum of the areas of the constituent triangles. For a regular polygon (equal sides and equal angles) let R = radius of circumscribed circle, r = radius of inscribed circle, a = length of a side, n = number of sides, 2α = central angle subtended by one side; then

$$A = 1/4\, na^2 \cot\alpha = 1/2\, nR^2 \sin 2\alpha = nr^2 \tan\alpha \quad a = 2R\sin\alpha = 2r\tan\alpha$$

Circle (Fig. 11). Circumference $L = 2\pi r = \pi d$. Area $A = \pi r^2 = 1/4\pi d^2$. Chord $C = 2r \sin 1/2\,\theta$. Rise of arc $h = r\,(1 - \cos 1/2\,\theta)$. Arc subtended by θ $a = \pi d \times (\theta \text{ in degrees})/360 = r\,(\theta \text{ in radians})$; when h is small compared to C, $a = 1/3(8c - C)$; this is approximate, but even when $h = r$, the error is less than 1-1/4 %. Sector $OMPN$: area = area of circle (θ in degrees)/360. Segment $MPNM$: area = area sector $- 1/2\, r^2 \sin\theta$; when h is small compared to C, area $= 2/3\, Ch$, also $h\,(8c + 6C)/15$, both approximate; when $h = 1/4\, r$, the first errs about 3-1/2 % and the second less than 1%.

To Find the Center of a Given Circle, draw a chord and then a perpendicular to the chord at its middle point; this line extended in both directions to the circumference is a diameter. To draw a circle through three points: join one of the points with the others, thus getting two chords; at the middle points of the chords erect perpendiculars; the intersection of these is the center of the circle. If, in the preceding problem, the center is inaccessible or the radius is too long for drawing instruments, proceed as follows, the three points being A, O, A', Fig. 10: draw arcs Aa' and $A'a$ with centers at A' and A respectively; extend AO to determine a and $A'O$ to determine a'; divide aA' into equal parts ab, bc, etc.; lay off $a'b'$, $b'c'$, etc., equal to ab; join A with any point as b and A' with the corresponding point b'; the intersection P of these joining lines is a point in the circle. If in the preceding problem it is desired to determine points in the circle by ordinates as NP, let r = radius of the circle, h = rise or center ordinate OQ, C = chord AA', y = any ordinate as NP, and $x = QN$ (distance of that ordinate from the center one), then $r = (4\,h^2 + C^2)/8\,h$, and

$$y = \sqrt{r^2 - x^2} - (r - h) = \sqrt{r^2 - x^2} - \sqrt{r^2 - C^2/4}$$

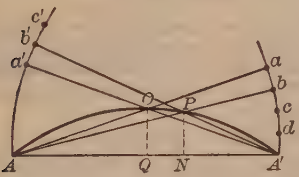

Fig. 10

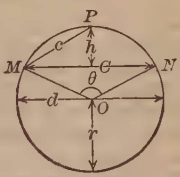

Fig. 11

Ellipse (see also Art. 11). The circumference of an ellipse whose semi-axes are a and b is $\pi(a + b)k$ where k is an abbreviation for $(1 + c^2/4 + c^4/64 + c^6/256 + \ldots)$ and $c = (a - b)/(a + b)$. The following table gives k for several values of c:

c	k	c	k	c	k	c	k	c	k
0.05	1.0006	0.25	1.0158	0.45	1.0516	0.65	1.1083	0.85	1.1903
.10	1.0025	.30	1.0215	.50	1.0635	.70	1.1267	.90	1.2154
.15	1.0054	.35	1.0311	.55	1.0768	.75	1.1466	.95	1.2430
.20	1.0100	.40	1.0404	.60	1.0922	.80	1.1677	1.00	1.2732

The area of an ellipse is given by πab; the area of any part as $ABQP$ (Fig. 12) is $xy + ab \sin^{-1} x/a$, in which x and y are the distances of P or Q from the axes.

Hyperbola (see also Art. 11). The area of any symmetrical segment like $PAQP$ (Fig. 13) is $xy - ab \log_\epsilon (x/a + y/b)$, where $2\,a$ and $2\,b$ are the lengths of the axes of the hyperbola and x and y the distances of P or Q from the axes.

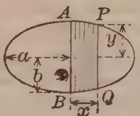

Fig. 12

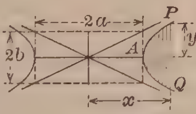

Fig. 13

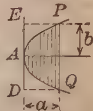

Fig. 14

Parabola (see also Art. 11). The length of any arc beginning at the vertex as AP (Fig. 14) is $(b^2/4\,a)\left[\sqrt{c(1 + c)} + \log_\epsilon\left(\sqrt{c} + \sqrt{1 + c}\right)\right]$, in which c is an abbreviation for $4\,a^2/b^2$, a and b being distances as marked. The area of a symmetrical segment like $PAQP$ is two-thirds that of the rectangle $PEDQ$.

Irregular Figure (Fig. 15). To find the area divide the figure into an even number of strips of equal width; the more numerous these strips the more accurate is the result given by the following formulas. The first of the ordinates bounding the strips is called y_0, the second y_1, etc., n the number of strips, w their common width and A the area. The following formulas give A approximately:

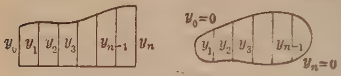

Fig. 15

$A = w(1/2\, y_0 + y_1 + y_2 + \ldots + y_{n-1} + 1/2\, y_n)$, Trapezoidal.

$A = 1/3\, w(y_0 + 4\,y_1 + 2\,y_2 + 4\,y_3 + \ldots + 2\,y_{n-2} + 4\,y_{n-1} + y_n)$, Simpson's.

$A = w(0.4\,y_0 + 1.1\,y_1 + y_2 + y_3 + \ldots + y_{n-2} + 1.1\,y_{n-1} + 0.4\,y_n)$, Durand's.

Simpson's rule applies only when n is odd; when n is even the area of $n - 3$ strips may be computed and then the area of the remaining 3 strips may be found by

$$A = 3/8\, w\, (y_0 + 3\,y_1 + 3\,y_3 + y_j), \text{ Cotes's.}$$

Prism. Let V = volume, h = altitude, A = base area; $V = Ah$. Truncated prism: V = the product of the length of line joining the centers of gravity of the bases and the area of section perpendicular to the line. Truncated triangular prism: V = product of one-third the sum of the parallel edges and the area of section perpendicular to edges.

Cylinder. Let V = volume, A = base area, h = altitude; $V = Ah$. Right circular cylinder: Let r = base radius, S = area cylindrical surface; $V = \pi r^2 h$, $S = 2\,\pi r h$, $A = \pi r^2$. Frustum right circular cylinder: H = greatest height, h = least height; $V = \pi r^2 (H + h)/2$, $S_{,} = \pi r(H + h)$. Wedge of right circular cylinder (Fig. 16); Let $2\,C$ = straight edge of base, p = altitude of the base (segment of circle), $2\,\phi$ = central angle of the arc of base in degrees, r = radius of base; $V = [C\,(3\,r^2 - C^2) + 3\,r^2\,(p - r)\,\phi\,\pi/180]\; h/3\,p$, $S = [C + (p - r)\phi\pi/180]\; 2\,rh/p$. Hollow right circular cylinder: R = outer radius of base, r = inner radius, t = thickness $= R - r$; $V = \pi(R^2 - r^2)h = \pi\,(2\,R - t)\,th = \pi\,(2\,r + t)\,th$.

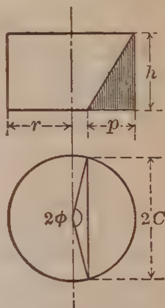

Fig. 16

Pyramid. Let V = volume, A = area base, h = altitude; $V = Ah/3$. Frustum of a pyramid: let A and a = areas of bases; $V = (A + a + \sqrt{Aa})\, h/3$.

Cone. Let V = volume, A = area of base, h = altitude; then $V = Ah/3$. Circular Cone: let r = radius of base; $V = \pi r^2 h/3$. Right circular cone: area of conical surface, $S = \pi r \sqrt{r^2 + h^2}$. Frustum of right circular cone: let R = radius larger and r = radius smaller base; $V = \pi\,(R^2 + Rr + r^2)h/3$; $S = \pi\,(R + r)\sqrt{(R - r)^2 + h^2}$.

Sphere. Let V = volume, A = area, r = radius, and d = diameter. Then $V = 4/3\,\pi r^3 = 1/6\,\pi d^3$; $A = 4\,\pi r^2 = \pi d^2$. Zone: let h = altitude, a = radius larger base, b = radius smaller base; $V = 1/2\,\pi h\,(a^2 + b^2 + h^2/3)$; convex $A = 2\,\pi r h$; $r^2 = a^2 + [(a^2 - b^2 - h^2)/2\,h]^2$. Segment (zone with one base): make $b = 0$ in formulas for zone. Sector with one conical surface (Fig. 17): $V = 2/3\,\pi r^2 h$, total $A = \pi r\,(2\,h + a)$.

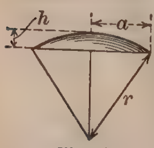

Fig. 17

Ellipsoid. Every section is a circle or an ellipse. Let a, b, and c = the semi-axes; then the volume $= 4/3\,\pi abc$.

Paraboloid of Revolution generated by revolving a parabola about its own axis (Fig. 18). Let r = radius of base and a = altitude of paraboloid;

then volume = $1/2\,\pi r^2 a$. Volume of a frustum, R and r being radii of bases and h = height (see Fig. 18) is $1/2\,\pi(R^2 + r^2)h$ (approx.).

Surface and Solid of Revolution. The first may be generated by revolving a plane curve about a line in its plane, or else by a straight line about an intersecting line; and the second by revolving a plane area about a line in its plane. The **axis** of the surface or solid is the line about which the revolution takes place. The area and volume can be determined by the principles of Pappus and Guldinus which are:

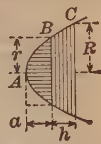

Fig. 18

(a) For area A: let l = length of generating curve (must not cut the axis of revolution) and x_1 = distance of its center of gravity from the axis; then $A = 2\,\pi x_1 l$.

(b) For volume V: let a = area of the generating figure (must not be intersected by the axis) and x_2 = distance of its center of gravity from the axis; then $V = 2\,\pi x_2 a$.

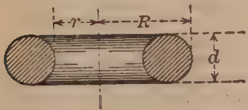

Fig. 19

The revolution of a circle about a line in its plane but not cutting the circle generates a "tore," or "torus" or anchor ring (Fig. 19). The preceding formulas give its area and volume thus: $A = 2\,\pi\,1/2\,(R + r)\,\pi\,d = \pi^2\,(R + r)\,d$, and $V = 2\,\pi\,1/2\,(R + r)\,1/4\,\pi\,d^2 = 1/4\,\pi^2\,(R + r)d^2$.

A Prismoid is a solid with plane faces two being parallel (and called ends) and each of the others containing a part of the perimeter of one end and at least a point in that of the other end. Let V = volume of the prismoid, A_1 and A_2 = areas of the ends, A_m = area of the cross-section of the prismoid midway between the ends and l = length of prismoid (distance between ends); then $V = 1/6\,(A_1 + A_2 + 4\,A_m)l$.

7. Interest and Sinking Fund

Interest. The notation in the formulas is: p = principal, the sum loaned; i = rate of interest, expressed 0.06 or 6/100 for 6%; n = time, term, or period of the loan in years; a = amount due after n years, that is, the sum of the principal and interest. When interest is paid only on the principal and not also on interest which may be due, the interest is simple; then

$$a = p(1 + in) \qquad i = (a - p)/pn \qquad p = a/(1 + in) \qquad n = (a - p/pi$$

The adjoining table gives the simple interest on \$100 at various rates for one year, one month (1/12 year), and one day (1/30 month, bank practice).

	2%	3%	4%	5%	6%	8%
1 year	\$2.00	\$3.00	\$4.00	\$5.00	\$6.00	\$8.00
1 month	0.16-2/3	0.25	0.33-1/3	0.41-2/3	0.50	0.66-2/3
1 day	0.0055-5/9	0.0083-1/3	0.0111-1/9	0.0138-8/9	0.0166-2/3	0.0222-2/9

When interest is not paid periodically but regarded as additions to the principal, also to draw interest during the remainder of the time, then the interest is compound. If additions of interest to principal are made annually, then

$$a = p(1+i)^n, \quad i = \sqrt[n]{(a/p)} - 1, \quad p = a/(1+i)^n, \quad n = (\log a - \log p)/\log\,(1+i)$$

For semiannual compound interest, $a = p(1 + i/2)^{2n}$. The adjoining table gives the amount of one dollar at interest compounded annually at various rates for various periods. Thus, \$2000 at 3% in 20 years amounts to 2000 × \$1.80611 = \$3612.22; the interest earning is \$3612.22 − \$2000, or \$1612.22.

Amount (Principal plus Interest) of $1 at Compound Interest

No. of Years	Rates of compound interest						
	2%	2-1/2%	3%	3-1/2%	4%	4-1/2%	5%
1	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000
2	1.04040	1.05062	1.06091	1.07122	1.08160	1.09202	1.10250
3	1.06121	1.07689	1.09273	1.01872	1.12486	1.14117	1.15762
4	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551
5	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628
10	1.21899	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889
15	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893
20	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330
25	1.64061	1.85394	2.09378	2.36324	2.66584	3.00543	3.38635
30	1.81136	2.09757	2.42726	2.80679	3.24340	3.74532	4.32194
35	1.99989	2.37321	2.81386	3.33359	3.94609	4.66735	5.51602
40	2.20804	2.68506	3.26204	3.95926	4.80102	5.81636	7.03999
45	2.43785	3.03790	3.78160	4.70236	5.84118	7.24825	8.98501
50	2.69159	3.43711	4.38391	5.58493	7.10668	9.03264	11.46740

An Annuity is a succession of equal sums paid regularly, generally annually; each payment is also called annuity, but more properly, installment. The amount of an annuity for a term of years is the sum of the installments plus their interest earnings, interest being generally considered as compound. Let A = amount; i = interest rate expressed decimally (as 0.05 for 5%); n = number of years in the period of the annuity; and I = (annual) installment, one at the end of each year. Then $A = I\,[(1 + i)^n - 1]/i$. The following table gives the amount of an annual annuity of $1 for various periods and various rates of interest. Thus, $200 invested at the end of each year of a 20-year period at 4% amounts to 200 × $29.77808, or $5955.62, at the end of that period; the interest earned is $5955.62 − (20 × $200) = $1955.62.

Amount (Installments plus Interest Earned) of an Annual Annuity of $1

No. of Years	Rates of compound interest						
	2%	2-1/2%	3%	3 1/2%	4%	4-1/2%	5%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.02000	2.02500	2.03000	2.03500	2.04000	2.04500	2.05000
3	3.06040	3.07562	3.09090	3.10622	3.12160	3.13702	3.15250
4	4.12161	4.15252	4.18363	4.21494	4.24646	4.27819	4.31012
5	5.20404	5.25633	5.30914	5.36247	5.41632	5.47071	5.52563
10	10.94972	11.20338	11.46388	11.73139	12.00611	12.28821	12.57789
15	17.29342	17.93193	18.59891	19.29568	20.02359	20.78405	21.57856
20	24.29737	25.54466	26.87037	28.27968	29.77808	31.37142	33.06595
25	32.03030	34.15776	36.45926	38.94986	41.64591	44.56521	47.72710
30	40.56808	43.90270	47.57542	51.62268	56.08494	61.00707	66.43885
35	49.99448	54.92821	60.46208	66.67401	73.65222	81.49662	90.32031
40	60.40198	67.40255	75.40126	84.55028	95.02552	107.03032	120.79977
45	71.89271	81.51613	92.71986	105.78167	121.02939	138.84996	159.70016
50	84.57940	97.48435	112.79687	130.99791	152.66708	178.50303	209.34800

A Sinking or Amortization Fund is one created to provide a definite sum (for canceling a debt, for example) at a certain time; it is usually created by equal and regular contributions or installments and their interest earnings, compounded, that is, by an annuity and its interest earnings. Let S = the amount of a sinking fund; n = number of years for its creation; i = interest rate expressed decimally (as 0.05 when the rate is 5%); and I = annual installment required, paid at the end of each year of the period. Then $I = Si/[(1 + i)^n - 1]$. The following table gives annual installments required to create a fund of one dollar (equal to I/S) in various periods at various interest rates. Thus the annual installment necessary to accumulate $22 000 in 25 years, at 4%, is 22 000 × $0.02401, or $528.22; the interest earning is 22 000 − (25 × 528.22), or $8794.50.

Annual Annuity Required to Accumulate $1 (Installments plus Interest Earnings)

No. of Years	Rates of compound interest						
	2%	2-1/2	3%	3-1/2%	4%	4-1/2%	5%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.49505	0.49382	0.49261	0.49140	0.49020	0.48900	0.48780
3	0.32675	0.32514	0.32353	0.32193	0.32035	0.31877	0.31721
4	0.24262	0.24082	0.23902	0.23725	0.23550	0.23374	0.23201
5	0.19218	0.19025	0.18835	0.18648	0.18463	0.18279	0 18098
10	0.09133	0.08926	0.08723	0.08524	0.08329	0.08138	0.07950
15	0.05782	0.05577	0.05380	0.05183	0.04994	0.04811	0.04634
20	0.04116	0.03915	0.03722	0.03536	0.03356	0.03187	0.03024
25	0.03122	0.02928	0.02743	0.02567	0.02401	0.02244	0.02095
30	0.02465	0.02278	0.02102	0.01937	0.01783	0.01639	0.01505
35	0.02000	0.01821	0.01654	0.01499	0.01358	0.01227	0.01107
40	0.01655	0.01484	0.01326	0.01183	0.01052	0.00934	0.00828
45	0.01391	0.01226	0.01080	0.00945	0.00826	0.00720	0.00626
50	0.01182	0.01026	0.00886	0.00763	0.00655	0.00560	0.00478

The Present Worth of a given sum due at a future time is such a sum which if now placed at interest, would amount to the given sum at that time. Let p = present worth, a = given sum due in n years from now, and i = interest rate; then on simple interest basis $p = a/(1 + in)$, and on annual compound $p = a/(1 + i)^n$. The following table gives present worths, computed at various rates of compound interest, of $1 due in various periods. For example, an addition to plant necessary 5 years hence will then entail a cost of $8000; how much can one afford to pay for such addition now, it being without use or expense for 5 years? As much as the present worth of $8000 due in 5 years at the current rate of interest, that is, if the rate is 5%, 8000 × $0.78353, or $6268.24.

Present Worth of $1 Due at a Future Date

No. of Years	Rates of compound interest						
	2%	2°1/2%	3%	3°1/2%	4%	4°1/2%	5%
1	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238
2	0.96117	0.95181	0.94260	0.93351	0.92456	0.91573	0.90703
3	0.94232	0.92860	0.91514	0.90894	0.88900	0.87630	0.86304
4	0.92385	0.90595	0.88849	0.87144	0.85480	0.83856	0.82270
5	0.90573	0.88385	0.86261	0.84197	0.82193	0.80245	0.78352
10	0.82035	0.78120	0.74409	0.70892	0.67556	0.64393	0.61391
15	0.74301	0.69147	0.64186	0.59689	0.55526	0.51672	0.48102
20	0.67297	0.61027	0.55368	0.50257	0.45639	0.41464	0.37689
25	0.60953	0.53939	0.47761	0.42315	0.37512	0.33273	0.29530
30	0.55207	0.47674	0.41199	0.35628	0.30832	0.26700	0.23138
35	0.50003	0.42137	0.35538	0.29998	0.25342	0.21425	0.18129
40	0.45289	0.37243	0.30656	0.25257	0.20829	0.17193	0.14205
45	0.41020	0.32917	0.26444	0.21266	0.17120	0.13796	0.11130
50	0.37153	0.29094	0.22811	0.17905	0.14071	0.11071	0.08720

The present Worth of an Annuity is the sum which now placed at compound interest will reach the same amount as will be reached by the annuity. Let $V =$ the present value of an annuity I; then $V = \frac{I\,[1 - (1 + i)^{-n}]}{i}$. The following table gives values of an annuity of $1 for various periods and various rates. Thus, the present worth of an annuity of $1 for 20 years at 4% is $13.59.

Present Value of an Annuity of $1

No. of Years	Rates of compound interest						
	2%	2-1/2%	3%	3-1/2%	4%	4-1/2%	5%
1	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238
2	1.94156	1.92742	1.91347	1.89969	1.88609	1.87267	1.85941
3	2.88388	2.85602	2.82861	2.80164	2.77509	2.74896	2.72325
4	3.80773	3.76197	3.71710	3.67308	3.62989	3.58753	3.54595
5	4.71346	4.64582	4.57971	4.51505	4.45182	4.38998	4.32948
10	8.98258	8.75206	8.53020	8.31660	8.11090	7.91272	7.72173
15	12.84926	12.38138	11.93793	11.51741	11.11839	10.73955	10.37966
20	16.35143	15.58916	14.87747	14.21240	13.59033	13.00794	12.46221
25	19.52346	18.42438	17.41315	16.48151	15.62208	14.82821	14.09394
30	22.39646	20.93029	19.60044	18.39204	17.29203	16.28889	15.37245
35	24.29862	23.14516	21.48722	20.00066	18.66461	17.46101	16.37419
40	27.35548	25.10277	23.11477	21.35507	19.79277	18.40158	17.15909
45	29.49016	26.83302	24.51871	22.49545	20.72004	19.15635	17.77407
50	31.42361	28.36231	25.72976	23.45562	21.48218	19.76201	18.25592

Annual Depreciation of a property is the necessary annual installment of the annuity which will amount to the first cost of that property at the expiration of its useful life; annual depreciation is also expressed in percentage of first cost. Thus, if the first cost of a part of a plant is $10 000 and its life is estimated at 40 years, then if money is worth 3%, the annual depreciation is. 10 000 × $0.01326, or $132.60, and the rate of depreciation is 1.326% **Capitalization** of a plant is the sum of the first cost C and the capital necessary to earn annually (a) the annual cost of operation O and (b) the annual depreciation D; that is, if i is the interest rate, capitalization equals $C + (O + D)/i$. Thus, if in the preceding illustration O = $500, the capitalization equals $10 000 + ($500 + ($132.06)/0.03 = $31 068.

Annual Annuity for Various Periods which $1 will Purchase

No. of Years	Rates of compound interest						
	2%	2-1/2%	3%	3-1/2%	4%	4-1/2%	5%
1	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000
2	0.51505	0.51883	0.52261	0.52640	0.53020	0.53400	0.53780
3	0.34675	0.35014	0.35353	0.35693	0.36035	0.36377	0.36721
4	0.26262	0.26582	0.26903	0.27225	0.27549	0.27874	0.28201
5	0.21216	0.21525	0.21835	0.22148	0.22463	0.22779	0.23097
10	0.11133	0.11426	0.11723	0.12024	0.12329	0.12638	0.12950
15	0.07783	0.08077	0.08377	0.08683	0.08994	0.09311	0.09634
20	0.06116	0.06415	0.06722	0.07036	0.07358	0.07688	0.08024
25	0.05122	0.05428	0.05743	0.06067	0.06401	0.06744	0.07095
30	0.04465	0.04778	0.05102	0.05437	0.05783	0.06139	0.06505
35	0.04000	0.04321	0.04654	0.05000	0.05358	0.05727	0.06107
40	0.03656	0.03984	0.04326	0.04683	0.05052	0.05434	0.05828
45	0.03391	0.03727	0.04079	0.04445	0.04826	0.05220	0.05626
50	0.03182	0.03526	0.03887	0.04263	0.04655	0.05060	0.05478

The formula for this table is $i/[1 - (1 + i)^{-n}]$.

ADVANCED MATHEMATICS

8. Differential Calculus

Definitions and Symbols. In a discussion of quantities, a constant is one regarded as having the same value throughout, and a variable is one supposed to take different values. When several variables are related, their values being interdependent, each is a function of the others; thus the area and the base of a triangle of constant altitude being definitely related, these quantities are functions of each other. Also any expression containing the symbol of a quantity is a function of that quantity; thus $ax^2 + bx - c$ is a function of x, and $ay^2 + bx$ is a function of x and y. The following abbreviations are used: $f(x)$, $F(x)$, $\phi(x)$, etc., for functions of x; $f(x, y)$ $F(x, y)$, $\phi(xy)$ etc., for functions of x and y, etc. The letters f, F, ϕ, etc., are functional symbols; and $f(x)$ and $f(y)$, or $F(x)$ and $F(y)$, denote the same functions of x and of y; thus if $f(x)$ denotes $x^3 + 6x - 7$, then $f(y)$ denotes $y^3 + 6y = -7$. When $y = f(x)$ as $y = x^3 + 4x$, the equation being solved for y, then y is an explicit function of x; but in $F(x, y) = 0$, as $xy + 4x^2 - y^2 = 0$, the equation not being solved for y, then y is an implicit function of x. If $y = f(x)$, and for each value of x there is only one value of y, then y is a single-valued function of x; if for each value of x there is more than one value of y, then y is a multiple-valued function.

If two variables x and y are related and x is regarded as taking on any values and then y its corresponding values, x is the independent and y the dependent variable. When any variable x changes from a value x_1 to a value x_2, the difference $x_2 - x_1$ is an increment of x; any increment of x is denoted by Δx, also δx. An increment of x may be positive or negative; when negative, the numerical value of the increment is called decrement. A variable regarded as taking on equal increments is an equicrescent variable. If in $y = f(x)$ all equal changes in x produce equal changes in y, then y is a uniform variable with respect to x, and the graph of $y = f(x)$ is straight. The rate of y with respect to x, or the x-rate of y, is the change in y per unit change in x. If $x_2 - x_1$, or Δx (Fig. 20), is any change in x, and $y_2 - y_1$, or Δy, is the corresponding change in y, then the x-rate of y is $(y_2 - y_1)/(x_2 - x_1) = \Delta y/\Delta x$. If in $y = f(x)$ all equal changes in x do not produce equal changes in y, then y varies non-uniformly with respect to x, or at a variable rate; the graph of $y = f(x)$ is a curve. The average rate of y with respect to x, or average x-rate of y, for any change in x, is that constant rate which would give the actual change in y due to the change in x. For the change $x_2 - x_1$ in x (Fig. 21) this constant rate is $(y_2 - y_1)/(x_1 - x_1)$, or $\Delta y/\Delta x$, and is represented by the slope of the chord AB. The actual x-rate of y when $x = x_1$, say, is the value which the average rate $\Delta y/(x_2 - x_1)$ approaches as x_2 is taken closer and closer to x_1; this limiting average rate, or actual rate, is represented by the slope of the tangent at A. As $y = f(x)$, the x-rate of y is also the x-rate of $f(x)$.

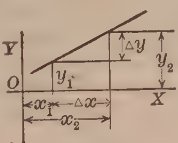

Fig. 20

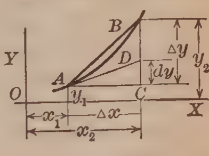

Fig. 21

The Derivative, the differential coefficient, and the derived function of y or $f(x)$, y being equal to $f(x)$, with respect to x are expressions which denote the x-rate of y or of $f(x)$; the first is the most common. There are several standard notations for this quantity: thus, $D_x y$ or $D_x f(x)$, y_x' or $f_x'(x)$ (the subscript is generally omitted), and dy/dx or $d/dx\, f(x)$. The last two are the most common symbols and have the advantage of suggesting how the rate is obtained in the first instance. If the curve in Fig. 21 is the graph of $y = f(x)$, $dy/dx = \tan\alpha$ (when $x = x_1$), α being the slope angle at A. The derivative

dy/dx is essentially one quantity, but may be regarded as a fraction provided that dy and dx is regarded so that their ratio equals $\tan\alpha$; thus if dx is regarded as a finite increment of x, equal to Δx, then dy must be taken as CD, not equal to Δy. So taken, dy is the **differential** of y with respect to x; it is an hypothetical increment of y, equal to the increment which occurs in y for a change in x from x_1 to $x_1 + dx$ on the supposition that the x-rate of y remains constant during the change in x and is equal to its actual value when $x = x_1$. Thus, $dy/dx = \phi(x)$ may also be written $dy = \phi(x)\cdot dx$.

The **fluxion** of a variable function is its time rate. This term was used by Newton but is now uncommon; a fluxion of a variable as y was denoted by him thus, $\dot{y}$.

Differentiation is the process of finding the differential or derivative of a function. The following formulas give differentials (and derivatives by division) of some simple functions. Differentials of many other functions can be obtained by combining formulas; thus according to the formula for $d\,(uv)$, $d\,(\sin x\cdot\cos x) = \sin x\cdot d\,(\cos x) + \cos x\cdot d\,(\sin x)$, and as given $d\,(\cos x) = -\sin x\,dx$ and $d\,(\sin x) = \cos x\,dx$; hence $d\,(\sin x\cdot\cos x) = -\sin^2 x\,dx + \cos^2 x\,dx$. In the formulas, a and n are constants, e is the base of the Napierian system of logarithms, and in each case the differential of the function is given with respect to x.

$$d\,(a+x) = dx \qquad d\,(ax) = a\,dx \qquad dx^n = nx^{n-1}\,dx$$
$$de^x = e^x dx \qquad da^x = a^x \log_e a\,dx \qquad d\log_e x = dx/x$$
$$d\sin x = \cos x\,dx \qquad d\csc x = -\csc x\cot x\,dx$$
$$d\cos x = -\sin x\,dx \qquad d\sec x = \sec x\tan x\,dx$$
$$d\tan x = \sec^2 x\,dx \qquad d\cot x = -\csc^2 x\,dx$$
$$d\,\mathrm{vers}\,x = \sin x\,dx \qquad d\,\mathrm{covers}\,x = -\cos x\,dx$$
$$d\sin^{-1} x = dx/\sqrt{1-x^2} = -d\cos^{-1} x$$
$$d\tan^{-1} x = dx/(1+x^2) = -d\cot^{-1} x$$
$$d\sec^{-1} x = dx/x\sqrt{x^2-1} = -d\csc^{-1} x$$
$$d\,\mathrm{vers}^{-1} x = dx/\sqrt{2x-x^2} = -d\,\mathrm{covers}^{-1} x$$
$$d\sinh x = \cosh x\,dx \qquad d\,\mathrm{csch}\,x = -\mathrm{csch}\,x\coth x\,dx$$
$$d\cosh x = \sinh x\,dx \qquad d\,\mathrm{sech}\,x = -\mathrm{sech}\,x\tanh x\,dx$$
$$d\tanh x = \mathrm{sech}^2 x\,dx \qquad d\coth x = -\mathrm{csch}^2 x\,dx$$
$$d\sinh^{-1} x = dx/\sqrt{x^2+1} \qquad d\,\mathrm{csch}^{-1} x = -dx/x\sqrt{1+x^2}$$
$$d\cosh^{-1} x = dx/\sqrt{x^2-1} \qquad d\,\mathrm{sech}^{-1} x = -dx/x\sqrt{1-x^2}$$
$$d\tanh^{-1} x = dx/(1-x^2) \qquad d\coth^{-1} x = -dx/(x^2-1)$$

In the four following, u, v, w, etc., are functions of x and all differentials are with respect to x:

$$d\,(u+v+w+\ldots) = du + dv + dw + \ldots \qquad d\,(uv) = u\,dv + v\,du$$
$$d\,(uvw\ldots) = [du/u + dv/v + dw/w + \ldots]\,uvw\ldots$$
$$d\,(u/v) = (v\,du - u\,dv)/v^2$$

Partial Derivatives and Differentials. The partial derivative of a function of two or more independent variables with respect to one of them is the derivation of the function obtained on the supposition that the others are for the time being constants; the partial x-derivation of a function u is written $\partial u/\partial x$, also (du/dx); for example, of if $u = y^3 + 4\,xy + x^2 + 2$, $\partial u/\partial x = 4\,y + 2\,x$ and $\partial u/\partial y = 3\,y^2 + 4\,x$. If u (or z) is a function of two independent variables, the partial derivatives have geometrical significance. The equation $z = f\,(x, y)$ represents a surface; suppose that APa, Fig. 22, is its intersection with a plane parallel to ZOX, and BPb its intersection with a plane parallel to ZOY; on these curves, y and x respectively are constant. PT' and PT'' are tangents to the curves at P as shown, and the slopes of these tangents represent $\partial z/\partial x$

and $\partial z/\partial y$ respectively. The partial differential of a function of two or more independent variables with respect to any one of them is the differential of the function obtained on the supposition that the other variables are, for the time being, constants. Thus if $z = y^3 + 4xy + x^2 + 2$, the x-partial differential of z is $4\,ydx + 2\,xdx$ and the y-partial differential is $3\,y^2dy + 4\,xdy$. If z is a function of only two independent variables the partial differentials have geometrical significance. If Pm and Pn are taken to represent the differentials dx and dy respectively, then mT' and nT'' represents the x- and the y-partial differentials of z. Also these partial differentials are equal to $[\partial z/\partial x]\,dx$ and $[\partial z/\partial y]\,dy$.

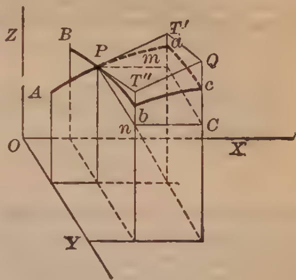

Fig. 22

Total Derivatives and Differentials. The total differential of a function of two or more variables is the differential obtained on the supposition that all change. It equals the sum of the partial differentials of the function with respect to the several variables; thus if $u = f(xyz)$,

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

If u is a function of only two variables, du has a geometrical significance. Thus if $z = (x, y)$ and Pm and Pn (Fig. 22) are taken as dx and dy respectively, then CQ represents dz. The differential dz then is a hypothetical and not the actual increment, Cc in z due to changes dx and dy in x and y. The total derivative of a function of two or more variables all dependent on a single variable is the rate at which the function changes per unit change of that variable; thus if $u = f(x, y, z)$ and x, y, and z are all functions of t,

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

and similarly for any number of variables. If the independent variable is x say, then

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx} + \frac{\partial u}{\partial z}\frac{dz}{dx}$$

Successive Differentiation is the process of finding derivatives or differentials of derivatives or differentials. The successive derivatives and differentials are called first, second, third, etc., derivatives and differentials, and of the first, second, third, etc., order; those of the second, third, etc., order are "higher derivatives" and higher differentials. For the successive x-derivatives of $y = f(x)$ the following symbols are used:

first,	$Dy, y', f'(x)$,	and	dy/dx
second,	$D(Dy)$ or $D^2y, y'', f''(x)$,	and	$d/dx\ dy/dx$ or d^2y/dx^2
third,	$D^3y, y''', f'''(x)$,	and	d^3y/dx^3

The first x-differential of y is dy, the second d^2y, the third d^3y, etc.

For the successive partial derivatives of $u = f(xy)$ the following symbols are used:

first partial x-derivative $\partial u/\partial x$, first partial y-derivative $\partial u/\partial y$
second partial x-derivative $\partial^2u/\partial x^2$, second partial y-derivative $\partial^2u/\partial y^2$

The first partial x-derivative may be taken with respect to x and the second with respect to y; this second is written $\partial^2u/\partial y\partial x$. Similarly $\partial^2u/\partial x\partial y$ means the x-partial derivative of the y-partial derivative of u. But the order of differentiation is immaterial, that is, $\partial^2u/\partial y\partial x = \partial^2u/\partial x\partial y$, and this independence is true for any number of successive differentiations with respect to any number of variables.

Maxima and Minima. A maximum value of a function is one which is algebraically greater than, and a minimum value is one algebraically less than its adjacent values. Thus, if the curve in Fig. 23 is the graph of $y = f(x)$, y_1, y_2, and y_3 are maximum values of y or $f(x)$ and y_4, y_5, and y_6 are minimum values; again, imagine $u = f(x, y)$ graphed, x and y independent variables plotted on horizontal axes and u vertically, thus defining in general an undulating surface; then the values of u corresponding to high and low points of the surface are maximum or minimum values of u or $f(x, y)$. Maximum and minimum values are also known as turning values; the values of the independent variable or variables corresponding to turning values are critical values. The following formulas refer to maxima and minima, like y_1 and y_4, where the tangent line is horizontal and not to those like y_3 and y_5 (rare), where the tangent line is not horizontal. At maxima and minima of the first kind the gradient dy/dx changes sign gradually, and at the peaks the change of sign is sudden. From this property peak values of a function can be obtained " by trial " with dy/dx.

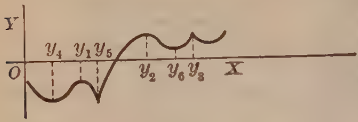

Fig. 23

(1) Turning value of $y = f(x)$: Solve $f'(x) = 0$; any root obtained, as x_1, is a critical value if the lowest x-derivative of y which does not vanish for $x = x_1$ is of an even order; the turning value, $f(x_1)$, is a maximum or minimum according as that lowest x-derivative is negative or positive for $x = x_1$.

(2) Turning value of y in $f(x, y) = 0$: Solve $\partial f(x, y)/\partial x = 0$ and $f(x, y) = 0$ simultaneously for x and y; the values obtained, x_1 and y_1, are critical and turning values respectively if $\partial f(x, y)/\partial y$ does not vanish for $x = x_1$ and $y = y_1$; y_1 is maximum or minimum according as $[-\partial^2 f(x, y)/\partial x^2] \div [\partial f(x, y)/\partial y]$ is negative or positive for $x = x_1$ and $y = y_1$.

(3) Turning values of $u = f(x, y)$: Solve $\partial f(x, y)/\partial x = 0$ and $\partial f(x, y)/\partial y = 0$ simultaneously for x and y; the values obtained, x_1 and y_1, are critical values if $[\partial^2 f(x, y)/\partial x^2]\,[\partial^2 f(x, y)/\partial y^2] - [\partial^2 f(x, y)/\partial x\partial y]^2$ is positive for $x = x_1$ and $y = y_1$;

$$\left.\begin{array}{l} u \text{ is a maximum} \\ u \text{ is a minimum} \end{array}\right\} \text{ if } \frac{\partial^2 f(x, y)}{\partial x^2} \text{ and } \frac{\partial^2 f(x, y)}{\partial y^2} \text{ are both } \left\{\begin{array}{l} \text{negative} \\ \text{positive} \end{array}\right.$$

Indeterminate Forms. In general a function of a variable has a definite value for any particular value of the variable; yet the values of some functions for special values of the variable lead to expressions like $0/0$, $0 \cdot \infty$ and the like, which are so-called indeterminate forms. For example, the function $(1 - \cos x)/x$ for $x = 0$ becomes $0/0$ and $(1/2\,\pi - x) \tan x$ for $x = \pi/2$ is $0 \times \infty$.

(1) The quotient $f(x) \div F(x)$ becomes $0/0$ for a particular value of x as a: the indeterminate form equals the value of $f'(x) \div F'(x)$ when $x = a$; if this ratio is also an indeterminate form, the original one equals $f''(x) \div F''(x)$ when $x = a$.

(2) The quotient $f(x) \div F(x)$ becomes ∞/∞ when $x = a$: the true value may be obtained as in case (1).

(3) The product $f(x) \times F(x)$ becomes $0.\infty$ when $x = a$: the product equals $f(x) \div [1/F(x)]$ which takes the form $0/0$ when $x = a$ and may be evaluated as in (1).

(4) The difference $f(x) - F(x)$ becomes $\infty - \infty$ when $x = a$: the difference equals $\phi(x) \div \psi(x)$, case (1), where $\phi(x) = [1/F(x)] - [1/f(x)]$ and $\psi(x) = 1/[f(x)\,F(x)]$.

(5) The function $[F(x)]^{f(x)}$ becomes 1^∞, 0^0, or ∞^0 when $x = a$: the function equals $e^{f(x) \log F(x)}$ (the logarithm being Napierian), and the value of this form can be got if $f(x) \log F(x)$ can be evaluated; but $f(x) \log F(x)$ falls under case (3).

Maclaurin's Theorem expresses the law for the expansion of a function of a variable in a series of ascending powers of the variable, thus,

$$f(x) = f(0) + \frac{x}{1} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots$$

wherein $f(0)$ means the value of $f(x)$ when $x = 0$ and $f'(0)$ is the value of $f'(x)$ when $x = 0$, etc.

As illustration, $\cos x$ is here expanded by Maclaurin's theorem: $f(x) = \cos x$, $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, $f''''(x) = \cos x$, etc.; $f(0) = 1$, $f'(0) = 0$, $f''(0) = -1$, $f'''(0) = 0$, $f''''(0) = 1$. Hence $\cos x = 1 - x^2/2! + x^4/4! + \ldots$

Taylor's Theorem expresses the law for the expansion of a function of a variable plus an increment in a series of ascending powers of the increment; thus h, denoting an increment of x,

$$f(x+h) = f(x) + \frac{h}{1} f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \ldots$$

To expand $\cos(x + h)$ by Taylor's theorem: $f(x) = \cos x$, $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, $f''''(x) = \cos x$, etc., and $\cos(x + h) = \cos x\,(1 - h^2/2! + h^4/4! - \ldots) - \sin x\,(h - h^3/3! + h^5/5! - \ldots)$.

9. Integral Calculus

Definitions and Notation. Integration is the process of determining a function from its derivative or differential; it is the inverse of differentiation. The function is called the integral of the derivative or differential; also anti-derivative and anti-differential. The symbol of integration is $\int$. Thus the x-derivative of $f(x)$ being written $f'(x)$, $\int f'(x) = f(x)$, but integration of the differential instead is generally indicated, as $\int f'(x)\,dx = f(x)$; dx indicates the "variable of integration," $f'(x)$ is called integrand, and the result of the integration, that is, the function determined $f(x)$, is the integral of the derivative or differential from which it is found. Strictly $\int f'(x)$ or $\int f'(x)dx = f(x) + C$, C being a constant, called constant of integration; $f(x)$ is called the indefinite integral and $f(x) + C$ the general integral. In a general integral the constant of integration is an arbitrary constant, but in a particular problem of integration it has a special value determinable from data of the problem. Thus, suppose in a given case it is known that when $x = a$, $\int f'(x) = A$ then $A = f(a) + C$ or $C = A - f(a)$; and $f(x) + A - f(a)$ is a particular integral.

Integral Forms. In the following formulas K is the constant of integration; $a, b, c, A, B, m,$ and n are constants; logarithms indicated are Napierian, and ϵ is the base of that system (Art. 2); Y is an abbreviation for $a + bx$, Z for $a + bx + cx^2$; and u and v are any functions of x.

$$\int a\,du = a \int du \qquad \int (u + v)\,dx = \int u\,dx + \int v\,dx$$

$$\int u\,dv = uv - \int v\,du \qquad \int Y^n dx = \frac{Y^{n+1}}{(n+1)b} + K$$

$$\int \frac{dx}{Y} = \frac{1}{b} \log Y + K \qquad \int \frac{x\,dx}{Y} = \frac{1}{b^2}(Y - a \log Y) + K$$

$$\int \frac{x\,dx}{Y^2} = \frac{1}{b^2}\left(\log Y + \frac{a}{Y}\right) + K$$

$$\int \frac{x^2\,dx}{Y} = \frac{1}{b^3}(1/2\,Y^2 - 2\,aY + a^2 \log Y) + K$$

$$\int \frac{x^2\,dx}{Y^2} = \frac{1}{b^3}\left(Y - 2\,a \log Y - \frac{a^2}{Y}\right) + K$$

$$\int \frac{dx}{xY} = -\frac{1}{a}\log\frac{Y}{x} + K \qquad \int \frac{dx}{xY^2} = \frac{1}{aY} - \frac{1}{a^2}\log\frac{Y}{x} + K$$

$$\int \frac{dx}{x^2Y} = -\frac{1}{ax} + \frac{b}{a^2}\log\frac{Y}{x} + K \qquad \int \sqrt{Y}\,dx = \frac{2}{3\,b}Y^{3/2} + K$$

$$\int x\sqrt{Y}\,dx = -\frac{(2\,a - 3\,bx)\,Y^{3/2}}{15\,b^2} + K$$

$$\int x^2\sqrt{Y}\,dx = \frac{2\,(8\,a^2 - 12\,abx + 15\,b^2x^2)\,Y^{3/2}}{105\,b^3} + K$$

$$\int \frac{dx}{\sqrt{Y}} = \frac{2\sqrt{Y}}{b} + K \qquad \int \frac{x\,dx}{\sqrt{Y}} = -\frac{2\,(2\,a - bx)\sqrt{Y}}{3\,b^2} + K$$

$$\int \frac{x^2\,dx}{\sqrt{Y}} = \frac{2\,(8\,a^2 - 4\,abx + 3\,b^2x^2)\sqrt{Y}}{15\,b^3} + K$$

$$\int \frac{dx}{x\sqrt{Y}} = \frac{1}{\sqrt{a}}\log\frac{\sqrt{Y} - \sqrt{a}}{\sqrt{Y} + \sqrt{a}} + K, \text{ if } a > 0$$

$$= \frac{2}{\sqrt{-a}}\tan^{-1}\frac{\sqrt{Y}}{\sqrt{-a}} + K, \text{ if } a < 0$$

$$\int \frac{\sqrt{Y}dx}{x}\,dx = 2\sqrt{Y} + a\int \frac{dx}{x\sqrt{Y}}$$

$$\int \frac{dx}{x^2\sqrt{Y}} = -\frac{\sqrt{Y}}{ax} - \frac{b}{2\,a}\int \frac{dx}{x\sqrt{Y}}$$

$$\int \frac{dx}{a + bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}\sqrt{\frac{b}{a}}\,x + K, \text{ if } a > 0 \text{ and } b > 0$$

$$\int \frac{dx}{a - bx^2} = \frac{1}{2\sqrt{ab}}\log\frac{\sqrt{ab} + bx}{\sqrt{ab} - bx} + K, \text{ if } a > 0 \text{ and } b > 0$$

$$= \frac{1}{\sqrt{ab}}\tanh^{-1}\sqrt{\frac{b}{a}}\,x + K, \text{ if } a > 0 \text{ and } b > 0$$

$$\int \frac{x^2\,dx}{a + bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a + bx^2}$$

$$\int \frac{dx}{x^2\,(a + bx^2)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{dx}{a + bx^2}$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\log\left(x + \sqrt{x^2 \pm a^2}\right) + K$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + K$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left(x + \sqrt{x^2 \pm a^2}\right) + K$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + K = -\cos^{-1}\frac{x}{a} + K$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\cos^{-1}\frac{a}{x} + K$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\log\frac{a + \sqrt{a^2 \pm x^2}}{x} + K$$

$$\int \frac{\sqrt{a^2 \pm x^2}}{x}\,dx = \sqrt{a^2 \pm x^2} + a^2\int\frac{dx}{x\sqrt{a^2 \pm x^2}}$$

$$\int \frac{\sqrt{x^2 - a^2}}{x}\,dx = \sqrt{x^2 - a^2} - a^2\int\frac{dx}{x\sqrt{x^2 - a^2}}$$

$$\int \frac{x\,dx}{\sqrt{a^2 \pm x^2}} = \pm\sqrt{a^2 \pm x^2} + K \qquad \int\frac{x\,dx}{\sqrt{x^2 - a^2}} = \sqrt{x^2 - a^2} + K$$

$$\int\frac{dx}{Z} = \frac{2}{\sqrt{4ac - b^2}}\tan^{-1}\frac{b + 2cx}{\sqrt{4ac - b^2}} + K, \text{ if } 4ac - b^2 > 0$$

$$\int\frac{dx}{Z} = \frac{1}{\sqrt{b^2 - 4ac}}\log\frac{\sqrt{b^2 - 4ac} - b - 2cx}{\sqrt{b^2 - 4ac} + b + 2cx} \text{ if } b^2 - 4ac > 0$$

$$= \frac{-2}{\sqrt{b^2 - 4ac}}\tanh^{-1}\frac{b + 2cx}{\sqrt{b^2 - 4ac}} + K \text{ if } b^2 - 4ac > 0$$

$$\int\frac{x\,dx}{Z} = \frac{1}{2c}\log Z - \frac{b}{2c}\int\frac{dx}{Z}$$

$$\int\frac{x^2dx}{Z} = \frac{x}{c} - \frac{b}{2c^2}\log Z + \frac{b - 2ac}{2c^2}\int\frac{dx}{Z}$$

$$\int\frac{(A + Bx)dx}{Z} = A\int\frac{dx}{Z}\int + B\int\frac{x\,dx}{Z}$$

$$\int\frac{dx}{\sqrt{Z}} = \frac{1}{\sqrt{c}}\log\left(b + 2cx + 2\sqrt{c}\sqrt{Z}\right) + K, \text{ if } c > 0$$

$$= \frac{1}{\sqrt{c}}\sinh^{-1}\frac{b + 2cx}{\sqrt{4ac - b^2}} + K, \text{ if } c > 0 \text{ and } 4ac - b^2 > 0$$

$$= \frac{1}{\sqrt{c}}\cosh^{-1}\frac{b + 2cx}{\sqrt{b^2 - 4ac}} + K \text{ if } c > 0 \text{ and } b^2 - 4ac - b^2 > 0$$

$$= \frac{-1}{\sqrt{-c}}\sin^{-1}\frac{b + 2cx}{\sqrt{b^2 - 4ac}} + K \text{ if } c > 0$$

$$\int\frac{x\,dx}{\sqrt{Z}} = \frac{\sqrt{Z}}{c} - \frac{b}{2c}\int\frac{dx}{\sqrt{Z}}$$

$$\int\frac{(A + Bx)\,dx}{\sqrt{Z}} = A\int\frac{dx}{\sqrt{Z}} + B\int\frac{x\,dx}{\sqrt{Z}}$$

$$\int\frac{x^2dx}{\sqrt{Z}} = \left(\frac{x}{2c} - \frac{3b}{4c^2}\right)\sqrt{Z} + \frac{3b^2 - 4ac}{8c^2}\int\frac{dx}{\sqrt{Z}}$$

$$\int\sqrt{Z}\,dx = \frac{b + 2cx}{4c}\sqrt{Z} + \frac{4ac - b^2}{8c}\int\frac{dx}{\sqrt{Z}}$$

$$\int x\sqrt{Z}\,dx = \frac{Z\sqrt{Z}}{3c} - \frac{b}{2c}\int\sqrt{Z}\,dz$$

$$\int x^2\sqrt{Z}\,dx = \left(x - \frac{5b}{6c}\right)\frac{Z\sqrt{Z}}{4c} + \frac{5b^2 - 4ac}{16c^2}\int\sqrt{Z}\,dx$$

$$\int \sin x\,dx = -\cos x + K \qquad \int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + K$$

$$\int \sin^2 x\,dx = -1/2\cos x \sin x + 1/2\,x + K$$

$$\int \sin^n x\,dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \cos x\,dx = \sin x + K \qquad \int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + K$$

$$\int \cos^2 x\,dx = 1/2\sin x \cos x + 1/2\,x + K$$

$$\int \cos^n x\,dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \sin x \cos x\,dx = 1/2\sin^2 x + K$$

$$\int \sin ax \cos bx\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + K$$

$$\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + K$$

$$\int \cos ax \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + K$$

$$\int \cos^n x \sin x\,dx = -\frac{\cos^{n+1} x}{n+1} + K \qquad \int \sin^n x \cos x\,dx = \frac{\sin^{n+1} x}{n+1} + K$$

$$\int \sin^m x \cos^n x\,dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x\,dx$$

$$= -\frac{\sin^{m-1} x \cos^{n+1}}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} \cos^n dx$$

$$\int x \sin x\,dx = \sin x - x\cos x + K$$

$$\int x^2 \sin x\,dx = 2\,x\sin x - (x^2-2)\cos x + K$$

$$\int x \cos x\,dx = \cos x + x\sin x + K$$

$$\int x^2 \cos x\,dx = 2\,x\cos x + (x^2-2)\sin x + K$$

$$\int \tan x\,dx = -\log\cos x + K \qquad \int \tan^2 x\,dx = \tan x - x + K$$

$$\int \cot x\,dx = \log\sin x + K \qquad \int \cot^2 x\,dx = -\cot x - x + K$$

$$\int \sec x\,dx = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + K \qquad \int \csc x\,dx = \log\tan 1/2\,x + K$$

$$\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2} + K$$

$$\int \cos^{-1} x\,dx = x\cos^{-1} x - \sqrt{1-x^2} + K$$

$$\int \tan^{-1} x\,dx = x \tan^{-1} x - 1/2 \log (1 + x^2) + K$$

$$\int \cot^{-1} x\,dx = x \cot^{-1} x + 1/2 \log (1 + x^2) + K$$

$$\int \operatorname{vers}^{-1} x\,dx = (x - 1) \operatorname{vers}^{-1} x + \sqrt{2x - x^2} + K$$

$$\int a^x\,dx = \frac{a^x}{\log a} + K \qquad \int \log x\,dx = x \log x - x + K$$

$$\int (\log x)^n\,dx = x (\log x)^n - n \int (\log x)^{n-1}\,dx$$

$$\int \frac{(\log x)^n}{x}\,dx = \frac{1}{n+1} (\log x)^{n+1} + K$$

$$\int \frac{dx}{x (\log x)^n} = -\frac{1}{n-1}\,\frac{1}{(\log x)^{n-1}} + K$$

$$\int \epsilon^{ax}\,dx = \frac{\epsilon^{ax}}{a} + K \qquad \int x \epsilon^{ax}\,dx = \frac{\epsilon^{ax}}{a^2} (ax - 1) + K$$

$$\int x^n \epsilon^{ax}\,dx = \frac{x^n \epsilon^{ax}}{a} - \frac{n}{a} \int x^{n-1} \epsilon^{ax}\,dx$$

$$\int \frac{\epsilon^{ax}}{x^n}\,dx = -\frac{\epsilon^{ax}}{(n-1)\,x^{n-1}} + \frac{a}{n-1} \int \frac{\epsilon^{ax}}{x^{n-1}}\,dx$$

$$\int \epsilon^{ax} \log x\,dx = \frac{\epsilon^{ax} \log x}{a} - \frac{1}{a} \int \frac{\epsilon^{ax}}{x}\,dx$$

$$\int \epsilon^x \sin x\,dx = \frac{\epsilon^x (\sin x - \cos x)}{2} + K$$

$$\int \epsilon^x \cos x\,dx = \frac{\epsilon^x (\sin x + \cos x)}{2} + K$$

Definite integrals result from integration between "limits"; thus $\int_a^b f'(x)\,dx = [f(x)]_b^a = f(b) - f(a)$; a and b are "lower" and "upper limits" respectively and $(b - a)$ is the "range" of integration. Interchanging the limits in a definite integral changes the sign of the result, and a definite integral can be expressed as the sum of several other integrals; thus

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \quad \text{and} \quad \int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Approximate Integration. An approximate value of $\int f(x)\,dx$ between any limits as x_0 and x_n can be obtained as follows: divide the range $x_n - x_0$ into an even number (n) of equal parts (w) and compute the values of $f(x)$ when $x = x_0$, $x_0 + w$, $x_0 + 2w$, . . ., x_n, and call these values $y_0, y_1, y_2, \ldots, y_n$. Then substitute in an approximate formula for the area of an irregular figure (Art. 6), as Simpson's for example; the value obtained for A is an approximate value of the integral. If the values of x and y are plotted and a smooth curve is drawn through the points obtained, then the area between the curve, the x axis and the end ordinates y_0 and y_n represents the integral; and the area according to the proper scale in a given case equals the integral. If the area is cut by the x axis, the parts above and below should be regarded as positive and negative respectively.

10. Plane Analytic Geometry

Plane Coordinate Systems. For rectangular coordinate systems, see Art. 3. An oblique coordinate system is like the rectangular except that the coordinate axes are not at right angles, and in the oblique the x and y coordinates of a point are measured parallel to the x and y axes respectively. Rectangular and oblique systems are called Cartesian. The **polar coordinates** of a point P (Fig. 24) are its distance from some point or pole O and the angle between a reference line as OX and the line joining P and O; the line OP is the radius vector of P. Change or transformation of coordinates from one system to another can be made from $x = r \cos \phi$. $y = r \sin \phi$, or $\sin \phi = y/r$, $\cos \phi = x/r$, $r = \sqrt{x^2 + y^2}$; the reference line and pole of the polar system and the axes of the rectangular system must be related as shown in the figure. For example, the equation of the circle (Fig. 25) with respect to axes OX and OY is $x^2 + y^2 = 2\,ay$, a being the radius; its polar equation with OX as reference line and O as pole is $r^2 \cos^2 \phi + r^2 \sin^2 \phi = 2\,ar \sin \phi$, or $r = 2\,a \sin \phi$.

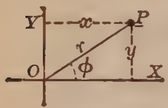

Fig. 24

Transformation from a set of rectangular axes to a parallel set: XOY (Fig. 26) is the original set, UQV the new, and (x_1, y_1) the coordinates of the new origin Q with respect to the original set; then P being any point whose coordinates with respect to the two sets of axes are (x, y) and (u, v) respectively, $x = u + x_1$ and $y = v + y_1$.

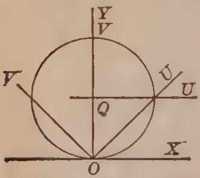

Fig. 25

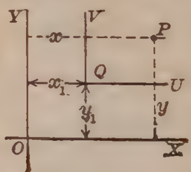

Fig. 26

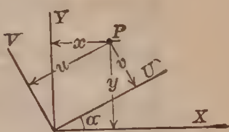

Fig. 27

For example, the equation of the circle (Fig. 25) with respect to the axes XOY being $x^2 + y^2 = 2\,ay$, its equation with respect to UQV is

$$(u + 0)^2 + (v + a)^2 = 2\,a\,(v + a), \text{ or } u^2 + v^2 = a^2$$

Transformation from a set of rectangular axes to another set, having the same origin but different directions: XOY (Fig. 27) is the original set, UOV the new, and α denotes the angle through which OX must be turned to bring it into OU, α being positive for counter-clockwise turning; then P being any point whose coordinates are as shown, $x = u \cos \alpha - v \sin \alpha$ and $y = u \sin \alpha + v \cos \alpha$. For example, the equation of the circle (Fig. 25) with respect to XOY being $x^2 + y^2 = 2\,ay$ its equation with respect to UOV is $(u \cos 45° - v \sin 45°)^2 + (u \sin 45° + v \cos 45°)^2 = 2\,a\,(u \sin 45° + v \cos 45°)$, or $u^2 + v^2 = a\sqrt{2}\,(u + v)$.

Straight Line. When a straight line lies in a coordinate plane (Fig. 28), the slope-angle is the angle through which XM must be turned to bring it into the line, the angle being regarded as positive or negative according as the turning is counter-clockwise or not, and by the slope or gradient of the line is meant the tangent of its slope-angle. OM and ON are the "intercepts" cut off by the line on the coordinate axes, and the intercepts are regarded as positive or negative according as they lie on the positive or negative parts of the coordinate axes. The equation of a straight line is of the first degree, its general form being $Ax + By + C = 0$; written in the form $y = mx + b$,

m is the slope or gradient of the line, and b the intercept on the y axis; written in the form $x/a + y/b = 1$, a and b are the intercepts on the x and y axes respectively. The equation of a line whose slope is m and containing the point (x_1, y_1) is $y - y_1 = m(x - x_1)$; the equation of a line containing the points (x_1, y_1) and (x_2, y_2) is $y - y_1 = (x - x_1)(y_2 - y_1)/(x_2 - x_1)$; the equation of a line whose angle with the x axis is α and distance from the origin is p, is $y \cos\alpha - x \sin\alpha = p$ (See also Art. 3.)

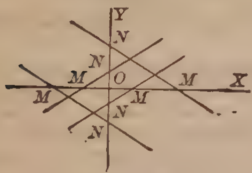

Fig. 28

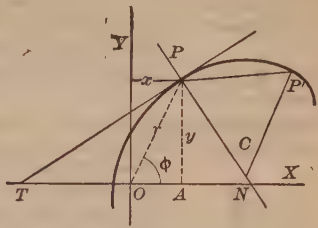

Fig. 29

Equation of a Curve. The equation of a curve referred to any coordinate reference frame is an equation between the coordinates of any and all points on the curve. When rectangular or oblique coordinates x and y are used, it is an equation between x and y; when polar coordinates r and ϕ are used, it is an equation between r and ϕ and is called the polar equation of the curve.

Tangents, Normals, Asymptotes. The **tangent** to a curve at P is the limiting position of a secant PP' (Fig. 29) as P' is taken nearer and nearer to P. The **normal** at P is a perpendicular to the tangent there, and in the plane of the curve. Generally these are understood to be lines of indefinite length, but when they are referred to as of definite lengths, the parts between the points of tangency and the x axis are meant, PT and PN. The subtangent and the subnormal are the projections of the definite tangent and normal on the x axis, AT and AN respectively. If a curve extends to a point of which one (or both) of the coordinates is infinitely great, then the tangent at that point is an **asymptote** of the curve. The equation of the tangent to a curve at a point P at (x_1, y_1) is $y - y_1 = (dy/dx)(x - x_1)$, and the equation of the normal at that point is $y - y_1 = -(dx/dy)(x - x_1)$. The definite tangent $PT = y_1 ds/dy$; the definite normal $PN = y_1 ds/dx$; the subtangent $AT = y_1 dx/dy$; and the subnormal $AN = y_1 dy/dx$.

Curvature. The total curvature of an arc PP' (Fig. 29) is the angle between the tangents to the curve at the points P and P'. The average curvature of the arc is the ratio of the curvature to the length of the arc, or $\Delta\alpha/\Delta s$, if $\Delta\alpha$ is the total curvature and Δs the length of arc. The curvature at a point P of the curve is the limiting value of the average curvature as P' approaches P, that is, $d\alpha/ds$; also curvature at a point is given by

$$\frac{d\alpha}{ds} = \frac{d^2y}{dx^2} \bigg/ \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

When for any arc the tangent lines are only slightly inclined to the x axis, then dy/dx is nearly zero for all points on the curve and the curvature is approximately equal to d^2y/dx^2

PC and $P'C$ are normals to the curve at P and P'; as P' is taken nearer and nearer to P, the intersection of the normals moves along the normal at ∞, approaching a definite point on that normal. This definite point is the center of curvature of the curve for the point P; the line joining the center and P is the radius of curvature of the curve at P, and a circle with that center and radius is the circle of curvature of the curve at P. If R denotes the radius of curvature, then

$$R = \frac{ds}{d\alpha} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \div \frac{d^2y}{dx^2}$$

and if x_1 and y_1 denote the coordinates of the center of curvature corresponding to any point (x, y) on a curve, then

$$x_1 = x - Rdy/ds \quad \text{and} \quad y_1 = y + Rdx/ds$$

and for R, dy/ds and dx/ds must be substituted, their values corresponding to the point (x, y).

Evolute and Involute. The line determined by the centers of curvature of a curve is the evolute of the curve, and the curve is an involute of the evolute. Thus C_1, C_2, and C_3 (Fig. 30) being centers of curvature for points on $P_1P_2P_3$, they lie on the evolute of $P_1P_2P_3$. The radii of curvature P_1C_1, P_2C_2, and P_3C_3 are tangent to the evolute; and the free end of a thread fastened at C_3, wound around the evolute and stretched into the position $C_3C_1P_1$, would, when unwound, describe $P_1P_2P_3$, an involute of $C_1C_2C_3$. The free end of a shorter cord, as $C_3C_1Q_1$, would, if unwound, describe $Q_1Q_2Q_3$, another involute of $C_1C_2C_3$. All points of the straight part of the thread describe parallel curves all of which are involutes of $C_1C_2C_3$, and $C_1C_2C_3$ is the evolute of all the parallel curves. The equation of the evolute of a given curve, $f(x, y) = 0$, can be obtained by eliminating x and y from $f(x, y) = 0$ and the equations for the coordinates of the center of curvature; then the final equation (freed from x's and y's) is the equation sought, x_1 and y_1 being regarded as the variables in it.

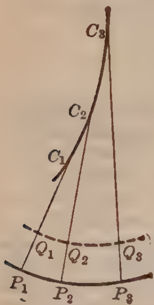

Fig. 30

Gradient, Convexity and Concavity. The slope-angle of a curve at any point of the curve is the angle which the tangent line to the curve makes with the positive x axis; the gradient or slope of the curve there is the tangent of the slope-angle; it is given by the value of dy/dx for that point of the curve. This derivative is positive or negative according as the curve extends upward and to right or left (see Fig. 31 for all possible cases). The second derivative d^2y/dx^2 relates to the bending of the curve. It is positive or negative according as the curve is concave or convex upward (see Fig. 31 for all possible cases).

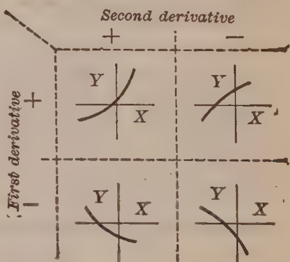

Fig. 31

Envelope. An equation of a curve contains one or more constants, called **parameters**; thus in $y = 4x + 2$, the equation of a straight line, 4 and 2 are parameters. The equation $y = mx + 2$ represents an infinitude of straight lines, one for each possible numerical value of m; a literal parameter, as m, regarded as capable of taking on different values is a variable parameter. The diagram representing an equation with a variable parameter is called a **family of curves**, and the parameter is the parameter of the family. In general the members of the family intersect; the successive intersections of members lie on some line, and the line containing the successive intersections of all members of a family is the envelope of the family. In general, the envelope of a family touches each member. If $f(x, y, a) = 0$ is the equation of a family of curves, the equation of the envelope may be obtained by eliminating a between $f(x, y, a) = 0$ and $\partial f(x, y, a)/\partial a = 0$.

Singular Points. An inflection point of a curve is one where the curve crosses the tangent line at the point and bends away from the tangent in opposite directions on opposite sides of the point (Fig. 32). (The tangent is an inflectional tangent.) At an inflection point $d^2y/dx^2 = 0$, and to locate an inflection point on a curve (having one) whose equation is $y = f(x)$, get

Fig. 32

Fig. 33

Fig. 34

Fig. 35

$f''(x)$, that is, dy^2/dx^2; equate to zero and solve for x; from that value of x and the equation of the curve get y; in general (x, y) is an inflection point. To make sure that it is, determine whether $f''(x)$ changes sign at the point; if so, it is an inflection point. When two or more branches of a curve intersect, or cross one another (Fig. 33), the point of intersection is a multiple point; if two branches cross, the intersection is a node. At a multiple point, dy/dx has two or more real unequal values and y at least two equal values. A cusp is a point of a curve where two branches have a common tangent; if the two branches stop at the point the cusp is single (Fig. 34); if not it is double (Fig. 35).

11. Conic Sections

A Conic is a curve traced by a point moving in a plane so that the distance of the point from a fixed point is in a constant ratio to its distance from a fixed line, the point and line lying in the plane. The fixed point is the focus, the fixed line the directrix, and the constant ratio the eccentricity of the conic. If the directrix is taken as a y axis, a line perpendicular to it and passing through the focus as the x axis, d to denote the distance between focus and directrix, and e the eccentricity, then the equation of the conic is $(x - d)^2 + y^2 = e^2x^2$. If

- $e > 1$, the equation represents a hyperbola
- $e = 1$, the equation represents a parabola
- $e < 1$, the equation represents an ellipse
- $e = 0$, the equation represents a circle

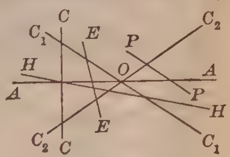

Fig. 36

Hence the circle is a special case of the ellipse, and the parabola is a limiting case for both hyperbola and ellipse.

The general second-degree equation between two coordinates x and y, $Ax^2 + 2\,Hxy + By^2 + 2\,Gx + 2\,Fy + C = 0$, represents a conic. It is an ellipse (or circle), parabola, or hyperbola according as $AB - H^2$ is positive, zero or negative; but if $ABC + 2\,FGH - AF^2 - BG^2 - CH^2 = 0$, the conic is a "degenerate," being a point, two intersecting straight lines, or two parallel straight lines in the three cases respectively.

A conic may be defined with reference to a double right cone with a circular base, whence the name conic. Let AA (Fig. 36) be the axis of such a cone, C_1C_1 and C_2C_2 elements of the surface in the plane of the paper, and let HH, PP, EE, and CC represent planes perpendicular to the paper; then HH, which cuts the elements on opposite sides of the vertex O, cuts a hyperbola from the surface; PP, which is parallel to an element cuts off a parabola; EE which cuts the elements on the same side of O and is not perpendicular to the axis, cuts off an ellipse: and CC, perpendicular to the axis, cuts off a circle.

For mensuration of the conic sections, see Art. 6.

Circle. The equation of a circle of radius r is, if the center is at the origin of rectangular coordinates, $x^2 + y^2 = r^2$; if the center is at a point (a, b) the equation is $(x - a)^2 + (y - b)^2 = r^2$. Any equation of the form $Ax^2 + Ay^2 + 2\,Gx + 2\,Fy + C = 0, A$ not being zero, is the equation of a circle.

An Ellipse is a curve such that the sum of the distances of any and every point on it from two fixed points is always the same (Fig. 37). The two fixed points are foci of the ellipse, and the distances of a point on the curve to the foci are focal distances, or focal radii, of the point. An ellipse has two lines of symmetry, called the axes of the ellipse; the longer is the major axis and the shorter the minor axis. The equation of an ellipse with respect to x and y axes coincident with the major and minor axes respectively, is $x^2/a^2 + y^2/b^2 = 1$, a and b denoting semi-major and semi-minor axis respectively. Any line through the center, terminating in the ellipse, is a diameter. Two diameters are conjugate to each other when either is parallel to tangents to the ellipse at the extremities of the other. The acute angles α and β which two conjugate diameters make with the major axis are related thus: $\tan\alpha \tan\beta = b^2/a^2$, and if A and B are semi-conjugate diameters, $A^2 + B^2 = a^2 + b^2$.

The definition of ellipse here given is in accordance with the one given in the present article under conic, but the focus and directrix there named are in case of the ellipse double; that is, there are two of each. As there, let e = eccentricity of the ellipse, d = distance from either focus to corresponding directrix, also c = distance from center to either focus; then $a = de/(1 - e^2)$, $b = de/\sqrt{1 - e^2}$, $c = de^2/(1 - e^2)$, $e = 1 - b^2/a^2$, $c = ae$, $c^2 = a^2 - b^2$. The latus rectum is the length of the focal chord perpendicular to the major axis; denoting it by p, then $p = 2\,de = 2\,a\,(1 - e^2) = 2\,b^2/a$. The polar equation of the ellipse with F as pole and FA as polar axis (Fig. 37) is $r = p/2\,(1 + e \cos\phi) = a\,(1 - e^2)/(1 + e\cos\phi)$.

Geometric Constructions of Ellipse. (1) To determine the foci of any ellipse, its axes given: from either end of the minor axis draw an arc whose radius equals the semi-major axis; the intersections of the arc and major axis are the foci. (2) To draw a tangent and a normal to the ellipse at any point of it: from the point draw focal radii;

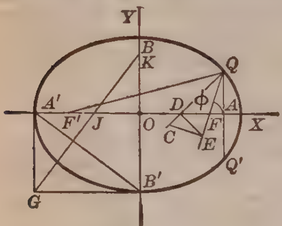

Fig. 37

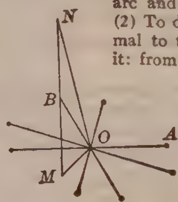

Fig. 38

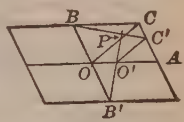

Fig. 39

the bisector of one of the angles between them is the tangent and that of the other angle is the normal. (3) To draw a tangent from a point P without the ellipse: join the point with either focus; on the joining line as diameter construct a circle; on the major axis as diameter construct a circle; the lines connecting the intersections of the two circles with P are tangents. (4) To find the center and the radius of curvature for any point of and ellipse: let Q (Fig. 37) be the point; at Q draw the normal and at its intersection with the major axis erect a perpendicular to the normal; at intersection of this perpendicular with either focal radius from Q erect a perpendicular to that radius; the intersection C of the last line with the normal is the center of curvature and QC is the radius of curvature. This construction fails for a point at either end of the major axis. For such points determine G, and through G draw a perpendicular to $A'B'$; the intersection J is the center of curvature for A' and K for the point B'. (5) To determine the axes of an ellipse from a pair of semi-diameters: from one end B (Fig. 38) of the shorter diameter draw a perpendicular to the longer and make BM and $BN = OA$; then the bisectors of the angles between the lines OM and ON are the directions of the axes; their lengths are $ON + OM$ and $ON - OM$. (6) Construction of an ellipse on its axes: draw auxiliary circles on the axes as diameters (Fig. 40); draw any diameter of these circles; through its intersections b with the small circle draw lines parallel to the major axis; through the intersections a with the other draw lines parallel to the minor axis; the intersections c of these lines are points on the ellipse. (7) Construction of an ellipse on a pair of coordinate diameters: construct a parallelogram on the diameters as medians (Fig. 39); divide OA and CA into proportional parts, beginning at O and at C, readily done by parallels to OC; join any point of division C' on AC with B and determine the intersection of BC' with the line joining B' and the corresponding division point O' on OA; this intersetion P is on the ellipse.

Mechanical Constructions of an ellipse from its axes: (a) First locate the foci; then take an inelastic string and fasten it at the foci so that the length of the loop between the fast points equals that of the major axis; then if the string is pulled out taut into any position, as FPF' (Fig. 40), P is a point on the ellipse, and a moving pencil, or other scribe, pressing against the string at P will describe the ellipse. (b) On a strip of stiff paper or other suitable material, from a point Q (Fig. 40) lay off QM = the semi-major axis and QN = the semi-minor; a pencil, or other scribe, at Q will trace the ellipse when the strip is moved so that M moves along the minor axis and N along the major.

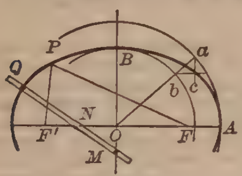

Fig. 40

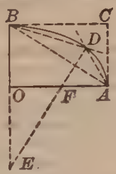

Fig. 41

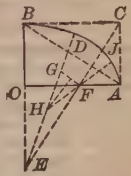

Fig. 42

Approximate Ellipse, consisting of arcs of circles; it is supposed that the axes of the ellipse are given. (1) Semi-ellipse with three centers: OA and OB (Fig. 41) are the given semi-axes; join A and B and bisect the angles CAB and CBA, thus determining D; through D draw a perpendicular to AB, thus determining F and E; from F with radius FA and from E with radius EB draw arcs; they meet tangentially at D. (2) Semi-ellipse with five centers: OA and OB (Fig. 42) are the given semi-axes; join A and B, and through C draw a perpendicular to AB, determining F and E, two of the centers; from E with EB as radius draw an arc BD as long as thought suitable, and join D with E; make $DG = AF$; join F and G; at the center of FG draw a perpendicular to FG and note its intersection H (the third center) with DE; from H with radius HD draw an arc to HF extended, and from F with FA as radius complete the curve.

The Hyperbola is a curve such that the difference between the distances of any and every point on the curve from two fixed points is always the same. The two fixed points are foci, the distances are focal distances, focal radiuses, or radius vectors. A hyperbola has two axes of symmetry (Fig. 43), one cut by the curve; the length AA' cut off by the curve is the transverse or real axis of the hyperbola, and the length BB' on the other line equal to

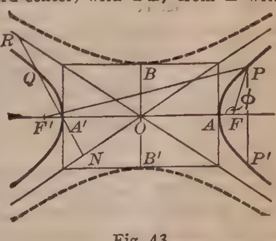

Fig. 43

$\sqrt{(FF')^2 - (AA')^2}$ is the conjugate or imaginary axis. The intersection of the axes is the center of the hyperbola, and any line through the center terminating on the hyperbola is a diameter. The equation of the hyperbola referred to the transverse and conjugate axes as x and y coordinate axes is

$x^2/a^2 - y^2/b^2 = 1$, a and b denoting semi-transverse and conjugate axes respectively. Two diameters are conjugate when each bisects all chords parallel to the other; either of two such diameters is parallel to the tangents at the extremities of the other. The acute angles α and β which two such diameters make with the transverse axis are related thus; $\tan\alpha \tan\beta = b^2/a^2$; if A and B denote semi-conjugate diameters, $A^2 - B^2 = a^2 - b^2$. The two hyperbolas $x^2/a^2 - y^2/b^2 = 1$ and $-x^2/a^2 + y^2/b^2 = 1$ are conjugate hyperbolas; the latter is shown by dotted lines in Fig. 43. They have the same asymptotes.

The definition of hyperbola given above is in agreement with the one given earlier in this article. As there, let e = eccentricity and a = distance between either focus and corresponding directrix; also let c = distance from center to either focus; then

$$a = de/(e^2 - 1) \qquad b = de/\sqrt{e^2 - 1} \qquad c = de^2/(e^2 - 1)$$
$$e^2 = 1 + b^2/a^2 \qquad c = ae \qquad c^2 = a^2 + b^2$$

The latus rectum of a hyperbola is the length of either focal chord perpendicular to the transverse axis; denoting it by p, then $p = 2\,de = 2\,a\,(e^2 - 1) = 2\,b^2/a$. The polar equation of the hyperbola with F as pole and FA as polar axis is

$$r = p/2\,(1 + e\cos\phi) = a\,(e^2 - 1)/(1 + e\cos\phi)$$

A hyperbola whose axes are equal is an equilateral or rectangular hyperbola; the asymptotes are at right angles. The equation of such a hyperbola referred to the axes is $x^2 - y^2 = a^2$, or $-x^2 + y^2 = a^2$; referred to the asymptotes, the equation is $xy = 1/2\,a^2$. The eccentricity is $\sqrt{2}$.

Geometric Constructions of Hyperbola. (1) To determine the foci when the axes are given: make OF and OF' (Fig. 43) equal to AB; then F and F' are the foci. (2) To draw a tangent and a normal at any point of a hyperbola: from the point draw the focal radii; the bisector of one of the angles between the radii is the tangent and that of the other is the normal. (3) To construct the asymptotes to a hyperbola, the axes being given: construct a rectangle on the axes as medians; diagonals of the rectangle extended are the asymptotes. (4) Construction of a hyperbola, its axes AA' and BB' (Fig. 43) being given: (a) Locate the foci F and F', mark any point M on the transverse axis extended but not between the foci; with MA and MA' as radii, strike arcs from F and F' respectively; their intersections are points on the hyperbola. Repeat for other points like M. (b) Draw the asymptotes; through A' draw any oblique line and note its intersections N and R with the nearer and remoter asymptote; lay off from R toward A' a length $RQ = A'N$; then Q is on the hyperbola. Repeat with other oblique lines through A', A, or through any other known point of the curve, as Q.

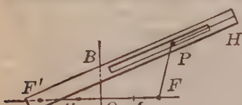

Fig. 44

Mechanical construction of a hyperbola from its axes AA' and BB': Determine the foci F and F' (Fig. 44); pivot one end of a strip of stiff paper or suitable material at F'; fasten an inextensible string HPF at H and F and so that the length HPF is less than HF' by AA'; place a pencil or other scribing point under the string and through the slot; keep the string taut with the pencil and then rotate the strip about F'; the pencil describes an arc of a hyperbola.

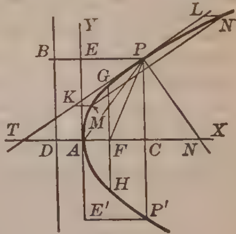

Fig. 45

Parabola. This is a curve such that any and every point of it is equidistant from a fixed point and a fixed line. The point is the focus, the line the directrix of the parabola, and the distance of any point of the curve to the focus is a focal radius, or focal distance. A parabola has a line of symmetry called the axis of the parabola (Fig. 45); it contains the focus F and is perpendicular to the directrix. The vertex is the intersection of the axis with the curve; the latus rectum is the length of the focal chord perpendicular to the axis.

The equations of the parabola referred to the axis as x axis and the vertex tangent as a y axis is $y^2 = px$ where p = latus rectum; the polar equation referred to F as pole and FA as plar axis is $r = p/2\ (1 + \cos\phi)$.

Geometric Properties and Constructions. The vertex is midway between the focus and the directrix. The length of the latus rectum equals four times the distance between focus and vertex. Any line parallel to the axis of a parabola bisects a system of parallel chords and is therefore a diameter of the parabola; the chords are parallel to the tangent at the end of the diameter terminating in the parabola. The extension of a diameter at P to the directrix equals the focal radius at P ($PB = PF$), and the distances from the focus to the ends of a definite tangent are equal ($FT = FP$). The tangent and the normal at any point bisect the angles between the focal radius and the diameter at that point; a subtangent is bisected by the vertex ($AC = AT$), and all subnormals equal one-half the latus rectum (CN = 1/2 GH). The projection of the radius of curvature R at any point on the axis of the parabola equals twice the focal distance of the point, and the extension of R to the directrix also equals twice the focal distance; R at the vertex equals the latus rectum. To construct a parabola: (a) Given the vertex (Fig. 46), the axis AX, and a point P. Join A and P, and through P draw a line parallel to the axis; draw any line as AQ; then perpendicular to the axis a line QR to AP; then parallel to the axis a line RS to AQ; S is a point on the parabola. Repeat for other points Q. Or, divide AB into equal parts and BP into the same number of equal parts; number the points of division, beginning at A on AB and at B on BP; draw lines as shown and note intersections of corresponding lines; they are on the parabola. (b) The vertex A and focus F (Fig. 47) are given. Join A and F and draw a perpendicular to AF at A; draw any line as FB and then BC perpendicular to FB; BC is a tangent to the parabola. Repeat with other lines like FB and BC and thus determine the parobola by means of tangents.

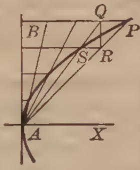

Fig. 46

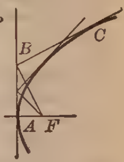

Fig. 47

12. Higher Curves

A Cycloid is the curve generated or traced by any point on the circumference of a circle which rolls on a straight line. Fig. 48 shows one half of one branch of a cycloid OA generated by the point P on the circle PQ rolled on OX; the other half of the branch is symmetrical with respect to AX. Let θ be the angle described by any line of the circle while it rolls from its original position OaY to any other as the one shown, r the radius of the circle, and x and y the coordinates of P relative to the axis shown; then $x = r\ (\theta - \sin\theta)$, $y = r\ (1 - \cos\theta)$, and the equation of the cycloid is $x = r \cos^{-1}(1 - y/r) \pm \sqrt{(2r - y)y}$. A cycloid may be constructed as follows: Draw the circle OaY of the chosen radius r, and make $OX = \pi r$; divide the arc OaY and OX into the same number of equal parts; at two corresponding points of division, as a and b, draw a horizontal and a vertical respectively to their intersection c and make $cd = ae$; then d is a point on the cycloid. Repeat for other points of division. To find the position of the generating circle corresponding to any point of the cycloid as P: make $Pf = gh$, and from f drop a perpendicular to OX; its foot Q is the lowest point of the circle desired. The normal at any point of the cycloid, as P, passes through the lowest point of the corresponding position of the generating circle; $PQ = 2r \sin 1/2\ \theta = \sqrt{2ry}$. The radius of curvature is twice PQ; at the highest point it equals $4r$ and at the lowest 0. The length of any arc as OP beginning at the lowest point is $4r\ (1 - \cos 1/2\ \theta)$

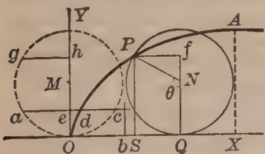

Fig. 48

$= 4r - 2\sqrt{2r(2r - y)}$, y being the ordinate of P; the length of one complete cycloid or $2\,OPA$ is $8r$. The area of any part as $OPSO$ is $r^2(3/2\,\theta - 2\sin\theta + 1/4\sin 2\theta) = 3/2\,rx - 1/2\,y\sqrt{(2r - y)y}$, x and y being the coordinates of P; the entire area between one cycloid and the track, that is $2\,OAXO$ is $3\pi r^2$.

When a circle rolls upon the outside of a fixed circle, each point of the circumference of the rolling circle describes or traces an epicycloid; and when it rolls upon the inside the curve described is a hypocycloid. A point without or within the rolling circle and fixed to it describes a trochoid, and epitrochoid, or a hypotrochoid according as the circle rolls on a straight line, on the outside of a fixed circle, or on the inside of a fixed circle.

The Spiral of Archimedes is a curve generated by a point moving at a constant speed in a given straight line which rotates at constant speed about a fixed point of the line. Fig. 49 shows such a spiral; O is the fixed point and OA the position of the line when the moving point is at the fixed point. The polar equation of the spiral is $r = (r'/2\pi)\phi$, in which r is the distance of the moving point P from O, ϕ the corresponding angle described by the moving line expressed in radians, and r' is the distance traveled by the moving point along the line while the line turns through 360°. To construct the spiral

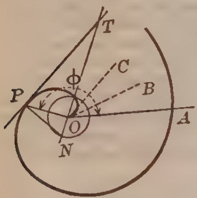

Fig. 49

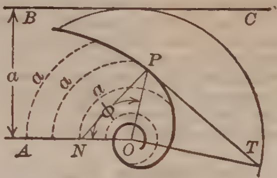

Fig. 50

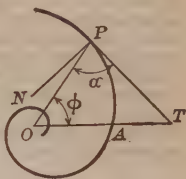

Fig. 51

having selected r': draw lines OA, OB, OC, etc., so that the successive angles between them is $360/n$, n being a convenient number; make $OB = r'/n$; $OC = 2\,r'/n$; $OD = 3\,r'/n$, etc.; B, C, D, etc., are on the spiral. To draw a normal and a tangent at any point P: draw a circle with center at O, and radius $r'/2\pi$; join P and O, and draw a perpendicular to OP at O, and note the intersection N as shown; then PN is the normal and PT (perpendicular to PN) the tangent. The length of any arc OP is $(r'/4\pi)\,[\phi\sqrt{1 + \phi^2} + \sinh^{-1}\phi]$; for many turns it equals approximately $r'\phi^2/4\pi$.

The Hyperbolic Spiral may be constructed as follows: draw a number of concentric circles and then a radius of the largest circle; from this radius measure off on the circles equal arcs all on the same side of the radius; the other ends of the arcs lie on the spiral (Fig. 50). The polar equation of the spiral is $r\phi = a$, a being the constant length of arc referred to, r the distance of any point of the curve as P from O and ϕ the angle between the radius OA and OP. The curve has an asymptote BC distant a from the reference radius. To draw a tangent and a normal at any point P: draw a circle with center at O and radius a, and join P and O; to OP at O draw a perpendicular, and note its intersection T with the circle on the side shown; then TP is the tangent and PN (perpendicular to PT) is the normal.

The Logarithmic or Equiangular Spiral is a curve such that the angles between its tangents and the corresponding radii drawn to the center of the curve are equal. Its polar equation is $r = a\epsilon^{m\phi}$; a is the value of r when $\phi = 0$ and $m = \cot\alpha$, α being the constant angle referred to (Fig. 51). To construct a curve for given values of a and α: compute m and then $m\phi$ for a number of values of ϕ in radians, as $20° = 0.349$ radian, $40° = 0.698$, $60° = 1.047$, etc.; from a table of Napierian logarithms (page 35) find the values of $\epsilon^{m\phi}$ (these are the numbers corresponding to the logarithms $m\phi$); compute values of $a\epsilon^{m\phi}$ or r; then lay off these values of r from a fixed point on radii making the different angles ϕ (20°, 40°, 60°, etc.) with a reference line through the fixed point. The length of any arc as $PO = r/\cos\alpha$.

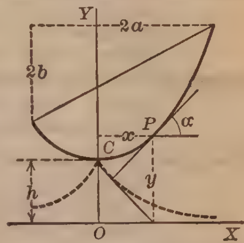

Fig. 52

The Catenary is the curve assumed by a chain suspended from two points. Its equation is $y = 1/2\ h\ (\epsilon^{x/h} + \epsilon^{-x/h}) = h \cosh x/,\ h$ or $x = h \log_\epsilon (y/h \pm \sqrt{(y/h)^2 - 1}) = h \cosh^{-1} y/h$; h is the distance from the lowest point of the curve to the origin (Fig. 52). The gradient at any point P whose coordinates are x and y is given by $\tan\alpha = \sinh x/h$ or $\cos\alpha = h/y$, from which the direction of the tangent line at any point can be computed. The radius of curvature R at any point P is $R = y^2/h = h/\cos^2\alpha$; it also equals the length cut off by the x axis from the normal through P. The length of any arc, as CP, is given by $s = h \sinh x/h = h \tan\alpha = \sqrt{y^2 - h^2}$, and the abscissa of P is

$$x = h \log_e [s/h + \sqrt{1 + (s/h)^2}] = h \sinh^{-1} s/h$$

To locate the origin of coordinates for a chain of length $2\,l$, supported from two points whose horizontal distance is $2\,a$ and vertical distance is $2\,b$: Let A and B denote the distances from the middle of the line joining the points of suspension to the y and x axis respectively; $A = \psi h$ and $B = l \cot\phi$, in which $h = a/\phi$ and $\psi = \tanh^{-1} b/l$, ϕ to be obtained by trial from $a \sinh\phi = \phi\sqrt{l^2 - b^2}$. The catenary is the evolute of the other curve shown in Fig. 52; its tangents are all equal to h. It is also called "anti-friction" curve, being the axial section of the surface of a vertical pivot of uniform wear.

13. Hyperbolic Functions

The Hyperbolic Functions are related to an equilateral or rectangular hyperbola much as the circular functions are related to a circle. Let P (Fig. 53) be a point on the hyperbola $x^2 - y^2 = a^2$; l = arc AP, positive or negative according as P is above or below OX; l/r = the mean of the reciprocals of all radii from O to AP; and $u = l\ (l/r)$. Then

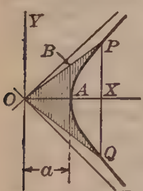

Fig. 53

hyperbolic sine of u, or $\sinh u = y/a$
hyperbolic cosine of u, or $\cosh u = x/a$
hyperbolic tangent of u, or $\tanh u = \sinh u \div \cosh u = y/x$
hyperbolic cotangent of u, or $\coth u = 1/\tanh u = x/y$
hyperbolic secant of u, or $\operatorname{sech} u = 1/\cosh u = a/x$
hyperbolic cosecant of u, or $\operatorname{csch} u = 1/\sinh u = a/y$.

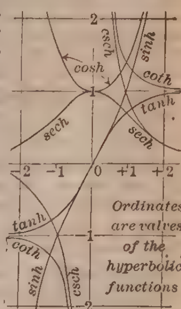

Fig. 54

If a is made 1, then the numerical values of double the area of the hyperbolic sector $OPAO$ equals l/r or u; also $\sinh u = XP$, $\cosh u = OX$; $\tanh u = AB$. See Sect. 1., Art. 22, for tables of hyperbolic functions. Fig. 54, plotted to scale, shows the relations between the functions for values of u from -2 to $+2$; it also indicates the sign of any function of u as dependent on the sign of u. Inverse hyperbolic sine of n means the l/r or u whose hyperbolic sign equals n; it is written $\sinh^{-1} n$; similarly, inverse hyperbolic cosine, tangent, etc.

The hyperbolic functions of u are closely related to the exponential function ϵ^u and ϵ^{-u}, ϵ being the Napierian base, thus

$$\sinh u = 1/2\,(\epsilon^u - \epsilon^{-u}), \quad \cosh u = 1/2\,(\epsilon^u + \epsilon^{-u})$$
$$\tanh u = (\epsilon^u - \epsilon^{-u})/(\epsilon^u + \epsilon^{-u})$$
$$\sinh u + \cosh u = \epsilon^u, \quad \cosh u - \sinh u = \epsilon^{-u}$$

Inverse hyperbolic functions as functions of Napierian logarithms:

$$\sinh^{-1} u = \log_\epsilon (u + \sqrt{u^2 + 1}), \quad \cosh^{-1} u = \log_\epsilon (u + \sqrt{u^2 - 1})$$
$$\tanh^{-1} u = 1/2 \log_\epsilon [(1 + u)/(1 - u)]$$

The hyperbolic functions are related to circular functions, thus ($i = \sqrt{-1}$):

$$i \sinh u = \sin iu \quad \sin u = -i \sinh iu$$
$$\cosh u = \cos iu, \quad \cos u = \cosh iu$$
$$i \tanh u = \tan iu, \quad \tan u = -i \tanh iu$$

The following are some of the relations between hyperbolic functions:

$$\cosh^2 u - \sinh^2 u = 1 \qquad 1 - \tanh^2 u = \operatorname{sech}^2 u$$
$$\coth^2 u - 1 = \operatorname{csch}^2 u$$
$$\sinh (u \pm v) = \sinh u \cosh v \pm \cosh u \sinh v$$
$$\cosh (u \pm v) = \cosh u \cosh v \pm \sinh u \sinh v$$
$$\tanh (u \pm v) = (\tanh u \pm \tanh v)/(1 \pm \tanh u \tanh v)$$
$$\sinh 2u = 2 \sinh u \cosh u$$
$$\cosh 2u = 1 + 2 \sinh^2 u = 2 \cosh^2 u - 1$$
$$\tanh 2u = 2 \tanh u/(1 + \tanh^2 u)$$
$$\sinh 1/2\, u = \sqrt{1/2\,(\cosh u - 1)}$$
$$\cosh 1/2\, u = \sqrt{1/2\,(\cosh u + 1)}$$
$$\tanh 1/2\, u = \sinh u/(1 + \cosh u) = (\cosh u - 1)/\sinh u$$

14. Probability of Errors

The Probability of an event is the ratio of the number of favorable chances of its occurrence to the total number of chances, favorable and unfavorable. Thus, if there are a white and b black balls in a jar, the probability of drawing a white ball at a single trial is $a/(a + b)$. If the probabilities of two independent events are p_1 and p_2, the probability of their concurrence in any single instance is $p_1 p_2$. Thus, suppose that there are two jars, J_1 and J_2, J_1 containing a_1 white and b_1 black balls, and J_2 containing a_2 white and b_2 black balls; the probability of drawing a white ball from J_1 is $a_1/(a_1 + b_1)$, the probability of drawing a white one from J_2 is $a_2/(a_2 + b_2)$, and that of drawing a pair of white balls, one from each jar, in a single trial is $a_1 a_2/(a_1 + b_1)\,(a_2 + b_2)$.

An Error of an Observation is the true value of a quantity minus the observed value. Errors are accidental or systematic; accidental errors are those which in the long run are as often negative as positive, and they affect the mean result but little; systematic errors due to the same cause affect the mean in the same sense, and do not tend to balance each other in the mean. Only accidental errors are referred to in the following. From the theory of probabilities, it has been shown that in a series comprising a great number of observations the relative frequency (proportionate number) of the errors whose values lie between x and $x + \Delta x$ is approximately $(h/\sqrt{\pi})\,\epsilon^{-h^2x^2}\,\Delta x$, and the relative frequency of the errors whose values lie between a and b is $(h/\sqrt{\pi}) \int_a^b \epsilon^{-h^2x^2}\,dx$; here ϵ is the Napierian base (Art. 1) and h a constant for any particular series of observations. The graph of $y = (h/\sqrt{\pi})\,\epsilon^{-h^2x^2}$ (Fig. 55) is a probability or "error curve"; the shaded area represents the relative frequency of the errors between a and b, and the whole area between the curve and the x axis is unity, according to the scale used, and represents the frequency of the whole number of errors that is 1 or 100 %. The dotted curve is also an error curve but the constant h for that curve is greater than for the first; also in the second the relative frequency of small errors is greater and that of large errors is smaller than in the first. Thus the constants h serve as a means for comparing the accuracies of several series of observations, and the value of h for any given series is called the measure of precision for that series.

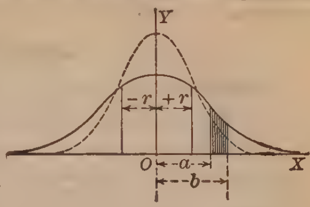

Fig. 55

Another index of accuracy is the so-called probable error of the series, an error of such value that the number of errors in the series less and greater than it are equal, the signs of the errors being disregarded in the count. It is the error r (Fig. 55) corresponding to the symmetrical ordinates which include one-half the whole area below the error curve. This middle error is more often called the **probable error** of a single observation to convey the idea that the error of any subsequent observation in such a series is just as apt to be less as greater than the middle error. The probable error and the measure of precision are always related thus, $rh = 0.4769$. The expression for relative frequency of the errors falling between $-a$ and $+a$ is

$$\frac{h}{\sqrt{\pi}}\int_{-a}^{+a}\epsilon^{-h^2x^2}\,dx = \frac{2}{\sqrt{\pi}}\int_0^{0.4769\,a/r}\epsilon^{-h^2x^2}\,d\,(hx)$$

The table below gives the values of the expressions for the various values of the arguments a/r given therein. Thus, in a large series of observations, the relative frequency of the errors falling between $a = -r$ and $a = +r$ is .500, or 50 %; between $a = -2r$ and $a = +2r$, 0.823, or 82.3 %; the relative frequency of the errors arithmetically greater than $4r$ is 0.007, or 0.7 %; etc.

Values of the Probability Integral

a/r	frqncy	a/r	frqncy	a/r	frqncy	a/r	frqncy	a/r	frqncy
0.	0.000	1.0	0.500	2.0	0.823	3.0	0.957	4.0	0.993
0.1	0.054	1.1	0.542	2.1	0.843	3.1	0.963	4.1	0.994
0.2	0.107	1.2	0.582	2.2	0.862	3.2	0.969	4.2	0.995
0.3	0.160	1.3	0.619	2.3	0.879	3.3	0.974	4.3	0.996
0.4	0.213	1.4	0.655	2.4	0.895	3.4	0.978	4.4	0.997
0.5	0.264	1.5	0.688	2.5	0.908	3.5	0.982	4.5	0.998
0.6	0.314	1.6	0.719	2.6	0.921	3.6	0.985	4.6	0.998
0.7	0.363	1.7	0.748	2.7	0.931	3.7	0.987	4.7	0.998
0.8	0.411	1.8	0.775	2.8	0.941	3.8	0.990	4.8	0.999
0.9	0.456	1.9	0.800	2.9	0.950	3.9	0.991	4.9	0.999
1.0	0.500	2.0	0.823	3.0	0.957	4.0	0.993	5.0	0.999

The probable error r for n observations of equal precision on the same quantity is computed as follows: (1) find the arithmetic mean by adding the observations and dividing by n, (2) subtract each observation from that mean thus obtaining n residuals v_1, v_2, v_3, etc., (3) square each residual and find their sum Σv^2. Then

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}} \quad \text{and} \quad r_0 = \frac{r}{\sqrt{n}}$$

the first formula giving the probable error of a single observation, while the second gives the probable error of the arithmetic mean. For a numerical example, see below.

14½. Method of Least Squares

An Observation is the result of a measurement, thus when 635.74 ft. is stated as the measured length of a line, the quantity 635.74 ft. is called an observation, it being understood that all constant errors have been removed therefrom; this quantity, however, is still affected by the result of accidental errors. The true value of a quantity cannot be found by measurement, but the best that can be done is to deduce from the observations the most probable value of the quantity. The Method of Least Squares teaches how to obtain the most probable values of observed quantities.

Direct Observations are those which result from measurements made directly upon a single quantity. When only one observation is at hand, it is the most probable value of the quantity. When two or more observations are made with equal precision on the same quantity, their arithmetic mean is the most probable value, and the probable error of this arithmetic mean can be computed by the formulas at the end of Art. 14.

Example: Let a line be measured eight times with equal care by a tape graduated in centimeters and the following results be found, M indicating an observation, v a residual found by subtracting M from the arithmetic mean, and v^2 the square of a residual:

M	789.7	788.1	789.1	789.9	788.3	788.0	788.1	788.8
v	0.95	0.65	.0.35	1.15	0.45	0.75	0.65	0.05
v^2	0.902	0.423	0.122	1.323	0.203	0.562	0.423	0.002

Here, the arithmetic mean, found by adding the observations and dividing by 8, is 788.75, which is the most probable length of the line in centimeters. The eight residuals are some positive, some negative, their sum being zero. The sum of the squares of the residuals is $\Sigma v^2 = 3.96$. Then by the formula at foot of last page $r = 0.51$ cm., which is the probable error of a single observation; also $r_0 = 0.18$ cm., which is the probable error of the arithmetic mean. The final result of this series of observations may then be written 788.75 ± 0.18; that is, the true length of the line is just as likely to be between 788.57 cm. and 788.93 cm. as it is to be outside of those limits.

Weighted Observations. Weights of Observations are numbers proportional to their degrees of precision, so that one observation of weight p is worth as much as p observations of weight unity. When there are n weighted observations, M, with weight p_1, M_2 with weight p_2, and so on, these being made directly upon the same quantity, then the most probable value of the quantity is the weighted mean z, or

$$z = \frac{p_1M_1 + p_2M_2 + \ldots + p_nM_u}{p_1 + p_2 + \ldots + p_n} = \frac{\Sigma pM}{\Sigma p}$$

To find the probable errors, let each observation be subtracted from this mean z, the difference being a residual v, square each residual, multiply each square by its weight, and find the sum Σpv^2, then

$$r = 0.6745\sqrt{\frac{\Sigma pv^2}{n-1}} \quad \text{and} \quad r_0 = \frac{r}{\sqrt{\Sigma p}}$$

the first being the probable error of an observation of the weight unity and the second being the probable error of the weighted mean Z.

Example: Let six observations on the same quantity be made, with weights as in the first line, the sum of these weights being 21. Multiplying each observation M by its weight p, gives the quantities in the third line the sum of which is 3741.36. Then the most probable value of the observed quantity is $z = 3741.36/21 = 178.16$. Subtracting each M from this gives the residuals in the fourth line. The weighted squares of the residuals are in the last line, their sum being $\Sigma pv^2 = 62.95$. Then the preceding formulas give the probable error of an observation of the weight unity as $r = 2.39$ and the probable error of the weighted mean as $r_0 = 0.52$. The final result then is $z = 178.16 \pm 0.52$.

p	5	4	1	4	3	4
M	178.26	176.30	181.06	177.95	176.20	180.85
pM	891.30	705.20	181.06	711.80	528.60	723.40
v	0.10	1.86	2.90	0.21	1.96	2.69
v^2	0.010	3.460	8.410	0.441	3.842	7.236
pv^2	0.05	13.84	8.41	0.18	11.53	28.94

Observation Equations arise when measurements are made to find the values of several quantities. For example, let O be a given bench and Z_1, Z_2, Z_3 three points whose elevations above O are to be determined. Let z_1, z_2, z_3 be the most probable elevations of Z_1, Z_2, Z_3; let five lines of levels be run giving

$$
\begin{array}{llllll}
Z_1 \text{ above } O & = 10 \text{ ft.} & \text{or} & z_1 & & = 10 \\
Z_1 \text{ above } Z_3 & = 2 \text{ ft.} & \text{or} & z_1 & - z_3 & = 2 \\
Z_2 \text{ above } O & = 18 \text{ ft.} & \text{or} & z_2 & & = 18 \\
Z_2 \text{ above } Z_3 & = 9 \text{ ft.} & \text{or} & z_2 & - z_3 & = 9 \\
Z_1 \text{ above } Z_3 & = 2 \text{ ft.} & \text{or} & - z_1 + z_2 & & = 7
\end{array}
$$

The problem is to determine the most probable values of z_1, z_2, z_3.

Normal Equations are derived from observation equations, when these are of equal weight, by the following rule: To find a normal equation for z_1, multiply each observation equation by the coefficient of z_1 in that equation and add the results; similarly a normal equation for each of the other unknown quantities is found; thus, for the above level observations the normal equation for z_1 is found by multiplying the first equation by $+1$, the second by $+1$, the third by 0, the fourth by 0, and the fifth by -1; the addition of these gives $3z_1 - z_2 - z_3 = 5$ as the normal equation for z_1. Similarly the normal equation for z_2 is $-z_1 + 3z_2 - z_3 = 34$, and that for z_3 is $-z_1 - z_2 + 2z_3 = -11$. These three normal equations contain three unknown quantities, and their solution gives $z_1 = 10\text{-}3/8$, $z_2 = 17\text{-}5/8$, $z_3 = 8\text{-}1/2$, which are the most probable elevations of the points Z_1, Z_2, Z_3 above the bench mark O.

General Method. Let the n observation equations be:

$$
\begin{array}{ll}
a_1 z_1 + b_1 z_2 + c_1 z_3 + \ldots\ldots = M_1 & \text{with weight } p_1 \\
a_2 z_1 + b_2 z_2 + c_2 z_3 + \ldots\ldots = M_2 & \text{with weight } p_2 \\
\cdots\cdots\cdots\cdots\cdots\cdots & \\
a_n z_1 + b_n z_2 + c_n z_3 + \ldots\ldots = M_n & \text{with weight } p_n
\end{array}
$$

in which $M_1, M_2 \ldots M_n$ are the observed quantities, z_1, z_2, z_3 are quantities required to be found, and a, b, c, are known coefficients. For each unknown quantity form a normal equation by multiplying each observation equation by the coefficient of that unknown quantity in that equation and also by its weight, and adding the results. If the number of unknown quantities is q the number of normal equations will also be q, thus:

$$
\begin{array}{l}
\Sigma pa^2.z_1 + \Sigma pab.z_2 + \Sigma pac.z_3 + \ldots = \Sigma paM \\
\Sigma pab.z_1 + \Sigma pb_2.z_2 + \Sigma pbc.z_3 + \ldots = \Sigma pbM \\
\Sigma pac.z_1 + \Sigma pbc.z_2 + \Sigma pc_2.z_3 + \ldots = \Sigma pcM
\end{array}
$$

in which $\Sigma pa^2 = p_1a_1^2 + p_2a_2^2 + \ldots p_na^2_n$ and similarly $\Sigma pab = p_1a_1b_1 + p_2a_2b_2 + \ldots p_na_nb_n$. The solution of these normal equations gives the most probable values of the unknown quantities $z_1, z_2, z_3 \ldots z_q$.

After these probable values are found, let them be inserted in the observation equations and the second members be subtracted from the first, thus giving small residuals $v_1, v_2 \ldots v_n$. Then

$$r = 0.6745\sqrt{\frac{\Sigma pv^2}{n-q}} \quad \text{and} \quad r' = \frac{r}{\sqrt{p'}}$$

the first being the probable error of an observation of weight unity, while the second is the probable error of an observation of weight p'.

For an example in which the weights are unequal, see the adjustment of angles at a station in Art. 50 of Sect. 6. When the weights are equal p is to be taken as unity.

Probable Error of a Line. When a line is measured the probable error of an observation increases as the square root of the length of the line. Thus $R = r\sqrt{l}$, where r is the probable error of a line one unit long, and R is the probable error of a line l units long.

The same rule holds good for discrepancies or apparent errors which are found in duplicate measurements of a line. The discrepancy r for a line one unit long may be found as follows: Let a line of length l_1 be measured twice, R_1 being the difference of the observed lenghts; let a second length, l_2, be measured twice, R_2 being the difference of the observed lengths. Then $r = (R_1 - R_2)/(\sqrt{l_1} - \sqrt{l_2})$.

The same rule holds for duplicate lines of levels. Thus, if r is the difference in the results for a line one unit long, then the difference R for a line of length l ought to be $R = r\sqrt{l}$. The practical rules stated at top of page 460 are derived from this principle.

STATICS

15. Forces and Moments

Force. An action of one body upon another which changes or tends to change the state of rest or motion of the body acted upon, is called force. A force has magnitude, direction, and place of application. When the extent of the place of application is negligible, and the force is regarded as applied or concentrated at a point, this is the point of application; and a line through the point parallel to the direction of the force is the line of action. The word " sense " as applied to forces refers to one of the two directions along the line of action of the force. The unit of force commonly used in America is the " pound," which is a force equal to the earth's attraction on the standard of weight, also called pound; this unit of force varies slightly from place to place on the earth, but the variation is negligible in most engineering calculations. Any number of forces considered collectively is a system of forces; a system is **concurrent**, or nonconcurrent, according as the lines of action of the forces do or do not intersect in a point, and it is **coplanar**, or noncoplanar, according as they do or do not lie in a plane. The resultant of a system of forces is the single force which is equivalent to that system, but if a system has no single force equivalent, then the simplest equivalent system may be called the resultant; a resultant never includes more than two forces. The process of determining the resultant is called composition (Art. 16).

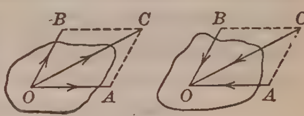

Fig. 56

Parallelogram Law. If magnitudes, lines of action, and senses of two concurrent forces acting on a rigid body are represented by OA and OB (Fig. 56), then the magnitude, line of action, and sense of their resultant is represented by the diagonal OC of the parallelogram $OABC$. The points of application of the forces may be anywhere on the body in the lines OA, OB, and OC, or their extensions. The arrowheads on the lines on AO, OB, and OC all point toward or all away from the point of concurrence O. **Triangle Law:** If the magnitudes and directions of two concurrent forces are represented by AB and BC (Fig. 57 or 58), then the magnitude and direction of the resultant is represented by the side AC of the triangle ABC. The arrowheads on the

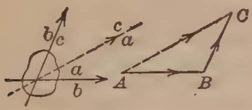

Fig. 57

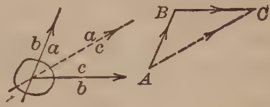

Fig. 58

sides AB and BC are confluent (point the same way around), but the arrowhead on AC is not confluent with the others. The two forces and their resultant being concurrent, the line of action of the resultant is ac, parallel to AC.

The resultant of two concurrent forces may be determined algebraically, thus: let P and Q be the forces and α that angle between their lines of action in which the resultant lies, R the resultant and θ the angle between R and P; then $R = (P^2 + Q^2 + 2\ PQ \cos\alpha)^{1/2}$ and $\tan\theta = (Q \sin\alpha)/(P + Q\cos\alpha)$. If the angle between the given forces is 90°, then $R = \sqrt{P^2 + Q^2}$ and $\tan\theta = Q/P$.

Parallelopiped Law. If the magnitudes and lines of action of three noncoplanar concurrent forces are represented by OA, OB, and OC, then the magnitude and the line of action of the resultant is represented by the diagonal OD of the parallelopiped $OABC - D$ (Fig. 59). This is not a practical device to get numerical results; for a better, see Art. 16. When the three forces are mutually at right angles then the parallelopiped is right-angled, and the resultant can be conveniently determined algebraically, thus: let P, Q, and S denote the three forces, R the resultant and α, β and γ the angles between $R^{1/2}$ and the three forces respectively; then $R = (P^2 + Q^2 + S^2)^{1/2}$, $\cos\alpha = P/R$, $\cos\beta = Q/R$ and $\cos\gamma = S/R$.

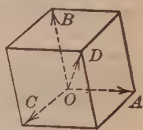

Fig. 59

Resolution of a Force. Two or more forces which together are equivalent to a given force are components of the force. The process of determining components of a force is called resolution; resolution of a force into two components at right angles to each is the common case; such components are "rectangular" and each is a "resolved part" of the given force. (1) Resolution into concurrent components. A force may be resolved into two concurrent components by applying the parallelogram law inversely; thus to resolve the 10 lb. (Fig. 60) into two components: lay off AB by scale to represent 10 lb.; construct a parallelogram on AB as diagonal; then AC and

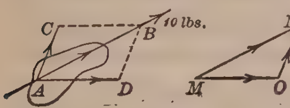

Fig. 60

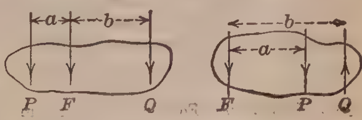

Fig. 61

AD represent components of the given force. Or, by means of the triangle law, thus: lay off MN by scale to represent the magnitude and direction of the given force; construct a triangle on MN as one side, then MO and ON represent magnitudes and directions of two components of the given force, the action lines being concurrent with that of the 10-lb. force. Two rectangular components of a force can readily be found algebraically; each equals the product of the force and the cosine of the acute angle between the force and that component. Three rectangular components of a force can be readily computed if the angles between the force and the desired components are known; each component equals the product of the force and the cosine of the acute angle between the force and that component. (2) Resolution into two components parallel to the force, their lines of action being specified: Let F (Fig. 61) be the force to be resolved into two components P and Q (values and senses unknown), a and b the distances from F to P and Q respectively, and c the distance between P and Q; the principle that the moment of F about a point on either force equals the moment of the other about the same point determines the senses of the components as shown, also their values, $P = Fb/c$ and $Q = Fa/c$. Or, graphically, suppose F (Fig. 62) to be

Fig. 62

the force and P and Q the components; resolve AB (representing magnitude and direction of F) into any two concurrent components AO and OB, action lines in ao and ob; at 1 resolve AO along P and the line 12, and at 2 resolve OB along Q and the line 12; the two components along 12 balance, and those along P and Q represented by AC and CB in value and direction are the desired components. (3) To resolve a force into three nonconcurrent nonparallel forces coplanar with the force: Let P (Fig. 63) be the force, the components to act in F_1, F_2, and F_3; join the intersection of P and F_1 with that of F_2 and F_3; draw AB to represent P and then resolve P into two components along F_1 and the line 12 (represented by AC and CB); then resolve CB into two components along F_2 and F_3 (represented by CD and DB); then AC, CD, and DB are the magnitudes and directions of the desired components.

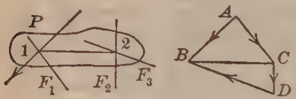

Fig. 63

The Moment of a Force with Respect to a Point is the product of the magnitude of the force and the perpendicular distance between the force and the point; this distance is the "arm" of the force with respect to the point, and the point is the origin or center of moments. Moments are commonly given signs, those corresponding to forces which tend to turn the body acted upon in one direction (clockwise) being given the same sign and all others the opposite. The moment of a force with respect to a point may also be computed by taking the algebraic sum of the moments of its two rectangular components with respect to that point; and it will often be convenient to resolve the force so that one of the components will act through the origin of moments, so that that component will have no moment.

The Moment of a Force with Respect to a Line is the product of its rectangular component perpendicular to the line (the other being parallel) and the distance between the line and the perpendicular component (or the force); the line is the "axis" of moments. If a force is parallel to the axis of moments or if it cuts the axis, then its moment with respect to that axis is zero. Moments of forces about the same axis are commonly given signs; those corresponding to the forces which tend to turn the body acted upon about the axis (regarded as a shaft) in the same direction (clockwise) being given the same sign and others the opposite. The moment of a force with respect to an axis may also be computed by resolving the force into three rectangular components, one being parallel to the axis, the other two perpendicular to it; then the moment of the given force equals the algebraic sum of the moments of the two perpendicular components. If the resolution is made so that one of the perpendicular components cuts the axis, then the moment of the given force equals the moment of the other perpendicular component.

The moment of the resultant of any coplanar forces about a point in their plane (or of any noncoplanar forces about a line) equals the algebraic sum of the moments of those forces about the point (or line). This is called the principle of moments for forces. The word **torque** is frequently used as synonymous with moment of a force, especially when the force is distributed around a circumference.

Couples. Two equal, parallel, and opposite forces are called a couple; the perpendicular distance between the forces is the " arm " of the couple. The moment or torque of a couple with respect to any point or origin in their plane is the algebraic sum of the moments of the two forces with respect to that point; this sum or moment, the same for all origins in the plane, always equals the product of one of the forces and the arm of the couple. Moments of couples whose planes are parallel are sometimes given signs; those corresponding to couples which tend to turn the body in the same direction are given the same sign, and the others the opposite sign. A couple may be represented sufficiently for statical purposes by means of a single vector; the vector is drawn perpendicular to the plane of the couple, and the arrowhead is so placed that it points toward the place from which the rotation appears, say counter-clockwise. Two couples whose vectors are equal—the same in length and direction—have equal moments (sign included) and their planes coincide or are parallel. Such are equivalent couples; that is, either may be substituted for the other without change of effect on the body acted upon if rigid. The resultant of a number of couples is a couple. If the planes of the given couples are parallel or coincident, the resultant couple is one whose plane is parallel to the others and whose moment (with sign) equals the algebraic sum of the moments of the given couples. If the planes of the given couples are not parallel or coincident, then the resultant can be determined from the vectors representing the different couples; thus, add the vectors, that is, find their resultant; this resultant vector represents the resultant couple. A couple can be resolved into component couples thus: resolve the vector of the given couple into component vectors (Art. 16) which are pendicular to the planes of the desired components; these component vectors represent the several component couples.

16. Composition of Forces

The Force Polygon for a system of forces is the figure formed by drawing consecutively lines representing the magnitudes and directions of the forces of a system; the order in which the lines are drawn is immaterial, but the arrowheads on the lines (to indicate the senses of the forces) must be confluent, that is, pointing the same way around. A force polygon, unlike a geometrical one, need not be a closed figure. For a given system, as many different force polygons may be drawn as there are orders of taking the forces; if the number of forces is n, then the number of possible force polygons is $n!$ (see Art. 1). In Fig. 64 there are three different force polygons for the system acting upon the body shown at the left. If a system is not coplanar, then its force polygon is not plane, but is called " gauche."

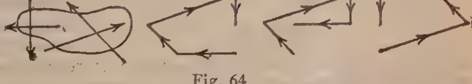

Fig. 64

Concurrent Systems. (1) **Graphic Method:** Draw a force polygon for the system; the magnitude of the resultant is represented by the length of the line joining the ends of the polygon; the sense of the resultant is represented by an arrowhead on that line not confluent with the other arrowheads, and the resultant is concurrent with the given forces. If the forces are noncoplanar, then this method is not practical but it can be used by drawing the force polygon in "plan and elevation." (2) **Algebraic Method:** If the forces F are coplanar, resolve each into components F_x and F_y along axes x and y at right angles to each other; get the algebraic sums of the x and y components, ΣF_x and ΣF_y; then the resultant being called R, and its angle with the x axis α, $R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2$ and $\tan \alpha = (\Sigma F_y)/(\Sigma F_x)$. The approximate direction of the resultant is apparent from the directions of its x and y components, which respectively equal ΣF_x and ΣF_y. If the forces F are noncoplanar, then each force should be resolved into components parallel to three rectangular axes, x, y, and z. $R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2$, $\cos \alpha = (\Sigma F_x)/R$, $\cos \beta = (\Sigma F_y)/R$, $\cos \gamma = (\Sigma F_z)/R$, α, β, and γ being the angles between R and the x, y, and z axes respectively.

Nonconcurrent Coplanar Systems. (1) **Graphic Method:** If the forces are not parallel or not nearly parallel, find the resultant R_1 of any two forces (Triangle Law, Art. 15), then the resultant R_2 of R_1 and the third force, etc., until all the forces have been compounded. If the forces are parallel or nearly so, the method just explained fails because the lines of action of the several resultants cannot be determined readily on account of inaccessible intersections. In this case, each force may be replaced by two components in a certain way, and then the resultant of these components can be found. Thus, to find the resultant of the three forces ab, bc, and cd acting on the body in Fig. 65: draw a force polygon, as $ABCD$, for the given forces; resolve AB into AO and OB; BC into BO and OC, etc.; O having been taken anywhere, all components except the first and last occur in pairs and the forces of each pair are equal and opposite, thus OB and BO, OC and CO, etc.; choose the action lines of these components so that those of any one pair shall be colinear; thus, insert the components of AB at pleasure as at oa and ob, the components of BC so that the component BO shall act in ob, and hence the component OC is oc, etc.; thus the pairs of components consisting of equal, colinear, and opposite forces each balance, leaving only the first and last components AO and OD acting in ao and od; and their resultant (which is also the resultant of the given forces) is AD (magnitude and direction) ad (action line). The point O (Fig. 65) is called the **pole,** lines OA, OB, OC, etc., are **rays**; oa, ob, oc, etc., are **strings**, and all the strings constitute the **string polygon** for the forces. This polygon is also called "funicular," "link," and "equilibrium polygon," the last being especially appropriate when the given forces are in equilibrium. The part of the drawing which represents the body and the lines of action of the forces is the space diagram; that representing force magnitudes, the vector diagram. If the force polygon closes, then the resultant is in general a couple, the forces of the couple acting in the first and last strings of the string polygon, the magnitude of the forces being represented by the corresponding ray. If it happens that the first and last strings coincide, then the string polygon is closed and the resultant vanishes.

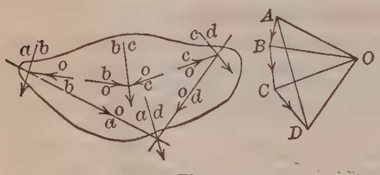

Fig. 65

(2) **Algebraic Method:** When the forces F are parallel, give to the forces in the one direction the same sign, and to the others the opposite; then the resultant R equals ΣF, the sense of R being indicated by the sign of ΣF. The position or line of action of R may be fixed by means of the arm of R, a, with respect to any origin O in the plane of the forces; thus if ΣM denotes the sum of the moments of the given forces with respect to O, then $a = (\Sigma M)/R$, a being measured in such direction from O that the sign of the moment of R will be the same as that of ΣM. If $\Sigma F = 0$, then the resultant of the system is a couple whose moment equals ΣM. When the parallel forces are two in number, P and Q (Fig. 66), then if P and Q act in the same direction, R cuts any line AB internally, and if P and Q are opposite, then externally on the side of the larger force; and in each case the segments of AB are inversely proportional to P and Q, that is, $AC/BC = Q/P$. When the forces are not parallel, compute the algebraic sums of the x and y components of the forces (ΣF_x and ΣF_y) and the algebraic sum of the moments of the forces with respect to any origin O in their plane (ΣM). Then the resultant $R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2$, its angle with the x axis $= \tan^{-1} (\Sigma F_y)/(\Sigma F_x)$ and its arm with respect to O is $a = \Sigma M/R$. The general direction of R is apparent from the directions of its components ΣF_x and ΣF_y; a must be measured in such a direction from O that the sign of the moment of R will be the same as that of ΣM. If $\Sigma F_x = \Sigma F_y = 0$, the resultant is in general a couple whose moment $= \Sigma M$.

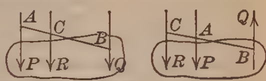

Fig. 66

Nonconcurrent Noncoplanar Systems. The graphic method is generally not advantageous; the algebraic is here given. (1) **Forces Parallel.** Give to the forces F acting in the same direction one sign and to the others the opposite sign; then the resultant $R = \Sigma F$, the sense of R being indicated by the sign of ΣF. Next compute the sums of the moments of the forces with respect to two axes (x and y, say) perpendicular to the forces; call these sums ΣM_x and ΣM_y and the arms of R with respect to those axes respectively a_x and a_y; then $a_x = (\Sigma M_x)/R$ and $a_y = (\Sigma M_y)/R$. The signs in these ratios may be disregarded; a_x and a_y have such positions that the moments of R with respect to the x and y axes have the same signs as those of ΣM_x and ΣM_y respectively. If $\Sigma F = 0$, the resultant in general is a couple which can be determined by finding the resultant of all the forces but one; the resultant and the omitted force constitute the resultant couple.

(2) **Forces Not Parallel.** In general, the resultant is not a single force, but the system can be reduced to a force R acting through any point of the body selected and a couple C; and if desired, R and C can in general be compounded into two nonparallel nonconcurrent forces. To determine R and C: select a set of coordinate axes (x, y, and z) in the body, the origin O being at the selected point referred to; determine the sums of the x, y, and z components of the given forces ($\Sigma F_x, \Sigma F_y$, and ΣF_z) and the algebraic sums of the moments of the forces with respect to the x, y, and z axes (ΣM_x, ΣM_y, and ΣM_z); ΣF_x, ΣF_y, and ΣF_z are the x, y, and z components of R, and ΣM_x, ΣM_y, and ΣM_z are the moments of the components of C perpendicular to the x, y, and z axes respectively.

$R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2$ and $C^2 = (\Sigma M_x)^2 + (\Sigma M_y)^2 + (\Sigma M_z)^2$; if α_1, α_2, and α_3 denote the angles between R and the x, y, and z axes, and θ_1, θ_2, and θ_3 the angles between the vector representing C (Art. 15) and the x, y, and z axes respectively, then

$$\cos \alpha_1 = (\Sigma F_x)/R \qquad \cos \alpha_2 = (\Sigma F_y)/R \qquad \cos \alpha_3 = (\Sigma F_z)/R$$
$$\cos \theta_1 = (\Sigma M_x)/C \qquad \cos \theta_2 = (\Sigma M_y)/C \qquad \cos \theta_3 = (\Sigma M_z)/C$$

The resultant force R and the resultant couple C can be compounded into two forces as follows: take the plane of the couple so that one of the forces of the couple intersects R; find the resultant of this force and R; this resultant and the other force of C are the two forces sought. In general the final two forces are skewed. If the plane of C is parallel to R, then C and R may be compounded into a single force as follows: take forces of the couple so that they are parallel to R; then find the resultant of those forces and R; this is a single force.

17. Principles of Equilibrium

Conditions of Equilibrium. A force exerted on a body (definite portion of matter) by another body is an external force with reference to the first body. A force exerted upon one part of a body by another part of the same body is an internal force. All the external forces applied to a body at rest constitute a system said to be in equilibrium. When a force system is in equilibrium, its resultant is zero; this is the general condition of equilibrium. Detailed conditions for the various kinds of force systems follow, the notation being: F denotes force, F_x, F_y, and F_z, x, y, and z components of F, M denotes moment of F, M_a, M_b and M_c moments of F with respect to points a, b, and c, M_x, M_y, and M_z moments of F with respect to x, y, and z axes respectively. (1) **Colinear System:** $\Sigma F = 0$ or $\Sigma M_a = 0$ (a is not to be taken on the forces). (2) **Coplanar Concurrent System:** $\Sigma F_x = 0$ and $\Sigma F_y = 0$; or $\Sigma F_x = 0$ and $\Sigma M_a = 0$ (the x axis must not be perpendicular to the line joining a and the point of concurrence of the forces); or $\Sigma M_a = 0$ and $\Sigma M_b = 0$ (a, b, and the point of concurrence must not be colinear). For the case of three forces: $F_1 : F_2 : F_3 : : \sin\alpha_1 : \sin\alpha_2 : \sin\alpha_3$; F_1, F_2, and F_3 denote the forces, α_1, α_2, and α_3 the acute angles between F_2 and F_3, F_3 and F_1, and F_1 and F_2 respectively. (3) **Coplanar Parallel System:** $\Sigma F = 0$ and $\Sigma M_a = 0$; or $\Sigma M_a = 0$ and $\Sigma M_b = 0$ (the line joining a and b must not be parallel to the forces). (4) **Coplanar Nonconcurrent Nonparallel System:** $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$; or $\Sigma F_x = 0$, $\Sigma M_a = 0$, and $\Sigma M_b = 0$ (the x axis must not be perpendicular to the line joining a and b); or $\Sigma M_a = 0$, $\Sigma M_b = 0$, and $\Sigma M_c = 0$ (a, b, and c must not be colinear). (5) **Noncoplanar Concurrent System:** $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$. (6) **Noncoplanar Parallel System:** $\Sigma F = 0$, $\Sigma M_x = 0$, and $\Sigma M_y = 0$ (the x and y axes are not parallel to the forces or to each other). (7) **Noncoplanar Nonconcurrent Nonparallel System:** $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$ and $\Sigma M_z = 0$.

Also, based on graphic methods, these conditions of equilibrium: For concurrent systems, the force polygon closes; for coplanar nonconcurrent systems, the force and string polygons close. Special Principles, applicable in either algebraic or graphic analysis. (1) If three forces are in equilibrium, then they are coplanar, and concurrent or parallel. (2) If four coplanar nonconcurrent nonparallel forces are in equilibrium, then the resultant of any two is concurrent with the other two.

Virtual Work. Any imaginary displacement of a body or system of bodies is a virtual displacement. The work done by a force during a virtual displacement of its point of application is called the virtual work of that force. Virtual works are computed according to the definitions and rules for computing real works (see Art. 28). The quantity here called virtual work is also called "virtual moment." (1) If a rigid body is in equilibrium, then for any infinitesimal virtual displacement the algebraic sum of the virtual works of the external forces equals zero. The work of a force for a displacement of its application point at right angles to the force is zero; and so in applying the principle of virtual work to determine a particular force of a system in equilibrium, it is generally advantageous to take a virtual displacement so that the displacements of the application points of as many forces (particularly unknowns but excepting the one in question) as possible shall be at right angles to the corresponding forces. (2) If any system of particles (constituting a rigid body, a deformable body, or a collection of such bodies) is in equilibrium, then for any infinitesimal virtual displacement of the system the algebraic sum of the virtual works of all external and internal forces acting upon it equals zero. Internal forces occur in pairs, and the forces of any pair are equal, colinear, and opposite. Let S denote the magnitude of either force of a pair (regarded as positive if they are pulls and negative if pushes), and let ds denote the change in the distance between the application points of the forces for infinitesimal virtual displacement of the points (regarded as positive or negative according as the distance is increased or decreased); then in such a displacement the work of the pair is $-Sds$. For any infinitesimal displacement of a rigid body ds is zero for all pairs of internal forces, and the work of each pair (and of all pairs) is zero. In applying the principle to a collection of rigid bodies which press against one another or are connected as by hinges or strings, the equation of virtual work must in general include, besides the external forces, those internal forces which the bodies exert upon each other. But the virtual works of these forces may be zero; thus, the virtual works of the pressures at a frictionless contact is zero for any virtual displacement which preserves the contact, and the virtual work of the binding forces of a string is zero for any virtual displacement which leaves the string taut and unchanged in length.

Stability. When a body (or collection of bodies) is in equilibrium and the state is such that if when displaced slightly in any way the body returns of itself to its original position, then the equilibrium is stable; if when displaced slightly the body moves farther from its original position, then the equilibrium is unstable; and if when displaced slightly it remains in that displaced position, the equilibrium is neutral, or indifferent. The body or collection is also said to be stable, unstable, or neutral (or indifferent) respectively. When a body or collection is stable, its potential energy is a minimum; when unstable, a maximum; and when neutral, constant (or stationary); the converse statements also are true. When the potential energy is gravitational, that is, due to weight, then when a body or collection is stable its center of gravity is in a lowest position; when unstable, in a highest position; and when neutral, at a uniform height, that is, it moves in a horizontal plane if the body or collection is slightly displaced. When a body rests on a number of points, the smallest polygon including all the points is called the supporting base or, simply, base. If the resultant of all forces acting on the body, including its own weight but not the supporting forces, cuts the base, the equilibrium is stable, and the moment of the resultant about the side of the base nearest the resultant is a measure of the stability.

Properties of the Equilibrium or String Polygon. (See also Art. 16.) In the following the force-systems are not assumed to be in equilibrium except where so stated. (1) The line of action of the resultant of any number of coplanar forces which are represented consecutively in a force polygon passes through the intersection of the two strings of the equilibrium polygon which are parallel to the two rays embracing those forces in the force polygon. Thus the resultant of AB, BC, and CD (Fig. 67), acting in ab, bc, and cd, acts through the intersection of ao and od;

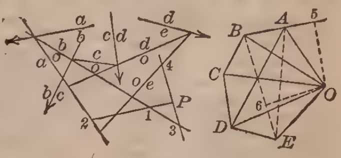

Fig. 67

the resultant of BC, CD, and DE acts through the intersection of ob and oe; etc. (2) If a pole is taken at the beginning of a force polygon for a given force-system, then each string of a corresponding equilibrium polygon is the action line of all the forces from (and including) the first up to that string. Thus, the string od (Fig. 68) is the line of action of the forces AB, BC, and CD, acting in ab, bc, and cd; and ae is the line of action of AB, BC, CD, and DE. (3) If two equilibrium polygons are drawn for a given force-system from the same force polygon but with different poles, then the intersections of corresponding strings will lie on a straight line parallel to that joining the poles. Thus in Fig. 69 there are represented forces AB, BC, and CD, acting in ab, bc, and cd, and two equilibrium polygons are shown corresponding to poles P and Q; corresponding strings intersect in points 1, 2, 3, and 4, all being in a line parallel to PQ. (4) If an equilibrium polygon for a system of forces in equilibrium is regarded as a series of links, jointed at the intersections of the segments of the polygon, by means of which the forces react upon each other, the series would remain at rest under the action of the forces; each link would be under tension or compression, and the ray corresponding to any particular link represents the amount of that tension or compression. In Fig. 70 each link is under compression.

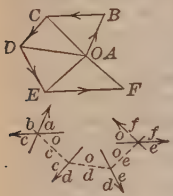

Fig. 68

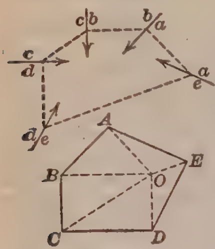

Fig. 69

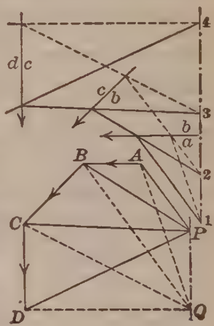

Fig. 70

An equilibrium polygon for a coplanar force-system furnishes a ready means of obtaining the moment of any one of the forces, and of the resultant of any of the forces consecutive in the force polygon. Thus the moment of any force with respect to any origin is the product of its "intercept" and "pole distance"; by intercept of a force is meant the distance (by the space scale) intercepted by the strings corresponding to the force from the line drawn parallel to the force through the origin of moments, and the pole distance of a force is the perpendicular distance (by the force scale) from the pole to the line representing the force in the force polygon. Thus the moment of AB (Fig. 67) acting in ab, about P, is $\overline{12} \times \overline{O5}$, and the moment of the resultant of BC, CD, and DE about P is $\overline{34} \times \overline{O6}$. Both moments are counter-clockwise, determined from the lines of action and senses of forces as related to the origin of moments.

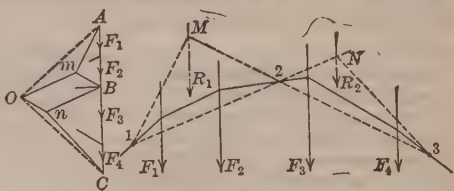

Fig. 71

An equilibrium polygon for a coplanar system of forces can be drawn so as to pass through any three points of the plane of the forces. Thus, to draw one through points 1, 2, and 3 for the forces F_1, F_2, F_3, F_4 (Fig. 71): find the resultants R_1 and R_2 of the forces whose lines of action are cut by the lines joining 1 and 2, and 2 and 3; the first acts in R_1 and the second in R_2 (construction for position not shown), and their values are represented by AB and BC respectively in the force polygon for the forces.

Extend lines R_1 and $\overline{32}$ to their intersection M, and lines R_2 and $\overline{12}$ to their intersection N. From A and B draw lines parallel to $\overline{M1}$ and $\overline{M2}$ to their intersection m, and from B and C draw lines parallel to $\overline{N2}$ and $\overline{N3}$ to their intersection n. Complete the parallelogram $VmnO$; then O is a pole and the corresponding equilibrium polygon, if started through one of the three specified points, will pass through the other two. (It is advisable to draw as first string the one through point 2, parallel to OB.)

18. Typical Problems

In each of the problems following, some of the forces of the system in equilibrium are unknown in sense. In writing an equilibrium equation for the system, senses may be assumed when unknown; if the computed value of a force comes out positive, the sense was guessed correctly; if negative, then incorrectly. In the figure, incorrectly assumed senses are indicated by a short line through the arrowhead.

A Coplanar Concurrent Force-System is in Equilibrium and the forces are all known except two whose action lines only are known; these two are to be determined completely. This is a common problem in the determination of the stresses of a roof or bridge truss, and the numerical illustration is from a truss, but the method of solution is as general as the statement of the problem.

(1) Algebraic Solution: Three sets of equilibrium equations are available (see Art. 17). No general rule covering all cases can be laid down as to which set is best in a particular case, but if a resolution equation ($\Sigma Fx = 0$ or $\Sigma Fy = 0$) is taken first, it is advantageous to take the resolution axis perpendicular to one of the unknown forces. If a moment equation ($\Sigma M = 0$) is taken first, it is advantageous to take the moment origin on the action line of one of the unknowns. Fig. 72 represents a joint of a truss under the action of a load of 1600 lb., a known pull of a member, 2000 lb., and two unknown forces F_1 and F_2; to find these two: Choosing $\Sigma Fx = 0$ and $\Sigma Fy = 0$, with the x axis horizontal, $\Sigma Fy = -1600 + F_1 \sin 30° = 0$, or $F_1 = +3200$ lb., the positive sign indicating that F_1 acts as assumed in the figure. Next $\Sigma Fx = -2000 + 3200 \cos 30° + F_2 = 0$, or $F_2 = -771$ lb., the negative sign indicating that F_2 acts toward the left. Or, beginning with $\Sigma M = 0$, the origin being on F_2, 10 ft. to the right of O, say, then $\Sigma M = -1600 \times 10 + F_1 \times 10 \sin 30° = 0$, or $F_1 = 3200$; F_2 may now be determined as before or from another moment equation with origin anywhere except on F_2. When there are only three forces in the system, then a special condition of equilibrium (Art. 17) may be applied thus: suppose that the three forces are 1600 lb., F_1, and F_2 (Fig. 72); $F_1/\sin 90° = F_2/\sin 60° = 1600/\sin 30°$, which equations furnish values of F_1 and F_2.

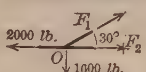

Fig. 72

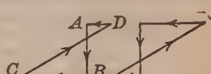

Fig. 73

(2) Graphic Solution: The condition of equilibrium is that the force polygon for the force-system must close; constructing the polygon and making it close will determine the unknown forces. The order in which the forces are represented in the polygon is immaterial, but the knowns must be drawn first of course. To construct a force polygon for the forces: draw AB (Fig. 73) to represent the 1600-lb. force, BC to represent the 2000-lb. force; then from A and C lines parallel to the other two forces; the intersection of these two lines is D, and CD and DA represent the values and directions of the two unknowns. The unlettered polygon in Fig. 73 is another possible force polygon, giving the same results as the one explained.

A Coplanar Parallel Force-System is in Equilibrium and all the forces are known except two whose action lines only are known; these two are to be determined completely. The determination of the reactions on a beam or truss on horizontal supports and under vertical loads is a problem of this sort. Such a beam is used as an illustration, but the method of solution is as general as the statement of the problem.

(1) Algebraic Solution: Either one of two sets of equilibrium equations is available (Art. 17). It will be well to use the two moment equations with origins on the action lines of the two unknown forces. Then after the unknowns have been determined one might test, as a check, whether ΣF is zero. Fig. 74 represents a beam supported at R_1

and R_2, the beam bearing a concentrated load at the left end, a uniform load as shown, and its own weight, 1800 lb.; to determine the reactions. With origin at R_2, the moment equation is $-10\,000 \times 18 - 40\,000 \times 6 - 1800 \times 9 + R_1 \times 10 = 0$, or $R_1 = 43\,620$ lb.; with origin at R_1, it is $-10\,000 \times 8 + 40\,000 \times 4 + 1800 \times 1 - R_2 \times 10 = 0$, or $R_2 = 8180$ lb. As algebraic sum of loads and reactions is zero, the computation checks.

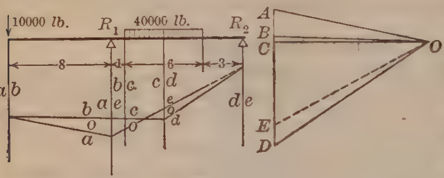

Fig. 74

(2) Graphic Solution. The conditions of equilibrium are that the force and string polygons close; if these are constructed and made to close, the problem will have been solved. Using the preceding illustration again, the 40 000-lb. load is regarded as concentrated at its center; and also the weight of the beam at the middle of the beam; first $ABCD$, the polygon for the known forces, is laid off; the right reaction will be taken next and called DE and the other EA, but E is as yet unknown; next a pole O is chosen, rays are drawn and then the strings or oa, ob, oc, and od; od should be extended to de, and oa to ea. The line oe is the closing string, and a ray parallel to it fixes the point E. Then DE and EA represent the magnitudes of the reactions.

A Coplanar Nonconcurrent Nonparallel Force-System is in Equilibrium, and all the forces are known except two; the action line of one of these and a point in that of the other are known; required to determine these unknowns completely. This problem occurs in the determination of the reactions on a roof truss sustaining wind pressures, the truss being fixed at one end and resting on rollers at the other. This case is used in illustrations below, but the solutions are as general as the statement of this problem.

(1) Algebraic Solution: Calling the first described unknown P and the second Q, imagine Q replaced by two rectangular components Q_x and Q_y acting at the given point of Q; then the unknowns of the system are the magnitudes and senses of P, Q_x, and Q_y. Any one of three sets of equilibrium equations may be used (Art. 17); generally it is advantageous to begin with a moment equation, the origin being at the known point of Q, as such an equation will furnish P directly. Q_x and Q_y can be determined from the other two equations of the set selected, and then Q itself from its components. Fig. 75 represents a roof truss whose span is 60 ft., rise 12 ft., resting on rollers at the left end and pinned to the support at the right; there are four loads as shown; required the reactions. The reaction at the roller end can be vertical only; that at the other end may have any direction. The first is P and the second Q, but Q is represented in the figure by its two unknown components Q_x and Q_y. The moment equation for the system with origin at 1 is $P \times 60 - 1500 \times 55.71 - 3000 \times 44.94 - 3000 \times 34.17 - 1500 \times 23.40 = 0$, or $P = 5933$. Then from $\Sigma M_2 = 0$, or $\Sigma F_y = 0$, Q_y may be found to be 2423, and from $\Sigma F_x = 0$, or $\Sigma M_3 = 0$, Q_x may be found to be 3342.

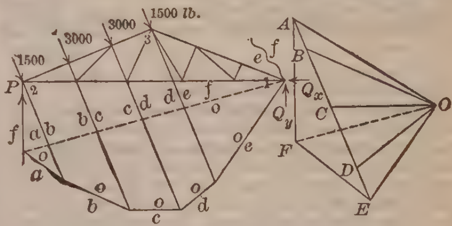

Fig. 75

(2) Graphic Solution: The conditions of equilibrium are that the force and string polygons must close; constructing them and making them close will determine the unknowns. In order to make the construction possible, the first string drawn must be one corresponding to the unknown force Q, one point of which is known, and it must be drawn through that point. The following special graphic solution is simpler in principle: First determine the resultant R of the known forces, and imagine the knowns replaced

by that resultant: then the system consists of three forces, namely that resultant and the two unknowns. If R and the unknown P whose action line is known are not parallel, then the three forces R, P, and Q are concurrent and the action line of Q is determined. The solution of the three-force system can then be made as explained in the first paragraph. If R and P are parallel, then Q is also parallel to P and R; P and Q can be determined most readily algebraically, and graphically by constructing the force and funicular polygons for the three forces. (When R, P, and Q are parallel, this special graphical method is no simpler than the general method first described.) As illustration of the general method the reactions on the truss shown in Fig. 75 are determined thus: The force polygon for the known forces is $ABCDE$; calling the reaction at the fixed end *ef* and the other *fa*, the first string drawn is *oe*; then *od*, *oc*, *ob*, *oa*, and *of* the closing string. Next the ray parallel to *of* is drawn and its intersection with AF determines F; EF and FA represent the magnitudes and directions of the reaction at the fixed and roller ends respectively.

A coplanar nonconcurrent nonparallel force-system is in equilibrium and all the forces are known except three whose action lines only are known. Required to determine these three completely. (This problem is indeterminate if the three unknowns are concurrent or parallel.)

(1) Algebraic Solution: Any one of three sets of equilibrium equations may be used (Art. 15). In general it is advantageous to use a moment equation first, the origin being at the intersection of two of the unknowns; the choice of the other two equations will depend on the particular problem under consideration. For example, consider the overhanging truss (Fig. 76) which sustains three loads as shown and is supported at 1 so that the reaction there acts along the line marked R_1 and at 2 by two forces R_2 and R_3 which are horizontal and vertical respectively. Required R_1, R_2, and R_3. $\Sigma M_1 = -800 \times 20 - 1500 \times 10 + R_2 \times 10 = 0$ or $R_2 = 3100$ lb.; $\Sigma M_3 = 1500 \times 10 + 1000 \times 20 - R_3 \times 20 = 0$, or $R_3 = 1750$ lb.; and $\Sigma F_x = R_1 \cos a - 3100 = 0$, or $R_1 = 3466$ lb.

(2) Graphic Solution: The conditions of equilibrium are that the force and equilibrium polygons must close. In order to construct the equilibrium polygon, the first string must be drawn through the intersection of two of the unknowns. Using the preceding illustration the force polygon for the three known forces is $ABCD$ (Fig. 76). The reaction at 1 is called *de* and the reactions at 2 are called *ef* and *fa*. The first string is *oa* and it is drawn through the intersection of *fa* and *ef*; then *ob*, *oc*, and *od* are drawn, the latter to its intersection with *de*. The closing string is *oe*, and a ray parallel to *oe* determines E, in the line through D parallel to *de*. Then completing the force polygon by line through A and E parallel to R_2 and R_3 respectively determines F, and $FA = R_2$ and $EF = R_3$. The senses of R_1, R_2 and R_3, apparent in this example are given by arrow heads on DE, EF and FA confluent with the senses of AB, BC and CD.

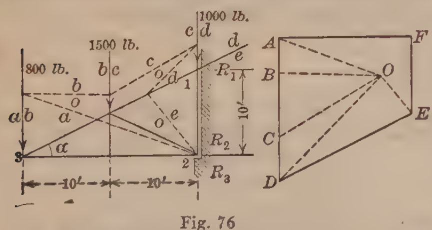

Fig. 76

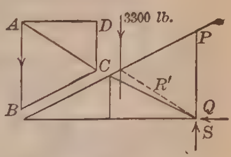

Fig. 77

The following special method is simpler than the foregoing: Determine the resultant R of the known forces, and imagine them replaced by their resultant; then the system consists of R and the three unknowns P, Q, and S. Note that R, P, and the resultant R' of Q and S are concurrent, and by a force triangle determine P and R'; finally resolve R' into its components Q and S. For example, in the preceding illustration, the resultant of the known forces is 3300 lb. as shown in Fig. 77. The resultant R' of Q and S is concurrent with R and P, and acts in the dotted line. The force triangle for R, P, and R' is $ABCA$, AB representing 3300 lb., BC representing P, and CA representing R'. Then resolving CA into Q and S, it is found that CD represents S and DA represents Q.

19. Shears and Moments in Beams

Beams and Trusses are usually subjected to vertical loads and reactions. In such cases, the **vertical shear** at any cross-section of the beam or truss is the algebraic sum of all the loads and reactions on either side of the section; if the shear is computed from the forces (loads and reactions) to the left of the section, then upward forces are given the positive sign, but if from those on the right, then the upward forces are taken as negative. The **bending moment** at any cross-section of a beam or truss is the algebraic sum of the moments of all the loads and reactions on either side of the section, the origin of moments being taken in the section; if the bending moment is computed from the forces to the left of the section, clockwise moments are regarded as positive, but if from those to the right, clockwise moments are taken as negative. V and M are used to denote shear and moment respectively.

Fig. 78*a* represents a cantilever sustaining a concentrated load of 1000 lb. at the free end and uniform load of 2000 lb. on half its length as shown. Fig. 78*b* is a shear diagram for the cantilever as loaded, showing how the external shear varies from section to section; at a section just to the right of the concentrated load $V = -1000$, at the wall $V = -3000$ lb. Fig. 78*c* is a moment diagram for the cantilever as loaded, showing how the bending moment varies from section to section; at the middle $M = 5000$ and at the wall $M = 15\,000$ ft-lb.

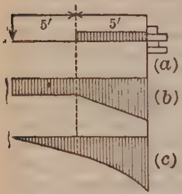

Fig. 78

Fig. 79*a* represents a beam resting on two supports A and B and bearing a uniform load of 1000 lb. per ft. between the supports, and a concentrated load of 2500 lb. at the right end; the reactions at A and B are respectively 4000 and 8500 lb. Fig. 79*b* is a shear diagram for the beam so loaded; just to the right of A, $V = +4000$ lb., just to the left of B, $V = -6000$ lb., and at any section to the right of B, $V = +2500$ lb. Fig. 79*c* is a moment diagram for the beam as loaded; at B, $M = -10\,000$ ft-lb. and the greatest positive value of M is 8000 ft-lb., at the section 4 ft. from the left end.

The equilibrium polygon is a moment diagram for the beam as loaded, the vertical ordinates in it being proportional to the bending moments at the corresponding sections. The bending moment at any particular section is the product of the corresponding ordinate in the equilibrium polygon (according to the scale of the drawing of the beam) and the distance from the pole to the " load line " (according to the scale of the force polygon).

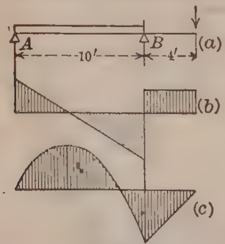

Fig. 79

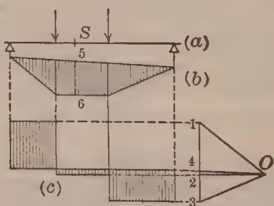

Fig. 80

For example, in Fig. 80, the beam represented is supported at each end and sustains two loads of values 12 and 23; the perimeter of the upper shaded part is an equilibrium polygon and constitutes a moment diagram for the beam as loaded. The bending moment at any section as S equals the product of the distance represented by the ordinate 5-6 and the force represented by the perpendicular from O to the line 1-2-3. The shear diagram (Fig. 80*c*) can be constructed from the force polygon by obvious means.

For a beam bearing a distributed load an approximate moment diagram may be constructed thus: draw an equilibrium polygon for the beam under an approximately equivalent series of concentrated loads obtained by imagining the uniform load divided into parts and each part replaced by a concentrated load equal to that part and applied at its center. See Fig. 81, in which the distributed load is divided into three parts. The true bending moment line is curved below the distributed load, and the curve is tangent to the equilibrium polygon at points immediately below the lines of division of the load.

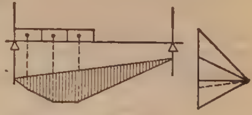

Fig. 81

Properties of Shear and Moment Diagrams. For any part of a beam bearing no load and weight of beam being disregarded, the shear line is horizontal and the moment line is generally inclined; for any part bearing a uniformly distributed load, the shear line is inclined downward to the right, and the moment line is a parabola, convex upward, with axis vertical. The shears on either side of a concentrated load differ by an amount equal to the load, and the moment line changes direction there suddenly. At each end of a beam the shear and moment are zero. Where the shear changes sign, there the moment has a maximum or minimum value.

20. Simple Frameworks

A Truss is a framework intended to carry loads, while each member of the truss is subjected only to longitudinal stress, either tensile or compressive. In this article it is assumed, except as otherwise noted, that (a) the truss under consideration is pin-jointed. that is, the members have " eyes " and are pinned together at the joints, (b) each member is continuous between two joints only, and (c) the loads and reactions are applied to the truss at the joints. When these assumptions are fulfilled the forces acting upon any member, consisting of loads, reactions or pin pressures, are applied at its ends only; and the resultants, R_1 and R_2, of the forces at each end act through both ends, that is, R_1 and R_2 are colinear, and they are equal and opposite. When they are pulls, the member is in tension, and any two parts exert pulls upon each other (Fig. 82); when they are pushes, the member is in compression, and any two parts exert pushes upon each other (Fig. 82). These internal pulls and pushes are each equal to the end pulls or pushes and colinear with them. By force or stress in a member is meant either of the forces which either of the two parts of the member exerts on the other part; the magnitude of a stress is the magnitude of one of the two forces referred to. By analysis of a truss for certain loads is meant the determination of the stresses in its members due to these loads.

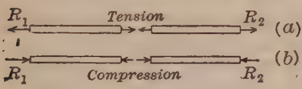

Fig. 82

The assumptions stated above are not realized in all actual trusses; yet if either (a) or (b) is not realized but the joints are properly made, then the methods of analysis here explained, or their equivalent, are used and without practical error; if (c) is not realized, the methods here given require amplification only.

To Determine the Stress in any particular member of a truss due to certain loads: First, determine the reactions on the truss due to the loads; second, imagine the truss separated into two distinct parts (that is, pass a section through the truss) so that the member under consideration is one of the members cut and so that the system of forces, including stresses, acting on either part of the truss is solvable for the desired stress; third, solve the system. (For plane trusses, the system will be coplanar and concurrent, or nonconcurrent; the first kind can be solved completely if it includes not more than two unknown stresses, and the second if not more than three except when these three are concurrent or parallel.)

To illustrate how to pass the section, suppose the stress in HI (Fig. 83) is required, the truss being supported at its ends and bearing five loads L and one P, and suppose the reactions determined. Trying section 1-1, the force system on the left part of the truss (Fig. 83b) is a nonconcurrent one of seven forces, and includes four unknown stresses, S_1, S_2, S_3, and S_4; it is not solvable for the desired stress S_1. Trying section

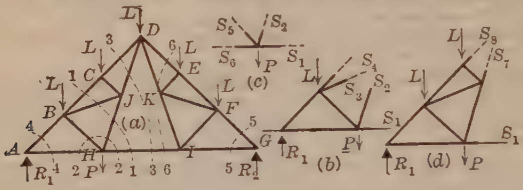

Fig. 83

2-2, the force system on the lower part (Fig. 83c) is a concurrent one, and includes four unknown stresses, S_1, S_2, S_5, and S_6; it is not solvable. Trying section 3-3, the force system on the left part (Fig. 83d) is nonconcurrent with three unknown stresses, S_1, S_7, and S_8; it is solvable. In some instances different sections may be used, each leading to a solution.

An Algebraic Analysis of a Truss is carried out by solving the various force-systems resulting from sections passed, as explained in the foregoing, by algebraic methods. The problems to be solved are generally like one of Art. 18, where methods for their solution are given. In the following the unknown stresses will be assumed to be pulls always; then positive computed stresses will be tension and negative ones will be compression. In a truss with horizontal chords, the stress in any diagonal equals the vertical shear in the panel in which the member is, multiplied by the secant of the angle which the member makes with the vertical. The stress in either chord member of any panel equals the bending moment at the point where the other chord member and the stressed web member of that panel intersect, divided by the height of the truss.

(1) **Roof Truss.** In Fig. 83 angles A and G are 45°, $AH = HI = IG =$ 16 ft.; AB, BC, CD, DE, EF, and FG are equal, also HJ, JD, IK, and KD. Loads L = 800 lb., load P = 1200 lb. The reaction R_1 is found thus: $R_1 \times 48 - 4000 \times 24 - 1200 \times 32 = 0$, or $R_1 = 2800$ lb.; also $R_2 = 2400$ lb.

To determine the stresses: Passing section 4-4, the force-system on the part within the section (Fig. 84a) is concurrent with two unknowns S_1 and S_2. Taking a vertical y axis, $\Sigma Fy = S_2 \sin 45° + 2800 = 0$, or $S_2 = -3960$ lb., the negative sign indicating that S_2 is compressive; next with the corrected direction of S_2, $\Sigma Fx = +S_1 - 3960 \cos 45° = 0$, or $S_1 = +2800$ lb., the positive sign indicating that S_1 is tensile. In a similar manner the stresses in GF and GI might be determined; they are respectively 3394 lb. compression and 2400 lb. tension. No other section than 4-4 or 5-5 can be passed so that the force-system acting on either part of the truss will be concurrent including only two unknown stresses; in fact the only sections leading to solvable force-systems are 3-3 and 6-6. On the left of 3-3 (Fig. 84b) the force-system is nonconcurrent with three unknowns S_3, S_4, and S_5. To determine S_3: $\Sigma M_D = -2800 \times 24 + 1200 \times 8 + 800 \times 8 + 800 \times 16 + S_3 \times 24 = 0$, or $S_3 = +1600$ lb. tension. One might now solve the sytem for S_4 and S_5 or pass a section around joint H or I (Fig. 83), each furnishing a solvable concurrent system. But continuing with Fig. 84b, $\Sigma M_H = -2800 \times 16 + 800 \times 8 - S_5 \times BH = 0$, or $S_5 = -3394$ lb. compression; and $\Sigma M_A = 0$ gives $S_4 = +2530$ lb. tension. Next a section may be passed about H or I as stated, or about C; section about H gives a concurrent system (Fig. 84c) with two unknowns, S_6 and S_7. Taking an x axis at right angles to S_7, $\Sigma Fx = -1200$

$\cos 45° - 2800 \cos 45° + 1600 \cos 45° + S_6 \cos (71° \ 34' - 45°) = 0$, or $S_6 = +1897$ lb. tension; and taking a vertical y axis, $\Sigma F_y = -1200 + S_7 \cos 45° + 1897 \sin 71° \ 34' = 0$, or $S_7 = -847$ lb. compression. Next, passing a section about C, the force-system is concurrent (Fig. 84d) with two unknowns S_8 and S_9. Two resolution equations, the axes being taken along the unknown stresses, are simple; they give $S_8 = -556$ lb. compression, and $S_9 = -3960$ lb. compression. Next passing a section about B gives a concurrent system with one unknown (Fig. 84e) S_{10}. Or, one might pass next a section about J and get a solvable system (Fig. 84f). In similar manner the stresses in the members of the right half of the truss may be determined. When, in the analysis, a force system is reached in which there are fewer unknown stresses than the number of conditions of equilibrium for the system, as in Figs. 83e and f, then a partial check on the preceding computations may be made, thus: determine the unknown stress or stresses and then test whether the force-system satisfies the superfluous or extra equation or equations of equilibrium. Thus for Fig. 84e with the x axis along the two equal stresses, $\Sigma F_x = S_{10} \times \cos 26° \ 34' - 800 \cos 45° = 0$, or $S_{10} = +633$ lb. tension, and $\Sigma F_y = 847 - 800 \sin 45° - 633 \sin 26° \ 34' = -2$, or nearly zero, and so the check is satisfactory.

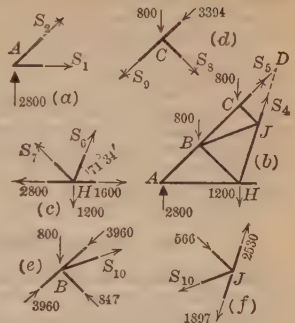

Fig. 84

(2) **Howe Bridge Truss.** The truss represented in Fig. 85 is generally constructed all of wood except the "verticals." The "diagonals" are not connected to other members at the various joints but simply butt up against bearing blocks at their ends, and so can be subjected only to compression. Two diagonals in any panel cannot be stressed at the same time; when the loading is symmetrical with respect to the middle of the truss, those diagonals represented are the ones stressed and they are the main diagonals. The others, not shown, may be stressed only when the truss is partially loaded, and they are counter diagonals, or braces, or simply counters.

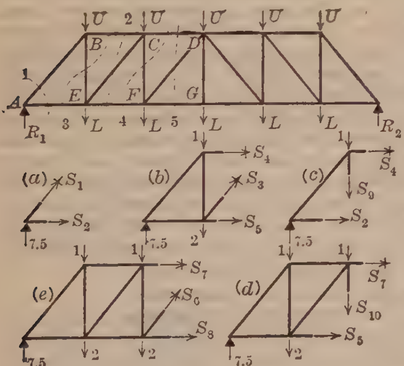

Fig. 85

In the following example the truss is supposed to be loaded symmetrically (see Fig. 85) and the counters are not mentioned. The span is 72 ft., height of truss 15 ft., each upper load U is 1 ton and each lower load L is 2 tons; then each reaction is 7·5 tons. The stress in any diagonal may be found by passing a vertical section through the truss so as to cut the member under consideration and then solving the force system acting on either part of the truss. Thus, to determine the stress in AB, pass the section 1, and solve the system shown in Fig. 85a for S_1; to determine the stress in EC, pass section 2 and solve the system shown in Fig. 85b for S_3; similarly determine the stress in FD, etc. It will be noticed that the vertical component of the stress in any diagonal equals the external shear at the section cutting the diagonal; thus (see Fig. 85b) $S_3 \times \cos \alpha = 7.5 - 2 - 1 = 4.5$. The stress in any vertical, except the middle one if there is one, may be found by passing a section so as to cut only that member and adjacent chord members and then solving the system of forces acting on either part of the truss for the desired stress. Thus, to determine the stress in *BE*, pass section 3, and from the system shown in Fig. 85*c* get $S_9 = 7.5 - 1 = 6.5$ tons; to determine the stress in *CF*, pass section 4, and from the system shown in Fig. 85*d* get $S_{10} = 7.5 - 2 - 1 - 1 = 3.5$ tons. The stress in the middle vertical equals the load at its lower end, 2 tons. The stress in any chord member may be found by passing a section cutting that member and the others in the same panel, and then solving the force-system acting on either part of the truss for the desired stress, the solution is easily made from a moment equation, the origin being taken at the intersection of the other two members cut. Thus to determine the stress in *CD*, pass section 5; the moment equation for the system shown in Fig. 85*e* with origin at the intersection of S_6 and S_8 is $7.5 \times 24 + 3 \times 12 - S_7 \times 15 = 0$, or $S_7 = 9.60$ tons.

21. Stress Diagrams for Trusses

Graphic Methods for Analyzing Trusses are especially well adapted for solving problems like the preceding. As in the algebraic method, the truss is imagined separated into two parts and then the attention is directed to the forces acting upon either part. Graphic instead of algebraic conditions of equilibrium are then applied to the system of forces to determine the unknowns. In making the imaginary separations of the truss, care should be taken to cut not more than three members in which the forces are unknown. It is advantageous to make the separation so that not more than two such members are cut. If that is done, a single force polygon will determine the two unknowns, while if three are cut, a force polygon and an equilibrium polygon, or the equivalent, are necessary for determining the three unknowns. In drawing the force polygon, it will be advantageous to represent the forces in the order in which they occur about the joint. A force polygon so drawn will be called a polygon for the joint; and for brevity, if the order taken is clockwise, the polygon will be called a clockwise polygon, and if counter-clockwise, it is called a counter-clockwise polygon. If the polygons for all the joints of a truss are drawn separately, then the stress in each member will have been represented twice. It is possible to combine the polygons so that it will not be necessary to represent the stress in any member more than once, thus reducing the number of lines to be drawn. Such a combination of force polygons is called a stress diagram. Each triangular space in the truss diagram is marked by a small letter, also the space between consecutive action lines of the loads and reactions. Then the two letters on opposite sides of any line serve to designate that line, and the same large letters are used to designate the magnitude of the corresponding force.

To construct a stress diagram for a truss under given loads:

(1) Determine the reactions. (2) Letter the truss diagram as directed. (3) Construct a force polygon for all the external forces applied to the truss (loads and reactions), representing them in the order in which their application points occur about the truss, clockwise or counter-clockwise. (4) On the sides of that polygon construct the polygons for all the joints. They must be clockwise or counter-clockwise according as the polygon for the loads and reactions was drawn clockwise or counter-clockwise. (The first polygon drawn must be for a joint at which but two members are fastened; the joints at the supports are usually such. Next, that joint is considered, and its polygon is drawn, at which not more than two stresses are unknown.)

(1) **Roof Truss.** Fig. 86 represents a truss sustaining loads of 600, 1000, 1200 and 1800 lb.; the right reaction is 2100 lb. and the left 2500 lb. *ABCDEFA* is a polygon for the loads and reactions, these being represented in the order in which their points of application occur about the truss. The polygon for joint 1 is *FABGF*; the force *BG* acts

toward the joint, hence bg is under compression, and GF acts away from the joint, hence gf is in tension. The polygon for joint 2 is $CDEHC$; the force EH acts away from the joint, hence eh is in tension; and HC acts toward the joint, hence hc is in compression. The polygon for joint 3 is $HEFGH$; the force GH acts away from the joint and hence gh is in tension. If the work has been correctly done, GH is parallel to gh. (In Fig. 86a the polygons are all clockwise, and in Fig. 86b counter-clockwise.)

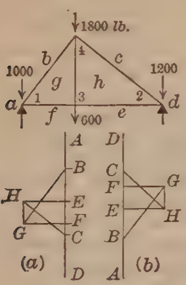

Fig. 86

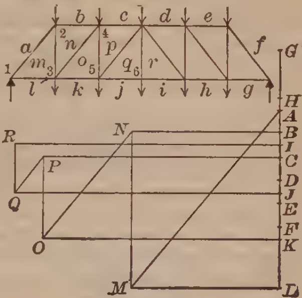
Fig. 87

(2) **Howe Truss.** Fig. 87 represents a Howe Truss, 72 ft. span and 15 ft. high, under five 1-ton loads on the upper chord and five 2-ton loads on the lower. $ABCDEFGHIJKLA$ is a force polygon for all the loads and reactions. Polygon for joint 1 is $LAML$; for joint 2, $MABNM$; for joint 3, $KLMNOK$; for joint 4, $ONBCPO$; for joint 5, $JKOPQJ$; and for joint 6, $IJQRI$.

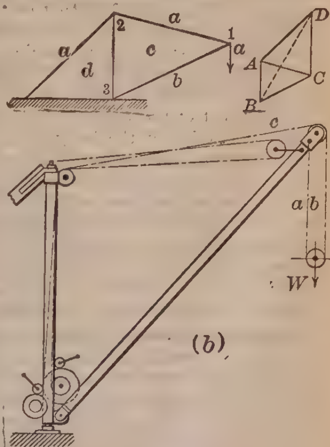

Fig. 88

(3) **Derrick.** Fig. 88a represents a stiff-leg derrick, only one stiff leg, da, shown, and the boom and shown stiff leg in the same vertical plane. For the analysis it is not necessary to determine the reactions first. The polygon for joint 1 may be drawn first; it is $ABCA$, and BC and CA represent compression and tension respectively. The polygon for joint 2 may be drawn next; it is $ACDA$, and CD and DA represent compression and tension respectively. The reaction at 3 equals the resultant of CB and DC, that is DB.

The foregoing analysis is imperfect because it assumes, in part, a single stay along ac, whereas such derricks usually have a multiple stay and a hoisting cable as shown in Fig. 88b. To determine the reaction P_1 at the top of the mast exerted by the stiff leg and that P_2 at its base: P_1 acts nearly along the axis of the stiff leg, and P_2 in a direction unknown as yet; these forces along with the weights of the mast, boom, and load, constitute a system in equilibrium, and it may be solved for P_1 and P_2 as explained in Art. 18. (If hoisting is accomplished not by a winze mounted on the derrick, as assumed, but by an engine, then there must be included in the system described the pull of the engine.) To determine the reaction Q_1 at the base of the boom and the pull Q_2 of the top stay: Q_2 acts along the stay and Q_1 in a direction unknown as yet; these forces along with the weight of the boom, a pull of $1/2W$ in line a, and a pressure at the top pulley pin identical with the resultant of the pulls $1/2W$ in the lines b and c, constitute a system in equilibrium which may be solved for Q_1 and Q_2. (Art. 18.)

GRAVITY AND INERTIA FUNCTIONS

22. Principles of Center of Gravity

Definitions. The resultant of the weights, or parallel forces of gravity, upon all the particles of a body, always passes through a certain particle or point fixed with reference to the body, no matter how the body is turned about; this particle or point is the **center of gravity** of the body. If the body can be supported at its center of gravity, then so supported it would remain at rest in any position. Center of gravity is also called center of mass and center of inertia, especially in discussions relating to the motion of the body. The center of gravity of a line (or length), surface (or area), solid (or volume) is the center of gravity of the line, surface, or solid imagined materialized, that is, conceived as a very slender wire, thin plate, or homogeneous body. Centroid is also used in place of center of gravity as applied to lines, surfaces, and solids. The center of gravity of a line, surface, solid or homogeneous body is a mean point; that is, the distance of the center of gravity from any reference plane is the mean of the distances of all the equal elementary parts of the line, surface, solid, or body from the plane. If the reference plane cuts the line, surface, solid, or body, distances on opposite sides of the plane must be regarded as opposite in sign. **Symmetry:** Two points are symmetrical with respect to a third point if the line adjoining the two is bisected by the third. Two points are symmetrical with respect to a line or a plane if the line joining them is perpendicular to the given line or plane and is bisected by it. A body, line, surface, or volume is symmetrical with respect to a point, a line, or a plane if all the points of the body, line, surface, or volume can be paired off so that each pair is symmetrical with respect to the point, line, or plane. If a line, surface, solid or homogeneous body is symmetrical with respect to a point, line, or plane, then its center of gravity is at the point in the line, or plane. The **statical moment** of a body (or weight), line (or length), surface (or area), or solid (or volume) with respect to any plane is the product of the weight, length, area, or volume and the distance of the center of gravity of the body, line, surface, or solid from the plane. The statical moment of a plane line (or length) or plane surface (or area) with respect to a straight line in the plane is the product of the length or area and the distance of the center of gravity of the line or surface from the reference line. A statical moment is regarded as positive or negative according as the corresponding center of gravity is on the positive or negative side of the reference plane or line. In the foregoing definitions and in the following, the words body, line, surface, and solid are used broadly to include what would ordinarily be described as a collection of bodies, lines, surfaces, or solids.

Methods for Locating Center of Gravity. The statical moment of a body, line, surface, or solid equals the algebraic sum of the statical moments of all the parts into which it is, or is imagined to be, divided. This principle is the basis of all formulas for locating centers of gravity. If the sum of the statical moments is zero, then the center of gravity is in the reference plane or line as the case may be. The position of a center of gravity is generally conveniently specified by its rectangular coordinates $\bar{x}$, $\bar{y}$, and $\bar{z}$.

(1) The formulas for the coordinates of the center of gravity of a body are:

$$W\bar{x} = \int x\,dW \qquad W\bar{y} = \int y\,dW \qquad W\bar{z} = \int \bar{z}\,dW,$$

in which W denotes the weight of the body, dW the weight of any elementary portion, and x, y, and z the coordinates of the center of gravity of that element; the limits of integration must be assigned so that the integration includes all elementary parts of the body. The formulas furnish values of $\bar{x}$, $\bar{y}$, and $\bar{z}$ in any case if the body is mathematically regular and such that the integrations can be performed. The following are corresponding formulas for the coordinates of the center of gravity of lines, surfaces, and solids:

$$L\bar{x} = \int x\,dL \qquad L\bar{y} = \int y\,dL \qquad L\bar{z} = \int z\,dL$$
$$A\bar{x} = \int x\,dA \qquad A\bar{y} = \int y\,dA \qquad A\bar{z} = \int z\,dA$$
$$V\bar{x} = \int x\,dV \qquad V\bar{y} = \int y\,dV \qquad V\bar{z} = \int z\,dV$$

in which L, A, and V denote length, area, and volume respectively.

(2) If a body consists of finite parts whose weights and centers of gravity are known, then the coordinates of the center of gravity of the body can be computed, without integration, from

$$W\bar{x} = W_1\bar{x}_1 + W_2\bar{x}_2 + \cdots \qquad W\bar{y} = W_1\bar{y}_1 + W_2\bar{y}_2 + \cdots$$
$$W\bar{z} = W_1\bar{z}_1 + W_2\bar{z}_2 + \cdots$$

in which W denotes the weight of the body, W_1, W_2, etc., the weights of its parts; $\bar{x}_1$, $\bar{y}_1$, z_1, the coordinates of the center of gravity of W_1; $\bar{x}_2$, $\bar{y}_2$, $\bar{z}_2$, those of the center of gravity of W_2; etc. The following are corresponding formulas for lines, surfaces, and solids:

$$L\bar{x} = L_1\bar{x}_1 + L_2\bar{x}_2 + \cdots \qquad L\bar{y} = L_1\bar{y}_1 + L_2\bar{y}_2 + \cdots \qquad L\bar{z} = L_1z_1 + L_2\bar{z}_2 + \cdots$$
$$A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2 + \cdots \qquad A\bar{y} = A_1\bar{y}_1 + A_2\bar{y}_2 + \cdots \qquad A\bar{z} = A_1\bar{z}_1 + A_2\bar{z}_2 + \cdots$$
$$V\bar{x} = V_1\bar{x}_1 + V_2\bar{x}_2 + \cdots \qquad V\bar{y} = V_1\bar{y}_1 + V_2\bar{y}_2 + \cdots \qquad V\bar{z} = V_1z_1 + V_2\bar{z}_2 + \cdots$$

in which the L's, A's, and V's denote lengths, areas, and volumes.

The center of gravity of two bodies, lines, surfaces, or solids is on the straight line joining the centers of gravity of the two, and the center of gravity of the two divides the joining line into segments inversely proportional to their weights, lengths, areas, or volumes. The center of gravity of three bodies, lines, surfaces, or solids is in the plane of the centers of gravity of the three.

If a center of gravity of the parts of a body, line, surface, or solid lies in a plane, the center of gravity of the whole may be found graphically. (1) Let a, b, c, etc., be the centers of gravity of the parts and A, B, C, etc., their weights, lengths, areas or volumes; then imagine parallel forces of values A, B, C, etc., to act through a, b, c, etc., and find the line of action of their resultant; repeat the operation for the parallel forces at an angle (as 90°) with their first positions. The intersection of the resultants is the center of gravity sought. (2) Choose any point O (Fig. 89) in the plane of the centers of gravity as origin; measure Oa, Ob, Oc, etc., and form the products $A\overline{Oa}$, $B\overline{Ob}$, $C\overline{Oc}$, etc. Imagine forces whose values equal the products respectively, to act from O in the lines Oa, Ob, Oc, etc., and find their resultant R (Art. 16). The center of gravity sought, r, lies in the line of action of R at a distance from O equal to $R \div (A + B + C + \cdots)$.

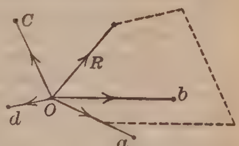

Fig. 89

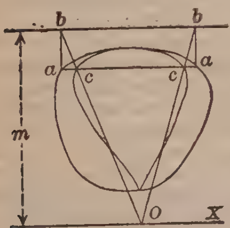

Fig. 90

The center of gravity of an irregular plane figure may be found by cutting out the figure from stiff paper and then determining the center of gravity of the paper experimentally by balancing (see below); this center of gravity gives that of the figure. Or, proceed as follows (Fig. 90): Take a point O and a line bb on opposite sides of the figure at any convenient distance m apart; project any width of the figure parallel to bb as aa on bb, connect the projections bb with O and note the intersections cc;

determine other points cc and draw a smooth curve through them as shown; measure the area A' within the curve cc; then $A'm$ is the statical moment of the given figure with respect to OX; if A is the area of the given figure and y the distance of its center of gravity from OX, $y = A'm/A$. In a similar way the distance of the center of gravity from a line perpendicular to OX can be determined.

Experimental Determination of center of gravity must be resorted to when the body is so irregular that the appropriate formulas given above cannot be applied. (1) Method of Suspension: The body is suspended from one point of it, and the direction of the suspending cord is then marked in some way on the body; the operation is repeated for another point of suspension. The center of gravity is at the intersection of the two lines or directions so fixed in the body. (2) Method of Balancing: The body is balanced on a straight-edge, and the vertical plane containing the edge is marked on the body; the operation is repeated for two more balancing positions of the body. The center of gravity is at the common point of the three planes so fixed in the body. This method is readily applied to a body in the form of a thin plane plate; practically only two balancings are necessary. (3) Method of Weighing: The weight W of the body is determined, and then it is supported on a knife-edge (Fig. 91) and on a point support which rests upon a platform scale; the reaction R of the point support is weighed, the horizontal distance a of the point from the knife-edge is measured; then the distance from center of gravity to knife-edge $\bar{x}$ is Ra/W.

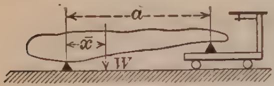

Fig. 91

23. Centers of Gravity of Some Lines and Areas

Circular Arc (Fig. 92): the center of gravity is on the axis of symmetry; its distance from the center is $x = rc/s = r \sin\alpha/\alpha$, the last α being expressed in radians (1 degree = 0.0175 radian). For a semicircle $\bar{x} = 2\,r/\pi + 0.6366\,r$; for a quadrant $\bar{x} = 2\,r\,\sqrt{2}/\pi = 0.9003\,r$, and the distance of the center of gravity from the radius drawn to either end of the arc is $2\,r/\pi = 0.6366\,r$. For flat arcs, α small, the distance from its center of gravity to the chord c is closely equal to $2/3\,h$; the error is less than 1/2% when $\alpha = 30°$, and less than 1.1% when $\alpha = 45°$.

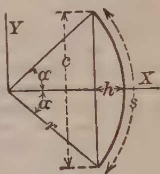

Fig. 92

Triangle: The center of gravity is at the intersections of the medians; its distance (perpendicular) from any side equals one-third the altitude of the triangle measured from that side. When x_1, x_2, x_3 are parallel distances from the vertexes to any plane, then the distance of the center of gravity is $1/3(x_1 + x_2 + x_3)$.

Trapezoid: Let a and z be the parallel bases and h the altitude. For the left-hand diagram in Fig. 92-1/2, where m is the overhang of the right side, the horizontal distance of the center of gravity from the left corner is $\bar{x} = 2/3\,b + 1/3\,m - 1/3\,a(a - m)/(a + b)$. For the right diagram the same formula applies if m is taken as negative. For all cases the distance of the center of gravity above the base b is $y = 1/3\,h(b + 2\,a)/(a + b)$.

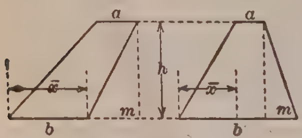

Fig. 92-1/2

When a equals b, then $x = 1/2(b \pm m)$ and $\bar{y} = 1/2\,h$. Geometric determinations of center of gravity: (1) Extend AB (Fig. 93) so that $BE = CD$, and in the opposite direction extend CD so that $DF = AB$; the intersection of FE and the median GH is the center of gravity. (2) Divide the trapezoid (Fig.

93) into triangles by a diagonal as AC; find the centers of gravity G_1 and G_2 of the triangles (construction indicated in the figure); the intersection of G_1G_2 with the median EF is the center of gravity sought.

Quadrilateral: (a) Divide the quadrilateral

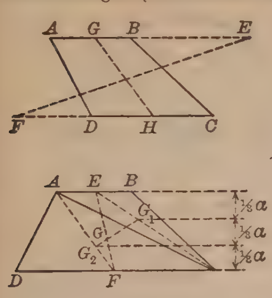

Fig. 93

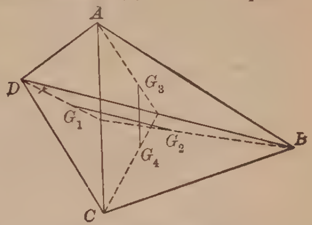

Fig. 94

into triangles by a diagonal AC (Fig. 94) and find their centers of gravity G_1 and G_2; divide it into triangles by the other diagonal and find their centers of gravity G_3 and G_4; the intersections of the lines G_1G_2 and G_3G_4 is the center of gravity sought. (b) Divide the sides into thirds (Fig. 96) and draw lines through the third points as shown; these lines form a parallelogram whose diagonals intersect at the center of gravity of the quadrilateral.

Circular Sector (Fig. 95*a*). The center of gravity is on the axis of symmetry at a distance from the center equal to $\bar{x} = 2/3\ rc/s = 2/3\ r \sin \alpha/\alpha$, the last α being expressed in radians (1 degree = 0.0175 radian). For a semicircle $\bar{x} = 4\,r/3\,\pi = 0.4244\,r$. For a quadrant $\bar{x} = 4\sqrt{2}\,r/3\,\pi = 0.6002\,r$, and distance of the center of gravity from each bounding radius is $4\,r/3\,\pi = 0.4244\,r$.

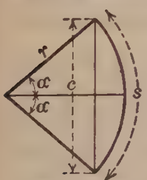

Fig. 95*a*

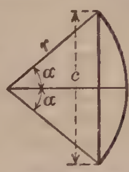

Fig. 95*b*

Circular Segment (Fig. 95*b*). The center of gravity is on the axis of symmetry at a distance from the center equal to $\bar{x} = c^3/12\,A = (2\,r^3 \sin^3 \alpha)/3\,A$, in which A is the area of the segment, or $1/2\,r^2(2\,\alpha - \sin 2\,\alpha)$, the first α being expressed in radians (1 degree = 0.0175 radian).

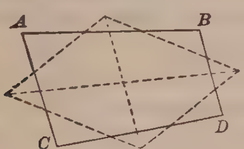

Fig. 96

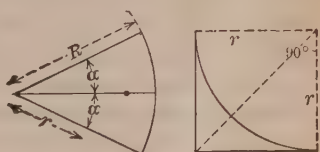

Fig. 97

Sector of a Circular Ring (Fig. 97). The center of gravity is on the axis of symmetry at a distance from the center of the circle which is given by $\bar{x} = 2/3\,(R^3 - r^3) \sin \alpha/(R^2 - r^2)\alpha$, the last α being in radians.

The Surface (Fig. 97) bounded by a circular quadrant and the tangents at its extremities. The center of gravity is on the axis of symmetry at a distance from each tangent equal to $0.223\,r$.

Parabolic Segment (Fig. 98): G_1 and G_2 are the centers of gravity of $OXCO$ and $OYCO$ respectively: $x_2 = 3/5\ a$, $y_1 = 3/8\ b$, $x_1 = 3/10\ a$, $y_2 = 3/4\ b$.

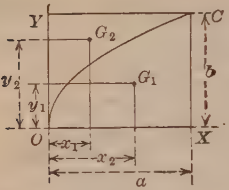

Fig. 98

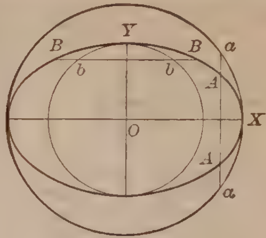

Fig. 99

Symmetric Elliptic Segment (Fig. 99). The center of gravity of $YBBY$ coincides with that of the circular segment $YbbY$, and the center of gravity of $XAAX$ coincides with that of the circular segment $XaaX$.

24. Centers of Gravity of Certain Volumes

Right Circular Cylinder (Fig. 100). The base XOA is normal to the axis of the cylinder, and the top makes an angle α with the base; the radius of the base is r and the mean height is h; then

$$\bar{x} = (r^2 \tan \alpha)/4\ h, \quad \bar{y} = 1/2\ h + (r^2 \tan^2 \alpha)/8\ h.$$

If the oblique top cuts the base in a diameter, $\bar{x} = 3/16\ \pi r$, and $\bar{y} = 3/32\ \pi a$.

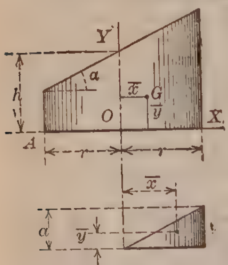

Fig. 100

Cone and Pyramid. The center of gravity of the surface (not including base) is on a line joining the apex with the center of gravity of the perimeter of the base at a distance two-thirds the length of that line from the apex. The center of gravity of the solid cone or pyramid is on the line joining the apex with center of gravity of the base three-fourths the length from the apex.

Frustum of a Circular Cone. Let R = radius of larger base, r = radius of smaller, a = altitude; then distance of center of gravity of the conical surface from larger base is $1/3\ a\ (R + 2\ r)/(R + r)$, from smaller base $1/3\ a(2\ R+r)/(R+r)$, from a plane midway between bases $1/6\ a\ (R-r)/(R+r)$. The distance from the center of gravity of the solid frustum to the larger base is $1/4\ a(R^2 + 2\ Rr + 3\ r^2)/(R^2 + Rr + r^2)$.

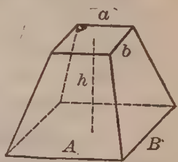

Fig. 101

Frustum of a Pyramid. If the pyramid has regular bases, let R and r be the lengths of sides of the larger and smaller bases, and h the altitude; then the distance from the center of gravity of the surface (not including bases) from the larger base is $1/3\ h(R + 2\ r)/(R + r)$. If A and a are the areas of the large and small bases of the frustum of any pyramid and h the altitude, the distance from the center of gravity of the solid from the larger base is $1/4\ h(A + 2\sqrt{Aa} + 3\ a)/(A + \sqrt{Aa} + a)$.

Obelisk and Wedge (Fig. 101). The distances from the center of gravity to the base AB is $1/2\ h(AB + Ab + aB + 3\ ab)/(2\ AB + Ab + aB + 2\ ab)$.

If $b = 0$, the solid is a wedge, and the distance from the center of gravity to the base is $1/2\ h(A + a)/(2\ A + a)$.

Sphere Parts. The center of gravity of any zone (surface) (Fig. 102) of a sphere is midway between the bases. Segment (solid): height h (Fig. 103),

Fig. 102

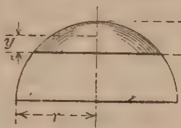

Fig. 103

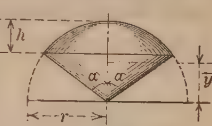

Fig. 104

$\bar{y} = 1/4\ h(4\ r - h)/(3\ r - h)$; when $h = r$ (hemisphere), $\bar{y} = 3/8\ r$. Sector (Fig. 104): $\bar{y} = 3/8\ (1 + \cos\alpha)\ r = 3/8\ (2\ r - h)$.

Ellipsoid. Let the three axes be taken as x, y, and z coordinate axes, and a, b, and c denote the semi-lengths of the corresponding axes of the ellipsoid; the center of gravity of one octant of the solid is given by $\bar{x} = 3/8\ a$, $\bar{y} = 3/8\ b$, and $\bar{z} = 3/8\ c$.

Paraboloid of Revolution formed by revolving a parabola about its axis. Let h = height of the paraboloid, the distance from its apex to the base; then the distance from the center of gravity of the solid to the base is $1/3\ h$.

25. Principles of Moment of Inertia

Definitions. The **moment of inertia** of a surface (figure or area) with respect to or about a line is the sum of the products obtained by multiplying the area of each element of the surface by the square of its distance from the line. Thus, if I denotes moment of inertia, A area, and r distance of any element dA from the line or axes with respect to which I is taken, then $I = \int r^2\, dA$; the limits of integration are to be so chosen that the integration will include all products like $r^2\, dA$ for the surface. The moment of inertia, obviously, of any surface or figure is the sum of the moments of inertia of its parts. The moment of inertia of a plane figure with respect to a line in the plane is called rectangular and one with respect to a line perpendicular to the plane is called polar; these are the only moments of inertia of surfaces that are of practical importance. A unit moment of inertia is four "dimensions" in length, and is called quadric inch, foot, etc., according as the inch or foot is used as unit length; the corresponding abbreviations are in^4, ft^4, etc. **The radius of gyration** of a surface (figure or area) with respect to a line is the length whose square multiplied by the area of the surface equals the moment of inertia of the surface with respect to the same line. Thus if k denotes radius of gyration, A area, and I moment of inertia, $k^2A = I$, or $k = \sqrt{I/A}$. The square of the radius of gyration of a figure with respect to a line is the mean of the squares of the distances of all the elementary parts of the figure from the line. (See also Art. 27.)

The Product of Inertia of a plane surface (figure or area) with respect to a pair of coordinate axes in the plane is the sum of the products obtained by multiplying the area of each element of the surface by its coordinates. Thus if J denotes product of inertia with respect to x and y axes, A area, and x and y the coordinates of any element-area dA, then $J = \int xy\, dA$; the limits of integration are to be so chosen that the integration will include all products like $xy\, dA$ for the surface. A unit product of inertia, like a unit moment of inertia (see foregoing), is four dimensions in length. Unlike moments of inertia, products of inertia may be zero or negative as well as positive. If a figure has an axis of symmetry, then its product of inertia with respect to that axis and one perpendicular thereto is zero.

Graphic Determination of Moment of Inertia. When the outline of a surface is so irregular that the integration in the expression for I cannot be performed, then the following may be resorted to: Let $aaaa$ (Fig. 105) be the outline and XX' the axis with respect to which the moment of inertia is desired; at any convenient distance m from XX' draw two parallels (but if XX' does not cut the figure, then only one parallel, the one on the opposite side of the figure from XX'); draw any line as aa parallel to XX' and project the points aa on the nearer parallel; join the projections bb to any point O in XX' and note the intersections cc on aa; project cc on the same parallel; join the projections dd with O and note the intersections ee on aa. In a similar manner determine points like ee for other widths like aa, and connect all points e as shown. Then measure the area of the loops OPO and OQO; denoting this combined area by A'', $I = A''m^2$. (There will be only one loop if only one parallel, bb, is used.)

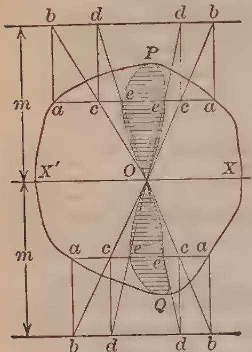

Fig. 105

Transformation Formulas. (1) Let I = moment of intertia of a figure with respect to any line or axis, $\bar{I}$ = that with respect to a parallel axis passing through the center of gravity of the figure, d = distance between the axes, k and $\bar{k}$ = the radii of gyration with respect to the same axes respectively, and A = area of the figure; then

$$I = \bar{I} + Ad^2 \qquad \text{and} \qquad k^2 = \bar{k}^2 + d^2$$

These show that with respect to all parallel axes the moment of inertia and the radius of gyration is least for the one passing through the center of gravity of the figure. (2) Let I_x, I_y, and I_z = the moments of inertia of a plane figure with respect to x, y, and z axes respectively, the axes being at right angle to each other and the x and y axes in the plane; and let k_x, k_y, and k_3 = the corresponding radii of gyration; then

$$I_x + I_y = I_z \qquad \therefore \qquad k_x^2 + k_y^2 = k_z^2$$

(3) Let J = the product of inertia of a plane figure with respect to a pair of coordinate axes in the plane, and $\bar{J}$ = that with respect to a parallel pair whose origin is at the center of gravity; $\bar{x}$, $\bar{y}$ the coordinates of the center of gravity referred to first pair, and A the area of the figure; then $J = \bar{J} + A\bar{x}\bar{y}$. (4) Let XOY and UOV (Fig. 106) be two sets of rectangular coordinate axes with a common origin and in a given plane figure; I_x, I_y, I_u, I_v = moments of inertia of the figure with respect to x, y, u, and v axes respectively, J_{xy} and J_{uv} = its products of inertia with respect to the sets of axes respectively; α = angle through which x axis must be rotated to bring it into u axis, regarded as positive or negative according as the turning is counter-clockwise or clockwise. Then $I_u + I_v = I_x + I_y$, and

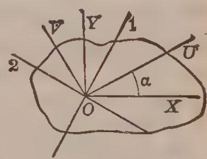

Fig. 106

$$I_u = I_x \cos^2\alpha + I_y \sin^2\alpha - J_{xy} \sin 2\alpha$$
$$J_{uv} = 1/2\,(I_x - I_y) \sin 2\alpha + J_{xy} \cos 2\alpha.$$

Principal Axes of Inertia. In general, the moments of inertia of a plane figure with respect to different lines through any point of the plane are unlike; for one axis the moment is greater and for another less than for any other axis through the point. The principal axes for a plane figure at a particular point of the plane are the two axes for which the moments of inertia are greater and less than for any other axis through the point and in the plane; the corresponding moments of inertia are called the **principal moments of inertia** of the figure at the point. The principal axes are always at right angles to each other.

With respect to the principal axes, the product of inertia is zero; from this principle the principal axes can readily be found in some cases. Thus, at a corner of a square the principal axes are the diagonal through that corner and a line perpendicular to it, for with respect to those lines the product of inertia of the square is zero (see under Product of Inertia). In any case, the principal axes of a figure at a point of it can be found from the following formula if the moments of inertia and the product of inertia of the figure with respect to two rectangular axes through the point and in the plane are known; thus let the two rectangular axes be OX and OY (Fig. 106), I_x, I_y the corresponding moments of inertia, J_{xy} the corresponding product of inertia, and θ the (unknown) angle through which OX must be turned counter-clockwise to bring it into either principal axis, $O1$ or $O2$; then $\tan 2\theta = 2J_{xy}/(I_y - I_x)$, which gives always two values of θ differeing by 90° unless Jxy and $Iy - Ix$ are both zero. In that case the figure has no principal axis at the point, and the moments of inertia with respect to the different lines through the point are all equal.

The Principal Moments of Inertia I_1 and I_2 can be computed from

$$I_1 = I_x \cos^2\theta_1 + I_y \sin^2\theta_1 - J_{xy} \sin 2\theta_1,$$

and

$$I_2 = I_x \cos^2\theta_2 + I_y \sin^2\theta_2 - J_{xy} \sin 2\theta_2$$

θ_1 and θ_2 being the two values of θ given by the formula above for principal axes. Or, after I_1 is determined, $I_2 = I_x + I_y - I_1$.

The Inertia-Circle and Ellipse. The inertia-circle is a device for determining the moment of inertia of a plane figure with respect to any line of the plane and the principal axes and principal moments of inertia at any point graphically. To construct the circle, it is necessary to know the moments of inertia and the product of inertia with respect to two rectangular axes through the point, in the plane figure. Thus if I_u and I_1 and I_2 at O (Fig. 107) are desired, I_x, I_y, and J_{xy} being known: By any scale, lay off $OX = I_x$, $OY = I_y$, and $YA = J_{xy}$, upwards or downwards from XY according as J_{xy} is positive or negative; bisect XY and from the middle point C as center describe a circle passing through A; this is the inertia-circle for axes xOy. Draw a secant through A parallel to the u axis, and from its intersection B with the circle draw a perpendicular BU to XY; then $OU = I_u$, by the scale used (and $BU - J_{uv}$). Lines through O parallel to AP and AQ are principal axes at O; and OP and OQ represent the corresponding principal moments of inertia respectively by the scale used.

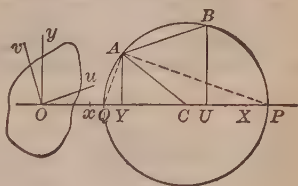

Fig. 107

The inertia-ellipse like the inertia-circle shows the relations between the moments of inertia of a plane figure for different axes through a point of the plane; for qualitative results the ellipse is preferable, generally, but for quantitative results, the circle. The inertia-ellipse for a plane surface at any particular point has its center at the point and is so drawn that the distance from any diameter to either parallel tangent equals the radius of gyration of the figure with respect to that diameter. An inertia-ellipse of a figure at its center of gravity is the central inertia-ellipse. In general, the ellipse is determined most readily thus: determine the principal axes at the point in question and the corresponding principal radii of gyration; from the point lay off on each axis in each direction distances equal (by some scale) to the radius of gyration with respect to the other axis; construct the ellipse on those two lengths as axes (Art. 11).

26. Moments of Inertia of Certain Plane Figures

Rectangle. Let b = base and h = altitude; about a line through center parallel to b, $I = 1/12\ bh^3$; about a line through the center parallel to h, $I = 1/12\ hb^3$; about side b, $I = 1/3\ bh^3$; about side h, $I = 1/3\ hb^3$; about a diagonal $I = 1/6\ b^3h^3/(b^2 + h^2)$; about a line through the center perpendicular to the diagonal $I = 1/12\ (bh^3 + hb^3)$.

Square. Make $b = h$ in foregoing. The moment of inertia is $1/12\ h^4$ for all axes in the plane of the square and passing through the center.

Hollow Rectangle. Let B and b = outer and inner breadths, and D and d = outer and inner depths; about an axis parallel to B and b and passing through the center $I = 1/12\ (BD^3 - bd^3)$.

Triangle. Let b = base and h = altitude; about the base $I = 1/12\ bh^3$; about a line through the center of gravity parallel to the base $I = 1/36\ bh^3$; about a line through the vertex parallel to the base $I = 1/4\ bh^3$.

Regular Polygon. Let A = area, R = radius of circumscribed circle, r = radius of inscribed circle, and s = length of a side; about any axis through the center and in the plane of the polygon $I = 1/24\ A\ (6\ R^2 - s^2) = 1/48 A\ (12\ r^2 + s^2)$; about a line perpendicular to the plane of the polygon passing through the center I = double the preceding I.

Trapezoid. Let B = long base, b = short base, h = altitude; about long base $I = 1/12\ (B + 3\ b)\ h^3$; about short base $I = 1/12\ (3\ B + b)h^3$; about a line through center of gravity and parallel to bases $I = 1/36\ (B^2 + 4\ Bb + b^2)\ h^3/(B + b)$.

Circle. Let d = diameter and r = radius; about a diameter $I = 1/64\ \pi d^4 = 1/4\ \pi r^4$, and $k^2 = 1/16\ d^2 = 1/4\ r^2$; about a line through the center and perpendicular to the circle $I = 1/32\ \pi d^4 = 1/2\ \pi r^4$, and $k^2 = 1/8\ d^2 = 1/2\ r^2$.

Semicircle. Let d = diameter and r = radius; about the bounding diameter or about the line of symmetry $I = 1/128\ \pi d^4 = 1/8\ \pi r^4$; about a line through the center of gravity parallel to the bounding diameter $I = (9\ \pi^2 - 64)\ d^4/1152\ \pi = 0.00686\ d^4 = 0.110\ r^4$.

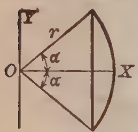

Fig. 108

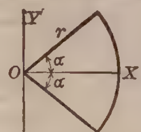

Fig. 109

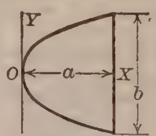

Fig. 110

Hollow Circle. Let D and d = outer and inner diameters, and R and r = outer and inner radii; about a diameter $I = 1/64\ \pi\ (D^4 - d^4) = 1/4\ \pi\ (R^4 - r^4)$, and $k^2 = 1/16\ (D^2 + d^2) = 1/4\ (R^2 + r^2)$; about a line through the center and normal to circle $I = 1/32\ \pi\ (D^4 - d^4) = 1/2\pi(R^4 - r^4)$, and $k^2 = 1/8\ (D^2 + d^2) = 1/2\ (R^2 + r^2)$.

Circular Segment (Fig. 108). Let A = area of the segment;

$$I_x = 1/4\ Ar^2\ [1 - 2/3\ (\sin^3\alpha\cos\alpha)/(\alpha - \sin\alpha\cos\alpha)],$$
$$I_y = 1/4\ Ar^2\ [1 + (2\sin^3\alpha\cos\alpha)/(\alpha - \sin\alpha\cos\alpha)].$$

Circular Sector (Fig. 109). Let A = area of the sector; $I_x = 1/4Ar^2\ (1 - \sin\alpha\cos\alpha/\alpha)$, $I_y = 1/4\ Ar^2\ (1 + \sin\alpha\cos\alpha/\alpha)$; with respect to a line through O perpendicular to the sector, $I = 1/2\ Ar^2$.

Parabolic Segment (Fig. 110). $Ix = 4/15\ ab^3$, $Iy = 4/7\ ba^3$.

Ellipse. Let $2\,a$ and $2\,b$ = lengths of the axes of the ellipse; about the $2\,a$ axis $I = 1/4\,\pi ab^3$; about the $2\,b$ axis $I = 1/4\,\pi ba^3$; about a line through the center and perpendicular to the ellipse $I = 1/4\,\pi ab\,(a^2 + b^2)$.

27. Moments of Inertia of Bodies

Definitions. The **moment of inertia of a body** with respect to or about a line is the sum of the products obtained by multiplying the mass of each elementary part by the square of its distance from the line. Thus I denoting moment of inertia, m mass, and p the distance of any element dm from the line of reference, $I = \int p^2\,dm$. The moment of inertia of a body is, obviously, the sum of the moments of inertia of its parts. A unit moment of inertia of a body is one dimension in mass and two in length. The **center of gyration** of a body with respect to a line is a point at such a distance from the line that if the entire mass of the body were concentrated there, the moment of inertia of the mass-point would be the same as that of the body; the distance of the center of gyration from the line is the radius of gyration with respect to the line. Thus k denoting radius of gyration, $k^2m = I$, or $k = \sqrt{I/m}$. The **product of inertia** of a body with respect to two coordinate planes is the sum of the products obtained by multiplying the mass of each element of the body by the two coordinates of the element with reference to those planes. Thus with respect to YOZ and ZOX (Fig. 111), ZOX and XOY, and XOY and YOZ planes, the products of inertia are respectively $\int xy\,dm$, $\int yz\,dm$, and $\int zx\,dm$. A unit product of inertia is, like a unit moment of inertia, one dimension in mass and two in length. Unlike moments, products of inertia may be zero or negative as well as positive.

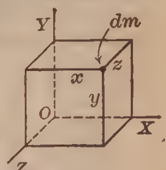

Fig. 111

Transformation Formulas. (1) If I denotes the moment of inertia of a body with respect to any line or axis, $\bar{I}$ that with respect to a parallel line through the center of gravity, d the distance between the axes, and m the mass of the body, then

$$I = \bar{I} + md^2; \quad \text{also} \quad k^2 = \bar{k}^2 + d^2$$

k and $\bar{k}$ denoting radii of gyration corresponding to the axes named respectively. (2) Let I_x, I_y, and I_z denote the moments of inertia of a body with respect to rectangular axes x, y, and z respectively; J_{xy}, J_{yz}, and J_{zx} its products of inertia with respect to yz and zx planes, zx and xy planes, and xy and yz planes respectively; I the moment of inertia of the body with respect to a line through the origin of coordinates having direction-angles α, β, and γ; then

$$I = I_x \cos^2\alpha + I_y \cos^2\beta + I_z \cos^2\gamma - 2\,J_{yz}\cos\beta\cos\gamma - 2\,J_{zx}\cos\gamma\cos\alpha - 2\,J_{xy}\cos\alpha\cos\beta$$

Principal Axes and Moments of Inertia. The values of moments of inertia of a body for all axes through a given point are in general unequal; for one axis the moment of inertia is greater and for another it is less than for any other axis through the point. These two axes are at right angles, and they together with one at right angles to their plane and passing through the point are principal axes of the body at the point; the corresponding moments of inertia are the principal moments of inertia of the body at the point. If the point is the center of gravity of the body, then the axes and moments are called central principal axes and central principal moments of inertia.

If $J_{xy} = J_{yz} = 0$, the y axis is a principal axis at the origin,
If $J_{yz} = J_{zx} = 0$, the z axis is a principal axis at the origin,
If $J_{xz} = J_{xy} = 0$, the x axis is a principal axis at the origin.

If a body has a plane of symmetry, then any perpendicular to the plane is a principal axis of the body at the point where the line pierces the plane. If a body has two planes of symmetry at right angles to each other, then their intersection is a principal axis at any point of the intersection, the other two being in the planes of symmetry. If a body has three planes of symmetry their lines of intersection are the central principal axes of the body.

Special Cases. The bodies are supposed to be homogeneous; m = the mass of the body in each instance and δ = its density, that is, its mass per unit of volume. In any system, like the C.G.S. or Engineers' (see Art. 29), $m = W/g$ and $\delta = w/g$, wherein W denotes the weight of the body, w its weight per unit volume, and g the acceleration of a freely falling body.

Straight Rod. Let l = its length; about a line making an angle α with the axis of the rod and passing through its center of gravity $I = 1/12\, ml^2 \sin^2 \alpha$; about a line through one end of the rod $I = 1/3\, ml^2 \sin^2 \alpha$.

Rod Bent into a Circular Arc (Fig. 112). $I_x = 1/2\, mr^2\, [1 - \sin \alpha \cos \alpha)/\alpha]$; $I_y = 1/2\, mr^2\, [1 + (\sin \alpha \cos \alpha)/\alpha]$; about a line perpendicular to the plane of the arc and through the center of the circle, $I = mr^2$.

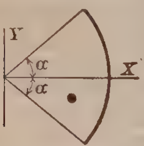

Fig. 112

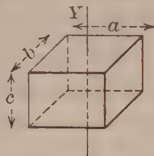

Fig. 113

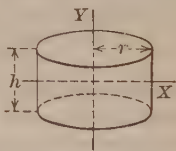

Fig. 114

Right Prism. Let h = its altitude and A = area of base; about any line perpendicular to the bases, $I = \delta h \times$ moment of inertia of a base about the same line; about a line through its center of gravity and perpendicular to the axis of the prism $I = 1/12\, \delta A h^3 + \delta h \times$ the moment of inertia of the central cross-section about the line.

Right Parallelopiped (Fig. 113). $I_y = 1/12\, m\, (a^2 + b^2) = 1/12\, abc\delta\, (a^2+b^2)$; the y axis passes through the center of gravity and is parallel to the edge c.

Right Circular Cylinder (Fig. 114). $I_y = 1/2\, mr^2 = 1/2\, \pi r^4 h\delta$; $I_x = 1/12\, m\, (3\, r^2 + h^2) = 1/12\, \pi r^2 h\delta\, (3\, r^2 + h^2)$. The y axis is the axis of the cylinder, and the x axis is perpendicular to it and passes through the center of gravity.

Hollow Right Circular Cylinder. Let R and r be the outer and inner radii, and axes taken as in Fig. 114. $I_y = 1/2\, m\, (R^2 + r^2) = 1/2\, \pi h\delta\, (R^4 - r^4)$, $I_x = 1/4\, m\, (R^2 + r^2 + 1/3\, h^2) = 1/4\, \pi\, (R^2 - r^2)\, h\delta\, (R^2 + r^2 + 1/3\, h^2)$.

Right Rectangular Pyramid (Fig. 115). $I_y = 1/20\, m\, (a^2 + b^2) = 1/60\, abh\delta\, (a^2 + b^2)$ and $I_x = 1/20\, m\, (3/4\, h^2 + b^2) = 1/60\, abh\delta\, (3/4\, h^2 + b^2)$; the y is the axis of the pyramid and x axis passes through the center of gravity and is parallel to side a.

Right Circular Cone (Fig. 116). $I_y = 3/10\, mr^2 = 1/10\, \pi r^4 h\delta$, $I_x = 3/20\, m\, (r^2 + 1/4\, h^2) = 1/20\, \pi r^2 h\delta\, (r^2 + 1/4\, h^2)$, $I_z = 3/20\, m\, (r^2 + 4\, h^2)$; the y is the axis of the cone, the x is parallel to the base and passes through the center of gravity, and the z is parallel to the base and passes through the apex.

Frustum of a Cone. Let R and r = radii of larger and smaller bases, h = altitude; about the axis of the frustum $I = 3/10\, m\, (R^5 - r^5)/(R^3 - r^3) = 1/10\, \pi h \delta\, (R^5 - r^5)/(R - r)$.

Sphere. Let r = its radius; about any diameter $I = 2/5\, mr^2 = 8/15\, \pi r^5 \delta$.

Hollow Sphere. Let R and r = the outer and inner radii; about any diameter $I = 2/5\, m\, (R^5 - r_5)/(R^3 - r^3) = 8/15\, \pi\delta\, (R^5 - r^5)$.

Ellipsoid. Let $2\,a$, $2\,b$, and $2\,c$ = the lengths of the axes; about the axis $2\,c$, $I = 1/5\, m\, (a^2 + b^2) = 4/15\, \pi abc\delta\, (a^2 + b^2)$.

Paraboloid generated by revolving a parabola about its axis. Let h = its height and r = radius of base; about the axis of revolution $I = 1/3\, mr^2 = 1/6\, \pi h r^4 \delta$.

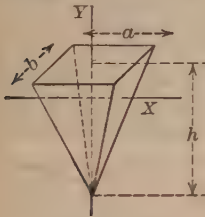

Fig. 115

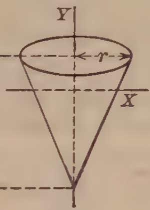

Fig. 116

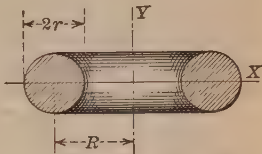

Fig. 117

Ring (Fig. 117). $I_y = m\,(R^2 + 3/4\, r^2) = 1/2\, \pi^2 R r^2\, (4\, R^2 + 3\, r^2)\, \delta$, $I_x = m\,(1/2\, R^2 + 5/8\, r^2) = \pi^2 R r^2\, (R^2 + 5/4\, r^2)\, \delta$; both axes, x and y, pass through the center of gravity as shown.

DYNAMICS

28. Dynamical Quantities

Weight and Mass. In common parlance the word weight is used in at least two senses: thus a body is said to be heavy, its weight is 500 lb., etc., the reference being to the earth-pull or gravity on the body; also a cask is said to contain much sugar, its weight is 500 lb., etc., the reference here being to quantity of material, matter or stuff of a certain kind (sugar). Doubtless the legal definitions of our standards of weight imply the second sense, since they were framed primarily to standardize weight measures of commodities made in trade. For the sake of clearness, many writers restrict the use of the word weight to one meaning, namely the first, that is, earth-pull; and to denote the second, quantity of substance, they employ the term mass. This usage is employed in the present chapter.

Force has been previously defined (Art. 15). Every so-called practical unit of force is a force equal to the earth-pull on a standard of mass, and the name given to such unit force is the same as the name given to the standard or unit of mass. Thus, a force equal to the weight or gravity of a pound mass is a unit of force, and that unit is called a pound force; a force equal to the weight or gravity of a kilogram mass is a unit of force and is called a kilogram force, etc. These units are called gravitational units.

Some writers seeking to make gravitational units of force absolute specify that the pound force, for example, is the weight or gravity of the pound mass at London or at sea level 45° latitude. However, the actual pound forces used in different places are the earth-pulls on pound masses at those places; hence gravitation units of force as used are not absolute; the magnitudes of the units for two places are as the values of g for those places. To make the unit force absolute and to simplify certain dynamical equations, units of force have been proposed based on the following: In any system of dynamical units the unit force is one which applied to the unit mass of that system produces the unit acceleration of that system. Thus in the C.G.S. system the unit of force is that force which applied to a gram mass gives it an acceleration of one cm. per sec. per sec.; this unit is called dyne. And, in the F.P (mass)S. system (never widely used), the unit is the force which applied to the pound mass gives it an acceleration of 1 ft. per sec. per sec.; it is called poundal.

Work. When the point of application of a force moves, so that the force has a component along the displacement of its application point, the force is said to do work; also the body exerting the force is said to do work. If the force is constant in magnitude and in direction and the displacement is straight, then the magnitude of the work is the product of the component of the force along the displacement and the displacement; if this component is in the direction of the displacement the work is regarded as positive; if opposite, the work is negative. Thus in the displacements of the body C (Fig. 118) from A to B, the works of the several forces are respectively $+F_1 s$, $-F_2 s$, $+F_3 \cos\theta \cdot s$, $-F_4 \cos\phi \cdot s$, and 0. If the force varies in magnitude or direction, or if the displacement of its application point is curved, the work must in general be computed by an integration; the general expression for work in

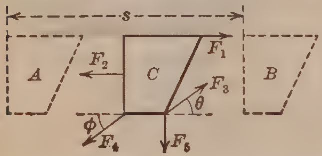

Fig. 118

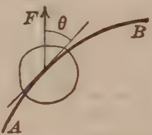

Fig. 119

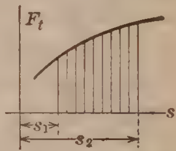

Fig. 120

this case is $\int F \cos\theta \cdot ds$, in which F denotes the force, ds elementary part of the displacement, and θ the angle between F and the direction of the elementary displacement (Fig. 119); the limits of integration must be assigned so as to include the entire displacement A to B. $F \cos\theta$ is the component of F along the tangent to the curve of displacement; denoting this component by F_t, the work is also given by $\int F_t ds$. The work can also be regarded as the product of the force and the component of the displacement along the action line of the force; this component is called the distance through which the force acts, and so work is also said to equal force times distance through which it acts. The work done by a force can be represented by a work diagram, which is constructed by plotting a line showing how the component of the force along the displacement Ft varies with the displacement (Fig. 120). The area included between the curve, the displacement axis, and two ordinates represents the work done by the force during the corresponding displacement $s_2 - s_1$. This diagram suggests also that the work done by a force equals the product of the average value of the tangential or working component and the displacement.

The Unit of Work depends on the units used for force and distance; thus, there are the foot-pound, the foot-ton, the dyne-centimeter (for which there is a special name, erg), etc. The joule is a practical electrical unit of work and equals 10 000 000 ergs; also the kilogrammeter, equal to 100 000 ergs. Work is also expressed in horsepower-hours, watt-hours, etc.; the horsepower-hour is the amount of work done in one hour at the rate of one horsepower, and the watt-hour is the amount of work done in one hour at the rate of one watt.

Power. By power of a force or agent doing work is meant the time-rate at which the work is done. Some units of power are: foot-pound per second;

dyne-centimeter (or erg) per second; horsepower (abbreviated hp.) = 550 foot-pounds per second, or 33 000 foot-pounds per minute; watt = one joule (10 000 000 ergs) per second; kilowatt = 1000 watts; the metric or French horsepower = 75 kilogram-meters per second.

1 ft-lb. per sec.	= 0.13820 kg-m. per sec.	1 kg-m. per sec.	= 7.233 ft-lb. per sec.
1 Engl. hp.	**= 746 watts**	**1 kilowatt**	**= 1.34 Engl. hp.**
1 Engl. hp.	**= 1.0136 Fr. hp.**	**1 Fr. hp.**	**= 0.98 3 Engl. hp.**

Energy. When the condition or state of a body, or system, is such that it can do work, it is said to possess energy; and by its amount or store of energy is meant the amount of work which the body can do in passing to some standard state. Thus, a body in motion has energy, and the amount of its energy is the amount of work it can do in coming to rest; also a stretched spring has energy, and the amount of its energy is the amount of work which it can do in assuming its natural unstretched state. Energy is expressed in the same units as work, foot-pound, foot-ton, dyne-centimeter (erg), etc. Energy which a system has in virtue of its velocity is called kinetic energy. The **kinetic energy** of a particle of mass m moving with velocity v is $1/2\ mv^2$, and the kinetic energy of any body is the sum of the kinetic energies of its particles. In translation: kinetic energy $= 1/2\ Mv^2 = 1/2\ (W/g)v^2$, M denoting mass of the body, W its weight, v its velocity, and g acceleration of gravity; the last form gives energy in foot-pounds if W is expressed in pounds, v in feet per second, and g is taken as 32.2. In rotation: kinetic energy $= 1/2\ I\omega^2 = 1/2\ Mk^2\omega^2 = 1/2\ (W/g)\ k^2\omega^2 = (W/g)\ k^2\ 2\ \pi^2 n^2$, I being the moment of inertia of the body with respect to the axis of rotation, k the corresponding radius of gyration, ω its angular velocity expressed in radians per unit time, W its weight, g the acceleration of gravity, and n the number of revolutions per unit time; the last form gives energy in foot-pounds if W is expressed in pounds, k in feet, n in revolutions per second, and g is taken as 32.2. **Potential energy** is that energy which a system has in virtue of its configuration. The earth and a body elevated above the earth's surface constitute a system having potential energy. Because the energy can be withdrawn only from the elevated body it is said to possess the energy. The amount of potential energy possessed by an elevated body (rigid or not) $= Wh$, W denoting the weight of the body and h the vertical distance through which its center of gravity can descend.

Kinetic energy and potential energy are also called mechanical energy. There are other forms of energy which may not be mechanical, as thermal, chemical, and electrical. But the thermal energy of a body is generally regarded as due to the motions of the ultimate particle constituents of the body, thus being kinetic; much of chemical energy is regarded as due to the relative positions of ultimate particles, thus potential; the nature of electric energy is even less understood than that of chemical and thermal energy.

The Impulse of a force which remains constant in magnitude and direction for any time is the product of the magnitude of the force and the time. The unit of impulse depends on the units used to express the force and the time; thus there are the pound-second, dyne-second, etc. If the force F varies in magnitude but is constant in direction, then its impulse for the interval of time $t_2 - t_1$ is $\int_{t_1}^{t_2} F\,dt$. If the force varies in magnitude and in direction, its impulse can be computed from the impulses of its x, y, and z components; thus if F_x, F_y, and F_z denote these components of F, then the component impulses are $\int F_x\,dt$, $\int F_y\,dt$, and $\int F_z\,dt$, and the impulse of F equals the square root of the sum of the squares of the components. The angular impulse of a force about any line for an element of time is the product of the moment of the force about the line and the element of time, that is, $M\,dt$, if M denotes the moment of the force; and the angular impulse for any interval of time $t_2 - t_1$ is $\int_{t_1}^{t_2} M\,dt$.

The Momentum of a particle is the product of its mass and velocity. The unit of momentum depends on the units of mass and velocity used; the dimensions of unit momentum are the same as those of unit impulse (see preceding). The momentum of a body is the resultant of the momentums of its particles. This resultant is not the scalar but the vector-sum of the momentums, that is, the resultant is to be computed as the resultant of a number of forces is, the separate momentums being regarded as having the directions of the velocities of the several particles and acting at the particles. In a translation the momentum is $Mv = (W/g)v$, M denoting the mass of the body, W its weight, v its velocity, and g the acceleration of gravity; the second form gives momentum in pound-seconds if W is expressed in pounds, v in feet per second, and g is taken as 32.2.

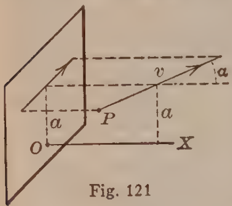

Fig. 121

The angular momentum of a particle about a line is the moment about that line of its momentum; thus, if the particle at P (Fig. 121) has a mass m and a velocity v, its momentum is mv acting in the line v and its angular momentum about OX is $(mv \sin \alpha)\,a$. The angular momentum of a body about a line is the algebraic sum of the angular momentums of its constituent particles. A rotating body has angular momentum about the axis of rotation equal to $I\omega = Mk^2\omega = (W/g)k^2\omega = (W/g)k^2\,2\pi n$, I denoting the moment of inertia of the body with respect to the axis, k its corresponding radius of gyration, W its weight, ω its angular velocity in radians per unit time and n in revolutions per unit time, and g the acceleration of gravity.

Dimensions of Units. All mechanical and nearly all physical units can be defined in terms of one or more of three units arbitrarily defined, that is, without reference to other units. These three are fundamental units and the others derived units. In theorectical mechanics and physics the fundamental units chosen are those of length, mass, and time; in applied mechanics, units of length, force, and time are generally more convenient. A statement of the way in which a derived unit depends on the fundamental ones is a statement of the dimensions of the derived. Thus a unit of velocity depends on the units of length and time used, but is independent of the third fundamental unit; and the magnitude of a unit of velocity varies directly as the unit of length and inversely as the unit of time used. Denoting the first set of fundamental units by L, M, and T respectively, and the unit of velocity by V, the dimensional statement is written thus, $V = L^1M^0T^{-1}$, and is called the dimensional formula for V.

The dimensional formulas may be used to test the accuracy of equations between mechanical quantities. Such an equation, if rationally and correctly deduced, is **homogeneous**; that is, its terms are of the same kind. To ascertain whether terms are the same in kind, substitute for each quantity the dimensional formula for its unit, then treating the symbols L, M or F, and T as quantities, reduce each term; if the reduced terms are alike, then the equation is correct dimensionally. Thus in the equation $24\,EIy = W\,(4l^3x - x^4)/l$, E denotes modulus of elasticity, I moment of inertia of an area, W a load, l, x, and y lengths, and the dimensional form is (dropping abstract members) $L^{-2}F^1L^4L^1 = F\,(L^3L^1 - L^4)/L^1$, which reduces to $L^3F = L^3F - L^3F$, and so the original equation is correct dimensionally.

29. Dynamical Principles

Laws of Motion. (1) The normal state of the center of mass of a body is one of rest or uniform rectilinear motion; a departure from this state is due to force applied to the body. (2) When a single force acts upon a body, the center of mass sustains an acceleration in the direction of the force and proportional to the force directly and to the mass of the body inversely; if several forces act on the body, the acceleration is given by the vector sum of the accelerations which would be produced by the forces acting singly. (By vector sum is meant the result reached by adding according to the parallelogram law.) (3) When one body exerts a force upon another the second also exerts one upon the other first, and the two forces are equal, colinear, and opposite. If F = force, m = mass, and a = acceleration, then the second law can be written $a \propto F/m$ or $a = kF/m$, k being a constant whose value depends on units used in F, m, and a. It is possible to make $k = 1$ by proper choice of units. Such choice was made in the so-called absolute systems of dynamical units; first units of length, mass, and time were selected and then as unit of F such a force which produces unit acceleration in unit-mass. For example, in the C.G.S. system these units are respectively the centimeter, gram, second, and dyne, the latter about 1/981 gram weight. The constant k may also be made unity in so-called gravitational systems; units of length, force, and time are first taken (that of force as the weight of something), and then the unit of m as that mass which sustains unit acceleration when acted on by unit force. For example, in the **engineers' system,** these units are respectively foot, pound force, second, and the unit of m a mass equal to about 32.2 pounds. In any system in which $F = ma$, then also $m = W/g$, m and W being the mass and weight of the body and g the acceleration of gravity, all three quantities being expressed in units of that system.

Laws of Impulse and Momentum. When external forces act upon a body or collection of bodies they produce in general a change in the momentum of the body or collection. In any interval, the change in the component of the linear momentum along any line equals the algebraic sum of the components of the impulses of the forces along that line for the same time; and the change in any interval in the angular momentum about any line equals the algebraic sum of the angular impulses of the forces about that line for the same time. When no external forces are acting, the component linear momentum along any line and the angular momentum about any line remain constant; these are the principles of conservation of linear and angular momentum.

Principles of Work and Energy. In general, the kinetic energy of a body changes value during a change of its position or form, and the external and the internal forces acting on the body do work. The increment in the kinetic energy equals the sum total of work done by all the external and internal forces. In applying this principle, signs must be given to the works of the various forces as explained on p. MM; then if the total work is positive, the increment is positive and there is a real gain in kinetic energy, and if negative then there is a loss. If the body is a rigid one, the internal forces do no work in any displacement of the body, and the increment in kinetic energy equals the total work done by the external forces acting on the body. If a body or system of bodies is isolated so that it neither receives nor gives out energy, then its total store of energy, all forms included, remains constant; there may be a transfer of energy from one part of the system to another, but the total gain or loss in one part is exactly equivalent to the loss or gain in the remainder. This is the principle of conservation of energy.

30. Rectilinear Motion

Motion of a Point. The velocity of a moving point at any instant is the time-rate at which it is describing distance then; the symbol is v. In uniform motion (equal distances described in all equal intervals of time) the rate is constant and $v = \Delta s/\Delta t$, Δs denoting the distance described in the interval Δt.

In non-uniform motion the rate varies, and $\Delta s/\Delta t$ is the average velocity for the interval Δt; the actual velocity at any instant is the value of ds/dt for that instant. The acceleration of a point at any instant is the rate at which its velocity is changing then; the symbol is a. If v changes uniformly the rate is constant, and $a = \Delta v/\Delta t$, Δv denoting the velocity-change occurring in the interval Δt. If v does not change uniformly $\Delta v/\Delta t$ is the average acceleration for the interval Δt; the actual acceleration at any instant is the value of dv/dt for that instant. The general differential equations of rectilinear motion are $v = ds/dt$, $a = dv/dt = d^2s/dt^2$, $vdv = ads$; their integral forms are

$$s_2 - s_1 = \int_{t_1}^{t_2} v\,dt \qquad v_2 - v_1 = \int_{t_1}^{t_2} a\,dt \qquad v_2^2 - v_1^2 = \int_{s_1}^{s_2} a\,ds$$

in which s_1, v_1, and t_1 are corresponding or simultaneous values of s, v, and t, likewise s_2, v_2, and t_2.

Space-time, velocity-time, acceleration-time, velocity-space, acceleration-space curves respectively are graphs showing the relations between s and t, v and t, a and t, v and s, a and s (Figs. 122–126; they do not correspond to the same motion). (1) In the s–t diagram, the slope of the curve at any point represents the corresponding

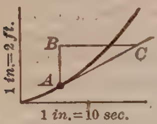

Fig. 122

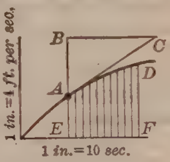

Fig. 123

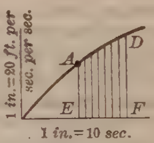

Fig. 124

velocity. If AB and BC are measured by the s and t scales of the drawing respectively, then the slope equals the velocity; thus if $AB = 0.2$ in. and $BC = 0.4$ in., $v = 0.4 \div 4 = 0.1$ ft. per sec. (2) In the v–t diagram, the slopes represent the accelerations; if AB and BC are measured by the v and t scales respectively, then the slope equals the acceleration; thus if $AB = 0.3$ in. and $BC = 0.5$ in., $a = 1.2 \div 5 = 0.24$ ft. per sec. per sec. The area included between any two ordinates (as AE and DF), the curve, and the t axis represents the displacement of the moving point in the time EF. If the area is computed by multiplying its average ordinate measured by the velocity scale (thus giving the average velocity) by EF measured by the time scale, then the product equals the displacement; thus if the average ordinate is 0.35 in., and EF is 4 sec., the displacement $= 1.4 \times 4 = 5.6$ ft. (3) In the a–t diagram the slope represents the rate at which the acceleration is changing. The area included between any two ordinates (as AE and DF), the curve, and the t axis represents the velocity change in the time EF. Thus if the average ordinate is 0.3 in. and EF is 2 sec., then the velocity change $= (0.3 \times 20) \times 2 = 12$ ft. per sec. (4) In the v–s diagram, the subnormals represent the accelerations. If the length of the subnormal is multiplied by the square of the velocity scale number and the product divided by the space scale number, the result will equal the acceleration; thus suppose that the subnormal $BC = 1/3$ in., then $a = (1/3 \times 25) \div 10 = 0.83$ ft. per sec per sec. (5) In the a–s diagram, the area included between two ordinates (as AC and BD), the curve, and the s axis represents the change in the velocity square. If the area is computed by multiplying the mean ordinate measured by the acceleration scale by CD measured by the space scale, the product equals the change in the velocity square; thus if the average ordinate $= 0.3$ in., and $CD = 0.4$, then the change $= 2.4 \times 4 = 9.6$.

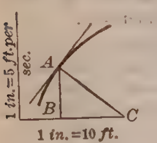

Fig. 125

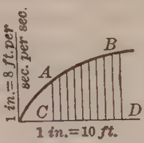

Fig. 126

Uniformly Accelerated Motion, constant acceleration. Let v_1 and v_2 denote the velocities at times t_1 and t_2, s_1 and s_2 the corresponding values of space, and

a = acceleration. Then the velocity change is $v_2 - v_1 = a\ (t_2 - t_1)$; the space traversed is $s_2 - s_1 = 1/2\ (v_1 + v_2)\ (t_2 - t_1) = v_1\ (t_2 - t_1) + 1/2\ a\ (t_2 - t_1)^2$; also $v_2^2 - v_1^2 = 2\ a\ (s_2 - s_1)$.

Falling Body. The acceleration due to gravity (denoted by g) is approximately 32.2 ft. per sec. per sec. It changes slightly with elevation above sea level and with latitude; denoting elevation in feet by e and latitude by ϕ, then in feet per second per second

$$g = 32.0894\ (1 + 0.0052375 \sin \phi)\ (1 - 0.0000000957\ e)$$

At the equator sea level $g = 32.0894$ and at the poles 32.254; the extreme values for the United states are roughly 32.19 (in latitude 49° sea level) and 32.09 (in latitude 25° and 10 000 ft. above sea).

A body dropped without initial velocity falls in a vacuum according to the laws

$$v = gt \qquad h = 1/2\ gt^2 \qquad v^2 = 2\ gh$$

wherein v = velocity and h the descent at the time t. A body projected downward in a vacuum with initial velocity v_1 moves according to

$$v = v_1 + gt, \qquad h = v_1 t + 1/2\ gt^2, \qquad v^2 = v_1^2 + 2\ gh.$$

These three formulas apply to upward projection if the sign + is changed to −; h then denotes ascent in the time t, after projection. The total ascent (to highest position) and the corresponding time are respectively $v_1^2/2\ g$ and v_1/g.

Simple Harmonic Motion. If a point moves in a circle with constant speed, then the motion of the projection of the point on any diameter of the circle is simply harmonic; other motions resembling this are also called simply harmonic. Simple harmonic motion may also be defined as any rectilinear motion in which the acceleration is always directed toward a fixed point in the path and is proportional to the distance between that and the moving point. Taking the first definition, let P (Fig. 127) be the point moving in the circle and regard the motion of its projection on the vertical diameter. The period is the time of one revolution of P, or one complete oscillation of Q; the frequency is the number of revolutions of P, or oscillations of Q, per unit time; the amplitude is half the length of the path of Q or the radius of the circle; the displacement at any time is the ordinate of Q from the center of the path; the phase is the angle at any time from OX to OP (thus when Q is at the top of its path the phase is 90°, when at the center going down, 180°, when at the lowest point, 270° or − 90°). If, in the description of a simple harmonic motion, time is reckoned from the instant when the phase angle is not zero, then that motion is said to have a lead or a lag according as the initial phase (angle) is between 0 and 180° or 0 and − 180°, and the value of the angle is called lead or lag as the case may be. Let ϵ = lead or lag, t = time elapsed after starting (motion from P_0 to P), ω = angular velocity of OP, T = period, n = frequency, r = amplitude, y = displacement, v = velocity of Q and a = its acceleration. The following are the general relations between the quantities involved:

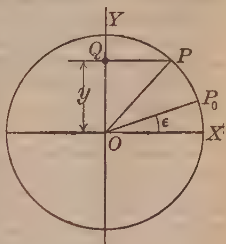

Fig. 127

$$n = 1/T \qquad \omega = 2\ \pi n = 2\ \pi/T \qquad y = r \sin (\omega t + \epsilon)$$

$$v = \omega r \cos (\omega t + \epsilon) = \omega \sqrt{r^2 - y^2} \qquad a = -\ \omega^2 r \sin (\omega t + \epsilon) = -\ \omega^2 y$$

If the time is reckoned from the instant when Q is in its mid-position, $\epsilon = 0$. The three curves (Fig. 130) OA, $O'B$, OC are the space-time, velocity-time, and acceleration-time curves for one complete period of a simple harmonic motion; $\epsilon = 0$; Ot represents the period; the values of y, v, and a marked are for position Q shown. In

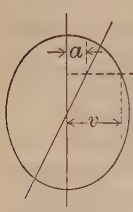

Fig. 128

Fig. 129

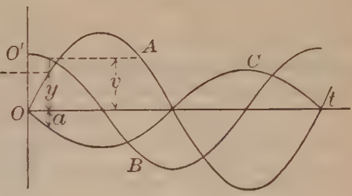

Fig. 130

Fig. 128 the curve is the velocity-space curve, and the inclined line the acceleration-space curve. They show how v and a vary with the displacement of the moving point; thus for the moving point Q v and a have values as marked.

Translation is a motion of a body such that the lines joining all pairs of points of the body remain fixed in direction. In any interval of time all points describe equal paths; at any instant all points have identical velocities and identical accelerations. Translations in which each point of the body describes straight lines are the common ones, and translation is defined sometimes as any rectilinear motion of a body. The resultant of all forces acting on the body passes through the center of gravity of the body; that force and the acceleration have at each instant the same direction, and $R = ma = (W/g)\,a$, wherein R = resultant force, a = acceleration, m and W = mass and weight of the body respectively, and g = the acceleration of gravity. Also if a_x = the x component of the acceleration at any instant and ΣF_x = the algebraic sum of the x components of all the external forces, then $\Sigma F_x = ma_x$; and similarly for y and z axes, the three axes being fixed and rectangular.

For example, consider the motion of a parallel rod of a locomotive, weighing 275 lb., running at constant velocity of 60 mi. per hr. on a level track, the driver diameter being 5.5 ft. and the crank length 1 ft. The forces acting on the rod are its weight and the pressures of the crank pins at its ends; the latter each are represented (Fig. 131) by their horizontal and vertical components. Since the resultant of all these forces acts through the center of gravity, $V_1 = V_2$; also $2\,V_1 - 275 = (W/g)\,ay = 8.55\,ay$ and $H_1 - H_2 = (W/g)\,ax = 8.55\,ax$. To determine ax and ay: The velocity of the center of either crank pin relative to the locomotive is $(88 \times 1)/2.75 = 32$ ft. per sec. (60 mi. per hr. = 88 ft. per sec.), and the relative motion of the pin being circular at constant velocity, the relative acceleration is toward the center of the crank-pin circle at all times and equals $32^2/1 = 1024$ ft. per sec. per sec. This is also the absolute acceleration of the crank pin, since the locomotive is assumed to have no acceleration. But the rod has the same acceleration as the crank pin; hence $ax = 1024 \sin\theta$ and $ay = 1024 \cos\theta$. In the lowest position of the rod $\theta = 0$, $ax = 0$, $ay = 1024$, $H_1 = H_2$, $V = 1/2\,(8755 + 275) = 4515$. In a mid-position when $\theta = 90°$, $ax = 1024$, $ay = 0$, $H_1 - H_2 = 8755$, and $V = 1/2\ 275 = 137.5$. In the highest position, $\theta = 180°$, $ax = 0$, $ay = -1024$, $H_1 = H_2$, and $V = 1/2\,(275 - 8755) = -4240$, the negative sign meaning that V acts downward on the rod.

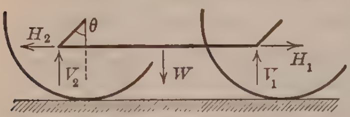

Fig. 131

If the translation is rectilinear and R is constant in value, then

$$Rt = mv_2 - mv_1 \quad \text{and} \quad Rs = 1/2\,mv_2^2 - 1/2\,mv_1^2$$

wherein v_1 and v_2 are the values of v at ends of any interval of time t, and s the distance described in that time. These formulas are the principles of impulse and momentum, and of work and energy for this special motion. If the translation is not rectilinear: According to the principle of impulse and momentum, $\Sigma\int_{t_1}^{t_2} Fxdt = m\,v''x - mv'x$ and similar equations for y and z directions; the left-hand member is the algebraic sum of the x components of the impulses of the forces acting on the body for the time $t_2 - t_1$, and $v'x$ and $v''x$ are the x components of the velocity at the times t_1 and t_2. According to the principle of work and energy, $\Sigma\int_{s_1}^{s_2} Ftds = 1/2\, mv_2^2 - 1/2\, mv_1^2$; the left-hand member is the algebraic sum of the works done by all the forces acting on the body while its velocity changes from v_1 to v_2, and the displacement is $s_2 - s_1$.

31. Curvilinear Motion

Curvilinear Motion of a Point. If the motion is uniform (equal distances traversed in all equal intervals of time), $v = \Delta s/\Delta t$, and if non-uniform, $v = ds/dt$, just as in rectilinear motion; the velocity of the moving point is directed along the tangent to the path at the point. But acceleration formulas for rectilinear motion do not hold for this case; here it is important to note that the velocity varies in direction continually, and that acceleration is the rate at which the velocity-changes (including magnitude and direction) occur. These complete velocity-changes for a given motion are exhibited by the **Hodograph** for the motion. A hodograph is constructed by drawing from any point O' (Fig. 132) vectors which represent the velocities of the moving point at a number of its successive positions and then joining the free ends of the vectors by a smooth curve. The velocity-change occurring in any motion as from A to B is represented by the vector $A'B'$. If Δt is the time in which this occurred, then the magnitude of the average acceleration for that time is (chord $A'B')/\Delta t$, $A'B'$ being measured by the scale of the hodograph diagram, and the direction of that average acceleration is that of the vector $A'B'$. The actual acceleration at A, say, is the limit of this average as B is taken closer and closer to A; that is, a at A is the value of ds'/dt at A' (s' denoting distance on the hodograph), and the direction of a is that of the tangent at A'. It will be noticed that the acceleration of the moving point is represented by the velocity of its corresponding point in the hodograph (A' is A's corresponding point).

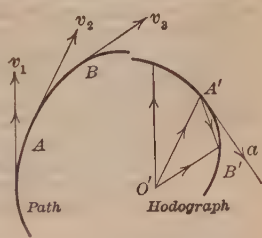

Fig. 132

Velocities and accelerations may be resolved; let α, β, and γ' denote the direction angles of v (with respect to x, y, and z axes), vx, vy, and vz the components of v parallel to these axes (axial components), x, y, and z the coordinates of the moving point. Then $vx = v\cos\alpha = dx/dt$, $vy = v\cos\beta = dy/dt$, $vz = v\cos\gamma = dz/dt$. Let ax, ay, and az denote the axial components of a, and λ, μ, and ν the direction angles of a; then $ax = a\cos\lambda = dvx/dt = d^2x/dt^2$, $ay = a\cos\mu = dvy/dt = d^2y/dt^2$, $az = a\cos\nu = dvz/dt = d^2z/dt^2$. Let at and an denote the components of a along the tangent and normal to the path, ϕ the angle between the tangent and a, θ that between normal and a, and r the radius of curvature at the moving point; then $at = a\cos\phi = dv/dt = d^2s/dt^2$, $an = a\cos\theta = v^2/r$. The tangential acceleration corresponds to change in value of velocity, and the normal acceleration to change in direction of velocity. When a point moves in a circle with constant speed v, then $at = 0$ and $a = an = v^2/r$, and a is directed along the radius toward the center of the circle.

Motion of a Projectile. In the following formulas air resistance is neglected; v_0 = velocity of projection, θ = angle of projection (Fig. 133), x and

y = coordinates of the projectile at any time t after projection, v = velocity, v_x and v_y = x and y components respectively of v, r = range on the horizontal plane through O, and h = greatest height attained. The path of the projectile, or the trajectory, is a parabola as represented, and its equation is $y = x \tan\theta - gx^2/2\,v_0^2\cos^2\theta$. Also

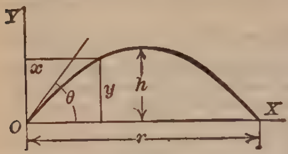

Fig. 133

$$v_x = v_0\cos\theta \quad v_y = v_0\sin\theta - gt \quad v = \sqrt{v_0^2 - 2\,gy}$$
$$x = v_0\cos\theta\cdot t \qquad y = v_0\sin\theta\cdot t - 1/2\,gt^2$$
$$h = \sin^2\theta\cdot v_0^2/2\,g \qquad r = \sin 2\,\theta\cdot v_0^2/g$$

If the direction of projection is horizontal, $\theta = 0$; the equation of the path is $y = -gx^2/2\,v_0^2$, and $x = v_0t$ and $y = -1/2\,gt^2$.

Relative Motion. Description of motion requires the use of coordinate axes or other equivalent means of reference for the specification of positions of the moving point. Motion relative to a body is described by means of coordinate axes fixed in or on the body. Thus, suppose that in a given time a body A (Fig. 134) moves from A' to A'', and a point B from B' to B'': In the first position the coordinates of B are 0.3 and 0.2 in., and in the second 0.5 and 0.4. Then the x and y components of the relative displacement of B with respect to A are $0.5 - 0.3 = 0.2$ and $0.4 - 0.2 = 0.2$, and the total relative displacement is $\sqrt{0.2^2 + 0.2^2} = 0.28$ in. Motion relative to a point is described by means of coordinate axes having the point as origin and their directions remaining constant (relative to the body to which the motion of the point itself is referred). Thus suppose that in a certain time a point A moves from A' to A'' and another B from B' to B'' (Fig. 135); the x and y component displacements of B relative to A are 0.2 and 0.1 in. respectively and the displacement of B relative to A is $\sqrt{0.2^2 + 0.1^2}$. The displacements, velocities, and accelerations of two points relative to each other at any instant are equal and opposite. The velocities of three points A, B, and C relative to each other are related as shown in Fig. 136; V_{ab} means velocity of A relative to B and V_{ba} means velocity of B relative to A. Among three points there are six relative velocities, and

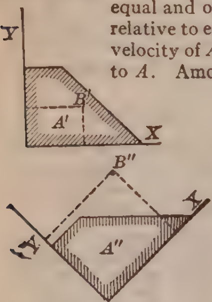

Fig. 134

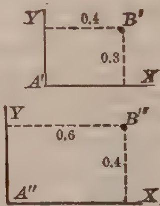

Fig. 135

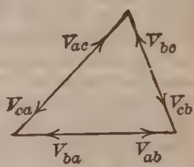

Fig. 136

when any two (except the velocities of two points with respect to each other) are known, the others can be computed by compounding as may be seen from the figure. Among three points there are six relative accelerations; their relations are similar to those of the velocities, and the figure shows the relations, V being now taken to mean acceleration.

Force and Acceleration. The forces acting on a body and the motion of its center of gravity are simply related; the algebraic sum of the components of all the external forces acting on the body along any line equals the product of the mass of the body and the component of acceleration of its center of gravity along that line. In general, the principle leads to

$$\Sigma F_x = m\bar{a}_x, \qquad \Sigma F_y = m\bar{a}_y, \qquad \Sigma F_z = m\bar{a}_z$$

where in m = mass = weight ÷ g; the left-hand members are the sums of the components of the forces along three rectangular axes, and $\bar{a}_x$, $\bar{a}_y$, and $\bar{a}_z$ are the corresponding components of the acceleration of the center of gravity. Or, if the directions of resolution are taken along the tangent to the path described by the center of gravity, the principal normal and a line perpendicular to the principal normal and tangent (the binormal), then

$$\Sigma F_t = m\bar{a}_t \qquad \Sigma F_n = m\bar{a}_n \qquad \Sigma F_b = 0.$$

The resultant of the normal components, ΣF_n, is called the centripetal force on the body; it is always directed inward toward the center of the curved path just as $\bar{a}_n$ is. The reaction corresponding to ΣF_n, exerted by the body on those that exert the centripetal force, is called centrifugal force.

For example, consider a car rounding a curve of radius r at a constant speed v. Imagine the rail pressure on each wheel resolved into three components, one parallel to the rails, one parallel to the ties, and one perpendicular to the track; call the resultants of these sets of components respectively R_1, R_2, and R_3 (Fig. 137). Also let P_1 and P_2 denote the pulls at the front and rear ends of the car. The components of the rail pressures parallel to the rails, P_1, and P_2, are practically parallel to the tangent to the path of the center of gravity of the car at that point. The velocity of the car is constant, so $\bar{a}_t = 0$; also $\bar{a}_n = v^2/r$. Hence

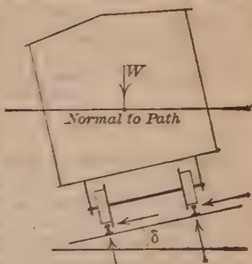

Fig. 137

$$\Sigma F_t = P_1 - P_2 - R_1 = (W/g)\,\bar{a}_t = 0$$
$$\Sigma F_n = R_2 \cos\delta + R_3 \sin\delta = (W/g)\,\bar{a}_n = (W/g)\,v^2/r$$
$$\Sigma F_b = R_3 \cos\delta - R_2 \sin\delta - W = 0.$$

The first equation shows that $P_1 - P_2 = R_1$, and the second and third solved simultaneously give $R_2 = (W/g)(v^2/r)\cos\delta - W\sin\delta$ and $R_3 = (W/g)(v^2/r)\sin\delta + W\cos\delta$. To make $R_2 = 0$, $(W/g)(v^2/r)\cos\delta = W\sin\delta$ or $\tan\delta = v^2/gr$. This value of tilt of track will not necessarily make each component rail pressure parallel to ties zero, but the resultant of all such components will be zero.

32. Rotation

Rotation of a Body is motion such that one straight line in the body or in its extension remains fixed. This line is the axis of the rotation, and all points of the body not on the line describe circles whose centers are on the line. All lines of the body perpendicular to the axis describe equal angles in equal times; angle described by the body means the angle described by any of the lines just mentioned. The angular velocity of the body at any instant is the rate at which it is describing angle then; the symbol is ω. In uniform rotation (equal angles described in all equal intervals of time) the rate is constant and $\omega = \Delta\theta/\Delta t$, $\Delta\theta$ denoting the angle described in the interval Δt. In non-uniform rotation, the rate varies and $\Delta\theta/\Delta t$ is the average angular velocity for the interval; the actual value of ω at any instant is the value of $d\theta/dt$ for that instant. The angular acceleration of a rotating body is the rate at which its angular velocity is changing; the symbol is α. If the ω changes uniformly the rate is constant and $\alpha = \Delta\omega/\Delta t$, $\Delta\omega$ denoting the velocity-change in the interval Δt. If ω does not change uniformly, then the rate varies, and $\Delta\omega/\Delta t$ is the average angular acceleration for the interval Δt; the actual value of α at any instant is the value of $d\omega/dt$ at that instant. The general differential equations of rotation are $\omega = d\theta/dt$, $\alpha = d\omega/dt = d^2\theta/dt^2$, $\omega d\omega = \alpha d\theta$; their integrated forms are

$$\theta_2 - \theta_1 = \int_{t_1}^{t_2} \omega \, dt \qquad \omega_2 - \omega_1 = \int_{t_1}^{t_2} \alpha \, dt \qquad \omega_2^2 - \omega_1^2 = 2\int_{\theta_1}^{\theta_2} \alpha \, d\theta$$

in which θ_1, ω_1, and t_1 are corresponding or simultaneous values of θ, ω, and t, likewise θ_2, ω_2, and t_2. If the agnular velocity is constant, the angular displacement in the interval $t_2 - t_1$ is $\theta_2 - \theta_1 = \omega\,(t_2 - t_1)$; if the angular acceleration is constant the velocity-change $\omega_2 - \omega_1 = \alpha\,(t_2 - t_1)$, the change in the velocity square, $\omega_2^2 - \omega_1^2 = 2\,\alpha\,(\theta_2 - \theta_1)$, and $\theta_2 - \theta_1 = 1/2\,(\omega_1 + \omega_2)(t_2 - t_1)$. The relations between θ, ω, and t can be represented by curves exactly analogous to the curves described on page 118. The corresponding variables are θ and s, ω and v, and α and a.

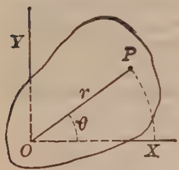

Fig. 138

Let the irregular line in Fig. 138 represent a body rotating about a line through O perpendicular to the paper, OP a line of the body, and OX and OY fixed axes of reference; also θ = angle XOP, $r = OP$, v and a velocity and acceleration respectively of P, a_t and a_n the components of a perpendicular and parallel to OP, and ω and α the angular velocity and acceleration of the body. If the radian is used in θ, ω, and α, then

$$v = r\omega, \quad a_t = ra, \quad a_n = r\omega^3, \quad a = r\,(\alpha^2 + \omega^4)1/2$$

any units of time and length may be used. If n = the angular velocity in revolutions per unit time, $\omega = 2\,\pi n$, and this value may be substituted for ω in the foregoing. If the rotation is uniform $\alpha = 0$, and $a = r\omega^2 = v^2/r = 4\,r\,\pi^2 n^2$.

Relation between Forces and Motion. The angular acceleration of a rotating body depends only on the moments of all the external forces acting on the body about the axis of rotation and on the moment of inertia of the body about the same axis. If ΣM = the algebraic sum of moments of the forces, I the moment of inertia, m the mass of the body, W its weight, and k its radius of gyration with respect to the axis of rotation, then

$$\Sigma M = I\alpha = mk^2\alpha = (W/g)\,k^2\,\alpha$$

Let ω_1 and ω_2 = the angular velocities of a rotating body at time t_1 and a later time t_2; then according to the principle of angular impulse and momentum $\Sigma \int_{t_1}^{t_2} M\,dt = I\omega_2 - I\omega_1$. The left-hand member is the algebraic sum of the angular impulses, about the axis of rotation, of all the external forces for the time $t_2 - t_1$. If the moments of all the forces are constant during the time, the left-hand member becomes $\Sigma M\,(t_2 - t_1)$. According to the principle of work and energy $\Sigma \int_{s_1}^{s_2} F_t ds = 1/2\,I\omega_2^2 - 1/2\,I\omega_1^2$. The left-hand member is the algebraic sum of the works done on the body by all the external forces for the interval $t_2 - t_1$.

Centrifugal Force. Suppose that P is a particle of a body rotating with angular velocity ω, and let dm = mass of P, r = radius of the circle described by P. The resultant of all the forces acting on P may be resolved into two components, one along the radius r and one along the tangent to the path of P; the radial or normal component $= dmr\omega^2$ and acts from P toward the axis of rotation; the tangential component $= dmr\alpha$. The normal component is called the centripetal force on P; it (and the other component also) is exerted on P by other things (neighboring particles, usually). P reacts on the other things with a force whose radial and tangential components equal the two above but in the opposite directions. The normal component, $dmr\omega^2$ outward, is called the centrifugal force of P; the resultant of the centrifugal forces of all the particles is the centrifugal force of the body; in general the resultant of the centrifugal forces of all the particles is not a single force, but in certain common cases there is a single force resultant. Thus if m = mass of the body ($= W/g$), $\bar{r}$ the radius of the circle described by the center of gravity G (Fig. 139), n = revolutions per unit time ($\omega = 2\pi n$), and the body is homogeneous, then (a) when the body is symmetrical about a plane perpendicular to the axis of rotation, the centrifugal force $= m\bar{r}\omega^2$ and acts in the line OG; (b) when the body is symmetrical about a line parallel to the axis of rotation, the centrifugal force $= m\bar{r}\omega^2$ and it acts in the line OG; (c) when the body is symmetrical about a plane containing the axis of rotation, the centrifugal force $= m\bar{r}\omega^2$, acts in the plane of symmetry, is parallel to OG at a distance from any XY plane equal to $\left(\int dmxz\right) \div m\bar{r}$. If ω is in radians per second, n in revolutions per minute, $\bar{r}$ in feet, W (weight of body) in pounds, $\bar{v}$ (velocity of center of gravity) in feet per second, then centrifugal force in pounds is

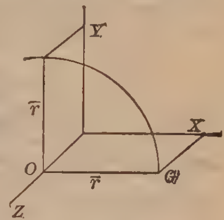

Fig. 139

$$\frac{W}{g}\bar{r}\omega^2 = \frac{W}{g}\frac{\bar{v}^2}{r} = \frac{W\bar{r}\,4\pi^2 n^2}{g\,3600} = \frac{W\bar{r}n^2}{2937} = 0.000341\,W\bar{r}n^2.$$

When a body is rotating with constant angular velocity, then the resultant of all the actual external forces acting on the body (weight, reactions of axle bearings, belt pulls, etc.) is equal and opposite to the resultant centrifugal force of the body. For example, in Fig. 140 PC is a rod supported by a pivot at P and by a cord AB so that the rod can be rotated about AP. During rotation it is under the action of the pivot pressure represented by components P', P'', and P''' (not shown), the pull of the cord T, the weight of the rod W, air resistance, and the driving force F (not shown). When the speed is constant, then the resultant of all these is as described under (c) above. If air resistance and pivot friction are negligible, no driving force is required to maintain speed, and $P''' = 0$; T, P', and P'' can be determined as follows (supposing $PC = 6$ ft., $PB = 5$ ft., $PG = 3$ ft., $ABP = 90°$, $APB = 30°$, $W = 100$ lb., and $n = 60$ rev. per min.): Since the resultant of T, P', P'', and W is horizontal, the sum of their vertical components $= 0$, or $T \sin 30 + P'' - 100 = 0$; the sum of their horizontal components $= m\bar{r}\omega^2$, or $T\cos 30° + P' = (100/32.2)1.5(2\pi\, 60/60)^2 = 184$; the resultant acts at a point Q whose distance above PX equals $\int dmxz \div m\bar{r} = 3.46$ ft., and $PQ = 4$ ft.; the moment sum of all the forces about $Q = 0$, or $- T \times 1 - 100 \times 1/2 + P' \times 3.46 + P'' \times 2 = 0$. The three equations solved simultaneously give $T = 157.5$, $P' = 47.5$, and $P'' = 22.5$ lb.

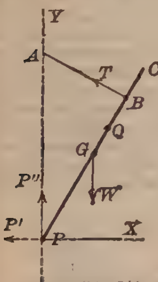

Fig. 140

33. Balancing

The reacting forces which support a rotating body depend in general not merely on the weight of the body and applied forces but also on the speed, the mass, and shape of the body; moreover, the components of the supporting forces which depend on the speed, mass, and shape change direction continually. Thus in the preceding illustration, when the rod is not rotating $T = 30$ lb., $P' = 26$ lb., and $P'' = 85$ lb., values quite different from those for motion. Or, consider a body weighing 100 lb. mounted on a shaft supported in bearings equally distant from the body and the center of gravity 2 in. from the axis of the shaft. When the system is at rest, the bearing pressures (due to the body) = 50 lb. each and act vertically, but when the system rotates at constant speed the bearing pressures have additional components each equal to 1/2 (100/32.2)(1/6) $4\pi^2 n^2$, n being the speed in revolutions per second. These components are said to be due to centrifugal action or centrifugal force, and will be called centrifugal components. If the center of gravity of the rotating system is in the axis of rotation, the system is said to have standing balance; such a system with shaft horizontal would not turn of itself from any position into which it might be placed (see Fig. 141). If the centrifugal force of the rotating system is zero, then the system is said to have or be in running balance. Standing balance is a necessary condition for running balance but not a sufficient one; thus in Fig. 141 there is standing but obviously not running balance. In fact the centrifugal forces of the two bodies are equal, (100/32.2) (1/6) $4\pi^2 n^2$ and (200/32.2) (1/12) $4\pi^2 n^2$ or $20.4\,n^2$, and opposite but not colinear and so they constitute a couple. The centrifugal components of the reactions of the bearings are therefore each $20.4\,n^2 \times 4 \div 9 = 9.07\,n^2$, and directed as shown, P and Q. If these bearings were not firmly supported, then the rotating system would wobble.

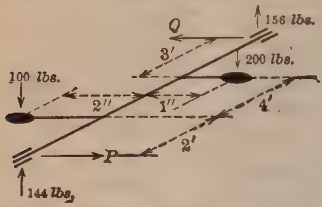

Fig. 141

Balancing a system out of running balance requires the rearrangement of the rotating material or the addition or removal of material. Car wheel-pairs are balanced experimentally by running them in spring-supported bearings and adding material here and there to the wheels, by trial, until steady running is secured. An unbalanced system can always be balanced by the addition of two bodies rotating about any two arbitrarily selected points on the axis of rotation; moreover the weights of these bodies and their exact positions relative to the rotating system can be computed provided the centrifugal force of the unbalanced system is known or if the unbalanced system can be divided up into parts the centrifugal forces of which are known. Thus suppose that it is desired to balance the body of weight W (Fig. 142 or 143), out of center a distance r (center of gravity to axis of shaft = r) by means of two bodies rotating about A and B.

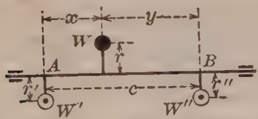

Fig. 142

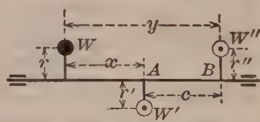

Fig. 143

In order that there may be running balance: (1) the centers of gravity of the three bodies and the axis of the shaft must be in a plane, and (2) the sun of moments of the centrifugal forces of the three bodies must be zero about any point in the plane. Condition (2) requires that the middle one of the three bodies, counting along the shaft, shall be alone on one side of the shaft, and further, that

$$W'r' = Wry/c \qquad \text{and} \qquad W''r'' = Wrx/c$$

where W' and W'' are the weights of the balancing bodies rotating about A and B respectively and r' and r'' are their distances out of center. Any unit of weight and of distance may be used in these formulas.

For example, consider the balancing of the cranks and crank pins on a pair of locomotive drive wheels. In Fig. 144 the cranks are set 90° apart, the planes of rotation of their centers of gravity are 62 in. apart, and the planes of rotation of the centers of gravity of the pins are 72 in. apart; for each crank pin $W = 25$ lb. and $r = 12$ in.; for the unbalanced part of each crank $W = 30$ lb. and $r = 5$ in.

The counter (or balancing) weights are to rotate about A and B with radii $= r$ inches say. To balance P_1 requires a_1 and b_1 (see figure), such that for a_1, $Wr = (25 \times 12)\ 66/60 = 330$, and for b_1, $Wr = (25 \times 12)\ 6/60 = 30$ in-lb.; similarly to balance P_4 requires a_4 and b_4, and for a_4, $Wr = 30$ and for b_4, $Wr = 330$ in-lb. To balance P_2 requires a_2 and b_2 and so that for a_2, $Wr = (30 \times 5)\ 61/60 = 152.5$ in-lb and for b_2, $Wr = (30 \times 5)\ 1/60 = 2.5$ in-lb.; similarly to balance P_3 requires a_3 and b_3, and for a_3, $Wr = 2.5$ and for b_3, $Wr = 152.5$ in-lb. These eight bodies a_1, b_1, a_2, b_2, etc., would balance the cranks and pins; it remains to find two substitutes for the weight. Now a_1 and a_2 can be combined, giving $Wr = 330 + 152.5 = 482.5$; a_3 and a_4, giving $2.5 + 30 = 32.5$; b_1 and b_2, giving 32.5; and b_3 and b_4, giving 482.5 in-lb. The centrifugal forces of a' and a'' (whose $Wr = 482.5$ and 32.5) have a simple resultant, and a single body whose centrifugal force is identical with that resultant can be found thus: treat 482.5 and 32.5 as though they were forces acting from A to a' and A to a'' and find their resultant; it is 484 in-lb. directed along Aa, the angle $a'Aa$ being nearly 4°; then any body the radius to whose center of gravity lies along Aa and whose $Wr = 484$ in-lb. will serve as one counterweight. In a similar manner the other counterweight may be determined; its $Wr = 484$ and its radius lies along Bb. (It should be noted that the effects of coupling and connecting rod are not considered.)

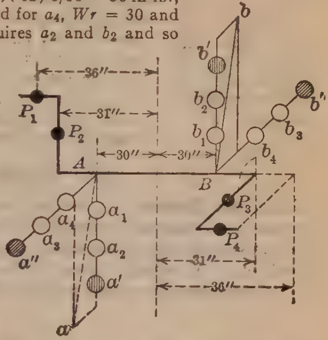

Fig. 144

34. Pendulums

The Compound or Physical Pendulum is a body suspended on a horizontal axis so it can oscillate freely under the influence of gravity. Let G (Fig. 145) be the center of gravity of the pendulum, O the center of suspension, $a = OG$, and $k =$ the radius of gyration of the pendulum with respect to its axis of suspension, and $T =$ the time of one oscillation (from one extreme position of the pendulum to the opposite extreme position); then if the oscillation is small, $T = \pi\sqrt{k^2/ag}$. If $\alpha =$ the maximum angle which OG makes with the vertical, that is, if 2α is the complete angle described by the pendulum in one oscillation, then more correctly,

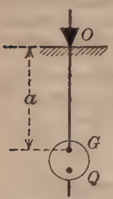

Fig. 145

$$T = \pi\sqrt{\frac{k^2}{ag}}\left[1 + (1/2)^2 \sin^2\frac{\alpha}{2} + (1/2\cdot 3/4)^2 \sin^4\frac{\alpha}{2} + \cdots\right]$$

If $\alpha = 8°$, the bracket is only 1.00122 and nearer unity for smaller values of α. T for a given pendulum is hence practically independent of α, if α is small; that is, for small angles a pendulum is practically isochronous. The point Q in the line OG extended and $OQ = k^2/a$ is called the center of oscillation corresponding to the center of suspension O; it is also called center of percussion (Art. 37). The centers of suspension and oscillation are interchangeable, that is, the times of oscillation about axes through O and Q are equal. This property is made use of in pendulum determinations of g as follows: the pendulum is made so that it can be suspended from two points O at a known distance d apart; then the times of oscillation for the two points O are compared and by trial are made equal by shifting a weight along the stem of the pendulum; either of these points then is the center of oscillation for the other as center of suspension, and $d = k^2/a$; then $g = \pi^2 d/T^2$.

A Simple Pendulum is a very small sphere supported by a cord; its length is the distance from the center of the sphere to the point of suspension. A physical and a simple pendulum whose times of oscillation are equal are said to be equivalent. A seconds pendulum is one whose time of oscillation is one second. The length of the equivalent seconds pendulum at New York is 39.101 in.

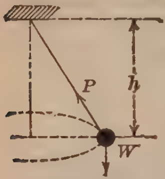

Fig. 146

The Conical Pendulum consists of a body suspended from a fixed point and so that it can rotate about a vertical axis through the point. The height of the pendulum h (Fig. 146) depends only on the angular velocity; thus for n revolutions per unit time $h = g/4\,\pi^2 n^2$. If g is taken as 32.2 ft. per sec. per sec., then n must be taken in rev. per sec.; h will be in feet. If n is less than $\sqrt{g/l} \div 2\,\pi$, the ball will not fly out or remain in any deflected position. If P = tension in the cord, l = length of pendulum, W = weight, then $P = Wl\,4\,\pi^2 n^2/g$.

35. Uniplanar Motion

Uniplanar Motion is a motion such that each point of the moving body remains at a constant distance from a fixed plane. Examples: motion of the connecting rod of a steam engine, rolling of a wheel, rotation of a fly wheel, etc. All straight lines of the body which are parallel to the plane of the motion describe equal angles during any displacement of the body, and

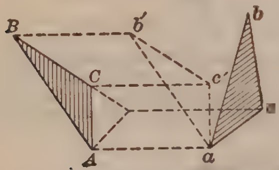

Fig. 147

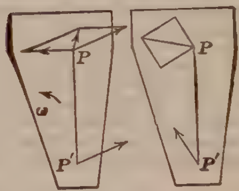

Fig. 148

by angle described by the body is meant the angle described by any of the lines mentioned. The angular velocity of the body at any instant is the rate at which it is describing angle then. The angular acceleration at any instant is the rate at which its angular velocity is changing then. Any displacement may be accomplished by a translation of the body which will bring one line of it which is perpendicular to the plane of the motion into final position followed by a rotation of the body about that line into final position. Thus the displacement of a body from ABC to abc (Fig. 147) may be accomplished by a translation from ABC to $ab'c'$ followed by a rotation about a into position abc. The amount of translation depends on the line of the body selected as axis of the rotation; the amount of the rotation does not. The actual state of (uniplanar) motion of a body at any instant may be regarded as consisting of two components, a translational and a rotational. And the velocity v of any point P of the body (Fig. 148) may be regarded as the resultant of two velocities, one the actual velocity v' of a second point P', and the other the velocity of P about (relative to) P', $r\omega$, where ω = angular velocity of the body and $r = PP'$. Likewise the acceleration a of P may be regarded as the resultant of the actual acceleration a' of P' and the acceleration of P about (relative to) P'.

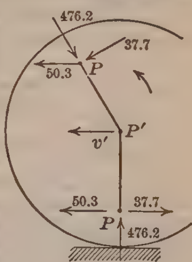

Fig. 149

For example, when a wheel 8 ft. in diameter rolls uniformly at 2 rev. per sec., or $\omega = 12.56$ radians per sec., then taking P' at the center (Fig. 149), $v' = \pi\,8 \times 2$

$= 50.3$ ft. per sec., and $a' = 0$; the velocity of a point P 3 ft. from the center relative to P' is 37.7 ft. per sec. directed as shown, and the actual velocity of P is the resultant of 50.3 and 37.7; the acceleration of P relative to P' is $3\ \omega^2 = 476.27$ ft. per sec. per sec. (see page 122) directed as shown, and since $a' = 0$, the acceleration of P is 476.2.

Uniplanar Displacement can also be accomplished by a single rotation about some line. Thus the displacement from ABC to abc (Fig. 147) can be made by a rotation about the intersection of the perpendiculars to Aa and Bb at their centers. Any uniplanar motion may be regarded as a continual rotation about a line which in general is continually shifting its position in space and in the body or its extension. This line is called **instantaneous axis** (of rotation), and any point of it the instantaneous center. In uniplanar motion, the velocity of any point of the body at any instant equals the product of the angular velocity of the body then and the distance of the point from the instantaneous center, and the direction of the velocity is perpendicular to the line joining the point and the instantaneous center; the velocities of different points of the body are as their distances from the instantaneous axis. The instantaneous center may be determined when the directions of the velocities of any two points of the body (the line joining which is parallel to the plane of the motion) are known, provided these directions are not parallel, as follows: at each of the two points draw a line parallel to the plane of motion and perpendicular to the direction of the velocity of that point, and determine their intersection; this is the instantaneous center.

For example, consider the motion of the connecting rod of an engine. The velocity of the crank end of the rod is perpendicular to the crank and the velocity of the other end is parallel to the guides; hence for any particular position of the rod the instantaneous center is at the intersection of the crank (radius) extended and a line perpendicular to the guides at the guide end of the rod.

The relation between the external forces and the motion produced is expressed thus: take x and y axes of reference parallel to the plane of motion; then

$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma M = I\alpha = mk^2\alpha = (W/g)k^2\alpha$$

wherein ΣF_x and ΣF_y denote the sums of the x and y components of all the external forces, $\bar{a}_x$ and $\bar{a}_y$ the x and y components of the acceleration of the center of gravity, α the angular acceleration of the body, m its mass, W its weight, ΣM the algebraic sum of the moments of external forces about an axis through the center of gravity and perpendicular to the plane of motion, I the moment of inertia, and k the radius of gyration of the body with respect to the same axis. For example, consider a cylinder of radius r and weight W rolling down an inclined plane. The forces acting on the cylinder are its weight W and the reaction of the plane regarded as replaced by its two components N and F (Fig. 150); taking the x axis along the plane and the y normal to it, the three formulas become $(W \sin\phi - F) = (W/g)\bar{a}$, $N - W\cos\phi = 0$, and $Fr = 1/2(W/g)r^2\alpha$, $1/2(W/g)r^2$ being the moment of inertia of the cylinder with respect to its axis. (It is assumed that there is no indentation of the roadway, that is, no rolling resistance.) If there is no slipping, then $\bar{a} = r\alpha$, which equation combined with the first and third above gives $\bar{a} = 2/3\ g \sin\phi$, $\alpha = 2/3\ (g/r) \sin\phi$, and $F = 1/3W \sin\phi$; also $N = W\cos\phi$.

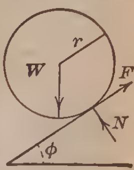

Fig. 150

The Kinetic Energy of translation and rotation combined is $1/2\ m\bar{v}^2 + 1/2\ \bar{I}\omega^2$, where m denotes the mass of the body, $\bar{v}$ the velocity of its center of gravity, $\bar{I}$ its moment of inertia with respect to a line through the center of gravity and perpendicular to the plane of motion, and ω its angular velocity. The first term is sometimes called the translational and the second the rotational part of the kinetic energy. It is also given by $1/2\ I\omega^2$, where I denotes the moment of inertia of the body with respect to its instantaneous axis (at the instant for which the energy is to be figured) and ω is the angular velocity.

For example, take a cylinder of radius r and weight W rolling at n revolutions per unit time: $m = W/g$, $v = 2\pi rn$, $\bar{I} = 1/2(W/g)r^2$ (see page 110), and $\omega = 2\pi n$; hence the kinetic energy $= 3(W/g)\,\pi^2 r^2 n^2$, one-third of which is rotational. Also $I = \bar{I} + (W/g)r^2 = 3/2(W/g)r^2$; hence according to the second formula the kinetic energy is $1/2(3W/2\,g)\,r^2\,4\,\pi^2 n^2 = 3\,(W/g)\,\phi^2 r^2 n^2$ as before.

36. Three-dimensional Motion

Any motion of a body may be regarded as consisting of two components: one, a translation equal to that of the center of gravity and the other a rotation about some axis through the center of gravity. These motions may be said to be produced independently by the forces acting on the body; thus (a) the acceleration of the center of gravity is the same as if the whole mass were concentrated at the center of gravity and acted upon by forces equal in magnitude to and same in direction as the actual external forces; and (b) the angular acceleration is the same as if the center of gravity were fixed and the actual external forces applied. The reasonableness of this will be seen from the following: imagine each force acting on the body replaced by a force acting at the center of gravity G and a couple; the resultant of all the forces acting at G is a single force R, and the resultant of all the couples is a single couple C; R cannot turn the body but gives it a motion of translation only, and C cannot move G but merely turns the body about some line through G. In general C does not cause turning about a line perpendicular to the plane of C, only so if the plane of C is perpendicular to one of the principal central axes of the body (see page 123). To determine the acceleration of the center of gravity, take fixed x, y, and z axes outside the body and resolve all external forces F_1, F_2, etc., into x, y, and z components; then

$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma F_z = m\bar{a}_z$$

m denoting the mass of the body. To determine the angular acceleration of the body, take moments of all the forces F_1, F_2, etc., about the three central principal axes; calling the sums of the moments about these axes ΣM_1, ΣM_2, and ΣM_3, the components of the angular acceleration α_1, α_2, and α_3, the components of the angular velocity ω_1, ω_2, and ω_3, then

$$\Sigma M_1 = I_1\alpha_1 + (I_3 - I_2)\,\omega_2\omega_3, \qquad \Sigma M_2 = I_2\alpha_2 + (I_1 - I_3)\,\omega_3\,\omega_1,$$
$$\Sigma M_3 = I_3\alpha_3 + (I_2 - I_1)\,\omega_1\,\omega_2$$

wherein I_1, I_2, and I_3 denote the three central principal moments of inertia of the body. In any motion of a body, the kinetic energy may be computed in two parts: (1) the kinetic energy of the whole body moving with a velocity equal to that of the center of gravity, and (2) the sum of the kinetic energies of the constituent particles of the bodies due to their velocities relative to the center of gravity.

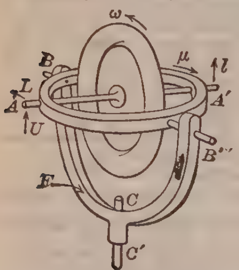

Fig. 151

Gyrostat (Fig. 151). The wheel ω and axle can rotate about AA', the supporting ring $ABA'B'$ can rotate about BB' and the supporting frame F can rotate about CC'. If the disk or wheel is spinning rapidly in the direction indicated by ω and an upward force U is applied at A, the axle AA' will not rotate about BB' but about CC' in the direction indicated by the arrowhead u; a downward force causes rotation in the opposite direction. A horizontal force L to the left would cause the axle AA' to rotate not about CC' but about BB' in the direction indicated by the arrow l; a force to the right would cause rotation in the opposite direction. These rotations of the axle of spin are called precessional motions, the wheel being said to precess. The precession of the axle of spin, AA', tends to convert the existing spin of the wheel into a spin about the axis of the applied moment and in the direction required by that moment.

A Mono-rail Car, as its name implies, runs on a single rail; and the Brennan car is wholly above the rail. Stability of this car is furnished by two gyrostats mounted in the car as shown in plan and elevation (Fig. 152). W is one of the gyrostat wheels (W' the other); it is mounted rigidly on the axle A, the bearings of which are on the frame or case C. (These cases wholly enclose the gyrostat wheels, and a vacuum is maintained in the cases to reduce the windage of the wheels; the wheels are driven electrically—indeed the wheels are motor armatures—in opposite directions.) The case C has trunnions T and T supported in bearings on the double frame F, which also supports the other gyrostat case C'. The double frame F is supported on an axle X parallel to the rail, on which the whole gyrostat system can turn. To the two upper trunnions T and T' are keyed two spur-toothed segments which mesh at G; thus the cases C and C' can rotate on their trunnions only together, in opposite directions, and equal amounts. R and R' are wheels or rollers which turn on bearings fixed to the cases C and C'. Rigidly fastened to the car are four shelves M, M', N, and N'; as shown in the plan these do not extend beyond the line OO'. When for any reason the axles A and A' are at OP and $O'P'$ say, or OQ and $O'Q'$, and the car is tipping either way relative to the gyrostat system, then one shelf, and only one, will come up against an axle end or roller; for instance when the axles are at OP and $O'P'$ and the car tips clockwise, then the shelf M comes up against the axle A.

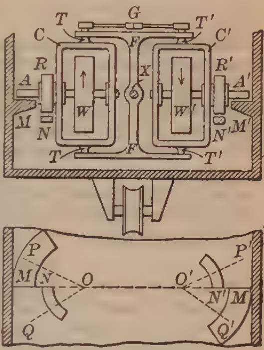

Fig. 152. (After Perry)

When the car is running straight, erect, and balanced, the axles A and A' are in line, across the car, and this is the normal position of the gyrostats. Now suppose a disturbance, as a wind pressure on the left side, tips the car clockwise; the shelf M comes up against the axle A and causes precession of A away from the reader, which continues as long as there is contact at A. The rapidly rotating axle slips on the shelf M and is itself subjected to a friction which tends to hurry the precession, the effect of which is to increase the pressure on the shelf. This pressure down arrests the tipping and pushes the car back against the wind to the neutral position, in which the wind and gravity moments are balanced. This neutral position is reached with a certain momentum, and the car is carried past it, the shelf M recedes from A with the axles of the gyrostats in positions as OP and $O'P'$, and eventually the shelf N' comes up under the roller R'. The upward pressure on the roller causes precession of A' toward the reader, which continues as long as there is contact at the shelf N'. The friction of the roller on its axle tends to retard this precession, and the correcting pressure of the roller on the shelf N' is not so great as that of the axle A on M (in similar circumstances), and the gyrostats are returned to the normal position with the car still tilted to the left of the neutral position. But the gyrostats reach the normal position with velocity and swing beyond it; the roller R' runs off the end of the shelf N' and the axle A' comes into contact with the shelf M'. The pressure of M' on A' causes precession of A' toward the reader, the friction on A' tends to hurry the precession, and A' reacts strongly on the shelf M', bringing it to its neutral position and beyond. Eventually M' recedes from A', the shelf N coming up against the roller R, with the gyrostats in the positions OQ and $O'P'$. The shelf N, like N', brings the gyrostats into normal position and then shelf M into play; and so the operations described are repeated. But the oscillations of the car and the swings of the gyrostats rapidly decay, and the car settles down into its neutral position quickly. Geared together and spinning in opposite directions, the two gyrostats work together in the above described actions. A single gyrostat might furnish the stability on a straight track, but two gyrostats are essential for running on curves. In Oct., 1909, Mr. Brennan operated a mono-rail car 40 ft. long, weighing 22 tons, and designed to carry a load of 10 to 15 tons; it is furnished with an 80 and a

20-hp. petrol-electric set for running the car and the gyrostats; each gyrostat weighs about 3/4 ton, is 3 ft. 6 in. in diameter, and is run at 3000 r.p.m.; the vacuum of the gyrostat cases is equivalent to about 1/2 to 5/8 in. of mercury. The car was run on a curve of 105-ft. radius at 7 mi. per hr., and over reverse curves of 35-ft. radius without "appreciable disturbance" of the level of the car floor.

37. Impact

Direct Central Impact is such that before collision the centers of gravity of the bodies move along the same straight line. Central impact is collision during which the forces which the bodies exert on each other are directed along the line joining the centers of gravity. In any collision, the forces which the two bodies exert on each other are equal and opposite at each instant; hence the total impulses of these forces during the collision are equal and opposite, and according to the principle of impulse and momentum the changes in the momentum of the bodies produced by the collision must be equal and opposite; or, otherwise stated, the total momentum of the two bodies is unchanged by the collision. Or, for direct central impact:

$$m_1 v_1 + m_2 v_2 = m_1 V_1 + m_2 V_2$$

wherein m_1 and m_2 = the masses of the bodies, v_1 and v_2 their velocities before and V_1 and V_2 their velocities after the collision; but in numerical substitution velocities in one direction are given the same sign and those in the other direction the opposite sign.

Experiments on direct central impact of spherical bodies have shown that the relative velocities of spheres after impact are always less than before the impact and that these relative velocities are opposite in direction. The ratio of the relative velocities after impact to that before impact is called coefficient of restitution; it seems to depend only on the material of the impinging spheres. For glass the coefficient is 15/16, for steel and cork 5/9, ivory 8/9, wood about 1/2, clay and putty 0. If e = the coefficient, then

$$(v_1 - v_2) = -e\,(V_1 - V_2)$$

This equation and the preceding one solved simultaneously show that

$$V_1 = v_1 - \frac{(1+e)\,m_2}{m_1 + m_2}(v_1 - v_2) \qquad V_2 = v_2 - \frac{(1+e)\,m_1}{m_1 + m_2}(v_2 - v_1)$$

During impact there is, in general, loss of kinetic energy; the loss is $1/2\,(v_1 - v_2)^2 (1 - e^2)\,m_1 m_2/(m_1 + m_2)$. Bodies for which $e = 0$ are said to be inelastic; and those for which e is nearly 1 are said to be nearly perfectly elastic. When a sphere is dropped on a horizontal surface of a large body from a height h, and if H = the height of rebound, then $H = e^2 h$. This equation furnishes a means of computing e.

Force of a Blow. By this is meant the actual pressure between two bodies in collision. This pressure or force varies during the collision from zero at the beginning up to a maximum value and then down to zero at the end. Curves showing how the force varies with respect to the time and with respect to the displacement of the point of contact during the collision are quite dissimilar. The average values of the force as shown by these two curves are unequal. The first is called the time-average and the second the space-average force of the blow. Let F_t and F_s denote these averages respectively and t and s the total time and displacement; then $F_t t$ = the impulse exerted on either body, and if the impact is direct and central, $F_s s$ = the work done on either body by the force of the blow. When any object is struct by a hand-hammer for example, the momentum of the hammer is changed; calling the change M, then $F_t t = M$, or $F_t = M/t$. That is, for a given change in momentum the time-average force of a blow varies inversely as the time of the impact. The energy of the hammer is also changed; calling this change E, if the energy dissipated (in vibration of the hammer) is negligible, $F_s s = E$, or $F_s = E/s$. That is. for a given change in energy the space-average force of a blow varies inversely as the space through which the force acts. (It is assumed in the preceding that no other force than that of the blow affects the momentum and the energy changes)

A pile-driver hammer of weight W falling a height h to a pile which it drives a distance s is given an amount of energy equal to $W(h + s)$ by gravity before the hammer is arrested. This energy is delivered to the pile and is wasted against resistance between earth and the pile, in vibration of the pile and adjacent earth. Calling the space-average resistance R and neglecting the energy lost in vibration, then $Rs = W(h + s)$, or $R = W(1 + h/s)$. This formula is in use for computing the bearing capacity of a pile; h being the drop of the hammer and s the penetration of the pile in the last blow (or average penetration for last few blows), W weight of hammer, and R ultimate bearing capacity, safe capacity being generally taken as 1/5 or 1/6 of R. As 1 is small compared to h/s, the formula may be written, safe load = 1/5 or 1/6 Wh/s.

Ballistic Pendulum. This is a device formerly used to measure the velocity of a projectile; it is essentially a pendulum with a massive bob into which the projectile may be fired and arrested (Fig. 153). Let v = velocity of the projectile, m its mass, M the mass of the pendulum ($= W/g$, W denoting its weight and g acceleration of gravity), k its radius of gyration with respect to the axis of suspension (for determination of which see below), a the distance from the axis to the center of gravity and r that to the path of the projectile, and θ deflection of pendulum caused by projectile; then

$$v = \sqrt{2\,ga\,(1 - \cos\theta)}\,(m + Mk^2/r^2) \div (mk/r)$$

If the weight of the bob is large compared to that of the suspending rod, then $k/r = 1$ nearly. If a pendulum in its vertical position is struck a horizontal blow through the center of gravity and perpendicular to the axis of suspension, the whole pendulum tends to move in the direction of the blow. But such translation is prevented by the support which exerts a sudden reaction opposite to the blow. If the pendulum is struck below the center of gravity the blow tends to produce a translation in its direction and a rotation about the center of gravity. There would still be a sudden reaction of the support on the pendulum to overcome the tendency to the motion there unless the translation and the rotational velocities of the center of suspension, which the blow tends to produce, are equal. That point of impact for which the blow produces no sudden reaction at the center of suspension is called **center of percussion**; it coincides with the center of oscillation (see page 125). Its distance from the center of suspension $= k^2/a$.

Fig. 153

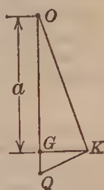

Fig. 154

The distance k^2/a can be determined experimentally thus: let the pendulum oscillate and ascertain the time T of one swing, extreme to extreme position; then $k^2/a = gT^2/\pi^2$; if g is taken in feet per second per second and T in seconds, then k^2/a is in feet. Let k_G be the radius of gyration of the pendulum with respect to a line through its center of gravity and parallel to the axis of suspension. When k_G is known the center of percussion Q (Fig. 154) can be located graphically thus: at G draw GK perpendicular to OG and make $OG = k_G$; join O and K and draw a perpendicular to OK at K; Q is the intersection of this perpendicular and OG extended.

The Moment of Inertia I of an irregular or nonhomogeneous body with respect to a specified line can be determined best experimentally; thus proceed as described above to determine k^2/a, then $I = (W/g)k^2 = WT^2/a\pi^2$. If the specified line passes through the center of gravity of the body then, since the body would not oscillate about such line, proceed thus: determine I about any parallel line distant a from the center of gravity, then the desired moment of inertia $= I - (W/g)a^2$. If it is impossible or inconvenient to suspend the body at the specified line, then suspend and oscillate it about any line parallel to the specified line, and determine I as explained about such parallel line; calling the distances of the parallel and specified lines from the center of gravity a and b respectively, then the desired moment of inertia $= I - (W/g)a^2 + (W/g)b^2 = I - (a^2 - b^2)W/g$.

FRICTION

38. Static Friction

Static Friction, or **Friction of Rest** is the friction between two bodies when there is a tendency to, but not actual, slipping of one relative to the other. Let P (Fig. 155) = a pull applied to a body A which is supported by a body B, R = the reaction of B on A; then the component F of R along the surface of contact is the friction, and the component N perpendicular to the surface is the normal pressure. So long as there is no slipping $F = P$, but the greatest value of F obtains when slipping impends; this value of F is called limiting friction. The coefficient of static friction for two bodies is the ratio of their limiting friction to the accompanying normal pressure; it is denoted by f. The angle between R and N changes as F changes, and its greatest value obtains when motion impends; this value is called the angle of friction for the two bodies, and it will be denoted by ϕ. When a body A is supported on an inclined plane B, and no forces act on A except its own weight and the supporting force, then that inclination of the plane to the horizontal which will just cause slipping of A is called the angle of repose for A and B; it also will be denoted by ϕ because the angles of repose and friction for two bodies are equal. Also the tangents of these angles equal the coefficient of friction,

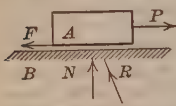

Fig. 155

$$F \leqq fN \qquad \therefore f = \tan\phi$$

Suppose the body A (Fig. 156) is supported by a body B, let Q denote the resultant of all the forces acting on A excepting the supporting force; and suppose that the line of action of Q cuts the surface of contact at a; then the cone with vertex at a, axis normal to the contact surface and apex angle equal to double the angle of friction, is called a cone of friction for the bodies A and B. If Q falls within the cone, motion will not ensue for any value of Q, but if without, then Q will move A.

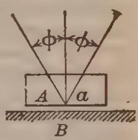

Fig. 156

The coefficient of static friction for two bodies A and B may be determined thus: (1) Place A on B as in Fig. 155 and determine the pull P which will just start A; then $f = P$ divided by the weight of A. Or (2) tilt B and determine the inclination at which gravity will start A down; then f = the tangent angle of that of inclination. In either method, several determinations must be made to obtain a fair average value of the coefficient.

Coefficients of Static Friction depend on the nature of the materials, character of rubbing surfaces, and kind of lubricant, if any is used. Early experimenters reported (Coulomb 1871, Rennie 1828, Morin 1834, and others) that the coefficient is independent of the intensity of normal pressure; although this announcement was clearly subject to the limitation of the range of the experiments performed, yet it was generalized and long accepted as a universal law of friction. But the universality of the law has been questioned; Morin himself pointed out that length of the time of contact of the two bodies influences the coefficient and obviously the coefficient changes when the intensities of pressure get so high as to affect the character of the surfaces in contact. Messiter and Hanson report practical constancy of coefficiency for yellow pine and spruce (Eng. News, 1895, vol. 33, p. 322), the first for a range from 100 to 1500 lb. per sq. in. and the second for a range from 100 to 800. They report for planed or sandpapered yellow pine: $f = 0.25$–0.32; average 0.29, for 100 to 1000 lb. per sq. in.; for spruce: $f = 0.18$–0.53; average 0.42, for 100 to 600 lb. per sq. in. The variation depends on relation of grain of wood to direction of slide.

Coefficients of Static Friction

Compiled by Rankine from Experiments by Morin and others

Dry masonry and brickwork	0.6 to 0.7	Masonry on dry clay	0.51
Masonry and Brickwork, with damp mortar	0.74	Masonry on moist clay	0.33
Timber on stone	about 0.4	Earth on earth	0.25 to 1.0
Iron on stone	0.7 to 0.3	Earth on earth, dry sand, clay, and mixed earth	0.38 to 0.75
Timber on timber	0.5 to 0.2	Earth on earth, damp clay	1.0
Timber on metals	0.6 to 0.2	Earth on earth, wet clay	0.31
Metals on metals	0.25 to 0.15	Earth on earth, shingle and gravel	0.81 to 1.11

Coil Friction. When a rope wholly or partially encircles a pulley or drum (Fig. 157) the tensions P_1 and P_2 may be quite different on account of the friction on the rope. The difference is greatest when slipping impends; if α = arc of contact in radians, ϵ = Napierian base = 2.718, the maximum ratio $P_2/P_1 = \epsilon^{f\alpha}$.

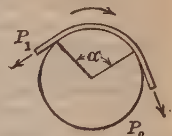

Fig. 157

Maximum Ratio P_2/P_1 (Slipping Impending)

$\alpha/2\pi$	f = 1/4	f = 1/3	f = 1/2	$\alpha/2\pi$	f = 1/4	f = 1/3	f = 1/2
0.1	1.17	1.23	1.37	0.7	3.00	4.33	9.00
0.2	1.37	1.51	1.87	0.8	3.51	5.34	12.34
0.3	1.60	1.87	2.57	0.9	4.11	6.58	16.90
0.4	1.87	2.31	3.51	1.0	4.81	8.12	23.14
0.5	2.19	2.85	4.81	2.0	23.1	66.0	535.5
0.6	2.57	3.51	6.59	3.0	111	535	1239.0

Belting. The formula $P_2/P_1 = \epsilon^{f\alpha}$ applies here also. Single (thickness) leather belt varies from 3/16 to 7/32 in. thick; double leather (two thicknesses glued) belt varies from 3/8 to 1/2 in. thick. Cement splices are nearly as strong as the belt; lace joints average 70% as strong as the belt; metal fastenings generally less. Working strengths of 400 lb. per sq. in. for endless (cement spliced) belts and 200 to 300 for laced belts are safe, but smaller values are advised by F. M. Taylor (Trans. A. S. M. E., 1894, vol. 15, p. 204). These figures correspond to 100 lb. per inch of width for cemented single belt, 50 to 75 for laced single belt, and twice these for double belt.

The horsepower which a belt transmits is given by $(T - Z)Cv/550$, wherein T = total tension in tight side of belt in pounds, v = velocity of belt travel in feet per second, Z = a quantity depending on centrifugal action (table below), C = a quantity depending on coefficient of friction and angle of wrap on the pulley (table below). The formula is Nagle's (Trans. A. S. M. E., 1881, vol. 2, p. 91) but modified by Kimball and Barr.

For a leather belt which weighs 0.035 lb. per cu. in. when:

v (ft. per sec.) =	50	60	70	80	90	100	110	120	130	140	150
Z (lb.) =	32.5	47.0	64.2	83.4	105	130	158	188	220	256	293

Values of C in Formula hp. = (T − Z)Cv/550

f \ α	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
0.15	0.210	0.230	0.250	0.270	0.288	0.307	0.325	0.342	0.359	0.376
0.25	.325	.354	.381	.407	.432	.457	.480	.503	.524	.544
0.35	.423	.457	.489	.520	.548	.575	.600	.624	.646	.667
0.45	.507	.544	.579	.610	.640	.667	.692	.715	.737	.757
0.55	.578	.617	.652	.684	.713	.739	.763	.785	.805	.822
1.00	.792	.825	.853	.877	.897	.913	.927	.937	.947	.956

39. Sliding Friction

Kinetic Friction. Friction of Motion, or sliding friction, is the friction between two bodies when sliding actually occurs. The coefficient of kinetic friction for two bodies is the ratio of the kinetic friction to the corresponding normal pressure between them. One of the so-called laws of friction states that the kinetic coefficient is less than the static coefficient and implies that there is a sudden or abrupt change in the values of the coefficients. Experiments by Jenkin and Ewing (Phil. Trans. Roy. Soc., 1877, vol. 167, Part 2) on the kinetic coefficients at speeds as low as 0.0002 ft. per sec., about 3/4 ft. per hour, lead them to conclude that "it is highly probable that the kinetic coefficient gradually increases when the velocity becomes extremely small, so as to pass without discontinuity into the static coefficient." Experiments by Kimball (Am. Jour. Sci., 1877, vol. 13, p. 353) also indicate that there is no abrupt change from static to kinetic coefficient. Moreover they show that the kinetic coefficient may be greater than the static. For dry surfaces, the kinetic coefficient probably decreases progressively from the value of the static coefficient, as the velocity increases as indicated by the following table from Galton and Westinghouse's experiments (Proc. Inst. Mech. Engrs., 1879).

Coefficients of Friction at Various Speeds for Cast-iron Brake Shoes and Steel-tired Wheels

Velocity		Coefficients			Number of tests
Mi. per hr.	Ft. per sec.	Maximum	Minimum	Mean	
0+	0+	....	....	0.330	
5−	7−	0.340	0.156	.273	20
7.5	11	.325	.123	.244	28
10	14.5	.281	.161	.242	54
15	22	.280	.131	.223	78
20	29	.240	.133	.192	69
25	36.5	.205	.108	.166	70
30	44	.196	.098	.164	94
35	51	.197	.087	.142	80
40	59	.194	.088	.140	70
45	66	.179	.083	.127	77
50	73	.153	.050	.116	55
55	81	.136	.060	.111	67
60	88	.123	.058	.074	12

The foregoing given coefficients were obtained from measurements taken very soon after application of brakes.

The rubbing of dry surfaces abrades them and decreases the kinetic coefficient. The following table shows how this coefficient changed with lapse of time after rubbing began.

Coefficient of Friction as Affected by Time of Rubbing

Cast-iron brake shows on steel-tired wheels

Miles per hour	Time after applying brakes				
	0+	5 sec.	10 sec.	15 sec.	20 sec.
20	0.182	0.152	0.133	0.116	0.099
27	.171	.130	.119	.081	.072
37	.152	.096	.083	0.69	
47	.132	.080	.070		
60	.072	.063	.058		

The discrepancies between the two tables are due in part to the fact that the values at time 0+ in the second table are averages based on comparatively few experiments. In the second table preceding there is a wide variation from the mean value of the coefficient for each velocity. The variation was due in part at least to differences in intensities of pressure in the various runs of each velocity. In general, the coefficients decrease with increase of intensity.

Coefficients of Friction for Wheels Skidded on Rails

Galton-Westinghouse Experiments

Wheels skidded on rails gave much smaller coefficients than brake shoes, as shown by the adjacent table.

Approximate velocity		Steel tire on	
Ft. per sec.	Mi. per hr.	Steel rail	Iron rail
0+	0+	0.242	0.247
10	6.8	0.88	.095
20	13.6	.072	.073
40	27.3	.070	
50	34.1	.065	.070
60	40.9	.057	
70	47.7	.040	.060
80	54.5	.038	
88	60	.027	

Kinetic Coefficients for Brake Shoes

From Experiments by Ernest Wilson (Eng. News, 1909, vol. 62, p. 736)

Materials	Pressure, Lb. per sq. in.	Vel., Mi. per hr.		Lubrication
		7	15	
Cast iron....	10	0.43	0.37	none
Cast iron....	40	.36	.30	none
Oak........	10	.60	.55	none
Oak........	40	.43	.40	none
Poplar......	10	....	.72	none
Poplar......	40	....	.53	none
Cast iron....	20	.32	.28	water
Cast iron....	80	.30	.26	water
Oak........	40	.037	.032	water
Oak........	120	.073	.055	water
Poplar......	40	.041	.038	water
Poplar......	120	.070	.053	water

The adjacent table furnishes additional information on the variation of the kinetic coefficient with the intensity of pressure and on the velocity; also on the influence of water lubrication.

Coefficients of Kinetic Friction (rough averages)

Compiled by Rankine from Experiments by Morin and others

Wood on wood, dry	0.25 to 0.50	Leather on oak	0.27 to 0.38
soapy	.2	Leather on metals, dry	.56
Metals on oak, dry	.5 to .6	wet	.36
wet	.24 to .26	greasy	.23
soapy	.2	oily	.15
Metals on elm, dry	.2 to .25	Metals on metals, dry	.15 to .2
Hemp on oak, dry	.53	wet	.3
wet	.33		

Pivots. Let W = load (lb.), μ = coefficient of kinetic friction, n = speed (r.p.m.), m = moment of the frictional resistance about the axis of the shaft (in-lb.), w = work done against friction per revolution (in-lb.), P = power lost through friction (hp.), dimensions as marked in figures (in.). Flat Pivot (Fig. 158): $m = 2/3\,\mu W r$; $w = 4/3\,\pi\mu W r = 4.187\,\mu W r$; $P = 4/3\,\pi\mu W r n/396\,000 = \mu W r n/94\,700$. Collar Bearing (Fig. 159): $m = 2/3\,\mu W\,(R^3 - r^3)/(R^2 - r^2)$; $w = 2\,\pi m$; $P = 2\,\pi m n/396\,000 = mn/63\,000$.

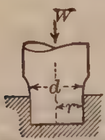

Fig. 158

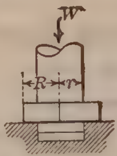

Fig. 159

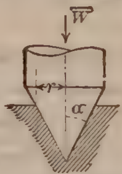

Fig. 160

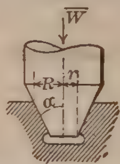

Fig. 161

Conical Pivot (Fig. 160): $m = 2/3\,\mu W r/\sin\alpha$; $w = 4/3\,\pi\mu W r/\sin\alpha$; $P = 4/3\,\pi\mu W r n/396\,000 \sin\alpha = \mu W r n/94\,700 \sin\alpha$. Frustrated Conical Pivot (Fig. 161): $m = 2/3\,\mu W\,(R^3 - r^3)/R^2 - r^2)\sin\alpha$; $w = 2\,\pi m$; $P = 2\,\pi m n/396\,000 = mn/63\,000$.

Journal Friction. The coefficient of journal friction is the ratio of the frictional resistance at the journal to the pressure between journal and bearing. The pressure is not uniformly distributed over the surface of contact between journal and bearing. By nominal (intensity) of pressure is meant the whole pressure divided by product of length and diameter of bearing.

For example, suppose a fly wheel and shaft weighing 6000 lb. are supported midway between two bearings, the diameter of the wheel is 8 ft. (or 96 in.), that of the journal is 6 in., and that a force of 10 lb. applied to the rim is necessary to overcome friction to maintain a constant speed. Then the frictional resistance is $(10 \times 48) \div 3 = 160$ lb., 80 at each journal; the pressure at each journal is 3000 lb., and the coefficient of journal friction is $160 \div 3000 = 0.053$. If the length of bearing is 10 in., the nominal (intensity) of pressure is $3000 \div (6 \times 10) = 50$ lb. per sq. in.

Energy Lost at Journals, Work Done Against Friction. Let W = total load on bearing (lb.), d = diameter of journal (in.), f = coefficient of journal friction, n = number of revolutions per minute; then

work done per revolution (ft-lb.) $= fW\pi d/12$
work done per minute (ft-lb.) $= fW\pi dn/12$
power lost (hp.) $= fW\pi dn/396\,000 = Wdn/126\,000$

Coefficients of Journal Friction depend on (1) the method of lubrication, (2) the lubricant, (3) its temperature, (4) the velocity of rubbing, and (5) the intensity of pressure (lb. per sq. in.) on the bearing.

(1) The relative intensities of friction for different methods of lubrication are as follows according to Tower (Proc. Inst. Mech. Engs.) and Goodmann (Mechanics Applied to Engineering):

Method	Tower	Goodman
Bath.	1.00	1.00
Saturated pad.	....	1.32
Ordinary pad.	6.48	2.21
Siphon.	7.06	4.20

Tower found that in bath lubrication the load is completely oil-borne, in other words that a continuous film separates journal and bearing. For any given pressure there seems to be a minimum velocity below which the oil film will not form. H. F. Moore also found (Am. Mach., 1903) that this pressure velocity relation is approximately $p = 7.47\sqrt{v}$, p being in lb. per sq. in. and v in ft. per min. Moore used mineral oil at 90° temperature. The friction then is wholly fluid friction and not solid. (2) In the following table the figures are based on mean actual frictional resistances at the surface of the journal at speed of 300 r.p.m. with all nominal loads from 100 to 310 lb. per sq. in.; the temperature was 90°; the journal was steel, 4 in. in diameter and 6 in. long; the bearing a gun-metal brass embracing somewhat less than half the circumference of the journal. The relative frictional resistances for various lubricants, according to Tower, are 1.00 for sperm oil, 1.06 for rape oil, 1.29 for mineral oil, 1.35 for lard oil and olive oil, 2.17 for mineral grease. Tower states also: "The numbers represent the relative thickness or body of the various oils, and also in their order, although perhaps not exactly in their numerical proportions, their relative weight-carrying power. Thus sperm oil, which has the highest lubricating power, has the least weight-carrying power; and, though the best oil for light loads, would be inferior to the thicker oils if heavy pressures or high temperatures were to be encountered." (3) The coefficient decreases

Coefficient of friction

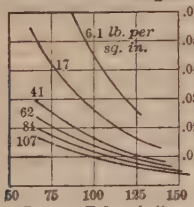

Fig. 162

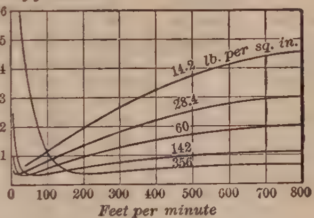

Fig. 163

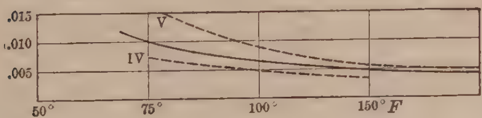

Fig. 164

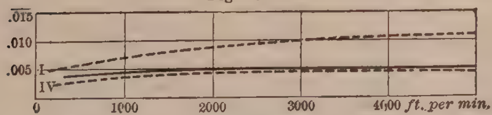

Fig. 165

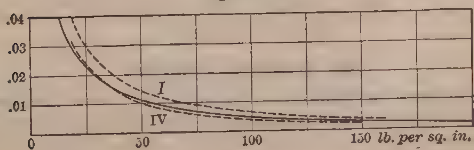

Fig. 166

with increased temperature (see Figs. 162 and 164). But if the temperature gets so high as greatly to lower the viscosity, the lubricant gets squeezed out and then the coefficient increases. (4) In general the coefficient increases with increase of speed (see Figs. 163 and 165 and accompanying explanations). But at the lower speeds the coefficient may decrease with increase of speed (see Fig. 163). (5) The coefficient decreases with increase of intensity of pressure (see Figs. 163 and 166). But the intensity may become so great that the lubricant gets squeezed out and then the coefficient increases and seizing occurs.

Figs. 162 and 163 show the relation between coefficient and temperature, and coefficient and velocity. Sellers bearing, length 13 in. and diameter 2.75 in.; ring oiler; "gas motor oil"; oil temperature 77°; velocity 846 ft. per min. (Stribeck, Zeit. Ver. Deutsch. Ing., vol. 46, p. 1341.) Figs. 164, 165, and 166 show respectively relations between coefficient and temperature, velocity, and pressure. In 164, p = 92.5 lb. per sq. in. and v = 197 ft. per min.; in 165, p = 92.5 lb. per sq. in. and t = 112°; in 166, t = 112°; and v = 197 ft. per min. The heavy line represents an average law for forced lubrication on five different journals as described below, and the other curves relate to the two journals which departed most widely from the average. (Lasche, Zeit. Ver. Deutsch. Ing., vol. 46.)

Number	Journal	Bearing	Number	Journal	Bearing
I	steel	white metal	IV	nickel steel	bronze
II	nickel steel	white metal	V	ingot iron	white metal
III	nickel steel	mercury alloy			

Coefficients of Journal Friction (Tower's Experiments)

Lubrication	Velocity, feet per minute	Nominal Pressure, Pounds per Square Inch					
		100	153	205	310	415	520
Olive oil by bath...	105	0.0036	0.0023	0.0018	....	....	....
	157	.0045	.003	.0021	0.0015	0.0012	0.0008
	471	.0089	.0057	.004	.0027	.0024	.0017
Lard oil by bath...	105	.0035	.0022	.0017	....	....	....
	157	.0042	.0027	.0020	.0014	.0012	.0009
	471	.009	.0052	.0042	.0029	.0021	.0017
Mineral grease by bath...........	105	.0054	.0028	.0026	.002	....	....
	157	.0076	.0038	.0034	.0022	.0016	.0014
	471	.0151	.0083	.0066	.004	.0027	.0022
Sperm oil by bath..	105	.0025	.0016	.0013	....	....	Seized
	157	.003	.0019	.0016	.0011	.0015	....
	471	.0064	.0037	.0027	.0019	.0021	....
Rape oil by bath...	105	.0028	.0016	.0011	....	....	....
	157	.0036	.0020	.0014	.0008	.0009	.0010
	471	.0071	.0040	.0024	.0016	.0016	.0015
Mineral oil by bath.	105	.0033	....	.0018	....	....	....
	157	.0041	....	.0020	.0014	.0012	.0012
	471	.0073	....	.0035	.0024	.002	.0018
Rape oil by siphon.	105	.0144	....	.0132	....	....	....
	157	.0125	....	.0098	.0056*	....	....
	314	.0163	....	.0082	.0068*	....	....
Rape oil by pad....	105	.0105	....	....	....	....	....
	157	.0099	.009	.0105	.0099	....	....
	314	.0133	.0105	.0078	.0099	....	....

* For 258 lb. per sq. in.

40. Rolling Friction

Rolling Resistance (or Friction). When a roller or wheel is made to roll, the reaction of the roadway has a component opposite to the direction of rolling, and this component is the rolling resistance. Thus R (Fig. 167) represents the reaction of the roadway, and its horizontal component is the rolling resistance. The distance c is called coefficient of rolling resistance. Let W = weight of roller or the load on the roadway, and P = driving force; when applied as in Fig. 167, then $Pr = Wc$; when at the top of the roller $P\,2\,r = Wc$. When a roller is interposed between a load and roadway so that there is rolling resistance at two places, then $P\,2\,r = W\,2\,c$. Few determinations of the coefficient c have been made; it has been reported as independent of W and r, and also as independent of W but varying with $\sqrt{r}$. "Laws" of rolling resistances certainly remain to be established. The following are some reported values of the coefficient of rolling resistance:

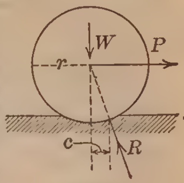

Fig. 167

Lignum vitæ roller on oak track, 0.019 in.
Elm roller on oak track, .032
Cast-iron wheel (20 in. diam.) on cast-iron rail, 0.018–0.019.
Railroad wheels (39.4 in. diam.) 0.020–0.022.
Iron or steel wheels on wood track, 0.06–0.10

Roller Bearings offer less resistance than ordinary bearings. C. H. Benjamin has made a comparison (Machinery, N. Y., Oct., 1905) between the friction in a flexible roller, a solid roller, and ordinary bearing. The speed was 560 revolutions per minute and the loads varied from 113 to 456 lb. Under 470-lb. load the flexible bearing developed end thrust of 13.5 lb. and the solid roller bearing one of 11 lb.

Coefficients of Journal Friction

Diameter of Journal	Flexible rollers			Solid Rollers			Babbitt bearing		
	Max.	Min.	Mean	Max.	Min.	Mean	Max.	Min.	Mean
1-1/16	0.032	0.012	0.018	0.033	0.017	0.022	0.074	0.029	0.043
2-3/16	.019	.011	.014	......	......	......	.088	.078	.082
2-7/16	.042	.025	.032	.028	.015	.021	.114	.083	.096
2-15/16	.029	.022	.025	.039	.019	.027	.125	.089	.107

Stribeck (Zeit. Ver. Deutsch. Ing., 1902, vol. 46, p. 1463) from experiments of four different makes of roller bearings, proposes the following: Let W = load on bearing (lb.), n = number of rollers, l = length of each (in.), r = radius of convex bearing surface for rollers (in.), d = diameter of rollers (in.), D = diameter of circle described by centers of rollers (in.), $2\,r + d = D$, c = coefficient of rolling resistance, f = coefficient of friction such that fW = total resistance to turning as though applied tangentially at surface of bearing and fWr = moment of resistance to turning; then $f = 1.2\,cD/rd$, and c has following values:

for $5\,W/lnd$ =	50	75	100	150	200 lb. per sq. in.
c =	0.0041	.0032	.0028	.0022	.0019 in.

He also states that the resistance of roller bearings is nearly independent of velocity; that the value of the highest roller pressure is 5 times the average, that is, $5\,W/n$; and he proposes as formula for safe load in lb. on the bearing $kldn/5$, where k may be taken from 85 to 155 lb. per sq. in. for unhardened rollers and bearing surfaces. F. R. Jones proposes (Machine Design, Part II) as formula for safe load for a well-made solid roller journal bearing of six or more rollers $100\,000\,d^2ln/3\,v$, where v is the velocity of the convex bearing surface in feet per minute; but values of v less than 50 should not be used in the formula; for such velocity the safe load is practically the same as for $v = 50$.

Ball Bearings. Stribeck reports extensive tests (Zeits. Ver. Deutsch. Ing., 1901, vol. 45, p. 121) from which the following is taken. The ball cases were grooves, cross-sections of which were arcs of circles of radius $2/3\,d$, thus aording two points of contact for each ball, $d = 7/8$ in. $D = 4$ in.

Coefficients of Friction for a Ball Bearing

Load in pounds	Revolutions per minute		
	65	385	780
840	0.0033	0.0035	0.0037
1 870	.0020	.0021	.0022
2 420	.0017	.0018	.0019
3 480	.0016	.0016	.00165
4 500	.0015	.0015	.0015
6 600	.0015	.0013	.0013
10 800	....	....	.0011

Stribeck also states that when the number of balls, n, in a race is between 10 and 20, the greatest pressure on any ball is about $5\ W/n$, where W is the load per race; then if P is the safe load for a single ball, the safe load per race is $Pn/5$. Of course, in a good thrust bearing the load is uniformly distributed among the balls. He also recommends as safe loads per ball $2100\ d^2$ for two-point bearing balls, and from $500\ d^2$ to $750\ d^2$ for three- or four-point bearings.

A Comparison of Some Bearings for Line Shafts

The table (from Trans. Am. Soc. Mech. Engrs., Vol. 35, page 593) gives relative amounts of power consumed in friction of three kinds of bearings for a 2-7/16-in. line shaft. Grease was used for the ball bearings and a mineral oil for the other two, applied by rings in the case of babbitt. Temperatures refer to lubricant and are Fahrenheit.

Bearings	100 ft. per min.		300 ft. per min.	
	77°	100°	77°	100°
Ball........	1	1	1	1
Flexible roller	2.2	2.5	2.7	3
Babbitt.....	3	3.6	4.5	4

41. Efficiency of Machines

Efficiency. A machine in operation receives and transmits or delivers energy. The energy received is called input, that transmitted or delivered is called output; the latter is also called the useful work of the machine. The output is always less than the input, some of the energy miscarrying as it were. The difference between output and input is the lost work, or simply loss. The efficiency of a machine is the ratio of output to input. The efficiency of a combination or succession of machines, the first receiving energy, transmitting to the second, the second to the third, etc., is the continued product of their separate efficiencies.

Efficiency of Some Machine Elements

(Kimball and Barr, Elements of Machine Design)

Common bearing, singly	96–98%
Common bearing, long lines of shafting	95
Roller bearing	98
Ball bearings	99
Spur gear cast teeth, including bearings	93
Spur gear cut teeth, including bearings	96
Bevel gear cast teeth, including bearings	92
Bevel gear cut teeth, including bearings	95
Belting	96–98
Pin-connected chains, as used on bicycles	95–97
High-grade transmission chains	97–99

In some simple machines, the energy is supplied by means of a single force called the effort; for example in a hoisting tackle, the pull applied is the effort. The force against which the useful work is done is called load; in the illustration the weight of the lifted body is the load. The mechanical advantage of a machine is the ratio of the load to the effort. While the effort works or acts through any particular distance, the load acts through a definite distance also; the ratio of the former to the latter distance is called the velocity ratio of the machine. Mechanical advantage depends on efficiency of the machine; velocity ratio does not. In any case mechanical advantage = velocity ratio × efficiency.

Inclined Plane and Wedge. Let W = weight of body, f = coefficient of static friction, ϕ = angle of repose = angle of friction and $f = \tan\phi$, α = inclination of plane and θ inclination of P (Fig. 168), θ being positive as shown and negative when the inclination of P to the horizontal is less than α. (1) To start the body up, $P = W \sin(\phi + \alpha)/\cos(\theta - \phi)$. P is least when $\theta = \phi$, its value then being $W \sin(\phi + \alpha)$. (2) When α is greater than ϕ, gravity would overcome friction and the body, if not prevented by some pull as P, would slip down. To prevent slipping

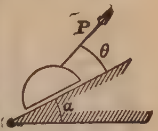

Fig. 168

$$P = W \sin(\alpha - \phi)/\cos(\theta + \phi)$$

P is least when $\theta = -\phi$, its value then being $W \sin(\alpha - \phi)$. When α is less than ϕ, there is no danger of slipping. To start the body down

$$P = W \sin(\phi - \alpha)/\cos(\theta - \phi)$$

this is least when $\theta = -\phi$, its value then being $W \sin(\phi - \alpha)$. When the body is being dragged up, let ϕ = angle of kinetic friction, that is, the angle whose tangent equals the kinetic coefficient, and e = efficiency. Then

$$e = \sin\alpha \cos(\theta - \phi)/\cos\theta \cdot \sin(\phi + \alpha)$$

e is maximum when P is minimum, that is, when $\theta = \phi$.

To start the wedge (Fig. 169) against the resistances R, $P = 2R\tan(\alpha + \phi)$, ϕ being the angle of static friction. If the forces R continue to act and P ceases, the wedge will be pushed up if α is greater than ϕ. If α is less than ϕ, the pull required to start the wedge out is $P = 2R\tan(\phi - \alpha)$. The efficiency of the wedge is $\tan\alpha/\tan(\alpha + \phi)$. In Fig. 170 there are three rubbing surfaces here assumed equally rough. To start the wedge in, $P = R\tan(\alpha + 2\phi)$ and the efficiency is $(\tan\alpha)/\tan(\alpha + 2\phi)$.

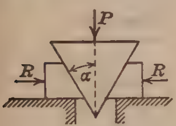

Fig. 169

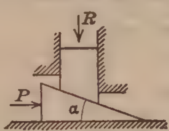

Fig. 170

To pull the wedge out, when α is less than 2ϕ, $P = W\tan(2\phi - \alpha)$.

Screw. Let p = pitch, r = mean radius of screw thread, d = diameter, α = angle of pitch ($\tan\alpha = p/\pi d$), f = static, and μ = kinetic coefficient, and ϕ_s and ϕ_k = the corresponding angles of friction ($f = \tan\phi_s$ and $\mu = \tan\phi_k$). The turning moment required to raise or lower a load W is

$$Wr(f\cos\alpha \pm \sin\alpha)/(\cos\alpha \mp f\sin\alpha)$$

the upper sign for raising and the lower for lowering the load. The efficiencies for raising and lowering are $(\tan\alpha)/\tan(\alpha + \phi_k)$ and $[\tan(\alpha - \phi_k)]/\tan\alpha$.

The following table gives highest and lowest efficiencies of some screws tested by Albert Kingsbury (Trans. A. S. M. E., vol. 17, p. 96). Mean diameter of thread = 1.352 in., pitch = 1/3 in., depth of nut 1-1/16 in., area of rubbing surface of thread about 1 sq. in.; the threads were cut carefully and worn to good condition before tested; speed about one-half r.p.m. There were 5 screws and 4 nuts as follows: S1 mild steel; S2 wrought iron; S3 cast iron; S4 cast bronze; S5 mild steel, case-hardened; N1 mild steel; N2 wrought iron; N3 cast iron; N4 cast bronze.

Efficiencies of Square Threaded Screws (Kingsbury)

Pressure on thread, Lb. per sq. in.	Lubrication	Highest	Lowest
10 000	machinery oil	0.20, S5 N4	0.11, S3 N3
10 000	lard oil	0.25, S4 N4	0.09, S3 N3
10 000	machinery oil and graphite	0.15, S5 N1	0.03, S5 N4
3 000	machinery oil	0.19, S5 N2	0.11, S2 N4

Tackle. Fig. 171 represents a fixed pulley, Fig. 172 a movable pulley lifting and Fig. 173 a movable pulley lowering the load. Let P = pull in lead line or off side, Q = that in following or on side; then $k = P/Q$, where k is a coefficient always greater than unity. Its value depends on the stiffness or rigidity of the rope and the pin friction; this has been proposed:

Fig. 171

Fig. 172

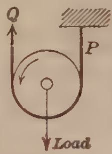

Fig. 173

$$k = 1 + Cd^2/a + 2\,fr/a$$

where d = diameter of rope, a = distance center of pin to center of rope, r = pin radius, f = coefficient of axle or pin friction, C an experimental coefficient. $C = 0.46$ has been recommended for hemp rope; with that value, and $a = 4\,d$, $r = d/2$, and $f = 0.08$,

for d =	1/2	3/4	1	1-1/2 in.
k =	1.08	1.11	1.13	1.19

Some experiments by the American Bridge Co. (Trans. Am. Soc. C. E., vol. 51, p. 161) indicate that C itself depends on rope diameter. The following table gives values of C adopted and k computed from the formula preceding with $f = 0.08$; values of d, a, and r are in inches.

	Hemp rope				Wire rope
d	1-1/4	1-1/2	1-3/4	2	3/4
a	3-7/8	4-9/16	5-3/8	6-/12	7-3/8
r	7/8	1	1-1/8	1-3/8	2-1/2
C	0.46	0.40	0.38	0.34	1.8
k	1.20	1.21	1.23	1.23	1.15

It may be noted that the pin friction contributes little to the value of k; in the cases above, 0.02 for the hemp and about 0.01 for the wire rope. The efficiencies for a single pulley are: if fixed $1/k$, movable lifting $(1 + k)/2\,k$, movable lowering $2/(1 + k)$. The following table gives the ratios of load to pull for tackles of manila rope, as determined by experiments of American Bridge Co. (see Engr. Record, vol. 48, p. 307).

Ratios of Load to Lead-line Pull for Manila Rope

No. of Parts	Diameter of rope, inches							
	3/4	7/8	1	1-1/4	1-1/2	1-3/4	2	2-1/4
2	1.93	1.92	1.93	1.92	1.91	1.91	1.91	1.90
3	2.73	2.68	2.74	2.68	2.67	2.64	2.65	2.63
4	3.48	3.37	3.50	3.37	3.36	3.30	3.32	3.28
5	4.12	3.95	4.16	3.95	3.93	3.84	3.87	3.80
6	4.71	4.48	4.77	4.48	4.45	4.33	4.37	4.28
7	5.23	4.92	5.30	4.92	4.89	4.72	4.78	4.65
8	5.71	5.32	5.80	5.32	5.28	5.08	5.14	5.00
9	6.12	5.66	6.23	5.65	5.61	5.37	5.45	5.27
10	6.50	5.96	6.63	5.96	5.91	5.64	5.72	5.52
11	6.83	6.22	6.98	6.21	6.15	5.85	5.94	5.72
12	7.14	6.45	7.30	6.44	6.38	6.04	6.15	5.90
13	7.40	6.64	7.58	6.63	6.56	6.20	6.31	6.04
14	7.64	6.82	7.85	6.81	6.73	6.34	6.46	6.17

Number of parts means number of runs of rope to movable block. Efficiency in per cent for any case = ratio divided by number of parts. Example: The fixed and movable blocks of a tackle are each double (two sheaves in each), and the rope is fastened to the fixed block; rope diameter is one inch. Then the number of parts is 4 and the load is 3.50 times the pull; the efficiency is $3.50/4 = 87.5\%$.

42. Muscular Exertion or Labor

Power and Work of Men and Animals. The power or rate of working of any agent (man or beast) depends on several factors, the principal ones being the duration and form of the exertion. For example, in raising his own weight (climbing a stairs) man has worked (against gravity) at the average rate of 1-1 2 hp. for 6 seconds; but only at the average rate of about 1/8 hp. for a day of 10 hours, and in this case the (necessary) descents were made without effort on his part. In one day a man can do about three times as much work pulling horizontally as in lifting weights.

Daily total work or performance depends on resistance overcome, velocity at which it is overcome, and length of the working day. The possible daily performance of any agent in any given form of exertion depends on proper choice or adjustment of these three factors. This is well illustrated by a celebrated example developed by Dr. J. F. Taylor: At a certain steel works the regular performance of men loading pig iron onto cars from piles on the ground was 12-1/2 tons per day; the men best adapted to such labor were induced to work according to a suitable program of working and resting, whereby their output reached 47-1/2 tons per day. It may be noted that this performance was not all work in the mechanical sense, but partly transport; see following paragraphs.

In trials, horses have overcome resistance equal to one-half their own weight through 100 ft. of distance. A draft or resistance of one-fourth the weight of the horse is regarded as heavy. For steady work during a day of 10 hours, a draft equal to 1/10 to 1/8 of the weight of the horse exerted at 2.5 miles per hour, is regarded as full demand on the animal. Under these circumstances a 1000-lb. horse works at 0.67 to 0.83 hp.

The two tables on next page in the main are from Rankine on the authority principally of Coulomb, Navier and Poncelet.

Explanation of Table. R = resistance overcome in pounds; v = effective velocity in feet per second, or distance through which R is overcome divided by total time occupied, including time of moving unloaded if any; Rv = effective power, foot-pounds per second; T = hours per working day; Rvt = daily work, in foot-pounds. In item 6, 132 is the weight of earth in the barrow, and .075 is the vertical velocity of the laborer when pushing the loaded barrow. Therefore Rv and Rvt (for this item) do not include all the mechanical work done by the laborer.

Transport by Men and Draft Animals. When a man stands and supports a load on his back he is subject to more or less fatigue and ordinarily he regards himself as working. But he is doing no work in the technical or mechanical sense, "overcoming resistance through distance." When he is walking under this load, he does some mechanical work, for he lifts the load slightly at each step, but even this is a small part of his exertion. In order to express amount of exertion involved in transporting loads horizontally, where the main effort is to support the load, Rankine uses product of load and distance transported, and he calls the product transport. Thus like work transport is product of force and distance but in the latter case the force and distance are at right angles to each other, whereas in the former they are directed along the same line. Transport then may be expressed in foot-pounds. Railroad traffic engineers use a similar concept, freight (hauled) or traffic, and measure it in ton-miles.

Work by Man and Horse

Kind of Exertion	R	v	Rv	T	Rvt
Man					
1. Raising own weight up stair or ladder	143	0.5	72.5	8	2 088 000
2. Hoisting weight with rope and pulley	40	0.75	30	6	648 000
3. Lifting weights by hand	44	0.55	24.2	6	522 720
4. Carrying weights upstairs, returning unloaded	143	0.13	18.5	6	399 600
5. Shoveling up earth to height 5 ft. 3 in.	6	1.3	7.8	10	280 800
6. Wheeling earth in barrow up slope 1 : 12, returning unloaded	132	0.075	9.9	10	356 400
7. Pushing or pulling horizontally (capstan or oar)	26.5	2.0	53	8	1 526 400
	12.5	5.0	62.5	?	
8. Turning a crank or winch	18	2.5	45	8	1 296 000
	20	14.4	288	2 min.	
9. Working a pump	13.2	2.5	33	10	1 188 000
10. Hammering	15	?	?	8?	480 000
Horse					
11. Cantering and trotting, drawing light railway carriage	22-1/2 30-1/2 50	14-2/3	447-2/3	4	6 444 000
12. Walking, drawing cart or boat	120	3.6	432	8	12 441 000
13. Walking, driving a gin or mill	100	3.0	300	8	8 640 000
14. Trotting, driving a gin or mill	66	6.5	429	4-1/2	6 950 000

Transport by Man and Horse

Kind of Exertion	L	v	Lv	T	Lvt
Man					
1. Walking unloaded, transferring own weight	140	5	700	10	25 200 000
2. Carrying burden, returning unloaded	140	1-2/3	233	6	5 032 800
3. Traveling with burden	90	2-1/2	225	7	5 670 000
	252	0	0		
4. Carrying burden, 30 seconds only	126	11.7	1574.2		
	0	23.1	0		
5. Wheeling load L in 2-wheeled barrow, returning unloaded	224	1-2/3	372	10	13 428 000
6. Wheeling load L in 1-wheel barrow, returning unloaded	132	1-2/3	220	10	7 920 000
Horse					
7. Carrying burden, walking	270	3.6	972	10	34 992 000
8. Carrying burden, trotting	180	7.2	1296	7	32 659 200

Explanation of Table. L is the weight or load transported in pounds; v is "effective velocity" as in preceding table; Lv = rate of transport in foot-pounds per second; T = hours per working day; and Lvt = transport in foot-pounds per day.

MATHEMATICAL TABLES

APPENDIX TO SECTION I

26. Six-Place Logarithms of Numbers

The small table on page 173 gives logarithms of the natural numbers from 1 to 100 with their characteristics.

The extended table on pages 147 to 173 gives the logarithms of the natural numbers from 100 to 10 000 without their characteristics which are to be supplied by the rules of Art. 24, Sect. 1. For example

$$\log 112 = 2.049218 \qquad \log 1126 = 3.051538$$

Logarithms of numbers with more than four figures are to be found by help of the last column giving differences and the proportional parts at the foot of each page. For example when the number 17 456 is given first find log 1745 = 241795 and the difference 249. Then the proportional part corresponding to 6 in the last place is 149 and log 17456 = 241795 + 149 = 241944 to which the characteristic is to be prefixed; hence log 17456 = 4.241944. Similarly log 174567 = 5.241795 + 149 + 17 = 5.242061.

When a logarithm is given to find the corresponding number: If log is 2.932220 the number is 855.5. When the logarithm is not found exactly in the table take the next smaller one and find the difference between it and the given logarithms; then divide that difference by the number in the column Diff. and add the quotient to the smaller logarithms. For example, given 2.932260, the next smaller logarithm is 2.932220 corresponding to the number 855.5; the logarithm of this is 40 units less than the given one and the Diff. is 51 then 40/51 = 8 so that the number corresponding to the logarithm 2.932260 is 855.58. The table of proportional parts at the foot of the page shows 8 immediately as the figure to be added.

26. Logarithms of Numbers

No. 100 L. 000.] [No. 109 L. 040.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415	424
3	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775	416
5	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	0195	0600	1004	1408	1812	2216	2619	3021	404
8	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	0207	0602	0998	397
	04										

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
433	43.3	86.6	129.9	173.2	216.5	259.8	303.1	346.4	389.7
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
431	43.1	86.2	129.3	172.4	215.5	258.6	301.7	344.8	387.9
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
429	42.9	85.8	128.7	171.6	214.5	257.4	300.3	343.2	386.1
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
427	42.7	85.4	128.1	170.8	213.5	256.2	298.9	341.6	384.3
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
425	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
423	42.3	84.6	126.9	169.2	211.5	253.8	296.1	338.4	380.7
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
421	42.1	84.2	126.3	168.4	210.5	252.6	294.7	336.8	378.9
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
419	41.9	83.8	125.7	167.6	209.5	251.4	293.3	335.2	377.1
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
417	41.7	83.4	125.1	166.8	208.5	250.2	291.9	333.6	375.3
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
415	41.5	83.0	124.5	166.0	207.5	249.0	290.5	332.0	373.5
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
413	41.3	82.6	123.9	165.2	206.5	247.8	289.1	330.4	371.7
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
411	41.1	82.2	123.3	164.4	205.5	246.6	287.7	328.8	369.9
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
409	40.9	81.8	122.7	163.6	204.5	245.4	286.3	327.2	368.1
408	40.8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
407	40.7	81.4	122.1	162.8	203.5	244.2	284.9	325.6	366.3
406	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4
405	40.5	81.0	121.5	162.0	202.5	243.0	283.5	324.0	364.5
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
403	40.3	80.6	120.9	161.2	201.5	241.8	282.1	322.4	362.7
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
401	40.1	80.2	120.3	160.4	200.5	240.6	280.7	320.8	360.9
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
399	39.9	79.8	119.7	159.6	199.5	239.4	279.3	319.2	359.1
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
397	39.7	79.4	119.1	158.8	198.5	238.2	277.9	317.6	357.3
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	356.4
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5

26. Logarithms of Numbers

No. 110 L. 041.] [No. 119 L. 078.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	0380	0766	1153	1538	1924	2309	2694	386
3	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320	379
5	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514	370
8	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
393	39.3	78.6	117.9	157.2	196.5	235.8	275.1	314.4	353.7
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
391	39.1	78.2	117.3	156.4	195.5	234.6	273.7	312.8	351.9
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
389	38.9	77.8	116.7	155.6	194.5	233.4	272.3	311.2	350.1
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
387	38.7	77.4	116.1	154.8	193.5	232.2	270.9	309.6	348.3
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
385	38.5	77.0	115.5	154.0	192.5	231.0	269.5	308.0	346.5
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
383	38.3	76.6	114.9	153.2	191.5	229.8	268.1	306.4	344.7
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
381	38.1	76.2	114.3	152.4	190.5	228.6	266.7	304.8	342.9
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
379	37.9	75.8	113.7	151.6	189.5	227.4	265.3	303.2	341.1
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
377	37.7	75.4	113.1	150.8	188.5	226.2	263.9	301.6	339.3
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
375	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
373	37.3	74.6	111.9	149.2	186.5	223.8	261.1	298.4	335.7
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
371	37.1	74.2	111.3	148.4	185.5	222.6	259.7	296.8	333.9
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
369	36.9	73.8	110.7	147.6	184.5	221.4	258.3	295.2	332.1
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
367	36.7	73.4	110.1	146.8	183.5	220.2	256.9	293.6	330.3
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
365	36.5	73.0	109.5	146.0	182.5	219.0	255.7	292.0	328.5
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
363	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
361	36.1	72.2	108.3	144.4	180.5	216.6	252.7	288.8	324.9
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0
359	35.9	71.8	107.7	143.6	179.5	215.4	251.3	287.2	323.1
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
357	35.7	71.4	107.1	142.8	178.5	214.2	249.9	285.6	321.3
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4

26. Logarithms of Numbers

No. 120 L. 079.] [No. 134 L. 130.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
120	079181	9543	9904								
				0266	0626	0987	1347	1707	2067	2426	360
1	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905										
		0258	0611	0963	1315	1667	2018	2370	2721	3071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681		
										0026	346
6	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916		
										0253	338
9	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915		
										0245	330
2	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690		
	13									0012	323

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
355	35.5	71.0	106.5	142.0	177.5	213.0	248.5	284.0	319.5
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
353	35.3	70.6	105.9	141.2	176.5	211.8	247.1	282.4	317.7
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
351	35.1	70.2	105.3	140.4	175.5	210.6	245.7	280.8	315.9
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
349	34.9	69.8	104.7	139.6	174.5	209.4	244.3	279.2	314.1
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
347	34.7	69.4	104.1	138.8	173.5	208.2	242.9	277.6	312.3
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
345	34.5	69.0	103.5	138.0	172.5	207.0	241.5	276.0	310.5
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
343	34.3	68.6	102.9	137.2	171.5	205.8	240.1	274.4	308.7
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
341	34.1	68.2	102.3	136.4	170.5	204.6	238.7	272.8	306.9
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
339	33.9	67.8	101.7	135.6	169.5	203.4	237.3	271.2	305.1
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
337	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
335	33.5	67.0	100.5	134.0	167.5	201.0	234.5	268.0	301.5
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
333	33.3	66.6	99.9	133.2	166.5	199.8	233.1	266.4	299.7
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
331	33.1	66.2	99.3	132.4	165.5	198.6	231.7	264.8	297.9
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
329	32.9	65.8	98.7	131.6	164.5	197.4	230.3	263.2	296.1
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
327	32.7	65.4	98.1	130.8	163.5	196.2	228.9	261.6	294.3
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
325	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
323	32.3	64.6	96.9	129.2	161.5	193.8	226.1	258.4	290.7
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8

26. Logarithms of Numbers

No. 135 L. 130.] [No. 149 L. 175.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879	0194	0508	0822	1136	1450	1763	2076	2389	2702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
1	9219	9527	9835	0142	0449	0756	1063	1370	1676	1982	307
2	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	0168	0469	0769	1068	301
5	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
321	32.1	64.2	96.3	128.4	160.5	192.6	224.7	256.8	288.9
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
319	31.9	63.8	95.7	127.6	159.5	191.4	223.3	255.2	287.1
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	254.4	286.2
317	31.7	63.4	95.1	126.8	158.5	190.2	221.9	253.6	285.3
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
315	31.5	63.0	94.5	126.0	157.5	189.0	220.5	252.0	283.5
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
313	31.3	62.6	93.9	125.2	156.5	187.8	219.1	250.4	281.7
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
311	31.1	62.2	93.3	124.4	155.5	186.6	217.7	248.8	279.9
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
309	30.9	61.8	92.7	123.6	154.5	185.4	216.3	247.2	278.1
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
307	30.7	61.4	92.1	122.8	153.5	184.2	214.9	245.6	276.3
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
305	30.5	61.0	91.5	122.0	152.5	183.0	213.5	244.0	274.5
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
303	30.3	60.6	90.9	121.2	151.5	181.8	212.1	242.4	272.7
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
301	30.1	60.2	90.3	120.4	150.5	180.6	210.7	240.8	270.9
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
299	29.9	59.8	89.7	119.6	149.5	179.4	209.3	239.2	269.1
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
297	29.7	59.4	89.1	118.8	148.5	178.2	207.9	237.6	267.3
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
295	29.5	59.0	88.5	118.0	147.5	177.0	206.5	236.0	265.5
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
293	29.3	58.6	87.9	117.2	146.5	175.8	205.1	234.4	263.7
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
291	29.1	58.2	87.3	116.4	145.5	174.6	203.7	232.8	261.9
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
289	28.9	57.8	86.7	115.6	144.5	173.4	202.3	231.2	260.1
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
287	28.7	57.4	86.1	114.8	143.5	172.2	200.9	229.6	258.3
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4

26. Logarithms of Numbers

No. 150 L. 176.] [No. 169 L. 230.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
1	8977	9264	9552	9839	0126	0413	0699	0986	1272	1558	287
2	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	0051	281
5	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	0029	0303	0577	0850	1124	274
9	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783	0051	0319	0586	0853	1121	1388	1654	1921	267
3	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887 23	8144	8400	8657	8913	9170	9426	9682	9938	0193	256

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
285	28 5	57.0	85.5	114 0	142.5	171 0	199.5	228.0	256.5
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
283	28.3	56.6	84.9	113.2	141.5	169.8	198.1	226.4	254.7
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
281	28.1	56.2	84.3	112 4	140.5	168.6	196.7	224.8	252.9
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0
279	27.9	55.8	83.7	111.6	139.5	167.4	195.3	223.2	251.1
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
277	27.7	55.4	83.1	110.8	138.5	166.2	193.9	221.6	249.3
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
275	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	247.5
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
273	27.3	54.6	81.9	109.2	136.5	163.8	191.1	218.4	245.7
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
271	27.1	54.2	81.3	108.4	135.5	162.6	189.7	216.8	243.9
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
269	26.9	53.8	80.7	107.6	134.5	161.4	188.3	215.2	242.1
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
267	26.7	53.4	80.1	106.8	133.5	160.2	186.9	213.6	240.3
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
265	26.5	53.0	79.5	106.0	132.5	159.0	185.5	212.0	238.5
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
263	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
261	26.1	52.2	78.3	104.4	130.5	156.6	182.7	208.8	234.9
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
259	25.9	51.8	77.7	103.6	129.5	155.4	181.3	207.2	233.1
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
257	25.7	51.4	77.1	102.8	128.5	154.2	179.9	205.6	231.3
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5

26. Logarithms of Numbers

No. 170 L. 230.] [No. 189 L. 278.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800			
									0050	0300	250
4	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932		
										0176	245
8	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980								
				0213	0446	0679	0912	1144	1377	1609	233
7	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
253	25.3	50.6	75.9	101.2	126.5	151.8	177.1	202.4	227.7
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
251	25.1	50.2	75.3	100.4	125.5	150.6	175.7	200.8	225.9
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
249	24.9	49.8	74.7	99.6	124.5	149.4	174.3	199.2	224.1
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
247	24.7	49.4	74.1	98.8	123.5	148.2	172.9	197.6	222.3
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
245	24.5	49.0	73.5	98.0	122.5	147.0	171.5	196.0	220.5
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
243	24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
241	24.1	48.2	72.3	96.4	120.5	144.6	168.7	192.8	216.9
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
239	23.9	47.8	71.7	95.6	119.5	143.4	167.3	191.2	215.1
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
237	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
235	23.5	47.0	70.5	94.0	117.5	141.0	164.5	188.0	211.5
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
233	23.3	46.6	69.9	93.2	116.5	139.8	163.1	186.4	209.7
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
231	23.1	46.2	69.3	92.4	115.5	138.6	161.7	184.8	207.9
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0
229	22.9	45.8	68.7	91.6	114.5	137.4	160.3	183.2	206.1
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205.2
227	22.7	45.4	68.1	90.8	113.5	136.2	158.9	181.6	204.3
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
190	278754	8982	9211	9439	9667	9895	0123	0351	0578	0806	228
1	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	0161	0378	0595	0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542	212
5	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	0008	0211	203
4	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
225	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
223	22.3	44.6	66.9	89.2	111.5	133.8	156.1	178.4	200.7
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
221	22.1	44.2	66.3	88.4	110.5	132.6	154.7	176.8	198.9
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
219	21.9	43.8	65.7	87.6	109.5	131.4	153.3	175.2	197.1
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
217	21.7	43.4	65.1	86.8	108.5	130.2	151.9	173.6	195.3
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
215	21.5	43.0	64.5	86.0	107.5	129.0	150.5	172.0	193.5
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
213	21.3	42.6	63.9	85.2	106.5	127.8	149.1	170.4	191.7
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
211	21.1	42.2	63.3	84.4	105.5	126.6	147.7	168.8	189.9
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
209	20.9	41.8	62.7	83.6	104.5	125.4	146.3	167.2	188.1
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
207	20.7	41.4	62.1	82.8	103.5	124.2	144.9	165.6	186.3
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
205	20.5	41.0	61.5	82.0	102.5	123.0	143.5	164.0	184.5
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
203	20.3	40.6	60.9	81.2	101.5	121.8	142.1	162.4	182.7
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8

26. Logarithms of Numbers

No. 215 L. 332.] [No. 239 L. 380.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
215	332438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849	0047	0246	199
9	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	0054	194
4	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	0025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	0143	0328	0513	0698	0883	185
5	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	0030	181
	38										

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
201	20.1	40.2	60.3	80.4	100.5	120.6	140.7	160.8	180.9
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
199	19.9	39.8	59.7	79.6	99.5	119.4	139.3	159.2	179.1
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
197	19.7	39.4	59.1	78.8	98.5	118.2	137.9	157.6	177.3
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
195	19.5	39.0	58.5	78.0	97.5	117.0	136.5	156.0	175.5
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
193	19.3	38.6	57.9	77.2	96.5	115.8	135.1	154.4	173.7
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
191	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.9
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
189	18.9	37.8	56.7	75.6	94.5	113.4	132.3	151.2	170.1
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
187	18.7	37.4	56.1	74.8	93.5	112.2	130.9	149.6	168.3
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
185	18.5	37.0	55.5	74.0	92.5	111.0	129.5	148.0	166.5
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
183	18.3	36.6	54.9	73.2	91.5	109.8	128.1	146.4	164.7
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
181	18.1	36.2	54.3	72.4	90.5	108.6	126.7	144.8	162.9
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
179	17.9	35.8	53.7	71.6	89.5	107.4	125.3	143.2	161.1

26. Logarithms of Numbers

No. 240 L. 380.] [No. 269 L. 431

N.	0	1	2	3	4	5	6	7	8	9	Diff.
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	0051	0228	0405	0582	0759	177
6	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
1	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228	173
2	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752 43	9914	0075	0236	0398	0559	0720	0881	1042	1203	161

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
177	17.7	35.4	53.1	70.8	88.5	106.2	123.9	141.6	159.3
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
175	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
173	17.3	34.6	51.9	69.2	86.5	103.8	121.1	138.4	155.7
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
171	17.1	34.2	51.3	68.4	85.5	102.6	119.7	136.8	153.9
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
169	16.9	33.8	50.7	67.6	84.5	101.4	118.3	135.2	152.1
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
167	16.7	33.4	50.1	66.8	83.5	100.2	116.9	133.6	150.3
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
165	16.5	33.0	49.5	66.0	82.5	99.0	115.5	132.0	148.5
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
163	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.7
162	16.2	32.4	48.5	64.8	81.0	97.2	113.4	129.6	145.8
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9

26. Logarithms of Numbers

No. 270 L. 431.] [No. 299 L. 476.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964						
						0122	0279	0437	0594	0752	158
6	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941		
										0095	154
2	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995						
						0146	0296	0447	0597	0748	151
9	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969									
			0116	0263	0410	0557	0704	0851	0998	1145	147
6	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
159	15.9	31.8	47.7	63.6	79.5	95.4	111.3	127.2	143.1
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
157	15.7	31.4	47.1	62.8	78.5	94.2	109.9	125.6	141.3
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4
155	15.5	31.0	46.5	62.0	77.5	93.0	108.5	124.0	139.5
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
153	15.3	30.6	45.9	61.2	76.5	91.8	107.1	122.4	137.7
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
151	15.1	30.2	45.3	60.4	75.5	90.6	105.7	120.8	135.9
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
149	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
147	14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132.3
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
145	14.5	29.0	43.5	58.0	72.5	87.0	101.5	116.0	130.5
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
143	14.3	28.6	42.9	57.2	71.5	85.8	100.1	114.4	128.7
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
141	14.1	28.2	42.3	56.4	70.5	84.6	98.7	112.8	126.9
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.0

26. Logarithms of Numbers

No. 300 L. 477.] [No. 339 L. 531.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958										
		0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962								
				0099	0236	0374	0511	0648	0785	0922	137
7	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874					
							0009	0143	0277	0411	134
4	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
1	9828	9959									
			0090	0221	0353	0484	0615	0745	0876	1007	131
2	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943		
										0072	128
9	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
139	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.2	125.1
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
137	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
135	13.5	27.0	40.5	54.0	67.5	81.0	94.5	108.0	121.5
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
133	13.3	26.6	39.9	53.2	66.5	79.8	93.1	106.4	119.7
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
131	13.1	26.2	39.3	52.4	65.5	78.6	91.7	104.8	117.9
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
129	12.9	25.8	38.7	51.6	64.5	77.4	90.3	103.2	116.1
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204	125
7	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106	123
5	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
1	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426	117
2	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
125	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
123	12.3	24.6	36.9	49.2	61.5	73.8	86.1	98.4	110.7
122	12 2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
121	12.1	24.2	36.3	48.4	60.5	72.6	84.7	96.8	108.9
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
119	11.9	23.8	35.7	47.6	59.5	71.4	83.3	95.2	107.1

26. Logarithms of Numbers

No. 380. L. 579.] [No. 414 L. 617.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	0012	0126	0241	0355	0469	0583	0697	0811	114
1	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
9	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953	
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	
8	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864	109
9	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
7	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554	
8	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
117	11.7	23.4	35.1	46.8	58.5	70.2	81.9	93.6	105.3
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
115	11.5	23.0	34.5	46.0	57.5	69.0	80.5	92.0	103.5
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6
113	11.3	22.6	33.9	45.2	56.5	67.8	79.1	90.4	101.7
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
111	11.1	22.2	33.3	44.4	55.5	66.6	77.7	88.8	99.9
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
109	10.9	21.8	32.7	43.6	54.5	65.4	76.3	87.2	98.1
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
107	10.7	21.4	32.1	42.8	53.5	64.2	74.9	85.6	96.3
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6

26. Logarithms of Numbers

No. 415 L. 618.] [No. 459 L. 662.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
415	618048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	0032	
7	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	0021	0123	0224	0326	
7	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	
6	9486	9586	9686	9785	9885	9984	0084	0183	0283	0382	
7	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	
6	9335	9432	9530	9627	9724	9821	9919	0016	0113	0210	
7	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	
7	9916	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6
103	10.3	20.6	30.9	41.2	51.5	61.8	72.1	82.4	92.7
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4	81.6	91.8
101	10.1	20.2	30.3	40.4	50.5	60.6	70.7	80.8	90.9
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
99	9.9	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.1

26. Logarithms of Numbers

No. 460 L. 662.] [No. 499 L. 698.

N	0	1	2	8	4	5	6	7	8	9	Diff.
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	
7	9317	9410	9503	9596	9689	9782	9875	9967			
									0060	0153	98
8	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
6	9428	9519	9610	9700	9791	9882	9973				
								0063	0154	0245	
9	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	
9	9309	9398	9486	9575	9664	9753	9841	9930			
									0019	0107	
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
97	9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
95	9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
93	9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
91	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
89	8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	80.1
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.8
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	
1	9838	9924	0011	0098	0184	0271	0358	0444	0531	0617	
2	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	0033	
3	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	0077	
5	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	0055	0136	0217	0298	0378	0459	0540	0621	0702	
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
85	8.5	17.0	25.5	34.0	42.5	51.0	59.5	68.0	76.5
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6

26. Logarithms of Numbers

No. 545 L. 736.] [No. 584 L. 767.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	
9	9572	9651	9731	9810	9889	9968					
							0047	0126	0205	0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	
2	9736	9814	9891	9968							
					0045	0123	0200	0277	0354	0431	
3	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	
5	9668	9743	9819	9894	9970						
						0045	0121	0196	0272	0347	
6	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.3
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6

26. Logarithms of Numbers

No. 585 L. 767.] [No. 629 L. 799.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
585	767156	7230	7304	7379	7453	7527	7601	7675	7749	7823	
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	0042	
9	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	
2	9596	9669	9741	9813	9885	9957	0029	0101	0173	0245	
3	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	
6	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215	
7	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
73	7.3	14.6	21.9	29.2	36.5	43.8	51.1	58.4	65.7
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
71	7.1	14.2	21.3	28.4	35.5	42.6	49.7	56.8	63.9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
69	6.9	13.8	20.7	27.6	34.5	41.4	48.3	55.2	62.1

26. Logarithms of Numbers

No. 630 L. 799.] [No. 674 L. 829.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	
1	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	
5	9560	9627	9694	9762	9829	9896	9964	0031	0098	0165	
6	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	9544	9610	9676	9741	9807	9873	9939	0004	0070	0136	
1	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
67	6.7	13.4	20.1	26.8	33.5	40.2	46.9	53.6	60.3
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6

26. Logarithms of Numbers

No. 675 L. 829.] No. 719 L. 857.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
675	829304	9368	9432	9497	9561	9625	9690	9754	9818	9882	
6	9947	0011	0075	0139	0204	0268	0332	0396	0460	0525	
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	0043	
2	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	
8	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
63	6.3	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.7
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8
61	6.1	12.2	18.3	24.4	30.5	36.6	42.7	48.8	54.9
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0

26. Logarithms of Numbers

No. 720 L. 857.] [No. 764 L. 883.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	0038	0098	0158	0218	0278	
5	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	
1	9818	9877	9935	9994	0053	0111	0170	0228	0287	0345	
2	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	
8	9669	9726	9784	9841	9898	9956	0013	0070	0127	0185	
9	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4

26. Logarithms of Numbers

No. 765 L. 883.] [No. 809 L. 908.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	
6	9862	9918	9974	0030	0086	0141	0197	0253	0309	0365	56
7	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	
4	9821	9875	9930	9985	0039	0094	0149	0203	0258	0312	
5	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
55	5.5	11.0	16.5	22.0	27.5	33.0	38.5	44.0	49.5
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

26. Logarithms of Numbers

No. 810 L. 908.] [No. 854 L. 931.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	0037	
3	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	
1	9601	9653	9706	9758	9810	9862	9914	9967	0019	0071	
2	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
1	9930	9981	0032	0083	0134	0185	0236	0287	0338	0389	
2	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
53	5 3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47.7
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
51	5.1	10.2	15.3	20.4	25 5	30 6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
855	931966	2017	2068	2118	2169	2220	2271	2322	2372	2423	
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	
5	7016	7066	7116	7167	7217	7267	7317	7367	7418	7468	
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	
1	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	
8	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	
1	9878	9926	9975	0024	0073	0121	0170	0219	0267	0316	
2	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
49	4.9	9.8	14.7	19.6	24.5	29.4	34.3	39.2	44.1
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2

26. Logarithms of Numbers

No 900 L. 954.] [No. 944 L. 975.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	
2	9995	0042	0090	0138	0185	0233	0280	0328	0376	0423	
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	
3	9882	9928	9975	0021	0068	0114	0161	0207	0254	0300	
4	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4

26. Logarithms of Numbers

No. 945 L. 975.] [No. 989 L. 995

N.	0	1	2	3	4	5	6	7	8	9	Diff.
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
5	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	
7	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294	
8	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
45	4.5	9.0	13.5	18.0	22.5	27.0	31.5	36.0	40.5
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
43	4.3	8.6	12.9	17.2	21.5	25.8	30.1	34.4	38.7

26. Logarithms of Numbers

No. 990 L. 995.] [No. 999 L. 999.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

Logarithms of Numbers from 1 to 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

	Value at 0°.	Sign in 1st Quad.	Value at 90°.	Sign in 2d Quad.	Value at 180°.	Sign in 3d Quad.	Value at 270°	Sign in 4th Quad.	Value at 360°.
Sin........	O	+	R	+	O	−	R	−	O
Tan.......	O	+	∞	−	O	+	∞	−	O
Sec	R	+	∞	−	R	−	∞	+	R
Versin....	O	+	R	+	2R	+	R	+	O
Cos	R	+	O	−	R	−	O	+	R
Cot.... ...	∞	+	O	−	∞	+	O	−	∞
Cosec.....	∞	+	R	+	∞	−	R	−	∞

R signifies equal to rad; ∞ signifies infinite; O signifies evanescent.

27. Six-Place Logarithmic Sines, Cosines, Tangents and Cotangents

In the following pages, 175 to 219, characteristics of logarithms are increased by 10 in order to avoid negative ones. Thus log sin 1° 10′ = 8.308794 or $\bar{2}$.308794, log cot 1° 10′ = 11.691116 or 1.691116, log tan 0° 30′ = 7.940858 or $\bar{3}$.940858.

Pages 175 to 219 give values of these functions to six decimal places for every minute of the first and second quadrants. The degrees are at the top and bottom of the pages and the minutes at the sides below or above the degrees. For example, on page 185 the angles 10° 26′ and 169° 34′ have log sin = 9.257898, while 79° 20′ and 100° 40′ have log cot = 9.274964.

The columns headed D. 1″ enable interpolation to be made for seconds; thus for 10° 26′ 15″ the D. 1″ is 11.42 for log sin, whence 11.42 × 15 = 171 and log sin for this angle is 9.257898 + 171 = 9.258069. Also for 163° 38′ 15″ the log tan is 9.467880 − 117 = 9.467763. The computed difference is to be added or subtracted according as the tabular values of the function increase or decrease with an increase in the angle.

The columns of D. 1″ are omitted on pages 175 and 176, except for log cos; while other columns are added which enable intermediate values of the other functions to be found for small angles more accurately than can be done by interpolation. Thus to find log sin A and log tan A, when A contains seconds, the equations

$$\log \sin A = S + \log A'', \qquad \log \tan A = T + \log A'',$$

are to be used, A'' signifying the number of seconds in the angle A. For example, let the angle A be 1° 6′ 33″ or 3993″; for 1° 6′ the value of S is taken from the fourth column on page 176 and log 3993 from Table 26. Then

$$\begin{array}{ll} \text{For } 1^\circ\ 6' & S = 4.685548 \\ \log 3993 & = 3.601299 \\ \hline \log \sin 1^\circ\ 6'\ 33'' & = 8.286847 \end{array}$$

Similarly for 0° 54′ 12″ or 3252″ the log tan is found as follows:

$$\begin{array}{ll} \text{For } 0^\circ\ 54' & T = 4.685611 \\ \log 3252 & = 3.512151 \\ \hline \log \tan 0^\circ\ 54'\ 12'' & = 8.197762 \end{array}$$

To find log cot for a small angle the equation log cot $A = C - \log A''$ is to be used where C is taken from the eighth column. For example, for 1° 0′ 16″ or 3616″ the value of C is 15.314381 and that of log 3616 is 3.558228, whence log cot 1° 0′ 16″ = 11.756153.

To find the angle from a given logarithmic function, the eye must run along the table until the tabular value nearest to it is found. Thus, when log tan is given as 9.516910 this is found on page 193 and the angle is either 18° 12′ or 161° 48′. Again, when log tan is given as 9.526004, this is found to lie between 9.525778 and 9.526197; to the first value corresponds the angle 18° 33′ and the D. 1″ is 6.98; the difference 9.526004 − 9.525778 is 226 and 226/6.98 = 32.4″, so that the required angle is 18° 33′ 32″.4.

When the given function falls on page 175 or 176, the number of seconds is found by the equations

$$\log A'' = \log \sin A - S, \quad \log A'' = \log \tan A - T, \quad \log A'' = C - \log \cot A.$$

For example, given log tan A as 8.465371 for which T is 4.685700, then log A'' = 8.465371 − 4.685700 = 3.779671 from which by Table 26 there is found A'' = 6021″ and hence A = 1° 40′ 21″.

27. Logarithmic Sines, etc.

0°		Sine.	q−l		Tang.	Cotang.	q+l	D1″	Cosine.	179°
″	′		4.685				15.314			′
0	0	Inf. neg.	575	575	Inf. neg.	Inf. pos.	425		ten	60
60	1	6.463726	575	575	6.463726	13.536274	425		ten	59
120	2	.764756	575	575	.764756	.235244	425		ten	58
180	3	6.940847	575	575	6.940847	13.059153	425		ten	57
240	4	7.065786	575	575	7.065786	12.934214	425		ten	56
300	5	.162696	575	575	.162696	.837304	425		ten	55
360	6	.241877	575	575	.241878	.758122	425	.02	9.999999	54
420	7	.308824	575	575	.308825	.691175	425	.00	.999999	53
480	8	.366816	574	576	.366817	.633183	424	.00	.999999	52
540	9	.417968	574	576	.417970	.582030	424	.00	.999999	51
600	10	.463726	574	576	.463727	.536273	424	.02	.999998	50
660	11	7.505118	574	576	7.505120	12.494880	424	.00	9.999998	49
720	12	.542906	574	577	.542909	.457091	423	.02	.999997	48
780	13	.577668	574	577	.577672	.422328	423	.00	.999997	47
840	14	.609853	574	577	.609857	.390143	423	.02	.999996	46
900	15	.639816	573	578	.639820	.360180	422	.00	.999996	45
960	16	.667845	573	578	.667849	.332151	422	.02	.999995	44
1020	17	.694173	573	578	.694179	.305821	422	.00	.999995	43
1080	18	.718997	573	579	.719003	.280997	421	.02	.999994	42
1140	19	.742478	573	579	.742484	.257516	421	.02	.999993	41
1200	20	.764754	572	580	.764761	.235239	420	.00	.999993	40
1260	21	7.785943	572	580	7.785951	12.214049	420	.02	9.999992	39
1320	22	.806146	572	581	.806155	.193845	419	.02	.999991	38
1380	23	.825451	572	581	.825460	.174540	419	.02	.999990	37
1440	24	.843934	571	582	.843944	.156056	418	.02	.999989	36
1500	25	.861662	571	583	.861674	.138326	417	.00	.999989	35
1560	26	.878695	571	583	.878708	.121292	417	.02	.999988	34
1620	27	.895085	570	584	.895099	.104901	416	.02	.999987	33
1680	28	.910879	570	584	.910894	.089106	416	.02	.999986	32
1740	29	.926119	570	585	.926134	.073866	415	.02	.999985	31
1800	30	.940842	569	586	.940858	.059142	414	.03	.999983	30
1860	31	7.955082	569	587	7.955100	12.044900	413	.02	9.999982	29
1920	32	.968870	569	587	.968889	.031111	413	.02	.999981	28
1980	33	.982233	568	588	.982253	.017747	412	.02	.999980	27
2040	34	7.995198	568	589	7.995219	12.004781	411	.02	.999979	26
2100	35	8.007787	567	590	8.007809	11.992191	410	.03	.999977	25
2160	36	.020021	567	591	.020044	.979956	409	.02	.999976	24
2220	37	.031919	566	592	.031945	.968055	408	.02	.999975	23
2280	38	.043501	566	593	.043527	.956473	407	.03	.999973	22
2340	39	.054781	566	593	.054809	.945191	407	.02	.999972	21
2400	40	.065776	565	594	.065806	.934194	406	.02	.999971	20
2460	41	8.076500	565	595	8.076531	11.923469	405	.03	9.999969	19
2520	42	.086965	564	596	.086997	.913003	404	.02	.999968	18
2580	43	.097183	564	598	.097217	.902783	402	.03	.999966	17
2640	44	.107167	563	599	.107203	.892797	401	.03	.999964	16
2700	45	.116926	562	600	.116963	.883037	400	.02	.999963	15
2760	46	.126471	562	601	.126510	.873490	399	.03	.999961	14
2820	47	.135810	561	602	.135851	.864149	398	.03	.999959	13
2880	48	.144953	561	603	.144996	.855004	397	.02	.999958	12
2940	49	.153907	560	604	.153952	.846048	396	.03	.999956	11
3000	50	.162681	560	605	.162727	.837273	395	.03	.999954	10
3060	51	8.171280	559	607	8.171328	11.828672	393	.03	9.999952	9
3120	52	.179713	558	608	.179763	.820237	392	.03	.999950	8
3180	53	.187985	558	609	.188036	.811964	391	.03	.999948	7
3240	54	.196102	557	611	.196156	.803844	389	.03	.999946	6
3300	55	.204070	556	612	.204126	.795874	388	.03	.999944	5
3360	56	.211895	556	613	.211953	.788047	387	.03	.999942	4
3420	57	.219581	555	615	.219641	.780359	385	.03	.999940	3
3480	58	.227134	554	616	.227195	.772805	384	.03	.999938	2
3540	59	.234557	554	618	.234621	.765379	382	.03	.999936	1
3600	60	8.241855	553	619	8.241921	11.758079	381	.03	9.999934	0
″	′		4.685				15.314			′
90°		Cosine.	q−l		Cotang.	Tang.	q+l	D1″	Sine.	89°

27. Logarithmic Sines,

1°		Sine.	q − l		Tang.	Cotang.	q + l	D1″	Cosine. 178°	
″	′		4.685				15.314			′
3600	0	8.241855	553	619	8.241921	11.758079	381		9.999934	60
3660	1	.249033	552	620	.249102	.750898	380	.03	.999932	59
3720	2	.256094	551	622	.256165	.743835	378	.05	.999929	58
3780	3	.263042	551	623	.263115	.736885	377	.03	.999927	57
3840	4	.269881	550	625	.269956	.730044	375	.03	.999925	56
3900	5	.276614	549	627	.276691	.723309	373	.05	.999922	55
3960	6	.283243	548	628	.283323	.716677	372	.03	.999920	54
4020	7	.289773	547	630	.289856	.710144	370	.03	.999918	53
4080	8	.296207	546	632	.296292	.703708	368	.05	.999915	52
4140	9	.302546	546	633	.302634	.697366	367	.03	.999913	51
4200	10	.308794	545	635	.308884	.691116	365	.05	.999910	50
4260	11	8.314954	544	637	8.315046	11.684954	363	.05	9.999907	49
4320	12	.321027	543	638	.321122	.678878	362	.03	.999905	48
4380	13	.327016	542	640	.327114	.672886	360	.05	.999902	47
4440	14	.332924	541	642	.333025	.666975	358	.05	.999899	46
4500	15	.338753	540	644	.338856	.661144	356	.03	.999897	45
4560	16	.344504	539	646	.344610	.655390	354	.05	.999894	44
4620	17	.350181	539	648	.350289	.649711	352	.05	.999891	43
4680	18	.355783	538	649	.355895	.644105	351	.05	.999888	42
4740	19	.361315	537	651	.361430	.638570	349	.05	.999885	41
4800	20	.366777	536	653	.366895	.633105	347	.05	.999882	40
4860	21	8.372171	535	655	8.372292	11.627708	345	.05	9.999879	39
4920	22	.377499	534	657	.377622	.622378	343	.05	.999876	38
4980	23	.382762	533	659	.382889	.617111	341	.05	.999873	37
5040	24	.387962	532	661	.388092	.611908	339	.05	.999870	36
5100	25	.393101	531	663	.393234	.606766	337	.05	.999867	35
5160	26	.398179	530	666	.398315	.601685	334	.05	.999864	34
5220	27	.403199	529	668	.403338	.596662	332	.05	.999861	33
5280	28	.408161	527	670	.408304	.591696	330	.05	.999858	32
5340	29	.413068	526	672	.413213	.586787	328	.07	.999854	31
5400	30	.417919	525	674	.418068	.581932	326	.05	.999851	30
5460	31	8.422717	524	676	8.422869	11.577131	324	.05	9.999848	29
5520	32	.427462	523	679	.427618	.572382	321	.07	.999844	28
5580	33	.432156	522	681	.432315	.567685	319	.05	.999841	27
5640	34	.436800	521	683	.436962	.563038	317	.05	.999838	26
5700	35	.441394	520	685	.441560	.558440	315	.07	.999834	25
5760	36	.445941	518	688	.446110	.553890	312	.05	.999831	24
5820	37	.450440	517	690	.450613	.549387	310	.07	.999827	23
5880	38	.454893	516	693	.455070	.544930	307	.05	.999824	22
5940	39	.459301	515	695	.459481	.540519	305	.07	.999820	21
6000	40	.463665	514	697	.463849	.536151	303	.07	.999816	20
6060	41	8.467985	512	700	8.468172	11.531828	300	.05	9.999813	19
6120	42	.472263	511	702	.472454	.527546	298	.07	.999809	18
6180	43	.476498	510	705	.476693	.523307	295	.07	.999805	17
6240	44	.480693	509	707	.480892	.519108	293	.07	.999801	16
6300	45	.484848	507	710	.485050	.514950	290	.07	.999797	15
6360	46	.488963	506	713	.489170	.510830	287	.05	.999794	14
6420	47	.493040	505	715	.493250	.506750	285	.07	.999790	13
6480	48	.497078	503	718	.497293	.502707	282	.07	.999786	12
6540	49	.501080	502	720	.501298	.498702	280	.07	.999782	11
6600	50	.505045	501	723	.505267	.494733	277	.07	.999778	10
6660	51	8.508974	499	726	8.509200	11.490800	274	.07	9.999774	9
6720	52	.512867	498	729	.513098	.486902	271	.08	.999769	8
6780	53	.516726	497	731	.516961	.483039	269	.07	.999765	7
6840	54	.520551	495	734	.520790	.479210	266	.07	.999761	6
6900	55	.524343	494	737	.524586	.475414	263	.07	.999757	5
6960	56	.528102	492	740	.528349	.471651	260	.07	.999753	4
7020	57	.531828	491	743	.532080	.467920	257	.08	.999748	3
7080	58	.535523	490	745	.535779	.464221	255	.07	.999744	2
7140	59	.539186	488	748	.539447	.460553	252	.07	.999740	1
7200	60	8.542819	487	751	8.543084	11.456916	249	.08	9.999735	0
″	′		4.685				15.314			′
91°		Cosine.	q − l		Cotang.	Tang.	q + l	D1″	Sine. 88°	

Cosines, Tangents, and Cotangents,

2°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	177°
0′	8.542819	60.05	9.999735	.07	8.543084	60.12	11.456916	60′
1	.546422	59.55	.999731	.08	.546691	59.62	.453309	59
2	.549995	59.07	.999726	.07	.550268	59.15	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.65	.446183	57
4	.557054	58.10	.999717	.07	.557336	58.20	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.72	.439172	55
6	.563999	57.20	.999708	.07	.564291	57.27	.435709	54
7	.567431	56.75	.999704	.08	.567727	56.83	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	.577566	55.43	.999689	.07	.577877	55.52	.422123	50
11	8.580892	55.02	9.999685	.08	8.581208	55.10	11.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.20	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.78	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.40	.999665	.08	.594283	53.48	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.62	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.85	.999645	.08	.606978	51.93	.393022	41
20	.609734	51.48	.999640	.08	.610094	51.58	.389906	40
21	8.612823	51.13	9.999635	.10	8.613189	51.22	11.386811	39
22	.615891	50.77	.999629	.08	.616262	50.85	.383738	38
23	.618937	50.42	.999624	.08	.619313	50.50	.380687	37
24	.621962	50.05	.999619	.08	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.10	.625352	49.80	.374648	35
26	.627948	49.38	.999608	.08	.628340	49.47	.371660	34
27	.630911	49.05	.999603	.10	.631308	49.13	.368692	33
28	.633854	48.70	.999597	.08	.634256	48.80	.365744	32
29	.636776	48.40	.999592	.10	.637184	48.48	.362816	31
30	.639680	48.05	.999586	.08	.640093	48.15	.359907	30
31	8.642563	47.75	9.999581	.10	8.642982	47.85	11.357018	29
32	.645428	47.43	.999575	.08	.645853	47.52	.354147	28
33	.648274	47.13	.999570	.10	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.10	.651537	46.92	.348463	26
35	.653911	46.52	.999558	.08	.654352	46.62	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.32	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.45	.334567	21
40	.667689	45.07	.999529	.08	.668160	45.17	.331840	20
41	8.670393	44.78	9.999524	.10	8.670870	44.88	11.329130	19
42	.673080	44.52	.999518	.10	.673563	44.60	.326437	18
43	.675751	44.23	.999512	.10	.676239	44.35	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	.681043	43.70	.999500	.12	.681544	43.80	.318456	15
46	.683665	43.45	.999493	.10	.684172	43.53	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	.693998	42.42	.999469	.10	.694529	42.53	.305471	10
51	8.696543	42.17	9.999463	.12	8.697081	42.27	11.302919	9
52	.699073	41.93	.999456	.10	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.12	.702139	41.78	.297861	7
54	.704090	41.45	.999443	.10	.704646	41.57	.295354	6
55	.706577	41.20	.999437	.10	.707140	41.30	.292860	5
56	.709049	40.97	.999431	.12	.709618	41.08	.290382	4
57	.711507	40.75	.999424	.10	.712083	40.85	.287917	3
58	.713952	40.52	.999418	.12	.714534	40.63	.285466	2
59	.716383	40.28	.999411	.12	.716972	40.40	.283028	1
60′	8.718800		9.999404		8.719396		11.280604	0′
92°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	87°

27. Logarithmic Sines,

3°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	176°
0′	8.718800	40.07	9.999404	.10	8.719396	40.17	11.280604	60′
1	.721204	39.85	.999398	.12	.721806	39.97	.278194	59
2	.723595	39.62	.999391	.12	.724204	39.73	.275796	58
3	.725972	39.42	.999384	.10	.726588	39.52	.273412	57
4	.728337	39.18	.999378	.12	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.12	.731317	39.10	.268683	55
6	.733027	38.78	.999364	.12	.733663	38.88	.266337	54
7	.735354	38.55	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.37	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.17	.999343	.12	.740626	38.27	.259374	51
10	.742259	37.95	.999336	.12	.742922	38.08	.257078	50
11	8.744536	37.77	9.999329	.12	8.745207	37.87	11.254793	49
12	.746802	37.55	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.48	.250260	47
14	.751297	37.18	.999308	.12	.751989	37.30	248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.80	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.60	.999287	.13	.758668	36.73	.241332	43
18	.760151	36.43	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.23	.999272	.12	.763065	36.35	.236935	41
20	.764511	36.07	.999265	.13	.765246	36.18	.234754	40
21	8.766675	35.88	9.999257	.12	8.767417	36.02	11.232583	39
22	.768828	35.70	.999250	.13	.769578	35.82	.230422	38
23	.770970	35.52	.999242	.12	.771727	35.65	.228273	37
24	.773101	35.37	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.17	.999227	.12	.775995	35.32	.224005	35
26	.777333	35.02	.999220	.13	.778114	35.13	.221886	34
27	.779434	34.83	.999212	.12	.780222	34.97	.219778	33
28	.781524	34.68	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.50	.999197	.13	.784408	34.63	.215592	31
30	.785675	34.35	.999189	.13	.786486	34.47	.213514	30
31	8.787736	34.18	9.999181	.12	8.788554	34.32	11.211446	29
32	.789787	34.02	.999174	.13	.790613	34.15	.209387	28
33	.791828	33.85	.999166	.13	.792662	33.98	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.55	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.38	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.25	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.07	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	.805852	32.78	.999110	.13	.806742	32.92	.193258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.77	11.191283	19
42	.809777	32.48	.999094	.13	.810683	32.63	.189317	18
43	.811726	32.35	.999086	.15	.812641	32.47	.187359	17
44	.813667	32.20	.999077	.13	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.13	.816529	32.20	.183471	15
46	.817522	31.90	.999061	.13	.818461	32.05	.181539	14
47	.819436	31.78	.999053	.15	.820384	31.90	.179616	13
48	.821343	31.62	.999044	.13	.822298	31.78	.177702	12
49	.823240	31.50	.999036	.15	.824205	31.63	.175795	11
50	.825130	31.35	.999027	.13	.826103	31.48	.173897	10
51	8.827011	31.22	9.999019	.15	8.827992	31.37	11.172008	9
52	.828884	31.08	.999010	.13	.829874	31.23	.170126	8
53	.830749	30.97	.999002	.15	.831748	31.08	.168252	7
54	.832607	30.82	.998993	.15	.833613	30.97	.166387	6
55	.834456	30.68	.998984	.13	.835471	30.83	.164529	5
56	.836297	30.55	.998976	.15	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.58	.160837	3
58	.839956	30.30	.998958	.13	.840998	30.45	.159002	2
59	.841774	30.18	.998950	.15	.842825	30.32	.157175	1
60′	8.843585		9.998941		8.844644		11.155356	0′
93°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	86°

Cosines, Tangents, and Cotangents

4°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	175°
0′	8.843585	30.03	9.998941	.15	8.844644	30.18	11.155356	60′
1	.845387	29.93	.998932	.15	.846455	30.08	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.57	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.30	.998887	.15	.855403	29.47	.144597	54
7	.856049	29.20	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.95	.998860	.15	.860686	29.12	.139314	51
10	.861283	28.85	.998851	.17	.862433	29.00	.137567	50
11	8.863014	28 73	9.998841	.15	8.864173	28.88	11.135827	49
12	.864738	28 62	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.17	.867632	28.65	.132368	47
14	.868165	28.38	.998813	.15	.869351	28.55	.130649	46
15	.869868	28.28	.998804	.15	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.17	.872770	28.32	.127230	44
17	.873255	28.05	.998785	.15	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.17	.876162	28.12	.123838	42
19	.876615	27.83	.998766	.15	.877849	28.00	.122151	41
20	.878285	27.73	.998757	.17	.879529	27.88	.120471	40
21	8.879949	27.63	9.998747	.15	8.881202	27.78	11.118798	39
22	.881607	27.52	.998738	.17	.882869	27.68	.117131	38
23	.883258	27 42	.998728	.17	.884530	27.58	.115470	37
24	.884903	27.32	.998718	.17	.886185	27.47	.113815	36
25	.886542	27.20	.998708	.15	.887833	27.38	.112167	35
26	.888174	27.12	.998699	.17	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.17	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.17	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	.894643	26.72	.998659	.17	.895984	26.87	.104016	30
31	8.896246	26 60	9.998649	.17	8.897596	26.78	11.102404	29
32	.897842	26 50	.998639	.17	.899203	26.67	.100797	28
33	.899432	26 42	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.32	.998619	.17	.902398	26.48	.097602	26
35	.902596	26 22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.28	.094430	24
37	.905736	26.02	.998589	.18	.907147	26.20	.092853	23
38	.907297	25 93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.85	.998568	.17	.910285	26.02	.089715	21
40	.910404	25.75	.998558	.17	.911846	25.92	.088154	20
41	8.911949	25.65	9.998548	.18	8.913401	25.83	11.086599	19
42	.913488	25.57	.998537	.17	.914951	25.73	.085049	18
43	.915022	25.47	.998527	.18	.916495	25.65	.083505	17
44	.916550	25 38	.998516	.17	.918034	25.57	.081966	16
45	.918073	25 30	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.17	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.28	.077381	13
48	.922610	25.03	.998474	.17	.924136	25.22	.075864	12
49	.924112	24.95	.998464	.18	.925649	25.12	.074351	11
50	.925609	24.85	.998453	.18	.927156	25.03	.072844	10
51	8.927100	24.78	9.998442	.18	8.928658	24.95	11.071342	9
52	.928587	24.68	.998431	.17	.930155	24 87	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.30	.059506	1
60′	8.940296		9.998344		8.941952		11.058048	0′
94°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	85°

27. Logarithmic Sines,

5°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	174°
0′	8.940296	24.03	9.998344	.18	8.941952	24.20	11.058048	60′
1	.941738	23.93	.998333	.18	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.18	.944852	24.05	.055148	58
3	.944606	23.80	.998311	.18	.946295	23.98	.053705	57
4	.946034	23.70	.998300	.18	.947734	23.90	.052266	56
5	.947456	23.63	.998289	.20	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.18	.950597	23.73	.049403	54
7	.950287	23.48	.998266	.18	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.20	.953441	23.58	.046559	52
9	.953100	23.32	.998243	.18	.954856	23.52	.045144	51
10	.954499	23.25	.998232	.20	.956267	23.45	.043733	50
11	8.955894	23.17	9.998220	.18	8.957674	23.35	11.042326	49
12	.957284	23.10	.998209	.20	.959075	23.30	.040925	48
13	.958670	23.03	.998197	.18	.960473	23.22	.039527	47
14	.960052	22.95	.998186	.20	.961866	23.15	.038134	46
15	.961429	22.87	.998174	.18	.963255	23.07	.036745	45
16	.962801	22.82	.998163	.20	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.20	.966019	22.92	.033981	43
18	.965534	22.65	.998139	.18	.967394	22.87	.032606	42
19	.966893	22.60	.998128	.20	.968766	22.78	.031234	41
20	.968249	22.52	.998116	.20	.970133	22.72	.029867	40
21	8.969600	22.45	9.998104	.20	8.971496	22.65	11.028504	39
22	.970947	22.37	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.32	.998080	.20	.974209	22.52	.025791	37
24	.973628	22.23	.998068	.20	.975560	22.43	.024440	36
25	.974962	22.18	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.25	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	.981573	21.83	.997996	.20	.983577	22.03	.016423	30
31	8.982883	21.77	9.997984	.20	8.984899	21.97	11.015101	29
32	.984189	21.70	.997972	.22	.986217	21.92	.013783	28
33	.985491	21.63	.997959	.20	.987532	21.83	.012468	27
34	.986789	21.57	.997947	.20	.988842	21.78	.011158	26
35	.988083	21.52	.997935	.22	.990149	21.70	.009851	25
36	.989374	21.43	.997922	.20	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.22	.992750	21.58	.007250	23
38	.991943	21.32	.997897	.20	.994045	21.53	.005955	22
39	.993222	21.25	.997885	.22	.995337	21.45	.004663	21
40	.994497	21.18	.997872	.20	.996624	21.40	.003376	20
41	8.995768	21.13	9.997860	.22	8.997908	21.33	11.002092	19
42	.997036	21.05	.997847	.20	8.999188	21.28	11.000812	18
43	.998299	21.02	.997835	.22	9.000465	21.22	10.999535	17
44	8.999560	20.93	.997822	.22	.001738	21.15	.998262	16
45	9.000816	20.88	.997809	.20	.003007	21.08	.996993	15
46	.002069	20.82	.997797	.22	.004272	21.03	.995728	14
47	.003318	20.75	.997784	.22	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.22	.006792	20.92	.993208	12
49	.005805	20.65	.997758	.22	.008047	20.85	.991953	11
50	.007044	20.57	.997745	.22	.009298	20.80	.990702	10
51	9.008278	20.53	9.997732	.22	9.010546	20.73	10.989454	9
52	.009510	20.45	.997719	.22	.011790	20.68	.988210	8
53	.010737	20.42	.997706	.22	.013031	20.62	.986969	7
54	.011962	20.33	.997693	.22	.014268	20.57	.985732	6
55	.013182	20.30	.997680	.22	.015502	20.50	.984498	5
56	.014400	20.22	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.18	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.07	.997628	.23	.020403	20.28	.979597	1
60′	9.019235		9.997614		9.021620		10.978380	0′
95°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	84°

Cosines, Tangents, and Cotangents

6°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	173°
0'	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60'
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.88	.997588	.23	.024044	20.12	.975956	58
3	.022825	19.85	.997574	.22	.025251	20.07	.974749	57
4	.024016	19.78	.997561	.23	.026455	20.00	.973545	56
5	.025203	19.72	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.68	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.22	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.80	.968763	52
9	.029918	19.52	.997493	.22	.032425	19.73	.967575	51
10	.031089	19.47	.997480	.23	.033609	19.70	.966391	50
11	9.032257	19.40	9.997466	.23	9.034791	19.63	10.965209	49
12	.033421	19.35	.997452	.22	.035969	19.58	.964031	48
13	.034582	19.32	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.37	.959349	44
17	.039197	19.08	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	.042625	18.95	.997341	.23	.045284	19.17	.954716	40
21	9.043762	18.88	9.997327	.23	9.046434	19.13	10.953566	39
22	.044895	18.85	.997313	.23	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.23	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.23	.049869	18.98	.950131	36
25	.048279	18.68	.997271	.23	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.25	.052144	18.88	.947856	34
27	.050519	18.60	.997242	.23	.053277	18.83	.946723	33
28	.051635	18.57	.997228	.23	.054407	18.80	.945593	32
29	.052749	18.50	.997214	.25	.055535	18.73	.944465	31
30	.053859	18.45	.997199	.23	.056659	18.70	.943341	30
31	9.054966	18.42	9.997185	.25	9.057781	18.65	10.942219	29
32	.056071	18.35	.997170	.23	.058900	18.60	.941100	28
33	.057172	18.32	.997156	.25	.060016	18.57	.939984	27
34	.058271	18.27	.997141	.23	.061130	18.50	.938870	26
35	.059367	18.22	.997127	.25	.062240	18.47	.937760	25
36	.060460	18.18	.997112	.23	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.25	.064453	18.38	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.32	.934444	22
39	.063724	18.03	.997068	.25	.066655	18.28	.933345	21
40	.064806	17.98	.997053	.23	.067752	18.23	.932248	20
41	9.065885	17.95	9.997039	.25	9.068846	18.20	10.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.85	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.82	.996994	.25	.072113	18.07	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.73	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.67	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.88	.923568	12
49	.074424	17.60	.996919	.25	.077505	17.85	.922495	11
50	.075480	17.55	.996904	.25	.078576	17.80	.921424	10
51	9.076533	17.50	9.996889	.25	9.079644	17.77	10.920356	9
52	.077583	17.47	.996874	.27	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.27	.082833	17.63	.917167	6
55	.080719	17.33	.996828	.27	.083891	17.60	.916109	5
56	.081759	17.30	.996812	.25	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.25	.086000	17.50	.914000	3
58	.083832	17.20	.996782	.27	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.25	.088098	17.43	.911902	1
60'	9.085894		9.996751		9.089144		10.910856	0'
96°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	83°

27. Logarithmic Sines,

7°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	172°
0′	9.085894	17.13	9.996751	.27	9.089144	17.38	10.910856	60′
1	.086922	17.08	.996735	.25	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.27	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.27	.092266	17.27	.907734	57
4	.089990	16.97	.996688	.25	.093302	17.23	.906698	56
5	.091008	16.93	.996673	.27	.094336	17.18	.905664	55
6	.092024	16.88	.996657	.27	.095367	17.13	.904633	54
7	.093037	16.83	.996641	.27	.096395	17.12	.903605	53
8	.094047	16.82	.996625	.25	.097422	17.07	.902578	52
9	.095056	16.77	.996610	.27	.098446	17.03	.901554	51
10	.096062	16.72	.996594	.27	.099468	16.98	.900532	50
11	9.097065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	.098066	16.65	.996562	.27	.101504	16.92	.898496	48
13	.099065	16.62	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.83	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.48	.996498	.27	.105550	16.77	.894450	44
17	.103037	16.47	.996482	.28	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.68	.892441	42
19	.105010	16.37	.996449	.27	.108560	16.65	.891440	41
20	.105992	16.35	.996433	.27	.109559	16.62	.890441	40
21	9.106973	16.30	9.996417	.28	9.110556	16.58	10.889444	39
22	.107951	16.27	.996400	.27	.111551	16.53	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.20	.996368	.28	.113533	16.47	.886467	36
25	.110873	16.15	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.28	.115507	16.40	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.35	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.33	882528	32
29	.114737	16.02	.996285	.27	.118452	16.28	.881548	31
30	.115698	15.97	.996269	.28	.119429	16.25	.880571	30
31	9.116656	15.95	9.996252	.28	9.120404	16.22	10.879596	29
32	.117613	15.90	.996235	.27	.121377	16.18	.878623	28
33	.118567	15.87	996219	.28	.122348	16.15	877652	27
34	.119519	15.83	996202	.28	.123317	16.12	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.75	.996168	.28	.125249	16.03	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.02	.873789	23
38	.123306	15.70	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.65	.996117	.28	.128130	15.95	.871870	21
40	.125187	15.63	.996100	.28	129087	15.90	.870913	20
41	9.126125	15.58	9.996083	.28	9.130041	15.88	10.869959	19
42	.127060	15.55	.996066	.28	.130994	15.83	.869006	18
43	.127993	15.53	.996049	.28	.131944	15.82	.868056	17
44	.128925	15.48	996032	.28	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.28	.133839	15.75	.866161	15
46	.130781	15.42	.995998	.30	.134784	15.70	.865216	14
47	.131706	15.40	.995980	.28	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.28	.136667	15.63	.863333	12
49	.133551	15.32	.995946	.30	.137605	15.62	.862395	11
50	.134470	15.28	.995928	.28	.138542	15.57	.861458	10
51	9.135387	15.27	9.995911	.28	9.139476	15.55	10.860524	9
52	.136303	15.22	.995894	.30	.140409	15.52	.859591	8
53	.137216	15.20	.995876	.28	.141340	15.48	.858660	7
54	.138128	15.15	.995859	.30	.142269	15.45	.857731	6
55	.139037	15.12	.995841	.30	.143196	15.42	.856804	5
56	.139944	15.10	.995823	.28	.144121	15.38	.855879	4
57	.140850	15.07	.995806	.30	.145044	15.37	.854956	3
58	.141754	15.02	.995788	.28	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.30	.146885	15.30	.853115	1
60′	9.143555		9.995753		9.147803		10.852197	0′
97°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	82°

Cosines, Tangents, and Cotangents

8°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	171°
0′	9.143555	14.97	9.995753	.30	9.147803	15.25	10.852197	60′
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.88	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.83	.995681	.28	.151454	15.15	.848546	56
5	.148026	14.82	.995664	.30	.152363	15.10	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.73	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.32	.155077	15.02	.844923	52
9	.151569	14.70	.995591	.30	.155978	14.98	.844022	51
10	.152451	14.65	.995573	.30	.156877	14.97	.843123	50
11	9.153330	14.63	9.995555	.30	9.157775	14.93	10.842225	49
12	.154208	14.58	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.55	.995501	.32	.160457	14.83	.839543	46
15	.156830	14.50	.995482	30	.161347	14.82	.838653	45
16	.157700	14.48	.995464	.30	.162236	14.78	.837764	44
17	.158569	14.43	.995446	.32	.163123	14.75	.836877	43
18	.159435	14.43	.995427	.30	.164008	14.73	.835992	42
19	.160301	14.38	.995409	.32	.164892	14.70	.835108	41
20	.161164	14.35	.995390	.30	.165774	14.67	.834226	40
21	9.162025	14.33	9.995372	.32	9.166654	14.63	10.833346	39
22	.162885	14.30	.995353	.32	.167532	14.62	.832468	38
23	.163743	14.28	.995334	.30	.168409	14.58	.831591	37
24	.164600	14.23	.995316	.32	.169284	14.55	.830716	36
25	.165454	14.22	.995297	.32	.170157	14.53	.829843	35
26	.166307	14.20	.995278	.30	.171029	14.50	.828971	34
27	.167159	14.15	.995260	.32	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.45	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	.169702	14.08	.995203	.32	.174499	14.38	.825501	30
31	9.170547	14.03	9.995184	.32	9.175362	14.37	10.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	14.00	.995146	.32	.177084	14.30	.822916	27
34	.173070	13.97	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.93	.995108	.32	.178799	14.27	.821201	25
36	.174744	13.90	.995089	.32	.179655	14.22	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.18	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.13	.817789	21
40	.178072	13.80	.995013	.33	.183059	14.13	.816941	20
41	9.178900	13.77	9.994993	.32	9.183907	14.08	10.816093	19
42	.179726	13.75	.994974	.32	.184752	14.08	.815248	18
43	.180551	13.72	.994955	.33	.185597	14.03	.814403	17
44	.181374	13.70	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.33	.187280	14.00	.812720	15
46	.183016	13.63	.994896	.32	.188120	13.97	.811880	14
47	.183834	13.62	.994877	.33	.188958	13.93	.811042	13
48	.184651	13.58	.994857	.32	.189794	13.92	.810206	12
49	.185466	13.57	.994838	.33	.190629	13.88	.809371	11
50	.186280	13.53	.994818	.33	.191462	13.87	.808538	10
51	9.187092	13.52	9.994798	.32	9.192294	13.83	10.807706	9
52	.187903	13.48	.994779	.33	.193124	13.82	.806876	8
53	.188712	13.45	.994759	.33	.193953	13.78	.806047	7
54	.189519	13.43	.994739	.32	.194780	13.77	.805220	6
55	.190325	13.42	.994720	.33	.195606	13.73	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.72	.803570	4
57	.191933	13.35	.994680	.33	.197253	13.68	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.67	.801926	2
59	.193534	13.30	.994640	.33	.198894	13.65	.801106	1
60′	9.194332		9.994620		9.199713		10.800287	0
98°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	81°

27. Logarithmic Sines,

9°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	170°
0′	9.194332	13.28	9.994620	.33	9.199713	13.60	10.800287	60′
1	.195129	13.27	.994600	.33	.200529	13.60	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.20	.994560	.33	.202159	13.53	.797841	57
4	.197511	13.18	.994540	.35	.202971	13.52	.797029	56
5	.198302	13.15	.994519	.33	.203782	13.50	.796218	55
6	.199091	13.13	.994499	.33	.204592	13.47	.795408	54
7	.199879	13.12	.994479	.33	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.35	.206207	13.43	.793793	52
9	.201451	13.05	.994438	.33	.207013	13.40	.792987	51
10	.202234	13.05	.994418	.33	.207817	13.37	.792183	50
11	9.203017	13.00	9.994398	.35	9.208619	13.35	10.791381	49
12	.203797	13.00	.994377	.33	.209420	13.33	.790580	48
13	.204577	12.95	.994357	.35	.210220	13.30	.789780	47
14	.205354	12.95	.994336	.33	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.35	.211815	13.27	.788185	45
16	.206906	12.88	.994295	.35	.212611	13.23	.787389	44
17	.207679	12.88	.994274	.33	.213405	13.22	.786595	43
18	.208452	12.83	.994254	.35	.214198	13.18	.785802	42
19	.209222	12.83	.994233	.35	.214989	13.18	.785011	41
20	.209992	12.80	.994212	.35	.215780	13.13	.784220	40
21	9.210760	12.77	9.994191	.33	9.216568	13.13	10.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.07	.781858	37
24	.213055	12.72	.994129	.35	.218926	13.07	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.65	.994087	.35	.220492	13.00	.779508	34
27	.215338	12.65	.994066	.35	.221272	13.00	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.58	.994024	.35	.222830	12.95	.777170	31
30	.217609	12.57	.994003	.35	.223607	12.92	.776393	30
31	9.218363	12.55	9.993982	.37	9.224382	12.90	10.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.85	.774071	27
34	.220618	12.48	.993918	.35	.226700	12.85	.773300	26
35	.221367	12.47	.993897	.37	.227471	12.80	.772529	25
36	.222115	12.43	.993875	.35	.228239	12.80	.771761	24
37	.222861	12.42	.993854	.37	.229007	12.77	.770993	23
38	.223606	12.38	.993832	.35	.229773	12.77	.770227	22
39	.224349	12.38	.993811	.37	.230539	12.72	.769461	21
40	.225092	12.35	.993789	.35	.231302	12.72	.768698	20
41	9.225833	12.33	9.993768	.37	9.232065	12.68	10.767935	19
42	.226573	12.30	.993746	.35	.232826	12.67	.767174	18
43	.227311	12.28	.993725	.37	.233586	12.65	.766414	17
44	.228048	12.27	.993703	.37	.234345	12.63	.765655	16
45	.228784	12.23	.993681	.35	.235103	12.60	.764897	15
46	.229518	12.23	.993660	.37	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.37	.236614	12.57	.763386	13
48	.230984	12.18	.993616	.37	.237368	12.53	.762632	12
49	.231715	12.15	.993594	.37	.238120	12.53	.761880	11
50	.232444	12.13	.993572	.37	.238872	12.50	.761128	10
51	9.233172	12.12	9.993550	.37	9.239622	12.48	10.760378	9
52	.233899	12.10	.993528	.37	.240371	12.45	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.45	.758882	7
54	.235349	12.07	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.00	.993440	.37	.243354	12.38	.756646	4
57	.237515	12.00	.993418	.37	.244097	12.37	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.33	.755161	2
59	.238953	11.95	.993374	.38	.245579	12.33	.754421	1
60′	9.239670		9.993351		9.246319		10.753681	0′
99°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	80°

Cosines, Tangents, and Cotangents

10°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	169°
0′	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60′
1	.240386	11.92	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.88	.993307	.38	.247794	12.27	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.23	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.23	.750736	56
5	.243237	11.83	.993240	.38	.249998	12.20	.750002	55
6	.243947	11.82	.993217	.37	.250730	12.18	.749270	54
7	.244656	11.78	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.77	.993149	.37	.252920	12.13	.747080	51
10	.246775	11.72	.993127	.38	.253648	12.10	.746352	50
11	9.247478	11.72	9.993104	.38	9.254374	12.10	10.745626	49
12	.248181	11.70	.993081	.37	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.02	.742731	45
16	.250980	11.62	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.60	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.57	.992944	.38	.259429	11.95	.740571	42
19	.253067	11.57	.992921	.38	.260146	11.95	.739854	41
20	.253761	11.53	.992898	.38	.260863	11.92	.739137	40
21	9.254453	11.52	9.992875	.38	9.261578	11.90	10.738422	39
22	.255144	11.50	.992852	.38	.262292	11.88	.737708	38
23	.255834	11.48	.992829	.38	.263005	11.87	.736995	37
24	.256523	11.47	.992806	.38	.263717	11.85	.736283	36
25	.257211	11.45	.992783	.40	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.38	.265138	11.82	.734862	34
27	.258583	11.42	.992736	.38	.265847	11.80	.734153	33
28	.259268	11.38	.992713	.38	.266555	11.77	.733445	32
29	.259951	11.37	.992690	.40	.267261	11.77	.732739	31
30	.260633	11.35	.992666	.38	.267967	11.73	.732033	30
31	9.261314	11.33	9.992643	.40	9.268671	11.73	10.731329	29
32	.261994	11.32	.992619	.38	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.40	.270077	11.70	.729923	27
34	.263351	11.27	.992572	.38	.270779	11.67	.729221	26
35	.264027	11.27	.992549	.40	.271479	11.65	.728521	25
36	.264703	11.23	.992525	.40	.272178	11.63	.727822	24
37	.265377	11.23	.992501	.38	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.20	.992454	.40	.274269	11.58	.725731	21
40	.267395	11.17	.992430	.40	.274964	11.57	.725036	20
41	9.268065	11.15	9.992406	.40	9.275658	11.55	10.724342	19
42	.268734	11.13	.992382	.38	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.52	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.07	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.03	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.42	.280488	11.43	.719512	12
49	.273388	11.02	.992214	.40	.281174	11.40	.718826	11
50	.274049	10.98	.992190	.40	.281858	11.40	.718142	10
51	9.274708	10.98	9.992166	.40	9.282542	11.38	10.717458	9
52	.275367	10.97	.992142	.40	.283225	11.37	.716775	8
53	.276025	10.93	.992118	.42	.283907	11.35	.716093	7
54	.276681	10.93	.992093	.40	.284588	11.33	.715412	6
55	.277337	10.90	.992069	.42	.285268	11.32	.714732	5
56	.277991	10.90	.992044	.40	.285947	11.28	.714053	4
57	.278645	10.87	.992020	.40	.286624	11.28	.713376	3
58	.279297	10.85	.991996	.42	.287301	11.27	.712699	2
59	.279948	10.85	.991971	.40	.287977	11.25	.712023	1
60′	9.280599		9.991947		9.288652		10.711348	0′
100°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	79°

27. Logarithmic Sines,

11°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	168°
0′	9.280599	10.82	9.991947	.42	9.288652	11.23	10.711348	60′
1	.281248	10.82	.991922	.42	.289326	11.22	.710674	59
2	.281897	10.78	.991897	.40	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.42	.290671	11.18	.709329	57
4	.283190	10.77	.991848	.42	.291342	11.18	.708658	56
5	.283836	10.73	.991823	.40	.292013	11.15	.707987	55
6	.284480	10.73	.991799	.42	.292682	11.13	.707318	54
7	.285124	10.70	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.70	.991749	.42	.294017	11.12	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.08	.705316	51
10	.287048	10.67	.991699	.42	.295349	11.07	.704651	50
11	9.287688	10.63	9.991674	.42	9.296013	11.07	10.703987	49
12	.288326	10.63	.991649	.42	.296677	11.03	.703323	48
13	.288964	10.60	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.60	.991599	.42	.298001	11.02	.701999	46
15	.290236	10.57	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.57	.991549	.42	.299322	10.97	.700678	44
17	.291504	10.55	.991524	.43	.299980	10.97	.700020	43
18	.292137	10.52	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.52	.991473	.42	.301295	10.93	.698705	41
20	.293399	10.50	.991448	.43	.301951	10.93	.698049	40
21	9.294029	10.48	9.991422	.42	9.302607	10.90	10.697393	39
22	.294658	10.47	.991397	.42	.303261	10.88	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.88	.696086	37
24	.295913	10.43	.991346	.42	.304567	10.85	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.85	.694782	35
26	.297164	10.40	.991295	.42	.305869	10.83	.694131	34
27	.297788	10.40	.991270	.43	.306519	10.82	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.35	.991218	.42	.307816	10.78	.692184	31
30	.299655	10.35	.991193	.43	.308463	10.77	.691537	30
31	9.300276	10.32	9.991167	.43	9.309109	10.75	10.690891	29
32	.300895	10.32	.991141	.43	.309754	10.75	.690246	28
33	.301514	10.30	.991115	.42	.310399	10.72	.689601	27
34	.302132	10.27	.991090	.43	.311042	10.72	.688958	26
35	.302748	10.27	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.23	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.63	.685753	21
40	.305819	10.18	.990934	.43	.314885	10.63	.685115	20
41	9.306430	10.18	9.990908	.43	9.315523	10.60	10.684477	19
42	.307041	10.15	.990882	.45	.316159	10.60	.683841	18
43	.307650	10.15	.990855	.43	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.43	.317430	10.57	.682570	16
45	.308867	10.12	.990803	.43	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.45	.318697	10.55	.681303	14
47	.310080	10.08	.990750	.43	319330	10.52	.680670	13
48	.310685	10.07	.990724	.45	.319961	10.52	.680039	12
49	.311289	10.07	.990697	.43	.320592	10.50	.679408	11
50	.311893	10.03	.990671	.43	.321222	10.48	.678778	10
51	9.312495	10.03	9.990645	.45	9.321851	10.47	10.678149	9
52	.313097	10.02	.990618	.45	.322479	10.45	.677521	8
53	.313698	9.98	.990591	.43	.323106	10.45	.676894	7
54	.314297	10.00	.990565	.45	.323733	10.42	.676267	6
55	.314897	9.97	.990538	.45	.324358	10.42	.675642	5
56	.315495	9.95	.990511	.43	.324983	10.40	.675017	4
57	.316092	9.95	.990485	.45	.325607	10.40	.674393	3
58	.316689	9.92	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.92	.990431	.45	.326853	10.37	.673147	1
60′	9.317879		9.990404		9.327475		10.672525	0′
101°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	78°

Cosines, Tangents, and Cotangents

12°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	167°
0'	9.317879	9.90	9.990404	.43	9.327475	10.33	10.672525	60'
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.85	.990324	.45	.329334	10.32	.670666	57
4	.320249	9.85	.990297	.45	.329953	10.28	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.47	.331187	10.27	668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.78	.990188	.45	.332418	10.25	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.22	.666967	51
10	.323780	9.77	.990134	.45	.333646	10.22	.666354	50
11	9.324366	9.73	9.990107	.47	9.334259	10.20	10.665741	49
12	.324950	9.73	.990079	.45	.334871	10.18	.665129	48
13	.325534	9.72	.990052	.45	.335482	10.18	.664518	47
14	.326117	9.72	.990025	.47	.336093	10.15	.663907	46
15	.326700	9.68	.989997	.45	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.47	.337311	10.13	.662689	44
17	.327862	9.67	.989942	.45	.337919	10.13	.662081	43
18	.328442	9.65	.989915	.47	.338527	10.10	.661473	42
19	.329021	9.63	.989887	.45	.339133	10.10	.660867	41
20	.329599	9.62	.989860	.47	.339739	10.08	.660261	40
21	9.330176	9.62	9.989832	.47	9.340344	10.07	10.659656	39
22	.330753	9.60	989804	.45	.340948	10.07	.659052	38
23	.331329	9.57	.989777	.47	.341552	10.05	.658448	37
24	.331903	9.58	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.55	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.55	.989693	.47	.343358	10.00	.656642	34
27	.333624	9.52	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.53	.989637	.45	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	.335337	9.48	.989582	.48	.345755	9.97	.654245	30
31	9.335906	9.48	9.989553	.47	9.346353	9.93	10.653647	29
32	.336475	9.47	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.93	.652455	27
34	.337610	9.43	.989469	.47	.348141	9.90	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.42	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.48	.349922	9.87	.650078	23
38	.339871	9.38	.989356	.47	.350514	9.87	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	.340996	9.37	.989300	.48	.351697	9.83	.648303	20
41	9.341558	9.35	9.989271	.47	9.352287	9.82	10.647713	19
42	.342119	9.33	.989243	.48	.352876	9.82	.647124	18
43	.342679	9.33	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.30	.989186	.48	.354053	9.78	.645947	16
45	.343797	9.30	.989157	.48	.354640	9.78	.645360	15
46	.344355	9.28	.989128	.47	.355227	9.77	.644773	14
47	.344912	9.28	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.25	.989071	.48	.356398	9.73	.643602	12
49	.346024	9.25	.989042	.47	.356982	9.73	.643018	11
50	.346579	9.25	.989014	.48	.357566	9.72	.642434	10
51	9.347134	9.22	9.988985	.48	9.358149	9.70	10.641851	9
52	.347687	9.22	.988956	.48	.358731	9.70	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.67	.640687	7
54	.348792	9.18	.988898	.48	.359893	9.68	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.65	.639526	5
56	.349893	9.17	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.13	.988782	.48	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.48	.362787	9.62	.637213	1
60'	9.352088		9.988724		9.363364		10.636636	0'
102°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	77°

27. Logarithmic Sines,

13°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	166°
0'	9.352088	9.12	9.988724	.48	9.363364	9.60	10.636636	60'
1	.352635	9.10	.988695	.48	.363940	9.58	.636060	59
2	.353181	9.08	.988666	.50	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.48	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.48	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.50	.366237	9.55	.633763	55
6	.355358	9.05	.988548	.48	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.50	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.48	.367953	9.52	.632047	52
9	.356984	9.00	.988460	.50	.368524	9.50	.631476	51
10	.357524	9.00	.988430	.48	.369094	9.48	.630906	50
11	9.358064	8.98	9.988401	.50	9.369663	9.48	10.630337	49
12	.358603	8.97	.988371	.48	.370232	9.45	.629768	48
13	.359141	8.95	.988342	.50	.370799	9.47	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.43	.628633	46
15	.360215	8.95	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.48	.372499	9.42	.627501	44
17	.361287	8.92	.988223	.50	.373064	9.42	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.88	.988163	.50	.374193	9.38	.625807	41
20	.362889	8.88	.988133	.50	.374756	9.38	.625244	40
21	9.363422	8.87	9.988103	.50	9.375319	9.37	10.624681	39
22	.363954	8.85	.988073	.50	.375881	9.35	.624119	38
23	.364485	8.85	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.82	.987953	.52	.378122	9.32	.621878	34
27	.366604	8.78	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.80	.987892	.50	.379239	9.30	.620761	32
29	.367659	8.77	.987862	.50	.379797	9.28	.620203	31
30	.368185	8.77	.987832	.52	.380354	9.27	.619646	30
31	9.368711	8.75	9.987801	.50	9.380910	9.27	10.619090	29
32	.369236	8.75	.987771	.52	.381466	9.23	.618534	28
33	.369761	8.72	.987740	.50	.382020	9.25	.617980	27
34	.370285	8.72	.987710	.52	.382575	9.23	.617425	26
35	.370808	8.70	.987679	.50	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.52	.383682	9.20	.616318	24
37	.371852	8.68	.987618	.50	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.52	.384786	9.18	.615214	22
39	.372894	8.67	.987557	.52	.385337	9.18	.614663	21
40	.373414	8.65	.987526	.50	.385888	9.17	.614112	20
41	9.373933	8.65	9.987496	.52	9.386438	9.15	10.613562	19
42	.374452	8.63	.987465	.52	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.52	.387536	9.13	.612464	17
44	.375487	8.60	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.12	.611369	15
46	.376519	8.60	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.57	.987310	.52	.389724	9.10	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.57	.987248	.52	.390815	9.08	.609185	11
50	.378577	8.53	.987217	.52	.391360	9.05	.608640	10
51	9.379089	8.53	9.987186	.52	9.391903	9.07	10.608097	9
52	.379601	8.53	.987155	.52	.392447	9.03	.607553	8
53	.380113	8.52	.987124	.53	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.03	.606469	6
55	.381134	8.48	.987061	.52	.394073	9.02	.605927	5
56	.381643	8.48	.987030	.53	.394614	9.00	.605386	4
57	.382152	8.48	.986998	.52	.395154	9.00	.604846	3
58	.382661	8.45	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.53	.396233	8.97	.603767	1
60'	9.383675		9.986904		9.396771		10.603229	0'
103° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	76°

Cosines, Tangents, and Cotangents

14°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	165°
0′	9.383675	8.45	9.986904	.52	9.396771	8.97	10.603229	60′
1	.384182	8.42	.986873	.53	.397309	8.95	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.42	.986809	.52	.398383	8.93	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.38	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.52	.399990	8.90	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.35	.986651	.53	.401058	8.88	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	.388711	8.33	.986587	.53	.402124	8.87	.597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.85	10.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.30	.986491	.53	.403718	8.85	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.82	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.83	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.80	.594692	44
17	.392199	8.27	.986363	.53	.405836	8.80	.594164	43
18	.392695	8.27	.986331	.53	.406364	8.80	.593636	42
19	.393191	8.23	.986299	.55	.406892	8.78	.593108	41
20	.393685	8.23	.986266	.53	.407419	8.77	.592581	40
21	9.394179	8.23	9.986234	.53	9.407945	8.77	10.592055	39
22	.394673	8.22	.986202	.55	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.53	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.55	.409521	8.73	.590479	36
25	.396150	8.18	.986104	.53	.410045	8.73	.589955	35
26	.396641	8.18	.986072	.55	.410569	8.72	.589431	34
27	.397132	8.15	.986039	.53	.411092	8.72	.588908	33
28	.397621	8.17	.986007	.55	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.53	.412137	8.68	.587863	31
30	.398600	8.13	.985942	.55	.412658	8.68	.587342	30
31	9.399088	8.12	9.985909	.55	9.413179	8.67	10.586821	29
32	.399575	8.12	.985876	.55	.413699	8.67	.586301	28
33	.400062	8.12	.985843	.53	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.08	.985778	.55	.415257	8.63	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.05	.985679	.55	.416810	8.60	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	.403455	8.05	.985613	.55	.417842	8.60	.582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.02	.985514	.57	.419387	8.57	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.57	.580099	16
45	.405862	7.98	.985447	.55	.420415	8.53	.579585	15
46	.406341	7.98	.985414	.55	.420927	8.55	.579073	14
47	.406820	7.98	.985381	.57	.421440	8.53	.578560	13
48	.407299	7.97	.985347	.55	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.57	.422463	8.52	.577537	11
50	.408254	7.95	.985280	.55	.422974	8.50	.577026	10
51	9.408731	7.93	9.985247	.57	9.423484	8.48	10.576516	9
52	.409207	7.93	.985213	.55	.423993	8.50	.576007	8
53	.409682	7.92	.985180	.57	.424503	8.47	.575497	7
54	.410157	7.92	.985146	.55	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.57	.425519	8.47	.574481	5
56	.411106	7.88	.985079	.57	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.57	.426534	8.45	.573466	3
58	.412052	7.87	.985011	.55	.427041	8.43	.572959	2
59	.412524	7.87	.984978	.57	.427547	8.42	.572453	1
60′	9.412996		9.984944		9.428052		10.571948	0′
104°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	75°

27. Logarithmic Sines,

15°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	164°
0'	9.412996	7.85	9.984944	.57	9.428052	8.43	10.571948	60'
1	.413467	7.85	.984910	.57	.428558	8.40	.571442	59
2	.413938	7.83	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.40	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.80	.984774	.57	.430573	8.37	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.80	.984706	.57	.431577	8.37	.568423	53
8	.416751	7.77	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.78	.984638	.58	.432580	8.33	.567420	51
10	.417684	7.77	.984603	.57	.433080	8.33	.566920	50
11	9.418150	7.75	9.984569	.57	9.433580	8.33	10.566420	49
12	.418615	7.73	.984535	.58	.434080	8.32	.565920	48
13	.419079	7.75	.984500	.57	.434579	8.32	.565421	47
14	.419544	7.72	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.28	.564424	45
16	.420470	7.72	.984397	.57	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.70	.984328	.57	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.27	.562437	41
20	.422318	7.67	.984259	.58	.438059	8.25	.561941	40
21	9.422778	7.67	9.984224	.57	9.438554	8.23	10.561446	39
22	.423238	7.65	.984190	.58	.439048	8.25	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.22	.560457	37
24	.424156	7.65	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.22	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.62	.984015	.57	.441514	8.20	.558486	33
28	425987	7.60	.983981	.58	.442006	8.18	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	.426899	7.58	.983911	.60	.442988	8.18	557012	30
31	9.427354	7.58	9.983875	.58	9.443479	8.15	10.556521	29
32	.427809	7.57	.983840	.58	.443968	8.17	.556032	28
33	.428263	7.57	.983805	.58	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.58	.444947	8.13	.555053	26
35	.429170	7.55	.983735	.58	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.60	.445923	8.13	.554077	24
37	.430075	7.53	.983664	.58	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.58	.446898	8.10	.553102	22
39	.430978	7.52	.983594	.60	.447384	8.10	.552616	21
40	.431429	7.50	.983558	.58	.447870	8.10	.552130	20
41	9.431879	7.50	9.983523	.60	9.448356	8.08	10.551644	19
42	.432329	7.48	.983487	.58	.448841	8.08	.551159	18
43	.432778	7.47	.983452	.60	.449326	8.07	.550674	17
44	.433226	7.48	.983416	.58	.449810	8.07	.550190	16
45	.433675	7.45	.983381	.60	.450294	8.05	.549706	15
46	.434122	7.45	.983345	.60	.450777	8.05	.549223	14
47	.434569	7.45	.983309	.60	.451260	8.05	.548740	13
48	.435016	7.43	.983273	.58	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.02	.547775	11
50	.435908	7.42	.983202	.60	.452706	8.02	.547294	10
51	9.436353	7.42	9.983166	.60	9.453187	8.02	10.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.38	.983058	.60	.454628	7.98	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.37	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.35	.982914	.60	.456542	7.95	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60'	9.440338		9.982842		9.457496		10.542504	0'
105°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	74°

Cosines, Tangents, and Cotangents

16°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	163°
0′	9.440338	7.33	9.982842	.62	9.457496	7.95	10.542504	60′
1	.440778	7.33	.982805	.60	.457973	7.93	.542027	59
2	.441218	7.33	.982769	.60	.458449	7.93	.541551	58
3	.441658	7.30	.982733	.62	.458925	7.92	.541075	57
4	.442096	7.32	.982696	.60	.459400	7.92	.540600	56
5	.442535	7.30	.982660	.60	.459875	7.90	.540125	55
6	.442973	7.28	.982624	.62	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.60	.460823	7.90	.539177	53
8	.443847	7.28	.982551	.62	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.62	.461770	7.87	.538230	51
10	.444720	7.25	.982477	.60	.462242	7.88	.537758	50
11	9.445155	7.25	9.982441	.62	9.462715	7.85	10.537285	49
12	.445590	7.25	.982404	.62	.463186	7.87	.536814	48
13	.446025	7.23	.982367	.60	.463658	7.83	.536342	47
14	.446459	7.23	.982331	.62	.464128	7.85	.535872	46
15	.446893	7.22	.982294	.62	.464599	7.83	.535401	45
16	.447326	7.22	.982257	.62	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.82	.533992	42
19	.448623	7.18	.982146	.62	.466477	7.80	.533523	41
20	.449054	7.18	.982109	.62	.466945	7.80	.533055	40
21	9.449485	7.17	9.982072	.62	9.467413	7.78	10.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.17	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.13	.981924	.63	.469280	7.77	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.75	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.63	.470676	7.75	.529324	32
29	.452915	7.12	.981774	.62	.471141	7.73	.528859	31
30	.453342	7.10	.981737	.62	.471605	7.73	.528395	30
31	9.453768	7.10	9.981700	.63	9.472069	7.72	10.527931	29
32	.454194	7.08	.981662	.62	.472532	7.72	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.70	.527005	27
34	.455044	7.08	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.62	.473919	7.70	.526081	25
36	.455893	7.05	.981512	.63	.474381	7.68	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.05	.981436	.62	.475303	7.67	.524697	22
39	.457162	7.03	.981399	.63	.475763	7.67	.524237	21
40	.457584	7.03	.981361	.63	.476223	7.67	.523777	20
41	9.458006	7.02	9.981323	.63	9.476683	7.65	10.523317	19
42	.458427	7.02	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.00	.981247	.63	.477601	7.63	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	7.00	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.63	.479432	7.62	.520568	13
48	.460946	6.97	.981057	.63	.479889	7.60	.520111	12
49	.461364	6.97	.981019	.63	.480345	7.60	.519655	11
50	.461782	6.95	.980981	.65	.480801	7.60	.519199	10
51	9.462199	6.95	9.980942	.63	9.481257	7.58	10.518743	9
52	.462616	6.93	.980904	.63	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.65	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.63	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.65	.483075	7.57	.516925	5
56	.464279	6.92	.980750	.63	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.65	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.63	.484435	7.53	.515565	2
59	.465522	6.88	.980635	.65	.484887	7.53	.515113	1
60′	9.465935		9.980596		9.485339		10.514661	0′
106°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	73°

27. Logarithmic Sines,

17°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	162°
0′	9.465935	6.88	9.980596	.63	9.485339	7.53	10.514661	60′
1	.466348	6.88	.980558	.65	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.52	.513758	58
3	.467173	6.87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.83	.980364	.65	.488043	7.48	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	.470046	6.82	.980208	.65	.489838	7.47	.510162	50
11	9.470455	6.80	9.980169	.65	9.490286	7.45	10.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.80	.980091	.65	.491180	7.45	.508820	47
14	.471679	6.78	.980052	.67	.491627	7.43	.508373	46
15	.472086	6.77	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.77	.979934	.65	.492965	7.42	.507035	43
18	.473304	6.77	.979895	.67	.493410	7.40	.506590	42
19	.473710	6.75	.979855	.65	.493854	7.42	.506146	41
20	.474115	6.73	.979816	.67	.494299	7.40	.505701	40
21	9.474519	6.73	9.979776	.65	9.494743	7.38	10.505257	39
22	.474923	6.73	.979737	.67	.495186	7.40	.504814	38
23	.475327	6.72	.979697	.65	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.67	.496073	7.37	.503927	36
25	.476133	6.72	.979618	.65	.496515	7.37	503485	35
26	.476536	6.70	.979579	.67	.496957	7.37	.503043	34
27	.476938	6.70	.979539	.67	.497399	7.37	.502601	33
28	.477340	6.68	.979499	.67	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.65	.498282	7.33	.501718	31
30	.478142	6.67	.979420	.67	.498722	7.35	.501278	30
31	9.478542	6.67	9.979380	.67	9.499163	7.33	10.500837	29
32	.478942	6.67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.32	.499519	26
35	.480140	6.65	.979220	.67	.500920	7.32	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.68	.502235	7.28	.497765	22
39	.481731	6.62	.979059	.67	.502672	7.28	.497328	21
40	.482128	6.62	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6.60	9.978979	.67	9.503546	7.27	10.496454	19
42	.482921	6.58	.978939	.68	.503982	7.27	.496018	18
43	.483316	6.60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6.58	.978858	.68	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.25	.494276	14
47	484895	6.57	.978737	.68	.506159	7.23	.493841	13
48	485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.67	.507027	7.22	.492973	11
50	.486075	6.53	.978615	.68	.507460	7.22	.492540	10
51	9.486467	6.55	9.978574	.68	9.507893	7.22	10.492107	9
52	.486860	6.52	.978533	.67	.508326	7.22	.491674	8
53	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	.68	.509191	7.18	.490809	6
55	.488034	6.50	.978411	.68	.509622	7.20	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	978329	.68	.510485	7.18	.489515	3
58	.489204	6.48	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60′	9.489982		9.978206		9.511776		10.488224	0′
107°	Cosine.	D 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	72°

Cosines, Tangents, and Cotangents

18°	Sine.	D. 1'.	Cosine.	D. 1'.	Tang.	D. 1'.	Cotang.	161°
0'	9.489982	6.48	9.978206	.68	9.511776	7.17	10.488224	60
1	.490371	6.47	.978165	.68	.512206	7.15	.487794	59
2	.490759	6.47	.978124	.68	.512635	7.15	.487365	58
3	.491147	6.47	.978083	.68	.513064	7.15	.486936	57
4	.491535	6.45	.978042	.68	.513493	7.13	.486507	56
5	.491922	6.43	.978001	.70	.513921	7.13	.486079	55
6	.492308	6.45	.977959	.68	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.68	.514777	7.12	.485223	53
8	.493081	6.42	.977877	.70	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.68	.515631	7.10	.484369	51
10	.493851	6.42	.977794	.70	.516057	7.12	.483943	50
11	9.494236	6.42	9.977752	.68	9.516484	7.10	10.483516	49
12	.494621	6.40	.977711	.70	.516910	7.08	.483090	48
13	.495005	6.38	.977669	.68	.517335	7.10	.482665	47
14	.495388	6.40	.977628	.70	.517761	7.08	.482239	46
15	.495772	6.37	.977586	.70	.518186	7.07	.481814	45
16	.496154	6.38	.977544	.68	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.37	.977461	.70	.519458	7.07	.480542	42
19	.497301	6.35	.977419	.70	.519882	7.05	.480118	41
20	.497682	6.37	.977377	.70	.520305	7.05	.479695	40
21	9.498064	6.33	9.977335	.70	9.520728	7.05	10.479272	39
22	.498444	6.35	.977293	.70	.521151	7.03	.478849	38
23	.498825	6.32	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.32	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.32	.977083	.70	.523259	7.02	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.00	.476320	32
29	.501099	6.28	.976999	.70	.524100	7.00	.475900	31
30	.501476	6.30	.976957	.72	.524520	7.00	.475480	30
31	9.501854	6.28	9.976914	.70	9.524940	6.98	10.475060	29
32	.502231	6.27	.976872	.70	.525359	6.98	.474641	28
33	.502607	6.28	.976830	.72	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.70	.526197	6.97	.473803	26
35	.503360	6.25	.976745	.72	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.70	.527033	6.97	.472967	24
37	.504110	6.25	.976660	.72	.527451	6.95	.472549	23
38	.504485	6.25	.976617	.72	.527868	6.95	.472132	22
39	.504860	6.23	.976574	.70	.528285	6.95	.471715	21
40	.505234	6.23	.976532	.72	.528702	6.95	.471298	20
41	9.505608	6.22	9.976489	.72	9.529119	6.93	10.470881	19
42	.505981	6.22	.976446	.70	.529535	6.93	.470465	18
43	.506354	6.22	.976404	.72	.529951	6.92	.470049	17
44	.506727	6.20	.976361	.72	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.92	.469219	15
46	.507471	6.20	.976275	.72	.531196	6.92	.468804	14
47	.507843	6.18	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.90	.467561	11
50	.508956	6.17	.976103	.72	.532853	6.88	.467147	10
51	9.509326	6.17	9.976060	.72	9.533266	6.88	10.466734	9
52	.509696	6.15	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.73	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.15	.975887	.72	.534916	6.87	.465084	5
56	.511172	6.13	.975844	.73	.535328	6.85	.464672	4
57	.511540	6.12	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.73	.536561	6.85	.463439	1
60'	9.512642		9.975670		9.536972		10.463028	0'
108°	Cosine.	D. 1'.	Sine.	D. 1'.	Cotang.	D. 1'.	Tang.	71°

27. Logarithmic Sines,

19°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	160°
0′	9.512642	6.12	9.975670	.72	9.536972	6.83	10.463028	60′
1	.513009	6.10	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.72	.538202	6.82	.461798	57
4	.514107	6.08	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6.82	.460980	55
6	.514837	6.08	.975408	.72	.539429	6.80	.460571	54
7	.515202	6.07	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6.80	.459347	51
10	.516294	6.05	.975233	.73	.541061	6.78	.458939	50
11	9.516657	6.05	9.975189	.73	9.541468	6.78	10.458532	49
12	.517020	6.03	.975145	.73	.541875	6.77	.458125	48
13	.517382	6.05	.975101	.73	.542281	6.78	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.02	.975013	.73	.543094	6.75	.456906	45
16	.518468	6.02	.974969	.73	.543499	6.77	.456501	44
17	.518829	6.02	.974925	.75	.543905	6.75	.456095	43
18	.519190	6.02	.974880	.73	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.73	.544715	6.73	.455285	41
20	.519911	6.00	.974792	.73	.545119	6.75	.454881	40
21	9.520271	6.00	9.974748	.75	9.545524	6.73	10.454476	39
22	.520631	5.98	.974703	.73	.545928	6.72	.454072	38
23	.520990	5.98	.974659	.75	.546331	6.73	.453669	37
24	.521349	5.97	.974614	.73	.546735	6.72	.453265	36
25	.521707	5.98	.974570	.75	.547138	6.70	.452862	35
26	.522066	5.97	.974525	.73	.547540	6.72	.452460	34
27	.522424	5.95	.974481	.75	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.75	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.73	.548747	6.70	.451253	31
30	.523495	5.95	.974347	.75	.549149	6.68	.450851	30
31	9.523852	5.93	9.974302	.75	9.549550	6.68	10.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.68	.449248	26
35	.525275	5.92	.974122	.75	.551153	6.65	.448847	25
36	.525630	5.90	.974077	.75	.551552	6.67	.448448	24
37	.525984	5.92	.974032	.75	.551952	6.65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.88	.973942	.75	.552750	6 65	.447250	21
40	.527046	5.90	.973897	.75	.553149	6.65	.446851	20
41	9.527400	5.88	9.973852	.75	9.553548	6 63	10.446452	19
42	.527753	5.87	.973807	.77	.553946	6.63	.446054	18
43	.528105	5.88	.973761	.75	.554344	6.62	.445656	17
44	.528458	5.87	.973716	.75	.554741	6.63	.445259	16
45	.528810	5.85	.973671	.77	.555139	6.62	.444861	15
46	.529161	5.87	.973625	.75	.555536	6.62	.444464	14
47	.529513	5.85	.973580	.75	.555933	6.60	.444067	13
48	.529864	5.85	.973535	.77	.556329	6.60	.443671	12
49	.530215	5.83	.973489	.75	.556725	6.60	.443275	11
50	.530565	5.83	.973444	.77	.557121	6.60	.442879	10
51	9.530915	5.83	9.973398	.77	9.557517	6.60	10.442483	9
52	.531265	5.82	.973352	.75	.557913	6.58	.442087	8
53	.531614	5.82	.973307	.77	.558308	6.58	.441692	7
54	.531963	5.82	.973261	.77	.558703	6.57	.441297	6
55	.532312	5.82	.973215	.77	.559097	6.57	.440903	5
56	.532661	5.80	.973169	.75	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.77	.559885	6.57	.440115	3
58	.533357	5.78	.973078	.77	.560279	6.57	.439721	2
59	.533704	5.80	.973032	.77	.560673	6.55	.439327	1
60′	9.534052		9.972986		9.561066		10.438934	0′
109°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	70°

Cosines, Tangents, and Cotangents

20°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	159°
0′	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60′
1	.534399	5.77	.972940	.77	.561459	6.53	.438541	59
2	.534745	5.78	.972894	.77	.561851	6.55	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.53	.437756	57
4	.535438	5.75	.972802	.78	.562636	6.53	.437364	56
5	.535783	5.77	.972755	.77	.563028	6.52	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.53	.436581	54
7	.536474	5.73	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.75	.972617	.78	.564202	6.52	.435798	52
9	.537163	5.73	.972570	.77	.564593	6.50	.435407	51
10	.537507	5.73	.972524	.77	.564983	6.50	.435017	50
11	9.537851	5.72	9.972478	.78	9.565373	6.50	10.434627	49
12	.538194	5.73	.972431	.77	.565763	6.50	.434237	48
13	.538538	5.70	.972385	.78	.566153	6.48	.433847	47
14	.538880	5.72	.972338	.78	.566542	6.50	.433458	46
15	.539223	5.70	.972291	.77	.566932	6.47	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.68	.972198	.78	.567709	6.48	.432291	43
18	.540249	5.68	.972151	.77	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.45	.431514	41
20	.540931	5.68	.972058	.78	.568873	6.47	.431127	40
21	9.541272	5.68	9.972011	.78	9.569261	6.45	10.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.67	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.65	.971870	.78	.570422	6.45	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.43	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.65	.971729	.78	.571581	6.43	.428419	33
28	.543649	5.63	.971682	.78	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.78	.572352	6.43	.427648	31
30	.544325	5.63	.971588	.80	.572738	6.42	.427262	30
31	9.544663	5.62	9.971540	.78	9.573123	6.40	10.426877	29
32	.545000	5.63	.971493	.78	.573507	6.42	.426493	28
33	.545338	5.60	.971446	.80	.573892	6.40	.426108	27
34	.545674	5.62	.971398	.78	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.80	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.78	.575044	6.38	.424956	24
37	.546683	5.60	.971256	.80	.575427	6.38	.424573	23
38	.547019	5.58	.971208	.78	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.80	.576193	6.38	.423807	21
40	.547689	5.58	.971113	.78	.576576	6.38	.423424	20
41	9.548024	5.58	9.971066	.80	9.576959	6.37	10.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.57	.970970	.80	.577723	6.35	.422277	17
44	.549027	5.55	.970922	.80	.578104	6.37	.421896	16
45	.549360	5.55	.970874	.78	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.35	.420752	13
48	.550359	5.55	.970731	.80	.579629	6.33	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.33	.419991	11
50	.551024	5.53	.970635	.82	.580389	6.33	.419611	10
51	9.551356	5.52	9.970586	.80	9.580769	6.33	10.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.52	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.50	.970394	.82	.582286	6.32	.417714	5
56	.553010	5.52	.970345	.80	.582665	6.32	.417335	4
57	.553341	5.48	.970297	.80	.583044	6.30	.416956	3
58	.553670	5.50	.970249	.82	.583422	6.30	.416578	2
59	.554000	5.48	.970200	.80	.583800	6.28	.416200	1
60′	9.554329		9.970152		9.584177		10.415823	0′
110° Cosine.		D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	69°

27. Logarithmic Sines,

21°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	158°
0′	9.554329	5.48	9.970152	.82	9.584177	6.30	10.415823	60′
1	.554658	5.48	.970103	.80	.584555	6.28	.415445	59
2	.554987	5.47	.970055	.82	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.80	.585686	6.27	.414314	56
5	.555971	5.47	.969909	.82	.586062	6.28	.413938	55
6	.556299	5.45	.969860	.82	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.25	.413185	53
8	.556953	5.45	.969762	.80	.587190	6.27	.412810	52
9	.557280	5.43	.969714	.82	.587566	6.25	.412434	51
10	.557606	5.43	.969665	.82	.587941	6.25	.412059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	.558258	5.42	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.43	.969518	.82	.589066	6.23	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.40	.969420	.83	.589814	6.23	.410186	45
16	.559558	5.42	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.83	.591308	6.22	.408692	41
20	.560855	5.38	.969173	.82	.591681	6.22	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.20	10.407946	39
22	.561501	5.38	.969075	.83	.592426	6.22	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.18	.406829	36
25	.562468	5.37	.968926	.82	.593542	6.20	.406458	35
26	.562790	5.37	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	.564075	5.35	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.33	9.968628	.83	9.595768	6.17	10.404232	29
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	28
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	27
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	23
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	22
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	21
40	.567269	5.30	.968178	.83	.599091	6.13	.400909	20
41	9.567587	5.28	9.968128	.83	9.599459	6.13	10.400541	19
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	18
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	17
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	16
45	.568856	5.27	.967927	.85	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.83	.601296	6.12	.398704	14
47	.569488	5.27	.967826	.85	.601663	6.10	.398337	13
48	.569804	5.27	.967775	.83	.602029	6.10	.397971	12
49	.570120	5.25	.967725	.85	.602395	6.10	.397605	11
50	.570435	5.27	.967674	.83	.602761	6.10	.397239	10
51	9.570751	5.25	9.967624	.85	9.603127	6.10	10.396873	9
52	.571066	5.23	.967573	.85	.603493	6.08	.396507	8
53	.571380	5.25	.967522	.85	.603858	6.08	.396142	7
54	.571695	5.23	.967471	.83	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.22	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.23	.967319	.85	.605317	6.08	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.20	.967217	.85	.606046	6.07	.393954	1
60′	9.573575		9.967166		9.606410		10.393590	0
111°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	68°

Cosines, Tangents, and Cotangents

22°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	157°
0'	9.573575	5.22	9.967166	.85	9.606410	6.05	10.393590	60'
1	.573888	5.20	.967115	.85	.606773	6.07	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.87	.607500	6.05	.392500	57
4	.574824	5.20	.966961	.85	.607863	6.03	.392137	56
5	.575136	5.18	.966910	.85	.608225	6.05	.391775	55
6	.575447	5.18	.966859	.85	.608588	6.03	.391412	54
7	.575758	5.18	.966808	.87	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.85	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.87	.609674	6.03	.390326	51
10	.576689	5.17	.966653	.85	.610036	6.02	.389964	50
11	9.576999	5.17	9.966602	.87	9.610397	6 03	10.389603	49
12	.577309	5.15	.966550	.85	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.87	.611120	6.00	.388880	47
14	.577927	5.15	.966447	.87	.611480	6.02	.388520	46
15	.578236	5.15	.966395	.85	.611841	6.00	.388159	45
16	.578545	5.13	.966344	.87	.612201	6.00	.387799	44
17	.578853	5.15	.966292	.87	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.87	.612921	6.00	.387079	42
19	.579470	5.12	.966188	.87	.613281	6.00	.386719	41
20	.579777	5.13	.966136	.85	.613641	5.98	.386359	40
21	9.580085	5.12	9.966085	.87	9.614000	5 98	10.386000	39
22	.580392	5.12	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.10	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.12	.965929	.88	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.08	.965772	.87	.616151	5.97	.383849	33
28	.582229	5.10	.965720	.87	.616509	5 97	.383491	32
29	.582535	5.08	.965668	.88	.616867	5.95	.383133	31
30	.582840	5.08	.965615	.87	.617224	5.97	.382776	30
31	9.583145	5.07	9.965563	.87	9 617582	5 95	10.382418	29
32	.583449	5.08	.965511	.88	.617939	5.93	.382061	28
33	.583754	5.07	.965458	.87	.618295	5 95	.381705	27
34	.584058	5.05	.965406	.88	.618652	5.93	.381348	26
35	.584361	5.07	.965353	.87	.619008	5.93	.380992	25
36	.584665	5.05	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.07	.965248	.88	.619720	5.93	.380280	23
38	.585272	5.03	.965195	.87	.620076	5 93	.379924	22
39	.585574	5.05	.965143	.88	.620432	5 92	.379568	21
40	.585877	5.03	.965090	.88	.620787	5.92	.379213	20
41	9.586179	5.05	9.965037	.88	9.621142	5.92	10.378858	19
42	.586482	5.02	.964984	.88	.621497	5 92	.378503	18
43	.586783	5.03	.964931	.87	.621852	5.92	.378148	17
44	.587085	5 02	.964879	.88	.622207	5.90	.377793	16
45	.587386	5 03	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.02	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.00	.964720	.90	.623269	5.90	.376731	13
48	.588289	5.02	.964666	.88	.623623	5.88	.376377	12
49	.588590	5.00	.964613	.88	.623976	5 90	.376024	11
50	.588890	5.00	.964560	.88	.624330	5.88	.375670	10
51	9.589190	4.98	9.964507	.88	9.624683	5.88	10.375317	9
52	.589489	5.00	.964454	.90	.625036	5 87	.374964	8
53	.589789	4.98	.964400	.88	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.88	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.90	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.88	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.90	.626797	5 87	.373203	3
58	.591282	4.97	.964133	.88	.627149	5.87	.372851	2
59	.591580	4.97	.964080	.90	.627501	5.85	.372499	1
60'	9.591878		9.964026		9.627852		10.372148	0'
112°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	67°

27. Logarithmic Sines,

23°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	156°
0′	9.591878	4.97	9.964026	.90	9.627852	5.85	10.372148	60′
1	.592176	4.95	.963972	.88	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.83	.371095	57
4	.593067	4.93	.963811	.90	.629255	5.85	.370745	56
5	.593362	4.93	.963757	.88	.629606	5.83	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.82	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.83	.368995	51
10	.594842	4.92	.963488	.90	.631355	5.82	.368645	50
11	9.595137	4.92	9.963434	.92	9.631704	5.82	10.368296	49
12	.595432	4.92	.963379	.90	.632053	5.82	.367947	48
13	.595727	4.90	.963325	.90	.632402	5.80	.367598	47
14	.596021	4.90	.963271	.90	.632750	5.82	.367250	46
15	.596315	4.90	.963217	.90	.633099	5.80	.366901	45
16	.596609	4.90	.963163	.92	.633447	5.80	.366553	44
17	.596903	4.88	.963108	.90	.633795	5.80	.366205	43
18	.597196	4.90	.963054	.92	.634143	5.78	.365857	42
19	.597490	4.88	.962999	.90	.634490	5.80	.365510	41
20	.597783	4.87	.962945	.92	.634838	5.78	.365162	40
21	9.598075	4.88	9.962890	.90	9.635185	5.78	10.364815	39
22	.598368	4.87	.962836	.92	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.90	.635879	5.78	.364121	37
24	.598952	4.87	.962727	.92	.636226	5.77	.363774	36
25	.599244	4.87	.962672	.92	.636572	5.78	.363428	35
26	.599536	4.85	.962617	.92	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.90	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.92	.637611	5.75	.362389	32
29	.600409	4.85	.962453	.92	.637956	5.77	.362044	31
30	.600700	4.83	.962398	.92	.638302	5.75	.361698	30
31	9.600990	4.83	9.962343	.92	9.638647	5.75	10.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.75	.360318	26
35	.602150	4.82	.962123	.93	.640027	5.73	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.75	.359629	24
37	.602728	4.82	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.80	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.82	.961902	.93	.641404	5.72	.358596	21
40	.603594	4.80	.961846	.92	.641747	5.73	.358253	20
41	9.603882	4.80	9.961791	.93	9.642091	5.72	10.357909	19
42	.604170	4.78	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.80	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.78	.961624	.92	.643120	5.72	.356880	16
45	.605032	4.78	.961569	.93	.643463	5.72	.356537	15
46	.605319	4.78	.961513	.92	.643806	5.70	.356194	14
47	.605606	4.77	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.78	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	.606465	4.77	.961290	.92	.645174	5.70	.354826	10
51	9.606751	4.75	9.961235	.93	9.645516	5.68	10.354484	9
52	.607036	4.77	.961179	.93	.645857	5.70	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.68	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.75	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.73	.960955	.93	.647222	5.67	.352778	4
57	.608461	4.73	.960899	.93	.647562	5.68	.352438	3
58	.608745	4.73	.960843	.95	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.93	.648243	5.67	.351757	1
60′	9.609313		9.960730		9.648583		10.351417	0′
113°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	66°

Cosines, Tangents, and Cotangents

24°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	155°
0'	9.609313	4.73	9.960730	.93	9.648583	5.67	10.351417	60'
1	.609597	4.72	.960674	.93	.648923	5.67	.351077	59
2	.609880	4.73	.960618	.95	.649263	5.65	.350737	58
3	.610164	4.72	.960561	.93	.649602	5.67	.350398	57
4	.610447	4.70	.960505	.95	.649942	5.65	.350058	56
5	.610729	4.72	.960448	.93	.650281	5.65	.349719	55
6	.611012	4.70	.960392	.95	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.93	.650959	5.63	.349041	53
8	.611576	4.70	.960279	.95	.651297	5.65	.348703	52
9	.611858	4.70	.960222	.95	.651636	5.63	.348364	51
10	.612140	4.68	.960165	.93	.651974	5.63	.348026	50
11	9.612421	4.68	9.960109	.95	9.652312	5.63	10.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.93	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.67	.959711	.95	.654674	5.62	.345326	42
19	.614665	4.65	.959654	.97	.655011	5.62	.344989	41
20	.614944	4.65	.959596	.95	.655348	5.60	.344652	40
21	9.615223	4.65	9.959539	.95	9.655684	5.60	10.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.65	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.63	.959368	.97	.656692	5.60	.343308	36
25	.616338	4.63	.959310	.95	.657028	5.60	.342972	35
26	.616616	4.63	.959253	.97	.657364	5.58	.342636	34
27	.616894	4.63	.959195	.95	.657699	5.58	.342301	33
28	.617172	4.63	.959138	.97	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.95	.658369	5.58	.341631	31
30	.617727	4.62	.959023	.97	.658704	5.58	.341296	30
31	9.618004	4.62	9.958965	.95	9.659039	5.57	10.340961	29
32	.618281	4.62	.958908	.97	.659373	5.58	.340627	28
33	.618558	4.60	.958850	.97	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.97	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.95	.660376	5.57	.339624	25
36	.619386	4.60	.958677	.97	.660710	5.55	.339290	24
37	.619662	4.60	.958619	.97	.661043	5.57	.338957	23
38	.619938	4.58	.958561	.97	.661377	5.55	.338623	22
39	.620213	4.58	.958503	.97	.661710	5.55	.338290	21
40	.620488	4.58	.958445	.97	.662043	5.55	.337957	20
41	9.620763	4.58	9.958387	.97	9.662376	5.55	10.337624	19
42	.621038	4.58	.958329	.97	.662709	5.55	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.55	.336958	17
44	.621587	4.57	.958213	.98	.663375	5.53	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.53	.336293	15
46	.622135	4.57	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.55	.958038	.98	.664371	5.53	.335629	13
48	.622682	4.57	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.52	.334965	11
50	.623229	4.55	.957863	.98	.665366	5.53	.334634	10
51	9.623502	4.53	9.957804	.97	9.665698	5.52	10.334302	9
52	.623774	4.55	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.53	.957687	.98	.666360	5.52	.333640	7
54	.624319	4.53	.957628	.97	.666691	5.50	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.52	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.50	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.52	.332318	3
58	.625406	4.52	.957393	.97	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60'	9.625948		9.957276		9.668673		10.331327	0'
114° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	65°

27. Logarithmic Sines,

25°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	154°
0'	9.625948	4.52	9.957276	.98	9.668673	5.48	10.331327	60'
1	.626219	4.52	.957217	.98	.669002	5.50	.330998	59
2	.626490	4.50	.957158	.98	.669332	5.48	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.50	.330339	57
4	.627030	4.50	.957040	.98	.669991	5.48	.330009	56
5	.627300	4.50	.956981	1.00	.670320	5.48	.329680	55
6	.627570	4.50	.956921	.98	.670649	5.47	.329351	54
7	.627840	4.48	.956862	.98	.670977	5.48	.329023	53
8	.628109	4.48	.956803	.98	.671306	5.48	.328694	52
9	.628378	4.48	.956744	1.00	.671635	5.47	.328365	51
10	.628647	4.48	.956684	.98	.671963	5.47	.328037	50
11	9.628916	4.48	9.956625	.98	9.672291	5.47	10.327709	49
12	.629185	4.47	.956566	1.00	.672619	5.47	.327381	48
13	.629453	4.47	.956506	.98	.672947	5.45	.327053	47
14	.629721	4.47	.956447	1.00	.673274	5.47	.326726	46
15	.629989	4.47	.956387	1.00	.673602	5.45	.326398	45
16	.630257	4.45	.956327	.98	.673929	5.47	.326071	44
17	.630524	4.47	.956268	1.00	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	325416	42
19	.631059	4.45	.956148	.98	.674911	5.43	.325089	41
20	.631326	4.45	.956089	1.00	.675237	5.45	.324763	40
21	9.631593	4.43	9.956029	1.00	9.675564	5.43	10.324436	39
22	.631859	4.43	.955969	1.00	.675890	5.45	.324110	38
23	.632125	4.45	.955909	1.00	.676217	5.43	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.42	.955789	1.00	.676869	5.42	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.43	.322480	33
28	.633454	4.42	.955609	.98	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	.633984	4.42	.955488	1.00	.678496	5.42	.321504	30
31	9.634249	4.42	9.955428	1.00	9.678821	5.42	10.321179	29
32	.634514	4.40	.955368	1.02	.679146	5.42	.320854	28
33	.634778	4.40	.955307	1.00	.679471	5.40	.320529	27
34	.635042	4.40	.955247	1.02	.679795	5.42	.320205	26
35	.635306	4.40	.955186	1.00	.680120	5.40	.319880	25
36	.635570	4.40	.955126	1.02	.680444	5.40	.319556	24
37	.635834	4.38	.955065	1.00	.680768	5.40	.319232	23
38	.636097	4 38	.955005	1.02	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.02	.681416	5.40	.318584	21
40	.636623	4.38	.954883	1.00	.681740	5.38	.318260	20
41	9.636886	4.37	9.954823	1.02	9.682063	5.40	10.317937	19
42	.637148	4.38	.954762	1.02	.682387	5.38	.317613	18
43	.637411	4.37	.954701	1.02	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.37	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4 35	.954518	1.02	.683679	5.37	.316321	14
47	.638458	4.37	.954457	1.02	.684001	5.38	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	.639242	4.35	.954274	1.02	.684968	5.37	.315032	10
51	9.639503	4.35	9.954213	1.02	9.685290	5.37	10.314710	9
52	.639764	4.33	.954152	1.03	.685612	5.37	.314388	8
53	.640024	4.33	.954090	1.02	.685934	5.35	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.37	.313745	6
55	.640544	4.33	.953968	1.03	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.33	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.02	.687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
60'	9.641842		9.953660		9.688182		10.311818	0'
115°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	64°

Cosines, Tangents, and Cotangents

26°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	153°
0′	9.641842	4.32	9.953660	1.02	9.688182	5.33	10.311818	60′
1	.642101	4.32	.953599	1.03	.688502	5.35	.311498	59
2	.642360	4.30	.953537	1.03	.688823	5.33	.311177	58
3	.642618	4.32	.953475	1.03	.689143	5.33	.310857	57
4	.642877	4.30	.953413	1.02	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5.33	.310217	55
6	.643393	4.28	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.30	.953228	1.03	.690423	5.32	.309577	53
8	.643908	4.28	.953166	1.03	.690742	5.33	.309258	52
9	.644165	4.30	.953104	1.03	.691062	5.32	.308938	51
10	.644423	4.28	.953042	1.03	.691381	5.32	.308619	50
11	9.644680	4.27	9.952980	1.03	9.691700	5.32	10.308300	49
12	.644936	4.28	.952918	1.05	.692019	5.32	.307981	48
13	.645193	4.28	.952855	1.03	.692338	5.30	.307662	47
14	.645450	4.27	.952793	1.03	.692656	5.32	.307344	46
15	.645706	4.27	.952731	1.03	.692975	5.30	.307025	45
16	.645962	4.27	.952669	1.05	.693293	5.32	.306707	44
17	.646218	4.27	.952606	1.03	.693612	5.30	.306388	43
18	.646474	4.25	.952544	1.05	.693930	5.30	.306070	42
19	.646729	4.25	.952481	1.03	.694248	5.30	.305752	41
20	.646984	4.27	.952419	1.05	.694566	5.28	.305434	40
21	9.647240	4.23	9.952356	1.03	9.694883	5.30	10.305117	39
22	.647494	4.25	.952294	1.05	.695201	5.28	.304799	38
23	.647749	4.25	.952231	1.05	.695518	5.30	.304482	37
24	.648004	4.23	.952168	1.03	.695836	5.28	.304164	36
25	.648258	4.23	.952106	1.05	.696153	5.28	.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.27	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	.649527	4.23	.951791	1.05	.697736	5.28	.302264	30
31	9.649781	4.22	9.951728	1.05	9.698053	5.27	10.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.20	.95160[illegible]	1.05	.698685	5.27	.301315	27
34	.650539	4.22	.951539	1.05	.699001	5.25	.300999	26
35	.650792	4.20	.951476	1.07	.699316	5.27	.300684	25
36	.651044	4.22	.951412	1.05	.699632	5.25	.300368	24
37	.651297	4.20	.951349	1.05	.699947	5.27	.300053	23
38	.651549	4.18	.951286	1.07	.700263	5.25	.299737	22
39	.651800	4.20	.951222	1.05	.700578	5.25	.299422	21
40	.652052	4.20	.951159	1.05	.700893	5.25	.299107	20
41	9.652304	4.18	9.951096	1.07	9.701208	5.25	10.298792	19
42	.652555	4.18	.951032	1.07	.701523	5.23	.298477	18
43	.652806	4.18	.950968	1.05	.701837	5.25	.298163	17
44	.653057	4.18	.950905	1.07	.702152	5.23	.297848	16
45	653308	4.17	.950841	1.05	.702466	5.25	.297534	15
46	.653558	4.17	.950778	1.07	.702781	5.23	.297219	14
47	.653808	4.18	.950714	1.07	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.07	.703409	5.22	.296591	12
49	.654309	4.15	.950586	1.07	.703722	5.23	.296278	11
50	.654558	4.17	.950522	1.07	.704036	5.23	.295964	10
51	9.654808	4.17	9.950458	1.07	9.704350	5.22	10.295650	9
52	.655058	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704976	5.23	.295024	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.22	.294397	5
56	.656054	4.13	.950138	1.07	.705916	5.20	.294084	4
57	656302	4.15	.950074	1.07	.706228	5.22	.293772	3
58	.656551	4.13	.950010	1.08	.706541	5.22	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.20	.293146	1
60′	9.657047		9.949881		9.707166		10.292834	0′
116°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	63°

27. Logarithmic Sines,

27°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	152°
0′	9.657047	4.13	9.949881	1.08	9.707166	5.20	10.292834	60′
1	.657295	4.12	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.13	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	.658284	4.12	.949558	1.07	.708726	5.18	.291274	55
6	.658531	4.12	.949494	1.08	.709037	5.20	.290963	54
7	.658778	4.12	.949429	1.08	.709349	5.18	.290651	53
8	.659025	4.10	.949364	1.07	.709660	5.18	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	.659517	4.10	.949235	1.08	.710282	5.18	.289718	50
11	9.659763	4.10	9.949170	1.08	9.710593	5.18	10.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.10	.949040	1.08	.711215	5.17	.288785	47
14	.660501	4.08	.948975	1.08	.711525	5.18	.288475	46
15	.660746	4.08	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.08	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.08	.712766	5.17	.287234	42
19	.661726	4.07	.948650	1.10	.713076	5.17	.286924	41
20	.661970	4.07	.948584	1.08	.713386	5.17	.286614	40
21	9.662214	4.08	9.948519	1.08	9.713696	5.15	10.286304	39
22	.662459	4.07	.948454	1.10	.714005	5.15	.285995	38
23	.662703	4 05	.948388	1.08	.714314	5.17	.285686	37
24	.662946	4.07	.948323	1.10	.714624	5.15	.285376	36
25	.663190	4.05	.948257	1.08	.714933	5.15	.285067	35
26	.663433	4.07	.948192	1.10	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.10	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.08	.715860	5.13	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.15	.283832	31
30	.664406	4.03	.947929	1.10	.716477	5.13	.283523	30
31	9.664648	4.05	9.947863	1.10	9.716785	5.13	10.283215	29
32	.664891	4.03	.947797	1.10	.717093	5.13	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.08	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.12	.718017	5.13	.281983	25
36	.665859	4.02	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.03	.947467	1 10	.718633	5.12	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.13	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	.666824	4.02	.947269	1.10	.719555	5.12	.280445	20
41	9.667065	4.00	9.947203	1.12	9.719862	5 12	10.280138	19
42	.667305	4.02	.947136	1.10	.720169	5 12	.279831	18
43	.667546	4.00	.947070	1.10	.720476	5.12	.279524	17
44	667786	4.02	.947004	1.12	.720783	5.10	.279217	16
45	.668027	4.00	.946937	1.10	.721089	5.12	.278911	15
46	.668267	3.98	.946871	1.12	.721396	5.10	.278604	14
47	.668506	4.00	.946804	1.10	.721702	5.12	.278298	13
48	.668746	4 00	.946738	1.12	.722009	5.10	.277991	12
49	.668986	3.98	.946671	1.12	.722315	5.10	.277685	11
50	.669225	3.98	.946604	1.10	.722621	5.10	.277379	10
51	9.669464	3 98	9.946538	1.12	9.722927	5.08	10.277073	9
52	.669703	3.98	.946471	1.12	.723232	5.10	.276768	8
53	.669942	3.98	.946404	1.12	.723538	5.10	.276462	7
54	.670181	3.97	.946337	1.12	.723844	5.08	.276156	6
55	.670419	3 98	.946270	1.12	.724149	5.08	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.10	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.97	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.95	946002	1.12	.725370	5.07	.274630	1
60	9.671609		9 945935		9.725674		10.274326	0′
117° Cosine.		D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	62°

Cosines, Tangents, and Cotangents

28°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	151°
0′	9.671609	3.97	9.945935	1.12	9.725674	5.08	10.274326	60′
1	.671847	3.95	.945868	1.13	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.13	.726892	5.08	.273108	56
5	.672795	3.95	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.93	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.95	.945464	1.13	.727805	5.07	.272195	53
8	.673505	3.93	.945396	1.13	.728109	5.05	.271891	52
9	.673741	3.93	.945328	1.12	.728412	5.07	.271588	51
10	.673977	3.93	.945261	1.13	.728716	5.07	.271284	50
11	9.674213	3.92	9.945193	1.13	9.729020	5.05	10.270980	49
12	.674448	3.93	.945125	1.12	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.93	.944990	1.13	.729929	5.07	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.03	.269767	45
16	.675390	3.90	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.92	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3.92	.944718	1.13	.731141	5.05	.268859	42
19	.676094	3.90	.944650	1.13	.731444	5.03	.268556	41
20	.676328	3.90	.944582	1.13	.731746	5.03	.268254	40
21	9.676562	3.90	9.944514	1.13	9.732048	5.05	10.267952	39
22	.676796	3.90	.944446	1.15	.732351	5.03	.267649	38
23	.677030	3.90	.944377	1.13	.732653	5.03	.267347	37
24	.677264	3.90	.944309	1.13	.732955	5.03	.267045	36
25	.677498	3.88	.944241	1.15	.733257	5.02	.266743	35
26	.677731	3.88	.944172	1.13	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.13	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.15	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.13	.734463	5.02	.265537	31
30	.678663	3.87	.943899	1.15	.734764	5.03	.265236	30
31	9.678895	3.88	9.943830	1.15	9.735066	5.02	10.264934	29
32	.679128	3.87	.943761	1.13	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.02	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.00	.264031	26
35	.679824	3.87	.943555	1.15	.736269	5.02	.263731	25
36	.680056	3.87	.943486	1.15	.736570	5.00	.263430	24
37	.680288	3.85	.943417	1.15	.736870	5.02	.263130	23
38	.680519	3.85	.943348	1.15	.737171	5.00	.262829	22
39	.680750	3.87	.943279	1.15	.737471	5.00	.262529	21
40	.680982	3.85	.943210	1.15	.737771	5.00	.262229	20
41	9.681213	3.83	9.943141	1.15	9.738071	5.00	10.261929	19
42	.681443	3.85	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.85	.943003	1.15	.738671	5.00	.261329	17
44	.681905	3.83	.942934	1.17	.738971	5.00	.261029	16
45	.682135	3.83	.942864	1.15	.739271	4.98	.260729	15
46	.682365	3.83	.942795	1.15	.739570	5.00	.260430	14
47	.682595	3.83	.942726	1.17	.739870	4.98	.260130	13
48	.682825	3.83	.942656	1.15	.740169	4.98	.259831	12
49	.683055	3.82	.942587	1.17	.740468	4.98	.259532	11
50	.683284	3.83	.942517	1.15	.740767	4.98	.259233	10
51	9.683514	3.82	9.942448	1.17	9.741066	4.98	10.258934	9
52	.683743	3.82	.942378	1.17	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.15	.741664	4.97	.258336	7
54	.684201	3.82	.942239	1.17	.741962	4.98	.258038	6
55	.684430	3.80	.942169	1.17	.742261	4.97	.257739	5
56	.684658	3.82	.942099	1.17	.742559	4.98	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60′	9.685571		9.941819		9.743752		10.256248	0′
118° Cosine.		D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	61°

27. Logarithmic Sines,

29°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	150°
0′	9.685571	3.80	9.941819	1.17	9.743752	4.97	10.256248	60′
1	.685799	3.80	.941749	1.17	.744050	4.97	.255950	59
2	.686027	3.78	.941679	1.17	.744348	4.95	.255652	58
3	.686254	3.80	.941609	1.17	.744645	4.97	.255355	57
4	.686482	3.78	.941539	1.17	.744943	4.95	.255057	56
5	.686709	3.78	.941469	1.18	.745240	4.97	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.77	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.18	.746132	4.95	.253868	52
9	.687616	3.78	.941187	1.17	.746429	4.95	.253571	51
10	.687843	3.77	.941117	1.18	.746726	4.95	.253274	50
11	9.688069	3.77	9.941046	1.18	9.747023	4.93	10.252977	49
12	.688295	3.77	.940975	1.17	.747319	4.95	.252681	48
13	.688521	3.77	.940905	1.18	.747616	4.95	.252384	47
14	.688747	3.75	.940834	1.18	.747913	4.93	.252087	46
15	.688972	3.77	.940763	1.17	.748209	4.93	.251791	45
16	.689198	3.75	.940693	1.18	.748505	4.93	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	.690098	3.75	.940409	1.18	.749689	4.93	.250311	40
21	9.690323	3.75	9.940338	1.18	9.749985	4.93	10.250015	39
22	.690548	3.73	.940267	1.18	.750281	4.92	.249719	38
23	.690772	3.73	.940196	1.18	.750576	4.93	.249424	37
24	.690996	3.73	.940125	1.18	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.20	.751167	4.92	.248833	35
26	.691444	3.73	.939982	1.18	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.18	.751757	4.92	.248243	33
28	.691892	3.72	.939840	1.20	.752052	4.92	.247948	32
29	.692115	3.73	.939768	1.18	.752347	4.92	.247653	31
30	.692339	3.72	.939697	1.20	.752642	4.92	.247358	30
31	9.692562	3.72	9.939625	1.18	9.752937	4.90	10.247063	29
32	.692785	3.72	.939554	1.20	.753231	4.92	.246769	28
33	.693008	3.72	.939482	1.20	.753526	4.90	.246474	27
34	.693231	3.70	.939410	1.18	.753820	4.92	.246180	26
35	.693453	3.72	.939339	1.20	.754115	4.90	.245885	25
36	.693676	3.70	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.18	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	.694564	3.70	.938980	1.20	.755585	4.88	.244415	20
41	9.694786	3.68	9.938908	1.20	9.755878	4.90	10.244122	19
42	.695007	3.70	.938836	1.22	.756172	4.88	.243828	18
43	.695229	3.68	.938763	1.20	.756465	4.90	.243535	17
44	.695450	3.68	.938691	1.20	.756759	4.88	.243241	16
45	.695671	3.68	.938619	1.20	.757052	4.88	.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.22	.757638	4.88	.242362	13
48	.696334	3.67	.938402	1.20	.757931	4.88	.242069	12
49	.696554	3.68	.938330	1.20	.758224	4.88	.241776	11
50	.696775	3.67	.938258	1.22	.758517	4.88	.241483	10
51	9.696995	3.67	9.938185	1.20	9.758810	4.87	10.241190	9
52	.697215	3.67	.938113	1.22	.759102	4.88	.240898	8
53	.697435	3.65	.938040	1.22	.759395	4.87	.240605	7
54	.697654	3.67	.937967	1.20	.759687	4.87	.240313	6
55	.697874	3.67	.937895	1.22	.759979	4.88	.240021	5
56	.698094	3.65	.937822	1.22	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.22	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.20	.760856	4.87	.239144	2
59	.698751	3.65	.937604	1.22	.761148	4.85	.238852	1
60′	9.698970		9.937531		9.761439		10.238561	0′
119°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	60°

Cosines, Tangents, and Cotangents

30°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	149°
0′	9.698970	3.65	9.937531	1.22	9.761439	4.87	10.238561	60′
1	.699189	3.63	.937458	1.22	.761731	4.87	.238269	59
2	.699407	3.65	.937385	1.22	.762023	4.85	.237977	58
3	.699626	3.63	.937312	1.23	.762314	4.87	.237686	57
4	.699844	3.63	.937238	1.22	.762606	4.85	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.62	.936946	1.23	.763770	4.85	.236230	52
9	.700933	3.63	.936872	1.22	.764061	4.85	.235939	51
10	.701151	3.62	.936799	1.23	.764352	4.85	.235648	50
11	9.701368	3.62	9.936725	1.22	9.764643	4.83	10.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.85	.235067	48
13	.701802	3.62	.936578	1.22	.765224	4.83	.234776	47
14	.702019	3.62	.936505	1.23	.765514	4.85	.234486	46
15	.702236	3.60	.936431	1.23	.765805	4.83	.234195	45
16	.702452	3.62	.936357	1.22	.766095	4.83	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	.703317	3.60	.936062	1.23	.767255	4.83	.232745	40
21	9.703533	3.60	9.935988	1.23	9.767545	4.82	10.232455	39
22	.703749	3.58	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.58	.935840	1.23	.768124	4.83	.231876	37
24	.704179	3.60	.935766	1.23	.768414	4.82	.231586	36
25	.704395	3.58	.935692	1.23	.768703	4.82	.231297	35
26	.704610	3.58	.935618	1.25	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.23	.769281	4.83	.230719	33
28	.705040	3.57	.935469	1.23	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.25	.769860	4.80	.230140	31
30	.705469	3.57	.935320	1.23	.770148	4.82	.229852	30
31	9.705683	3.58	9.935246	1.25	9.770437	4.82	10.229563	29
32	.705898	3.57	.935171	1.23	.770726	4.82	.229274	28
33	.706112	3.57	.935097	1.25	.771015	4.80	.228985	27
34	.706326	3.55	.935022	1.23	.771303	4.82	.228697	26
35	.706539	3.57	.934948	1.25	.771592	4.80	.228408	25
36	.706753	3.57	.934873	1.25	.771880	4.80	.228120	24
37	.706967	3.55	.934798	1.25	.772168	4.82	.227832	23
38	.707180	3.55	.934723	1.23	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	.707606	3.55	.934574	1.25	.773033	4.80	.226967	20
41	9.707819	3.55	9.934499	1.25	9.773321	4.78	10.226679	19
42	.708032	3.55	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.55	.934349	1.25	.773896	4.80	.226104	17
44	.708458	3.53	.934274	1.25	.774184	4.78	.225816	16
45	.708670	3.53	.934199	1.27	.774471	4.80	.225529	15
46	.708882	3.53	.934123	1.25	.774759	4.78	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.78	.224954	13
48	.709306	3.53	.933973	1.25	.775333	4.80	.224667	12
49	.709518	3.53	.933898	1.27	.775621	4.78	.224379	11
50	.709730	3.52	.933822	1.25	.775908	4.78	.224092	10
51	9.709941	3.53	9.933747	1.27	9.776195	4.78	10.223805	9
52	.710153	3.52	.933671	1.25	.776482	4.77	.223518	8
53	.710364	3.52	.933596	1.27	.776768	4.78	.223232	7
54	.710575	3.52	.933520	1.25	.777055	4.78	.222945	6
55	.710786	3.52	933445	1.27	.777342	4.77	.222658	5
56	.710997	3.52	.933369	1.27	.777628	4.78	.222372	4
57	.711208	3.52	.933293	1.27	.777915	4.77	.222085	3
58	.711419	3.50	.933217	1.27	.778201	4.78	.221799	2
59	.711629	3.50	.933141	1.25	.778488	4.77	.221512	1
60′	9.711839		9.933066		9.778774		10.221226	0′
120°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	59°

27. Logarithmic Sines,

31°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	148°
0'	9.711839	3.52	9.933066	1.27	9.778774	4.77	10.221226	60'
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.48	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.77	.220368	57
4	.712679	3.50	.932762	1.28	.779918	4.75	.220082	56
5	.712889	3.48	.932685	1.27	.780203	4.77	.219797	55
6	.713098	3.50	.932609	1.27	.780489	4.77	.219511	54
7	.713308	3.48	.932533	1.27	.780775	4.75	.219225	53
8	.713517	3.48	.932457	1.28	.781060	4.77	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.75	.218654	51
10	.713935	3.48	.932304	1.27	.781631	4.75	.218369	50
11	9.714144	3.47	9.932228	1.28	9.781916	4.75	10.218084	49
12	.714352	3.48	.932151	1.27	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.48	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.27	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.47	.931768	1.28	.783626	4.73	.216374	43
18	.715602	3.45	.931691	1.28	.783910	4.75	.216090	42
19	.715809	3.47	.931614	1.28	.784195	4.73	.215805	41
20	.716017	3.45	.931537	1.28	.784479	4.75	.215521	40
21	9.716224	3.47	9.931460	1.28	9.784764	4.73	10.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.73	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.73	.214668	37
24	.716846	3.45	.931229	1.28	.785616	4.73	.214384	36
25	.717053	3.43	.931152	1.28	.785900	4.73	.214100	35
26	.717259	3.45	.931075	1.28	.786184	4.73	.213816	34
27	.717466	3.45	.930998	1.28	.786468	4.73	.213532	33
28	.717673	3.43	.930921	1.30	.786752	4.73	.213248	32
29	.717879	3.43	.930843	1.28	.787036	4.72	.212964	31
30	.718085	3.43	.930766	1.30	.787319	4.73	.212681	30
31	9.718291	3.43	9.930688	1.28	9.787603	4.72	10.212397	29
32	.718497	3.43	.930611	1.30	.787886	4.73	.212114	28
33	.718703	3.43	.930533	1.28	.788170	4.72	.211830	27
34	.718909	3.42	.930456	1.30	.788453	4.72	.211547	26
35	.719114	3.43	.930378	1.30	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.28	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.72	.210415	22
39	.719935	3.42	.930067	1.30	.789868	4.72	.210132	21
40	.720140	3.42	.929989	1.30	.790151	4.72	.209849	20
41	9.720345	3.40	9.929911	1.30	9.790434	4.70	10.209566	19
42	.720549	3.42	.929833	1.30	.790716	4.72	.209284	18
43	.720754	3.40	.929755	1.30	.790999	4.70	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.70	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.72	.208437	15
46	.721366	3.40	.929521	1.32	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.30	.792128	4.70	.207872	13
48	.721774	3.40	.929364	1.30	.792410	4.70	.207590	12
49	.721978	3.38	.929286	1.32	.792692	4.70	.207308	11
50	.722181	3.40	.929207	1.30	.792974	4.70	.207026	10
51	9.722385	3.38	9.929129	1.32	9.793256	4.70	10.206744	9
52	.722588	3.38	.929050	1.30	.793538	4.68	.206462	8
53	.722791	3.38	.928972	1.32	.793819	4.70	.206181	7
54	.722994	3.38	.928893	1.30	.794101	4.70	.205899	6
55	.723197	3.38	.928815	1.32	.794383	4.68	.205617	5
56	.723400	3.38	.928736	1.32	.794664	4.70	.205336	4
57	.723603	3.37	.928657	1.32	.794946	4.68	.205054	3
58	.723805	3.37	.928578	1.32	.795227	4.68	.204773	2
59	.724007	3.38	.928499	1.32	.795508	4.68	.204492	1
60'	9.724210		9.928420		9.795789		10.204211	0'
121°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	58°

Cosines, Tangents, and Cotangents

32°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	147°
0′	9.724210	3.37	9.928420	1.30	9.795789	4.68	10.204211	60′
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.37	.928263	1.33	.796351	4.68	.203649	58
3	.724816	3.35	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.37	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.35	.928025	1.32	.797194	4.67	.202806	55
6	.725420	3.37	.927946	1.32	.797474	4.68	.202526	54
7	.725622	3.35	.927867	1.33	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	.726225	3.35	.927629	1.33	.798596	4.68	.201404	50
11	9.726426	3.33	9.927549	1.32	9.798877	4.67	10.201123	49
12	.726626	3.35	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.33	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.35	.927310	1.32	.799717	4.67	.200283	46
15	.727228	3.33	.927231	1.33	.799997	4.67	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.67	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.65	.199443	43
18	.727828	3.32	.926991	1.33	.800836	4.67	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.67	.198884	41
20	.728227	3.33	.926831	1.33	.801396	4.65	.198604	40
21	9.728427	3.32	9.926751	1.33	9.801675	4.67	10.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.65	.198045	38
23	.728825	3.32	.926591	1.33	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.33	.802513	4.65	.197487	36
25	.729223	3.32	.926431	1.33	.802792	4.67	.197208	35
26	.729422	3.32	.926351	1.35	.803072	4.65	.196928	34
27	.729621	3.32	.926270	1.33	.803351	4.65	.196649	33
28	.729820	3.30	.926190	1.33	.803630	4.65	.196370	32
29	.730018	3.32	.926110	1.35	.803909	4.63	.196091	31
30	.730217	3.30	.926029	1.33	.804187	4.65	.195813	30
31	9.730415	3.30	9.925949	1.35	9.804466	4.65	10.195534	29
32	.730613	3.30	.925868	1.33	.804745	4.63	.195255	28
33	.730811	3.30	.925788	1.35	.805023	4.65	.194977	27
34	.731009	3.28	.925707	1.35	.805302	4.63	.194698	26
35	.731206	3.30	.925626	1.35	.805580	4.65	.194420	25
36	.731404	3.30	.925545	1.33	.805859	4.63	.194141	24
37	.731602	3.28	.925465	1.35	.806137	4.63	.193863	23
38	.731799	3.28	.925384	1.35	.806415	4.63	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	.732193	3.28	.925222	1.35	.806971	4.63	.193029	20
41	9.732390	3.28	9.925141	1.35	9.807249	4.63	10.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.27	.924979	1.37	.807805	4.63	.192195	17
44	.732980	3.28	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.62	.191639	15
46	.733373	3.27	.924735	1.35	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.37	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.35	.809193	4.63	.190807	12
49	.733961	3.27	.924491	1.37	.809471	4.62	.190529	11
50	.734157	3.27	.924409	1.35	.809748	4.62	.190252	10
51	9.734353	3.27	9.924328	1.37	9.810025	4.62	10.189975	9
52	.734549	3.25	.924246	1.37	.810302	4.63	.189698	8
53	.734744	3.25	.924164	1.35	.810580	4.62	.189420	7
54	.734939	3.27	.924083	1.37	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.37	.811134	4.60	.188866	5
56	.735330	3.25	.923919	1.37	.811410	4.62	.188590	4
57	.735525	3.23	.923837	1.37	.811687	4.62	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.62	.188036	2
59	.735914	3.25	.923673	1.37	.812241	4.60	.187759	1
60′	9.736109		9.923591		9.812517		10.187483	0′
122°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	57°

27. Logarithmic Sines,

33°	Sine.	D. 1'.	Cosine.	D. 1'.	Tang.	D. 1'.	Cotang.	146°
0'	9.736109	3.23	9.923591	1.37	9.812517	4.62	10.187483	60'
1	.736303	3.25	.923509	1.37	.812794	4.60	.187206	59
2	.736498	3.23	.923427	1.37	.813070	4.62	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.60	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.38	.813899	4.62	.186101	55
6	.737274	3.22	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.38	.814452	4.60	.185548	53
8	.737661	3.23	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	.738048	3.22	.922768	1.37	.815280	4.58	.184720	50
11	9.738241	3.22	9.922686	1.38	9.815555	4.60	10.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.60	.184169	48
13	.738627	3.22	.922520	1.37	.816107	4.58	.183893	47
14	.738820	3.22	.922438	1.38	.816382	4.60	.183618	46
15	.739013	3.22	.922355	1.38	.816658	4.58	.183342	45
16	.739206	3.20	.922272	1.38	.816933	4.60	.183067	44
17	.739398	3.20	.922189	1.38	.817209	4.58	.182791	43
18	.739590	3.22	.922106	1.38	.817484	4.58	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.60	.182241	41
20	.739975	3.20	.921940	1.38	.818035	4.58	.181965	40
21	9.740167	3.20	9.921857	1.38	9.818310	4.58	10.181690	39
22	.740359	3.18	.921774	1.38	.818585	4.58	.181415	38
23	.740550	3.20	.921691	1.40	.818860	4.58	.181140	37
24	.740742	3.20	.921607	1.38	.819135	4.58	.180865	36
25	.740934	3.18	.921524	1.38	.819410	4.57	.180590	35
26	.741125	3.18	.921441	1.40	.819684	4.58	.180316	34
27	.741316	3.20	.921357	1.38	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.40	.820234	4.57	.179766	32
29	.741699	3.17	.921190	1.38	.820508	4.58	.179492	31
30	.741889	3.18	.921107	1.40	.820783	4.57	.179217	30
31	9.742080	3.18	9.921023	1.40	9.821057	4.58	10.178943	29
32	.742271	3.18	.920939	1.38	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.18	.920688	1.40	.822154	4.58	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.15	.920436	1.40	.822977	4.57	.177023	22
39	.743602	3.17	.920352	1.40	.823251	4.55	.176749	21
40	.743792	3.17	.920268	1.40	.823524	4.57	.176476	20
41	9.743982	3.15	9.920184	1.42	9.823798	4.57	10.176202	19
42	.744171	3.17	.920099	1.40	.824072	4.55	.175928	18
43	.744361	3.15	.920015	1.40	.824345	4.57	.175655	17
44	.744550	3.15	.919931	1.42	.824619	4.57	.175381	16
45	.744739	3.15	.919846	1.40	.824893	4.55	.175107	15
46	.744928	3.15	.919762	1.42	.825166	4.55	.174834	14
47	.745117	3.15	.919677	1.40	.825439	4.57	.174561	13
48	.745306	3.13	.919593	1.42	.825713	4.55	.174287	12
49	.745494	3.15	.919508	1.40	.825986	4.55	.174014	11
50	.745683	3.13	.919424	1.42	.826259	4.55	.173741	10
51	9.745871	3.15	9.919339	1.42	9.826532	4.55	10.173468	9
52	.746060	3.13	.919254	1.42	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.40	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	5
56	.746812	3.12	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.53	.171830	3
58	.747187	3.12	.918745	1.43	.828442	4.55	.171558	2
59	.747374	3.13	.918659	1.42	.828715	4.53	.171285	1
60'	9.747562		9.918574		9.828987		10.171013	0'
123°	Cosine.	D. 1'.	Sine.	D. 1'.	Cotang.	D. 1'.	Tang.	56°

Cosines, Tangents, and Cotangents

34°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	145°
0'	9.747562	3.12	9.918574	1.42	9.828987	4.55	10.171013	60'
1	.747749	3.12	.918489	1.42	.829260	4.53	.170740	59
2	.747936	3.12	.918404	1.43	.829532	4.55	.170468	58
3	.748123	3.12	.918318	1.42	.829805	4.53	.170195	57
4	.748310	3.12	.918233	1.43	.830077	4.53	.169923	56
5	.748497	3.10	.918147	1.42	.830349	4.53	.169651	55
6	.748683	3.12	.918062	1.43	.830621	4.53	.169379	54
7	.748870	3.10	.917976	1.42	.830893	4.53	.169107	53
8	.749056	3.12	.917891	1.43	.831165	4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	.749429	3.10	.917719	1.42	.831709	4.53	.168291	50
11	9.749615	3.10	9.917634	1.43	9.831981	4.53	10.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.08	.917462	1.43	.832525	4.52	.167475	47
14	.750172	3.10	.917376	1.43	.832796	4.53	.167204	46
15	.750358	3.08	.917290	1.43	.833068	4.52	.166932	45
16	.750543	3.10	.917204	1.43	.833339	4.53	.166661	44
17	.750729	3.08	.917118	1.43	.833611	4.52	.166389	43
18	.750914	3.08	.917032	1.43	.833882	4.53	.166118	42
19	.751099	3.08	.916946	1.45	.834154	4.52	.165846	41
20	.751284	3.08	.916859	1.43	.834425	4.52	.165575	40
21	9.751469	3.08	9.916773	1.43	9.834696	4.52	10.165304	39
22	.751654	3.08	.916687	1.45	.834967	4.52	.165033	38
23	.751839	3.07	.916600	1.43	.835238	4.52	.164762	37
24	.752023	3.08	.916514	1.45	.835509	4.52	.164491	36
25	.752208	3.07	.916427	1.43	.835780	4.52	.164220	35
26	.752392	3.07	.916341	1.45	.836051	4.52	.163949	34
27	.752576	3.07	.916254	1.45	.836322	4.52	.163678	33
28	.752760	3.07	.916167	1.43	.836593	4.52	.163407	32
29	.752944	3.07	.916081	1.45	.836864	4.50	.163136	31
30	.753128	3.07	.915994	1.45	.837134	4.52	.162866	30
31	9.753312	3.05	9.915907	1.45	9.837405	4.50	10.162595	29
32	.753495	3.07	.915820	1.45	.837675	4.52	.162325	28
33	.753679	3.05	.915733	1.45	.837946	4.50	.162054	27
34	.753862	3.07	.915646	1.45	.838216	4.52	.161784	26
35	.754046	3.05	.915559	1.45	.838487	4.50	.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.47	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.52	.160703	22
39	.754778	3.03	.915210	1.45	.839568	4.50	.160432	21
40	.754960	3.05	.915123	1.47	.839838	4.50	.160162	20
41	9.755143	3.05	9.915035	1.45	9.840108	4.50	10.159892	19
42	.755326	3.03	.914948	1.47	.840378	4.50	.159622	18
43	.755508	3.03	.914860	1.45	.840648	4.48	.159352	17
44	.755690	3.03	.914773	1.47	.840917	4.50	.159083	16
45	.755872	3.03	.914685	1.45	.841187	4.50	.158813	15
46	.756054	3.03	.914598	1.47	.841457	4.50	.158543	14
47	.756236	3.03	.914510	1.47	.841727	4.48	.158273	13
48	.756418	3.03	.914422	1.47	.841996	4.50	.158004	12
49	.756600	3.03	.914334	1.47	.842266	4.48	.157734	11
50	.756782	3.02	.914246	1.47	.842535	4.50	.157465	10
51	9.756963	3.02	9.914158	1.47	9.842805	4.48	10.157195	9
52	.757144	3 03	.914070	1.47	.843074	4.48	156926	8
53	.757326	3.02	.913982	1.47	.843343	4.48	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.50	.156388	6
55	.757688	3.02	.913806	1.47	.843882	4.48	.156118	5
56	.757869	3.02	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.00	.913630	1.48	.844420	4.48	.155580	3
58	.758230	3.02	913541	1.47	.844689	4.48	.155311	2
59	.758411	3.00	.913453	1.47	.844958	4.48	.155042	1
60'	9.758591		9.913365		9.845227		10.154773	0'
124° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	55°

27. Logarithmic Sines,

35°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	144°
0′	9.758591	3.02	9.913365	1.48	9.845227	4.48	10.154773	60′
1	.758772	3.00	.913276	1.48	.845496	4.47	.154504	59
2	.758952	3.00	.913187	1.47	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.47	.846302	4.47	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.48	.153430	55
6	.759672	3.00	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.98	.912744	1.48	.847108	4.47	.152892	53
8	.760031	3.00	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.98	.912566	1.48	.847644	4.48	.152356	51
10	.760390	2.98	.912477	1.48	.847913	4.47	.152087	50
11	9.760569	2.98	9.912388	1.48	9.848181	4.47	10.151819	49
12	.760748	2.98	.912299	1.48	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.48	.848717	4.48	.151283	47
14	.761106	2.98	.912121	1.50	.848986	4.47	.151014	46
15	.761285	2.98	.912031	1.48	.849254	4.47	.150746	45
16	.761464	2.97	.911942	1.48	.849522	4.47	.150478	44
17	.761642	2.98	.911853	1.50	.849790	4.45	.150210	43
18	.761821	2.97	.911763	1.48	.850057	4.47	.149943	42
19	.761999	2.97	.911674	1.50	.850325	4.47	.149675	41
20	.762177	2.98	.911584	1.48	.850593	4.47	.149407	40
21	9.762356	2.97	9.911495	1.50	9.850861	4.47	10.149139	39
22	.762534	2.97	.911405	1.50	.851129	4.45	.148871	38
23	.762712	2.95	.911315	1.48	.851396	4.47	.148604	37
24	.762889	2.97	.911226	1.50	.851664	4.45	.148336	36
25	.763067	2.97	.911136	1.50	.851931	4.47	.148069	35
26	.763245	2.95	.911046	1.50	.852199	4.45	.147801	34
27	.763422	2.97	.910956	1.50	.852466	4.45	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.47	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	.763954	2.95	.910686	1.50	.853268	4.45	.146732	30
31	9.764131	2.95	9.910596	1.50	9.853535	4.45	10.146465	29
32	.764308	2.95	.910506	1.52	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.50	.854069	4.45	.145931	27
34	.764662	2.93	.910325	1.50	.854336	4.45	.145664	26
35	.764838	2.95	.910235	1.52	.854603	4.45	.145397	25
36	.765015	2.93	.910144	1.50	.854870	4.45	.145130	24
37	.765191	2.93	.910054	1.52	.855137	4.45	.144863	23
38	.765367	2.95	.909963	1.50	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.52	.855671	4.45	.144329	21
40	.765720	2.93	.909782	1.52	.855938	4.43	.144062	20
41	9.765896	2.93	9.909691	1.50	9.856204	4.45	10.143796	19
42	.766072	2.92	.909601	1.52	.856471	4.43	.143529	18
43	.766247	2.93	.909510	1.52	.856737	4.45	.143263	17
44	.766423	2.92	909419	1.52	.857004	4.43	.142996	16
45	.766598	2.93	.909328	1.52	.857270	4.45	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.43	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.43	.142197	13
48	.767124	2.93	.909055	1.52	.858069	4.45	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.43	.141664	11
50	.767475	2.90	.908873	1.53	.858602	4.43	.141398	10
51	9.767649	2.92	9.908781	1.52	9.858868	4.43	10.141132	9
52	.767824	2.92	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.90	.908599	1.53	.859400	4.43	.140600	7
54	.768173	2.92	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.90	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.92	.908324	1.52	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.42	.139270	2
59	.769045	2.90	.908049	1.52	.860995	4.43	.139005	1
60′	9.769219		9.907958		9.861261		10.138739	0′
125°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	54°

Cosines, Tangents, and Cotangents

36°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	143°
0′	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	60′
1	.769393	2.88	.907866	1.53	.861527	4.42	.138473	59
2	.769566	2.90	.907774	1.53	.861792	4.43	.138208	58
3	.769740	2.88	.907682	1.53	.862058	4.42	.137942	57
4	.769913	2.90	.907590	1.53	.862323	4.43	.137677	56
5	.770087	2.88	.907498	1.53	.862589	4.42	.137411	55
6	.770260	2.88	.907406	1.53	.862854	4.42	.137146	54
7	.770433	2.88	.907314	1.53	.863119	4.43	.136881	53
8	.770606	2.88	.907222	1.55	.863385	4.42	.136615	52
9	.770779	2.88	.907129	1.53	.863650	4.42	.136350	51
10	.770952	2.88	.907037	1.53	.863915	4.42	.136085	50
11	9.771125	2.88	9.906945	1.55	9.864180	4.42	10.135820	49
12	.771298	2.87	.906852	1.53	.864445	4.42	.135555	48
13	.771470	2.88	.906760	1.55	.864710	4.42	.135290	47
14	.771643	2.87	.906667	1.53	.864975	4.42	.135025	46
15	.771815	2.87	.906575	1.55	.865240	4.42	.134760	45
16	.771987	2.87	.906482	1.55	.865505	4.42	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.42	.134230	43
18	.772331	2.87	.906296	1.53	.866035	4.42	.133965	42
19	.772503	2.87	.906204	1.55	.866300	4.40	.133700	41
20	.772675	2.87	.906111	1.55	.866564	4.42	.133436	40
21	9.772847	2.85	9.906018	1.55	9.866829	4.42	10.133171	39
22	.773018	2.87	.905925	1.55	.867094	4.40	.132906	38
23	.773190	2.85	.905832	1.55	.867358	4.42	.132642	37
24	.773361	2.87	.905739	1.57	.867623	4.40	.132377	36
25	.773533	2.85	.905645	1.55	.867887	4.42	.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.40	.131848	34
27	.773875	2.85	.905459	1.55	.868416	4.40	.131584	33
28	.774046	2.85	.905366	1.57	.868680	4.42	.131320	32
29	.774217	2.85	.905272	1.55	.868945	4.40	.131055	31
30	.774388	2.83	.905179	1.57	.869209	4.40	.130791	30
31	9.774558	2.85	9.905085	1.55	9.869473	4.40	10.130527	29
32	.774729	2.83	.904992	1.57	.869737	4.40	.130263	28
33	.774899	2.85	.904898	1.57	.870001	4.40	.129999	27
34	.775070	2.83	.904804	1.55	.870265	4.40	.129735	26
35	.775240	2.83	.904711	1.57	.870529	4.40	.129471	25
36	.775410	2.83	.904617	1.57	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	23
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	.776090	2.82	.904241	1.57	.871849	4.38	.128151	20
41	9.776259	2.83	9.904147	1.57	9.872112	4.40	10.127888	19
42	.776429	2.82	.904053	1.57	.872376	4.40	.127624	18
43	.776598	2.83	.903959	1.58	.872640	4.38	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.40	.127097	16
45	.776937	2.82	.903770	1.57	.873167	4.38	.126833	15
46	.777106	2.82	.903676	1.58	.873430	4.40	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.38	.126306	13
48	.777444	2.82	.903487	1.58	.873957	4.38	.126043	12
49	.777613	2.80	.903392	1.57	.874220	4.40	.125780	11
50	.777781	2.82	.903298	1.58	.874484	4.38	.125516	10
51	9.777950	2.82	9.903203	1.58	9.874747	4.38	10.125253	9
52	.778119	2.80	.903108	1.57	.875010	4.38	.124990	8
53	.778287	2.80	.903014	1.58	.875273	4.40	.124727	7
54	.778455	2.82	.902919	1.58	.875537	4.38	.124463	6
55	.778624	2.80	.902824	1.58	.875800	4.38	.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.78	.902539	1.58	.876589	4.38	.123411	2
59	.779295	2.80	.902444	1.58	.876852	4.37	.123148	1
60′	9.779463		9.902349		9.877114		10.122886	0′
126°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	53°

27. Logarithmic Sines,

37°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	142°
0′	9.779463	2.80	9.902349	1.60	9.877114	4.38	10.122886	60′
1	.779631	2.78	.902253	1.58	.877377	4.38	.122623	59
2	.779798	2.80	.902158	1.58	.877640	4.38	.122360	58
3	.779966	2.78	.902063	1.60	.877903	4.37	.122097	57
4	.780133	2.78	.901967	1.58	.878165	4.38	.121835	56
5	.780300	2.78	.901872	1.60	.878428	4.38	.121572	55
6	.780467	2.78	.901776	1.58	.878691	4.37	.121309	54
7	.780634	2.78	.901681	1.60	.878953	4.38	.121047	53
8	.780801	2.78	.901585	1.58	.879216	4.37	.120784	52
9	.780968	2.77	.901490	1.60	.879478	4.38	.120522	51
10	.781134	2.78	.901394	1.60	.879741	4.37	.120259	50
11	9.781301	2.78	9.901298	1.60	9.880003	4.37	10.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.38	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	46
15	.781966	2.77	.900914	1.60	.881052	4 37	.118948	45
16	.782132	2.77	.900818	1.60	.881314	4.38	.118686	44
17	.782298	2.77	.900722	1.60	.881577	4.37	.118423	43
18	.782464	2.77	.900626	1.62	.881839	4.37	.118161	42
19	.782630	2.77	.900529	1.60	.882101	4 37	.117899	41
20	.782796	2.75	.900433	1.60	.882363	4.37	.117637	40
21	9.782961	2.77	9.900337	1.62	9.882625	4.37	10.117375	39
22	.783127	2.75	.900240	1.60	.882887	4.35	.117113	38
23	.783292	2.77	.900144	1.62	.883148	4.37	.116852	37
24	.783458	2.75	.900047	1.60	.883410	4.37	.116590	36
25	.783623	2.75	.899951	1.62	.883672	4.37	.116328	35
26	.783788	2.75	.899854	1.62	.883934	4.37	.116066	34
27	.783953	2.75	.899757	1.62	.884196	4.35	.115804	33
28	.784118	2.73	.899660	1.60	.884457	4.37	.115543	32
29	.784282	2.75	.899564	1.62	.884719	4.35	.115281	31
30	.784447	2.75	.899467	1.62	.884980	4.37	.115020	30
31	9.784612	2.73	9.899370	1.62	9.885242	4.37	10.114758	29
32	.784776	2.75	.899273	1.62	.885504	4.35	.114496	28
33	.784941	2.73	.899176	1.63	.885765	4.35	.114235	27
34	.785105	2.73	.899078	1.62	.886026	4.37	.113974	26
35	.785269	2.73	.898981	1.62	.886288	4.35	.113712	25
36	.785433	2.73	.898884	1.62	.886549	4.37	.113451	24
37	.785597	2.73	.898787	1.63	.886811	4.35	.113189	23
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	22
39	.785925	2.73	.898592	1.63	.887333	4.35	.112667	21
40	.786089	2.72	.898494	1.62	.887594	4.35	.112406	20
41	9.786252	2.73	9.898397	1.63	9.887855	4.35	10.112145	19
42	.786416	2.72	.898299	1.62	.888116	4.37	.111884	18
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622	17
44	.786742	2.73	.898104	1.63	.888639	4.35	.111361	16
45	.786906	2.72	.898006	1.63	888900	4.35	.111100	15
46	.787069	2.72	.897908	1.63	.889161	4.33	.110839	14
47	.787232	2.72	.897810	1.63	.889421	4.35	.110579	13
48	.787395	2.70	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2.72	.897614	1.63	.889943	4.35	.110057	11
50	.787720	2.72	.897516	1.63	.890204	4.35	.109796	10
51	9.787883	2.70	9.897418	1.63	9.890465	4.33	10.109535	9
52	.788045	2.72	.897320	1.63	.890725	4.35	.109275	8
53	.788208	2.70	.897222	1.65	.890986	4.35	.109014	7
54	.788370	2.70	.897123	1.63	.891247	4.33	.108753	6
55	.788532	2.70	.897025	1.65	.891507	4.35	.108493	5
56	.788694	2.70	.896926	1.63	.891768	4.33	.108232	4
57	.788856	2.70	.896828	1.65	.892028	4.35	.107972	3
58	.789018	2.70	.896729	1.63	.892289	4.33	.107711	2
59	.789180	2.70	.896631	1.65	.892549	4.35	.107451	1
60′	9.789342		9.896532		9.892810		10.107190	0′
127°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	52°

Cosines, Tangents, and Cotangents

38°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang. 141°	
0′	9.789342	2.70	9.896532	1.65	9.892810	4.33	10.107190	60′
1	.789504	2.68	.896433	1.63	.893070	4.35	.106930	59
2	.789665	2.70	.896335	1.65	.893331	4.33	.106669	58
3	.789827	2.68	.896236	1.65	.893591	4.33	.106409	57
4	.789988	2.68	.896137	1.65	.893851	4.33	.106149	56
5	.790149	2.68	.896038	1.65	.894111	4.35	.105889	55
6	.790310	2.68	.895939	1.65	.894372	4.33	.105628	54
7	.790471	2.68	.895840	1.65	.894632	4.33	.105368	53
8	.790632	2.68	.895741	1.67	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	.790954	2.68	.895542	1.65	.895412	4.33	.104588	50
11	9.791115	2.67	9.895443	1.67	9.895672	4.33	10.104328	49
12	.791275	2.68	.895343	1.65	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.65	.896192	4.33	.103808	47
14	.791596	2.68	.895145	1.67	.896452	4.33	.103548	46
15	.791757	2.67	.895045	1.67	.896712	4.32	.103288	45
16	.791917	2.67	.894945	1.65	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.67	.897231	4.33	.102769	43
18	.792237	2.67	.894746	1.67	.897491	4.33	.102509	42
19	.792397	2.67	.894646	1.67	.897751	4.32	.102249	41
20	.792557	2.65	.894546	1.67	.898010	4.33	.101990	40
21	9.792716	2.67	9.894446	1.67	9.898270	4.33	10.101730	39
22	.792876	2.65	.894346	1.67	.898530	4.32	.101470	38
23	.793035	2.67	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.65	.894146	1.67	.899049	4.32	.100951	36
25	.793354	2.67	.894046	1.67	.899308	4.33	.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.68	.899827	4.33	.100173	33
28	.793832	2.65	.893745	1.67	.900087	4.32	.099913	32
29	.793991	2.65	.893645	1.68	.900346	4.32	.099654	31
30	.794150	2.63	.893544	1.67	.900605	4.32	.099395	30
31	9.794308	2.65	9.893444	1.68	9.900864	4.33	10.099136	29
32	.794467	2.65	.893343	1.67	.901124	4.32	.098876	28
33	.794626	2.63	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.63	.893142	1.68	.901642	4.32	.098358	26
35	.794942	2.65	.893041	1.68	.901901	4.32	.098099	25
36	.795101	2.63	.892940	1.68	.902160	4.33	.097840	24
37	.795259	2.63	.892839	1.67	.902420	4.32	.097580	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.70	.902938	4.32	.097062	21
40	.795733	2.63	.892536	1.68	.903197	4.32	.096803	20
41	9.795891	2.63	9.892435	1.68	9.903456	4.30	10.096544	19
42	.796049	2.62	.892334	1.68	.903714	4.32	.096286	18
43	.796206	2.63	.892233	1.68	.903973	4.32	.096027	17
44	.796364	2.62	.892132	1.70	.904232	4.32	.095768	16
45	.796521	2.63	.892030	1.68	.904491	4.32	.095509	15
46	.796679	2.62	.891929	1.70	.904750	4.30	.095250	14
47	.796836	2.62	.891827	1.68	.905008	4.32	.094992	13
48	.796993	2.62	.891726	1.70	.905267	4.32	.094733	12
49	.797150	2.62	.891624	1.68	.905526	4.32	.094474	11
50	.797307	2.62	.891523	1.70	.905785	4.30	.094215	10
51	9.797464	2.62	9.891421	1.70	9.906043	4.32	10.093957	9
52	.797621	2.60	.891319	1.70	.906302	4.30	.093698	8
53	.797777	2.62	.891217	1.70	.906560	4.32	.093440	7
54	.797934	2.62	.891115	1.70	.906819	4.30	.093181	6
55	.798091	2.60	.891013	1.70	.907077	4.32	.092923	5
56	.798247	2.60	.890911	1.70	.907336	4.30	.092664	4
57	.798403	2.62	.890809	1.70	.907594	4.32	.092406	3
58	.798560	2.60	.890707	1.70	.907853	4.30	.092147	2
59	.798716	2.60	.890605	1.70	.908111	4.30	.091889	1
60′	9.798872		9.890503		9.908369		10.091631	0′
128° Cosine.		D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	51°

27. Logarithmic Sines,

39°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	140°
0′	9.798872	2.60	9.890503	1.72	9.908369	4.32	10.091631	60′
1	.799028	2.60	.890400	1.70	.908628	4.30	.091372	59
2	.799184	2.58	.890298	1.72	.908886	4.30	.091114	58
3	.799339	2.60	.890195	1.70	.909144	4.30	.090856	57
4	.799495	2.60	.890093	1.72	.909402	4.30	.090598	56
5	.799651	2.58	.889990	1.70	.909660	4.30	.090340	55
6	.799806	2.60	.889888	1.72	.909918	4.32	.090082	54
7	.799962	2.58	.889785	1.72	.910177	4.30	.089823	53
8	.800117	2.58	.889682	1.72	.910435	4.30	.089565	52
9	.800272	2.58	.889579	1.70	.910693	4.30	.089307	51
10	.800427	2.58	.889477	1.72	.910951	4.30	.089049	50
11	9.800582	2.58	9.889374	1.72	9.911209	4.30	10.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.73	.911725	4.28	.088275	47
14	.801047	2.57	.889064	1.72	.911982	4.30	.088018	46
15	.801201	2.58	.888961	1.72	.912240	4.30	.087760	45
16	.801356	2.58	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.73	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.28	.086986	42
19	.801819	2.57	.888548	1.73	.913271	4.30	.086729	41
20	.801973	2.58	.888444	1.72	.913529	4.30	.086471	40
21	9.802128	2.57	9.888341	1.73	9.913787	4.28	10.086213	39
22	.802282	2.57	.888237	1.72	.914044	4.30	.085956	38
23	.802436	2.55	.888134	1.73	.914302	4.30	.085698	37
24	.802589	2.57	.888030	1.73	.914560	4.28	.085440	36
25	.802743	2.57	.887926	1.73	.914817	4.30	.085183	35
26	.802897	2.55	.887822	1.73	.915075	4.28	.084925	34
27	.803050	2.57	.887718	1.73	.915332	4.30	.084668	33
28	.803204	2.55	.887614	1.73	.915590	4.28	.084410	32
29	.803357	2.57	.887510	1.73	.915847	4.28	.084153	31
30	.803511	2.55	.887406	1.73	.916104	4.30	.083896	30
31	9.803664	2.55	9.887302	1.73	9.916362	4.28	10.083638	29
32	.803817	2.55	.887198	1.75	.916619	4.30	.083381	28
33	.803970	2.55	.887093	1.73	.916877	4.28	.083123	27
34	.804123	2.55	.886989	1.73	.917134	4.28	.082866	26
35	.804276	2.55	.886885	1.75	.917391	4.28	.082609	25
36	.804428	2.55	.886780	1.73	.917648	4.30	.082352	24
37	.804581	2.55	.886676	1.75	.917906	4.28	.082094	23
38	.804734	2.53	.886571	1.75	.918163	4.28	.081837	22
39	.804886	2.55	.886466	1.73	.918420	4.28	.081580	21
40	.805039	2.53	.886362	1.75	.918677	4.28	.081323	20
41	9.805191	2.53	9.886257	1.75	9.918934	4.28	10.081066	19
42	.805343	2.53	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4.28	.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.52	.885627	1.75	.920476	4.28	.079524	13
48	.806254	2.53	.885522	1.77	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.75	.920990	4.28	.079010	11
50	.806557	2.53	.885311	1.77	.921247	4.27	.078753	10
51	9.806709	2.52	9.885205	1.75	9.921503	4.28	10.078497	9
52	.806860	2.52	.885100	1.77	.921760	4.28	.078240	8
53	.807011	2.53	.884994	1.75	.922017	4.28	.077983	7
54	.807163	2.52	.884889	1.77	.922274	4.27	.077726	6
55	.807314	2.52	.884783	1.77	.922530	4.28	.077470	5
56	.807465	2.50	.884677	1.75	.922787	4.28	.077213	4
57	.807615	2.52	.884572	1.77	.923044	4.27	.076956	3
58	.807766	2.52	.884466	1.77	.923300	4.28	.076700	2
59	.807917	2.50	.884360	1.77	.923557	4.28	.076443	1
60′	9.808067		9.884254		9.923814		10.076186	0′
129°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	50°

Cosines, Tangents, and Cotangents

40°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	139°
0′	9.808067	2.52	9.884254	1.77	9.923814	4.27	10.076186	60′
1	.808218	2.50	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.52	.884042	1.77	.924327	4.27	075673	58
3	.808519	2.50	.883936	1.78	.924583	4.28	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.78	.925352	4.28	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.28	.074135	52
9	.809419	2.50	.883297	1.77	.926122	4.27	.073878	51
10	.809569	2.48	.883191	1.78	.926378	4.27	.073622	50
11	9.809718	2.50	9.883084	1.78	9.926634	4.27	10.073366	49
12	.809868	2.48	.882977	1.77	.926890	4.28	.073110	48
13	.810017	2.50	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.48	.882764	1.78	.927403	4.27	.072597	43
15	.810316	2.48	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.78	.928427	4.28	.071573	42
19	.810912	2.48	.882229	1.80	.928684	4.27	.071316	41
20	.811061	2.48	.882121	1.78	.928940	4.27	.071060	40
21	9.811210	2.47	9.882014	1.78	9.929196	4.27	10.070804	39
22	.811358	2.48	.881907	1.80	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.78	.929708	4.27	.070292	37
24	.811655	2.48	.881692	1.80	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.78	.930220	4.25	.069780	35
26	.811952	2.47	.881477	1.80	.930475	4.27	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.27	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.27	.069013	32
29	.812396	2.47	.881153	1.78	.931243	4.27	.068757	31
30	.812544	2.47	.881046	1.80	.931499	4.27	.068501	30
31	9.812692	2.47	9.880938	1.80	9.931755	4.25	10.068245	29
32	.812840	2.47	.880830	1.80	.932010	4.27	.067990	28
33	.812988	2.45	.880722	1.82	.932266	4.27	.067734	27
34	.813135	2.47	.880613	1.80	.932522	4.27	.067478	26
35	.813283	2.45	.880505	1.80	.932778	4.25	.067222	25
36	.813430	2.47	.880397	1.80	.933033	4.27	.066967	24
37	.813578	2.45	.880289	1.82	.933289	4.27	.066711	23
38	.813725	2.45	.880180	1.80	.933545	4.25	.066455	22
39	.813872	2.45	.880072	1.82	.933800	4.27	.066200	21
40	.814019	2.45	.879963	1.80	.934056	4.25	.065944	20
41	9.814166	2.45	9.879855	1.82	9.934311	4.27	10.065689	19
42	.814313	2.45	.879746	1.82	.934567	4.25	.065433	18
43	.814460	2.45	.879637	1.80	.934822	4.27	.065178	17
44	.814607	2.43	.879529	1.82	.935078	4.25	.064922	16
45	.814753	2.45	.879420	1.82	.935333	4.27	.064667	15
46	.814900	2.43	.879311	1.82	.935589	4.25	.064411	14
47	.815046	2.45	.879202	1.82	.935844	4.27	.064156	13
48	.815193	2.43	.879093	1.82	.936100	4.25	.063900	12
49	.815339	2.43	.878984	1.82	.936355	4.27	.063645	11
50	.815485	2.45	.878875	1.82	.936611	4.25	.063389	10
51	9.815632	2.43	9.878766	1.83	9.936866	4.25	10.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.27	.062879	8
53	.815924	2.42	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.83	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.82	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.27	.061858	4
57	.816507	2.42	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.43	.877999	1.82	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60′	9.816943		9.877780		9.939163		10.060837	0′
130°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	49°

27. Logarithmic Sines,

41°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	138°
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60′
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.43	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.40	.877340	1.83	.940183	4.27	.059817	56
5	.817668	2.42	.877230	1.83	.940439	4.25	.059561	55
6	.817813	2.42	.877120	1.83	.940694	4.25	.059306	54
7	.817958	2.42	.877010	1.85	.940949	4.25	.059051	53
8	.818103	2.40	.876899	1.83	.941204	4.25	.058796	52
9	.818247	2.42	.876789	1.85	.941459	4.23	.058541	51
10	.818392	2.40	.876678	1.83	.941713	4.25	.058287	50
11	9.818536	2.42	9.876568	1.85	9.941968	4.25	10.058032	49
12	.818681	2.40	.876457	1.83	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.85	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.83	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.23	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.38	.875682	1.85	.944007	4.25	.055993	41
20	.819832	2.40	.875571	1.87	.944262	4.25	.055738	40
21	9.819976	2.40	9.875459	1.85	9.944517	4.23	10.055483	39
22	.820120	2.38	.875348	1.85	.944771	4.25	.055229	38
23	.820263	2.38	.875237	1.85	.945026	4.25	.054974	37
24	.820406	2.40	.875126	1.87	.945281	4.23	.054719	36
25	.820550	2.38	.875014	1.85	.945535	4.25	.054465	35
26	.820693	2.38	.874903	1.87	.945790	4.25	.054210	34
27	.820836	2.38	.874791	1.85	.946045	4.23	.053955	33
28	.820979	2.38	.874680	1.87	.946299	4.25	.053701	32
29	.821122	2.38	.874568	1.87	.946554	4.23	.053446	31
30	.821265	2.37	.874456	1.87	.946808	4.25	.053192	30
31	9.821407	2.38	9.874344	1.87	9.947063	4.25	10.052937	29
32	.821550	2.38	.874232	1.85	.947318	4.23	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.25	.052428	27
34	.821835	2.37	.874009	1.88	.947827	4.23	.052173	26
35	.821977	2.38	.873896	1.87	.948081	4.23	.051919	25
36	.822120	2.37	.873784	1.87	.948335	4.25	.051665	24
37	.822262	2.37	.873672	1.87	.948590	4.23	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.25	.051156	22
39	.822546	2.37	.873448	1.88	.949099	4.23	.050901	21
40	.822688	2.37	.873335	1.87	.949353	4.25	.050647	20
41	9.822830	2.37	9.873223	1.88	9.949608	4.23	10.050392	19
42	.822972	2.37	.873110	1.87	.949862	4.23	.050138	18
43	.823114	2.35	.872998	1.88	.950116	4.25	.049884	17
44	.823255	2.37	.872885	1.88	.950371	4.23	.049629	16
45	.823397	2.37	.872772	1.88	.950625	4.23	.049375	15
46	.823539	2.35	.872659	1.87	.950879	4.23	.049121	14
47	.823680	2.35	.872547	1.88	.951133	4.25	.048867	13
48	.823821	2.37	.872434	1.88	.951388	4.23	.048612	12
49	.823963	2.35	.872321	1.88	.951642	4.23	.048358	11
50	.824104	2.35	.872208	1.88	.951896	4.23	.048104	10
51	9.824245	2.35	9.872095	1.90	9.952150	4.25	10.047850	9
52	.824386	2.35	.871981	1.88	.952405	4.23	.047595	8
53	.824527	2.35	.871868	1.88	.952659	4.23	.047341	7
54	.824668	2.33	.871755	1.90	.952913	4.23	.047087	6
55	.824808	2.35	.871641	1.88	.953167	4.23	.046833	5
56	.824949	2.35	.871528	1.90	.953421	4.23	.046579	4
57	.825090	2.33	.871414	1.88	.953675	4.23	.046325	3
58	.825230	2.35	.871301	1.90	.953929	4.23	.046071	2
59	.825371	2.33	.871187	1.90	.954183	4.23	.045817	1
60′	9.825511		9.871073		9.954437		10.045563	0′
131°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	48°

Cosines, Tangents, and Cotangents

42°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	137°
0′	9.825511	2.33	9.871073	1.88	9.954437	4.23	10.045563	60′
1	.825651	2.33	.870960	1.90	.954691	4.25	.045309	59
2	.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955708	4.22	.044292	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.92	.956215	4.23	.043785	53
8	.826631	2.32	.870161	1.90	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.90	.956723	4.23	.043277	51
10	.826910	2.32	.869933	1.92	.956977	4.23	.043023	50
11	9.827049	2.33	9.869818	1.90	9.957231	4.23	10.042769	49
12	.827189	2.32	.869704	1.92	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.92	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.90	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.92	.958247	4.22	.041753	45
16	.827745	2.32	.869245	1.92	.958500	4.23	.041500	44
17	.827884	2.32	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.32	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.32	.868900	1.92	.959262	4.23	.040738	41
20	.828301	2.30	.868785	1.92	.959516	4.22	.040484	40
21	9.828439	2.32	9.868670	1.92	9.959769	4.23	10.040231	39
22	.828578	2.30	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.32	.868440	1.93	.960277	4.22	.039723	37
24	.828855	2.30	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.93	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.22	.038708	33
28	.829407	2.30	.867862	1.92	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.22	.038201	31
30	.829683	2.30	.867631	1.93	.962052	4.23	.037948	30
31	9.829821	2.30	9.867515	1.93	9.962306	4.23	10.037694	29
32	.829959	2.30	.867399	1.93	.962560	4.22	.037440	28
33	.830097	2.28	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.30	.867167	1.93	.963067	4.22	.036933	26
35	.830372	2.28	.867051	1.93	.963320	4.23	.036680	25
36	.830509	2.28	.866935	1.93	.963574	4.23	.036426	24
37	.830646	2.30	.866819	1.93	.963828	4.22	.036172	23
38	.830784	2.28	.866703	1.95	.964081	4.23	.035919	22
39	.830921	2.28	.866586	1.93	.964335	4.22	.035665	21
40	.831058	2.28	.866470	1.95	.964588	4.23	.035412	20
41	9.831195	2.28	9.866353	1.93	9.964842	4.22	10.035158	19
42	.831332	2.28	.866237	1.95	.965095	4.23	.034905	18
43	.831469	2.28	.866120	1.93	.965349	4.22	.034651	17
44	.831606	2.27	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.23	.034145	15
46	.831879	2.27	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.28	.865653	1.95	.966362	4.23	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.28	.865419	1.95	.966869	4.23	.033131	11
50	.832425	2.27	.865302	1.95	.967123	4.22	.032877	10
51	9.832561	2.27	9.865185	1.95	9.967376	4.22	10.032624	9
52	.832697	2.27	.865068	1.97	.967629	4.23	.032371	8
53	.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.95	.968136	4.22	.031864	6
55	.833105	2.27	.864716	1.97	.968389	4.23	.031611	5
56	.833241	2.27	.864598	1.95	.968643	4.22	.031357	4
57	.833377	2.25	.864481	1.97	.968896	4.22	.031104	3
58	.833512	2.27	.864363	1.97	.969149	4.23	.030851	2
59	.833648	2.25	.864245	1.97	.969403	4.22	.030597	1
60′	9.833783		9.864127		9.969656		10.030344	0′
132°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	47°

27. Logarithmic Sines,

43°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	136°
0′	9.833783	2.27	9.864127	1.95	9.969656	4.22	10.030344	60′
1	.833919	2.25	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.23	.029838	58
3	.834189	2.27	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.98	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.23	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.23	.863183	1.98	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	.835134	2.25	.862946	1.98	.972188	4.22	.027812	50
11	9.835269	2.23	9.862827	1.97	9.972441	4.23	10.027559	49
12	.835403	2.25	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.23	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.25	.862471	1.97	.973201	4.22	.026799	46
15	.835807	2.23	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.23	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.23	.025534	41
20	.836477	2.23	.861758	2.00	.974720	4.22	.025280	40
21	9.836611	2.23	9.861638	1.98	9.974973	4.22	10.025027	39
22	.836745	2.22	.861519	1.98	.975226	4.22	.024774	38
23	.836878	2.23	.861400	2.00	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.98	.975732	4.22	.024268	36
25	.837146	2.22	.861161	2.00	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.98	.976238	4.22	.023762	34
27	.837412	2.23	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	.837812	2.22	.860562	2.00	.977250	4.22	.022750	30
31	9.837945	2.22	9.860442	2.00	9.977503	4.22	10.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.22	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.22	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.22	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.20	.859842	2.02	.978768	4.22	.021232	24
37	.838742	2.22	.859721	2.00	.979021	4.22	.020979	23
38	.838875	2.20	.859601	2.02	.979274	4.22	.020726	22
39	.839007	2.22	.859480	2.00	.979527	4.22	.020473	21
40	.839140	2.20	.859360	2.02	.979780	4.22	.020220	20
41	9.839272	2.20	9.859239	2.00	9.980033	4.22	10.019967	19
42	.839404	2.20	.859119	2.02	.980286	4.20	.019714	18
43	.839536	2.20	.858998	2.02	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.22	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.22	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.22	.018450	13
48	.840196	2.20	.858393	2.02	.981803	4.22	.018197	12
49	.840328	2.18	.858272	2.02	.982056	4.22	.017944	11
50	.840459	2.20	.858151	2.03	.982309	4.22	.017691	10
51	9.840591	2.18	9.858029	2.02	9.982562	4.20	10.017438	9
52	.840722	2.20	.857908	2.03	.982814	4.22	.017186	8
53	.840854	2.18	.857786	2.02	.983067	4.22	.016933	7
54	.840985	2.18	.857665	2.03	.983320	4.22	.016680	6
55	.841116	2.18	.857543	2.02	.983573	4.22	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.22	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.22	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.20	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.22	.015416	1
60′	9.841771		9.856934		9.984837		10.015163	0′
133°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	46°

Cosines, Tangents, and Cotangents

44°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	135°
0'	9.841771	2.18	9.856934	2.03	9.984837	4.22	10.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.22	.014910	59
2	.842033	2.17	.856690	2.03	.985343	4.22	.014657	58
3	.842163	2.18	.856568	2.03	.985596	4.20	.014404	57
4	.842294	2.17	.856446	2.05	.985848	4.22	.014152	56
5	.842424	2.18	.856323	2.03	.986101	4.22	.013899	55
6	.842555	2.17	.856201	2.05	.986354	4.22	.013646	54
7	.842685	2.17	.856078	2.03	.986607	4.22	.013393	53
8	.842815	2.18	.855956	2.05	.986860	4.20	.013140	52
9	.842946	2.17	.855833	2.03	.987112	4.22	.012888	51
10	.843076	2.17	.855711	2.05	.987365	4.22	.012635	50
11	9.843206	2.17	9.855588	2.05	9.987618	4.22	10.012382	49
12	.843336	2.17	.855465	2.05	.987871	4.20	.012129	48
13	.843466	2.15	.855342	2.05	.988123	4.22	.011877	47
14	.843595	2.17	.855219	2.05	.988376	4.22	.011624	46
15	.843725	2.17	.855096	2.05	.988629	4.22	.011371	45
16	.843855	2.15	.854973	2.05	.988882	4.20	.011118	44
17	.843984	2.17	.854850	2.05	.989134	4.22	.010866	43
18	.844114	2.15	.854727	2.07	.989387	4.22	.010613	42
19	.844243	2.15	.854603	2.05	.989640	4.22	.010360	41
20	.844372	2.17	.854480	2.07	.989893	4.20	.010107	40
21	9.844502	2.15	9.854356	2.05	9.990145	4.22	10.009855	39
22	.844631	2.15	.854233	2.07	.990398	4.22	.009602	38
23	.844760	2.15	.854109	2.05	.990651	4.20	.009349	37
24	.844889	2.15	.853986	2.07	.990903	4.22	.009097	36
25	.845018	2.15	.853862	2.07	.991156	4.22	.008844	35
26	.845147	2.15	.853738	2 07	.991409	4 22	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.20	.008338	33
28	.845405	2.13	.853490	2.07	.991914	4.22	.008086	32
29	.845533	2.15	.853366	2.07	.992167	4.22	.007833	31
30	.845662	2.13	.853242	2.07	.992420	4.20	.007580	30
31	9.845790	2.15	9.853118	2.07	9.992672	4.22	10.007328	29
32	.845919	2.13	.852994	2.08	.992925	4.22	.007075	28
33	.846047	2.13	.852869	2.07	.993178	4.22	.006822	27
34	.846175	2.15	.852745	2.08	.993431	4.20	.006569	26
35	.846304	2.13	.852620	2.07	.993683	4.22	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.22	.006064	24
37	.846560	2.13	.852371	2.07	.994189	4.20	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.22	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.22	.005306	21
40	.846944	2.12	.851997	2.08	.994947	4.20	.005053	20
41	9.847071	2.13	9.851872	2.08	9.995199	4.22	10.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.22	.004548	18
43	.847327	2.12	.851622	2.08	.995705	4.20	.004295	17
44	.847454	2.13	.851497	2.08	.995957	4.22	.004043	16
45	.847582	2.12	.851372	2.10	.996210	4.22	.003790	15
46	.847709	2.12	.851246	2.08	.996463	4.20	.003537	14
47	.847836	2.13	.851121	2.08	.996715	4.22	.003285	13
48	.847964	2.12	.850996	2.10	.996968	4.22	.003032	12
49	.848091	2.12	.850870	2.08	.997221	4.20	.002779	11
50	.848218	2.12	.850745	2.10	.997473	4.22	.002527	10
51	9.848345	2.12	9.850619	2.10	9.997726	4.22	10.002274	9
52	.848472	2.12	.850493	2.08	.997979	4.20	.002021	8
53	.848599	2.12	.850368	2.10	.998231	4.22	.001769	7
54	.848726	2.10	.850242	2.10	.998484	4.22	.001516	6
55	.848852	2 12	.850116	2.10	.998737	4.20	.001263	5
56	.848979	2.12	.849990	2.10	.998989	4.22	.001011	4
57	.849106	2.10	.849864	2.10	.999242	4.22	.000758	3
58	.849232	2.12	.849738	2.12	.999495	4.20	.000505	2
59	.849359	2.10	.849611	2.10	.999747	4.22	.000253	1
60'	9.849485		9 849485		10.000000		10.000000	0'
134° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1"	Tang.	45°

28. Natural Sines and Cosines

′	0° Sine	0° Cosin	1° Sine	1° Cosin	2° Sine	2° Cosin	3° Sine	3° Cosin	4° Sine	4° Cosin	′
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	89° Cosin	89° Sine	88° Cosin	88° Sine	87° Cosin	87° Sine	86° Cosin	86° Sine	85° Cosin	85° Sine	′

28. Natural Sines and Cosines

′	5° Sine	5° Cosin	6° Sine	6° Cosin	7° Sine	7° Cosin	8° Sine	8° Cosin	9° Sine	9° Cosin	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

28. Natural Sines and Cosines

′	10° Sine	10° Cosin	11° Sine	11° Cosin	12° Sine	12° Cosin	13° Sine	13° Cosin	14° Sine	14° Cosin	′
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

28. Natural Sines and Cosines

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.24093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

28. Natural Sines and Cosines

′	20° Sine	20° Cosin	21° Sine	21° Cosin	22° Sine	22° Cosin	23° Sine	23° Cosin	24° Sine	24° Cosin	′
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

28. Natural Sines and Cosines

′	25° Sine	25° Cosin	26° Sine	26° Cosin	27° Sine	27° Cosin	28° Sine	28° Cosin	29° Sine	29° Cosin	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	43051	.90259	.44620	.89493	.46175	88701	.47716	.87882	.49242	87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

28. Natural Sines and Cosines

′	30° Sine	30° Cosin	31° Sine	31° Cosin	32° Sine	32° Cosin	33° Sine	33° Cosin	34° Sine	34° Cosin	′
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

28. Natural Sines and Cosines

′	35° Sine	35° Cosin	36° Sine	36° Cosin	37° Sine	37° Cosin	38° Sine	38° Cosin	39° Sine	39° Cosin	′
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.82310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

28. Natural Sines and Cosines

′	40° Sine	40° Cosin	41° Sine	41° Cosin	42° Sine	42° Cosin	43° Sine	43° Cosin	44° Sine	44° Cosin	′
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

29. Natural Tangents and Cotangents

′	0° Tang	0° Cotang	1° Tang	1° Cotang	2° Tang	2° Cotang	3° Tang	3° Cotang	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang 89°	Tang 89°	Cotang 88°	Tang 88°	Cotang 87°	Tang 87°	Cotang 86°	Tang 86°	′

29. Natural Tangents and Cotangents

′	4° Tang	4° Cotang	5° Tang	5° Cotang	6° Tang	6° Cotang	7° Tang	7° Cotang	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	85° Cotang	85° Tang	84° Cotang	84° Tang	83° Cotang	83° Tang	82° Cotang	82° Tang	′

29. Natural Tangents and Cotangents

′	8° Tang	8° Cotang	9° Tang	9° Cotang	10° Tang	10° Cotang	11° Tang	11° Cotang	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

29. Natural Tangents and Cotangents

′	12° Tang	12° Cotang	13° Tang	13° Cotang	14° Tang	14° Cotang	15° Tang	15° Cotang	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

29. Natural Tangents and Cotangents

′	16° Tang	16° Cotang	17° Tang	17° Cotang	18° Tang	18° Cotang	19° Tang	19° Cotang	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	73° Cotang	73° Tang	72° Cotang	72° Tang	71° Cotang	71° Tang	70° Cotang	70° Tang	′

29. Natural Tangents and Cotangents

′	20° Tang	20° Cotang	21° Tang	21° Cotang	22° Tang	22° Cotang	23° Tang	23° Cotang	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	69° Cotang	69° Tang	68° Cotang	68° Tang	67° Cotang	67° Tang	66° Cotang	66° Tang	′

29. Natural Tangents and Cotangents

′	24° Tang	24° Cotang	25° Tang	25° Cotang	26° Tang	26° Cotang	27° Tang	27° Cotang	′
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91552	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91414	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91288	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91142	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91017	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

29. Natural Tangents and Cotangents

′	28° Tang	28° Cotang	29° Tang	29° Cotang	30° Tang	30° Cotang	31° Tang	31° Cotang	′
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	61°		60°		59°		58°		

29. Natural Tangents and Cotangents

′	32° Tang	32° Cotang	33° Tang	33° Cotang	34° Tang	34° Cotang	35° Tang	35° Cotang	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70170	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.63314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	57° Cotang	57° Tang	56° Cotang	56° Tang	55° Cotang	55° Tang	54° Cotang	54° Tang	′

29. Natural Tangents and Cotangents

′	36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	53°		52°		51°		50°		

29. Natural Tangents and Cotangents

′	40° Tang	40° Cotang	41° Tang	41° Cotang	42° Tang	42° Cotang	43° Tang	43° Cotang	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12657	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang 49°	Tang 49°	Cotang 48°	Tang 48°	Cotang 47°	Tang 47°	Cotang 46°	Tang 46°	′

29. Natural Tangents and Cotangents

′	44° Tang	44° Cotang	′	′	44° Tang	44° Cotang	′	′	44° Tang	44° Cotang	′
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	45° Cotang	45° Tang	′	′	45° Cotang	45° Tang	′	′	45° Cotang	45° Tang	′

SECTION 3

PHYSICS, METEOROLOGY, WEIGHTS AND MEASURES

ORIGINALLY WRITTEN * BY

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CHEMISTRY

PHYSICS

METEOROLOGY

WEIGHTS AND MEASURES

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CHEMISTRY

1. The Chemical Elements

In the following table, in every case the atomic number is given as well as the atomic weight. The list is the same as that in International Critical Tables, Vol. I, pp. 43–45. The atomic weights are those of 1929.

Name *	Symbol	Atomic number	Atomic weight	Nature
Actinium	Ac	89		
Aluminum	Al	13	26.97	Metal
Antimony (stibium)	Sb	51	121.77	Metal
Argon	A	18	39.94	Inert gas
Arsenic	As	33	74.96	Metalloid
Barium	Ba	56	137.36	Metal
Beryllium (glucinum)	Be	4	9.02	Metal
Bismuth	Bi	83	209.00	Metal
Boron	B	5	10.82	Metalloid
Bromine	Br	35	79.916	Liquid
Cadmium	Cd	48	112.41	Metal
Calcium	Ca	20	40.07	Metal
Carbon	C	6	12.000	Metalloid
Cassiopeium (see lutecium)	Cp	71		
Celtium (see hafnium)	Ct	72		
Cerium	Ce	58	140.13	Metal
Cesium	Cs	55	132.81	Metal
Chlorine	Cl	17	35.457	Gas
Chromium	Cr	24	52.01	Metal
Cobalt	Co	27	58.94	Metal
Columbium (niobium)	Cb	41	93.1	Metal
Copper (cuprum)	Cu	29	63.57	Metal
Dysprosium	Ds or Dy	66	162.46	Metal
Erbium	Er	68	167.64	Metal
Europium	Eu	63	152.0	Metal
Fluorine	F	9	19.00	Most active gas
Gadolinium	Gd	64	157.26	Metal
Gallium	Ga	31	69.72	Metal
Germanium	Ge	32	72.60	Metal
Glucinum (same as beryllium)	Gl	4		
Gold (aurum)	Au	79	197.2	Metal
Hafnium (celtium)	Hf	72	178.6	Metal
Helium	He	2	4.002	Inert gas
Holmium	Ho	67	163.5	Metal
Hydrogen	H	1	1.0078	Lightest gas
Indium	In	49	114.8	Metal
Iodine	I	53	126.932	Metalloid
Iridium	Ir	77	193.1	Metal
Iron (ferrum)	Fe	26	55.84	Metal
Krypton	Kr	36	82.9	Inert gas
Lanthanum	La	57	138.90	Metal
Lead (plumbum)	Pb	82	207.22	Metal
Lithium	Li	3	6.940	Metal
Lutecium (cassiopeium)	Lu	71	175.0	Metal
Magnesium	Mg	12	24.32	Metal
Manganese	Mn	25	54.93	Metal
Masurium	Ma	43		
Mercury (hydrargyrum)	Hg	80	200.61	Metal
Molybdenum	Mo	42	96.0	Metal

* Names of the more abundant elements are in bold-face type

Name	Symbol	Atomic number	Atomic weight	Nature
Neodymium	Nd	60	144.27	Metal
Neon	Ne	10	20.183	Inert gas
Nickel	Ni	28	58.69	Metal
Niobium (see columbium)	Nb	41		
Niton (see radon)	Nt	86		
Nitrogen	N	7	14.008	Gas
Osmium	Os	76	190.8	Metal
Oxygen	O	8	16.000	Gas
Palladium	Pd	46	106.7	Metal
Phosphorus	P	15	31.02	Metalloid
Platinum	Pt	78	195.23	Metal
Polonium	Po	84	(210)	
Potassium (kalium)	K	19	39.10	Metal
Praseodymium	Pr	59	140.92	Metal
Protoactinium	Pa	91	°	
Radium	Ra	88	225.97	Metal
Radium emanation	Em	86	222.	
Radon	Rn	86	222.	
Rhenium	Re	75	188.7	
Rhodium	Rh	45	102.91	Metal
Rubidium	Rb	37	85.44	Metal
Ruthenium	Ru	44	101.7	Metal
Samarium	Sa or Sm	62	150.43	Metal
Scandium	Sc	21	45.10	Metal
Selenium	Se	34	79.2	Metalloid
Silicon	Si	14	28.06	Metalloid
Silver (argentum)	Ag	47	107.880	Metal
Sodium (natrium)	Na	11	22.997	Metal
Strontium	Sr	38	87.63	Metal
Sulfur	S	16	32.06	Metalloid
Tantalum	Ta	73	181.5	Metal
Tellurium	Te	52	127.5	Metalloid
Terbium	Tb	65	159.2	Metal
Thallium	Tl	81	204.39	Metal
Thorium	Th	90	232.12	Metal
Thulium	Tm or Tu	69	169.4	Metal
Tin (stannum)	Sn	50	118.70	Metal
Titanium	Ti	22	47.90	Metal
Tungsten (wolfram)	W	74	184.0	Metal
Uranium	U	92	238.14	Metal
Uranium-X_2(isotope of protoactinium)	UX_2	91	(234)	
Vanadium	V	23	50.96	Metal
Xenon	Xe	54	130.2	Inert gas
Ytterbium	Yb	70	173.6	Metal
Yttrium	Y or Yt	39	88.92	Metal
Zinc	Zn	30	65.38	Metal
Zirconium	Zr	40	91.22	Metal

A Chemical Element is a substance that can not by any ordinary means be separated into two or more different substances. This definition does not exclude the possibility of decomposing an element into others by radioactive or similar agencies. **Chemical Compounds** are pure substances which can, by ordinary chemical means, be separated or decomposed into different elements, or which can be made by combining different elements. The **molecules** of a compound are regarded as aggregates of smaller particles of the elements, known as **atoms,** into which the compound can be broken up. The atoms are regarded the smallest mass-elements which occur separately in the structure of the molecules of either compounds or elementary substances, so far as can be determined by ordinary chemical means. The molecule of an element consists of a definite (usually small) number of its atoms. The molecule of a compound consists of one or more atoms of each of its several elements, the numbers of the various kinds of atoms and their arrangement being definite and fixed, and determining the character of the compound.

Although the atom is the smallest mass of any element that can exist either alone or in a compound, it is itself complex, being made up of charges of positive and negative electricity. Various theories concerning the structure of the atom have been proposed. Bohr, whose ideas have at present the most supporters, believes that in the atom there is a central nucleus made up of one or more charges of positive electricity, the **protons** in close connection with charges of negative electricity, the **electrons.** Around the nucleus revolve other electrons in definite orbits. By deductions from, and amplification of, Bohr's theory, it has been found possible to account for the more important chemical and physical properties of the elements in a consistent and logical manner. The property of an element which is in many ways the most important is its **atomic weight** or the mass of one atom compared with the mass of an atom of oxygen, which by definition equals 16,000. The **atomic number** of an element represents the position of the element in the series of elements arranged in the order of their atomic weights from hydrogen = 1.0078 to uranium = 238.14. It is equal to the number of electrons outside of the nucleus.

Radioactivity is one of the consequences of the structure of the atom, and is evidence of the loss of protons or electrons. When such loss occurs, not only are the various types of rays emitted, but the complex which remains is found to be an element of lower atomic number. Thus when uranium (at. no. 92) breaks down, it yields helium and lead (at. nos. 2 and 82). Ordinary lead has an atomic weight of 207.22, while that from uranium has the atomic weight 206.06. Thorium (at. no. 90) in breaking down yields helium and lead which has the atomic weight 208.00. The name **isotope** is applied to the different forms of lead and of other elements which are formed in a similar way. A third of the elements may exist in only one form; the other two-thirds must exist in two or more forms. Little has been done on the separation of the elements into their isotopes.

2. Combustion and Fuels

Carbon is the principal constituent of solid, liquid, and gaseous fuels; it is either free, as in charcoal and coke, or combined with hydrogen, oxygen, or with both. In burning, the carbon and hydrogen combine with oxygen from the air, yielding, when combustion is complete, carbon dioxide and water vapor. When insufficient air is admitted over the bed of fuel, or into the fire-box when liquid or gaseous fuels are used, much of the fuel may be lost, either as free carbon (smoke, soot) or in the form of unburned gases, especially carbon monoxide. It is almost impossible to attain complete combustion of solid fuels by forcing air under the grate and through the bed of coals, because part of the carbon dioxide formed near the grate is reduced to carbon monoxide on passing through the overlying layers of hot fuel. This monoxide, which has a high heating value, can be burned completely only when a plentiful supply of air is admitted over the bed of fuel. The blue flames seen when anthracite, coke, or charcoal is burned are due to carbon monoxide. It is not economical to supply sufficient air by forced draft through the fuel, because the excessive temperatures rapidly burn out the grate bars, and also because of the increased mechanical wear on the boiler tubes due to the greater number of fine particles of coal and ash driven through them.

Wood consists mainly of lignin and cellulose, compounds of carbon with hydrogen and oxygen, together with varying amounts of water and mineral matter. The latter largely remains behind in the ash.

Charcoal is made by piling wood into heaps which are covered with earth, leaving a few small openings to admit a limited amount of air and allow the products of combustion to escape when the wood is ignited. When sufficient wood has burned to insure thorough charring ("destructive distillation") of the remainder, the openings are closed and the pile allowed to cool completely. By this method of making charcoal only a little tar is obtained and all the volatile constituents are allowed to escape. When wood is heated in closed retorts, large amounts of tar, creosote, wood or methyl alcohol, acetone, and pyroligneous (acetic) acid, etc., are obtained. The yield of charcoal is also nearly doubled. Charcoal consists of carbon and the mineral matter of the wood. Its value in metallurgy is due to its low content of phosphorus and sulfur. The calorific value of charcoal is about 75% that of anthracite.

Peat is formed by the partial decay of mosses and other bog plants under water. Even when compressed and dried it contains much water and its mineral content may be high. Its calorific value is 3000 to 4000 cal. per kg.

Lignite or brown coal, is a stage beyond peat in the formation of coal. It contains much moisture and ash and is often high in sulfur content. Its calorific value is 4000 to 5500 cal. per kg. Owing to its large amount of volatile matter, lignite burns with a long, smoky flame.

Bituminous Coal is formed by the further transformation of lignite by heat and pressure. It comprises many varieties, including gas, coking, steam, and cannel coals. They differ principally in their content of volatile matter, the "fat" coals having at times as high as 50% of compounds of carbon and hydrogen, which are readily driven off by heating. The length of flame of burning bituminous coal depends on the percentage of volatile matter

Anthracite Coal is produced by the further action of heat and pressure upon bituminous coal, whereby nearly all the volatile constituents are driven off, leaving mainly carbon and mineral matter. These coals burn with little flame and smoke, and do not cake. Their calorific value may be 9000 to 9500 cal. per kg.

Coke. As charcoal is the residue left by heating wood in retorts or partially burning it with a limited air supply, so coke is made by heating bituminous coals. In some types of coke ovens the gaseous and liquid products formed by the destructive distillation are allowed to escape into the air; with other types of ovens this loss is not permitted and valuable by-products are obtained, such as ammonia, fuel and illuminating gas, and coal tar. Coke is mainly carbon, but contains also the mineral constituents of the coal. It is low in volatile matter and sulfur. Upon this and its infusibility and resistance to crushing depends its value as a fuel in blast furnaces. Its calorific value is about 90% that of the much more expensive anthracite.

Chemical Examination of Coal and Coke. The heating value of any fuel can be determined conveniently by means of one of the numerous forms of bomb calorimeters. But this leaves unanswered many questions which have a very practical bearing, for example, the percentages of volatile matter, sulphur, and ash, and the amount of coke the coal will yield. In general, the lower the percentage of ash the better the quality of the fuel. The only mineral constituent that has any heating value is the sulfur of pyrites. But as sulfur is injurious in practically all metallurgical operations and the oxides of sulfur have a corroding effect upon boiler tubes, etc., a coal high in either pyritic or organic sulfur is undesirable. Gas coals and coal for use in certain metallurgical operations requiring long reducing flames should be high in volatile matter. Coals that are to be worked economically for coke may be low in volatile matter, but must possess the property of partially fusing or caking together when heated in the ovens.

In taking samples, for calorimetric determinations or for chemical analysis, the accepted method is to select a large number of pieces, representing as nearly as possible in size, etc., the whole load. These must be chosen from all parts of the coal to be tested and taken from the top and down through the whole mass. These lumps should then be broken into nearly uniform size, thoroughly mixed together, and a smaller sample obtained by "quartering." If this sample is still too large it should be further broken up and again quartered until a sample that can readily be enclosed in air-tight jars is obtained. In sampling at the mine, the coal should be cut from a freshly exposed face, and the sample, after quartering to suitable size, should fairly represent not only the actual coal but also the interpenetrating veins of shale, etc., if these are regularly mined with the coal. It is very important to prepare the sample, not only with great care but also as rapidly as possible, to minimize the inevitable loss of moisture in breaking up the lumps. This explains the necessity of placing the fuel sample in air-tight receptacles, such as fruit jars with rubber rings. The volatile matter in coal is highly explosive when mixed with air in the proper proportion; therefore care should be taken to ensure thorough ventilation of all places where coal, especially bituminous, is stored.

3. Liquid and Gaseous Fuels

Crude Petroleum is the most important of the liquid fuels. It owes its importance not only to its comparative cheapness but also to the ease with which it is handled and its high efficiency, which is two or more times that of anthracite. It is usually burned in the form of a spray obtained by means of a blast of air or superheated steam. Petroleum residues and coal-tar residues are also burned to some extent. Their calorific value is not as great as that of crude petroleum, but may run as high as 16 000 cal.

Gasoline is the lowest boiling distillate from crude petroleum, or from the "cracking" of the higher boiling fractions by high temperature, usually under excess pressure, and sometimes in the presence of catalysts which facilitate the breaking down of the more complex hydrocarbon molecules into simpler ones. **Casinghead gasoline** is condensed from natural gas by compression and cooling, before the gas enters the mains through which it is distributed. It is too volatile to be used alone for ordinary purposes, but large quantities of it are mixed with the other forms of gasoline. Each of the kinds of gasoline is a mixture of hydrocarbons containing different percentages of carbon and hydrogen. When mixed with the proper amount of air the vapors form a mixture which is readily ignited and burns with explosive violence. If the vapor is largely in excess of the proportion needed for complete combustion the force of the explosion is weakened, so that, apart from the actual loss of unburned gases, the fuel is not used economically. There is a similar loss in power when too much air is present. For the complete combusion of one cubic foot of the vapor of the hydrocarbon hexane,

C_6H_{14}, 45.2 cu. ft. of air are required, whereas the same volume of heptane (C_7H_{16}) vapor requires 52.4 cu. ft. of air, or 16% more.

The chief use for gasoline is as the fuel for internal-combustion engines. Most of it is used without the addition of anything else, but there are on the market various mixtures of gasoline with benzol from coal tar, and with alcohol. The value of a motor fuel depends not only upon its volatility and calorific value, but also upon its "anti-knocking" characteristics, upon the degree to which the mixture of its vapor with air can be compressed without pre-ignition, upon its relative freedom from sulfur compounds, which yield sulfuric acid when burned, and upon other factors.

Natural Gas is the most efficient as well as the cheapest of all fuels, though its use is of course limited by the distance to which it can be economically piped. It consists mainly of methane, CH_4, with 10% or less of hydrogen and other gases. Methane is also known as marsh gas, from its abundant formation when vegetable matter decays under water. The name fire damp refers to its occurrence in coal mines, where it is one of the causes of explosions.

Coal Gas which is made by distilling bituminous coal in retorts, contains 80 to 85% of a mixture of nearly equal parts of hydrogen and methane, with much smaller amounts of oxygen, nitrogen, carbon monoxide and dioxide, etc. It is used to some extent in gas engines, and as a fuel.

Water Gas is formed by the action of superheated steam upon white-hot coal or coke. The steam gives up its oxygen to the carbon of the fuel, forming carbon monoxide, CO, and leaving hydrogen, thus, $C + H_2O = CO + 2\,H$. The reaction is endothermic, that is, it requires the addition of heat, so that it is necessary to cut off the steam every few minutes and reheat the fuel by an air blast. Water gas consists of about 45% each of hydrogen and carbon monoxide, with small percentages of oxygen, nitrogen, carbon dioxide, etc. The first two gases burn with very hot, non-luminous flames. For use as an illuminant is must be "enriched" with naphtha or other similar oil.

Producer Gas is made in much the same way as water gas, except that only air and no steam is passed through the incandescent coal or coke. The carbon is burned to carbon monoxide, which makes about 25% of the gas. Small amounts of hydrogen, methane, and carbon dioxide are present. There is also nearly 65% of nitrogen from the air which is used. This is unavoidable, though the presence of such a large amount of inert gas reduces the thermal efficiency. The reaction whereby carbon is burned to carbon monoxide is accompanied by the evolution of about one-third the total heating value of the fuel. It is evident that if the gas can be burned without allowing it to cool, a great saving of heat can be effected. This is not always feasible and it is the practice, with some forms of producers, to pass some steam with the air, thus making a mixed water-producer gas. The heat which would otherwise be lost is used up in forming water gas, and the resultant fuel gas has an increased fuel value. It is much more economical to convert the fuel into producer gas and use it in explosion engines than to burn it under steam boilers.

4. Explosives

The Fundamental Property of an explosive is that when ignited or subjected to a sudden shock it shall decompose, or its components react, suddenly yielding a relatively large volume of highly heated gas. This definition includes not only gunpowder, nitroglycerin, and similar substances, but mixtures of inflammable gases and vapors with air; or even coal dust, fine sawdust, or flour suspended in the air. The last three have all been the cause of disasters, the reason being that when some of the particles are ignited the flame is rapidly communicated to adjacent ones, yielding large volumes of highly heated gaseous products of combustion, in addition to which the surrounding air is also heated. Thus, one gram of anthracite, of specific gravity 1.5, occupies a volume equal to only 2/3 cubic centimeter. If it contains 95% of carbon, it will yield when burnt about 1761 cc., or 2642 times its own volume, of carbon dioxide measured at 0° C. and 760 mm. pressure. If suspended as dust in a large volume of air and burned in a fraction of a second, it is evident that the large amount of hot gases must expand with explosive violence.

Gunpowder is a mixture of 75 parts by weight of saltpeter, or potassium nitrate, 15 parts of charcoal, and 10 parts of sulfur, made by grinding the ingredients together with enough water to moisten the mass. It is then compressed into a cake and broken into grains, which are glazed by revolving with graphite and sorted into sizes by sieves. The larger grains are used for blasting, and the smaller ones for small arms. **Blasting powder** is frequently made with the cheaper Chile saltpeter, or sodium nitrate, which produces a cheaper and less powerful powder than that made from ordinary saltpeter. Chile saltpeter, however, has the disadvantage of absorbing moisture from the air, and powder made from it cannot be kept too long or stored in a damp place. The proportions used are 73 parts of Chile saltpeter, 16 parts of charcoal, and 11 parts of sulfur.

Guncotton, Nitrocellulose, typical of the second class of explosives, is made by the action of a mixture of nitric and sulfuric acids upon cotton. When only moderately strong acids are allowed to act on the cotton for a short time, the product is pyroxylin, or soluble nitrocellulose, used for making collodion and celluloid. By longer action with more concentrated acids, guncotton is formed. It is then washed in a machine of the kind used for making paper pulp to remove all traces of acids that might cause spontaneous explosions. While still moist, it is compressed into blocks or sticks. Guncotton is usually stored and transported in a moist condition, and can be exploded without drying. It is comparatively safe to handle, as ordinary shocks do not explode it readily. In the open, it burns with extreme rapidity.

Nitroglycerin is made by the cautious addition of glycerin to a well-stirred and cooled mixture of the strongest nitric and sulfuric acids. The oily product is washed to remove all traces of acids that might cause spontaneous explosion. Under the most favorable conditions, nitroglycerin is not safe to handle. The fact that it is a liquid with consequent liability to leakage from containers greatly increases the danger of transportation and storage. For this reason, it is commonly mixed with some absorbent or transformed into a gelatinous mass.

Dynamite is a mixture of nitroglycerin with infusorial earth, powdered "rottenstone," or similar porous material, known as "dope." Instead of these inactive dopes that take no part in the explosion, explosive mixtures are often used to absorb the nitroglycerin. Gunpowder is one of these. Dynamite consisting of 40% nitroglycerin, 44% sodium nitrate, 15% wood pulp, and 1% calcium carbonate, is an example of dynamite with an active dope.

Explosive gelatin is a jelly-like mass made from a solution of soluble nitrocellulose in nitroglycerin. Too powerful for common work, it is used with success for very hard rock in tunnels. Gelatin dynamite is a mixture of explosive gelatin with a dope such as sodium nitrate and wood pulp; it is not so powerful as the straight gelatin. Smokeless powder is a general term covering many modifications of explosive gelatin, and mixtures of nitrocellulose with nitrobenzene, etc.; they are usually given fanciful names, as ballistite, cordite, indurite, and so forth. Nitroglycerin and mixtures containing it are all liable to freeze at moderately low temperatures. They cannot be used satisfactorily in that condition, and should not be thawed by placing them near a fire or on steam pipes but by leaving them in a warm chamber kept at a temperature not over 90° F.

Picric Acid or trinitrophenol, is made by the action of nitric and sulfuric acids upon phenol (carbolic acid). It is a yellow, crystalline substance, formerly used only as a dye for silks and so forth. For years it was not known as an explosive, but it is now known that it will explode with great violence when detonated. If ignited, it usually burns without exploding and is not very susceptible to shock. Lyddite, melinite, and shimose are composed of picric acid. Some of the salts, or picrates, are exploded by slight blows.

Nitrocellulose, Nitroglycerin, and Nitrostarch are true nitrates, as they all contain the atomic group NO_3. They are chemically quite different from the true nitro-compounds, such as picric acid, which contain the atomic group NO_2. Benzene, toluene, naphthalene, and other substances obtained from coal tar yield nitro-compounds when treated with nitric acid. The best known of these is trinitrotoluene, or "TNT," which was used in such enormous quantities in the Great War. These are all more or less unstable and are used as components of explosives, mixed with either ammonium nitrate or other nitrates, or with chlorates, which are good oxidizing agents, or they may be used in dynamite because they lower the freezing point of the nitroglycerin. Rack-a-rock, roburite, bellite, and securite are typical of the explosives made from these nitro-compounds and oxidizing agents.

A Detonator contains a high explosive, too powerful and unstable to be employed alone, which by its sudden disruptive force brings about the instantaneous explosion of a large amount of a more stable explosive. The ones commonly in use consist of copper capsules containing a definite amount of a mixture of chlorate of potash and mercury fulminate, which is exploded either by a fuse or a wire heated electrically. The fulminate is made by mixing a solution of mercury in strong nitric acid with alcohol. The gray, crystalline powder which is precipitated must be well washed to remove all acid. It is sensitive to shock and may explode even when wet.

Explosives must be selected with reference to the character of the work. For quarrying building stone, those that act slowly, with little shattering effect, must be chosen. When the stone is to be crushed after quarrying, or for breaking up rock so that it can be handled by a steam shovel, a quick shattering effect is desired. In all open work, the character of the gases arising from the explosion may be disregarded, but in tunnels or mines, especially if not well ventilated, this factor is of great importance. No explosive is absolutely safe in this respect. In coal mines, where the presence of fire damp (methane) is a menace, no explosive giving a long flame or a high heat of detonation should be used. Even in the absence of gas, there is danger of igniting the coal dust.

Explosives should be stored in a dry place so that the sodium or ammonium nitrates will not take up enough moisture to lessen their power. But if in too dry a place, they may lose the moisture they naturally contain, which will change their speed of explosion and thus modify the character of the results obtained. Explosives should not be stored for a longer time than absolutely necessary, on account of the possibility of chemical changes taking place in the nitro-compounds most of them contain.

PHYSICS

5. Physical Properties of Solids

The properties of substances nominally the same differ so widely that it would be misleading to give more than two or three significant figures in most cases without such detailed specification of conditions as would make the tables too voluminous. Lower and upper limits of values found for different specimens are given when warranted by the data available.

Physical Properties of Rocky Materials

Substance	Specific gravity or density	*Coefficient of linear expansion (Mean 0–100° C.)	Specific heat (Mean 0–100° C.)
Asphaltum....	0.9	....	....
	1.7	....	....
Basalt........	2.7	....	0.20
	3.2	....	0.24
Brick.........	1.4	....	....
	2.3	6.0	0.22
Cement,† loose.	1.3	....	....
	2.0	....	....
Cement,† set...	2.7	10.0	....
	3.2	14.0	0.2 +
Coal, anthracite	1.4	....	....
	1.8	2.0	....
Coal, bituminous......	1.2	....	....
	1.5	....	....
Concrete......	1.8	10.0	....
	2.5	14.0	....
Glass.........	2.4	5.0	....
	5.9	10.0	0.19±
Glass (Quartz)...	2.6	0.5	0.18
Glass (Jena 16III).	....	8.0	....
Gneiss........	2.4	....	0.20
	2.7	....	....

Substance	Specific gravity or density	*Coefficient of linear expansion (Mean 0–100° C.)	Specific heat (Mean 0–100° C.)
Granite...	2.5	....	....
	3.1	9.0	0.19
Graphite..	2.3	....	....
	2.7	2.5	0.20
Greenstone	2.9	....	....
	3.0	....	....
Limestone...	2.7	9.0	0.21
Marble....	2.6	5.0	....
	2.8	16.0	0.22
Porcelain..	2.3	2.0	....
	2.5	20.0	....
Sandstone.	2.1	6.0	....
	2.4	12.0	0.22
Serpentine.	2.5	....	....
	2.7	....	0.26
Slate.....	2.6	....	0.27
	3.3	10.0	0.65
Soapstone.	2.6	....	....
	2.8	....	....
Terra cotta..	1.9	....	....
Trap.......	2.7	....	....

* Divide each number in this column by 1 000 000. † Portland.

Mohs' Scale of Hardness: 1. Talc; 2. Gypsum; 3. Calc spar; 4. Fluorspar; 5. Apatite; 6. Feldspar; 7. Quartz; 8. Topaz; 9. Sapphire; 10. Diamond.

Density. The terms True Density and Apparent Density are used in describing certain porous bodies to distinguish between the density of the substance and the average density of the substance plus the pores. The terms Density and Specific Gravity are synonymous in engineering work.

Thermal Conductivity. The value given in the tables is the number of gram-calories that will pass per second through every square centimeter of a plane section within the substance when the temperature is uniform over the section and falls along the normal to it at the rate of 1° C. per cm.

1 gm.-cal. per sec. per sq. cm. for a temperature gradient of 1° C. per cm. = 360 kg.-cal. per hr. per sq. m. for a temperature gradient of 1° C. per m. = 2.90×10^3 B.t.u. per hr. per sq. ft. for a temperature gradient of 1° F. per in.

Electrical Resistivity. Each value given in the table of metals and alloys is the resistance in microhms between the opposite faces of a cube 1 cm. on each edge when at 18° C. (64.4° F.). This increases bR for every degree C., or $5/9\, bR$ for every degree F., that the temperature of the substance exceeds 18° C. 1 microhm to the sq. cm. of cross-section per cm. of length = 6.015 ohms to the circular mil of cross-section per foot of length.

Physical Properties of Woods

Kind	Specific gravity or density: Dry	Specific gravity or density: Green	* Coefficient of linear expansion 2°–34° C.: Parallel to fibers	* Coefficient of linear expansion 2°–34° C.: Perpendicular to fibers
Acacia	0.58	0.75	.....	....
	0.85	1.00	.....	....
Alder	0.42	0.63	.....	....
	0.68	1.01	.....	....
Ash	0.57	0.70	.....	....
	0.94	1.14	9.5	....
Beech	0.62	0.85	2.6	61.0
	0.90	1.25	6.0	....
Birch	0.51	......	.....	....
	0.77	......	.....	....
Blue gum	0.84	......	.....	....
Box	0.91	1.20	.....	....
	1.16	1.26	2.6	61.0
Butternut	0.38	......	.....	....
Cedar	0.49	......	.....	....
	0.57	......	.....	....
Cherry	0.70	1.05	.....	....
	0.90	1.18	.....	....
Chestnut	0.58	......	6.5	33.0
Cork	0.22	......	.....	....
	0.26	......	.....	....
Ebony	1.11	......	.....	....
	1.33	......	9.7	....
Elm	0.54	0.78	.....	....
	0.82	1.18	5.7	44.0
Fir	0.31	0.38	.....	....
	0.85	1.08	3.7	58.0
Hickory	0.60	......	.....	....
	0.93	......	.....	....
Lancewood	0.68	......	.....	....
	1.00	......	.....	....
Larch	0.47	......	.....	....
	0.56	0.81	.....	....
Lignum Vitæ	1.17	......	.....	....
	1.33	......	.....	....
Linden or lime	0.32	0.58	.....	....
	0.59	0.87	.....	....
Locust	0.67	......	.....	....
	0.71	......	.....	....
Mahogany	0.56	......	.....	....
	1.06	......	3.6	40.0
Maple	0.53	0.83	.....	....
	0.81	1.05	6.4	48.0
Oak	0.60	0.93	.....	....
	1.07	1.28	4.9	54.0
Pear	0.61	0.96	.....	....
	0.73	1.23	.....	....
Pine	0.35	0.40	.....	....
	0.85	1.07	5.4	34.0
Poplar	0.35	0.61	.....	....
	0.59	1.07	3.9	37.0
Satinwood	0.95	......	.....	....
Spruce	0.48	......	.....	....
	0.70	......	.....	....
Sycamore	0.40	......	.....	....
	0.60	......	.....	....
Teak	0.66	......	.....	....
	0.98	......	.....	....
Walnut	0.60	0.91	.....	....
	0.81	0.92	6.6	48.0
Willow	0.40	......	.....	....
	0.60	0.79	.....	....

* Numbers in these columns to be divided by 1 000 000.

Physical Properties of Metals and Alloys

Substance	Specific gravity or density	Hardness (Mohs)	Melting point (Centigrade)	* Coefficient of linear expansion 20° C.	Specific heat (Mean 0–100° C.)	Thermal conductivity (Mean 0–100° C.)	Electrical resistivity at 18° C.	
							Microhm per cm. cu.	Temperature coefficient C. %
Aluminum..Al	2.7	3−	660	23.0	0.22	0.48	2.8	0.36
Antimony..Sb	6.7	3+	630	11.4	0.050	0.042	42.	0.41
Bismuth....Bi	9.8	2+	271	13.3	0.030	0.018	119.	0.42
Brass.........	8.2 8.7	3+	900±	18. 21.	0.092	0.15 0.30	6. 9.	......
Bronze........	8.7 8.9	3+	900±	19.	0.086	....	....	....
Cobalt.....Co	8.7	6	1480	12.3	0.103	....	10.	....
Constantan.... ("Advance")	8.8	....	....	15.	0.10	0.05	47. 50.	−0.003 +0.005
Copper.....Cu	8.9	3	1083	16.6	0.093	0.91	1.7	0.40
German silver..	8.3 8.8	3+	1000±	18.	0.095	0.07 0.09	16. 49.	0.06 0.023
Gold.......Au	19.3	3−	1063	14.2	0.032	0.70	2.3	0.35
Iron.......Fe	7.9	4	1535	11.7	0.113	0.11 0.14	9. 15.	up to 0.6
Iron (cast).....	7.0 7.7	6 8	1100 1300	11.	0.11 0.13	....	56. 114.	....
Iridium....Ir	22.4	....	2350±	6.5	0.032	0.14	7.	0.41
Lead......Pb	11.3	2−	327	29.1	0.031	0.082	24.	0.40
Manganin......	8.5	....	....	18.	....	0.6	39. 46.	<0.003
Mercury...Hg	13.6	....	−39	1.82†	0.033	0.018	95.8	0.092
Molybdenum Mo	10.2	....	2620±	5.	0.065	0.346	6.	....
Nichrome......	8.2	....	1500±	....	....	....	96.	0.043
Nickel.....Ni	8.9	4+	1452	12.8	0.109	0.14	9.	up to 0.6
Osmium....Os	22.5	....	2700	6.1	0.031	....	60.	0.4
Palladium..Pd	12.0	....	1555	11.8	0.059	0.17	10.7	0.38
Platinum...Pt	21.4	4+	1755	8.9	0.032	0.17	8. 16.	0.36
Rhodium...Rh	12.5	....	1955±	8.4	0.058	0.21	6.0	0.44
Silver......Ag	10.5	2+	961	18.9	0.056	1.00	1.6	0.37
Steel..........	7.9	4 9	1300 1450	10. 14.	0.114 0.117	0.06 0.14	10. 50.	....
Tantalum..Ta	16.6	....	2850	7.0	0.036	0.13	15.	0.33
Tin........Sn	7.3	2	232	20.0	0.056	0.15	....	....
Tungsten..W	19.3	....	3370	4.3	0.034	....	6.	....
Zinc.......Zn	7.1	3+	419	33.	0.094	0.26	6.	0.37

* Numbers in this column to be divided by 1 000 000.
† Liquid (cubical expansion).

6. Properties of Water and Gases

1 Standard Atmosphere is the pressure that will support a column of mercury 76 cm. = 29.921 in. high at 0° C. at a place where $g = g_0 = 980.665$ cm. per sec. per sec.

Specific Heat of Water. (From Kohlrausch, 1910)

0° C.	1.008	40°	0.9990	140°	1.025	7°	1.0033	15°	1.0000
5	1.0044	50	1.0000	160	1.036	8	1.0028	16	0.9998
10	1.0018	60	1.0017	180	1.048	9	1.0023	17	0.9995
15	1.0000	70	1.0034	200	1.062	10	1.0018	18	0.9993
20	0.9989	80	1.005	220	1.077	11	1.0014	19	0.9991
25	0.9984	90	1.007	240	1.094	12	1.0010	20	0.9989
30	0.9983	100	1.010	260	1.113	13	1.0007	21	0.9987
35	0.9985	120	1.017	280	1.133	14	1.0003	22	0.9986
40	0.9990	140	1.025	300	1.155	15	1.0000	23	0.9985

Specific Volume of Water. (From Kohlrausch, 1910)

°C.	Cu. cm. per gm.	°C.	Cu. cm. per gm.	°C.	Cu. cm. per gm.	°C.	Cu. cm. per gm.	°C.	Cu. cm. per gm.
0	1.000 13	45	1.009 85	90	1.035 90	140	1.079	230	1.215
4	1.000 00	50	1.012 07	95	1.039 59	150	1.090	240	1.236
10	1.000 27	55	1.014 48	99	1.042 65	160	1.102	250	1.26
15	1.000 87	60	1.017 05	100	1.043 43	170	1.114	260	1.28
20	1.001 77	65	1.019 79	101	1.044 22	180	1.128	270	1.30
25	1.002 94	70	1.022 70	102	1.045 01	190	1.143	280	1.34
30	1.004 35	75	1.025 76	110	1.051	200	1.159	290	1.38
35	1.005 98	80	1.028 99	120	1.060	210	1.177	300	1.42
40	1.007 82	85	1.032 37	130	1.069	220	1.195	310	1.46

Boiling Point of Water in Centigrade Degrees. (From Wiebe, 1910)

	Height of mercurial barometer in millimeters											
	680	690	700	710	720	730	740	750	760	770	780	790
0	96.92	7.32	7.71	8.11	8.49	8.88	9.26	9.63	100.00	0.37	0.73	1.09
1	6.96	7.36	7.75	8.14	8.53	8.91	9.29	9.67	0.04	0.40	0.76	1.12
2	7.00	7.40	7.79	8.18	8.57	8.95	9.33	9.70	0.07	0.44	0.80	1.16
3	7.04	7.44	7.83	8.22	8.61	8.99	9.37	9.74	0.11	0.48	0.84	1.19
4	7.08	7.48	7.87	8.26	8.65	9.03	9.41	9.78	0.15	0.51	0.87	1.23
5	7.12	7.52	7.91	8.30	8.69	9.07	9.44	9.82	0.18	0.55	0.91	1.26
6	7.16	7.56	7.95	8.34	8.72	9.10	9.48	9.85	0.22	0.58	0.94	1.30
7	7.20	7.60	7.99	8.38	8.76	9.14	9.52	9.89	0.26	0.62	0.98	1.33
8	7.24	7.63	8.03	8.42	8.80	9.18	9.56	9.93	0.29	0.66	1.02	1.37
9	7.28	7.67	8.07	8.45	8.84	9.22	9.59	9.96	0.33	0.69	1.05	1.41

Common Gases. (From Kohlrausch, 1910)

Gas	Specific gravity or density *	Molecular mass	Specific heat (0°–200° C.) constant pressure	$\frac{c_p}{c_v}$ ‡	Melting point ° C.	Boiling point ° C.	Water dissolves cu. cm. † At 0° C.	Water dissolves cu. cm. † At 20° C.	Symbol
Air (free of CO_2)....	1.2928	28.98	0.238	1.40	....	−193	29	19	Air
Acetylene..	1.1759	24.02	....	1.26	− 81.5	− 83.6	1730	1030	C_2H_2
Ammonia..	0.7708	17.03	0.52	1.32	− 78	− 33.5	(12×10^5)	(7×10^5)	NH_3
Carbon dioxid....	1.9768	44.00	0.218	1.30	− 57	− 78.2	(1800)	(900)	CO_2
Carbon monoxid.	1.2503	28.00	0.243	1.41	−207	−190.0	35.4	23.2	CO
Chlorine...	3.2197	70.92	0.121	1.32	−102	− 33.4	(4600)	(2300)	Cl_2
Hydrogen..	0.08985	2.016	3.41	1.41	−259	−252.6	21.1	18.1	H_2
Nitrogen..	1.2507	28.02	0.244	1.41	−210.5	−195.7	23.5	15.4	N_2
Nitrous oxid....	1.9777	44.02	0.225	1.28	−103	−90	1300	650	N_2O
Oxygen....	1.4292	32.00	0.220	1.40	−227	−182.8	48.9	31.0	O_2

* Numbers in this column to be divided by 1000. † These columns contain the number of cubic centimeters of gas that will be dissolved at a barometric pressure of 76 centimeters in one liter of water. ‡ This column gives the ratio of the specific heat at constant pressure to that at constant volume.

Freezing Mixtures. (From Hütte)

Mixture	Parts by mass	Temperature falls ° C. From	Temperature falls ° C. To	Parts by mass	Temperature falls ° C. From	Temperature falls ° C. To
Common salt (NaCl).............. Snow..............................	1 3	0	−17.7	1 1	0	−18
Calcium chlorid ($CaCl_2$)............ Snow..........................	3 2	0	−33	2 1	0	−42
Sal ammoniac (NH_4Cl)........... Saltpeter (KNO_3).................. Water..........................	5 5 16	+10	−12	1 1 1	+8	−24
Ammonium nitrate (NH_4NO_3)...... Water..........................	1 1	+10	−16	......	......	......
Potassium hydroxide (KOH)........ Snow..........................	4 3	0	−37	......	......	......

7. Light and Illumination

The Unit of Light Intensity is the **international candle,** as agreed upon by England, France and the United States in July, 1909. The **Hefner,** which is the legal unit in Germany and certain other European countries, is 9/10 of the international candle. The international candle is maintained by the National Bureau of Standards at Washington, by means of groups of carbon filament incandescent electric lamps.

The relations between other units of light intensity are: 1 international candle = 1 pentane candle = 1 bougie decimale = 0.104 Carcel unit = 1.11 Hefner Kerze.

Candlepower is luminous intensity expressed in candles. Light sources are measured in horizontal candles, in spherical candles, or in **lumens.** One spherical candle is equal to 4π (12.57) lumens.

The Color of Light is determined by its spectral distribution and is the subjective evaluation by the eye of the quality of light flux. The easily visible spectrum extends from about $\lambda = 380$ mμ, to $\lambda = 780$ mμ, violet to red.

Wave-lengths, λ, are commonly expressed in microns (μ) or millimicrons (mμ). 1 micron (μ) = 10^3 millimicrons (mμ) = 10^{-3} mm.

The Speed of Light in a vacuum is approximately 300,000 kilometers per second (186,000 miles per second.) (The most recent measurements, made by Michelson in 1926, give a value 299,796 km. per sec.)

When a ray of light strikes the plane surface of a transparent body, in passing to an optically denser medium, it is refracted toward the normal to that plane. The ratio of the sine of the angle between the original ray and the normal to the sine of the angle between the refracted ray and the normal is called the index of refraction.

The ratio of the speed of light in a vacuum to that in any substance is equal to the index of refraction of the substance.

Incandescent Electric Lamps are the generally used light sources. These lamps are rated commercially in total watts at a definite voltage or current, in lumens, or in spherical candles, depending on size and type. Lamps for ordinary lighting service on multiple circuits are designated in watts; lamps for street lighting service on series circuits. in lumens; and lamps for automobile service, in spherical candles. Inside-frosted lamps have replaced clear and frosted lamps up to and including the 100-watt size for general lighting service.

Standard specifications for the purchase of incandescent lamps have been issued by the Bureau of Standards, Washington, D. C.

Illumination is measured in foot-candles. A point source of one candle (4π or 12.57 lumens) produces an illumination of one foot-candle on a surface 1 ft. from the source. A foot-candle is equivalent to one lumen per square foot. The light output, or luminous flux of lamps should be redirected by shades and reflectors for good illumination. Illuminations of from 10 to 15 foot-candles are considered good practice in lighting offices and drafting rooms.

Direct sunlight from the noon-day sun gives an illumination of from 6000 to 10 000 foot-candles; full moon about 0.02 foot-candle. Daylight illuminations vary very greatly.

The Absorption, Transmission and Reflection of materials used as enclosing, diffusing and reflecting devices, and the color and surface of walls, ceilings and surroundings affect interior illumination obtained from light sources of definite light output.

Polished silver reflects approximately 92% of the incident light flux, aluminum 68%, nickel 65%, silver-backed glass 88%. Enclosing and diffusing glassware, as commonly used on lighting units, transmits from 75% to 85% of the incident light.

8. Microscopes and Telescopes

Magnification, increasing visibility of detail, is secured by bringing the image of an object nearer to the eye, or by any other means of increasing the visual angle which the object subtends. Vision with the unaided normal eye is, however, most distinct at a distance of 25 to 30 cm. because the accommodating mechanism of the eye is unable to focus sharply on the retina points nearer than this. A magnifying optical system produces in effect the required increase in the visual angle while forming an image (real or virtual) farther from the eye than the least distance of distinct vision. Often the arrangement is such that the eye views a virtual image at an infinite distance, so that the muscles of accommodation may be completely relaxed.

The Simple Microscope (Fig. 1). A single converging lens if placed closer to an object than the principal focal length produces an enlarged virtual image, which is seen on looking through the lens. The magnification produced is $1 + d/f$ for an eye whose least distance of distinct vision is d. A simple plano-convex lens, with the plane side toward the eye, gives good images for magnification less than eight diameters, that is, with focal lengths greater than about 3 cm. The image may be much improved, especially where the magnification is considerable, by the use of special combinations of lenses designed to reduce spherical and chromatic aberration so as to give a fairly large field of view approximately free from distortion and color.

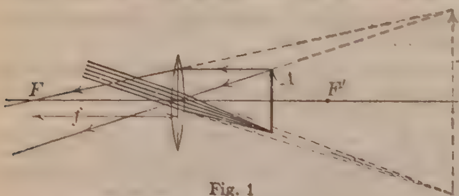

Fig. 1

The Ramsden or Positive Eyepiece (Fig. 2) consists of two converging lenses, usually plano-convex, with their convex surfaces facing each other, of equal focal length, and separated by 2/3 the focal length of either. A virtual image of the object or real image A is formed by the field-lens L_1 at A'. The eye-lens L_2 forms an image of this at infinity. This eyepiece is fairly, but not quite, achromatic.

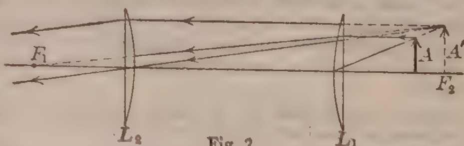

Fig. 2

Huyghens or Negative Eyepiece (Fig. 3). Two converging lenses, usually plano-convex, with the plane surfaces toward the eye, are so arranged as to divide equally between them the

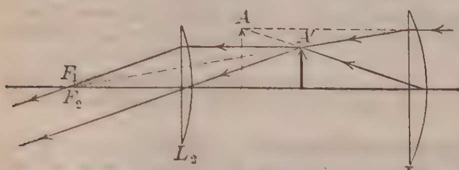

Fig. 3

deviation produced on incident light parallel to and close to the axis. The field-lens L_1 has three times the focal length of the eye-lens L_2, and the two are separated by the difference in their focal lengths. Light which if unhindered would converge at A is deviated by L_1 to form an image at A', of which L_2 forms an image at infinity. This eyepiece is highly achromatic and free from disturbing spherical aberration.

For Measuring Microscopes and Telescopes in which the eyepiece is fitted with cross-hairs, the positive eyepiece is far more suitable than the negative because the image of the hairs being formed by both lenses is corrected for both chromatic and spherical aberration, and because the cross-hairs can be easily adjusted to suit different eyes by altering their distance from the eyepiece.

The Compound Microscope (Fig. 4) in its simplest form consists of two converging lenses. The objective L_1 forms within the tube a real, inverted, magnified image A' of the object A. This image is viewed through the eyepiece L_2 and further magnified. A microscope is usually fitted with either a Huyghens or a Ramsden eyepiece, according to the purpose for which it is to be used. The objective is also generally a combination of several lenses to overcome spherical and chromatic aberration while admitting as much light as possible. In microscopes of the highest power a drop of oil of cedar is placed between the side and the objective; this is known as "immersion." The smallest interval that can be optically resolved is about 0.00005 mm., and the limit of resolution of the microscope is attained when the total magnification is about 1200.

Fig. 4

The Astronomical Refracting Telescope differs from the compound microscope in that the objective forms a reduced image of a distant object. The objective is generally a compound lens consisting of a convex lens of crown and a concave lens of flint glass. A Huyghens or a Ramsden eyepiece is ordinarily used; but the best instruments employ eyepieces embodying later improvements.

The Terrestrial Telescope (Fig. 5) produces an erect image by an inverting system between the eyepiece and the inverted image formed by the objective. One form of inverting system consists of two converging lenses of equal focal length so placed that the inverted image A formed by the objective is at the principal focus of the first lens. An erect image A' is then formed at the principal focus of the second lens, and is magnified by an eyepiece.

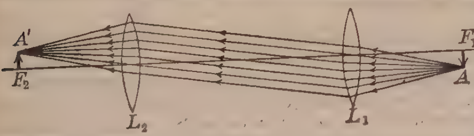

Fig. 5

Galileo's Telescope (Fig. 6) consists of a convex lens L_1 for objective and a concave lens L_2 for eyepiece. The light from L_1 converging so that if unhindered it would form at A' a real, inverted image of the distant object A, is intercepted by L_2 and rendered parallel or slightly divergent as if it came from A'', which is a virtual, erect

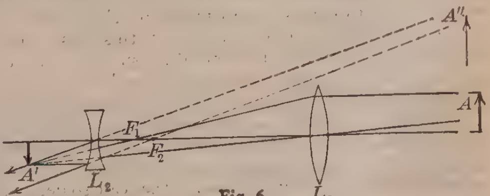

Fig. 6

image of A. The use of the diverging eye-lens limits considerably the angular field of view. Ordinary field-glasses and opera-glasses are Galilean telescopes.

The Prism Binocular (Fig. 7) secures the wide field of view that accompanies the use of a converging eyepiece, while at the same time it avoids the inconvenient length of the ordinary terrestrial telescope. This is accomplished by employing four total reflections within two right-angled prisms to invert the image formed by the objective. Otherwise the construction is the same as that of the astronomical telescope. This prism construction permits a considerable shortening of the telescope by separating the prisms, since the light traverses the distance between them three times. In addition the stereoscopic effect due to binocular vision may be greatly increased by placing the centers of the objectives much farther apart than the pupils of the eye. The increased field of view is obtained at a sacrifice of illumination.

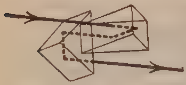

Fig. 7

In Reflecting Telescopes the object lens is replaced by a concave mirror. Being strictly achromatic, reflecting telescopes are valuable for certain classes of astronomical work. Mirrors as large as 100 in. in aperture have been made. The mirror is of glass, upon which a thin film of silver has been deposited.

9. Temperature and Temperature Scales

The Temperature of a body may be defined as its thermal state considered from the standpoint of its ability to communicate heat to other bodies. When two bodies are placed in thermal communication, the one which loses heat to the other is said to be at the higher temperature.

Certain thermal states or " temperatures " may be reproduced and recognized by the fact that definite physical phenomena occur at these temperatures. Such thermal states are called " fixed points " and may, quite apart from any temperature scale, be specified by the physical phenomena characteristic of those temperatures. For example we may speak of the temperature of freezing water (ice point), the boiling point of water under a specified pressure (steam point), the freezing point of gold (gold point), etc. The purpose of establishing a temperature scale is to assign a number to every thermal state or temperature, and to provide a means for determining the temperature of any particular body.

A Temperature Scale may be defined by (1) selecting definite numbers for certain fixed points, (2) selecting some physical property of a definite substance which varies with temperature, and (3) selecting a mathematical law expressing temperatures on the scale in question in terms of the selected property of the thermometric substance. For example, on the centigrade mercury-in-glass scale, the ice and steam points are numbered 0 and 100 respectively, the relative or " apparent " expansion of a volume of mercury enclosed in glass of a definite kind is the property used, and the mathematical relation used to express temperature on this scale is that equal increments of apparent volume of the mercury in this glass correspond to equal increments of temperature. If some other substance is substituted for mercury or if glass of a different kind is used, another scale would be obtained which would agree with it at 0 and 100 but not at other temperatures.

Although, in general, a temperature scale depends on the thermometric substance as well as on the expression for the temperature in terms of some property of this substance, Lord Kelvin has shown that if the property selected is the availability of energy, the scale so defined is wholly independent of the substance and depends only on the mathematical relation chosen. Any scale so defined is known as a thermodynamic scale.

The Kelvin Scale finally chosen by Lord Kelvin, is the one on which the temperature interval from the ice point to the steam point is 100° and the ratio of the values of any two temperatures is equal to the ratio of the heat taken in to the heat rejected by a reversible thermodynamic engine working with a source and refrigerator at the higher and lower temperatures respectively. On this scale, which is also known as the absolute thermodynamic scale, the lowest attainable temperature is 0 and the ice point is found experimentally to be 273.10°. The steam point therefore is 373.10° or 100° higher.

The Absolute Fahrenheit Scale is identical with the Kelvin scale except that the interval from the ice to the steam point is taken as 180°. Any temperature on this scale is therefore 1.8 times the same temperature expressed on the Kelvin scale.

The Thermodynamic Centigrade Scale is derived by subtracting from the Kelvin scale a constant number of the proper magnitude to make the ice point 0°. On this scale, therefore, the ice and steam points are 0° and 100° respectively, and the so-called " absolute zero " is − 273.10°.

While other thermodynamic scales may be devised, such as the first one proposed by Lord Kelvin on which temperatures ranged from $-\infty$ to $+\infty$, the three described above are the only ones that have ever been widely used.

The International Temperature Scale for national and international use was adopted at the September 1927 meeting of the General Conference of Weights and Measures at Sèvres, France.

The international temperature scale, given below in detail, does not purport to replace the thermodynamic scale. It is, in fact, a practical representation of the thermodynamic centigrade scale to a degree of accuracy as great as is possible with present-day apparatus and methods. It is therefore to be regarded as susceptible of revision and amendment as improved and more accurate methods of measurement are evolved.

Definition of International Temperature Scale

1. The thermodynamic Centigrade scale, on which the temperature of melting ice and the temperature of condensing water vapor, both under the pressure of one standard atmosphere, are numbered 0° and 100° respectively, is recognized as the fundamental scale to which all temperature measurements should ultimately be referable.

2. The experimental difficulties incident to the practical realization of the thermodynamic scale have made it expedient to adopt for international use a practical scale, designated as the International Temperature Scale. This scale conforms with the thermodynamic scale as closely as is possible with present knowledge, and is designed to be definite, conveniently and accurately reproducible, and to provide means for uniquely determining any temperature within the range of the scale, thus promoting uniformity in numerical statements of temperature.

3. Temperatures on the International Scale will ordinarily be designated as " ° C." but may be designated as " °C. (Int.) " if it is desired to emphasize the fact that this scale is being used.

4. The International Temperature Scale is based upon a number of fixed and reproducible equilibrium temperatures to which numerical values are assigned and upon the indications of interpolation instruments, calibrated according to a specified procedure at the fixed temperatures.

5. The basic fixed points and the numerical values assigned to them for the pressure of one standard atmosphere are given in the following table, together with formulas which represent the temperature (t_p) as a function of vapor pressure (p) over the range 680 mm. to 780 mm. of mercury.

6. Basic Fixed Points of the International Temperature Scale:

(a) Temperature of equilibrium between liquid and gaseous oxygen at the pressure of one standard atmosphere (oxygen point), −182.97° C.

$$t_p = t_{760} + 0.0126\,(p - 760) - 0.0000065\,(p - 760)^2$$

(b) Temperature of equilibrium between ice and air saturated water at normal atmospheric pressure (ice point), 0.000° C.

(c) Temperature of equilibrium between liquid water and its vapor at the pressure of one standard atmosphere (steam point), 100.000° C.

$$t_p = t_{760} + 0.0367\,(p - 760) - 0.000023\,(p - 760)^2$$

(d) Temperature of equilibrium between liquid sulfur and its vapor at the pressure of one standard atmosphere (sulfur point), 444.60° C.

$$t_p = t_{760} + 0.0909\,(p - 760) - 0.000048\,(p - 760)^2$$

(e) Temperature of equilibrium between solid silver and liquid silver at normal atmospheric pressure (silver point), 960.5° C.

(f) Temperature of equilibrium between solid gold and liquid gold at normal atmospheric pressure (gold point), 1063° C.

Standard atmospheric pressure is defined as the pressure due to a column of mercury 760 mm. high, having a mass of 13.5951 grams per cm.^{3}, subject to a gravitational acceleration of 960.665 cm./sec.^{2} and is equal to 1.013.250 dynes/cm.^{2}

It is an essential feature of a practical scale of temperature that definite numerical values shall be assigned to such fixed points as are chosen. It should be noted, however, that the last decimal place given for each of the values in the table is significant only as regards the degree of reproducibility of that fixed point on the International Temperature Scale. It is not to be understood that the values are necessarily known on the Thermodynamic Centigrade Scale to the corresponding degree of accuracy.

7. The means available for interpolation lead to a division of the scale into four parts.

8. **From the ice point to 660° C.** the temperature t is deduced from the resistance R_t of a standard platinum resistance thermometer by means of the formula

$$\mathbf{R_t = R_0\,(1 + At + Bt^2)}$$

The constants R_0, A, and B of this formula are to be determined by calibration at the ice, steam, and sulfur points, respectively.

The purity and physical condition of the platinum of which the thermometer is made should be such that the ratio R_t/R_0 shall not be less than 1.390 for t = 100° and 2.645 for t = 444.6°.

9. **From −190° C. to the ice point** the temperature t is deduced from the resistance R_t of a standard platinum resistance thermometer by means of the formula

$$\mathbf{R_t = R_0\,[1 + At + Bt^2 + C\,(t - 100)\,t^3]}$$

The constants R_0, A, and B are to be determined as specified above, and the additional constant C is determined by calibration at the oxygen point.

The standard thermometer for use below 0° C. must in addition have a ratio R_t/R_0 less than 0.250 for t = −183°.

10. **From 660° C. to the gold point,** the temperature t is deduced from the electromotive force e of a standard platinum vs. platinum-rhodium thermocouple, one junction of which is kept at a constant temperature of 0° C. while the other is at the temperature t defined by the formula

$$\mathbf{e = a + bt + ct^2}$$

The constants a, b, and c are to be determined by calibration at the freezing point of antimony, and at the silver and gold points.

11. **Above the gold point** the temperature t is determined by means of the ratio of the intensity J_2 of monochromatic visible radiation of wavelength λ cm. emitted by a black body at the temperature t to the intensity J_1 of radiation of the same wavelength emitted by a black body at the gold point, by means of the formula

$$\log_e \frac{J_2}{J_1} = \frac{c_2}{\lambda}\left[\frac{1}{1336} - \frac{1}{(t + 273)}\right]$$

The constant c_2 is taken as 1.432 cm. degrees. The equation is valid if $\lambda(t + 273)$ is less than 0.3 cm. degrees.

Recommended Experimental Procedure

1. Oxygen. The temperature of equilibrium of liquid and gaseous oxygen has been best realized experimentally by the static method, the oxygen vapor-pressure thermometer being compared with the thermometer to be standardized in a suitable low temperature bath.

2. Ice. The temperature of melting ice is realized experimentally as the temperature at which pure finely divided ice is in equilibrium with pure, air-saturated water, under standard atmospheric pressure. The effect of increased pressure is to lower the freezing point to the extent of 0.007° C. per atmosphere.

3. Steam. The temperature of condensing water vapor is realized experimentally by the use of a hypsometer so constructed as to avoid superheat of the vapor around the thermometer, or contamination with air or other impurities. If the desired conditions have been attained, the observed temperature should be independent of the rate of heat supply to the boiler, except as this may affect the pressure within the hypsometer, and of the length of time the hypsometer has been in operation.

4. Sulfur. For the purpose of standardizing resistance thermometers, the temperature of condensing sulfur vapor is realized by adherence to the following specifications relating to boiling apparatus, purity of sulfur, radiation shield and procedure.

The boiling-tube is of glass, fused silica or similar material, and has an internal diameter of not less than 4 nor more than 6 cm. The vapor column must be sufficiently long that the bottom of the radiation shield is not less than 6 cm. above the free liquid surface and its top is not less that 2 cm. below the top of the heat insulating material surrounding the tube. Electric heating is preferable although gas may be used, but the source of heat and all good conducting material in contact with it must terminate at least 4 cm. below the free surface of the liquid sulfur. Above the source of heat the tube is surrounded with insulating material. Any device used to close the end of the tube must allow a free opening for equalization of pressure.

The sulfur should contain not over 0.02% of impurities. Selenium is the impurity most likely to be present in quantities sufficient to affect the temperature of the boiling point.

The radiation shield is cylindrical and open at the lower end, and is provided with a conical portion at the top, to fit closely to the protecting tube of the thermometer. The cylindrical part is 1.5 to 2.5 cm. larger in diameter than the protecting tube of the thermometer and at least 1 cm. smaller in diameter than the inside of the boiling tube. The cylinder should extend at least 1.5 cm. beyond each end of the thermometer coil. There should be ample opening at the top of the cylindrical and below the conical portion to permit free circulation of vapor. The inner surface of the shield should be a poor reflector. The shield may be made of sheet metal, graphite, etc.

In standardizing a thermometer the sulfur is heated to boiling and the heating so regulated that the condensation line is at least 1 cm. above the top of the insulating material. The thermometer with its radiation shield is inserted in the vapor, and when the line of condensation again reaches its former level simultaneous observations of resistance and barometric pressure are made. In all cases care should be taken to prove that the temperature is independent of vertical displacements of the thermometer and shield.

5. Silver and Gold. For standardizing a thermocouple, the metal to be used at its freezing point is contained in a crucible of pure graphite, refractory porcelain, or other material which will not react with the metal so as to contaminate it to an appreciable extent.

Silver must be protected from access of oxygen while heated.

The crucible and metal are placed in an electric furnace capable of heating the contents to a uniform temperature.

The metal is melted and brought to a uniform temperature a few degrees above its melting point, then allowed to cool slowly with the thermocouple immersed in it as described in the next paragraph.

The thermocouple, mounted in a porcelain tube with porcelain insulators separating the two wires, is immersed in the molten metal through a hole in the center of the crucible cover. The depth of immersion should be such that during the period of freezing the thermocouple can be lowered or raised at least 1 cm. from its normal position without altering the indicated e.m.f. by as much as one microvolt. During freezing, the e.m.f. should remain constant within one microvolt for a period of at least 5 minutes.

As an alternative to displacing the couple, as a means of testing the absence of the influence of external conditions upon the observed temperature, both freezing and melting points may be observed, and if these do not differ by more than 2 microvolts the observed freezing point may be considered satisfactory.

6. **The Standard Platinum Resistance Thermometer**. The diameter of the wire should not be smaller than 0.05 or larger than 0.2 mm.

The platinum wire of the thermometer must be so mounted as to be subject to the minimum of mechanical constraint, so that dimensional changes accompanying changes of temperature may result in a minimum of mechanical strain being imposed upon the platinum.

The design of the thermometer should be such that the portion, the resistance of which is measured, shall consist only of platinum, and shall be at the uniform temperature which is to be measured. This may be accomplished by either of the accepted systems of current and potential, or compensating leads.

After completion the thermometer should be annealed at a temperature of at least 660°.

7. **The Standard Thermocouple.** The platinum of the standard couple shall be of such purity that the ratio R_t/R_0 is initially not less than 1.390 for $t = 100°$. The alloy is to consist of 90% platinum with 10% rhodium. The completed thermocouple must develop an electromotive force, when one junction is at 0° and the other at the freezing point of gold, not less than 10 200 nor more than 10 400 international microvolts. The diameter of the wires used for standard thermocouples should lie between the values 0.35 mm. and 0.65 mm.

The freezing point of antimony, specified for the standardization of the thermocouple, lies within the range of 0° to 660° where the International Scale is fixed by the indications of the standard resistance thermometer, and the numerical value of this temperature is therefore to be determined with the resistance thermometer. In the appendix the result of such determinations is given as 630.5°, but the temperature of any particular lot of antimony which is to be used for standardizing the thermocouple is to be determined with a standard resistance thermometer.

The procedure to be followed in using the freezing point of antimony as a fixed temperature is substantially the same as that specified for silver. Antimony has a marked tendency to undercool before freezing. The undercooling will not be excessive if the metal is heated only for degrees above its melting point and if the liquid metal is stirred. During freezing, the temperature should remain constant within 0.1° for a period of at least 5 minutes.

1. **Secondary points.** In addition to the basic fixed points the temperatures of a number of other points are available and may be used in the calibration of secondary temperature measuring instruments. These points and their temperatures on the International Scale are listed below. The temperatures given are those corresponding to a pressure of one standard atmosphere. The formulas for the variation of vapor pressure with temperature are valid for the range from 680 to 780 mm.

Secondary Points

	Deg. C.
Temperature of equilibrium between solid and gaseous carbon dioxide.....	−78.5
$t_p = t_{760} + 0.1443\ (t_p + 273.2) \log_{10}(p/760)$	
Temperature of freezing mercury..........	−38.87
Temperature of transition of sodium sulfate..........	32.38
Temperature of condensing naphthalene vapor..........	217.96
$t_p = t_{760} + 0.208\ (t_p + 273.2) \log_{10}(p/760)$	
Temperature of freezing tin..........	231.85
Temperature of condensing benzophenone vapor..........	305.9
$t_p = t_{760} + 0.194\ (t_p + 273.2) \log_{10}(p/760)$	

Temperature of freezing cadmium	320.9
Temperature of freezing lead	327.3
Temperature of freezing zinc	419.45
Temperature of freezing antimony	630.5
Temperature of freezing copper in a reducing atmosphere	1083.
Temperature of freezing palladium	1555.
Temperature of melting tungsten	3400.

10. Temperature-measuring Equipment

The Platinum vs. Platinum-Rhodium Thermocouple may be used to measure temperatures up to 1600° C. At this high temperature, however, the life of a couple is very short. When suitably protected, a couple of this kind may be used for long periods up to 1300° C. In a reducing atmosphere such couples are contaminated by the reduction to silicon of the silica in porcelain protection tubes. These couples can be used at any lower temperatures, of course, but below about 1000° C. base metal couples often are to be preferred, being less expensive and giving a higher electromotive force.

Base Metal Couples of copper vs. constantan may be used for temperatures below 500° C. For extreme precision this couple should not be used above 350° C. Iron vs. constantan couples may be used up to 900° C., while chromel vs. alumel couples may be used for continuous service up to 1100° C. In the accompanying table are given the calibration data for representative couples of these four types with the cold junctions at 0° C. If the cold junctions are not at 0° C., the e.m.f. in the table corresponding to the temperature of the cold junction must be added to the observed e.m.f. before using table.

Example. Chromel-alumel couple with cold junction at 12° C. yields at e.m.f. of 24.5 millivolts.

24.5 millivolts observed e.m.f.
.5 millivolt e.m.f. at 12° C.
25.0 millivolts

Therefore the temperature of the hot junction is 601° C.

The e.m.f. may be measured by means of a portable potentiometer or millivoltmeter. If a millivoltmeter is used it should have a high resistance, preferably 300 ohms or more.

Mercury-in-Glass Thermometers may be used for measuring temperatures from − 38° C. up to about 500° C. Fused silica thermometers filled with gallium are usable up to 1000° C.

Pyrometric Cones are used for the control of numerous industrial processes, notably in the ceramic industry. The temperatures determined by the Bureau of Standards at which the American cones (made by the Standard Pyrometric Cone Co. of Columbus, Ohio) start to bend and the temperatures at which the tip falls to the level of the base or "end points" are shown for two different rates of heating in the second table herewith. The Bending Interval is the difference between the "end point" and the temperature at which bending starts.

The temperatures in the table on page 265, as determined under standardized and reproducible conditions (controlled rate of heating in clean air) will apply with fair approximation for the cones when heated in kilns where the gases are normally oxidizing and free from sulfur oxides. If the cones show a visible hardening of the surface or a skin effect resulting from exposure to kiln gases containing insufficient oxygen, small corrections may be necessary. Such surface action is most frequently noticed with cones 015 to 01. In kilns fired with fuel containing appreciable amounts of sulfur, or where the ware gives off sulfur dioxide the end points may be noticeably different from those given in the table.

Calibration Data of Representative Couples with One Junction at 0° C. and the Other at T° C.

(E.m.f. is in millivolts)

Platinum vs. Platinum–10% Rhodium		Copper vs. Constantan		Iron vs. Constantan		Chromel vs. Alumel	
E.m.f.	T° C.	E.m.f.	T° C.	E.m.f.	T° C.	E.m.f.	T° C.
0	0	0	0	0	0	0	0
1	147	1	25	5	100	5	120
2	265	2	49	10	195	10	242
3	374	3	72	15	288	15	365
4	478	4	94	20	380	20	482
5	578	5	115	25	470	25	601
6	675	6	136	30	560	30	721
7	769	7	156	35	647	35	842
8	861	8	175	40	731	40	966
9	950	9	194	45	814	45	1099
10	1037	10	213	50	897	50	1232
11	1122	11	232	55	980	55	1369
12	1206	12	250	60	1062	60	
13	1289	13	268				
14	1372	14	285				
15	1455	15	302				
16	1537	16	319				
17	1621	17	336				
18	1704	18	353				

Note. This table may be used to get approximate temperatures. For accurate work each individual couple must be calibrated.

Optical and Radiation Pyrometers may be used to measure any temperature above 700° C. The indications of a radiation pyrometer depend on the whole spectrum of radiant energy emitted by a body, while the optical pyrometer indicates temperature by measuring the brightness of the light of one particular wavelength emitted by an incandescent body.

Radiation and optical pyrometers are usually calibrated to read correctly when sighted into a uniformly heated hollow enclosure such as a furnace with an opening small in comparison with the walls of the furnace. Under such conditions, called "black body" conditions, the pyrometer receives not only the energy radiated directly by the part of the wall or other surface sighted upon, but also the energy which is emitted by other portions of the furnace and reflected by this surface into the pyrometer. Surfaces which have a high radiating power have a low reflecting power, the lack of energy directly emitted being in all cases exactly compensated for by the energy reflected. Under black body conditions, therefore, the energy received is independent of the radiating characteristics of the surfaces and depends only on the temperature. When an optical or radiation pyrometer is sighted upon a surface in the open, it receives only the light directly emitted and consequently always reads low. The temperature indicated by a pyrometer when sighted upon a surface in the open, the pyrometer being calibrated to read correctly when sighted into a black body, is called the " apparent " or " black body " temperature of the object viewed. The difference between the true and apparent temperatures for any surface is between four and five times as great in the case of measurements with a radiation pyrometer. This fact renders the radiation pyrometer unsuitable for the measurement of

Rate °C. per hr. Cone No.	20° C. End point	150° C. End point	20° C. Bending interval	150° C. Bending interval	20° C. Cone interval	150° C. Cone interval
022	585° C.	605° C.	45° C.	55° C.	10° C.	10° C.
021	595	651	45	45	30	35
020	625	650	30	25	5	10
019	630	660	30	20	40	60
018	670	720	30	30	50	50
017	720	770	30	30	15	25
016	735	795	35	55	35	10
015	770	805	30	45	25	25
014	795	830	45	40	30	30
013	825	860	45	50	15	15
012	840	875	50	85	35	30
011	875	905	65	65	15	−10
010	890	895	30	25	40	35
09	930	930	35	40	15	20
08	945	950	55	60	30	40
07	975	990	35	50	30	25
06	1005	1015	25	35	25	25
05	1030	1040	30	30	20	20
04	1050	1060	40	40	30	55
03	1080	1115	40	35	15	10
02	1095	1125	35	35	15	20
01	1110	1145	50	45	15	15
1	1125	1160	30	45	10	5
2	1135	1165	30	45	10	5
3	1145	1170	30	40	20	20
4	1165	1190	40	35	15	15
5	1180	1205	40	50	10	25
6	1190	1230	40	35	20	20
7	1210	1250	40	60	15	10
8	1225	1260	45	55	25	25
9	1250	1285	65	115	10	20
10	1260	1305	40	95	25	20
11	1285	1325	70	80	25	10
12	1310	1335	80	45	40	15
13	1350	1350	70	55	40	50
14	1390	1400	100	70	20	35
15	1410	1435	85	115	40	30
16	1450	1465	70	125	15	10
17	1465	1475	50–75 ?	125	20	15
18	1485	1490	90	85	30	30
19	1515	1520	100	70	5	10
20	1520	1530	....	60	....	50
23		1580		30	....	15
26	In	1595		10	....	10
27	Arsem	1605		15	....	10
28	furnace	1615	Cones	10	....	25
29	at	1640	23 to 38	30	....	10
30	600° C.	1650	heated	25	....	30
31	per	1680	at	25	....	20
32	hour	1700	100° C.	15	....	45
33		1745	per	30	....	15
34	1755	1760	hour	15	....	25
35	1755	1785		15	....	25
36	1810	1810		25	....	10
37	1830	1820		5	....	15
38	1850	1835		15		
39	1865					
40	1885					
41	1970					
42	2015					

the temperature of bodies in the open. The radiation pyrometer finds its best application under black body conditions where a fixed installation is possible and an automatic record is desired. When used as a portable instrument it is not satisfactory; the optical pyrometer should be used in such cases. In the table are noted the relations between the true and apparent temperatures of various surfaces for optical and radiation pyrometry.

True Temperature Versus Apparent Temperature Measured by Optical Pyrometers

Using Red Light ($\lambda = 0.65\ \mu$), When Sighted upon the Materials in the Open

Observed temperature, degrees Centigrade	True temperature, degrees Centigrade				
	Molten copper	Molten iron	Solid iron oxide	Solid nickel oxide	Nichrome or chromel
700	....	....	700	701	702
800	....	....	801	802	804
900	....	....	902	904	906
950	1088	....	953	955	958
1000	1150	....	1004	1007	1010
1050	1213	....	1055	1058	1063
1100	1277	1183	1106	1110	1116
1150	1341	1239	1158	1162	1170
1200	1405	1296	1210	1215	1224
1250	1470	1353	....	1267	
1300	1536	1410	....	1320	
1400	....	1525			
1500	....	1641			
1600	....	1758			
1700	....	1876			
1750	....	1935			

True Temperature Versus Apparent Temperature Measured by Radiation Pyrometers

When Sighted upon the Materials in the Open

Observed temperature, degrees Centigrade	True temperature, degrees Centigrade				
	Molten iron	Molten copper	Copper oxide	Iron oxide	Nickel oxide
600	....	1130	720	630	710
650	....	1210	775	....	755
700	....	1290	830	735	800
750	....	....	890	....	845
800	1200	....	945	840	895
850	1270	....	1000	....	940
900	1340	....	1060	945	985
950	1410	....	1115	....	1030
1000	1475	....	1170	1050	1075
1050	1550	....	....	....	1120
1100	1610	....	....	1155	1165
1150	1680	....	....	....	1210
1200	1750	....	....	1260	1255

Melting Points of the Chemical Elements *

Element	C.	F.
Helium	<−271	<−456
Hydrogen	−259	−434
Neon	−253?	−423
Fluorine	−223	−369
Oxygen	−218	−360
Nitrogen	−210	−346
Argon	−188	−306
Krypton	−169	−272
Xenon	−140	−220
Chlorine	−101.5	−150.7
Mercury	− 38.87	− 37.97
Bromine	− 7.3	+ 18.9
Caesium	+ 26	79
Gallium	30	86
Rubidium	38	100
Phosphorus	44	111
Potassium	62.3	144.1
Sodium	97.5	207.5
Iodine	113.5	236.3
Sulfur S_I	112.8	235.0
Sulfur S_{II}	119.2	246.6
Sulfur S_{III}	106.8	224.2
Indium	155	311
Lithium	186	367
Selenium	217–220	423–428
Tin	231.9	449.4
Bismuth	271	520
Thallium	302	576
Cadmium	320.9	609.6
Lead	327.4	621.3
Zinc	419.4	786.9
Tellurium	452	846
Antimony	630.0	1166.0
Cerium	640	1184
Magnesium	651	1204
Aluminum	660	1220
Radium	700	1292
Calcium	810	1490
Lanthanum	810 ?	1490
Strontium	> Ca < Ba?	....

Element	C.	F.
Neodymium	840 ?	1544
Arsenic	850	1562
Barium	850	1562
Praseodymium	940	1724
Germanium	958	1756
Silver	960.5	1760.9
Gold	1063.0	1945.5
Copper	1083.0	1981.4
Manganese	1230	2246
Beryllium (Glucinum)	1280	2336
Samarium	1300–1400	2370–2550
Scandium	?	....
Silicon	1420	2588
Nickel	1452	2646
Cobalt	1480	2696
Yttrium	1490	2714
Iron	1530	2786
Palladium	1550	2822
Chromium	1615	2939
Zirconium	1700 ?	3090
Columbium (Niobium)	1700 ?	3090
Thorium	>1700 <Mo	>3090 <Mo
Vanadium	1720	3128
Platinum	1755	3191
Ytterbium	?	....
Titanium	1800	3272
Uranium	<1850	<3360
Rhodium	1950	3542
Boron	2200–2500?	4000–4500
Iridium	2350 ?	4260
Ruthenium	2450 ?	4440
Molybdenum	2550	4620
Osmium	2700	4890
Tantalum	2900	5250
Tungsten	3400	6152
Carbon	> 3600	> 6500

* From U. S. Bureau of Standards Circular 35, 4th edition.

11. Heat

Heat is defined as energy in the process of transfer from one body to another by a thermal process, i.e., by radiation, convection, or conduction.

The Gram-Calorie is a unit of heat, being the heat per degree centigrade required to raise the temperature of 1 gram of water at some specified temperature. The 15° calorie, the 20° calorie, and the mean calorie (0° to 100° C.) are in common use.

The B. t. u. (British thermal unit) is the quantity of heat per degree Fahrenheit required to raise the temperature of one pound of water, at some specified temperature.

The Specific Heat of a substance is the number of calories per degree centigrade required to raise the temperature of one gram of the substance.

In the case of gases, this depends largely upon how the pressure and volume change during the heating. Two special values are of importance in the case of a gas: C_p, the specific heat at constant pressure, and C_v, the specific heat at constant volume.

The Latent Heat of Fusion is the amount of heat that must be added to unit mass of a solid substance to change it to a liquid without any change in temperature.

The Latent Heat of Vaporization is the amount of heat required to change unit mass of a substance from a liquid to saturated vapor without any change in temperature.

12. Heat Transfer

By Radiation. The energy radiated per unit area per unit time from a substance is approximately proportional to the fourth power of the absolute temperature. (For a black body the proportionality is exact.) The net loss per unit of time depends on the surroundings, being the difference between the energy radiated and the energy received by radiation from surrounding bodies.

By Convection. Transfer of heat by convection occurs in liquids and gases, the transfer being accomplished by the motion of the fluid from a locality where it receives heat to a locality where it gives up heat.

By Conduction. Transfer of heat by conduction occurs through continuous materials in which energy is transferred directly between adjacent molecular aggregates without mass motion of the materials.

When heat flows in a solid, homogeneous body in only one direction, the time rate of heat flow, after a steady state has been reached, across a given area will be proportional to the temperature drop per unit length in the direction of the flow of heat. The factor of proportionality is, by definition, the thermal conductivity.

If $\frac{dQ}{dt}$ = the time rate of heat flow;

A = the area measured perpendicular to the direction of flow;

$\frac{dT}{dx}$ = the temperature gradient;

and K = the thermal conductivity;

$$\frac{dQ}{dt} = KA\frac{dT}{dx}$$

for a steady state where $\frac{dT}{dt} = 0$ at all points.

13. Thermal Insulation

The heat conductivity of a material is a measure of the insulating value of that material; the lower the conductivity, the greater the insulating value. Obviously the best conductor of heat is the poorest heat insulator. The customary measure of the conductivity K of a material is the amount of heat in B.t.u. (British thermal units) which will flow in one hour through a layer of the material one square foot in area when the temperature difference between the surfaces of the layer is 1° F., per inch of thickness.

The thermal conductivity K is a property of a material itself, and does not depend upon the size or shape of a particular piece of the material in question. The insulating value of a layer of material of any thickness

T depends upon the thickness as well as upon the thermal conductivity of the material of which the layer is composed. In general the insulating value, i.e., the resistance to heat flow, R, of a flat layer of any material is equal to the thickness of the layer divided by the thermal conductivity of the material of which the layer is composed.

The same principles are involved in what is sometimes called "insulation against cold" as in "insulation against heat." The only difference is the point of view, with regard to the direction of heat flow. The insulation of a building against the outside cold is merely a question of reducing the heat flow from the inside to the outside. The insulating value of a material depends somewhat upon the temperature of the material, but this effect is small over the small temperature ranges occurring in buildings. The same layer of material would be somewhat more effective as house insulation than as oven insulation. . . .

The first of the two tables following gives the thermal conductivities and weights per cubic foot of various materials which have been tested at the Bureau of Standards. The weights do not include the paper or other coverings confining loose materials. The second table is a more practical table for general use. It gives the conductances of commercial thicknesses of various materials, and in another column gives the insulating value of the commercial thicknesses. The insulating value is merely the reciprocal of the conductance, in accordance with the definition of insulating value previously stated, namely, the thickness of the material divided by its thermal conductivity. The weights per square foot given include the surface coverings, if any are present. In all cases the tabulated values are the average of tests on a number of samples of each material. Since materials of this general class are not very uniform, differences in conductivity amounting to one or two in the last figure have no particular significance.

The figures in both tables correspond to an average temperature of 90° F., e.g., 110° F. on one surface of the insulating layer, and 70° F. on the other. At lower temperatures, corresponding more nearly to actual conditions in practice, the conductivities are a few per cent lower. For practical comparative purposes, the values at 90° F. are sufficiently good.

Thermal Conductivity of Materials *

D = Weight in pounds per cubic foot.
K = Thermal conductivity in B.t.u. per hour, square foot, and temperature gradient of 1° F. per inch of thickness. The lower the conductivity, the greater the insulating values.

Soft Flexible Materials in Sheet Form

		D	K
Dry zero	Kapok between burlap or paper	1.0	0.24
		2.0	0.25
Cabots quilt	Eel grass between kraft paper	3.4	0.25
		4.6	0.26
Hair felt	Felted cattle hair	11.0	0.26
		13.0	0.26
Balsam wool	Chemically treated wood fiber	2.2	0.27
Hairinsul	75% hair, 25% jute	6.3	0.27
	50% hair, 50% jute	6.1	0.26
Linofelt	Flax fibers between paper	4.9	0.28
Thermofelt	Jute and asbestos fibers, felted	10.0	0.37
	Hair and asbestos fibers, felted	7.8	0.28

* From Bureau of Standards LC 227.

Thermal Conductivity of Materials—Continued

		D	K
	Loose Materials		
Rock wool	Fibrous material made from rock; also made in sheet form, felted and confined with wire netting	6.0	0.26
		10.0	0.27
		14.0	0.28
		18.0	0.29
Glass wool	Pyrex glass, curled	4.0	0.29
		10.0	0.29
Sil-O-Cel	Powdered diatomaceous earth	10.6	0.31
Regranulated Cork	Fine particles	9.4	0.30
	About 3/16 in. particles	8.1	0.31
Thermofil	Gypsum in powdered form	26.	0.52
		34.	0.60
Sawdust	Various	12.0	0.41
	Redwood	10.9	0.42
Shavings	Various, from planer	8.8	0.41
Charcoal	From maple, beech and birch, coarse	13.2	0.36
	6-mesh	15.2	0.37
	20-mesh	19.2	0.39
	Semi-Flexible Materials in Sheet Form		
Flaxlinum	Flax fiber	13.0	0.31
Fibrofelt	Flax and rye fiber	13.6	0.32
	Semi-Rigid Materials in Board Form		
Corkboard	No added binder; very low density	6.4	0.25
	low density	7.0	0.27
	medium density	10.6	0.30
	high density	14.0	0.34
Eureka	Corkboard with asphaltic binder	14.5	0.32
Rock cork	Rock wool block with binder; also called "Tu-cork"	16.7	0.37
Lith	Board containing rock wool, flax and straw pulp	14.3	0.40
	Stiff Fibrous Materials in Sheet Form		
Insulite	Wood pulp	16.2	0.34
		16.9	0.34
Celotex	Sugar cane fiber	13.2	0.34
		14.8	0.34
	Cellular Gypsum		
Insulex or Pyrocell		8	0.35
		12	0.44
		18	0.59
		24	0.77
		30	1.00
	Woods (Across Grain)		
Balsa		7.3	0.33
		8.8	0.38
		20	0.58
Cypress		29	0.67
White pine		32	0.78
Mahogany		34	0.90
Virginia pine		34	0.98
Oak		38	1.02
Maple		44	1.10

Conductance and Insulating Value of Sheet Materials in Thicknesses as Sold *

W = Weight in pounds per square foot;
T = Thickness in inches;
$K \div T = C$ = Conductance in B.t.u. of a given thickness, per hour, per square foot, and per degree F.;
$R = 1/C$ = Resistance or insulating value.

Soft Flexible Materials

		W	T	C	R
Cabots quilt	Single ply	0.14	0.35	0.72	1.39
	Double ply	0.18	0.48	0.54	1.85
	Triple ply	0.31	0.67	0.39	2.56
Balsam wool	1/2-in. house insulation; smooth paper	0.16	0.55	0.48	2.10
	1/2-in. refrigerator insulation, creped paper	0.24	0.66	0.41	2.47
	1-in. refrigerator insulation, creped paper	0.32	1.13	0.25	4.08
Harinsul	75% hair, 25% jute	0.46	0.55	0.49	2.05
	50% hair, 50% jute	0.42	0.51	0.51	1.96
Carinsul	Hairfelt between asbestos paper	0.58	0.60	0.46	2.19
Salamander	Hairfelt paper, asbestos and cheesecloth; paper between plys: 2-ply	0.54	0.61	0.42	2.40
	3-ply	0.69	0.70	0.36	2.75
Thermofelt	Jute and asbestos	0.42	0.51	0.72	1.39
	Hair and asbestos	0.42	0.63	0.45	2.22
Nycinsul	Hair felt between cheesecloth, the latter treated with magnesite solution	0.97	0.45	0.82	1.21
Linofelt	1/2 in.	0.41	0.67	0.42	2.40
Resisto	Similar to Nycinsul: Single	0.56	0.40	0.75	1.34
	Double	0.77	0.62	0.49	2.05
Semi-Flexible Materials					
Flaxlinum		0.61	0.56	0.56	1.80
Fibrofelt		0.66	0.58	0.56	1.80
Stiff Fibrous Materials					
Insulite	Wall board	0.66	0.49	0.69	1.46
	Insulation board	0.80	0.56	0.60	1.67
Celotex	Building board	0.58	0.47	0.72	1.38
	Railroad insulation board	0.64	0.58	0.59	1.71
Plaster and Wall Boards					
Gyplap	Gypsum between layers of heavy paper	2.23	0.50	2.6	0.38
Sheet rock	Gypsum mixed with sawdust between layers of heavy paper	1.97	0.39	3.6	0.27

* From Bureau of Standards LC 227.

Thermal Conductivity of Materials

(Taken from various sources)

(Report of A.S.R.E. Insulation Committee, Revised to 1924)

D = Weight in pounds per cubic foot;
t = Temperature of specimen in degrees F.;
K = Thermal conductivity in B.t.u. per hour, square foot, and temperature gradient of 1° F. per inch.

Materials	D	t	K	Authority
Asbestos, sheet	48	....	0.27–0.31	Willard and Lichty
Asbestos paper in layers with organic binder	31	86	0.49	Bureau of Standards
Asbestos paper	......	....	1.25	Lees and Chorlton
Asbestos fire-felt	7.2	50–392	0.31	Randolph
		50–572	0.46	
		50–752	0.55	
Mineral wool	12–21	86	0.26–0.30	Bureau of Standards
Mineral wool	27	50–932	0.48	Randolph
Magnesia	......	....	0.46–1.31	Hutton–Blard
Magnesia, 85% and asbestos 15% (rigid)	19	86	0.50	Bureau of Standards
Quartz sand, fine	......	....	0.38	Forbes
Quartz	......	....	1.05	Hutton–Blard
Magnesium carbonate (safe temperature about 572° F.)	28	212	0.67	Skinner
		392	0.73	
		572	0.73	
Paraffin	56	86	1.60	Bureau of Standards
Boiler clinkers (dry)	47	68	1.13	Hencky
Asphalt roofing	55	86	0.70	Bureau of Standards
Gravel, loose, dry	116	68	2.58	Groeber
Plaster of paris	......	....	2.03	Lees and Chorlton
Plaster of paris powder	......	....	7.55	Lees and Chorlton
Soil, dry	......	....	1.0	Lees and Chorlton
Soil, wet	......	....	4.6	Lees and Chorlton
Asphalt for streets	132	68	4.8	Poensgen
Cinder concrete	54	68	2.0	Hencky
Concrete (1 portland cement, 2 sand, 2 gravel)	136	73	5.3	Groeber
Slate	......	201	10.4	Lees and Chorlton
Glass, soda	162	68	5.0	Barratt
		212	5.3	
Fire Brick (Schamotte)	107	50	3.95	Poensgen
	....	77	4.03	
	....	104	4.11	
	....	140	4.27	
	....	392	4.11	Van Rinsum
	....	1112	5.32	
	....	1832	6.62	
Silica brick	.....	392	4.5	Van Rinsum
		1112	7.1	
		1832	9.6	
Magnesite brick	.....	392	9.3	Van Rinsum
		1112	10.4	
		1832	11.5	
Gas retort brick	....	212–1963	11.0	Wologdine
Fire clay brick	.....	257	9.28	Wologdine
		2164	15.7	

Thermal Conductivity of Materials—Continued

Material	D	t	K	Authority
Carborundum brick	.....	302 2128	9.28 78.4	Wologdine
Graphite brick	....	572–1228	69.7	Wologdine
Gas carbon	87	68 212	24.7 27.6	Barratt
Chalk	....	....	6.4	H–L–D
Boiler scale, No. 1	....	124–180	9.1	Ernst
Boiler scale, No. 2	....	98–167	22.3	Ernst
Serpentine	....	....	12.8	H–L–D
Bricks, very porous, dry	44	68	1.21	Hencky
Same, masonry, allowing for joints	....	68	2.0	Hencky
Hollow tile, dry, laid flat	....	68	1.5	Knoblauch
Limestone, fine grained, dry	104	77	4.8	Poensgen
Limestone, coarse grained, dry	124	77	6.45	Poensgen
Marble	....	....	14–16	H–L–D
Granite	....	212–932	12–28	Poole
Steel wool, No. 2	9.5 6.3 4.7	50–212 50–212 50–212	0.56 0.61 0.63	Randolph
Mercury	.....	32 122	43 55	H. F. Weber
Antimony	.....	32 212	128 115	Lorenz
Lead	.....	−297 10	314 267	Macchia
		212	222	Lorenz
Steel, Bessemer	....	59	280	Kirchhoff and Hanseman
Steel, puddled	....	59	399	
Nickel, 99%	.....	−256 64.4	374 406	Lees
Tin	.....	32 212	443 413	Lorenz
Bronze (85.7% copper, 7.15% zinc, 6.39% tin, 0.58% nickel	....	64.4 212	414 492	Jaeger and Diesselhorst
Brass, yellow	.....	32 212	593 738	Lorenz
Brass, red	.....	32 212	714 820	Lorenz
Aluminum	.....	32 212	997 1050	Lorenz
Magnesium	....	32–212	1091	Lorenz
Copper, pure	.....	64 212	2588 2547	Jaeger and Diesselhorst
Silver, 999.8 fine	.....	32	3182	H. F. Weber
		64.4 212	2920 2880	Jaeger and Diesselhorst

The insulating value, except at elevated temperatures of an ordinary enclosed air space of width greater than 1/2 in. is equivalent to approximately 1/3 in. of good insulating material. At high temperatures air spaces have but little insulating value because of the rapid increase of radiation with increase in temperature.

Factors

Thermal Conductivity

	Cal. sec.$^{-1}$ cm.$^{-1}$ deg. C.$^{-1}$	Watt cm.$^{-1}$ deg. C.$^{-1}$	Cal. hr.$^{-1}$ cm.$^{-1}$ deg. C.$^{-1}$	B.t.u. hr.$^{-1}$ ft.$^{-2}$ in. deg. F.$^{-1}$	B.t.u. day^{-1} ft.$^{-2}$ in. deg. F.$^{-1}$
1 cal. sec.$^{-1}$ cm.$^{-1}$ deg. C.$^{-1}$ .... =	1	4.183	3600	2903	69670
1 watt cm.$^{-1}$ deg. C.$^{-1}$ .... =	0.2391	1	860.6	694.0	16655
1 cal. hr.$^{-1}$ cm.$^{-1}$ deg. C.$^{-1}$ .... =	0.0002778	0.001162	1	0.8064	19.35
1 B.t.u. hr.$^{-1}$ ft.$^{-2}$ in. deg. F.$^{-1}$...... =	0.0003445	0.001441	1.240	1	24
1 B.t.u. day^{-1} ft.$^{-2}$ in. deg. F.$^{-1}$...... =	0.00001435	0.00006004	0.05167	0.04167	1

Thermal Conductance

	Cal. sec.$^{-1}$ cm.$^{-2}$ deg. C.$^{-1}$	Watt cm.$^{-2}$ deg. C.$^{-1}$	Cal. hr.$^{-1}$ cm.$^{-2}$ deg. C.$^{-1}$	B.t.u. hr.$^{-1}$ ft.$^{-2}$ deg. F.$^{-1}$	B.t.u. day^{-1} ft.$^{-2}$ deg. F.$^{-1}$
1 cal. sec.$^{2-}$ cm.$^{-2}$ deg. C.$^{-1}$........ =	1	4.183	3600	7373	176960
1 watt cm.$^{-2}$ deg. C.$^{-1}$ .... =	0.2391	1	860.6	1763	42304
1 cal. hr.$^{-1}$ cm.$^{-2}$ deg. C.$^{-1}$........ =	0.0002778	0.001162	1	2.048	49.16
1 B.t.u. hr.$^{-1}$ ft.$^{-2}$ deg. F.$^{-1}$........ =	0.0001356	0.0005673	0.4882	1	24
1 B.t.u. day^{-1} ft.$^{-2}$ deg. F.$^{-1}$........ =	0.000005651	0.00002364	0.02034	0.04167	1

Heat Flow

	Cal. sec.$^{-1}$ cm.$^{-2}$	Watt cm.$^{-2}$	Cal. hr.$^{-1}$ cm.$^{-2}$	B.t.u. hr.$^{-1}$ ft.$^{-2}$	B.t.u. day^{-1} ft.$^{-2}$
1 cal. sec.$^{-1}$ cm.$^{-2}$ =	1	4.183	3600	13272	318530
1 watt cm.$^{-2}$ =	0.2391	1	860.6	3173	76147
1 cal. hr.$^{-1}$ cm.$^{-2}$ =	0.0002778	0.001162	1	3.687	88.48
1 B.t.u. hr.$^{-1}$ ft.$^{-2}$ =	0.00007534	0.0003152	0.2712	1	24
1 B.t.u. day ft.$^{-2}$ =	0.000003139	0.00001313	0.01130	0.04167	1

14. Changes Due to Heating

Heat and Expansion. Although most substances expand as they are warmed, or if hindered from expanding freely will exert pressure upon their containers, water between 0° and 4° C., quartz-glass below −84° C., and some other substances are exceptions to the general rule. When heating causes a body to change its state of aggregation (melt, vaporize, etc.) there is also change in volume. This may be either an expansion or a contraction, and may be accompanied by little or no change of temperature until after the process is completed.

Thermal Hysteresis. When a solid has its temperature changed, especially by rapid cooling, slow changes in its dimensions continue long after it has attained the same temperature throughout its entire extent. These changes are often so minute as to require extremely delicate means for detection; but in such cases as glass they are very marked. The gradual contraction of thermometer bulbs has been observed to continue for over a quarter of a century, causing the zero reading to rise. The zero point of even a good thermometer may be shifted many degrees in a few minutes by heating to several hundred degrees. (Bul. Bureau Standards, vol. 2, 1906, p. 189.) Most of this after-effect disappears in a few hours or days, and the process is considerably accelerated by prolonged heating at a high temperature followed by slow annealing.

Laws of Change. As long as a physically homogeneous body is not subjected to treatment that causes more or less permanent alterations in its structure, its volume appears to be a definite function of its temperature and pressure, when these are uniform throughout its entire extent. The amount of expansion under constant pressure caused by a given change of temperature depends upon the material of the body. Solids and liquids differ considerably among themselves and follow no regular law. Gases, on the other hand, show remarkable uniformity, all expanding about 100/273 = 0.367 of their volumes at 0° C. when warmed from this temperature to 100° C. (Law of Gay-Lussac.) Besides, all gases expand very nearly alike throughout great ranges of temperature; and the particular pressure under which a gas is warmed makes very little difference.

Coefficients of Expansion. If the volume at $t°$ is V and at 0° is V_0, then the rate at which unit volume changes under constant pressure with change of temperature is $(dV/dt)/V$ which is called the **true** coefficient of cubical expansion at $t°$; while $(V - V_0)/V_0t = \alpha$ is called the **mean-zero** coefficient from 0° to $t°$. Similarly, if L represents the length of a solid, then $(L - L_0)/L_0t = \beta$ is the mean-zero coefficient of **linear** expansion from 0° to $t°$. These definitions give $V = V_0 (1 + \alpha t)$ and $L = L_0 (1 + \beta t)$. Usually α and β are so small that one may assume $\alpha = 3\beta$.

Values of the coefficient of linear expansion for solids are given above in Art. 7. The coefficient of cubical expansion is three times the linear. Art. 7 gives values for the Centigrade degree.

Dalton's Law. In a mixture of several gases or vapors that do not react upon each other chemically the pressure exerted is approximately the same as if each constituent exerted the full pressure it would exert if it alone filled the entire volume; that is to say, if volumes V_1, V_2, V_3, . . . , of different gases and vapors, all under the same pressure p and at the same temperature, when mixed fill a volume V, then $P = p_1 + p_2 + p_3 + \ldots$, approximately, where $p_1 = pV_1/V$; $p_2 = pV_2/V$; etc. This relation is more accurately fulfilled the farther all the vapors are from the conditions under which they liquefy and the less their liquids dissolve one another. p_1, p_2, etc., are spoken of as the partial pressures exerted by the different components.

15. Sound and Acoustics

Velocity of Sound in air at 0° F. is 1050 ft. per sec., and it increases 1.1 ft. per sec. for each 1° F. rise in temperature. A few values are

Temperature, F.	−10°	+10°	+30°	+50°	+70°	+90°
Velocity, ft. per sec.	1039	1061	1083	1105	1127	1149

At 0° C. the velocity of sound in dry air is **331 meters per sec.** and it increases 0.609

meters per sec. for a rise of each degree of the centigrade scale. At +30° C. the velocity is about 349 meters per sec

The velocity of sound in air is independent of the barometric pressure, and hence is the same on high mountains as at sea level, the temperature being the same in both cases. It is said to be little influenced by fog or rain but materially altered by aqueous vapor and also by wind.

A formula for velocity of sound in any solid or liquid is $v = \sqrt{Eg/w}$, where E = modulus of elasticity; w = weight per cubic unit of the substance; and g = acceleration of gravity. Mean values of v are as follows in ft. per sec.:

Fresh water	4 700	Copper	11 700
Salt water	4 765	Cast iron	12 400
Timber	13 200	Wrought iron	15 500
Silver	8 550	Steel	17 200

Stress is transmitted through a solid with the same velocity as sound.

The velocities of sound in different gases at the same pressure and temperature vary inversely as the square roots of their densities. Thus, using densities of air and hydrogen as given on p. 254, the velocity in hydrogen at 0° F. is 3980 ft. per sec.

In open air the intensity of sound varies inversely as the square of the distance of the source from the ear. For air in a pipe this law does not hold, but the intensity remains the same for considerable distances, being diminished only by internal frictional resistances. The intensity also depends on the medium; the ticking of a watch is heard 2-1/2 times as far under water as in air. It also depends on the volume of sound produced by the sonorous body; the explosion of a volcano has been heard at a distance of 300 miles.

When a sonorous body approaches the ear the tone perceived is higher than the true, and when it recedes the tone is lower. This is because high tones are produced by short waves, and in the case of approach to the ear, the sound waves are crowded together or shortened.

Architectural Acoustics. Acoustical properties of rooms are readily adjusted by the use of materials of proper absorbing values. The optimum values to produce satisfactory reverberation will depend upon the purpose for which the room is to be used. Excessive absorption gives the open or out-of-doors type of acoustical defect.

In new or remodeling operations proper wall and ceiling designs may be used to enhance or reduce the acoustical values as desired to a more satisfactory condition.

The following paragraphs and tables are from Bureau of Standards Circular 300, Architectural Acoutics (Available from the Supt. of Documents, Washington, D. C., at 5 cents a copy).

Different materials differ considerably in their absorbing powers for sound. The most complete absorber known is an open window. It is theoretically possible that a small amount of sound may be sent back by diffraction from the edges of the window, but this quantity is so small that it is permissible to say that an open window is a perfect absorber. The next most perfect absorber of sound is probably hair felt, which may absorb, perhaps, half as much sound as an equal area of open window. In other words, if it may be said that an open window absorbs (or transmits) all the sound that falls upon it, its coefficient of absorption is unity, while that of the sample of hair felt quoted would be 0.50.

In like manner, every substance may be said to have its own absorption coefficient. This constant was measured by Sabine for a number of common materials, and later workers have extended the list.

The first of the following tables gives the absorption coefficients for a number of substances. Strictly speaking, these coefficients will vary somewhat with the frequency of the incident sound, and the values given are for a frequency of 512 (Watson).

In the second table are given values of the total absorption of individual objects, and in the third table absorption coefficients of various substances for different sound frequencies, as determined at the Bureau of Standards.

Sound Absorption Coefficients

Akoustolith (artificial stone)	0.36
Brick wall, 18 in. thick	.032
Brick wall, painted	.017
Brick, set in portland cément	.025
Carpets, unlined	.15
Carpets, lined	.20
Carpets, heavy, with lining	.25
Carpet rugs	.20
Celotex, 1/2 in. thick	.31
Cheesecloth	.019
Cocoa matting, lined	.17
Concrete	.015
Cork tile	.03
Cretonne cloth	.15
Curtains, chenille	.23
Curtains. in heavy folds	0.5 to 1.0
Flax, 1 in. thick, with unpainted membrane	.55
Glass, single thickness	.027
Hair felt, 1 in. thick, with unpainted membrane	.55
Hair felt, 1 in. thick, with painted membrane	.25 to .45
Hair felt, 2 in. thick, with unpainted membrane	.70
Hair felt, 2 in. thick, with painted membrane	.40 to .60
Insulite, 1/2 in. thick	.31
Linoleum	.03
Marble	.01
Oil paintings, including frames	.28
Open windows	1.00
Oriental rugs, extra heavy	.29
Plaster on wood lath	.034
Plaster on wire lath	.033
Plaster on tile	.025
Stage opening, depending on stage furnishing	.25 to .40
Varnished wood	.03
Ventilators (50% open space)	.50
Wood sheathing	.061
Wood, varnished	.03

Total Absorption by Individual Objects *

Audience	per person	4.7
Church pews	per seat	.2
House plants	per cubic foot	.0031
Seats, upholstered, depending on material and lining	per seat	1.0 to 2.5
Seat cushions, cotton, covered with corduroy	per seat	2.16
Seat cushions, hair covered with canvas and light damask	per seat	2.27
Settees, upholstered in hair and leather, seat and back	per seat	3
Wood seats, for auditoriums	per seat	.100

* The values given under this heading must be regarded as relative values. For complete explanation of their significance and use reference should be made to the original article.

Absorption Coefficients of Several Materials Showing Variations with Variations in Pitch

Material	297	581	1095	2190	2890
Glass	0.021	....	0.020	0.010	....
5/8-in. celotex	.058	....	.056	.072	0.097
Celotex B	.178	....	.399	.734	.708
Sound absorbing tile No. 1	.079	....	.116	.401	.35
Same, No. 2	.078	....	.127	.201	.213
Hair felt, 1 in.	.330	....	.62	*.94	*.90
Brick	.019	....	.019	.056	.021
Pine wood	.012	0.009	.016	.016	.009
Oak wood	.011	.007	.011	.006	.005
Akoustolith	....	.301	.434	.380	.272
Brass	.021	.015	.023	.004	.001
1/2-in. insulite	.056	.102	.182	.256	.172
1/2-in. flax-li-num	....	.229	.312	.503	.336
1-in. flax-li-num	.182	.422	.456	.569	.407

* The absorption at 2190 and at 2890 cycles was measured upon a different piece of material from that used at 297 and at 1095 cycles.

The values 297, 581 etc. represent the cycles per second or pitch of the sound.

Selected Bibliography on Acoustics as related to Architecture:

1. Sabine, Collected Papers on Acoustics, Harvard University Press; 1922.
2. Watson, Acoustics of Buildings, John Wiley & Sons, New York; 1923.
3. Eckhardt, The Acoustics of Rooms, Jour. Franklin Inst.: June, 1923.
4. Swan, Architectural Acoustics, Jour. Am. Inst. Architects; December, 1919.
5. Watson, The Absorption of Sound by Materials, Bulletin 172 Engineering Experiment Station, University of Illinois.
6. Architectural Acoustics, Circular of the Bureau of Standards, No. 300.

METEOROLOGY

16. Signals of United States Weather Bureau

Flag Signal for cold wave warning is shown in Fig. 8. Color Key

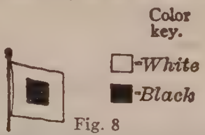

Fig. 8

Storm and Hurricane Warnings are given by the flags in Fig. 9.

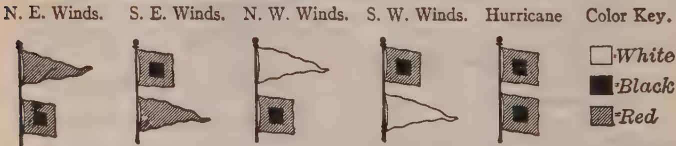

Fig. 9. Storm and Hurricane Flags

A red flag with a black center indicates that a storm of marked violence is expected. The pennants displayed with the flags indicate the direction of the wind; red, easterly (from northeast to south); white, westerly (from southwest to north). The pennant above the flag indicates that the wind is expected to blow from the northerly quadrants; below from the southerly quadrants. By night a red light indicates easterly winds, and a white light below a red light, westerly winds.

Two red flags with black centers, displayed one above the other, indicate the expected approach of a tropical hurricane, or one of those extremely severe and dangerous storms which occasionally move across the Lakes and northern Atlantic coast. No night hurricane warnings are displayed

Wind Pressure on Flags. Tests at the U. S. Navy Yard, Washington, D. C., using the largest flags that could be handled in the wind tunnel, form the basis for an empirical formula for determining the pressure of wind on flags, for use in designing flagpoles. Two sizes of flags were used—one 3 × 5-1/2 ft. and the other 2-1/2 × 4-1/4 ft.—and velocities varying from 20 to 60 miles per hour were applied. The following formula was found to represent the results, the constant varying but slightly with size of flags, being less for the larger flag:

$$R = 0.0003 A V^{1.9}$$

in which R is the resistance in pounds, A is the area of flag in square feet, and V is the velocity of wind in miles per hour. This formula is for steady wind pressure. It was not found practicable to measure the forces produced by wind gusts.

17. Local Weather Predictions

The wind and barometer indications for the United States are generally summarized in the following table (E. B. Garriott, U. S. Dept. Agriculture).

Wind direction	Barometer reduced to sea level	Character of weather indicated
SW to NW	30.10 to 30.20 and steady.......	Fair, with slight temperature changes, for 1 to 2 days
SW to NW	30.10 to 30.20 and rising rapidly	Fair, followed within 2 days by rain
SW to NW	30.10 to 30.20 and falling slowly	Warmer, with rain within 24 to 36 hours
SW to NW	30.10 to 30.20 and falling rapidly	Warmer, with rain within 18 to 24 hours
SW to NW	30.20 and above and stationary	Continued fair, with no decided temperature change
SW to NW	30.20 and above and falling slowly	Slowing rising temperature and fair for 2 days
S to SE...	30.10 to 30.20 and falling slowly	Rain within 24 hours
S to SE...	30.10 to 30.20 and falling rapidly	Wind increasing in force, with rain within 12 to 24 hours
SE to NE.	30.10 to 30.20 and falling slowly	Rain in 12 to 18 hours
SE to NE.	30.10 to 30.20 and falling rapidly	Increasing wind, and rain within 12 hours
E to NE..	30.10 and above and falling slowly	In summer, with light winds, rain may not fall for several days. In winter, rain within 24 hours
E to NE..	30.10 and above and falling rapidly	In summer, rain probable within 12 to 24 hours. In winter, rain or snow, with increasing winds, will often set in when the barometer begins to fall and the wind sets in from the NE
SE to NE.	30.00 or below and falling slowly	Rain will continue 1 to 2 days
SE to NE.	30.00 or below and falling rapidly	Rain, with high winds, followed, within 36 hours, by clearing, and in winter by colder
S to SW...	30.00 or below and rising slowly	Clearing within a few hours, and fair for several days
S to E....	29.80 or below and falling rapidly	Severe storm imminent, followed, within 24 hours, by clearing, and in winter by colder
E to N....	29.80 or below and falling rapidly	Severe northeast gale and heavy precipitation; in winter, heavy snow, followed by a cold wave
Going to W	29.80 or below and rising rapidly	Clearing and colder

As a rule winds from the east quadrants and falling barometer indicate foul weather; and winds shifting to the west quadrants indicate clearing and fair weather. The rapidity of the storm's approach and its intensity are indicated by the rate and the amount in the fall of the barometer.

The indications afforded by the wind and the barometer are the best guides for determining future weather conditions. As low barometer readings usually attend stormy weather, and high barometer readings are generally associated with clearing or fair weather, it follows that falling barometer indicates precipitation and wind, and rising barometer, fair weather or the approach of fair weather. As atmospheric areas of high barometer and areas of low barometer are, by natural laws, caused to assume circular or oval forms, the wind directions with reference to areas of low barometer are spirally and contraclockwise inward toward the region of lowest atmospheric pressure, as indicated by readings of the barometer. The areas of high barometer, on the contrary, show winds flowing spirally clockwise outward from the region of highest barometric pressure.

The wind directions thus produced give rise to, and are responsible for, all local weather signs. The south winds bring warmth, the north winds cold, the east winds, in the middle latitudes, indicate the approach from the westward of a low barometer, or storm, area, and the west winds show that the storm area has passed to the eastward. The indications of the barometer generally forerun the shifts of the wind. This much is shown by local observations.

During the colder months, when the land temperatures are below the water temperatures of the ocean, precipitation will begin along the seaboards when the wind shifts and blows steadily from the water over the land without regard to the height of the barometer. In such cases the moisture in the warm ocean winds is condensed by the cold of the continental area. During the summer months, on the contrary, the onshore winds are not necessarily rain winds, for the reason that they are cooler than the land surfaces and their capacity for moisture is increased by the warmth that is communicated to them by the land surface. In such cases thunderstorms commonly occur when the ocean winds are intercepted by mountain ranges or peaks. If, however, the easterly winds of summer increase in force, with falling barometer, the approach of an area of low barometric pressure from the west is indicated and rain will follow within a day or two.

From the Mississippi and Missouri valleys to the Atlantic coast, and on the Pacific coast, rain generally begins on a falling barometer, whereas in the Rocky Mountain and Plateau districts, and on the eastern Rocky Mountain slope, precipitation seldom begins until the barometer begins to rise, after a fall. This is true as regards the eastern half of the country, however, only during the colder months, and in the presence of general storms that may occur at other seasons. In the warmer months summer showers and thunderstorms usually come about the time the barometer turns from falling to rising. It is important to note the fact that during practically the entire year precipitation on the great western plains and in the mountain regions that lie between the plains and the Pacific coast districts does not begin until the center of the low barometer area has passed to the eastward or southward, and the wind has shifted to the north quadrants, with rising barometer.

18. Weather Observations

Mean Temperature. When maximum and minimum readings of temperature are taken, the mean of the two may be taken as the mean temperature of the day. The mean of all the daily means in a month is the mean temperature of the month. When a recording thermometer is used the area between the curve and an axis of abscissas is to be divided by the length of that axis in order to obtain the mean temperature for the elapsed time.

Rainfall. The rain gage used by voluntary observers consists of a cylindrical receiver 8 in. in diameter which has a funnel-shaped bottom that discharges into a tube 2.53 in. in diameter. The cross section of tube is one-tenth that of the receiver, and hence height of water in tube is ten times as

Temperature in the United States to Jan. 1, 1928

Prepared by the Weather Bureau, U. S. Department of Agriculture

States and territories	Stations	Temperature F. Mean Jan.	Mean July	Extremes Highest	Extremes Lowest
Ala.	Birmingham	45	80	106	−10
	Mobile	52	81	103	−1
Ariz.	Flagstaff	27	65	93	−25
	Phoenix	51	90	119	12
	Yuma	54	91	120	22
Ark.	Fort Smith	40	82	108	−15
	Little Rock	41	81	106	−12
Calif.	Fresno	46	82	115	17
	Los Angeles	55	70	109	28
	Sacramento	46	73	114	19
	San Diego	54	67	110	25
	San Francisco	50	58	101	29
Col.	Denver	30	72	105	−29
	Grand Junc.	24	78	105	−21
	Pueblo	30	74	104	−27
Conn.	New Haven	28	72	101	−14
D.C.	Washington	33	77	106	−15
Fla.	Jacksonville	55	82	104	10
	Key West	70	84	100	41
	Pensacola	52	81	103	7
	Tampa	60	81	98	19
Ga.	Atlanta	43	78	102	−8
	Augusta	47	81	106	3
	Savannah	51	82	105	8
Idaho	Boise	30	73	121	−28
Ill.	Cairo	35	80	106	−16
	Chicago	24	72	103	−23
	Springfield	26	76	107	−24
Ind.	Indianapolis	28	76	106	−25
Iowa	Des Moines	20	75	110	−30
	Dubuque	19	74	106	−32
	Keokuk	25	77	108	−27
Kan.	Dodge City	29	78	108	−26
	Wichita	31	79	107	−22
Ky.	Louisville	34	79	107	−20
La.	New Orleans	54	82	102	7
	Shreveport	47	83	110	−5
Me.	Portland	22	68	103	−21
Md.	Baltimore	34	77	105	−7
Mass.	Boston	28	72	104	−14
Mich.	Detroit	24	72	104	−24
	Marquette	16	65	108	−27
	Port Huron	22	69	104	−25
Minn.	Duluth	8	64	99	−41
	St. Paul	13	72	104	−41
Miss.	Vicksburg	48	81	104	−1
Mo.	Kansas City	28	78	108	−22
	St. Louis	31	79	107	−22
	Springfield	34	77	106	−29
Mont.	Helena	20	66	103	−42
Mont.	Kalispell	20	64	99	−34
	Miles City	14	73	111	−49
Neb.	N. Platte	23	73	107	−35
	Omaha	22	77	110	−32
Nev.	Winnemucca	29	71	104	−28
N. C.	Charlotte	41	78	103	−5
	Hatteras	47	78	93	8
	Wilmington	46	79	103	5
N. D.	Bismarck	8	70	108	−45
N. H.	Concord	22	68	102	−23
N. J.	Atlantic City	32	72	104	−7
	Cape May	34	73	100	−7
N. Mex.	Santa Fe	29	69	97	−13
N. Y.	Albany	23	73	104	−24
	Binghamton	24	70	99	−28
	Buffalo	25	70	95	−14
	N. Y. City	31	74	102	−13
	Oswego	24	70	100	−23
Ohio	Cincinnati	30	75	105	−17
	Columbus	29	75	104	−20
	Toledo	26	73	103	−16
Okla.	Oklahoma	36	81	108	−17
Oreg.	Portland	39	67	104	−2
Pa.	Erie	27	71	96	−16
	Phila	33	76	106	−6
	Pittsburgh	31	75	103	−20
R. I.	Block Island	31	68	92	−6
S. C.	Charleston	50	81	104	7
S. D.	Huron	11	72	108	−43
	Yankton	17	74	108	−36
Tenn.	Chattanooga	41	78	104	−10
	Memphis	41	81	104	−9
	Nashville	39	79	104	−13
Tex.	Abilene	44	83	110	−6
	Amarillo	35	77	106	−16
	El Paso	45	81	113	−5
	Galveston	54	83	100	8
	San Antonio	52	84	108	4
Utah	Salt Lake City	29	76	103	−20
Vt.	Burlington	19	70	100	−28
Va.	Lynchburg	38	78	105	−7
	Norfolk	41	79	105	2
Wash.	Seattle	40	63	98	3
	Spokane	28	69	104	−30
	Walla Walla	33	74	113	−17
W. Va.	Elkins	30	70	99	−28
	Parkersburg	32	75	106	−27
Wis.	La Crosse	16	73	104	−43
	Milwaukee	21	70	102	−25
Wyo.	Cheyenne	26	67	100	−38

great as actual rainfall. The depth in the tube is measured by a stick which is so graduated as to read the true rainfall in inches. The tube is 20 in. long so that a precipitation of 2 in. or less can be measured without emptying it.

Self-registering rain gages are used at main stations of U. S. Weather Bureau. Snowfall is caught, melted, and then measured; roughly 10 in. of snow make 1 in. of water. A rain gage at the top of a building gives a less rainfall than one on the ground.

Voluntary observers of the Weather Bureau record maximum and minimum temperature, precipitation, wind direction, general character of day, and miscellaneous phenomena such as halos, dates of frost, hail, sleet, auroras, and tornadoes. The general character of the day is recorded "clear" when the sky is 3/10 or less obscured, "partly cloudy" when from 4/10 to 7/10 is obscured, and "cloudy" when more than 7/10 is obscured.

19. Rainfall and Evaporation

Mean Annual and Monthly rainfall at many stations are given in Sect. 9, Art. 40. The term Rainfall includes both rain and melted snow.

Measurements of Evaporation are made by placing water-tight pans at

Place	Position of pan	Diameter of pan, ft.	Annual evaporation in.
Salton Sea, Calif	1500 ft. inland	2	164.50
Salton Sea, Calif	500 ft. at sea	4	108.65
Salton Sea, Calif	7500 ft. at sea	4	106.45
Indio, Calif	15 miles from Salton Sea	6	119.33
Mecca, Calif	1/2 mile from Salton Sea	6	107.81
Brawley, Calif	20 miles from Salton Sea	6	103.55
Mammoth, Calif	40 miles from Salton Sea	6	125.53
N. Yakima, Wash	1/2 mile west of city	4	67.96
Hermiston, Ore	On raft in reservoir	4	68.05
	On ground	3	97.29
Granite Reef, Ariz	Floating in Salt River	4	97.74
	On ground	4	115.18
California, Ohio	Floating in reservoir	4	45.99
Birmingham, Ala	Floating in reservoir	4	51.74
Dutch Flats, Neb	A few miles from Mitchell	4	65.67
Deer Flat, Idaho	On raft near water edge	4	77.43
	On ground of embankment	3	79.00
Ady, Ore	Floating in borrow pit	4	53.45
Fallon, Nev	Floating in canal	4	53.65
Lake Tahoe, Calif	2 ft. above water	4	42.21
Elephant Butte, N. Mex	Near Rio Grande River	4	86.95
Carlsbad, N. Mex	In the city	4	107.25
	In an alfalfa field	4	94.35
Lake Avalon, N. Mex	A few miles from Carlsbad	4	94.51

The evaporation from a pan 2 ft. in diameter is about 75%, that from a pan 4 ft. in diameter is about 50%, and that from a pan 6 ft. in diameter is about 30% greater than the evaporation from a large pond or lake. The above figures may be roughly corrected by using these percentages; thus, at Birmingham, Ala., the true annual evaporation is 34.50 in.

The U. S. Weather Bureau maintains at selected locations hook gage measurements of evaporation losses from cylindrical pans 10 in. deep and 4 ft. in diameter, exposed on wood frames, bottom of pans 1 in. above the ground, generally on level ground, and in full sunshine. Detailed description published in Monthly Weather Review, December, 1916, or in Circular L, Instrument Division, Weather Bureau, No. 559. From comparisons available, it appears that losses from bodies of water of considerable area are 50 to 60% as great as from pans exposed as above. The records are published in detail in the reports issued by the state section directors of the U. S. Weather Bureau, and in the Annual Reports of the Chief of the Weather Bureau.

the level of the ground and noting daily the variations in depth, together with the rainfall. On a water surface similar measurements may be made by floating boxes. It is found that the evaporation from water surfaces is greater than that from land, that it is greater in dry and desert regions than in cultivated ones, that it is less in low lands than on mountains, that it decreases as the humidity of the air increases, and that it increases with the temperature of the air and the velocity of the wind. In the North Atlantic states the annual evaporation from land surfaces is, on the average, about 40%, while that from water surfaces is about 60% of the annual rainfall. In low and level localities these percentages are decreased, while for high regions and steep slopes they are increased. In some arid localities west of the Rocky Mountains nearly all the rainfall evaporates from land surfaces, while the evaporation from water surfaces may be several times as great as the rainfall.

Experiments made in 1909-10 by the U. S. Department of Agriculture gave the following figures for the annual evaporation at twenty places in the United States, the evaporating pan being at or very near the surface of the ground or water.

Maximum Intensity of Rainfall

Station	Inches per hour for			Station	Inches per hour for		
	5 minutes, in.	10 minutes, in.	60 minutes, in.		5 minutes, in.	10 minutes, in.	60 minutes, in.
Bismarck, N. D. .	9.00	6.00	2.00	Chicago, Ill.	6.60	5.92	1.60
St. Paul, Minn. . .	8.40	6.00	1.30	Galveston, Tex. . . .	6.48	5.58	2.55
New Orleans, La. .	8.16	4.86	2.18	Omaha, Neb.	6.00	4.80	1.55
Milwaukee, Wis. .	7.80	4.20	1.25	Dodge City, Iowa.	6.00	4.20	1.34
Kansas City, Mo.	7.80	6.60	2.40	Norfolk, Va.	5.76	5.46	1.55
Washington, D. C.	7.50	5.10	1.78	Cleveland, Ohio. . .	5.64	3.66	1.12
Jacksonville, Fla. .	7.44	7.08	2.20	Atlanta, Ga.	5.46	5.46	1.50
Detroit, Mich. . . .	7.20	6.00	2.15	Key West, Fla. . . .	5.40	4.80	2.25
New York City. . .	7.20	4.92	1.60	Philadelphia, Pa. .	5.40	4.02	1.50
Boston, Mass.	6.72	4.98	1.68	St. Louis, Mo.	4.80	3.84	2.25
Savannah, Ga. . . .	6.60	6.00	2.21	Cincinnati, Ohio. .	4.56	4.20	1.70
Indianapolis, Ind.	6.60	3.90	1.60	Denver, Colo.	3.60	3.30	1.18
Memphis, Tenn . .	6.60	4.80	1.86	Duluth, Minn.	3.60	2.40	1.35

This table has been compiled from all the available records at stations of the U. S. Weather Bureau which are equipped with self-registering rain gages.

20. Speed of Winds in the United States

U. S. Weather Bureau Records of the average speed of wind in miles per hour at selected stations, and the highest speeds ever reported for a period of five minutes.

The Beaufort Scale is used by seamen. In following table the corresponding velocity per hour in statute miles and in nautical miles is added.

Station	Average	Highest	Station	Average	Highest
Abilene, Texas	10	66	Louisville, Ky	8	74
Albany, N. Y	7	76	Lynchburg, Va	5	63
Alpena, Mich	10	72	Memphis, Tenn	8	75
Atlanta, Ga	10	66	Miles City, Mont	6	62
Bismarck, N. D	10	78	Montgomery, Ala	6	54
Boise, Idaho	5	55	Moorehead, Minn	10	75
Boston, Mass	10	72	Nashville, Tenn	9	75
Buffalo, N. Y	13	96	New Orleans, La	8	86
Charlotte, N. C	6	72	New York, N. Y	17	96
Chattanooga, Tenn	7	64	North Platte, Neb	9	96
Chicago, Ill	15	84	Omaha, Neb	9	66
Cincinnati, Ohio	7	54	Palestine, Tex	7	60
Cleveland, Ohio	12	73	Philadelphia, Pa	10	75
Denver, Colo	7	75	Pittsburgh, Pa	11	70
Detroit, Mich	11	87	Portland, Me	8	61
Dodge City, Kan	11	75	Red Bluff, Calif	6	60
Dubuque, Iowa	6	60	Rochester, N. Y	9	78
Duluth, Minn	13	78	St. Louis, Mo	11	80
Eastport, Me	11	78	St. Paul, Minn	11	102
El Paso, Tex	11	78	Salt Lake City, Utah	7	68
Fort Smith, Ark	7	74	San Diego, Calif	6	54
Galveston, Texas	11	93	San Francisco, Calif	10	64
Havre, Mont	9	74	Sante Fe, N. M	7	53
Helena, Mont	7	70	Savannah, Ga	8	88
Huron, S. D	11	72	Spokane, Wash	6	52
Jacksonville, Fla	8	75	Toledo, Ohio	11	84
Kansas City, Mo	10	74	Vicksburg, Miss	6	62
Keokuk, Iowa	8	63	Washington, D. C	7	68
Knoxville, Tenn	6	84	Wilmington, N. C	8	68

Intensity or force of wind, Beaufort's scale	Velocity	
	Statute miles per hour	Nautical miles per hour
0. **Calm.** Full-rigged ship, all sail set, no headway	0 to 3	0 to 2.6
1. **Light Air.** Just sufficient to give steerage-way	8	6.9
2. **Light Breeze.** Speed of 1 or 2 knots, "full and by"	13	11.3
3. **Gentle Breeze.** Speed of 3 or 4 knots, "full and by"	18	15.6
4. **Moderate Breeze.** Speed of 5 or 6 knots, "full and by"	23	20.0
5. **Fresh Breeze.** All plain sail, "full and by"	28	24.3
6. **Strong Breeze.** Topgallant sails over single-reefed topsails	34	29.5
7. **Moderate Gale.** Double-reefed topsails	40	34.7
8. **Fresh Gale.** Treble-reefed topsails (or reefed upper topsails and courses)	48	41.6
9. **Strong Gale.** Close-reefed topsails and courses (or lower topsails and courses)	56	48.6
10. **Whole Gale.** Close-reefed main topsail and reefed foresail (or lower main topsail and reefed foresail)	65	56.4
11. **Storm.** Storm staysails	75	65.1
12. **Hurricane.** Under bare poles	90 and over	78.1 and over

The words "intensity" and "force," used in connection with this scale, have no direct relation to pressure, but refer to speed or velocity.

21. Barometric Observations

Whenever pressure is specified in terms of the height of a column of mercury, it is always tacitly understood that it is the height the column balancing the pressure would have under "standard conditions," that is, at 0° C. and where g has the standard value $g_0 = 980.665$ cm. per sec. per sec. adopted by the International Committee of Weights and Measures. In all cases where the mercury column (if of average barometric height and at ordinary atmospheric temperature) is to be read closer than 2 or 3 mm. (0.1 in.) one or more of the corrections described below must be made or the accuracy of the reading will be imaginary.

Corrections. Let l be the height of the mercury column as read at $t°$ C. with a scale correct at $t_0°$ C. whose coefficient of linear expansion is β. Let ϕ be the latitude and H the elevation in meters above sea level.

(1) Temperature of the Mercury. Subtract $0.000182\ lt$.

(2) Temperature of the Measuring Scale. Add $\beta\ (t - t_0)\ l$. For brass $\beta = 0.000019$; for glass $\beta = 0.000008$. If, as is usual, the scale is correct at 0° C., the complete correction, (1) and (2), for the expansion of both the mercury and the scale may be made by subtracting from the observed reading $(0.000182 - \beta)\ lt$, which gives

$0.000163\ lt$ for a brass scale, and
$0.000174\ lt$ for a glass scale.

Under ordinary conditions the correction may amount to as much as 4 mm.

(3) Capillary Depression in a Cistern Barometer. Add to the reading of the top of the meniscus the amount given in the table below corresponding to the internal diameter of the tube and the height of the meniscus. This somewhat uncertain correction can be avoided by using a tube at least 25 mm. in diameter.

Capillary Depression of Mercury

(After Mendeléeff and Gutkowsky. Kohlrausch, 1910)

Diameter	Height of the meniscus in mm.							
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
4	0.83	1.22	1.54	1.98	2.37	....	....	....
5	0.47	0.65	0.86	1.19	1.45	1.80	....	....
6	0.27	0.41	0.56	0.78	0.98	1.21	1.43	....
7	0.18	0.28	0.40	0.53	0.67	0.82	0.97	1.13
8	....	0.20	0.29	0.38	0.46	0.56	0.65	0.77
9	....	0.15	0.21	0.28	0.33	0.40	0.46	0.52
10	....	....	0.15	0.20	0.25	0.29	0.33	0.37
11	....	....	0.10	0.14	0.18	0.21	0.24	0.27
12	....	....	0.07	0.10	0.13	0.15	0.18	0.19
13	....	....	0.04	0.07	0.10	0.12	0.13	0.14

(4) Pressure of the Mercury Vapor. This causes a slight depression at high temperatures, but is less than 0.01 mm. under 40° C.

(5) Variation of weight with Latitude and Elevation. Multiply by $g/g_0 = (1 - 0.0026 \cos 2\phi - 0.000\,000\,2\ \mathrm{H})$, the local height obtained by applying the above corrections to the reading. The correction for elevation is only 0.1 mm. at 700 m., but the correction for latitude may amount to as much as 2 mm.

Mean Barometer-height b at an Elevation of H Meters above Sea Level. (Kohlrausch)

Air at 10° C. (50° F.) b_0 = 760 mm.

H =	0	100	200	300	400	500	600	700	800	900	1000 m.
b =	760	751	742	733	724	716	707	699	690	682	674 mm.
H =	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000 m.
b =	674	666	658	650	642	635	627	620	612	605	598 mm.

The international meteorological formula * for reducing height b of mercurial barometer at $t°$ C. and latitude ϕ to height b_0 at 0° C. and latitude 45° is

$$\log_\epsilon b_0 = \log_\epsilon b + \frac{H\,(1 - 3/8\ \epsilon/b)}{(18\,429 + 67.5\,t + 0.003\,H)\,(1 + 0.0026\cos 2\phi)}$$

22. Humidity

There is always present in the atmosphere a greater or less quantity of invisible water vapor, mixed with the other gases. The **absolute humidity** is the mass of water vapor present per unit volume; and the gas pressure which this water vapor is exerting is called the **vapor pressure.** The ratio of the actual absolute humidity to the maximum quantity of water vapor that could be present at the existing temperature is the **relative humidity.** By means of suitable tables, the relative humidity may be computed from the difference between simultaneous readings of an ordinary thermometer and a thermometer with a wetted and amply ventilated bulb. The temperature at which the actual absolute humidity would be the maximum possible—i.e., the temperature at which the existing quantity of water vapor would produce saturation—is called the **dew point.**

23. Protection against Lightning

The following discussion and rules have been abstracted from a Safety Code for Protection against Lightning which has been prepared for the American Engineering Standards Committee under the sponsorship of the Bureau of Standards and the American Institute of Electrical Engineers. The material included here is intended to indicate the general principles to be followed. For details as to materials and methods reference should be made to the Code.

Protection of Buildings and Miscellaneous Property

Fundamental Principles of Protection. The fundamental theory of lightning protection for buildings is to provide means by which a discharge may enter or leave the earth without passing through a nonconducting part of the structure, as for example, parts which are made of wood, brick, tile or concrete. Damage is caused by the heat and mechanical forces generated in such nonconducting portions by the discharge, whereas in metal parts the heat and mechanical forces are known to be of negligible effect if the metal has sufficient cross-sectional area. It is further known that there is a strong tendency for lightning discharges on structures to travel on the metal parts, especially those which extend in the general direction of the discharge. Hence, if metal parts are provided, of proper proportions and distribution, damage can be largely prevented. However, because lightning has such a wide range of characteristics, it is difficult to provide any practical means which will afford absolute protection under

* For a more complete treatment and application of this formula and for other forms of this equation see: (a) Smithsonian Meteorological Tables, 4 ed., pp. XXXXIX, also pp. 85-158; or (b) W. J. Humphreys, Physics of the Air, chap. V.

Annual Means of Relative Humidity and Precipitation for Many Years

Stations		Annual percentage of relative humidity at 8 a.m. 75th meridian time	Annual amount precipitation, in.
Ala.	Birmingham	79	49.48
	Mobile	84	61.61
Ariz.	Flagstaff	*	22.80
	Phoenix	54	7.87
	Yuma	59	3.47
Ark.	Fort Smith	80	38.87
	Little Rock	81	49.89
Calif.	Fresno	74	9.68
	Los Angeles	78	15.64
	Sacramento	82	18.08
	San Diego	79	10.30
	San Francisco	87	22.02
Col.	Denver	63	14.05
	Grand Junc.	62	8.30
	Pueblo	64	11.95
Conn.	New Haven	75	47.19
D. C.	Washington	76	43.50
Fla.	Jacksonville	83	53.25
	Key West	78	38.11
	Pensacola	80	57.85
	Tampa	84	53.13
Ga.	Atlanta	79	48.35
	Augusta	82	44.90
	Savannah	81	47.23
Idaho	Boise	70	12.71
Ill.	Cairo	81	41.71
	Chicago	78	32.86
	Springfield	79	36.96
Ind.	Indianapolis	77	39.90
Iowa	Des Moines	80	32.04
	Dubuque	81	32.90
	Keokuk	79	32.64
Kan.	Dodge City	79	20.51
	Wichita	78	30.61
Ky.	Louisville	76	44.33
La.	New Orleans	83	57.46
	Shreveport	83	43.37
Me.	Portland	75	42.51
Md.	Baltimore	72	43.18
Mass.	Boston	73	40.14
Mich.	Detroit	80	32.16
	Marquette	78	32.63
	Port Huron	80	30.65
Minn.	Duluth	81	29.93
	St. Paul	80	28.68
Miss.	Vicksburg	82	51.93
Mo.	Kansas City	77	37.37
	St. Louis	77	37.44
	Springfield	81	44.57
Mont.	Helena	68	12.77
Mont.	Kalispell	82	16.94
	Miles City	79	13.17
Neb.	N. Platte	81	18.86
	Omaha	78	27.77
Nev.	Winnemucca	66	8.40
N. C.	Charlotte	78	46.05
	Hatteras	87	60.85
	Wilmington	81	51.05
N. D.	Bismarck	80	16.34
N. H.	Concord	77	37.53
N. J.	Atlantic City	80	40.56
	Cape May	*	40.75
N. Mex.	Santa Fe	58	14.49
N. Y.	Albany	78	36.38
	Binghamton	80	32.94
	Buffalo	77	35.82
	N. Y. City	75	44.63
	Oswego	79	36.18
Ohio	Cincinnati	76	38.33
	Columbus	79	36.34
	Toledo	79	32.03
Okla.	Oklahoma	80	31.69
Oreg.	Portland	86	43.24
Pa.	Erie	76	36.93
	Philadelphia	74	40.41
	Pittsburgh	77	36.35
R. I.	Block Island	81	44.36
S. C.	Charleston	79	52.07
S. D.	Huron	83	21.10
	Yankton	80	25.30
Tenn.	Chattanooga	80	51.61
	Memphis	79	50.34
	Nashville	80	47.20
Tex.	Abilene	77	24.74
	Amarillo	76	22.55
	El Paso	54	9.16
	Galveston	84	44.77
	San Antonio	81	26.83
Utah	Salt Lake City	60	16.03
Vt.	Burlington	76	31.61
Va.	Lynchburg	77	43.42
	Norfolk	80	49.54
Wash.	Seattle	87	33.11
	Spokane	77	16.36
	Walla Walla	74	17.67
W. Va.	Elkins	86	42.75
	Parkersburg	81	40.22
Wis.	La Crosse	81	30.81
	Milwaukee	78	30.08
Wyo.	Cheyenne	65	14.99

* Not computed

all conditions, although the degree of protection afforded by present practice is high if the installation is properly made.

The required condition that there be a metallic path for the part of the discharge which is intercepted is met most fully by a grounded metal or metal-covered structure which presents what might be thought of as an infinite number of parallel conductors from the uppermost part of the structure to earth. It is substantially met by a steel-framed structure, which, though faced with brick, terra-cotta, or other building material, usually has, or at relatively small cost can be equipped with, a sufficient number of metal terminals or receiving points on the upper portions which connect with the frame to protect it thoroughly.

For a structure which is built wholly or partly of non-conducting materials, one of the best defenses against direct hits by lightning is to surround it with a ring of grounded metallic masts or poles of sufficient height. Or, if the structure is not large, a single mast erected nearby may be sufficient. Experiments have indicated that, under certain assumed test conditions, such a vertical conductor will generally divert to itself all direct hits which might otherwise fall within a cone-shaped space of which the apex is the top of the conductor, and the base a circle of radius two to four times the height of the conductor. This agrees with theoretical deductions. Incidentally, any metallic structure, or adequately protected structure, will function in the same manner as a mast. Thus a tall steel windmill or water-tower or rodded steeple will tend to protect nearby structure of less height, although before relying upon such protection care should be taken to see that the structure lies well within the cone-shaped space mentioned above.

Generally, however, on account of architectural considerations, the mast type of protection is not feasible. More suitable protection is provided by the installation of lightning conductors. Here the required conditions of protection are closely approximated by placing air terminals or receiving points on the uppermost parts of the building, with interconnecting and grounding conductors attached to the building itself. By this means a relatively small amount of metal properly proportioned and distributed is made to afford a satisfactory degree of protection and at the same time, if necessary, to afford a minimum of interference with the contour of the structure. It should be stated, however, that this type of protection is to be considered only for structures in which very small induced sparks do not present an appreciable element of danger, as they do in oil tanks, cotton warehouses and powder storage houses. The latter require much more elaborate precautions to insure their safety than do the general run of buildings.

When designing and installing a system of protection of the lightning-rod type the following principles should be followed:

(a) The structure should be examined and all points or parts most likely to be struck by lightning should be noted, with the view of erecting air terminals thereon for the reception of the discharge. The object is to intercept the discharge immediately above the parts liable to be struck rather than to attempt to divert it in a direction it is not likely to take. The receiving points should be placed high enough above the structure to obviate danger of fire from the arc; the more inflammable the roof material the higher the points should be placed.

(b) Conductors should be installed with the view of offering the least possible obstruction to the passage of a stroke between air terminals and ground. The most direct path is in general the best, and there should be no sharp bends or loops for the lightning to jump across. The obstruction is practically inverse to the number of widely separated paths, so from each air terminal there should be at least two paths to ground and more if practicable. The number of paths is increased and the obstruction lessened by connecting the conductors to form a cage enclosing the building.

(c) When a stroke is about to take place to earth the surrounding surface of the ground for a radius of several miles carries an electric charge. As the discharge takes place this surface charge moves radially toward the ground end of the air path, forming an electric current in the ground. Near the point where the discharge enters the ground the current density becomes high, and if the flow takes place through the foundation wall of a building damage may result. Ground connections should therefore be distributed more or less symmetrically about the circumference of a structure rather than grouped on one side. With ground connections properly distributed the current will be collected at the outer extremities and a flow underneath the building minimized. In every case, for the foregoing reason, at least two ground connections should be made at opposite extremities of the structure. Satisfactory ground connections are made in the majority of cases by extending the rod into the earth to a distance of 6 to 10 ft. Driven rods or plates may be used as alternatives. If there is a water pipe nearby connection should be made to it

(d) If a lightning conductor system is placed on a building within or about which are metal objects of considerable size within a few feet of the conductor, there will be a strong tendency for sparks, or sideflashes, to jump from the conductor to the metal at its nearest point. To prevent damage an interconnecting conductor should be provided at all places where sideflashes are likely to occur.

(e) Within buildings where metallic objects may be liable to a dangerous rise of potential due to a lightning flash, the metal, if not interconnected with the lightning-rod system, should under some circumstances be independently grounded.

(f) Since a lightning condutor system as a general rule is expected to remain in working condition for long periods with little attention, the mechanical construction should be strong and the materials used such as to offer high resistance to corrosion.

(g) The minimum permissible weight of copper conductor for all ordinary buildings is 187-1/2 lb. per 1000 ft. The foregoing general principles are embodied in the detailed specifications mentioned at the beginning of this section, which correspond to the approval requirements of underwriters' laboratories. An approved lightning conductor, therefore, will meet the requirements of the Safety Code.

Protection of Structures Containing Inflammable Liquids and Gases from Lightning

Lightning is responsible for a majority of the tank fires of the petroleum industry. In a Report on Records of Oil Tank Fires in the United States, 1915-1925, published by the American Petroleum Institute, it is stated that lightning caused 55% of the fires recorded.

1. Reduction of Damage. Certain types of structures used for the storage of inflammable liquids and gases are essentially self-protecting. Protection, of a greater or less degree, may be secured in the case of others through the installation of various types of protective equipment, such as screens, rods, protective towers and by other means.

2. Fundamental Principles of Protection. Protection of structures and their contents from lightning involve the following principles.

(a) The storage of inflammable liquids and gases in all-metal structures essentially gas-tight.

(b) The use in all necessary breathing vents of efficient flame arresters.

(c) The maintenance of containers in good condition, so far as potential hazards from electrical discharges are concerned.

(d) The avoidance, so far as possible, of the accumulation of explosive mixtures, in and about such structures.

(e) The avoidance of spark gaps in metallic conductors or between metallic conductors at points where there may be an accumulation of explosive mixtures or an escape of inflammable vapors or gases to the air.

(f) In connection with structures not inherently self-protecting, the establishment of cones of protection through the use of grounded screen, rods or towers, or the equivalent.

(g) The location of structures containing inflammable liquids and gases not inherently self-protecting, in positions of lesser exposure with regard to lightning. Thus elevated positions should be avoided.

24. Wind Pressure on Structures

Air Resistance of Automobiles and Trains; Laws of Flight of Airplanes

Windstorm Damage. According to the statistics of one of the large insurance companies * well over one-half of the total windstorm damage is caused

* Associated Factory Mutual Fire Insurance Co., Boston, Mass. Handbook on Windstorms.

by the high winds that accompany ordinary storms and thunder-storms. Much of the loss is directly attributable to the omission of anchorage of buildings to foundations, of roofs to the walls, to inadequate fastening of tiles and other roof coverings and to similar neglect of good practice in construction details. It is not customary to design such details for definite wind loads and aside from exceptional cases any reasonable provision gives ample strength.

Protection against very severe storms can be accomplished only by adequate engineering design of the whole structure.

Wind Velocity Measurement. When it is desired to design a structure to withstand high winds, great difficulty is experienced in determining the maximum wind velocity to which the structure will be exposed. The records of the Weather Bureau give the maximum average velocity indicated for a five-minute period, known as the "**maximum**" and the velocity of the fastest mile, known as the "**extreme.**" At 100 miles per hour average velocity the **extreme** velocity is really the average over a period of 36 seconds. The maximum gust velocities are known to be much greater. What then shall be the speed for which the design is to be made?

Fig. 10

When the records at any one place are examined, it is found that the high velocities occur very infrequently. Therefore in the case of such structures as telephone or power lines an attempt is made to balance the cost of replacement against the cost of insurance. In other words, it is cheaper to rebuild occasionally than to build the structure strong enough initially to withstand the maximum wind speeds. In buildings and other structures the element of human safety enters and the considerations of cost are not the only ones. Yet so far as tornadoes are concerned, it is usually felt to be impracticable to build strong enough to withstand the maximum speeds. The cost of insurance is far less than the increased construction costs.

Robinson Anemometers. The official Weather Bureau Instrument is a Robinson type cup-anemometer. Prior to January 1, 1928, a four-cup driving unit was used but on that date a change was made to a three-cup unit because of the large errors of the older instrument at high speeds.

Pitot Tubes. The standard instrument adopted by the Bureau of Standards, National Physical Laboratory and other scientific organizations for the measurement of air velocities is a Pitot-static tube of proper design. The instrument consists of an open tube facing into the wind and a concentric tube closed to the air stream except for several small holes drilled radially and sufficiently far from the nose that the air flows by smoothly when the tube is in line with the wind direction. A pressure gage is used to measure the difference in the pressure between the two tubes. The second tube gives the static pressure, i.e., the pressure that would be shown by a gage moving with the air. The first gives the static pressure plus the increased pressure produced by reducing the velocity to zero at the mouth of the tube. The differential pressure is $\frac{1}{2}\, dV^2$, where d is the density of the air and V the

Indicated Wind Speeds by Robinson Cup Anemomentors

True speed miles per hour	Indicated speed old four-cup standard	Indicated speed new three-cup standard	True speed miles per hour	Indicated speed old four-cup standard	Indicated speed new three-cup standard
5	5	5	60	78	63
10	11	10	65	85	68
15	17	15	70	91	73
20	23	20	75	98	79
25	30	25	80	105	84
30	37	31	85	112	89
35	44	36	90	118	95
40	50	41	95	125	100
45	57	47	100	132	105
50	64	52	105	138	111
55	71	57	110	145	116

Note. Values above a true speed of 75 miles per hour are extrapolated, but are probably correct to the precision given, namely, 1 mile per hour.

wind speed, and is known as the velocity pressure. Pitot-static tubes of poor design usually indicate a higher pressure because of a failure to give the true static pressure. Values of the velocity pressure at various wind speeds are given in the table below for an air density of 0.07651 lb. per sq. ft. corresponding to 15° C., 760 mm. Hg.

True wind speed, miles per hour	Velocity pressure, lb. per sq. ft.	True wind speed, miles per hour	Velocity pressure, lb. per sq. ft.	True wind speed, miles per hour	Velocity pressure, lb. per sq. ft.	True wind speed, miles per hour	Velocity pressure, lb. per sq. ft.
5	.06	35	3.13	65	10.80	95	23.08
10	.26	40	4.09	70	12.53	100	25.57
15	.58	45	5.18	75	14.38	105	28.19
20	1.02	50	6.39	80	16.36	110	30.94
25	1.60	55	7.73	85	18.47	115	33.82
30	2.30	60	9.21	90	20.71	120	36.82

Note. The formula becomes $\frac{1}{2g}\,dV^2$ if the pressure is measured in pounds per square foot and the density in pounds per cubic foot.

The dimensions of several standard Pitot tubes, which may be used without calibration if the supporting bracket is at least 20 diameters behind the static holes, are given in the figure.

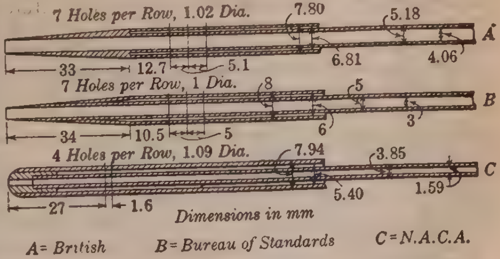

Fig. 11

Dines Tube Anemometer. The laboratory standard Pitot tubes are not suitable for use in field measurements of wind velocity but an instrument of this general type is in occasional use, namely, the Dines Tube Anemometer. In this instrument, the impact tube is kept pointed into the wind by means of a weather vane while the static tube is vertical. Because of this arrangement of the static tube a calibration factor is necessary and it has recently been shown that the factor depends to some extent on the wind direction and on the exact constructional details of the instrument.*

The pressures developed are transmitted to a float whose motion is recorded by a pen on a chart. Gusts are shown in the records and average velocities over comparatively short intervals of time can be estimated by a consideration of the inertia effects.

Electrical Anemometers. Recently several instruments giving an indication and record of wind velocity have been developed. the most widely known being the Burton anemometer. The instrument consists of an electric generator driven by either a four-cup or three-cup Robinson anemometer wheel. The voltage is indicated or recorded, the scale being graduated in terms of wind velocity. Gust velocities are indicated and can be evaluated for comparatively short time intervals by a consideration of the inertia effects.

Bridled Cup Anemometer. Still another type of instrument available for field indication of wind velocity is the bridled cup anemometer, available in an instrument developed by Julien P. Friez. A multi-blade turbine wheel is rotated against a spring system to a position of equilibrium such that the spring torque balances the wind torque on the wheel. The spring system is arranged to give a uniform wind velocity scale on the indicating dial.

Interpretation of Records. The only records available in sufficient quantity to be of any service to the engineer are those of the Weather Bureau for the maximum velocity over a five-minute period and the velocity of the fastest mile. The use and interpretation of these records in connection with any particular engineering project must be left to the judgment of the engineer in charge. If it is desired to build so strong that the structure will not fail under any possible circumstances, it is believed that the design should be made for a wind speed of 150 miles per hour.

It is commonly believed that the natural gusty wind imposes severe racking stresses which are greater than those imposed by a steady wind of the same speed. This is probably true only in rare instances for most of the puzzling effects of high winds in lifting roofs, causing walls to fail outwards, etc., are readily explained by the nature of the pressure distribution in a steady wind.

Design Practice. The current practice of most structural engineers is to consider neither the shape of the structure, the nature of the exposure, nor the probable maximum wind velocity. All of these variable factors are lumped together in a design wind load of usually 20 to 30 lb. per sq. ft. In addition, the usual factors of safety for the material are often waived, and the allowable working stresses increased 25, 50, or even 100%; a procedure described by Fleming† as giving an intellectual assent to the theories of the textbook and ignoring them in actual practice. It would seem to be more logical to retain the same working stresses and to design for a properly selected wind velocity. To that end we attempt to give experimental data known to apply with sufficient accuracy for design purposes on the wind pressures on various engineering structures. From these data the wind load actually encountered at a selected wind velocity may be computed.

* Annual Report of the National Physical Laboratory, 1926, p. 217.

† Wind Pressure on Structures. Robin Fleming (Engineering News Co., 1915). This book gives a good summary of current design practice.

Method of Giving Wind Pressure Data.* The nature of the reaction between the wind and an obstacle to its progress is extremely complicated. Two characteristics should be emphasized. First, the reaction consists of a surface distribution of pressure and the representation of the action by a single resultant force or by an average wind pressure is only a convenient device useful for certain purposes. Second, when there is no wind there is a distribution of pressure over the surface due to the normal atmospheric pressure of approximately 14.7 lb. per sq. in. The effect of the wind is a modification of this normal pressure, at some points an increase, at others a decrease in pressure. The magnitude of these changes is only a small percentage of the normal atmospheric pressure, and the words " suction " or " vacuum " as commonly used in this connection do not imply any large change in density or pressure. The changes are usually less than 2% of the normal atmospheric pressure (0.3 lb. per sq. in. or 42 lb. per sq. ft.).

The maximum increase in pressure produced by the wind is equal to $\frac{1}{2}\, dV^2$, where d is the density of the air, and V the wind speed. This pressure, usually termed the velocity pressure or impact pressure, is indicated by a Pitot-static tube of standard design. Values have already been given in a preceding table.

It is found convenient to express all observed pressure differences including the average wind pressure as ratios to the velocity pressure. Although the maximum increase in pressure is equal to the velocity pressure, pressure decreases of greater amount frequently occur and the force resulting from the surface distribution over a structure is frequently greater than the velocity pressure times the projected area. We may regard the ratio, or coefficient as it is sometimes called, as a shape factor. Thus if the shape factor is 1.5, we may readily find the average pressure at any wind speed by multiplying by 1.5 the velocity pressure taken from the table.

The shape factor for a thin square flat plate normal to the wind is 1.12 and therefore the wind force on such a plate is 1.12 times the velocity pressure times the area. On an infinitely long thin rectangular plate the shape factor is 2.0. Other values are given later.

In the above examples the shape factor is very nearly independent of size or wind speed. In general, the shape factor depends to some degree on the size and speed, the variations being said to be due to scale effect. For bodies with flat surfaces and sharp edges, the scale effect is in general so small as to be negligible for the present purpose. For spheres and cylinders the variation is great and critical regions of speed and size occur where the shape factor decreases very rapidly. In these cases it is difficult to predict the average pressure on a full-scale structure from model tests. The important practical structure of this type is the tall chimney.

It should be especially emphasized that there is no single value of the wind pressure applicable to all types of structures for a given wind velocity. The variations in shape factor are sufficiently large to demand individual treatment.

Flat Plates, Signs, etc. The shape factor for a thin flat plate depends on the ratio of length to breadth. With the wind normal to the plate, the shape factor varies from 1.12 for a square or round plate to 1.33 for a rectangular plate of length-breadth ratio equal to 6. The wind force also varies with the angle of the plate to the wind. The direction of the resultant force is very nearly at right angles to the plate, and if the force is expressed as an average pressure over the area of the plate (*not* projected normal to the wind), the

* From Sci. Paper 523, Bureau of Standards.

maximum shape coefficients observed are 1.75 for a square plate, 1.00 for a rectangular plate of length-breadth ratio equal to 3 and 1.34 for a rectangular plate of length-breadth ratio equal to 6. The detailed values for several angles to the wind are given in International Critical Tables, Vol. 1, p. 406. For approximately square signs, a shape factor of 1.75 should be used. For rectangular signs, the factor may be reduced as indicated by the above values.

Structural Shapes, Bridge Trusses. The shape factor for a very long thin flat plate is 2. In attempting to apply this value to built-up structures such as bridge trusses or radio towers difficulty is experienced due to shielding. Experiments on built-up members have recently been published in Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, Vol. III, 1927; and experiments are in progress at the National Physical Laboratory of Great Britain. Reference should be made to the original publication for details as to the variation of the shape factors with angle.

On simple structural steel shapes such as angles, I beams and built-up columns, the wind pressure is referred to the product of the length of the section by the greater dimension of the cross-section. The resultant force is resolved parallel to this dimension and perpendicular to it and two shape factors are given. We consider here only the shape factor for the component perpendicular to the greater dimension of the cross section; i.e., the component giving the higher stress. The maximum values observed ranged from 1.6 to 2.2, in most cases being about 2 and occurring when the wind was normal to the face of greater area. We may therefore conclude that a safe value for isolated structural steel shapes is 2.

A number of model bridge trusses were tested. When used singly, the shape factors were from 1.4 to 1.6 referred to the projected area of the truss. When two trusses were placed one behind the other with wind normal to the plane of the truss, the shape factor for the front truss was from 1.4 to 1.6 for all positions of the rear truss, and for all models, whereas that for the rear truss varied, with the distance apart from 0.0 to 1.2, depending on the ratio of open to closed area.

Further experiments were made on one model at a given spacing (2.75 times the vertical dimension of truss) by varying the wind direction in both horizontal and vertical directions. Variation in the vertical plane is most important, the shielding effect disappearing at an angle of 15°.

A further experiment was made on a model consisting of two trusses spaced a distance equal to the maximum vertical dimension of the truss with floor between. In this case the shape factor referred to the area of *one* truss reached the value 1.85, and at varying angles in the vertical plane a large lifting force was found. Bridges across deep canyons or in other locations where variations of the wind direction in a vertical direction are possible may need some provision to take care of possible lifting forces.

It appears that in general a shape factor of 2 on the area of one truss is sufficient, although if the trusses are farther apart than the maximum vertical dimension of one truss or if the wind direction may vary in the vertical plane, a factor of at least 2.5 applied to the area of one truss should be used. Under exceptional conditions a factor of 3 applied to one truss may be demanded. In any large project where wind forces are an important item, wind tunnel tests of a model will more than pay their cost.

Buildings, Skyscrapers. For tall buildings of usual proportions it appears that a shape factor in the neighborhood of 1.5 is required.* For slender

* Scientific Paper 523, Bureau of Standards.

towers the factor probably approaches 2.0 as for a long prism. For buildings of approximately cubical form, the factor is from 0.8 to 1.0. As in the case of bridge structures, model tests are justified in important projects. Otherwise an estimate should be made from the known shape factors given in the table below:

Cube	0.8
Thin square flat plate	1.1
Prism 1x1x3	1.5
Very long prism	2.0

In many cases, questions arise as to the strength of individual walls. Here the load depends on the pressure in the interior of the building which in turn depends on the number, size, and location of openings. It seems probable that in many cases the average pressure on a wall may equal twice the velocity pressure. The average pressure over small areas may reach three times the velocity pressure. It should especially be noted that the force may be in either direction depending on the pressure within the building, and the outward collapse of walls in high winds is readily explained without the necessity of assuming a very low barometric pressure.

Roofs. The force on a roof also depends on the pressure within the building, which is somewhat indeterminate. The distribution on the outer surface is such as to give ordinarily a lifting force provided the pitch of the roof does not exceed 30 deg. There seems to be no occasion for providing for a wind load acting downward on the roof under these conditions. There is every reason to provide for substantial lifting forces which are present even when the wind does not penetrate beneath the roof covering. If the interior is not subject to the full velocity pressure through broken windows or otherwise, the upward loading is of the order of 0.7 to 1 times the velocity pressure. Under special circumstances such as in a factory type building with windows broken on the windward side, the loading may reach twice the velocity pressure. The roof should accordingly be securely anchored to the walls and the walls in turn to the foundations.

When the pitch of the roof exceeds 30 deg., regions of increased pressure appear as shown by the experiments of T. E. Stanton (Proc. Inst. C. E., London, 156, 1903-04, p. 78).

Recently a large number of measurements of wind pressure on roofs and buildings have been published in Jahrbuch der Deutschen Gesellschaft für Bauingenieurwesen, 1927, p. 87.

Volume 97 of Engineering News-Record contains analyses of the effects of the Florida hurricane of 1926 which show that pressures of 55 to 65 lb. per sq. ft. were obtained. The wind velocity was independently estimated as 132 miles per hour which would give the observed pressures on structures having a shape factor of 1.5.

Additional model experiments on an approximately cubical structure are given in Volume 100, No. 13, p. 508, by Professor Dawley of the Kansas State Agricultural College. These give shape factors from 0.7 to 0.85, depending on the presentation of the model to the wind and the presence or absence of windows and doors.

Chimneys, Standpipes, Gas-holders, Flag Poles, Transmission Lines. If the product of diameter in feet by wind speed in miles per hour is less than about 40, the shape factor for cylindrical structures may be taken from the table at the top of the next page.

The flow about cylinders in this region in a steady wind is very definitely periodic, the frequency in cycles per second being equal to about 0.3 times

Ratio of length to diameter	Shape factor	Ratio of length to diameter	Shape factor
1	0.63	10	0.83
2	0.69	20	0.92
3	0.75	40	1.00
5	0.74	Infinite	1.20

the ratio of the wind speed in miles per hour to the diameter in feet. Care should be taken that the natural period of vibration of the structure does not correspond to the above eddy frequency at high wind speeds. It is understood that in some cases transmission lines have been observed to vibrate (in segments) with the eddy frequency and in certain conditions cause fatigue failures.

When the product of diameter in feet by wind speed in miles per hour exceeds 40, the shape factor is found to decrease to a comparatively low value. Experiments on full-scale cylinders in a natural wind indicate a value as low as 0.3 to 0.4. The flow is no longer definitely periodic but is very irregular and often unsymmetrical. Sufficient information is not available to justify such a radical reduction in the shape factor. Until the flow about large cylinders is fully understood, it is recommended that the same shape factors be used as for small cylinders.

In transmission lines, the problem of shielding again appears since individual wires are near each other. It is common practice to compute the wind force not on the actual wire but on the wire assumed coated with ice 1/2 in. in thickness. The shielding depends on the spacing of the wires, and on the wind speed. With a spacing of about 1 ft., the force on the shielded wires is about 50% of that on the front unshielded wire. The factor for an unshielded wire may be taken from the table above.

If the wind blows at an angle to a single wire, the force remains nearly normal to the wire and the magnitude of the force falls off approximately as the square of the sine of the angle of the wire to the wind.

The only measurements known of the wind pressure on flags gave a shape factor of approximately 0.1 applied to the area of the flag. The measurements were made at the Washington Navy Yard wind tunnel and the actual empirical formula given was $P = 0.0003\,A\,V^{1.9}$ where P is the pull of the flag in pounds, A the area of the flag in square feet, and V the wind speed in miles per hour.

General Remarks on the Choice of Shape Coefficients. It has been assumed in the foregoing treatment that the winds of high velocity might come from any direction and the maximum shape coefficients only are given. Under certain special conditions it may be found that high winds are only likely to prevail from certain directions. In such a case some further study than that given here is necessary.

In any extensive construction where wind loads are an important factor, the shape factor should be determined by wind tunnel experiments on a model.

Air Resistance of Automobiles. The observed shape coefficients found for automobiles of conventional design vary from about 0.75 to 1.35 * depending on the exact shape. Special stream line automobiles and racing cars have

* Public Roads, Vol. 6, No. 9, Nov. 1925, p. 203. Zeitschrift fur Flugtechnik und Motorluftschiffahrt, 15, 1924, p. 22; also 13, 1922, p. 201. Motorwagen, 26, 1923, p. 355.

been designed for which the shape coefficient is as low as 0.4. The area to which the shape coefficients apply is the area of projection on a plane normal to the usual direction of motion.

An average value is of doubtful service since in most cases the resistance of a particular automobile is desired for certain specified conditions of car speed, wind speed, and wind direction. This is most readily found by wind tunnel experiments on a model, since full-scale conditions are not easily controlled. Some unpublished experiments at the Bureau of Standards show that when the relative wind strikes the car at an angle of only 5 deg. there is a lateral force tending to skid the car equal to about 50% of the head resistance. An angle of 5 deg. requires a side wind of a speed equal to only 11% of the automobile speed. The force is much greater for a closed car than for an open car. The effect of natural winds especially as regards direction is therefore a very important one and it is regrettable that more information is not available.

Air Resistance of Trains. A few papers have appeared on the determination of the air resistance of trains.* It appears from the experiments of Goss that the shape factor for an isolated car is about 0.5, applied to the area of projection normal to the usual direction of motion. For the first car in a train, the factor is about 0.4, second car about 0.036, other intermediate car about 0.04, last car 0.1.

The French experiments were carried out on a train consisting of locomotive, tender, and two cars, the dimensions of which are not given. The coefficients appear to be of the same order of magnitude as those found by Goss, except in the case of the intermediate car. Those interested should consult the original papers for details as to the shapes of the cars.

No information is available on the effect of angle of the train to the relative wind. It seems probable that as in the case of automobiles a comparatively small side wind will introduce a fairly large side force.

Laws of Flight of an Airplane.† Airplanes are supported by the dynamic reaction of the air on large fixed surfaces called wings. The necessary relative motion is provided by driving the airplane forward by means of a motor and propeller. Other types of heavier-than-air craft have been proposed, namely, the helicopter, which depends for its lift on the direct vertical thrust of air propellers, and the ornithopter, in which the wings are made to flap like the wings of a bird. So far only the airplane can be said to be successful.

In all instances in which a force is obtained by dynamical means certain fundamental principles apply and operate to limit the possible efficiency. According to Newton's laws of motion, a force, F, can be secured by dynamical means only by continuously imparting a velocity, V, to a mass, M, each second such that $\mathbf{MV} = \mathbf{F}$. Thus if F is the lift of an airplane, M is the mass of air deflected downward each second with a velocity V. Or if F is the thrust of an airplane propeller, M is the mass of air passing through the propeller each second and having its velocity increased by an amount V.

* Atmospheric Resistance to the Motion of Railway Trains, W. F. M. Goss; Proceedings Western Railway Club, 1897-98, Vol. 10, p. 347. Mésure de la resistance de l'air sur le matériel des chemins de fer; Ch. Maurain, A. Touissant, R. Pris, Comptes Rendus, 177, July 1923, p. 308.

† The development of aeronautics in recent years has been so great that many textbooks and handbooks are now available. An attempt is made here to give a few fundamental principles, but the reader is referred to books such as Aerodynamics by E. P. Warner (McGraw-Hill, 1927) or Applied Aerodynamics by Leonard Bairstow (Longmans, Green, 1920) for more complete technical information.

There is as a consequence an amount of energy imparted to the air each second equal to $\frac{1}{2}\, MV^2$ or $\frac{1}{2}\, FV$. It is apparent that some energy is always necessary to produce a force, F, and that the energy, $\frac{1}{2}\, FV$, for a given force, $F = MV$, increases as V increases, or as M decreases, since MV is constant, which means usually as the area of the wings or propeller decreases. The advantage of an airplane depends partly on this fact, for successful helicopters find it necessary to use propellers comparable in size to airplane wings. Another advantage of an airplane is that to produce a given lift, the required towing force is equal to less than one-seventh or one-eighth, in some cases only one-twentieth, the lift, so that a propeller of moderate size is able to develop the required force with a high efficiency. Finally, the airplane moving forward very rapidly is able to come in contact with a large mass of undisturbed air each second, thus making M very large and V small.

The relations given above express the essential physical relations but are of little quantitative value because of the difficulty of determining M and V and of the added energy losses due to friction and to turbulent motion. Recourse is had to experiment on surfaces of various shapes. In some early airplanes, the wing surfaces were merely flat plates and the laws of the air force received great attention. These formulas are now of only historic interest since all airplane wings are curved and of some thickness. Such curved surfaces are technically known as airfoils.

The direction and magnitude of the air force at different angles to the wind and at different air speeds is determined in an artificial air-stream in a wind tunnel. For convenience the force is resolved into a lift component perpendicular to the wind and a drag component parallel to the wind. The results are then given in the form of lift and drag coefficients, C_L and C_D, defined by the relations $\mathbf{L} = \mathbf{C_L\, A}\, \frac{1}{2}\, \mathbf{dV^2}$, $\mathbf{D} = \mathbf{C_D\, A}\, \frac{1}{2}\, \mathbf{dV^2}$, where L is the lift, D the drag, A the wing area, d the air density and V the speed, all expressed in a consistent system of units in which case C_L and C_D are independent of the particular system of units employed. Non-dimensional quantities are used throughout. The system of units used must however be self-consistent, such as force in pounds, mass in slugs, distance in feet, time in seconds, etc. These coefficients vary very greatly with angle of attack and to a small extent with size of model and speed of test. Engineers sometimes prefer to write $\mathbf{L} = \mathbf{K_l AV^2}$, $\mathbf{D} = \mathbf{K_d AV^2}$ where L and D are in pounds, A in square feet, and V in miles per hour. For standard air density, $K_l = .00255\ C_L$, $K_d = .00255\ C_D$. The wind tunnel test also gives the point of intersection of the line of action of the force with the chord-line of the airfoil which is usually expressed as the ratio of the distance of the point of intersection x from the nose to the chord length c.

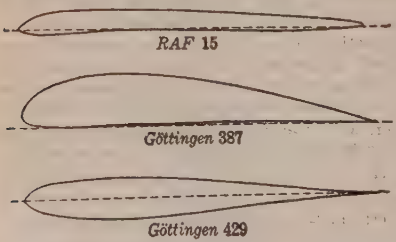

Fig. 12

There are now available atlases of airfoil characteristics containing data on over 600 sections.* It is manifestly impossible to even abstract these data. For illustrative

* Aerodynamic Characteristics of Airfoils, Technical Reports 93, 124, 182, 244 and 286 of the National Advisory Committee for Aeronautics (Government-Printing Office).

purposes, data are given on three representative airfoil sections. The shapes of the sections are given by Cartesian coordinates of the upper and lower surfaces from the chord line and vertical through the nose. The models were 6-in. chord (along wind) by 36-in. span (across the wind) and were tested in a 7.5-ft. wind tunnel at 40 miles per hour.

Characteristics of R. A. F. 15 Airfoil Section

Form Ordinates—per cent chord			Aerodynamical characteristics				
Station	Upper	Lower	Angle of attack	C_L	C_D	Lift/Drag	x/c
0.00	0.30	0.30	− 4°	−0.18	0.025	− 7.3	.....
1.25	1.90	− .35	− 2°	− .04	.014	− 2.8	.....
2.50	2.85	− .70	− 1°	+ .03	.013	+ 2.6	0.966
5.00	3.95	−1.05	0°	.14	.013	10.7	.479
7.50	4.65	−1.15	1°	.24	.013	18.8	.407
10.00	5.05	−1.20	2°	.32	.016	20.0	.367
15.00	5.55	− .85	4°	.46	.023	20.0	.321
20.00	5.78	− .55	6°	.61	.033	18.4	.302
30.00	5.80	− .10	8°	.76	.047	16.2	.297
40.00	5.60	− .03	10°	.89	.061	14.7	.288
50.00	5.23	− .24	12°	1.00	.083	12.1	.281
60.00	4.65	− .50	14°	1.02	.124	8.2	.298
70.00	4.05	− .65					
80.00	3.30	− .65					
90.00	2.30	− .30	Note. Angles measured from tangent to lower surface.				
95.00	1.68	.00					
100.00	.65	.34					

Characteristics of Göttingen 387 Airfoil Section

Form Ordinates—per cent chord			Aerodynamical characteristics				
Station	Upper	Lower	Angle of attack	C_L	C_D	Lift/Drag	x/c
0.00	3.61	3.61	− 8°	−0.07	0.071	− 0.9	.....
1.25	6.74	1.35	− 6°	+ .08	.031	+ 2.6	1.410
2.50	7.98	.80	− 4°	.22	.024	9.4	.684
5.00	9.86	.35	− 2°	.37	.026	14.3	.507
7.50	11.32	.18	0°	.51	.031	16.4	.436
10.00	12.40	.09	2°	.66	.039	16.9	.396
15.00	13.83	0	4°	.81	.051	15.9	.369
20.00	14.77	.07	6°	.96	.067	14.3	.348
30.00	15.36	.21	8°	1.10	.084	13.0	.337
40.00	14.88	.37	10°	1.23	.104	11.8	.323
50.00	13.47	.54	12°	1.33	.125	10.6	.307
60.00	11.59	.54	14°	1.42	.148	9.6	.312
70.00	9.27	.54	16°	1.43	.182	7.9	.315
80.00	6.57	.49	18°	1.42	.213	6.7	.327
90.00	3.61	.27	20°	1.41	....	....	....
95.00	1.99	.16					
100.00	.36	.00					

Characteristics of Göttingen 429 Airfoil Section

Form Ordinates—per cent chord			Aerodynamical characteristics				
Station	Upper	Lower	Angle of attack	C_L	C_D	Lift/Drag	x/c
0.00	0.00	0.00	− 4°	−0.26	0.014	......	......
1.25	2.02	−1.65	− 2°	−0.10	0.012	− 8.8	.....
2.50	2.71	−2.45	0°	+0.04	0.013	+ 3.1	0.197
5.00	3.67	−3.46	2°	0.18	0.015	12.4	0.224
7.50	4.47	−4.10	4°	0.33	0.020	17.2	0.229
10.00	4.95	−4.57	6°	0.50	0.028	17.5	0.241
15.00	5.37	−5.27	8°	0.65	0.040	16.2	0.242
20.00	5.69	−5.58	10°	0.78	0.054	14.6	0.244
30.00	5.69	−5.69	12°	0.88	0.076	11.6	0.246
40.00	5.32	−5.27	14°	0.73	0.170	4.3	0.234
50.00	4.68	−4.52	16°	0.70	0.239	2.9	0.382
60.00	3.72	−3.56					
70.00	2.61	−2.39					
80.00	1.60	−1.44					
90.00	0.69	−0.74					
95.00	0.37	−0.43					
100.00	0.16	−0.16					

It is evident that the mass of air, M, affected by an airplane wing is proportional to the air density, d, the area of the wing, A, and the forward velocity of the wing, V. Prandtl has computed the constant of proportionality to be $\pi/4$ times the ratio of span b (dimension across wind) to the chord, c (dimension along wind). The ratio b/c, known as aspect ratio, R, plays an important part in determining the air forces on an airfoil. We may write $M = \frac{\pi}{4}\frac{b}{c}dAV = \frac{\pi}{4}RdAV$. The lift, L, equals M times the downward velocity, v, imparted to the air and hence $v = \frac{4L}{\pi RdAV} = \frac{2V}{\pi R}C_L$. The required energy to produce the lift is $\frac{1}{2}Mv^2 = \frac{dAV^3}{2\pi R}C_L^2$. If we regard this as due to a drag D, we may write the energy equal to DV or $C_D = \frac{D}{\frac{1}{2}dV^2} = \frac{C_L^2}{\pi R}$. As stated before, C_D, C_L, and R are dimensionless quantities, provided a self-consistent system of units is used. This drag coefficient, known as the induced drag coefficient, accounts for nearly one-half the drag at large lift coefficients. It is the minimum drag coefficient possible for a given lift coefficient. Its value decreases as the aspect ratio increases; hence the typical plan form of airplane wings.

The production of the induced drag may be regarded as due to a change in the angle at which air strikes the airfoil, some of the downward velocity beginning before the airfoil is reached. If the change in angle is called α, we may regard the drag as a component, $L\alpha$, of the lift in the direction of motion. It is easily verified that $\alpha = \frac{C_L}{\pi R}$ and that the downward velocity at the airfoil is one-half the final downward velocity. This angle is called the induced angle of attack.

To a first approximation we may regard the total drag coefficient as the sum of the variable induced drag coefficient and a constant profile drag coefficient. We may therefore write $C_D = C_{D_0} + \frac{C_L^2}{\pi R}$ and $\alpha = \alpha_0 + \frac{C_L}{\pi R}$ and compute from a test of one aspect ratio the values for all other aspect ratios. If R is greater than 3, the formulas are found to give results as accurate as the experimental data.

In the case of level horizontal flight, neglecting the forces on other parts of the airplane, we may write the weight, W, as equal to the lift. Hence $W = L = C_L \frac{1}{2} dV^2A$ or $V = \sqrt{\frac{2}{d}\frac{W}{A}\frac{1}{C_L}}$. When C_L reaches its maximum, V reaches a minimum, and hence the maximum value of C_L determines the landing speed. The speed of a given airplane can be varied only by varying C_L which is done by moving the elevator to trim the airplane to a new angle of attack. The power does not enter into this relation. If it is not sufficient to give horizontal flight, the airplane loses altitude in a glide but the speed and angle of attack remain nearly the same. If the power is greater than that required, the airplane will climb, but again the speed and angle of attack will be nearly the same as before.

The power required for level flight equals the drag, D, times the forward speed, V, or $C_D A \frac{1}{2} d V^3$; or substituting the value of V above, $\frac{C_D}{C_L^{3/2}}\sqrt{\frac{2}{d}\frac{W^3}{A}}$. The ratio $\frac{C_D}{C_L^{3/2}}$ for a given airfoil is a measure of the power required for a given weight and wing area of the airplane.

The reader is referred to some of the textbooks on aeronautics for a further account of performance prediction, choice of airfoils, control and stability, etc.

Skin Friction. Even as late as Langley's experiments, skin friction in air was regarded as a negligible quantity, but the extensive experiments of Zahm showed that it was appreciable. Wieselsberger * gives the following formula for the surface friction on well varnished surfaces:

$$R_F = 0.0375\ AqR^{-0.15}$$

where R_F is the frictional force on the area, A, exposed to the stream, q is the dynamic pressure, $\frac{1}{2}\,dV^2$, d being the air density, and V the air speed, and R the Reynolds number $\frac{VLd}{m}$, L being the length parallel to the wind, and m the viscosity. Any self-consistent system of units may be used. The accompanying table was computed from this equation for standard density (0.002378 slugs per cubic foot) and viscosity $\left(\frac{\text{viscosity}}{\text{density}} = 0.0001535 \text{ sq. ft. per sec.}\right)$.

The values given in the table on next page differ somewhat from the older values of Zahm but are believed to be more reliable because of improvements in experimental methods. In any case the values apply strictly to plane surfaces parallel to the wind and application to curved surfaces can be expected to give only approximate results. The effect of roughness is comparatively large. The following papers may be consulted in addition to the one referred to:

Rept. Brit. Assoc. Advancement Sci., 1872, 1874. W. Froude.
Phil. Mag. 8 (6th Series) 1904, p. 58ff. A. F. Zahm.
Rept. 6, Part II, Nat. Advisory Comm. Aeronautics, W. A. Gibbons.
Engineering, May 30, 1924, p. 718. T. E. Stanton.

* Ergeb. Aerodyn. Versuchsanstalt, Göttingen, I, 1921.

Frictional Force per Square Foot for Various Speeds and Lengths of Surface

Wind speed, miles per hour	Average frictional force in pounds per square foot of exposed area (both sides) length, ft.					
	1	2	4	8	16	32
5	0.000476	0.000429	0.000387	0.000349	0.000314	0.000282
10	0.00172	0.00155	0.00140	0.00126	0.00113	0.00102
15	0.00364	0.00328	0.00295	0.00266	0.00240	0.00215
20	0.00619	0.00558	0.00503	0.00453	0.00408	0.00366
25	0.00935	0.00843	0.00760	0.00685	0.00617	0.00554
30	0.0131	0.0118	0.0107	0.00960	0.00865	0.00776
35	0.0174	0.0157	0.0142	0.0128	0.0115	0.0103
40	0.0223	0.0201	0.0181	0.0163	0.0147	0.0132
50	0.0337	0.0304	0.0274	0.0247	0.0223	0.0200
60	0.0473	0.0426	0.0384	0.0346	0.0312	0.0280
70	0.0629	0.0566	0.0510	0.0460	0.0415	0.0372
80	0.0805	0.0725	0.0654	0.0589	0.0531	0.0476
90	0.1000	0.0902	0.0813	0.0732	0.0660	0.0592
100	0.1216	0.1096	0.0987	0.0890	0.0802	0.0720

WEIGHTS AND MEASURES

25. The Metric System

Units of Length and Mass. The **Unit of Length** in the metric system is the meter, a length very nearly equal to the one ten-millionth of the distance between the equator and the pole. The original meter was a platinum bar constructed and deposited in the Archives of the French Republic in 1799. At the same time a platinum weight was constructed which was made as nearly as possible equal to the mass of a cube of pure water at 4° C., the sides of the cube being one-tenth the length of the meter. This weight, which is equal to one thousand units in the metric system, is called the kilogram.

The use of the meter as the basis of geodetic surveys had become so general throughout Europe that a conference was called in Paris, France, in 1870, for the purpose of establishing a central bureau where the standards of the different countries could be intercompared. As a result of this conference an International Bureau of Weights and Measures was established near Paris in 1875, by the concurrent action of the principal nations of the world. One of the first tasks undertaken by the Bureau was the construction of exact copies of the meter and kilogram deposited in the Archives. Thirty-one standard meters of iridio-platinum and forty kilograms of the same alloy were constructed and carefully compared with the standards of the Archives and with one another. This great work was completed in 1889, and the meter and kilogram which agreed most nearly with the original standards were called international prototypes, and were deposited at the International Bureau, where they are maintained to-day subject to the authority of the International Committee on Weights and Measures. The remaining meters and kilograms were distributed by lot to the different nations which contributed to the support of the Bureau. The United States secured two copies of the meter and two copies of the kilogram, which are in the custody of the Bureau of Standards at Washington. One of the meters, known as No. 27, and one kilogram, No. 20, were selected as the United States standards, while the other meter and kilogram are used as secondary standards. It was the declared intention of the International Committee that the various national prototypes should be returned to the International Bureau at regular intervals for the purpose of recomparing them with the international standards and with one another. In this way all measurements based upon metric standards throughout the world are ultimately referred to the international meter and kilogram.

The Unit of Capacity in the metric system is the liter, which is defined as the volume of one kilogram of pure water at the temperature of maximum density (4° C.) under a pressure of 76 cm. of mercury. For all practical purposes the cubic decimeter and the kilogram of water may be regarded as identical, the difference between them being less than three parts in one hundred thousand, the kilogram of water having the larger volume.

26. The English System

Units of Length and Mass. While it is the common impression that the customary system of weights and measures in use in the United States is identical with the system in everyday use in Great Britain, there are important differences in some of the units. For all practical purposes the units of length and mass are the same, although the fundamental standards are quite different in the two countries. In Great Britain the primary standard of length is the yard, a bronze bar in the custody of the Board of Trade, and preserved in the Standards Office, Westminster. In the United States the primary standard of length is the National prototype meter, the length of which is known in terms of the International prototype meter. The United States yard is defined in terms of the meter. In terms of the International meter, in England, one yard equals 36/39.370113 meters. In the United States the yard is defined as 3600/3937 meters. The difference amounts to only a little more than one part in 400 000 which is about the difference found in comparing the Imperial yard with its authentic copies at different periods. A similar condition exists with respect to the avoirdupois pound; in England a certain platinum cylinder deposited in the Standards Office is regarded as the standard pound, the legal equivalent of which is 453.59243 grams, whereas in the United States the pound is defined as 453.5924277 grams.

The Capacity Measures of the two countries are, however, quite different. The United States gallon of 231 cu. in. and the Winchester bushel of 2150.42 cu. in., the United States Standards, have not been legal measures in Great Britain since 1825, having been superseded by the present Imperial gallon, which is defined as the volume of 10 lb. of pure water at 62° F., weighed against brass weights in air at the same temperature as the water, and with the barometer at 30 in., and by the Imperial bushel which is defined as the volume of 80 lb. of water weighed under the same conditions as the gallon. The bushel of Great Britain is, therefore, exactly equal to eight gallons. The volume of the Imperial gallon, according to the most reliable data available at this time, is 277.420 cu. in. and the bushel 2219.36 cu. in., the first being larger than the United States gallon by approximately 20% and the latter larger than the United States bushel by 3.2%.

27. Chinese and Other Foreign Weights and Measures

In China in 1915 a new weights and measures law was promulgated, making the meter and kilogram the fundamental standards of length and mass, and recognizing two systems of weights and measures, namely, a native system and the metric system. Of the native system, the Ying Chao Ch'ih (builder's foot) is the unit of length, and the Kuping tael (or liang) is the unit of weight. The Ministry of Agriculture and Commerce is charged with the enforcement of the law and has authority to restrict the use of the native system whenever necessary.

Measures of length

Chinese		Metric	
1 Hao	=	0.0032	Centimeter
1 Li	=	0.032	Centimeter
1 Fen	=	0.32	Centimeter
1 Ts'un	=	3.2	Centimeters
1 Ch'ih	=	0.32	Meter
1 Pu	=	1.6	Meters
1 Chang	=	3.2	Meters
1 Li	=	576.	Meters

Measures of surface

Chinese		Metric	
1 Hao	=	0.006144	Are
1 Li	=	0.06144	Are
1 Fen	=	0.6144	Are
1 Mou	=	6.144	Ares
1 Ch'ing	=	614.4	Ares

Measures of capacity

Chinese		Metric	
1 Shao	=	0.0104	Liter
1 Ko	=	0.1035	Liter
1 Sheng	=	1.0355	Liters
1 Tou	=	10.355	Liters
1 Hu	=	51.7734	Liters
1 Tan or Picul	=	103.5469	Liters

Measures of weight

Chinese		Metric	
1 Hao	=	0.0037301	Gram
1 Li	=	0.037301	Gram
1 Fen	=	0.37301	Gram
1 Ch'ien	=	3.7301	Grams
1 Liang	=	37.301	Grams
1 Chin	=	596.816	Grams

Note. A common commercial unit in the foreign trade of China is the catty, equivalent to 1-1/3 pounds, avoirdupois.

In Japan. The metric system was adopted by Japan in 1921. The law is to be put into force by Imperial Ordinances, and it appears that this is being done gradually.

Following are equivalents of the native units of weights and measures:

The Kwan, equal to 3.75 kg. or 8.267 lb. avoirdupois, is the unit of mass. It is divided into 1000 mommes, the momme into 10 funs and the fun into 10 rins, the rin into 10 mos, and the mo into 10 shis.

The Shaku is the unit of length, equal to 10/33 (0.30303) meter, or 0.994 U. S. ft. The shaku is decimally divided into sun, bu, rin, mo, shi. Multiples of the shaku are the ken, equal to 6 shaku; the cho equal to 60 ken, and the ri, equal to 36 cho, or 12 960 shaku.

The shaku (land measure) is equal to 1/3025 are or 0.3558 sq. ft. One bu. or tsubu is equivalent to 100/3025 are or 35.583 sq. ft. Thirty bu. equal one sé, ten sé equal one tan and ten tan equal one cho.

The shaku (capacity measure) equals 0.01804 liter, or 0.01924 U. S. liquid quart, ten shaku equal one go, ten go one sho, ten sho one to, and ten to one koku.

In Russia the metric system has been adopted and is gradually being put into force.

Following are equivalents of the native units of weights and measures:

The Funt, equal to 0.4095 kg., or 0.9028 lb. avoirdupois, is the basis of Russian weights. The funt is divided into 96 zolotniks or 32 loths, and the zolotnik is divided into 96 dolias; 40 funts equal one pood.

The Archine, equal to 28 in. or 0.71120 meter, is the basis of the length measure. The archine contains 16 verskops; three archines equal one sagene and 500 sagenes equal one verst.

The dessiatine, equal to 2400 square sagenes or 2.6997 acres, is the unit for land measure.

The tchetvert, equal to 2.099 hectoliters, or 5.9568 U. S. bushels, is the unit for dry measure. The tchetvert is divided into 8 tchetveriks, and the tchetverik into 8 garnetz. Twelve tchetverts equal one last.

The vedro, equal to 12.2994 liters, or 3.2492 U. S. gallons, is used for liquids.

28. Special Commercial Units

For Logs the Doyle rule, known in some sections as the Connecticut River Rule, the St. Croix Rule, the Thurber Rule, the Moore and Beeman Rule, and the Scribner Rule, is more generally employed than any other: Deduct 4 in. from the diameter of the log, as an allowance for slab; square one-quarter of the remainder and multiply the result by the length of the log in feet. It is the usual custom to measure the diameter inside the bark at the small end.

The Miner's Inch is defined as the quantity of water that will pass through an orifice one square inch in cross-section under a given head. The head has been fixed in various localities as from 4 to 6-1/2 in. to the center of the orifice.

Bushel (dry measure) = 4 pecks = 32 quarts, 1 struck bushel = 1.24446 cu. ft. = 2150.42 cu. in.

Barrel. The crude oil barrel is 42 gallons; while the refined product is bought and sold on the basis of 50 gallons to the barrel. The large lime barrel contains 280 lb.; the small one 180 lb. Legal net weight of a barrel of flour = 196 lb. For natural cement barrels contain 150 lb. to 400 lb., for portland cement 376 lb. or 4 sacks of 94 lb. each.

A legal standard barrel for fruits, vegetables and other dry commodities, except cranberries, contains 7056 cu. in. Its outside dimensions are: Length of stave, 28-1/2 in.; distance between heads, 26 in.; diameter of head, 17-1/8 in.; circumference of bulge, 64 in.; outside measurement, and thickness of staves not greater than 0.4 in. For cranberries the corresponding dimensions of the barrel are 28-1/2 in., 25-1/4 in., 16-1/4 in., 58-1/2 in.

Firewood is mostly sold by the cord of 128 cu. ft.

Old Foreign Units, still sometimes used, with their U. S. equivalents, are as follows:

Central America: libra = 1.014 lb., arroba = 25.36 lb., quintal = 101.41 lb., vara = 32.87, 32.91, and 33 in.

Cuba: libra = 1.0143 lb., arroba = 25.36 lb., vara = 33.38 in.

Virgin Islands: pund = 1.023 lb., fod = 1.029 ft., tönde = 3.948 bu., pot = 0.2552 gal., mil = 4.68 miles.

Mexico: libra = 1.015 lb., arroba = 25.367 lb., vara = 32.992 in., quartillo = 1.999 liquid quarts.

South Africa: leaguer = 153.64 gal., muid = 3.095 bu., morgen = 2.116 acres.

South America: libra = 1.014 lb., arroba = 25.36 lb., quintal = 101.4 lb., vara = 32.91 and 33.38 in.

Spain: libra = 1.0143 lb., arroba = 25.36 lb., vara = 32 9096 in., fanega = 1.52 bu.

Turkey: old oke = 2.828 lb., old kilé = 1.135 bu., pic = about 27 in.

29. Equivalents of Units *

In the following tables the quantities in the same line are equivalents. For example, 1 kilogram equals 2.20462 pounds or 0.980665 megadynes.

Numbers in **boldface** type are exact values by definition. The notation 0.(³)1259 indicates that the (³) is to be replaced by three ciphers.

Density

Gm. per cu. cm.	Lb. per cu. ft.	Short tons per cu. yd.	Lb. per U. S. gal.
1	62.4281 1.79538*	0.84278 $\bar{1}$.92572*	8.3454 0.92145*
0.01602 $\bar{2}$.20462*	**1**	0.0135 $\bar{2}$.13033*	0.13368 $\bar{1}$.12607*
1.18655 0.07428*	74.074 1.86967*	**1**	9.9023 0.99574*
0.11983 $\bar{1}$.07855*	7.4805 0.87393*	0.10099 $\bar{1}$.00428*	**1**

* Logarithm of the number immediately above.

Force

Megadynes	Kilograms	Pounds
1	1.01972 0.008481*	2.248089 0.351813*
0.980665 $\bar{1}$.991490*	**1**	2.20462 0.343334*
0.444822 $\bar{1}$.648186*	0.453592 $\bar{1}$.656666*	**1**

1 Megadyne = 10^6 dynes

* Logarithm of the number immediately above.

* Arts. 28–30 have been taken from the previous edition without recomputing the values of the various units.

Length

1 meter = 10 decimeters = 100 centimeters = 1000 millimeters = 10^6 microns = 0.1 dekameter = 0.01 hectometer = 0.001 kilometer = 0.0001 myriameter.

1 U. S. yard = 3600/3937 meters (by definition); log = $\bar{1}$.9611371.

Meters	Inches	Feet	Yards	Links	Rods, poles, or perches	Chains, Gunter's	Statute miles U. S.	Nautical miles U. S.
1	39.37 1.59517*	3.2808 0.51598*	1.0936 0.03886*	4.971 0.69644*	0.1988 $\bar{1}$.29850*	0.04971 $\bar{2}$.69644*	0.$(^{3})$6214 $\bar{4}$.79335*	0.$(^{3})$5396 $\bar{4}$.73207*
0.0254 $\bar{2}$.40483*	1	0.08333 $\bar{2}$.92082*	0.02778 $\bar{2}$.44370*	0.1263 $\bar{1}$.10127*	0.005051 $\bar{3}$.70333*	0.001263 $\bar{3}$.10127*	0.$(^{4})$1578 $\bar{5}$.19818*	0.$(^{4})$1371 $\bar{5}$.13690
0.3048 $\bar{1}$.48402*	12 1.07918*	1	0.3333 $\bar{1}$.52288*	1.515 0.18046*	0.06061 $\bar{2}$.78252*	0.01515 $\bar{2}$.18046*	0.$(^{3})$1894 $\bar{4}$.27737*	0.$(^{3})$1645 $\bar{4}$.21608*
0.9144 $\bar{1}$.96114*	36 1.55630*	3 0.47712*	1	4.545 0.65758*	0.1818 $\bar{1}$.25964*	0.04545 $\bar{2}$.65758*	0.$(^{3})$5682 $\bar{4}$.75449*	0.$(^{3})$4934 $\bar{4}$.69320*
0.2012 $\bar{1}$.30356*	7.92 0.89873*	0.66 $\bar{1}$.81954*	0.22 $\bar{1}$.34242*	1	0.04 $\bar{2}$.60206*	0.01 $\bar{2}$.00000*	0.$(^{3})$1250 $\bar{4}$.09691*	0.$(^{3})$1086 $\bar{4}$.03564
5.029 0.70150*	198 2.29667*	16.5 1.21748*	5.5 0.74036*	25 1.39794*	1	0.25 $\bar{1}$.39794*	0.$(^{2})$3125 $\bar{3}$.49485*	0.$(^{2})$2714 $\bar{3}$.43357*
20.12 1.30356*	792 2.89873*	66 1.81954*	22 1.34242*	100 2.00000*	4 0.60206*	1	0.0125 $\bar{2}$.09691*	0.01086 $\bar{2}$.03564*
1609.3 3.20665*	63360 4.80182*	5280 3.72263*	1760 3.24551*	8000 3.90309*	320 2.50515*	80 1.90309*	1	0.8684 1.93872
1853.25 3.26793*	72962 4.86310*	6080.2 3.78392*	2026.73 3.30680*	9212 3.96437*	368.5 2.56643*	92.12 1.96437*	1.1516 0.06128*	1

1 nautical mile of the British admiralty = 6080 ft. 1 furlong = 1/8 mile = 660 feet. 1 league = 3 miles = 24 furlongs. 1 fathom = 2 yards = 6 feet.

* Logarithm of the number immediately above.

Area

1 hectare = 100 ares = 10 000 centares or square meters.

Square meters	Square inches	Square feet	Square yards	Square rods	Square chains	Acres	Square miles or sections
1	1550 3.19033*	10.764 1.03197*	1.1960 0.07773*	0.03954 $\bar{2}$.59700*	0.$(^{2})$2471 $\bar{3}$.39288*	0.$(^{3})$2471 $\bar{4}$.39288*	0.$(^{6})$3861 $\bar{7}$.38670*
0.$(^{3})$6452 $\bar{4}$.80967*	1	0.006944 $\bar{3}$.84164*	0.$(^{3})$7716 $\bar{4}$.88740*	0.$(^{4})$2551 $\bar{5}$.40667*	0.$(^{5})$1594 $\bar{6}$.20255*	0.$(^{6})$1594 $\bar{7}$.20255*	0.$(^{9})$2491 $\overline{10}$.39637*
0.09290 $\bar{2}$.96803*	144 2.15836*	1	0.1111 $\bar{1}$.04576*	0.$(^{2})$3673 $\bar{3}$.56503*	0.$(^{3})$2296 $\bar{4}$.36091*	0.$(^{4})$2296 $\bar{5}$.36091*	0.$(^{7})$3587 $\bar{8}$.55473*
0.8361 $\bar{1}$.92227*	1296 3.11260*	9 0.95424*	1	0.03306 $\bar{2}$.51927*	0.$(^{2})$2066 $\bar{3}$.31515*	0.$(^{3})$2066 $\bar{4}$.31515*	0.$(^{6})$3228 $\bar{7}$.50898*
25.29 1.40300*	39204 4.5933*	272.25 2.43497*	30.25 1.48072*	1	0.0625 $\bar{2}$.79588*	0.00625 $\bar{3}$.79588*	0.$(^{5})$9766 $\bar{6}$.98970*
404.69 2.60712*	627264 5.79745*	4356 3.63909*	484 2.68484*	16 1.20412*	1	0.1 $\bar{1}$.00000*	0.$(^{3})$1562 $\bar{4}$.19382*
4046.9 3.60712*	6272640 6.79745*	43560 4.63909*	4840 3.68484*	160 2.20412*	10 1.00000*	1	0.001562 $\bar{3}$.19382*
2589998 6.41330*		27878400 7.44527*	3097600 6.49102*	102400 5.01030*	6400 3.80618*	640 2.80618*	1

* Logarithm of the number immediately above.

Mass (Physicists' system) or **Force** (Engineers' system)

1 kilogram = 1000 grams = 0.001 metric ton. 1 gm. = 10 decigrams = 100 centigrams = 1000 milligrams = 0.1 dekagram = 0.01 hectogram = 0.001 kilogram = 0.0001 myriagram.

1 U. S. avoirdupois pound = 0.4535924277 kg. = (by definition) 7000/5760 troy pounds.

Kilograms	Grains	Ounces		Pounds		Tons		
		Avoir.	Troy and apoth.	Troy and apoth.	Avoir.	Short, 2000 lb.	Long, 2240 lb.	Metric, 1000 kg.
1	15432. 4.18843*	35.274 1.54745*	32.151 1.50719*	2.6792 0.42801*	2.20461 0.34333*	0.001102 $\bar{3}$.04230*	0.$(^{3})$9842 $\bar{4}$.99309*	**0.001** $\bar{3}$.00000*
0.$(^{4})$6480 $\bar{5}$.81157*	1	0.$(^{2})$2286 $\bar{3}$.35902*	0.002083 $\bar{3}$.31876*	0.$(^{3})$1736 $\bar{4}$.23958*	0.$(^{3})$142 $\bar{4}$.15490*			
0.028349 $\bar{2}$.45255*	**437.5** 2.64098*	1	0.91146 $\bar{1}$.95974*	0.075955 $\bar{2}$.88056*	**0.0625** $\bar{2}$.79588*	**0.$(^{4})$3125** $\bar{5}$.49485*	0.$(^{4})$2790 $\bar{5}$.44563*	0.$(^{4})$2835 $\bar{5}$.45255*
0.031103 $\bar{2}$.49281*	**480** 2.68124*	1.0971 0.04026*	1	0.083333 $\bar{2}$.92082*	0.068571 $\bar{2}$.83614*	0.$(^{4})$3429 $\bar{5}$.53511*	0.$(^{3})$3061 $\bar{4}$.56508*	0.$(^{4})$3110 $\bar{5}$.49281*
0.37324 $\bar{1}$.57199*	**5760** 3.76042*	13.166 1.11944*	**12** 1.07918*	1		0.$(^{3})$4114 $\bar{4}$.61429*	0.$(^{3})$3673 $\bar{4}$.56508*	0.$(^{3})$3732 $\bar{4}$.57199*
0.45359 $\bar{1}$.65667*	**7000** 3.84510*	**16** 1.20412*	14.583 1.16386*	1.2153 0.08468*	1	**0.0005** $\bar{4}$.69897*	0.$(^{3})$4464 $\bar{4}$.64975*	0.$(^{3})$4536 $\bar{4}$.65667*
907.18 2.95770*		**32000** 4.50515*	29167. 4.46489*	2430.6 3.38570*	**2000** 3.30103*	1	0.89286 $\bar{1}$.95078*	0.90718 $\bar{1}$.95770*
1016.1 3.00691*		**35840** 4.55437*	32667 4.51410*	2722.2 3.43492*	**2240** 3.35025*	**1.12** 0.04922*	1	1.0160 0.00691*
1000 3.00000*		35274 4.54745*	32151 4.50719*	2679.2 3.42801*	2204.6 3.34333*	1.1023 0.04230*	0.98421 $\bar{1}$.99309*	1

1 quarter = 28 lb. avoir. 1 pennyweight = 24 gr. = 0.05 oz. troy. 1 oz. avoir. = 16 drams avoir. = 437.5 gr. 1 stone = 14 lb. 1 cental = 100 lb. 1 hundredweight = 112 lb. 1 apothecaries' oz. = 8 apoth. drams = 24 scruples = 480 grains.

* Logarithm of the number immediately above.

Power

1 kilowatt = 1000 watts = 1000 joules per second.
1 horsepower = 550 foot-pounds per second.
1 cheval-vapeur = 75 kilogram-meters per second.

Kilowatts	Cheval-vapeur	Poncelet	Horse-power	M-kg. per sec.	Ft-lb. per sec.	Kg. cal. per sec.	B.t.u. per sec.
1	1.3600 0.13341*	1.0197 0.00848*	1.341 0.12743*	101.97 2.00848*	737.5 2.86780*	0.2388 $\bar{1}$.37803*	0.9475 $\bar{1}$.97660*
0.7355 $\bar{1}$.86659*	1	**0.75** $\bar{1}$.87506*	0.9863 $\bar{1}$.99402*	**75** 1.87506*	542.5 2.73438*	0.1756 $\bar{1}$.24456*	0.6969 $\bar{1}$.84318
0.980665 $\bar{1}$.99152*	1.333 0.12493*	1	1.3151 0.11896*	**100** 2.00000*	723.3 2.85932*	0.2342 $\bar{1}$.36951*	0.9292 $\bar{1}$.96812*
0.7457 $\bar{1}$.87257*	1.0139 0.00598*	0.7604 $\bar{1}$.88104*	1	76.04 1.88104*	**550** 2.74036*	0.1780 $\bar{1}$.25055*	0.7066 $\bar{1}$.84916*
0.009807 $\bar{3}$.99152*	0.01333 $\bar{2}$.12493*	**0.01** $\bar{2}$.00000*	0.01315 $\bar{2}$.11896*	1	7.233 0,85932*	0.002342 $\bar{3}$.36951*	0.009292 $\bar{3}$.96812*
0.001356 $\bar{3}$.13220*	0.001843 $\bar{3}$.26562*	0.00138 $\bar{3}$.14068*	0.001818 $\bar{3}$.25964*	0.1383 $\bar{1}$.14068*	1	0.0003237 $\bar{4}$.51016*	0.001285 $\bar{3}$.10880*
4.188 0.62201*	5.694 0.75542*	4.271 0.63049*	5.616 0.74945*	427.1 2.63049*	**3089** 3.48984*	1	3.968 0.59861*
1.055 0.02340*	1.435 0.15682*	1.076 0.03188*	1.415 0.15084*	107.62 2.03188*	778.4 2.89120*	0.2520 $\bar{1}$.40139*	1

* Logarithm of the number immediately above.

Speed and Velocity

Cm. per sec.	Km. per hour	Ft. per sec.	Ft. per min.	Miler per hour	Knots
1	0.036 $\bar{2}$.55630*	0.03281 $\bar{2}$.51598*	1.9685 0.29413*	0.02237 $\bar{2}$.34965*	0.01942 $\bar{2}$.28825*
27.7778 1.44370*	1	0.91134 $\bar{1}$.95968*	54.6806 1.73783*	0.62137 $\bar{1}$.79335*	0.53960 $\bar{1}$.73207*
30.4801 1.48402*	1.0973 0.40432*	1	60 1.77815*	0.68182 $\bar{1}$.83367*	0.59209 $\bar{1}$.77238*
0.5080 $\bar{1}$.70586*	0.01829 $\bar{2}$.26217*	0.01667 $\bar{2}$.22185*	1	0.01136 $\bar{2}$.05553*	0.009868 $\bar{3}$.99423*
44.7041 1.65035*	1.6093 0.20670*	1.46667 0.16633*	88 1.94448*	1	0.86839 $\bar{1}$.93872*
51.4971 1.71178*	1.8532 0.26793*	1.68894 0.22761*	101.337 2.00577*	1.15155 0.06128*	1

1 knot = 1 nautical mile per hour.

* Logarithm of the number immediately above.

Volume and Capacity

1 liter = 1 cubic decimeter = 1000 cubic centimeters = 10 deciliters = 100 centiliters = 1000 milliliters = 0.1 dekaliter = 0.01 hectoliter = 0.01 kiloliter = 0.001 cubic meters or steres.

Cubic inches	Cubic feet	Cubic yards	U. S. Quarts		U. S. Gallons		U. S. Bushels	Liters
			Liquid	Dry	Liquid	Dry †		
1	0.$^{(3)}$57870 $\bar{4}$.76246*	0.$^{(4)}$2143 $\bar{5}$.33109*	0.017316 $\bar{2}$.23845*	0.014881 $\bar{2}$.17263*	0.004329 $\bar{3}$.63639*	0.003720 $\bar{3}$.57057*	0.$^{(3)}$4650 $\bar{4}$.66748*	0.016387 $\bar{2}$.21450*
1728 3.23754*	1	0.037037 $\bar{2}$.56864*	29.922 1.47599*	25.714 1.41017*	7.4805 0.87393*	6.4285 0.80811*	0.80356 $\bar{1}$.90502*	28.317 1.45205*
46656 4.66891*	27 1.43136*	1	807.90 2.90736*	694.28 2.84153*	201.97 2.30530*	173.57 2.23948*	21.696 1.33638*	764.56 2.88341*
57.75 1.76155*	0.033420 $\bar{2}$.52401*	0.001238 $\bar{3}$.09026*	1	0.85937 $\bar{1}$.93418*	0.25 $\bar{1}$.39794*	0.21484 $\bar{1}$.33212*	0.026855 $\bar{2}$.42903*	0.94636 $\bar{1}$.97606*
67.201 1.82737*	0.038889 $\bar{2}$.58983*	0.001440 $\bar{3}$.15847*	1.1637 0.06582*	1	0.29091 $\bar{1}$.46376*	0.25 $\bar{1}$.39794*	0.03125 $\bar{2}$.49485*	1.1012 0.04188*
231 2.36361*	0.13368 $\bar{1}$.12607*	0.004951 $\bar{3}$.69471*	4 0.60206*	3.4375 0.53624*	1	0.85937 $\bar{1}$.93418*	0.10742 $\bar{1}$.03109*	3.7854 0.57812*
268.80 2.42943*	0.15556 $\bar{1}$.19189*	0.005761 $\bar{3}$.76053*	4.6546 0.66788*	4 0.60201*	1.1637 0.06582*	1	0.125 $\bar{1}$.09691*	4.4049 0.64394*
2150.4 3.33253*	1.2445 0.09498*	0.046091 $\bar{2}$.66362*	37.237 1.57097*	32 1.50515*	9.3092 0.96891*	8 0.90309*	1	35.239 1.54703*
61.023 1.78550*	0.035315 $\bar{2}$.54795*	0.001308 $\bar{3}$.11659*	1.0567 0.02394*	0.90808 $\bar{1}$.95812*	0.26417 $\bar{1}$.42188*	0.22702 $\bar{1}$.35606*	0.028377 $\bar{2}$.45297*	1

1 U. S. liquid quart = 2 pints = 8 gills = 32 fluid ounces = 256 fluid drams = 768 fluid scruples. 1 bushel = 4 pecks.

1 Imperial gallon = 1.201 U. S. gallons = 0.1605 cu. ft. = 4.5460 liters.

1 U. S. gallon = 0.8327 Imperial gallon. 1 cubic foot = 6.229 Imperial gallons.

1 British bushel = 1.2837 cubic feet.

Shipping Measure: 1 register ton = 100 cu. ft. I U. S. shipping ton = 40 cu. ft. 1 British shipping ton = 42 cu. ft.

* Logarithm of the number immediately above.

† Given for purposes of comparison only. The gallon is not a legal unit of dry measure.

Pressure

Kilograms per sq. cm.	Pounds: Per sq. in.	Pounds: Per sq. ft.	Short tons, per sq. ft.	Atmospheres	Columns of mercury ‡: Meters	Columns of mercury ‡: Inches	Columns of water †: Meters	Columns of water †: Feet
1	14.223 1.15300*	2048.2 3.31137*	1.0241 0.01034*	0.96781 $\bar{1}$.98579*	0.73553 $\bar{1}$.86660*	28.958 1.46177*	10.009 1.00038*	32.837 1.51636*
0.070307 $\bar{2}$.84700*	1	144 2.15836*	0.072 $\bar{2}$.85733*	0.06804 $\bar{2}$.83279*	0.051713 $\bar{2}$.71360*	2.0359 0.30876*	0.70368 $\bar{1}$.84738*	2.3087 0.36336*
0.(3)4882 $\bar{4}$.68863*	0.006944 $\bar{3}$.84164*	1	0.0005 $\bar{4}$.69897*	0.(3)4725 $\bar{4}$.67442*	0.(3)3591 $\bar{4}$.55524*	0.014138 $\bar{2}$.15040*	0.004887 $\bar{3}$.68901*	0.016032 $\bar{2}$.20500*
0.97648 $\bar{1}$.98966*	13.889 1.14267*	2000 3.30103*	1	0.94504 $\bar{1}$.97545*	0.71823 $\bar{1}$.85627*	28.277 1.45143*	9.7734 0.99004*	32.065 1.50603*
1.0333 0.01421*	14.697 1.16722*	2116.3 3.32558*	1.0582 0.02955*	1	0.76 $\bar{1}$.88081*	29.921 1.47598*	10.342 1.01459*	33.929 1.53058*
1.3596 0.13340*	19.338 1.28640*	2784.6 3.44476*	1.3923 0.14373*	1.3158 0.11919*	1	39.37 1.59517*	13.607 1.13378*	44.644 1.64976*
0.034533 $\bar{2}$.53823*	0.49118 $\bar{1}$.69124*	70.729 1.84960*	0.035365 $\bar{2}$.54857*	0.033421 $\bar{2}$.52402*	0.025400 $\bar{2}$.40484*	1	0.34563 $\bar{1}$.53861*	1.1340 0.05460*
0.099913 $\bar{2}$.99962*	1.4211 0.15262*	204.64 2.31099*	0.10232 $\bar{1}$.00996*	0.096697 $\bar{2}$.98541*	0.073489 $\bar{2}$.86622*	2.8933 0.46139*	1	3.2808 0.51598*
0.030453 $\bar{2}$.48364*	0.43315 $\bar{1}$.63664*	62.374 1.79500*	0.031187 $\bar{2}$.49397*	0.029473 $\bar{2}$.46942*	0.022399 $\bar{2}$.35024*	0.88187 $\bar{1}$.94540*	0.30480 $\bar{1}$.48402*	1

* Logarithm of the number immediately above. † At 15° C. and $g = g_0$. ‡ At 0° C.

Energy

Joules $= 10^7$ erg	Meter-kilograms	Foot-pounds	Kilowatt-hours	Cheval-vapeur-hours	Horse-power-hours	British thermal units
1	0.10197 $\bar{1}$.00848*	0.73756 $\bar{1}$.86780*	0.(6)27778 $\bar{7}$.44370*	0.(6)37767 $\bar{7}$.57711*	0.(6)37251 $\bar{7}$.57113*	0.(3)9475 $\bar{4}$.97660*
9.80665 0.9915207*	1	7.2330 0.85932*	0.(5)27241 6.43522*	0.(5)37937 $\bar{6}$.56863*	0.(5)36530 $\bar{6}$.56265*	0.009292 $\bar{3}$.96812*
1.3558 0.13220*	0.13826 $\bar{1}$.14068*	1	0.(6)37662 $\bar{7}$.57590*	0.(6)51206 $\bar{7}$.70932*	0.(6)50505 $\bar{7}$.70333*	0.001285 $\bar{3}$.10880*
3.6×10^6 6.55630*	3.6710×10^5 5.56478*	2.6552×10^6 6.42410*	1	1.3596 0.13342*	1.3410 0.12743*	3411. 3.53290*
2.6478×10^6 6.42288*	270000 5.43136*	1.9529×10^6 6.29068*	0.73550 $\bar{1}$.86658*	1	0.98631 $\bar{1}$.99401*	2509. 3.39948*
2.6845×10^6 6.42887*	2.7375×10^5 5.43735*	1.98×10^6 6.29667*	0.74571 $\bar{1}$.87257*	1.0139 0.00598*	1	2544. 3.40547*
1055. 3.02340*	107.6 2.03188*	778.4 2.89120*	$0.0_3 2932$ $\bar{4}$.46710*	$0.0_3 3986$ $\bar{4}$.60051*	$0.0_3 3931$ $\bar{4}$.59453*	1

* Logarithm of the number immediately above.

30. Conversion Tables

The following tables give multiples for transferring English to metric measures or metric to English measures. For example, to reduce 39.37 in. to centimeters, take two numbers from the table at foot of page 314 and add them together, thus 39 in. = 99.06 cm., and 0.37 in. = 0.94 cm.; then 39.37 in. = 99.06 + 0.94 = 100.00 cm.

INDEX TO CONVERSION TABLES

LENGTH

AREA

VOLUME AND CAPACITY

WEIGHT AND MASS

PRESSURE

VELOCITY AND SPEED

ENERGY AND WORK

POWER

Cubic Feet to U. S. Liquid Gallons

Cu. Ft.	0	1	2	3	4	5	6	7	8	9
		7.4805	14.961	22.442	29.922	37.403	44.883	52.364	59.844	67.325
10	74.805	82.286	89.766	97.247	104.73	112.21	119.69	127.17	134.65	142.13
20	149.61	157.09	164.57	172.05	179.53	187.01	194.49	201.97	209.45	216.94
30	224.42	231.90	239.38	246.86	254.34	261.82	269.30	276.78	284.26	291.74
40	299.22	306.70	314.18	321.66	329.14	336.62	344.10	351.58	359.06	366.55
50	374.03	381.51	388.99	396.47	403.95	411.43	418.91	426.39	433.87	441.35
60	448.83	456.31	463.79	471.27	478.75	486.23	493.71	501.19	508.68	516.16
70	523.64	531.12	538.60	546.08	553.56	561.04	568.52	576.00	583.48	590.96
80	598.44	605.92	613.40	620.88	628.36	635.84	643.32	650.81	658.29	665.77
90	673.25	680.73	688.21	695.69	703.17	710.65	718.13	725.61	733.09	740.57

U. S. Liquid Gallons to Cubic Feet

U.S.Gal.	0	1	2	3	4	5	6	7	8	9
		0.1337	0.2674	0.4010	0.5347	0.6684	0.8021	0.9358	1.0694	1.2031
10	1.3368	1.4705	1.6042	1.7378	1.8715	2.0052	2.1389	2.2726	2.4063	2.5399
20	2.6736	2.8073	2.9410	3.0747	3.2083	3.3420	3.4757	3.6094	3.7431	3.8767
30	4.0104	4.1441	4.2778	4.4115	4.5451	4.6788	4.8125	4.9462	5.0799	5.2135
40	5.3472	5.4809	5.6146	5.7483	5.8819	6.0156	6.1493	6.2830	6.4167	6.5503
50	6.6840	6.8177	6.9514	7.0851	7.2188	7.3524	7.4861	7.6198	7.7535	7.8872
60	8.0208	8.1545	8.2882	8.4219	8.5556	8.6892	8.8229	8.9566	9.0903	9.2240
70	9.3576	9.4913	9.6250	9.7587	9.8924	10.026	10.160	10.293	10.427	10.561
80	10.694	10.828	10.962	11.095	11.229	11.363	11.497	11.630	11.764	11.898
90	12.031	12.165	12.299	12.532	12.566	12.700	12.833	12.967	13.101	13.234

Imperial Gallons to U. S. Liquid Gallons

Imp.Gal.	0	1	2	3	4	5	6	7	8	9
		1.2010	2.4019	3.6029	4.8038	6.0048	7.2057	8.4067	9.6076	10.809
10	12.010	13.210	14.411	15.612	16.813	18.014	19.215	20.416	21.617	22.818
20	24.019	25.220	26.421	27.622	28.823	30.024	31.225	32.426	33.627	34.828
30	36.029	37.230	38.430	39.631	40.832	42.033	43.234	44.435	45.636	46.837
40	48.038	49.239	50.440	51.641	52.842	54.043	55.244	56.445	57.646	58.847
50	60.048	61.249	62.450	63.650	64.851	66.052	67.253	68.454	69.655	70.856
60	72.057	73.258	74.459	75.660	76.861	78.062	79.263	80.464	81.665	82.866
70	84.067	85.268	86.469	87.669	88.870	90.071	91.272	92.473	93.674	94.875
80	96.076	97.277	98.478	99.679	100.88	102.08	103.28	104.48	105.68	106.88
90	108.09	109.29	110.49	111.69	111.89	114.09	115.29	116.49	117.69	118.89

U. S. Liquid Gallons to Imperial Gallons

U.S.Gal.	0	1	2	3	4	5	6	7	8	9
		0.8327	1.6653	2.4980	3.3307	4.1634	4.9960	5.8287	6.6614	7.4941
10	8.3267	9.1594	9.9921	10.825	11.657	12.490	13.323	14.155	14.988	15.821
20	16.653	17.486	18.319	19.151	19.984	20.817	21.649	22.482	23.315	24.148
30	24.980	25.813	26.646	27.478	28.311	29.144	29.976	30.809	31.642	32.474
40	33.307	34.140	34.972	35.805	36.638	37.477	38.303	39.136	39.968	40.801
50	41.634	42.466	43.299	44.132	44.964	45.797	46.630	47.462	48.295	49.128
60	49.960	50.793	51.626	52.458	53.291	54.124	54.956	55.789	56.622	57.454
70	58.287	59.120	59.952	60.785	61.618	62.450	63.283	64.116	64.948	65.781
80	66.614	67.446	68.279	69.112	69.944	70.777	71.610	72.443	73.275	74.108
90	74.941	75.773	76.606	77.439	78.271	79.104	79.937	80.769	81.602	82.435

Kilograms to Pounds Avoirdupois

Kg.	0	1	2	3	4	5	6	7	8	9
		2.20	4.41	6.61	8.82	11.02	13.23	15.43	17.64	19.84
10	22.05	24.25	26.46	28.66	30.86	33.07	35.27	37.48	39.68	41.89
20	44.09	46.30	48.50	50.71	52.91	55.12	57.32	59.52	61.73	63.93
30	66.14	68.34	70.55	72.75	74.96	77.16	79.37	81.57	83.78	85.98
40	88.18	90.39	92.59	94.80	97.00	99.21	101.41	103.62	105.82	108.03
50	110.23	112.44	114.64	116.84	119.05	121.25	123.46	125.66	127.87	130.07
60	132.28	134.48	136.69	138.89	141.10	143.30	145.51	147.71	149.91	152.12
70	154.32	156.53	158.73	160.94	163.14	165.35	167.55	169.76	171.96	174.17
80	176.37	178.57	180.78	182.98	185.19	187.39	189.60	191.80	194.01	196.21
90	198.42	200.62	202.83	205.03	207.23	209.44	211.64	213.85	216.05	218.26

Pounds Avoirdupois to Kilograms

Lbs.	0	1	2	3	4	5	6	7	8	9
		0.4536	0.9072	1.361	1.814	2.268	2.722	3.175	3.629	4.082
10	4.536	4.990	5.443	5.897	6.350	6.804	7.257	7.711	8.165	8.618
20	9.072	9.525	9.979	10.433	10.886	11.340	11.793	12.247	12.701	13.154
30	13.608	14.061	14.515	14.969	15.422	15.876	16.329	16.783	17.237	17.690
40	18.144	18.597	19.051	19.504	19.958	20.412	20.865	21.319	21.772	22.226
50	22.680	23.133	23.587	24.040	24.494	24.948	25.401	25.855	26.308	26.762
60	27.216	27.669	28.123	28.576	29.030	29.484	29.937	30.391	30.844	31.298
70	31.751	32.205	32.659	33.112	33.566	34.019	34.473	34.927	35.380	35.834
80	36.287	36.741	37.195	37.648	38.102	38.555	39.009	39.463	39.916	40.370
90	40.823	41.277	41.731	42.184	42.638	43.091	43.545	43.998	44.452	44.906

Metric Tons to Short Tons (2000 Pounds)

Metric tons	0	1	2	3	4	5	6	7	8	9
		1.102	2.205	3.307	4.409	5.512	6.614	7.716	8.818	9.921
10	11.023	12.125	13.228	14.330	15.432	16.535	17.637	18.739	19.842	20.944
20	22.046	23.149	24.251	25.353	26.455	27.558	28.660	29.762	30.865	31.967
30	33.069	34.172	35.274	36.376	37.479	38.581	39.683	40.786	41.888	42.990
40	44.092	45.195	46.297	47.399	48.502	49.604	50.706	51.809	52.911	54.013
50	55.116	56.218	57.320	58.422	59.525	60.627	61.729	62.832	63.934	65.036
60	66.139	67.241	68.343	69.446	70.548	71.650	72.753	73.855	74.957	76.059
70	77.162	78.264	79.366	80.469	81.571	82.673	83.776	84.878	85.980	87.083
80	88.185	89.287	90.390	91.492	92.594	93.696	94.799	95.901	97.003	98.106
90	99.208	100.31	101.41	102.51	103.62	104.72	105.82	106.92	108.03	109.13

Short Tons (2000 Pounds) to Metric Tons

Short tons	0	1	2	3	4	5	6	7	8	9
		0.907	1.814	2.722	3.629	4.536	5.443	6.350	7.257	8.165
10	9.072	9.979	10.886	11.793	12.701	13.608	14.515	15.422	16.329	17.237
20	18.144	19.051	19.958	20.865	21.772	22.680	23.587	24.494	25.401	26.308
30	27.216	28.123	29.030	29.937	30.844	31.751	32.659	33.566	34.473	35.380
40	36.287	37.195	38.102	39.009	39.916	40.823	41.731	42.638	43.545	44.452
50	45.359	46.266	47.174	48.081	48.988	49.895	50.802	51.710	52.617	53.524
60	54.431	55.338	56.245	57.153	58.060	58.967	59.874	60.781	61.689	62.596
70	63.503	64.410	65.317	66.224	67.132	68.039	68.946	69.853	70.760	71.668
80	72.575	73.482	74.389	75.296	76.204	77.111	78.018	78.925	79.832	80.739
90	81.647	82.554	83.461	84.368	85.275	86.183	87.090	87.997	88.904	89.811

Metric Tons to Long Tons (2240 Pounds)

Metric tons	0	1	2	3	4	5	6	7	8	9
		0.984	1.968	2.953	3.937	4.921	5.905	6.889	7.874	8.858
10	9.842	10.826	11.810	12.795	13.779	14.763	15.747	16.732	17.716	18.700
20	19.684	20.668	21.653	22.637	23.621	24.605	25.589	26.574	27.558	28.542
30	29.526	30.510	31.495	32.479	33.463	34.447	35.431	36.416	37.400	38.384
40	39.368	40.352	41.337	42.321	43.305	44.289	45.273	46.258	47.242	48.226
50	49.210	50.195	51.179	52.163	53.147	54.131	55.116	56.100	57.084	58.068
60	59.052	60.037	61.021	62.005	62.989	63.973	64.958	65.942	66.926	67.910
70	68.894	69.879	70.863	71.847	72.831	73.815	74.800	75.784	76.768	77.752
80	78.737	79.721	80.705	81.689	82.673	83.658	84.642	85.626	86.610	87.594
90	88.579	89.563	90.547	91.531	92.515	93.500	94.484	95.468	96.452	97.436

Long Tons (2240 Pounds) to Metric Tons

Long tons	0	1	2	3	4	5	6	7	8	9
		1.016	2.022	3.048	4.064	5.080	6.096	7.112	8.128	9.144
10	10.160	11.177	12.193	13.209	14.225	15.241	16.257	17.273	18.289	19.305
20	20.321	21.337	22.353	23.369	24.385	25.401	26.417	27.433	28.449	29.465
30	30.481	31.497	32.514	33.530	34.546	35.562	36.578	37.594	38.610	39.626
40	40.642	41.658	42.674	43.690	44.706	45.722	46.738	47.754	48.770	49.786
50	50.802	51.818	52.834	53.850	54.867	55.883	56.899	57.915	58.931	59.947
60	60.963	61.979	62.995	64.011	65.027	66.043	67.059	68.075	69.091	70.107
70	71.123	72.139	73.155	74.171	75.187	76.204	77.220	78.236	79.252	80.268
80	81.284	82.300	83.316	84.332	85.348	86.364	87.380	88.396	89.412	90.428
90	91.444	92.460	93.476	94.492	95.508	96.524	97.541	98.557	99.573	100.59

Fractions of an Inch to Millimeters

16ths	32nds	64ths	Millimeters	16ths	32nds	64ths	Millimeters	16ths	32nds	64ths	Millimeters
		1	0.397			23	9.128			45	17.859
	1	2	0.794	6	12	24	9.525		23	46	18.256
		3	1.191			25	9.922			47	18.653
1	2	4	1.588		13	26	10.319	12	24	48	19.050
		5	1.984			27	10.716			49	19.447
	3	6	2.381	7	14	28	11.113		25	50	19.844
		7	2.778			29	11.509			51	20.241
2	4	8	3.175		15	30	11.906	13	26	52	20.638
		9	3.572			31	12.303			53	21.034
	5	10	3.969	8	16	32	12.700		27	54	21.431
		11	4.366			33	13.097			55	21.828
3	6	12	4.763		17	34	13.494	14	28	56	22.225
		13	5.159			35	13.891			57	22.622
	7	14	5.556	9	18	36	14.288		29	58	23.019
		15	5.953			37	14.684			59	23.416
4	8	16	6.350		19	38	15.081	15	30	60	23.813
		17	6.747			39	15.478			61	24.209
	9	18	7.144	10	20	40	15.875		31	62	24.606
		19	7.541			41	16.272			63	25.003
5	10	20	7.938		21	42	16.669	16	32	64	25.400
		21	8.334			43	17.066				
	11	22	8.731	11	22	44	17.463				

Millimeters to Decimals of an Inch

Millimeters	0	1	2	3	4	5	6	7	8	9
		.0394	.0787	.1181	.1575	.1968	.2362	.2756	.3150	.3543
10	0.3937	.4331	.4724	.5118	.5512	.5905	.6299	.6693	.7087	.7480
20	0.7874	.8268	.8661	.9055	.9449	.9842	1.0236	1.0630	1.1024	1.1417
30	1.1811	1.2205	1.2598	1.2992	1.3386	1.3780	1.4173	1.4567	1.4961	1.5354
40	1.5748	1.6142	1.6534	1.6929	1.7323	1.7716	1.8110	1.8504	1.8898	1.9291
50	1.9685	2.0079	2.0472	2.0866	2.1260	2.1654	2.2047	2.2441	2.2835	2.3228
60	2.3622	2.4016	2.4409	2.4803	2.5197	2.5590	2.5984	2.6378	2.6772	2.7165
70	2.7559	2.7953	2.8346	2.8740	2.9134	2.9528	2.9921	3.0315	3.0709	3.1102
80	3.1496	3.1890	3.2283	3.2677	3.3071	3.3464	3.3858	3.4252	3.4646	3.5039
90	3.5433	3.5827	3.6220	3.6614	3.7008	3.7402	3.7795	3.8189	3.8583	3.8976

Hundredths of an Inch to Millimeters

100ths of an inch	0	1	2	3	4	5	6	7	8	9
		0.254	0.508	0.762	1.016	1.270	1.524	1.778	2.032	2.286
10	2.540	2.794	3.048	3.302	3.556	3.810	4.064	4.318	4.572	4.826
20	5.080	5.334	5.588	5.842	6.096	6.350	6.604	6.858	7.112	7.366
30	7.620	7.874	8.128	8.382	8.636	8.890	9.144	9.398	9.652	9.906
40	10.160	10.414	10.668	10.922	11.176	11.430	11.684	11.938	12.192	12.446
50	12.700	12.954	13.208	13.462	13.716	13.970	14.224	14.478	14.732	14.986
60	15.240	15.494	15.748	16.002	16.256	16.510	16.764	17.018	17.272	17.526
70	17.780	18.034	18.288	18.542	18.796	19.050	19.304	19.558	19.812	20.066
80	20.320	20.574	20.828	21.082	21.336	21.590	21.844	22.098	22.352	22.606
90	22.860	23.114	23.368	23.622	23.876	24.130	24.384	24.638	24.892	25.146

Centimeters to Inches

Centimeters	0	1	2	3	4	5	6	7	8	9
		0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543
10	3.937	4.331	4.724	5.118	5.512	5.906	6.299	6.693	7.087	7.480
20	7.874	8.268	8.661	9.055	9.449	9.843	10.236	10.630	11.024	11.417
30	11.811	12.205	12.598	12.992	13.386	13.780	14.173	14.567	14.961	15.354
40	15.748	16.142	16.535	16.929	17.323	17.717	18.110	18.504	18.898	19.291
50	19.685	20.079	20.472	20.866	21.260	21.654	22.047	22.441	22.835	23.228
60	23.622	24.016	24.409	24.803	25.197	25.591	25.984	26.378	26.772	27.165
70	27.559	27.953	28.346	28.740	29.134	29.528	29.921	30.315	30.709	31.102
80	31.496	31.890	32.283	32.677	33.071	33.465	33.858	34.252	34.646	35.039
90	35.433	35.827	36.220	36.614	37.008	37.402	37.795	38.189	38.583	38.976

Inches to Centimeters

Inches	0	1	2	3	4	5	6	7	8	9
		2.54	5.08	7.62	10.16	12.70	15.24	17.78	20.32	22.86
10	25.40	27.94	30.48	33.02	35.56	38.10	40.64	43.18	45.72	48.26
20	50.80	53.34	55.88	58.42	60.96	63.50	66.04	68.58	71.12	73.66
30	76.20	78.74	81.28	83.82	86.36	88.90	91.44	93.98	96.52	99.06
40	101.60	104.14	106.68	109.22	111.76	114.30	116.84	119.38	121.92	124.46
50	127.00	129.54	132.08	134.62	137.16	139.70	142.24	144.78	147.32	149.86
60	152.40	154.94	157.48	160.02	162.56	165.10	167.64	170.18	172.72	175.26
70	177.80	180.34	182.88	185.42	187.96	190.50	193.04	195.58	198.12	200.66
80	203.20	205.74	208.28	210.82	213.36	215.90	218.44	220.98	223.52	226.06
90	228.60	231.14	233.68	236.22	238.76	241.30	243.84	246.38	248.92	251.46

Meters to Feet

Meters	0	1	2	3	4	5	6	7	8	9
		3.28	6.56	9.84	13.12	16.40	19.68	22.97	26.25	29.53
10	32.81	36.09	39.37	42.65	45.93	49.21	52.49	55.77	59.06	62.34
20	65.62	68.90	72.18	75.46	78.74	82.02	85.30	88.58	91.86	95.14
30	98.42	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.95
40	131.23	134.51	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164.04	167.32	170.60	173.88	177.16	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.41	206.69	209.97	213.25	216.54	219.82	223.10	226.38
70	229.66	232.94	236.22	239.50	242.78	246.06	249.34	252.62	255.90	259.19
80	262.47	265.75	269.03	272.31	275.59	278.87	282.15	285.43	288.71	291.99
90	295.28	298.56	301.84	305.12	308.40	311.68	314.96	318.24	321.52	324.80

Feet to Meters

Feet	0	1	2	3	4	5	6	7	8	9
		0.305	0.610	0.914	1.219	1.524	1.829	2.134	2.438	2.743
10	3.048	3.353	3.658	3.962	4.267	4.572	4.877	5.182	5.486	5.791
20	6.096	6.401	6.706	7.010	7.315	7.620	7.925	8.230	8.534	8.839
30	9.144	9.449	9.754	10.058	10.363	10.668	10.973	11.278	11.582	11.887
40	12.192	12.497	12.802	13.106	13.411	13.716	14.021	14.326	14.630	14.935
50	15.240	15.545	15.850	16.154	16.459	16.764	17.069	17.374	17.678	17.983
60	18.288	18.593	18.898	19.202	19.507	19.812	20.117	20.422	20.726	21.031
70	21.336	21.641	21.946	22.250	22.555	22.860	23.165	23.470	23.774	24.079
80	24.384	24.689	24.994	25.298	25.603	25.908	26.213	26.518	26.822	27.127
90	27.432	27.737	28.042	28.346	28.651	28.956	29.261	29.566	29.870	30.175

Kilometers to U. S. Statute Miles

Km	0	1	2	3	4	5	6	7	8	9
		0.621	1.243	1.864	2.485	3.107	3.728	4.350	4.971	5.592
10	6.214	6.835	7.456	8.078	8.699	9.321	9.942	10.563	11.185	11.806
20	12.427	13.049	13.670	14.292	14.913	15.534	16.156	16.777	17.398	18.020
30	18.641	19.262	19.884	20.505	21.127	21.748	22.369	22.991	23.612	24.233
40	24.855	25.476	26.098	26.719	27.340	27.962	28.583	29.204	29.826	30.447
50	31.068	31.690	32.311	32.933	33.554	34.175	34.797	35.418	36.039	36.661
60	37.282	37.904	38.525	39.146	39.768	40.389	41.010	41.632	42.253	42.875
70	43.496	44.117	44.739	45.360	45.981	46.603	47.224	47.845	48.467	49.088
80	49.710	50.331	50.952	51.574	52.195	52.816	53.438	54.059	54.681	55.302
90	55.923	56.545	57.166	57.787	58.409	59.030	59.652	60.273	60.894	61.516

U. S. Statute Miles to Kilometers

Miles	0	1	2	3	4	5	6	7	8	9
		1.61	3.22	4.83	6.44	8.05	9.66	11.27	12.87	14.48
10	16.09	17.70	19.31	20.92	22.53	24.14	25.75	27.36	28.97	30.58
20	32.19	33.80	35.41	37.01	38.62	40.23	41.84	43.45	45.06	46.67
30	48.28	49.89	51.50	53.11	54.72	56.33	57.94	59.55	61.16	62.76
40	64.37	65.98	67.59	69.20	70.81	72.42	74.03	75.64	77.25	78.86
50	80.47	82.08	83.69	85.30	86.90	88.51	90.12	91.73	93.34	94.95
60	96.56	98.17	99.78	101.39	103.00	104.61	106.22	107.83	109.44	111.04
70	112.65	114.26	115.87	117.48	119.09	120.70	122.31	123.92	125.53	127.14
80	128.75	130.36	131.97	133.58	135.19	136.79	138.40	140.01	141.62	143.23
90	144.84	146.45	148.06	149.67	151.28	152.89	154.50	156.11	157.72	159.33

Square Centimeters to Square Inches

Sq. cm.	0	1	2	3	4	5	6	7	8	9
		0.155	0.310	0.465	0.620	0.775	0.930	1.085	1.240	1.395
10	1.550	1.705	1.860	2.015	2.170	2.325	2.480	2.635	2.790	2.945
20	3.100	3.255	3.410	3.565	3.720	3.875	4.030	4.185	4.340	4.495
30	4.650	4.805	4.960	5.115	5.270	5.425	5.580	5.735	5.890	6.045
40	6.200	6.355	6.510	6.665	6.820	6.975	7.130	7.285	7.440	7.595
50	7.750	7.905	8.060	8.215	8.370	8.525	8.680	8.835	8.990	9.145
60	9.300	9.455	9.610	9.765	9.920	10.075	10.230	10.385	10.540	10.695
70	10.850	11.005	11.160	11.315	11.470	11.625	11.780	11.935	12.090	12.245
80	12.400	12.555	12.710	12.865	13.020	13.175	13.330	13.485	13.640	13.795
90	13.950	14.105	14.260	14.415	14.570	14.725	14.880	15.035	15.190	15.345

Square Inches to Square Centimeters

Sq. in.	0	1	2	3	4	5	6	7	8	9
		6.45	12.90	19.35	25.81	32.26	38.71	45.16	51.61	58.06
10	64.52	70.97	77.42	83.87	90.32	96.77	103.23	109.68	116.13	122.58
20	129.03	135.48	141.94	148.39	154.84	161.29	167.74	174.19	180.65	187.10
30	193.55	200.00	206.45	212.90	219.36	225.81	232.26	238.71	245.16	251.61
40	258.07	264.52	270.97	277.42	283.87	290.32	296.77	303.23	309.68	316.13
50	322.58	329.03	335.48	341.94	348.39	354.84	361.29	367.74	374.19	380.65
60	387.10	393.55	400.00	406.45	412.90	419.36	425.81	432.26	438.71	445.16
70	451.61	458.07	464.52	470.97	477.42	483.87	490.32	496.78	503.23	509.68
80	516.13	522.58	529.03	535.48	541.94	548.39	554.84	561.29	567.74	574.19
90	580.65	587.10	593.55	600.00	606.45	612.90	619.36	625.81	632.26	638.71

Square Meters to Square Yards

Sq. m.	0	1	2	3	4	5	6	7	8	9
		1.196	2.392	3.588	4.784	5.980	7.176	8.372	9.568	10.764
10	11.960	13.156	14.352	15.548	16.744	17.940	19.136	20.332	21.528	22.724
20	23.920	25.116	26.312	27.508	28.704	29.900	31.096	32.292	33.488	34.684
30	35.880	37.076	38.272	39.468	40.663	41.859	43.055	44.251	45.447	46.643
40	47.839	49.035	50.231	51.427	52.623	53.819	55.015	56.211	57.407	58.603
50	59.799	60.995	62.191	63.387	64.583	65.779	66.975	68.171	69.367	70.563
60	71.759	72.955	74.151	75.347	76.543	77.739	78.935	80.131	81.327	82.523
70	83.719	84.915	86.111	87.307	88.503	89.699	90.895	92.091	93.287	94.483
80	95.679	96.875	98.071	99.267	100.46	101.66	102.85	104.05	105.25	106.44
90	107.64	108.83	110.03	111.23	112.42	113.62	114.81	116.01	117.21	118.40

Square Yards to Square Meters

Sq. yd.	0	1	2	3	4	5	6	7	8	9
		0.836	1.672	2.508	3.345	4.181	5.017	5.853	6.689	7.525
10	8.361	9.197	10.034	10.870	11.706	12.542	13.378	14.214	15.050	15.886
20	16.723	17.559	18.395	19.231	20.067	20.903	21.739	22.576	23.412	24.248
30	25.084	25.920	26.756	27.592	28.428	29.265	30.101	30.937	31.773	32.609
40	33.445	34.281	35.117	35.954	36.790	37.626	38.462	39.298	40.134	40.970
50	41.807	42.643	43.479	44.315	45.151	45.987	46.823	47.659	48.496	49.332
60	50.168	51.004	51.840	52.676	53.512	54.348	55.185	56.021	56.857	57.693
70	58.529	59.365	60.201	61.038	61.874	62.710	63.546	64.382	65.218	66.054
80	66.890	67.727	68.563	69.399	70.235	71.071	71.907	72.743	73.580	74.416
90	75.252	76.088	76.924	77.760	78.596	79.432	80.269	81.105	81.941	82.777

Square Kilometers to Square Miles

Sq. km.	0	1	2	3	4	5	6	7	8	9
		0.386	0.772	1.158	1.544	1.931	2.317	2.703	3.089	3.475
10	3.861	4.247	4.633	5.019	5.405	5.792	6.178	6.564	6.950	7.336
20	7.722	8.108	8.494	8.880	9.266	9.653	10.039	10.425	10.811	11.197
30	11.583	11.969	12.355	12.741	13.127	13.514	13.900	14.286	14.672	15.058
40	15.444	15.830	16.216	16.602	16.988	17.375	17.761	18.147	18.533	18.919
50	19.305	19.691	20.077	20.463	20.849	21.236	21.622	22.008	22.394	22.780
60	23.166	23.552	23.938	24.324	24.710	25.097	25.483	25.869	26.255	26.641
70	27.027	27.413	27.799	28.185	28.571	28.958	29.344	29.730	30.116	30.502
80	30.888	31.274	31.660	32.046	32.432	32.819	33.205	33.591	33.977	34.363
90	34.749	35.135	35.521	35.907	36.293	36.680	37.066	37.452	37.838	38.224

Square Miles to Square Kilometers

Sq. miles	0	1	2	3	4	5	6	7	8	9
		2.59	5.18	7.77	10.36	12.95	15.54	18.13	20.72	23.31
10	25.90	28.49	31.08	33.67	36.26	38.85	41.44	44.03	46.62	49.21
20	51.80	54.39	56.98	59.57	62.16	64.75	67.34	69.93	72.52	75.11
30	77.70	80.29	82.88	85.47	88.06	90.65	93.24	95.83	98.42	101.01
40	103.60	106.19	108.78	111.37	113.96	116.55	119.14	121.73	124.32	126.91
50	129.50	132.09	134.68	137.27	139.86	142.45	145.04	147.63	150.22	152.81
60	155.40	157.99	160.58	163.17	165.76	168.35	170.94	173.53	176.12	178.71
70	181.30	183.89	186.48	189.07	191.66	194.25	196.84	199.43	202.02	204.61
80	207.20	209.79	212.38	214.97	217.56	220.15	222.74	225.33	227.92	230.51
90	233.10	235.69	238.28	240.87	243.46	246.05	248.64	251.23	253.82	256.41

Hectares to Acres

Ha	0	1	2	3	4	5	6	7	8	9
		2.47	4.94	7.41	9.88	12.36	14.83	17.30	19.77	22.24
10	24.71	27.18	29.65	32.12	34.59	37.07	39.54	42.01	44.48	46.95
20	49.42	51.89	54.36	56.83	59.31	61.78	64.25	66.72	69.19	71.66
30	74.13	76.60	79.07	81.54	84.02	86.49	88.96	91.43	93.90	96.37
40	98.84	101.31	103.78	106.26	108.73	111.20	113.67	116.14	118.61	121.08
50	123.55	126.02	128.49	130.97	133.44	135.91	138.38	140.85	143.32	145.79
60	148.26	150.73	153.21	155.68	158.15	160.62	163.09	165.56	168.03	170.50
70	172.97	175.44	177.92	180.39	182.86	185.33	187.80	190.27	192.74	195.21
80	197.68	200.15	202.63	205.10	207.57	210.04	212.51	214.98	217.45	219.92
90	222.39	224.86	227.34	229.81	232.28	234.75	237.22	239.69	242.16	244.63

Acres to Hectares

Acres	0	1	2	3	4	5	6	7	8	9
		0.405	0.809	1.214	1.619	2.023	2.428	2.833	3.237	3.642
10	4.047	4.452	4.856	5.261	5.666	6.070	6.474	6.880	7.284	7.689
20	8.094	8.498	8.903	9.308	9.712	10.117	10.522	10.927	11.331	11.736
30	12.141	12.545	12.950	13.355	13.759	14.164	14.569	14.973	15.378	15.783
40	16.187	16.592	16.997	17.402	17.806	18.211	18.616	19.020	19.425	19.830
50	20.234	20.639	21.044	21.448	21.853	22.258	22.662	23.067	23.472	23.877
60	24.281	24.686	25.091	25.495	25.900	26.305	26.709	27.114	27.519	27.923
70	28.328	28.733	29.137	29.542	29.947	30.352	30.756	31.161	31.566	31.970
80	32.375	32.780	33.184	33.589	33.994	34.398	34.803	35.208	35.612	36.017
90	36.422	36.827	37.231	37.636	38.041	38.445	38.850	39.255	39.659	40.064

Cubic Meters to Cubic Feet

Cu. m.	0	1	2	3	4	5	6	7	8	9
		35.3	70.6	105.9	141.3	176.6	211.9	247.2	282.5	317.8
10	353.1	388.5	423.8	459.1	494.4	529.7	565.0	600.3	635.7	671.0
20	706.3	741.6	776.9	812.2	847.5	882.9	918.2	953.5	988.8	1024.1
30	1059.4	1094.7	1130.1	1165.4	1200.7	1236.0	1271.3	1306.6	1341.9	1377.3
40	1412.6	1447.9	1483.2	1518.5	1553.8	1589.2	1624.5	1659.8	1695.1	1730.4
50	1765.7	1801.0	1836.4	1871.7	1907.0	1942.3	1977.6	2012.9	2048.2	2083.6
60	2118.9	2154.2	2189.5	2224.8	2260.1	2295.4	2330.8	2366.1	2401.4	2436.7
70	2472.0	2507.3	2542.6	2578.0	2613.3	2648.6	2683.9	2719.2	2754.5	2789.8
80	2825.2	2860.5	2895.8	2931.1	2966.4	3001.7	3037.0	3072.4	3107.7	3143.0
90	3178.3	3213.6	3248.9	3284.2	3319.6	3354.9	3390.2	3425.5	3460.8	3496.1

Cubic Feet to Cubic Meters

Cu. ft.	0	1	2	3	4	5	6	7	8	9
		0.0283	0.0566	0.0850	0.1133	0.1416	0.1699	0.1982	0.2265	0.2549
10	0.2832	0.3115	0.3398	0.3681	0.3964	0.4248	0.4531	0.4814	0.5097	0.5380
20	0.5663	0.5947	0.6230	0.6513	0.6796	0.7079	0.7362	0.7646	0.7929	0.8212
30	0.8495	0.8778	0.9061	0.9345	0.9628	0.9911	1.0194	1.0477	1.0760	1.1044
40	1.1327	1.1610	1.1893	1.2176	1.2459	1.2743	1.3026	1.3309	1.3592	1.3875
50	1.4159	1.4442	1.4725	1.5008	1.6291	1.5574	1.5858	1.6141	1.6424	1.6707
60	1.6990	1.7273	1.7557	1.7840	1.8123	1.8406	1.8689	1.8972	1.9256	1.9539
70	1.9822	2.0105	2.0388	2.0671	2.0955	2.1238	2.1521	2.1804	2.2087	2.2370
80	2.2654	2.2937	2.3220	2.3503	2.3786	2.4069	2.4353	2.4636	2.4919	2.5202
90	2.5485	2.5768	2.6052	2.6335	2.6618	2.6901	2.7184	2.7468	2.7751	2.8034

Liters to U. S. Liquid Gallons

Liters	0	1	2	3	4	5	6	7	8	9
		0.2642	0.5283	0.7925	1.0567	1.3209	1.5850	1.8492	2.1134	2.3775
10	2.6417	2.9059	3.1700	3.4342	3.6984	3.9626	4.2267	4.4909	4.7551	5.0192
20	5.2834	5.5476	5.8118	6.0759	6.3401	6.6043	6.8684	7.1326	7.3968	7.6609
30	7.9251	8.1893	8.4535	8.7176	8.9818	9.2460	9.5101	9.7743	10.038	10.303
40	10.567	10.831	11.095	11.359	11.624	11.888	12.152	12.416	12.680	12.944
50	13.209	13.473	13.737	14.001	14.265	14.528	14.794	15.058	15.322	15.586
60	15.850	16.114	16.379	16.643	16.907	17.171	17.435	17.699	17.964	18.228
70	18.492	18.756	19.020	19.284	19.549	19.813	20.077	20.341	20.605	20.869
80	21.134	21.398	21.662	21.926	22.190	22.454	22.719	22.983	23.247	23.511
90	23.775	24.040	24.304	24.568	24.832	25.096	25.360	25.625	25.889	26.153

U. S. Liquid Gallons to Liters

U.S. Gal.	0	1	2	3	4	5	6	7	8	9
		3.7854	7.5709	11.356	15.142	18.927	22.713	26.498	30.283	34.769
10	37.854	41.640	45.425	49.211	52.996	56.781	60.567	64.352	68.138	71.923
20	75.709	79.494	83.279	87.065	90.850	94.636	98.421	102.21	105.99	109.78
30	113.56	117.35	121.13	124.92	128.70	132.49	136.28	140.06	143.85	147.63
40	151.42	155.20	158.99	162.77	166.56	170.34	174.13	177.92	181.70	185.49
50	189.27	193.06	196.84	200.63	104.41	208.20	211.98	215.77	219.55	223.34
60	227.13	230.91	234.70	238.48	242.27	246.05	249.84	253.62	257.41	261.19
70	264.98	268.77	272.55	276.34	280.12	283.91	287.69	291.48	295.26	299.05
80	302.83	306.62	310.41	314.19	317.98	321.76	325.55	329.33	333.12	336.90
90	340.69	344.47	348.26	352.04	355.83	359.62	363.40	367.19	370.97	374.76

Hectoliters to U. S. Bushels

Hl.	0	1	2	3	4	5	6	7	8	9
		2.84	5.68	8.51	11.35	14.19	17.03	19.86	22.70	25.54
10	28.38	31.22	34.05	36.89	39.73	42.57	45.40	48.24	51.08	53.92
20	56.75	59.59	62.43	65.27	68.11	70.94	73.78	76.62	79.46	82.29
30	85.13	87.97	90.81	93.65	96.48	99.32	102.16	105.00	107.83	110.67
40	113.51	116.35	119.19	122.02	124.86	127.70	130.54	133.37	136.21	139.05
50	141.89	144.72	147.56	150.40	153.24	156.08	158.91	161.75	164.59	167.43
60	170.26	173.10	175.94	178.78	181.62	184.45	187.29	190.13	192.97	195.80
70	198.64	201.48	204.32	207.16	209.99	212.83	215.67	218.51	221.34	224.18
80	227.02	229.86	232.69	235.53	238.37	241.21	244.05	246.88	249.72	252.56
90	255.40	258.23	261.07	263.91	267.75	269.59	272.42	275.26	278.10	280.94

U. S. Bushels to Hectoliters

Bu.	0	1	2	3	4	5	6	7	8	9
		0.352	0.705	1.057	1.410	1.762	2.114	2.467	2.819	3.172
10	3.524	3.876	4.229	4.581	4.933	5.286	5.638	5.991	6.343	6.695
20	7.048	7.400	7.753	8.105	8.457	8.810	9.162	9.515	9.867	10.219
30	10.572	10.924	11.277	11.629	11.981	12.334	12.686	13.039	13.391	13.743
40	14.096	14.448	14.800	15.153	15.505	15.858	16.210	16.562	16.915	17.267
50	17.620	17.972	18.324	18.677	19.029	19.382	19.734	20.086	20.439	20.791
60	21.144	21.496	21.848	22.201	22.553	22.906	23.258	23.610	23.963	24.315
70	24.667	25.020	25.372	25.725	26.077	26.429	26.782	27.134	27.487	27.839
80	28.191	28.544	28.896	29.249	29.601	29.953	30.306	30.658	31.011	31.363
90	31.715	32.068	32.420	32.773	33.125	33.477	33.830	34.182	34.534	34.887

Centimeters per Second to Feet per Minute

Cm. per sec.	0	1	2	3	4	5	6	7	8	9
		1.97	3.94	5.91	7.87	9.84	11.81	13.78	15.75	17.72
10	19.69	21.65	23.62	25.59	27.56	29.52	31.50	33.46	35.43	37.40
20	39.37	41.34	43.31	45.28	47.24	49.21	51.18	53.15	55.12	57.09
30	59.06	61.02	62.99	64.96	66.93	68.90	70.87	72.83	74.80	76.77
40	78.74	80.71	82.68	84.65	86.61	88.58	90.55	92.52	94.49	96.46
50	98.43	100.39	102.36	104.33	106.30	108.27	110.24	112.20	114.17	116.14
60	118.11	120.08	122.05	124.02	125.98	127.95	129.92	131.89	133.86	135.83
70	137.80	139.76	141.73	143.70	145.67	147.64	149.61	151.57	153.54	155.51
80	157.48	159.45	161.42	163.39	165.35	167.32	169.29	171.26	173.23	175.20
90	177.17	179.13	181.10	183.07	185.04	187.01	188.98	190.94	192.91	194.88

Feet per Minute to Centimeters per Second

Ft. per min.	0	1	2	3	4	5	6	7	8	9
		0.508	1.016	1.524	2.032	2.540	3.048	3.556	4.064	4.572
10	5.080	5.588	6.096	6.604	7.112	7.620	8.128	8.636	9.144	9.652
20	10.160	10.668	11.176	11.684	12.192	12.700	13.208	13.716	14.224	14.732
30	15.240	15.748	16.256	16.764	17.272	17.780	18.288	18.796	19.304	19.812
40	20.320	20.828	21.336	21.844	22.352	22.860	23.368	23.876	24.384	24.892
50	25.400	25.908	26.416	26.924	27.432	27.940	28.448	28.956	29.464	29.972
60	30.480	30.988	31.496	32.004	32.512	33.020	33.528	34.036	24.544	35.052
70	35.560	36.068	36.576	37.084	37.592	38.100	38.608	39.116	39.624	40.132
80	40.640	41.148	41.656	42.164	42.672	43.180	43.688	44.196	44.704	45.212
90	45.720	46.228	46.736	47.244	47.752	48.260	48.768	49.276	49.784	50.292

Feet per Second to Miles per Hour

Feet per sec.	0	1	2	3	4	5	6	7	8	9
		0.682	1.364	2.045	2.727	3.409	4.091	4.773	5.455	6.136
10	6.818	7.500	8.182	8.864	9.545	10.227	10.909	11.591	12.273	12.946
20	13.636	14.318	15.000	15.682	16.364	17.045	17.727	18.409	19.091	19.773
30	20.455	21.136	21.818	22.500	23.182	24.864	24.545	25.227	25.909	26.591
40	27.273	27.955	26.636	29.318	30.000	30.682	31.364	32.045	32.727	33.409
50	34.091	34.773	35.455	36.136	36.818	37.500	38.182	38.864	39.545	40.227
60	40.909	41.591	42.273	42.955	43.636	44.318	45.000	45.682	46.364	47.045
70	47.727	48.409	49.091	49.773	50.455	51.136	51.818	52.500	53.182	53.864
80	54.545	55.227	55.909	56.591	57.273	57.955	58.636	59.318	60.000	60.682
90	61.364	62.045	62.727	63.409	64.091	64.773	65.455	66.136	66.818	67.500

Miles per Hour to Feet per Second

Miles per hr.	0	1	2	3	4	5	6	7	8	9
		1.47	2.93	4.40	5.87	7.33	8.80	10.27	11.73	13.20
10	14.67	16.13	17.60	19.07	20.53	22.00	23.47	24.93	26.40	27.87
20	29.33	30.80	32.27	33.73	35.20	36.67	38.13	39.60	41.07	42.53
30	44.00	45.47	46.93	48.40	49.87	51.33	52.80	54.27	55.73	57.20
40	58.67	60.13	61.60	63.07	64.53	66.00	67.47	68.93	70.40	71.87
50	73.33	74.80	76.27	77.73	79.20	80.67	82.13	83.60	85.07	86.53
60	88.00	89.47	90.93	92.40	93.87	95.33	96.80	98.27	99.73	101.20
70	102.67	104.13	105.60	107.07	108.53	110.00	111.47	112.93	114.40	115.87
80	117.33	118.80	120.27	121.73	123.20	124.67	126.13	127.60	129.07	130.53
90	132.00	133.47	134.93	136.40	137.87	139.33	140.80	142.27	143.73	145.20

Radians per Second to Revolutions per Minute

Radians per sec.	0	1	2	3	4	5	6	7	8	9
		9.55	19.10	28.65	38.20	47.75	57.30	66.85	76.39	85.94
10	95.49	105.04	114.59	124.14	133.69	143.24	152.79	162.34	171.89	181.44
20	190.99	200.54	210.08	219.63	229.18	238.73	248.28	257.83	267.38	276.93
30	286.48	296.03	305.58	315.13	324.68	334.23	343.77	353.32	362.87	372.42
40	381.97	391.52	401.07	410.62	420.17	429.72	439.27	448.82	458.37	467.92
50	477.46	487.01	496.56	506.11	515.66	525.21	534.76	544.31	553.86	563.41
60	572.96	582.51	592.06	601.61	611.15	620.70	630.25	639.80	649.35	658.90
70	668.45	678.00	687.55	697.10	706.65	716.20	725.75	735.30	744.85	754.39
80	763.94	773.49	783.04	792.59	802.14	811.69	821.24	830.79	840.34	849.89
90	859.44	868.99	878.54	888.08	897.63	907.18	916.73	926.28	935.83	945.38

Revolutions per Minute to Radians per Second

Rev. per min.	0	1	2	3	4	5	6	7	8	9
		0.1047	0.2094	0.3142	0.4189	0.5236	0.6283	0.7330	0.8378	0.9425
10	1.0472	1.1519	1.2566	1.3614	1.4661	1.5708	1.6755	1.7802	1.8850	1.9897
20	2.0944	2.1991	2.3038	2.4086	2.5133	2.6180	2.7227	2.8274	2.9322	3.0369
30	3.1416	3.2463	3.3510	3.4558	3.5605	3.6652	3.7699	3.8746	3.9794	4.0841
40	4.1888	4.2935	4.3982	4.5029	4.6077	4.7124	4.8171	4.9218	5.0265	5.1313
50	5.2360	5.3407	5.4454	5.5501	5.6549	5.7596	5.8643	5.9690	6.0737	6.1785
60	6.2832	6.3879	6.4926	6.5973	6.7021	6.8068	6.9115	7.0162	7.1209	7.2257
70	7.3304	7.4351	7.5398	7.6445	7.7493	7.8540	7.9587	8.0634	8.1681	8.2729
80	8.3776	8.4823	8.5870	8.6917	8.7965	8.9012	9.0059	9.1106	9.2153	9.3201
90	9.4248	9.5295	9.6342	9.7389	9.8437	9.9484	10.053	10.158	10.263	10.367

Kilograms per Square Centimeter to Pounds per Square Inch

Kg. per sq. cm.	0	1	2	3	4	5	6	7	8	9
		14.2	28.4	42.7	56.9	71.1	85.3	99.6	113.8	128.0
10	142.2	156.5	170.7	184.9	199.1	213.4	227.6	241.8	256.0	270.2
20	284.5	298.7	312.9	327.1	341.4	355.6	369.8	384.0	398.3	412.5
30	426.7	440.9	455.1	469.4	483.6	497.8	512.0	526.3	540.5	554.7
40	568.9	583.2	597.4	611.6	625.8	640.1	654.3	668.5	682.7	696.9
50	711.2	725.4	739.6	753.8	768.1	782.3	796.5	810.7	825.0	839.2
60	853.4	867.6	881.9	896.1	910.3	924.5	938.7	953.0	967.2	981.4
70	995.6	1009.9	1024.1	1038.3	1052.5	1066.8	1081.0	1095.2	1109.4	1123.6
80	1137.9	1152.1	1166.3	1180.5	1194.8	1209.0	1223.2	1237.4	1251.7	1265.9
90	1280.1	1294.3	1308.6	1322.8	1337.0	1351.2	1365.4	1379.7	1393.9	1408.1

Pounds per Square Inch to Kilograms per Square Centimeter

Lb. per sq. in.	0	1	2	3	4	5	6	7	8	9
		0.0703	0.1406	0.2109	0.2812	0.3515	0.4218	0.4921	0.5625	0.6328
10	0.7031	0.7734	0.8437	0.9140	0.9843	1.0546	1.1249	1.1952	1.2655	1.3358
20	1.4061	1.4764	1.5467	1.6171	1.6874	1.7577	1.8280	1.8983	1.9686	2.0389
30	2.1092	2.1795	2.2498	2.3201	2.3904	2.4607	2.5310	2.6014	2.6717	2.7420
40	2.8123	2.8826	2.9529	3.0232	3.0935	3.1638	3.2341	3.3044	3.3747	3.4450
50	3.5153	3.5856	3.6559	3.7263	3.7966	3.8669	3.9372	4.0075	4.0778	4.1481
60	4.2184	4.2887	4.3590	4.4293	4.4996	4.5699	4.6402	4.7105	4.7809	4.8512
70	4.9215	1.9918	5.0621	5.1324	5.2027	5.2730	5.3433	5.4136	5.4839	5.5542
80	5.6245	5.6948	5.7651	5.8355	5.9058	5.9761	6.0464	6.1167	6.1870	6.2573
90	6.3276	6.3979	6.4682	6.5385	6.6088	6.6791	6.7494	6.8197	6.8901	6.9604

Kilograms per Square Meter to Pounds per Square Foot

Kg. per sq. m.	0	1	2	3	4	5	6	7	8	9
		0.205	0.410	0.614	0.819	1.024	1.229	1.434	1.639	1.843
10	2.048	2.253	2.458	2.663	2.867	3.072	3.277	3.482	3.687	3.892
20	4.096	4.301	4.506	4.711	4.916	5.120	5.325	5.530	5.735	5.940
30	6.145	6.349	6.554	6.759	6.964	7.169	7.373	7.578	7.783	7.988
40	8.193	8.397	8.602	8.807	9.012	9.217	9.422	9.626	9.831	10.036
50	10.241	10.446	10.650	10.855	11.060	11.265	11.470	11.675	11.879	12.084
60	12.289	12.494	12.699	12.903	13.108	13.313	13.518	13.723	13.928	14.132
70	14.337	14.542	14.747	14.952	15.156	15.361	15.566	15.771	15.976	16.181
80	16.385	16.590	16.795	17.000	17.205	17.409	17.614	17.819	18.024	18.229
90	18.434	18.638	18.843	19.048	19.253	19.458	19.662	19.867	20.072	20.277

Pounds per Square Foot to Kilograms per Square Meter

Lb. per sq. ft.	0	1	2	3	4	5	6	7	8	9
		4.88	9.76	14.65	19.53	24.41	29.29	34.18	39.06	43.94
10	48.82	53.71	58.59	63.47	68.35	73.24	78.12	83.00	87.88	92.77
20	97.65	102.53	107.41	112.30	117.18	122.06	126.94	131.83	136.71	141.59
30	146.47	151.35	156.24	161.12	166.00	170.88	175.77	180.65	185.53	190.41
40	195.30	200.18	205.06	209.94	214.83	219.71	224.59	229.47	234.36	239.24
50	244.12	249.00	253.89	258.77	263.65	268.53	273.41	278.30	283.18	288.06
60	292.94	297.83	302.71	307.59	312.47	317.36	322.24	327.12	332.00	336.89
70	341.77	346.65	351.53	356.42	361.30	366.18	371.06	375.95	380.83	385.71
80	390.59	395.48	400.36	405.24	410.12	415.00	419.89	424.77	429.65	434.53
90	439.42	444.30	449.18	454.06	458.95	463.83	468.71	473.59	478.48	483.36

Meter-kilograms to Foot-pounds

M-kg.	0	1	2	3	4	5	6	7	8	9
		7.23	14.47	21.70	28.93	36.16	43.40	50.63	57.86	65.10
10	72.33	79.56	86.80	94.03	101.26	108.49	115.73	122.96	130.19	137.43
20	144.66	151.89	159.13	166.36	173.59	180.82	188.06	195.29	202.52	209.76
30	216.99	224.22	231.46	238.69	245.92	253.15	260.39	267.62	274.85	282.09
40	289.32	296.55	303.79	311.02	318.25	325.48	332.72	339.95	347.18	354.42
50	361.65	368.88	376.12	383.35	390.58	397.81	405.05	412.28	419.51	426.75
60	433.98	441.21	448.45	455.68	462.91	470.14	477.38	484.61	491.84	499.08
70	506.31	513.54	520.78	528.01	535.24	542.47	549.71	556.94	564.17	571.41
80	578.64	585.87	593.11	600.34	607.57	614.80	622.04	629.27	636.50	643.74
90	650.97	658.20	665.44	672.67	679.90	687.13	694.37	701.60	708.83	716.07

Foot-pounds to Meter-kilograms

Ft.-lb.	0	1	2	3	4	5	6	7	8	9
		0.138	0.277	0.415	0.553	0.691	0.830	0.968	1.106	1.244
10	1.383	1.521	1.659	1.797	1.936	2.074	2.212	2.350	2.489	2.627
20	2.765	2.903	3.042	3.180	3.318	3.456	3.595	3.733	3.871	4.009
30	4.148	4.286	4.424	4.562	4.701	4.839	4.977	5.115	5.254	5.392
40	5.530	5.668	5.807	5.945	6.083	6.221	6.360	6.498	6.636	6.775
50	6.913	7.051	7.189	7.328	7.466	7.604	7.742	7.881	8.019	8.157
60	8.295	8.434	8.572	8.710	8.848	8.987	9.125	9.263	9.401	9.540
70	9.678	9.816	9.954	10.093	10.231	10.369	10.507	10.646	10.784	10.922
80	11.060	11.199	11.337	11.475	11.613	11.752	11.890	12.028	12.166	12.305
90	12.443	12.581	12.719	12.858	12.996	13.134	13.273	13.411	13.549	13.687

Gram-calories to Joules

Gm.-cal.	0	1	2	3	4	5	6	7	8	9
		4.19	8.38	12.56	16.75	20.94	25.13	29.32	33.50	37.69
10	41.88	46.07	50.26	54.44	58.63	62.82	67.01	71.20	75.38	79.57
20	83.76	87.95	92.14	96.32	100.51	104.70	108.89	113.08	117.26	121.45
30	125.64	129.83	134.02	138.20	142.39	146.58	150.77	154.96	159.14	163.33
40	167.52	171.71	175.90	180.08	184.27	188.46	192.65	196.84	201.02	205.21
50	209.4	213.6	217.8	222.0	226.2	230.3	234.5	238.7	242.9	247.1
60	251.3	255.5	259.7	263.8	268.0	272.2	276.4	280.6	284.8	289.0
70	293.2	297.3	301.5	305.7	309.9	314.1	318.3	322.5	326.7	330.9
80	335.0	339.2	343.4	347.6	351.8	356.0	360.2	364.4	368.5	372.7
90	376.9	381.1	385.3	389.5	393.7	397.9	402.0	406.2	410.4	414.6

Joules to Gram-calories

Joules	0	1	2	3	4	5	6	7	8	9
		0.239	0.478	0.716	0.955	1.194	1.433	1.671	1.910	2.149
10	2.388	2.627	2.865	3.104	3.343	3.582	3.820	4.059	4.298	4.537
20	4.776	5.014	5.253	5.492	5.731	5.969	6.208	6.447	6.686	6.925
30	7.163	7.402	7.641	7.880	8.118	8.357	8.596	8.835	9.074	9.312
40	9.551	9.790	10.029	10.267	10.506	10.745	10.984	11.223	11.461	11.700
50	11.939	12.178	12.416	12.655	12.894	13.133	13.372	13.610	13.849	14.088
60	14.327	14.565	14.804	15.043	15.282	15.521	15.759	15.998	16.237	16.476
70	16.714	16.953	17.192	17.431	17.670	17.908	18.147	18.386	18.625	18.863
80	19.102	19.341	19.580	19.819	20.057	20.296	20.535	20.774	21.012	21.251
90	21.490	21.729	21.968	22.206	22.445	22.684	22.923	23.161	23.400	23.639

Kilogram-calories to Meter-kilograms

Kg.-cal.	0	1	2	3	4	5	6	7	8	9
		427	854	1 281	1 708	2 135	2 562	2 989	3 416	3 844
10	4 271	4 698	5 125	5 552	5 979	6 406	6 833	7 260	7 687	8 114
20	8 541	8 968	9 395	9 822	10 249	10 676	11 103	11 531	11 958	12 385
30	12 812	13 239	13 666	14 093	14 520	14 947	15 374	15 801	16 228	16 655
40	17 082	17 509	17 936	18 363	18 791	19 218	19 645	20 072	20 499	20 926
50	21 353	21 780	22 207	22 634	23 061	23 488	23 915	24 342	24 769	25 196
60	25 623	26 050	26 478	26 905	27 332	27 759	28 186	28 613	29 040	29 467
70	29 894	30 321	30 748	31 175	31 602	32 029	32 456	32 883	33 310	33 738
80	34 165	34 592	35 019	35 446	35 873	36 300	36 727	37 154	37 581	38 008
90	38 435	38 862	39 289	39 716	40 143	40 570	40 997	41 425	41 852	42 279

Meter-kilograms to Kilogram-calories

M.-kg.	0	1	2	3	4	5	6	7	8	9
		.00234	.00468	.00702	.00937	.01171	.01405	.01639	.01873	.02107
10	0.02342	.02576	.02810	.03044	.03278	.03512	.03747	.03981	.04215	.04449
20	0.04683	.04917	.05152	.05386	.05620	.05854	.06080	.06322	.06556	.06791
30	0.07025	.07259	.07493	.07727	.07961	.08196	.08430	.08664	.08898	.09132
40	0.09366	.09601	.09835	.10069	.17303	.10537	.10771	.11006	.11240	.11474
50	0.11708	.11942	.12176	.12411	.12645	.12879	.13113	.13347	.13581	.13815
60	0.14050	.14284	.14518	.14752	.14986	.15220	.15455	.15689	.15923	.16157
70	0.16391	.16625	.16860	.17094	.17328	.17562	.17796	.18030	.18265	.18499
80	0.18733	.18967	.19201	.19435	.19669	.19904	.20138	.20372	.20606	.20840
90	0.21074	.21309	.21543	.21777	.22011	.22245	.22479	.22714	.22948	.23182

British Thermal Units to Foot-pounds

B.t.u.	0	1	2	3	4	5	6	7	8	9
		778	1 557	2 335	3 114	3 892	4 670	5 449	6 227	7 006
10	7 784	8 562	9 341	10 119	10 897	11 676	12 454	13 233	14 011	14 789
20	15 568	16 346	17 125	17 903	18 681	19 460	20 238	21 017	21 795	22 573
30	23 352	24 130	24 909	25 687	26 465	27 244	28 022	28 800	29 579	30 357
40	31 136	31 914	32 692	33 471	34 249	35 028	35 806	36 584	37 363	38 141
50	38 920	39 698	40 476	41 255	42 033	42 811	43 590	44 368	45 147	45 925
60	46 703	47 482	48 260	49 039	49 817	50 595	51 374	52 152	52 931	53 709
70	54 487	55 266	56 044	56 823	57 601	58 379	59 158	59 936	60 714	61 493
80	62 271	63 050	63 828	64 606	65 385	66 163	66 942	67 720	68 498	69 277
90	70 055	70 834	71 612	72 390	73 169	73 947	74 726	75 504	76 282	77 061

Foot-pounds to British Thermal Units

Ft.-lb.	0	1	2	3	4	5	6	7	8	9
		.00128	.00257	.00385	.00514	.00642	.00771	.00899	.01028	.01156
10	0.01285	.01413	.01542	.01670	.01799	.01927	.02056	.02184	.02312	.02441
20	0.02569	.02698	.02826	.02955	.03083	.03212	.03340	.03469	.03597	.03726
30	0.03854	.03983	.04111	.04240	.04368	.04496	.04625	.04753	.04882	.05010
40	0.05139	.05267	.05396	.05524	.05653	.05781	.05910	.06038	.06167	.06295
50	0.06424	.06552	.06680	.06809	.06937	.07066	.07194	.07323	.07451	.07580
60	0.07708	.07837	.07965	.08094	.08222	.08351	.08479	.08608	.08736	.08864
70	0.08993	.09121	.09250	.09378	.09507	.09635	.09764	.09892	.10021	.10149
80	0.10278	.10406	.10535	.10663	.10791	.10920	.11048	.11177	.11305	.11434
90	0.11562	.11691	.11819	.11948	.12076	.12205	.12333	.12462	.12590	.12719

Kilowatts to Horsepower

Kw.	0	1	2	3	4	5	6	7	8	9
		1.341	2.682	4.023	5.364	6.705	8.046	9.387	10.728	12.069
10	13.410	14.751	16.092	17.433	18.774	20.115	21.456	22.797	24.138	25.479
20	26.820	28.161	29.502	30.843	32.184	33.525	34.866	36.208	37.549	38.890
30	40.231	41.572	42.913	44.254	45.595	46.936	48.277	49.618	50.959	52.300
40	53.641	54.982	56.323	57.664	59.005	60.346	61.687	63.028	64.369	65.710
50	67.051	68.392	69.733	71.074	72.415	73.756	75.097	76.438	77.779	79.120
60	80.461	81.802	83.143	84.484	85.825	87.166	88.507	89.848	91.189	92.530
70	93.871	95.212	96.553	97.894	99.235	100.58	101.92	103.26	104.60	105.94
80	107.28	108.62	109.96	111.30	112.65	113.99	115.33	116.67	118.01	119.35
90	120.69	122.03	123.37	124.71	126.06	127.40	128.74	130.08	131.42	132.76

Horsepower to Kilowatts

Hp.	0	1	2	3	4	5	6	7	8	9
		0.746	1.491	2.237	2.983	3.729	4.474	5.220	5.966	6.711
10	7.457	8.203	8.948	9.694	10.440	11.186	11.931	12.677	13.423	14.168
20	14.914	15.660	16.405	17.151	17.897	18.643	19.388	20.134	20.880	21.625
30	22.371	23.117	23.862	24.608	25.354	26.100	26.845	27.591	28.337	29.082
40	29.828	30.574	31.319	32.065	32.811	33.557	34.302	35.048	35.794	36.539
50	37.285	38.031	38.776	39.522	40.268	41.014	41.759	42.505	43.251	43.996
60	44.742	45.488	46.233	46.979	47.725	48.471	49.216	49.962	50.708	51.453
70	52.199	52.945	53.691	54.436	55.182	55.928	56.673	57.419	58.165	58.910
80	59.656	60.402	61.148	61.893	62.639	63.385	64.130	64.876	65.622	66.367
90	67.113	67.859	68.605	69.350	70.096	70.842	71.587	72.333	73.079	73.824

Cheval-vapeur to Horsepower

Cheval-vapeur	0	1	2	3	4	5	6	7	8	9
		0.986	1.973	2.959	3.945	4.932	5.918	6.904	7.891	8.877
10	9.863	10.849	11.836	12.822	13.808	14.795	15.781	16.767	17.754	18.740
20	19.726	20.713	21.699	22.685	23.672	24.658	25.644	26.631	27.617	28.603
30	29.590	30.576	31.562	32.548	33.535	34.521	35.507	36.494	37.480	38.466
40	39.453	40.439	41.425	42.412	43.398	44.384	45.371	46.357	47.343	48.330
50	49.316	50.302	51.289	52.275	53.261	54.247	55.234	56.220	57.206	58.193
60	59.179	60.165	61.152	62.138	63.124	64.111	65.097	66.083	67.070	68.056
70	69.042	70.029	71.015	72.001	72.988	73.974	74.960	75.946	76.933	77.919
80	78.905	79.892	80.878	81.864	82.851	83.837	84.823	85.810	86.796	87.782
90	88.769	89.755	90.741	91.728	92.714	93.700	94.687	95.673	96.659	97.645

Horsepower to Cheval-vapeur

Horse-power	0	1	2	3	4	5	6	7	8	9
		1.014	2.028	3.042	4.055	5.069	6.083	7.097	8.111	9.125
10	10.139	11.153	12.166	13.180	14.194	15.208	16.222	17.236	18.250	19.264
20	20.277	21.291	22.305	23.319	24.333	25.347	26.361	27.375	28.388	29.402
30	30.416	31.430	32.444	33.458	34.472	35.486	36.499	37.513	38.527	39.541
40	40.555	41.569	42.583	43.596	44.610	45.624	46.638	47.652	48.666	49.680
50	50.694	51.707	52.721	53.735	54.749	55.763	56.777	57.791	58.805	59.818
60	60.832	61.846	62.860	63.874	64.888	65.902	66.916	67.929	68.943	69.957
70	70.971	71.985	72.999	74.013	75.027	76.040	77.054	78.068	79.082	80.096
80	81.110	82.124	83.137	84.151	85.165	86.179	87.193	88.207	89.221	90.235
90	91.248	92.262	93.276	94.290	95.304	96.318	97.332	98.346	99.359	100.373

31. Values of Foreign Coins

The following equivalents in terms of the U. S. gold dollar were established by the Secretary of the Treasury on July 1, 1929, for use in estimating the value of all foreign merchandise exported to the United States and expressed in such metallic currencies.

Country	Monetary Unit	Value in terms of U. S. money
Argentine Rep'b'c	Peso	$0.9648
Austria	Schilling	.1407
Belgium	Belga	.1390
Bolivia	Boliviano	.3650
Brazil	Milreis	.5462
British Colonies in Australasia and Africa	Pound sterling	4.8665
British Honduras	Dollar	1.0000
Bulgaria	Lev	.0072
Canada	Dollar	1.0000
Chile	Peso	.1217
China	Tael*: Amoy	.6513
	Tael*: Canton	.6494
	Tael*: Chefoo	.6230
	Tael*: Chin Kiang	.6363
	Tael*: Fuchau	.6025
	Tael*: Haikwan	.6627
	Tael*: Hankow	.6094
	Tael*: Kiaochow	.6312
	Tael*: Nanking	.6445
	Tael*: Niuchwang	.6108
	Tael*: Ningpo	.6262
	Tael*: Peking	.6350
	Tael*: Shanghai	.5949
	Tael*: Swatow	.6017
	Tael*: Takau	.6555
	Tael*: Tientsin	.6312
	Dollar*: Yuan	.4221
	Dollar*: Hong Kong, British	.4284
	Dollar*: Mexican	.4315
Colombia	Peso	.9733
Costa Rica	Colon	.4653
Cuba	Peso	1.0000
Denmark	Krone	.2680
Dominican Republic	Dollar	1.0000
Ecuador	Sucre	.2000
Egypt	Pound (100 piasters)	4.9431
Estonia	Kroon	.2680
Finland	Markka	.0252
France	Franc	.0392
Germany	Reichsmark	$.2382
Great Britain	Pound sterling	4.8665
Greece	Drachma	.0130
Guatemala	Quetzal	1.0000
Haiti	Gourde	.2000
Honduras	Lempira	.5000
Hungary	Pengö	.1749
India (British)	Rupee	.3650
Indo-China	Piaster*	.4291
Italy	Lira	.0526
Japan	Yen	.4985
Latvia	Lat	.1930
Liberia	Dollar	1.0000
Lithuania	Litas	.1000
Mexico	Peso	.4985
Netherlands	Guilder (florin)	.4020
Newfoundland	Dollar	1.0000
Nicaragua	Cordoba	1.0000
Norway	Krone	.2680
Panama	Balboa	1.0000
Paraguay	Peso (Argentine)	.9648
Persia	Kran*	.0731
Peru	Libra	4.8665
Philippine Islands	Peso	.5000
Poland	Zloty	.1122
Portugal	Escudo	1.0805
Rumania	Leu	.0060
Russia	Ruble	.5146
Salvador	Colon	.5000
Siam	Baht (Tical)	.4424
Spain	Peseta	.1930
Straits Settlements	Dollar	.5678
Sweden	Krona	.2680
Switzerland	Franc	.1930
Turkey	Piaster	.0440
Uruguay	Peso	1.0342
Venezuela	Bolivar	.1930
Yugoslavia	Dinar	.1930

*Silver standard; other countries have the gold standard.

SECTION 4

THERMODYNAMICS AND FUEL POWER GENERATION

The following outlines of fundamental principles and facts have been prepared, not for mechanical but for civil engineers whose knowledge of the subject is limited.—EDITOR-IN-CHIEF.

BY

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THERMODYNAMICS

1. Preliminary Statements and Definitions

Heat Energy. The mechanical theory of heat asserts that heat is a form of energy due to the motion or configuration of the molecules of a body. Like mechanical energy, heat energy may of the kinetic or of the potential form. The summation $\Sigma\ 1/2\ mv^2$ extended to the moving molecules of a system gives the thermal kinetic energy of the system. Considerations derived from the kinetic theory of gases show that the temperature of a body is a measure of its thermal kinetic energy. When the temperature rises, it is inferred that the thermal kinetic energy is increased and vice versa. Thermal potential energy is due to the position or configuration of the molecules of a body. Thus when the volume of the body is increased, work is required to separate the molecules against their mutual attractions, and this work is stored as potential energy. Again, when the state of aggregation is changed, as in fusion or vaporization, work is required to break down the molecular structure, and this work is stored in the system as potential energy. In the case of gases, like air and nitrogen, the attractive forces between the molecules are so small that the thermal potential energy is practically zero. The internal energy of a gas is therefore assumed to be wholly of the kinetic form.

Units of Energy. The conventional units used in this chapter are: (1) for **mechanical energy,** the foot-pound and the horsepower-hour (**hp.-hr.**), which is equal to 1 980 000 ft.-lb.; (2) for **heat energy,** the British thermal unit (**B.t.u.**), which is defined as the heat required to raise the temperature of one pound of water from 63° to 64° F.

Mechanical Equivalent of Heat. The numerical relation between the unit of heat and the unit of work has been determined very accurately from experiments. The accepted relations are

$$1 \text{ B.t.u.} = 777.64 \text{ ft.-lb.} \qquad 1 \text{ hp.-hr.} = 2546.2 \text{ B.t.u.}$$

In ordinary calculations the **integral values 778 and 2546** are sufficiently accurate. The mechanical equivalent is denoted by the symbol J and the reciprocal of it by A; thus, $J = 778$ and $A = 1/J = 1/778$.

State of a System. A thermodynamic system is defined as a body or group of bodies capable of receiving and giving out heat or other forms of energy. Examples of such systems are the media used in heat engines, as water vapor, air, ammonia, etc. In order that a system may receive or give up energy, its state must change; hence the magnitudes that determine the state must change. It is assumed ordinarily that the system is homogeneous and of uniform density and temperature throughout, also that it is subjected to uniform pressure. Then the magnitudes that describe the state of a unit mass are: the **pressure** p, the **temperature** t, and the **volume** v. In the case of a homogeneous mixture of vapor and liquid, as wet steam, a fourth variable is required; this is the **ratio** x of the **weight of vapor** to the **weight of the mixture.** These magnitudes, p v t and x are called the **coordinates** of the system.

Absolute Temperature. Many of the equations of thermodynamics are simplified by taking the temperatures from absolute zero instead of from the zero of the F. or C. scale. The position of the absolute zero, as determined by experiments on actual gases, is about 273.1° below zero C., or 459.6° below zero F. Hence, denoting ordinary temperatures by t and absolute temperatures by T,

$$T = t + 273.1 \text{ for the C. scale,}$$
$$T = t + 459.6 \text{ for the F. scale.}$$

Characteristic Equations. Of the three coordinates, p, v, T, any two may be, in general, taken as independent and the third is then a function of these two. Thus in the case of a confined gas, the pressure p may be kept at any desired value and by the addition of heat the temperature T may be raised to any predetermined point. The volume v must, however, depend upon the values given to p and T; that is, v is a function of p and T, as $\mathbf{v} = \mathbf{f}\ (\mathbf{pT})$. Likewise, taking v and T as independent, a functional relation $\mathbf{p} = \phi(\mathbf{v}, \mathbf{T})$ must exist. For any substance there is such a functional relation between the coordinates; this relation is called the characteristic equation of the substance. The simplest characteristic equation is that which applies to an ideal perfect gas, namely.

$$\mathbf{pv} = \mathbf{BT}$$

For certain imperfect gases van der Waal's equation

$$\mathbf{p} = \frac{\mathbf{BT}}{\mathbf{v} - \mathbf{b}} - \frac{\mathbf{a}}{\mathbf{v}^2}$$

represents the relation between p, v, and T. The behavior of superheated steam is, within certain limits, represented accurately by the empirical equation

$$\mathbf{p}(\mathbf{v} - \mathbf{c}) = \mathbf{BT} - \mathbf{p}(1 + \mathbf{a}\mathbf{p}^{1/2})\frac{\mathbf{m}}{\mathbf{T}^{\mathbf{n}}}.$$

The characteristic equation $\phi(\mathbf{p}, \mathbf{v}, \mathbf{T}) = 0$ having three variables may be represented geometrically by a surface. Any possible state of the substance is represented by a point on the surface and a change of state is represented by a path lying in the surface. In the case of a mixture of liquid and vapor (as wet steam), the pressure is a function of the temperature only, and the volume depends upon the temperature and the ratio $\mathbf{x}$. The characteristic surface of such a mixture is a cylindrical surface cutting the pt-plane in the curve $\mathbf{p} = \mathbf{f}(\mathbf{t})$.

Changes of State. Certain changes of state of a thermodynamic system are of special importance.

1. **Constant Volume.** The volume remaining constant, the pressure varies with the temperature and, in general, heat is absorbed or rejected by the system.

2. **Constant pressure.** A change of volume involves a change of temperature. Heat is absorbed or rejected.

3. **Isothermal.** If the temperature remains constant during a change of state, the change is said to be isothermal. In general, heat is absorbed or rejected.

4. **Adiabatic.** An adiabatic change is one in which the system neither receives nor gives out heat.

5. **Isodynamic.** In an isodynamic change of state the internal energy of the system remains constant.

Specific Heat. The heat required to raise the temperature of a unit weight of a body one degree under given external conditions is called the **thermal capacity** of the body. The ratio of the thermal capacity of a substance at the temperature **t** to the thermal capacity of water at a chosen standard temperature (17.5° C. or 63.5° F.) is the specific heat of the substance. Since at 63.5° F. the thermal capacity of water is 1 B.t.u., it follows that the specific heat of a substance at temperature t is numerically equal to the thermal capacity at the same temperature. In the case of gaseous substances two specific heats are of special importance: the specific heat at constant volume denoted by $\mathbf{c_v}$, and that at constant pressure denoted by $\mathbf{c_p}$. For solids and liquids, these specific heats are practically identical, but for gases they are considerably different; thus in the case of air, $c_p = 1.4\ c_v$.

Latent Heat. During a change of state of aggregation the heat added to a substance is expended in performing work of disgregation, and none of it is used in raising temperature; that is, the heat absorbed goes to increase the potential energy of the system. Heat thus absorbed during fusion or vaporization is called latent heat. The heat required to vaporize a unit weight of a liquid depends upon the pressure under which the vapor is formed. The following are latent heats of fusion for various substances in British thermal units per pound:

Ice	144	Cast iron, grey	41.4
Lead	9.66	Cast iron, white	59.4
Tin	25.65	Zinc	50.6
Silver	37.93	Sulfur	16.86

2. Fundamental Laws of Thermodynamics

Transformations of Energy. All transformations of energy are subject to two far-reaching general laws: (1) The law of **conservation of energy,** of which the following is a statement: The total energy of an isolated system remains constant and cannot be increased or diminished by any physical process whatever. (2) The law of **degradation of energy.** According to this law, the result of any transformation of energy is to reduce the quantity of energy that may be usefully transformed into mechanical work.

Examples of the law of degradation are abundant. Work is transformed into heat through friction, and only a small part can be recovered; electrical energy is rendered unavailable when transformed into heat in the conducting system; heat flows from a body of higher temperature to one of lower temperature and as a result a smaller fraction of it becomes available for transformation into work; in the transformation of work into electrical energy or of electrical energy into work, some of the high-grade energy is transformed into heat and is rendered unavailable. The term thermodynamic degeneration is sometimes applied to the inevitable loss of available energy in any transformation of energy; and the law of degradation may be stated as follows: Every natural process is accompanied by thermodynamic degeneration.

The First Law of Thermodynamics is merely the law of conservation applied to the transformation of heat into work. It may be stated as follows:

> When work is expended in producing heat the quantity of heat generated is equivalent to the work done; and conversely, when heat is employed to do work, a quantity of heat precisely equivalent to the work done disappears.

Denoting by **Q** the heat converted into work, and by **W** the work thus obtained, the first law is expressed symbolically by the equations $\mathbf{W} = \mathbf{JQ}$ and $\mathbf{Q} = \mathbf{AW}$.

The Energy Equation. Let heat be absorbed by a body, at a given mass of gas or a saturated vapor. If the volume of the body remains constant, the energy absorbed must be added to the intrinsic energy of the system; as a result the temperature will rise, or, in the case of a liquid, vaporization will ensue. If, however, the volume of the system changes, external work will be done, and the heat absorbed will in part be used in doing this work, whereas the remainder will increase the intrinsic energy. In general, therefore,

heat absorbed = increase of energy + external work.

The change of energy depends upon the initial and final states of the system only. The external work depends, however, on the relation between p and v

during the change of state, that is, upon the path; hence the heat absorbed also depends upon the path.

If a system passes through a closed cycle of processes and returns to its initial state, the change of energy for the cycle is zero; hence for a closed cycle the heat absorbed by the system is the equivalent of the external work. If the change of state is adiabatic, the heat absorbed is zero and the external work is gained at the expense of the intrinsic energy of the system.

The Second Law of Thermodynamics is essentially the law of degradation of energy. Whereas the first law gives a relation that must be satisfied in any transformation of energy, it is the second law that gives information regarding the possibility of transformation and the availability of a given form of energy for transformation into work. A general statement of the second law is

> No change in a system of bodies that takes place of itself can increase the available energy of the system.

A more concrete statement is that of Kelvin: It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of surrounding objects. In effect, Kelvin's statement denies the possibility of deriving work directly from the heat contained in the atmosphere.

Carnot's Cycle. The availability of a given quantity of heat energy for transformation into work is given by the efficiency of the ideal Carnot engine. In the ideal cycle described by Carnot, the medium is subjected to four processes: (1) It is placed in contact with a source of heat at a higher temperature T_1 and expands isothermally while absorbing heat Q_1 from the source. (2) It is removed from the source and expands adiabatically until the temperature reaches the lower value T_2. (3) It is placed in contact with a refrigerator at temperature T_2, is compressed isothermally, and rejects to the refrigerator the heat Q_2. (4) It is compressed adiabatically and arrives at the initial state. The heat $Q_1 - Q_2$ is transformed into work, and the efficiency of the cycle is therefore the fraction $e = \frac{Q_1 - Q_2}{Q_1}$. According to Carnot's principle, which may be proved by the second law, all ideal reversible engines working between the same temperature limits T_1 and T_2 have the same efficiency; that is, the efficiency is independent of the working medium and depends upon T_1 and T_2 only. Hence $e = f(T_1, T_2)$. Kelvin proposed as a definition of temperature the relation $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$. From this definition, $e = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$. Assume a quantity of heat Q in a body having the temperature T, and let T_0 denote the lowest available temperature (e.g., the temperature of the atmosphere). No device can transform a greater part of Q into work than the ideal Carnot engine; hence the available part of Q is $Q\left(1 - \frac{T_0}{T}\right)$ and the remainder $\frac{QT_0}{T}$ must inevitably be wasted. For example, if the absolute temperature of the source is 900° and that of the atmosphere is 540°, the available energy is $1 - \frac{540}{900} = 0.4$ of the total energy; therefore of 1000 B.t.u. taken from the source not more than 400 B.t.u. can by any possible means be transformed into work and at least 600 B.t.u. is unavailable.

Entropy. Experience shows that any actual physical process, as the change of state of a system, is irreversible and is accompanied by frictional effects. A strictly reversible frictionless process is an ideal that may be approached but never attained. In the case of the ideal reversible process, there is no change in the quantity of available energy; but an actual irreversible process is always accompanied by a decrease of the amount of energy available for transformation, or, what is the same thing, an increase of unavailable energy. An investigation of special cases shows that the increase of unavailable energy is the product of two factors: one is T_0, the lowest absolute temperature available in a refrigerator, the other is a term of the form $\frac{Q}{T}$ or $\int T^{-1} dQ$. This second factor is called the increase of entropy of the system under consideration.

When the conception of increase of entropy is applied to the system composed of all the bodies involved in a change, that is, to an isolated system, it appears that the increase of entropy is a measure of the thermodynamic degeneration produced by the change. According to the law of degradation every natural change is accompanied by thermodynamic degeneration, therefore it is accompanied by an increase of entropy. The following important principle is evident: The direction of a process, physical or chemical, that occurs of itself is such as will bring about an increase of entropy of the system. This principle is the foundation of the application of thermodynamics to chemistry.

The Change of Entropy of a body which is in thermal communication with other bodies (as a pound of air, or the steam in an engine cylinder) is illustrated as follows: Let the ordinate $A_1 A$ (Fig. 1) represent the absolute temperature T_1 of the body in its initial state. Suppose the body to receive heat from external sources while its state changes reversibly; and let the ordinate A_1A increase in length so as to represent the successive values of T during the heating and at the same time move horizontally in such a way that the area swept over represents the heat absorbed. The end of the ordinate must move in a definite path, as AB. The horizontal distance A_1B_1 through which the ordinate moves represents the increase of entropy. If OA_1 is chosen arbitrarily to represent the entropy S_1 of the body in the initial state, then OB_1 represents the entropy S_2 in the final state. The area A_1ABB_1 represents the heat Q absorbed (provided the process is reversible), and the following relations between Q and the change of entropy exist:

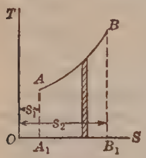

Fig. 1

$$Q = \int_{S_1}^{S_2} T\, dS, \qquad S_2 - S_1 = \int_{T_1}^{T_2} \frac{dQ}{T}.$$

The entropy of a system, as thus defined, depends upon the state only; hence S may be used with p, v, and T as a coordinate. Graphical representations on the TS-plane are specially advantageous, as the areas involved represent heat entering or leaving the system. If the process in question is not reversible (for example, as in the case of the flow of steam in a nozzle), this graphical representation fails. The area in this case does not represent the heat entering the system.

An isothermal process is represented on the TS-plane by a straight line parallel to the S-axis, as AB and CD, Fig. 2; reversible adiabatic processes by lines parallel to the T-axis, as BC and DA. The closed cycle $ABCD$ is the ideal Carnot cycle. Area A_1ABB_1 represents the heat Q_1 absorbed from the source during the isothermal expansion AB, area B_1CDA_1 represents the heat Q_2 rejected to the refrigerator during the isothermal compression CD, and the cycle area $ABCD$ represents the heat $Q_1 - Q_2$ transformed into work. The expression for efficiency $e = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$.

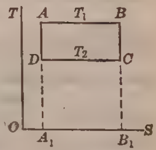

Fig. 2

follows from the geometry of the figure. When the order of events in the cycle of Fig. 2 is reversed, the cycle represents the operation of a refrigerating machine under ideal conditions. During the isothermal expansion DC heat Q_2 is absorbed from the cold body, and during the isothermal compression heat Q_1 is rejected to a body at temperature T_1. The cycle area represents the equivalent of the work W that must be furnished from external sources; and $Q_1 = Q_2 + AW$.

Heat Content. The functions $u + pv$ plays an important part in many technical applications of thermodynamics. The energy u is measured in mechanical units (foot-pounds) and the product pv has necessarily the same unit. The heat equivalent $A(u + pv)$ is denoted by i and is called the heat content of the system (per unit weight). Evidently i, like u and s is a function of the state only. The heat absorbed by a unit weight of a substance at constant pressure is the change in heat content $i_2 - i_1$. It is frequently convenient to represent changes of state on a plane having i and s as coordinates. In this scheme of representation, quantities of heat are represented by segments of lines instead of by areas.

3. Properties of Gases

Experimental Laws. The so-called permanent gases obey quite closely the laws of Boyle, Charles, and Joule, namely:

Boyle's Law. At constant temperature, the volume of a given mass of gas varies inversely as the pressure. That is, the product pv is constant when T is constant.

Charles's Law. At constant pressure, the change of volume of a gas is proportional to the change of temperature. That is, $(\Delta v/\Delta t)\,p = \text{constant}$.

Joule's Law. The intrinsic energy of a gas is independent of the volume of the gas and depends upon the temperature only. Denoting the energy of unit weight of gas by u, Joule's law is expressed symbolically by the relations: $u = f(t)$, $\partial u/\partial v = 0$.

Characteristic Equation. By a combination of the laws of Boyle and Charles the relation $pv = BT$ is obtained as the characteristic equation of a gas that strictly obeys those laws. Here v denotes the volume of unit weight. If V is used to denote the volume of M pounds of gas, the equation takes the useful form $pV = MBT$. The homogeneous form $p_1V_1/T_1 = p_2V_2/T_2$ is advantageous in the solution of problems that involve two states of the gas. The numerical value of the constant B is determined from the equation by inserting known values of p, v, and T corresponding to some assumed standard state. The following are the values of B for certain gases (English units):

Air	53.34	Hydrogen	765.86
Oxygen	48.25	Carbon dioxide	35.09
Nitrogen	55.10	Carbon monoxide	55.14

The gas equation may be written in the form $p = B\gamma T$ where $\gamma = 1/v$ denotes the weight of unit volume. For a chosen pressure and temperature, the product $B\gamma$ is the same for all gases to which the equation applies; and since γ is directly proportional to the molecular weight m, the product Bm is likewise the same for such gases. This product is denoted by R and it is called the universal gas constant. For English units the numerical value of $R = 1544$; hence the gas equation may be written in the form $pv = \left(\frac{1544}{m}\right)T$.

It is frequently desirable to refer the specific volume or the specific weight of a gas to a standard state, namely, atmospheric pressure and a temperature of 32° C. In this state v and γ are expressed in terms of the molecular weight m of the gas by the relations $v_0 = 358.65/m$ and $\gamma_0 = 0.002788\ m$.

Specific Heat of Gases. The specific heat of a gas that obeys the law $pv = BT$ must be independent of the volume and also of the pressure, but it may vary with the temperature. For moderate ranges of temperature the specific heats c_p and c_v may be assumed constant without serious error. The following are mean values for the range 0° to 200° C. (32° to 392° F.):

	c_p	c_v	$k = c_p/c_v$
Air	0.242	0.173	1.4
Hydrogen	3.424	2.446	1.4
Nitrogen	0.2438	0.174	1.4
Oxygen	0.2175	0.155	1.4
Carbon monoxide	0.2426	0.162	1.3
Ammonia	0.5106	0.387	1.32

When the temperature range is large, as exemplified in the internal combustion engine, the assumption of constant specific heat is no longer justified. The experiments of Mallard and Le Chatelier, Langen, and others, show that both c_p and c_v increase with the temperature according to a law that is expressed sufficiently well by the linear relation $c = a + bt$. The variation of specific heat with temperature is represented by the following equations, temperatures being Fahrenheit and m being the molecular weight of the gas:

1. **Diatonic Gases** N_2, O_2, H_2, CO, and **air**.

$$c_p = \frac{1}{m}\left[6.93 + 0.12\left(\frac{T}{1000}\right)^2\right]$$

$$c_v = \frac{1}{m}\left[4.945 + 0.12\left(\frac{T}{1000}\right)^2\right]$$

T = absolute temperature fahr.

m = molecular weight.

2. **Carbon Dioxide** CO_2

For $T < 2900$

$$c_p = 0.1625 + 0.0866\frac{T}{1000} - 0.01364\left(\frac{T}{1000}\right)^2$$

$$c_v = 0.1174 + 0.0866\frac{T}{1000} - 0.01364\left(\frac{T}{1000}\right)^2$$

For $T > 2900$

$$c_p = 0.2772 + 0.0955\frac{T}{1000}$$

$$c_v = 0.2321 + 0.0955\frac{T}{1000}$$

3. **Water Vapor** H_2O

$$c_p = 0.4628 - 0.01533\frac{T}{1000} + 0.0235\left(\frac{T}{1000}\right)^2$$

$$c_v = 0.3525 - 0.01533\frac{T}{1000} + 0.0235\left(\frac{T}{1000}\right)^2$$

Changes of State. For any change of state of a permanent gas the following relations hold. Denoting by M the weight of gas under consideration, the change of energy is

$$U_2 - U_1 = JMc_v(T_2 - T_1) = (p_2V_2 - p_1V_1)/(k - 1).$$

the change of heat content is

$$I_2 - I_1 = Mc_p(T_2 - T_1) = A\,(p_2V_2 - p_1V_1)k/(k-1),$$

and the change of entropy per unit weight is

$$s_2 - s_1 = c_p \log_\epsilon \frac{v_2}{v_1} + c_v \log_\epsilon \frac{p_2}{p_1}.$$

For certain important special changes of state the following relations exist: (W = external work, Q = heat absorbed.)

(a) **Constant Volume:**

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}. \qquad W = 0. \qquad Q = Mc_v(T_2 - T_1) = (U_2 - U_1)A.$$

$$S_2 - S_1 = Mc_v \log_\epsilon \frac{T_2}{T_1}.$$

(b) **Constant Pressure:**

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}. \qquad W = p(V_2 - V_1) = MB(T_2 - T_1).$$

$$U_2 - U_1 = \frac{p(V_2 - V_1)}{k-1} = \frac{W}{k-1}. \quad Q = Mc_p(T_2 - T_1) = I_2 - I_1.$$

$$S_2 - S_1 = Mc_p \log_\epsilon \frac{T_2}{T_1}.$$

(c) **Isothermal Change of State:**

$$p_1V_1 = p_2V_2. \quad U_2 - U_1 = 0. \quad W = MBT \log_\epsilon \frac{V_2}{V_1} = p_1V_1 \log_\epsilon \frac{V_2}{V_1}.$$

$$Q = AW. \qquad S_2 - S_1 = \frac{Q}{T} = ABM \log_\epsilon \frac{V_2}{V_1}.$$

(d) **Adiabatic Change of State:** In the adiabatic change, the equation of the expansion curve is pv^k = const. Combining this with the equation $pv = BT$,

$$Tv^{k-1} = \text{const., or } \left(\frac{V_2}{V_1}\right)^{k-1} = \frac{T_1}{T_2}; \quad \frac{p^{\frac{k-1}{k}}}{T} = \text{const., or } \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \frac{T_2}{T_1}.$$

$$Q = 0. \quad W = U_1 - U_2 = JMc_v\,(T_1 - T_2) = \frac{p_1V_1 - p_2V_2}{k-1}. \quad S = \text{const.}$$

(e) **Polytropic Change of State:** This change is defined by the relation pv^n = **a constant** in which n is a constant.

$$\left(\frac{V_2}{V_1}\right)^{n-1} = \frac{T_1}{T_2}. \quad \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1}.$$

$$W = \int_{v_1}^{v_2} p\,dV = p_1V_1{}^n \int_{v_1}^{v_2} V^{-n}\,dV = \frac{p_2V_2 - p_1V_1}{1-n}. \quad U_2 - U_1 = \frac{p_2V_2 - p_1V_1}{k-1}.$$

$$JQ = U_2 - U_1 + W = \frac{k-n}{(k-1)(1-n)}\,p_2V_2 - p_1V_1).$$

$$W : U_2 - U_1 : JQ = k-1 : 1-n : k-n.$$

For example, let air be compressed according to the law $pV^{1.3}$ = a constant. Taking $k = 1.4$, $W : U_2 - U_1 : JQ = 0.4 : -0.3 : 0.1$. That is, three-fourths of the work of compression is stored in the air and is manifested by a rise of temperature and one-fourth of it is taken away by the water jacket.

The specific heat corresponding to the polytropic change is constant and is given by the relation $c_n = c_v(k - n/(1 - n)$. For values of n lying between 1 and k, c_n is negative. The work W and heat absorbed may be expressed in terms of c_n; thus

$$\mathbf{W} = \mathbf{JM}\,(\mathbf{c_n} - \mathbf{c_v})(\mathbf{T_2} - \mathbf{T_1}) = \frac{\mathbf{MB}}{\mathbf{n} - 1}(\mathbf{T_2} - \mathbf{T_1}). \quad \mathbf{Q} = \mathbf{Mc_n}(\mathbf{T_2} - \mathbf{T_1}).$$

Also

$$\mathbf{S_2} - \mathbf{S_1} = \mathbf{Mc_n} \log_\epsilon \frac{\mathbf{T_2}}{\mathbf{T_1}}.$$

To determine the value of n for an experimental curve, which is assumed to follow the law pV^n = const., measure to any convenient scale p_1, V_1 and p_2, V_2 at two assumed points A and B. Then compute $n = \dfrac{\log P_2 - \log P_1}{\log V_1 - \log V_2}$.

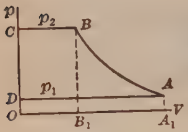

Fig. 3

Air Compression. The ideal indicator diagram of an air compressor without clearance is shown in Fig. 3. The curve AB represents the change of state of the air during compression. Without a water jacket, curve AB may be taken as an adiabatic. If a water jacket is used, AB is represented by an equation pV^n = a constant, and the value of n lies between 1.4 and 1.0. In practice, n is usually about 1.3.

The work represented by area $ABCD$ is given by the expression $W = \dfrac{n(p_1V_1 - p_2V_2)}{n - 1}$, in which V_1 and V_2 denote the volumes at points A_1 and B_1 respectively. The heat absorbed by the water in the jacket during the compression AB is given by $Q = \dfrac{AW(k - n)}{k - 1}$. The work of compression is reduced: (a) by water-jacketing; (b) by compression in two or more stages with cooling between the stages.

4. Saturated and Superheated Vapors

Characteristics of Vapors. When sufficient heat is applied to a liquid the state of aggregation is changed from the liquid to the gaseous. The process is called vaporization, and the resulting gaseous product is called a vapor. As long as the vapor is in contact with the liquid from which it is formed it is said to be saturated. The leading characteristic of a saturated vapor is that its temperature depends upon the pressure only, that is, $\mathbf{t} = \mathbf{f}(\mathbf{p})$. If the vapor is removed from the liquid and is further heated at constant pressure, the temperature rises above the saturation temperature given by $\mathbf{t} = \mathbf{f}(\mathbf{p})$, and the specific volume increases. The vapor is then said to be superheated, and the difference between the temperature of the vapor and the saturation temperature is the degree of **superheat.** In the case of saturated vapor, the volume of one pound is a function of the pressure only ($\mathbf{v} = \phi(\mathbf{p})$), while in the case of a superheated vapor the volume is a function of pressure and temperature ($\mathbf{v} = \mathbf{F}(\mathbf{p}, \mathbf{t})$). The so-called permanent gases, air, nitrogen, etc., are in reality superheated vapors far removed from the saturation state.

Vapor and Liquid Mixtures. The process of vaporization and superheating is shown graphically on the TS-plane in Fig. 4. Liquid at 32° F. represented by point A ($OA = 491.6$) is heated under constant pressure. The process is represented by AB. At B the saturation temperature is reached, vaporization begins and proceeds till all the liquid is changed to vapor, as indicated by point C. Further application of heat superheats the vapor, as indicated by CD. Point B represents liquid at the boiling temperature, point C, saturated vapor, and point D, superheated vapor. For different pressures, points B and C will move along curves s' and s'', called the liquid and saturation curves, respectively. In raising the temperature from 32° to the boiling point, heat represented by area $OABB_1$ is absorbed; this is the heat of the liquid (q'). To vaporize the liquid, heat represented by area B_1BCC_1 is required; this is the heat of vaporization (r). The sum of these is the total heat of the vapor (q''). The area C_1CDD_1 represents the heat absorbed during the superheating. OB_1 represents the increase of entropy (s') during the heating of the liquid, and B_1C_1 the further increase of entropy during vaporization. A point M between B and C represents a mixture of vapor and liquid, the ratio **BM/BC** being the ratio of the weight of vapor to the weight of the mixture. This ratio is denoted by x and is called the **quality** of the mixture.

The latent heat of vaporization r is separable into two parts: (1) the external latent heat, that is, the heat equivalent of the work done during the vaporization process; (2) the internal latent heat ρ, which is the heat required to break up the molecular structure and is stored in the steam in the form of potential energy. The total heat of steam q'' is practically equal to the heat content i'', and in recent steam tables i'' rather than q'' is given.

For a mixture having the quality x (as indicated by point M, Fig. 4) the total heat is $q' + xr$ B.t.u. per lb., and the energy above that of water at 32° is $q' + x\rho$ B.t.u. per lb. The entropy of the mixture per pound is $s' + xr/T$.

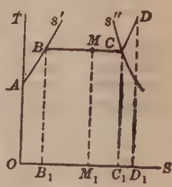

Fig. 4

Thermal Properties of Saturated and Superheated Steam. The magnitudes i', i'', r, ρ, v'', s', and s'', defined in the preceding articles, are dependent on the temperature of vaporization. For several vapors of technical importance these properties have been determined experimentally with more or less accuracy, and empirical formulas have been deduced by means of which the properties are expressed as functions of the temperature. In the case of water vapor, the older tables based on Regnault's experiments are now known to be inaccurate.

The relation between pressure and temperature of saturated steam is given very accurately by the formula

$$\log p = 10.5688080 - \frac{4876.643}{T} - 0.0155 \log T - 0.00406258\, T$$
$$+ 0.00000140055\, T^2 - 0.00002\left[10 - 10\left(\frac{t-370}{100}\right)^2 + \left(\frac{t-370}{100}\right)^4\right].$$

The relation between the volume, pressure and temperature of superheated steam is given by the equation

$$v - 0.017 = 0.59495\,\frac{T}{p} - (1 + 0.05129\, p^{1/2})\frac{66834 \times 10^6}{T^4},$$

in which p is in pounds per square inch.

The following formulas give the heat content and entropy of superheated steam.

$$i = 0.320\,T + 0.000063\,T^2 - \frac{23583}{T} - p(1 + 0.0342\,p^{1/2})\,\frac{6188 \times 10^7}{T^4} + 948.5,$$

$$s = 0.73683 \log T + 0.000126\,T - \frac{11792}{T^2}$$

$$- 0.254 \log p - (1 + 0.03420\,p^{1/2})\,\frac{49504 \times 10^6}{T^5} - 0.0811.$$

With **p** in pounds per square inch the energy **u** is obtained from the relation

$$u = i - 0.1852\,pv.$$

If in the preceding formulas corresponding values of **p** and **T** at the saturation state are inserted, the volume, heat content, and entropy (v'', i'', s'') of saturated steam are obtained.

Properties of Saturated Steam

(Condensed from Goodenough's "Properties of Steam and Ammonia")

Absolute pressure, inches of mercury	Temperature Fahr.	Heat content, B.t.u.		Latent heat, B.t.u.		Entropy			Volume of 1 lb. cu. ft.
		Of liquid	Of vapor	Total	Internal	Of liquid	Of vaporization	Of vapor	
p	t	i′	i″	r	ρ	s′	r/T	s″	v″
1.0	79.06	47.1	1095.0	1047.9	988.7	0.0915	1.9455	2.0370	652
2.0	101.2	69.2	1105.1	1036.0	974.3	0.1316	1.8474	1.9790	338.9
3.0	115.1	83.0	1111.4	1028.3	965.2	0.1561	1.7893	1.9474	231.4
4.0	125.4	93.4	1115.9	1022.5	958.3	0.1739	1.7478	1.9217	176.5
5.0	133.8	101.7	1119.6	1017.9	952.8	0.1880	1.7154	1.9034	143.2
10	161.5	129.4	1131.4	1002.1	934.1	0.2336	1.6134	1.8470	74.8
15	179.1	147.0	1138.8	991.7	922.0	0.2617	1.5526	1.8143	51.1
20	192.4	160.3	1144.1	983.8	912.7	0.2822	1.5089	1.7912	39.1
25	203.1	170.1	1148.3	977.3	905.2	0.2986	1.4747	1.7733	31.7
Lb. per sq. in.									
14.7	212.0	180.0	1151.7	971.7	898.8	0.3100	1.4469	1.7589	26.81
15	213.0	181.0	1152.2	971.2	898.1	0.3135	1.4438	1.7573	26.20
16	216.3	184.3	1153.4	969.1	895.8	0.3184	1.4337	1.7521	24.76
17	219.4	187.5	1154.6	967.1	893.5	0.3230	1.4242	1.7473	23.40
18	222.4	190.5	1155.7	965.2	891.4	0.3274	1.4153	1.7427	22.18
19	225.2	193.3	1156.7	963.4	889.3	0.3316	1.4068	1.7384	21.09
20	228.0	196.0	1157.7	961.7	887.3	0.3356	1.3987	1.7343	20.10
22	233.1	201.2	1159.6	958.4	883.6	0.3430	1.3837	1.7267	18.38
24	237.8	206.0	1161.3	955.3	880.1	0.3499	1.3698	1.7197	16.95
26	242.2	210.4	1162.8	952.4	876.8	0.3563	1.3570	1.7133	15.73
28	246.4	214.6	1164.3	949.7	873.7	0.3622	1.3452	1.7074	14.67
30	250.3	218.6	1165.7	947.1	870.7	0.3679	1.3340	1.7019	13.76
32	254.0	222.4	1166.9	944.6	867.9	0.3731	1.3236	1.6967	12.95
34	257.6	225.9	1168.1	942.2	865.2	0.3781	1.3137	1.6918	12.24
36	260.9	229.4	1169.2	939.9	862.7	0.3829	1.3044	1.6873	11.60
38	264.2	232.6	1170.3	937.7	860.2	0.3874	1.2956	1.6830	11.03

Properties of Saturated Steam—Concluded

Absolute pressure, lb. per sq. in.	Temperature Fahr.	Heat content, B.t.u.		Latent heat, B.t.u.		Entropy			Volume of 1 lb. cu. ft.
		Of liquid	Of vapor	Total	Internal	Of liquid	Of vaporization	Of vapor	
p	t	i′	i″	r	ρ	s′	r/T	s″	v″
40	267.2	235.8	1171.3	935.5	857.8	0.3917	1.2871	1.6788	10.51
42	270.2	238.8	1172.2	933.5	855.5	0.3958	1.2791	1.6749	10.04
44	273.0	241.7	1173.2	931.5	853.3	0.3998	1.2714	1.6712	9.61
46	275.8	244.5	1174.0	929.6	851.2	0.4036	1.2640	1.6676	9.22
48	278.4	247.2	1174.8	927.7	849.1	0.4072	1.2570	1.6642	8.86
50	281.0	249.8	1175.6	925.9	847.1	0.4108	1.2501	1.6609	8.53
55	287.1	255.9	1177.5	921.5	842.3	0.4190	1.2342	1.6532	7.80
60	292.7	261.7	1179.1	917.4	837.8	0.4267	1.2195	1.6462	7.18
65	298.0	267.1	1180.6	913.5	833.5	0.4338	1.2058	1.6397	6.66
70	302.9	272.2	1182.0	909.8	829.5	0.4405	1.1931	1.6336	6.22
80	312.0	281.6	1184.4	902.8	821.9	0.4527	1.1700	1.6227	5.48
85	316.3	286.0	1185.5	899.6	818.4	0.4583	1.1595	1.6178	5.18
90	320.3	290.1	1186.5	896.4	815.0	0.4636	1.1495	1.6131	4.905
95	324.1	294.1	1187.5	893.4	811.7	0.4687	1.1400	1.6087	4.663
100	327.8	297.9	1188.4	890.5	808.6	0.4736	1.1309	1.6045	4.442
105	331.4	301.6	1189.2	887.6	805.5	0.4782	1.1222	1.6004	4.240
110	334.8	305.1	1190.0	884.8	802.6	0.4827	1.1138	1.5965	4.057
115	338.1	308.6	1190.7	882.1	799.7	0.4870	1.1058	1.5928	3.889
120	341.3	311.9	1191.4	879.5	796.9	0.4911	1.0982	1.5893	3.735
125	344.4	315.1	1192.0	876.9	794.2	0.4950	1.0908	1.5858	3.593
130	347.4	318.2	1192.6	874.4	791.6	0.4989	1.0836	1.5825	3.461
140	353.1	324.2	1193.7	869.6	786.4	0.5062	1.0700	1.5762	3.226
150	358.5	329.8	1194.7	864.9	781.6	0.5131	1.0573	1.5704	3.020
160	363.6	335.2	1195.7	860.5	776.9	0.5196	1.0453	1.5649	2.839
170	368.5	340.3	1196.5	856.2	772.4	0.5258	1.0339	1.5597	2.679
180	373.1	345.2	1197.2	852.0	768.0	0.5316	1.0231	1.5547	2.536
190	377.6	350.0	1197.9	847.9	763.9	0.5372	1.0128	1.5500	2.408
200	381.9	354.5	1198.5	844.0	759.8	0.5426	1.0030	1.5456	2.292
210	386.0	358.8	1199.0	840.2	755.9	0.5477	0.9936	1.5413	2.186
220	390.0	363.0	1199.5	836.5	752.1	0.5526	0.9846	1.5372	2.090
230	393.8	367.1	1199.9	832.8	748.3	0.5573	0.9760	1.5333	2.002
240	397.5	371.0	1200.3	829.3	744.7	0.5619	0.9676	1.5295	1.921
250	401.1	374.9	1200.6	825.8	741.2	0.5663	0.9595	1.5258	1.846
260	404.5	378.6	1201.0	822.4	737.7	0.5706	0.9517	1.5223	1.777
270	407.9	382.2	1201.2	819.1	734.4	0.5747	0.9442	1.5189	1.713
280	411.2	385.7	1201.5	815.8	731.1	0.5787	0.9369	1.5156	1.654
290	414.4	389.1	1201.7	812.6	727.9	0.5826	0.9298	1.5123	1.598
300	417.5	392.4	1201.9	809.4	724.7	0.5863	0.9229	1.5092	1.545
350	431.9	408	1202.5	794.5	709.7	0.6036	0.8912	1.4949	1.327
400	444.8	422	1202.5	780.6	695.9	0.6190	0.8631	1.4821	1.162

Properties of Superheated Steam

(Condensed from Goodenough's "Properties of Steam and Ammonia")

i = heat content, B.t.u. per pound; s = entropy; v = volume, cubic feet per pound

Absolute pressure, lb. per sq. in.		Temperature Fahr. Sat	300	350	400	450	500	550	600	650	700
20 [228.0]*	i	1157.7	1192.7	1216.5	1240.2	1263.8	1287.4	1311.1	1335.0	1359.1	1383.3
	s	1.7343	1.7827	1.8131	1.8414	1.8681	1.8934	1.9175	1.9406	1.9628	1.9841
	v	20.10	22.37	23.91	25.44	26.96	28.47	29.97	31.47	32.97	34.47
40 [267.2]	i	1171.3	1188.0	1212.9	1237.3	1261.5	1285.6	1309.7	1333.8	1358.1	1382.4
	s	1.6788	1.7013	1.7331	1.7624	1.7897	1.8155	1.8399	1.8633	1.8856	1.9071
	v	10.51	11.05	11.85	12.64	13.42	14.19	14.95	1.571	16.46	17.22
60 [292.7]	i	1179.1	1183.0	1209.0	1234.3	1259.1	1283.7	1308.1	1332.5	1357 0	1381.6
	s	1.6462	1.6513	1.6845	1.7148	1.7429	1.7692	1.7940	1.8176	1.8402	1.8618
	v	7.18	7.27	7.83	8.37	8.90	9.42	9.94	10.45	10.96	11.47
80 [312.0]	i	1184.4	.....	1205.0	1231.1	1256.6	1281.7	1306.5	1331.2	1355.9	1380.7
	s	1.6227	.....	1.6487	1.6801	1.7089	1.7357	1.7609	1.7848	1.8076	1.8294
	v	5.48	.....	5.81	6.23	6.64	7.04	7.43	7.82	8.21	8.59
100 [327.8]	i	1188.4	.....	1200.8	1227.8	1254.0	1279.6	1304.8	1329.8	1354.8	1379 7
	s	1.6045	.....	1.6199	1.6523	1.6820	1.7093	1.7349	1.7592	1.7822	1.8042
	v	4.44	.....	4.60	4.95	5.28	5.61	5.93	6.24	6.55	6.86
120 [341.3]	i	1191.4	.....	1196.4	1224.4	1251.3	1277.4	1303.0	1328.4	1353.6	1378.7
	s	1.5893	.....	1.5955	1.6291	1.6594	1.6874	1.7134	1.7379	1.7612	1.7833
	v	3.74	.....	3.79	4.09	4.38	4.65	4.92	5.19	5.45	5.71
140 [353.1]	i	1193.7	.....	.....	1220.9	1248.5	1275.2	1301.2	1326.9	1352.4	1377.7
	s	1.5762	.....	.....	1.6087	1.6400	1.6685	1.6949	1.7198	1.7433	1.7656
	v	3.23	.....	.....	3.48	3.73	3.97	4.21	4.44	4.66	4.89
160 [363.3]	i	1195.7	.....	.....	1217.3	1245.6	1272.9	1299.3	1325.4	1351.1	1376.7
	s	1.5649	.....	.....	1.5906	1.6226	1.6518	1.6787	1.7039	1.7276	1.7501
	v	2.84	.....	.....	3.01	3.24	3.46	3.67	3.87	4.07	4.27
180 [373.1]	i	1197.2	.....	.....	1213.6	1242.7	1270.5	1297.4	1323.8	1349.8	1375.6
	s	1.5547	.....	.....	1.5741	1.6070	1.6368	1.6641	1.6896	1.7136	1.7364
	v	2.54	.....	.....	2.65	2.86	3.06	3.25	3.43	3.61	3.79
200 [381.9]	i	1198.5	.....	.....	1209.8	1239.7	1268.1	1295.5	1322.2	1348.5	1374.5
	s	1.5456	.....	.....	1.5589	1.5927	1.6231	1.6509	1.6768	1.7010	1.7239
	v	2.29	.....	.....	2.36	2.56	2.74	2.91	3.08	3.24	3.40
220 [390.0]	i	1199.5	.....	.....	1205.9	1236.6	1265.6	1293.5	1320.6	1347.1	1373.4
	s	1.5372	.....	.....	1.5447	1.5794	1.6105	1.6388	1.6650	1.6895	1.7126
	v	2.09	.....	.....	2.13	2.31	2.47	2.64	2.79	2.94	3.09
240 [397.5]	i	1200.3	.....	.....	1202.0	1233.5	1263.1	1291.4	1318.9	1345.8	1372.3
	s	1.5295	.....	.....	1.5314	1.5670	1.5987	1.6275	1.6541	1.6789	1.7022
	v	1.92	.....	.....	1.93	2.10	2.26	2.41	2.55	2.69	2.83
260 [404.5]	i	1201.0	.....	.....	.....	1230.3	1260.5	1289.4	1317.2	1344.4	1371.1
	s	1.5223	.....	.....	.....	1.5533	1.5877	1.6170	1.7439	1.6690	1.6925
	v	1.777	.....	.....	.....	1.922	2.071	2.212	2.347	2.477	2.605
280 [411.2]	i	1201.5	.....	.....	.....	1227.0	1257.9	1287.2	1315.5	1342.9	1369.9
	s	1.5156	.....	.....	.....	1.5442	1.5773	1.6071	1.6344	1.6597	1.6835
	v	1.654	.....	.....	.....	1.770	1.911	2.045	2.172	2.295	2.414
300 [417.5]	i	1201.9	.....	.....	.....	1223.7	1255.2	1285.1	1313.7	1341.5	1368.7
	s	1.5092	.....	.....	.....	1.5336	1.5674	1.5977	1.6254	1.6510	1.6750
	v	1.545	.....	.....	.....	1.638	1.773	1.900	2.020	2.136	2.249

* The number in brackets is the saturation temperature corresponding to the pressure.

Example 1. Let a mixture of steam and water having a quality $x = 0.96$ expand adiabatically from a pressure of 120 lb. per sq. in. absolute to atmospheric pressure. Required the properties of the mixture in the initial and final states and the work of expansion.

In the initial state the heat content is $i_1' + x_1 r = 311.9 + 0.96 \times 879.5 = 1156.2$ B.t.u., and the energy of the mixture is $i_1' + x_1 \rho_1 = 311.9 + 0.96 \times 796.9 = 1076.8$ B.t.u. The entropy is $s_1' + x_1 r_1/T_1 = 0.4911 + 0.96 \times 1.0982 = 1.5454$. The volume of 1 lb. of dry steam at 120-lb. pressure is 3.735 cu. ft., and the volume of a pound of the mixture is $3.735 \times 0.96 = 3.586$ cu. ft.

In the adiabatic expansion the entropy remains constant, and in the final state, therefore, $s_2' + x_2 r_2/T_2 = 1.5454$, whence $x_2 = 0.852$; that is at the end of the expansion the mixture contains nearly 15% water. The heat content in the second state is $180 + 0.852 \times 971.7 = 1007.9$ B.t.u., the energy is $180 + 0.852 \times 898.9 = 945.9$ B.t.u., and the volume of 1 lb. is $26.81 \times 0.852 = 22.84$ cu. ft. The external work is the equivalent of the decrease in energy, or $778\ (1076.8 - 945.9) = 101\ 840$ ft.-lb. per pound of mixture. The difference $i_1 - i_2 = 1156.2 - 1007.9 = 148.3$ B.t.u., is the available heat, that is, the heat that may be transformed into work in an ideal engine working between the pressure limits under consideration. It is also the equivalent of the kinetic energy $w^2/2\,g$ of a jet flowing from a region of 120 lb. pressure into a region of atmospheric pressure.

Example 2. Let steam at 200 lb. per sq. in. absolute pressure superheated to 560° expand adiabatically to a pressure of 3 in. of mercury. Required the final quality, the change in heat content, and the change of energy.

By interpolation the following values are obtained for the steam in the initial state: $i = 1300.9$, $s = 1.7562$, $v = 2.95$. Consequently the energy per pound is $1300.9 - 0.1852 \times 200 \times 2.95 = 1191.6$ B.t.u. $s_2 = 1.6562 = s_2' + x_2 r_2/T_2 = 0.1561 + 1.7893 x_2$, whence $x_2 = 0.838$, the final quality. $i_2 = 83 + 0.838 \times 1028.3 = 945.1$, $u_2 = 83 + 0.838 \times 965.2 = 892.2$ B.t.u. The change in energy is therefore $1191.6 - 892.2 = 299.4$ B.t.u., and the change in heat content is $1300.9 - 945.1 = 355.8$ B.t.u.

Adiabatic Changes. An adiabatic expansion of a saturated or superheated vapor may be represented approximately by the equation $\mathbf{pv^m} =$ **const.** For saturated steam m depends upon the initial quality and pressure and is given by the equation $m = 1.059 + 0.000315\,p + 0(0.0706 + 0.000376\,p)x$. For superheated steam m may be taken as 1.31, for superheated ammonia at 1.333, and for superheated sulfur dioxide as 1.282. The final volume V_2 is given by the relation $\mathbf{V_2 = V_1(p_1/p_2)^{\frac{1}{m}}}$ and the external work is then approximately $\mathbf{W = (p_1V_1 - p_2V_2)/(m - 1)}$.

5. Flow of Elastic Fluids

General Equation of Flow. Let an elastic fluid, as air or steam, flow along a horizontal tube, Fig. 5. At an assumed cross-section $\mathbf{F_1}$ the pressure of the fluid is $\mathbf{p_1}$ and the mean velocity is $\mathbf{w_1}$; at a second section $\mathbf{F_2}$ the pressure is $\mathbf{p_2}$ and the velocity is $\mathbf{w_2}$. The principal equation of flow is $\mathbf{(w_2^2 - w_1^2)/2\,g = J\,(i_1 - i_2)}$, in which $\mathbf{i_1}$ and $\mathbf{i_2}$ denote, respectively, the heat content per pound of fluid at sections F_1 and F_2. If section F_1 is taken in the boiler or reservoir from which the fluid is flowing, then $w_1 = 0$, and the equation becomes

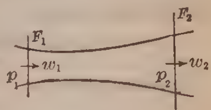

Fig. 5

$$\mathbf{w^2/2\,g = J\,(i_1 - i_2) \quad \text{or} \quad w_2 = 223.7\sqrt{i_1 - i_2}.}$$

In the flow of elastic fluids, the heat content i plays the same part as the head h in the flow of liquids.

Example. Steam at 120 lb. per sq. in., quality 0.96, flows into the atmosphere through a properly proportioned nozzle. Required the velocity of flow. The heat content i_1 is 1156.2 B.t.u. per lb., and if the flow is adiabatic and frictionless the final heat content i_2 is 1007.9 B.t.u. Hence $w = 223.7\sqrt{1156.2 - 1007.9} = 2724$ ft. per sec.

Formulas of Discharge. The weight M of fluid flowing past a cross-section per second is given by the equation of continuity $Fw = Mv$ in which F denotes the area of the section and v the specific volume of the fluid at this section. If the flow is through an orifice or short tube, it is found that the pressure in the plane of the orifice never falls below a certain value p_m called the **critical pressure.** The ratio p_m/p_1 (p_1 being the pressure in the boiler or reservoir) is called the **critical ratio.** For air its value is about 0.53; for saturated steam, about 0.57. If the pressure p_2 of the region into which the fluid is flowing is greater than p_m, the pressure at the orifice is p_2, but if p_2 is less than p_m, then the pressure at the orifice is p_m and the discharge M is the same whatever the value of p_2. The following formulas for discharge are in current use:

(a) **Fliegner's equations for air.**

When $\quad p_2 < 0.53\, p_1 \qquad M = 0.53\, F \dfrac{p_1}{\sqrt{T_1}}.$

When $\quad p_2 > 0.53\, p_1 \qquad M = 1.06\, F \sqrt{\dfrac{p_2(p_1 - p_2)}{T_1}}.$

(b) **Grashof's Equation for Steam.** Taking the area of the orifice in square inches and the pressure p_1 in lb. per sq. in.,

$$M = 0.0165\, F p_1^{0.97}. \qquad p_2 < 0.57\, p_1.$$

(c) **Rateau's Equation.** This equation is empirical and is based on experimental data.

$$M = \frac{F p_1}{1000}(16.367 - 0.96 \log p_1). \quad p_2 < 0.57\, p_1.$$

(d) **Napier's Equations.** These are also empirical and rather inaccurate.

When $\quad p_1 > \dfrac{5}{3} p_2, \quad M = \dfrac{F p_1}{70}.$

When $\quad p_1 < \dfrac{5}{3} p_2, \quad M = \dfrac{F p_2}{42}\sqrt{\dfrac{3(p_1 - p_2)}{2\, p_2}}.$

Example. Required the discharge in pounds per minute of saturated steam at 100 lb. pressure (absolute) through an orifice having an area of 0.4 sq. in. The back pressure is less than the critical pressure, 57 lb. per sq. in.

By Grashof's formula:

$$M = 60 \times 0.0165 \times 0.4 \times 100^{0.97} = 34.493 \text{ lb.}$$

By Rateau's formula:

$$M = \frac{60 \times 0.4 \times 100}{1000}(16.367 - 0.96 \times 2) = 34.673 \text{ lb.}$$

By Napier's formula:

$$M = \frac{0.4 \times 100}{70} \times 60 = 34.286 \text{ lb.}$$

The discharge may be found from the two fundamental formulas:

$$w = 223.7\sqrt{i_1 - i_2}, \text{ and } M = Fw/v.$$

The critical pressure p_m is 57 lb. per sq. in. From the steam table, i_1 (for 100 lb.) = 1188.4 B.t.u.; i_m (for 57 lb.) = 1144.1 B.t.u.; $x_m = 0.963$; $v_m = 7.26$ cu. ft. Then $w = 223.7\sqrt{1188.4 - 1144.1} = 1490$ ft. per sec., and $M = 60 \times \dfrac{0.4}{144} \times \dfrac{1490}{7.26} =$ 34.18 lb.

Flow in Nozzles. When the pressure p_2 of the region into which a jet is flowing is less than the critical pressure p_m, which is the pressure existing in the emerging fluid, the difference $p_m - p_2$ gives rise to a lateral spreading of the jet. This spreading may be prevented by the addition of a properly proportioned tube (Fig. 6). The tube must diverge so as to permit the expansion of the fluid required by the drop in pressure from p_m at the smallest section a to the external pressure p_2 at the end section b. The effect of the diverging nozzle is to cause a solid jet to emerge with a velocity w_2 given by the fundamental relation $w_2 = \sqrt{2\,gJ\,(i_1 - i_2)} = 223.7\sqrt{i_1 - i_2}$. If the external pressure p_2 is greater than p_m the diverging tube is unnecessary and a simple orifice is used. In the Rateau turbine, for example, the drop in pressure from cell to cell is so arranged that $p_2/p_1 > 0.57$ and the steam flows through orifices rather than nozzles.

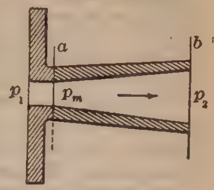

Fig. 6

On account of the length of a nozzle there is necessarily some loss of energy due to frictional resistances. The work of friction is converted into heat, which enters the flowing fluid and increases the final heat content i_2 to a larger value i_2'. It is customary to take as a friction coefficient the ratio of the loss of energy to the kinetic energy of the jet without friction. Denoting the coefficient by y, its formula is

$$y = \frac{i_2' - i_2}{i_1 - i_2}$$

and the actual velocity of exit w_2' is given by the relation

$$w_2' = \sqrt{2\,gJ\,(1 - y)\,(i_1 - i_2)} = 223.7\sqrt{(1 - y)\,(i_1 - i_2)}.$$

The quality of the steam at exit is increased by the heat generated through friction and is given by the equation

$$x_2' = x_2 + \frac{(i_1 - i_2)y}{r_2},$$

in which x_2 denotes the quality on the assumption of frictionless adiabatic expansion and is given by $s_1' + \frac{x_1 r_1}{T_1} = s_2' + \frac{x_2 r_2}{T_2}$. Experiments indicate that y may vary from 0.08 to 0.20, depending on the size of nozzle.

Throttling. The throttling or wiredrawing of a fluid is merely a special case of flow, in which the loss by friction is excessive. The fluid in the region of higher pressure p_1 passes through a valve or constricted passage into a region of lower pressure p_2. The velocity is increased from w_1 to w_2, but the increased kinetic energy is dissipated as the fluid passing the orifice enters and mixes with the fluid in the second region. Ultimately the velocity w_2 is sensibly equal to the original velocity w_1, and therefore $i_2 = i_1$. For a mixture of saturated vapor and liquid, the equation of throttling is $i_1' + x_1 r_1 = i_2' + x_2 r_2$. From this equation the value of x_2 is found, and having x_2, the increase of entropy is determined; and the increase of entropy multiplied by T_0 gives the loss of available energy resulting from the throttling.

If steam at the higher pressure p_1 is nearly dry, it becomes superheated when throttled to a much lower pressure. The equation, in this case, becomes $i_1' + x_1 r_1 = i_2'' + c_p(t_2' - t_2)$, in which t_2 denotes the saturation temperature for the pressure p_2, t_2' the temperature of the superheated steam, and c_p the mean specific heat for the range $t_2' - t_2$. By means of this equation the initial quality x_1 may be determined from observed values of p_2 and t_2'.

6. Steam Boilers

Types of Boilers. Steam boilers may be divided into two general classes: (1) **fire tube boilers,** in which the hot gases from the furnace pass through the tubes and the water surrounds the tubes; (2) **water-tube boilers,** in which the water flows through the tubes. A second classification takes account of the location of the furnace. In an **externally fired** boiler, the furnace is placed in a brick chamber external to the boiler. The **internally fired** boiler has its furnace enclosed in the steel shell of the boiler itself. Marine and locomotive boilers are examples of the latter type.

Horsepower. The capacity of a boiler for the generation of steam is measured in terms of an arbitrary unit called horsepower. As defined by the American Society of Mechanical Engineers, a **boiler horsepower** is the equivalent of the evaporation of 34.5 lb. of water per hour from and at 212° F. The latent heat of steam at 212° F. and atmospheric pressure is 971.7 B.t.u.; hence a horsepower is equivalent to the development of **33 523 B.t.u. per hour.**

Let **M** denote the weight of feed water actually evaporated per hour. Also let i_1' denote the heat content of the feed water at the temperature at which it enters the boiler, and i_2 the heat content of the steam in its final condition (wet, saturated, or superheated). Then the boiler horsepower is given by the formula

$$\text{Boiler hp.} = \frac{M\,(i_2 - i_1')}{33\,523}$$

Boiler Capacity. The **rated** boiler horsepower depends upon the heating surface, which is the total surface of the shell and tubes that is exposed to hot gases and has water on the opposite side. According to recent practice stationary boilers are rated on the basis of 10 sq. ft. of heating surface per boiler horsepower. By this system of rating, boilers having the same heating surface are rated at the same horsepower. The actual capacity of one boiler may, however, be greater than that of another boiler of the same horsepower due to a more advantageous arrangement of heating surface. Most power plant boilers can develop from 200% to 400% of their normal rating, and many are operating for long periods at 200% rating. However, the efficiency of the boiler may be decreased when it is forced to develop much more than its rated capacity. The decrease in efficiency is not significant until the overload reaches 25% (for fire tube) to 100% (for water tube).

Boiler Performance. In estimating boiler performance the following items may receive consideration: 1. Efficiency. 2. Rate of combustion. 3. Evaporation per pound of fuel. 4. Heat absorbed per square foot of grate surface per hour. 5. Percentage of rated horsepower developed.

The overall **boiler efficiency** is the ratio of the heat absorbed to that supplied: thus

$$E_b = \frac{M\,(i_2 - i_2')}{M_c\,H}$$

in which E_b = efficiency of boiler, furnace, and grate;
M_c = total weight of coal fired per hour;
H = heat of combustion of 1 lb. of coal.

The **rate of combustion** may be expressed as the weight of fuel burned per hour (*a*) per square foot of projected gate surface, or (*b*) per cubic feet of furnace volume. The boiler horsepower that can be developed is dependent on the rate of combustion; that is, upon the kind of fuel, size and kind of grate, and the quantity of air supplied. Rates of expansion as high as 200 lb. per hour per square feet of grade surface have been obtained in locomotive boilers, the high capacity accompanied by a lowered efficiency. With ordinary chimney draft the grate area should be 1/4 sq. ft. or more per rated horsepower. With forced draft, the area may be considerably smaller. The rate of combustion is ordinarily 10 to 20 lb. of coal per hour per square foot of grate for anthracite coal and 20 to 30 lb. for bituminous coal. With forced draft these rates may be largely exceeded.

Small **hand-fired** boilers may show overall efficiencies varying from 0.40 to 0.65; **stoker-fired** boilers may show efficiencies from 0.60 to 0.80, depending on the size of the plant, and the amount of heat saving equipment in operation. Large **pulverized coal installations** have given efficiencies of 80 to 85%. With **superheaters, air pre-heaters,** and **economizers** in operation the efficiency of a large installation with mechanical stokers, or with pulverized coal may be pushed up to nearly 90%. It may be noted that the average all-year boiler efficiency is well below the high value obtained for short periods under test conditions.

Heat Losses. The principal losses of heat incurred in the combustion of fuel in the boiler furnace are:

1. Heat carried away by the dry chimney gases.
2. Heat required to evaporate moisture in coal.
3. Heat required to evaporate moisture formed by burning hydrogen in fuel.
4. Heat required to superheat water vapor in air supplied.
5. Loss of heat due to incomplete combustion of fuel.
6. Heating value of unconsumed carbon in ash and refuse.
7. Heat radiated; also heat not accounted for.

A tabulation of the various heat losses, along with the heat absorbed by the boiler constitutes a **heat balance** for the boiler and furnace. From such a heat balance a comparison of the various losses may be made.

The **data required** for the determination of the various losses are: Chemical analysis of fuel, of chimney gases, and of ash; heat of combustion of fuel; relative humidity of air supplied; temperature of air supplied and temperature of chimney gases.

An indication of the boiler operating conditions as regards economy is furnished by the composition of the **chimney gases.** With insufficient air supply some of the carbon content of the fuel may be burned to CO instead of to CO_2. The result will be a loss of heat developed amounting to about 10 150 B.t.u. per pound of carbon. The CO_2 content of the chimney gases is of special significance. Under good conditions of operation the CO_2 content will vary from 9 to 13% by volume. A value lower than 9% usually indicates either excessive air supply to the fuel bed or leakage of air through the furnace walls. More than 13% of CO_2 may indicate an insufficient supply of air, and a small amount of CO may be present in the gases. Many boiler plants are provided with automatic CO_2 recorders.

Boiler Tests. The object of a boiler test may be:

1. To determine the capacity and efficiency of the boiler.
2. To compare different kinds of fuels.
3. To compare different conditions and methods of operation.

Boiler tests should be conducted in accordance with the boiler test code prepared by the Power Test Code Committee of the American Society of Mechanical Engineers.

7. Steam Engines

Types of Steam Engines. Reciprocating steam engines may be grouped in classes in several ways. The type of valve gear furnishes one basis of division; thus engines may be classed as slide-valve, Corliss, four-valve, etc. Slide-valve engines may be either throttling or automatic, according as the governor regulates the work in the cylinder by throttling the steam or by changing the point of cutoff. Engines in which the complete expansion of steam takes place in one cylinder are **simple engines**; if the expansion requires two, three, or four cylinders, the engines are termed **compound, triple,** and **quadruple,** respectively. Engines are **condensing** when the exhaust steam passes into a condenser, **non-condensing** when it passes directly into the atmosphere. Most steam engines are **double-acting,** that is, work is done during both strokes of the piston.

Ideal Cycle. The ideal cycle of a steam engine without clearance may be represented most advantageously on the temperature-entropy plane. The point A (Fig. 7) represents the state of one pound of water entering the boiler; curve AB represents the heating of the water to the boiling temperature T_1, BC represents vaporization, CD adiabatic expansion in the engine cylinder, and DA condensation of the exhaust steam. The cycle area $ABCD$ represents the available heat, that is, the part of the total heat supplied that may be transformed into work. Denoting by i_1 the heat content of the steam in state C and by i_2 the heat content after adiabatic expansion to D, the available heat is the difference $i_1 - i_2$. The **weight of steam consumed** per horsepower-hour is **$2546/(i_1 - i_2)$**.

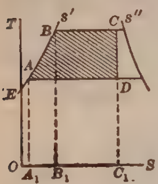

Fig. 7

Example. Referring to the examples on p. 341, it was shown that in the case of an engine using steam at 120-lb. pressure, quality 0.96, and exhausting at atmospheric pressure, the available heat was 148.3 B.t.u. per lb. For an engine using steam at 200-lb. pressure superheated to 560° and with a condenser pressure of 3 in. of mercury, the available heat was 355.8 B.t.u. per lb. In the first case the ideal steam consumption is 2546/148.3 = 17.2 lb., and in the second case it is 2546/355.8 = 7.16 lb. per hp. hr.

This ideal cycle is called the **Rankine cycle** and is used as a standard by which to measure engine performance. With given limiting pressures p_1 and p_2 and initial quality x_C, the available heat is the maximum amount of heat that can be transformed into work under these conditions, and the ratio of the heat actually transformed to the available heat indicates the real efficiency of the engine from the point of view of thermodynamics.

Indicator Diagram. The actual cycle of the steam engine differs from the ideal Rankine cycle in several particulars: (1) The metal of the cylinder walls and piston conducts heat, and there is thus an active interchange of heat between the metal and steam, which makes adiabatic expansion impossible. (2) The valves do not act instantly and there is wiredrawing of steam in passing through the ports. (3) The cylinder must have clearance. The result of these modifications is a cycle having a smaller area than the ideal Rankine cycle.

On the pv-plane the actual cycle of the engine is the diagram drawn by the indicator (Fig. 8). In this diagram, OV represents the line of **zero pressure,** XX the atmospheric line, OX representing atmospheric pressure and YY the line of boiler pressure. The length GH represents the volume swept through by the piston, and XG the clearance volume V_c. AB is the **steam line,** B the point of cutoff, BC the **expansion curve**, C the **point of release**, CD the **exhaust line**, DE the **back-pressure line**, E the **point of exhaust closure**, EF the **compression curve**, and FA the **admission line.** A single indicator diagram shows graphically the pressures exerted by the steam on one side of the piston during two strokes. To determine the net steam pressure transmitted to the piston rod, diagrams taken simultaneously from opposite ends of the cylinder must be superimposed.

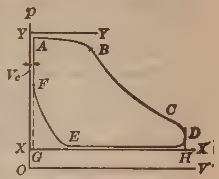

Fig. 8

The expansion curve BC of the actual diagram may be represented by an equation of the form $pV^n =$ a constant. The usual statement is that the curve is an equilateral hyperbola, in other words, that $n = 1$, has no foundation. Experiments show that the value of n depends upon the quality of the mixture in the cylinder at cut off; if there is a large percentage of water in the mixture, n falls as low as 0.85 or 0.9, whereas if there is little or no water present, due to use of superheated steam, n may rise to 1.2.

The indicator diagram has **two principal uses:** (1) It shows the distribution of the steam to the cylinder, and reveals errors in the setting of the valves. (2) It gives the horsepower developed in the cylinder, the so-called indicated horsepower (i.hp.).

Indicated Horsepower. The **area** of the indicator diagram is the integral $\int pdV$ taken around the closed cycle; hence it represents the **work of the cycle.** The mean ordinate of the diagram (see Fig. 8) is found by any convenient method. Ordinarily the area is determined by a planimeter, and this area divided by the length GH gives the mean ordinate. Multiplying by the scale of the indicator spring (30, 40, or 60 lb. per sq. in.), the resulting product is the **mean effective pressure** (m.e.p.) in pounds per square inch.

Let **p** denote the mean effective pressure, **a** the area of piston in square inches, **l** the length of stroke in feet, and **n** the revolutions per minute. Then the indicated horsepower is

$$\text{hp.} = \frac{2\ \mathbf{plan}}{33\,000}$$

For a **single-acting engine,** the factor 2 is dropped. For accurate calculations, the area a should be reduced by one-half of the cross-section area of the piston rod if the rod passes through one head and by the whole area if the rod passes through two heads. The piston speed S, is given by $2\,ln$; hence i.h.p. $= paS/33\,000$.

Brake Horsepower is the horse-power of the engine delivered at the flywheel. The difference between the i.hp. and b.hp. is the **friction horsepower,** and the ratio $\frac{\text{b.hp.}}{\text{i.hp.}}$ the **mechanical efficiency** of the engine. The brake horsepower is determined experimentally by the use of a band brake or an absorption dynamometer.

Losses in the Steam Engine. The following are the principal sources of loss of available heat in the operation of the engine: (1) Wiredrawing in ports and valves, and friction in pipes. (2) Leakage past the piston and valves. (3) Loss due to clearance. (4) Radiation and conduction from the cylinder. (5) Initial condensation.

The losses indicated in the first four items may be reduced to a minimum by careful design and construction. The most serious loss is due to the interchange of heat between steam and metal, resulting in initial condensation and subsequent evaporation. The magnitude of this loss depends upon the conditions of operation. It is greater the slower the rotative speed of the engine, the earlier the cutoff, the greater the temperature range in the cylinder, and the greater the ratio of cylinder-wall surface to cylinder volume. The following are the means used to reduce the loss due to initial condensation.

(a) **Steam Jackets.** The cylinder is surrounded by an annular space containing steam at boiler pressure. The condensation takes place in this jacket instead of in the cylinder. The jacket is most effective in slow-speed engines having large ratios of expansion.

(b) **Compounding.** The large temperature range that accompanies a large ratio of expansion may be divided between 2, 3, or 4 cylinders, thus giving a relatively small range in each cylinder. The total loss from condensation in all the cylinders is less than the loss would be if the steam were expanded in a single counter-flow cylinder. The use of two or more cylinders is also advantageous mechanically; the pressures on cranks, shafts, etc., are reduced, and by setting the cranks at proper angles a more uniform turning moment on the shaft is obtained.

(c) **Superheating.** The most effective means of reducing initial condensation is the use of steam initially superheated. By a sufficient degree of superheat dry steam at cutoff may be insured, and as the rate of absorption of heat by a gas is small, the activity of heat interchange between steam and metal is greatly reduced.

(d) **Uni-directional Flow.** Steam flows directly from the inlet to the exhaust. The engine piston serves as a valve at the exhaust ports, which are located in the cylinder walls midway between the heads. The inlet ports are not chilled by the low temperature exhaust steam; and the cushion steam is compressed to a pressure and temperature nearly equal to those of the supply steam. The expansion of the steam through a wide temperature range in a single cylinder with reduced cylinder condensation results in excellent economy under all conditions of load.

Efficiency Standards. The term efficiency when applied to the steam engine may mean any one of several ratios. Let q = heat supplied to an engine per pound of steam, q_a = available heat, the maximum quantity of heat that can be transformed into work in the ideal Rankine cycle, q_e = heat transformed into work in actual engine, $W_e = Jq_e$ the indicated work per lb. of steam, W_b = the work obtained at the brake per pound of steam. Then $e_a = q_a/q$ = thermal efficiency of Rankine engine,

$$e_e = \frac{q_e}{q} = \text{thermal efficiency of actual engine,}$$

$$e_i = \frac{e_e}{e_a} = \frac{q_e}{q_a} = \text{efficiency ratio (sometimes called potential efficiency),}$$

$$e_b = \frac{W_b}{Jq_e} = \text{brake efficiency ratio (based on work at break),}$$

$$e_m = \frac{W_b}{W_a} = \text{mechanical efficiency.}$$

Two other standards are coming into use. These are B.t.u. consumed per indicated horsepower-hour and B.t.u. per brake horsepower-hour. Since 1 hp.-hr. = 2546 B.t.u., the first ratio is equal to 2546 divided by the indicated thermal efficiency; thus if the thermal efficiency is 0.20, the engine consumes 2546/0.20 = 12 730 B.t.u. per hour per indicated horsepower.

The really useful criterion of engine performance is the ratio e_b which is equal to the product $e_t \times e_m$. The ratio e_t is a measure of the extent to which the engine transforms into work the heat q_a that is available for transformation, and the ratio e_m measures the mechanical perfection of the engine. The product $e_t \times e_m$, therefore, is an indication of the quality of the engine both thermodynamically and mechanically.

The Rankine efficiency e_a depends upon the pressure limits used and may vary from 0.08 to 0.30. The thermal efficiency of the actual engine varies from 0.03 to as high as 0.25. The efficiency ratio e_e usually lies between 0.60 and 0.75, but in exceptional cases the extremely high value 0.88 has been attained. At full load the mechanical efficiency e_m should lie between 0.80 and 0.95. Since the engine friction remains nearly constant at all loads, e_m may fall as low as 0.50 at light loads.

Performance of Steam Engines. The following gives approximate values of steam consumption of various classes of steam engines (Allen and Bursley's Heat Engines, third edition, p. 246):

Steam consumption.	Pounds per indicated horsepower-hour
Simple throttling engine, non-condensing	33–49
Simple automatic engine, non-condensing	27–34
Simple Corliss engine, non-condensing	21.5–26
Simple automatic engine, condensing	17–20
Simple Corliss engine, condensing	19.3–21.7
Compound automatic engine, non-condensing	25–30
Compound automatic engine, condensing	18–20
Compound Corliss engine, condensing	12–15
Triple Corliss engine, condensing	12-1/4–13
Uniflow engine, simple, condensing, superheat	11-1/4–12

The Economy of Pumping Engines is usually expressed in terms of **duty**, which is defined as follows: The duty is the number of foot-pounds of work obtained from the pump cylinders per million B.t.u. furnished to the engine by the boilers. The duty is sometimes expressed as the foot-pounds of work per 1000 lb. of dry steam or per 100 lb. of coal. The energy supplied by 1000 lb. of dry steam is somewhat indefinite, but, roughly, the work done by 1000 lb. of dry steam is 10 to 25% greater than the work done per million B.t.u. In view of the difference in quality and heat of combustion of different coals, duty expressed in 100 lb. of coal is practically meaningless. The duty that may be expected of various forms of pumping engines is as follows (Allen and Bursley's Heat Engines, p. 248, third edition):

Small duplex non-condensing pumps	10 000 000
Large duplex non-condensing pumps	25 000 000
Small simple flywheel pumps, condensing	50 000 000
Large simple flywheel pumps, condensing	65 000 000
Small compound flywheel pumps, condensing	85 000 000
Large compound flywheel pumps, condensing	120 000 000
Large triple-expansion flywheel pumps, condensing	165 000 000
Large triple-expansion flywheel pumps, condensing, of exceptional economy	180 000 000

The efficiency ratio of a good steam engine should lie between 0.62 and 0.75. Under exceptional conditions it may rise to 0.80. In general, the steam consumption per horsepower-hour and the efficiency ratio vary with the load. At light loads the efficiency decreases rapidly as the load is decreased; for loads in excess of the engine's rating, the efficiency also decreases but at a slower rate.

Steam Engine Testing. The object of the test is to determine the steam consumption of the engine and the efficiency ratio. If a surface condenser is used the steam consumption is determined by weighing the steam condensed;

otherwise, the water fed to the boilers must be measured, care being taken that all the steam produced from the feed water goes to the engine. The length of the engine test should be at least 5 hours, and if the test includes the boilers, it should continue 24 hours. The indicated horsepower is determined from indicator diagrams taken at intervals of 10 or 15 minutes; and the brake horsepower is obtained by the use of a brake or dynamometer. The weight of water reduced in pounds per hour is divided by the average indicated horsepower, and the quotient is the steam consumption in pounds per horsepower-hour. The steam consumption for the ideal Rankine engine is readily found for the same conditions and the ratio of these gives the efficiency ratio.

A code of rules for conducting steam-engine tests is contained in the Transactions of the A. S. M. E., vol. xxiv. The following are extracts from the report of the committee:

From Introduction to Report. The heat consumption of a steam engine plant is ascertained by measuring the quantity of steam consumed by the plant, calculating the total heat of the entire quantity, and crediting this total with that portion of the heat rejected by the plant which is utilized and returned to the boiler. The term engine plant as here used should include the entire equipment of the steam plant which is concerned in the production of the power, embracing the main cylinder or cylinders; the jackets and reheaters; the air, circulating, and boiler-feed pumps, if steam driven; and any other steam-driven mechanism or auxiliaries necessary to the working of the engine.

X. Measurement of Feed Water. The method of determining the steam consumption applicable to all plants is to measure all the feed water supplied to the boilers, and deduct therefrom the water discharged by separators and drips, as also the water and steam which escape on account of leakage of the steam main and branches connecting the boiler and engine. In plants where the engine exhausts into a surface condenser the steam consumption can be measured by determining the quantity of water discharged by the air pump, corrected for any leakage of the condenser, and adding thereto the steam used by jackets, reheaters, and auxiliaries as determined independently.

XIII. Indicated Horsepower. The indicated horsepower should be determined from the average m.e.p. of diagrams taken at intervals of 20 minutes, and at more frequent intervals if the nature of the test makes this necessary, for each end of the cylinder.

XXI. Standards of Economy and Efficiency. The hourly consumption of heat . . . divided by the indicated and brake horsepower, that is, the number of heat units consumed per i.hp. and per b.hp. per hour, are the standards of engine efficiency recommended by the committee.

XXIV. Ratio of Economy of an Engine to That of an Ideal Engine. The ideal engine recommended for obtaining this ratio is . . . one which follows the Rankine cycle, where steam at constant pressure is admitted into the cylinder with no clearance, and after the point of cutoff is expanded adiabatically to the back pressure. In obtaining the economy of this engine the feed water is assumed to be returned to the boiler at exhaust temperature.

8. Steam Turbines

Types of Steam Turbines. Steam turbines may be divided into two general classes: (1) The **impulse** or **velocity** turbine, which is analogous to the Pelton water wheel. Steam expands in a nozzle until the pressure drops to the pressure in the region in which the turbine wheel rotates, and the jet issuing with relatively high velocity is directed against the blades of the wheel. (2) The **reaction** or **pressure** turbine, which is analogous to the water turbine. In this type of turbine the steam flows through alternate guide blades and moving blades and the pressure gradually falls through both sets of blades. The essential differences between the two types are shown by the following comparison:

Impulse or Velocity	Reaction or Pressure
1. Drop of pressure in nozzles only.	1. Drop of pressure continuous in guides and moving blades.
2. Jet may fill only part of wheel circumference.	2. Turbine runs full.
3. Blades may be symmetrical.	3. Blades necessarily unsymmetrical.
4. Jet velocity high, 1000 to 3500 ft. per sec.	4. Steam velocities low, 300 to 900 ft. per sec.
5. Speed of blade nearly one-half of jet velocity for highest efficiency.	5. Speed of blade nearly equal to velocity of steam.

Compounding. The high velocity of the steam jet resulting from a considerable drop of pressure renders desirable some methods of compounding in order that the peripheral speed of the turbine wheels may be kept within reasonable limits. In most turbines pressure compounding is used. The total drop of pressure $p_1 - p_2$ is divided among several wheels, thus reducing the velocity of the jet at each wheel. The arrangement is shown in Fig. 9. Steam passes through orifices m_1, m_2, etc., in the partition which divide the interior of the turbine into wheel chambers. The pressure drops from p_1 to p_2 in passing through the first orifices, then from p_2 to p_3, etc. The curves p and w show roughly the pressure and velocity changes. The principle of velocity compounding is shown in Fig. 10. The steam is expanded in a nozzle to the back pressure p_2, thus giving a jet of relatively high velocity. The jet passes into the first moving wheel, then through a fixed guide, where its direction is reversed, then into a second moving wheel. A second guide and a third wheel are sometimes added. The curves p and w show the changes of pressure and velocity.

Leading Commercial Turbines. The **De Laval turbine** is a velocity turbine having a single stage. The steam expands in a diverging nozzle to the final pressure and the jet passes through a single wheel.

The **Rateau turbine** is a velocity turbine with pressure compounding, Fig. 9. The ratio of pressures between successive stages is kept above the critical ratio, so that diverging nozzles are not necessary. The number of pressure stages is relatively large. The larger turbines constructed by the General Electric Co. are of the Rateau type.

The **Parsons turbine** is a reaction or pressure turbine. The moving blades are mounted on a drum or rotor and the rows of moving blades alternate with rows of stationary blades attached to the casing. The entire annular space between rotor and casing is filled with steam and the pressure drops continuously as the steam passes through the stages. The radial length of the blades is increased from stage to stage to provide for the increasing specific volume of the steam. The Parsons turbine is made in the United States by the Westinghouse Company and by the Allis-Chalmers Company.

The **Westinghouse double-flow turbine** is a combination of a velocity and pressure turbine. The steam at boiler pressure is expanded in a nozzle to a much lower pressure and the jet at high velocity is passed through two sets of moving blades with intermediate guide blades (see Fig. 10). The steam is then led to two rotors on opposite ends of the shaft, and the remainder of the expansion is the same as in the Parsons turbine.

Performance of Steam Turbines. The economy of a steam turbine measured in steam consumption per horsepower-hour is not much different from the economy of a reciprocating engine under the same conditions. With a good vacuum (28 to 29 in. of mercury) and with superheated steam the consumption per brake horsepower-hour should lie between 10.5 and 14 lb. The efficiency ratios shown by numerous tests lie between 0.60 and 0.78.

Steam Turbine Speeds. Turbines of practically equal efficiency may be designed within wide limits of rotative speed. The peripheral velocity is the controlling condition so that a lower rotative speed requires an impeller of correspondingly larger diameter and results in greater cost. Formerly units having rotative speeds up to 20 000 r.p.m. were used in small turbines, whereas large units direct connected to electric generators were designed for speeds as low as 750 r.p.m. The tendency during recent years has been toward greatly

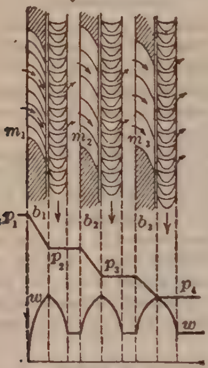

Fig. 9

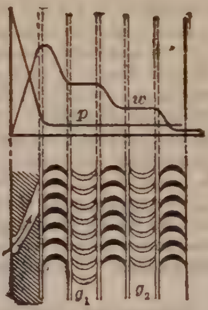

Fig. 10

increased rotative speeds in the larger units which are being designed with speeds up to 3600 in this country and considerably higher in foreign countries. Units of ordinary size up to about 2500 hp. are commonly built with speeds of 4500 to 6000 according to manufacture. Peripheral velocities now range from about 200 ft. per sec. for smaller units up to 750 or more for larger units. Foreign practice is considerably in advance of American practice in the use of these high velocities, but American designers are following rapidly.

Because of high speeds, the turbine is often used with **reduction gears.** The **double helical** type is used for reduction ratios up to about 15 to 1 and **worm** gears for higher ratios. Reductions up to 100 to 1 may be obtained by single worm gear reductions and a ratio of about 900 has been used in at least one case by a **compound worm** gear reduction. Obviously this ratio could have been increased up to 10 000 if conditions demanded. These gears have been developed to a high degree of efficiency, durability and freedom from noise. The better helical gears absorb less than 2% of the power transmitted and the worm gears less than 10%. The considerable cost and the space occupied by these reduction gears are the main drawbacks to their use.

When the steam turbine is used for driving **centrifugal pumps** direct drive or gear drive is used according to circumstances, more often direct for the smaller units, though at sacrifice of efficiency. The general prevalence of alternating current power has caused paramount development of centrifugal pumps at the various **synchronous electric motor speeds** particularly in 60-cycle current, namely, approximately 900, 1100, 1400, 1800 and 3300 r.p.m., the latter being rarely used on account of lesser pump efficiency. For use with pumps of about 900 to 1400 r.p.m. steam turbine must be geared. For pumps of about 1800 r.p.m. the turbine is not infrequently used direct connected for simplicity and cheapness, but at a considerable sacrifice of steam economy, as illustrated in Fig. 11. In this figure curves A and B, Fig. 11, represent approximate performance of two commercial makes, one having normal speed of 6000 and the other of 4500 r.p.m.

EFFECT OF SPEED ON STEAM ECONOMY USING COMMERCIAL TYPES OF TURBINES, SIZES UP TO ABOUT 2500 H. P.

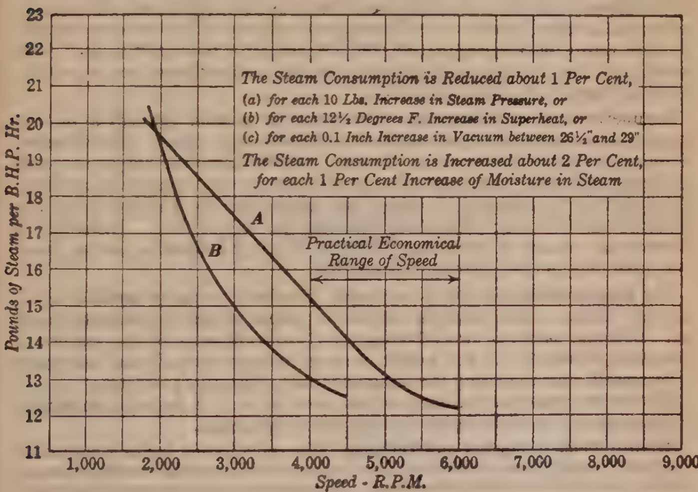

Fig. 11

Higher Steam Pressure and Superheat. Modern tendencies are toward greatly increased steam pressures. Plants of ordinary size are being designed for pressures of about 400 lb., medium sized utility plants for pressures of about 600 lb., and the largest plants for pressure up to 1300 lb. Super-heat of from 100° to 150° F. is being used for plants of ordinary size and 200° for the high pressure installations. For pressures up to about 400 lb. steam use by turbines is decreased roughly 10% for every 100 lb. increase in pressure and 8% for every 100 degrees of superheat. The increase in efficiency at higher pressures is at a lesser rate.

In recent installations steam at a pressure of 1200 lb. is passed through a turbine and is discharged at a pressure of about 450 lb. into the steam main. The high-pressure turbine thus acts as a reducing valve and adds 5000 to 7000 kw. to the capacity of the plant.

The initial temperature of the steam has been increased along with the pressure. At present about 750° Fahr. is the upper limit.

Low-pressure Steam Turbines. Because of the limitation of cylinder volume, the reciprocating engine is unable to make effective use of an extremely high vacuum. No such restriction applies to the steam turbine, as the buckets in the final stages may be made long enough to pass the required volume of steam at the lowest pressures obtainable. In general, the reciprocating engine is more economical than the turbine at high pressures, but the reverse is true at low pressures; hence a combination of reciprocating engine, to work between boiler pressure and atmospheric pressure, and a turbine to take steam from the engine and expand it to condenser pressure, is more efficient than the engine alone or turbine alone. Such combinations have found favor in marine engineering. In many cases low-pressure turbines have been installed with excellent results in existing reciprocating engine plants (see paper by Stott and Pigott, Trans. A. S. M. E., 1910). Rateau has applied the low-pressure turbine in cases where the steam supply is intermittent, as in rolling mills, mine hoists, etc. The engine exhausts at about atmospheric pressure into a heat accumulator, which is simply a tank partly filled with water, and the turbine is supplied from this accumulator (see paper by Rateau, Trans. A. S. M. E., vol. 25).

Resuperheating and Bleeding. The steam in passing through a turbine loses its superheat and begins to condense in the later stages. The presence of water is detrimental to stage efficiency and therefore the stages near the condenser end suffer more and more as the water in the steam accumulates. This loss may be obviated by taking the steam from the turbine before the saturation point is reached and superheating it a second time. Reheating is effective as a means of increasing efficiency, but it is expensive and is, therefore, used only in the larger central stations.

The **regenerative** principle is employed effectively in many turbine installations. Steam is bled from the turbine at various stages to feed-water heaters. Bleeding presents two advantages: (1) the efficiency of the turbine is somewhat increased; (2) the weight of steam passing through the last stages is decreased, thus rendering the design of those stages more effective.

Bleeder or extraction turbines are also used where it is desirable to extract low pressure steam from the turbine for various processes.

Relative Fields of Engines and Turbines. The reciprocating engine is most effective in any service of an irregular character, where varying speed is required, or where the direction of rotation must be reversed. The moderate rotative speed of the engine is also a decided advantage in many classes of service. The field in which the turbine is preëminent is in central power-station service. The high rotative speed of the turbine facilitates direct connection to electric generators. The turbine requires small floor space and comparatively inexpensive foundations. For these reasons, as well as low first cost, the turbine has been more generally adopted than the reciprocating engine in central-station service. The turbine has also been applied successfully to marine service both alone and in combination with reciprocating engines.

9. Internal Combustion Engines

Heating by Internal Combustion. An air engine is one that uses air as the working fluid. The term hot-air engine is usually applied, however, to an engine in which the air is separated from the furnace by a metal wall. Such engines have been failures for the reason that the air absorbs heat slowly and it is impossible to maintain a high temperature in the working fluid. By the method of heating by internal combustion the rapid chemical action supported by the medium itself permits the rapid heating of large quantities of air to a very high temperature. The medium and the furnace being within the cylinder, the metal walls can be kept at a sufficiently low temperature by a water jacket, and the inner surface may be exposed to high temperature without danger of destruction. Engines that make use of the principle of internal combustion are in reality air engines, since the working medium is principally air, but they are generally known as gas engines and oil engines.

Types of Gas Engines. Internal combustion engines fall under two chief classes: (1) The **explosion** type, in which the mixture of fuel and air is drawn into the cylinder and ignited, thus producing an explosion and a sudden rise of pressure at nearly constant volume. (2) The **slow-burning** type, in which the fuel is introduced gradually and burned quietly without increase of pressure. The Otto engine is a representative of the first type, the Diesel engine of the second type.

Another classification is based on the number of strokes of the piston required for the completion of the cycle. In the **four-cycle** engine four strokes are required, in the **two-cycle** engine, two strokes. Engines of small power are usually **single-acting; large engines, are, however, usually double-acting.**

According to the class of service, internal combustion engines may be grouped as follows: (1) Small gas and gasoline engines for miscellaneous service. (2) Automotive engines for automobiles and aviation. (3) Large gas and oil engines for power plants. For descriptions of various types, see Streeter's "Internal Combustion Engines."

The Otto Cycle. In Fig. 12 is shown the ideal indicator diagram of the four-cycle Otto engine. The operations are as follows: The explosive mixture is drawn into the cylinder, as represented by *ED*, and compressed adiabatically (*DA*). The mixture is ignited, causing a rise of pressure, as shown by *AB*. The gases in the cylinder, which are products of combustion with an excess of air, expand adiabatically, as shown by *BC*. Finally, the burned gases are in part expelled from the cylinder, as indicated by *DE*. As a first approximation it is assumed that the medium throughout the cycle has the properties of air, that the specific heat is constant, and that the medium during the process *AB* receives a quantity of heat equal to that developed by the combustion of the fuel in the actual cycle. With this assumption, the ideal efficiency of the engine is

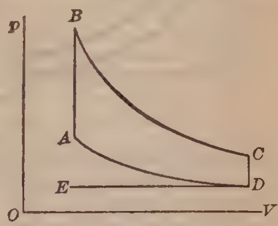

Fig. 12

$$e = 1 - \frac{T_d}{T_a} = 1 - \left(\frac{V_a}{V_d}\right)^{k-1} = 1 - \left(\frac{p_d}{p_a}\right)^{\frac{k-1}{k}}.$$

It is seen that the higher the compression pressure p_a, the greater the ideal efficiency.

For the ideal case the temperature and pressure at the point *B* are determined as follows: If q_1 is the heat absorbed per pound of air during the process *AB*,

$$q_1 = c_v(T_b - T_a), \quad \text{whence} \quad \frac{T_b}{T_a} = \frac{q_1}{c_v T_a} + 1;$$

and since $V_a = V_b$,

$$\frac{p_b}{p_a} = \frac{T_b}{T_a}, \qquad \text{or} \qquad p_b = p_a\left(\frac{q_1}{c_v T_a} + 1\right).$$

The temperature and pressure as thus calculated are never realized in practice. In the first place, part of the heat q_1 is absorbed by the water jacket; secondly, the properties of the actual fluid in the cylinder are not the same as the properties of air, and the specific heat of the fluid is not constant but increases with the temperature. The efficiency of the cycle when the calculation is made with the actual contents of the cylinder taken into consideration is not more than 0.8 of the efficiency deduced for the cycle with air as the medium; and the efficiency of the actual engine is still smaller.

Diesel Cycle. In the Diesel oil engine, air without fuel is compressed to a pressure of about 500 lb. per sq. in. The fuel is introduced into this air by the aid of a separate small air compressor, or by spraying under high pressure and burns without explosion. No igniting device is required, as the temperature of the air is raised by compression above the temperature of ignition. The fuel may be cut off early or late, depending on the load.

In Fig. 13 is shown the ideal indicator diagram of the Diesel engine in which air is compressed adiabatically from D to A, fuel is introduced and burns from A to B, with constant pressure maintained and, after cutoff at B at from one-

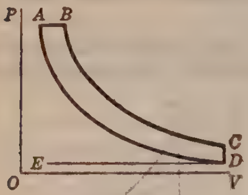

Fig. 13

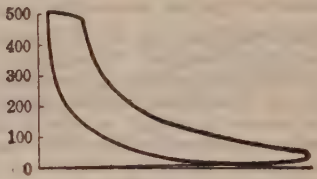

Fig. 14

tenth to one-sixth of the effective stroke, the gases of the combustion expand adiabatically from B to C, followed by their release at D with drop in pressure and rejection along DE. With the same assumptions as for the Otto Cycle, the heat added during the combustion of the fuel from A to B is $q_1 = c_p(T_b - T_a)$; that rejected from C to D is $q_2 = c_v(T_c - T_d)$. The ideal efficiency then is

$$e = \frac{q_1 - q_2}{q_1} = 1 - \frac{1}{k}\,\frac{(T_c - T_d)}{(T_b - T_a)},$$

where $k = \frac{C_p}{C_v}$. The work obtainable per pound of combustible is

$$W = J\,[c_p(T_b - T_a) - c_v(T_c - T_d)]$$

Fig. 14 is a typical diagram from a 4-cycle Diesel engine. The compression of the air to 500 lb. per sq. in. more or less raises it to a temperature sufficient to ignite the atomized fuel and the cycle is completed as indicated.

In the Diesel engine the pressure during combustion is limited, whereas in gas engines (Otto cycle) the combustibles enter the cylinder already mixed, under a limited pressure to prevent precombustion; then when the electric spark is discharged, combustion is practically instantaneous without change in volume and is accompanied by an enormous pressure before expansion can begin. The Diesel is more efficient than the gas engine at both full and fractional loads, uses less and cheaper fuel and lubricating oil, and has a longer life due to freedom from explosive shocks.

Semi-Diesel Engines. In the so-called semi-Diesel engine compression is brought only to about 250-lb. pressure, the ignition of the fuel depending on a "hot-bulb" heated from an external source. This type of engine is now practically obsolete and may be dismissed from further consideration.

The two prevalent methods of introducing the fuel into the combustion space are **air-injection** and **solid-injection.** The former requires a multistage air compressor, built as an integral part of the engine, which delivers air at from 800 to 1300 lb. pressure, depending on the load. A metered amount of fuel delivered to the fuel nozzle is atomized by the high pressure air blown through the nozzle at the proper time, and is carried into the combustion space. In the solid-injection method, fine atomization into the cylinder is obtained by forcing oil under 4000 to 4500 lb. through fine spiral grooves or through very small holes about .016 in. in diameter. Mean effective pressures are less with solid than with air injection. Solid injection engines are somewhat cheaper to build than those requiring compressor equipment. They do not require as high compression—only 450 lb.—for ignition temperature, because there is no cooling effect due to injection air.

Diesel engines are either **two-cycle** or **four-cycle.** The former have exhaust ports in the cylinder bore, which are uncovered near the end of the power stroke, and either scavenging air ports still lower down in the cylinder bore or else scavenging valves in the cylinder head. The scavenging air pressure, ranging from 3 to 6 lb., is supplied usually by casting a compression chamber in the crank case, the main piston also functioning as a compression piston on the down stroke, but in a few makes there is a separate scavenging pump. Four-cycle engines, on the other hand, require air and exhaust valves in the cylinder head, and the four strokes are for suction, compression, power and exhaust respectively, so that only one stroke in each two revolutions is effective. Scavenging, however, is more complete, compression is effective throughout the full stroke, and cylinder heating is more readily controlled. The air and exhaust valves and their operating mechanism, together with the larger cylinder required for a given output, make the four-cycle engine more expensive to build. The **Busch-Sulzer** engine, for example, is designed for four cycles up to 600 b.hp., with trunk pistons. Above 600 b.hp. this company makes the two-cycle type with cross-head and guides and having a capacity up to 4000 b.hp. Most makers employ one type exclusively. **The McIntosh & Seymour** engine, for example, is made only in the four-cycle type, up to 8000 b.hp. in 8 cylinders. Three to six cylinders are usually assembled in one unit.

Efficiency. The Diesel engine is the most efficient of all prime movers depending on the combustion of fuel, and its efficiency is nearly as high in a small unit as in a large one (which does not hold true of other prime movers), efficiencies ranging from 30% to 35% at full load; that of a two-cycle engine is slightly less than that of a four-cycle. The efficiency is expressed in terms of weight of fuel per brake horsepower per hour at full and fractional loads, the one shown in Fig. 15 being characteristic. The efficiency throughout fractional loads is better maintained than for any other prime mover. Manufacturers of Diesel engines generally rate them at or only a little below full load capacity, a 10% overload being a not uncommon limit. Lost heat appears in the cooling water rejected at from 100° to 140° F., 20% to 34%; in the exhaust gases, at from 425° to 700° F., 32% to 28%; in friction and radiation, 7% to 14%. The first two are recoverable in part for heating or for generating low-pressure steam. Ratings are based on sea level, capacity decreasing 1.5%, 5% and 14.5% at elevation of 1000, 2000 and 5000 ft. above sea level. **Lubrication** of Diesel engines of best make is based on one gallon of oil per 2500 b. hp.-hr. for small engines to 3000 or even more for large ones. The engine maker's guarantee should be imperatively required. **Cooling water** required depends on its initial and discharge temperatures, 6 to 10 g.p.m. being ordinarily provided (up to 30 g.p.m. in the tropics) for full load with 3000 B.t.u. per horsepower per hour assumed as the heat to be carried off by the water. The discharge temperature limit of the water ranges from 110° F. for brackish water to 125° F. for water of 12 deg. relative hardness, and to 150° F. for soft water. The amount of water is thus one-sixth to one-tenth that required by a condensing steam plant under similar load conditions.

Fuel Oils most suitable for Diesel engines are topped crude oils from which benzene, gasoline and other light oils have been distilled, if made reasonably free from sulfur, dirt and water. Run of oil from the still and not "compounded" from heavy residue and lighter oils is preferable. Oils with a

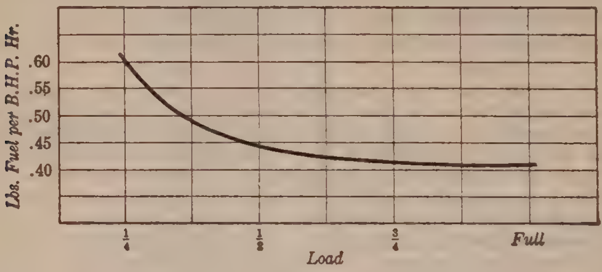

Fig. 15

specific gravity between 24° and 34° Baumé can be used; lighter than 34° oils are not satisfactory, very light ones introducing fire risk; also, light oils contain appreciably less B.t.u's per gallon, which increase the fuel cost. Oils of 20° Baumé and heavier do not flow readily and even those up to 24° may require heating in winter to reduce viscosity if kept in exposed storage tanks. A lighter oil is generally used for half an hour at the beginning and end of each run where Mexican crudes and bunker oils are used. One pound of fuel oil contains 18 000 to 19 500 B.t.u's per pound, 18 500 being assumed as the average value.

Fuel oil is quoted by either the gallon or the barrel of 42 gal. Oil with s. g. .8998, or 26° Baumé (nearly), weighs 7.5 lb. per gal., and 0.5 lb. per brake horsepower, 1 gal. = 15 b.hp.

With cost of fuel at	**3**	**4**	**5**	**6**	**7**	**8**	**cents per gal.**
the equivalent cost is	**\$1.26**	**\$1.68**	**\$2.10**	**\$2.52**	**\$2.94**	**\$3.36**	**per bbl.**

Costs. Diesel engines are heavy due to the high pressures generated in the cylinders, and they are high in first cost compared with a steam engine or a steam turbine; but in establishing comparative costs, boilers, condensers, and other auxiliary apparatus including piping and pump drives, coal storage and handling and the much larger buildings required, increase the cost of steam plants to such an amount that the investment, for example in a reciprocating triple expansion condensing pumping plant, is considerably more than for a Diesel engine plant; but the latter will cost more than a turbine-driven centrifugal pumping station and still more than an electric-motor-driven centrifugal pumping plant. The type of equipment is governed by the relative cost of fuel oil, coal and electric power in the locality of the plant.

Life and Maintenance. Experience shows the **life of Diesel engines** to be at least 25 years.

The brake horsepower capacity of a Diesel engine should be chosen to take the station peak load without exceeding its rated capacity. It has been found that by relieving the engine by from one-eighth to one-sixth of its rated full load, the maintenance charge has been materially reduced. In well managed plants, regular inspections are scheduled. One shut-down of several hours only per year is necessary for pulling pistons and cleaning rings and adjusting piston rods; three very short ones, four months apart, are advisable for inspection of air, exhaust and fuel valves; provided good oil is used and engine is not overloaded, they run continuously.

Maintenance of a Diesel engine is often based on a percentage of its cost, say 2% per annum. It is, however, higher for small engines and it is higher under continuous 24-hour operation than under daily shut-down conditions, where $1.00 per 2500 and 5000 b.hp.-hr. will cover small and large engines respectively. It should not exceed $5.00 per 5000 b.hp.-hr. under continuous full-load conditions. A skilled chief engineer is as good an investment for a Diesel as for a steam plant to keep maintenance at a minimum.

The comparative pumping costs for a city pumping plant requiring an average of 10 m.g.d. pumped against a head of 360 ft. with Diesel-driven triplex pumps, vertical triple-expansion steam pumps, turbine-driven centrifugal pumps, and electric-motor-driven centrifugal pumps, are indicated in an extract from "Power," shown in the accompanying table.

Horsepower of Gas Engines. The indicated horsepower of a gas engine is given by the formula i.hp. = $plan/33\,000$, in which n denotes the number of explosions per minute. The mean effective pressure that may be obtained depends upon the kind of fuel. It may vary from 60 to 70 lb. per sq. in. for producer and blast-furnace gas, 85 to 95 lb. for natural gas and gasoline, and 95 to 110 lb. for alcohol. The brake horsepower of automobile engines is given approximately by the rule of the Association of Automobile Manufacturers, namely: b.hp. = $d_2N/2.5$, in which d denotes the cylinder diameter in inches and N the number of cylinders.

Performance of Gas Engines. The internal-combustion engine has intrinsically a higher thermal efficiency than the steam engine. Based on coal used, the consumption of a gas engine with producer gas is about 30% less than the consumption of a steam engine. This result is obtained under most favorable conditions for both engines.

A large number of tests of a gas engine and gas producer were made by the U. S. Geological Survey at the St. Louis exhibition. The producer was rated at 250 hp. capacity and the engine at 235 hp. at 200 r.p.m. Tests were made with a large number of coals. With high-grade bituminous coal the weight of coal consumed by the producer per brake horsepower delivered by the engine ranged from 0.95 lb. to 1.32 lb. A steam engine of the same power would consume about double the weight of coal.

The following are the results of tests of gas and oil engines of the larger sizes:

Engine	Fuel	Brake hp.	Thermal efficiency	
			Per i.hp.	Per b.hp.
Westinghouse	Natural gas	606	28.6	25.5
Snow	Natural gas	595	29.4	23.7
Borsig-Oechelhaeuser	Coke-oven gas	628	33.0	27.5
Premier	Producer gas	368	33.7	25.6
Koerting	Producer gas	341	34.0	24.1
Westinghouse	Producer gas	500	30.1	25.2
Cockerill	Blast-furnace gas	725	31.5	26.0
Nürnberg	Blast-furnace gas	1186	33.9	28.2
Diesel	Oil	297	45.8	32.2

Comparison of Municipal Pumping Costs, Oil, Steam, and Electricity

Comparative plant and operating costs for a station pumping an average of 10 m.g.d. against 350-ft. total head, with oil, steam and electric equipment as indicated.

(From "Power," Dec. 18, 1923, Vol. 58, pp. 984-987)

Plant equipped with	Deisel engines, triplex pumps		Triple-expansion steam pumps	
Building:				
Pumproom (basement to eaves), ft	66 × 86 × 42		54 × 88 × 52	
Boiler room, ft	....		44 × 58 × 30	
Fuel storage, ft	30 × 50 × 12		25 × 58 × 12	
Cost of above items		$ 69 000		$100 000
Equipment installed:	3-750 b.hp. Diesels	$213 600	1 12-m.g.d. unit	$160 000
	3-10 m.g.d. pumps	123 750	1-10 m.g.d. unit	140 000
			1 8-m.g.d. unit	125 000
			3 300-hp. boilers	33 000
Foundations for pump sets		11 250		4 000
Suction and discharge piping		50 000		51 000
Total investment		$467 600		$613 000
Yearly operating costs:				
Labor:	1 chief engineer	$3 000	1 chief engineer	$3 000
	3 assistant engineers	5 400	3 assistant engineers	5 400
	3 oilers	4 500	3 oilers	4 500
			3 firemen	4 500
Fuel:				
Oil at $1.70 per bbl.*	9010 bbl. oil	15 317	22 163 bbl. oil or 5700 tons coal	37 678
Coal at $6.75 per ton				
Lubricating oil and waste		1 000		300
Maintenance and repairs:				
Building, foundations, piping	$130 250 at 0.5%	652	$155 000 at 0.5%	775
Machinery and equipment	$337 350 at 2.0%	6 747	$458 000 at 1.5%	6 870
Total		$36 616		$63 023
Fixed charges:				
Interest at 5% on sinking fund	$467 600 or	$23 380	$613 000 or	$30 650
Building (assumed life 50 years)	0.5% on $69 000	345	0.5% on $100 000	500
Equipment	25 year life or 2.1% on $398 600	8 371	35 year life or 1.1% on $513 000	5 643
Total		$32 096		$36 793
Total yearly expense		$68 712		$99 816

* **Note by Author.** Oil at $1.70 per bbl. or 4.05 cents per gal. is low for small eastern plants at present (1929). An increase to 6-1/2 cents would increase Diesel engine costs from $68 712 to $78 000. A reduction to $5.00 per ton for coal and to 1 cent per kilowatt-hour for electric current would reduce the cost of triple-expansion steam pumping to $90 000, steam turbine centrifugal pumping to $82 800, and electric pumping to $80 000.

Comparison of Municipal Pumping Costs, Oil, Steam, and Electricity

Comparative plant and operating costs for a station pumping an average of 10 m.g.d. against 350-ft. total head, with oil, steam and electric equipment as indicated.

(From "Power," Dec. 18, 1923, Vol. 58, pp. 984-987)

Plant equipped with	Steam turbines centrifugal pumps		Electric motors, centrifugal pumps	
Building:				
Pumproom (basement to eaves), ft.	50 × 84 × 42		50 × 60 × 32	
Boiler room, ft.	44 × 58 × 30		....	
Fuel storage, ft.	25 × 58 × 12		....	
Cost of above items		$83 000		$48 000
Equipment installed:	1 12-m.g.d. unit	$44 500	1 12-m.g.d. unit	$23 200
	1 10-m.g.d. unit	40 500	1 10-m.g.d. unit	20 200
	1 8-m.g.d. unit	37 500	1 8-m.g.d. unit	13 200
	3 350 hp. boilers	38 500	Switch and wiring	400
			Transformers	6 500
Foundations for pump sets	....	1 400		1 400
Suction and discharge piping	....	50 000		50 000
Total investment	....	$295 400		$162 900
Yearly operating costs:				
Labor:	1 chief engineer	$3 000	1 chief engineer	$3 000
	3 assistant engineers	5 400	3 assistant engineers	5 400
	3 oilers	4 500	3 operators	4 500
	3 firemen	4 500		
Fuel:				
Oil at $1.70 per bbl.*	33 204 bbl. oil or	56 447	Electric energy at 1-1/4 cents per kwh.	$68 219
Coal at $6.75 per ton	8 360 tons coal			
Lubricating oil and waste	....	600		200
Maintenance and repairs:				
Building, foundations, piping	$134 400 at 0.5%	672	$99 400 at 0.5%	497
Machinery and equipment	$161 000 at 1.7%	2 737	$63 500 at 1.7%	1 080
Total	....	$77 856		$82 896
Fixed charges:				
Interest at 5% on sinking fund	$295 400 or	$14 770	$162 900 or	$8 145
Building (assumed life 50 years)	0.5% on $83 000	415	0.5% on $48 000	240
Equipment	25 year life or 2.1% on $212 000	4 460	25 year life or 2.1% on $114 900	2 413
Total	....	$19 645		$10 798
Total yearly expense	....	$97 501		$93 694

* **Note by Author.** Oil at $1.70 per bbl. or 4.05 cents per gal. is low for small eastern plants at present (1929). An increase to 6-1/2 cents would increase Diesel engine costs from $68 712 to $78 000. A reduction to $5.00 per ton for coal and to 1 cent per kilowatt-hour for electric current would reduce the cost of triple-expansion steam pumping to $90 000, steam turbine centrifugal pumping to $82 800, and electric pumping to $80 000.

Comparative Costs of Pumping by Various Methods. An engineering company having charge of a number of water-works has worked out the following comparison of pumping costs. The unit cost of pumping including cost of labor, maintenance, depreciation and fixed charges is about the same when:

(a) For electric pumping, electric power costs.......	\$0.01 per kw. hr.
(b) For steam pumping by triple-expansion engines or steam turbines, coal of 14 000 B.t.u. costs.....	\$5.00 per ton
(c) For Diesel engine pumping, fuel oil of 19 000 B.t.u. costs..............................	\$0.06-1/2 per gal.
(d) For gas engines, natural gas of 1150 B.t.u. costs..	\$0.40 per 1000 cu. ft.
(e) For gasoline engines, gasoline of 20 100 B.t.u. costs..................................	\$0.08 per gal.

Note. The above unit prices are for comparative purposes only and do not represent the market values of the fuels mentioned. The high cost of operation for a gasoline unit makes it uneconomical except for standby service.

The above comparisons are respectively based on:

(a) Combined pump and motor efficiency of 73.5%.
(b) Pump duty of 130 million ft.-lb. per 1000 lb. of steam for triple-expansion steam pumps, a water rate of 12.8 lb. of steam per brake horsepower per hour for steam turbine, and a boiler efficiency of 72%.
(c) 0.066 gal. of fuel oil per brake horsepower per hour.
(d) 10.9 cu. ft. of natural gas per brake horsepower per hour.
(e) 0.1 gal. of gasoline per brake horsepower per hour.

SECTION 5

ELECTRICAL ENGINEERING

The following outlines of fundamental principles have been prepared not for electrical engineers but for civil engineers whose knowledge of the subject is limited.—EDITOR-IN-CHIEF.

BY

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ELECTRICITY

1. Definitions, Units, Fundamental Laws

Systems of Units. There are two systems of electrical units, both of which are derived from the fundamental or centimeter-gram-second (**C. G. S.**) system. The **electrostatic system** is based on the force exerted between two charges of electricity; the **electromagnetic system** is based on the force exerted between a magnetic field and a conductor carrying a current placed in that field. Many of these units are inconvenient to use in practice, and certain modifications, known as practical units, have been adopted and legalized in a number of countries by international agreement. These practical units are used in all engineering work; **C. G. S.** units are frequently used in physical and research work.

Electromotive Force (symbols **E, e,** and **e. m. f.**) is that which causes electricity to flow or tend to flow, and is analogous to hydrostatic head. The practical unit is the **Volt**, that electromotive force which, when applied to a circuit of one ohm resistance, will cause a current of one ampere to flow. The standard volt is represented by the Clark and Weston standard cells. The legal value of the **Clark cell**, when made according to certain prescribed directions, is **1.434** volts at 15° C. The value of the **Weston cell**, adopted by the U. S. Bureau of Standards, Jan. 1, 1911, is **1.0183** volts at 20° C.

The **Volt** is equal to 10^8 **C. G. S. units.** When very small electromotive forces or potentials are to be measured, the unit millivolt, one thousandth of a volt, is frequently used. High potentials are often expressed in kilovolts or thousands of volts. **Difference of Potential** is the difference in electrical pressure between two points in an electric circuit and is measured in volts. This term is often incorrectly used instead of electromotive force.

Current (**I, i**) is the rate of flow of electricity. The practical unit is the **Ampere,** which is the current flowing in a circuit of one ohm resistance when an electromotive force of one volt is impressed on it. The legal standard ampere is that steady current which will deposit **0.001118 gram of silver per second** when passed through a silver nitrate solution under certain prescribed conditions.

The **Ampere** is equal to 10^{-1} **C. G. S. units.** A **milliampere** is one thousandth part of an ampere.

Resistance (**R, r**) is that property of a material that opposes the flow of electricity through it. It varies directly with the length and inversely with the cross-sectional area. The practical unit is the **Ohm,** which is the resistance of a circuit in which a current of one ampere flows when subjected to an electromotive force of one volt. The legal standard ohm is the resistance of a column of mercury 106.3 cm. long and 14.4521 grams mass at 0° C. Working standard resistances are made of resistance wire or ribbon carefully adjusted and standardized by comparison with the legal standard. Manganin, a copper alloy, is most extensively used because of its permanency. The usual sizes range from 100 000 ohms to 0.00001 ohm, and always in powers of 10.

The **Ohm** is equal to 10^9 **C. G. S. units.** Insulation resistances are usually expressed in terms of the **megohm** or **one million ohms** and very small resistances in **microhms,** or **millionths of an ohm.** **Resistivity** (ρ) or specific resistance of a material is the resistance between opposite faces of a centimeter cube or an inch cube. **Conductance** is the reciprocal of resistance, and **conductivity** (γ) is the reciprocal of resistivity. **Relative conductivity** (usually incorrectly abbreviated to "**conductivity**") is the term most used in engineering. It is the percentage ratio of the true conductivity (i.e., reciprocal of resistivity) of the material to that of copper of a certain specific gravity and specific resistance known as the International Annealed Copper Standard. Materials are classed as conductors or insulators according as they are of low or high resistivity, respectively. **Insulation resistance** refers to the resistance of insulating material measured in megohms. The **dielectric strength** of an insulating material is the high-potential voltage which ruptures that material and is usually expressed in **volts per mil or per millimeter.**

Quantity of Electricity (Q, q) is the product of current and time. The practical unit is the **Coulomb,** which is one ampere flowing in a circuit for one second.

The Coulomb is equal to 10^{-1} **C. G. S. units.** The unit in more common use commercially is the ampere-hour, equal to 3600 ampere-seconds or coulombs.

Electrical Energy (W) is the work done in a circuit or an apparatus by current flowing through it. The practical unit is the **Joule** or **Watt-second,** which is the work done when one ampere flows through a resistance of one ohm for one second.

The commercial unit is the **watt-hour,** equal to **3600 joules;** also the **kilowatt-hour** equal to 1000 watt-hours.

Electrical Power (P) is the rate of expending electrical energy or the rate of doing work per unit of time. The practical unit is the **Watt,** which is the work done in one second when one ampere flows in a circuit under a pressure of one volt, or $\mathbf{P = I \times E = I^2R}$.

The commercial unit is the **kilowatt** (one thousand watts). **One horsepower is equal to 746 watts.**

Capacitance or **Electrostatic Capacity** (C, c) of a circuit or apparatus is the property by virtue of which it can hold a charge of electricity. The practical unit is the **Farad,** which is the capacity of a circuit or apparatus that will be charged to a potential of one volt by one coulomb of electricity.

The farad is so large a quantity that commercially the **microfarad** is generally used. The farad is equal to 10^{-9} **C. G. S. units;** the microfarad to 10^{-15} **C. G. S. units.**

Self-induction of a circuit is that property which opposes any change in the value of the current flowing through it. It is analogous to inertia and is due to the magnetic field which surrounds a conductor carrying current. When the current is established or changes, the field increases or decreases and the lines of force cut the conductor, inducing a counter e. m. f. which opposes the change in the current.

Inductance (L, l), or coefficient of self-induction, of a circuit is the constant by which the time rate of change of the current in the circuit must be multiplied to give the e. m. f. induced in the circuit by such change. The practical unit is the **Henry,** which is the inductance of a circuit where a change of one ampere per second will induce one volt e. m. f.

A henry is equal to 10^9 **C. G. S. units.** The unit ordinarily used is the **millihenry** (one-thousandth of a henry).

Impedance (Z) of a circuit is the resistance offered to the passage of alternating current. It depends on the frequency of the current and the resistance, inductance, and capacity of the circuit. It is measured in ohms and is equal to $\sqrt{\mathbf{R}^2 + \mathbf{X}^2}$.

Reactance (X) of a circuit is that part of the resistance offered to the passage of alternating current which is due to the inductance and capacity of the circuit. It is measured in ohms and is equal to $2\,\pi \mathbf{n} L - \frac{1}{2\,\pi \mathbf{n} C}$.

Frequency (n) of an alternating-current circuit is the number of complete reversals, or cycles, of the current per second. It is equal to the product of half the number of poles on the generator and the revolutions per second. An alternation is half a cycle.

The standard frequencies in this country are 25 and 60 cycles, although 40-cycle systems are in operation. A very few of the early 133-cycle systems are also to be found, and 15 cycles has been advocated for alternating current railways.

Phase refers to the time relation between the current and potential in an alternating-current circuit, or to the time relation between the potentials in two ro more circuits.

Thus, if the current and potential in a circuit reverse at the same instant, they are in phase. When the current reverses after the potential, it is said to be out of phase, and a lagging current; when it reverses before the potential, it is out of phase, and a leading current. A two-phase generator has two circuits in its armature and the e. m. f.'s generated are 1/4 cycle or 90 electrical degrees apart. Similarly, a three-phase generator has three circuits and the e. m. f.'s are 1/3 of a cycle or 120° apart.

Power Factor of a circuit is the ratio of the true power passing through it to the product of the volts and amperes. It is equal to the cosine of the time angle at any instant between the potential and current.

When the time angle is zero, that is, when the potential and current are in phase, the power factor is 1.00 (cosine 0°), often expressed 100%. If they are out of phase by 90°, the power factor will be zero (cosine 90°).

Load Factor. The daily or yearly (or other period) load factor of a machine, plant or system is the ratio of the average power to the maximum power during the period indicated.

The **demand factor** of an installation is the ratio of the maximum load actually taken by the installation to the total connected load (that is, the sum of the ratings of all apparatus in the installation).

The **diversity factor** of an installation is the ratio of the sum of the maximum demands of the various loads in the installation to the maximum demand of the whole installation.

Hysteresis is that property of magnetic material which causes the induction corresponding to a given magnetizing force to be greater when the latter is decreasing than when it is increasing.

Hysteresis loss is the energy expended in the material because of hysteresis when the magnetizing force is changed from one direction to the other and back again. It appears in the form of heat.

Eddy Currents are stray local currents induced in those metal parts of electrical apparatus which are in rapidly changing magnetic fields.

Eddy-current loss which appears in the form of heat, is the power expended by eddy currents flowing in these metal parts, and is equal to the product of the square of the current and the resistance. It is one of the largest losses in alternating current apparatus, and therefore parts in which this loss occurs are laminated in order to reduce the length of the current paths, thereby increasing the resistance and decreasing the currents.

A Series Circuit is one in which the total current passes through each part of the circuit. A **Parallel** or **Multiple Circuit** is one in which the total current is divided among the various parts of the circuit.

FUNDAMENTAL LAWS. The following are the more important laws and principles of electricity and magnetism underlying the practical applications of electricity.

Ohm's Law. The current in a circuit containing no source of **e. m. f.** is equal to the potential applied at the terminals of the circuit divided by the resistance, or $I = \frac{E}{R}$. This law applies to alternating current circuits only when they are non-inductive, that is, contain resistance only. Otherwise the current is equal to the potential divided by the impedance, or $I = \frac{E}{Z}$.

Laws of Series and Parallel Circuits Carrying Direct Current. In a series circuit the total resistance is equal to the sum of the resistances of its component parts. In a parallel circuit the current in each circuit is inversely proportional to the resistance of each branch of the circuit, and the reciprocal of the total resistance of the circuit is equal to the sum of the reciprocals of the branch resistances.

Capacitances. When two or more capacitances (condensers) are connected in series, the reciprocal of the total capacitance is equal to the sum of the reciprocals of the capacitances. If they are connected in parallel, the total capacitance is equal to the sum of the capacitances. It is to be noted that the law is just the reverse of that for resistances.

Conductor in a Magnetic Field. When a free conductor is placed in a magnetic field and current is passed through it, the conductor will move. The force exerted will be proportional to the field strength and the current. This is the fundamental principle of **motors.**

Electromagnetic Induction. When a conductor is moved through a magnetic field or a magnetic field is moved through a conductor, an e. m. f. will be induced in it which will be proportional to the field strength, the rate of motion, and the length of conductor cutting the field. This is the fundamental principle of **generators.**

Fundamental Equation for the Generation of an E. M. F. The following is the equation for the e. m. f. generated in (*a*) a coil rotating in a magnetic field at a uniform speed or (*b*) a coil which incloses an alternating magnetic flux varying in such a manner that a sine wave e. m. f. is generated (see Art. 2).

$$E = 4.44\frac{n\phi f}{10^8}$$

where E = e. m. f., mean effective value (see Art. 2) in volts;
n = number of complete turns in coil;
ϕ = total magnetic flux (maximum instantaneous value if produced by alternating current) inclosed by the circuit. If the magnetic flux density is uniform (the usual case) throughout the space occupied by the coil, $\phi = BA$ where B = flux density in lines per square centimeter and A = area inclosed by coil in square centimeters;
f = revolutions of coil per second or frequency in cycles per second of magnetizing current.

Law of Magnetic Circuits. In a magnetic circuit the total magnetic flux is equal to the magnetomotive force divided by the reluctance of the circuit, the relation being similar to Ohm's law. The **total magnetic flux** (**lines of force**) corresponds to **current** (**amperes**), the **magnetomotive force** (**gilberts**) to **electromotive force** (**volts**), and **magnetic reluctance** (**oersteds**) to **resistance** (**ohms**). In practice it is customary to deal with the magnetizing force per unit length of magnetic circuit, which, when the magnetic circuit is of non-magnetic material, is equal to $H = 4\,\phi NI/10\,l$, where H = **field strength** (**gausses**) NI = **ampere turns** and l = **length.** When the material is magnetic, **the magnetic induction** (**gausses**) is $B = kH$, where k = **permeability,** a quantity which varies with the material and with B.

Galvanic Electricity. If two unlike metals are immersed in an acid or salt solution an electromotive force will be generated between them. The magnitude of the e. m. f. will depend upon the metals and the solution used. This phenomenon is the basis of all electric batteries.

Electrolysis. When electric current (direct) is passed through a solution (called electrolyte), it is decomposed. According to **Faraday's law,** the amount of decomposition is directly proportional to the quantity of electricity (product of current and time) passed through the solution.

Thermoelectric Effect. When a circuit is made up of two dissimilar metals and the two junctions are maintained at different temperatures, an e. m. f. will be established which will be very nearly proportional to the difference in temperature between the two junctions. Conversely if current is passed through a circuit consisting of two dissimilar metals, heat will be developed at one junction and absorbed at the other junction. This is known as Peltier effect.

Heating Effect. When an electric current is passed through a solid conductor, the electric energy expended in the conductor is entirely converted to heat energy. The amount of heat developed is proportional to the resistance, the square of the resistance and the time (Joule's law). That is,

$$H = 0.2928\, I^2RT,$$

where H = heat in British thermal units;
I = current in amperes;
R = resistance in ohms;
T = time in hours.

2. Commercial Electrical Measurements

Commercial and Ordinary Engineering Measurements are made with various indicating instruments which are calibrated by various more or less direct methods against the primary standard or standards representing the quantity indicated by the instrument.

Direct Current and Potential are measured with ammeters and voltmeters respectively. They are almost universally of the D'Arsonval type, in which a small fine wire coil is free to move in the field of a permanent magnet. In ammeters this coil is connected to a specially made low resistance called a "shunt." Its deflections are proportional to the fall of potential across this shunt and therefore to the current flowing through it. Voltmeters differ from ammeters only in that the moving coil is connected in series with a high resistance and across the circuit to be measured instead of in parallel to a low resistance which is in series with the circuit.

For moderate currents (25 amperes and less) the shunt is contained in the instrument case, but for larger currents it is external to the instrument and connected to the latter by small flexible wires. Thus, shunts as high as 25 000 amperes capacity can be placed in the circuit at the most convenient point. Instruments permanently located on switchboards are called switchboard or station instruments, as distinguished from portable instruments used for testing purposes.

Alternating Current is measured with ammeters of several types, the more usual being the iron-disk, inclined-coil dynamometer, and hot-wire types. The principle of iron-disk instruments is the repulsion between the eddy currents induced in a very small iron disk, and the field which produces them. The principle of the inclined-coil and dynamometer instruments is the attraction and repulsion between two coils carrying current. In hot-wire instruments the expansion of a taut wire, when heated by the passage of a current, is utilized to measure the current. Some of the types are made self-contained up to 300 amperes, but current transformers are usually used with larger currents. In all high-tension circuits (over 440 volts) current transformers are used irrespective of the size of the current in order to insulate the instrument from the circuit.

The several **values of an alternating current** (and e. m. f.) are (*a*) **instantaneous value** or value at any instant during a cycle, (*b*) **mean effective value,** which is the square root of the sum of the squares of the instantaneous values during one cycle (also called effective value and root-mean-square value), (*c*) **average value**, which is the arithmetical average of the instantaneous values during one alternation or half cycle, (*d*) **maximum value**, which is the maximum of the instantaneous values during one cycle. When the wave form is a sine curve (that is, when the variation of the instantaneous values during one cycle with respect to time follows the sine law) the relations between these various values are as follows:

Maximum value = $\sqrt{2}$ × effective value;

$$\text{Maximum value} = \frac{\pi}{2} \times \text{average value;}$$

$$\text{Average value} = \frac{2\sqrt{2}}{\pi} \times \text{effective value;}$$

$$\frac{\text{Effective value}}{\text{Average value}} = \frac{\pi}{2\sqrt{2}} = 1.11 \text{ (= form factor);}$$

$$\frac{\text{Maximum value}}{\text{Effective value}} = \sqrt{2} = 1.414 \text{ (= crest, peak or amplitude factor).}$$

All of these values are employed in electrical engineering but the **effective value** is the one ordinarily measured and used. This particular value is used because it is the value which will produce the same heating effect as a direct current of the same magnitude, and it is the value indicated by all alternating-current instruments used in all ordinary measurements.

Alternating Potentials are measured with voltmeters of the same types as the ammeters, being wound with many turns of small wire instead of a few turns of relatively large wire. Voltage transformers which step-down the voltage to the standard value of 110 volts are used when measuring high voltages.

The secondaries of current transformers are usually 5 amperes capacity, but the instrument scales are marked to read the primary current. Similarly, the scale of the voltmeter may be marked to indicate the line potential although the voltage on the instrument is usually about 110 volts.

Power in direct current circuits is usually measured with an ammeter and a voltmeter, the power in watts being the product of the amperes and the volts. In alternating current circuits, however, power is usually measured with a **wattmeter**. It is a two-circuit instrument, one circuit being a fixed coil of a relatively few turns of relatively large wire, connected in series with the circuit to be measured. The other circuit consists of a coil of many turns of fine wire so mounted as to be free to move within the magnetic field produced by the fixed coil. It is connected in series with a certain amount of resistance and across the circuit to be measured.

In a single-phase alternating-current circuit, having a power factor of 1.00 (which is practically the case where the load is incandescent lamps), the power may be measured with an ammeter and a voltmeter because in that case the power is equal to the product of the amperes and the volts. If, however, the power factor is less than 1.00 (which is always the case where the load is all or partially motors, arc lamps, etc.), the power is equal to the product of amperes, volts and power factor, and a wattmeter is used. (The power may be measured by the less convenient "three-ammeter" or "three-voltmeter" method indicated in Figs. 2 and 3).

Wattmeter Connections. The following paragraphs show the methods of connecting wattmeters in various kinds of alternating current circuits. Current and potential transformers are used with wattmeters where necessary, the same set of transformers being often used for all three instruments (wattmeter, ammeter and voltmeter) at the same time.

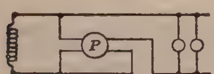

Fig. 1

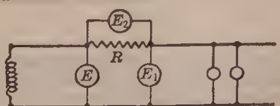

Fig. 2

Single-phase Circuit. One wattmeter connected as shown in Fig. 1 will read true watts. The power may also be measured with three voltmeters or three ammeters.

In the three volt-meter method, a known non-inductive resistance, R, is connected in series with the load as shown in Fig. 2, where E, E_1, and E_2 are points where volt-meter readings are to be taken. The power in watts is

$$\mathbf{W} = \frac{\mathbf{E}^2 - \mathbf{E}_1^2 - \mathbf{E}_2^2}{2\,\mathbf{R}} \qquad \text{(watts)}$$

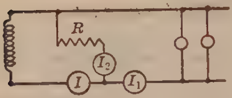

Fig. 3

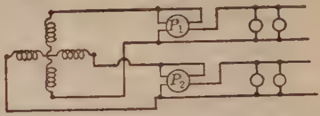

Fig. 4

Similarly, in the three-ammeter method, Fig. 3, the power in watts is

$$\mathbf{W} = \mathbf{R}\left(\frac{\mathbf{I}^2 - \mathbf{I}_1^2 - \mathbf{I}_2^2}{2}\right) \qquad \text{(watts)}$$

Two-phase Four-wire Circuit (**not interconnected**). Two wattmeters, connected as shown in Fig. 4, are sufficient, these conditions being equivalent to two single-phase circuits. The total power is obviously the arithmetical sum of the readings of the two instruments.

Two-phase Three-wire Circuit. Two wattmeters should be connected as shown in Fig. 5, the total power being the algebraic sum of the two readings. This connection

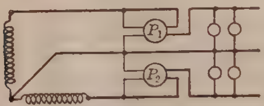

Fig. 5

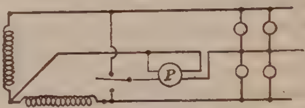

Fig. 6

is correct for all conditions of load, balance and power factor. One wattmeter may be used as in Fig. 6, provided there is no load across the outer conductors and the phases are balanced as to load and power factor.

Two-phase Four-wire Interconnected Circuit. Three wattmeters should be used, connected as in Fig. 7, the total power being the algebraic sum of the three readings. This connection is correct for all conditions of load, balance and power factor. Two wattmeters, one in each phase, will give the true power only when the load is balanced.

Three-phase Three-wire Circuits. Two wattmeters should be used, connected as

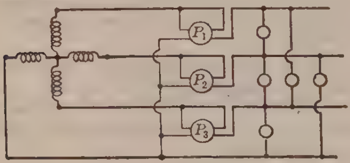

Fig. 7

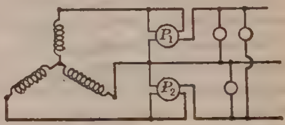

Fig. 8

indicated in Fig. 8, the total power being the algebraic sum of the two readings. With a balanced load, each instrument will indicate half the total power at unity power factor, and at 50% power factor one instrument will indicate the total power, the other instrument indicating zero. At less than 50% power factor, one instrument will read negative.

Three-phase Three-wire Circuits Balanced Load. When the load is balanced, the power may be measured with one wattmeter by the following methods:

(a) With "star" box or artificial neutral as shown in Fig. 9. The total power is three times the reading of the wattmeter. The resistance in each leg of the star box should be non-inductive and small compared with that of the potential circuit of the wattmeter, so that the current taken by the latter will not disturb the potential at the neutral point.

(b) With "Y" box as shown in Fig. 10. The total power is three times the wattmeter reading. This arrangement is similar to (a), one leg of the star box being replaced

Fig. 9

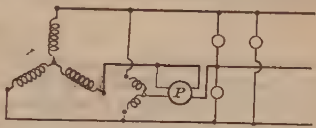

Fig. 10

with the potential circuit of the wattmeter itself. The other two legs have the same resistance as the potential circuit of the wattmeter.

(c) With a "T" reactance coil as shown in Fig. 11. The total power is twice the wattmeter reading. The impedance of the reactance coil must be small compared with that of the potential circuit of the wattmeter, so that the current taken by the potential circuit will not disturb the potential at *O*.

Three-phase Four-wire Circuits. Three wattmeters are used as shown in Fig. 12. The total power is the algebraic sum of the three readings. This method is correct

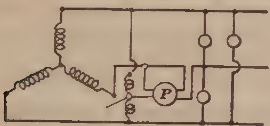

Fig. 11

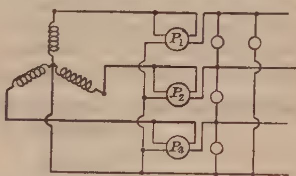

Fig. 12

for all conditions of load, balance and power factor. A three-phase "star" system with a grounded neutral is virtually a four-wire system and the power should be measured with three wattmeters. Obviously, if the load is balanced, one wattmeter can be used, the total power being the indication of the wattmeter multiplied by three. In that case, the current coil should be connected in series with one conductor or phase wire and the potential coil between that conductor and the neutral.

The Power Factor of a single-phase circuit and of each phase of a two-phase circuit is the ratio of the watts, indicated by a wattmeter, to the product of the volts and amperes. In a three-phase circuit it is the ratio of the total watts to the product of the average volts across the three phases, the average current in the three conductors and **1.73** ($\sqrt{3}$).

Electrical Energy is measured with watt-hour meters. They are essentially small motors in which the speed is proportional to the power and the total revolutions is proportional to the energy being consumed. The moving element is geared to a suitable registering mechanism (similar to that of gas and water meters) which indicates the kilowatt-hours of energy passed through the meter. On direct-current circuits the whole current usually passes through the instrument. Alternating current meters can be used with potential and current transformers, and in such cases the meter readings are multiplied by the product of the ratios of these transformers. Single-phase energy is measured with one meter, two-phase with one meter in each phase, or with a polyphase meter, and three-phase energy is measured either with two meters connected in the same manner as two indicating wattmeters or with a polyphase watt-hour meter which consists essentially of two single-phase meters with a common shaft.

Watt-hour meters are connected into the circuit just the same as wattmeters, as indicated in Figs. 1 to 12 inclusive. A polyphase meter has separate terminals for each element and is connected just as two wattmeters would be connected.

3. Conductors

Electrical Conductors are materials of comparatively low resistance, through which electricity will flow in appreciable quantities, as distinguished from high-resistance materials called **insulators,** through which electricity will not pass except in minute quantities. There is no sharp distinction between the two, there being good and poor conductors, and good and poor insulators. Most metals are conductors, but only a few are used commercially. Silver is the best conductor, but copper is very nearly as good, and being comparatively plentiful, cheap, and otherwise very suitable, it is by far the most generally used metal for conductors. Aluminum, which has about 62% of the conductivity of copper, is being used to some extent in high-voltage transmission and on heavy current switchboards where minimum weight is desirable. Iron and steel have 10 to 20% of the conductivity of copper and are used in telephone and telegraph lines and in special cases where the current is small and low cost or greater strength is important. A bimetallic conductor consisting of steel wire with a shell of copper welded on the outside is used to a considerable extent. This material has a conductivity of 30 to 40% of that of copper alone, and a much greater tensile strength. Poor conductors, such

Specific Resistance, Conductivity and Resistance-Temperature Coefficient of Conductors

Material	Specific resistivity, microhms per cm. cube at 0° C.	Ohms resistance per mil-foot at 0° C.	Relative conductivity, per cent *	Temperature coefficient, increase per degree C. from 0° C.
Silver	1.47	8.84	108.2	0.40
Copper, soft †	1.59	9.56	100.0	0.43
Copper, hard drawn	1.63	9.80	97.6	....
Copper, cast ‡	1.8 to 16	10.8 to 96	88.5 to 100	....
Gold	2.22	13.35	71.6	0.37
Aluminum	2.62	15.76	60.6	0.42
Zinc	5.75	34.6	27.6	0.40
Platinum	10.96	65.92	14.5	0.37
Iron	8.85	53.2	18.0	0.63
Iron, soft cast	75.0	451.0	2.1	....
Iron, hard cast	98.0	589.0	1.6	....
Steel, soft	15.9	95.6	10.0	0.42
Steel, glass hard	46	2.76	3.5	0.16
Nickel	6.93	41.7	22.9	0.62
Tin	13.0	78.2	12.2	0.46
Lead	20.4	122.7	7.8	0.43
Mercury	94.1	566	1.7	0.09

* In terms of the International Annealed Copper Standard (see "Resistance," Art 1.)

† International Annealed Copper Standard.

‡ In order to make sound copper castings, it is necessary to add small amounts of other materials and these invariably reduce the conductivity.

Bare Copper Wire

(Principally from Circular No. 31, Bureau of Standards)

Size B.& S. gage *	Diameter mils	Area, circular mils	Weight, lb.		Length, ft. per lb.	Resistance, ohms at 25° C. (77° F.)	
			Per 1000 ft.	Per mile		Per 1000 ft.	Per mile
......	1152	1 000 000	3090	16 320	0.323	0.0108	0.0570
......	1031	800 000	2470	13 040	0.405	0.0135	0.0713
......	964	700 000	2160	11 400	0.463	0.0154	0.0813
......	893	600 000	1850	9 770	0.540	0.0180	0.0950
......	814	500 000	1540	8 130	0.650	0.0216	0.114
......	728	400 000	1240	6 550	0.806	0.0270	0.1425
......	575	250 000	772	4 075	1.295	0.0431	0.2275
0000	528	212 000	653	3 448	1.532	0.0509	0.269
000	470	168 000	518	2 734	1.93	0.0642	0.339
00	418	133 000	411	2 170	2.435	0.0811	0.428
0	373	106 000	326	1 721	3.07	0.102	0.539
1	332	83 700	258	1 362	3.88	0.129	0.681
2	292	66 400	205	1 082	4.88	0.162	0.855
3	260	52 600	163	861	6.14	0.205	1.082
4	232	41 700	129	686	7.75	0.259	1.365
5	206	33 100	102	538.5	9.80	0.326	1.720
6	184	26 300	81	427.5	12.4	0.410	2.165
7	164	20 800	64.3	339.5	15.6	0.519	2.74
8	128.5	16 500	50.0	264	20.0	0.641	3.38
9	114.4	13 100	39.6	209	25.2	0.808	4.27
10	101.9	10 380	31.4	165.5	31.8	1.02	5.39
11	90.7	8 234	24.9	131.5	40.1	1.28	6.76
12	80.8	6 530	19.8	104.5	50.6	1.62	8.55
13	72.0	5 178	15.7	82.9	63.8	2.04	10.75
14	64.1	4 107	12.4	65.5	80.4	2.58	13.62

* Down to an including No. 7 B. & S. is stranded; No. 8 B. & S. and smaller is solid. The mass and resistance of stranded wire are assumed to be 2% greater than those of the equivalent solid wire.

These data apply to bare copper wire of 100% relative conductivity (International Annealed Copper Standard). The resistance of hard-drawn wire may be considered as about 2.5% higher than these values. The corresponding data for aluminum wire of 61% conductivity may be obtained by multiplying weights by 0.304, feet per pound by 3.29 and resistance by 1.64.

The data for sizes larger than No. 0000 B. & S. refer to stranded cable and are approximate. They may be in error from 1% to 3%.

as nickel, nickel alloys of various kinds, and platinum, are used to a great extent where the comparativly high resistance is desirable, as in rheostats and heating devices. Wire made of these materials is known as resistance wire.

The cross-section of electrical conductors is usually measured in circular mils. A circular mil is the area of a circle of 0.001 in. diameter. The specific resistance is often expressed in terms of a mil-foot, which is the resistance of a wire one circular mil in cross-section and one foot long. Telephone and telegraph engineers use the term pounds per mile-ohm, which is the weight in pounds of a conductor one mile long and having a resistance of one ohm. The resistance of conductors varies with the temperature, usually increasing in direct proportion to the temperature up to at least 100° C. The increase in resistance per ohm per degree increase in temperature above a given standard (0° or 20° C.) is the resistance temperature coefficient. The preceding tables give data for the more common conductor materials and copper wire of standard sizes.

Resistance Wires. The following materials are representative of this class of conductors:

German Silver is one of the first materials used for resistance purposes. Alloy of copper, nickel, and zinc (the percentage in the table refers to percentage of nickel). Tends to become brittle if repeatedly heated and cooled and has comparatively high temperature coefficient (0.0002 to 0.0004 per degree C.). Resistance per mil-foot, 195 and 290 ohms, respectively.

Manganin. Alloy of copper, nickel, and manganese. Very low temperature coefficient (order of 0.00001 per degree C.). Permanent if not heated excessively. Used extensively for precision instruments. Resistance per mil-foot, 270 ohms at 20° C. Thermal e. m. f., nil.

"**Nichrome.**" Trade name for a nickel-chromium alloy. Practically non-corrosive and has extremely high melting point (2800° F.). Used extensively in heating appliances and small electric furnaces. Resistance per mil-foot, 660 ohms at 20° C.

"**Advance.**" Trade name for a copper-nickel alloy. Temperature coefficient practically nil. Durable at moderate temperatures. Used extensively in electrical instruments and temperature measuring instruments because of high thermal e. m. f. Resistance per mil-foot, 295 ohms at 20° C. Thermal e. m. f., 0.043 mv. per degree C.

Pure Nickel. High temperature resistance coefficient. Suitable for moderately high temperatures. Used extensively for resistance type pyrometers. Resistance per mil-foot, 60 ohms at 20° C. Temperature coefficient, 0.0048 average in range 0–100° C.

Resistance Wire Table

Size, B. & S.	Diameter, mils	Ohms per 1000 ft.					
		18% German silver	30% German silver	Manganin	"Nichrome"	"Advance"	Pure Nickel
14	64.1	47	71	65	161	71	14.6
16	50.8	75	112	103	254	113	23.1
18	40.3	119	179	168	412	184	37.4
20	32.0	190	285	263	645	287	58.7
22	25.3	302	453	421	1 031	460	93.7
24	20.1	480	720	668	1 634	726	150
26	15.9	764	1 140	1 068	2 610	1 160	237
28	12.6	1 210	1 820	1 700	4 160	1 850	378
30	10.0	1 930	2 890	2 700	6 600	2 940	600
32	8.0	3 070	4 610	4 220	10 315	4 600	937
34	6.3	4 880	7 330	6 800	16 625	7 400	1 510
36	5.0	7 770	11 600	10 800	21 020	11 760	2 400
38	4.0	12 300	18 500	16 870	41 240	18 375	3 750
40	3.1	19 600	29 400	30 000	73 330	32 660	6 250

4. Direct-current Generators and Motors

Definitions. A **Generator** is a machine for converting mechanical power into electrical power, and a **Motor** is a machine which converts electrical power into mechanical power. A direct-current generator can also be operated as a motor, but commercially it is not customary to make motors and generators interchangeable because there are minor differences in design.

When a loop of wire is revolved in a magnetic field, an electromotive force is induced, the value of which, at any instant, depends upon the speed of rotation, the strength of the magnetic field, and the size of the loop. The direction of the e. m. f. in such a loop will reverse twice each revolution, hence to obtain direct current a **commutator** is used to reverse the connection between the external circuit and this coil at the proper moment, thus keeping the polarity of the potential at the terminals the same. A number of such loops wound on an iron or steel **core** together with a suitable commutator constitute the **armature** of a generator. The magnetic field is produced by two, or any multiple of two, electromagnets, called **poles**, which surround the armature. These poles and the supporting frame which completes the magnetic circuit constitute the **field** of the machine.

Types of Generators. The various types of direct-current generators may be classified according to the method of exciting the field. **Series Generators** have the field windings connected in series with the armature, and hence the voltage depends upon the load. This type can be designed to give potentials of 2000 to 6000 volts, and a special form was formerly used extensively for series arc-lighting systems. **Shunt generators** have their field windings connected in parallel with the armature. The field contains many turns of relatively small wire and requires only a small current. The voltage is regulated independently of the load by increasing or decreasing the field current by means of a variable resistance (rheostat) which is in series with the field This type is little used, the **compound generator** being the standard type. In this, both a shunt field and a series field are provided, one winding being placed over the other.

In the plain shunt generator the voltage decreases somewhat as the load increases unless the field current is increased. By winding a few turns of heavy conductor on the poles and passing the load current through them, the field is automatically increased so that the voltage can be not only kept automatically constant but even raised to compensate for the drop in voltage in the feeders and mains. This form of compound generator is said to be over-compounded. The compound generator is universally used for lighting and power at 125 and 250 volts, and for railway purposes at 550 to 1200 volts. Any number of compound generators can be operated in parallel, but a special connection between the series fields of the several machines, called an **equalizer** is necessary to obtain satisfactory operation.

A Separately Excited Generator is a generator in which the field current is obtained from a source other than the generator itself. It is used only in special cases, such as low-voltage generators for electroplating and for storage-battery charging. **Boosters** are separately excited low-voltage generators in which the armature is connected in series with a circuit in which it is desired to raise the voltage. They are frequently used on circuits where the voltage is below normal on account of the excessive drop due to heavy loads or long length. **Magneto generators** have permanent magnets and are only built in small sizes for telephone signals, gas-egnine ignition, and similar purposes.

Types of Motors. There are three general classes of motors,—**series, shunt,** and **compound**; like generators, they are distinguished by the method of exciting the field. The field of the **series motor** is in series with the armature and the characteristic feature is a large torque at low speed, which makes it particularly adapted for railways, hoists, and cranes where a high torque is required at starting. **Shunt motors** have the fields in parallel with the armature and are essentially shunt generators operated as motors. This is the type in most common use for general power purposes, as its speed decreases only a few per cent from no load to full load. Fig. 13 shows the characteristic relation between speed and torque, also between current and torque for both series and shunt motors. **Compound motors** have both shunt and series fields, and are either accumulative or differential, according as the series field strengthens or weakens the total field when the load increases. The accumulative form is ordinarily used, for it combines the high-starting-torque feature of the series motor with the constant-speed feature of the shunt motor. Where very uniform speed or an increase in speed is desired, the differential form is used.

To prevent excessive sparking in motors subject to rapidly changing loads over wide ranges, interpoles are provided. These are small poles placed between the regular poles and provided with a winding which is in series with the armature.

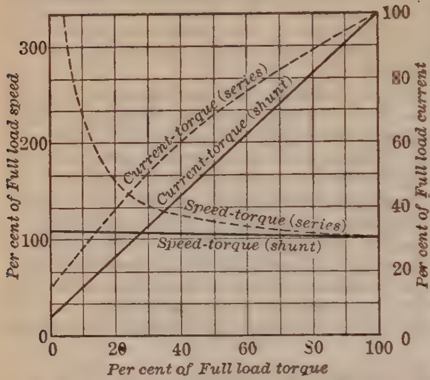

Fig. 13

Speed Regulation can be effected by changing the voltage applied to the armature or the amount of magnetic flux passing through it. The former is done either with a regulating rheostat in series with the armature or by having several different voltages available to which the armature can be connected. The usual method of changing the magnetic flux is to change the field current by means of a rheostat. There are also in commercial use, in small motors, methods in which the distance between the pole pieces and the armature can be readily changed with a hand wheel thus altering the field strength.

Starting. Motors are not connected directly to the line when starting because the resistance is so low that an excessive current would flow which would injure the motor. When the motor is running, a generator or **counter e. m. f.** is produced which opposes the line voltage, and limits the current to the amount required by the load. A motor is started by reducing the line voltage to a low value by inserting resistance (starting rheostat) in series with the armature, the resistance being gradually cut out as the motor comes up to speed. These rheostats are usually provided with attachments which automatically cause the lever to return to the starting position whenever the voltage is cut off the line.

Motor Starters, Controllers and Switches. Shunt motors up to 1/4 hp. and series motors up to 4 or 5 hp. may be started by connecting directly to the line. For larger sizes, starting resistance must be used as stated above.

Shunt and compound motors up to 50 hp. which are started infrequently (i.e., two or three times an hour or less often) and without load use the face-plate type of starting rheostat rated for " starting duty " only. If the motor must start with a load, a heavier duty size must be used. Where the speed is to be regulated, a similar style of rheostat is used for moderate size motors designated as " starting and speed-control " rheostat. For frequent starting and frequent adjustment of speed as in crane, hoisting, and elevator service and many industrial machines, the drum-controller type of rheostat is preferable because it is of more substantial construction which is necessary for this more severe use and reversing the direction of rotation motor is more easily accomplished. Large motors are started with multiple-switch starters where the resistance is cut out of circuit in successive steps by separate switches manipulated individually but interlocked so that the proper sequence must be observed. In the case of very large motors, these switches are operated electrically by means of magnets. Provision can be made for remote control of starting or for automatic starting controlled by any ordinary condition met with in industry such as water level in a tank, pressure, temperature, etc.

It is customary to install a knife switch (double-pole) with each motor by means of which the motor is totally connected to and disconnected from the supply circuit even though all starting rheostats, on the first step, automatically disconnect the motor. Above about 50 hp. it is desirable to use oil switches where the circuit is broken under oil instead of in the open air as in the ordinary knife switch.

In addition to the switch and starting device, automatic protection of the motor against overloads which would burn out the motor must be provided. Fuses, consisting of short pieces of fusible alloy wire or ribbon which melt and open the circuit at a given current, are used for small and moderate size motors. Above 50 hp. circuit breakers are preferable. These are quick-opening switches which operate automatically when a predetermined current is reached.

Note. The Board of Fire Underwriters has an elaborate code of rules with which electric installations, including motors, must comply where the property is insured by fire insurance companies.

The Capacity of generators and motors is practically always determined by the temperature that their insulation will withstand continuously without injury. The rating of such machines is therefore more or less indefinite, but the American Institute of Electrical Engineers prescribes in its Standardization Rules that, at rated load and with a room temperature not exceeding 40° C., the maximum rise in temperature above the room of any part of the insulation shall not exceed certain stipulated values for the various types of insulation in general use. These range from 55° to 85° C.

It is obvious that since temperature is the usual limiting factor which determines the capacity and therefore the rating of a motor, ventilation has a great influence on the rating. Ordinary applications permit the use of motors designed to provide maximum ventilation (known as open-type) but there are many industrial applications where the motor must be screened or partly enclosed for mechanical protection (semi-enclosed type) or entirely enclosed on account of dust, etc. (totally-enclosed type). A semi-enclosed motor will have approximately three-quarters and a totally-enclosed motor approximately one-half of the capacity of an open-type motor of the same weight. In other words, for the same load, semi-enclosed and totally-enclosed motors must be larger and therefore are more expensive than the open-type motors.

Selection of Motors. (See discussion of alternating current motors under this caption.) Each application of an electric motor must be considered by itself with due regard to the many factors which govern the best selection of type and size for the particular application. In general, as in the application of other engineering equipment, the best choice is that which will result in the lowest total annual cost for the life desired. Too large a motor results in a high investment cost and, because of low efficiency, a high operating cost. On the other hand, too small a motor means a high depreciation cost because of shortened life and a high maintenance cost because of repairs and frequent attention required.

Among the many factors that must be considered are:

(a) Character of Load. The torque and speed characteristics of the load dictate the class of motor best suited. Thus, series motors having a large starting torque which decreases as speed increases and are suited to railway, crane, hoisting and similar applications. Shunt motors are substantially constant-speed and are suitable for the great majority of ordinary applications. Cumulative compound motors are most suitable for applications requiring heavy starting torque but also a more constant speed with variations in load than the series motor such as elevators, large metal-working tools like power hammers, punch presses, etc.

The character of the load as to fluctuation influences the size. If the load is constant in magnitude, the rating of a motor which accurately fits the load is easily selected. If the load varies, the best selection will depend upon the load cycle, i.e., magnitude of the maximum load and its duration, the magnitude of the minimum load and its duration, regularity of the cycles, etc. Sparking at the brushes (with resulting rapid wear of commutator and brushes) may be the limiting factor rather than temperature. In well designed standard motors this may be taken as about one-third above the normal rating. In other words with an intermittent load, the normal rating of the motor may be 75% of the maximum load provided the load cycle is such that the prescribed temperature limit is not exceeded. Interpole motors can withstand higher overloads so that a still smaller rating can be used.

In general, the manufacturer of the device, machine or apparatus to which a motor is to be applied can recommend the most suitable class, type and size of motor or at least furnish the power characteristic. In driving groups of machines or apparatus, the diversity of use should be considered, because this may mean a motor as small as only one-third the sum of the powers required by the individual units. However, allowance usually has to be made for line shafts, counter shafts, etc. In installations of any considerable size, it is always best, where at all possible, to determine the actual power requirements by a test with a temporarily-installed motor.

(b) **General Conditions** as to danger of foreign objects getting into the motor and prevalence of dust, moisture, gas, acid fumes, etc. Standard lines of motors are manufactured for each of these general conditions being designated, for example, as dust-proof where an accumulation of dust will not interfere with successful operation, or dust-tight where dust is totally excluded, weather-proof, water-proof, gas-proof, gas-tight, etc. Obviously, the open-type motor being the cheapest, should be used if possible.

(c) **Ambient Temperature.** Where the temperature of the surrounding air is generally higher than the standard at which motors are rated (40° C.) a motor must be used which has a higher normal rating than that corresponding to the load to be carried. An approximate allowance would be about 2% increase in rating for one degree Centigrade increase in temperature above 40° C. In very high temperature situations a motor with so-called class "B"

Average Efficiency and Cost of Direct-current Generators and Motors

Generators			Motors		
Size, kw.	Efficiency at full load, per cent	Cost, dollars per kw.	Size, hp.	Efficiency at full load, per cent	Cost, dollars per hp.
5	83	60	1	75	90
10	85	45	2	80	70
25	87	35	3	81	60
50	89	25	5	82.5	50
100	91	20	7.5	84	45
200	92	17	10	85	40
500	93	15	15	86.5	35
1000	94	14	20	87.5	30
1500	95	13.5	50	90	25
2000	95	13	100	91	20

For a standard starting rheostat, add for example about 10% for a 1-hp. motor and about 5% for a 100-hp. motor.

insulation (heat-resistant) or the ordinary type with forced ventilation should be considered.

Specifications for generators should state the class of service on which the machine is to operate; type of machine; kind of prime mover; and whether direct-connected or belted; kilowatt capacity; voltage; speed; allowable temperature rise of commutator, field, and armature under continuous operation at full load and two hours at 25% overload; that no sparking should occur at any load from no load to full load with brushes stationary; that the machine should carry 50% overload two hours without injury or serious sparking; efficiency at 25, 50, 75, 100, and 125% loads; and change in voltage from no load to full load. Specifications for motors include similar clauses, except that speed regulation replaces voltage regulation.

Cost and Efficiency of generators and motors will vary with the type and conditions of use. In the table on p. 378 are given approximate figures for standard high-speed machines for ordinary service.

The cost of motors and generators varies greatly with the speed. Standard slow-speed machines are 30 or 40% more expensive than standard high-speed machines of the same capacity. The efficiency falls off, with decrease in load, about 1% at 75% load, 2 to 3% at 50% load, and 7 to 15% at 25% load.

5. Alternating-current Generators and Motors

Alternating-current Generators, frequently called alternators, are essentially direct-current machines without commutators, the armature circuit being connected directly to the outside circuit. The connection between the armature and the outside circuit is not reversed every time the winding passes from a north pole to a south pole, and vice versa, as in a direct-current generator. Thus an e. m. f. is produced at the machine terminals which rises from zero to a maximum value in one direction, decreases to zero, reverses, increases to a maximum in the other direction, and again decreases to zero in the time that the winding passes under one north pole and one south pole. Such a cyclic period is called a **cycle,** and the **frequency** of the current is the number of cycles per second.

Types of Generators. The most common forms of alternating-current generators may be divided into two types. In the **revolving-armature** type, the armature is the moving member and the field is stationary, current being delivered to the circuit by means of **brushes** resting on the revolving collector rings. In the **revolving-field** type, the field is the moving member, while the armature is stationary. It is wound in the frame which surrounds the field.

The revolving-armature type is confined to generators of less than 200 or 300 kw. capacity. The stationary-armature type permits the direct generation of potentials up to 15 000 volts and higher, and the economical construction and operation of large capacity generators for a wide range of speeds, from that of the low-head water-wheel to that of the steam turbine.

Windings. There are three kinds of alternators, based on the number of circuits in the armature. A **single-phase** generator has only one circuit in its armature and two terminals. In a **two-phase** generator there are two circuits which generate e.m.f.'s 90 electrical degrees apart. When each circuit or phase is brought out separately there are four terminals, for connection to a two-phase four-wire circuit. Frequently, the two phases are connected to each other and to the middle one of three terminals, for connection to a two-phase three-wire circuit. In a **three-phase** generator there are three circuits or phases in which three e. m. f.'s 120 electrical degrees apart are generated. These circuits are interconnected with either a delta or a star connection requiring three or four terminals respectively.

The accompanying diagrams indicate the relation between the currents and voltages in these various forms of generators and the circuits to which they are ordinarily connected. **I** and **i** are the amperes per phase, and **E** and **e** the volts per phase in armature and circuit respectively.

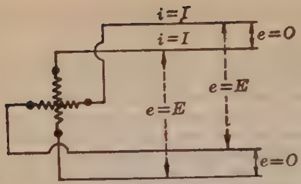

Fig. 14. Two-phase, Four-wire Circuit

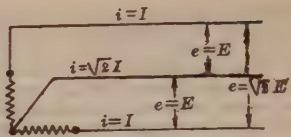

Fig. 15. Two-phase, Three-wire Circuit

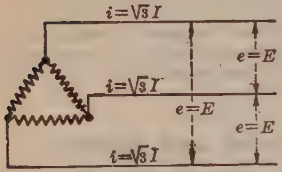

Fig. 16. Three-phase, Three-wire Circuit, Delta Connected

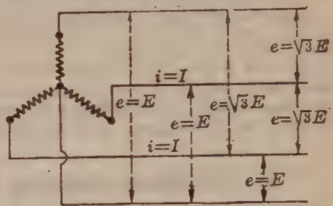

Fig. 17. Three-phase, Four-wire Circuit, Star Connected

Field Excitation. Alternators are usually separately excited. Small machines are sometimes provided with a commutator device by means of which sufficient current is rectified to excite the field. A common practice is to provide a separate generator or exciter for each alternator. This machine is driven from the alternator shaft by a belt or by direct mechanical connection. In large stations, a source of excitation common to all the generators is often provided.

Efficiency and Cost. The following table gives average conservative figures for the efficiency and cost of polyphase alternators. The cost figures, in particular, are necessarily approximate. Slow-speed machines cost more than high-speed machines, and high-voltage more than low-voltage machines. Hence there may be a difference of at least 25% in the cost of generators of the same capacity.

Efficiency and Cost of Polyphase Alternators

Size, kw. *	Efficiency			Cost, dollars per kw.
	50% load	75% load	100% load	
50	83	86	87	35
100	87	89	90	25
200	88	90	91	20
300	88.5	91	92	17.5
500	89.5	92	93	15
1 000	90.5	93	94	12
2 000	92.5	94	95	9
3 000	92.5	94.5	95.5	7.5
5 000	92.5	94.5	95.5	7
10 000	93	95	96	6
20 000	93.5	95.5	96.5	6

* Ratings are in kilowatts at 0.8 power factor. The capacity at a power factor of 1.0 is 25% greater. The power factor of commercial and industrial loads is nearer 0.8, hence ratings on that basis are more in accord with usual practical conditions.

Types of Motors. There are two general types of alternating-current motors—the synchronous and the induction. The **synchronous motor** is essentially an alternator operated as a motor. It has to be brought up to synchronous speed (the speed at which it would run if operated as a generator at the line frequency) by external means before it can be connected to the line. It will operate only at synchronous speed.

Synchronous motors are used as a rule only in large sizes (200 hp. and over) and where it is not necessary to stop the machine frequently. Such cases are in substations where power is converted from one frequency to another or from alternating current to direct current, and in large factories.

Induction Motors are, theoretically, transformers in which the core and the secondary winding are free to move, and the force which the windings of a static transformer exert on each other is utilized to produce mechanical power. When the rotating member (rotor) runs at synchronous speed, no magnetic flux from the stationary member (stator), which is connected to the line, cuts the conductors, and hence no torque is exerted. When a load is applied to the motor, the speed falls below synchronism (called slip), and a current is induced in the rotor conductors which will produce the necessary torque to carry the load. Fig. 18 shows the characteristic relation between speed and torque, also between current and torque, of a polyphase induction motor.

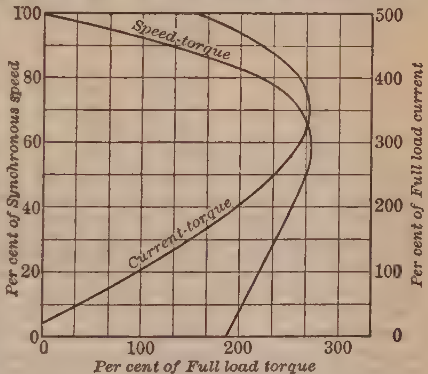

Fig. 18

Polyphase Induction Motors (both two- and three-phases) are the most commonly used type. Their characteristics are similar to those of the shunt motor, in that they have a comparatively low starting torque and operate at approximately constant speed. Polyphase motors are built in all sizes from fan motors to 1000 hp. and over, and are used for all power purposes. There are two forms—the **squirrel cage** in which the winding of the rotor consists of heavy copper bars short-circuited on each other, and with no external connections; and the **slip-ring** form, in which the rotor has a regular polyphase distributed winding, into the phases of which an external variable resistance is connected through collector rings on the shaft. The slip-ring form has a larger torque at starting than the squirrel-cage form, but its efficiency is somewhat lower. When induction motors are overloaded a certain definite amount they will stop. This load is called the maximum torque, or " stalling " load and is two or three times normal load.

Single-phase Induction Motors are infrequently used (and then only in small sizes) because special devices are necessary to make them self-starting. They are more expensive and less efficient than polyphase motors and they unbalance the supply system, which is usually polyphase.

The Series Alternating-current Motor is essentially a direct-current series motor with certain modifications which are necessary for operation on alternating current. It is a single-phase motor and is used in traction work where variable speed and variable torque are required. The repulsion motor is one of several forms of the series alternating-current motor.

Operating Data and Cost. In the following table are given approximate average values for the slip, power factor, efficiency, and cost of standard 60-cycle polyphase motors of the squirrel-cage type. The efficiency and cost of 25-cycle motors are slightly higher. As with all rotating electrical apparatus, the cost varies greatly with the speed—as much as 50% in some cases.

Polyphase Induction Motor Data

(Squirrel-cage type)

Standard sizes, hp.	Slip, per cent	Power factor, per cent	Efficiency, per cent	Cost, dollars per hp.
1	10	80	80	70
2	8	85	82	55
5	6	88	85	40
10	6	89	87	30
20	5.5	90	89	20
50	4.5	90	89	12
100	3.5	90	90	11
200	3.0	91	91	9
500	3.0	91	91	8

Starting. If an induction motor is connected directly to the line when starting it will take a current two to four times the normal full-load running current. To avoid the disturbance which this produces in the system, it is customary to start at a reduced voltage. This is obtained by means of a **compensator**, a single-winding transformer with several taps on it to which the motor is successively connected. The slip-ring type is started with full-line voltage on the motor but with resistance in the rotor circuits. This resistance is gradually cut out as the motor speed increases.

Speed Variation. The inherent characteristics of induction motors are such that the speed is practically constant at a value depending on the frequency and the number of poles in the stator winding. Certain fixed changes in speed are sometimes obtained by providing a special winding by means of which the number of poles can be changed. Change in speed may also be obtained with a variable resistance in the rotor circuits of a slip-ring motor, but the efficiency is materially decreased.

Rating. Alternating-current apparatus, like direct-current apparatus, is rated on the basis of the load which it will carry without overheating. In alternators, this rating is the output in kilovolt-amperes, and in motors it is horsepower. The recommendations of the American Institute of Electrical Engineers for the limiting temperature rise above a room temperature not exceeding 40° C., vary from 55° C. to 85° C., depending upon the class of insulation used.

The discussion of the capacity of direct-current motors (p. 377) applies also to alternating-current motors.

Selection of Motors. The discussion of selection of direct-current motors (p. 377) applies also to alternating-current motors.

Induction motors of the squirrel-cage type have essentially constant speed much like shunt-type direct-current motors and like them are the most generally used type of alternating-current motor. Where higher starting torques and variable speeds are required as with elevators, cranes, etc., the slip-ring type is more suitable. Synchronous motors are preferable only in large sizes where constant speed is required.

As far as surrounding conditions such as dust, dampness, fumes, etc., are concerned, it is obvious that the induction motor of the squirrel-cage type is applicable to a much wider range of conditions than direct-current motors because of the absence of the commutator and sparking. The open-type is therefore very generally used with perhaps special treatment of the insulation for conditions where moisture, acid fumes, oil, etc., are present in unusual amounts.

Specifications for alternators should include, in addition to a description of the type of machine desired and the class of service in which it is to be used, certain performance requirements as follows: Efficiency at various loads from 50% to 125% normal load; voltage regulation (per cent rise in voltage when the load is reduced from normal value to zero) at 100% and 80% power factor; maximum voltage required to excite the field with 125% normal load at 80% power factor; temperature rise of the various parts after continuous operation at full load and 25% overload; ability to withstand 50% overload for one hour without injury; and ability to withstand running at 100% overspeed without injury. Specifications for **induction motors** should include: description of type of motor desired; purpose for which they are to be used; efficiency; power factor and slip at various loads up to 25% overload; maximum torque at starting when connected directly to the line; maximum current required when starting; maximum torque and maximum load which motor will carry without stopping; temperature rise after continuous operation at full load and after two hours at 25% overload; and ability to carry 50% overload one hour without injury.

6. Power Plants

The term Central Station usually refers to a plant operating under a franchise for the manufacture and sale of electricity to the general public. It is therefore a semipublic enterprise as distinguished from a private plant, in which electricity is generated for private purposes. Central stations vary in size from 50-kw. plants in small villages to the large stations in New York and Chicago of several hundred thousand kilowatts capacity. Private plants range from single-unit plants for small office buildings and industrial establishments to the large traction and industrial plants of over 50 000 kw. capacity.

Location. The location of **hydraulic plants** is obviously at the source of water power. Many conditions govern the location of a **steam-driven** central station. The more important are: kind of service to be rendered, whether alternating or direct current; concentrated or scattered area of distribution; size of plant; coast of real estate; and transportation facilities. Large steam plants must be located where an ample supply of condensing water is available (100 to 500 gal. per kw-hr. required). Where a waterway is not available, cheap land, railroad facilities, and water free from scale-forming impurities are the more important considerations. On the other hand, it might be more economical for a smaller plant to be located as near the center of distribution as possible, regardless of other disadvantages. **Gas-driven** plants are usually private plants, and hence are located on the same premises as the load. Private plants in buildings primarily used for other purposes should be located with due regard to the following points: effect of vibration and noise, ventilating facilities, facilities for handling coal and ashes, value of the space for other purposes, and coal storage facilities.

Prime Movers. The prime movers in common use for electric power generation are reciprocating steam engines, high- and low-pressure steam turbines, internal-combustion engines, and hydraulic turbines.

The Reciprocating Steam Engine is the most commonly used small and moderate size steam-driven prime mover. The principal types used for electric power generation are the simple and compound high-speed engines. Small single-cylinder engines receive steam at 75 to 100 lb. pressure per sq. in., and exhaust it at about atmospheric pressure. Larger engines operate condensing; receiving steam at 100 to 200 lb. pressure and discharging it into a vacuum at 2 to 4 lb. absolute pressure.

Steam Turbines are rapidly superseding reciprocating engines above a few hundred horsepower because they are simpler, occupy much less space, cost less per kilowatt, are more efficient, can be used with steam at higher pressures and with a much higher degree of superheat than is permissible with reciprocating engines, and can be built in much larger units—machines up to 150 000 kw. capacity are now in daily operation. Furthermore, high-speed generators, which are cheaper than low-speed generators, can be used.

Low-pressure turbines receive steam at about atmospheric pressure and discharge it into a vacuum. A much greater percentage of the energy in steam below atmospheric pressure can be utilized in a turbine than in the low-pressure cylinder of a reciprocating engine. The capacity of existing plants using condensing engines can be increased 10 to 30% by exhausting from the engines at atmospheric pressure into low-pressure turbines. This additional capacity is obtained without additional fuel consumption. At the same time the range of loads at which maximum economy is obtained is greatly increased.

Internal-Combustion Engines are operated with natural, illuminating, producer, and blast-furnace gases, with crude petroleum, kerosene oil and with gasoline. Where liquid fuels are used, an auxiliary device vaporizes the liquid before it is admitted to the cylinders. This very efficient class of prime movers has been handicapped for electric power plant work because of non-uniform angular velocity, unreliability of operation, limited overload capacity, and high maintenance cost. These faults are rapidly being overcome and gas engines are being used quite extensively, particularly where very cheap fuel is available, such as the waste gases from blast furnaces. Engines of several thousand horsepower are in successful operation on this fuel. The thermal efficiency of the gas engine is from 20 to 30% and is the highest of all heat engines. The cost of power is therefore very low even in small sizes, especially in connection with producer-gas plants, where cheap grades of coal can be utilized.

Hydraulic Turbines in power plants where the head is less than 150 ft. are usually of the pressure type. The turbines in the older plants are of the vertical type, but the lower cost and more reliable operation of horizontal generators has led to the general use of horizontal turbines. Where the head is high the impulse type of turbine is used.

Approximate Steam and Fuel Consumption of Prime Movers

	Pounds of steam per kw-hr.	Pounds of coal per kw-hr.
Steam Reciprocating Engines:		
Simple, non-condensing, 25 to 200 hp........	100 to 40	25 to 8
High-speed, automatic, compound, condensing, 200 to 1000 hp........................	60 to 30	8 to 5
Simple, Corliss, condensing, 150 to 500 hp......	40 to 25	6 to 4
Compound, Corliss, condensing, 500 to 5000 hp.	30 to 18	4 to 2
Compound and triple, four-cylinder, condensing, 2 000 to 10 000 hp...................	20 to 15	2 to 1.5
Steam Turbines, high pressure:		
Non-condensing, 10 to 50 kw................	75 to 50	10 to 6
Condensing, 100 to 500 kw..................	30 to 15	4 to 2
Condensing, 1 000 to 5 000 kw..............	15 to 13	1.8 to 1.5
Condensing, 5 000 to 50 000 kw.............	13 to 10	1.3 to 1.0
Steam Turbines, low pressure, combined with reciprocating engines:		
One 1000-kw. engine with one turbine.......	18	2.2
8000-kw. engine plant with one turbine......	12	1.3
One 7500-kw. engine with one turbine.......	10	1.1
Gas Engines:		
Natural gas, 50 to 200 hp..................	....	25 to 15 cu. ft.
Producer gas, 50 to 200 hp.*...............	....	3.0 to 2.0
Illuminating gas, 10 to 75 hp..............	....	35 to 25 cu. ft.
Gasoline, 10 to 75 hp......................	....	2.0 to 1.2 pints
Oil Engines:		
100 to 500 hp..............................	....	1.5 to 1.0 lb. oil

* Complete plant

Boilers for the production of steam are built in several different types, the most commonly used for power-plant purposes being the water-tube and fire-tube. The **water-tube boiler** is the safest, most efficient, and can be operated at the highest pressure. The small **fire-tube boiler** is cheaper and easier to operate than the small water-tube boiler, and hence is preferable for small plants. Boilers are arranged to reduce the piping to a minimum and make firing as convenient as possible. They are usually placed side by side in one row or in two rows facing each other. All are piped to a common pipe or header, but in such a way that each boiler and group of boilers can be cut off by conveniently located valves.

Rating. Boilers are rated in horsepower, an arbitrary unit equal to the evaporation of 34.5 lb. of water per hour from water at 212° F. to steam at 212° F. Boilers properly rated should develop their rated horsepower when fired by an ordinary fireman and with ordinary steam coal but boilers in modern power plants are regularly operated at two to four times their rating.

Steam Piping should be well lagged and free from pockets where condensed steam can collect. All valves should be easily accessible. The following table gives the approximate size of piping required to deliver any quantity of steam at any distance from the boiler. This table is based on Babcock's formula, which is as follows:

$$p = 0.0001321 \left(1 + \frac{36}{d}\right) \frac{M^2L}{Dd^5}$$

where p = difference in pressure between the two ends of the pipe, pounds per square inch;
d = internal diameter of pipe, inches;
M = amount of steam, pounds per minute;
L = length of pipe, feet;
D = mean density of steam, pounds per cubic foot.

It is to be noted that authorities do not agree on what is the correct formula for steam flow so that for exact calculations reference should be made to the more extensive discussions of the subject in Lucke's "Engineering Thermodynamics," Marks' "Mechanical Engineer's Handbook" (p. 360), etc.

Flow of Steam through Pipes (Babcock)

Pounds per minute with 1 lb. loss of pressure

Initial gage pressure, lb. per sq. in.	Diameter of pipe, inches. Length, 240 diameters								
	1	1-1/2	2	3	4	5	6	8	10
50	4.04	11.2	20.01	49.48	91.34	150.8	226.0	412.2	665.0
60	4.32	11.9	21.38	52.87	97.60	161.1	241.5	440.5	710.6
70	4.58	12.6	22.65	56.00	103.37	170.7	255.8	466.5	752.7
80	4.82	13.3	23.82	58.91	108.74	179.5	269.0	490.7	791.7
90	5.04	13.9	24.92	61.62	113.74	187.8	281.4	513.3	828.1
100	5.25	14.5	25.96	64.18	118.47	195.6	293.1	534.6	862.6
120	5.63	15.5	27.85	68.87	127.12	209.9	314.5	573.7	925.6
150	6.14	17.0	30.37	75.09	138.61	228.8	343.0	625.5	1009.2

For any other loss of pressure, multiply by the square root of the proposed loss. For any other length of pipe, divide 240 by the given length expressed in diameters, and multiply the table figures by the square root of this quotient to get the flow for one pound loss of pressure.

The resistance due to steam entering pipe = 60 diameters additional length; to a globe valve = 60; to an elbow = 40, or two-thirds of a globe valve.

Furnaces are designed for the particular class of fuel to be used. The object of a furnace is to transfer the heat in the fuel to the water in the boiler with the least loss; hence there is a certain kind of grate and arrangement of fire-walls, baffles, flues, etc., best suited for each fuel, i.e., lump coal, powdered coal, fuel oil or gas. The average grate surface provided per boiler horsepower is about 1/4 to 1/2 sq. ft. Furnaces for oil, gas fuel, and powdered coal are the same as coal-burning furnaces, except that special burners are substituted for grates.

Great progress has recently been made in improving furnace efficiencies through the use of water-cooled walls, preheated air, higher combustion temperatures, etc.

Combustion. The sources of inefficiency in a furnace are radiation and conduction losses, coal passing through the grates unburnt (in lump coal furnaces), and incomplete combustion. The unburnt coal in the ashes should not exceed 2 or 3%. Complete combustion means that the carbon—the source of heat in the fuel—is completely oxidized to carbon dioxide. If oxidized only to the monoxide, about two-thirds of the available heat is wasted. Incomplete combustion is usually caused by insufficient air supply. Too much air, on the other hand, not only lowers the temperature of the furnace but absorbs heat which is carried up the chimney. Frequent analysis of the flue gases and ash is necessary to insure that the furnace is being properly operated.

Draft. Air is supplied to the furnaces by natural draft through chimneys or by forced draft. **Natural draft** is more commonly used and various formulas have been developed to express the relation between the internal area and height of chimney required per horsepower of boilers. Kent's formula, based on the rather liberal allowance of 5 lb. of coal per boiler hp., is $\mathbf{HP = 3.33\,(A - 0.6\sqrt{A})H}$, where HP = horsepower of boiler plant, A = inside area in square feet, and H = height in feet. Self-supporting steel chimneys cost about one-third as much as brick chimneys, and are therefore replacing brick for plants of 100-hp. capacity and less. **Forced draft**, either by pressure in the ash pit made air-tight or by suction in the flues, is used to a considerable extent in large power plants. The maximum capacity of the plant can be greatly increased with forced draft, and greater economy, under fluctuating load conditions, is obtained.

Mechanical Stokers are devices for automatically carrying the coal under the boilers, properly burning it, and depositing the ashes in the ash pit. The principal forms are the **rocking grate**, **endless chain grate**, and the **screw feed**. The many advantages in large plants include economy in labor, economy in fuel because cheaper fuels may be used, and more uniform fires. Their use is not advisable in comparatively small plants where the total load varies through wide ranges, hand firing under such conditions being more flexible and economical.

Approximate Cost and Heating Value of Fuels (1928)*

Fuel	Cost, dollars	B.t.u.	B.t.u. per dollar
Anthracite (rice size)	5.50 per ton	13 000 per lb.	4 750 000
Bituminous	4.00 per ton	14 000 per lb.	7 000 000
Coke	8.00 per ton	13 100 per lb.	3 300 000
Wood, hard, dry	15.00 per cord	6 250 per lb.	1 850 000
Crude petroleum	0.03 per gal.	19 000 per lb.	4 700 000
Kerosene	0.15 per gal.	17 000 per lb.	880 000
Illuminating gas	1.25 per M. cu. ft.	750 per cu. ft.	600 000
Natural gas	0.35 per M. cu. ft.	1 000 per cu. ft.	2 900 000
Gasoline	0.20 per gal.	18 000 per lb.	475 000

* Prices are obviously greatly affected by market conditions, distance from source of supply, quantities purchased, etc.

Generators. The type of generator used in a power plant varies with the conditions. Where the load is concentrated, near at hand, and mixed lighting and power, direct-current generators are used. Since, however, the cost of power production decreases with the size of the units and the size of the plant, the trend of modern engineering is toward the concentration of power production in large stations with large alternating-current generator units (single units of over 150 000 kw. capacity of the compound type are in service), the power being distributed at high tension to distributing centers called substations. The selection of the size and number of units for a plant requires careful consideration. Both prime movers and generators decrease rapidly in efficiency below 50 or 60% load, hence the sizes should be such that those in use will always be well loaded. There should be enough units to carry the maximum load, with one as a reserve. For example, a 200-kw. plant ought to have three 100-kw. units, a 600-kw. plant four 200-kw. units, and a 1000-kw. plant three 500-kw. units.

The Foundations for all apparatus should be separate from that of the building. They should be of ample dimensions and never less than those recommended by the manufacturer. Direct-connected generators, motor generators, and rotary converters, although self-contained, should have substantial foundations in order to reduce the vibration and consequent wear on the bearings and insulation. Concrete foundations are the best and cheapest: they will safely carry loads of 7500 lb. per sq. ft. The safe bearing loads of various soils are as follows: clay 4000 lb., coarse gravel and sand 2500 to 3500 lb., rock 10 000 to 30 000 lb.

Switchboards are designed to provide not only independent, easy and quick control of each generator, exciter, and feeder, but to show by means of instruments mounted on them the amount and character of the load. They are made up in panels of slate, marble, and artificial stone placed side by side, one panel for each machine and feeder. The board is usually installed along one side of the generator room with ample space in the front and rear. In low-tension alternating-current and direct-current plants, all circuits are brought to the switches and instruments on the board. **Buses** or heavy copper bars are mounted on the back of the board (often in duplicate) and connected to the various panels so that any number of generators and feeders may be connected in parallel. The circuits of high-tension plants are not brought to the switchboard but are controlled by switches under oil which are operated mechanically or electrically from the switchboard, the instruments being connected in the secondary circuits of instrument transformers.

The very large modern stations generate high-tension alternating current and all generators and feeders are controlled from a miniature remote-control switchboard. The main switches are operated by motors controlled by small switches on this control board. One operator can thus handle all of the outgoing power of a very large plant.

Direct-current generator and feeder panels are usually provided with knife switches, an automatic overload circuit breaker, a voltmeter and ammeter. Low-tension alternating-current panels are similarly equipped, with the addition of a wattmeter and a power-factor meter.

Cost of Power Plants. The cost of power plants depends upon location, type of plant, class of service, size, etc. **Hydraulic plants** formerly, on the whole, cost less than steam plants, but now the reverse is generally the case because the after-war increase in costs has been compensated by marked improvements in efficiency and economy of steam plants, whereas no such improvement has been made in hydraulic power machinery. The average investment per kilowatt of hydraulic plant will range from $125 to $250 per kilowatt, but some plants cost $400 per kilowatt. This is divided, in a typical case, about as follows: hydraulic works 60%, wheels and fittings 12%, building 5%, generators, exciters, and switchboard 15%, and step-up transformers 10%. **Steam plants** cost from $75 to $125 per kilowatt, and an average approximate division of the cost is: boilers and piping 15%, engines and condensers 25%, The following distribution of items which make up the cost of power in various kinds of plants was made by H. G. Stott (Power-Plant Economies, vol. xxv, Trans. A. I. E. E., 1906). This analysis was prepared nearly 25 years ago and obviously does not accurately apply to present day conditions. No more recent figures are available, but these are retained because they give a general "picture" of the relative distribution of power costs.

Relative Cost of Power

Items	Reciprocating engines	Steam turbines	Reciprocating engines and steam turbines combined	Gas-engine plants	* Gas engines and steam turbines combined
Maintenance:					
1. Engine, room, mechanical	2.57	0.51	1.54	2.57	1.54
2. Boiler room or producer room	4.61	4.30	3.52	1.15	1.95
3. Coal- and ash-handling apparatus	0.58	0.54	0.44	0.29	0.29
4. Electrical apparatus	1.12	1.12	1.12	1.12	1.12
Operation:					
5. Coal- and ash-handling labor	2.26	2.11	1.74	1.13	1.13
6. Removal of ashes	1.06	0.94	0.80	0.53	0.53
7. Dock rental	0.74	0.74	0.74	0.74	0.74
8. Boiler-room labor	7.15	6.68	5.46	1.79	3.03
9. Boiler-room oil, waste, etc.	0.17	0.17	0.17	0.17	0.17
10. Coal	61.30	57.30	46.87	26.31	25.77
11. Water	7.14	0.71	5.46	3.57	2.14
12. Engine-room, mechanical labor	6.71	1.35	4.03	6.71	4.03
13. Lubrication	1.77	0.35	1.01	1.77	1.06
14. Waste, etc.	0.30	0.30	0.30	0.30	0.30
15. Electrical labor	2.52	2.52	2.52	2.52	2.52
Relative cost of maintenance and operation	100.00	79.64	75.72	50.67	46.32
Relative investment	100.00	82.50	77.00	100.00	91.20

* A proposed combination where the exhaust gases from the gas engine would be used to generate low-pressure steam for low-pressure turbines. Both of these machines have a high efficiency and each would be used under the most favorable conditions.

pumps and other auxiliaries 5%, generators and switchboard 20%, building, foundations, and smoke-stack 25%, coal-handling plant 5%, and engineering 5%.

The cost of Producing Energy depends upon a great many factors, principal among which are the kind and size of plant, the cost of fuel, labor, and supplies, and the load factor. In hydraulic plants it will range from 0.4 to 1.0 cent per kilowatt-hour delivered to the transmission line. An average division of cost will be about as follows: interest, taxes and depreciation 65%; labor 15%; maintenance and supplies 20%. The average cost per kilowatt-hour at the bus-bars in steam plants may be taken at about 3 cents for 250-kw. plants, 1.75 cents for 500-kw. plants, 1.5 cents for 1000-kw. plants, and 1.25 for 2000-kw. plants. Very large plants of 50 000 kw. and over probably produce power for less than 0.5 cent per kilowatt-hour. The cost in gas-engine plants will be less than in steam plants of the same size. This is particularly true of the smaller sizes. Producer-gas plants of 400 or 500 kw. can produce energy at a cost of 1 cent per kilowatt-hour.

7. Transmission of Power

Losses of Electrical Power occur in transmission, the principal one of which is that due to the resistance of the conductors. The amount of loss which may be economically allowed will depend upon several factors such as cost of producing the power, the line investment, size of the load, load factor and the value of the power at the point of delivery. Much greater losses may be allowable and much larger distances covered under certain conditions that would be financially permissible under others.

When current flows through a conductor, a loss occurs which is equal to the product of the current and the fall in potential between the ends of the conductor. From Ohm's law it follows that with a given percentage loss the distance that power may be transmitted will be proportional to the square of the voltage. High-voltage direct current is not commercially practicable, for although a moderately high voltage may be produced by connecting generators in series, it cannot be readily reduced to low voltage for general distribution. On the other hand, alternating current can be transformed from low to very high voltage and vice versa with very simple and efficient apparatus called transformers.

It is now standard practice in large concentrated systems to generate all of the power in one station and transmit it (usually underground) at 2200 to 25 000 volts to various sub-stations from which it is distributed at low potentials. The distance in such cases is, however, comparatively short. Where the electricity is generated by water power, it is usually necessary to transmit long distances to reach a sufficiently large market. In such systems it is customary to generate at 2200 to 15 000 volts and transform to 6600 to 220 000 volts, depending upon the distance. About 220 000 volts is at present the highest voltage in commercial use and about 500 miles the greatest distance of transmission.

Transformers are essentially large induction coils without interrupters. Two coils are wound on a common core built up of laminated steel. They are thoroughly insulated from the core and from each other with varnished cambric and similar materials. When alternating voltage is applied to one coil, the primary winding, an alternating flux is produced in the core. This flux induces an e. m. f. in the other coil, the secondary winding. The ratio of the voltages will be practically the same as the ratio of the number of turns of wire in the two coils.

Classification. Transformers may be classified according to their application, whether for power purposes or for use with instruments. Power transformers may be either constant potential or constant current. The **constant-potential** transformer is the most usual form and is used for all general power transmission at constant potential. Power from two-phase generators is transmitted with two transformers, one in each phase. Power from three-phase generators is transmitted with three transformers connected delta or star and with three-phase transformers in which three transformers are combined by placing three sets of windings on three parts of a common magnetic circuit. The three-phase transformer is cheaper than three single transformers, but the latter method is more flexible and permits operation with only two transformers if one burns out. **Constant current** transformers convert power from low-voltage constant potential to constant current at variable high voltage. They are used for series lighting systems. **Instrument transformers** are used only to insulate instruments from the line and have only 10 to 200 watts power capacity. **Voltage transformers** are connected to voltmeters and wattmeters. They transform the high voltage of the line to about 110 volts. **Current transformers** are connected to ammeters and wattmeters. They insulate the instrument from the line, and also transform the current to a small value, usually 5 amperes.

Rating. The capacity of transformers is determined by the maximum temperature reached in operation. This is fixed at values which will not cause deterioration of the insulating materials. The American Institute of Electrical Engineers recommends a temperature limit corresponding to 55° to 85° C. rise above a standard room temperature of not exceeding 40° C. after continuous operation, the limit of permissible rise depending upon the class of insulation employed. Various methods are used to dissipate the heat and thus increase the capacity. In **self-cooled** transformers the case is filled with oil. The oil, in addition to increasing the insulation, carries the heat by natural circulation from the core and coils to the sides of the tanks, from which it is dissipated by radiation and convection. The surface of the tank is made as large as possible by corrugating and by the addition of external pipes or reservoirs connected to the tank at the top and bottom. The **water-cooled type** is the same as the oil-cooled type, with the addition of coils of pipe at the top through which water flows and carries off the heat. Transformers up to 500 or 600 kw. capacity are usually self-cooled; larger transformers may be either self-cooled or water-cooled.

Losses and Efficiency. The losses in transformers are of two kinds: **Copper loss** is due to the resistance of the windings and varies with the square of the load. The **core loss** is due to the rapid reversal of the magnetic flux in the steel core and is constant at all loads. In well-designed transformers these losses are about equal, except in distribution transformers (50 kw. and under) where the iron loss is usually not over half the copper loss. The iron loss in a transformer is constant irrespective of the load and consequently is made a smaller proportion in distribution transformers for light and power (see Art. 8) where the load is on only a few hours a day but the iron loss is constant for 24 hours. In other words, the "all-day" efficiency becomes more important. The efficiency of transformers is comparatively high, as indicated by the following table:

Efficiency of Power Transformers

Size, kw.	Per cent of normal load				
	125	100	75	50	25
5	95	95.5	96	95.5	93.5
50	97	97.5	98	98.0	97.25
100	97.5	98.0	98.25	98.25	97.5
500	97.75	98.25	98.5	98.25	97.5
1000	98.25	98.75	99.0	98.75	98.0
3000	98.5	99.0	99.25	99.25	98.5
5000	98.5	99.0	99.25	99.25	98.5

These are approximate average values for three-phase, 2200-volt distribution transformers up to 500 kw. and single-phase, 44 000-volt transmission transformers over 500 kw. The efficiency is slightly lower for 25 cycles than for 60 cycles. Also the efficiency is lower for high voltages than for low voltages.

Regulation. With a constant primary voltage, the secondary voltage of a transformer will increase as the load decreases due to the resistance and inductance of the windings. The ratio of this increase to the voltage at full load is the **regulation** of the transformer. In designing, the aim is to keep the regulation as small and the efficiency as high as possible. The regulation at 100% power factor of well-designed transformers will range from 3% for a 5-kw. transformer to 1.5% for a 1000-kw. It will increase very rapidly as the power factor decreases.

Specifications usually include the following: purpose for which the transformer is to be used, capacity, voltage, kind of cooling, efficiency at various 100% power-factor loads, regulation at 100% and 80% power factor, temperature after continuous operation at full load and after 2 hours at 25% overload, high-potential test of insulation at double normal voltage and an over-potential test by operating at double voltage for 5 minutes.

Cost. The cost of transformers of a given rating will depend upon the high-tension voltage, the frequency and the method of cooling. For example, the approximate cost of 5-kw., 2200-volt distribution transformers is of the order of $15 per kw. but for 22 000 volts the cost is $50 or $60. The cost of a 5000-kw. water-cooled, three-phase transformer will vary from about $1.50 per kw. at 22 000 volts to the order of $2.00 at 66 000 volts. Self-cooled transformers cost 25 to 50% more than water-cooled. Sixty-cycle transformers are slightly cheaper than 25-cycle.

The Transmission Line is the weakest part of a transmission system, and therefore, where continuity of service is at all important, requires great care in designing. It is essential to have ample insulation under all weather conditions; ample factor of safety in poles, conductors and insulators under all temperature, wind, and sleet conditions; effective protection against lightning.

Moderate voltage lines are run on wood poles with wood cross-arms. They are set 100 to 200 ft. apart and the conductors are carried on rigid porcelain pin-type insulators. High-voltage lines, also important heavily loaded moderate voltage lines, are carried on towers built up of steel shapes throughout and set 400 to 800 ft. apart. The conductors are suspended from the cross-arms by insulators made up of 3 to 12 or 15 porcelain disks fastened together with flexible joints. On large systems these towers are either rectangular, square, or triangular in shape and are very substantially built. The base will occupy a space 10 to 15 ft. square, the height will vary from 40 to 60 ft., and they will withstand loads of 10 000 to 15 000 lb., applied at the top of the tower. A two-legged flexible steel tower is being extensively used on transmission lines where a low investment charge is essential.

A very high voltage transmission line is usually run over a private right-of-way 50 to 100 ft. wide with all tall and dead trees cut away on each side. To guard against complete shut-down due to an accident to the line, large systems are frequently provided with a duplicate line. This line may be run beside the other, but is preferably run through a different part of the country so that both lines would not ordinarily be subjected to a lightning storm at the same time.

The kind of material to be used for the conductors depends upon various factors such as topography (rough country may require some long spans where the strength of the conductors would be unusually important), climate (excessive wind and sleet storms would require high-strength conductors), importance of continuity of service, length of line and the economics of the proposition. Copper, aluminum, steel, iron, and copper-clad steel are used, but hard-drawn copper is by far the most generally employed.

Size of Conductor. The size of the conductors will depend upon many conditions including most economical energy loss, load factor, character of the installation (that is, whether temporary or permanent), strength required and voltage regulation necessary. With very high voltage, especially at high altitudes, loss due to corona (which is greater with smaller conductors) must also be considered. So far as the energy loss is concerned, Lord Kelvin propounded the law that "the most economical area of conductor will be that for which the annual interest on capital outley equals the annual cost of energy wasted."

Line Drop, Efficiency and Regulation. The **line drop** is the difference between the voltage at the sending end of the line and that at the receiver end under prescribed load conditions. It will depend upon the resistance and reactance of the line. The **line efficiency** is the ratio of the power delivered by the line (that is, receiver load) to the power input at the sending end under prescribed conditions at the receiver end. **Line regulation** is the change in voltage at the receiver end where the prescribed load changes from no load to full load, voltage being maintained constant at the generator or sending end. It will depend upon, in addition to the resistance, the inductive reactance and capacitance reactance, which will vary with the frequency, and with the size and spacing of the conductors.

For calculations of these quantities for long lines and very high voltages, reference should be made to the numerous reference books on the subject. The following formulas will, however, be approximately correct for short lines of moderate voltage where capacitance reactance and capacitance current is negligible. E_S = voltage, sending end; E_r = voltage, receiver end; θ = angle of lag between current and voltage, receiver end; I = current in one conductor in amperes; R = total resistance of one conductor in ohms, x = total reactance of one conductor in ohms.

$$E_s = \sqrt{(E_r \cos\theta + IR)^2 + (E_r \sin\theta + IX)^2} \text{ (volts)}$$

$$\text{Line drop} = E_s - E_r \text{ (volts)}$$

$$\text{Regulation} = \frac{E_s - E_r}{E_r} \times 100\ (\%)$$

$$\text{Power, receiver end} = E_r I \cos\theta \text{ (kw.)}$$

$$\text{Power, sending end} = E_r I \cos\theta + I^2R \text{ (kw.)}$$

$$\text{Power factor, sending end} = \frac{E_r \cos\theta + IR}{E_s} \times 100\ (\%)$$

$$\text{Efficiency} = \frac{E_r I \cos\theta}{E_r I \cos\theta + I^2R} \times 100\ (\%)$$

In application, it is more convenient to use conductor-to-neutral rather than conductor-to-conductor voltage and the power in one wire only, converting back again in the final results. In a single-phase circuit this means 1/2 line voltage and 1/2 total power respectively, and in three-phase circuits, $\frac{1}{\sqrt{3}}$ line voltage and 1/3 power respectively. Resistance and reactance tables will be found in various handbooks.

Conductor Spacings are fixed by experience, but for moderate voltages the distance between conductors in inches may be made 20+ voltage in kilovolts between conductors.

Conductor Stresses. The general formula usually applied to the spans assumes that the conductor takes the shape of a parabolic curve. It is ordinarily approximately true. This formula is

$$T = \frac{L^2w}{8\,d}$$

where T = total tension at center of span in pounds;
L = distance between supports (assumed same height) in feet;
w = weight of conductor in pounds per foot;
d = sag at center of span in feet.

The tension at the point of support is:

$$T' = T + wd.$$

The length of the conductor between supports is approximately

$$L' = L + \frac{8\,D^2}{3\,L}.$$

The important factor of added load due to wind, sleet, snow and temperature changes depends of course upon the climate, and for data, reference should be made to electrical engineering handbooks. This added load is taken as high as 0.75 in. of ice plus 11 lb. wind pressure per square foot of projected area (including ice).

Lightning Storms are the cause of most interruptions to the service on high-tension transmission lines. The poles, conductors, and insulators are often struck by direct discharges. The potential of the conductors may be raised an excessive amount above that of the earth by electrostatic induction from the thunder cloud. This may break down insulators, cause arcing over and grounding of the circuit, or set up waves of high potential which travel along the conductors to the power station or substation and seriously damage apparatus. **Lightning arresters** are devices for allowing excessive voltages to be quietly dissipated without interfering with the service. A common, but not universal, form of protection for a transmission line is a steel wire along the topmost points of the poles or towers and connected to the gound at frequent intervals. This is supplemented with special arresters wherever apparatus is connected to the line.

The voltage at the receiver end of the line will depend upon the inherent regulation of the line as mentioned above and the character of the load. With a low-power-factor load, it is customary to connect either a static or a synchronous condenser (a form of synchronous motor operated without load and with a field current above the normal value) to the receiver end of the line, thus neutralizing the lagging current and improving the voltage regulation by raising the power factor.

Substations are stations at the various centers of distribution where the high-tension power is transformed by "step-down" transformers to low-tension power. It is then distributed either as alternating or direct current, by any of the methods described in Art. 8.

With alternating-current distribution, the substation contains only the transformers which lower the voltage to the required value, and switchboards controlling the incoming and outgoing lines. For direct-current distribution, the alternating current is transformed to direct current either by **synchronous** converters or motor generators. A **synchronous converter** is essentially a double-current generator, a machine which, when driven as a generator, will give direct current at one end and alternating current at the other end. If the alternating-current end is connected to an alternating-current supply and operated as a motor, the machine becomes a rotary converter and direct current will be produced at the other end. The relation between the impressed alternating-current voltage and the direct-current voltage is fixed. Variation of the direct-current voltage must therefore be made with auxiliary apparatus which will alter the alternating-current voltage. The principal devices of this kind are the **synchronous booster,** a generator in series with the alternating-current supply; the **induction regulator,** a single-winding transformer with a movable secondary coil by means of which the supply of voltage can be raised or lowered in small steps; and the **split-pole field** by means of which the wave form of the machine and therefore the alternating-current voltage can be altered. A **motor generator** is a synchronous or induction motor mechanically connected directly to a direct-current generator. This method gives complete electrical separation of the direct-current from the alternating-current system, and complete control of the direct-current supply. The rotary converter, on the other hand, has a higher efficiency, especially at loads less than full load, occupies less floor space, and is somewhat lower in cost. The switchboards in substations, like those in central stations, are designed to give control over each machine and outgoing feeder. Control of the incoming high-tension current is provided by means of remote-control oil switches.

Rectifiers. Alternating currents of relatively small magnitudes are transformed to unidirectional current by several other methods. In the **mercury-arc rectifier** the operation depends upon the fact that if a glass (or steel) tube is exhausted to a low pressure, filled with mercury vapor and has a pool of mercury at one end with a metallic electrode at the other, the resistance to the flow of current is low in one direction and high in the other. Thus a valve action is obtained which in the simple case and with alternating current, would permit current to flow only every other half cycle. It is necessary to start the action by first producing a momentary arc between the mercury and the other electrode, which is usually accomplished by tipping the tube for an instant. In commercial rectifiers there are two positive electrodes (the mercury being the negative electrode) which are so connected to a transformer, suitable reactances and the direct-current load, that both halves of the wave are utilized and a continuous but pulsating unidirectional current is obtained. They are built for use on two- and three-phase circuits as well as single-phase.

Mercury-arc rectifiers are available having current capacities up to 40 or 50 amperes with glass tubes and order of 1000 kw. with steel tubes. The most common application of the smaller sizes is for charging storage batteries in electric vehicles, operating moving picture arc lamps, etc. The larger rectifiers are used in electric railway work, and high-voltage rectifiers are employed in conjunction with constant-current transformers to operate series, direct-current arc-lighting circuits (see Art. 9).

The "**Tungar rectifier**" employs the principle of the thermionic valve which is as follows: If one of two electrodes in a vacuum tube is heated to a red heat, current will flow between the electrodes if they are connected to an e.m.f. and the hot electrode is negative. When the hot electrode is positive, no current will flow. These rectifiers are limited to about 5 or 6 amperes capacity at voltages from 7.5 to 75.

Mechanical rectifiers of various types have been developed in which the connection between the alternating-current circuit and the direct-current load is reversed each half cycle by means of a commutator driven by a synchronous motor having the same number of poles as the commutator has segments, or by means of alternating-current electro-magnets and a pivoted, contact-making, polarized magnet which changes the connection every half cycle. The former type is used with X-ray apparatus and the latter for charging automobile ignition batteries which require small currents at low voltages.

Auxiliary Steam Plants. Many water-power transmission systems have auxiliary steam plants connected to the system. These plants, in addition to assisting the water-power plant at times of low water, are kept under a low head of steam, so that they will be available on short notice in case of failure of the transmission line.

8. Power Distribution

Distribution Systems. Electrical power is distributed to consumers in two forms, direct current and alternating current. **Direct current** systems have been considered the most suitable for general public service purposes in closely settled districts where the distances involved are short. Direct current motors are somewhat more flexible in application because of the more convenient speed control. With a direct current system, continuity of service to customers in case of accidental interruption of the supply for short periods can be more readily maintained because of the applicability of large "stand-by" storage batteries which can be kept constantly connected to the system and automatically discharge into the system in case of failure of the supply. But the great increase in the use of electricity in recent years, and the development of equipment and methods for the distribution and utilization of alternating current power as effectively as direct current power, together with development of applications of electricity where alternating current is the more suitable, are resulting in direct current being gradually supplanted by **alternating current** even in congested districts because of the great economy and flexibility of distribution of the latter. Even in congested districts of the largest cities, electricity is being furnished in the alternating current form — being distributed at 2200 to 6600 volts by means of underground cables in the streets and stepped down to 110 or 220 volts at convenient points for service to consumers. **Alternating-current** systems are always used where scattered districts are served because the distribution can be made at a higher voltage and thus greatly reduce the investment in copper. The power is usually distributed at 2200 to 6600 volts and reduced to a lower voltage at the consumers' premises by small transformers called distribution transformers at each building or group of buildings. Direct current is distributed by series two-wire and three-wire systems. Alternating current is distributed by the series single-phase; two- and three-wire single-phase; three- and four-wire two-phase; and three- and four-wire three-phase systems.

Series Systems. In both direct- and alternating-current systems all apparatus is connected in series and the same current passes through all. This system is used only for street lighting, where the smaller investment in copper required is a great advantage. The current is kept constant by automatic apparatus in the station and the voltage varies with the number of lamps in the circuit. Open-arc lamps require 45 to 50 volts each, inclosed-arc lamps 75 to 80 volts each, and series incandescent lamps 15 to 30 volts each. Forty to one hundred lamps may be burned on one circuit, requiring a total of 2000 to 6000 volts. Provision is made in each lamp for automatically keeping the circuit closed when the electrodes of an arc lamp become too short or the filament of an incandescent lamp breaks. Incandescent lamps of different current ratings (and therefore different candlepower ratings) are used on the same alternating-current circuit by employing specially designed individual current transformers.

Two- and Three-wire Systems. The series system is not feasible for general distribution because lamps cannot be turned on and off individually without affecting the others on the circuit, series motors are not suitable for general use, and the high voltage of the series system would be dangerous to life and property if carried into buildings. Distribution to and within buildings is therefore made with constant-potential multiple circuits, either two-wire or three-wire.

In a Two-wire Direct-current System, pairs of cables (feeders) radiate from the central station, each pair dividing and subdividing into other pairs (mains) running to the premises of the consumers. The voltage varies from 100 to 125 volts and each lamp and motor is connected directly across these lines, each being independent of the others. **Single-phase alternating current** is similarly distributed at high voltage to transformers just outside of the consumers' premises and from the transformers secondary distribution is furnished at 100 to 125 volts. The two-wire system requires a large copper investment to keep the potential drop within reasonable limits. Excessive drop in potential due to too high resistance in the circuit means not only lost power but flickering of incandescent lamps, since the candlepower of the latter changes 4 to 6% with 1% change in voltage.

The Three-wire System was developed to decrease the amount of copper required. Two direct-current generators are connected in series and feeders are run from each outer terminal of the two machines and the common connection between them. The distribution to the consumers' premises is made with three instead of two feeders and mains. The lighting load is divided, half being connected between each outer wire and the middle or neutral wire. Motors are connected to the outer wires in order to decrease the flickering of lamps which fluctuating motor loads produce. Furthermore, 250-volt motors are somewhat cheaper than 125-volt motors. In the three-wire system the advantages of the two-wire system are retained and at the same time only 38% as much copper is required for the same percentage loss. By keeping the load fairly evenly divided, the neutral feeders and mains may be made smaller and an additional saving effected.

Secondary distribution on single-phase alternating-current systems is made by the same method by winding the secondary of transformers for 200 to 250 volts, and connecting the third wire to the center of the winding.

Two-phase Systems. The two-phases system of distributing alternating current is very generally used where the load is both lamps and motors, the lamps being connected across each phase separately and motors across both phases together. Single-phase motors are not practicable except in small sizes, hence the two-phase system combines the advantages of a polyphase system for motors with those of the two- and three-wire systems for lamps. The current is generated with two-phase alternators and is distributed over four or three wires. In the four-wire system, each phase is distributed independently through two wires. In the three-wire system, one wire of each phase is common.

Three-phase Systems of distribution are used only where the load is principally motors and where the power has to be transmitted considerable distances. About 25% less copper is required than for two-phase, four-wire, systems under the same conditions. The current is generated in three-phase alternators, usually at 25 or 60 cycles, and may be distributed directly to consumers up to 440 volts or transformed to a higher voltage for transmission to distributing centers, or directly to the consumer's premises, where it is transformed to a low voltage. The primary distribution is always three-wire, but the secondary distribution may be made with four wires where there is a considerable lighting load. In such cases, lamps are connected between each of the three main wires and the neutral, while motors are connected to the three main wires.

Distribution Methods. The power is delivered to the consumers' premises from the power plant or substation through feeders and mains carried on poles or placed underground. The former method is very much cheaper and is the general practice outside of large cities. In congested districts, the crowded conditions of the streets, the unsightly appearance of pole lines, and the handicapping of firemen have caused the development of the underground method.

Overhead Distribution is made on wood, built-up steel, cast-iron, or reinforced-concrete poles 30 to 60 ft. high and spaced 50 to 200 ft. apart, depending upon the local conditions. The wires are carried on insulators attached to wood cross-arms. Glass insulators are used for low-voltage circuits and glass or porcelain insulators for high-voltage circuits. Wire with weather-proof covering is commonly used. The cost of overhead distribution depends upon the kind, quality, height, and spacing of the poles, upon the number of circuits, condition of the streets, and so forth. It varies from $750 to $2000 per mile exclusive of the cost of the wire.

Underground Distribution is largely carried out in this country by the conduit system—that is, tubes of various kinds, 3 to 5 in. diameter, are laid in the ground and insulated conductors (cables) are pulled into them. In some special cases such as park and parkway lighting, armored cable is laid directly in the ground.

Conduits are made of wood impregnated with creosote, sheet-iron pipe lined with cement, fiber, and vitrified clay. Wood conduit and sometimes fiber conduit is laid directly in the soil, but the other conduits mentioned are usually encased in concrete or cement mortar, a number of conduits being laid side by side. The conduit materials most in use are vitrified clay and fiber tubing. Clay conduits are made in single-duct pieces about 18 in. long, and in multiduct pieces, with any number of round or square holes (from two to sixteen), about 30 in. long. Fiber conduits are made up in pipes about 5 ft. long of treated wood pulp. Where used for multiple-duct conduits they are embedded in and surrounded by a casing of concrete. On account of the greater strength and smaller number of joints, the multiduct clay conduit is frequently laid directly in the ground without cement encasement, the joints being wrapped in burlap, and well asphalted. Manholes built of brick or concrete are provided every 100 to 500 ft. where service connections may be made, cables pulled in and out for repairs, and splices made. If distribution is made at high voltage, the step-down distribution transformers are installed in these manholes.

Interior Wiring. The system used for the distribution is usually continued through the building. The materials and methods used in wiring buildings on which insurance is carried must conform to the rules of the National Board of Fire Underwriters. The building departments of all large cities also have rules which must be observed. The standard methods are open or cleat work, molding, rigid steel conduit, flexible steel conduit, non-metallic tubing and armored cable.

In Cleat Work, the wires are exposed and supported on walls and ceilings by porcelain cleats or knobs. Porcelain tubes are used where the wires pass through walls or ceilings. This class of wiring is the cheapest but is entirely satisfactory when the work is properly done and appearance is of secondary importance.

In Molding Work the wires are embedded in grooved strips of wood or metal, and covered with a thin strip of the same material. Special porcelain fittings are used where apparatus is connected. (Wood molding is no longer allowed in some of the larger cities.)

Rigid Steel Conduit Work consists of special soft-steel pipe run throughout the building and through which the wires are pulled. Steel junction boxes are provided through which the wires are pulled into the pipe and defective ones pulled out. This is the most expensive form of wiring, in first cost, but it completely protects the wires from mechanical injury, reduces the fire risk on the building to a minimum, and the maintenance cost is small, for old and defective wires can be readily replaced without injury to the conduit.

Flexible Steel Conduit is a tubing of steel ribbon wound spirally in such a manner as to produce a very flexible steel tubing which is reasonably tight but not water-tight. Steel outlet boxes and junction boxes are used as with the rigid conduit. Although this is a cheaper construction than the rigid pipe system it eliminates the unsafe features of molding work. It is particularly applicable in wiring old buildings.

Non-Metallic Tubing is a stiff though sufficiently flexible hose of fiber composition which is a cheap substitute for the metal conduit. It is obviously not waterproof.

Armored Cable is similar to flexible steel conduit with the conductors included. The conductors are covered with suitable insulation and the steel ribbon wound directly over this insulation. This form of wiring is rapidly replacing the flexible conduit method for old buildings because conduit and wire are installed together.

Wires and Cables. There are three general classes of wires and cables used in electrical work: weatherproof, rubber insulated, and lead-covered.

Weather-proof Wire is insulated with two or three layers of cotton braid impregnated with insulating compounds. It is used only for exterior wiring, principally for overhead distribution. It is the cheapest form of insulation, but also the least efficient. Although sufficient protection is provided for pole-line purposes where the wires are supported on insulators, it is entirely inadequate for interior wiring.

Rubber-covered Wire forms the greatest percentage of wire used for electrical purposes. It is used almost exclusively for wiring buildings and extensively for high-tension cables. There are three principal grades of rubber insulation: National Electric Code Standard, 30% Para, and Navy Standard All grades consist of rubber, ground and mixed with various kinds of powdered mineral matter together with a certain amount of sulfur. After being applied to the wire by running through dies, it is vulcanized by subjection to a high temperature. The principal difference in the grades is in the quality and amount of rubber used. National electric Code insulation is made with various kinds of old and reclaimed rubber and rubber substitutes but must contain about 20% Para rubber. Before it can be used, it must be approved by the National Board of Fire Underwriters. Thirty per cent Para insulation contains at least 30% of new Para rubber gum. It is more durable than the National Electric Code wire and is used in all first-class interior-wiring work. Navy Standard insulation contains not less than 40% of new Para gum. It is used by the government on all battleships and is the highest grade of wire regularly manufactured.

Lead-covered Wires and Cables include various forms of insulation and are made up in a great variety of ways to suit any requirement for insulation and mechanical protection. Lead-covered cables are used where protection from mechanical or chemical injury is essential, as in underground distribution and under water. One, two, or three conductors may be in one cable. The conductors in two- or three-conductor cables may be side by side or one inside the other (concentric). The standard insulations are 30% rubber, paper, and varnished cambric. Paper insulation is applied in a narrow strip wound spirally on the copper to any desired thickness and saturated with suitable oil to increase the insulation and render the cable flexible. Varnished-cambric insulation is applied in the same manner. The capacity of wires and cables is fixed by the temperature at which the life of the insulation will be affected

The accompanying table shows the carrying capacity of various kinds of wires and cables based on safe temperature limits. The second column of this table gives capacity for wire in still air, at 125° F. The third and fifth columns give the capacities allowed by the Fire Insurance Underwriters (National Electric Code). The limits given for rubber are those beyond which deterioration (when exposed to air) may begin. The seventh column gives capacity at 125° F.; the eighth column gives capacity at 175° F. for still air at about 70° F. These data apply substantially to the conditions existing in a standard single-duct, underground conduit. If the conduit is multiduct with a loaded cable in each duct, the capacity of each cable will be reduced from 5 to 25 or 30%, depending on the number and arrangement of ducts and cables.

The temperature of paper cables will be about 10% higher than that of rubber cables for the same current and thickness of coverings. Paper cables should not be operated over 190° F. and rubber cables over 160° F. Cables immersed in water will carry, for the same temperature rise, 40 to 50% more current than indicated in the table, and if buried in moist earth, 10 to 25% more current.

The capacity of each conductor in a two-conductor, non-concentric cable is about 85% of that of a single-conductor cable; in a two-conductor, concentric cable, 80%; in a three-conductor, non-concentric cable, 75%; and in a three-conductor, concentric cable, 60%.

The voltage drops given in the table are per 1000 ft. of wire (500 ft. of two-wire cir-

Current Capacity of Wires and Cables, Amperes

Size, A. W. G.*	Bare copper wire	Interior wiring				Low-tension underground cable, lead-covered	
		"Code" rubber insulation	Corresponding drop, volts per 1000 ft. (approx.)	Varnished cambric insulation	Corresponding drop, volts per 1000 ft. (approx.)	Rubber insulation	Paper or cambric insulation
14	12	15	38.5	18	46.5	....	....
12	17	20	32.5	25	40.5	....	....
10	24	25	25.5	30	30.6	20	24
8	42	35	22.5	40	25.6	30	36
6	59	50	20.2	60	24.2	50	60
4	83	70	17.7	85	21.5	75	95
2	118	90	14.3	110	17.5	110	130
0	170	125	12.5	150	15.0	160	200
00	202	150	11.9	180	14.3	190	240
000	240	175	11.0	210	13.2	235	285
0000	286	225	11.3	270	13.5	280	340
300 000	373	275	9.9	330	11.9	370	450
400 000	463	325	8.8	390	10.5	460	560
500 000	549	400	8.6	480	10.4	550	660
600 000	631	450	8.1	540	9.7	625	760
700 000	708	500	7.7	600	9.2	700	860
800 000	781	550	7.4	660	8.9	770	960
1000 000	922	650	7.0	780	8.4	900	1150

* Solid wire assumed up to and including No. 0000 although in practice stranded wire is usually used in sizes over No. 6. Sizes above No. 0000 are assumed stranded and are expressed in circular mils. Stranded wire has about 2% greater resistance than same size solid wire.

cuit) and are calculated from the resistance at 25° C. The copper temperature will be actually higher than 25° C. by an amount which will depend upon conditions so that the actual drop may be as much as 5% greater than these figures.

Choice of Wiring. The selection of the type of wiring is largely a matter of cost from among those allowed by the Underwriters for a particular type of building (certain types are not permitted under certain conditions). The rigid-conduit, flexible-conduit and armored-cable types offer about equal protection from fire due to an accidental arc in circuits of moderate size. In the larger sizes where an arc would be heavier, the rigid conduit is preferable. The relative cost varies, of course, with the conditions but the following gives a general idea of the range for new buildings * :

Rigid steel conduit, exposed 125%
Rigid steel conduit, concealed (i.e., installed during construction of building) 100%
Armored cable 65%
Open wiring 50%
Knob or tube wiring 40%

Before the size of wire can be decided the current loads must be determined. In lighting systems all branch circuits are limited to 660 watts by the Under-

* "Interior Wiring," A. L. Cook, p. 244. John Wiley & Sons, Inc.

writers. Feeders and mains are calculated on a basis of 600 watts for each branch circuit—the corresponding current being the total watts in each case divided by the nominal voltage (usually 120 volts). In the case of power systems, each motor over one-quarter horsepower is supplied by a separate branch circuit, the current being taken as the full load running current as marked on the nameplate. The diversity in the use of the several motors (over 5 or 6) may be taken advantage of in estimating feeder and main currents and may be taken as low as 50% of the total of the nameplate currents where there are more than 10 or 12 motors.

The best size of wire is determined for each part of the circuit between the source of supply (entrance to building if from public utility service or switchboard if private plant) and the consuming equipment (lamps, motors, etc.), i.e., feeders, mains and branches, respectively. There are two limitations on the minimum size: temperature rise and voltage drop. The former is automatically provided for by the Underwriters' rules which limit the current for each size of wire (see interior wiring section of table on p. 398) and the latter is determined by calculation (see wiring table, p. 399). The voltage drop is usually limited to about 5% at the lamp in the case of lighting systems and 10% at the motor in power systems (about one-quarter of this in branches, one-quarter in mains and one-half in feeders). If the size which would be allowed by voltage drop is less than that allowed by the Underwriters for that current, the latter size should be used.

The Sale of Power. Systems of selling power are based on the direct cost of manufacturing; on the investment in power-station and distribution equipment; and on the reserve capacity of the plant required to meet unusual demands of the consumer. Among the numerous systems in use, the principal ones are **flat rate, straight meter basis**, and **maximum demand**. In the **flat-rate** method a fixed price, depending

Wiring Table

Distances in feet which power at 100 volts may be transmitted with various sizes of wire at various percentages of loss.

Size, B. & S. gage, per cent loss			Current in amperes							
2	5	10	2	4	8	12	20	40	60	100
......	......	0000	48 900	24 450	12 225	8150	4890	2445	1630	978
......	......	000	38 750	19 400	9 700	6460	3875	1940	1290	775
......	......	00	30 800	15 400	7 700	5130	3080	1540	1025	616
......	0000	0	24 450	12 220	6 110	4075	2445	1220	815	489
......	000	1	19 450	9 720	4 860	3240	1945	972	648	389
......	00	2	15 450	7 720	3 860	2575	1545	772	515	309
......	0	3	12 250	6 120	3 060	2040	1225	612	408	245
0000	1	4	9 700	4 850	2 425	1615	970	485	323	194
000	2	5	7 700	3 850	1 925	1280	770	385	257	154
00	3	6	6 100	3 040	1 520	1015	610	304	203	122
0	4	7	4 850	2 420	1 210	810	485	242	161	97
1	5	8	3 845	1 920	960	640	385	192	128	76
2	6	9	3 050	1 520	760	507	305	152	102	61
3	7	10	2 420	1 210	605	403	242	121	81	48
4	8	11	1 920	960	480	320	192	96	64	38
5	9	12	1 520	760	380	253	152	76	51	30
6	10	13	1 210	605	302	200	121	60	40	24
8	12	15	760	380	190	127	76	38	25	15
10	14	......	480	240	120	80	48	24	16	10
12	16	......	302	151	75	50	30	15	10	6

upon the size of the installation, is charged per month, irrespective of the hours of use. This is used only where power is cheap or the loads small. The **meter system** is the usual system employed by central stations. Various discount methods, based on the total kilowatt-hour consumption per month and on the size of the installation, are used. The **maximum demand** system makes the price paid depend upon the maximum power taken during any given period, as for instance during any 5-minute period in 24 hours. This system is extensively used where large quantities of energy are being consumed and is designed to take care of the investment in equipment not used except for a short time during each day.

Meters. Where the charge for electricity is based on the energy consumed, the energy is measured with watt-hour meters (see p. 371). If the maximum demand is involved in the charge it may be estimated or actually measured. The former practice is used extensively for small consumers because the expense of metering the maximum demand is not justifiable. Various systems are used, such as total number of active rooms, floor area or number of outlets in the case of a residence and total connected load (from nameplate data of connected motors, etc.) or trial test in the case of an industrial plant.

Maximum-demand meters are, however, used for large customers. They function with the watt-hour meter and may be combined with the latter or in a separate case but electrically connected with the watt-hour meter. Some types merely indicate, by means of a pointer, the largest amount of energy which has been used in the fixed interval agreed upon (usually 15 or 30 minutes) since the pointer was previously set back. When the watt-hour meter is read, this indication is also noted and the pointer reset at zero. Other types show a graphic record on a continuous tape or chart on which the demand is shown continuously for successive intervals and from which the maximum is determined by observation.

In the rate for a large industrial plant, there is sometimes included a factor which depends upon the power factor since low power factor is objectionable to the supply company because generation and transmission capacity is used without a corresponding increase in energy consumption. Meters which record the volt-amperes (as distinguished from watts) are used for this purpose. Sometimes special rates for certain applications are made by the power company if the energy is taken during certain " off-peak " hours when the plant equipment is not loaded—the " off-peak " energy being measured by a separate watt-hour meter which is cut in and out of circuit by means of a clock.

9. Electric Lighting and Illumination

Lighting by electricity is accomplished by three methods: by the arc between two carbon pencils automatically kept a short distance apart, by the passage of current through rarefied gases, and by the heating of a refractory material to incandescence.

The intensity of a source of light is measured in terms of a unit source called a **candlepower.** The four principal standard sources of light are (a) the British standard candle, a spermaceti candle, (b) the English Harcourt lamp, which burns pentane vapor mixed with air in an Argand burner, (c) the French Carcel lamp, which burns colza oil with a wick, and (d) the German amylacetate lamp, which burns amylacetate with a wick. The British candle is nominally the standard in this country but the amylacetate lamp is probably the most generally used standard lamp. All of these standards are of course arbitrary, and are made and used in strict accordance with very detailed specifications in order to insure reliable and reproducible results.

Illuminants are measured with a **photometer,** an instrument which gives the value of the unknown source in terms of a standard lamp. Because of the much greater convenience in commercial work, this standard is usually an electric incandescent lamp that has been carefully standardized against one or more of the above primary standards.

Rating of Illuminants. The theoretical standard of intensity of light, the candlepower, is a point of light giving a candlepower in each direction. **Mean horizontal candlepower** is the average candlepower in all directions in a horizontal plane through the center of the source. **Mean spherical candlepower** is the average candlepower in all directions from the center of the source. **Mean hemispherical candlepower** is the average candlepower in all directions in a hemisphere with the source as the center.

Illuminants are now being rated in terms of the lumen which is the unit of light radiation. A **lumen** is equal to the light flux emitted by a source of one candlepower in unit solid angle. A source of one candlepower emits a total light flux of 4π lumens.

All commercial illuminants have a more or less irregular distribution curve, that is, the candlepower is greater in some directions than in others, and in making comparisons between different illuminants it is therefore necessary to be specific. Formerly, it was customary to rate electric lamps on a basis of the maximum candlepower obtainable. Later incandescent lamps were rated on a mean horizontal candlepower basis and other illuminants on a mean spherical or mean hemispherical candlepower basis. The present practice, however, is to rate all electric illuminants on the basis of the nominal watts consumed. This applies particularly to incandescent lamps which are all labeled and described on this basis, the candlepower (or lumens) being omitted. This practice is the result of the wide use of reflectors, each form of which gives a different apparent candlepower.

Arc Lamps are used for lighting streets, large interiors, and similar places where the illumination must be general but not necessarily high. Arc lamps are, however, being rapidly superseded by large incandenscent lamps even for these applications.

In the **open arc,** the earliest form of electric light, an arc is maintained between solid carbon rods in the open air. They are usually operated 40 to 100 in series, on both direct and alternating constant-current circuits. The standard sizes are 6.6- and 9.6-ampere lamps, requiring 45 to 50 volts. Although this type is relatively very efficient as a source of light, it has been largely superseded by the **enclosed-arc** lamp, a lamp which is less efficient but also much less expensive to operate and gives a much better distribution of light. The arc is surrounded by a small, close-fitting glass globe which excludes the air and retards the consumption of the carbons, thereby increasing their life eight to ten times. These lamps are operated on both direct and alternating current, but more often on multiple circuits at 110 volts than on series circuits. The voltage per lamp is 75 to 80 and the current varies from 3.5 to 7.5 amperes. The **magnetite-arc** lamp is a direct-current open arc in which the electrodes are metallic copper and an oxide of iron. Its special features are long life of electrodes (about 150 hours), high efficiency, and light distribution particularly suited for street lighting. The **flaming-arc** lamp is an alternating and direct-current open arc with small-diameter carbons impregnated with sodium or other similar salts which enormously increase the light emitted. They are the highest candlepower units in general use, 2500 to 4000 cp. being given in the maximum direction, and are extensively used for spectacular and show purpose.

Lamps with Luminous Rarefied Gases are not used for general illumination but only where the special characteristics of the light are advantageous.

The Cooper Hewitt lamp consists of an arc in a tube 2 to 4 ft. long, in which the cathode is metallic mercury and the arc passes through rarefied mercury vapor. The light is not due to the temperature of the arc, as in the ordinary arc lamps, but to the luminescence of the vapor. The light is therefore green in color, red and yellow being almost entirely absent from the spectrum. The actinic value of the light is high, hence it is used for photographing and blueprinting. It is also said to be less fatiguing to the eyes than white light, and is therefore desirable in drafting rooms. On account of the unnatural greenish tint of the light it is not suitable for general use. When a quartz tube is used instead of glass, the lamp can be operated at a much higher current density, and a higher efficiency as well as a whiter light is obtained. The Cooper Hewitt lamp operates on either direct or alternating current.

The various forms of tube light employ the principle of the Geissler tube, that is, a continuous and silent electrical discharge takes place through gas at a low pressure in a sealed glass tube. The tubes are operated from small high-tension transformers. The color depends upon the kind of gas, neon giving the familiar pink light and nitrogen, a white light.

Incandescent Lamps are almost exclusively used for interior illumination where relatively high and uniformly distributed illumination is required.

An Incandescent Lamp is an illuminant in which a refractory material in the form of a long thin wire in a glass bulb is heated to incandescence by the passage of electric current. The types are distinguished by the material of which the filament is made, the carbon filament being the oldest. The others in the order of development are metallized (specially treated) carbon, tantalum, and tungsten. The essential difference between these is in the temperature at which the materials may be operated, the temperature of carbon being the lowest and that of tungsten the highest. For the same amount of light, thinner filaments may be used as the permissible temperature increases, and therefore less power is required. Where the power required per candle-power in carbon lamps is about 3.1 watts, that for gas-filled tungsten lamps is 1 watt. or less.

The tungsten lamp has now about completely supplanted the other kinds. It is made in two types. In one type (known commercially as type " B ") the filament operates in a vacuum while in the other type the bulb is filled with nitrogen, argon or other inert gas at about atmospheric pressure (" type C "). The latter is operated at a slightly higher temperature and therefore a slightly higher efficiency than the vacuum type. The gas-filled type is made principally in the larger sizes where the increase in efficiency is most pronounced.

The accompanying table shows the approximate lumens, power consumption in watts per lumen, and cost of tungsten lamps. The sizes given are those most commonly used. The prices are approximate for not less than 5000 lamps purchased per year.

Luminous Output, Efficiency, and Cost of Tungsten (Mazda) Lamps

Nominal watts	Vacuum (Type B)			Gas-filled (Type C)		
	Nominal lumens	Lumens per watt	Approximate cost	Nominal lumens	Lumens per watt	Approximate cost
15	126	8.4	$0.20	....	....	....
25	235	9.4	0.20	....	....	....
40	396	9.9	0.20	....	....	....
50	....	....	....	515	10.3	$0.22
60	....	....	....	666	11.1	0.22
100	....	....	....	1320	13.2	0.35

Operation of Incandescent Lamps. Incandescent lamps are used on series as well as multiple circuits, but series lamps are a small per cent of the total. Series lamps have short thick filaments, requiring 15 to 30 volts, and are often connected in the same series circuit with arc lamps and placed in side streets, alleys, and similar places where high illumination is not required. Lamps of different sizes, that is, different current ratings, can be operated from the same series circuit by employing a special type of current transformer. The continuity of the series circuit is maintained when a filament burns out by means of an automatic short-circuiting device in the lamp socket.

The life and candle-hours of incandescent lamps are seriously affected by changes in voltage for although raising the voltage on a lamp increases its efficiency, the life is reduced at a much greater rate. The most economical operating voltage is therefore that where a balance is obtained between the decreased cost of energy consumption and the increased cost of lamp replacements. Fluctuating voltage also produces disagreeable flickering, consequently it is essential that the system be well regulated if satisfactory service is to be obtained. The following table shows the effect of change in voltage on the candlepower and life of tungsten lamps (vacuum type).

Effect of Voltage Variation on Tungsten Incandescent Lamps

Per cent normal voltage	Per cent normal lumens	Per cent normal life *
90	68	435
95	83	203
100	100	100
105	120	50
110	142	26

* Both vacuum and gas-filled types

The rating, efficiency and approximate life of the principal electric illuminants are shown in the following table.

Rating, Efficiency and Life of Electric Illuminants

Kind and type	Rating	Lumens per watt	Approximate life, hours
Multiple Burning:			
Incandescent, carbon..........	10– 60 watts	2.5– 3.5	500
Incandescent, tungsten (vacuum)	10– 40 watts	7.2–10.0	1000
Incandescent, tungsten (gas filled)	50–1000 watts	10.3–21.0	1000
Cooper Hewitt (mercury vapor).	500 scp.* (rated max.)	15.7	†
Moore Type, tube light........	8 scp. per foot	5.0	†
Enclosed arc, d. c.............	150–250 scp.	4.2	100–125
Series Burning:			
Incandescent, tungsten gas filled,			
6.6 amp...	600– 6 000 lumens	13.8–17.8	1350
7.5 amp...	600– 6 000 lumens	13.9–17.8	1350
15 amp...	4 000 lumens	17.7	1350
20 amp...	6000–25 000 lumens	18.7–20.4	1350
Open arc, carbon, 6.6 amp...	260 scp.	9.3	8
Open arc, carbon, 9.6 amp...	410 scp.	11.5	8
Open arc, magnetite, 4.0 amp...	250 scp.	9.7	150
Open arc, magnetite, 6.6 amp...	720 scp.	18.0	100
Open arc, flaming.............	700–1000 scp.	25–18	15–100
Enclosed arc, a. c., 6.6 amp...	130 scp.	3.8	100–125
Enclosed arc, a. c., 7.5 amp...	155 scp.	4.1	100–125
Enclosed arc, d. c., 6.6 amp...	260 scp.	6.6	100–125

* scp. = mean spherical candlepower.
† Manufacturers claim several thousand hours.

The Relative Cost of Lighting depends not only on the amount of light produced per watt of power consumed, but also upon the cost of power, hours of burning, initial investment, repairs, life, and cost of maintenance (cleaning glassware, replacing incandescent lamps, renewing carbons, glowers, and so forth). Hence the relative standing of the various illuminants will vary with conditions applying to each case, and no general statement can be made that will be universally applicable. For instance, where the cost of energy is low or the lamps are burned only a small fraction of the time, it might cost less to use lamps of relatively low efficiency.

Illumination. Illumination is expressed in terms of the **foot-candle**, which is the illumination produced on a surface normal to, and one foot from, a source of one candlepower. The intensity of illumination varies inversely as the square of the distance from the source.

Illumination of Interiors is usually effected by direct lighting, indirect, and semi-indirect lighting. In direct lighting the illumination is received directly from the light sources; in the indirect method the light sources are concealed from direct view and the light is reflected into the room from a highly reflecting surface, usually the ceiling. In the semi-indirect method, a part of the illumination is obtained by reflection and part by direct lighting usually by means of an inverted, translucent reflector or bowl. The two latter methods have the physiological advantage of removing the light sources from the field of vision of the eye, but they usually require somewhat more energy for the same amount of illumination. In direct lighting, the intrinsic brilliancy of the sources is (or should be) reduced by enclosing the lamp or by some other means. This entails a loss of the order of 15%.

The amount of illumination which is considered desirable in various applications is steadily increasing as the value of better artificial lighting becomes more appreciated. The following tabulation lists the values of the illumination considered desirable at the present time in a few typical applications.* The ranges given provide for the different conditions, such as the kind of room in the case of a building or the kind of work in the case of a factory. For example, in a residence the minimum value given is sufficient for halls and similar places but the highest value is for rooms where plenty of light is desirable, such as the living room. The Illuminating Engineering Society has prepared complete, detailed codes for lighting factories and school buildings,

The candlepower required in the lamps can be readily computed for a given case if the distance to the lamps and the curve of distribution of light around the lamps are known together with the approximate reflection factors of the ceiling and walls.

Typical Illumination Values

Class of service	Illumination, foot-candles	Class of service	Illumination, foot-candles
Auditoriums	2–4	Stores, miscellaneous	4–8
Theatres	3–10	Stores, clothing	6–12
Churches	3–6	Drafting	10–20
Reading rooms	3–10	Engraving	10–50
Residences	0.5–8	Machine shops	4–50
Desk lighting	8–10	Foundries	3–12

Street Lighting. The best method of illuminating streets is that which will produce the most uniform illumination for the same average illumination. This lies between the two extremes, large units placed far apart and smaller units placed close together. Cost considerations have dictated the very general use of the former method, the lamps being of such a size and so spaced that the minimum illumination midway between lamps will be at least that of average moonlight or about 0.02 foot-candle. Obviously, the amount of illumination may be as much greater than this minimum as cost considerations will permit. The prices obtained for street lighting by electric companies vary from $50 to $125 per lamp per year, and the cost per mile of street will range from $500 per year in villages to $5000 for the important avenues in cities. The cost varies greatly even for the same illumination because of the varying conditions such as hours of burning, whether all night or only a part, and whether every night or on a "moonlight schedule"; type of construction, whether ornamental iron poles or wood poles, and overhead or underground distribution; cost of producing energy and so forth.

Reflectors. Efficient artificial illumination requires that as much as possible of the light emitted from the source fall upon the surface or objects to be

* From "Light, Photometry and Illuminating Engineering," W. E. Barrows, McGraw-Hill Book Co., pp. 168–177, where nearly 300 applications are listed.

illuminated. Care should be exercised to select the proper type and size of lamps for the particular case and to locate them where they will be most effective. Reflectors placed back of the lamps can be advantageously used to reflect the light from directions where it is useless to the object to be illuminated. Reflectors are now scientifically designed according to the laws of reflection and refraction. They may be obtained with various forms of distribution curves, either extreme concentration, moderate concentration, or wide diffusion.

10. Batteries

Primary Batteries. If two metals are placed in a liquid, an **e. m. f.** is generated at the plates, the value of which will depend upon the metals and the liquid. Such a combination constitutes a **voltaic couple.** Those couples which produce the greatest potentials are used to a large extent as sources of electricity for telegraphing, signals, telephones, call-bell systems, and other purposes where very small amounts of power are required. They are called **primary cells,** and a number connected together form a primary battery. The various kinds may be divided into two classes: closed circuit and open circuit. Batteries in the first class always use a fluid electrolyte. Those in the second class use either fluid or nonfluid electrolyte.

Closed-circuit Batteries are those which can be discharged continuously. The most common battery of this class is the gravity battery, with electrodes of copper and zinc and a copper-sulfate electrolyte. Its construction is such that it operates best when kept connected to a closed circuit. The voltage of the gravity battery and the similar Daniell battery is about 1 volt per cell, but the current capacity is only a few hundredths of an ampere. The gravity battery is used extensively for telegraph and railway signal work. **Potassium-bichromate** batteries use carbon and zinc electrodes with a solution of bichromate of potash. These cells have a potential of about 2 volts each and a comparatively large current capacity. Frequent renewal of the elements and electrolyte is necessary. The **Edison-Lelande** battery consists of zinc and copper electrodes and caustic-soda electrolyte. Its potential is about 1 volt per cell and the largest size will deliver about 7 amperes for nearly 100 hours on one charge of material.

Open-circuit Batteries are intended to be used only on intermittent service, such as call bells and short telephone lines. Very large quantities are used for pocket flash lamps and radio sets. Practically all batteries in this class are carbon and zinc in a sal-ammoniac solution. The **e. m. f.** per cell is about 1.5 volts. **Dry batteries** are carbon-zinc-sal-ammoniac batteries made nonspillable by filling the space between the electrodes with sawdust or similar material which has been saturated with sal-ammoniac solution. They cannot be renewed when exhausted.

Standard Cells are cells used as standards of **e. m. f.** The two types in most general use are the Clark cell and the Weston cell. The **Clark cell** is made with zinc and mercury electrodes and a solution of zinc and mercurous sulfate. Its potential when made up in accordance with certain conditions is 1.434 volts at 15° C. The **Weston cell** is made with cadmium instead of zinc and the potential is 1.0183 volts at 20° C. Both will remain constant for years if no appreciable current is drawn. The Weston cell has a much lower temperature coefficient and a longer life than the Clark cell.

Storage Batteries are those in which the chemical process is reversible, and which, after being discharged, can be restored to the original chemical condition by sending current through them in the opposite direction. In the best-known class, the elements or electrodes are metallic sponge lead (negative plate) and lead peroxide (positive plate) in a sulfuric-acid electrolyte. While discharging, both elements partially change to lead sulfate and when current is passed through the cell in the opposite direction (charging) they are converted back again to the original state. The two important methods of making the plates are the Planté method and the pasted or Faure method. In the

Planté process the lead peroxide and the sponge lead are formed by chemical or electrochemical means directly on lead plates from the plates themselves. In the **Faure process** various oxides of lead are mechanically applied to the plates and then reduced electrochemically to sponge lead and lead peroxide, respectively. The Planté type batteries are usually used for stationary work where weight and volume are relatively unimportant. For motor vehicles, train lighting, and similar purposes where the maximum capacity per unit of weight and volume is essential, the pasted or Faure type is used.

The Capacity of a battery is measured in ampere-hours. A capacity of one ampere-hour means the ability to deliver one ampere continuously for one hour. The capacity of a battery depends upon the amount of active material; the area of the plates; the amount, temperature, and specific gravity of the electrolyte; and the current at which discharge is taking place. In order to get as great capacity as possible per unit volume, the effective surface of the plates is increased by cutting or casting grooves and ridges in the plates and by laminating. Batteries of any capacity are obtained by putting additional plates in the same cells and connecting all positive plates together and all negative plates together. The volts per cell, however, are the same irrespective of the size of the plates or the number connected in parallel, the total voltage required being obtained by connecting cells in series.

The capacity of a battery will vary with the temperature, increasing as the temperature of the electrolyte rises. The change per degree varies with the type of battery. The capacity is greater with currents below the normal rate and smaller with currents above the normal rate. Lyndon gives the following approximate figures for the capacity of lead batteries at high rates: 200% normal rate, 75% normal capacity; 300% normal rate, 58% normal capacity; 400% normal rate, 50% normal capacity.

Rating. The standard method of rating a storage battery is the current which it will give continuously during 8 hours for stationary batteries, and 5 hours for vehicle batteries. Automobile lighting and starting batteries are rated on two bases. One, the lighting rating, is the ampere-hour capacity obtained when discharging at 5 amperes; the other, the starting rating, is the current the battery will deliver continuously for 20 minutes.

Voltage. During discharge at normal rate the voltage of a lead battery falls rapidly at first from about 2.10 or 2.15 to 2.0 volts per cell, and then drops slowly to 1.90 or 1.85 volts. Beyond this point the voltage drops rapidly. In order to maintain constant voltage on a load being supplied by a battery **end cell switches** are often used by means of which an additional cell is cut in when the total voltage drops 2 volts.

Efficiency. There are two efficiencies of storage batteries, the watt-hour and the ampere-hour. **Watt-hour efficiency** is the ratio of the total energy delivered during discharge to the total energy received during charge. **Ampere-hour efficiency** is the ratio of the product of amperes and hours during discharge to that of amperes and hours during charge. The watt-hour efficiency of lead batteries averages 75 to 80% and the ampere-hour efficiency about 85 to 90%.

Application. Storage batteries are used extensively in a great variety of applications—railway and other signalling purposes, automobile propulsion, automobile starting and lighting, railway train lighting, emergency lighting, etc. All modern power plants use storage batteries as a source of power for operating switches which is independent of the system power.

They are used in connection with the power supply developed in power plants of all sizes. In small, isolated, or private plants, a battery is often used to carry the load during parts of the day when the load is too small to warrant running a generator. In the largest direct-current central stations and substations, large batteries are often installed for the sole purpose of carrying the load in case of an accident necessitating the shutting down of the regular source of power. Such a battery is designed to deliver very large currents for short periods of time. It is often connected across the buses all the time, so that it will deliver current automatically when the bus voltage falls below the normal value.

Care. Storage batteries require intelligent care and use if reasonable life is to be obtained. High rates of charge and discharge, repeated discharging to voltage below 1.70 volts per cell, and charging longer than necessary will rapidly disintegrate the active material and decrease the life. The specific gravity should be kept within the proper limits, the battery should be charged immediately after discharge, and should be charged at occasional intervals when not in regular use.

The most reliable indication of the state of charge or discharge is the specific gravity. hence in large plants readings of the specific gravity are taken at frequent intervals on certain representative cells called pilot cells. Autographic hydrometers have been devised which indicate and record the specific gravity at a point some distance from the cells, as for instance the switchboard.

Cost and Depreciation. The cost of stationary type storage batteries, installed, depends on the size and particularly the kind of service. For continuous service involving frequent discharging, a long life battery with heavier plates and therefore more expensive may be the most economical. On the other hand for infrequent demands, the lighter plate battery with high discharge capacity and shorter life, but cheaper, may be the more economical. Costs will range from about $1.25 per ampere per cell for small batteries of 5 amperes capacity to order of $1.00 for 500-ampere batteries. The cost will decrease slightly with larger capacities. Small portable batteries cost from $4 or $5 to $1.00 per ampere per cell. The depreciation of batteries varies enormously with the care and amount of use which they receive. The rate of depreciation for stationary batteries varies from 8 to 15% and for vehicle batteries from 20 to 50% per year.

The Edison Battery is an alkaline storage battery quite extensively used for vehicle and other purposes where weight is specially important. The elements are nickel and iron, and the electrolyte is sodium hydrate. It is considerably lighter than a lead battery and, according to the manufacturers, requires less care and attention and has a considerably larger life. On the other hand it is considerably more expensive. The output per pound of cell is about 2.5 watts and 16 watt-hours, compared with about 1.25 watts and 12 watt-hours respectively for the vehicle-type lead battery. The efficiency of the Edison battery is, however, lower than that of the lead battery by about 15 to 20%.

SECTION 6

SURVEYING, GEODESY, RAILROAD LOCATION

BY

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INSTRUMENTS and MAPS

1. Chains and Tapes

The Engineer's Chain has 100 links, each one foot long. Gunter's chain, formerly much used in land surveys, is 66 ft. in length and has 100 links, each 7.92 in. long. The metric chain is 20 meters in length and has 100 links, each 20 cm. long.

Each end link is provided with a handle, the outside edge of which is the zero point. Every tenth link counting from either end is marked by a brass tag having one, two, three or four points on the tag corresponding to the number of tens of links it marks; the middle of the chain is marked by a round tag. The long links are connected by two short links for flexibility. This introduces about 600 wearing surfaces which causes the chain to lengthen with use. All chains should therefore be frequently compared with a Standard; the length is adjusted by means of a screw and nut in one handle which allows the length of the end link to be changed, thus putting all of the error in length into the end link. Chains have been quite generally displaced by the heavy steel tapes, which will stand rough usage and are not subject to change of length from wearing.

Cloth, Metallic, and Steel Tapes are in common use. Cloth tapes stretch so readily that they are useless for surveying. Metallic tapes are cloth tapes with fine brass wires woven into them to prevent stretching. They are usually graduated into feet, tenths, and half-tenths and are made in lengths of from 25 ft. to 100 ft. It is not uncommon for metallic tapes when subjected to rough usage and to alternate wetting and drying to become 0.2 to 0.5 ft. in error in a length of 50 ft. Metallic tapes should not be used except for measurement of short lines where great precision is not required.

Steel Tapes may be obtained in lengths up to 500 ft., but the most common lengths are 50 ft., 100 ft., 200 ft. and 300 ft. The shorter tapes are usually made of thin ribbons of steel; the longer tapes are made heavier in cross-section so that they will withstand rougher usage and will not become kinked so readily. The light tapes are graduated throughout their length into feet, tenths, and hundredths; the heavier tapes are generally marked only at every foot, the first and last foot being divided into tenths; sometimes it is an additional foot beyond the zero point that is divided into tenths. In most tapes the zero point is at the outer edge of the ring attached to the end of the ribbon of steel; some tapes are graduated so that the zero point comes on the tape 0.2 or 0.3 ft. from the end ring. Light tapes are not conveniently handled in rough work; they become easily kinked and broken, but can be mended by riveting to the back of the tape a piece of an old tape of the same width. A new tape is considered to be ot standard length by the U. S. Bureau of Standards at 68° F. with a pull of 10 lb. and supported throughout its length. Although the steel tape varies in length slightly with variations in temperature and pull, still in much of the work of surveying it is accurate enough to use it without regard to its change in length, and for surveys which require great accuracy the change in length can be determined and proper corrections applied to measured distances.

Special steel tapes fitted with a thermometer and with a spring-balance handle for registering the amount of pull can be obtained from the manufacturers, also tapes graduated in feet and inches for use in building construction. Pocket Steel Tapes from 3 ft. up are very useful in the field.

Tapes made of nickel steel, called "Invar" tapes, change in length only slightly with change in temperature. See Art. 13.

Steel tapes are broken usually by jerking them when they are looped or kinked or by stepping on them or allowing a vehicle to pass over them in soft ground. A tape properly cared for to prevent rusting and kept intact will last until the graduations are worn off.

To change measurements recorded in tenths and hundredths of a foot into inches and fractions, the following equivalents may be used:

Decimal of Foot =	.01	.08	.17	.25	.50	.75
Inches =	1/8−	1−	2+	3	6	9

If these values are memorized, decimals of a foot may be quickly transposed into inches as follows: 0.44 ft. = 0.50 ft. − 0.06 ft. = 6 in. − 3/4 in. = 5-1/4 in.

The Odometer is an instrument attached to a wheel of a vehicle which records the number of revolutions the wheel makes in traversing between two given points. If the circumference of the wheel is known the approximate distance can be computed. Since the wheel travels on sloping roads in hilly country the distances recorded may be slightly larger than horizontal distances.

The Pedometer is an instrument which records the number of steps of a walker carrying it. A modern form records the distance traversed by a person carrying the pedometer, after it has been adjusted for the length of pace of the walker.

2. Verniers

The Vernier is a device for determining the subdivision of the smallest division of a scale more accurately than can be done by simply estimating the fractional part. Its accuracy depends upon the fact that one can judge more accurately when two lines coincide than he can estimate a fractional part of a space. The simplest form of vernier (Fig. 1) is used on some kinds of leveling rods. It is constructed by making the entire length of the vernier equal the length of 9 divisions on the scale, and subdividing this vernier length into 10 equal parts. In this way one division on the vernier equals 9/10 of a division on the scale, so that *ab* is 1/10 and *cd* is 2/10 of a scale division. If the vernier is raised until *a* becomes opposite *b* then the reading will be 701; if raised until *c* comes opposite *d* the reading is 702. The number of the line on the vernier that coincides with some line on the scale is the number of tenths of the smallest division on the scale that the index point lies above the scale division just below it.

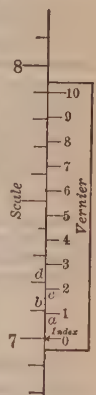


Fig. 1

Transit Verniers are usually made double, one on each side of each index, so that angles can be read in either direction. For transits reading to minutes the vernier scale is made by dividing a space equal to 29 half-degrees of arc into 30 parts, so that the difference in length of one division of the circle and one division of the vernier is equal to one minute of arc. If the vernier is placed so that its zero point (or the index) coincides with the 0° mark on the circle, then the first lines on either side of the zero of the vernier will fail to coincide with the corresponding lines on the circle by just one minute, the second lines on each side of the zero of the vernier will fail to coincide with their corresponding marks on the circle by just 2 minutes, and so on. If the vernier is moved along the circle so that the first line next to the zero of the vernier exactly coincides with its corresponding line on the circle the reading will be 0° 01′; and if the vernier is moved one minute more the second lines will coincide and the reading will be 0° 02′. Therefore in reading an angle on a transit, read the position of the vernier index on the circle, reading the angle in the same direction that the telescope has moved and estimate roughly the number of minutes the index has passed over from the graduation next back of it on the circle, then follow along the vernier in the same direction and find the vernier line which coincides with some line on the circle and add the number of this line to the circle reading. Thus, Fig. 2 represents a one-minute transit circle with double vernier which reads, if the telescope (which always moves with the vernier) has moved clockwise, 274° 23′. If it has moved anticlockwise it reads 85° 37′. For a one-minute instrument the circle is divided into 30′ spaces and there are 30 divisions on the vernier; a 30-second instrument has the circle divided into 20′ spaces with 40 divisions on the vernier; a 20-second instrument has the circle divided usually into 20′ spaces with 60 divisions on the vernier;

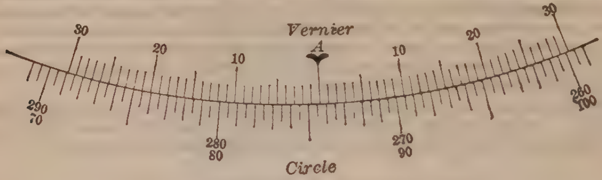


Fig. 2. Transit Circle with Double Vernier

and a 10-second instrument has the circle divided into 10′ spaces with 60 divisions on the verniers.

Single Verniers which are to be read in either direction must be numbered from both ends, and the index at either one end or the other is set at 0° on the circle depending upon which direction the angle is to be measured. These single verniers are used on transits reading to 10 seconds or 20 seconds where a double vernier would be so long as to interfere with the standards. Another type of single vernier called the **folded vernier** is sometimes used on the vertical circle of transits and on alidades of plane tables. It is like any single vernier except that the index is the middle division of the vernier. In reading it, if a coincidence is not reached by passing along the vernier in the proper direction, it is necessary to pass to the other end of the vernier and continue in the same direction, toward the center, until the coincidence is found.

Retrograde Vernier. The verniers described above are all **direct** verniers in which the smallest division of the vernier is smaller than the smallest division of the circle. The **retrograde** vernier is one in which the smallest division of the vernier is larger than the smallest division of the circle. It is seldom used on surveying instruments.

3. The Compass

The Surveyor's Compass is an instrument for determining the difference in direction between any horizontal line and a magnetic needle. The needle is balanced on a pivot in the center of a compass-box so that it can swing free in a horizontal plane, the pivot being the center of a circle which is graduated usually to half-degrees and numbered from 0° to 90° in both directions from the N and S end of the compass-box. The pivot and the circle are secured to a brass frame on which are two vertical sights placed on the line passing through the pivot and the N and S points of the compass-box. When not in use the needle should be kept raised from the pivot by the screw and the lever provided for that purpose so as not to dull the pivot-point. The compass is connected to the tripod by a ball-and-socket joint which is clamped in position after the instrument has been leveled by means of two spirit levels attached to the frame at right angles to each other. The frame has a spindle which fits into the ball-and-socket joint so that after the instrument has been leveled it can be swung around in a horizontal plane. Since in the northern hemisphere the N end of the needle if not counterbalanced would dip downward, a little counterweight is attached to the S end of the needle.

To Use the Compass it is first set up over the proper point and leveled, and then sighted along the line whose direction is desired. Since the needle stands still and the box turns under it, the letters E and W on the box have been reversed from their natural position so that the reading of the needle will give the proper quadrant. To obtain the **bearing** of the line in the direction it is sighted it is important to follow this rule: When the N point of the compass-box is toward the station whose bearing is desired, read the N end of the needle (the end to which the counterbalance is not attached in northern hemisphere). When the S point of the box is toward the station, read the south end of the needle. The term " station " as here used means any point on the survey line. Bearings are usually read to the nearest quarter of a degree. The bearing looking in the opposite direction along the line is called the **reverse bearing.** To avoid a parallax error the needle should be read by looking along the needle, not across it.

Precautions. Before the bearing is read the glass should be tapped lightly over the end of the needle to be sure that it is not touching the under side of the glass. If the needle appears to cling to the glass it probably indicates that the glass has become electrified, and this difficulty can be removed by placing the moistened finger on the glass.

In the use of the compass great care should be taken that no iron or steel is near the instrument to attract the needle from its true position. The chain, pins, ax, pocket knife, iron wire in a stiff hat are fruitful sources of errors. Electric currents affect the needle so seriously that it is of little use in cities for obtaining even an approximate magnetic meridian. It is customary to take the forward and reverse bearing of lines so that any local attraction of the needle may be detected. If the bearing of AB taken from Station A and the bearing of BA from Station B do not agree it indicates that at either A or B there is local attraction. To determine at which station it exists, take the bearing of BC with the compass at B, and then with the compass at C determine the bearing of CB. If these agree it indicates that there is no local attraction at B.

Adjustments of the Compass. (1) The bubbles are adjusted like those on the horizontal plate of a transit (Art. 4). (2) Straightening needle and centering pivot-point: Bent needle may be detected by reading both N and S ends when N end is held opposite a given graduation, and then by reading both ends again when S end is held opposite same graduation; take needle out and bend it with pliers at the middle until the end reads the same in either position. Pivot may be centered by finding (by trial) position of maximum difference between end readings on different parts of the circle, and then bending pivot so that end readings agree. (3) Remagnetizing needle: rub with a bar magnet from center toward ends, using N end of magnet for S end of needle, and vice versa.

The Pocket Compass is a hand instrument for obtaining roughly the bearing of a line. There are two kinds, the **plain** and the **prismatic**. The former is much like the the surveyor's compass, except that it has no sights. In the prismatic compass the graduations, instead of being on the compass-box, are on a card which is fastened to the needle (like a mariner's compass) and which moves with it. This compass is generally provided with sights. The bearing can be read, by means of a prism, at the same instant that the compass is sighted along the line.

4. The Transit and Its Adjustments

The Engineer's Transit, a view of which is shown in Fig. 3, has two spindles one inside the other, to each of which is attached a horizontal circular plate, the outer spindle being attached to the lower plate and the inner one to the upper plate. Attached to this upper plate are two standards supporting the horizontal axis of the telescope. The motion of this horizontal axis and of the two spindles is controlled by clamps and slow-motion (tangent) screws.

On the upper plate are two spirit-levels, which are used in leveling the instrument. Most transits have four leveling screws; but those made for triangulation work usually have only three. When the instrument is provided with a telescope bubble and a vertical arc it is called an **Engineer's** (or **Surveyor's**) **transit**; if it does not have these attachments it is called a **plain transit.**

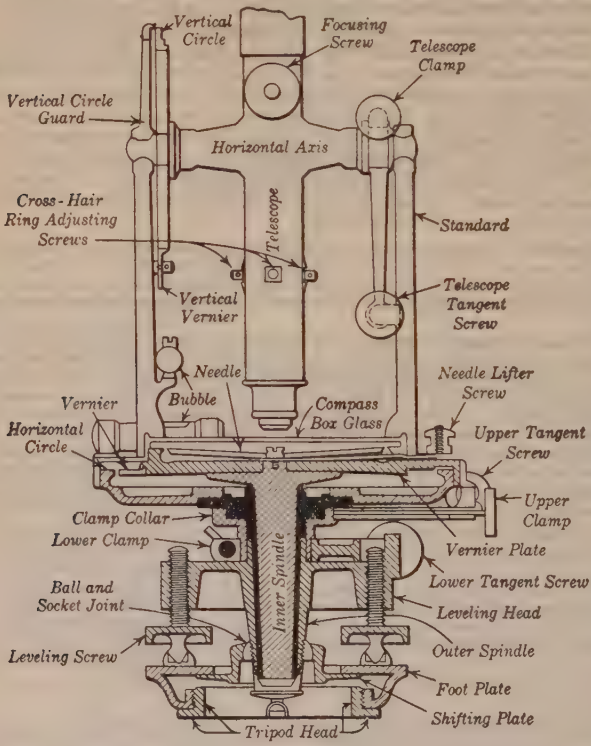


Fig. 3

The **Telescope** has two lenses cemented together for its objective, and an eyepiece of four lenses if the instrument is an erecting transit, two lenses if inverting. An inverting instrument gives a much more illuminated field. Between the objective and the eyepiece is the cross-hair ring. The objective is moved away from or toward the cross-hair to focus. There is a growing tendency (prevalent in Europe) toward the use of an interior system of focusing, which was introduced as "Porro's Telescope." This has a movable lens or lenses between the object glass and cross-hair, thus making the telescope barrel dust-proof; the stadia constant is zero. Its disadvantage lies in the fact that the additional lenses cut out light and the adjustment is not so completely in the hands of the operator. The **line of sight,** or **line of collima-**

tion, is the straight line drawn through the optical center of the objective and the point of intersection of the cross-hairs. **Focusing** is the adjustment of the eyepiece and of the objective so that the cross-hairs and image of the object can be clearly seen at the same time. The **magnifying power** of a telescope is the amount by which an object is increased in apparent size. It is equal to tan $1/2\ A$/tan $1/2\ a$ (or nearly A/a), A being the angle subtended by the object as seen through the telescope and a the angle as seen by the unaided eye. It may be found approximately as follows: Hold a rod a short distance from the instrument and view it through the telescope with one eye; at the same time look at it directly with the other eye. It will be noticed that one space as viewed through the telescope will appear to cover several spaces as seen with the naked eye. This number is approximately the magnifying power of the telescope.

Common Special Attachments are: (1) Diagonal or prismatic eyepiece for sighting high altitudes. (2) A reflector attached to the telescope for illuminating the cross-hairs when working in the dark. (3) A gradienter screw for measuring rates of grades directly. (4) Stadia hairs, which are two extra horizontal cross-hairs used in measuring distances by stadia. (5) Solar attachment, for determining a meridian by observation on the sun.

By Observing a Few Important Precautions much of the wear of instruments and accidents to them can be avoided. Neither the tripod legs nor the shoes should be allowed to become loose. In taking the instrument out of its box always lift it by placing the hands beneath the leveling base. When about to move the transit from one point to another be sure that the four leveling screws are properly bearing, that the needle is lifted, and that the clamps are set just firmly enough so that any slight shock will allow motion. A waterproof bag should be carried at all times to cover the instrument if it rains or when the instrument is exposed to dust. When the transit must stand out in the rain the cap should be put over the object glass; if water should get into the telescope, take out the eyepiece, cover the open end of telescope tube with cloth, and allow it to dry out. Any parts of the instrument that have been wet should be wiped dry, especially the vertical arc, but be careful not to touch the edges of the arcs. One should always avoid placing the hands on exposed graduations, as it will tarnish the metal. In cleaning the lenses use a fine camel-hair brush, and for screw threads use a stiff tooth brush and apply a very little watch oil after cleaning, but do not apply oil to exposed screws.

Important Adjustments of the Transit are: (1) to make the plane of the plate bubbles perpendicular to the vertical axis of the instrument. (2) To make the line of sight perpendicular to the horizontal axis. (3) To make the horizontal axis perpendicular to the vertical axis of the instrument. These three adjustments are made to depend on the principle of reversion, the effect of an error being doubled by a reversal of the instrument. Each adjustment should be repeated to test its accuracy.

(1) **Adjustment of the Bubble Tubes.** Bring plate bubbles in center of respective tubes by means of leveling screws. Turn 180° in azimuth; half the apparent error in bubbles is the rear error; so bring bubble half-way back by screws on the bubble tubes. Adjust each bubble independently.

(2) **Adjustment of the Line of Sight.** First adjust the vertical cross-hair so it will lie in a plane perpendicular to the horizontal axis. Set up and level the instrument. Leveling the instrument is not necessary for this part of the adjustment, but as it is an essential part of the second portion of this adjustment it is well to level at this time. Sight the vertical hair on a well-defined point, clamp both plates, rotate telescope about horizontal axis. If point does not appear to travel along the vertical cross-hair, loosen screws holding cross-hair ring, and by tapping lightly on one screw, rotate ring until

above condition is fulfilled. Then tighten screws and proceed with the second part of the adjustment as follows: sight telescope at point A (200 to 300 ft. away), and clamp vertical axis; revolve telescope on horizontal axis and set a point B in line of sight and same distance approximately from instrument as A but in the opposite direction. Points A and B should be at about the same elevation. Loosen clamp, turn in azimuth and sight A again, clamp, revolve telescope on horizontal axis and set point C in line of sight beside point B. Mark or note a point D one-fourth the distance between C and B measured from C. To adjust, move the cross-hair ring until D is sighted by loosening the screw on one side of the telescope and tightening that opposite. If the transit has an erecting eyepiece move the ring in the direction D to C; if an inverting eyepiece, move ring in the same direction C to D.

(3) **Adjustment of the Standards.** Set up and level the transit. Sight on some high point A and clamp vertical axis. Lower telescope and set point B in line of sight about level with telescope. Reverse telescope and turn in azimuth 180° and sight on B and clamp. Raise telescope until point A is visible and note point C in line of sight and at same height at A. If C does not fall on A, loosen pivot cap screws at adjustable end of horizontal axis and raise or lower the end of the axis by means of capstan-headed screw under axis. The adjusting screw should be brought into position by a right-hand turn, otherwise the block on which the horizontal axis rests may stick and not follow the screw. The cap screws should then be tightened just enough to avoid looseness of the bearing.

If the instrument is badly out of adjustment it is better to bring it as a whole gradually into adjustment rather than to attempt complete adjustment of one part at a time. In this way the adjustment of any part will not disturb the preceding adjustments, the parts are not subjected to strains, and the instrument will remain in adjustment longer.

To Adjust the Vernier of the Vertical Circle so it will read 0° when the telescope bubble is in the center, loosen the screws holding the vernier, and tap lightly until the zeros coincide. This adjustment eliminates index correction in measuring vertical angles, provided the line of sight is parallel to the bubble tube.

To Adjust the Objective Slide so it will move parallel to the line of sight. Adjust the line of sight as described under (2) above, but use very distant objects, then repeat this same adjustment using points very near by, and if in this last test there is an apparent error, it indicates that the objective slide does not move parallel to the line of sight. The adjustment is made by moving the adjusting screws of the objective slide so as apparently to increase the error by one-quarter of the apparent error. Then test adjustment of line of sight again by using distant points. The adjustments of the objective slide—centering the eyepiece tube, centering the circles—are usually done by the instrument makers.

To Eliminate Effects of Errors in Adjustment and still obtain accurate results the instrument must be used as follows. To avoid errors in the plate bubble, level up, turn 180° in azimuth and bring bubbles half-way back by means of leveling screws. This makes the vertical axis truly vertical, and the bubbles should remain in the same parts of their respective tubes as the instrument is turned about the vertical axis. Errors in light of sight and horizontal axis are avoided by using the instrument with telescope in its direct and then in its reversed position and taking the mean of the results, whether the work is running lines or measuring angles. Errors of eccentricity are eliminated by taking the mean of the readings of the two opposite verniers, and errors of graduation of the circle are nearly eliminated by reading the angle in different parts of the circle or by measuring the angle by repetition. Where only one vernier is read in determining an angle always read the one that was set.

5. The Solar Attachment

The Solar Attachment is a small instrument which is sometimes attached to a transit and with which a true meridian can be established by observation on the sun. The commonest form of solar attachment (**Saegmuller's**) consists of a small axis called the polar axis attached at right angles to the telescope and to its horizontal axis; on this axis is mounted a small telescope. Another form of solar attachment (**Burt's**) has the small telescope replaced by a lens and a screen on which the sun's image can be thrown. This instrument is provided with a special arc for setting off the sun's declination.

Astronomic Terms. In Fig. 4, which represents half of a celestial sphere, the circle $NWSE$ is the observer's horizon, $SZPN$ is the observer's meridian

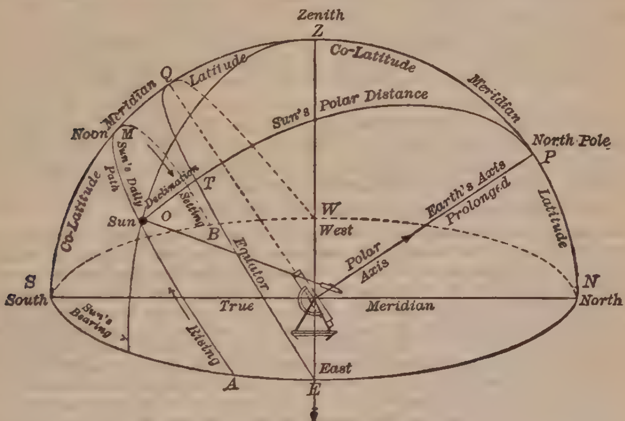


Fig. 4. Celestial Sphere

(a vertical circle through the pole, P). The circle EQW is the celestial equator, and AMB, parallel to the equator, is a parallel of declination, or the path of the sun on a certain day. The sun's **declination** is its angular distance north or south of the equator, or arc OT; it is + when north and − when south. The **polar distance** of the sun is the complement of the declination, the arc OP; when the declination is minus (in winter) the polar distance is more than 90°. If the polar axis points toward the pole, the small telescope can be made to following the sun's daily path by giving it an inclination to the polar axis equal to the sun's polar distance and then revolving it about the polar axis.

To Find the True Meridian by an observation on the sun with a Saegmuller solar: (1) Make the angle between the polar axis and the solar telescope equal to the sun's polar distance at the time of the observation. This is done by turning the solar telescope into the same plane as the main telescope by sighting both on some distant object, and then making the angle between the two telescopes equal to the sun's declination. Incline the main telescope until the reading of the vertical circle equals the declination, and clamp; then level the solar telescope by means of the attached level. The angle between the polar axis and the solar telescope is then 90° plus or minus the reading of the vertical circle.

(2) By means of the vertical circle of the transit incline the polar axis to the vertical by an angle equal to the colatitude of the place, which is 90° minus the latitude. The polar axis will then have the same angle of elevation as the celestial pole.

(3) If the observation is in the forenoon, place the solar telescope on the left of the main telescope (on the right if in the afternoon); then, by moving the whole instrument about the vertical axis and the solar telescope about the polar axis, point the solar telescope at the sun. The sun's image is brought to the center of the square which is formed by four cross-hairs in the solar telescope. The final setting is made by the tangent screw controlling the horizontal motion of the transit and the one controlling the motion of the solar about the polar axis. Only one position can be found where the solar telescope will point to the sun. In this position the vertical axis points to the zenith, the polar axis to the pole, and the solar telescope to the sun. Since the solar telescope is pointing to the sun the main telescope must be in the plane of the meridian. If all of the work has been correctly done it will be observed that the sun's image will remain between the cross-hairs set parallel to the equator, and therefore the sun can be followed in its path by a motion of the solar telescope alone revolving about the polar axis. If it is necessary to move the instrument about the vertical axis to point the solar telescope again at the sun, this shows that the main telescope was not truly in the meridian. For good results observations should not be made near noon, or near sunrise or sunset when the altitude is less than 10°.

The Sun's Polar Distance may be obtained from the "American Ephemeris and Nautical Almanac," published by the Government. The polar distance is not given directly, but its complement, the sun's "apparent declination," is given for each day and for the instant of 0^h or Greenwich mean midnight. The rate of change of the declination, that is, variation per hour, is also given. In order to use these data for any given locality, it is first necessary to find the local or the standard time corresponding to midnight (0^h) at Greenwich. In the United States, where standard time is in use, the relation to Greenwich time is simple. In the Eastern time belt the time is exactly 5 hours earlier than at Greenwich; in the Central belt, 6 hours earlier; in the Mountain belt, 7 hours earlier; and in the Pacific belt, 8 hours earlier. If a certain declination corresponds to 0^h Greenwich time, the same declination corresponds to 7^h p.m. in the Eastern belt, 6^h p.m. in the Central, etc., of the preceding date. The declination for any other hour in the day may be found by applying the "variation per hour" multiplied by the number of hours from 7^h p.m. (or 6 p.m., etc.). Declinations marked North must be regarded as positive and those marked South as negative. An examination of the values of the declination for successive days will show which way the correction is to be applied. It will be useful to remember that the declination is 0° about March 21, and increases to about June 22, when it is approximately 23° 27′ North; it then decreases, passing the 0° point about September 22, until about December 21, when it is approximately 23° 27′ South; it then goes North and is again 0° on March 21.

Atmospheric Refraction. After the correct declination is found it has still to be corrected for refraction of the atmosphere. The effect of refraction is to make the sun appear higher up in the sky than it actually is. In the northern hemisphere, when the declination is North this correction must be added, when South, subtracted; or algebraically it is always added. The amount of refraction can readily be found by **Comstock's method** as follows. Set the vertical hair on one edge (or limb) of the sun and note the instant by a watch. Set the vernier of plate 10′ ahead and note the time when the same edge meets the cross-hair. If n is the number of seconds of time between the observations and h is the altitude in degrees, then the refraction in minutes equals $2000/hn$ approximately.

The Colatitude which must be set off on the vertical circle may be obtained from a map, or may be determined by an observation as follows. Set off the sun's declination for noon, as for any other observation, the two telescopes being in the same vertical plane, and point the small telescope at the sun. By varying the angle of elevation of the main telescope, keep the solar telescope pointing at the sun until the maximum altitude is reached. The angle read on the vertical circle is the colatitude. This observation necessarily comes near noon, but in order to be sure of the maximum altitude it is necessary to begin the observation some time before noon, for the maximum does not occur at exactly apparent noon because the declination is continually changing, the interval between apparent and Standard noon may not be known, and the watch may not be exactly right.

6. The Level and Its Adjustments

A Level Instrument is a telescope with a delicate spirit-level attached to it so that when the bubble is in the center the line of sight is horizontal, or tangent to a level surface, which is a curve every point of which is perpendicular to the direction of gravity. The two common types are the **wye** and the **dumpy** levels. In both of these instruments the telescope is mounted on a vertical axis about which the telescope can swing, and is leveled by means of four leveling screws.

In the Wye Level the spirit-level is attached to the telescope which rests in two Y-shaped supports, which in turn are fastened to a horizontal bar to which the vertical axis is attached. The telescope can be taken out of the Y's, turned end for end and replaced, when testing the bubble for adjustment.

The Dumpy Level has its vertical axis, the horizontal bar and the supports of the telescope all in one piece, to which the spirit-level is attached. The dumpy level will stand much rougher usage than the wye level, has fewer wearing parts and allows fully as precise work. Practically the only advantage the wye level has over the dumpy is that the adjustment of the line of sight can be a little more conveniently tested. The precautions in use of the level and its care are similar to those described for the transit in Art. 4.

The Locke Hand Level is simply a metal tube with plain glass covers at its ends and with a small spirit-level on top. When looking through the tube the bubble is seen on one side of the tube in a mirror through a lens; on the other side the landscape is viewed. When the bubble is in the center of the tube the observer can note where the horizontal line which appears in the center of the bubble tube cuts a rod and in this way do approximate leveling (see Art. 28).

Principal Adjustments of the Wye Level are: (1) to make the line of sight coincide with the axis of the pivots, or parallel to it. (2) To make the line of sight and axis of bubble tube parallel. (3) To make the axis of the bubble tube perpendicular to the vertical axis. As in the transit instrument, most of the adjustments depend upon the principle of reversion.

(1) **Adjustment of Line of Sight.** First make the horizontal cross-hair truly horizontal when the instrument is level. This adjustment is tested by sighting, after leveling, on a point and noting if the cross-hair appears to remain on the point as the telescope is revolved about its vertical axis. If an adjustment is necessary it is done by rotating the cross-hair ring as described in the case of the second adjustment of the transit. After this adjustment is made loosen the clips which hold the telescope in the wyes. Sight the intersection of the cross-hairs at a point and clamp. Rotate the telescope 180° in wyes. If the cross-hairs do not remain on the point they must be moved half-way back to the point by means of screws on the cross-hair ring, each cross-hair being adjusted separately by means of its proper ring screws.

(2) **Adjustment of Bubble by Indirect Method.** Bring the bubble in the center of its tube and clamp in that position. Rotate telescope in wyes a few degrees around its horizontal axis; if the bubble moves, correct the entire

error by means of the horizontal capstan screws at one end of the bubble tube. Then clamp the telescope over the pair of leveling screws and bring the bubble into the center of the tube, lift the telescope from the wyes, turn it end for end, and replace in the wyes without jarring the instrument. Correct half the apparent error by the vertical adjusting screw of the bubble tube.

(3) **Adjustment of the Wyes.** Level the instrument, then bring the bubble exactly to the middle over a pair of leveling screws. Then turn the telescope 180° about its vertical axis and correct half the apparent error by means of the adjusting screw of the wye support. Since the bubble is brought to the center of the tube at each rod-reading this last adjustment in no way affects the accuracy of the leveling work but is a convenience.

The Adjustments of the Dumpy Level are the same in purpose as for the wye level, but are, owing to the construction of the instrument, done in a different order and by a different procedure in some cases. They are: (1) To make the horizontal cross-hair truly horizontal when the instrument is level. (2) To make the axis of the bubble tube perpendicular to the vertical axis. (3) To make the line of sight parallel to the axis of the bubble.

(1) **The Adjustment of the Cross-hairs** is done as described under the first adjustment of the wye level.

(2) **Adjustment of Bubble Tube.** Level the instrument and carefully center the bubble over a pair of leveling screws. Turn the telescope 180° in azimuth and correct half the apparent error in the bubble by means of the adjusting screws of the level tube.

(3) **Adjustment of Line of Sight by Direct or "Peg" Method.** Select points A and B 200 ft. or more apart. Set up the level beside A so that when a rod is held on A the eyepiece will swing and just clear the rod. Look through the telescope wrong end to at the rod and find the reading opposite the center of the field. Turn the telescope toward B and take a rod-reading in the usual manner, being sure that the bubble is in the middle of the tube. Then set up the level at B and repeat the above operation. These two sets of observations give two independent determinations of the difference in elevation between the two points. The true difference in elevation is the mean of these two results. Leaving the instrument at B, set the rod at A so that it will read the height the instrument is above B plus or minus the true difference in elevation between A and B. Then if the level is sighted on the target of the rod it will define a level line. While the bubble is in the center of its tube the line of sight should be brought to coincide with the target by moving the cross-hair ring by means of its adjusting screws. The following example illustrates the method:

Instrument at Sta. A		Instrument on Sta. B	
Rod-reading on Sta. A	= 3.971	Rod-reading on Sta. B	= 5.064
Rod-reading on Sta. B	= 4.937	Rod-reading on Sta. A	= 4.036
Diff. in elev. of A and B	= 0.966	Diff. in elev. of B and A	= 1.028

Mean of two diff. in elev. = 1/2 (0.966 + 1.028) = 0.997, true diff. in elev.
Instrument is now 5.064 above Sta. B.
Rod-reading on Sta. A should be 5.064 − 0.997 = 4.067 to give a level sight.

This "peg" adjustment may be used for adjusting the line of sight of the wye level except that in the wye level after the target at A has been set at the correct elevation to define a level line the line of sight is made to coincide with the target by means of the leveling screws and then the bubble is brought to its mid position by means of its adjusting screws. This "peg" adjustment is also used for the hand level. In using the "peg" method for transits the adjustment may be made either by moving the cross-hair ring or by moving one end of the level tube. If the cross-hair is moved the adjustment of the line of collimation must be tested; if the bubble tube is moved the vertical arc must be adjusted.

To eliminate the effect of errors in adjustment of the line of sight, of the bubble tube or the wyes the observer must be sure that the bubble is in the center of the tube at the instant that the rod is read, and the length of the backsights and foresights should be made equal.

7. Leveling Rods

Two General Types of Rods, target and self-reading, are in common use. The **target** rods are read only by the rodman, while the **self-reading** rods are read directly by the level man. The commonest forms of target rod are the Boston, the New York and the Philadelphia rods; the latter may be used also as a self-reading rod.

The Boston Rod is an extension target rod made of two strips one of which slides in a groove in the other, and provided with clamps to hold the two parts in any desired position. There is a scale on each side of the rod graduated to hundredths of a foot, each scale being provided with a vernier for reading to thousandths of a foot. The target is fastened to one of the strips, the other one is held on the ground and the target strip raised to the proper reading, the highest reading being 5.8 ft.; these are called "short rod-readings." For "long rod-readings" the rod is turned end for end and the target strip is raised, its highest reading being 11.4 ft. A serious objection to this kind of rod is that in reversing it any error in the position of the target with reference to the zero of the scale is doubled.

The New York Rod has two strips arranged similarly to those of the Boston rod, but the circular target is movable and for short rod the target is moved up and down on the rod, the scale graduated to hundredths being in the face of the rod; it is read by means of a vernier on the target. The scale for long rod is on the side of one of the strips and the vernier is on the other strip. When used as a long rod the target is clamped to the face of the rod at the reading corresponding to the lowest reading on the side scale; then the target is raised just as in the Boston rod.

The Philadelphia Rod is marked on its face throughout its entire length (extended) with red numbers to designate the feet and black numbers for the tenths so that it can be used as a self-reading rod. The red figures are 0.1 ft. high and the black figures 0.06 ft. When it is used as a target rod, it is operated just as the New York rod; the only difference being that the short scales at the center of the metal target and on the back of the rod (for long rod-readings) are not verniers, but these scales are graduated to 0.005 ft. and are used in estimating the readings to thousandths of a foot.

Special Designs of self-reading rods are in use, the figures on the face being as a rule made of some definite height (0.06 or 0.08 ft.) and of a thickness of 0.01 or 0.02 ft. to aid in estimating the readings. Some of these rods are in three parts, giving an extended length of 16 ft. In nearly every respect self-reading rods are preferable to target rods.

The Tape-rod is a wooden rod made in one piece with a metal roller near each end around which is a continuous movable steel band 20 ft. long and about 0.1 ft. wide, on the outside of which the scale is painted so that it can be used as a self-reading rod; the band has a clamp to fasten it to the rod at any desired position. Unlike the ordinary rod the scale reads down instead of up. In using this rod the band is set so that the level sights the reading of the elevation of the bench-mark. For example, if the B.M. elevation is 142.36 the band is moved and clamped so that the level sights 12.36 on the scale. When the rod is held at any other point the rod-reading plus 130 gives the elevation of the point. Such a rod is of great convenience in cross-section work.

Self-reading rods for Precise Leveling are used by the U. S. Coast and Geodetic Survey and are made of single pieces of wood soaked in paraffin to prevent changes in length due to moisture; they are made in the form of a + or a T in cross-section. The graduations are marked in the metric system on a strip of invar metal attached to the face of the rod (Art. 25). For plumbing the rod in precise work special devices are used such as spirit-levels attached to brass angles or watch levels held on the corner of the rod.

8. Drafting Instruments and their Uses

It is assumed that the reader is familiar with the ordinary drafting instruments; only a few of the more uncommon will be described.

The Pantograph is a jointed framework of several pieces (or arms) of wood or metal so joined as to form a parallelogram; and used for enlarging or reducing maps. It rests upon three points, one of which, *A*, is fixed and the other two, *B* and *C*, are movable. There are other bearing points but they simply support the instrument and are not essential to its principle. The two movable points *B* and *C* are in such positions that they will trace exactly similar figures. The instrument is used for copying plans either to the same or different scales; the latter is accomplished by varying the positions of the points *B* and *C* on the arms. Thus two points *B* and *C* can be attached to their respective arms at any desired position, but the essential condition is that *A*, *B*, and *C* shall lie in a straight line and each of the three points must be attached to one of three different sides (or sides produced) of the parallelogram. Any one of the three points can be the fixed point. These instruments are usually provided with scales on the arms indicating the proper settings for various reductions or enlargements. Because of lost motion in the joints very accurate results cannot as a rule be reached, but the best metal pantographs are sufficiently accurate for most topographic maps.

The Planimeter is an instrument for determining the area of a figure by moving the tracing point of the instrument around the perimeter of the plotted area however irregular its shape. The most common form is the **Amsler polar planimeter,** which has two arms, one fixed in length, at the end of which is the anchor point which has a needle point to attach the instrument to the paper. The length of the other arm, the tracing arm, can be changed to give results in different units. At the end of the tracing arm is a point which is moved along the outline of the area to be measured. This arm passes through a collar to which the fixed arm is attached by a pivot. Connected with this collar is a graduated wheel which, together with a little disk which records the revolution of the wheel, gives the area in units depending upon the length of the movable arm. The planimeter rests on three points, the anchor, the tracing point and the periphery of the wheel. To measure an area, press the anchor point into the paper at a position outside of the perimeter of the figure, if it is not too large, and start the tracer from a definite point on the periphery of the area, preferably such as will bring the two arms approximately at right angles to each other. Read the disk, wheel and its vernier, giving four figures. The tracing point is then moved carefully around the outline of the area until the starting point is again reached, when the disk and the wheel area again read. The difference of the two readings gives the area in the unit depending upon the length of the movable arm. The setting of the scale marked on the movable arm for different units is given by the maker. If the area is so large that the anchor point cannot be set outside its limits it can be divided into smaller parcels and the area of each determined separately; or the anchor can be placed inside the area provided the area of a **correction circle** is added to the result, which value is also given by the maker. Results correct to within 1% may be easily obtained with this instrument.

The Rolling Planimeter is not anchored to the drawing. It has a tracing point at the end of an adjustable pivoted arm which is fastened to a frame supported on two rollers. The whole instrument is rolled forward and backward in a straight line while the tracing point traverses the outline of the area. Results correct to 0.01% are easily reached with this instrument. The rolling planimeter is much more expensive than the polar planimeter.

The Three-armed Protractor is used to solve the "three-point problem" graphically. It is similar to the ordinary protractor except that it has three arms, the middle one fixed at 0° of the circle and the other two movable. On either side of the 0° point angles can be laid off. If a point is located by two angles taken between signals *A* and *B* and between *B* and *C* this point sought can be located by laying off the two angles one on either side of the zero point of the protractor and then moving the instrument about on the plan until the plotted points *A*, *B*, and *C* lie on the beveled edges of their respective arms. When this position is found the center of the protractor locates the point sought.

9. Drawing Papers and Blueprinting

Drawing Papers for Working Plans are of all grades from manila detail paper, which costs from 6 to 20 cents per yard, to well-seasoned mounted paper, which will not change greatly with changes in moisture, and which costs from \$1.50 to \$4.00 per yard, depending upon the width and quality. Mounted paper comes in 10-, 20-, and 30-yd. rolls and in widths of 36, 42, 56, 58, 62 and 72 in. The best grades of drawing papers can also be obtained in sheets, either plain or mounted; the common sizes are Cap, 13 by 17; Demy, 15 by 20; Medium, 17 by 22; Royal, 19 by 24; Super Royal, 19 by 27; Imperial, 22 by 30; Atlas, 26 by 34; Double Elephant, 27 by 40; Antiquarian, 31 by 53 in.

Transparent Paper, similar to bond paper, is used largely for studies and for temporary copies of plans. But for more permanent copies tracing cloth is used. Tracing paper comes in sheets of the same standard sizes as mounted paper; it also comes in 20- and 50-yd. rolls and in widths 30, 36, 39, 42, 57 and 60 in., which cost from 7 to 30 cents per yard.

Tracing Cloth is a very uniform quality of fine linen coated with a preparation to make it transparent. This material is manufactured in 24-yd. rolls in widths, 24, 30, 36, 38, 42, 48 and 54 in., and costs from 75 cents to \$2.00 per yard. Most tracing cloth has to be rubbed with powdered chalk before it will take ink. It has a smooth and a rough side; draftsmen should work on the rough side because it will take pencil lines and will not show erasures as much as the smooth side when process prints are made from them. In making a tracing of another tracing place white paper under the under tracing. From one tracing any number of process prints can be made.

Cross-section and Profile Papers can be procured with colored lines, both on heavy paper and on transparent paper or cloth. Cross-section paper, divided into 4, 8, 10, 12 and 16 squares to the inch, is printed in orange, green, red, blue and black in sheets 16 by 20, 17 by 22 and 18 by 23 in., and also on smaller sheets called "coordinate paper." Cross-section papers can be procured in 20- and 50-yd. rolls with the engraving 20, 24 and 30 in. wide, and also in metric units 50 cm. and 75 cm. wide. Profile papers are printed in green, orange and red in three scales, called Plates A, B, and C. Plate A has 4 spaces horizontally to the inch and 20 vertically; Plate B has 4 horizontally and 30 vertically; and Plate C has 5 horizontally and 25 vertically. This material is mostly manufactured in 20- and 50-yd. rolls with the engraving 9, 10 and 20 in. wide and costs from 15 to 25 cents per yard on paper and \$1.00 to \$1.35 a yard on tracing cloth.

Blueprint Paper is the most common of the pcocess papers. It is a white paper coated on one side with a solution which is sensitive to light. When fresh it is a yellowish green color. The following is a formula for this solution: solution (*a*) citrate of iron and ammonia 1 part (by weight) to 5 parts of water; solution (*b*) red prussiate of potash (recrystallized), 1 part (by weight) to 5 parts of water. These two solutions should be mixed in equal parts in the dark or in subdued light. The mixed solution is applied to the paper by means of a flat brush or sponge in a dark room or in subdued light, care being taken to coat the paper uniformly. Then the paper is hung in a dark room to dry and is stored there until used. The above coating will require an exposure of about 5 minutes in bright sunlight; for quick printing paper use a larger proportion of citrate of iron and ammonia. Prepared blueprint paper and cloth can be procured in 10- and 50-yd. rolls in widths of 24, 30, 36, 42, and 54 in. The paper costs from 10 to 25 cents and the cloth 50 cents to $1.50 per yard. In making the print expose the tracing in a printing frame (with the blueprint paper under it) to the sunlight a proper length of time depending upon the sensitiveness of the paper and the brilliancy of the light. Then remove the paper and wash thoroughly in water. Great care should be exercised so that the tracing may not become wet, for it is impossible to eradicate such spots on tracing cloth.

Vandyke Paper is a sensitized paper which is printed in the same manner as blueprint paper, except that the tracing is put into the frame with the ink lines adjacent to the sensitive side of the paper. After an exposure of about 5 minutes in bright sunlight the print is washed for about 5 minutes in clear water and then put into a solution of half an ounce of hyposulphite of soda to a quart of water. It is left in this solution about 5 minutes and then put into clear water again for 20 minutes. This process gives a negative of the drawing, a dark brown paper with white transparent portions where the ink lines of the tracing covered the paper in printing. This Vandyke print can then be used like a tracing to make other positive prints on Vandyke paper, giving brown lines on white background, or to make blueprints with blue lines and white background. This paper can be obtained in 10 and 50-yd. rolls and in widths of 30, 36 and 42 in.; it costs from 15 to 30 cents a yard. **Blackprint papers** are used to some extent; they require a chemical developing bath, and give a blackline copy of the original.

Process-printing Machines, equipped with a brilliant artificial light, so that the prints may be made at any time of day or night, are common. In one of the best types several horizontal rollers are provided, with the light so arranged that as the tracing and process paper pass from one roller to another the exposure is made. The speed of the machine is controllable and the length of the tracing that can be printed is limited only by the length of the roll of paper; thus long plans or profiles can be printed without the necessity of frequent splicing which is required with other types of printing frame; furthermore the print is of uniform color throughout. These machines are provided with an apparatus for washing and drying the prints as fast as the come out.

Photostats are used to a large extent by engineering offices for copying, enlarging and reducing plans, government topographical maps, notes, deeds, and other records. They make a faithful copy. Even faint pencil lines of field notes can be reproduced by this process. Enlargements up to about twice the original size can usually be made in one step; but since enlargements of the enlargements can be made, a plan of any desired scale can be produced by this process. A photostat of a tracing will show white lines on a black background, but a positive of the same scale can be made by making a photostat of the negative photostat. Photostats of blue prints show black lines on white background. Since all photostats go through a fixing bath and a washing process they shrink unevenly, as do blueprints; it is therefore advisable to draw a graphical scale or scales on plans which are to be photostated.

Engineers can obtain excellent process prints, at lower cost than they can be privately made, from process print companies located in all large cities. The prevailing prices per square foot are: for blueprints, 4 cents; for Vandyke negatives, 12 cents; for blue lines from Vandykes, 7 cents; for brown lines from Vandykes, 12 cents; for red lines on white paper, called "direct process," "ozalid," or "red lines," 7 cents; for direct process copies of tracings on tracing cloth, called "lithoprints," 35 cents per square foot; for photostats, 9 × 14 in. and under, 25 cents, and 14 × 18 in., 75 cents, and 18 × 22 in., $1.00.

10. Methods of Plotting Traverses

The Precision of Plotting depends upon the purpose of the survey. If it has been made simply to produce a map the angles may be laid off by means of a protractor and the distances scaled, but it is advisable to plot the triangulation system or the traverses on which the entire survey is based by a more exact method, leaving the details only to be plotted with protractor and scale. A traverse that is closed in the field should close on the plan; any error of closure in the plot indicates an error in scaling some distance or in laying off an angle, one or both. The bearing of each line of the traverse can always be computed from the bearing of some one line, assumed to be correct, and then any line can be extended till it meets the plotted meridian and its direction checked independently. Such checks are absolutely necessary if the traverse is not closed; and also the scaling of each line of the traverse should be checked. In the case of a closed traverse, if it closes on the plan, it is a good indication that it has been accurately plotted.

Plotting by Rectangular Coordinates is a common method for accurate results. It consists in referring all the angle points in the traverse to a pair of coordinate axes; for convenience these axes are the same ones used in calculating the area if the latter is required. The advantages of this method are that each point is plotted independently of the others, that all plotting is done by means of the scale only and that the plotting can be readily checked. First, compute the ordinate and abscissa, called **total latitude** and **total departure**, of each angle point of the traverse (Art. 16). If a meridian through the most westerly point and a perpendicular through the most southerly point are chosen for axes there will be no negative ordinates or abscissas, for east departures and north latitudes are always regarded plus values. The total latitude of any point is the algebraic sum of all the latitudes of the intervening courses beginning with the most southerly point, and similarly for the total departure but beginning with the most westerly point. Construct a rectangle whose height equals the difference in latitude of the most northerly and southerly points and whose width equals the difference in departure of the most westerly and easterly points. This rectangle should be plotted very accurately by straight-edge and reliable triangles or by use of a beam compass, and checked by scaling the diagonals. To plot any point lay, off its total latitude on both the easterly and westerly sides of the rectangle from the southerly side. The point will lie on a line connecting these two points. Along this line scale off the total departure of the point, thus plotting the position of the point. Similarly all of the other points are plotted. To check a plot of this character scale the lengths of the traverse lines. This method is particularly useful in plotting closed traverses where the traverse encloses a field whose area is desired, for in this case the latitude and departure of the courses have to be computed in determining the area; but in the case of traverses that do not close the Tangent Method or the Chord Method is generally preferable.

The Tangent Method consists in laying off the angles by constructing at each vertex a right triangle whose base is 10 in., or any other unit, and whose other leg is the natural tangent of the angle laid off in the same unit as the base. The traverse should first be roughly plotted with protractor and scale so as to determine its extent and shape and so that the first line of the plot can be laid in its proper place on the paper. This first line, AB (Fig. 5), is then drawn to the proper scale. The forward end is extended, say 10 in., at which point f a perpendicular is erected by using triangles.

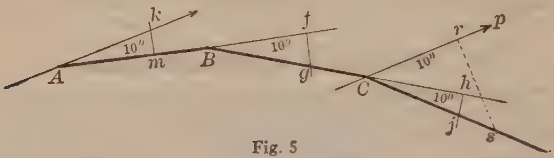


Fig. 5

On this perpendicular the natural tangent fg of the deflection angle is scaled off and line Bg is drawn. This gives the direction of the second line of the traverse on which the proper length BC is laid off, which is in turn extended 10 in. further to form the base of a new right triangle in which the acute angle hCj will be the deflection angle measured at that station. To check such a traverse, calculate the bearing of all the lines referred to any line as a meridian and draw such a meridian line accurately by plotting the bearing angle kAm from the first line by the tangent method. Then draw a line Cp parallel to this meridian line so that it will intersect the course Cj whose direction is to be checked, and measure the angle between this course and the meridian by laying off a 10-in base and erecting a perpendicular rs and scaling rs to determine the tangent of the angle pCj.

When the angle much exceeds 45° it is more accurate to plot the complement of the deflection angle rather than the angle itself, in which case a right angle is erected at the station point and this is used as the base of the right triangle; in such a case and throughout this method it is important that the triangles used shall be true.

In the Chord Method of plotting a traverse the angles are laid off by scaling off a chord across an arc so that it will subtend the required angle at the center of the arc. In detail, plot the first line AB to scale (Fig. 6). With B as a center swing an arc

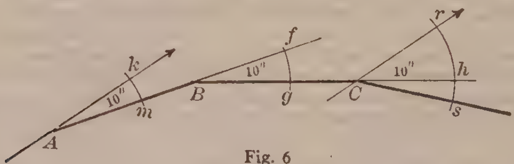


Fig. 6

with radius of say 10 in. Extend AB till it cuts the arc at f, from which point scale off the chord fg along the arc in the proper direction. Chords for different angles have been published, but any natural sine table is practically as convenient, since the chord distance equals twice the sine distance of half the angle. The necessity for multiplying by 20 can be avoided by plotting the natural sine of half the angle directly with a scale of 10 ft. to an inch, while the radius is scaled with a 20-ft. scale. The direction of any course Cs may be checked by calculating the bearings of all the lines and measuring the angle between a meridian line rC and the course in question Cs by drawing an arc rs as in plotting the angle, measuring the chord rs and finding the angle rCs corresponding.

For Maps of Large Areas, such as a state or portion of a country, it is not sufficiently accurate to draw the meridians and parallels of latitude as rectangles. The most common form of projection used is the **polyconic,** in which the surface of the sphere representing the earth is developed on a series of cones. In the U. S. Coast Survey Report for 1884, Appendix No. 6, p. 135, is a table giving coordinates of curvature for plotting the parallels of latitude and meridian lines for this projection.

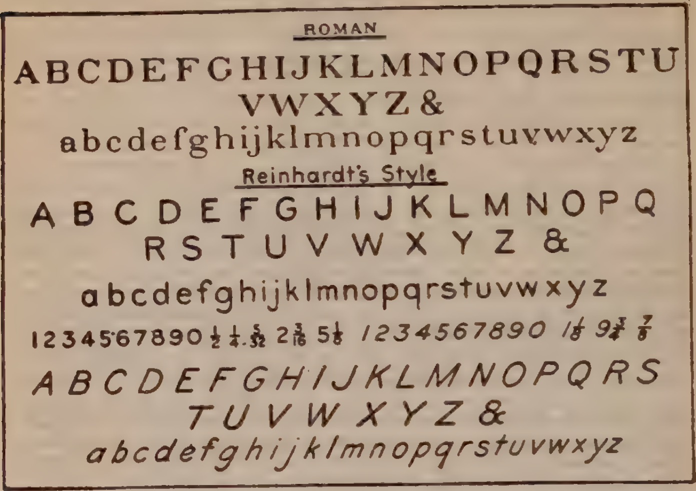
11. Finishing the Plan

The Style of a drawing and the data which it should give depend upon the use to which it is to be put. On every plan, however, there should be a complete title which should be a brief description of the drawing, the owner's name, the location of property, the scale, the date of the survey, the surveyor's name and address. Besides these, if the plan is a land plan, a meridian line should be drawn and some designation as to whether it is the true or magnetic meridian, the limits of abutting property and the names of their present owners. Notes are frequently added to give such further information as is necessary for a correct intepretation of the plan. All essential dimensions are lettered in their proper places. In the case of a land plan the area is usually expressed in square feet or acres and lettered in the middle of the parcel; the lengths of the sides are lettered in the middle of each line, and sometimes the bearings of each side or the angles between them are given, also any stone bounds, iron pipes or other physical boundaries which may exist are represented by abbreviations such as S.B. for stone bound, I.P. for iron pipe, sp. for spike, and stk. for stake. It is the practice of most surveyors to omit from the plan the calculated bearings of the lines or the angles. The bearings or angles are frequently of no use to the owner, but they are of great value to any other surveyor who may have occasion to rerun the lines of the property, and to fail to give them on the plan is in some instances at least withholding data for which the owner of the property has paid and to which he is rightfully entitled. Oftentimes the fact that the angles have been omitted from a drawing necessitates a resurvey which would have been unnecessary had the first surveyor given these data on his plan.

On Working Drawings and sometimes on finished plans the traverse line is drawn, usually as a colored full line, the angle points being shown either by very small circles the center of which marks the exact point or else by very short lines drawn through the angle points so as to bisect the angles. Triangulation stations are represented by small equilateral triangles with the point in the center, stadia stations by small squares, and other auxiliary stations by circles. The location of bench-marks is frequently represented by a small cross and figures thus, B.M. $\times$ 427.62.

The boundaries of the property and the physical features, such as streets or buildings, are usually drawn as full black ink lines. Shore lines are represented by black or by Prussian blue lines, and they should as a rule be the heaviest lines on the plan unless they are a very unimportant part of the plan. Whenever colors are used better results can always be obtained with water colors than with bottled inks. The following, burnt sienna, raw sienna, yellow ocher, scarlet, vermilion, carmine, sepia, cadmium yellow, gamboge, Hooker's green, Prussian blue and indigo, are the colors commonly used by surveyors. All of these except gamboge may be used on tracing linen without danger of running, but if sun process prints are to be made from the tracing it will be found that Prussian blue and indigo will not print well. Some colors which do not give good prints may be made to do so by adding a little of some color to give it body (such as Chinese white or cadmium yellow) but not in quantity enough to change the original color.

Lettering on a drawing should be plain so as to be easily read. Except in titles, the Reinhardt erect or inclined lettering is in general use. Gothic and Roman letters are appropriate for titles of published plans. All lettering should read from left to right when the plan lies with its title horizontal; lettering running perpendicular to the bottom should read from the bottom upward. Put title in lower right-hand corner of plan so that it can be read by turning up the corner of the sheets that lie above it in the drawer; put the number of the plan in the same corner. A rolled plan should have a brief title on the back at both ends. Put a graphical scale on all plans to be reproduced photographically.



Border lines are not as a rule used on construction drawings, but on finished land plans a single line drawn 3/4 in. to 2 in. from the edge of the drawing gives a good appearance if the line is not too heavy. A fine line just inside of a heavier one makes a good border line; it is the tendency of most draftsmen to make the border lines too heavy.

The Meridian should be represented on all land plans as a full arrow if it is a true meridian and as a half arrow if it represents the magnetic meridian, the half arrow head being on the side toward the declination. It should be of simple design, not too conspicuous in size or weight of lines. It is good practice to show both the true and the magnetic meridian and to letter on the magnetic the declination at the place and time the survey was made.

On Mounted Plans accurate scaling may be difficult on account of shrinkage of the paper. It is hence well to draw lines parallel to the borders, forming sides of 100 or 1000 ft., so that allowance may be made in scaling after the paper shrinks.

To Clean the Drawing use a soft pencil eraser, a sponge eraser or dry bread crumbs. For tracings, gasoline or benzine will remove pencil marks without affecting the ink lines.

12. Topographic Signs

Conventional Signs, which have come to be used so generally that they are practically accepted as standard, are used on topographic maps to show certain physical features. A sheet of standard topographic symbols adopted (1925) by the Board of Surveys and Maps of the federal government is on sale by the U. S. Geological Survey for 40 cents. A few of the most common are shown in Fig. 7. They are executed with a free-hand pen except that the horizontal lines on the " salt marsh " signs and the straight lines in " cultivated land " are ruled, also fences, railroads and buildings. In all cases the symbol for grass and marsh land should be parallel to the bottom of the map. In the

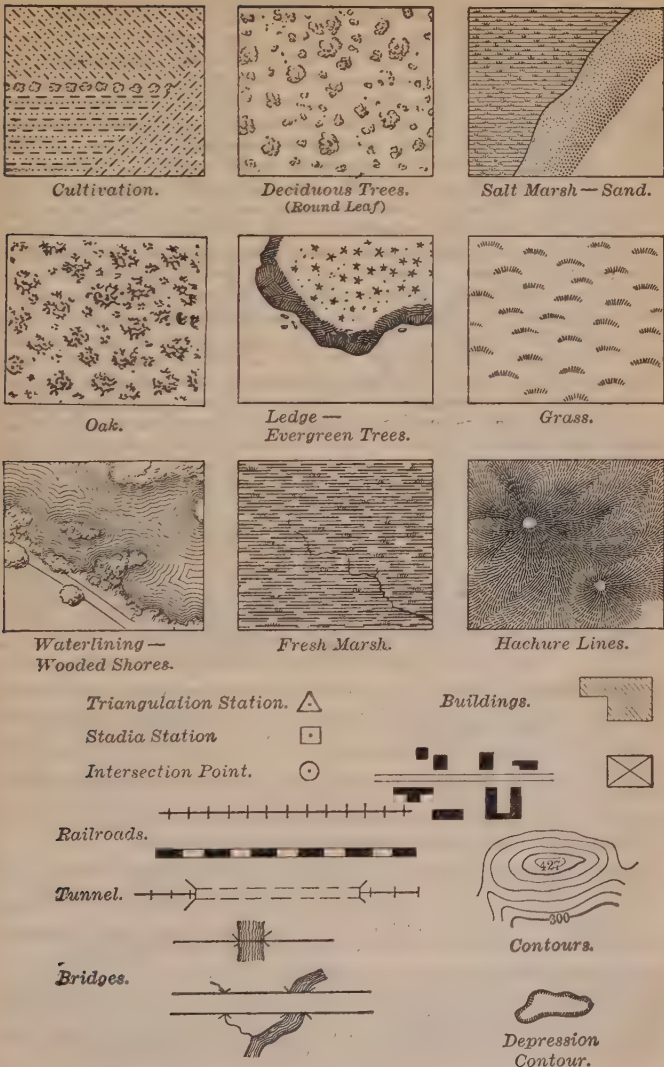


Fig. 7. Conventional Topographic Signs for Maps

grass symbol the little lines which represent a tuft of grass should all appear to radiate from some point below the base of the group. It is usually composed of five or seven short lines, the tops of the lines forming a curve. They are made beginning at the left-hand side with a short dash and increasing in length up to the middle line, which is vertical, and then diminishing and ending with a short dash or dot at the right-hand side. In executing "water-lining" the first line outside the shore line should be a light full line drawn just as close to the shore line as possible, and should follow very carefully every irregularity. The next line should be drawn parallel to the first but with a little more space between them than was left between the shore line and the first line. Then the third line should be spaced a little farther out and so on; five to ten lines are sufficient. The aim should be to change the spacing of the lines so gradually that no one place can be observed where the spacing suddenly increases. Water-lining, fresh marsh and salt marsh symbols are often represented in Prussian blue. A distinction is generally made between evergreen and deciduous trees by using different symbols, and a further distinction is made by using a special symbol for oak trees. The symbol for an evergreen tree is made up of five or six short free-hand radial lines of uniform weight, giving a star-like appearance to each individual symbol. They are made of different weight of lines, some much lighter than others. To avoid the tendency of making them in rows it is well to make irregular groups of the symbols in different parts of the area to be covered and to fill in between these groups with smaller or lighter symbols. In the symbol for deciduous trees it is attempted to represent the plan of a tree in foliage, with a slight shading toward the lower right-hand side. These little symbols are placed regularly over the area if they represent cultivated trees and are given a smoother outline than those representing wild growth, which are scattered irregularly over the area. On some topographic maps, such as the U. S. Geological Survey maps, most of the topographic signs are represented in colors, while the U. S. Coast and Geodetic maps are all produced in black. When colors are used the trees should be green; the grass, a light green tint (Hooker's green No. 2); water, a light Prussian blue tint; cultivated land, yellow ocher.

Contour Lines are almost always drawn in burnt sienna water-color. Every fifth or tenth contour is usually represented by a line slightly heavier and a little darker in color. These may be drawn either with a Gillott's No. 303 pen or with a contour pen. In numbering the contours just enough numbers should be marked so that the elevation of any contour can be found without difficulty. The numbers on the contours should be small figures in burnt sienna. A common mistake is to make the contours too heavy so that they subordinate some of the more important features on the map.

Hachure Lines instead of contours are sometimes used to represent the shape of the surface of the ground (Fig. 7). Contour lines are first sketched lightly in pencil as guides in drawing the hachure lines, which are drawn normal to the contours from the summit downward in rows, each row touching the next preceding; the steeper the slope the heavier and shorter the line. All of the lines are qually spaced.

Sub-aqueous Contours are usually represented on hydrographic maps by dot-and-dash black lines, the shallowest contour having one dot between the dashes, the next contour in depth having two dots between the dashes and so on. In some cases contours representing fathoms of water are shown as single dots for the first fathom, two dots and then a space for the second fathom, dots in threes for the next and so on.

LAND AND TOWN SURVEYING

13. Measurements of Length

Variations in Tape Measurements are due to erroneous length of tape, improper pull on the tape, careless plumbing, incorrect alignment, wind, changes in temperature, and sag of tape. The erroneous length of a tape can be discovered by comparing it with some standard; most cities have a standard which has been established with more or less care. The United States Bureau of Standards at Washington will, for a nominal charge, standardize tapes. The tape should be tested at intermediate points as well as for its total length; its temperature and pull should be noted, and whether it is suspended or supported. If the tape is too long the measurements made with it are recorded too short, and the proper corrections should be added.

The Amount of Pull on a tape will have a very appreciable effect upon its length; ordinary light 100-ft. tapes will stretch 0.01 to 0.02 ft. with an increase of 10 lb. over the ordinary pull. This increase will be different for different tapes; so it is well to investigate it by fastening the ring end of a tape to a nail in the floor, and with the tape supported on the floor to exert different amounts of tension on the tape and measure them with a spring balance. In this manner the variations in length due to different tensions can readily be determined for any tape.

Plumbing is the operation of transferring any point on a horizontal tape to the ground under the tape by means of a plumb-bob. Inaccuracies due to this process can only be avoided by using great care. It should be borne in mind, however, that if a line must be measured by plumbing, more accurate results can be obtained by measuring downhill than uphill. Even when great care is taking in plumbing, so much error is introduced that for accurate results it is better to measure the inclined distance from the horizontal axis of the instrument to the station mark ahead, measure with the vertical arc the angle of inclination of the tape, and compute the horizontal distances by means of the versine of the angle of inclination; horizontal distance = inclined distance minus (inclined distance × versine angle). This computation can usually be done with sufficient accuracy by means of a slide rule. This method requires a set-up of the transit at every other tape-length. Another way to measure these inclined distances is to tape directly from stake to stake in one tape-length; set the instrument up at every other stake and measure the vertical angle to a point above the two station points on either side of the instrument equal to the distance the horizontal axis is above the stake under the transit. This will give the inclination of the tape. Still another method is to measure the inclined distance from stake to stake and obtain the difference in elevation by leveling. When much of this inclined work is to be done it is more expeditious to use a 200-ft. or 300-ft. tape.

Alignment Errors are not likely to be large; lining in the tape by eye is for most measurements exact enough. If in measuring 100-ft. tape-lengths a point is 1 ft. out of line it will introduce an error of only 0.005 ft. in that tape-length. The error due to poor alignment may be computed by the formula $c - a = h^2/2\,c$, where c is the tape-length or fractional tape-length; h is the offset from the correct line; and a is the correct length. Thus, if one end of the tape is on line and the other 0.8 ft. off line the error in that one tape-length is $0.8^2/200 = 0.0032$ ft. Of course there will be a similar error in the next tape-length, making a total error of 0.0064 ft. The shorter the tape-length the greater is the error due to poor alignment.

Errors Due to Wind can be avoided only by making measurements in calm weather, because it is impossible to determine accurately the amount of these errors under such variable conditions as exist when the wind is blowing.

Temperature Changes of the ordinary 100-ft. steel tapes are about 0.01 ft. per 16° F. Tapes are usually made to be of standard lengths at 68° F. The coefficient of expansion of steel is about 0.00000645 for 1° F. Special tapes are made of alloys whose coefficient of expansion is very small; "Invar" tapes of nickel steel have been used in most government surveys and by many other engineers, the coefficient of expansion being about 1/25th that of ordinary steel tapes. By their use the temperature correction is eliminated except for very exact measurements. When ordinary steel tapes are used the temperature of the tape must be obtained and the temperature correction applied if exact results are desired. Small tape-thermometers are made for this purpose, but unless the thermometer is in contact with the tape and protected from the direct sunlight it will not register the tape temperature. Such an attachment is at best awkward. If the tape can be compared with a standard in sunlight and also in shade and the air temperature taken in the shade at both tests, then the correction to be made in measurements on account of temperature change can be readily determined if the air temperature in the shade is observed when the measurements are taken, for we have an empirical relation between the air temperatures and tape corrections. This is on the assumption, however, that the ratio between the tape temperature when lying in the sunlight and the air temperature in the shade is a constant.

Sag. Unless the tape is supported throughout its length, which is often impracticable, a correction must be applied for the sag of the tape due to its own weight, or, what is more commonly done, the pull is increased so as to stretch the tape an amount equal to the shortening due to sag. If supported at both ends it will hang in a curve of the form of a catenary, and the ends of the tape will therefore be less than its length apart, the amount of error depending upon the weight of the tape, the distance apart of the points of suspension, and the pull. With a 12-lb. pull on a 100-ft. ribbon steel tape supported at its ends the effect of sag will be about 0.01 ft. This may be found for any particular tape by the formula, Shortening due to Sag $= (L/24)\,(wl/t)^2$, where w = weight of tape in pounds per foot of length; t = tension in pounds; l = length of tape in feet between supports; and L = total length of tape in feet. The result will be in feet.

If a tape can be compared in a suspended condition with a standard, the shortening due to different amounts of sag may be determined, and then the proper approximate corrections applied to any measurements by judging the amount of sag.

With a steel tape, if ordinary care is taken in plumbing and in alignment and with rough corrections made for temperature, an accuracy of 1 in 5000 can easily be obtained. For accuracy greater than 1 in 10 000 it is necessary to apply corrections for pull, temperature, alignment, and sag.

In all measurements it is of utmost importance to distinguish between errors which tend to balance and those which continually accumulate, the latter being far more important. For those which tend to balance, the number of errors which will probably remain uncompensated, according to the Method of Least Squares, will be the square root of the total number of opportunities for error. For example, if a 100-ft. tape is 0.02 ft. too long an accumulative error of about 1 ft. will be made in measuring a line one mile long. If, on the other hand, the tape-lengths are not marked closer than 0.05 ft., the total error made by this compensating error of 0.05 will only be $\sqrt{52} \times 0.05 = 0.36$ ft.

14. Magnetic Variations

Surveys with Compass and Chain cannot be relied upon to be closer than about 1 part in 500, since the bearings are read only to the nearest quarter of a degree. This method is therefore adapted only to rough surveys of woodlands where the land is cheap and where there is little danger from local attraction of the needle.

For Transit and Tape Surveys it is desirable in some instances to determine the direction of the true meridian by observations on the sun or on Polaris (the "north star"), as described in Art. 44. In many cases exact bearings of the lines are not needed and it is sufficiently accurate to determine the magnetic bearings by reading the needle. The approximate true bearings may then be determined by applying the declination of the needle to the observed bearings; but in most transit and tape surveys in cities the magnetic bearings are so unreliable that when taken at all they are used merely for the purpose of plotting a needle on the map to show approximately the "points of the compass."

The Declination of the needle is the angle which it makes with the true meridian. The needle rarely points in the true meridian; if it points east of the true meridian it is called an east declination. The declination of the needle at places east of Ohio and the Carolinas is West (in 1925), being a maximum of 22-1/2° in Maine; in the western part of the country it is East, being a maximum of 25° in the state of Washington. The average declination in Alaska is 20-1/2° E; Porto Rico 4-1/2° W; Canal Zone 5° E; Hawaiian Islands 10-1/2° E; Philippine Islands 1° E. The declination continually changes; these changes are called variations.

The Secular Variation is a long slow swing, periodic in its character, and covering many years. In the United States all east declinations are now (1925) gradually decreasing and all west declinations are increasing at an average rate of 2 to 3 min. per year.

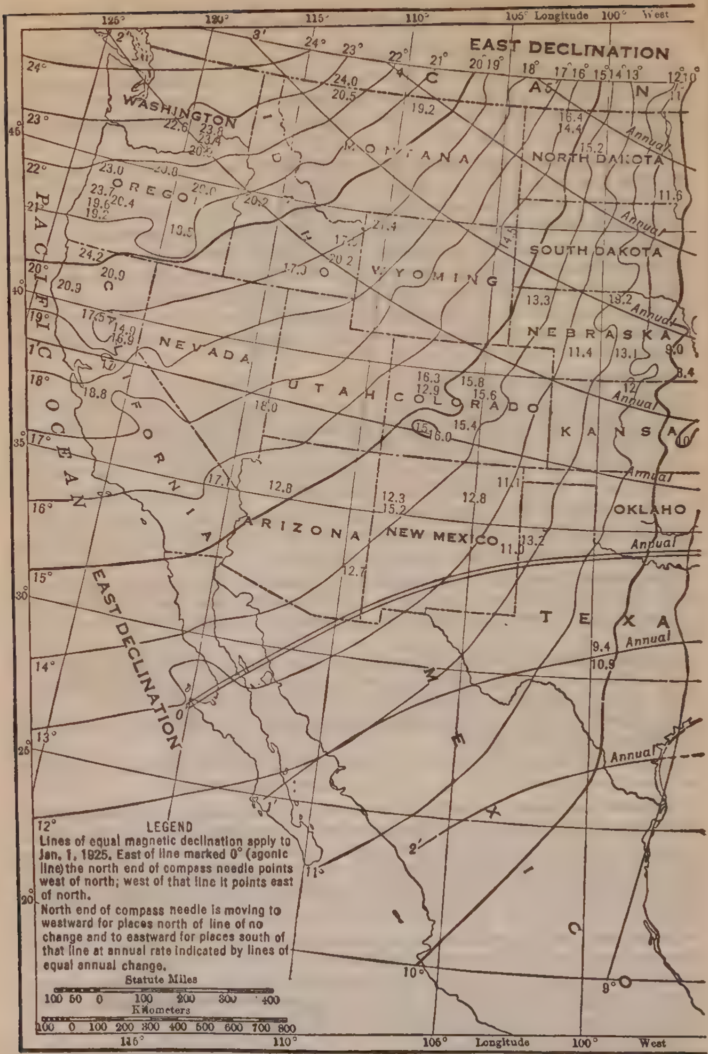
The Daily Variation consists of a swing from the extreme easterly position at about 8 a.m. to its most westerly position at about 1.30 p.m. It is in its mean position at about 10 a.m. and 5 p.m. This daily variation is from 5 to 15 min. of arc.

The Annual Variation (about one minute per year) is so small that it need not be considered in surveying work.

Irregular Variations, due to so-called magnetic storms, are uncertain in character and cannot be predicted. These variations are sometimes large.

Isogonic Lines are lines drawn on a map so as to connect all places where the declination of the needle is the same at a given time. The U. S. Coast and Geodetic Survey has constructed isogonic charts of the United States and these can be obtained from Washington. These charts do not give results with great precision, but are useful in finding approximate values of the declination. They are prepared by plotting upon the map the observed declination at magnetic stations located throughout the country and interpolating results at intermediate places. Observed declinations at these interpolated places frequently show results quite different from those given on these isogonic charts, so that it is necessary whenever the declination of the needle is desired with precision to make observations for finding the true meridian.

Changes in Declination make it necessary in rerunning old lines to modify the given bearings an amount equal to the change in declination which has taken place since the lines were first run. To determine the declination for some past date, records of the U. S. Coast Survey are of assistance provided they had a magnetic station in the vicinity of the place in question. A better way of finding the difference in declination is to take the magnetic bearing of any well-defined line of the old survey, such as be-



Isogonic Chart of United

States for 1925

tween two identified stone heaps, and compare the present bearing of this line with its original bearing.

An Isogonic Chart for Jan. 1, 1925, copied from one issued by the U. S. Coast and Geodetic Survey, will be found on pages 434 and 435. The full lines are the isogonic lines, or lines of equal magnetic declination, the heavy one marked 0° passing through the places in the United States where the north end of the magnetic needle points to the true north. East of the 0° line the north end of the magnetic needle points west of true north, west of that line it points east of true north. Thus, at Boston, Mass., the declination in 1925 was about 14-1/2° W, and at Helena, Mont., it was about 20-1/2° E.

These isogonic lines are constantly shifting; the 0° line is moving westward at a rate between 1′ and 2′ per year. On the chart a double line is seen marked "Annual Change 0"; at all places on that double line there was no change in declination in 1925, at all places east of it the west declination was increasing, at all places west of it the east declination was decreasing. Thus, near Denver, Colo., the east declination decreased nearly 2′ in 1925.

In 1926 the U. S. Coast and Geodetic Survey published a pamphlet (Serial No. 360) entitled "Magnetic Declination in the United States in 1925," which gives tables showing the magnetic declination as observed for many places since 1750. The following table, showing the secular change for eight places is taken from that publication.

Secular Change of the Magnetic Declination

Longitude Latitude..	70° 44°	72° 42°	74° 44°	76° 36°	84° 32°	84° 36°	84° 38°	84° 40°
State....	Maine	Conn.	New York	N. Car.	Ga.	Tenn.	Ky.	Ohio
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
1750	9 34 W	6 46 W	8 00 W	1 18 W	3 04 E	1 11 E	....	....
1760	9 14	6 16	7 19	0 43	3 40	1 50	....	....
1770	9 02	5 55	6 46	0 13 W	4 15	2 28	....	....
1780	9 02	5 43	6 18	0 09 E	4 47	3 02	....	....
1790	9 09	5 40	6 05	0 23	5 12	3 29	....	....
1800	9 29	5 48	6 03	0 27	5 30	3 49	4 28 E	4 03 E
1810	9 56	6 04	6 11	0 24	5 40	3 59	4 39	4 15
1820	10 31	6 29	6 31	0 12 E	5 41	4 00	4 41	4 17
1830	11 12	7 02	7 01	0 09 W	5 35	3 53	4 34	4 09
1840	11 57	7 43	7 40	0 37	5 20	3 36	4 17	3 51
1850	12 44	8 26	8 21	1 12	4 58	3 12	3 53	3 26
1860	13 29	9 08	9 02	1 48	4 28	2 42	3 23	2 56
1870	14 00	9 43	9 41	2 28	3 55	2 07	2 48	2 21
1880	14 30	10 19	10 26	3 04	3 16	1 27	2 08	1 40
1890	14 50	10 45	10 54	3 37	2 37	0 46	1 26	0 57
1900	15 16	11 18	11 29	4 13	2 07	0 14	0 53	0 22
1905	15 33	11 37	11 47	4 31	2 01	0 04 E	0 42	0 09 E
1910	16 03	12 07	12 16	4 51	1 59	0 02 W	0 33	0 02 W
1915	16 33	12 35	12 45	5 10	1 59	0 09	0 25	0 14
1920	16 50	12 55	13 05	5 25	2 01	0 13	0 17	0 24
1925	17 15 W	13 20 W	13 30 W	5 40 W	2 00 E	0 20 W	0 05 E	0 40 W
Annual change in in 1925	5′.0 incr.	5′.0 incr.	5′.0 incr.	3′.0 incr.	0′.0	1′.5 incr.	2′.4 decr.	3′.2 incr.

These figures show that the variation in different decades is not the same and that the values of the declination are represented by a curve. The curve of sines approximately represents these values for a given place.

15. Traversing with Transit and Tape

The Transit and Tape Method is commonly used for running traverses. The instrument is set up at every angle point in the traverse and the angle measured with the transit. The distance between the angle points is measured with the tape. The distances are sometimes measured and recorded as separate distances from angle point to angle point, the angle points being designated by letters or by numbers. Another method is to measure the entire traverse from the first point, which is called " Station 0," continuously throughout the traverse, each 100-ft. point being called a station. If the second transit point is 672.43 ft. from Station 0 it is called " Sta. 6 + 72.43," if the next point is 350 ft. farther it would be recorded " 10 + 22.43 " and so on, so that the station of any point is its distance from the point of beginning of the traverse measured along the traverse.

The Precision with which the measurements should be taken will depend upon the object of the survey. If it is a city survey of valuable property the distances would be measured to the nearest hundredth of a foot, and the angles to 15 seconds; whereas in a farm survey the angles to the nearest minute and the distances to a tenth of a foot or in some cases even to a foot (by the stadia method) is exact enough. Transits used for general surveying work read to minutes or to half-minutes. It is therefore necessary in accurate surveys to measure the angles by repetition, as explained at the end of this article, to obtain results consistent with the accuracy of the tape measurements. A proper appreciation of the relation of distances and angles may be had if it is borne in mind that 0.03 ft. a hundred feet away subtends one minute.

The Angles Measured may be the deflection angle (the angle between the last course produced and the next course), the interior angle, or the azimuth angle. In measuring a deflection angle the telescope has to be inverted, and any error in the line of collimation will therefore be introduced into the deflection angle. This error may be eliminated if the angle is measured first with the telescope direct and then, without changing the circle reading, invert the telescope and " double " the angle; half the final angle is the correct angle. To avoid this necessity of reversing the telescope and repeating the angles some surveyors prefer to measure the interior angles directly.

For Azimuths the instrument is first set up at the point *A* with the circle set at 0°, and the lower clamp loose. The telescope is turned so as to point toward magnetic south or true south (true meridian is the better reference direction) and lower motion clamped. Then the upper plate is unclamped, *B* sighted, and the angle read in a clockwise direction. This gives the azimuth of the line *AB*, an azimuth angle being the total angle read in a clockwise direction from the south around to 360°. The instrument is then taken to *B* so as to obtain the azimuth of *BC*. Invert the telescope and backsight on *A*, the vernier remaining at the same reading it had when azimuth *AB* was read, clamp the lower motion, turn the telescope to its direct position, loosen the upper clamp and sight *C*. This will give the azimuth of *BC* referred to the same meridian as *AB*. Evidently this method does not eliminate any error in the line of collimation adjustment. By the azimuth method, when the survey is about to be closed the azimuth of the first line *AB* can be obtained by setting up again on *A* and using the last course as a backsight. The difference between the first determination of the azimuth of *AB* and its final determination gives directly the total error in the angular work.

As a check against large errors in the angles the magnetic bearing of each line should be read when practicable and compared with the calculated bearing. The calculated bearing of the first line is assumed to be its observed bearing; the calculated bearing of any line is obtained from the calculated bearing of the line next preceding combined with the measured angle between the two lines.

Deflection Angles should be recorded as R (right) or L (left). The algebraic sum of the deflection angles of a closed traverse should equal 360°. If the interior angles are read, the sum of all the interior angles of a closed traverse should be $(n - 2) \times 180°$, where n is the number of lines.

The Error of Closure of a traverse in which the angles are measured to the nearest minute and the distances to a tenth of a foot should not exceed 1/5000. But where the angles are measured to 15 seconds and the distances to hundredths of a foot, a closing error of not more than 1 in 20 000 to 40 000 may be obtained.

Checks on Traverses. In traverses which enclose an area there is a mathematical check on the distances provided the angles are correct, as shown in Art. 16. But in traverses which do not close there is no check on the distances other than by remeasurement of the lines (preferably in the opposite direction from the first measurement), except by cut-off lines which form closed traverses of portions of the survey. The angles can be checked by determining the true bearing of the first line of the traverse by solar or stellar observation, and whenever it is desired to check the angles the true bearing of any course can be determined by another meridian observation. The meridian can be determined to the nearest minute by solar observation and much closer by observation on the pole star.

To Measure an Angle by Repetition, set up the transit at A, set the verniers at 0°, sight B, and clamp lower clamp. Loosen upper clamp and sight C, read and record the angle. Leaving the two plates clamped together, unclamp the lower clamp, and sight B, unclamp upper clamp and sight C, and read and record the angle. Half this angle should check the first angle read, and the result obtained is more exact than the first angle. It is evident that by repeating this process with a one-minute instrument for six times and dividing by 6 an angle to ten seconds can be obtained. This method of repetition might be carried even further, but ordinary transits are not good for results much closer than ten seconds.

This method of repetition can be readily applied to laying off an angle. The angle is first laid off and a point set, and then the angle which has been laid off is measured by repetition as described in the previous paragraph. If it is found to be in error, say 20 sec., the point on the stake is moved in the proper direction a distance equal to the distance from the point set to the instrument $\times$ tan 20 sec.

16. Areas of Fields

Corrections of Field Notes are made before computation for area is begun. Errors in tape measurements due to erroneous length of tape are corrected. Errors in angles, provided they are not large enough to indicate mistakes, are eliminated by "balancing the angles," which means altering the value of those angles which were taken from short sights or those angles where the error is most likely to lie. The calculated bearing of each course is computed starting from one course whose bearing is either known or assumed.

The Double Meridian Distance Method of computing an area is as follows: The data are usually tabulated as shown in the computation below. The **latitude** of a course equals the distance times the cosine of the bearing; the **departure** is the distance times the sine of the bearing. North latitudes and East departures are regarded as positive values and South latitudes and West departures are negative. After these have been computed the error in latitude and in departure is found, which is the difference between the plus and the minus latitudes and the plus and minus departures. If these errors are not large enough to indicate a mistake in the measurements or computations then they are distributed among the latitudes and departures according to the following rule if it is a transit and tape survey. The correction applied to the $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ of any course is to the total error in $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ as the $\left\{\begin{matrix}\text{latitude}\\ \text{departure}\end{matrix}\right\}$ of that course is to the sum of all the $\left\{\begin{matrix}\text{latitudes}\\ \text{departures}\end{matrix}\right\}$ (without regard to algebraic signs). Any knowledge of difficulties met in the field which would lead the surveyor to suspect that the error lay in certain lines should take precedence over this rule. Furthermore it is more probable that on account of sag of the tape and small obstacles on the line the recorded distances are too long rather than too short, and therefore it is not good practice to apply the above rule when it lengthens any of the distances. In the example here given, however, the rule is rigidly followed in balancing this survey in order merely to show its application. The algebraic sum of balanced latitudes and the departures should be zero.

The next column to compute is the double meridian distance (D.M.D.) of each course. The D.M.D. of the first course equals the departure of the first course. The D.M.D. of any other course equals the D.M.D. of the course preceding plus the departure of the preceding course plus the departure of the course itself. The D.M.D. of the last course should be numerically equal to the departure of the last course but should have the opposite algebraic sign. The algebraic signs must be carefully observed not only in computing the D.M.D.'s but in all of this computation for area. If the D.M.D.'s are computed beginning at the most westerly point in the traverse all the D.M.D.'s will be plus. The last column of positive and negative double areas is found by multiplying each D.M.D. by its corresponding latitude. Half the algebraic sum of the double areas gives the area of the field.

Computation for Area by Double Meridan Distance Method

Station	Bearings	Distance, ft.	Latitudes	Departures	Balanced Latitudes	Balanced Departures	D.M.D.	Double areas
A	N 56° 25′ E	540.91	+ 299.20	+ 450.62	+299.19	+450.66	+ 450.66	+134 833
B	S 64° 27.5′ E	198.44	− 85.56	+ 179.05	− 85.56	+179.06	+1080.38	− 92 437
C	S 67° 50′ E	212.46	− 80.16	+ 196.76	− 80.16	+196.78	+1456.22	−116 731
D	S 68° 34′ E	98.13	− 35.86	+ 91.34	− 35.86	+ 91.35	+1744.35	− 62 553
E	S 29° 17′ E	186.75	− 162.89	+ 91.35	−162.90	+ 91.36	+1927.06	−313 918
F	S 61° 26.5′ E	651.70	− 311.55	+ 572.41	−311.57	+572.45	+2590.87	−807 234
G	S 66° 50′ W	910.92	− 358.37	− 837.46	−358.39	−837.40	+2325.92	−833 583
H	N 45° 21′ W	1046.25	+ 735.28	− 744.32	+735.25	−744.26	+ 744.26	+547 217
		3845.56	+1034.48 −1034.39 error +0.09	+1581.53 −1581.78 −0.25				2)1 544 406 772 203 sq. ft.

This result should be roughly checked by determining the area by use of the planimeter (Art. 8) or by dividing it into triangles, scaling the bases and altitudes and computing the area. The agreement between the sum of the + and − latitudes and + and − departures is a check on the multiplication of the distance by the cosine and sine of the bearing. The D.M.D.'s are checked as shown above. But there is no check on the double areas. A good exact check on the area is to compute from the latitudes the double parallel distances (D.P.D.) just as the D.M.D.'s were computed from the departures, and then obtain double areas by multiplying each D.P.D. by its departure. If l is the error in latitudes and d the error in departures, the error of closure of the survey equals $\sqrt{l^2 + d^2}$. Thus, the error of closure of the survey shown above is 0.266 ft., and

$$\frac{\sqrt{0.09^2 + 0.25^2}}{3845.56} = \frac{0.266}{3846} = \frac{1}{14\,500} = \text{relative error of closure}$$

Sometimes it is not possible to follow the perimeter of the field with the traverse, in which case the fractional areas to be added to or subtracted from the area of the traverse are computed by dividing them into triangles, rectangles or trapezoids.

Large mistakes made in the fieldwork will be detected in the computation by the D.M.D. method. If the latitudes and departures do not balance within reasonable limits, the error in departure divided by the error in latitude equals the tangent of the bearing of the line which would represent the error of closure, and if only one mistake has been made in measuring the distances it probably lies in a line having this angle for its bearing.

Compass Surveys should be balanced by the following rule, because the errors are more likely to be due to the rough results obtained in the angles than in the distances. **The correction to be applied to the $\left\{\begin{array}{l}\text{latitude}\\\text{departure}\end{array}\right\}$ of any course is to the total error in $\left\{\begin{array}{l}\text{latitude}\\\text{departure}\end{array}\right\}$ as the length of the course is to the perimeter of the field.**

Area of a Field by Rectangular Coordinates. Where the coordinates of the corners of a property are known the coordinate method of computing its area is more direct than the D.M.D. method. Let x_a, x_b, x_c, etc., and y_a, y_b, y_c, etc., represent the abscissas and ordinates of points A, B, C, etc., of the same area computed above by the D.M.D. method. By the rectangular coordinate method the area is found as follows: Every abscissa is multiplied by the difference between the following and preceding ordinates, always subtracting the following from the preceding. The algebraic sum of these products divided by 2 gives the area. Expressed as a formula, this rule is

$$\text{Area} = 1/2\left\{x_a(y_b - y_h) + x_b(y_c - y_a) + x_c(y_d - y_b) + x_d(y_e - y_c) + \ldots\right\}$$

Below is the computation by coordinates of the same area as that above. The origin of coordinates is 100 ft. W and 2 000 ft. S of the initial point.

Computation for Area by Coordinate Method

Station	x	y	Difference between adjacent y's	Double areas +	Double areas −
A	100.00	2000.00	−1034.44	....	103 444
B	550.66	2299.19	− 213.63	....	117 637
C	729.72	2213.63	+ 165.72	120 929	....
D	926.50	2133.47	+ 116.02	107 493	....
E	1017.85	2097.61	+ 198.76	202 308	....
F	1109.21	1934.71	+ 474.47	526 287	....
G	1681.66	1623.14	+ 669.96	1 126 645	....
H	844.26	1264.75	− 376.86	....	318 168
				2 083 662	539 249

Area = 1/2 (2 083 662 − 539 249) = 772 206 sq. ft.

17. Crooked Boundaries

Curved Boundaries, such as brooks, are often located by measuring the perpendicular offsets from the traverse line which has been run alongside of the brook. The curvature of the boundary may be so flat and uniform that it will be feasible to take the offsets at regular intervals, while in cases where the boundary changes in direction abruptly it is necessary to take the offsets only at the points where these changes occur. In either case there will be the areas of several trapezoids to compute to find the area between the traverse line and the boundary, but if the offsets are taken at equal intervals then the computation may be made by one of the following methods:

By the Trapezoidal Rule, Area $= 1/2\, d\,(h_e + 2\,\Sigma h + h_e')$, where d = common interval between offsets; h_e and h_e' = end offsets of the series of trapezoids; and Σh = sum of intermediate offsets. This rule assumes that the boundary is a straight line between adjacent offsets.

Simpson's One-third Rule assumes that between adjacent offsets the boundary is a curve (a parabola). By this rule **Area** $= \frac{d}{3}\,(h_e + 2\,\Sigma h_{odd} + 4\,\Sigma h_{even} + h_e')$, where d = common interval between offsets; h_e and h_e' = end offsets of the series; $2\,\Sigma h_{odd}$ = twice the sum of the odd offsets except the first and last (the 3rd, 5th, 7th, etc.); $4\,\Sigma h_{even}$ = four times the sum of the even offsets (the 2nd, 4th, 6th, etc.). For this rule to apply there must be an odd number of offsets; if there is an even number, compute the area of one end trapezoid separately.

To Straighten a Crooked Boundary is to run a straight line cutting the crooked boundary so as to make the lots on either side of the straight line have the same areas as they have with the crooked boundary for the division line. To do this, first run a trial line AB (Fig. 8), then measure proper offsets to the crooked boundary and compute the areas X, Y and Z on each side of AB between it and the crooked boundary. The sum of X and Z should equal Y, for $X + Z$ is the amount taken from Lot M, and Y is the amount taken from Lot N. If $X + Z$ does not equal Y the difference is the amount the trial line has taken from one lot more than it has taken from the other, so this whole difference must be returned. For example, suppose the original area of Lot M = 50 000 and Lot N = 40 000. When AB is run it is found that the area of $X = 300$, $Y = 1000$, and $Z = 500$, that is, the trial line AB has made Lot M = 50 000 − 800 + 1000 = 50 200, and Lot M = 40 000 − 1000 + 800 = 39 800. An area 1000 − (300 + 500) = 200 must therefore be taken from Lot M and added to Lot N by running the final line AD so as to make the area ADB equal to 200.

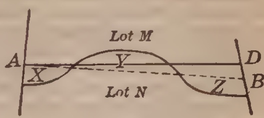


Fig. 8

18. Old Lines

In Rerunning Old Property Lines the surveyor must first determine where the original boundaries of the property lie and then survey those boundary lines. He should not attempt to correct the original lines even though he may be sure that errors exist in them. He must first of all find the physical evidence of the location of the boundaries, and failing in this he should base his judgment as to their location on such evidence as occupancy or dimensions given in deeds or the word of competent witnesses. It must not be assumed that a boundary is missing because it is not at once visible. Stone bounds are often buried two or three feet deep; the top of a stake soon rots off, but evidences of the existence of the stake are often found many years after the top has disappeared, and the supposed location should be carefully dug over to find traces of the old stake.

In Interpreting a Deed it is assumed that it was intended to convey property the boundaries of which will form a closed traverse. Therefore, if it is found that the omission of a whole chain-length or the reversing of the direction of a bearing will make a deed description close, this change in dimensions may be made, for it is assumed that the description is of a closed field. Where the record of the original survey does not close, the deeds of adjoining property may be of assistance. Where artificial features are mentioned as boundaries, these always take precedence over the recorded measurements or angles, but these marks must be mentioned in the deed in order to have the force or authority of monuments. When the area does not agree with the boundaries as described in the deed those boundaries control. All distances unless otherwise specified are to be taken as straight lines; but distances given as so many feet along a wall or highway are supposed to follow these lines even if they are not straight. When a deed refers to a plan the dimensions on this plan become a part of the description of the property.

Legal Boundaries. Where property is bounded by a highway the abutters own to the center line, but where it is an accepted street each abutter yields his portion of the street for public use; if, however, the street is abandoned the land reverts to the original owners. If a street has been opened and used for a long period, bounded by walls or fences, and there has been no protest regarding them, these lines usually hold as legal boundaries. In the case of a line between private owners acquiescence in the location of the boundary will, in general, make it the legal line; but if there is a mistake in its location and it has not been brought to the attention of the interested parties or the question of its position raised, then occupancy for many years does not make it a legal line.

Where property is bounded by a non-navigable stream it extends to the thread of the stream. If the property is described as running to the bank of the river it is interpreted to mean to the low-water mark unless otherwise stated. Where original ownership ran to the shore line of a navigable river and the water has subsequently receded, the proper sub-division is one that gives to each owner along the shore his proportional share of the channel of the river; these lines will therefore run, in general, perpendicular to the channel of the stream from the original intersection of division lines and shore lines.

A more complete statement of the principles mentioned above, particularly with reference to the U. S. Public Land Surveys, will be found in an address on "The Judicial Functions of Surveyors," by Chief-Justice Cooley of the Michigan Supreme Court. See Proceedings Mich. Association of Engineers and Surveyors, 1882, pp. 112–122.

Should all evidence of artificial boundaries of the property be missing the surveyor will have to use the deed description as the best evidence. Where the directions of the lines are given as magnetic bearings it is necessary first to determine the declination of the needle at the date of the survey. The declination should be stated in the deed or on the original plan, but it seldom does appear in either. If the date can be established the declination for that year and place may be obtained from the records of local surveyors or from past U. S. Coast Survey records. If one line can be identified as a boundary the difference between its present magnetic bearing and its original bearing gives the difference in declination directly, and all the rest of the deed lines can be run out by correcting all the bearings by this amount. The chain used in the original survey may have been of different length from the one now used; this can be readily determined by measuring the length of any of the well-defined lines of the property.

19. Obstacles and Inaccessible Distances

A Random Line is sometimes required on account of obstacles on line. If it be required to run the line AB (Fig. 9), neither point being visible from the other, a random line AX can be run. Measure the perpendicular distance BC and also AC. The length of AB may then be computed and any desired point may be set on the line AB as follows. Suppose a point D is to be set on AB and that a perpendicular from D meets AC at E, then $AE = AD \times AC/AB$, and $ED = BC \times AD/AB$. By this method, then, it may not be necessary to run out and actually measure the line AB. The angle CAB can be found from CB and AC.

If a straight line such as AX cannot be run, a traverse composed of several straight lines and angles may be made to connect A and B, and using AB as the closing side of a closed traverse, its length and direction may be computed (Art. 16), and also the coordinates of any point in the traverse may be found referred to AB so that distances may be laid off from the traverse which will define any desired points on AB. This traverse method, since it involves the measurement of angles, will not lend itself as readily to accurate results as the case first described where the random line is a straight line. In running a straight line where it is necessary to produce the line by reversing the telescope it should always be done by taking the mean result of a double reversal, the telescope being erect in taking the first backsight and inverted when taking the second backsight, so as to eliminate any error in the line of collimation.

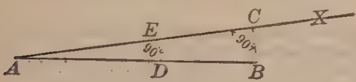


Fig. 9

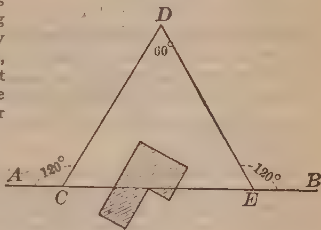


Fig. 10

An Equilateral Triangular Traverse about an obstacle is a special case of the above method. Let the obstacle be on the line AB (Fig. 10); the instrument is set up at C, an angle of 120° is laid off, and a stake set a sufficient distance away at D, and CD is measured. The transit is then set up at D and an interior angle of 60° laid off so as to run a line that will cut AB; this line is made equal to CD and stake E is set. With the instrument at E an angle DEB is laid off equal to 120° which should be sighting along AB if the work has been done accurately. Evidently $CD = ED = CE$. This method is weak in accuracy owing to its dependence upon angle measurements. A slight error in any angle will introduce an appreciable error in CE. If CD is short the error in CE is small, but the error in producing EB by an angle laid off from the short side ED is likely to be large.

Running Parallel Line Past Obstacle. One of the most exact ways of producing a straight line past an obstacle of limited size, such as a house, is to run an offset parallel transit line past the obstacle as follows. The instrument is set up at C (Fig. 11) and a right angle ACC' laid off with the transit.

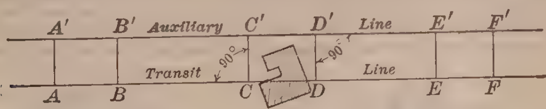


Fig. 11. Running Parallel Line past Obstacle

CC' is made any convenient distance which will bring the auxiliary line beyond the obstacle. Similarly, point A' is set opposite point A, and sometimes a second point B' opposite B. These points A' and B' need not be set by means of a transit set up at A and B if AA' is short. The instrument is then set up at C' and backsighted on A', the sight is checked on B', the telescope inverted, and points D', E' and F' set on line. Leaving the telescope inverted, another backsight is taken on A', and the process repeated to eliminate any error in the line of collimation. Then the transit is moved to point D', and a right angle turned off, and point D set, the distance $D'D$ being made equal to $C'C$. Then by setting up at D and sighting ahead on F ($FF' = DD'$), and checking on point E ($EE' = DD'$), the transit line is again run forward in its original location. The distance $C'D'$ is carefully measured, which gives the distance CD, and thus it appears why it is so necessary that the lines CC' and $D'D$ shall be laid off at right angles by means of the transit. The other offsets AA', BB', EE' and FF' are not in any way connected with the measurement along the line; they simply define the direction of the line, so that if convenient it is often only necessary to show these distances as swing offsets for the transitman to sight on. The swing offset is given by swinging a tape in a horizontal arc about A as a pivot. The zero point of the tape is held at A, and at the distance AA' out on the tape a pencil is held vertically on which the transitman sights when the pencil appears to be swung out farthest from A so that his line of sight will be tangent to the arc. The offsets BB' and EE' are not absolutely necessary, but they serve as desirable checks on the work, and in first-class surveying they should not be omitted. For obvious reasons the offsets AA' and FF' should be taken as far back from the obstacle as is practicable.

Should the house be in a hollow so that it is possible to see over it with the instrument at A, the point F, or a foresight of some sort, should be set on line beyond the house to be used as a foresight when the transit is set up again on the original line. The distance may be obtained by an offset line around the house. Sometimes it is possible to place exactly on line on the ridgepole of the house a nail, which gives an excellent backsight when extending the line on the other side of the building.

If the building has a flat roof and it is possible to set a point on the roof exactly on line, move the instrument to this point on the roof, and prolong the line in this way. Under these conditions the transitman will have to be extremely careful in the use of his instrument on account of its insecure foundation. If he walks around the transit he will find that it affects the level bubbles and the position of the line of sight; it is therefore well for him if possible to stand in the same tracks while he backsights and foresights. Sometimes two men, one in front and one behind the transit, can carry on the work under these conditions more accurately and conveniently. This method insures an accurate prolongation of the line, but the distance through the building must be found by an offset method, by plumbing from the edge of the flat roof, or by inclined measurements.

To Measure an Inaccessible Distance where the line is visible, as across a pond, several methods may be employed:

(1) Lay off from the transit line AC (Fig. 12) a line AB which passes by the end of the pond so that it can be taped and set stake B; then set up the instrument at B and lay off $ABC = 90°$, point C being obtained by intersecting the main transit line. Measure angle CAB and side AB, from which $AC = AB/\cos CAB$; or, better, $AC = AB + AB \operatorname{exsec} CAB$. The measurement of the angle at C and of CB will serve as checks.

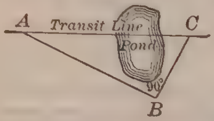


Fig. 12

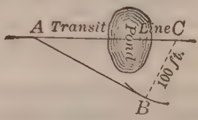


Fig. 13

(2) With the instrument at A (Fig. 13) and a swing offset of 100 ft. from C (some point on the main traverse line on the other side of the pond) measure the angle between CA and a tangent AB to the swing offset. $AC = 100/\sin CAB$.

(3) Any line AB (Fig. 14) may be laid off along the shore of a river, and a point C set on the main traverse line across the river, measure angles A and B, and C as a

check, and from AB as a base compute AC by trigonometry. If AB is run perpendicular to CA and made some number of hundred feet long the computation is greatly simplified.

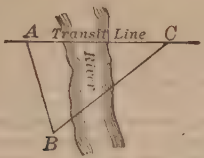


Fig. 14.

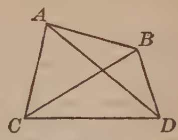


Fig. 15

To Obtain the Distance between Two Inaccessible Points A and B (Fig. 15) by observations from two accessible points C and D, measure DC and angles ADC, ADB, ACB and BCD. Compute CB in triangle CBD, AC in triangle ACD, and then in triangle ACB compute AB.

To Obtain the Inaccessible Distance AB (Fig. 15) between two accessible points by observation on two inaccessible points C and D when distance CD is known. Measure angles CAD, DAB, CBD and ABC. Assume $AB = 1$. Then compute CD by the process described in the above paragraph. This gives a ratio between AB and CD, and since CD is known, the actual length of AB may be computed.

20. City Surveying

Staking Out Line and Grade for streets, curbs, sewers, and pavements is constantly done by a city engineering department. This class of work as a rule calls for lines and grades to the nearest hundredth of a foot. For this reason it is customary to use transits with the horizontal arcs graduated so as to read 30″, 20″ or even 10″, and to use steel tapes graduated to hundredths of a foot. In some cases the spring balance and the thermometer should be used so as to make the proper tension and temperature corrections (Art. 13).

A Standard of Length is established as a rule by carefully transferring the length of some other standard by means of different tapes and under different weather conditions, or by means of tapes which have been standardized by the U. S. Bureau of Standards. This standard should be placed where it will not be exposed to the direct rays of the sun. When the tape is tested it should be stretched out at full length beside the standard and left there until it acquiries the same temperature as the standard before the comparison is made.

Street Lines are usually marked by monuments, but the best practice requires that besides these monuments, which may become misplaced, accurate offsets to the underpinning of buildings along the line shall be measured and recorded so that if the monuments become disturbed, due to building operations or paving of the sidewalks, they can readily be replaced in their exact location. These monuments are usually ordinary stone bounds 3 or 4 ft. long and 4 to 8 in. square on top. The bound should be long enough so as to rest on ground below frost. A drill-hole in the center of the top marks the point; a more exact method of marking is to fill this drill-hole with lead and drive a very small copper tack in the lead at the exact point. A copper bolt, with a punch hole marking the exact point, is sometimes inserted in the drill-hole.

To Set a Stone Bound in the place of a stake marking a corner, first drive four temporary stakes around the corner stake 3 or 4 ft. from it and in such a way that a line stretched from two opposite stakes will pass over the tack in the head of the corner stake. Then tacks are carefully set in the tops of these temporary stakes in such positions that a stretching line running from the tack on one stake to the tack on the opposite stake will pass exactly over the tack in the corner stake. Then the corner stake is removed and the hole dug for the stone bound. Care should be taken not to dig the hole any deeper than is necessary so that the bound may be set on firm earth. When the hole for the bound has been dug to the proper depth the monument is dropped into the hole in such a manner that it will be plumb. The fill around the bound should be placed with considerable care, the material being properly rammed as the filling proceeds and the bound kept in such a position that the drill-hole in the top of it shall be exactly under the intersection of the strings. It is sometimes desirable to put in a foundation of concrete and to fill with concrete around the monument to within a foot of the surface where a very substantial bound is required, or where the ground is so soft as to furnish an insecure foundation.

Curved Street Lines are as a rule composed of simple or compound circular curves and are laid out by the method of deflection angles as explained in Art. 56. But the lengths of these curves are the actual length of arcs; this calls for a slight modification in the computations and in measuring the lengths from that used in railroad practice. The length of the curve in city practice equals the radius × the circular measure of the central angle. Since the tape must be stretched along chords in making the measurements the station points to be set by the deflection angle method will be located by measuring the chord subtending the arc of say 50 or 100 ft. depending upon how frequently the points are to be located on the curve. A convenient formula for determining the difference in length between the arc and its chord is $\mathbf{L - C = L^3/24\,R^2}$, where L is the arc, C is the chord, and R the radius, all in the same units. This formula is correct to the nearest hundredths of a foot for chords of 50 ft. or less where the radius is 150 ft. or greater, or for chords of 25 ft. or less and radii 25 ft. or greater. It is sufficiently exact to compute the results by such a formula with the ordinary slide rule. The deflection angle to any point on the curve equals half the total central angle multiplied by the ratio of its distance from the instrument to the total length of curve (see Art. 56).

In Staking Out Street Grades for setting curbs, for pavement or sewer construction, self-reading rods, such as the Philadelphia rod, are the most practicable, although target rods are used to some extent. The target rod is particularly useful in taking an individual exact reading at any considerable distance from the instrument, which is required in city surveying more than in topographic surveying.

A target rod is also useful in "shooting in grades," for curbs or sewers as follows. If it is a straight grade from Sta. 0 to 5 and grade stakes have been set at 0 and 5 the instrument is set up just to one side of Sta. 0, the rod held on the grade stake at Sta. 0, and target clamped at the height of the telescope, then the rod is held on the grade stake at Sta. 5 and the telescope inclined so as to sight the target and clamped. The line of sight is then parallel to the grade line, and any intermediate stake may be set by driving it until the target coincides with the horizontal hair of the telescope when the rod is held on the stake.

It often happens that the pavement is so hard that stakes cannot be driven or that it is not advisable to use stakes on account of the danger to pedestrians, in which case the grades are marked on fences and buildings, and frequently a convenient place is not found at the grade height. In such cases the grade is marked one or two feet higher or lower than the grade height and marked on the building thus $\underline{\diagdown^{2'}\diagup}$ meaning that the correct grade is 2 ft. lower than this mark, or $\overline{\diagup_{1'}\diagdown}$ meaning that the grade is 1 ft. above this mark; the arrow points in the direction and the number gives the distance to measure to reach the proper elevation.

When the grades are marked in hard pavements it is customary to drive stout spikes flush with the ground and take the elevation of the tops of these spikes and make a record of the amount the several spikes are above or below grade to be given to the foreman.

Both curb and sewer grades are as a rule laid out as above described. The lines given for such construction are usually parallel lines, 3 to 6 ft. to one side of the center line, and marked by stakes or spikes.

Parabolic Vertical Curves are used to connect grades of different slope, the same system of computing elevations for points on the curve being used as explained in Art. 60.

A Rectangular Coordinate System based upon plane surveying, disregarding the effect of curvature of the earth, is used in the survey of some of the large cities, such as Baltimore and Boston. Two arbitrary lines at right angles to each other are chosen as axes of a coordinate system, and all important points, such as street corners, are located with reference to these coordinates. One of the chief advantages of this system is that any number of lost points can be easily replaced because their relation to the remaining located points in the city is known.

In laying out a system of this sort a base-line is measured and a triangulation scheme computed as a basis, secondary triangles being formed from the larger ones, and then traverses are run from these triangulation points and closed on other triangulation points. All of the triangles are considered to be plane triangles in the computation. The origin of the coordinate axes is either taken so as to lie outside of the city limits or else, if it is inside, it is given a value such as 20 000, 20 000, so that negative values for coordinates are avoided.

21. Special Problems

Setting Batter-boards for a Building. One of the most common tasks of the surveyor is to set the batter-boards for the excavation and construction of the cellar of a new building. For a brick or stone building the lines to be defined are the outside lines of the building, and the elevation desired is usually the top of the first floor. In the case of a wooden building the line usually given is the outside line of the brick or stone underpinning, and the elevation given is the top of this underpinning on which the sill of the house is to rest. Sometimes the outside line of the sill is desired instead of the outside line of the underpinning. There should be a definite understanding in regard to these points before the work of staking out is begun.

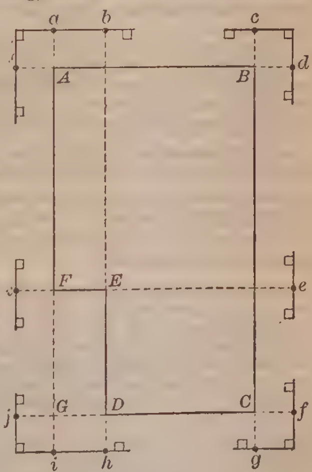


Fig. 16

The first work is to stake out the location of the building by accurately setting temporary stakes at all of the corners of the building—in Fig. 16, at A, B, C, D, E and F. A stake should be set at G also so that the entire work can be checked by measuring the diagonals AC and BG, and FD and EG. These checks should always be applied where possible. Then the posts for the batter-boards are driven into the ground 3 or 4 ft. outside the line of the cellar so that they will not be disturbed when the walls are being constructed. On these posts, which are usually of 2 × 4-in. scantling, 1-in. boards are nailed. These boards are set by the surveyor so that their top edges are level with the grade of the top of the underpinning or for whatever other part of the building the grades are required. After the batter-boards are all in place they should be checked roughly by sighting across them; they should all appear at the same level. Sometimes, however, on account of the slope of the ground some of them have to be set a definite number of feet above or below grade.

Then the lines are to be marked by nails driven in the top of these batter-boards. The transit is set up on one of the corner stakes of the house at *B* (Fig. 16), for example, and a sight is taken on *C*. This line is then marked on the batter-board beyond (at *g*) and on the one near the transit (at *c*). Then a sight is taken along *BA* and this line is produced both ways and nails set on the batter-boards at *l* and *d*. In a similar manner all of the lines are marked on the batters. These points should be marked with nails driven in the top-edges of the batter-boards, and there should be some lettering on the boards to make clear which lines have been given. It is well for the surveyor also to show these marks to the builder or inspector and have it clearly understood just what parts of the structure these lines and grades govern.

Supplying Closing Side of a Traverse. If the latitudes and departures of the several courses of a traverse which does not close are computed, the algebraic sum of the latitudes give the latitude of the closing side and the algebraic sum of the departures gives the departure of the closing side (Art. 16). The square root of the sum of the squares of these two elements gives the length of the closing side, and the tangent of its bearing is its departure divided by its latitude.

To Cut Off an Area by a straight line from a point on a side: Plot the field and the known point; draw a trial line from this point to an angle in the other side of the field so as to lay off approximately the required area. Then the sides and bearings of all the lines except the trial line across the field are known; and its length and bearing can be computed as explained in the previous article, and the area of the portion cut off computed by the D.M.D. method (Art. 16). The difference between this area and the area required is a triangle whose base is the trial line and whose altitude can be readily computed, from which the distance along the side of the field from the end of the trial line to the correct point can be computed. Then the closing side can again be calculated and the area cut off computed by the D.M.D. method, which should this time give the desired result.

To find the area cut off by a line in a given direction from a given point, let the cut-off line be *AC*. This problem may be readily solved by drawing a line from *A* in the traverse to the corner *B* which lies nearest the other extremity *C* of the cut-off line. The area of the portion of the traverse cut off by *AB* is then computed (Art. 16), and to this area is added or from it is subtracted the area of the triangle *ABC*.

Athletic Grounds. In the layout of athletic grounds the matter of orientation, so that the sun will not shine in the eyes of the players or in the eyes of the spectators, is of great importance. A baseball, football or tennis ground should therefore be laid out with its length running in a north and south direction, and with the grand-stand on the west side because most games are played in the afternoon.

Running Tracks should be as a rule one-eighth or one-quarter mile in length, because on tracks of these lengths a full number of laps (or half laps) are required for the ordinary dashes of 220 yd., 440 yd. and 880 yd., thus bringing both the start and the finish of the race in front of the grand-stand. A good running track should be 15 ft. wide on all portions except in front of the grand-stand, where it should be made 20 ft. wide for use in 100-yd. dashes, in which race the contestants are usually "bunched." For these short dashes the straightaway portion of the track is usually lengthened beyond the curve so as to allow at least 40 ft. beyond the finish line. The curves at the ends of an oval field-track should not be sharper than 100-ft. radius if it is possible to avoid it. The line for measurement of the length of a running track is 18 in. from the pole, and in a horse race-track it is 3 ft. from the pole. A good running track can be made in three layers: 8 in. of rock, 2 in. of cinders, and 2 in. of 3 parts screened cinders to 2 parts screened clay loam. A quarter-mile track with curves of 100-ft. radius at its ends will allow placing a football gridiron inside the oval with 20 ft. of turf between the side lines of the football field and the running track, which should be about the minimum distance. One of the best example of this arrangement is the quarter-mile track shown in Fig. 17. The Harvard Stadium is another example of this arrangement, but there the side lines of the football field are too near the running track.

Football Grounds should be 360 ft. × 160 ft., and on level ground; the goal lines are 300 ft. apart, but the goal posts are 360 ft. apart. Lines 5 yd. apart and parallel to the goal lines are laid out to aid in estimating distances made in the plays. The goals are placed in the center of the ends of the field; the goal poles are 18-1/2 ft. apart, and the horizontal cross-bar 10 ft. above the ground. It is of importance, if a good football field is to be made, to screen the surface material through a half-inch screen.

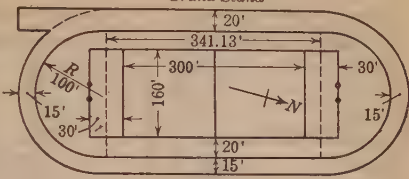


Fig. 17

A Baseball Diamond is laid out as shown in Fig. 18. The size of the field itself is limited only by the space available, except that the shortest distance from home plate to any fence between the foul lines should be 235 ft. The ground should be as near level as possible; there must not be more than 15 in. fall from the pitcher's box to the base lines or to home plate, and the base lines should be level. Where possible to obtain it, there should be 90 ft. behind the home plate, with a minimum of 50 ft. The official junior diamond for boys under 16 years is 82 ft. square and has a pitching distance of 50 ft.

Fig. 18

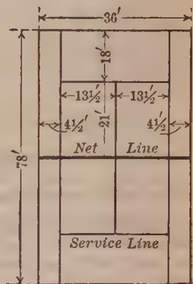


Fig. 19

A Tennis Court is laid out as shown in Fig. 19. An area of 120 ft. × 66 ft. is about the minimum for a good tennis court. It should have surface drainage by grading the court on a slight slope (about 6 in. per 100 ft.) from the center toward the sides. The net posts are placed 3 ft. outside the side lines, and are 3.5 ft. high; the net should be 3 ft. high at the center.

Field Hockey grounds are laid out as shown in Fig. 20. The length is always 300 ft., but the width must be not less than 165 ft., nor more than 180 ft.; always 180 ft., if possible.

In Lacrosse, the field is rectangular, 110 ft. long between goals which are set 6 ft. apart and 6 ft. high in the middle of the ends of the rectangle. The width should be not less than 210 ft. nor more than 255 ft.; there should be 60 to 100 ft. of level ground behind each goal. Each pair of goals is in the middle of a rectangle which measures

18 ft. crosswise of the field and 12 ft. lengthwise. In the center of the field is a circle of 10-ft. radius.

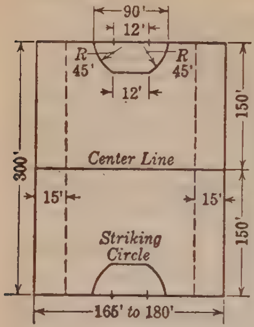


Fig. 20

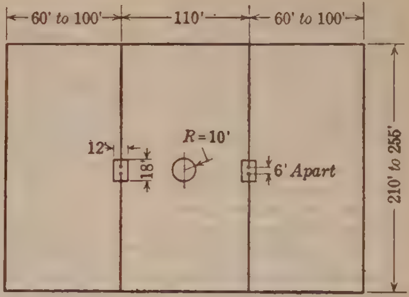


Fig. 20a

In Soccer the field is rectangular, not less than 165 ft. nor more than 195 ft. wide, and not less than 300 ft. nor more than 330 ft. long. The field is marked and goals located as in Fig. 21.

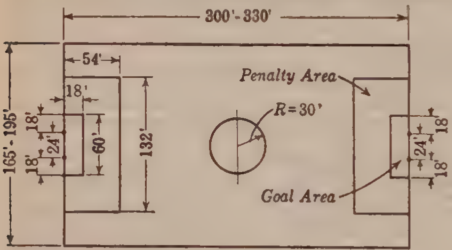


Fig. 21

In Basketball the rectangular court is not less than 35 ft. nor more than 50 ft. in width, and not less than 60 ft. nor more than 94 ft. in length. The field is marked and the goals located as in Fig. 22.

Badminton Courts courts are rectangular and are laid out as in Fig. 23. The net extends 8 ft. on each side of the central line of the courts and at right angles to it; it is 5 ft. high at the center and 5 ft. 1 in. at the posts.

In Quoits the rink should be about 80 ft. long and 25 ft. wide. The pitching distance is 54 ft. Stiff by sticky pottery clay should be used around each iron pin so that the quoit will stay where it lands.

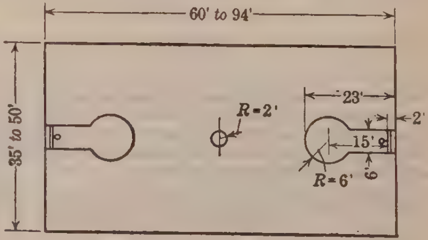


Fig. 22

22. United States Public Lands

The System. The United States system of surveying public lands, inaugurated in 1784 and since modified by various acts of Congress, requires that public lands "shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles so as to form townships six miles square," and that corners of townships thus surveyed "must be marked with progressive numbers from the beginning." Also, that townships shall be subdivided into 36 sections, each of which shall contain 640 acres, as nearly as may be, by a system of two sets of parallel lines, one governed by true meridians and the other by parallels of latitude, the latter intersecting the former at right angles, at intervals of a mile. Since meridians converge, it is evident that the requirement that the lines shall conform to true meridians, and also that townships shall be six miles square, is mathematically impossible.

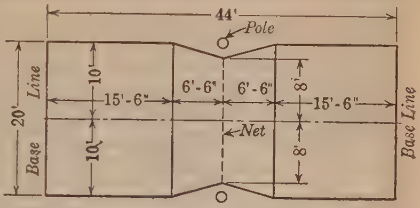


Fig. 23

A surveyor whose work lies in a district formerly a part of the U. S. public lands should procure from General Land Office, at Washington, a copy of "Manual of Surveying Instructions for Survey of the Public Lands of the United States." In this will be found all general and many specific instructions regarding the system, only the general scheme of which is described below.

Subdivision work is carried on as follows: First, the establishment of an **initial point**, a **principal meridian**, by astronomical observations, which is a true meridian through the initial point; and a **base-line** which is a true parallel of latitude through the intial point (Fig. 24).

These operations are performed in different localities as a basis for surveys in those regions. The principal meridian is a straight line and the base-line a curve, being at every point at right angles to the meridian through that point. The base-line is laid out by first running a straight line and measuring offsets from it to locate points on the parallel of latitude. Two methods are used for this, called the secant method and the tangent method, complete explanations of which, with tables, will be found in the manual issued by the General Land Office.

Second, the division of the area into tracts approximately 24 miles square, by establishment of **standard parallels,** sometimes called **correction lines,** which are true parallels of latitude extending east and west through 24-mile points on the principal meridian; and also establishment of **guide meridians,** which are true meridians through 24-mile points on base-line and on standard parallels, and extending north to the intersection of the next standard parallel or base-line.

Since these guide meridians converge, the tracts will be 24 miles on their southern and less on their northern boundaries (Fig. 24). The southerly end of these guide meridians, where they leave the standard parallel or base-line, are called **standard corners,** and points where they meet the next standard parallel to the north are called **closing corners.**

Third, the division of each 24-mile tract into **townships** (Fig. 24), each approximately 6 miles square, by establishment of **meridional lines,** or **range lines,** which are true meridians through standard township corners, established at intervals of 6 miles on the base-line and on the standard parallels, and extending north to an intersection with the next standard parallel or base-line; also establishment of **latitudinal lines,** or **township lines,** joining township corners previously established at intervals of 6 miles on principal meridian, guide meridians, and range lines.

Neglecting discrepancies in fieldwork, the east and west boundaries of all townships will be 6 miles in length, but the north and south will vary in length, being a maximum of 6 miles at standard parallel or base-line forming the southern limit of a 24-mile tract, and a minimum at that forming its northern limit. Townships are designated by numbering them in order north and south from the base-line and east and west from the principal meridian. Any series of contiguous townships or sections situated north and south of each other constitutes a **range,** while such a series in an east and west direction constitutes a **tier.** Thus "Township 2 North, range 4 west of the sixth principal meridian" locates its position; usually abbreviated to "T 2 N, R 4 W, 6th P M" (Fig. 24). The half-mile intervals on range lines are made full 40 chains for the entire 24 miles except the most northerly half-mile, into which all excess or deficiency due to irregularities of measurement is thrown. Similarly, corners on east and west lines are so marked that excess or deficiency of measurements is thrown into the most westerly half-mile of a 24-mile tract.

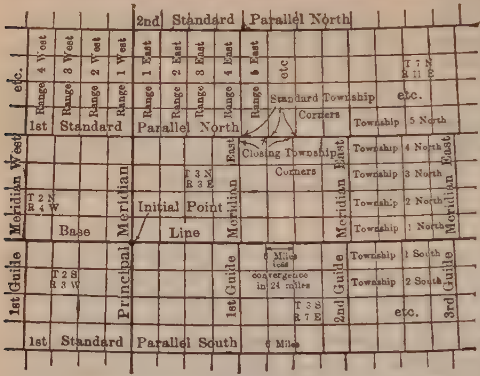


Fig. 24. Subdivision of 24-mile Tract into Townships

Fourth, the division of each township into **sections** (Fig. 25), each approximately 1 mile square (640 acres), by establishing **section lines,** both meridional and latitudinal, parallel to and at intervals of 1 mile from eastern and southern boundaries of township. The sections in all townships are numbered.

In staking out section corners, the surveyor sets up the instrument at the southeast corner of the township, observes the meridian, and retraces range line northward 1 mile, and township line westward the same distance. This is for comparing his own meridian and needle observations, and length of his chain with those of the surveyor who laid off township exteriors. Then from the southwest corner of section 36 he runs north on a line parallel with the east boundary of the township, setting a quarter-section corner at 40 chains and a section corner at 80 chains. Then from the section corner just set he runs east on a random line, parallel to the south boundary of the section, setting a temporary quarter-section corner at 40 chains. On intersecting the range line he notes the falling of his random line and also the distance it overruns or falls short of length of the south boundary of the section. If the falling is not more than 50 links (33 ft. representing an angular deviation of 21 min.) and if the distance overrun for falls short of length of the southern boundary of section 36 by not more than the same amount, a return course which will join the two section corners is calculated; this new line is then run toward the west, the permanent quarter-section corner being set at its middle point.

From the section corner just regained, the survey is now continued north between sections 25 and 26, the direction being changed slightly to east or west according to whether the latitudinal section line just completed exceeded or fell short of the desired length. At 40 and 80 chains on this line quarter-section and section corners, respectively, are set; from the section corner a random line is run across to the range line, a return course being calculated and run as before. This process is continued until 5 of the 6 sections in the series are enclosed. Then, if the north boundary of the township is not a correction line, from the section corner last established a random is run north to the township boundary, and from data thus secured a true line is calculated and run from section corner on township line back to initial corner. If the north boundary of township is a correction line, however, the point at which the random intersects this boundary is established as a **closing corner,** and its distance from the nearest **standard corner** is measured and recorded. In either case the permanent quarter-section corner is established at 40 chains north of the initial corner, the excess or deficiency being thrown into the most northerly half-mile.

In a similar manner the succeeding ranges of sections are enclosed, randoms being run eastward to section corners previously established, and true lines corrected back. But, from the fifth series of section corners thus established, random lines are projected to westward also, and are closed on corresponding section corners in the range line forming the western boundary of township. In correcting these lines back, however, the permanent quarter-section corners are established at points 40 chains from initial corners of the randoms, thereby throwing all fractional measurements into the most westerly half-miles. Reference to Fig. 25 will assist in understanding this method of subdivision.

Subdivision of Sections. When public lands were parceled out to settlers, the quarter-section was usually the unit area granted as a "homestead"; this required establishment of the quarter-section corner at the center of a section. Also, the subsequent division of original "quarters" into "eighties," "forties," or other minor subdivisions, has necessitated the location of numerous corners in addition to those originally established by the government.

The interior quarter-section corner of a section is always located at the intersection of straight lines joining quarter corners on opposite sides of the section. This holds wherever the section may be located within a township; that is, it applies to those in north and west sides as well as to other sections of township. A modification of this method is required in west and north sides of a township; in case of sections lying in west range the meridional line from the quarter corner on north or south boundary of section is run parallel to the east line of the section; similarly, in case of sections lying in the north tier, the latitudinal interior line initiated at the quarter corner on east or west boundary is run parallel to the south line of section. The reasons for this procedure in these special cases are apparent from consideration of methods previously described for establishment of original quarter-section corners in such sections. For subdivisions smaller than quarter-sections the same general methods are employed.

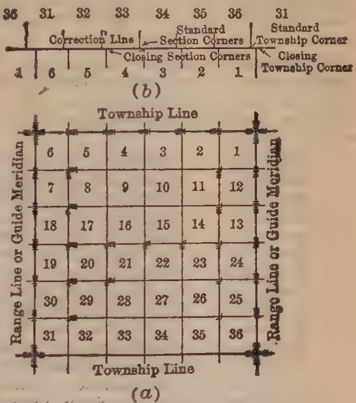


Fig. 25. Subdivision of Township into Sections and Method of Marking Corners

Corners are marked on the ground by various kinds of monuments, depending upon the character and importance of the corner to be perpetuated, the soil, the materials available, and upon other local and special conditions. Where stone is plentiful, stone monuments are usually set. In timbered districts, where suitable stones are difficult to obtain, posts are driven to mark the points. In prairie regions, where neither stone nor timber is available, a mound of earth may be raised over the corner, a small marked stone, a charred stake, a quart of charcoal, or some other permanent and distinguishable mark being deposited beneath it. Occasionally, in timber lands, the corner falls on a spot occupied by a tree, in which case the tree itself may stand as a monument.

Stones or posts are marked with horizontal notches, to indicate their respective positions in the township. Section corners on range lines, including under this term principal and guide meridians, are marked with notches on their north and south faces, the number of notches being equal to the number of miles to the next adjacent township corner north or south. Similarly, section corners on township lines, including baselines and standard parallels, are notched on their east and west faces. Township corners, being located on both range and township lines, are marked with six notches on each of the four sides. Besides being notched, corners on correction lines are marked SC on their northern or CC on their southern faces, depending upon whether they are standard or closing corners. Section corners in interior of a township are given notches on their east and south faces, corresponding to the number of miles to east and south boundaries of township. Thus, the corner common to sections 20, 21, 28, and 29 would have two notches on south and four on east face, as indicated in Fig. 25. Quarter-section corners are marked with the fraction "1/4," those on meridional lines on their west and those on latitudinal lines on their north faces. Wherever possible, a monument set at a corner is witnessed by several nearby objects which may easily be found, are not readily moved or obliterated, and are comparatively permanent. In timbered country the stone or post is usually witnessed by "bearing trees," situated near the corner.

Field Notes of public land surveys must be returned to the General Land Office, giving in narrative form complete data of alignment and topographic features crossed or near the lines. Original notes and plats of these surveys in the following states have been transferred to the state authorities, from whom may be obtained copies on request.

Alabama:	Secretary of State, Montgomery.
Arkansas:	Commissioner of State Lands, Little Rock.
Florida:	Commissioner of Agriculture, Tallahassee.
Illinois:	Auditor of State, Springfield.
Indiana:	Auditor of State, Indianapolis.
Iowa:	Secretary of State, Des Moines.
Kansas:	Auditor of State and Register of State Lands, Topeka.
Louisiana:	Register of the State Land Office, Baton Rouge.
Michigan:	Commissioner of State Land Office, Lansing.
Minnesota:	Secretary of State, St. Paul.
Mississippi:	Commissioner of State Lands, Jackson.
Missouri:	Secretary of State, Jefferson City.
Nebraska:	Commissioner of Public Lands and Buildings, Lincoln.
North Dakota:	State Engineer, Bismarck.
Ohio:	Auditor of State, Columbus.
Oklahoma:	Commissioner of General Land Office, Washington, D.C.
Wisconsin:	Commissioners of Public Lands, Madison.

In many, if not all, of the states named either the original records, or copies of them, have been distributed among the various counties of the state, and are kept for reference and inspection in the office of the County Register of Deeds, County Surveyor, or other official.

Photo-lithographic copies of township plats and field notes of surveys of the area covered by public lands surveys in the above states may also be obtained from the General Land Office, Washington, at nominal prices. In other public-land states, copies of records can be procured on application to the Surveyor General at the State Capitol, except in California and Oregon, whose Surveyors General are at San Francisco and Portland respectively.

Relocating Lost Corners. An act of Congress specifically provides that corners actually located in the field shall be established as proper corners of sections or quarter-sections which they were intended to designate, irrespective of whether or not they were properly located in the first place. A further provision is that "the boundary lines actually run and marked" (in the field) "shall be established as the proper boundary lines of the sections, or subdivisions for which they were intended, and the length of such lines as returned by . . . the surveyors aforesaid shall be held and considered as the true length thereof." The principles upon which present practice in relocating corners of original surveys are based are given in "Circular on the Restoration of Lost or Obliterated Corners and Subdivisions of Sections," published by the General Land Office, Washington.

23. Leveling for Profile

In Leveling to Determine a Profile the line is first stationed, every 100-ft. point, or such other interval as is desired, being distinctly marked. The level is set up and a rod-reading called a **backsight** (B.S., or +S) taken on a point called a **bench-mark** (B.M.) whose elevation is known. When this B.S. is added to the elevation of the B.M. it gives the **height of instrument** (H.I.). Rod-readings called **foresights** (F.S., or −S) are then read on as many station points on the line as can conveniently be seen from the instrument, and the elevation of the point on which the rod rests when a F.S. is taken is found by subtracting the F.S. from the H.I. Rod-readings are taken at all distinct changes in slope which occur on the line which is being profiled, whether these changes come at the full station points or not, and the intermediate stations are determined by tape measurements (sometimes by pacing) and are recorded as plus stations as shown in the sample notes below.

Field Notes in Running a Profile

Profile of Hudson St. Curb (West side). June 17, 1908. Brown, level. Jones, rod.

Sta.	+S	H.I.	−S	Elev.	B.M. & T.P. Elev.	Descriptions.
B.M.₁.	7.218	104.912	......	....	97.694	N. E. cor. top granite post S.W. cor. Town Hall
0	......	....	4.24	100.67	....	
1	......	....	4.28	100.63	....	
2	......	....	4.33	100.58	....	
3	......	....	4.38	100.53	....	
T.P.	6.473	106.623	4.762	....	100.150	Rim M.H. opp. house #73.
4	......	....	6.12	100.50	....	
4+60	......	....	6.11	100.51	....	
5	......	....	6.13	100.49	....	
6	......	....	6.15	100.47	....	
6+85	......	....	6.17	100.45	....	
7	......	....	6.14	100.48	....	
8	......	....	6.10	100.52	....	
B.M.₂.	6.071	107.977	4.717	....	101.906	Top S.B. cor. Burrill St. Elev. 101.912
9	......	....	7.43	100.55	....	

When it is necessary to move the level to a new position to proceed along the line to be profiled a **turning point** (T.P.) is selected and its elevation determined to use at the next set-up of the level in finding the new H.I. When this new H.I. is found, F.S. readings are taken at the proper stations on the line as far as is consistent with the precision required and then a new T.P. is established, and so on. The readings on the B.M.'s and T.P.'s should be taken to one more decimal place than those taken for the profile. In the notes shown, for example, the readings were taken on the curb to the nearest 0.01 ft., while those on the T.P.'s were read to 0.001 ft. The B.M.'s are all carefully described in the notes; as a rule T.P.'s are not described, but where they are taken on points easily identified it is well to describe them also. The distances from the level to the rod when held on the T.P. or B.M. should be about equal for a backsight and its corresponding foresight, so that any errors in the adjustment of the line of sight or wyes may be eliminated.

If possible the levels should form a circuit, returning to the original B.M. so as to check the intermediate work. A common method is to check the work by readings on other B.M.'s which are passed in progressing along the line. When this is done those B.M.'s should be used as T.P.'s.

The calculation of the level notes may be checked by finding the difference between the sum of the B.S.'s and F.S.'s which were taken on the first B.M., on all the intermediate T.P.'s, and the final B.M. This gives the difference in elevation.

In taking levels for a profile it is necessary to start from some datum to which to refer the levels, and a B.M. whose elevation above some datum plane (such as mean sea level) is used if one is near the locality of the work. Or a B.M. of assumed elevation may be used, in which case all of the levels will refer to an assumed datum. At a later period, if necessary, these levels may be connected to a sea-level datum by running an accurate line of levels between the assumed B.M. and some other B.M. whose elevation referred to sea level is known.

The Proper Length of Sights will depend upon the distance at which the rod appears distinct and upon the precision required. Under ordinary conditions sights should not exceed 300 ft. where elevations are required to the nearest 0.01 ft., and even at a much shorter distance the boiling of the air may prevent a precision of this degree.

Curvature and Refraction Correction. Since a level line is a curved line which at every point is perpendicular to the direction of gravity and the line of sight of a level is along a tangent to this curve it is necessary to take this into account in the more precise leveling work. This correction is usually combined with that due to the refraction of the atmosphere; and the combined correction, for sights of 300 ft., is about 0.002 ft.; for 500 ft., 0.005 ft.; for 1000 ft., 0.020 ft. These corrections are to be applied to any single rod-reading by subtracting from the reading; but if the rod is equally distant from the instrument on the foresight and backsight the effect of curvature and refraction is eliminated from the result.

To Establish a Datum from tidal observations, set up a vertical staff, graduated to feet and tenths, in such a manner that the high and low water can be read. Read the positions of high and low water for each day for as long a period as practicable. The mean value obtained from an equal number of high and low water observations will give the approximate value of mean sea level. If the observations extend over one lunar month the result will be fairly good; to determine this accurately will require observations extending over about eighteen years.

Double Rodded Lines are frequently used when it will be impracticable to run a circuit or when a profile is to be made through a new country where no intermediate B.M.'s have been previously established. Instead of taking a foresight on a single T.P., foresights are taken on two different T.P.'s near together and varying in elevation by a foot at least. When the level is set up again a backsight is taken on both T.P.'s and two H.I.'s computed, which should agree within whatever limits of error are allowable for the class of work at hand. This method of carrying on a check on the leveling is particularly applicable to such cases as running preliminary surveys for a railroad.

24. Leveling for Cross-sections

Cross-sections for Borrow-pits. When it is desired to determine the shape of the ground with a nicety such as is required for landscape architect studies or for computing the material taken from a borrow-pit, the area may be divided into squares (or rectangles) and elevations taken at the corners of these squares and at as many intermediate points as are necessary to determine the shape of the ground. These surface elevations are usually taken to the nearest tenth of a foot, and the squares, which are anywhere from 10 ft. to 100 ft. on a side, are laid out with transit and tape. The corners may be designated by a system of letters for the lines running in one direction and numbers for the system perpendicular to the lettered system; for example, point D 7 lies at the intersection of lines D and 7, and intermediate point $E + 8$, $6 + 4$ lies 8 ft. from line E toward F and 4 ft. from line 6 toward line 7. The notes are kept like profile notes (Art. 23) except that the stations are designated as just described. The tape-rod, described in Art. 7, is of particular use in work of this sort.

When cross-sections are taken to determine the amount of material taken out of a borrow-pit, readings are taken, as described above, before the excavation is started; after the excavation is completed the same system of cross-section lines is again run out and the new elevations at corners, and intermediate points if necessary, are determined. The quantity of material removed is computed as explained in Art. 68.

Road Cross-sections are taken as a basis for estimating the quantity of earthwork in railroad or highway construction, but by an entirely different method from the borrow-pit method. From the plan of the proposed road its alignment is staked out and a profile is taken along the center line, which is subsequently plotted. On this profile the grade line is drawn, which corresponds to the finished surface of the road. Roads are usually first finished to subgrade, which is below the computed surface by an amount equal to the thickness of the road covering, the pavement of a highway or the ballast in the case of a railroad. The width of the base of the road and the inclination of the side slopes are known. For ordinary gravel the slope is usually 1-1/2 ft. horizontal to 1 ft. vertical, called " a slope of 1-1/2 to 1."

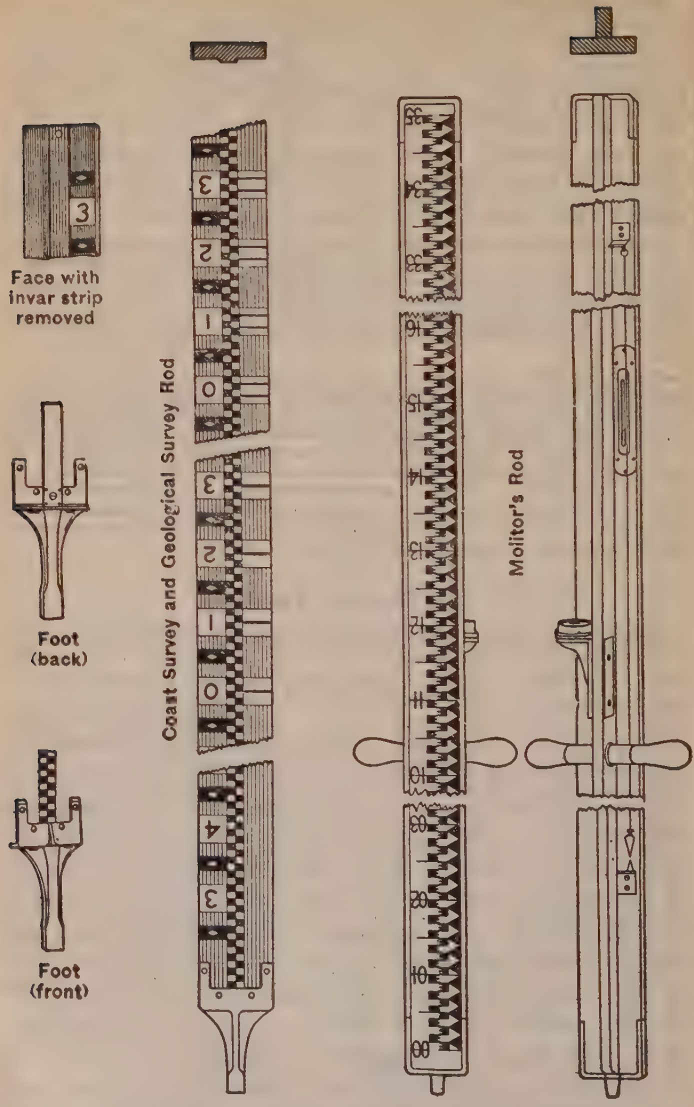
For construction work the engineer sets grade stakes at every full station or oftener on the center line and at both sides where the finished slope intersects the surface of the ground. All three of these stakes are marked, giving the amount of " cut " or " fill " to be made at these points. The cut or fill marked on the stakes where the slopes meet the original surface is the vertical distance from the base of the road to the surface of the ground at these points. The process of setting these slope stakes is described in Art. 64.

Cross-sections for dams, for canals and other engineering structures are common problems for the surveyor. This work consists of making a short profile at right angles to the line of the structure, at intervals of from 10 to 100 ft., depending upon various conditions, and extending far enough on either side to include the possible location of the structure. These cross-section lines representing the surface are plotted for each station, and on them are also plotted the proposed cross-sections of the structure at the respective stations. This gives a complete record of the conditions before the work is started; and as it progresses, by the use of different colored lines, a continual graphical record of the work may be kept from month to month, which is the customary interval for making estimates for partial payments to contractors.

25. Precise Leveling

Precise Spirit-leveling is like ordinary leveling except that certain refinements are introduced into the field methods and the construction and manipulation of the instrument. The characteristic features of a precise level are an inverting telescope of high power and good definition, provided with one vertical and three horizontal hairs, and a sensitive spirit-level of uniform curvature, and a mirror or a set of prisms so adjusted that the observer can see the bubble while he is looking at the rod. The instrument rests on three leveling screws. There is a box level (or two small levels at right angles to each other) for approximate leveling, and a micrometer screw by means of which one end of the telescope tube can be raised and lowered when centering the large bubble. Some of these instruments are of the wye and some of the dumpy type. The present Coast and Geodetic Survey level is a dumpy. This instrument differs from the earlier patterns in that it is made of nickel-iron alloy, which has a low coefficient of expansion; the bubble is placed close to the line of sight so that the adjustment will not be disturbed by local temperature variations; and the reflecting prisms are contained in a second tube at the left of the telescope, so that the observer can view the bubble with his left eye at the same instant that he observes the rod with the right eye. The level vial is provided with an air chamber for regulating the length of the bubble. With this instrument readings can be taken with great rapidity. (See Coast Survey Reports for 1900, p. 521, and 1903, p. 200.)

The Rods used in recent precise leveling work are of the non-extensible self-reading pattern. They are sometimes made in the form of a cross or a T, but the latest rods are flat, about 1 in. × 4 in. in section. The graduations



Precise Leveling Rods

are on a strip of invar fastened to the wood. The rods are usually graduated in the metric scale, the centimeter being the smallest division. The decimeter marks and the figures are painted on the wood at the side of the invar strip. Since the telescope is inverting the figures are for convenience painted upside-down. Sometimes rods are graduated to yards and hundredths. Such a rod is used by the U. S. Geological Survey. The foot of the rod carries a flat steel shoe instead of the rounded form previously used. These rods are provided with a spirit level and a thermometer.

The greater part of the **precise leveling** is done on railway lines. The turning point is ordinarily a nail driven into a tie; sometimes the top of a rail is used. For leveling on highways or whenever the ordinary turning points cannot be used steel pins are driven into the ground.

While being carried from one station to another the level should be protected from the sun's rays. The cam protecting the micrometer should be turned so that the point of the screw is not in contact with the telescope, the nuts at the top of the tripod should be loose, and the central clamp should be tight. The level should be held so that air will not escape into or from the air chamber and change the length of the bubble. When the level is set up the tripod nuts are tightened and the tension on the central clamp screw released so that the leveling screws are free. The instrument is first leveled approximately by means of the circular level and the leveling screws. The telescope is then turned toward the rod and the long bubble centered exactly by using the micrometer screw.

The observations are made in such a manner as to eliminate so far as possible the recognized common errors in leveling work, namely (1) settling of instrument or rod in soft ground; (2) unequal expansion and contraction of different parts of instrument due to temperature changes; (3) irregular refraction near surface of ground; (4) unequal length of foresights and backsights (faulty adjustment of instrument); (5) use of poor turning points; (6) rod not held plumb, and (7) bubble not centered in tube.

Errors due to settling of tripod or rod may be eliminated by taking two backsights and two foresights at each set-up, in the following order: 1, backsight; 2, foresight; 3, foresight; and 4, backsight. The same result is accomplished by taking the backsight first on one set-up and the foresight first on the next set-up. The latter is the method used by the Coast Survey. It is based upon the assumption that the rising or settling takes place at a uniform rate. Temperature errors are avoided by shading the level by an umbrella and by taking the readings in rapid succession. Rapidity also tends to eliminate error in the settling of tripod or turning point. Errors due to refraction may be partly avoided by using a high tripod, always keeping the line of sight away from the ground as much as possible. The lowest reading should be selected when the reading of the lower hair varies on account of refraction. Observations during the middle of the day are more likely to be free from such errors, but in practice leveling is carried on throughout the day. The lengths of backsights and foresights are balanced (nearly) on each sight by reading all three hairs and finding the stadia intervals from the upper and lower ones. The sums of the foresight intervals and the backsight intervals should be shown in the notes at each set-up. The rod is plumbed by use of the spirit-level and the temperature is read frequently.

On the U. S. Coast and Geodetic Survey leveling work two rods are used, the same rod being held on any T.P. first for a foresight and then for a backsight. This results in the same rod being read first at every set-up. The three hairs are read, and the mean and the difference computed, the latter being the stadia interval. The lines of levels are divided (roughly) into 1 kilometer sections, and each section is run forward and then backward, the allowable error of closure being that stated below. The longest sight allowable is 150 meters, which can be used only under exceptional conditions. The difference between foresight and backsight distances must not exceed 10 meters on any one sight, or 20 meters in the total at any time. The adjustment of the instrument is tested daily, and corrected if it exceeds a specified amount.

The **allowable errors** in closing a circuit in precise leveling work of various Surveys are as follows: U. S. Coast and Geodetic Survey, $4^{mm}\sqrt{\textbf{kilometers}}$; U. S. Lake Survey, $10^{mm}\sqrt{\textbf{kilometers}}$; Mississippi River Survey, $5^{mm}\sqrt{\textbf{kilometers}}$; U. S. Geological Survey, **0.017 ft.**$\sqrt{\textbf{miles}}$. The results actually reached fall well within these extreme limits. A high grade of leveling, nearly equal to that of precise leveling, has been done many times with an ordinary wye or dumpy level of good construction, but usually at a greater cost than would have been the case if a precise level were used.

TOPOGRAPHIC SURVEYING

26. Barometric Leveling

The Barometer may be used as a means of measuring differences in elevation, since one inch in the height of the mercury column is equivalent to about 900 ft. in altitude. The atmospheric pressure varies also with changes of temperature and humidity, so that it is necessary in measuring differences in altitudes with the barometer to determine the amounts of these variations and to make proper allowance for them. The ordinary cistern mercurial barometer and the aneroid are used in surveying. The latter on account of its compactness and sensitiveness is more generally employed, but it is liable to derangements if subjected to great ranges of pressure or if roughly handled; it is also subject to errors due to temperature changes.

Barometric Leveling is useful in making a reconnoissance or in determining the contours for a map of very small scale. It is approximate at best, but with a mercurial barometer results as close as 5 ft. may be obtained by careful handling of the instrument, or with an aneroid in good adjustment results correct to 5 or 10 ft. may be obtained but not without repeating the observations and taking great care in the use of the aneroid. The error in height determined by a barometer is approximately a constant, so that the percentage error is much smaller for large differences in elevation than for small differences. The barometer does not give actual heights, but the difference in its readings will be a function of the difference in elevation of the two points provided the atmospheric conditions have not changed so as to affect the readings.

To Read a Mercurial Barometer the screw at the bottom of the barometer is turned until the ivory point is just in contact with the mercury. The vernier is then raised or lowered until its lower edge is just at the height of the mercury column. More than one reading should be taken, and the ivory point should be set in contact with the mercury each time. The temperature of the air and of the mercury are both read when the mercury height is recorded. In carrying the mercurial barometer from place to place the bottom screw should be turned up until the mercury just fills the tube, then the barometer should be inverted and carried in its case upside down.

On the Aneroid Barometer are two scales, the inner one corresponding to inches of mercury and the outer one to feet of altitude, the zero of the altitude scale being in most instruments at 31 in. on the mercury scale. The outer scale should not be movable with reference to the inner scale, for the number of feet of altitude corresponding to one inch of mercury is different in different parts of the scale. Aneroids marked "compensated" are supposed to be adjusted so that changes in temperature of the instrument will not affect the readings. The instrument should be handled carefully in order to avoid disturbing its delicate mechanism. When it is to be read, tap the case lightly to be sure the instrument has adjusted itself to the changed pressure. It should stand a few minutes before it is read so as to allow it to come to the true reading; if should not be heated by the sun's rays or the body; and it may be held in either a vertical or horizontal position when being read, but as the readings in these two positions are different it should be held in the same position at all stations. As accurate results may be obtained from small as from large aneroids.

The Best Field Method requires the use of two barometers, one to remain at some fixed station whose elevation is known, and a continuous record of this instrument will give the atmospheric changes. It is assumed, since the other aneroid will be in use in the same general locality, that the atmospheric changes will be the same at both instruments. It is well to use a mercurial barometer at the fixed station. The second instrument (an aneroid) is read at the start at the fixed station. The difference between the readings of the two barometers at this station is an index correction to be applied to all readings of the moving barometer. The aneroid is then carried to the next station whose elevation is required, and read, and so on to as many points as are desired, making the time of travel between stations as short as possible. At each point the pressure, time and air temperature are read and recorded. Upon the return trip with the aneroid it will be well, as a check, to take readings again at the stations as they are passed, and upon arriving at the fixed station the aneroid is read again. At intervals during the day, every half hour, or oftener if the atmospheric conditions are rapidly changing, the barometer at the fixed station is read and the time, air temperature, and mercury temperature (if a mercurial) are recorded. The interpolated readings of the fixed barometer are entered in the notes of the moving barometer and the index correction applied, thereby eliminating all changes in reading except those due to changes in altitude. When only one barometer is used a reading is taken at the first station when starting on the trip and again at the end of the trip. The difference between these two readings represents the total change due to weather conditions, and interpolated readings for the times when the barometer was read at the other stations must be made for the first station, as illustrated by the example below.

Calculating the Altitude. Differences in elevation are calculated from the differences in pressure either by formula or by table, or it may be found with an aneroid by reading directly the altitude scale which is based on one of the formulas. **Laplace's formula** is one of the most accurate; it is

$$\mathbf{D = 60\,158.6\,(\log h - \log H)\left\{1 + (t_a' + t_a - 64°)/900\right\}},$$

where D = difference in elevation in feet; h = height of mercury at lower station in inches; H = height of mercury at upper station in inches; t_a' and t_a are the observed air temperatures in Fahrenheit degrees. The correction due to air temperature is often a large amount. In case a mercurial barometer is used H is the height of mercury at the upper station reduced to the temperature of the mercury at the lower station by

$$\mathbf{H = h'\left\{1 + 0.00008967\,(t_m - t_m')\right\}},$$

For Determining Difference in Elevation by Barometer

Barom., in.	Ft.	Barom., in.	Ft.	Barom., in.	Ft.	Barom., in.	Ft.	Barom., in.	Ft.
16.00	12 280	20.00	18 110	22.75	21 466	25.50	24 457	28.25	27 133
16.10	12 442	20.05	18 175	22.80	21 533	25.55	24 508	28.30	27 179
16.20	12 604	20.10	18 240	22.85	21 590	25.60	24 559	28.35	27 225
16.30	12 765	20.15	18 305	22.90	21 647	25.65	24 610	28.40	27 271
16.40	12 925	20.20	18 370	22.95	21 704	25.70	24 661	28.45	27 317
16.50	13 084	20.25	18 434	23.00	21 761	25.75	24 712	28.50	27 362
16.60	13 242	20.30	18 499	23.05	21 818	25.80	24 762	28.55	27 409
16.70	13 398	20.35	18 563	23.10	21 874	25.85	24 813	28.60	27 454
16.80	13 554	20.40	18 627	23.15	21 931	25.90	24 864	28.65	27 500
16.90	13 709	20.45	18 691	23.20	21 987	25.95	24 914	28.70	27 545
17.00	13 864	20.50	18 755	23.25	22 044	26.00	24 964	28.75	27 591
17.10	14 017	20.55	18 818	23.30	22 100	26.05	25 014	28.80	27 637
17.20	14 169	20.60	18 882	23.35	22 156	26.10	25 065	28.85	27 682
17.30	14 321	20.65	18 945	23.40	22 212	26.15	25 115	28.90	27 727
17.40	14 471	20.70	19 008	23.45	22 267	26.20	25 164	28.95	27 772
17.50	14 621	20.75	19 071	23.50	22 323	26.25	25 214	29.00	27 817
17.60	14 770	20.80	19 134	23.55	22 378	26.30	25 264	29.05	27 862
17.70	14 918	20.85	19 197	23.60	22 434	26.35	25 314	29.10	27 907
17.80	15 065	20.90	19 260	23.65	22 489	26.40	25 363	29.15	27 952
17.90	15 211	20.95	19 322	23.70	22 544	26.45	25 412	29.20	27 997
18.00	15 357	21.00	19 384	23.75	22 599	26.50	25 462	29.25	28 041
18.10	15 502	21.05	19 446	23.80	22 654	26.55	25 511	29.30	28 086
18.20	15 646	21.10	19 508	23.85	22 709	26.60	25 560	29.35	28 131
18.30	15 789	21.15	19 570	23.90	22 764	26.65	25 609	29.40	28 175
18.40	15 931	21.20	19 632	23.95	22 818	26.70	25 658	29.45	28 220
18.50	16 073	21.25	19 694	24.00	22 873	26.75	25 707	29.50	28 264
18.55	16 143	21.30	19 755	24.05	22 927	26.80	25 755	29.55	28 308
18.60	16 214	21.35	19 816	24.10	22 982	26.85	25 805	29.60	28 352
18.65	16 284	21.40	19 877	24.15	23 036	26.90	25 853	29.65	28 396
18.70	16 354	21.45	19 938	24.20	23 090	26.95	25 902	29.70	28 440
18.75	16 423	21.50	19 999	24.25	23 144	27.00	25 950	29.75	28 484
18.80	16 493	21.55	20 060	24.30	23 198	27.05	25 999	29.80	28 528
18.85	16 562	21.60	20 120	24.35	23 251	27.10	26 047	29.85	28 572
18.90	16 632	21.65	20 181	24.40	23 305	27.15	26 095	29.90	28 616
18.95	16 701	21.70	20 241	24.45	23 358	27.20	26 143	29.95	28 659
19.00	16 769	21.75	20 301	24.50	23 412	27.25	26 191	30.00	28 703
19.05	16 838	21.80	20 361	24.55	23 465	27.30	26 239	30.05	28 746
19.10	16 907	21.85	20 421	24.60	23 518	27.35	26 287	30.10	28 790
19.15	16 975	21.90	20 481	24.65	23 571	27.40	26 334	30.15	28 833
19.20	17 043	21.95	20 540	24.70	23 624	27.45	26 382	30.20	28 877
19.25	17 111	22.00	20 600	24.75	23 677	27.50	26 430	30.25	28 920
19.30	17 179	22.05	20 659	24.80	23 730	27.55	26 477	30.30	28 963
19.35	17 246	22.10	20 718	24.85	23 782	27.60	26 524	20.35	29 006
19.40	17 314	22.15	20 777	24.90	23 835	27.65	26 572	30.40	29 049
19.45	17 381	22.20	20 836	24.95	23 887	27.70	26 619	30.45	29 092
19.50	17 448	22.25	20 894	25.00	23 940	27.75	26 666	30.50	29 135
19.55	17 516	22.30	20 954	25.05	23 992	27.80	26 713	30.55	29 178
19.60	17 582	22.35	21 012	25.10	24 044	27.85	26 760	30.60	29 220
19.65	17 648	22.40	21 071	25.15	24 096	27.90	26 807	30.65	29 263
19.70	17 715	22.45	21 129	25.20	24 148	27.95	26 854	30.70	29 306
19.75	17 781	22.50	21 187	25.25	24 199	28.00	26 900	30.75	29 348
19.80	17 847	22.55	21 245	25.30	24 251	28.05	26 947	30.80	29 391
19.85	17 913	22.60	21 303	25.35	24 303	28.10	26 994	30.85	29 433
19.90	17 979	22.65	21 360	25.40	24 354	28.15	27 040	30.90	29 475
19.95	18 044	22.70	21 418	25.45	24 406	28.20	27 086	30.95	29 518

where t_m = the temperature of mercury at lower station and t_m' = the temperature of mercury at upper station. The following table condensed from Guyot is based on this formula.

Example using one aneroid barometer: find difference in elevation between Overlook and Bald Mt.

Station	Time	Barometer, inches	Air temperature
Overlook	9.25 a.m.	29.42	47° F.
Bald Mt.	10.30 a.m.	27.15	30°
Overlook	10.57 a.m.	29.36	43°

Here 10.30 − 9.25 = 1.05 = 65 min., time going up, and 10.57 − 9.25 = 1.32 = 92 min., total time. Also 29.42 − 29.36 = 0.06 in., diff. in barometer height at Overlook, whence 29.42 − (0.06 × 65/92) = 29.38, probable reading at Overlook at 10.30 a.m.

From Barometric table, 29.38 = 28 157
27.15 = 26 095

2 062

Temp. Corr. $= 2\,062\left[\left\{1/2\,(47+43)+30\right\}-64\right]/900$ = + 25

Difference in elevation = 2 087 feet.

The various **barometers** constructed on what is known as the "Paulin System" are capable of giving precise results. These instruments have the following characteristics: They have no inertia due to friction; they are free from lag, and come promptly to the correct reading; they can be locked so that ordinary shocks have no effect and the instrument can be safely transported. They are fully compensated for temperature.

Barometers of this type are made for leveling work, for altimeters of airplanes, and for general use. The ranges vary from 3300 ft. for leveling instruments to 20 000 ft. for altimeters for airplanes.

The principle involved in their construction is as follows: The force exerted by the vacuum chamber is opposed by that of a heavy spiral spring. When a reading is to be taken the spring is compressed until the vacuum box is forced back to its normal position. The force required to do this is read on the scale, either as inches of mercury or as feet of altitude. The vacuum box is always in the same condition when a reading is being taken, instead of being deformed and strained as with the ordinary aneroid. The device for determining when this balance is attained is extremely sensitive and the instrument can be made to read elevations directly to about 2 ft., and by estimation, somewhat closer than this. When the barometer remains in a fixed position this device shows at once the slightest change in atmospheric pressure. On altimeters it shows at once any variation in the altitude of the plane from the altitude to which it was set.

This instrument cannot of course overcome variations in pressure due to atmospheric changes, any more than any other barometer can. It is necessary, therefore, to use two barometers if the work extends over a period and changes are taking place. It is also necessary to make allowance for air temperature, as usual.

27. Trigonometric Leveling

Finding the Difference in Elevation of two points by means of the horizontal distance between them and the vertical angle is called **trigonometric leveling.** It is used chiefly in determining the elevation of triangulation stations and in obtaining the elevation of a plane-table station from any visible triangulation point of known elevation. In triangulation work the vertical angles are usually measured at the same time the horizontal angles are measured, so as to obtain the elevations of triangulation points as well as their horizontal positions. The vertical angle is measured to some definite point on the signal whose height above the center mark of the station was determined when the signal was erected, and the height of the instrument above its station should be measured and recorded. In the most exact work the angles are measured with a special vertical circle instrument. In less precise work an ordinary theodolite whose vertical arc reads by verniers to 30″ or to 20″ may be used, but with such instruments only single readings can be made. The best results with such an instrument are obtained by taking the average of several independent readings half of which are taken with the telescope direct and the other half with the telescope inverted. In every case the index correction, or reading of the vertical arc when the telescope is level, must be recorded.

Simultaneous Observations. The chief difficulty in obtaining accurate results by trigonometric leveling is due to the uncertainty of the angle of refraction, that is, the angular deviation of the line of sight on account of the refraction of the air. This varies with the locality, the temperature and the atmospheric pressure, so that the only way its effect can be practically eliminated is by taking simultaneous observations between two stations, in which case

$$\mathbf{h_1 - h_2 = K \tan \tfrac{1}{2} (\alpha_2 - \alpha_1) \left\{ 1 + \frac{h_1 + h_2}{2\,R} + \frac{K^2}{12\,R^2} \right\}}$$

where $h_1 - h_2$ = difference in elevation of stations in feet; K = arc of the earth subtended between plumb lines through the two station points, approximately equal to the distance between the two stations in feet; α_1 and α_2 are the simultaneously observed vertical angles; R = radius of earth at latitude of the stations; for most work it is close enough to use log R (in feet) = 7.32068.

Observation at One Station. If the observation is made at only one station the **refraction coefficient** must be known and applied to the vertical angle. The refraction coefficient is $m = r/c$, whence $r = mc$, where r = the angle of refraction and c = the angle at the center of the earth between the two stations. For a single observation

$$\mathbf{h_2 - h_1 = K \tan \left[\alpha_1 + (1/2 - m) \frac{K}{R \sin 1''} \right] \left[1 + \frac{h_1 + h_2}{2\,R} + \frac{K^2}{12\,R} \right]}$$

In this formula the letters have the same significance as in the formula for simultaneous observations above. In solving for $h_2 - h_1$, approximate values are first obtained by omitting the last factor; the approximate values are then substituted in the last factor and corrected values are computed.

From a large number of observations the U. S. Coast Survey has determined values of m as follows:

For lines crossing the sea	0.078
Between primary points (high elevation)	0.071
For interior of the country, about	0.065

Clarke, in his "Geodesy," gives:

For rays crossing the sea	0.0809
For rays not crossing the sea	0.0750

A Rough Determination of the difference in height may be found by multiplying the horizontal distance by the tangent of the vertical angle and applying a single correction for curvature and refraction by the formula $\mathbf{h} = \mathbf{K^2/1.7426}$, where K is the distance in miles between the stations and h is the correction in feet. If K is expressed in units of 1000 ft., then $h = 0.02 \times K^2$ (nearly). This correction is applied so as to increase the difference in elevation if the vertical angle is plus, and it should decrease the difference in elevation if the vertical angle is minus. This method is used for the stadia and plane table except that for ordinary sights or for rough work the curvature and refraction correction is omitted.

28. Contours

A Contour Line is the intersection of a level surface with the surface of the ground. For example, a shore line of a pond is a contour line, and if the pond level were raised a foot its new shore line would be the contour line of 1 ft. greater elevation. Contours are used to represent the shape of the ground, and give a means of reading elevations directly from the map. It is customary to locate them a whole number of feet above the datum, such as the 50-ft. contour, and at regular intervals, at, say, every 2 ft. or every 5 ft., the number of the contour is its elevation above the datum. Since the contours are taken equidistant vertically their horizontal distance apart gives a measure of the steepness of slopes.

Characteristics of Contours are: (1) All points on any one contour have the same elevation. (2) Every contour closes on itself, either within or beyond the limits of the map. (3) A contour which closes within the limits of the map encloses either a summit or a depression. In depressions there will usually be found a pond or a lake; but where there is no water the contours are usually marked in some way to indicate a depression. (4) One contour can never cross another except where there is an overhanging cliff, in which case there must be two intersections. (5) On a uniform slope contours are spaced equally. (6) On a plane surface they are straight and parallel. (7) In crossing a valley the contours run up the valley on one side and, turning at the stream, run back on the other side. Since the contours are always at right angles to the lines of steepest slope they are at right angles to the thread of the stream at the point of crossing. (8) Contours cross the ridge lines (divides) at right angles.

Contours and Profiles. If a line is drawn across a contour map, the profile of the surface along that line may be constructed, since the points where the contours are cut by the line are points of known elevation and the horizontal distances between these points can be scaled or projected from the map. The profile shown in Fig. 26 is constructed by first drawing, as a basis for the profile, equidistant lines, corresponding to the contour interval, and parallel to XY. From the points where XY cuts the contours, lines are projected to the corresponding line on the profile. Conversely, if the profiles of a sufficient number of lines on the map are given it is possible to plot these lines on the map, mark the elevations, and then to sketch the contours.

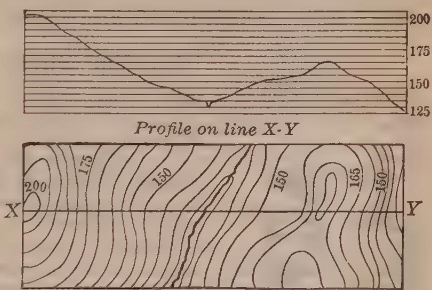


Fig. 26

Locating Contours. Since the topography of much of the country is due to stream erosion the position and fall of streams gives much information regarding the location of contours. These, together with a few ridge elevations, give information from which contours may be readily sketched. It should be borne in mind in sketching contours that they cross the stream at right angles and curve around from the hill on either side so as to represent the valley; they are usually farther apart at the top and bottom of a natural eroded slope (where no wave action is present) than in the middle; a stream is usually steeper near its source than in its lower portion.

Where the contour interval is small and much detail is desired the ground may be cross-sectioned (Art. 24) and from the elevation determined the contours may be sketched. Or the contours may be actually traced out and located by stadia. In this case the rodman moves up or down the slope until a level rod-reading shows that the foot of his rod is on a contour. The position of the rod is then located by distance and azimuth.

The hand level is used to a considerable extent on railroad preliminary surveys to locate contours, and is satisfactory for such work because the contours are located only a short distance from the transit line so that the inaccuracies of hand-level work cannot become very great. In using a hand level for locating contours, first measure the distance from the ground to the levelman's eye, which may be, say, 5.2 ft. Then from a reading of the rod held on a point of known elevation the elevation of the eye of the levelman is determined. The leveler then directs the rodman uphill or downhill until the rod-reading is such as will correspond to the foot of the rod being on a contour. When this point is found it is located by tape or by pacing. Then, if the contours are being located on a sidehill, the levelman moves uphill past the located point on which the rodman stands until the levelman reads the contour interval plus 5.2 on the rod. When this point is reached the levelman is standing on a contour which is then located. The levelman stands on this point and the rodman travels uphill until the levelman reads 5.2 minus the contour interval on the rod, when the rod is resting on the next higher contour. In going downhill the rodman holds his rod on a contour and the levelman will back down the hill until he reads 5.2 minus the contour interval on the rod, at which point the levelman will be standing on the contour next below the rodman; then the rodman passes the levelman and backs down the hill until the rod-reading is 5.2 plus the interval, which determines the next lower contour. Sometimes profiles are run and the elevations plotted on the plan give a basis for sketching the contours. Profiles of valleys and ridges as a rule give the best data.

29. The Stadia

The Stadia Method is one in which distances are measured by observing through the telescope of a transit the space, on a graduated rod, included between two horizontal hairs called **stadia hairs**. If the rod is held at different distances from the instrument different intervals on the rod are included between the stadia hairs, the spaces on the rod being proportional to the distances from the instrument to the rod, so that the intercepted space is a measure of the distance to the rod. This method furnishes a rapid means of measuring distances in filling in details of topographic and hydrographic surveys. It is used either with a transit or a plane table which is provided with the two additional horizontal cross-hairs. It has the great advantage that the intervening country does not have to be traveled, provides a means of measuring inaccessible distances such as across water surfaces, the errors of measurements are compensating rather than cumulative, and it affords an accuracy sufficient for many kinds of work, being applicable even to surveying for area of such lands as wood lots or farms, for an accuracy of one part in 500 may be attained with the stadia. The **rod** used may be of any desired pattern of self-reading rod, the Philadelphia rod being a good type for short distances. It is well to use a rod with the graduations represented by some

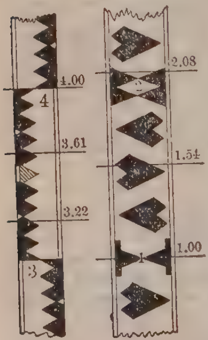


Fig. 27

diagram which can be seen distinctly a long distance away, 1000 to 2000 ft. Portions of two of these rods are shown in Fig. 27, with the reading of the three cross-hairs marked in the figure.

The Fundamental Principle of the stadia is the simple geometric proposition that in two similar triangles homologous sides are proportional. The vertex of these triangles is not the center of the transit but is at a point in front of the objective a distance equal to F, the focal length of the objective, so that the distance from the center of the instrument to the rod $= \frac{\mathbf{F}}{\mathbf{i}}(\mathbf{s}) + (\mathbf{F} + \mathbf{c})$, where i is the distance between the upper and lower stadia hairs; s is the intercepted space on the rod; and c is the distance from the center of the instrument to the objective. Evidently $(F + c)$ is practically a constant for any given instrument; for transits it varies from 0.75 to about 1.35, depending upon the style of instrument. For all ordinary purposes it may be taken as 1 ft. for transits and 2 ft. for plane-table alidades. The other function $(F/i)\,s$ is a variable, but as it is almost always customary to have the stadia hairs spaced so that $F/i = 100$ this reduces the equation to Distance $= 100\,s + 1$. Every hundredth of a foot on the rod is then a measure of one foot of distance.

Inclined Sights. Since it is the horizontal distance which is required it is necessary to reduce all inclined readings to the horizontal, and when the difference in elevation between the instrument station and the rod station is desired the vertical distance also must be computed from the inclined sight. In practice it is customary to hold the rod plumb rather than perpendicular to the line of sight, so that it is evident that if the line of sight is inclined it will subtend an interval on the rod which is too great depending upon the angle of inclination, α, and the distance the rod is from the instrument. By trigonometric proof, with one slight assumption, it may be shown that the

$$\textbf{Vertical Distance} = (\mathbf{F/i})\,\mathbf{s} \cdot \mathbf{1/2 \sin 2\alpha} + (\mathbf{F} + \mathbf{c}) \sin \alpha$$
$$\textbf{Horizontal Distance} = (\mathbf{F/i})\,\mathbf{s} \cdot \mathbf{\cos^2 \alpha} + (\mathbf{F} + \mathbf{c}) \cos \alpha$$

For the ordinary conditions (when $F/i = 100$ and $F + c = 1$) and making the approximations that $\sin\alpha = 1/2 \sin 2\alpha$ and $\cos\alpha = \cos^2\alpha$ in the last terms of the above formulas, which are nearly correct for small angles,

$$\textbf{Vertical Distance} = (100 \times \text{rod interval} + 1)\ 1/2 \sin 2\alpha$$
$$\textbf{Horizontal Distance} = (100 \times \text{rod interval} + 1) \cos^2 \alpha$$

These are in the form used for ordinary transit work. The reduction of the field-notes can be best accomplished, however, by the use of tables, diagrams or stadia slide rules, all of which are based upon these formulas.

The rod intervals are usually read to the nearest hundredth of a foot, giving the distance to the nearest foot only, so that in most topographic work, where the distances are not required to within a foot, the constant $(F + c)$ may be neglected, and in such work there will be no need of applying the horizontal correction for vertical angles under 3 degrees.

30. Stadia Fieldwork

Points are Located by (1) the azimuth, (2) the distance, and (3) the angle of elevation or depression. If elevations are not required the vertical angle is read only to the nearest 10 minutes, for that is close enough for determining the horizontal distance, but if elevations are required the vertical angle should be read to the nearest minute and sometimes to the half-minute and the index correction also read for each vertical angle. The distance and azimuth angle are read with whatever refinement is necessary for the work in hand;

as a rule the distances are recorded to the nearest foot and the azimuth to the nearest minute, although for single "shots" which are to be plotted by use of the ordinary protractor the azimuth to the nearest 5 minutes is close enough. Traverse lines which control the survey are measured by tape or by stadia as the accuracy of the survey may demand. For large-scale maps it may be necessary to tape the distances of the control, but for small-scale maps the stadia method is sufficiently accurate. In the latter case the distances should be read both on the foresight and backsight.

Azimuths. In starting the survey, if the true azimuth of any line is known all of the azimuths of the survey may refer to that meridian, but if no meridian has been established, as is frequently the case, the azimuths may be referred to the magnetic meridian. Azimuth angles are read from the south right-handed for 360°. The horizontal circle is set on 0°, the telescope turned so that it points toward magnetic S and the lower clamp tightened. Then if the upper clamp is loosened and the telescope sighted toward the next station the horizontal arc, if read right-handed, will give the azimuth of the first line referred to the magnetic meridian. The azimuth of any other line may be obtained by simply resetting the upper clamp as each sight is taken. When it is necessary to move to the next instrument station the telescope is oriented, by the method explained in Art. 15, so that it will read 0° when pointing to the south. The bearings of the traverse lines will give a rough check on the azimuths of those lines.

Distances are read by setting one of the stadia hairs on a whole footmark, by means of the vertical arc clamp and tangent screw, and counting the feet, tenths and hundredths of a foot between the stadia hairs. Great care should be taken not to mistake the middle horizontal cross-hair for a stadia hair. This can be avoided by always making a mental estimate of the distance. Occasionally a half-interval may be read when something obstructs the view of the whole interval.

The Vertical Angles are taken by sighting the middle cross-hair on a point on the rod whose distance above the foot of the rod is equal to the distance from the horizontal axis of the telescope to the station beneath the transit. The distance is known as the **height of instrument** (H.I.); it is not the same as the H.I. in ordinary leveling, which is the distance of the telescope above the datum. When the middle hair is sighted at the H.I. point on the rod the line of sight is parallel to a line drawn from the station under the transit to the foot of the rod, and the vertical angle read is the inclination of this line, so that the difference in elevation of the two stations can be directly computed.

It is sometimes impossible to set on the H.I. point on the rod owing to obstacles in the line of sight. In such a case the angle is taken when sighting at some convenient footmark on the rod, and this reading of the middle hair on the rod is recorded in the notes with the vertical angle. The H.I. point is sometimes marked by a red cloth band; its position being changed at each new set-up of the instrument.

The Order of Fieldwork is as follows: First read the distance and record it, then set the middle hair on the H.I. point of the rod and the vertical hair on the rod. Signal the rodman to go to the next point, and while he is moving to the next point read the horizontal circle, the vertical arc and its index correction if necessary. Unless elevations are required, the H.I. is not used, and the vertical arc will not be read except for angles of 3° or more.

Stadia Notes require a large amount of sketching and description of details in order to convey sufficient information to enable a draftsman who may not be familiar with the locality to plot the notes. Where the notes are merely for flat topography, with no elevations, the difficulties are not great. The side shots are numbered, and these numbers should be put on the sketch in such a manner as not to be mistaken for any measured distance which may also be recorded on the sketch. Where the survey is for a contour map the accuracy of the map depends in a large measure upon the completeness of the descriptions given in the notes regarding the slope of the ground between located points. The elevation of points on ridges, valleys, knolls, and depressions will be determined in the field, but without a full description of the slopes of the surface the correct interpretation is often next to impossible. One method is to describe the shape of the ground in the notes; another method is to sketch contours in the notebook, showing approximately the form of the surface.

31. Stadia Computations

Reduction of Stadia Field Notes is usually done by means of tables, diagrams or stadia slide rule rather than by direct use of the formulas. The **stadia slide rule** is the most convenient when the reductions are required in the field as in plane-table surveys. The difference in elevation can be most rapidly obtained by means of a diagram or stadia slide rule, while the horizontal distances can be rapidly computed by use of a table of **horizontal corrections** which gives the distances to be subtracted from the inclined readings to obtain the horizontal distance, to which must be added the instrument constant $(F + c)$. The stadia reduction tables presented here give, for vertical angles up to 20°, the difference in elevation for an inclined distance of 100 ft. If the inclined reading is 612, the instrument constant 1 ft., and the vertical angle 4° 42′, the difference in elevation = 6.13 × 8.17 = 50.1 ft. Since the rod intervals are read as a rule to only three significant figures this multiplication can be accomplished with sufficient accuracy by means of the ordinary 10-inch slide rule. The 50.1 ft. is recorded in the notes and this difference in elevation is applied to the elevation of the station over which the transit is set to give the elevation of the point desired. It will be seen from an inspection of the tables that a half-minute greater vertical angle would have increased the difference in elevation by a little less than 0.1 ft., which shows the importance of reading the vertical angle carefully and of applying its index correction.

Even without this table vertical heights may be computed if a table of horizontal corrections is available. A condensed table of horizontal corrections can be made which can be pasted on a leaf in the back of the notebook. After the horizontal distance has been found by the use of this table the vertical height can be computed by multiplying the horizontal distance by the tangent of the vertical angle, which can be conveniently accomplished by use of the tangent scale on an ordinary slide rule.

Where the vertical angle is not taken to the H.I. mark on the rod this must be taken into account in the working up of the column of differences in elevation as follows. Suppose at a point the distance read was 135, vertical angle −8° 50′ on 2.5, H.I. 4.5; then −(1.36 × 15.17) + 2 = −18.6 ft. Here the 2 ft. is + because the vertical angle was taken to a point 2 ft. lower than the H.I. point. Had the vertical angle been a + angle the 2 ft. would still have been applied so as to increase the difference in elevation.

Beaman's Stadia Arc. Another method of simplifying the calculations of elevations in the field consists in using only those vertical angles for which the differences in elevation are simple multiples of the rod interval. This principle is used in the attachment devised by Beaman for the vertical arc of the transit or plane table. On this attachment are marked the numbers giving multiples of the rod interval, the zero graduation being marked 50 instead of 0 so as to make it easier to determine which are plus and which minus angles. In using a transit with this attachment the observer reads the distance, turns the tangent screw of the telescope until the index line is opposite some

Reductions of Stadia Observations

Vertical Heights for 100 Ft. Inclined Distance

Minutes	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0	0.00	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45
2	0.06	1.80	3.55	5.28	7.02	8.74	10.45	12.15	13.84	15.51
4	0.12	1.86	3.60	5.34	7.07	8.80	10.51	12.21	13.89	15.56
6	0.17	1.92	3.66	5.40	7.13	8.85	10.57	12.26	13.95	15.62
8	0.23	1.98	3.72	5.46	7.19	8.91	10.62	12.32	14.01	15.67
10	0.29	2.04	3.78	5.52	7.25	8.97	10.68	12.38	14.06	15.73
12	0.35	2.09	3.84	5.57	7.30	9.03	10.74	12.43	14.12	15.78
14	0.41	2.15	3.90	5.63	7.36	9.08	10.79	12.49	14.17	15.84
16	0.47	2.21	3.95	5.69	7.42	9.14	10.85	12.55	14.23	15.89
18	0.52	2.27	4.01	5.75	7.48	9.20	10.91	12.60	14.28	15.95
20	0.58	2.33	4.07	5.80	7.53	9.25	10.96	12.66	14.34	16.00
22	0.64	2.38	4.13	5.86	7.59	9.31	11.02	12.72	14.40	16.06
24	0.70	2.44	4.18	5.92	7.65	9.37	11.08	12.77	14.45	16.11
26	0.76	2.50	4.24	5.98	7.71	9.43	11.13	12.83	14.51	16.17
28	0.81	2.56	4.30	6.04	7.76	9.48	11.19	12.88	14.56	16.22
30	0.87	2.62	4.36	6.09	7.82	9.54	11.25	12.94	14.62	16.28
32	0.93	2.67	4.42	6.15	7.88	9.60	11.30	13.00	14.67	16.33
34	0.99	2.73	4.48	6.21	7.94	9.65	11.36	13.05	14.73	16.39
36	1.05	2.79	4.53	6.27	7.99	9.71	11.42	13.11	14.79	16.44
38	1.11	2.85	4.59	6.33	8.05	9.77	11.47	13.17	14.84	16.50
40	1.16	2.91	4.65	6.38	8.11	9.83	11.53	13.22	14.90	16.55
42	1.22	2.97	4.71	6.44	8.17	9.88	11.59	13.28	14.95	16.61
44	1.28	3.02	4.76	6.50	8.22	9.94	11.64	13.33	15.01	16.66
46	1.34	3.08	4.82	6.56	8.28	10.00	11.70	13.39	15.06	16.72
48	1.40	3.14	4.88	6.61	8.34	10.05	11.76	13.45	15.12	16.77
50	1.45	3.20	4.94	6.67	8.40	10.11	11.81	13.50	15.17	16.83
52	1.51	3.26	4.99	6.73	8.45	10.17	11.87	13.56	15.23	16.88
54	1.57	3.31	5.05	6.79	8.51	10.22	11.93	13.61	15.28	16.94
56	1.63	3.37	5.11	6.84	8.57	10.28	11.98	13.67	15.34	16.99
58	1.69	3.43	5.17	6.90	8.63	10.34	12.04	13.73	15.40	17.05
60	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45	17.10

Horizontal Corrections

Distance	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
100	0.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.5
200	0.0	0.1	0.2	0.5	1.0	1.5	2.2	3.0	3.9	4.9
300	0.0	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.8	7.4
400	0.0	0.1	0.5	1.1	2.0	3.0	4.4	6.0	7.8	9.8
500	0.0	0.2	0.6	1.4	2.5	3.8	5.5	7.5	9.7	12.3
600	0.0	0.2	0.7	1.6	2.9	4.6	6.5	8.9	11.6	14.7
700	0.0	0.2	0.8	1.9	3.4	5.3	7.6	10.4	13.6	17.2
800	0.0	0.2	1.0	2.2	3.9	6.1	8.7	11.9	15.5	19.6
900	0.0	0.3	1.1	2.4	4.4	6.8	9.8	13.4	17.5	22.1
1000	0.0	0.3	1.2	2.7	4.9	7.6	10.9	14.9	19.4	24.5

Reductions of Stadia Observations—Continued

Vertical Heights for 100 Ft. Inclined Distance

Minutes	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
0	17.10	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78
2	17.16	18.78	20.39	21.97	23.52	25.05	26.55	28.01	29.44	30.83
4	17.21	18.84	20.44	22.02	23.58	25.10	26.59	28.06	29.48	30.87
6	17.26	18.89	20.50	22.08	23.63	25.15	26.64	28.10	29.53	30.92
8	17.32	18.95	20.55	22.13	23.68	25.20	26.69	28.15	29.58	30.97
10	17.37	19.00	20.60	22.18	23.73	25.25	26.74	28.20	29.62	31.01
12	17.43	19.05	20.66	22.23	23.78	25.30	26.79	28.25	29.67	31.06
14	17.48	19.11	20.71	22.28	23.83	25.35	26.84	28.30	29.72	31.10
16	17.54	19.16	20.76	22.34	23.88	25.40	26.89	28.34	29.76	31.15
18	17.59	19.21	20.81	22.39	23.93	25.45	26.94	28.39	29.81	31.19
20	17.65	19.27	20.87	22.44	23.99	25.50	26.99	28.44	29.86	31.24
22	17.70	19.32	20.92	22.49	24.04	25.55	27.04	28.49	29.90	31.28
24	17.76	19.38	20.97	22.54	24.09	25.60	27.09	28.54	29.95	31.33
26	17.81	19.43	21.03	22.60	24.14	25.65	27.13	28.58	30.00	31.38
28	17.86	19.48	21.08	22.65	24.19	25.70	27.18	28.63	30.04	31.42
30	17.92	19.54	21.13	22.70	24.24	25.75	27.23	28.68	30.09	31.47
32	17.97	19.59	21.18	22.75	24.29	25.80	27.28	28.73	30.14	31.51
34	18.03	19.64	21.24	22.80	24.34	25.85	27.33	28.77	30.19	31.56
36	18.08	19.70	21.29	22.85	24.39	25.90	27.38	28.82	30.23	31.60
38	18.14	19.75	21.34	22.91	24.44	25.95	27.43	28.87	30.28	31.65
40	18.19	19.80	21.39	22.96	24.49	26.00	27.48	28.92	30.32	31.69
42	18.24	19.86	21.45	23.01	24.55	26.05	27.52	28.96	30.37	31.74
44	18.30	19.91	21.50	23.06	24.60	26.10	27.57	29.01	30.41	31.78
46	18.35	19.96	21.55	23.11	24.65	26.15	27.62	29.06	30.46	31.83
48	18.41	20.02	21.60	23.16	24.70	26.20	27.67	29.11	30.51	31.87
50	18.46	20.07	21.66	23.22	24.75	26.25	27.72	29.15	30.55	31.92
52	18.51	20.12	21.71	23.27	24.80	26.30	27.77	29.20	30.60	31.96
54	18.57	20.18	21.76	23.32	24.85	26.35	27.81	29.25	30.65	32.01
56	18.62	20.23	21.81	23.37	24.90	26.40	27.86	29.30	30.69	32.05
58	18.68	20.28	21.87	23.42	24.95	26.45	27.91	29.34	30.74	32.09
60	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78	32.14

Horizontal Corrections

Distance	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
100	3.0	3.6	4.3	5.1	5.9	6.7	7.6	8.5	9.5	10.6
200	6.0	7.3	8.6	10.1	11.7	13.4	15.2	17.1	19.1	21.2
300	9.1	10.9	13.0	15.2	17.6	20.1	22.8	25.6	28.6	31.8
400	12.1	14.6	17.3	20.2	23.4	26.8	30.4	34.2	38.2	42.4
500	15.1	18.2	21.6	25.3	29.3	33.5	38.0	42.7	47.7	53.0
600	18.1	21.8	25.9	30.4	35.1	40.2	45.6	51.3	57.3	63.6
700	21.1	25.5	30.2	35.4	41.0	46.9	53.2	59.8	66.8	74.2
800	24.2	29.1	34.6	40.5	46.8	53.6	60.8	68.4	76.4	84.8
900	27.2	32.8	38.9	45.5	52.7	60.3	68.4	76.9	85.9	95.4
1000	30.2	36.4	43.2	50.6	58.5	67.0	76.0	85.5	95.5	106.0

line on the attached arc, and then notes the rod reading of the middle cross-hair. The difference in elevation between the telescope and the reading on the rod is (50 minus the reading of the arc) times the rod interval. From this result must be subtracted the rod reading to give the difference in elevation between the telescope and the foot of the rod. Evidently no stadia tables or slide rule are needed when this attachment is used.

32. The Plane Table and Its Use

The Plane Table is an instrument by means of which points are located in the field by graphical methods directly on the map, which is fastened to a table top supported on a tripod. The accuracy of the map is usually limited by the accuracy of plotting rather than by the field measurements, for it is used mostly for small-scale maps. In the field the map must be protected from becoming distorted by moisture, so as to preserve the accuracy of the plot. The plane table is the only surveying instrument admitting of a rapid solution of the Three-point Problem in the field; this makes it practicable to locate stations independently of each other, so that errors cannot accumulate as they do in traversing. The most important advantage of the plane-table method over other topographic methods, however, is that all of the sketching is done in the field, where the topographer can see the form of the ground that he is mapping. He can sketch details at once in their proper position, without burdening his memory and without making elaborate notes. For this reason the details may be accurately sketched from a much small number of located points than would be required, for instance, by the transit and stadia method.

The plane-table method has the disadvantage of requiring more time for the field-work than other methods, and it is also more dependent upon favorable weather than a method where the map is not exposed. But taking into account both the field and office work the plane table will prove to be much more economical than the transit and stadia for work in open country, and at the same time the results obtained will be sufficiently accurate for most topographic work.

The Plane Table itself consists of a board, usually about 24 by 30 in., mounted on a tripod with a device for leveling and clamping the board, the Johnson ball-and-socket leveling device and clamp being the most commonly employed. The **alidade** consists of a telescope mounted on a horizontal axis resting in wye supports, which are connected to a metal column at the base of which is attached a flat piece of metal about 18 in. long having both edges straight and parallel to the telescope. On this base are two spirit-levels for leveling the table. The telescope has a vertical motion only and a vertical arc. The entire instrument is moved about in azimuth on the board and sighted as desired.

Locating Points by Intersection. In order to begin a survey with a plane table it will in general be necessary to have on the map two plotted points corresponding to two points on the ground the distance between which is known and at least one of which can be occupied with the table. The simplest method of locating points by means of the plane table without measuring any distance is as follows: The base-line ab is plotted on the plane-table sheet, representing, to some scale, the measured base AB. The table is set so that a on the map is vertically above A on the ground and the table is leveled; then one edge of the alidade is placed along the base-line ab drawn on the map, the table is then turned in azimuth until the telescope sights the signal B, and the horizontal motion clamped. The line ab is now parallel to AB and the table is said to be oriented. The alidade is then placed so that the straight-edge passes through a, the telescope is sighted to some signal C, and an indefinite line drawn toward C. The point c on the map (representing the signal C)

is somewhere on this line. If the table is now moved to B (b being set vertically over B) and the process of orienting the table and sighting toward C is repeated, the point c is located on an indefinite line through b; hence it lies at the intersection of these two lines ac and bc. The triangle abc is similar to the triangle ABC, and each line on the map is parallel to the corresponding line on the ground. In a similar manner any number of points may be located.

Locating Points by Direction and Distance. The simplest way of locating points by the plane table is by obtaining the direction with the alidade and measuring the distance by stadia. This is the method most commonly used for filling in the details of a plane-table survey after the table has been oriented.

Locating Points by Resection. It sometimes happens that it is desired to locate a plane-table station from a base only one end of which can be occupied with the table. In such a case we may proceed as follows: Let A and B represent the points on the ground at the ends of the base-line; C is the signal which is to be located, and ab represents the base-line plotted on the plane-table sheet. Set up at A, the end of the base which is accessible, and orient the table by sighting B with the alidade along ab. Then, centering the alidade on a, draw an indefinite line toward C. This line should be drawn the full length of the alidade. The table is then taken to C and oriented by means of the indefinite line just drawn. Since the position of c on the indefinite line is not known it is necessary to estimate its position on the map and to use this point in placing the table over the point C. If the alidade is now centered on b and sighted toward B, a resection line may be drawn, and this line will cut the first indefinite line, thus locating the point c desired. Thus, without going to station B, the same line has been drawn on the map that would have been obtained by an intersection from B. It is evident that this resection method may be advantageous even if B could be occupied, for the point C has been located without taking the time required to go to station B. The position of c found by this method should be checked if possible by resection lines from other points whose positions are known to be correct.

The Three-point Problem. One of the great advantages of the plane table is that it may be set up at any place where three triangulation points (plotted on the sheet) can be seen and the position of this plane-table station located on the sheet simply by observations from this point. The position of the plane table is found by means of the so-called Three-point Problem, which, in this case, is an application of the principle of resection. The two graphical solutions of this problem chiefly used in plane-table surveying are known as (1) **Lehmann's** method, or the **Triangle-of-error** method; and (2) **Bessel's** method, or the **Inscribed Quadrilateral.** The former is a trial method, but it is the more rapid of the two for ordinary work and is used in practice far more than the latter method. Bessel's method gives a direct solution and consequently requires less experience than the former. It has the disadvantage, however, that in certain positions of the signals a part of the required geometric construction falls outside the limits of the plane-table sheet, in which case the solution is not practicable.

Lehmann's Method. If three signals A, B, and C, have their plotted positions at a, b, and c, and if the table be set up at D and oriented correctly, the resection lines drawn from a, b, and c will all pass through d, the plotted position of D. Since there is no means of accurately orienting the table, the position of d being unknown at the start, the table must be oriented approximately by estimation. If the plane table is not oriented exactly the three resection lines will not ordinarily pass through a common point but will form a triangle known as the triangle of error (Fig. 28). From this triangle of error the true position of d may be estimated, and by a second trial a new triangle of error may be obtained which is smaller than the former. By successive trials this triangle may be made so small that it is almost a point. In practice very few trials are necessary, the triangle often being reduced to a point in the second trial, so that the method is in reality a rapid one.

If the table is on the circumference of the circle through the three signals its position is indeterminate. When point D is inside the triangle ABC it is in a favorable position for an accurate location. If the table is outside this triangle there are certain positions of the signals which are not favorable, especially when the angles subtended by the sides of the triangle formed by the signals are small and the middle signal is farthest from D, but if the middle signal is near D the location of d is strong.

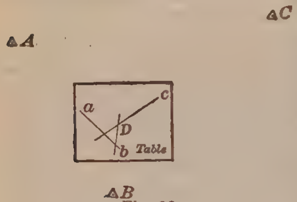


Fig. 28

If D lies inside the triangle ABC then d will lie inside the triangle of error and vice versa. If a circle is passed through a, b and the intersection of the resection lines from a and b it will pass through the true position of d; similarly a circle through b, c and resection lines from b and c may be sketched and in this manner a close estimate of the position of d made for the second trial. The distance of d from any resection line is proportional to the distance of the table from the signal from which that line was drawn.

33. Hydrographic Surveying

Hydrographic Surveying is the term applied to the processes used in surveying any body of water. The determination of the topography of the bottom of a lake, harbor, or other body of water is one of the common problems. In connection with such surveys the character of the material composing the bottom is often desired. The fieldwork is usually done by first establishing certain points on shore, by triangulation or traverse, to which the hydrographic survey may be referred, and then measuring (usually from a boat) the depth of the water at various points and determining the position of these points. Besides the ordinary transit and tape outfit, a sounding-pole or lead-line and a boat with the necessary equipment will be required. A tide gage should be set up in tidal waters and in lakes where the water level changes rapidly. The sounding-poles are made similar to self-reading rods graduated to tenths of a foot. A shoe is sometimes attached to the bottom, provided with a cup-shaped cavity which, if smeared with tallow or soap, enables samples of material to be collected. The lead-line consists of a long chain or a hemp or cotton line at the end of which is attached a lead weight. A brass sash-chain gives very satisfactory results, with cloth tags of various colors for foot-marks. Where there is not much current a 6- to 10-lb. weight will suffice for depths up to 40 ft.

Methods of Locating Soundings. There are six general methods of locating soundings: (1) The boat is rowed on a range at a uniform rate of speed and the soundings are located by time intervals. (2) The boat is rowed on a range line and the positions of the soundings are " cut in " by a transit angle taken on shore or by an angle taken with a sextant from the boat. (3) The boat may or may not be rowed on any definite range, and its position is located by angles taken simultaneously by two transits on shore or by angles taken simultaneously to shore points from the boat by means of two sextants. (4) The positions of the soundings are located by the stadia method. (5) The positions of the soundings are defined by the intersection of fixed ranges. (6) A wire or line is stretched across a stream from shore to shore and soundings are taken at different points along this wire and located by measured distances from one end of it.

Locating a Sounding by a Range and an Angle. In still water where there is no difficulty in keeping the boat in any desired position soundings may be conveniently located by keeping the boat on a range line of known position marked by two objects on shore, such as range poles, and then " cutting in " the position of the leadsman by means of a transit angle taken from shore at the instant the sounding is made. The ranges may be fan-shaped if the pivot-point of the system of ranges is some steeple or other object located far enough back from the shoré so that the lines will not diverge too rapidly. The recorder writes in his notebook the depths as they are called off by the leadsman and also the times when the soundings are made. He also notifies the leadsman by calling out " Sound " about 5 seconds before each sounding is desired. The leadsman takes the soundings as quickly as possible, usually within two or three seconds of the desired time. In the hydrographic work of the Corps of Engineers, U. S. A., where nearly all of the soundings are located by two angles taken with transits on shore, soundings are usually taken at 15, 20, or 30-second intervals, and a location made each minute by the instrument men at the instant the signal is given by the signalman in the boat. The chief of party usually acts as signalman and directs the work in the boat, and sees that the boat is kept on the ranges (if any are used) and that the boat is so propelled as properly to cover the area to be sounded. The signal is given by holding up a flag for about 10 seconds and dropping it suddenly the instant the sounding is taken, at which moment the transitmen on shore take angles to the leadsman or to his hand if visible. In the work of the U. S. Engineers, white, red, and sometimes black flags are used for signaling. Both the recorder's and the instrument men's notes should show the colors of the signal as well as the time for each located sounding, thus giving two means of identifying the angles and the corresponding soundings. This double check is of particular value where the lines of soundings are long. In tidal waters the time record is required also as a means of reducing the soundings to **Mean Low Water** or to whatever other datum is used. The tide gage readings and times are recorded throughout the day.

Where the soundings are taken with a view to obtaining every slight change in the slope of the bottom, instead of taking the soundings at a given interval of time, it is desirable to take them as frequently as the leadsman can conveniently handle the pole. In this method the boat is usually rowed on a range and the instrument men on shore " cut in " only those soundings that are designated by the signalman in the boat. The time in this case is recorded to the nearest second. As a rule every sixth or eighth sounding is located.

In plotting notes like these where the soundings are quite close together the points which were " cut in " are located on the plan and the intermediate readings are interpolated between them; the soundings are assumed to be equally spaced between the located ones.

Locating a Sounding by Two Angles from a Boat. A common method of locating soundings when ranges are not used is by taking two angles simultaneously from the boat to three signals, or any previously determined points, A, B, and C, on shore. This is an application of the Three-point Problem that is frequently used in plane-table work. It is essential that the signals or points should be fixed stations; such points as buoys or floats will not give satisfactory locations.

In measuring angles from a boat it is necessary to employ some instrument which does not require a steady support like the transit. For this reason the angles are usually taken with two sextants. These two angles are sufficient to locate the position of the soundings, except in the one case where the boat happens to be on the circumference of the circle passing through the three signals between which the angles are measured.

This method is less frequently employed than the " range and angle " method or the " two angles from shore " method, because it often happens that the two angles taken with the sextant do not give good intersections; this is especially true when the soundings extend far from shore. Furthermore, if the signal happens to be high above the shore, and consequently not at the level of the boat, the angle measured will be enough different from the horizontal angle to introduce serious error into some parts of the work.

The Sextant is an instrument adapted to measuring angles in any plane. Owing to the fact that it can be used by an observer who is on a moving object, such as a boat, it is especially valuable for hydrographic work. It is employed not only for taking angles from a boat in locating soundings but is also in common use for making astronomical observations which are necessary in determining the latitude, longitude, and time at sea. The **frame** *ABI* (Fig. 29) is usually of brass, on the under side of which is attached the wooden handle *D*. The index arm *IE* is pivoted at *I*, the center of the arc *AB*, and

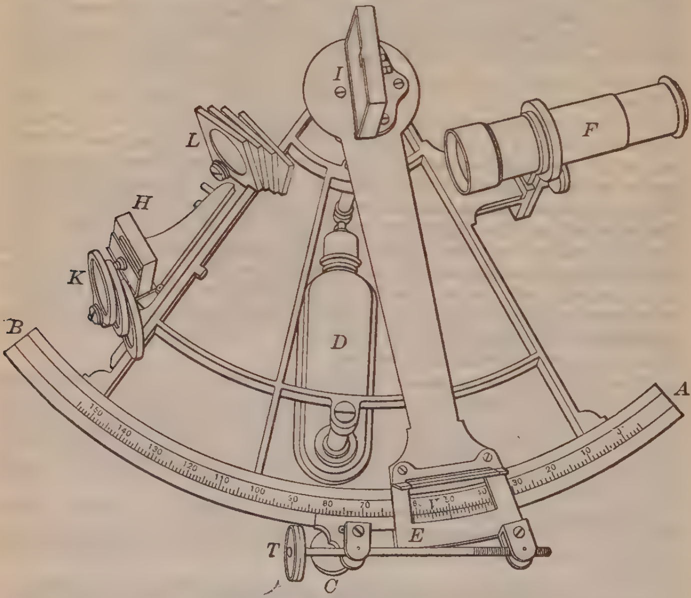


Fig. 29. The Sextant

this arm can be swung around *I* as a center so that the vernier *V* can pass from *A* to *B* on the arc (or limb) and can be set at any position on the arc by means of the clamp *C* and tangent screw *T*. At *I* is a plane glass mirror called the index glass; it is attached rigidly to the index arm and perpendicular to the plane of the sextant, its reflecting surface being over the pivot about which the index arm revolves. Rigidly attached and perpendicular to the frame of the sextant is the plane horizon glass *H*, the upper half of which is transparent and the lower half is a mirror; this glass is set so that its plane is parallel to the index glass when the vernier is set at 0°. The telescope is at *F*. *K* and *L* are colored glasses which are hinged so that they can be swung around the pivot into the path of the rays of light to protect the eye of the observer in making observations on the sun.

It should be borne in mind that the angle measured with the sextant is not the horizontal angle, but is the angle lying in a plane defined by the two objects and the eye of the observer; the objects sighted should therefore be near the shore level unless the sextant is far from shore.

The vertex of the angle is not a fixed point; it moves farther away from the instrument as the angles grow smaller for the lines defining the angle pass from the two objects through the center of the mirrors *I* and of the glass *H*.

The limb *AB* is graduated into spaces which are really half-degrees, but on account of the construction of the instrument each of these is marked as a whole degree, so that the scale has an extent of 120 degrees. The graduations are so subdivided that the angles can be read in most instruments to 10 seconds; in some of the smaller instruments the vernier reads only to half-minutes. In the ordinary sextant the arm is from 5 to 8 in. in length. A pocket sextant having an arm about 2 in. long is very convenient for reconnaissance surveys and for filling in the details of more accurate surveys.

To Measure an Angle with the sextant, hold the instrument by its handle in the right hand and turn it so that the plane of the sextant coincides with the plane through the two objects to be observed, with the telescope on the upper side of the sextant if the angle is horizontal, or on the left-hand side in the case of a vertical angle. Without changing the plane of the sextant twist it in the hand so as to turn the telescope toward the left-hand object and observe it through the upper (transparent) portion of the horizon glass. Then, holding the instrument as steady as possible, turn the index arm with the left hand until the other object appears in the silvered portion of the horizon glass opposite the first point. Bring the second point exactly opposite the first one by means of the clamp and tangent screw of the index arm. The coincidence of the images should be tested by twisting the instrument a little so as to make the reflected image move back and forth across the direct image. Read the vernier and apply the index correction. The telescope is not used for rapid work such as locating soundings; the sight is simply made through the ring in which the telescope fits.

34. Photographic Surveying

A rapid method of locating topographic details for construction of small-scale maps (say 2 or 3 miles to 1 in.) is afforded by photographic surveying. Best results are obtained where the country has characteristic shapes, and is not too thickly wooded to afford good positions for taking the views. To locate a point, it is photographed from two stations the positions of which are known. It is necessary to know the direction in which the camera was pointed when each photograph was taken, and the focal length f of the lens.

The **Instrument** used for this work sometimes consists of a transit, so constructed that standards and telescope can be removed and a camera mounted in their place, so that azimuth angles can be measured, or laid off, by the horizontal circle. The instrument is leveled and the horizontal angles between certain well-defined points are measured while the telescope is on the transit; when the camera is substituted, the same angles are laid off on the circle and the exposures are made. Such instruments cost about $500.

Notches on the four sides of the camera opening yield corresponding points on the edges of every picture; lines are then drawn across photograph connecting these four points. The vertical line is known as the **principal** and the other as the **horizontal** line; their intersection is called the **principal point.**

If the photograph is held at distance f in front of the eye, and normal to the line of sight, points a, b, and c on the picture will appear to cover corresponding points A, *B*, and *C* in the landscape. The photograph, when held in this position, thus gives a measure of the angles between various points in the view, a fact utilized in plotting.

To Plot Map, first locate positions of camera stations *X* and *Y* (Fig. 30). At each station draw a circle of radius *f*, and a radial line representing direction in which camera was pointed when photograph was taken, as *Xm* and *Yn*. Line *op* represents the trace of the photograph, tangent to arc at *m*, its principal line, and *qr* represents photograph taken at *Y*, its principal line passing through *n*. To plot point *a*, first measure on the photograph *op* the distance from *a* to the principal line, and plot that distance on line *po* as *ma'*. Similarly, the distance *na''* is measured on the other photograph and plotted along trace *rq*. The plotted position of *A* thus lies at the intersection of *Xa'* and *Ya''*, or at *a*. In the same manner are plotted as many points as desired.

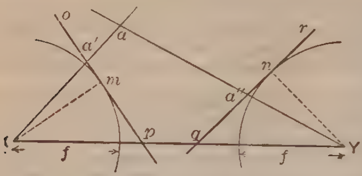


Fig. 30. Plotting Camera Stations

Differences in Elevation are computed as follows: Scale the distance of point *a* above the horizontal line on the photograph. This distance divided by distance from camera station to point *a'* (scaled from map) gives the natural tangent of angle of elevation or depression. The actual horizontal distance *Xa* is then scaled from map, after the point has been plotted, and is multiplied by tangent of vertical angle, to give difference in elevation between camera station and point *a*; or it can be found graphically. Details of the map are now sketched from a study of the photograph, after characteristic points have been plotted.

35. Aerial Surveying

(For further details, see "Higher Surveying," by Breed and Hosmer, 1926 Ed., Chap. 12.)

In photographic surveying from land stations the sensitive plate is always vertical when exposed; photographs of fixed objects are taken from fixed stations, the positions of which have been determined by survey. When photographing from an airplane, only the terrain is fixed; the camera is moving, and the plate or film is held in an approximately horizontal position.

Airplane Mapping is now often used for surveying: electric lines, reservoirs for water power, irrigation and water supply, harbor improvements, city planning, traffic studies, flood control, timber estimates. It is peculiarly adapted to regions difficult to travel. If controlled by triangulation, or by known distances between fixed objects on the ground, airplane mapping is in general accurate within narrow limits; but details may suffer considerably, especially where there are sudden and considerable changes of elevation.

The pictures are taken looking directly downward. To keep the axis of the lens as nearly vertical as possible, the camera is mounted on gimbals; the axis is exactly vertical only when the airplane is horizontal. If the terrain is level, pictures are true perspectives; if hilly, the perspective is true only of points in line with axis if the lens at moment of exposure.

Making Exposures. The airplane flies along nearly straight, parallel courses, so spaced that pictures taken at short intervals completely cover the area to be mapped, and with sufficient overlapping to enable a draftsman to construct a relatively accurate map.

There are 3 steps: (*a*) Piloting the plane in straight courses and at a proper elevation. This requires experience, for if the elevation of the plane varies the area of ground shown on the plate also varies and the scale of photograph is changed—the result is similar in passing over hilly ground; (*b*) taking the photographs, which are timed automatically, according to speed of plane and its elevation; (*c*) making the map.

Since the plane's fuel capacity limits the time it can remain in flight, the distance from the flying field influences the cost of the work. Atmospheric conditions are important; satisfactory pictures can be taken only when the air is free from low clouds, moisture, dust, smoke and haze; generally, about one day in seven is favorable.

Elevation of Airplane is determined by an altimeter. The height that the pilot endeavors to maintain depends upon focal length of camera and scale of map required. Exposures are so timed that pictures overlap about 60% in the direction of flight, and the distance between parallel flights is so chosen that pictures overlap about 50% along their sides. Theoretically, this causes each object to be photographed four times; some of them oftener.

This large amount of overlap is essential to make accurate maps, as the point on the ground corresponding to the projection of the center line of lens must appear on edges of adjacent photographs, so that their relative position will be known. Only the middle portion of each photograph is useful for accurate maps, the edges being subject to errors due to tilting of the camera, and high objects on the ground do not appear in true perspective if near the edges of the pictures. Ample overlap permits elimination of defective exposures, caused by excessive tilting of the camera or cloud shadows; it also affords opportunity to view obscure objects from more than one angle, which is especially valuable when a stereoscope is used in studying details.

A Stereoscope is an instrument in which two views of the same object are seen, one with each eye. Two photographs are placed side by side in the stereoscope, so that the binocular vision is clear. Objects in the picture stand out in sharp relief, and their relative heights and depths are shown. Thus the ground can be examined, relative elevations determined, and contour maps made. An instrument called a **stereograph** is used for tracing contours from the photographs.

Ground Control. Before beginning flying, base lines can be measured on the ground; white rings of cheesecloth, 3 ft. wide by 30 to 50 ft. in diameter, or white crosses, being placed at the ends of the base.

If these lines are laid out on uneven ground, or measured on slopes, and intermediate points are marked upon them, they serve to control details of the pictures near them; any photograph or part of a photograph appearing out of scale can be reproduced in the office to the scale indicated by the base line, provided both ends of that line appear on the photographs. Stations on the larger triangulation system controlling the whole map may also be marked by white circles or crosses, which show up distinctly in the pictures. For maps of large areas, the position of certain well-defined objects, which will appear in the photographs, are determined astronomically.

Choice of Scale of Photographs. A scale of 1 in. = 400 ft. is about as large as can be readily obtained from an airplane; for larger scale, the plane must fly so low as to require excessive speed of the camera.

Maps are readily made to scale of 1 in. = 800 ft., and enlarged by photographic process. The smaller the scale of actual exposures, the fewer the pictures needed. Parts of a map where details are required may be made on a scale of, say, 1 in. = 400 ft., and the outlying portions, 1 in. = 800 ft., the latter to be enlarged to 1 in. = 400 ft. in the office; expense is thus saved.

Office Processes. After developing films and making prints, any prints out of scale are rephotographed to proper scale. The entire set is then assembled upon a large board, on which the triangulation lines are plotted, and the photographs are carefully matched one to another, only the middle portions being retained. The matched plan is then mounted, forming what is known as a mosaic map (see below). For more accurate maps, errors in scale are corrected as stated above; errors of displacement due to objects being near the edge of picture instead of directly under the camera, and errors of distortion due to tilting of the camera, are corrected by geometric processes.

Kinds of Airplane Maps. 1. **Mosaic,** a term applied when pictures are fitted together from the contact prints, with no attempt to bring them to a particular scale (a contact print is made in direct contact with the negative, so that the image is the same size as that on the negative). Mosaics are useful where an exact scale is not needed. In estimating timber in a forestry survey a mosaic gives the required information clearly, as the scale is unimportant. In some traffic studies, the mosaic is also satisfactory. 2. **Photographic map** results from correcting the scale errors of the contact prints as well as possible, by rephotographing the whole strip of connected prints, to bring all to the desired scale. It is often an enlargement of the original. It has been much used for city maps, tax maps, harbor surveys, and locating electric transmission lines. 3. **Line map** is one in which all errors of scale, displacement, and distortion have been corrected so far as possible.

As a matter of convenience, line maps are usually made on tracing cloth, mainly by tracing the photographic details, for reproduction by blueprint or other process. For engineering projects, where contours are important, this is usually the best form of map. Contours placed directly on the photographs are likely to be less satisfactory than if kept separate, as they cover some of the detail in the photographs, and are themselves more difficult to read. A line map showing the main features and the contours, supplemented by numbered and indexed photographs, usually gives completer information than when all is crowded on one map. In some cases, however, photographic maps showing contours are quite effective, and more readily understood by non-technical men than a contour map without photographic detail.

Aerial Cameras: 1. **Eastman,** single-lens, focal-plane shutter type; focal length, 10–20 in. 2. **Air Service,** devised by Major J. W. Bagley. Has 3 and 4 lenses; focal length, 6.5–7.5 in., with between-lens automatic shutter (see instructions, T1 Aircraft camera, Eng. Div., Air Service, Dayton, Ohio). 3. **Fairchild,** single-lens, focal length, 6–28 in. 4. **Brock,** for plates instead of films; the pictures are used in a special stereoscope for correcting errors of tilt and scale.

Cost of Aerial Surveys. So much depends upon extent of the survey, distance from landing field to the site, and frequency of clear days, that the cost varies greatly. Cost of the map itself also depends largely upon the amount of control and of the office work, which is much greater for a line map than for a mosaic.

A mosaic map of an area of 25 sq. miles, in which the middle half of the area is photographed at 1 in. = 400 ft., and the fringes comprising the other half at 1 in. = 800 ft., the landing field being not over 5 miles distant, should cost \$2 000–\$4 000 for one large mosaic, made up of the middle portions of the photographs. Copies cost \$100–\$300 each.

36. Mine Surveying

In Mine Surveys, all accurate measurements are made with the transit and steel tape. Transists used in this work are provided with an auxiliary telescope attached to one side of or above the main telescope, so that it is possible to sight down a vertical shaft. The instrument known as the **eccentric bearing transit** has one telescope which can be used either in its center bearings as in an ordinary transit or in bearings built out over the limb of the instrument so as to take vertical sights.

Transferring a Meridian into the mine is one of the important steps. This is usually done by setting up the transit at the mouth of the shaft and after taking a backsight on a surface station to take a foresight down the shaft, the line of sight being made as much inclined as possible, and the inclined distance measured. The transit is then set up at the bottom of the shaft, a backsight taken on the top station and the traverse carried into the mine. The utmost care must be taken to eliminate all errors of adjustment of the transit, for the accuracy of all the underground survey depends upon this process. Another and probably more accurate method of transferring a meridian into a mine is by means of two heavy plumb-bobs hung from the staging above the shaft. The transit is set up both above and below on this line and thus an azimuth line is established between the surface and the workings. In order to have as long a base as possible the plumbs should be hung near the shaft casing.

Underground Traverses are run through the passages in the mine. It is often necessary to introduce very short lines into the traverse, and since the azimuth is transferred to distant parts of the mine through these short lines great care must be taken to eliminate all instrumental errors by reversing the telescope and using the mean of the two results. The positions of walls and workings are located by measurements from the traverse line. The station points are often placed in the roof; a nail in a wooden plug is a good station mark.

Notes of Mine Surveys are kept as a rule in the form of sketches, especially the details, such as the location and extent of the stopes. The different station points of the survey are numbered consecutively. For convenience, stations on the first level are numbered 101, 102, etc., on the second level, 201, 202, and so on. A column for vertical angles is required, for most of the traverse lines are inclined.

To Plot the Survey the three sets of coordinates must be computed, which give all the data necessary to plot the mine in plan, longitudinal section, and transverse section. The horizontal and vertical distances equal the inclined distance multiplied by the cosine and sine respectively of the vertical angle, and the azimuth or bearings together with the horizontal distances give a means of computing the latitudes and departures of the courses.

Surveying for Patent. By patent proceedings are meant the proceedings necessary to obtain from the government a fee-simple deed of a mining claim. Title to metalliferous land as granted by the United States, conveys the right to all minerals included in the downward prolongation of the portion of vein cut off by the vertical end boundaries of the claim. The federal law allows a claim to cover 1500 ft. located along the direction of a vein and 300 ft. of surface ground on either side of it. These are maximum dimensions and may be reduced by local laws. For further information see the "Manual of Instructions for the Survey of the Mineral Land of the United States," issued by the General Land Office, Washington, D. C.

ASTRONOMICAL OBSERVATIONS

37. Definitions

Practical Astronomy treats of the theory and use of astronomical instruments and the methods of computing the results obtained by observation. In this branch of astronomy only the relative directions of heavenly bodies are considered, not the actual distances, hence it is convenient to regard all such bodies as situated on the surface of a sphere, called the **celestial sphere,** Fig. 31, the radius of which is infinite and the center of which is the center of the earth. The **vertical** at any point on the earth's surface is the direction of the force of gravity at that point. This direction is shown by the plumb-line. The vertical produced upward pierces the celestial sphere in a point called the **zenith.** The point where this line pierces the sphere on the opposite side is the **nadir.** A plane through the center of the earth perpendicular to the vertical cuts the sphere in a great circle called the **horizon.** It is sometimes called the true horizon to distinguish it from the

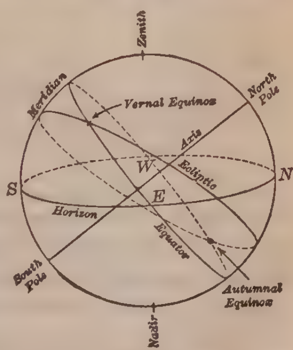


Fig. 31

sea horizon. The latter is a small circle, below the true horizon, the plane of which is parallel to the plane of the true horizon.

The Earth's Rotation upon its axis causes all celestial bodies to appear to revolve in the opposite direction at the same rate. To an observer looking along the axis from north to south all bodies appear to revolve in a right-handed, or clockwise, direction. The **axis** of the celestial sphere is the earth's axis produced. The **poles** are the points where this axis pierces the celestial sphere. The **celestial equator** is a great circle of the celestial sphere midway between the poles; its plane coincides with the plane of the earth's equator. The **meridian** of an observer is that great circle which passes through his zenith and the poles; its plane contains the observer's vertical. The intersection of the plane of the meridian and the plane of the horizon is the **meridian line.** This line cuts the horizon in the north and south points. The **prime vertical** is a great circle through the zenith the plane of which is perpendicular to the meridian. It cuts the horizon in the east and west points.

The **ecliptic** is a great circle of the celestial sphere which the sun appears to describe during the year. It is inclined to the equator at an angle of nearly 23° 27′. The points of intersection of the equator and the ecliptic are the **eqinoxes,** and the points on the equator midway between the equinoxes are the **solstices.**

38. Astronomical Coordinates

Two Spherical Coordinates may be used to designate the position of any point on the sphere. The circles of reference in any system are first, a great circle called the primary, and second, a system of great circles perpendicular to the primary, called secondaries. The following systems are commonly used.

In the Horizon System the circles of reference are the horizon and great circles perpendicular to the horizon, called **vertical circles** (Fig. 32). The

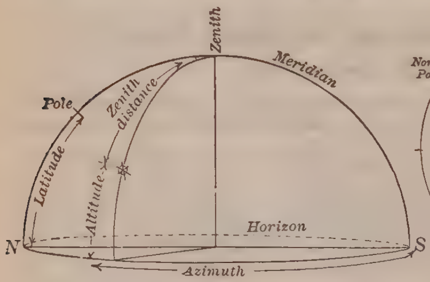


Fig. 32

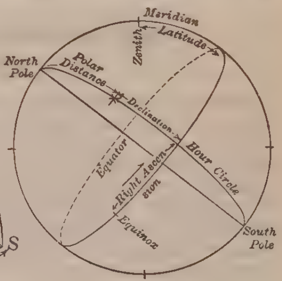


Fig. 33

coordinates are called **altitude** and **azimuth.** The altitude of a point is its angular distance above the horizon measured on a vertical circle through the point. The complement of the altitude is called the **zenith distance.** The azimuth of a body is the arc of the horizon included between the meridian and the vertical circle through the body. It is commonly measured from the south point westward from 0° to 360°; it is sometimes measured from the north point.

In the Equator System (Fig. 33) the circles of reference are the equator and great circles at right angles to the equator, called **hour circles.** The coordinates are called **declination** and **right ascension.** The declination of a point is its angular distance north or south of the equator. Declinations are considered positive when north and negative when south. The complement of the declination is called the **polar distance.** The right ascension of a point is the arc of the equator between the vernal equinox and the hour circle through the point. It is reckoned eastward from the equinox from 0° to 360° or from 0^h to 24^h. The position of a point may also be designated by its **declination** and **hour angle.** The hour angle is the arc of the equator between the meridian and the hour circle through the body. It is reckoned from the meridian westward from 0° to 360° or from 0^h to 24^h. It will be seen that declination and right ascension are independent of the observer's position, whereas the coordinates of the horizon system are dependent upon the observer's position.

Coordinates of the Observer. The observer's position is defined by his latitude and longitude. The **latitude** is the angular distance of the observer's zenith north or south of the equator. The **longitude** is the arc of the equator between the meridian of Greenwich (or other primary meridian) and the meridian of the observer.

The American Ephemeris and Nautical Almanac contains the right ascension and declination of the sun, moon, principal planets and many of the stars, given in most cases for the instant of Greenwich Midnight each day. For any other instant these coordinates must be obtained by interpolation, the variation per hour being given for that purpose. The Almanac also contains other data required in reducing astronomical observations.

Transformation of Coordinates. For a **body on the meridian** let Fig. 34 represent the plane of the meridian, and *EZ* the distance of the zenith north

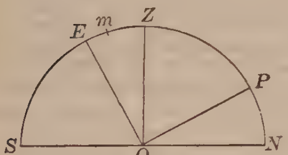


Fig. 34

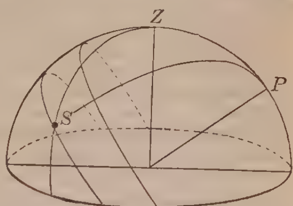


Fig. 35

of the equator, or the latitude. *PN* is the altitude of the pole. The arc *PN* = arc *EZ*. Hence the altitude of the pole equals the latitude of the observer. If *m* is a point on the meridian the relation between the altitude *h*, the declination *D*, and the Latitude *L*, is shown by the equation $90^0 - L = h - D$. If *m* is south of the equator the equation still holds true if *D* is considered to be negative. For a point between the zenith and the pole the relation is expressed by $L = h - p$, where *p* is the polar distance of the point. For a point below the pole the equation becomes $L = h + p$. For a **body off the meridian** Fig. 35 shows the relation between the altitude, declination, azimuth and hour angle of point *S*, and the latitude of *Z*, found by solving the spherical triangle *PZS*. The sides of the triangle are the colatitude *PZ*, the zenith distance *ZS*, and the polar distance *PS*. The angles are *P* the hour angle, *Z* the azimuth, and *S* the parallactic angle. The solution may be effected by means of the ordinary formulas of spherical trigonometry, but it is usually more convenient to apply special formulas derived from the fundamental relations; these will be given in the following paragraphs when required.

39. Measurement of Time

Measurement of Time. Every celestial body passes the meridian twice each day in describing its apparent path around the earth, once on the upper side, that containing the zenith, and once on the lower side. These are called the **upper** and **lower transits** or **culminations.** A **sidereal day** is the interval of time between two successive upper transits of the vernal equinox over the same meridian. A **solar day** is the interval of time between two successive lower transits of the sun's center over the same meridian. The **sidereal time** at any instant is the hour angle of the vernal equinox. The relation between sidereal time S, hour angle t, and right ascension R is given by the equation $\mathbf{S = R + t}$. If the body is on the meridian, $t = 0$, and for this instant $S = R$. The **solar time** at any instant is the hour angle of the sun's center plus 12^h, 0^h occurring at midnight.

Mean and Apparent Time. The apparent angular motion of the sun is not uniform, hence for ordinary purposes it is more convenient to use the time kept by a fictitious sun, or **mean sun,** which moves along the equator uniformly at the average speed of the true sun. Time kept by this mean sun is **mean solar time.** Time kept by the true sun is **apparent solar time.** The difference between the two is the **equation of time,** which is given in the Nautical Almanac for Greenwich Midnight each day.

Example. Given the local apparent time $8^h\ 51^m\ 52^s$ a.m. on Jan. 9, 1928, in longitude $4^h\ 44^m\ 18^s$ west. At Greenwich 0^h the equation of time is $-6^m\ 36^s.77$; the variation per hour is $-1^s.063$. The Greenwich time is $8^h\ 51^m\ 52^s + 4^h\ 44^m\ 18^s = 13^h\ 36^m\ 10^s$. The correction to the equation of time is $-1^s.063 \times 13^h.6 = -14^s.41$, giving $-6^m\ 51^s.18$ for the corrected value. The mean local time is therefore $8^h\ 51^m\ 52^s - (-6^m\ 36^s.77) = 8^h\ 58^m\ 28^s.77$ a.m.

Astronomical and Civil Time. In astronomical reckoning the day began at noon, or when the sun's center was on the meridian. This mode of reckoning was **discontinued in 1925.** The civil day begins at midnight, and, now, for astronomical purposes is numbered continuously from 0^h to 24^h. For ordinary purposes the civil day is divided into two parts of 12^h each. From midnight to noon is called a.m.; from noon to midnight is called p.m. Thus the time of 3 hours after noon may be expressed either 15^h or 3^h p.m.

Longitude and Time. The hour angle of the sun at any meridian is the local solar time p.m. at that meridian. The hour angle of the sun at Greenwich at that instant is the Greenwich time p.m. The difference is the longitude of the place expressed in hours, minutes and seconds. Similarly, for the vernal equinox the result is the sidereal time at the two places and the difference of time is the longitude of the place.

An angle expressed in hours, minutes and seconds of either solar or sidereal time may be expressed in degrees, minutes and seconds by multiplying by 15. Every circle may be divided into 24 hours, or into 360 degrees; hence $15° = 1^h$ for either solar or sidereal time; also $15' = 1^m$ and $15'' = 1^s$. In changing from degrees to hours the following relations are also convenient: $1° = 4^m$, and $1' = 4^s$.

Example. To convert a west longitude of 166° 20′ 05″ into time: $166°\ 20'\ 05'' = 165° + 1° + 15' + 05' + 05'' = 11^h + 04^m + 01^m + 20^s + 0^s.33 = 11^h\ 05^m\ 20^s.33$. To convert $4^h\ 44^m\ 18^s.0$ west longitude into degrees: $4^h\ 44^m\ 18^s.0 = 60° + 11° + 04'\ 30'' = 71°\ 04'\ 30''$.

Solar and Sidereal Intervals of Time. On account of the earth's motion in its orbit the sun appears to move eastward among the stars nearly 1° per day. This causes the sun to reach the meridian later each day as compared with the time of transit of the equinox. For this reason the mean solar day is nearly 4^m longer than the sidereal day. All intervals of mean solar time are correspondingly longer than those of sidereal time. This retardation is just sufficient to bring the sun back to its starting point at the end of a year, so that the number of sidereal days in the year is just one greater than the number of solar days. The year contains 365.2422 solar days and 366.2422 sidereal days. Therefore 1 sidereal day = 0.99726957 solar day and 1 solar day = 1.00273791 sidereal days. Solar time might be converted into sidereal time and vice versa by means of these two equations, but a more convenient way is to calculate the correction to be added in the one case and subtracted in the other. This correction is nearly 10^s per hour. Values of these corrections have been worked out for all hours and minutes from 0^h to 24^h and will be found in the Nautical Almanac.

Example. Convert $9^h\ 26^m\ 32^s.120$ of solar time into sidereal time. From Table III Appendix to Nautical Almanac,

For $9^h\ 26^m$, reduction is	$1^m\ 32^s.979$
For $32^s.120$, reduction is	$.088$
	$1^m\ 33^s.067$
	$9^h\ 26^m\ 32^s.120$
Corresponding sidereal interval	$= 9^h\ 28^m\ 05^s.187$

To convert $9^h\ 28^m\ 05^s.187$ into mean solar time. Table II, Nautical Almanac, gives:

For $9^h\ 28^m$	$1^m\ 33^s.053$
For $05^s.187$	$.014$
	$1^m\ 33^s.067$
	$9^h\ 28^m\ 05^s.187$
Corresponding mean solar interval	$= 9^h\ 26^m\ 32^s.120$

These results may be obtained with sufficient accuracy for many purposes by the following approximate rule. Express the given interval in hours and decimals, multiply by 10 and call the result seconds. Then subtract from this result 1^s for every 6^h in the interval. The result is the desired correction, correct within a few tenths of a second in ordinary cases. In the above example $9^h\ 26^m\ 32^s = 9^h.44$, which gives $94^s.4$. The amount to be subtracted from this is $1^s.6$, leaving $1^m\ 32^s.8$ as the correction to reduce the given interval to sidereal units.

Sidereal and Civil Time at any Instant. The relation between sidereal time S, the right ascension of the mean sun, R_S, and the civil time T is given by the equation $\mathbf{S = R_s + 12^h + T}$. From the Nautical Almanac we may obtain the value of $R_S + 12^h$ at Greenwich midnight. By increasing this by the change for the number of hours in the west longitude it is reduced to the right ascension for local midnight. The above equation may be used more conveniently in the form $\mathbf{S = R_s + 12^h + T + C}$, in which C is the correction to reduce T to a sidereal interval, and R_S is the right ascension for local midnight. If it is desired to find the civil time the equation is Civil Time = $\mathbf{S - (R_s + 12^h) - C'}$, C' being the correction to reduce $S - (R_S + 12^h)$ to a solar interval. It will be noticed that C and C' represent the increase in R_S since midnight.

Example. To find the local sidereal time when the civil time is $9^h\ 22^m\ 18^s.6$ (a.m.), Jan. 7, 1928, in longitude $4^h\ 44^m\ 18^s.0$ west.

$R_S + 12^h$ at Greenwich 0^h =	$7^h\ 01^m\ 17^s.59$
Increase in $4^h\ 44^m\ 18^s$ =	$46\ .70$
R_S at local 0^h =	$7^h\ 02^m\ 04^s.29$
T =	$9\ \ 22\ \ 18^s.60$
C =	$1\ \ 32\ .37$
Sidereal time = S =	$16^h\ 25^m\ 55^s.26$

Whenever the resulting sidereal time exceeds 24^h the actual sidereal time is found by subtracting 24^h. When $R_S + 12^h$ is larger than S, 24^h may be added to S before making the subtraction indicated by the equation for civil time.

Example. To reduce $3^h\ 10^m\ 00^s$ sidereal time, Jan. 7, 1928, to the corresponding civil time in longitude $4^h\ 44^m\ 18^s.0$ west. S must be taken as $27^h\ 10^m\ 00^s$; then $S - (R_S + 12^h) = 27^h\ 10^m\ 00^s - 7^h\ 02^m\ 04^s.29 = 20^h\ 07^m\ 55^s.71$. Subtracting the correction C', we have for the civil time $20^h\ 03^m\ 37^s.82$ (or $8^h\ 03^m\ 37^s.82$ p.m.).

Standard Time. In the United States the territory is divided into time belts, all places in the same belt using the local time at one of the meridians named below. Eastern Time is that of the 75th meridian, Central Time that of the 90th, Mountain Time the 105th and Pacific Time the 120th meridian. By this arrangement the minutes and seconds of time are the same in all parts of the country and also at Greenwich, the clocks in the various belts differing by whole hours. For example, when it is noon at Greenwich it is 7^h a.m. in the Eastern belt, 6^h a.m. in the Central belt, 5^h a.m. in the Mountain belt and 4^h a.m. in the Pacific belt of the United States.

To convert local civil time into Standard Time, take the difference in longitude between the meridian of the place and the Standard Meridian, convert this into hours, minutes and seconds, and add it to the local time if the place is west of the Standard Meridian, subtract if east.

Example 1. Local Civil time = $9^h\ 26^m\ 14^s$ a.m. Longitude 71° 04′ west. Find eastern time. $75°\ 00' - 71°\ 04' = 3°\ 56' = 15^m\ 44^s$. $9^h\ 26^m\ 14^s - 15^m\ 44^s = 9^h\ 10^m\ 30^s$ a.m. eastern time.

Example 2. Central time 8^h p.m. $= 20^h$. Longitude = 91° west. Find the local civil time. $91° - 90° = 1° = 4^m$. Then $20^h - 04^m = 19^h\ 56^m$ ($= 7^h\ 56^m$ p.m.). local civil time.

40. Altitudes

Correcting Measured Altitudes. When the altitude of a heavenly body is measured with a transit or a sextant it is necessary to apply certain corrections to the observed altitude in order to obtain the true altitude. **Refraction** is the bending downward of rays of light from a celestial body upon entering the atmosphere. This causes the object to appear too high above the horizon. To reduce an observed altitude to the true altitude it is necessary to subtract a correction for this refraction, the amount of the correction depending upon the altitude, upon the temperature of the air and upon the barometric pressure. For accurate work the correction must be taken from refraction tables. For rough computations it will be close enough to take the refraction correction in minutes equal to the natural cotangent of the altitude, provided the altitude is not less about about 10°. For low altitudes the refraction correction is uncertain.

Refraction Correction

(In minutes)

True altitude = measured altitude − refraction correction

Altitude	Refraction	Altitude	Refraction	Altitude	Refraction	Altitude	Refraction
5°	9.9	11°	4.9	17°	3.1	35°	1.4
6	8.5	12	4.5	18	3.0	40	1.2
7	7.4	13	4.1	19	2.8	45	1.0
8	6.6	14	3.8	20	2.6	50	0.8
9	5.9	15	3.6	25	2.1	55	0.7
10	5.3	16	3.3	30	1.7	60	0.6

Parallax. An observer on the earth's surface sees an object at a lower altitude than he would if he were at the center of the earth. This is called the error of parallax. The displacement of the object is greatest when it is at the horizon and is zero when the body is in the zenith. The parallax for a body on the horizon is called its horizontal parallax. The correction for any altitude equals the horizontal parallax times the cosine of the altitude. The sun's horizontal parallax is about 9″; that of the moon is nearly a degree.

Dip. If the altitude is measured at sea it must be taken with a sextant and measured from the sea horizon. In this case the observed altitude must be referred to the true horizon by subtracting the angle of dip. The amount of the correction varies with the height of the eye above the surface of the sea. For ordinary observations the dip in minutes may be taken as the square root of the height of the eye in feet above sea level.

Semidiameter. If the object observed is the sun, the moon or a planet, the edge (or limb) of the disk may be observed and the altitude reduced to the center by adding or subtracting the semidiameter. The semidiameter is given in the Nautical Almanac for every day at Greenwich 0^h. For approximate work the sun's semidiameter may be taken as 16′ in March and September, 15′ 45″ in June and 16′ 15″ in December.

41. Constellations Near the Pole

Polaris, the Pole-Star, is a second-magnitude star situated about 1° 05′ from the pole. Its right ascension is about $1^h\ 36^m$; consequently at the instant when the sidereal time is $1^h\ 36^m$ Polaris is vertically above the pole, on the meridian. Directly above Polaris at this time ($1^h\ 36^m$) is the constellation Cassiopeia, shaped like an inverted letter W. Below Polaris is the great dipper, Ursa Major. A line through the two stars in the outer side of the dipper bowl points nearly to Polaris, and these stars are called the "pointers." As this constellation is a conspicuous one the pointers are easily recognized and may be used to identify the pole-star. The positions of these constellations are shown in Fig. 36. When the star is vertically above or below the pole it is said to be **at culmination** (**upper** or **lower**); when the star is at its extreme eastern or western position it is said to be at its greatest **elongation** (**east** or **west**). The figure shows the constellations as they appear when Polaris is at its upper culmination; by looking at it inverted they are seen as they appear at its lower culmination. By looking at the figure from the left and right margins of the page, the positions are seen for the western and eastern elongations of Polaris.

Fig. 36

42. Observations for Latitude

By the Pole-Star. If the altitude of the pole-star (Polaris) is observed when the star is above the pole (upper culmination) and its maximum altitude found by trial, then this will be the meridian altitude. The latitude is then found by the equation $\mathbf{L = h - p}$. The measured value of the altitude h

must be corrected for refraction as well as for any index error of the instrument. The polar distance p is found by looking up the declination of Polaris in the Nautical Almanac and subtracting it from 90°. If the star is at the lower culmination the polar distance must be added.

Example. Observed altitude = 43° 37′ 00″, index error = + 30″, and refraction correction = − 1′ 01″; then true altitude = 43° 36′ 29″. Since declination of star was 88° 44′ 35″, polar distance = 1° 15′ 25″, and hence latitude = 42° 21′ 04″.

By the Sun at Noon. If the maximum altitude of the sun is found at noon this may be taken as the meridian altitude. This altitude is found with the transit by setting the horizontal cross-hair on the lower edge of the sun and following it as long as it continues to rise. When the sun drops below the cross-hair the altitude is read from the vertical arc. This altitude should be corrected for index error, refraction, semidiameter and parallax. The latitude is then found from the equation $\mathbf{L} = 90° - (\mathbf{h} - \mathbf{D})$. This method applies also to a star when crossing the meridian.

Example. In longitude $4^h.74$ west, the observed altitude of the sun's lower limb was 25° 06′.0; index error = + 01′.0; refraction correction = − 02′.0; sun's semidiameter = 16′.3. The corrected altitude = 25° 21′.3. On this day the sun's declination at Greenwich 0^h was − 22° 18′ 21″, the variation for 1 hour being + 19″.75. The change in $16^h.74$ = + 5′ 40′. The corrected declination is therefore −22° 12′ 41″. The formula for the latitude then gives 42° 26′.0 N.

43. Observations for Time

By Transit of Star. If the line of sight of a transit instrument be placed in the plane of the meridian the time can be accurately determined by noting the instant by a watch or a chronometer when some known star passes the vertical cross-hair. The sidereal time at the instant is the same as the star's right ascension. If a sidereal chronometer is used the difference between the chronometer reading and the right ascension is the error of the chronometer on local sidereal time. If a watch is used, then this sidereal time must be converted into solar time of the meridian for which the watch is regulated. This is done by subtracting from the right ascension of the star the right ascension of the mean sun + 12^h corrected to local 0^h. This result reduced to solar interval is the local civil time. This is reduced to standard time by taking the difference between the longitude of the place and that of the standard meridian, converting this into time, and adding it if the place is west of the standard meridian, subtracting if east.

Example. Transit of Star over Meridian. Longitude, $5^h\ 20^m\ 10^s.0$ west. Date, Oct. 5, 1928. Observed transit of β Aquarii = $8^h\ 50^m\ 20^s.0$ p.m. Right ascension of mean sun + 12^h, for Greenwich 0^h, = $0^h\ 53^m\ 40^s.66$. Increase in $5^h\ 20^m\ 10^s.0 = 52^s.60$. Corrected $R_S + 12^h = 0^h\ 54^m\ 33^s.26$.

Right ascension β Aquarii	=	21^h	27^m	$48^s.39$
$R_S + 12^h$ at local 0^h	=	0	54	33 .26
		20^h	33^m	$15^s.13$
Reduction to solar time	=		3	22 .04
		20^h	29^m	$53^s.09$
Reduction to eastern time	=		20	10 .00
Eastern time of transit	=	20^h	50^m	$03^s.09$
Observed time of transit	=	20	50	20 .00
Watch fast	=			$16^s.91$

This method is the one used in the most precise observations for time except that corrections are introduced to allow for the non-adjustment of the instrument, and the observations are recorded with great precision on an automatic register called a chronograph.

By Transit of the Sun. A similar observation may be made on the sun. The watch times of transit of the west and east edges of the sun across the meridian should be noted and the mean taken as the time for the center. This instant is $12^h\ 00^m\ 00^s$ local apparent time. This must be reduced to civil time by applying the equation of time and the result converted into standard time as already explained. The difference between this and the watch reading is the error of the watch on standard time.

Example. Transit of the Sun across the Meridian. Longitude, 71° 04-1/2′. Observed time of west and east edges of sun, $11^h\ 30^m\ 08^s$ and $11^h\ 32^m\ 26^s$.

Local apparent time of transit of center	$= 12^h\ 00^m\ 00^s$
Equation of time	$= \quad -12 \quad 07$
Local civil time	$= 11^h\ 47^m\ 53^s$
Reduction to eastern time	$= \quad -15 \quad 42$
Eastern standard time	$= 11^h\ 32^m\ 11^s$
Mean of observed times	$= 11 \quad 31 \quad 17$
Watch slow	$= 54^s$

Time by Altitude of Sun. The time may be determined by measuring the altitude of the sun's lower edge and noting the watch reading at the same instant. This altitude must be corrected for refraction, parallax, and semidiameter. The hour angle of the sun is then found by solving the *PZS* triangle for the angle at the pole. A convenient formula for this is

$$\sin 1/2\, t = \sqrt{\frac{\cos s \sin (s - h)}{\cos L \sin p}}$$

where t is the hour angle and $s = 1/2\,(L + h + p)$. The latitude L must be known in order to solve the triangle; the polar distance p is found from the Nautical Almanac by subtracting the declination from 90°; h is the corrected altitude. If it is in the forenoon the resulting hour angle (converted into time) must be subtracted from 12^h in order to obtain the apparent time. This apparent time must be corrected for the equation of time and then reduced to standard time as before.

Example. Altitude of Sun's Lower Limb for Time. Latitude, 42° 21′.0 N. Longitude, $4^h\ 44^m\ 18^s.0$ west. Date, Jan. 9, 1928. Mean of watch readings, $8^h\ 45^m\ 01^s$ a.m. Mean of altitudes, 12° 19′.6.

Observed altitude	= 12° 19′.6
Refraction	= −4 .4
	12° 15′.2
Semidiameter	= +16 .3
	12° 31′.5
Parallax	= .1
True altitude	= 12° 31′.6

L =	42° 21′.0	log sec.	= 0.13133
h =	12 31 .6		
p =	112 13 .8	log csc.	= 0.03354
$2s$ =	166° 66′.4		
s =	83 33 .2	log cos	= 9.05032
$s - h$ =	71 01 .6	log sin	= 9.97574
			2)9.19093
		$\log \sin\frac{t}{2}$	= 9.59546
		$\frac{t}{2}$	= 23° 12′.1
		t	= 46° 24′.2
			$= 3^h\ 05^m\ 36^s.8$

Declination at Greenwich 0^h	= −22° 18′ 21″
$19''.8 \times 13^h.7$ =	+4 32
Declination at $8^h\ 45^m$	= −22° 13′ 49″
p =	112° 13 .8

Apparent time	$= 8^h\ 54^m\ 23^s.2$
Equa. of time	$= \quad -6 \quad 51 .3$
Civil time	$= 9^h\ 01^m\ 14^s.5$
	$15 \quad 42 .0$
Eastern time	$= 8^h\ 45^m\ 32^s.5$
Watch time	$= 8 \quad 45 \quad 01 .0$
Watch slow	$= 31^s.5$

Equa. of time at Greenwich 0^h	$= -6^m\ 36^s.77$
$-1^s.06 \times 13^h.7$	$= - \quad 14 .52$
Equa. of time at $8^h\ 45^m$	$= -6^m\ 51^s.29$

By Altitude of a Star. If a star is observed in this manner the calculation is the same except that the resulting hour angle must be added to the star's right ascension to obtain the sidereal time. The sidereal time may then be converted into standard time as already explained. The sun or star should not be near the meridian, or within 10° of the horizon, if an accurate result is sought.

44. Determination of Longitude

If the local time is found at two places and these times are compared, the difference is the difference in longitude expressed in hours. The most precise method of making this comparison is by means of the telegraph, signals being recorded simultaneously on the two chronographs which are used to record the observations for local time at the two places, or, by radio, the 10^h p.m. time signal being received by a special receiving set and recorded on the chronograph. Longitude may also be found by carrying a chronometer or a watch from one place to the other, determining its error on the local time of each place and also the rate at which the timepiece gains or loses, in order to allow for the variation between the two observations. If M = the difference in longitude, e the error of the watch at the first station, e' the error at the second (+ when slow, − when fast), r the gain or loss per day (+ when losing, − when gaining), and d the number of days between the observations, then $M = e + dr - e'$.

Example At a place $4^h\ 44^m\ 18^s$ west of Greenwich a watch was $15^m\ 42^s$ slow on local mean time. At a place farther west the same watch was $14^m\ 10^s$ slow. The watch was gaining $6^s.5$ daily. The second observation was made 26 hours after the first. The difference in longitude is then $15^m\ 42^s - 26/24 \times 6^s.5 - 14^m\ 10^s = 1^m\ 25^s$ and the longitude of the second place is $4^h\ 45^m\ 43^s$.

45. Observations for Azimuth

By a Circumpolar Star at Elongation. The simplest and most accurate method of determining an azimuth or the direction of the true meridian is by observing the direction of a circumpolar star (such as Polaris) when at its greatest eastern or western elongation. An ordinary transit is used and it should be put in good adjustment. About half an hour before the computed time of elongation the transit should be set up over one end of the line whose azimuth is to be found, or at the point through which the meridian is desired. The cross-hairs must be illuminated by means of a flashlight or a lantern, preferably held by an assistant. Set the vertical hair on the star and clamp the plates. It will be observed that the star moves very slowly in azimuth, to the left if approaching western elongation; to the right if eastern elongation. As the star nears its elongation it no longer moves in azimuth but will move vertically, downward if at western elongation, upward if at eastern elongation. This shows that the position of elongation has been reached. Now lower the telescope without disturbing its azimuth and set a point on a stake some 300 ft. north of the transit, in line with the vertical hair. Next quickly reverse the telescope, re-level the plate levels if necessary, point again at the star, and set another point on the stake. If these two points do not agree take a point midway between them. To obtain the meridian lay off from this last point the azimuth of the star at elongation, which may be computed as explained later, or in case of Polaris, may be taken from the table on p. 491. This gives a point exactly north of the transit station.

The azimuth of any line may be found by measuring from this north point to the line in question. Evidently the azimuth might be found by measuring directly from the star to the line in question, since the azimuth of the star is known, provided the vernier was set to read 0° before pointing on the star.

Azimuth of Polaris at Elongation

Latitude	1928	1929	1930	1931	1932	1933	1934	1935
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
10	1 05.9	1 05.6	1 05.3	1 05.0	1 04.7	1 04.4	1 04.1	1 03.8
11	1 06.1	1 05.8	1 05.5	1 05.2	1 04.9	1 04.6	1 04.3	1 04.0
12	1 06.4	1 06.0	1 05.7	1 05.4	1 05.1	1 04.8	1 04.5	1 04.2
13	1 06.6	1 06.3	1 06.0	1 05.7	1 05.3	1 05.0	1 04.7	1 04.4
14	1 06.9	1 06.6	1 06.3	1 05.9	1 05.6	1 05.3	1 05.0	1 04.7
15	1 07.2	1 06.9	1 06.6	1 06.2	1 05.9	1 05.6	1 05.3	1 05.0
16	1 07.5	1 07.2	1 06.9	1 06.6	1 06.2	1 05.9	1 05.6	1 05.3
17	1 07.9	1 07.5	1 07.2	1 06.9	1 06.6	1 06.2	1 05.9	1 05.6
18	1 08.2	1 07.9	1 07.6	1 07.3	1 07.0	1 06.6	1 06.3	1 06.0
19	1 08.6	1 08.3	1 08.0	1 07.7	1 07.3	1 07.0	1 06.7	1 06.4
20	1 09.1	1 08.7	1 08.4	1 08.1	1 07.8	1 07.4	1 07.1	1 06.8
21	1 09.5	1 09.2	1 08.9	1 08.5	1 08.2	1 07.9	1 07.6	1 07.2
22	1 10.0	1 09.7	1 09.3	1 09.0	1 08.7	1 08.4	1 08.0	1 07.7
23	1 10.5	1 10.2	1 09.8	1 09.5	1 09.2	1 08.9	1 08.5	1 08.2
24	1 11.0	1 10.7	1 10.4	1 10.0	1 09.7	1 09.4	1 01.0	1 08.7
25	1 11.6	1 11.3	1 10.9	1 10.6	1 10.3	1 09.9	1 09.6	1 09.3
26	1 12.2	1 11.9	1 11.5	1 11.2	1 10.9	1 10.5	1 10.2	1 09.9
27	1 12.8	1 12.5	1 12.2	1 11.8	1 11.5	1 11.1	1 10.8	1 10.5
28	1 13.5	1 13.2	1 12.8	1 12.5	1 12.1	1 11.8	1 11.4	1 11.1
29	1 14.2	1 13.9	1 13.5	1 13.2	1 12.8	1 12.5	1 12.1	1 12.8
30	1 14.9	1 14.6	1 14.2	1 13.9	1 13.5	1 13.2	1 12.8	1 12.5
31	1 15.7	1 15.4	1 15.0	1 14.6	1 14.3	1 13.9	1 13.6	1 13.2
32	1 16.5	1 16.2	1 15.8	1 15.4	1 15.1	1 14.7	1 14.4	1 14.0
33	1 17.4	1 17.0	1 16.7	1 16.3	1 15.9	1 15.6	1 15.2	1 14.9
34	1 18.3	1 17.9	1 17.5	1 17.2	1 16.8	1 16.4	1 16.1	1 15.7
35	1 19.2	1 18.9	1 18.5	1 18.1	1 17.7	1 17.4	1 17.0	1 16.6
36	1 20.2	1 19.8	1 19.5	1 19.1	1 18.7	1 18.3	1 18.0	1 17.6
37	1 21.3	1 20.9	1 20.5	1 20.1	1 19.7	1 19.4	1 19.0	1 18.6
38	1 22.4	1 22.0	1 21.6	1 21.2	1 20.8	1 20.4	1 20.0	1 19.7
39	1 23.5	1 23.1	1 22.7	1 22.3	1 21.9	1 21.6	1 21.2	1 20.8
40	1 24.7	1 24.3	1 23.9	1 23.5	1 23.1	1 22.7	1 22.3	1 22.0
41	1 26.0	1 25.6	1 25.2	1 24.8	1 24.4	1 24.0	1 23.6	1 23.2
42	1 27.3	1 26.9	1 26.5	1 26.1	1 25.7	1 25.3	1 24.9	1 24.5
43	1 28.7	1 28.3	1 27.9	1 27.5	1 27.1	1 26.7	1 26.3	1 25.8
44	1 30.2	1 29.8	1 29.4	1 29.0	1 28.5	1 28.1	1 27.7	1 27.3
45	1 31.8	1 31.4	1 30.9	1 30.5	1 30.1	1 29.6	1 29.2	1 28.8
46	1 33.4	1 33.0	1 32.5	1 32.1	1 31.7	1 31.2	1 30.8	1 30.4
47	1 35.2	1 34.7	1 34.3	1 33.8	1 33.4	1 32.9	1 32.5	1 32.0
48	1 37.0	1 36.5	1 36.1	1 35.6	1 35.2	1 34.7	1 34.3	1 33.8
49	1 38.9	1 38.5	1 38.0	1 37.5	1 37.1	1 36.6	1 36.1	1 35.7
50	1 41.0	1 40.5	1 40.0	1 39.5	1 39.1	1 38.6	1 38.1	1 37.7

This table was computed using the mean declination of Polaris for the beginning of each year. A more accurate result will be obtained by applying to the tabular values the following corrections, which depend on the difference between the mean and apparent place of the star.

The Azimuth at Elongation may be computed by the equation **sin Z = cos D/cos L,** where D is the declination of the star as found from the Nautical Almanac, L the latitude of the place, and Z the required azimuth. For example, let declination of a star be 88° 55′ 21″ and the latitude of the place be 42° 21′, then azimuth of the star at elongation is 1° 27′ 28″. For an eastern elongation lay off this angle to the left and for a western elongation lay it off to the right in order to determine the true meridian.

Finding the Time of Elongation of Polaris. The circumpolar star generally used for this observation is Polaris (Art. 41). Its azimuths at elongation are given in the table on p. 491 with sufficient precision for the purposes of common surveying, while the tables on pp. 492 and 493 give the time of culmination and the interval by which elongation precedes or follows culmination. These tables are reprinted from "Magnetic Declination in the United States in 1925," issued by the U. S. Coast and Geodetic Survey.

Month	Correction	Month	Correction	Month	Correction
January.....	−0′.3	May.......	+0′.2	September..	0′.0
February....	−0 .3	June.......	+0 .3	October....	−0.2
March......	−0 .2	July.......	+0 .3	November..	−0 .5
April.......	0 .0	August.....	+0 .2	December..	−0 .7

Local Civil Time * of Upper Culmination of Polaris in the Year 1926

(Computed for 90°, or 6 hours west of Greenwich)

Date, 1926	Civil time of upper culmination			Variation per day		Date, 1926	Civil time of upper culmination			Variation per day	
	h.	m.	s.	m.	s.		h.	m.	s.	m.	s.
Jan. 1...	18	51	29	−3	57	July 10......	6	24	11	−3	55
Jan. 11...	18	11	59	3	57	July 20......	5	45	03	3	55
Jan. 21...	17	32	28	3	57	July 30......	5	05	55	3	55
Jan. 31...	16	52	58	3	57	Aug. 9......	4	26	46	3	55
Feb. 10...	16	13	28	3	57	Aug. 19......	3	47	37	3	55
Feb. 20...	15	33	58	3	57	Aug. 29......	3	08	27	3	55
Mar. 2...	14	54	30	3	57	Sept. 8......	2	29	16	3	55
Mar. 12...	14	15	04	3	57	Sept. 18......	1	50	04	3	55
Mar. 22...	13	35	40	3	56	Sept. 28......	1	10	50	3	55
Apr. 1...	12	56	17	3	56	Oct. 8......	0	31	35	3	56
Apr. 11...	12	16	57	3	56	Oct. 17......	23	52	18	3	56
Apr. 21...	11	37	39	3	56	Oct. 27......	23	12	59	3	56
May 1...	10	58	23	3	55	Nov. 6......	22	33	39	3	56
May 11...	10	19	09	3	55	Nov. 16......	21	54	17	3	56
May 21...	9	39	56	3	55	Nov. 26......	21	14	52	3	57
May 31...	9	00	45	3	55	Dec. 6......	20	35	26	3	57
June 10...	8	21	35	3	55	Dec. 16......	19	55	58	3	57
June 20...	7	42	26	3	55	Dec. 26......	19	16	30	3	57
June 30...	7	03	19	3	55	Jan. 5, 1927..	18	37	00	3	57

* Begins at midnight and is counted from 0 to 24 hours.

Eastern elongation precedes and western elongation follows upper culmination by the time interval given in table on p. 493. Lower culmination precedes or follows upper culmination by $11^h\ 58^m.0$. It should be noted that there are two upper culminations on one day in October (16th in 1926) and

Mean Time Interval between Upper Culmination and Elongation

Latitude	Time interval	Latitude	Time interval	Latitude	Time interval	Latitude	Time interval
°	h. m.	°	h. m.	°	h. m.	°	h. m.
10	5 58.2	35	5 56.0	48	5 54.2	58	5 52.1
15	5 57.8	40	5 55.4	50	5 53.8	60	5 51.5
20	5 57.4	42	5 55.1	52	5 53.4	62	5 50.8
25	5 57.0	44	5 54.8	54	5 53.0	64	5 50.1
30	5 56.5	46	5 54.5	56	5 52.6		

two lower culminations in April (15th in 1926). There are also two western elongations on one day in January and two eastern elongations on one day in July.

(*a*) To refer the times in table on p. 492 to other years:

		m.
1927	add	1.3
1928, up to March 1	add	2.6
1928, on and after March 1	subtract	1.3
1929	add	0.1
1930	add	1.6
1931	add	3.0
1932, up to March 1	add	4.4
1932, on and after March 1	add	0.5
1933	add	2.1
1934	add	3.7
1935	add	5.3

(*b*) To refer to other than the tabular days: Subtract from the time for the preceding tabular day the product of the variation per day and the days elapsed, as given below:

Days elapsed	Variation per day: 3m. 57s.	3m. 56s.	3m. 55s.	Days elapsed	Variation per day: 3m. 57s.	3m. 56s.	3m. 55s.
	m. s.	m. s.	m. s.		m. s.	m. s.	m. s.
1	3 57	3 56	3 55	6	23 42	23 36	23 30
2	7 54	7 52	7 50	7	27 39	27 32	27 25
3	11 51	11 48	11 45	8	31 36	31 28	31 20
4	15 48	15 44	15 40	9	35 33	35 24	35 15
5	19 45	19 40	19 35				

(*c*) To refer to any other than the tabular longitude (90°): Add 0.1^m for each 10° east of the ninetieth meridian or subtract 0.1^m for each 10° west of the ninetieth meridian.

(*d*) To refer to standard time: Add to the quantities in table on p. 492 four minutes for every degree of longitude the place of observation is west of the standard meridian (60°, 75°, 90°, etc.). Subtract when the place is east of the standard meridian.

Example. Illustrating the Use of the Polaris Tables. Find the eastern standard time when the eastern elongation of Polaris occurs in latitude 36° N, longitude 77° W, on Aug. 12, 1928. For Aug. 9, 1926, the table gives for the upper culmination 4^h 26^m 46^s (after midnight). Four days later (Aug. 13) it occurs 15^h 40^m earlier, or at 4^h 11^m 06^s. For the year 1928 we subtract $1.3,^m$ giving 4^h 09^m 48^s. The correction for longitude is $+ 1/10\ (90° - 77°) \times 0.1^m = + 0.13^m = + 8^s$. The correction to obtain eastern standard time is $+8^m$. The standard time of upper culmination on Aug. 13 is therefore $4^h\ 17^m\ 56^s$. Eastern elongation *precedes* culmination by $5^h\ 55^m.9$, giving $22^h\ 22^m\ 02^s$ (or $10^h\ 22^m\ 02^s$ p.m.) for the eastern standard time of eastern elongation on Aug. 12, 1928.

Meridian by Culmination of Polaris. The time of a culmination of Polaris having been found by help of the above table, a rough determination of the meridian may be made by pointing the telescope upon Polaris at that exact instant. Such a determination is liable to be in error 1′ or 2′, since the star moves most rapidly in azimuth when at its culmination. It is always preferable to observe Polaris at elongation as explained above.

By the Altitude of the Sun. If the azimuth of a line *A B* is to be determined, set the transit over *A* and set the vernier to read 0°. Sight the vertical hair on *B* and clamp the lower motion. Loosen the upper clamp, place a dark glass over the eyepiece, and observe the sun as follows: In the forenoon sight first on the right and lower edges of the sun (simultaneously) using the vertical hair and the **middle** horizontal hair, and read the watch, the vertical angle and the horizontal angle; take two more observations in the same way. Then sight on the left and upper edges, taking the same readings as before, and the same number of observations as before. If the transit has a complete vertical circle it is advisable to reverse the telescope between these two half sets of observations. The index error should be determined after the sights on the sun are finished. The error of the watch should have been determined previously by comparing with railroad time or by radio. After completing the pointings on the sun the point *B* should again be sighted and the 0° reading verified. (If the observation is being made in the afternoon sight the left and lower edges and then the right and upper edges.) Before calculating the azimuth the intervals between readings should be examined for errors. The change in azimuth per minute of altitude should be nearly the same as found from the four intervals. The means of the six altitudes, the six horizontal angles, and the six watch readings, correspond to a single observation on the center of the sun in some position. To obtain the true altitude the mean of the vertical circle readings must be corrected for index error and must be diminished by the amount of the refraction. (See table on p. 486.) It should also be increased by the parallax, which is but 9″ times the cosine of the altitude. The latitude of the place must be known. This may be scaled from a reliable map to the nearest minute, or found by observation. (See Art. 42.) The declination of the sun must be taken from the Nautical Almanac. The azimuth of the sun's center (from the north point) is then computed by the formula

$$\cos 1/2\, Z = \sqrt{\frac{\cos s \cos (s - p)}{\cos L \cos h}}$$

in which $s = 1/2\,(L + h + p)$, and L, h, and p stand for the latitude, altitude, and polar distance, respectively. In the forenoon Z will be measured toward the east, in the afternoon toward the west. Avoid taking observations when the sun is near the meridian. Best results are obtained when the sun's bearing is about east or west.

The following form may be found convenient for recording and computing sun observations for azimuth.

Suggested Form

OBSERVATION ON SUN FOR AZIMUTH

On Line from.................................to...........................

Lat....................Long.................   Date................19....

Object	Horizontal	Vertical	Watch a.m. p.m.
	° ′	° ′	h. m. s.
Mark......			
○			
○			
Mark......			
Mean......			

I. C. = error =

Ref. =

h = G. C. T. =

L = log sec =

h = log sec =

p =

$2s$ =

s = log cos =

$s - p$ = log cos =

2)

cos 1/2 Z =

1/2 Z =

Z =

Hor. angle =

Azimuth =

Example. Corrected mean of observed altitudes = 15° 22′.5. Mean of horizontal angles = 74° 07′.5. Mean of watch readings = $8^h\ 43^m\ 47^s$ a.m., Eastern time = $13^h\ 43^m\ 47^s$ Greenwich civil time. Latitude = 42° 21′.0 N. Sun's declination at Greenwich 0^h (or 7^h p.m.) = − 21° 14′ 01″. Variation per hour = − 26″.86. Corrected declination = − 21° 20′ 10″. North polar distance = 111° 20′.2.

L =	42° 21′.0	log sec =	0.13133
h =	15 22 .5	log sec =	0.01583
p =	111 20 .2		
$2s$ =	168° 63′.7		
s =	84 31 .8	log cos =	8.97920
$s - p$ =	−26 48 .4	log cos =	9.95062
			2)9.07698
		cos 1/2 Z =	9.53849
		1/2 Z =	69° 47′.1
		Z =	139 34 .2
		Horizontal angle =	74 07 .5
		Azimuth of mark =	N 65° 26′.7 E

The azimuth of a star may be found in exactly the same way, except that the star is bisected with both cross-hairs and it is not necessary to note the time. The star observed should not be near the meridian, nor within 10° of the horizon, if an accurate result is desired.

GEODETIC SURVEYING

46. Definitions

Geodetic Surveying may be defined as that branch of surveying in which, on account of the extent of the survey and the required precision of the results, it is necessary to consider the spheroidal form of the earth's surface. The object of an extensive geodetic survey is usually twofold: first, to locate certain points with great accuracy in order to connect different topographic or hydrographic maps and furnish an accurate control of the whole survey; second, to furnish data for perfecting our knowledge of the form and dimensions of the earth.

Triangulation. The most economical way of locating points with the accuracy required for these purposes is to establish a system of triangulation extending over the area. A triangulation system consists of a series of triangles formed by lines connecting observing stations built on prominent mountains or hills, or other points which it is desirable to locate. In this system of triangles the length of one side of some triangle must be known; then if all of the angles in each triangle are measured the lengths of all other lines in the system may be calculated by trigonometry. There are three types of systems which may be recognized: first, a series of approximately equilateral triangles; second, a series of central polygons, for example a row of hexagons each with an interior station; third, a series of quadrilaterals with both diagonals drawn. The first system is the cheapest when it is desired to extend the survey along a narrow belt, as in surveying a river. It has the disadvantage of having but few " checks," that is, few geometric conditions which must be satisfied by the measurements. The second system is adapted to surveying an area of greater width than the first. The third is the most accurate. Traverses of a high order of accuracy are used in flat and wooded country where triangulation would be difficult and expensive.

Triangulation is divided into four grades, according to the accuracy finally reached in the computed lengths of lines. First order triangulation is that in which the average error of closure of a triangle is about 1″ and the discrepancy between measured and computed bases does not exceed 1 in 25 000. In second order triangulation the average closure is 3″ and the check on a base-line 1 in 10 000. Triangulation of the third order is that in which the average closure of a triangle is 5″ and the check on a base is 1 in 5000. Triangulation with ordinary transits or with a plane table is of the fourth order.

Traverses are also divided into four grades, as follows: First order, position check within 1 in 25 000; second order, position check within 1 in 10 000; third order, position check within 1 in 5 000; fourth order, stadia, transit and tape, or wheel.

Subdivision of Fieldwork. The process of carrying out a triangulation scheme may be divided as follows: (*a*) Reconnaissance and Preliminary Work. (*b*) Base-line Measurement. (*c*) The Measurement of the Angles. (*d*) The Astronomical Observations.

47. Reconnaissance

The Reconnaissance includes: (1) Making a rough sketch map of the region and studying the general scheme with regard to the shape of the triangles and the general distribution of stations. Very acute angles should not be chosen if this can be avoided. (2) Testing the lines to see that in each triangle the points are visible, each from the others, and that sights will not be seriously interfered with by excessive atmospheric refraction or by smoke of nearby cities. (3) Making notes in regard to condition of roads, approaches to station, transportation, the lumber available, and any other details that will be useful when the work of measuring the angles is being carried out.

Elevation of Signals. In ascertaining whether one station is visible from another it is always preferable to test the line by sighting directly over it. In some cases this is not practicable and it becomes necessary to obtain the result by calculation. Suppose for example that station A (Fig. 37) has an elevation of 900 ft. and it is desired to sight to station B which is 60 miles distant and

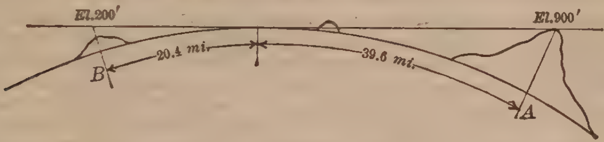


Fig. 37

has an elevation of 200 ft. In this case the sight cannot be taken on account of the curvature of the earth's surface, and it becomes necessary to calculate the height of a tower which when built on B will be visible from A. The distance from a tangent line to the curve may be calculated from the given horizontal distance and the dimensions of the earth. The distance so calculated must be decreased (by about one-seventh part) on account of the refraction of the atmosphere. The combined curvature and refraction is given with sufficient accuracy by the formula $\mathbf{h} = \mathbf{k}^2/\mathbf{1.743}$, where h is the offset from the tangent in feet and k is the horizontal distance in statute miles. In the above example the distance which an observer at A could see over a surface at sea level is given by $\sqrt{\mathbf{h} \times \mathbf{1.743}} = 39.6$ miles. The remaining distance, 20.4 miles, corresponds to $(20.4)^2/1.743 = 238.7$ ft.; that is, the lowest point which an observer at A can see has an elevation of 238.7 ft. above sea level or 38.7 ft. above B. Hence a tower 38.7 ft. high must be erected at B to make the sight possible. In practice it would be necessary to raise the line at least 10 ft. higher in order to avoid the errors due to high refraction of the air near the surface of the ground.

In the above solution it has been assumed that the intervening ground is at sea level, but if there are summits between the two stations it will be necessary to determine how much they rise above the sight line. Suppose that there is a hill 30 miles from A (on line AB) whose elevation is 75 ft. The hill is 9.6 miles from the tangent point and therefore the curvature correction is 53.0 ft. The top of this hill is $75 - 53.0 = 22.0$ ft. above the computed sight line, and the line must be raised 22.0 ft. at this point either by increasing the elevation of both stations by 22.0 ft. or by building the tower at B 44.0 ft. higher.

48. Base-line

Base-Line. To compute the lengths of the triangle sides it is necessary to choose some base-line, usually not the side of a principal triangle, and to measure its length with great accuracy, and then to connect this base-line with a line of the main system by means of a special system of triangles called the **base-net** or **expansion.** The position of the base should be chosen both with regard to its accurate connection with the main system and to convenience in the measurement of the length itself; but the accuracy of the connection should not be sacrificed to convenience in the measurement.

Measurement of the Base-Line may best be made by the invar tape apparatus. This tape is made of an alloy of nickel and steel which has a very low coefficient of expansion (from 1/25 to 1/30 that of steel tapes) and is in consequence much less affected by errors in the determination of the temperature of the tape during the measurements. Invar is softer than steel and is easily bent, but if it is wound on a reel of not less than 16 in. in diameter there is practically no error caused by bending the tape. The tape used in this work is usually 50 meters in length, and is suspended at the ends and at the middle point on stakes or tripods. A definite tension is given by means of a spring balance mounted on a special tension apparatus. The temperature of the tape is determined by means of thermometers clamped on to the tape. The points of support are carefully lined in and brought to a uniform grade. The rate of grade of each tape-length is determined by direct leveling or else by measuring the vertical angles. The positions of the end points of the tape on the intermediate stakes are marked by means of lines scratched on copper or zinc strips tacked to the tops of the stakes. The ends of the base-line are marked by means of bronze markers set in heavy stones or concrete blocks sunk into the ground. The transfer of the end marks to the tape, or vice versa, is made by means of a transit or by a special form of plumb bob.

In making the measurement the tape is hung on the first set of supports and carefully lined in; the tension is then applied and the zero mark is set over the end bolt and the position of the 50-meter end of the tape is marked on the metal strip. The temperature is noted at the same time. The tape is then carried forward to the second set of supports, the zero end being set on the mark previouly made, and the operation is repeated. This process is continued until the final mark is reached. The base is measured at least twice to verify the work and to give a value of the probable error of the measurements. The error of the base-line should not be greater than 1/500 000th part of its length for the main scheme of triangulation.

Corrections. In order to reduce the field measurements to the true length of base it is necessary to apply the following corrections: (1) Grade; (2) alignment; (3) variation of tension from normal; (4) sag (if number of supports is changed); (5) temperature; (6) error in absolute length of tape at standard temperature and tension; (7) reduction to sea level.

Correction for Grade may be made by the formula

$$\mathbf{C_g = -1/2\, L\, (a^2 + 1/4\, a^4)}, \quad \text{where } a = h/L,$$

in which L = the length of the tape or any section of it and h = the fall in the distance L. The second term $1/4\, a^4$ is negligible for ordinary grades, say less than 5%. The correction for errors of alignment may be made by means of the same formula.

The Temperature Correction is made by adding to the length of the tape the correction $\mathbf{C_t = L \times k \times (t_1{}^\circ - t_0{}^\circ)}$, where k is the coefficient of expansion for the tape used, $t_0{}^\circ$ standard temperature for which the length of the tape has been determined, $t_1{}^\circ$ the actual observed temperature, and L = length of tape.

Absolute Length. The error in the length will be determined by the United States Bureau of Standards for a fee depending upon the nature of the test. It is advisable to have the comparison made under exactly the same conditions, as regards the manner of support, tension, etc., in order to eliminate all uncertainty in computed corrections for sag, tension, etc.

Reduction to Sea Level. In order that the whole triangulation system shall be referred to the spheroid selected to represent the figure of the earth it is necessary to reduce the length of the base-line to what it would be if originally measured at sea level. If the elevation h of the base above sea level be determined by leveling, then the reduction is approximately equal to $-\mathbf{Bh/R}$, where B is the length of base and R the earth's radius at the point in question; h and R must be in the same units. (Average value of log R (in meters) = 6.80470; (in feet) = 7.32068.)

Marking the Stations. The position of the triangulation station should be marked by a bronze marker set in a drill-hole, or in concrete, or else by a stone monument set in the ground. In addition to the center mark several witness marks should be set near by, and their azimuths and distances from the center accurately determined so that the center mark could be replaced in case it is disturbed. When a stone monument is used as a mark the precaution is sometimes taken to set another ma.k several feet below the surface to be used in case the surface marks are destroyed.

49. Angle Measurements

Signals. Before the angular measurements are begun signals of some sort are placed over the selected stations to be used for sighting when the angles are measured. In some cases these consist of a mast made of 4 by 4-in. lumber supported by a tripod and marked by black and white stripes, sometimes also by flags (Fig. 38). The foot of the mast is set about 7 or 8 ft. above the ground so that the instrument may be set beneath the signal when measuring angles. Where it is necessary to build a high signal on account of woods this may be done by raising a tall mast made of two or three poles spliced together and bracing it by wire guys. Such signals are suitable for short lines, say up to 10 or 15 miles. For longer lines heliotropes or electric lights are used. A heliotrope is an apparatus for reflecting sunlight to a distant station, and consists of a movable mirror and a device for pointing it with sufficient accuracy so that the light will reach the station. In cases where it is necessary for the triangulation instrument to be raised in order to see over intervening obstacles, towers are built over the station, consisting of an inner tripod for the instrument and an outer structure for the observer, the two being disconnected so that the observer will not disturb the instrument when moving about on the tower. The latest form of tower is built of steel; these steel towers may be taken down and used over and over.

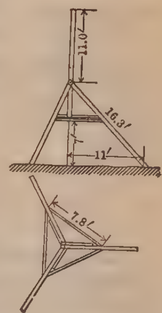


Fig. 38

Measuring the Angles. Instruments designed for triangulation differ from the ordinary transit chiefly in their size, in the fineness of the graduation and in the power of the telescope. Circles of very large diameter were formerly used for triangulation, but the present practice is to use circles 8 to 12 in. in diameter. The instrument is usually designed with three leveling screws, and rests either on a pier or on a tripod, according to its size. There are two types of instrument used in this work, the **repeating** instrument and the **direction** instrument, the latter being used for the most refined geodetic work.

The Repeating Instrument. The repeating instrument is constructed like the ordinary transit except that it has three leveling screws. The telescope is of high magnifying power and the cross-hairs are usually of the X pattern instead of vertical and horizontal. The verniers read either to 10″ or to 5″. In measuring angles in first-order triangulation the instrument is mounted on a tower or a heavy wooden support, and is protected from sun and wind by a tent or a temporary building. For work of a less precise character the instrument may be used on its tripod and sheltered by an umbrella. A common method of observation is to measure the angle six times by repetition, the left-hand signal being sighted first and then the right-hand signal, each time, the telescope being in the direct position; when six repetitions have been made, invert the telescope and measure the angle again six times, beginning with the right-hand object. (The U. S. C. & G. S. practice is to measure from the right-hand object right-handed around to the left-hand object.) It is not necessary to reset the vernier on zero for the second half of the set of angles, but it may be left at the reading it had at the end of the first six repetitions, then at the end of the second half of the set it should read approximately zero again. Both verniers should be read at the beginning and at the end of each half-set. The twelve repetitions made in this manner constitute a "set." For the main triangulation 5 or 6 sets should be measured; for second order work, 2 to 4 sets; for third order, one or more sets according to the size of the instrument. In order to eliminate errors of graduation the first set may be taken starting with vernier A at 0°, the second set starting at 360° mn and so on, m being the number of sets proposed to be taken and n the number of verniers on the instrument.

Record for a Repeating Instrument

Sta. Blue Hill. Date, May 21, 1929. Observer, A. L. Instr. No. 1960.

Station	Time	Tel.	Rep.	Ver. A.	B.	Mean	Angle	Final Angle
Prospect to Shaw	$4^h\ 00^m$ p.m.	D	0	0° 00′ 00″	00″	00″		
			1	123 28 10	20	15		
			6	380 49 40	40	40	123° 28′ 16″.7	
								123° 28′ 15″.8
		R	0	20 49 40	40	40		
			6	0 00 10	10	10	123 28 15 .0	

The Direction Instrument. In the direction instrument the horizontal circle and the upper portion of the instrument can be moved independently. The method of repetition is not used, but the directions of signals are read in order of azimuth around the circle, the angles being derived from the differences in these directions. The degrees and 5-minute spaces are read directly from the circle by means of one of the microscopes. For obtaining the minutes and seconds several micrometers (sometimes three but often two) are used. The angular space between the zero point of any micrometer and the last preceding graduation of the circle passed over by that micrometer is measured by means of the micrometer screw. The precision is increased by reading all the microscopes and taking the mean. To read a direction the circle is clamped in any desired position, the telescope is turned so that the cross-hairs (usually two vertical hairs) point at the signal; the degrees and 5-minute spaces are then read on the first microscope and the minutes, seconds and fractions of seconds are read on all of the microscopes. The mean of all the micrometer readings added to the reading of the index is the direction of the signal referred to the zero graduation of the circle.

Use of Direction Instrument. At each pointing the first microscope is read for the degrees and 5-minute spaces and all the micrometers are read for the seconds and fractions. After the directions of all signals have been read the telescope is reversed in its bearing and the series is repeated in the reverse order. This completes one "position." Sixteen of these positions are taken on first order work; on second order triangulation 4 to 8 position are used. In order to distribute the readings over all parts of the 360° the circle is shifted each time by an amount depending upon the number of positions to be used. Lists of initial settings are given in the following tables.

Theodolite Settings when 16 Positions of the Circle are Used

With a 3-micrometer theodolite		With a 2-micrometer theodolite		With a 3-micrometer theodolite		With a 2-micrometer theodolite	
	° ′ ″		° ′ ″		° ′ ″		° ′ ″
1	0 00 40	1	0 00 40	9	128 00 40	9	90 00 40
2	15 01 50	2	11 01 50	10	143 01 50	10	101 01 50
3	30 03 10	3	22 03 10	11	158 03 10	11	112 03 10
4	45 04 20	4	33 04 20	12	173 04 20	12	123 04 20
5	64 00 40	5	45 00 40	13	192 00 40	13	135 00 40
6	79 01 50	6	56 01 50	14	207 01 50	14	146 01 50
7	94 03 10	7	67 03 10	15	222 03 10	15	157 03 10
8	109 04 20	8	78 04 20	16	237 04 20	16	168 04 20

Theodolite Settings when 8 Positions of the Circle are Used

With a 3-micrometer theodolite		With a 2-micrometer theodolite		With a 3-micrometer theodolite		With a 2-micrometer theodolite	
	° ′ ″		° ′ ″		° ′ ″		° ′ ″
1	0 00 40	1	0 00 40	5	52 00 40	5	90 00 40
2	15 01 50	2	22 01 50	6	67 01 50	6	112 01 50
3	30 03 10	3	45 03 10	7	82 03 10	7	135 03 10
4	45 04 20	4	67 04 20	8	97 04 20	8	157 04 20

50. Computations

Reducing the Notes. The reduction of the notes and the calculation of results include the following steps: (1) Station adjustment. (2) Correction of angles for spherical excess. (3) Distribution of error of closure of triangle. (4) Computation of lengths of sides of triangles. (5) Reduction to center when angles are measured at eccentric station, and a repetition of steps 2, 3 and 4. (6) Computation of the geodetic positions of the stations. (7) Figure adjustment. (8) Final recomputation of 4 and 6.

The Station Adjustment is made by applying the Method of Least Squares. The following illustrates the method when the angles are measured by a repeating instrument. Let four lines OA, OB, OC, OD, radiate from the station O, thus giving 12 angles that can be measured. Of these only three are independent, and when the probable values of these are known, those of the others are found at once. First, for each of the measured angles an observation equation is written; second, from these are derived three normal equations; third, the solution of the normal equations gives the most probable values of the three independent angles; fourth, the most probable values of all other angles are then found by simple addition or subtraction.

Example. Let the measured angles and their weights be as follows, the weights being numbers proportional to the numbers of repetitions:

$$
\begin{aligned}
AOB &= 55^\circ\ 57'\ 58''.68 \text{ with weight } 3\\
BOC &= 48\ \ 49\ \ 13\ .64 \text{ with weight } 19\\
AOC &= 104\ \ 47\ \ 12\ .66 \text{ with weight } 17\\
COD &= 54\ \ 38\ \ 15\ .53 \text{ with weight } 13\\
BOD &= 103\ \ 27\ \ 28\ .99 \text{ with weight } 6
\end{aligned}
$$

Let O_1, O_2, O_3 designate the observed values of AOB, BOC, COD, and let z_1, z_2, z_3 be corrections to be applied to O_1, O_2, O_3, in order to give the most probable values of those angles. Then result the five simpler observation equations:

$$z_1 = 0 \quad \text{with weight } 3 \qquad z_3 = 0 \quad \text{with weight } 13$$
$$z_2 = 0 \quad \text{with weight } 19 \qquad z_2 + z_3 = -0''.18 \text{ with weight } 6$$
$$z_1 + z_2 = +0''.34 \text{ with weight } 17$$

Next multiply each observation equation by the coefficient of z_1 in that equation and also by its weight, and add the results, thus finding the normal equation for z_1; similarly form normal equations for z_2 and z_3. The three normal equations are

$$\begin{aligned} 20\,z_1 + 17\,z_2 \qquad\qquad &= +5''.78 \\ 17\,z_1 + 42\,z_2 + 6\,z_3 &= +4\ .70 \\ 6\,z_2 + 19\,z_3 &= -1\ .08 \end{aligned}$$

and the solution of these gives $z_1 = +0''.28$, $z_2 = +0'.01$, $z_3 = -0''.06$ as the most probable corrections. The following are hence the adjusted values of the five observed angles:

$$\begin{aligned} O_1 + z_1 &= 55° \ 57' \ 58''.96 \\ O_2 + z_2 &= 48 \ \ 49 \ \ 13\ .65 \\ O_1 + O_2 + z_1 + z_2 &= 104 \ \ 47 \ \ 12\ .61 \\ O_3 + z_3 &= 54 \ \ 38 \ \ 15\ .47 \\ O_2 + O_3 + z_2 + z_3 &= 103 \ \ 27 \ \ 29\ .12 \end{aligned}$$

which satisfy all the geometric requirements and from which the other 7 angles are readily found.

Spherical Excess. The spherical excess is the amount by which the sum of the three angles of a spherical triangle exceeds 180°. The amount of this correction depends upon the area of the triangle and upon the latitudes of the points. It is nearly equal to one second for each 75 square miles of area. It may be computed from the formula $\mathbf{e''} = \mathbf{bc \sin A/2\, R^2 \sin 1''}$, in which b and c are the sides and A the included angle of the triangle, and R the radius of the earth at the point; the mean value of log R (in meters) = 6.80470. For a more precise value use $\mathbf{e''} = \mathbf{bc \sin A/2\, RN \sin 1''}$, in which R is the radius of curvature of the meridian and N the normal for the given latitude. One third of the spherical excess of the triangle is to be subtracted from each angle, which reduces it to the corresponding plane angle.

Error of Closure. The sum of the three plane angles will not be 180°, however, unless the measurements were perfectly made. A test of the accuracy of the work is found in the error of the sum of these angles. In the triangulation of the U. S. Coast Survey the average error of closure for first order triangles is about 1″; for second order, 3″; and for third order, 5″. After the spherical excess has been calculated the remaining error is due to

Computation of Triangle Sides

Stations	Observed angles	Correction	Spherical angles	Spherical excess	Plane angles and distances	Logarithms
P to B					22 723m.08	4.356 4673
O	61° 47′ 18″.8	0″.4	18″.4	0″.3	61° 47′ 18″.1	0.054 9218
P	82 27 27 .9	0 .4	27 .5	0 .3	82 27 27 .2	9.996 2261
B	35 45 15 .4	0 .4	15 .0	0 .3	35 45 14.7	9.766 6415
O to B					25 563.20	4.407 6152
O to P					15 067.13	4.178 0306

Note:—In this triangle the spherical excess = 0″.9.

errors of measurement and is distributed equally among the three angles of the triangle.

Computation of Triangles. After the triangle has been adjusted for error of closure the two unknown sides of the triangle are computed from the formula **a/b = sin A/sin B.** The plane angles are used for this calculation because for any triangle which occurs in practice it is sufficiently exact to calculate it as a plane triangle and it is simpler than to compute the spherical triangle.

Reduction to Center. If owing to obstructions it is impossible to place the instrument over the center mark when observing the angles, it may be placed at some other point, called the **eccentric station,** and the angle observed with the same accuracy as for an angle at the center. These angles may then be reduced to the values they would have at the center mark provided the distance and direction to the center from the instrument are accurately determined. This reduction is made by computing the direction of each line reckoned from the center mark as the zero degree point, and then calculating the difference in direction of each distant signal as seen from the instrument and from the center mark. This is done by solving the triangle (Fig. 39) formed by joining the instrument, the center mark, and any distant signal, the result being approximately $s'' = d \sin \alpha / D \sin 1''$, where s'' is the correction in seconds, d is the eccentric distance (measured with a tape), α is the "direction" of the distant station, D is the approximate calculated distance to the distant station. The "direction" α may have any value from 0° to 360°; careful attention should be given to the algebraic signs of $\sin \alpha$. After the corrections (s'') have been calculated they are added algebraically to the corresponding directions previously found. The differences of the corrected directions then give the true angles at the center.

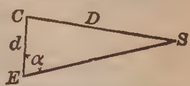


Fig. 39

Example of Reduction to Center

Stations	A	B	C	D	E
Observed direction	42° 14′ 20″.0	46° 01′ 59″.1	104° 47′ 30″.1	161° 10′ 06″.2	205° 10′ 03″.6
Log sin	9.8275	9.8572	9.9853	9.5090	9.6286 *n*
Log $\frac{1}{D}$	6.1052	6.1025	6.0640	6.2672	6.0909
Log $\frac{1.342^m}{\sin 1''}$	5.4422	5.4422	5.4422	5.4422	5.4422
Log S	1.3749	1.4019	1.4915	1.2184	1.1617
S	+ 23″.7	+ 25″.2	+ 31″.0	+ 16″.5	− 14″.5
Corrected direction	42° 14′ 43″.7	46° 02′ 24″.3	104° 48′ 01″.1	161° 10′ 22″.7	205° 09′ 49″.1

51. Dimensions of the Earth

Elements of the Spheroid deduced by Clarke in 1866, and used since 1880 in geodetic work in the United States, are:

Semi-major axis = 6 378 278 meters = 20 926 062 feet
Semi-minor axis = 6 356 654 meters = 20 855 121 feet
Meridian quadrant = 10 001 997 meters = 32 814 885 feet
Eccentricity = 0.082 271 Ellipticity = 1/294.98

The radius of a sphere which is equal in volume to this spheroid is 6 371 062 meters or 20 902 416 ft. or 3959 statute miles. The radius 3957 statute miles corresponds to a sphere whose quadrant equals the elliptical quadrant.

Hayford's discussion of 1909 gives 6 356 909 meters for the semi-minor axis and 1/297.0 for the ellipticity of the spheroid which best fits the North American continent.

Lengths of One Degree, in Meters, for the Clarke Spheroid

Latitude	On meridian	On parallel	Latitude	On meridian	On parallel
0°	110 568	111 322	45°	111 132	78 850
5°	110 577	110 901	50	111 231	71 699
10	110 602	109 642	55	111 326	63 997
15	110 644	107 553	60	111 416	55 803
20	110 700	104 650	65	111 497	47 178
25	110 769	100 953	70	111 567	38 189
30	110 850	96 489	75	111 624	28 904
35	110 939	91 291	80	111 666	19 395
40	111 034	85 397	85	111 692	9 735
45	111 132	78 850	90	111 701	0

RAILROAD LOCATION

52. Reconnaissance Survey

Three Types of Survey are employed in laying out a new railroad: first, the **reconnaissance,** or an examination of the country through which the railroad is to run for the purpose of choosing the general route (or routes) which it seems worth while to investigate further; second, a topographic or **preliminary survey** of a belt of country chosen by the reconnaissance; and third, the laying out or **location** of the proposed railroad within the belt of the preliminary. If the reconnaissance is carelessly made the subsequent work will not rectify this defect. It is in the reconnaissance that most of the bad errors in railroad location have occurred. Although financial considerations frequently determine that a road must pass through certain towns between its termini, the location of the road depends largely upon the topography, and the choice of route upon the locating engineer.

For a Reconnaissance the first step is to obtain the best available maps of the country in question. The U. S. Geological Survey maps are particularly valuable for such purposes, as their contour lines show elevations. On such maps routes may be sketched and a very good idea of the maximum grades of the road obtained. Routes which are obviously unsatisfactory are eliminated from further consideration and only those which look favorable need be thoroughly examined by traveling over them. The map will save much time and expense and minimize the amount of actual field examination. Unfortunately large portions of the country have never been surveyed and it is not uncommon for the engineer to find himself without any map of the country he is to examine.

The engineer will travel over the most favorable routes on foot, horseback, or by vehicle. Distances are obtained by pacing or by odometer. Elevations are obtained by the use of the aneroid barometer and a hand level, and such data incorporated into a rough map of the proposed road. The engineer must choose not only a route which can be built at a reasonable cost but one that can be maintained and operated economically. He must therefore take into consideration the grades, curvature, length of line. earthwork and character of the soil, bridging, tunneling and general drainage. To give proper weight to these matters he must be familiar with the economic principles of location, the relative economic value of additional length of line, of grade and curvature. The traffic to be accommodated by the proposed road may be heavy in one direction and light in the other, in which case a grade of 1% may not be as serious in one direction as a grade of 0.5% will be in the opposite direction. A large traffic also justifies an expensive line in order to save operating expense.

A ridge location is advantageous because of the small amount of drainage and bridging to be provided. But it more often happens that the termini and the intermediate towns to be tapped are located on rivers, so that a valley location is required. Valley locations are very common; they frequently allow low grades but often require many structures to bridge the lateral streams and are more subject to washouts. Where it is necessary to cross from one bank of a river to another especial care in the selection of the bridge site must be exercised. In valley locations the grade is practically determined by the rate of fall of the river, but in mountainous countries the rivers may be so steep as to be prohibitive for a railroad grade, in which case the road is purposely lengthened by passing up some lateral valley so as to keep the ruling grade down to the desired amount. In routes which cross the drainage of a country it is of the utmost importance to discover the lowest pass and to use this point as one of the governing points on the location. In very mountainous country it may be feasible in getting over the pass to introduce a short stretch of must steeper grade than the working maximum grade and to operate this grade by the use of one or more helper engines.

In early railroading in this country crossings of highways at grade were not avoided, but now practically every state requires that new railroads shall be constructed with no highway grade crossings, a requirement which introduces another important consideration into the choice of location.

In making the reconnaissance the engineer should not only choose routes from the standpoint of cost of construction, maintenance and operation, but he should also gather data of the industries and natural resources of the country through which the road is to pass.

53. Preliminary Survey

The Preliminary Survey is a transit and tape traverse run as near as practicable to the probable location of the railroad from one governing point to another. A governing point may be a town, a river crossing, a pass or other fixed point on the route. Between these points every effort will be made to maintain the lowest practicable and economical rate of grade. The ruling grade of each engine division should be adjusted with reference to those of adjoining divisions and to conditions of local traffic, so as to avoid breaking and making up of trains. All surveys should be made with regard to future permanent construction and every effort used to reduce the amount of temporary construction to the lowest limits. The preliminary should be run with considerable accuracy, for the location survey is to be checked against the preliminary. The distances are usually measured to tenths of a foot, and the deflection angles to the nearest minute.

The Purpose of the Preliminary is to serve as a basis of a topographic survey of a belt of country, varying in width from 300 ft. to 1000 ft. depending upon the character of the country, a belt in which the located railroad will probably lie. The organization of the preliminary survey party is usually as follows: chief of party, transitman, two chainmen, two flagmen, levelman and rodman, topographer and assistants, and axmen. A stake is driven at every 100-ft. station and marked with the station number. Each transit point is carefully marked by a nail in the head of the stake, but intermediate points are set to the nearest tenth only and are marked merely by stakes. A preliminary party in fairly open country will make 5 to 8 miles a day. The level party follows immediately behind the transit party, taking readings for a profile of the preliminary line.

Topographic Details are sketched to scale on cross-section paper, using the elevations obtained by the level party as a basis. As a rule a 5-ft. contour interval is used and the contours are located by means of the hand level and metallic tape, as described in Art. 28. The stadia is employed in some cases to obtain topographic details, and a small plane table is well adapted for this work in open country. The plot of the preliminary map and topographic details together with the profile of the preliminary line should be kept up-to-date so as to give the locating engineer information as regards grade and curvature to aid him in judging whether to push ahead with his line or to "back up" and run the line over somewhat different ground. A common scale for preliminary maps is 200 or 400 ft. to an inch.

The preliminary survey practically fixes the general alignment and grades, and where more than one preliminary has been run the best one can be chosen from a study of the plans and profiles and from other data accumulated when the preliminary is made. The kinds of materials in the excavations, the bridge and tunnel sites, and an estimate of the cost are data which the preliminary should include. The map and profile of the best preliminary line form a basis for the location.

54. Location Survey

The Located Railroad is composed of straight track, called **tangents,** and circular cuves. On most roads a spiral is introduced between the tangents and curves in the trackwork, but this refinement need not enter into the location line. First a trial location line is plotted on the preliminary map and its profile constructed from the data given by the contours on the preliminary. This trial line will pass through the governing points, such as bridge sites or passes, and will be laid out so as to make the total quantity of excavations and embankment a minimum, due regard being given to the grades and the amount of curvature in the alignment. On all curves the rate of grade should be flattened about 0.04% per degree of curve to neutralize the additional train resistance; this is called **curve compensation.** The drainage of the road is of utmost importance; it is good practice to lay out the line across flat country a little higher than the surrounding land. The line through cuts which are more than 500 ft. long should be on a grade for proper drainage, which requires about 0.2 ft. per 100 ft. It should be borne in mind that on account of the necessity for side ditches the cuts are wider than the fills and that this should be taken into account when judging from the profile of the relative amounts of cut and fill.

Several shifts of the location line will probably be made and profiles constructed for each position until the best line is determined. This final line will probably cross and recross the preliminary traverse line many times if the preliminary has been skilfully run. Ties from the location line to the preliminary line are then scaled from the map and are used in the field in laying out the located line as determined by the office study. This method is called **paper location.** Sometimes, in a canyon for example, where there is no choice of location, the preliminary survey is omitted and the located line is run out in the field. This is called **field location.**

After the location line has been actually run it may be found advisable in some instances not to follow exactly the line laid out on the plan. Minor changes and modifications may be made, for example, in fixing with care the location of stations, water tanks, or coaling plants and in adjusting the grades near such points so as to reduce the cost and disadvantage of train stops to the minimum. When train stops at or near the foot of a grade cannot be avoided the rate of grade should be flattened to facilitate the starting of trains. It is not infrequently found that stream diversions, even when of considerable magnitude, prove cheaper than bridging, both in first cost and in maintenance, particularly when the excavated material can be used in embankments. The locating engineer will give special attention to the determination of the necessary length of bridges and size of culverts, character and area of waterway. A cross-section of streams showing along the center line of the railroad, the river bottom, flood height and surface indications of rock should be plotted for each crossing. Thorough drainage is a maxim to be impressed on the mind, and the engineer must not be misled in so-called "rainless districts." In ravines carrying mountain torrents the openings must be left much larger than the appearance of the banks would seem to make necessary.

In Staking Out the center line of location the direction of the tangents is first defined and run to an intersection with adjacent tangents which locate the vertices of the curves. The curves are then staked out as explained in Art. 57. For location work a one-minute transit and 100-ft. steel tape are usually employed, the measurements being made to tenths of a foot. A stake is driven at every full station point and a "hub" with a nail is set at all transit stations together with a proper witness stake. The stakes at the beginning and end of each curve should be marked "P.C." and "P.T." respectively and should be carefully referenced by stakes set far enough on each side of the location to lie outside of the construction or filling. The true bearings of tangents should be accurately determined occasionally by solar or stellar observation so as to check the fieldwork and for the purpose of describing the loca-

Radius and Degree of Curve

Degree D	Radius R, ft.	Log R
0° 0′	∞	∞
10	34377.5	4.536274
20	17188.8	4.235244
30	11459.2	4.059154
40	8594.42	3.934216
50	6875.55	3.837308
1° 0′	5729.65	3.758128
10	4911.15	3.691183
20	4297.28	3.633194
30	3819.83	3.582044
40	3437.87	3.536289
50	3125.36	3.494900
2° 0′	2864.93	3.457115
10	2644.58	3.422356
20	2455.70	3.390176
30	2292.01	3.360217
40	2148.79	3.332193
50	2022.41	3.305869
3° 0′	1910.08	3.281051
10	1809.57	3.257576
20	1719.12	3.235305
30	1637.28	3.214122
40	1562.88	3.193925
50	1494.95	3.174627
4° 0′	1432.69	3.156151
10	1375.40	3.138430
20	1322.53	3.121404
30	1273.57	3.105022
40	1228.11	3.089236
50	1185.78	3.074005
5° 0′	1146.28	3.059290
10	1109.33	3.045059
20	1074.68	3.031281
30	1042.14	3.017927
40	1011.51	3.004972
50	982.64	2.992393
6° 0′	955.37	2.980170
10	929.57	2.968282

Degree D	Radius R, ft.	Log R
6° 20′	905.13	2.956711
30	881.95	2.945442
40	859.92	2.934459
50	838.97	2.923747
7° 0′	819.02	2.913295
10	800.00	2.903089
20	781.84	2.893118
30	764.49	2.883371
40	747.89	2.873840
50	732.01	2.864514
8° 0′	716.78	2.855385
10	702.18	2.846446
20	688.16	2.837687
30	774.69	2.829102
40	661.74	2.820685
50	649.27	2.812428
9° 0′	637.27	2.804327
10	625.71	2.796374
20	614.56	2.788566
30	603.80	2.780897
40	593.42	2.773361
50	583.38	2.765955
10° 0′	573.69	2.758674
10	564.31	2.751514
20	555.23	2.744471
30	546.44	2.737541
40	537.92	2.730721
50	529.67	2.724008
11° 0′	521.67	2.717397
10	513.91	2.710887
20	506.38	2.704473
30	499.06	2.698154
40	491.96	2.691926
50	485.05	2.685788
12° 0′	478.34	2.679735
30	459.28	2.662074
13° 0′	441.68	2.645111
30	425.40	2.628794

Degree D	Radius R, ft.	Log R
14° 0′	410.28	2.613076
30	396.20	2.597914
15° 0′	383.06	2.583272
30	370.78	2.569116
16° 0′	359.26	2.555415
30	348.45	2.542140
17° 0′	338.27	2.529268
30	328.68	2.516774
18° 0′	319.62	2.504638
30	311.06	2.492839
19° 0′	302.94	2.481361
30	295.25	2.470186
20° 0′	287.94	2.459300
30	280.99	2.448688
21° 0′	274.37	2.438337
30	268.06	2.428235
22° 0′	262.04	2.418371
30	256.29	2.408734
23° 0′	250.79	2.399315
30	245.53	2.390103
24° 0′	240.49	2.381091
30	235.65	2.372270
25° 0′	231.01	2.363633
30	226.55	2.355173
26° 0′	222.27	2.346882
30	218.15	2.338755
27° 0′	214.18	2.330785
30	210.36	2.322967
28° 0′	206.68	2.315295
30	203.13	2.307764
29° 0′	199.70	2.300370
30	196.38	2.293108
30° 0′	193.19	2.285974
30	190.09	2.278963

tion line. Connections should be made in the location survey to all property lines crossed and to all government land lines and corners.

Immediately behind the transit party comes the level party, which runs a profile of the center line, reading the surface elevations to 0.1 ft. and T.P.'s and B.M.'s to 0.01 ft. A substantial B.M. should be established every 2000 ft. and plainly marked. When the final alignment has been staked out the level party, using the grade line on the location profile as a basis, take the cross-sections (see Art. 64) at each full station or oftener to determine the amount of earthwork and at the same time to give stakes properly marked as guides to the contractor in excavating and filling. These grades refer to the sub-grade, on top of which are to be placed the ballast and track.

55. Simple Circular Curves

A Simple Curve is a circular curve connecting two tangents. The points where the curve is tangent to the tangents are called the **point of curvature P.C.** and the **point of tangency P.T.**, these points being at the beginning and at the end respectively of the curve. The tangents meet at the **vertex V.** The deflection angle at *V* between the two tangents is the **intersection angle I,** which is equal to the **central angle** between radii drawn to the P.C. and P.T. In railroad practice the **length of curve** is the distance from P.C. to P.T. measured by 100-ft. chords (a sub-chord may occur at one or both ends of the measurement). The **degree of curve D** is the angle at the center of the curve subtended by a chord of 100 ft. A chord of less than 100 ft. is called a **sub-chord** and its central angle is a **sub-angle.** The relation between *D* and R is expressed by the formula **$\sin 1/2\ D = 50/R$.**

In the Metric System the degree of curve is usually defined as the angle at the center subtended by a chord of 20 meters. The first and second columns of the following table give degree of curve and radius according to this definition, while the third column gives the equivalent degree of curve corresponding to the definition stated on the preceding page.

Degree of Curve and Radius for Metric System

Degree of curve is angle at center subtended by a chord of 20 meters.

Degree	Radius, meters	Equivalent U. S. degree	Degree	Radius, meters	Equivalent U. S. degree	Degree	Radius, meters	Equivalent U. S. degree
0° 0′	8	0° 0′	6° 0′	191.07	9° 09′	12° 0′	95.67	18° 20′
20	3437.8	0 30	20	181.03	9 40	20	93.09	18 51
40	1718.9	1 01	40	171.98	10 10	40	90.65	19 21
1° 0′	1145.9	1° 31′	7° 0′	163.80	10° 41′	13° 0′	88.34	19° 52′
20	859.46	2 02	20	156.37	11 11	20	86.14	20 23
40	687.57	2 32	40	149.58	11 42	40	84.05	20 54
2° 0′	572.99	3° 03′	8° 0′	143.36	12° 12′	14° 0′	82.06	21° 24′
20	491.14	3 33	20	137.63	12 43	20	80.16	21 55
40	429.76	4 04	40	132.35	13 13	40	78.34	22 26
3° 0′	382.02	4° 34′	9° 0′	127.45	13° 44′	15° 0′	76.61	22° 57′
20	343.82	5 05	20	122.91	14 15	20	74.96	23 28
40	312.58	5 35	40	118.68	14 45	40	73.37	23 59
4° 0′	286.54	6° 06′	10° 0′	114.74	15° 16′	16° 0′	71.85	24° 29′
20	264.51	6 36	20	111.05	15 47	20	70.40	25 0
40	245.62	7 07	40	107.58	16 17	40	69.00	25 31
5° 0′	229.26	7° 37′	11° 0′	104.33	16° 48′	17° 0′	67.66	26° 02′
20	214.94	8 08	20	101.28	17 19	18 0	63.92	27 35
40	202.30	8 38	40	98.39	17 49	19 0	60.59	29 08

56. Elements of a Simple Curve

In Fig. 40 let I be the central angle between radii drawn to the P.C. and P.T. of the simple curve; this is equal to the intersection angle between the tangents. Also let

Oc = Radius = R,
ac = Chord = C,
Vc = Tangent distance = T,
bd = Middle ordinate = M,
Vb = External distance = E,
ka = Offset from tangent = t.

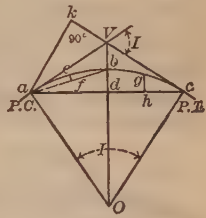


Fig. 40

The following relations between these elements are true when I is less than 180°. The trigonometric function exsec 1/2 I, used in the formulas, is called the external secant, and vers 1/2 I is called the versed sine (see Sect. 2, Art. 4).

Central Angle I $= D \times L/100$ (railroad practice only)
$= \text{arc } abc/R$ (in circular measure) $= (\text{arc } abc/2\pi R) \times 360°$

$\sin 1/2\, I = 1/2\, C/R = (E + M)/T \qquad \tan 1/2\, I = T/R$
$\tan 1/4\, I = M/ 1/2\, C = E/T \qquad \text{vers } 1/2\, I = M/R$
$\cos 1/2\, I = 1/2\, C/T = M/E \qquad \text{exsec } 1/2\, I = E/R$

Radius R $= 50/\sin 1/2\, D = 5730/D$ (approx.) $= 1/2\, C/\sin 1/2\, I = T \cot 1/2\, I$ $= E/\text{exsec } 1/2\, I = M/\text{vers } 1/2\, I = (\text{arc } abc/I)\ (180°/\pi)$.

Degree of Curve D = twice the angle whose sine is $50/R$.

$$D_x° = \frac{5730}{R_x°}\text{(approx.)} = \frac{T_1°}{T_x°}\text{ (approx.)} = \frac{E_1°}{E_x°}\text{ (approx.)}.$$

Chord $C = 2\,R \sin 1/2\, I = 2\,T \cos 1/2\, I = 2\,M \cot 1/4\, I$
$= 2\sqrt{M\,(2\,R - M)} = 2\,E \sin 1/2\, I/\text{exsec } 1/2\, I$
$= \text{arc } abc - (\text{arc } \overline{abc}^3/ 24\,R^2)$ (approx.).

Arc abc $= \pi RI/180° = C + (C^3/24\,R^2)$ approximately
$= R \times$ (Length of arc for radius 1). See Tables 5 and 21, Sect. 1.

Length of Curve L $= 100\, I/D$ (railroad practice only).

Tangent Distance T $= R \tan 1/2\, I = 1/2\, C/\cos 1/2\, I = E \cot 1/4\, I$
$= \sqrt{E\,(2\,R + E)} \doteq T_1°/D$ approximately.

External Distance E $= R \text{ exsec } 1/2\, I = T \tan 1/4\, I = \sqrt{T^2 + R^2} - R$
$= 1/2\, C \text{ exsec } 1/2\, I/\sin 1/2\, I = M/\cos 1/2\, I$
$= RM/(R - M) = E_1°/D$ (approx.).

Middle Ordinate M $= R \text{ vers } 1/2\, I = R - \sqrt{(R + 1/2\, C)\,(R - 1/2\, C)}$
$= E \cos 1/2\, I = 1/2\, C \tan 1/4\, I = T \cot 1/2\, I \times \text{vers } 1/2\, I$
$= C^2/8\,R$ (approx.) $= 4\, ef$ (approx.) $= 1/4\, t$ (approx.).

The chord for M must be the same as the chord for t (see Fig. 40).

Middle Ordinate ef $= 1/4\, M$ (approx.).

Ordinate gh $= \sqrt{(R^2 - \overline{dh}^2)} - \sqrt{R^2 - 1/4\, C^2}$
$= \sqrt{(R - dh)\,(R - dh)} - \sqrt{(R + 1/2\, C)\,(R - 1/2\, C)}$
$= M - (dh^2/2\,R)$ (approx.) $= (ah \times hc)/2\,R$ (approx.).

Offset from Tangent t $= C^2/2\,R = C \sin 1/2\, I = 4\,M$ (approx.). The chord for t must be the same as the chord for M (see Fig. 40).

57. Laying Out Curves

Four Methods of laying out circular curves are: (1) by deflection angles, (2) by offsets from the tangent, (3) by chord deflections, (4) by middle ordinates. It is assumed that the two tangents which are to be connected by a curve have been run out to an intersection and the vertex V set and I measured. The degree of curve being known, T is computed and the P.C. and P.T. stakes are set. The station of P.C. is thus established and the station of P.T. = Sta. P.C. + L. Approximately $L = 100\ I/D$.

The Deflection Angle Method is by far the most common and for nearly all railroad work it is sufficiently exact. To lay out a curve by this method set up the transit at the P.C., vernier reading 0°, and sight on V (Fig. 41). Lay off deflection angle $V\ Aa$ and measure chord Aa, thus locating point a. Angle $VAa = 1/2\ \alpha_a$, and $\alpha_a = D \times Aa/100$. Leaving the lower motion of transit clamped loosen the upper clamp and lay off $VAb = 1/2\alpha_b$ and measure chord ab, and so on, setting each stake and checking at the end of the curve on the P.T. stake.

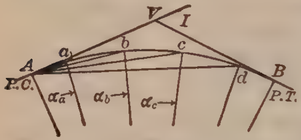


Fig. 41

The total deflection angle to any point on the curve is evidently half the central angle from the P.C. to that point; and where the stations are 100 ft. apart the deflections increase by 1/2 D for each successive station.

$$\text{Deflection angle for any sub-chord} = cD/200 \text{ (in degrees)}$$
$$= c \times 0.3 \times D \text{ (result in minutes when } D \text{ is in degrees)}$$

Example. Find deflection angles of a 6° curve connecting P.C. 8 + 41.7 and P.T. 12 + 73.4, the curve being to the right.

$$\begin{aligned}
58.3 \times 0.3 \times 6 = 105' &= 1^\circ\ 45' \text{ to Sta. 9} \\
1^\circ\ 45' + 3^\circ &= 4^\circ\ 45' \text{ to Sta. 10} \\
4^\circ\ 45' + 3^\circ &= 7^\circ\ 45' \text{ to Sta. 11} \\
7^\circ\ 45' + 3^\circ &= 10^\circ\ 45' \text{ to Sta. 12} \\
73.4 \times 0.3 \times 6 = 132' &= \underline{2^\circ\ 12'} \text{ to Sta. } 12 + 73.4 \\
& \ 12^\circ\ 57'
\end{aligned}$$

$$\text{P.T. } 12 + 73.4 - \text{P.C. } 8 + 41.7 = 431.7 = L$$
$$4.317 \times 6^\circ = 25^\circ.902 = 25^\circ\ 54' = I \text{ and } 12^\circ\ 57' = 1/2\ I.$$

This checks the deflection angle from P.C. to P.T. As a rule the deflection angles should be computed to the nearest half-minute.

It is better practice to lay the curves out backward, starting the chord measurements from the P.C., for example, while the instrument is at the P.T. In this case the circle is set at 0°, a sight taken on the P.C., and the same deflections used for each station as would be used in laying out the curve if the instrument were at the P.C. If the instrument were set up at the P.C. the circle should be set at 1/2 I, a sight taken on P.T., and chords measured beginning with the P.T., using the same deflection angles as before.

Intermediate Set-ups on Curve are necessary when the curve cannot be laid out for its entire length from either the P.C. or P.T. Set the transit up on the intermediate point, then apply one of these two methods: 1. Set the circle at 0°, reverse the telescope and sight P.C., using lower clamp. Turn telescope direct and lay off the same deflection angle to the next station on the curve that was computed for it when transit was at P.C. 2. Set the circle beyond 0° so as to read the same angle that was used in setting the intermediate station, reverse telescope, sight P.C., and clamp lower clamp. Loosen upper clamp and turn circle to read roughly 0° and see if it appears to be sighting along a tangent to the curve through the intermediate point. (If the deflection angle has been laid off on the correct side of the circle the telescope will be sighting along this auxiliary tangent when the circle reads 0°.) Turn telescope direct and lay off the proper deflection angle for the next point ahead on the curve, which will be 1/2 D if the next station is a full station from the intermediate set-up. The curve beyond the intermediate set-up point is therefore laid out like a new curve, starting at the intermediate point. This second method is the one to be used in laying out compound and reversed curves with the instrument set up at the P.C.C. or P.R.C. (Arts. 59 and 60.)

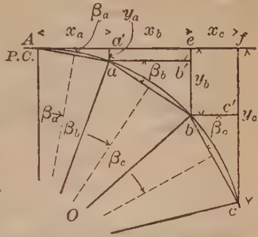


Fig. 42

Offsets-from-Tangent Method. Here no angles are used, every point on the curve being set by measured distances. The first step is to compute the rectangular coordinates of each point, the origin of coordinates being the P.C. and the axes being the tangent and the radius through the P.C. (Fig. 42). In the triangle $Aa'a$, $ab'b$, $bc'c$, etc., the acute angles are equal to the central angle from the P.C. to the middle of the chord forming the hypotenuse of the respective triangles.

$$x_a = Aa \cos \beta_a \qquad y_a = Aa \sin \beta_a$$
$$x_b = x_a + ab \cos \beta_b \qquad y_b = y_a + ab \sin \beta_b$$
$$x_c = x_b + bc \cos \beta_c \qquad y_c = y_b + bc \sin \beta_c$$

As a check, $x_c = R \sin AOc \qquad y_c = R \operatorname{vers} AOc$

In the fieldwork the transit is set up at P.C., sighted along the tangent, and points a', e, f, etc., set and marked by stake and nail. Point a is located by measuring with one tape y_a and with another tape Aa; the intersection of these two measurements locates a Point b is set by intersecting the measurement y_b with ab, etc. This method allows greater precision than the deflection angle method. It is not often used except in setting points where there are obstacles on the curve which prevent sighting across it as is required in the deflection angle method. An isolated point, such as c, can be set by laying off 90° with transit at f and measuring fc.

Deflection Distance Method. (Fig. 43.) This method is employed to lay out a curve by the use only of tapes, plumb-lines, and lining-poles. From the chord or sub-chord Aa, the distances ea and Ae are first computed by

$$\mathbf{ea} = \overline{\mathbf{Aa}}^2/2\,\mathbf{R} \quad \text{and} \quad \mathbf{Ae} = \mathbf{Aa} - (\overline{\mathbf{ae}}^2/2\,\mathbf{Aa}).$$

Sight from A to V and set point e. Set a by measuring ea and Aa. If Aa is a sub-chord, set point d by making $Ad = ea$ and $ad = Ae$. Then daf is a tangent to the curve through a. Compute $bf = \overline{ab}^2/2\,R$, and $af = ab - (\overline{bf}^2/2\,ab)$. Then sight along da and set the point f. Set point b by the ties fb and ab. If $ab = bc$, produce ab to g, making $bg = bc$. Set point c by the ties bc and gc. The chord deflection $gc = 2 \times fb$. The chord deflections can be used, then, whenever the chords on both sides of the last station are equal. When they are not equal an auxiliary tangent must be erected as at a. If chord Aa is very short, point b should also be set by a tangent offset from the main tangent so that the curve will not be produced from too short a base. The chords ab, bc, etc., are often taken less than 100 ft. This method is particularly useful in resetting a stake which has been knocked out of place.

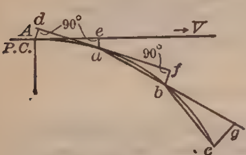


Fig. 43

Middle Ordinate Method. This is employed only for short sharp curves which are to be laid out without the use of a transit. In Fig. 44, the

P.C. and P.T. stakes being in place, chord AB is measured and stake e set by lining it in by eye in the middle of AB. Middle ordinate eb is computed ($M = C^2/8\,R$ approximate) and laid off perpendicular to AB by eye. Similarly points a and c are located, af and $cg = bc/4$.

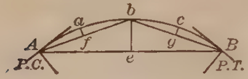


Fig. 44

Fig. 45

58. Curve Problems

Parallel Tangent Problems. In Fig. 45, both curves are of same degree.

$$AA' = VV' = BB' = d/\sin I$$

Fig. 46. Both curves have the same P.C.

$$R - R' = d/\text{vers } I$$

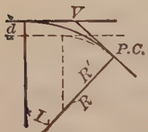


Fig. 46

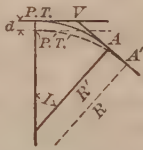


Fig. 47

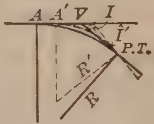


Fig. 48

Fig. 47. Both curves have P.T.'s on same radial line.

$$R - R' = d/\text{exsec } I \text{ and } AA' = (R - R')\tan I$$

Miscellaneous Problems. Fig. 48. The direction of the forward tangent to be changed at the P.T.

$$R' = R \text{ vers } I/\text{vers } I' \text{ and } AA' = R \sin I - R' \sin I'$$

Fig. 49. To find the station of B on a simple curve from which a tangent will pass through C; the radius is known. Measure B and AC. In triangle ACO, compute OC and AOC. In right triangle OBC, compute BOC. Then $AOB = AOC - BOC$, and Sta. B = Sta. $A + 100\ AOB/D$.

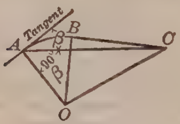


Fig. 49

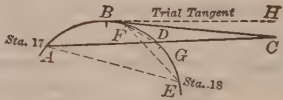


Fig. 50

Fig. 50. To find by approximate field-method the station of B from which a tangent line will pass through C. First find by approximate method the station of D, where line AC cuts the curve. Station A is the nearer full station to B. (If E were nearer than A then CE should be produced until it cuts the curve near A.) To find where CA cuts the curve, set point F in the middle of arc ABE by method of ordinates (Art. 57). If the arc FE is practically a straight line find the intersection of CA and FE. If the arc and chord do not practically coincide then set point G in the middle of arc FE by the method

of ordinates, and then find the intersection of AC and FG. The station D being known, assume point B so as to make AB slightly larger than BD and locate B carefully for a transit point. Set up the transit at B and lay off a tangent to the curve at that point. If this tangent does not pass through C, measure the angle between the tangent and the line BC, angle HBC in this case. Then move point B, forward or backward on the curve as the case requires, a distance $= 100 \times HBC$/degree of curve. A tangent from this new point B should pass almost exactly through C.

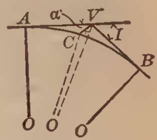


Fig. 51

Fig. 51. To find R of curve which shall join tangents AV and BV and pass through a given point C. Measure angle α and VC. Then $\sin VCO = \cos(1/2\,I + \alpha)/\cos 1/2\,I$. In triangle VOC, $CVO = 90° - 1/2\,I - \alpha$, VCO and VC are known; solve for radius OC. If angle VOC is very small find radius by $VC \sin \alpha/\text{vers}\,AOC$.

Fig. 51. To find distance VC, given R, I and α. Solve triangle VOC for VC; $OC = R$, $OV = R/\cos 1/2\,I$, $OVC = 90° - 1/2\,I - \alpha$.

Obstacles on Curves. When obstacles occur, as frequently happens, so that the entire curve cannot be laid out by deflection angles from one position of the instrument, the difficulty can often be overcome by running out the curve as far as possible and then setting up the transit at the last located point on the curve and running the curve ahead from this intermediate set-up (Art. 57). This method is applicable when the obstacle is not large or when it does not lie directly on the curve. If a large obstacle lies on the curve proper, the curve may be run out until it meets the obstacle, both from the P.C. and P.T., or such obstacles may be passed by use of the Offset-from-Tangent Method, the offset being measured from the main tangent or from an auxiliary tangent thrown off from some intermediate point on the curve. Sometimes an obstacle on a curve can be passed by laying off the deflection angle for a point beyond the obstacle, computing and measuring the long chord to that point, thus locating a station without making a new set-up of the transit, and regular chord measurements may start again from this station.

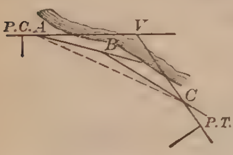


Fig. 52

If the P.C. is inaccessible lay off the curve from the P.T. as far as possible toward the P.C. and check the last point by the Method of Offset from Tangent. In applying this latter method, compute the distance along the tangent from the P.C. and subtract this distance from T, which gives the distance from V to the point on the tangent from which the tangent offset is laid off perpendicular to the tangent.

The vertex V is frequently inaccessible. The central angle I, however, can be found by connecting the tangents through the P.C. and P.T. by a traverse (ABC, Fig. 52). Compute AC, then in triangle AVC compute AV and VC; also determine the value of I. Then the P.C. and P.T. stakes are set by measuring $(T - AV)$ and $(T - VC)$ respectively from points A and C.

59. Compound Curves

Compound Curves are formed by two simple circular curves which run in the same general direction and lie upon the same side of their common tangent at their point of junction. The point where these curves join is called the **Point of Compound Curvature** (P.C.C.). Where more than two curves are in succession the curve is called a **multiple compound curve.** In laying out compound curves in the field, each simple curve portion is laid out separately. From the P.C. the simple curve will be laid out to the P.C.C., and the common tangent is used as a basis for laying out the simple curve to the next

P.C.C. and so on, the methods being the same as those employed in laying out simple curves (Art. 57).

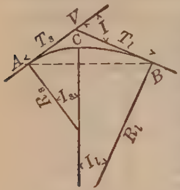


Fig. 53

Elements of Compound Curve. The following apply only to compound curves composed of two simple curves. Where there are more than two simple curves the formulas become quite long. In Fig. 53 it will be observed that all the elements marked with the subscript s refer to the curve of shorter radius, and those marked l refer to the curve of longer radius. Any compound curve composed of two simple curves may be defined by any four of the seven elements marked on Fig. 53. These four elements having been chosen the remainder may be computed by using the following formulas:

Given R_s, R_l, I_s, I_l; required I, T_s, T_l. Here $I = I_l + I_s$ and

$$T_s = R_s \tan 1/2\, I_s + \frac{(R_s \tan 1/2\, I_s + R_l \tan 1/2\, I_l) \sin I_l}{\sin I}$$

$$T_l = R_l \tan 1/2\, I_l + \frac{(R_s \tan 1/2\, I_s + R_l \tan 1/2\, I_l) \sin I_s}{\sin I}$$

Given T_s, R_s, R_l, I; required T_l, I_l, I_s.

$$\text{vers } I_l = \frac{T_s \sin I - R_s \text{ vers } I}{R_l - R_s} \qquad I_s = I - I_l$$

$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_s \cos I.$$

Given T_l, R_s, R_l, I; required T_s, I_l, I_s.

$$\text{vers } I_s = \frac{R_l \text{ vers } I - T_l \sin I}{R_l - R_s} \qquad I_l = I - I_s$$

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I.$$

Given T_s, R_s, I_s, I; required T_l, R_l, I_l. Here $I_l = I - I_s$ and

$$R_l = R_s + \frac{T_s \sin I - R_s \text{ vers } I}{\text{vers } I_l}$$

$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_l \cos I$$

Given T_l, R_l, I_l, I; required T_s, R_s, I_s. Here $I_s = I - I_l$ and

$$R_s = R_l - \frac{R_l \text{ vers } I - T_l \sin I}{\text{vers } I_s}$$

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I$$

Given T_l, T_s, R_s, I; required R_l, I_l, I_s.

$$\tan 1/2\, I_l = \frac{T_s \sin I - R_s \text{ vers } I}{T_l + T_s \cos I - R_s \sin I} \qquad I_s = I - I_l$$

$$R_l = R_s + \frac{T_l + T_s \cos I - R_s \sin I}{\sin I_l}$$

Given T_l, T_s, R_l, I; required R_s, I_l, I_s.

$$\tan 1/2\, I_s = \frac{R_l \text{ vers } I - T_l \sin I}{R_l \sin I - T_l \cos I - T_s} \qquad I_l = I - I_s$$

$$R_s = R_l - \frac{R_l \sin I - T_l \cos I - T_s}{\sin I_s}$$

Given AB, VAB, VBA, either R_s or R_l; required either R_l, or R_s, I_l, I_s, I. Solve the triangle AVB for AV and VB, which gives T_l and $T_s \cdot I = VAB + VBA$. Then the four elements T_l, T_s, I and either R_s or R_l are given. The remaining elements may be computed by formulas given above.

Compound Curve Problems. Given a simple curve between two tangents. Curve is to be compounded so as to end in a parallel tangent (Fig. 54). In case the new tangent is inside the old tangent, as in Fig. 54, the radius of the original simple curve becomes R_l, and vers $I_s = d/(R_l - R_s)$. If the new tangent falls outside the old tangent, then the original simple curve becomes R_s, and vers $I_l = d/(R_l - R_s)$.

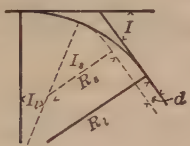


Fig. 54

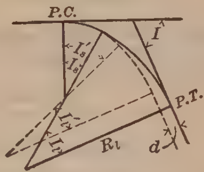


Fig. 55

Given a compound curve between two tangents. Required to retain the same radii but to change the P.C.C. so as to end in a parallel tangent.

(*a*) When the new tangent lies inside the old tangent and the curve of larger radius is at the P.T. end (Fig. 55).

$$\text{vers } I_l' = \text{vers } I_l - \frac{d}{R_l - R_s}$$

(*b*) When the new tangent lies outside the old tangent and the curve of larger radius is at the P.T. end.

$$\text{vers } I_l' = \text{vers } I_l + \frac{d}{R_l - R_s}$$

(*c*) When the new tangent lies inside the old tangent and the curve of shorter radius is at the P.T. end.

$$\text{vers } I_s' = \text{vers } I_s + \frac{d}{R_l - R_s}$$

(*d*) When the new tangent lies outside the old tangent and the curve of shorter radius is at the P.T. end.

$$\text{vers } I_s' = \text{vers } I_s - \frac{d}{R_l - R_s}$$

In the above formulas the angles with the prime mark are the new angles.

60. Reversed Curves

Reversed Curves are formed by two simple circular curves which run in the same general direction but lie on opposite sides of a common tangent at their point of junction. Where these curves meet is called the **Point of Reversed Curvature (P.R.C.)**.

Connecting Parallel Tangents (Fig. 56). In this case the central angles are equal, the P.R.C. lies on the straight line AB, and $BAE = 1/2\ I$.

$$\text{vers } I = \frac{d}{R_1 + R_2} \text{ and } AB = \sqrt{2\,d\,(R_1 + R_2)}$$

If both curves have the same radius, then

$$\text{vers } I = \frac{d}{2R} \text{ and } AB = 2\sqrt{dR}$$

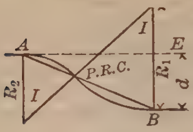


Fig. 56

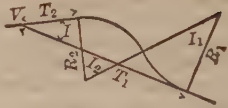


Fig. 57

Connecting Nonparallel Tangents (Fig. 57). As in the compound curve, the choice of four of its elements defines the reversed curve. Given T_1, I, R_1, R_2; required I_1, I_2, T_2.

$$\text{vers } I_2 = \frac{R_1 \text{ vers } I + T_1 \sin I}{R_1 + R_2} \qquad I_1 = I_2 - I$$

$$T_2 = T_1 \cos I + R_1 \sin I - (R_1 + R_2) \sin I_2$$

Given T_2, I, R_1, R_2; required I_1, I_2, T_1.

$$\text{vers } I_1 = \frac{R_2 \text{ vers } I + T_2 \sin I}{R_1 + R_2} \qquad I_2 = I_1 + I$$

$$T_1 = T_2 \cos I + R_2 \sin I + (R_1 + R_2) \sin I_1$$

The two tangents through the P.C. and P.T. are frequently so nearly parallel that the intersection V cannot be readily found, in which case a line V_1V_2 (Fig. 58) may be chosen as the common tangent and angles I_1 and I_2 measured, and the line V_1V_2. To connect these tangents by curves having equal radii:

$$R = \frac{V_1V_2}{\tan 1/2\, I_1 + \tan 1/2\, I_2}$$

Another way to define a reversed curve is, having chosen the P.C. and P.T. (Fig. 59), to measure the distance AB between them and also angles α and β. To connect these tangents by curves having equal radii:

$$\sin C = 1/2\,(\cos\alpha + \cos\beta) \qquad R = \frac{AB}{\sin\alpha + \sin\beta + 2\cos C}$$

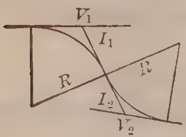


Fig. 58

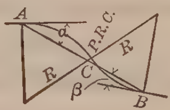


Fig. 59

In reversed curves with nonparallel tangents at the P.C. and P.T. the line AB (Fig. 59) does not pass through the P.R.C.

61. Easement Curves

The Outer Rail on a curve should be elevated to overcome the effect of centrifugal force, but on straight track the rails should be level across. Hence at the P.C. and P.T. conflicting conditions are present; the outer rail cannot be elevated and at the same time have the rails level across. The old practice was to begin to elevate the outer rail for a hundred feet or more along the tangent from the P.C. and P.T. and to reach its proper elevation either at the P.C. or a short distance beyond on the curve. This method was a make-shift, and considerable shock was given to the rolling stock at each end of the curve. To obviate this difficulty an **Easement Curve** is introduced between the tangent and the circular curve. This is a curve of varying radius which leaves the tangent as a very flat curve and grows sharper and sharper until it has the same radius as the circular curve. While the easement curve is developing from a very flat curve to one as sharp as the circular curve the outer rail is gradually elevated so that at any point from the beginning to the end of the easement curve the outer rail is elevated to the proper height.

Cubic Spiral Easement Curve (Fig. 60). AV is a tangent, ABC a **Spiral** connecting the tangent and the circular curve CF, CE is the circular curve produced backward to E, where it is parallel to tangent AV. Let D_c = degree of curcular curve, and R_c = its radius; l_c = total length of spiral AC; l = length of spiral from **P.S.** to any point on spiral; s_c = spiral angle, angle between radius at P.S., and at P.C.C. = angle EOC; s = angle between radius at P.S. and radius at any point on spiral; $q = AD$; $p = DE$; i = deflection angle (instrument at P.S.) to any point on spiral; i_c = deflection angle to P.C.C.; x = offset from tangent to any point on the spiral; x_c = offset GC; y = distance from P.C. to any point on the spiral measured along the tangent; $y_c = AG$. Point B is on the easement curve opposite D.

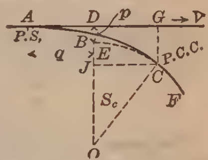


Fig. 60

The equation of the **Cubic Parabola** is $\mathbf{x = y^3/6\ R_c l_c}$.

The equation of the **Cubic Spiral** is $\mathbf{x = l^3/6\ R_c l_c}$. Since y practically equals l for the flat spirals used in railroad practice there is only slight difference between these two curves. The properties of the cubic spiral are:

(1) Degree of curve varies directly with its length from P.S.
(2) Deflection angle varies as square of length from P.S.
(3) Offset from tangent varies as cube of length from P.S.
(4) $x_c = l_c^3/6\ R_c l_c$ (the equation of cubic spiral).
$y_c = l_c - l_c^3/40\ R_c^2$ (this latter term is in most cases negligible).
$x_c = GC = 4/3\ EJ = 4/3\,R_c \text{ vers } s_c = 4\,p$. $EJ = 3\ DE = 6\ DB = 3\ p$.
$q = y_c - R_c \sin s_c$ and $p = x_c - R_c \text{ vers } s_c$.
(5) $s_c = l_c D_c/200$.
(6) Deflection angle to any point $= i = s/3$, $\therefore\ i_c = s_c/3$.
(7) The back deflection is equal to twice the forward deflection.

Example. Given $D = 4°$ and $l_c = 300$ ft. (*a*) Find data to lay out spiral by offsets from tangent, every 50-ft. point to be set. (*b*) Find data to lay out spiral by deflection angles, every 50-ft. point to be set.

(*a*) From (5), $s_c = 300 \times 4/200 = 6°$. From (4), $x_c = 4/3\ R_c \text{ vers } 6° = 10.47$ and $y_c = 300 - 300^3/40\ R_c^2 = 300 - 0.3 = 299.7$ From (3), offset to 50-ft. point $= 50^3/300^3 \times 10.47 = 0.05$, offset to 100-ft. point $= 10.47/27 = 0.39$, to 150-ft. point $= 10.47/8 = 1.31$, to 200-ft. point $= 8 \times 10.47/27 = 3.11$, to 250-ft. point $125 \times 10.4/216 = 6.06$. The y distances to measure along the tangent may be computed by the formula $y = l - l^3/40\ R^2$, in which R for any point is found from (1). But this computation is seldom necessary; it is sufficiently accurate to compute y from (4), or $q = 299.7 - R_c \sin s_c = 149.9$, and to call the y for the 50-ft. point = 50.0. Also for the 100-ft. point $y = 100.0$, for the 150-ft. point $y = q = 149.9$, for the 200-ft. point $y = 199.9$, for the 250-ft. point $y = 249.8$.

(*b*) From (5), $s_c = 300 \times 4/200 = 6°$. From (6), $i_c = 6°/3 = 2°$. From (2), deflection to 50-ft. point = $1/36 \times 2° = 0°\ 03'\ 20''$, to 100-ft. point = $1/9 \times 2° = 0°\ 13'\ 20''$, to 150-ft. point = $1/4 \times 2° = 0°\ 30'$, to 200-ft. point = $4/9 \times 2° = 0°\ 53'\ 20''$, to 250-ft. point = $25/36 \times 2° = 1°\ 23'\ 20''$. Evidently the amount of computation necessary for the deflection angle method is very small.

On many railroads it is the practice to use an easement on all curves sharper than 1° 30′, and the sharper the curve and greater the speed of trains the longer the spiral should be. See Report of Committee on Track of Am. Ry. Eng. & M. W. Assoc., Vol. 10, Part 1, 1909.

Example. Given $D_c = 4°$ and $p = 4$ ft. (*a*) Find data to lay out spiral by offsets, setting every quarter point on spiral. (*b*) Find data to lay out spiral by deflection angles, setting every quarter point on spiral.

(*a*) From (4), $x_c = 16.00$; for $1/4\ l_c$, $x = 1/64 \times 16 = 0.25$; for $1/2\ l_c$, $x = 1/8 \times 16 = 2.00$; for $3/4\ l_c$, $x = 27/64 \times 16 = 6.75$. Then vers $s_c = 3\ p/R_c = 12/R_c$, and $s_c = 7°\ 25'.3$.

From (5), $l_c = (s/D) \times 200 = 371.1$,

From (4), $y_c = 371.1 - (371.1^3/40\ R_c^2) = 370.5$,

$q = 370.5 - R_c \sin 7°\ 25'.3 = 185.6$.

The other values of y are interpolated between these values of y_c and q.

(*b*) Find s as in above $= 7°\ 25'.3$. Then $i_c = 7°\ 25'.3/3 = 2°\ 28'.4$, and

for 1/4 point, $i = 1/16 \times 2°\ 28'.4 = 0°\ 09'.3$,
for 1/2 point, $i = 1/4 \times 2°\ 28'.4 = 0°\ 37'.1$,
for 3/4 point, $i = 9/16 \times 2°\ 28'.4 = 1°\ 23'$.

Fieldwork. In new locations the tangent is usually run as far as point D (Fig. 61), then an offset $p = DC$ is measured, point C set and the circular curve run out from C to E. $DV = (R + p) \tan 1/2\ I$. Later on, when the track is to be laid the P.S. is located by measuring the distance q back from D. Then with the transit at P.S. the spiral is laid out by deflection angles to the P.C.C., where a new set-up of the transit is made. The back deflection (twice the forward deflection) is laid off to establish the auxiliary tangent at P.C.C. from which the circular curve is run to the second P.C.C. The central angle of the circular curve between the P.C.C.'s = I − sum of spiral angles on both ends. If both spirals have the same length, which is the usual case, than $I_c = I - 2\ s = I - l_c D_c/100$. Then the transit is taken to the second P.S. and the spiral at that end is run back and checked on the second P.C.C.

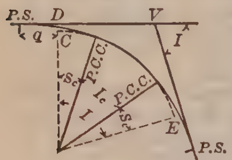


Fig. 61

Compound Curves. Easement curves are required between the two circular curves forming a compound curve for the same reason that calls for their use between a tangent and a simple curve. In case of a compound curve the sharper curve D_s must be at an offset p inside the flatter curve D_l at the point where these two curves are parallel. First determine points A and B (Fig. 62) where the curves will be parallel. $AB = p$, which defines l_c and q. Locate the ends of the spirals C and F by measuring $AC = q$, and $BF = l_c - q$. Set up the transit at C, lay off an angle (clockwise) which equals the deflection angle $XCA = 1/2\ D_l \times q$, and sight on A. If the lower motion is left clamped and the circle turned to 0° the telescope will be sighting along the auxiliary tangent CX. Then lay out the spiral by deflection angles as usual.

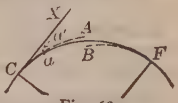


Fig. 62

The deflection angle $XCa = a'Ca + XCa'$. The angle $a'Ca$ is the same as the deflection angle to the first point on a spiral that connects a tangent and a circular curve of degrees $D_s - D_l = D_x$. Find the deflection angles for a spiral of the given length which will join a tangent and a D_x curve.

Then the deflection angle for any point on the spiral to use in laying out the spiral between the compound curves = the deflecting angle found for that point for the spiral to connect D_x + the deflection from the auxiliary tangent to a point on the circular curve CA opposite the required spiral point.

In Revising Old Simple Curve Alignment to introduce spirals, the many existing conditions will as a rule limit the choice of spiral and introduce special problems, one of the most common of which is the following: In Fig. 63, half of a circular curve AB is shown. One of the methods of introducing a spiral into this alignment is to move this simple curve 2 to 4 ft. toward its center and then introduce a spiral on each end. But this requires more track shifting than is allowable under many conditions and another expedient is resorted to as shown in Fig 63: the circular curve is thrown slightly outward in its middle portion and sharpened so that it will fall inside the tangent, making $ED = p$. Calling the distance $BC = k$, R_1 the original curve and R_2 the new curve, then

$$R_1 - R_2 + k = (k + p)/\text{vers } 1/2\ I \text{ and}$$

$$AF = q - (R_1 - R_2 + k) \sin 1/2\ I$$

If the requirement is that the track shall not be thrown at any point more than 1 ft., then since the greatest shift comes along the circular curve between the P.C.C.'s assume $k = 1$; and then assume p about $2\ k$. From the first equation above R_2 may be found, and then AF, which defines the P.S., while D_2 and p determine l_c.

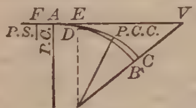


Fig. 63

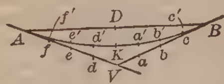


Fig. 64

A Vertical Parabolic Curve is used at the vertex of a grade to avoid the sudden change of direction in passing from one grade to another. If the change of grade is very slight a vertical curve may not be needed, but it is customary to introduce one wherever the change in rate of grade amounts to about 0.2%. In Fig. 64, AV and BV represent the straight grade lines, AKB is the parabolic curve, and Elev. D = 1/2 (Elev. A + Elev. B). In a parabola, point K is always midway between D and V, so that Elev. K = 1/2 [1/2 (Elev. A + Elev. B) + Elev. V]. Since the offsets from the tangent vary as the squares of the distances out along the tangent, $cc' = ff' = 1/16\ KV$, also $bb' = ee' = 1/4\ KV$ and $aa' = dd' = 9/16\ KV$. To find the elevations of points f', e', d', a', b' and c', compute on the straight grade lines the elevation of f, e, d, a, b and c, and then apply the distances ff', ee', and dd'.

EARTHWORK COMPUTATIONS

62. Classification of Grading

Grading usually includes all excavations and embankments, ditching, diversion of roads and streams, foundation pits.

Grading usually has three classifications: Solid rock, loose rock, common excavation; the latter is often called "earth excavation." Additional special classifications are sometimes introduced into the contract.

Solid rock usually comprises rock in solid beds or layers in its original position which may be best removed by blasting; and boulders or detached rocks having a volume of 1 cu. ft. or over (in some specifications 1/2 cu. ft. is this limit).

Loose rock comprises all detached masses of rock or stone greater than 1 cu. ft. and less than 1 cu. yd. (or 1/2 cu. yd.) and all rock which can be properly removed by pick and bar and without blasting, although steam shovel or blasting may be resorted to on favorable occasions in order to facilitate the work

Common excavation comprises all material that does not come under the above two classifications.

Where material is a mixture of loose rock and common excavation it is customary for the engineer to determine frequently (day by day) the percentage of each as the grading progresses and receive approval of his determination by the contractor, if possible.

Where the quantity of excavation is not sufficient to form embankments this deficiency is made up from material within the limits of the job by widening the excavations (as in a railroad), sometimes called **roadway excavation.** Such widening is included under the grading classification for excavation. An illustration of side excavation is the construction of an embankment for railroad or highway across the prairie, the embankment being made by excavating ample ditches at the side.

Borrow is a special classification usually applied to material taken from some pit near an embankment when there is insufficient excavation nearby on the job to form the embankment. Borrow pit excavation is therefore a special classification, usually bid upon as a special item in contracts. It frequently involves the cost of land, or a royalty (2 to 10 cents per cu. yd.) for material taken from the land, where the borrow pit is located; it also often requires the construction of a suitable road to the pit. This type of excavation therefore usually runs higher in cost than ordinary excavation.

Where the quantity of excavation exceeds the quantity required for embankment the surplus is often used to raise the height of an embankment or is deposited in a waste bank.

As a rule the grading is bid upon in the form of price per cubic yard for materials in excavation, no payment being made for embankment quantities.

Prices submitted are: (1) solid rock; (2) loose rock; (3) common excavation; (4) borrow; and the contractor is paid on this basis. He is required to haul the excavated material to any place designated by the engineer and if this haul is beyond the limit of free haul he is paid for the haul (see Art. 67).

Cross-sections in excavations are therefore taken to determine pay quantities. Cross-sections in embankment are taken to determine whether or not excavated material will make the embankment.

In some special cases both excavations and embankment are paid for as separate items; this is termed **two-way payment.**

Excavations in excess of authorized dimensions are usually not paid for unless the causes are beyond the control of the contractor; this applies to slipping of slopes and of trench sides. In trench excavation it is well to specify definitely the limits to which the measurement of pay quantity will be made, so as to avoid dispute by the contractor. This also tends to make the contractor careful, with the result that there is less "over-break," the latter term referring to the amount that the excavation passes outside of the dimensions required.

Another special classification not uncommon is **dry excavation** and **wet excavation,** the dry being defined as excavation above a certain elevation and wet as referring to that below a certain elevation. It is important to state this fixed elevation in the contract so that there may be no dispute as to what is meant by wet and by dry material.

Clearing and grubbing is sometimes included in the item of grading. It involves removing all trees, stumps, brush, rubbish, etc., over the area where construction is to be accomplished. Such work is often paid for at a price per acre. It does not involve any excavation of earth or rock materials.

The removal of the over-burden, that is, the removal of sod or loam, is excavation that is sometimes required to be done separately from the rest of the work. This is the case where loam is to be used for other purposes or where the excavated material would not be suitable if loam were mixed with it. The over-burden is usually a thin layer and where it is required to be removed separately the price for it is sometimes bid upon as a special item because its quantity is small and methods used for removal are different from those used for the rest of the grading.

63. Profiles and Sections

A Profile on which the grade line has been drawn will show the excavated portions above and the embankment portions below the grade line. In a cut, on account of the necessity of providing for drainage, the base of the roadbed is made wider than in fill, so that the area of cut as shown on profile represents more earthwork than an area of the same shape and size in the fill portion of the profile. Although it is true that the depth of the cut or fill and the shape of the ground on either side of the center line have much to do with the amount of earthwork, still it is found satisfactory, before the cross-sections have been taken, to make rough comparisons between the relative amounts of cut and fill by comparing the area of the cut and of the fill as represented on the profile; these may be determined by use of the planimeter.

Cross-sections for the exact computation of earthwork for pay quantities are determined at each full station, and oftener where the shape of the ground demands it. The usual form of these cross-sections (Fig. 65) is level across the base *ab*, with a straight slope extending from the ends of the base until it intersects the surface of the ground *e* and *f*, while its fourth side is formed by the ground surface *ef*. In determining cross-sections, enough dimensions must be taken to define the shape of the surface *ef*. Every slight elevation or hollow in the ground surface is not required; dimensions are taken so as to represent the quantity in the section and its general shape rather than its exact shape, so that there is an opportunity for the exercise of good judgment.

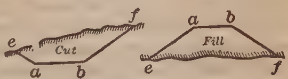


Fig. 65

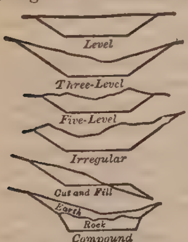


Fig. 66

Ordinary Cross-sections shown in Fig. 66, are: (1) **Level section** in which the ground service is parallel to the base. (2) **Three-level section** in which the ground service is estimated to be straight from the center stake to where each side slope intersects the surface. (3) **Five-level section** in which the surface is estimated to be straight from the center stake to points over the ends of the base, and from these two points to run straight to where the side slopes intersect the surface. (4) **Irregular section,** where the surface is very broken, requiring it to be divided into several straight slopes, the levels being taken where these slopes intersect and also where the two side slopes meet the surface. (5) **Cut-and-fill section** where part of the section is in cut and part in fill. These will be either in the shape of triangles or of shapes similar to a portion of one of the above-named sections. (6) **Compound sections** are formed when materials of different classification occur in the same section, such as rock covered by earth. The ordinary classifications are earth, loose rock and solid rock.

64. Fieldwork

Cross-sections are taken not only at every full station but wherever there is a distinct change in slope along the center line and also where the surface on either side of the center line demands an intermediate section in order to represent properly the volume included between the cross-sections. Cross-sections are taken perpendicular to the center line of the road, and radially on curves. Before this work is begun the following data should be at hand: (1) The center line, marked with alignment stakes at every full station and properly numbered. (2) Notes of alignment and profile. (3) Record of B.M.'s established by the preliminary or location survey party. (4) Width of base for cut and for fill and side slopes to use for each class of material. A slope of 1-1/2 to 1 means 1-1/2 ft. horizontal to 1 vertical.

To find the Cut or Fill at the Center set up the level and find the H.I. to the nearest hundredth of a foot. From the profile obtain the grade elevation at the given station. The H.I. minus the grade elevation gives the **rod-reading for grade,** which is computed to the nearest tenth of a foot. This grade is usually the subgrade on top of which the ballast is to be placed. Take a rod-reading on the ground at the center stake; the difference between the rod-reading for grade and the surface rod-reading gives the cut or fill at that point. It is customary to record cuts as + and fills −. The center cut or fill could be found by determining the elevation of the ground and subtracting it from the H.I., but by using the rod-reading for grade these computations can be readily made mentally. The surface elevation is obtained by adding the cut to the grade elevation, or by subtracting in the case of fill. A grade stake is driven on the center line and marked with the cut or fill as follows, "C 3.2" or "F 6.7."

Slope Stakes are set at the points where the side slopes meet the ground. These stakes are also marked, giving the amount the ground is above or below the base of the section; it is called the cut or fill at the side slope, but strictly speaking there is no cut or fill at the slope stakes. The position of a slope stake is found by trial as follows. In the case of a cut, estimate from the center cut and slope of the surface what the probable side cut will be. The distance from the center stake to a point on the side slope having this cut equals (1/2 base + cut × slope). Make this computation roughly and measure out this distance from the center stake and take a rod-reading at that point. The rod-reading for grade (distance from H.I. to base of section) minus this surface rod-reading gives the cut at the trial point. Compute the distance out from the center stake to a point on the side slope having this cut. If this computed distance equals the measured distance from the center to the rod the trial point was the correct point; if not, then a second trial must be made by holding the rod on another point and repeating the operation. The difference between the measured and calculated distances is an aid in judging where the rod should be held for the second trial. After a little practice it will be possible in most cases to set the slope stake at the second trial. When the correct point is found the stake is marked "C," followed by the feet and tenths of height of this surface point above the base.

The cut and the distance from the center to the slope stakes are entered in the notes. The same process is then repeated for the slope stake on the other side at the center. Rod-readings are taken at intermediate points if they are needed to define the shape of the surface; their positions are located by measuring the distances from the center stake, and the cut at these points is the difference between the rod-reading for grade and the surface rod-reading.

A slope-board is sometimes used in setting slope stakes and in obtaining intermediate elevations. It consists of a long straight wooden board with a spirit-level mounted in its upper edge. After the center cut has been obtained by means of the level instrument, the leveling for the side stakes is done by means of the level-board and a rod. It is particularly useful where there is considerable difference in elevation between the center and side stakes because it obviates the necessity of making a new set-up of the level to obtain the side heights.

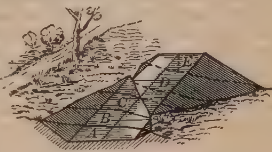


Fig. 67

In Passing from Cut to Fill it is customary to take three intermediate sections: (1) at the point where the cut runs out (is zero) on the down-hill side line, (2) where the cut is zero on the center line, and (3) where the fill begins on the up-hill side line, as shown by sections *B*, *C* and *D* in Fig. 67.

65. Methods of Computation

To Compute the Volume of earthwork between two parallel cross-sections, let A_1 and A_2 be the end areas, A_m the area of a section midway between the ends, c_1 and c_2 the center heights at the end sections, D_1 and D_2 the sums of the side distances d_r and d_l (Fig. 68), l the length of the solid and V its volume.

By Average End Areas, $V = 1/2\ l\ (A_1 + A_2)$,

By Prismoidal Formula, $V = 1/6\ l\ (A_1 + 4\ A_m + A_2)$,

By Average End Areas with prismoidal correction,

$$V = 1/2\ l\ (A_1 + A_2) - 1/12\ l\ (c_1 - c_2)\ (D_1 - D_2).$$

When the given quantities are in feet V is in cubic feet and must be divided by 27 to obtain cubic yards.

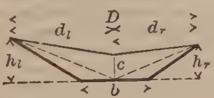


Fig. 68

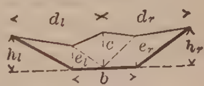


Fig. 69

The Areas of Sections must be found in order to use the above formula. Let b = width of base, c = center height, s = slope. Let h_r and h_l be side heights, d_r and d_l be side distances, and $D = d_r + d_l$. Then

For a Level Section, Fig. 66, $A = c\ (b + sc)$.

For a Three-Level Section, Fig. 68, $A = 1/2 \left\{ 1/2\ b\ (h_r + h_l) + cD \right\}$

For a Five-Level Section, Fig. 69, $A = 1/2\ (cb + e_r d_r + e_l d_l)$

Irregular Sections may be divided into geometrical figures as in Fig. 71, or into triangles by drawing diagonals from where the verticals meet the base (or base produced) to the surface end of the next vertical out, beginning at the center vertical, as shown in Fig. 70. If the latter rule is followed it will be found that the dimensions of the pairs of triangles chosen will be readily taken from the cross-section notes.


Fig. 70

It is the practice of some engineers to plot irregular sections on cross-section paper and to obtain their areas by use of the planimeter. This is not so rapid nor so accurate a method as to compute the area as shown. Where the cross-sections are plotted, however, it gives an excellent opportunity to record on the cross-sections by different colored ink lines the progress of the work from month to month; this is particularly useful where the cutting is deep or for cross-sections of dams or canals. The dimensions of the middle-section area, used in the three-term prismoidal formula, are found by proportion from the dimensions of the two bases.

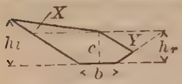


Fig. 71

The prismoidal formula gives an exact volume for a prismoid, which may be defined as "a solid having for its two ends any dissimilar parallel figures of the same number of sides, and all the sides of the solids plane figures." It applies to prisms, wedges and pyramids bounded by planes and to similar solids bounded by warped surfaces. Any solid met with in earthwork computation may be divided into these solids, therefore the formula applies to all ordinary cases of earthwork. It consumes considerable time to compute earthwork by the prismoidal formula method, which is for many cases more exact than is required, consequently the method of **average end areas** has become the most common. The end area method, however, gives results almost always too large. To obtain an exact result by the most rapid means, compute by the average end area method and then apply the prismoidal correction, which gives the same value as the prismoidal formula.

A general method of computing areas applicable to those that are quite irregular was used considerably on I.C.C. valuation work. It is an application of the principle of Art. 16. The cross-section notes should show a distance and a cut (or fill) for every angle point in the section. Write the cuts above grade as + and those below as −. Write the distances to right of center as + and those to left as −. Start at any cut and proceed clockwise around the figure multiplying every cut by (the distance for point next in advance minus distance for point next back) always using algebraic signs. The algebraic values of these quotients divided by 2 is the area.

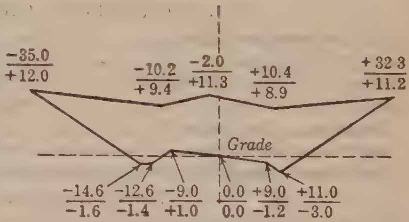


Fig. 72

Example:

	+	−
$+12.0[-10.2-(-14.6)] =$	52.8	
$+9.4[-2.0-(-35.0)] =$	310.2	
$+11.3[+10.4-(-10.2)] =$	232.8	
$+8.9[+32.3-(-2.0)] =$	305.3	
$+11.2[+11.0-(+10.4)] =$	6.7	
$-3.0[+9.0-(+32.3)] =$	69.9	
$-1.2[0.0-(+11.0)] =$	13.2	
$+1.0[-12.6-(0.0)] =$		12.6
$-1.4[-14.6-(-9.0)] =$	7.8	
$-1.6[-35.0-(-12.6)] =$	35.8	
	1034.5	12.6
	− 12.6	
	2)1021.9	
	510.9 sq. ft.	

Correction for Curvature. In making the field measurements the cross-sections on curves are taken along radial lines AB, CD and EF (Fig. 73). But when the earthwork is computed each solid is assumed to have a length equal to the chord (GH or HJ) and to have end sections whose planes are perpendicular to the chord. For example, solids $KGL - MHN$ and $OHP - QJR$ are computed. It will be seen that the solid PMH has been included twice and that WOH has been omitted. If the cross-section taken at H is symmetrical about the center line, then these discrepancies balance, but if at H there is an unsymmetrical cross-section, then a correction must be applied. In Fig. 74 DF has been drawn so as to form a symmetrical figure with DE. For the cross-section $EDFBA$ there is no correction, but for the portion FDC a correction must be applied. This correction may be found by an application of Pappus' Theorem, " If a plane area lying wholly on the same side of a straight line in its own plane revolves about that line and thereby generates a solid of revolution, the volume of that solid thus generated is equal to the product of the revolving area and the path described by the center of gravity of the plane area during the revolution." For the case of Fig. 74 there applies Correction $= (1/2\, b + sc)\,(h_r - h_l)\,(d_r + d_l) \times 0.00291\,\beta$

Fig. 73

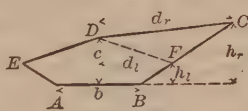


Fig. 74

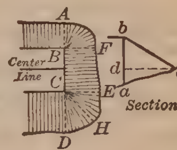


Fig. 75

in which β = sum of half the central angle under the chords GH and HJ, so that, if GH and HJ are each 100 ft., then β = Degree of curve. In above formula β should be taken in degrees. If the greater area is on the outside of the curve the correction should be added, and subtracted when the greater area is on the inside of the curve.

When Openings are Left in Embankments for bridge sites the mass $ABCDEF$ (Fig. 75) must be computed separately from the rest of the earthwork. It is composed of the wedge $BCEF$ and of the quarter cones ABF and DCE, while abc represents a section of the toe slope along the lines BA, BF, CE or CD. The volume by end-area method of the wedge $BCEF = 1/2$ (Area CE+Area BF) $\times$ BC. The volumes of the cones are found by the Pappus Theorem, or Volume of cone ABF = area $abc \times 1/3\, dc \times 1/2\, \pi$.

The prismoidal correction should be added to the volume obtained by the average-end-area method. If the prismoidal correction comes out minus, the numerical value is to be subtracted. This formula for prismoidal correction applies to complete sections of earthwork whose bases are of equal width and bounded at their ends by 3 level sections, triangles or lines. The prismoidal correction for a pyramid or cone is one-third the volume by end-area method and is always subtracted. In applying the prismoidal correction to a solid bounded by irregular sections, however, it is customary to find the dimensions of level sections having the same areas respectively as the two irregular sections and determine the prismoidal correction by using the dimensions of these two level sections. This is an approximation which is consistent with the accuracy of the original measurements taken in the field to define the irregular sections.

66. Earthwork Tables

Level-Section Tables are given below for bases from 12 to 42 ft., and for slopes of 1 to 1 and 1-1/2 to 1. They are especially useful in making preliminary estimates, but may also be used for other sections by finding the heights of level sections which give equivalent areas.

The Prismoidal Correction Tables presented here give quantities to be subtracted from volumes obtained by the method of average end areas. Here

$c_1 - c_2$ and $D_1 - D_2$ must be in feet, and then the quantity found will be in cubic yards. See pp. 532 and 533.

Triangular Prism Tables for prisms with triangular bases and 100 ft. in length are given on pp. 534–537. To use this table each end section is divided into triangles, and the base a and altitude b of each triangle determined in feet. Then $1/2\ ab$ is the area of the triangle and $100/54\ ab$ is the volume in cubic yards of the triangular prism 100 ft. long. The tables here given have been compiled from the triangular prism tables computed by C. Frank Allen. His tables, however, were computed for prisms 50 ft. in length, and for some methods of earthwork computation they are more serviceable than tables for triangular prisms 100 ft. long.

67. Haul; and Mass Diagram

Haul. It is the practice with many engineers to pay the contractor for hauling earthwork as well as for excavation. This item of haul is usually computed as so many cubic yards hauled 100 ft. It is obviously impracticable to trace each cubic yard of excavation to its place of deposit in embankment, so the average haul is computed. The **average length of haul** is the distance between the center of gravity of a mass of earth in cut and the center of gravity of the same mass after it has been deposited in fill. This distance (in feet) times the quantity hauled gives the haul in units of cubic yards hauled one foot. The best way to treat this computation is to consider it in two parts: (1) compute the haul of the cut portion to a vertical plane where the cut ends and the fill begins, (2) compute the haul of the fill portion beginning at the same plane. The practice is growing to allow no payment for any haul found necessary and thus avoid this troublesome item. If, therefore, there will be considerable haul the prices bid for excavation will run relatively high.

To Find the Center of Gravity of a large mass of earthwork composed of several individual volumes of different shape and size it is necessary either to find the position of the center of gravity of each solid and treat the computation of haul as separate loads comprising the volumes contained in a length of 100 ft. (or shorter), or else to assemble these 100-ft. solids in some manner so as to obtain the position of the center of gravity of a series of solids. The latter method may be applied when the individual solids are of equal lengths. Where the lengths are not alike, such as for example where sections have been taken at plus stations, the haul of the material in these volumes of odd length must be computed as a separate item as follows:

If the solid had end sections of equal area its center of gravity would be midway between the ends, but where the end sections are unequal the center of gravity of the solid will lie nearer the larger than the smaller end. An expression for the distance from the mid-section of a volume of earthwork to its center of gravity is

$$x_l = l^2\,(A_1 - A_2)/12\,V_f \text{ for solid of length } l$$

$$\text{or}\quad x_{100} = 100\,(S_1 - S_2)/6\,V_y \text{ for solid 100 ft. long,}$$

where x and x_{100} = distance in feet from mid-section to the center of gravity of solids of length l and 100 ft. respectively; A_1 and A_2 = area of end sections in square feet; l = length of solid in feet; S_1 and S_2 = volume in cubic yards of prisms each 50 ft. long whose end sections are A_1 and A_2 respectively; V_f = volume in cubic feet in a solid of length l and with A_1 and A_2 for end sections; and V_y = same volume in cubic yards of a 100-ft. solid with A_1 and A_2 for end sections. If the solid is less than 100 ft. in length then $x_l = x_{100} \times l/100$. The distance of the center of gravity from the end of the volume being known, its distance from any station is easily found, and the haul to that station will equal this distance times the cubic yards in the solid.

The Mass Diagram gives the best means of measuring haul and of making studies of the comparative economy of different schemes of haul and of excavation and filling. Fig. 76 represents a portion of a railroad profile and its mass diagram which is plotted as follows: At Sta. 0 the ordinate is zero, at Sta. 1 the ordinate represents the quantity of earth between Sta. 0 and Sta. 1, at Sta. 2 it represents the quantity from Sta. 0 to Sta. 2. So that at any station the ordinate represents, the algebraic sum of all the quantities from Sta. 0 up to that station, excavation being considered positive and embankment negative. This diagram (Fig. 76) shows that from Sta. 0 to 28 the cut and fill balance and that from Sta. 0 to 40 there is a little excess of cut.

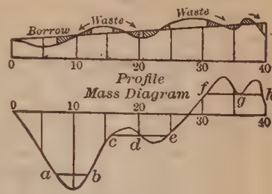


Fig. 76

The Properties of a Mass Diagram are: (1) High and low points in the mass diagram curve occur at points of no cut and fill on the profile. (2) In the mass diagram descending lines denote embankment and ascending lines excavation. (3) The difference in length (algebraically) between any two ordinates is a measure of the total quantity of material between the stations at which the ordinates are drawn. (4) Excavation equals embankment between any two points where a horizontal line intersects the mass diagram. (5) The area between a horizontal line and the curve of mass diagram is a measure of the haul between the two stations where the horizontal line cuts the diagram.

Referring to Fig. 76, the horizontal lines *ab*, *ce*, and *fh* are drawn in a manner explained below. Vertical lines projected from *a*, *b*, *c*, *d*, etc., to the profile will show between which stations the material must be hauled, borrowed or wasted. From Sta. 7 to Sta. 12, for example, the material is to be hauled. The earth in excavation just beyond Sta. 10 is thrown over into fill, each load being carried a short distance at first, the distance the material is hauled increasing until material at Sta. 12 is hauled to Sta. 7, which is called the limit of economical haul. If the cost of earthwork in cut and in fill and the cost of haul are known the length of haul beyond which it is more economical to waste excavated material and to borrow embankment may be readily calculated. This assumes, of course, that the material in cut can be wasted and that filling may be taken from a nearby borrow-pit, both without any appreciable haul or cost for material. Assuming that the economical limit of haul is 1000 ft., then the line *ab* should be drawn 1000 ft. long. Line *fgh* is drawn tangent to the curve at *g*, for had it been drawn lower than *g*, *fg* would be greater than 1000 ft., and had it been drawn a little higher than its present position then point *f* would fall further to the right and point *h* further to the left, with the result that more material would be wasted between Sta. 24 and 31 and more borrowed beyond Sta. 39 than is here shown, and this extra material would be wasted and borrowed while it was more economical to haul it. The line *fgh* as drawn, then, represents the best economy. Similarly the line *cde* is in the most economical position. It is drawn so as to make the sum of the areas above *cd* and below *de* a minimum, and these are a minimum when $cd = de$. This is the most economical condition as regards the cost of earthwork because, had *cd* been drawn any lower, it would have decreased the waste at Sta. 16 and increased the waste at Sta. 25 by exactly the same amount, but it would have made the sum of the two areas cut off by the horizontal line larger than at present, and this would have meant an increase in the amount of haul which is measured by these areas. Evidently the amount borrowed from Sta. 0 to 7 is the ordinate at *a*, the waste from 12 to 16 is the difference between the ordinates at *b* and *c*, and the waste from Sta. 25 to 31 is the sum (algebraic difference) of the ordinates at *e* and *f*. The beginning of a borrow is shown at Sta. 40.

Free Haul. It is sometimes provided that no payment shall be made for earth hauled less than a specified distance. The haul of any material carried

Level Sections, Slopes 1 to 1. Cubic Yards for 100 Ft. in Length

Depth in feet	Base in feet 12	14	16	18	20	22	24	26
1	48	56	63	70	78	85	93	100
2	104	119	133	148	163	178	193	207
3	167	189	211	233	256	278	300	322
4	237	267	296	326	356	385	415	444
5	315	352	389	426	463	500	537	574
6	400	444	489	533	578	622	667	711
7	493	544	596	648	700	752	804	856
8	593	652	711	770	830	889	948	1007
9	700	767	833	900	967	1033	1100	1167
10	815	889	963	1037	1111	1185	1259	1333
11	937	1019	1100	1181	1263	1344	1426	1507
12	1067	1156	1244	1333	1422	1511	1600	1689
13	1204	1300	1396	1493	1589	1685	1781	1878
14	1348	1452	1556	1659	1763	1867	1970	2074
15	1500	1611	1722	1833	1944	2056	2167	2278
16	1659	1778	1896	2015	2133	2252	2370	2489
17	1826	1952	2078	2204	2330	2456	2581	2707
18	2000	2133	2267	2400	2533	2667	2800	2933
19	2181	2322	2463	2604	2744	2885	3026	3167
20	2370	2519	2667	2815	2963	3111	3259	3407
21	2567	2722	2878	3033	3189	3344	3500	3656
22	2770	2933	3096	3259	3422	3585	3748	3911
23	2981	3152	3322	3493	3663	3833	4004	4174
24	3200	3378	3556	3733	3911	4089	4267	4444
25	3426	3611	3796	3981	4167	4352	4537	4722
26	3659	3852	4044	4237	4430	4622	4815	5007
27	3900	4100	4300	4500	4700	4900	5100	5300
28	4148	4356	4563	4770	4978	5185	5393	5600
29	4404	4619	4833	5048	5263	5478	5693	5907
30	4667	4889	5111	5333	5556	5778	6000	6222
31	4937	5167	5396	5626	5856	6085	6315	6544
32	5215	5452	5689	5926	6163	6400	6637	6874
33	5500	5744	5989	6233	6478	6722	6967	7211
34	5793	6044	6296	6548	6800	7052	7304	7556
35	6093	6352	6611	6870	7130	7389	7648	7907
36	6400	6667	6933	7200	7467	7733	8000	8267
37	6715	6989	7263	7537	7811	8085	8359	8633
38	7037	7319	7600	7881	8163	8444	8726	9007
39	7367	7656	7944	8233	8522	8811	9100	9389
40	7704	8000	8296	8593	8889	9185	9481	9778
41	8048	8352	8656	8959	9263	9567	9870	10174
42	8400	8711	9022	9333	9644	9956	10267	10578
43	8759	9078	9396	9715	10033	10352	10670	10989
44	9126	9452	9778	10104	10430	10756	11081	11407
45	9500	9833	10167	10500	10833	11167	11500	11833
46	9881	10222	10563	10904	11244	11585	11926	12267
47	10270	10619	10967	11315	11663	12011	12359	12707
48	10667	11022	11378	11733	12089	12444	12800	13156
49	11070	11433	11796	12159	12522	12885	13248	13611
50	11481	11852	12222	12593	12963	13333	13704	14074
51	11900	12278	12656	13033	13411	13789	14167	14544
52	12326	12711	13096	13481	13867	14252	14637	15022
53	12759	13152	13544	13937	14330	14722	15115	15507

Level Sections, Slopes 1 to 1. Cubic Yards for 100 Ft. in Length

Depth in feet	Base in feet 28	30	32	34	36	38	40	42
1	107	115	122	130	137	144	152	159
2	222	237	252	267	281	296	311	326
3	344	367	389	411	433	456	478	500
4	474	504	533	563	593	622	652	681
5	611	648	685	722	759	796	833	870
6	756	800	844	889	933	978	1022	1067
7	907	959	1011	1063	1115	1167	1219	1270
8	1067	1126	1185	1244	1304	1363	1422	1481
9	1233	1300	1367	1433	1500	1567	1633	1700
10	1407	1481	1556	1630	1704	1778	1852	1926
11	1589	1670	1752	1833	1915	1996	2078	2159
12	1778	1867	1956	2044	2133	2222	2311	2400
13	1974	2070	2167	2263	2359	2456	2552	2648
14	2178	2281	2385	2489	2593	2696	2800	2904
15	2389	2500	2611	2722	2833	2944	3056	3167
16	2607	2726	2844	2963	3081	3200	3319	3437
17	2833	2959	3085	3211	3337	3463	3589	3715
18	3067	3200	3333	3467	3600	3733	3867	4000
19	3307	3448	3589	3730	3870	4011	4152	4293
20	3556	3704	3852	4000	4148	4296	4444	4593
21	3811	3967	4122	4278	4433	4589	4744	4900
22	4074	4237	4400	4563	4726	4889	5052	5215
23	4344	4515	4685	4856	5026	5196	5367	5537
24	4622	4800	4978	5156	5333	5511	5689	5867
25	4907	5093	5278	5463	5648	5833	6019	6204
26	5200	5393	5585	5778	5970	6163	6356	6548
27	5500	5700	5900	6100	6300	6500	6700	6900
28	5807	6015	6222	6430	6637	6844	7052	7259
29	6122	6337	6552	6767	6981	7196	7411	7626
30	6444	6667	6889	7111	7333	7556	7778	8000
31	6774	7004	7233	7463	7693	7922	8152	8381
32	7111	7348	7585	7822	8059	8296	8533	8770
33	7456	7700	7944	8189	8433	8678	8922	9167
34	7807	8059	8311	8563	8815	9067	9319	9570
35	8167	8426	8685	8944	9204	9463	9722	9981
36	8533	8800	9067	9333	9600	9867	10133	10400
37	8907	9181	9456	9730	10004	10278	10552	10826
38	9289	9570	9852	10133	10415	10696	10978	11259
39	9678	9967	10256	10544	10833	11122	11411	11700
40	10074	10370	10667	10963	11259	11556	11852	12148
41	10478	10781	11085	11389	11693	11996	12300	12604
42	10889	11200	11511	11822	12133	12444	12756	13067
43	11307	11626	11944	12263	12581	12900	13219	13537
44	11733	12059	12385	12711	13037	13363	13689	14015
45	12167	12500	12833	13167	13500	13833	14167	14500
46	12607	12948	13289	13630	13970	14311	14652	14993
47	13056	13404	13752	14100	14448	14796	15144	15493
48	13511	13867	14222	14578	14933	15289	15644	16000
49	13974	14337	14700	15063	15426	15789	16152	16515
50	14444	14815	15185	15556	15926	16296	16667	17037
51	14922	15300	15678	16056	16433	16811	17189	17567
52	15407	15793	16178	16563	16948	17333	17719	18104
53	15900	16293	16685	17078	17470	17863	18256	18648

Level Sections, Slopes 1-1/2 to 1. Cubic Yards for 100 Ft. in Length

Depth in feet	Base in feet							
	12	14	16	18	20	22	24	26
1	50	57	65	72	80	87	94	102
2	111	126	141	156	170	185	200	215
3	183	206	228	250	272	294	317	339
4	267	296	326	356	385	415	444	474
5	361	398	435	472	509	546	583	620
6	467	511	556	600	644	689	733	778
7	583	635	687	739	791	843	894	946
8	711	770	830	889	948	1007	1067	1126
9	850	917	983	1050	1116	1183	1250	1317
10	1000	1074	1148	1222	1296	1370	1444	1519
11	1161	1243	1324	1406	1487	1569	1650	1731
12	1333	1422	1511	1600	1689	1778	1867	1956
13	1517	1613	1709	1806	1902	1998	2094	2191
14	1711	1815	1919	2022	2126	2230	2333	2437
15	1917	2028	2139	2250	2361	2472	2583	2694
16	2133	2252	2370	2489	2607	2726	2844	2963
17	2361	2487	2613	2739	2865	2991	3117	3243
18	2600	2733	2867	3000	3133	3267	3400	3533
19	2850	2991	3131	3272	3413	3554	3694	3835
20	3111	3259	3407	3556	3704	3852	4000	4148
21	3383	3539	3694	3850	4005	4161	4317	4472
22	3667	3830	3993	4156	4318	4481	4644	4807
23	3961	4131	4302	4472	4642	4813	4983	5154
24	4267	4444	4622	4800	4978	5156	5333	5511
25	4583	4769	4954	5139	5324	5509	5694	5880
26	4911	5104	5296	5489	5681	5874	6067	6259
27	5250	5450	5650	5850	6050	6250	6450	6650
28	5600	5807	6015	6222	6430	6637	6844	7052
29	5961	6176	6391	6606	6820	7035	7250	7465
30	6333	6556	6778	7000	7222	7444	7667	7889
31	6717	6946	7176	7406	7635	7865	8094	8324
32	7111	7348	7585	7822	8059	8296	8533	8770
33	7517	7761	8006	8250	8494	8739	8983	9228
34	7933	8185	8437	8689	8941	9193	9444	9696
35	8361	8620	8880	9139	9398	9657	9917	10176
36	8800	9067	9333	9600	9867	10133	10400	10667
37	9250	9524	9798	10072	10346	10620	10894	11169
38	9711	9993	10274	10556	10837	11119	11400	11681
39	10183	10472	10761	11050	11339	11628	11917	12206
40	10667	10963	11259	11556	11852	12148	12444	12741
41	11161	11465	11769	12072	12376	12680	12983	13287
42	11667	11978	12289	12600	12911	13222	13533	13844
43	12183	12502	12820	13139	13457	13776	14094	14413
44	12711	13037	13363	13689	14015	14341	14667	14993
45	13250	13583	13917	14250	14583	14917	15250	15583
46	13800	14141	14481	14822	15163	15504	15844	16185
47	14361	14709	15057	15406	15754	16102	16450	16798
48	14933	15289	15644	16000	16356	16711	17067	17422
49	15517	15880	16243	16606	16968	17331	17694	18057
50	16111	16481	16852	17222	17592	17963	18333	18704
51	16717	17094	17472	17850	18228	18606	18983	19361
52	17333	17719	18104	18489	18874	19259	19644	20030
53	17961	18354	18746	19139	19531	19924	20317	20709

Level Sections, Slopes 1-1/2 to 1. Cubic Yards for 100 Ft. in Length

Depth in feet	Base in feet							
	28	30	32	34	36	38	40	42
1	109	117	124	131	139	146	154	161
2	230	244	259	274	289	304	319	333
3	361	383	406	428	450	472	494	517
4	504	533	563	593	622	652	681	711
5	657	694	731	769	806	843	880	917
6	822	867	911	956	1000	1044	1089	1133
7	998	1050	1102	1154	1206	1257	1309	1361
8	1185	1244	1304	1363	1422	1481	1541	1600
9	1383	1450	1517	1583	1650	1717	1783	1850
10	1593	1667	1741	1815	1889	1963	2037	2111
11	1813	1894	1976	2057	2139	2220	2302	2383
12	2044	2133	2222	2311	2400	2489	2578	2667
13	2287	2383	2480	2576	2672	2769	2865	2961
14	2541	2644	2748	2852	2956	3059	3163	3267
15	2806	2917	3028	3139	3250	3361	3472	3583
16	3081	3200	3319	3437	3556	3674	3793	3911
17	3369	3494	3620	3746	3872	3998	4124	4250
18	3667	3800	3933	4067	4200	4333	4467	4600
19	3976	4117	4257	4398	4539	4680	4820	4961
20	4296	4444	4592	4741	4889	5037	5185	5333
21	4628	4783	4939	5094	5250	5406	5561	5717
22	4970	5133	5296	5459	5622	5785	5948	6111
23	5324	5494	5665	5835	6006	6176	6346	6517
24	5689	5867	6044	6222	6400	6578	6756	6933
25	6065	6250	6435	6620	6806	6991	7176	7361
26	6452	6644	6837	7030	7222	7415	7607	7800
27	6850	7050	7250	7450	7650	7850	8050	8250
28	7259	7467	7674	7881	8089	8296	8504	8711
29	7680	7894	8109	8324	8539	8754	8969	9183
30	8111	8333	8555	8778	9000	9222	9444	9667
31	8554	8783	9013	9243	9472	9702	9931	10161
32	9007	9244	9482	9719	9956	10193	10430	10667
33	9472	9717	9962	10206	10450	10694	10939	11183
34	9948	10200	10452	10704	10956	11207	11459	11711
35	10435	10694	10954	11213	11472	11731	11991	12250
36	10933	11200	11467	11733	12000	12267	12533	12800
37	11443	11717	11991	12265	12539	12813	13087	13361
38	11963	12244	12526	12807	13089	13370	13652	13933
39	12494	12783	13072	13361	13650	13939	14228	14517
40	13037	13333	13630	13926	14222	14519	14815	15111
41	13591	13894	14198	14502	14806	15109	15413	15717
42	14156	14467	14778	15089	15400	15711	16022	16333
43	14731	15050	15369	15687	16006	16324	16643	16961
44	15319	15644	15970	16296	16622	16948	17274	17600
45	15917	16250	16583	16917	17250	17583	17917	18250
46	16526	16867	17207	17548	17889	18230	18570	18911
47	17146	17494	17843	18191	18539	18887	19235	19583
48	17778	18133	18489	18844	19200	19556	19911	20267
49	18420	18783	19146	19509	19872	20235	20598	20961
50	19074	19444	19815	20185	20556	20926	21296	21667
51	19739	20117	20494	20872	21250	21628	22006	22383
52	20415	20800	21185	21570	21956	22341	22726	23111
53	21102	21494	21887	22280	22672	23065	23457	23850

Prismoidal Corrections in Cubic

$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9
$D_1 - D_2$									
0.1	0.03	0.06	0.09	0.12	0.15	0.19	0.22	0.25	0.28
0.2	0.06	0.12	0.19	0.25	0.31	0.37	0.43	0.49	0.56
0.3	0.09	0.19	0.28	0.37	0.46	0.56	0.65	0.74	0.83
0.4	0.12	0.25	0.37	0.49	0.62	0.74	0.86	0.99	1.11
0.5	0.15	0.31	0.46	0.62	0.77	0.93	1.08	1.23	1.39
0.6	0.19	0.37	0.56	0.74	0.93	1.11	1.30	1.48	1.67
0.7	0.22	0.43	0.65	0.86	1.08	1.30	1.51	1.73	1.94
0.8	0.25	0.49	0.74	0.99	1.23	1.48	1.73	1.98	2.22
0.9	0.28	0.56	0.83	1.11	1.39	1.67	1.94	2.22	2.50
1.0	0.31	0.62	0.93	1.23	1.54	1.85	2.16	2.47	2.78
1.1	0.34	0.68	1.02	1.36	1.70	2.04	2.38	2.72	3.06
1.2	0.37	0.74	1.11	1.48	1.85	2.22	2.59	2.96	3.33
1.3	0.40	0.80	1.20	1.60	2.01	2.41	2.81	3.21	3.61
1.4	0.43	0.86	1.30	1.73	2.16	2.59	3.02	3.46	3.89
1.5	0.46	0.93	1.39	1.85	2.31	2.78	3.24	3.70	4.17
1.6	0.49	0.99	1.48	1.98	2.47	2.96	3.46	3.95	4.44
1.7	0.52	1.05	1.57	2.10	2.62	3.15	3.67	4.20	4.72
1.8	0.56	1.11	1.67	2.22	2.78	3.33	3.89	4.44	5.00
1.9	0.59	1.17	1.76	2.35	2.93	3.52	4.10	4.69	5.28
2.0	0.62	1.23	1.85	2.47	3.09	3.70	4.32	4.94	5.56
2.1	0.65	1.30	1.94	2.59	3.24	3.89	4.54	5.19	5.83
2.2	0.68	1.36	2.04	2.72	3.40	4.07	4.75	5.43	6.11
2.3	0.71	1.42	2.13	2.84	3.55	4.26	4.97	5.68	6.39
2.4	0.74	1.48	2.22	2.96	3.70	4.44	5.19	5.93	6.67
2.5	0.77	1.54	2.31	3.09	3.86	4.63	5.40	6.17	6.94
2.6	0.80	1.60	2.41	3.21	4.01	4.81	5.62	6.42	7.22
2.7	0.83	1.67	2.50	3.33	4.17	5.00	5.83	6.67	7.50
2.8	0.86	1.73	2.59	3.46	4.32	5.19	6.05	6.91	7.78
2.9	0.90	1.79	2.69	3.58	4.48	5.37	6.27	7.16	8.06
3.0	0.93	1.85	2.78	3.70	4.63	5.56	6.48	7.41	8.33
3.1	0.96	1.91	2.87	3.83	4.78	5.74	6.70	7.65	8.61
3.2	0.99	1.98	2.96	3.95	4.94	5.93	6.91	7.90	8.89
3.3	1.02	2.04	3.06	4.07	5.09	6.11	7.13	8.15	9.17
3.4	1.05	2.10	3.15	4.20	5.25	6.30	7.35	8.40	9.44
3.5	1.08	2.16	3.24	4.32	5.40	6.48	7.56	8.64	9.72
3.6	1.11	2.22	3.33	4.44	5.56	6.67	7.78	8.89	10.00
3.7	1.14	2.28	3.43	4.57	5.71	6.85	7.99	9.14	10.28
3.8	1.17	2.35	3.52	4.69	5.86	7.04	8.21	9.38	10.56
3.9	1.20	2.41	3.61	4.81	6.02	7.22	8.43	9.63	10.83
4.0	1.23	2.47	3.70	4.94	6.17	7.41	8.64	9.88	11.11
4.1	1.27	2.53	3.80	5.06	6.33	7.59	8.86	10.12	11.39
4.2	1.30	2.59	3.89	5.19	6.48	7.78	9.07	10.37	11.67
4.3	1.33	2.65	3.98	5.31	6.64	7.96	9.29	10.62	11.94
4.4	1.36	2.72	4.07	5.43	6.79	8.15	9.51	10.86	12.22
4.5	1.39	2.78	4.17	5.56	6.94	8.33	9.72	11.11	12.50
4.6	1.42	2.84	4.26	5.68	7.10	8.52	9.94	11.36	12.78
4.7	1.45	2.90	4.35	5.80	7.25	8.70	10.15	11.60	13.06
4.8	1.48	2.96	4.44	5.93	7.41	8.89	10.37	11.85	13.33
4.9	1.51	3.02	4.54	6.05	7.56	9.07	10.50	12.10	13.61
5.0	1.54	3.09	4.63	6.17	7.72	9.26	10.80	12.35	13.89
$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9

Yards for a Solidity 100 Ft. Long

$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9
$D_1 - D_2$									
5.1	1.57	3.15	4.72	6.30	7.87	9.44	11.02	12.59	14.17
5.2	1.60	3.21	4.81	6.42	8.02	9.63	11.23	12.84	14.44
5.3	1.64	3.27	4.91	6.54	8.18	9.81	11.45	13.09	14.72
5.4	1.67	3.33	5.00	6.67	8.33	10.00	11.67	13.33	15.00
5.5	1.70	3.40	5.09	6.79	8.49	10.19	11.88	13.58	15.28
5.6	1.73	3.46	5.19	6.91	8.64	10.37	12.10	13.83	15.56
5.7	1.76	3.52	5.28	7.04	8.80	10.56	12.31	14.07	15.83
5.8	1.79	3.58	5.37	7.16	8.95	10.74	12.53	14.32	16.11
5.9	1.82	3.64	5.46	7.28	9.10	10.93	12.75	14.57	16.39
6.0	1.85	3.70	5.56	7.41	9.26	11.11	12.96	14.81	16.67
6.1	1.88	3.77	5.65	7.53	9.41	11.30	13.18	15.06	16.94
6.2	1.91	3.83	5.74	7.65	9.57	11.48	13.40	15.31	17.22
6.3	1.94	3.89	5.83	7.78	9.72	11.67	13.61	15.56	17.50
6.4	1.98	3.95	5.93	7.90	9.88	11.85	13.83	15.80	17.78
6.5	2.01	4.01	6.02	8.02	10.03	12.04	14.04	16.05	18.06
6.6	2.04	4.07	6.11	8.15	10.19	12.22	14.26	16.30	18.33
6.7	2.07	4.14	6.20	8.27	10.34	12.41	14.48	16.54	18.61
6.8	2.10	4.20	6.30	8.40	10.49	12.59	14.69	16.79	18.89
6.9	2.13	4.26	6.39	8.52	10.65	12.78	14.91	17.04	19.17
7.0	2.16	4.32	6.48	8.64	10.80	12.96	15.12	17.28	19.44
7.1	2.19	4.38	6.57	8.77	10.96	13.15	15.34	17.53	19.72
7.2	2.22	4.44	6.67	8.89	11.11	13.33	15.56	17.78	20.00
7.3	2.25	4.51	6.76	9.01	11.27	13.52	15.77	18.02	20.28
7.4	2.28	4.57	6.85	9.14	11.42	13.70	15.99	18.27	20.56
7.5	2.31	4.63	6.94	9.26	11.57	13.89	16.20	18.52	20.83
7.6	2.35	4.69	7.04	9.38	11.73	14.07	16.42	18.77	21.11
7.7	2.38	4.75	7.13	9.51	11.88	14.26	16.64	19.01	21.39
7.8	2.41	4.81	7.22	9.63	12.04	14.44	16.85	19.26	21.67
7.9	2.44	4.88	7.31	9.75	12.19	14.63	17.07	19.51	21.94
8.0	2.47	4.94	7.41	9.88	12.35	14.81	17.28	19.75	22.22
8.1	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50
8.2	2.53	5.06	7.59	10.12	12.65	15.19	17.72	20.25	22.78
8.3	2.56	5.12	7.69	10.25	12.81	15.37	17.93	20.49	23.06
8.4	2.59	5.19	7.78	10.37	12.96	15.56	18.15	20.74	23.33
8.5	2.62	5.25	7.87	10.49	13.12	15.74	18.36	20.99	23.61
8.6	2.65	5.31	7.96	10.62	13.27	15.93	18.58	21.23	23.89
8.7	2.69	5.37	8.06	10.74	13.43	16.11	18.80	21.48	24.17
8.8	2.72	5.43	8.15	10.86	13.58	16.30	19.01	21.73	24.44
8.9	2.75	5.49	8.24	10.99	13.73	16.48	19.23	21.97	24.72
9.0	2.78	5.56	8.33	11.11	13.89	16.67	19.44	22.22	25.00
9.1	2.81	5.62	8.43	11.23	14.04	16.85	19.66	22.47	25.28
9.2	2.84	5.68	8.52	11.36	14.20	17.04	19.88	22.72	25.56
9.3	2.87	5.74	8.61	11.48	14.35	17.22	20.09	22.96	25.83
9.4	2.90	5.80	8.70	11.60	14.51	17.41	20.31	23.21	26.11
9.5	2.93	5.86	8.80	11.73	14.66	17.59	20.52	23.46	26.39
9.6	2.96	5.93	8.89	11.85	14.81	17.78	20.74	23.70	26.67
9.7	2.99	5.99	8.98	11.98	14.97	17.96	20.96	23.95	26.94
9.8	3.02	6.05	9.07	12.10	15.12	18.15	21.17	24.20	27.22
9.9	3.06	6.11	9.17	12.22	15.28	18.33	21.39	24.44	27.50
10.0	3.09	6.17	9.26	12.35	15.43	18.52	21.60	24.69	27.78
$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9

Triangular Prisms. Cubic Yards for 100 Ft. in Length

Height in feet	Width in feet 1	2	3	4	5	6	7	8	9
0.2	0.4	0.7	1.1	1.5	1.9	2.2	2.6	3.0	3.3
0.4	0.7	1.5	2.2	3.0	3.7	4.4	5.2	5.9	6.7
0.6	1.1	2.2	3.3	4.4	5.6	6.7	7.8	8.9	10.0
0.8	1.5	3.0	4.4	5.9	7.4	8.9	10.4	11.9	13.3
1.0	1.9	3.7	5.6	7.4	9.3	11.1	13.0	14.8	16.7
1.2	2.2	4.4	6.7	8.9	11.1	13.3	15.6	17.8	20.0
1.4	2.6	5.2	7.8	10.4	13.0	15.6	18.2	20.7	23.3
1.6	3.0	5.9	8.9	11.9	14.8	17.8	20.7	23.7	26.7
1.8	3.3	6.7	10.0	13.3	16.7	20.0	23.3	26.7	30.0
2.0	3.7	7.4	11.1	14.8	18.5	22.2	25.9	29.6	33.3
2.2	4.1	8.1	12.2	16.3	20.4	24.4	28.5	32.6	36.7
2.4	4.4	8.9	13.3	17.8	22.2	26.7	31.1	35.6	40.0
2.6	4.8	9.6	14.4	19.3	24.1	28.9	33.7	38.5	43.3
2.8	5.2	10.4	15.6	20.7	25 9	31.1	36.3	41.5	46.7
3.0	5.6	11.1	16.7	22.2	27.8	33.3	38.9	44.4	50.0
3.2	5.9	11.9	17.8	23.7	29.6	35.6	41.5	47.4	53.3
3.4	6.3	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.7
3.6	6.7	13.3	20.0	26.7	33.3	40.0	46.7	53.3	60.0
3.8	7.0	14.1	21.1	28.1	35.2	42.2	49.3	56.3	63.3
4.0	7.4	14.8	22.2	29.6	37.0	44.4	51.9	59.3	66.7
4.2	7.8	15.6	23.3	31.1	38.9	46.7	54.4	62.2	70.0
4.4	8.1	16.3	24.4	32.6	40.7	48.9	57.0	65.2	73.3
4.6	8.5	17.0	25.6	34.1	42.6	51.1	59.6	68.1	76.7
4.8	8.9	17.8	26.7	35.6	44.4	53.3	62.2	71.1	80.0
5.0	9.3	18.5	27.8	37.0	46.3	55.6	64.8	74.1	83.3
5.2	9.6	19.3	28.9	38.5	48.1	57.8	67.4	77.0	86.7
5.4	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
5.6	10.4	20.7	31.1	41.5	51.9	62.2	72.6	83.0	93.3
5.8	10.7	21.5	32.2	43.0	53.7	64.4	75.2	85.9	96.7
6.0	11.1	22.2	33.3	44.4	55.6	66.7	77.8	88.9	100.0
6.2	11.5	23.0	34.4	45.9	57.4	68.9	80.4	91.9	103.3
6.4	11.9	23.7	35.6	47.4	59.3	71.1	83.0	94.8	106.7
6.6	12.2	24.4	36.7	48.9	61.1	73.3	85.6	97.8	110.0
6.8	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.7	113.3
7.0	13.0	25.9	38.9	51.9	64.8	77.8	90.7	103.7	116.7
7.2	13.3	26.7	40.0	53.3	66.7	80.0	93.3	106.7	120.0
7.4	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
7.6	14.1	28.1	42.2	56.3	70.4	84.4	98.5	112.6	126.7
7.8	14.4	28.9	43.3	57.8	72.2	86.7	101.1	115.6	130.0
8.0	14.8	29.6	44.4	59.3	74.1	88.9	103.7	118.5	133.3
8.2	15.2	30.4	45.6	60.7	75.9	91.1	106.3	121.5	136.7
8.4	15.6	31.1	46.7	62.2	77.8	93.3	108.9	124.4	140.0
8.6	15.9	31.9	47.8	63.7	79.6	95.6	111.5	127.4	143.3
8.8	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.7
9.0	16.7	33.3	50.0	66.7	83.3	100.0	116.7	133.3	150.0
9.2	17.0	34.1	51.1	68.1	85.2	102.2	119.3	136.3	153.3
9.4	17.4	34.8	52.2	69.6	87.0	104.4	121.9	139.3	156.7
9.6	17.8	35.6	53.3	71.1	88.9	106.7	124.4	142.2	160.0
9.8	18.1	36.3	54.4	72.6	90.7	108.9	127.0	145.2	163.3

For explanation see p. 526. Continued on next page

Triangular Prisms. Cubic Yards for 100 Ft. in Length

Height in feet	Width in feet 1	2	3	4	5	6	7	8	9
10.0	18.5	37.0	55.6	74.1	92.6	111.1	129.6	148.2	166.7
10.2	18.9	37.8	56.7	75.6	94.4	113.3	132.2	151.1	170.0
10.4	19.3	38.5	57.8	77.0	96.3	115.6	134.8	154.1	173.3
10.6	19.6	39.3	58.9	78.5	98.2	117.8	137.4	157.0	176.7
10.8	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
11.0	20.4	40.7	61.1	81.5	101.9	122.2	142.6	163.0	183.3
11.2	20.7	41.5	62.2	83.0	103.7	124.4	145.2	165.9	186.7
11.4	21.1	42.2	63.3	84.4	105.6	126.7	147.8	168.9	190.0
11.6	21.5	43.0	64.4	85.9	107.4	128.9	150.4	171.9	193.3
11.8	21.9	43.7	65.6	87.4	109.3	131.1	153.0	174.8	196.7
12.0	22.2	44.4	66.7	88.9	111.1	133.3	155.6	177.8	200.0
12.2	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.7	203.3
12.4	23.0	45.9	68.9	91.9	114.8	137.8	160.7	183.7	206.7
12.6	23.3	46.7	70.0	93.3	116.7	140.0	163.3	186.7	210.0
12.8	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
13.0	24.1	48.1	72.2	96.3	120.4	144.4	168.5	192.6	216.7
13.2	24.4	48.9	73.3	97.8	122.2	146.7	171.1	195.6	220.0
13.4	24.8	49.6	74.4	99.3	124.1	148.9	173.7	198.5	223.3
13.6	25.2	50.4	75.6	100.7	125.9	151.1	176.3	201.5	226.7
13.8	25.6	51.1	76.7	102.2	127.8	153.3	178.9	204.4	230.0
14.0	25.9	51.9	77.8	103.7	129.6	155.6	181.5	207.4	233.3
14.2	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
14.4	26.7	53.3	80.0	106.7	133.3	160.0	186.7	213.3	240.0
14.6	27.0	54.1	81.1	108.2	135.2	162.2	189.3	216.3	243.3
14.8	27.4	54.8	82.2	109.6	137.0	164.4	191.9	219.3	246.7
15.0	27.8	55.6	83.3	111.1	138.9	166.7	194.4	222.2	250.0
15.2	28.1	56.3	84.4	112.6	140.7	168.9	197.0	225.2	253.3
15.4	28.5	57.0	85.6	114.1	142.6	171.1	199.6	228.2	256.7
15.6	28.9	57.8	86.7	115.6	144.4	173.3	202.2	231.1	260.0
15.8	29.3	58.5	87.8	117.0	146.3	175.6	204.8	234.1	263.3
16.0	29.6	59.3	88.9	118.5	148.2	177.8	207.4	237.0	266.7
16.2	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
16.4	30.4	60.7	91.1	121.5	151.9	182.2	212.6	243.0	273.3
16.6	30.7	61.5	92.2	123.0	153.7	184.4	215.2	245.9	276.7
16.8	31.1	62.2	93.3	124.4	155.6	186.7	217.8	248.9	280.0
17.0	31.5	63.0	94.4	125.9	157.4	188.9	220.4	251.9	283.3
17.2	31.9	63.7	95.6	127.4	159.3	191.1	223.0	254.8	286.7
17.4	32.2	64.4	96.7	128.9	161.1	193.3	225.6	257.8	290.0
17.6	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.7	293.3
17.8	33.0	65.9	98.9	131.9	164.8	197.8	230.7	263.7	296.7
18.0	33.3	66.7	100.0	133.3	166.7	200.0	233.3	266.7	300.0
18.2	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
18.4	34.1	68.1	102.2	136.3	170.4	204.4	238.5	272.6	306.7
18.6	34.4	68.9	103.3	137.8	172.2	206.7	241.1	275.6	310.0
18.8	34.8	69.6	104.4	139.3	174.1	208.9	243.7	278.5	313.3
19.0	35.2	70.4	105.6	140.7	175.9	211.1	246.3	281.5	316.7
19.2	35.6	71.1	106.7	142.2	177.8	213.3	248.9	284.4	320.0
19.4	35.9	71.9	107.8	143.7	179.6	215.6	251.5	287.4	323.3
19.6	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
19.8	36.7	73.3	110.0	146.7	183.3	220.0	256.7	293.3	330.0

Continued on next page

Triangular Prisms. Cubic Yards for 100 Ft. in Length

Height in feet	Width in feet 1	2	3	4	5	6	7	8	9
20.0	37.0	74.1	111.1	148.2	185.2	222.2	259.3	296.3	333.3
20.2	37.4	74.8	112.2	149.6	187.0	224.4	261.9	299.3	336.7
20.4	37.8	75.6	113.3	151.1	188.9	226.7	264.4	302.2	340.0
20.6	38.1	76.3	114.4	152.6	190.7	228.9	267.0	305.2	343.3
20.8	38.5	77.0	115.6	154.1	192.6	231.1	269.6	308.2	346.7
21.0	38.9	77.8	116.7	155.6	194.4	233.3	272.2	311.1	350.0
21.2	39.3	78.5	117.8	157.0	196.3	235.6	274.8	314.1	353.3
21.4	39.6	79.3	118.9	158.5	198.2	237.8	277.4	317.0	356.7
21.6	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
21.8	40.4	80.7	121.1	161.5	201.9	242.2	282.6	323.0	363.3
22.0	40.7	81.5	122.2	163.0	203.7	244.4	285.2	325.9	366.7
22.2	41.1	82.2	123.3	164.4	205.6	246.7	287.8	328.9	370.0
22.4	41.5	83.0	124.4	165.9	207.4	248.9	290.4	331.9	373.3
22.6	41.9	83.7	125.6	167.4	209.3	251.1	293.0	334.8	376.7
22.8	42.2	84.4	126.7	168.9	211.1	253.3	295.6	337.8	380.0
23.0	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.7	383.3
23.2	43.0	85.9	128.9	171.9	214.8	257.8	300.7	343.7	386.7
23.4	43.3	86.7	130.0	173.3	216.7	260.0	303.3	346.7	390.0
23.6	43.7	87.4	131.1	174.8	218.5	262.2	305.9	349.6	393.3
23.8	44.1	88.2	132.2	176.3	220.4	264.4	308.5	352.6	396.7
24.0	44.4	88.9	133.3	177.8	222.2	266.7	311.1	355.6	400.0
24.2	44.8	89.6	134.4	179.3	224.1	268.9	313.7	358.5	403.3
24.4	45.2	90.4	135.6	180.7	225.9	271.1	316.3	361.5	406.7
24.6	45.6	91.1	136.7	182.2	227.8	273.3	318.9	364.4	410.0
24.8	45.9	91.9	137.8	183.7	229.6	275.6	321.5	367.4	413.3
25.0	46.3	92.6	138.9	185.2	231.5	277.8	324.1	370.4	416.7
25.2	46.7	93.3	140.0	186.7	233.3	280.0	326.7	373.3	420.0
25.4	47.0	94.1	141.1	188.2	235.2	282.2	329.3	376.3	424.3
25.6	47.4	94.8	142.2	189.6	237.0	284.4	331.9	379.3	426.7
25.8	47.8	95.6	143.3	191.1	238.9	286.7	334.4	382.2	430.0
26.0	48.2	96.3	144.4	192.6	240.7	288.9	337.0	385.2	433.3
26.2	48.5	97.0	145.6	194.1	242.6	291.1	339.6	388.1	436.7
26.4	48.9	97.8	146.7	195.6	244.4	293.3	342.2	391.1	440.0
26.6	49.3	98.5	147.8	197.0	246.3	295.6	344.8	394.1	443.3
26.8	49.6	99.3	148.9	198.5	248.1	297.8	347.4	397.0	446.7
27.0	50.0	100.0	150.0	200.0	250.0	300.0	350.0	400.0	450.0
27.2	50.4	100.7	151.1	201.5	251.9	302.2	352.6	403.0	453.3
27.4	50.7	101.5	152.2	203.0	253.7	304.4	355.2	405.9	456.7
27.6	51.1	102.2	153.3	204.4	255.6	306.7	357.8	408.9	460.0
27.8	51.5	103.0	154.4	205.9	257.4	308.9	360.4	411.9	463.3
28.0	51.9	103.7	155.6	207.4	259.3	311.1	363.0	414.8	466.7
28.2	52.2	104.4	156.7	208.9	261.1	313.3	365.6	417.8	470.0
28.4	52.6	105.2	157.8	210.4	263.0	315.6	368.1	420.7	473.3
28.6	53.0	105.9	158.9	211.9	264.8	317.8	370.7	423.7	476.7
28.8	53.3	106.7	160.0	213.3	266.7	320.0	373.3	426.7	480.0
29.0	53.7	107.4	161.1	214.8	268.5	322.2	375.9	429.6	483.3
29.2	54.1	108.2	162.2	216.3	270.4	324.4	378.5	432.6	486.7
29.4	54.4	108.9	163.3	217.8	272.2	326.7	381.1	435.6	490.0
29.6	54.8	109.6	164.4	219.3	274.1	328.9	383.7	438.5	493.3
29.8	55.2	110.4	165.6	220.7	275.9	331.1	386.3	441.5	496.7

Continued on next page

Triangular Prisms. Cubic Yards for 100 Ft. in Length

Height in feet	Width in feet 1	2	3	4	5	6	7	8	9
30.0	55.6	111.1	166.7	222.2	277.[illegible]	333.3	388.9	444.4	500.0
30.2	55.9	111.9	167.8	223.7	279.6	335.6	391.5	447.4	503.3
30.4	56.3	112.6	168.9	225.2	281.5	337.8	394.1	450.4	506.7
30.6	56.7	113.3	170.0	226.7	283.3	340.0	396.7	453.3	510.0
30.8	57.0	114.1	171.1	228.2	285.2	342.2	399.3	456.3	513.3
31.0	57.4	114.8	172.2	229.6	287.0	344.4	401.9	459.3	516.7
31.2	57.8	115.6	173.3	231.1	288.9	346.7	404.4	462.2	520.0
31.4	58.2	116.3	174.4	232.6	290.7	348.9	407.0	465.2	523.3
31.6	58.5	117.0	175.6	234.1	292.6	351.1	409.6	468.1	526.7
31.8	58.9	117.8	176.7	235.6	294.4	353.3	412.2	471.1	530.0
32.0	59.3	118.5	177.8	237.0	296.3	355.6	414.8	474.1	533.3
32.2	59.6	119.3	178.9	238.5	298.1	357.8	417.4	477.0	536.7
32.4	60.0	120.0	180.0	240.0	300.0	360.0	420.0	480.0	540.0
32.6	60.4	120.7	181.1	241.5	301.9	362.2	422.6	483.0	543.3
32.8	60.7	121.5	182.2	243.0	303.7	364.4	425.2	386.0	546.7
33.0	61.1	122.2	183.3	244.4	305.6	366.7	427.8	488.9	550.0
33.2	61.5	123.0	184.4	245.9	307.4	368.9	430.4	491.9	553.3
33.4	61.9	123.7	185.6	247.4	309.3	371.1	433.0	494.8	556.7
33.6	62.2	124.4	186.7	248.9	311.1	373.3	435.6	497.8	560.0
33.8	62.6	125.2	187.8	250.4	313.0	375.6	438.1	500.7	563.3
34.0	63.0	125.9	188.9	251.9	314.8	377.8	440.7	503.7	566.7
34.2	63.3	126.7	190.0	253.3	316.7	380.0	443.3	506.7	570.0
34.4	63.7	127.4	191.1	254.8	318.5	382.2	445.9	509.6	573.3
34.6	64.1	128.2	192.2	256.3	320.4	384.4	448.5	512.6	576.7
34.8	64.4	128.9	193.3	257.8	322.2	386.7	451.1	515.6	580.0
35.0	64.8	129.6	194.4	259.3	324.1	388.9	453.7	518.5	583.3
35.2	65.2	130.4	195.6	260.7	325.9	391.1	456.3	521.5	586.7
35.4	65.6	131.1	196.7	262.2	327.8	393.3	458.9	524.4	590.0
35.6	65.9	131.9	197.8	263.7	329.6	395.6	461.5	527.4	593.3
35.8	66.3	132.6	198.9	265.2	331.5	397.8	464.1	530.4	596.7
36.0	66.7	133.3	200.0	266.7	333.3	400.0	466.7	533.3	600.0
36.2	67.0	134.1	201.1	268.2	335.2	402.2	469.3	536.3	603.3
36.4	67.4	134.8	202.2	269.6	337.0	404.4	471.9	539.3	606.6
36.6	67.8	135.6	203.3	271.1	338.9	406.7	474.4	542.2	610.0
36.8	68.2	136.3	204.4	272.6	340.7	408.9	477.0	545.2	613.3
37.0	68.5	137.0	205.6	274.1	342.6	411.1	479.6	548.1	616.7
37.2	68.9	137.8	206.7	275.6	344.4	413.3	482.2	551.1	620.0
37.4	69.3	138.5	207.8	277.0	346.3	415.6	484.8	554.1	623.3
37.6	69.6	139.3	208.9	278.5	348.1	417.8	487.4	557.0	626.7
37.8	70.0	140.0	210.0	280.0	350.0	420.0	490.0	560.0	630.0
38.0	70.4	140.7	211.1	281.5	351.9	422.2	492.6	563.0	633.3
38.2	70.7	141.5	212.2	283.0	353.7	424.4	495.2	565.9	636.7
38.4	71.1	142.2	213.3	284.4	355.6	426.7	497.8	568.9	640.0
38.6	71.5	143.0	214.4	285.9	357.4	428.9	500.4	571.9	643.3
38.8	71.9	143.7	215.6	287.4	359.3	431.1	503.0	574.8	646.7
39.0	72.2	144.4	216.7	288.9	361.1	433.3	505.6	577.8	650.0
39.2	72.6	145.2	217.8	290.4	363.0	435.6	508.1	580.7	653.3
39.4	73.0	145.9	218.9	291.9	364.8	437.8	510.7	583.7	656.7
39.6	73.3	146.7	220.0	293.3	366.7	440.0	513.3	586.7	660.0
39.8	73.7	147.4	221.1	294.8	368.5	442.2	515.9	589.6	663.3

For explanation see p. 526.

beyond this free-haul limit is called **overhaul.** The limits of free haul and the method of computing overhaul have been defined by the Amer. Ry. Engrs. and M. of W. Assoc. in its 1921 Manual.

"The limits of free haul shall be determined by fixing on the profile two points, one on each side of the neutral grade point, one in excavation and the other in embankment, such that the distance between them equals the specified free-haul limit and the included quantities of excavation and embankment balance. All haul on material beyond this free-haul limit will be estimated and paid for on the basis of the following method of computation:

"All material within this limit of free haul will be eliminated from further consideration.

"The distance between the center of gravity of the remaining mass of excavation and center of gravity of the resulting embankment, less the limit of free haul as above described, shall be the overhaul distance.

"Overhaul shall be computed in units of one cubic yard moved 100 ft. and compensation to be rendered therefor shall be computed on such units.

"In case material is obtained from borrow-pits along the embankment and runways constructed, the haul shall be determined by the distance the team necessarily travels. The overhaul shall be determined by multiplying the number of cubic yards so hauled by one-half the round distance made by the team, less than freehaul distance. The runways shall be established by the Engineer."

68. Borrow-pits

In railroad construction it is often more economical when the length of haul becomes great to borrow material from near-by borrow-pits to form embankments. To calculate the amount of earth taken from a borrow-pit it is customary to divide the surface of the ground where the pit is to be excavated into squares and to take levels at each corner of these squares, and at intermediate points when the slope of the surface is not straight from corner to corner. Then when the excavation is completed the same system of square cross-section is again staked out and elevations are taken at the corners and at the necessary intermediate points. Care should be taken when the first system is staked out that it extends far enough to include the entire area which may possibly be covered by the borrow-pit. The field-work of laying out these cross-sections is done as explained in Art. 24. The difference between the original and the final elevations at the corners gives the lengths of the vertical edges of a series of vertical truncated rectangular prisms. Toward the edges of the borrow-pits, where the slopes occur, there probably will be several triangular and trapezoidal truncated prisms. Let A = area of right section of truncated prism, h_1, h_2, h_3, etc. = the lengths of the vertical edges of the prism, and V = volume of prism; then

For Truncated Triangular Prism, $\mathbf{V = A \times (h_1 + h_2 + h_3)/3.}$

For Truncated Rectangular Prism, $\mathbf{V = A \times (h_1 + h_2 + h_3 + h_4)/4.}$

When Additional Heights have been taken at intermediate points the computation is made as follows: (1) When an intermediate level is taken in the center of the square or rectangle, the volume of the rectangular prism is computed by the formula given above, and to it is added algebraically the volume of a pyramid having the rectangle for its base and the center height minus the average of the four corner heights for its altitude. The volume of this entire solid would then be expressed

$$\mathbf{V = A\,(h_1 + h_2 + h_3 + h_4)/4 + A\left\{h_c - (h_1 + h_2 + h_3 + h_4)/4\right\}/3}$$

in which h_c = altitude of prism along its center vertical line.

(2) When an intermediate level is taken at the middle of one side of a rectangle, the volume of the rectangular prism is computed as usual, and to it is added the volume of a pyramid having the rectangle for its base and the difference between the additional height and the average of the heights at the two corners which lie at the extremity of the side on which the additional height has been taken. If the additional height is h_5 and it is taken on the side where the end heights are h_3 and h_4 respectively, then an expression for the entire volume would be

$$V = A\,(h_1 + h_2 + h_3 + h_4)/4 + A\left\{h_5 - 1/2\,(h_3 + h_4)\right\}/3$$

(3) When intermediate heights are taken at any other point in the rectangle except in the center of the rectangle or in the middle of one of its sides, then the rectangular prism should be divided into four triangular prisms; the additional height forms the vertical edge at one of the corners of each of these four triangular prisms.

In arranging the data for computation it is customary to lay out the borrow-pit in plan on cross-section paper and to mark at each corner the depth of excavation, and to sketch any lines representing the proper subdivision of the rectangles or trapezoids as given in the field notes. A plan of a portion of a borrow-pit is shown in Fig. 77, and by the above method the volume of excavation will be found to be 1057 cu. yd. Where there are several rectangular prisms having the same area of cross-section A the computation of the quantity of earthwork may be simplified as follows: Volume of Assembly of Truncated Rectangular Prisms $= A\ (p_1 + 2\ p_2 + 3\ p_3 + 4\ p_4)/4$ in which p_1 = sum of heights common to one prism; p_2 = sum of heights common to two prisms; p_3 = those common to three; p_4 = those common to four prisms.

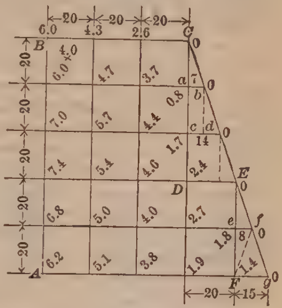


Fig. 77

Measurements of Dredged Material are taken in two ways, (1) measurement in place, and (2) scow measurement. For the **measurement-in-place** method soundings are taken before and again after the dredging work is done, and the volume of the material which has been removed is computed either by the Average End Area Method or by the Borrow-Pit Method. If the dredging covers a large area, contours may be drawn showing the shape of the bottom before and after the dredging has been done, and these may be used as bases of vertical solids. Where a channel has been dredged, sometimes it is more convenient to compute the volume as a series of horizontal solids with vertical end sections similar to a railroad excavation. In channel dredging the contractor is usually required to form the bottom at a given grade and the side slopes at a given slope, in which case it is not necessary to take cross-sections after the work is done, but soundings are taken as the dredging progresses to insure that the excavation is of proper depth.

When Scow Measurements are used to determine the quantity dredged each pocket of the scow is carefully measured and its capacity computed. When the scows have been loaded with the dredged material the surveyor or inspector makes a note of the number of full pockets; if they are not full he measures the distance down from the top of the coaming of the scow to where he estimates that the material in a pocket would come if it were leveled off. The volume of this small rectangular prism is calculated and deducted from the capacity of the corresponding pocket. For each scow in use tables giving the quantity in each pocket at various distances below the top of coaming are usually prepared for convenience. These scow measurements should be taken just before the tow starts for the dumping ground. When scows remain moored for a day or so before being towed to the dumping ground some of the material in the pockets leak out through the bottom doors if they are not tightly closed, and much material may find its way back again into the dredged portion of the channel. In the case of a deck scow where the material is piled on the deck any practical and convenient method may be used to determine the volume, the measurements depending upon the shape of the pile. For **rock** the amount taken out can be calculated by obtaining its weight, and this is ascertained by determining the displacement of the scow before and after loading.

Earthwork Computation from Contours is used in many landscape problems for determining an approximate value of the quantity of earth to be moved in a proposed grading project or in connection with a grading problem to form the shape of the final surface of a certain locality after a definite quantity of material has been removed from it. The use of contours in earthwork computations has been confined almost exclusively to preliminary estimates, but the principles involved in such computations are sufficiently sound so that as accurate determinations may be made in this way as by any of the other more common forms of earthwork computation, provided the contours are determined in the field with sufficient accuracy. It is frequently necessary to use a map, however, in which the contours have been determined by means of the stadia method or by some method of sketching their positions, using as a basis several characteristic points which have been located by the transit and the elevation of which has been determined by leveling. It is not customary or economical to define the contours accurately enough to admit of computing earthwork for payment of contracts because the borrow-pit method (Art. 68) is more readily adapted to this problem. For preliminary studies of all grading problems, however, a contour map is invaluable; its use can be extended to determining approximate quantities of earthwork without the necessity of any additional fieldwork, so that it is adapted to preliminary estimates.

There are three general methods of computing earthwork from data given on a contour map: (1) by computing directly the amount of cut or fill between successive contours; (2) by assuming a horizontal plane below the lowest part of any of the earthwork and first computing the volume of earth between that plane and the original surface, and afterward computing the volume between the same plane and the proposed surface; the difference between these two volumes will be the amount of excess cut or fill; (3) by drawing on the plan first a line of no cut or fill, a second line representing, say, 2-ft. cut or fill, a third line of 4-ft. cut or fill, and so on, and finally computing the volume between these successive 2-ft. layers. In all three cases the most practical way of determining the areas of the end sections is by means of a planimeter. The volumes are preferably computed by the end-area method.

Some surveyors use the prismoidal formula in problems of this nature by either interpolating by the eye the **middle area** or by treating every other area at a contour plane as a "middle area." The error in sketching the contours, in scaling the map, in the shrinkage of the paper on which the map is drawn, and the uncertainty in the amount of shrinkage in the earthwork are large enough to offset any advantage which the prismoidal formula may have over the end-area method in point of accuracy. The end-area method therefore is in practically all instances not only as exact as is the map upon which the computations are based, but is also sufficiently exact for preliminary estimates.

Method (1). The simplest application of this problem is illustrated by Fig. 78, in which the full lines are contours representing the shape of the existing surface, while the dash-line contours represent the proposed shape of the ground; the dotted lines are construction lines marking the limits of the solids to be computed. Such solids as *ANB-FQE*, *EQF-JTI*, *ITJUK-MVL* represent excavation. and solids like *BOC-GRF* and *FRG-J* are embankment. The solid *ANB-FQE* is substantially an inclined prismoid having two parallel horizontal bases whose areas are *ANB* and *FQE*, the perpendicular distance between these being the contour interval, in this case 2 ft. To compute the volume of solid *ANB-FQE*, first determine the area of the parallel bases *A BN* and *EFQ* by planimeter. The perpendicular distance between these bases is the contour interval, 2 ft. The volume is therefore computed by averaging the end areas and multiplying by the contour interval. All the other six excavation solids here shown should be computed in a similar manner; solid *CD-HSG* is a wedge. The solid *FRG-J* is a pyramid whose base is *FRG* and whose altitude is 2 ft.

Method (2). Where the problem is so complicated that the quantities of earthwork cannot be readily separated into prisms, wedges, and pyramids, as was done in Fig. 78, it can be solved by computing the quantity of earth by method (2). Such a treatment is particularly applicable to the problem of

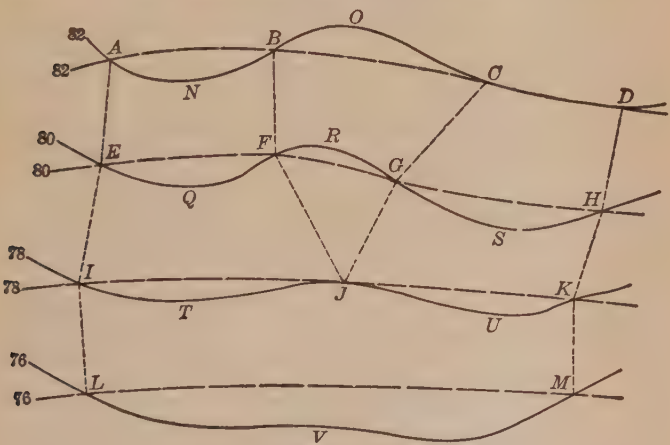


Fig. 78

determining the excess of cut or fill in a given grading project as represented by a set of proposed contours. It can be applied to determining the actual amount of cut and of fill, but for this purpose it is not so convenient of application as method (1).

Method (3). This method is particularly applicable when the original ground is very irregular and the proposed surface is to have an entirely different shape, so that the proposed contours do not cut many of the original contours of the same elevation. The first step in this process is to mark at every intersection of a new with an old contour the cut (or fill) and then to connect by means of a smooth curve the successive points of equal cut. These new curves will enclose areas which are the horizontal projections of irregular surfaces which are parallel to the final surface of the land and which are (if the contour interval is 2 ft.) at the line of no cut or fill, at 2 ft., 4 ft., etc., above the final surface. The solids included between these 2-ft. irregular surfaces are layers of earth each 2 ft. thick, and their volumes are computed by determining the areas of their horizontal projections and applying the usual end area method for volumes. To find the volume of the solid included between the curves of 2-ft. cut and the curves of 4-ft cut, for example, take for the upper surface the area within the dotted 2-ft. line as a plane surface; for the lower surface, the area within the dotted 4-ft. line; then multiply the mean of these two areas by 2 ft. in this case (the difference between the 2-ft. and 4-ft. lines). The solids at the top or bottom of these layers may be treated as wedges or pyramids.

SECTION 7

MATERIALS OF CONSTRUCTION

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MECHANICS OF SIMPLE STRESSES

BEAMS, COLUMNS, SHAFTS

TESTING AND INSPECTION

IRON, STEEL, AND OTHER METALS

NON-METALLIC MATERIALS

STANDARD MARKET SIZES

MECHANICS OF SIMPLE STRESSES

1. Tension, Compression, Shear

Stress is the internal force which, when a body is subjected to external forces, tends to hold the molecules in their original relation and to preserve the integrity of the body. Stresses are measured by the same units as forces, namely, in pounds, tons, kilograms.

Unit Stress is the measure of intensity of a stress. It is the quotient obtained by dividing a total uniform stress by the number of units of area over which the stress is distributed. Unit stresses are expressed in pounds per square inch, tons per square foot, kilograms per square centimeter, and the like.

Physicists usually use the term **stress** in place of unit stress, and measure stress in pounds per square inch or kilograms per square centimeter. Confusion between the two uses of the term will be avoided if the units used are always stated.

Ultimate Stress. If the external forces acting on a body are increased to such an extent that the internal force or stress is no longer able to preserve the integrity of the body, rupture occurs. Ultimate stress is the greatest stress which can be produced in a body before rupture occurs. Ultimate unit stress is the ultimate stress on one unit of area. It is customary to compute it on the basis of load and the original size of cross-section. Ultimate stress and ultimate strength are interchangeable terms. The ultimate stress in tension is called the **tensile strength** of a material.

Tension is the name for the stress which tends to keep two adjoining planes of a body from being pulled apart under the influence of two forces acting away from each other. If a bar of uniform cross-section is hung vertically from a rigid support, and a load of 2000 lb. suspended at the lower end, a tensile stress is developed in every section of the bar. In order that equilibrium may obtain, the internal stress or tension in the bar due to the load must equal the external force or load, or 2000 lb.

Compression is the name for the stress which tends to keep two adjoining planes of a body from being pushed together under the influence of two forces acting toward each other. A brick pier of uniform cross-section supporting a column carrying a load of 40 short tons will be subjected to a compressive stress due to the load. When equilibrium obtains this stress must equal 40 short tons. If the area of the pier is four square feet, the unit stress in the pier due to the load will be 10 short tons per square foot.

Shear is the name for the stress which tends to keep two adjoining planes of a body from sliding one on the other under the influence of two equal and parallel forces acting in opposite directions. The forces which induce shearing stresses in a body are termed **Shearing Forces**, and usually are but slightly separated, so that their action is similar to that of a pair of shears, whence the name Shear. When two flat plates, secured together by means of rivets, are subjected to forces which tend to pull the plates apart by sliding one on the other, as in the case of the plates of a boiler under internal steam pressure, shearing stresses are induced in the rivets which hold the plates together. Unless otherwise noted, shearing stress is assumed to be distributed uniformly over the section upon which its acts, although this is rarely, if ever, exactly true.

Axial Forces and Axial Stresses. When forces producing tension or compression act on a body of symmetrical form in such a way that their resultant coincides with the axis of the body, they are termed **Axial Forces**, and the stresses induced thereby, **Axial Stresses.** In simple axial tension or compression alone, the stress is usually assumed to be distributed uniformly over every section of the body normal to the direction of the force, although this is rarely exactly true.

Axial tension and compression are the two commonest forms in which stress is met with in engineering structures. An example of simple axial tension is to be found in a vertical eyebar in a bridge truss, where the only force acting on the bar is along its axis. A horizontal eyebar or one inclined to the vertical, while acted upon by axial forces transmitted through the end pins, is also subjected to a force acting normal to or at an angle with the axis, due to the weight of the bar, which produces stresses not axial, so that the resultant stress in the bar is not uniformly distributed over each section, and the case is not one of simple axial tension.

A short vertical prism, whose length does not exceed about six times the least side or diameter, is subjected to simple axial compression under a load the resultant of which passes through the centers of gravity of the end sections. If the length of the prism is more than eight or ten times its least side or diameter it becomes a **Column,** and although the resultant force is still axial as regards the end sections, some bending is assumed to have taken place in the shaft of the column, resulting in an unequal distribution of stress over a given section, and the case is not considered one of simple axial compression.

Formulas for Simple Stresses. Let P be any force producing tension, compression or shear, A the area over which the induced stress is uniformly distributed, and S the unit stress; then

$$P = SA \qquad S = \frac{P}{A} \qquad A = \frac{P}{S} \tag{1}$$

which are expressions usually assumed to apply to all cases of simple axial tension, compression, or shear. Two quantities being given the other one can be found.

Factor of Safety, Working Stress. In order that the safety of a structure shall be assured, there must be no danger of rupture in any of its members. To secure this assurance of safety the stress induced in any member by any load which the member will be called upon to carry must never approach the ultimate strength of the material. The **Working Stress** for any material is the unit stress which, by experiment, has been found safe to allow in that material and still give a proper degree of security against structural failure, and is the unit stress to be used in determining the sizes of structural members of that material. The **Factor of Safety** is the number by which the ultimate stress must be divided to give the working stress. If in formula (1) S is taken as the ultimate stress, n a factor of safety, and S_w the safe allowable stress or working stress, then $S_w = S/n$.

The purpose of the factor of safety and the working stress is twofold: first, to guard against undiscoverable defects in the structural material employed, which defects reduce the ultimate strength of the material; and second, to provide against the possibility of an increase in the load to be carried due to unforeseen circumstances. The selection of the working stress and the factor of safety therefore depends, first, upon the degree of definiteness with which the ultimate stress for the particular material is known. Thus, the factor of safety for steel or iron is smaller than for timber, since the ultimate strength of each of the various grades of steel or iron is very nearly a constant quantity, while there is a wide variation in the values for the ultimate strength of timber obtained from different specimens of the same variety. Second, upon the definiteness with which the loads to be carried can be computed. Thus, when "dead" load capable of exact calculation forms a large proportion of the total load to be carried, as in the truss members of long and heavy bridges, the factor of safety is smaller than when live or moving loads form a large proportion of the total load, as in the case of girders supporting traveling cranes or heavy moving machinery.

Riveted Joints, such as the longitudinal joints in boilers, are examples of simple stress and are usually designed or investigated by formula (1).

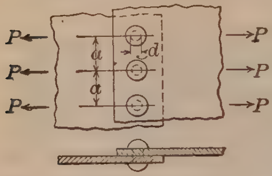


Fig. 1. Single Lap Joint

Case I. Lap joint with single riveting. (Fig. 1.) Here the shear in each rivet comes in one cross-section only and is termed "single shear." Let a = the pitch or rivet spacing, d = the diameter of the rivets, t = the thickness of the plates, P = the load to be transmitted from one plate to the other in the distance a, S_t = the unit tensile stress produced in the plates by the load P, S_c = the unit compressive stress produced in the plates where the rivets bear against them (commonly called "bearing"), S_s = the unit shear produced in the rivets; then

$$S_t = \frac{P}{t(a-d)} \qquad S_c = \frac{P}{td} \qquad S_s = \frac{P}{1/4\,\pi d^2}$$

Case II. Lap joint with double riveting. (Fig. 2.) As in Case I, the rivets are in single shear. Using the same symbols as before,

$$S_t = \frac{P}{t(a-d)} \qquad S_c = \frac{P}{2\,td} \qquad S_s = \frac{P}{2 \cdot 1/4\,\pi d^2}$$

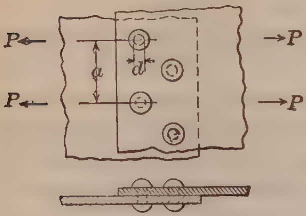


Fig. 2. Double Lap Joint

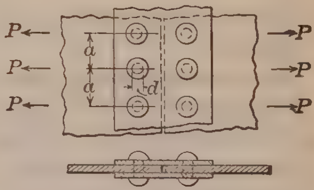


Fig. 3. Butt Joint

Case III. Butt joint with single riveting. (Fig. 3.) In this case the shear in the rivets comes in two cross-sections and is termed "double shear." Using the same symbols as before,

$$S_t = \frac{P}{t(a-d)} \qquad S_c = \frac{P}{td} \qquad S_s = \frac{P}{2 \cdot 1/4\,\pi d^2}$$

2. Deformation under Stress

Deformation is the amount of the change in the shape of a body caused by the action of an external force. Deformations are measured by the same unit as the linear dimensions of a body, namely, inches and millimeters. If a weight is suspended at the end of a steel bar, the effect of the weight is to increase slightly the length of the bar. This increase is the deformation.

Hooke's Law. Whenever a body is subjected to an external force a stress and an accompanying deformation result. From experiment it has been found that when the unit stress does not exceed a certain limit, which limit varies with each particular material, the stress bears a constant ratio to the accompanying deformation. Thus, if a weight W, suspended from a steel bar produces an elongation of 1/100 in., a weight of twice W will produce an elongation of 2/100 in., and a weight of three times W will produce an elongation of 3/100 in.

Stress and Strain. The word **strain** has been frequently used for the internal force in a body, or stress. It is present practice, however, to use the word strain as a synonym for deformation only. Thus the expression " stress and strain " is equivalent to the expression " stress and deformation." Owing to the conflicting meanings of the word Strain it has been suggested that it be avoided, the word **Deformation** being used in its place. Physicists usually define strain as the deformation per unit length.

Kinds of Deformation. Deformation may be of three kinds: elongation, or increase in length, due to tension; shortening, or decrease in length, due to compression; detrusion, or the slipping of one plane on another, due to shear.

Elasticity and Set. It has been found by experiment that up to a certain limit, a body deformed under stress will return to its original shape when the stress is removed. This ability to return to its original form after deformation is termed **Elasticity.** If the unit stress is increased, however, beyond such limit, it is found that the material undergoes a **Set**, or certain amount of permanent change, and is no longer capable of returning to its original shape when the stress is removed.

Elastic Limit. The elastic limit of a material is the highest unit stress to which that material may be subjected and still return to its original shape when the stress is removed.

Proportional Limit. The proportional limit of a material is the highest unit stress for which the deformation is proportional to the stress, or, in other words, the highest unit stress for which Hooke's law holds.

With actual materials it is probable that some very slight inelastic action occurs under any stress whatever, and that Hooke's law is a very close approximation rather than a rigid statement of fact. For the common structural steels and the usual degree of precision in laboratory tests the elastic limit and the proportional limit are nearly coincident, and the terms are, for practical purposes, interchangeable.

Various methods of determining the proportional limit or other practical " limits " closely related to it are in use. Some of these are discussed on p. 608.

Modulus of Elasticity. The modulus of elasticity of a material is the constant which, within the proportional limit, expresses the ratio between unit stress and unit deformation. Thus, let E = the modulus of elasticity, P = an axial force, A = the cross-sectional area of a bar, S = the unit stress produced by the force P, or P/A, d = the deformation produced by the force P in a bar of length l, and $d/l = e$; then

$$\mathbf{E} = \frac{\frac{\mathbf{P}}{\mathbf{A}}}{\frac{\mathbf{d}}{l}} \quad \text{or} \quad \mathbf{E} = \frac{\mathbf{S}}{\mathbf{e}}$$

Since l and d are linear dimensions, e is an abstract number, and E is expressed in the same units as S, such as pounds per square inch, tons per square foot, or kilograms per square centimeter.

Coefficient of Elasticity, and **Young's Modulus** are expressions sometimes used for the Modulus of Elasticity. They are not know in general use in the United States, Modulus of Elasticity being the generally accepted expression for the quantity E.

Deformation within Elastic Limit. The moduli of elasticity of materials measure their relative ability to resist deformation under unit stresses within their proportional limits. The above formula may be written in the form

$d = Sl/E$. For the same unit stress S, d decreases when E increases. It will thus be seen from a comparison of the various moduli of elasticity (Art. 3) that for the same unit stress timber in tension will be deformed ten times as much as cast iron, and cast iron about twice as much as steel. This formula may be employed to determine the total deformation of a bar of a length l when S and E are known.

From the values of the proportional limit and the moduli of elasticity in tension (Art. 3) the unit elongation at the proportional limit may be computed for each of the common structural materials, as follows:

For timber $\frac{1}{500} = 0.0020$ For wrought iron $\frac{1}{900} = 0.0011$

For medium steel $\frac{1}{857} = 0.0012$

The ultimate elongation, or the elongation at rupture, cannot be so readily computed, since beyond the proportional limit the ratio of the unit stress to the unit deformation is a constantly varying quantity.

3. Phenomena of Stress

Stress-deformation Diagrams. The action of a bar under stress may be represented by a curve the ordinates of which represent unit stresses and the abscissas the corresponding unit deformations. For all unit stresses below the

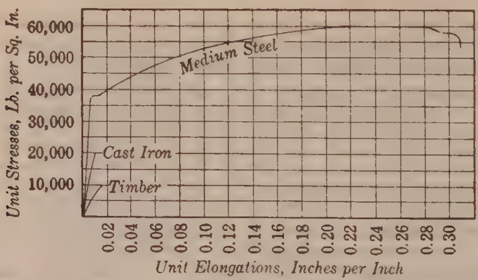


Fig. 4. Stress-elongation Diagram

proportional limit the stress-deformation curve will have the form of a straight line, since the formula $S = Ee$ is an equation which represents a straight line through the origin of coordinates, the slope of which is represented by E. Beyond the proportional limit the curve will take various forms and radii of curvature, depending upon the material. The end of the curve will represent the point of rupture, and the greatest ordinate will represent the ultimate strength of the material. The stress-deformation curve constructed from information derived from actual experiment, a sufficient number of points of the curve being plotted to determine accurately its curvature, affords in most cases the most satisfactory method available for determining the proportional limit of the material, the proportional limit being represented by the ordinate of the point of tangency between the straight line and the rest of the curve. Fig. 4 is a typical stress-deformation diagram, being that for medium steel in tension. The curves for cast iron and timber are shown for comparison.

The stress-deformation curve has various forms depending upon the material in question and whether the stress is tension or compression. In some cases the curve is of such form as to make it difficult to locate the proportional limit with accuracy. In the case of medium steel in tension (Fig. 4) the point of tangency between the straight line and the rest of the curve is fairly well defined, but in the corresponding curve for medium steel in compression the transition from the straight line to the curve is more gradual, and the point of tangency is more difficult to locate. Thus again, timber and cast iron have curves entirely different from medium steel or wrought iron, in that they are curved throughout their length and, for cast iron especially, the proportional limit cannot be at all definitely fixed.

Yield Point. In tests for the determination of the elastic properties and ultimate strength in tension of such metals as structural steel and wrought iron it is noted that after the elastic and proportional limits have been exceeded a point is reached where the unit elongations increase rapidly without any increase in the unit stresses. The unit stress at this point is termed the yield point.

The yield point is often confused with the elastic limit and the former is given for the latter in many reports of tests, owing to the facility with which the yield point may be determined with some testing machines. It is that point where the scale beam of the testing machine drops and remains down until the operating screws have stretched the metal a certain amount. After an interval the bar again rises and the stress in the specimen increases. The yield point is beyond the elastic limit—for structural steel from 3000 to 6000 lb. per sq. in. beyond it. Yield points are observed only in ductile metals, such as wrought iron and steel.

Elasticity and Set. The portion of the stress-deformation curve between the origin of coordinates and the elastic limit represents the range of unit stress and unit deformation within which the material will return to its original shape upon the removal of the stress. Beyond the elastic limit a body under stress only partly recovers its original shape upon the removal of the stress, the body being more or less permanently deformed. The amount of this permanent deformation is termed the **Permanent Set.** Thus a bar of wrought iron subjected to a tensile stress of 30 000 lb. per sq. in. shows a unit elongation of 0.003 in. per in. Upon the removal of the stress the bar will show a recovery of, say, 0.001 in. per in., having a permanent set of 0.002 in. per in.

Ultimate Strength and Stress at Rupture. The ultimate strength of a material is represented by the greatest ordinate in the stress-deformation diagram and is the greatest nominal unit stress to which the material may be subjected before rupture. The ultimate strength of a material as indicated by the stress-deformation diagram is not necessarily at the point of rupture. The stress-deformation curve for such materials as steel and wrought iron shows a decided drop after the ultimate strength has been reached, and the end of the curve corresponds to a nominal unit stress in some cases considerably below the ultimate strength.

To determine the unit stress in direct tension or compression divide the applied load by the area of cross-section of the original bar before the loads were applied. This is not strictly correct, since when the bar is approaching rupture the area of cross-section is considerably reduced. If after the ultimate strength has been reached the actual load, as indicated by the scale beam of the testing machine, could be divided by the actual area at the moment that particular load is acting, it would probably be found that the curve, if properly constructed, would curve upward beyond the point of ultimate strength.

Ultimate Deformation. Since there can be no general expression for the relation between stress and deformation beyond the elastic limit, it is not possible to determine the ultimate deformation, or deformation at the point of rupture, save by actual experiment. Ultimate deformations are rarely determined save in the case of tension tests, where the ultimate deformation becomes the ultimate elongation. It is usually expressed in per cent, and is

Average Properties of Structural Materials

Material	Weight per cubic foot, pounds	Modulus of elasticity: Tension compression	Modulus of elasticity: Shear	Elastic limit and proportional limit: Tension compression	Elastic limit and proportional limit: Shear	Ultimate strength: Tension	Ultimate strength: Compression	Ultimate strength: Shear
		Pounds per square inch						
Cast iron	450	15 000 000	6 000 000	*	*	20 000	80 000	†
Wrought iron	480	27 000 000	10 000 000	30 000	18 000	50 000	31 000‡	40 000
Medium steel	490	30 000 000	12 000 000	35 000	21 000	60 000	40 000‡	48 000
Nickel steel as rolled (3.5% nickel)	490	30 000 000	12 000 000	42 000	25 000	85 000	48 000‡	68 000
Timber	35	1 500 000	300 000 along grain	3 000	......	8 000	8 000	500 along grain
Stone	165	5 000 000	2 700 000	*	*	......	6 000	1 500
Brickwork	125	....	....	*	*	......	1 200	......
Terracotta masonry	120	....	....	*	*	......	3 000	......
Portland cement concrete (1 : 2 : 4)	150	2 500 000	....	*	*	150	2 000	†
Gypsum	80	1 000 000	....	*	*	......	1 400	......

* No well-defined elastic limit or proportional limit.

† Strength in shear greater than strength in tension; in shear tests a tension failure on an oblique plane takes place.

‡ No well-defined ultimate in compression. The yield point, which is slightly higher than the proportional limit, is the practical ultimate.

found by determining the amount that a measured length of bar has elongated after rupture and dividing the amount of this elongation by the original measured length. Thus, if a measured length of 8 in. of a bar before testing increases to 10-1/2 in. at rupture, there would be 31.2 per cent ultimate elongation. The elongation after fracture is considered to be an index of the ductility of the material.

4. Methods of Fracture

Brittleness. A material which can be only slightly deformed without rupture is termed brittle. Brittleness is relative, no material being perfectly brittle, that is, capable of no deformation before rupture. Many materials are brittle to a greater or less degree, glass being one of the most brittle of materials. Brittle materials have relatively short stress-deformation curves, the deformations of which they are capable before rupture being relatively very small. Of the common structural materials cast iron, brick, and stone are to be considered brittle in comparison with steel. Brittle materials rupture without appreciable reduction of area, and show a clean, sharp fracture, no flow of material taking place before rupture occurs.

Plasticity is the ability of a solid to change shape without fracture. A perfectly plastic material is one in which an applied load however small produces a permanent deformation.

Brittleness and plasticity are opposite terms. Materials which have a high degree of plasticity have no brittleness, and rupture with considerable reduction of area. The reduction of area at rupture may be considered the measure of the plasticity or brittleness of a material, a large reduction of area indicating a high degree of plasticity and little or no reduction of a area indicating a high degree of brittleness.

Ductility is the ability tó undergo great stretch without fracture.

Toughness is the ability to withstand high stress together with great deformation. Toughness is an index of the ability of a material to withstand impact without complete fracture. Toughness is measured by the area under the stress-deformation diagram for a material, or, less exactly, by the product of ultimate strength and elongation after fracture.

Fracture under Tension. Figs. 5, 6, 7 represent typical fractures under tension. Fig. 5 shows the fracture of a brittle material like hard steel, with a very small reduction of area. Figs. 6 and 7 show the fracture of more plastic materials, such as wrought iron and steel, with a typical cup fracture as for structural steel (Fig. 6), and a fibrous fracture as for wrought iron (Fig. 7), the reduction of area in both cases being marked.


Fig. 5


Fig. 6


Fig. 7


Fig. 8


Fig. 9

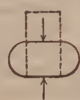


Fig. 10

Fracture under Compression. Figs. 8, 9, 10 represent the rupture of materials under compression. For very brittle materials, such as brick (Fig. 8) fracture frequently occurs along planes nearly parallel to the direction of the applied load with little or no increase in area. This is probably due to lateral deformation. For more plastic materials, such as timber (Fig. 9), fracture occurs along planes inclined to the direction of the load. A perfectly plastic material (Fig. 10) under its ultimate stress has a large increase in area without any shearing planes of fracture.

Relation of Shear to Tension and Compression. A body subjected to a force producing direct tension or compression has induced in it shearing stresses along certain planes of that body. Thus if an axial force P be applied to a bar (Fig. 11) along any plane $a-a$ inclined to the direction of the force, P may be resolved into two components, P_1 and P_2, acting respectively normal and parallel to the plane $a-a$. The component P_2 parallel to $a-a$ produces a shearing stress over the plane $a-a$. For all planes making angles 0° or 90° with the direction of the force P the shearing unit stress will be zero. It will be of maximum intensity along planes inclined 45° to the direction of P, and its value will be one-half of the direct tensile or compressive unit stress. Thus, if S be the tensile or compressive unit stress, the maximum shearing unit stress S_s along planes inclined 45° to the direction of P will be $S_s = 1/2\, S$.

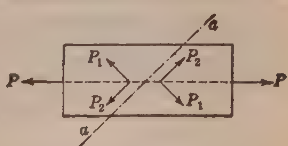


Fig. 11

In a body subjected to shearing stress, such as a shaft in torsion, there are set up tensile or compressive stresses on oblique planes. If in the body shown in Fig. 12 there is set up shearing stress along the plane *mn* there is set up shearing stress of equal intensity along planes at right angles to *mn* and tensile or compressive stress, along an inclined plane *pp*. If *pp* makes an angle of 45° with *mn* the shearing unit stress on *pp* becomes zero and the tensile or compressive unit stress is a maximum and its intensity is equal to that of the shearing unit stress along *mn*. The strength of a body is sometimes determined by stresses along oblique planes. Many brittle materials have great strength in compression, less strength in shear, and still less strength in tension. Under torsion such materials fail by tension along inclined planes; under compression they fail by shearing along inclined planes.

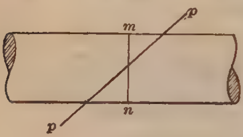


Fig. 12

5. Work and Resilience

External Work. When a load P is applied to a bar and a deformation d results, a force has acted over a certain path in producing this deformation and therefore a certain amount of work has been expended. When the force P is applied in small increments or is increased in amount gradually from zero up to P, the elastic limit of the material not being exceeded, the mean force which has acted to produce the deformation is $1/2\,P$, and the path over which the force has acted is the deformation d. If K is the amount of work expended, $\mathbf{K = 1/2\,Pd}$. Or, if A is the cross-section of the bar, l its length, S the unit stress produced, and e the unit elongation under the unit stress S, then this formula becomes $\mathbf{K = 1/2\,SeA}l$, which is an expression for the external work required to produce deformation within the elastic limit. The factor $1/2\,Se$ is the external work per unit of volume, the volume of the bar being Al.

Resilience is the amount of work which may be stored up in a body in the form of stress energy, and which may be recovered when the force producing the stress is removed. When the force is within the elastic limit, and it has been applied gradually so that no energy has been converted into heat, from the law of the conservation of energy the resilience must equal the external work. The resilience of a bar may be expressed, therefore, by the last formula, which, after making the necessary substitutions, may be written $\mathbf{K = 1/2\,(S^2/E)A}l$. When S is the elastic limit of the material, the factor $1/2\,S^2/E$ is termed the **Modulus of Resilience.**

The above formulas apply to a bar under tension, compression, or uniform shear. In the case of tension and compression the deformation or path of the force is normal to the planes of the body over which the stress is distributed, and parallel to the length, l; while in the case of shear the deformation is parallel to the planes over which the stress is distributed and normal to the length l. It should be further noted that the formulas apply only to elastic resilience, that is, to the resilience within the elastic limit of the material.

Work Required for Rupture. Since beyond the elastic limit the deformations are not proportional to the stresses, $1/2\,P$ does not express the mean value of the force acting. The formula $K = 1/2\,(S^2/E)Al$ therefore does not express the work required for deformations after the elastic limit of the material has been passed, and cannot express the work required for rupture. The work per unit volume required to produce deformations beyond the elastic limit or for rupture may, however, be determined from the stress-deformation diagram, as it is measured by the area included between the axis of abscissas, and the stress-deformation curve up to the deformation in question.

6. Cylinders and Rollers

Thin Cylinders. Under the internal pressure of water or steam, a pipe or a cylindrical boiler tends to rupture longitudinally along an element. The internal force or pressure acts normally to the inner surface and with equal intensity at all points. The tendency to rupture is resisted by the tensile strength of the material. When the thickness of the material is very small compared with the diameter, the stress may be considered uniformly distributed over the thickness without appreciable error and the case is considered one of a thin cylinder. Ordinary pipes and boilers are considered thin cylinders.

Let R be the unit pressure (Fig. 13), d the diameter of the pipe or boiler, l its length, t the thickness of the shell, and S the unit stress in the material. From a principle of hydrostatics the force which tends to produce rupture is ldR. The total resisting stress is $2\,ltS$. In order that equilibrium may obtain, the resisting stress must equal the pressure; hence

$$2\,tS = dR \qquad \text{or} \qquad \frac{S}{R} = \frac{d}{2\,t} \tag{1}$$

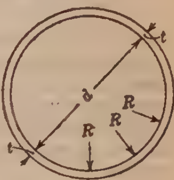


Fig. 13

This is the formula commonly used in investigating pipes, or the cylindrical shells of boilers, under internal pressure.

The Head of a Cylindrical Boiler under pressure tends to tear away from the cylindrical shell by transverse rupture under a force of $1/4\,\pi d^2 R$. This tendency is resisted by the tensile stress in the material distributed over an area equal to πdt. The stress S being considered uniformly distributed over the thickness t,

$$\pi dtS = 1/4\,\pi d^2 R \qquad \text{or} \qquad \frac{S}{R} = \frac{d}{4\,t} \tag{2}$$

A comparison of formulas (1) and (2) shows that in a thin cylinder the resistance to transverse rupture is twice the resistance to longitudinal rupture.

Cylinders under External Pressure, such as fire tubes in a boiler, fail by collapsing, the pressure tending to distort the cross-section of the cylinder from a true circle to an ellipse. While the formula $2\,tS = Rd$ applies to cylinders under external pressure as long as the cross-section remains a true circle, actual experience shows that irregularities of manufacture result in distortion, which the continued pressure tends to increase. No rational method being available for the investigation of cylinders under external pressure, recourse has been made to empirical methods. For tubes having a ratio of length to diameter greater than 6 and a ratio of thickness to diameter greater than 0.03, Carman and Carr found that the collapsing pressures are given by the following empirical formula for lap-welded steel pipe:

$$R = \frac{83\,290\,t}{d} - 1025 \tag{3}$$

in which R is the external pressure in pounds per square inch, d is the external diameter in inches, and t is the thickness of the tube in inches. (Univ. of Ill. Eng. Expt. Sta. Bulletins 5 and 99.)

Thick Cylinders. When the thickness of metal in a pipe or cylinder is such that the difference between the internal and the external radius is large compared with the mean radius, the stresses due to the internal pressure cannot be considered as uniformly distributed over the sectional area of the annulus, and formula (1) therefore does not apply. The most widely used formula for the design and investigation of thick cylinders is Lamé's formula as modified by Clavarino. Let r_1 and r_2 (Fig. 14) be respectively the internal and external radii, R_1 the pressure per square inch on the inside of the cylinder, R_2 the pressure per square inch on the outside of the cylinder, S the equivalent tangential unit stress (E times unit deformation) at a distance x from the axis of the cylinder. Then assuming the factor of the lateral contraction, or Poisson's ratio, to be 1/4,

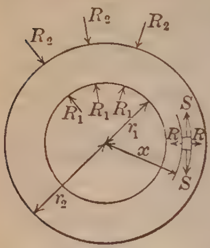


Fig. 14

$$S = \frac{\left[2\,r_1^2\,R_1 - 2\,r_2^2\,R_2 + \dfrac{5\,r_1^2\,r_2^2}{x^2}(R_1 - R_2)\right]}{4\,(r_2^2 - r_1^2)} \tag{4}$$

It will be noted that S decreases as x increases, wherefore S will be a maximum when $x = r_1$, or at the inner surface, and will be a minimum when $x = r_2$, or at the outer surface. When S is positive the stress is tension; when negative, compression. Under ordinary conditions R_2 may be neglected in comparison with R_1, being usually only the atmospheric pressure of 15 lb. per sq. in. Usually only the maximum pressure, or that on the inside of the cylinder, is required. Therefore, making $R_2 = 0$ and $x = r_1$, formula (4) becomes

$$S = \frac{R_1(2\,r_1^2 + 5\,r_2^2)}{4\,(r_2^2 - r_1^2)} \tag{5}$$

which is generally employed for common cases of investigation or design.

In a thick cylinder, in addition to the tangential stress S, which may be either tension or compression depending on the relative values of r_1, r_2, R_1 and R_2, there is a radial compressive stress R which will have its maximum value at the inner surface, where it will equal R_1, and its minimum value at the outer surface, where it will equal R_2.

Cylindrical Rollers are commonly used in providing expansion bearings at the ends of long girders and trusses, to provide for the difference in length due to temperature changes. These rollers are designed to travel between steel plates and to transmit the load from the upper to the lower plate, each roller taking its proper proportion of the total load (Fig. 15).

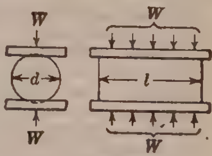


Fig. 15

Let W be the load carried by one roller of a length l and a diameter d, and let S be the maximum compressive stress in the roller and E the modulus of elasticity for the material; then

$$S = \left(\frac{9\,W^2E}{8\,d^2l^2}\right)^{1/3}$$

or

$$dl = \left(\frac{3\,W}{2\,S}\right)\left(\frac{E}{2\,S}\right)^{1/2}$$

The first formula is generally used for the investigation of cylindrical rollers, the load W, the length l, and the diameter d being given; while the second is the proper formula for the design of rollers with a given load, working stress and modulus of elasticity. If w is the load per unit of length of roller, or

$w = W/l$, then this formula becomes $w = 2/3\, dS(2\, S/E)^{1/2}$. Substituting $S = 25\,000$ and $E = 30\,000\,000$ there results $w = 680\, d$ for steel. The rule for bridge rollers given in recent specifications is $w = 600\, d$.

From tests by McDaniel at the University of Illinois the safe load in pounds per inch of length for hardwood rollers rolling on hardwood planks would seem to be about 25 times the diameter in inches.

These formulas are deduced under the assumption that the plates are not deformed. Experiments seem to indicate, however, that the plates are deformed as well as the rollers. The formulas err on the side of safety.

Ball Bearings. The permissible load on a hardened steel ball as determined by Stribeck from experimental data is given by the equation

$W = 565\, d^2$ for balls bearing against flat, hardened steel surfaces, and

$W = 1400\, d^2$ for balls supported in a hardened steel race in the shape of a groove with a radius $= 2/3\, d$

In the above W is the load in pounds, and d the diameter in inches.

7. Repetitive and Impact Stresses

Fatigue of Metals. It is a well-known fact based on experiment and general experience that stresses which can be applied to a body a few times without causing apparent structural damage may, if applied a great many times, cause failure. If a polished surface on a heavily stressed part of a body is examined under a microscope as stress is repeatedly applied, minute flaws will be seen to develop and spread, finally developing cracks. The phenomenon of failure under repeated stress is known as Fatigue of Metals. Progressive Fracture would be a better term.

A view formerly common was that under repeated stress metal "crystallized." This view became common because it seemed to explain the fact that under repeated stress even ductile materials fail by snapping sharply off, and the fracture has a crystalline appearance. The gradual spread of minute cracks would cause the same result, the effect of the crack being to reduce the available area resisting stress and to set up localized stresses of high intensity at the ends of these cracks. The crystallization theory of failure under repeated stress has given way to the progressive fracture theory.

The Endurance (or Fatigue) Limit. Tests on specimens of various metals subjected to repeated stress in specially designed testing machines show that

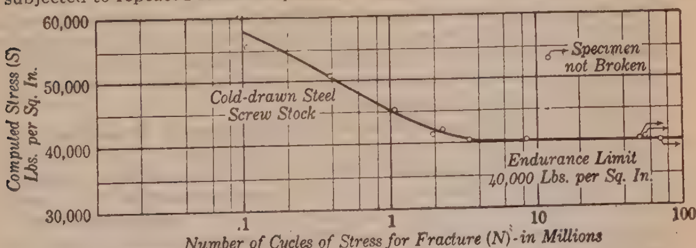


Fig. 16.

for nearly every metal tested there is a fairly well-defined limiting unit stress below which the metal can withstand hundreds of millions of repetitions of stress without fracture. This limit can best be determined for any metal by a series of tests of specimens using various unit stresses. The results of this series is plotted in a **stress-cycle**, or $S-N$ diagram. Fig. 16 shows a typical

$S-N$ diagram. Values of unit stress are plotted as ordinates and number of cycles of stress for fracture are plotted as abscissas (in Fig. 16 the abscissas are plotted to logarithmic coordinates). The unit stress for which the $S-N$ diagram becomes horizontal locates the **Endurance Limit** (or **Fatigue Limit**) for the metal tested. In Fig. 16 the endurance limit is 40 000 lb. per sq. in.

The endurance limit most commonly determined is that under cycles of completely reversed flexure. The table on p. 558 gives values of endurance limit under reversed flexure for a number of structural metals.

Endurance Limit for Various Ranges of Stress. For cycles of stress involving a complete reversal of axial stress (tension followed by compression) the endurance limit is found to be nearly the same as for tests under reversed flexure. In this connection it should be noted that it is exceedingly difficult in machine or structural parts to obtain pure axial stress free from flexure, and also that small localized flexural stresses which have no practical effect on static strength of ductile metals may have a marked effect on the strength under repeated stress.

For cycles of flexural or axial stress involving partial but not complete reversal the following formula has been found to give safe values for rolled or forged steel and iron:

$$(\mathbf{FL})' = \frac{3\,(\mathbf{FL})_{-1}}{(2 - \mathbf{r})} \tag{1}$$

in which $(FL)_{-1}$ is the endurance limit under cycles of completely reversed stress, r is the ratio of the (numerical) minimum stress during a cycle to the maximum stress (r is $-$ for partially or completely reversed stress; for completely reversed stress $r = -1.0$), and $(FL)'$ is the endurance limit under cycles of stress with the range r. If $r = 0$ (stress varying from 0 to a maximum), $(FL)' = 3/2\,(FL)_{-1}$. If during a cycle the stress varies from a given value in tension to 1/4 that value in compression, $r = -1/4$, and $(FL)' = 4/3\,(FL)_{-1}$.

Formula (1) may be used safely for cycles of non-reversed stress (r, positive) but **if it gives values above the elastic limit of the metal,** the danger of failure by inelastic distortion is greater than the danger of failure by progressive fracture, and the elastic limit is the criterion of strength rather than the endurance limit.

Endurance Limit under Repeated Shearing Stress. Test data for tests under cycles of shearing stress (such as would be set up in a shaft under torsion) are rather meager, but available data indicate that the endurance limit under cycles of shearing stress may safely be taken as 50% of the endurance limit under cycles of flexural stress.

Correlation of Endurance Limit with Other Strength Properties. The endurance limit of a metal seems to be more closely correlated with the tensile strength than with the elastic limit. This is not surprising in view of the fact that fatigue failure consists of a gradually spreading **fracture** of the metal. For rolled or forged steel or iron the endurance limit is usually about 50% of the tensile strength, or 250 times the Brinell Hardness Number. For sound steel castings the ratio of endurance limit to tensile strength is about 0.40, for cast iron about 0.33, and for non-ferrous metals the ratio varies from 0.25 to 0.50.

Localized Stress and Fatigue Failure. In designing parts to be subjected to non-repeated loading many localized stresses are commonly neglected (for example, the localized stress at the edge of a rivet hole, which may be two or three times the value computed by the formulas on p. 546). This practice is justified for static loading of ductile metals because the localized overstress does not cause appreciable distortion of the part as a whole. However, under repeated stress a minute area of overstressed metal may serve as a starting point for a spreading crack which ultimately causes failure of the piece, and such failure, if it occurs, comes suddenly without warning, just as if the piece were gradually sawed through with a hacksaw.

Designers should pay especial attention to avoiding notches, sharp shoulders, small holes, and other sources of localized stress in parts subjected to repeated loading. If such localized stress cannot be avoided (for example, in the case of a screw thread) allowance should be made for the localized stress.

The Stress-concentration Factor for localized stress is the ratio of the stress actually existing at the point of localized stress to the value of the stress as computed by the ordinary formulas of mechanics of materials, which take no account of such localized stress. The accompanying table gives safe values for typical stress-concentration factors.

Stress-concentration Factors for Typical Localized Stresses

r = radius at bottom of groove or at shoulder.
d = diameter or thickness of piece.

Location	Stress-concentration factor
Hole in plate, width not greater than one-sixth width of plate	3.0
Bottom of U. S. Standard Screw Thread	4.0
Shoulder on piece: $r/d = 0.5$	1.6
$r/d = 0.3$	1.9
$r/d = 0.2$	2.2
$r/d = 0.1$	2.8
$r/d = 0.05$	3.2
Groove in piece: $r/d = 2.0$	2.6
$r/d = 1.0$	3.0
$r/d = 0.5$	3.7
$r/d = 0.2$	5.4
$r/d = 0.1$	8.0

Corrosion-fatigue. Tests by McAdam and others show that under the **simultaneous** action of a corroding agent and repeated stress the endurance limit of a metal may be greatly reduced. Under the corroding action of a stream of fresh water at ordinary room temperature the endurance limit of heat-treated alloy steel may be reduced as much as 50%. Structural steel as rolled is less affected, and the various "stainless" steels and irons are even less affected than structural steel.

Detection of Fatigue Cracks before Final Failure. Whenever it is feasible to make a periodic examination of a structural member subjected to repeated loading there is a fair chance of detecting a fatigue crack before disastrous failure occurs. Such cracks have been detected in practice for boiler plates, car axles, and bridge members. A careful visual examination will sometimes disclose the crack. Examination with a magnifying glass is still better. A method in successful use consists in rubbing the suspected part with oil, wiping the oil off the surface, coating the surface with a wash of whiting, and when the whiting is dry striking the piece so as to set up slight bending stresses. Some of the oil which has seeped into any fatigue cracks is forced out, discoloring the white coating and locating the fatigue cracks.

Endurance Limits of Common Structural Metals under Cycles of Reversed Flexural Stress

From test results obtained with rotating-beam fatigue testing machines

The values given in this table were obtained from tests of small specimens, not over 1/2 in. in diameter. For large pieces made of materials of the same chemical composition and with the same nominal heat treatment the stresses developed at the endurance limit of the piece would be lower than those shown here. The use of a factor of safety (more accurately a factor of uncertainty) is necessary with these values for determining working stress in machine parts subjected to repeated stress.

Metal	Approximate chemical composition, per cent	Heat treatment	Tensile strength, lb. per sq. in.	Endurance limit, lb. per sq. in.	Endurance ratio *	Brinell hardness number
Armco	0.02 carbon	Annealed	42 400	26 000	0.61	69
Wrought iron	....	As rolled	46 900	23 000	0.49	105
Structural steel	0.18 carbon	As rolled	61 500	28 000	0.46	....
Machine steel	0.37 carbon	Normalized	71 900	33 000	0.46	132
		Water quench, draw at 1050° F.	102 600	57 000	0.56	209
Machine steel	0.52 carbon	Normalized	98 000	42 000	0.43	193
		Water quench, draw at 1200° F.	115 000	56 000	0.49	227
Spring steel	0.93 carbon	Oil quench, draw at 850° F.	188 000	98 000	0.52	380
High-carbon steel	1.20 carbon	Oil quench	220 000	105 000	0.48	444
Nickel steel	0.41 carbon 3.60 nickel	Oil quench, draw at 1200° F.	111 800	67 000	0.60	248
Chrome-nickel steel	0.24 carbon 3.33 nickel 0.87 chromium	Oil quench, draw at 1200° F.	114 200	67 000	0.59	246

Chrome-vanadium steel	0.55 carbon 0.99 chromium 0.19 vanadium	Water quench, draw at 900° F	201 000	94 500	0.47	....
Chrome-molybdenum steel	0.39 carbon 0.76 chromium 0.18 molybdenum	Water quench, draw at 900° F	136 600	68 500	0.50	....
Silico-manganese steel	0.51 carbon 0.66 manganese 1.96 silicon	Oil quench	157 500	62 000	0.39	....
Stainless iron	86 Iron 13 Chromium	As hot rolled	112 200	48 000	0.43	208
Cast steel	0.25 carbon	As cast Normalized	67 200 76 600	27 000 35 000	0.40 0.46	119 136
Cast iron	2.76 graph. carbon 0.68 comb. carbon	As cast	31 600	11 000	0.35	148
Aluminum	Commercially pure	As rolled	22 600	10 500	0.46	45
Duralumin	3.25 copper 0.70 manganese 0.28 iron rem. aluminum	Water quench, draw at 925° F	51 200	12 000	0.24	100
Magnesium	Commercially pure	As extruded	32 500	7 800	0.24	64
Copper	Commercially pure	Annealed	32 400	10 000	0.31	47
Brass	Copper, 60 Zinc, 40	Annealed	54 200	22 000	0.41	72
Bronze	Copper, 95 Tin, 5	Annealed	45 700	23 000	0.50	74
Nickel	Commercially pure	Annealed	69 900	28 000	0.40	90
Monel metal	Copper, 24 Nickel, 73	Annealed	89 800	32 000	0.36	166

* Ratio of endurance limit to tensile strength.

Static and Dynamic Loads. A load at rest, producing no change in the unit stress S, or a load which is increased gradually by increments from 0 up to P, is termed a **Static Load**, and unless otherwise noted is the load usually understood in the discussion of structural members. Many structural members, however, are subject to load applied so rapidly that the momentum of deformation causes strains and stresses beyond the "static" values. Such loads are termed **Dynamic Loads.** It is obvious that the effect of a dynamic load may be much greater than the effect of the same load applied in small increments.

Impact is a word used to denote the effect of a moving load. The blow of a hammer is a good example of impact, the velocity with which the weight of the hammer or the load is moving when it strikes being an important factor in the effect produced. If P is a load in motion with a velocity V at the moment of striking a horizontal bar, the energy due to the velocity, or the kinetic energy, could be expressed by $PV^2/2g$, in which g is the acceleration due to gravity. If h is the height through which a load must fall in order to acquire the velocity V, then $V^2/2g = h$, and the kinetic energy of the moving load may be expressed by Ph. The load P striking the bar produces a stress which increases from 0 up to Q, with a corresponding deformation increasing from 0 up to d_1. The energy stored in the bar is evidently expressed by $1/2\,Qd_1$, which must be equal to the external work provided no energy has been expended as heat or in giving velocity to the bar, or $1/2\,Qd_1 = Ph$. If d is the deformation produced by a static load P, then $d_1/d = Q/P$, whence by combining and solving for Q and d_1 there results

$$Q = P\left(\frac{2h}{d}\right)^{1/2} \qquad d_1 = d\left(\frac{2h}{d}\right)^{1/2}$$

from which it appears that Q and d_1 increase with h or, in other words, with the velocity with which the load is moving when impact occurs. If the bar is vertical instead of horizontal the external work expended is expressed by $P\,(h + d_1)$. Substituting this value for the external work above,

$$Q = P + P\left(\frac{1 + 2h}{d}\right)^{1/2} \qquad d_1 = d + d\left(\frac{1 + 2h}{d}\right)^{1/2}$$

It is obvious that these formulas are valid only when the stresses do not exceed the elastic limit of the material.

Compared with actual experiments the above formulas give values somewhat too large. This is due to the fact that some of the external work is not effective in producing stress, it being expended in giving motion to the bar and in producing heat, the heat being caused by the friction between the displaced molecules. For light bars, however, the values given by the formulas give results approximately correct.

Rupture from Impact. Since the stresses caused by moving loads increase with the velocity of the load, it is obvious that rupture may be caused by impact provided the load has the requisite velocity. The above formulas, however, do not apply, since they are valid only for stresses within the elastic limit. There being no rational formulas for rupture due to impact, the only information available has been obtained through experiment. The relation between the work required for rupture from impact and the work required under static loads has been determined by Hatt. From nearly 200 experiments he determined that the work required for rupture from impact was about 30% greater than that required by static loads. From available test data it does not seem that the ultimate elongation under impact (dynamic) loads is widely different from that under static loads.

Live Load Stresses, Coefficient of Impact. In computing stresses in structural members two classes of loads are usually considered; the dead load, or the weight of the various parts of the structure, and the live loads or the superimposed loads which the structure has been designed to carry. The effects of these two loads are usually computed separately. The stresses due to the dead loads are computed from static loads in the ordinary manner, the actual stresses increasing gradually as the structure is built. Stresses due to the live loads, however, may often be considerably greater than those due to corresponding static loads, since under some circumstances the loads are dynamic loads, as when a heavy, rapidly moving train runs onto the floor system of a bridge. It is evident, therefore, that in computing stresses due to various live loads proper allowance should be made for the dynamic effect. The **Coefficient of Impact** is the factor by which the corresponding static stress must be multiplied in order to give the amount that the live-load stress exceeds the corresponding static stress. Thus let S be the corresponding static unit stress due to a live load W, and i the coefficient of impact. Then the amount that the live-load stress exceeds the corresponding static stress would be iS and the total stress $S + iS$. Values of the coefficient of impact i have been determined by empirical methods, the values varying with existing conditions. The coefficient of impact varies widely for different parts of a structure. For example, the coefficient of impact for the floor-beams of a railroad bridge would be much higher than the coefficient of impact for the chord members of the trusses. At present no general rule can be given for determining values of this coefficient for structural parts. See Sect. 8, Art. 10.

It should be noted that the meaning of the word impact as here used differs somewhat from its strict theoretical meaning. The use of the terms "impact" and "coefficient of impact" in connection with live-load stresses is, however, very general.

8. Combined Stresses

Combinations of Axial Forces. Assume a bar of a cross-section A acted upon at the same time by a number of axial forces some of which produce tension and some compression. Assuming those producing tension to be positive and those producing compression negative, five forces may be represented by P_1, P_2, P_3 and $-P_1$, $-P_2$. Let the unit stress in the bar be S. It is obvious that the unit stress in the bar is equivalent to the stress produced by a load represented by the algebraic sum of the several loads acting, or $S = (P_1 + P_2 + P_3 - P_1 - P_2)/A$. If the forces represented by P_1 and $-P_1$ and P_2 and $-P_2$ have the same numerical values, then $S = P_3/A$. P_3 represents the algebraic sum of the forces acting, and being positive indicates tension.

Change in Cross-section. A body subjected to a simple axial load suffers lateral deformation. If the load is tensile the cross-section is diminished; if compressive, the cross-section is increased. When the elastic limit of the material is not exceeded, experiments show that the lateral unit deformation, or change in diameter or other lateral dimensions, bears a constant ratio to the linear unit deformation. Let this ratio be expressed by p. Let a bar of a diameter d and a length l under a unit stress S within the elastic limit have a linear unit deformation e then the total linear deformation under the unit stress S will be represented by el, and the total lateral deformation, or change in diameter, will be expressed by ped. If d' is the diameter and l' the length of the bar while under the stress S, then

$$\text{For tension,} \quad l' = (1 + e)l \quad \text{and} \quad d' = (1 - pe)d$$
$$\text{For compression,} \quad l' = (1 - e)l \qquad d' = (1 + pe)d$$

The quantity p is sometimes called the "factor of lateral contraction" but more commonly **Poisson's Ratio.** Recent tests by Jasper indicate that for rolled metals Poisson's ratio varies from 0.19 to 0.27; 0.25 seems to be a fairly representative value. For concrete Poisson's ratio is about 0.15.

Shear under Tension or Compression. A body under tensile or compressive forces is subjected to tensile or compressive stresses in planes normal to the direction of the forces, and to shearing stresses between planes inclined to the directions of the forces, the intensity of the shear varying with the inclination of the planes. Assume two forces P_1 and P_2 acting on a bar at right angles to each other, producing apparent normal unit stresses of S_1 and S_2. Let S' represent the maximum shearing stress. Then from the relation existing between the maximum shear and tension or compression, if S_1 and S_2 have the same sign,

$$S' = 1/2\ S_1 \text{ or } 1/2\ S_2, \text{ whichever is greater.}$$

If S_1 and S_2 have opposite signs,

$$S' = 1/2\ (S_1 - S_2)$$

The planes in which the maximum shearing stress occur are found to be those which make angles of 45° with the directions of the two forces.

A bar of cast iron 1 sq. in. in cross-section under a compressive load of 2400 lb. is subjected to unit stresses $S_1 = 2400$ and $S_2 = 0$. The maximum shearing stress is $S' = 2400/2 = 1200$.

Effect of Lateral Deformation. Under the action of two (or three) stresses at right angles, or under the combined action of axial stress and shearing stress, the lateral deformations set up have no effect on the stresses developed, but they do affect the deformations at any point. For example, a boiler shell is subjected to circumferential tensile stress due to the bursting action of the steam pressure, and also to axial stress due to the pull of the boiler heads. The lateral contraction due to the axial pull does not affect the **circumferential stress,** but does affect the **circumferential stretch** due to the bursting pressure, and the resulting **circumferential unit deformation** is less than the circumferential unit deformation due to bursting pressure alone. If the shell were subjected to axial compression the unit circumferential unit deformation would be greater than that due to bursting pressure alone.

Four Theories of Failure of Materials. When structural damage is done to the material in a member is it caused by stress or by deformation? Or is it caused by shearing stress on some oblique plane? Or is it caused by the energy stored up in the stressed material?

The Maximum Strain Theory (**St. Venant's**) holds that **deformation** is the vital factor in causing failure of material. Using this theory as a basis for calculation it is necessary to take account of the lateral deformation accompanying axial stress; Poisson's ratio must be considered.

The Maximum Stress Theory (**Rankine's**) holds that **stress** is the vital factor in causing failure. This is the theory on which the common formulas for strength are based. Using this theory no account is taken of lateral deformation accompanying axial stress; Poisson's ratio is not considered. The weight of experimental evidence seems to show that the maximum stress theory is not strictly true, yet in most cases it is sufficiently accurate for everyday use. Its use is most doubtful for members subjected to tension in one direction and compression at right angles to the tension.

The Maximum Shear Theory (**Guest's**) holds that **shearing stress** is the vital factor in causing failure. If this theory is used a bar under a simple axial load is in danger of failure from the shearing stresses on a plane making an angle of 45° with the axis of the bar. The value of the shearing unit stress on this 45° plane is one-half that of the axial unit stress.

The Maximum Energy Theory (**Haigh's**) holds that the **energy** per unit volume stored up in the stressed part is the vital factor in causing failure. This theory necessitates the consideration of axial stresses, lateral deformations, and shearing stresses.

Experimental Evidence for the Various Theories. Recent experimental studies of these theories have been made by Becker (Univ. of Ill. Eng. Expt. Sta. Bull. 85), by Matsumura and Hamabe (Memoirs of the Coll. of Eng., Kyoto Imperial University, Feb., 1915), and by Haigh (Proc. British Assn., 1919 and 1921 Report of Comm. on Complex Stress). A combination of the maximum strain theory with a consideration of shearing stress seems to fit the test results fairly well. The maximum energy theory seems to fit test data except for the case of axial stresses of the same sign and of nearly equal magnitude acting at right angles to each other. The maximum shear theory seems to give results on the safe side for nearly all cases. For nearly all cases of structural design the common Rankine formulas seem safe.

The different theories give the same results for simple axial stress and for simple shearing stress. For the combinations of axial stress and shearing stress usually found in practice the different theories do not yield widely differing values. For combinations of tension in one direction with compression at right angles to it there is some question as to the safety of the Rankine formulas.

9. Miscellaneous Cases

Eccentric Loads in a Rectangular Bar. An eccentric load is one whose line of action does not coincide with the axis of the bar upon which it acts. In concentric or axial loading the load, or if there are several loads all acting at the same time, their resultant, coincides with the axis of the bar. The result of an eccentric load is an uneven distribution of stress over the area of cross-section, the unit stress in one portion of the cross-section being considerably greater than in another portion, the actual variation depending upon the position of the resultant load with reference to the axis.

Let P be a load acting on a rectangular bar (whose area is A) at a distance e from the axis of the bar, e being measured in the direction of the width d (Fig. 17). Were the load P axial the unit stress S would be the same all over the area of cross-section and would be $S = P/A$. The load P being eccentric, P/A expresses only the average unit stress, the maximum unit stress and the minimum unit stress varying from the average an amount dependent upon the position of the load. Let mn be any section and S_1 be unit stress along edge nearer to P and S_2 unit stress along opposite edge. It has been found that the intermediate stresses between S_1 and S_2 will vary uniformly or as a straight line, so that the plane figure (Fig. 17), of which S_1 and S_2 are two parallel sides, is a trapezoid. In order that equilibrium may obtain, the resultant stress must equal P, and its line of action must be in the same line as P. From these two conditions of equilibrium it results that

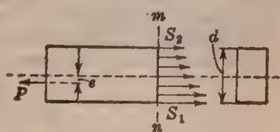


Fig. 17

$$S_1 = \frac{P}{A}\left(1 + \frac{6e}{d}\right) \qquad S_2 = \frac{P}{A}\left(1 - \frac{6e}{d}\right)$$

and these formulas are applicable alike to cases of tension or compression. The same result might have been obtained by considering the bar subject to an axial load P and bending moment Pe.

Making $e = o$ in the formulas, $S_1 = P/A$, and $S_2 = P/A$, which is a case of axial loading. When $e = d/6$, $S_1 = 2\ P/A$ and $S_2 = 0$, which indicates that when the resultant force is at the edge of the middle-third the maximum stress is twice the average stress and the minimum stress is zero. Making e greater than $d/6$, the sign of the stress S_2 changes, indicating a change from tension to compression or vice versa. A brick pier 5 ft. × 4 ft. in cross-section, loaded at a point 1 ft. from the center of the top in the direction of the width, with a load of 16 000 lb., would have a maximum unit compression of $S_1 = (16\,000/20)(1 + 6 \times 1/4) = 2000$ and a stress on the opposite edge of the pier of $S_2 = (16\,000/20)(1 - 6 \times 1/4) = -400$. The minus sign indicates a change in stress, or tension.

Centrifugal Stress in a Revolving Bar. If a weight is secured to an axis of revolution by means of a cord or a bar, and is made to revolve about that axis at a certain radius, a tensile stress is generated in the cord or bar. Let P be a weight revolving about an axis B (Fig. 18) at a radius r from the axis to the center of gravity of P and with a velocity V. Let Q be the centrifugal tension generated. Then from mechanics $Q = PV^2/gr$, in which g is the acceleration due to gravity, the mean value of which is usually taken as 32.16 ft. per second per second. Let n be the number of revolutions per second; then $V = 2\ \pi nr$ and $Q = (4\ P\pi^2n^2r)/g$, which gives the centrifugal stress in the member securing the weight to the axis when that member has no appreciable weight, as would be the case were it a cord. If the connecting member is a bar of a length l and weight W, then

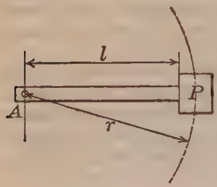


Fig. 18

$$Q = \frac{(4\ P\pi^2n^2r)}{g} + \frac{(2\ W\pi^2n^2l)}{g} \qquad \text{or} \qquad Q = \frac{4\ (Pr + 1/2\ Wl)\ \pi^2n^2}{g}$$

In this formula a portion of the stress Q is due to the weight P revolving with its center of gravity at a distance r from the axis, and a portion is due to the weight of the bar revolving with its center of gravity at a distance $1/2\ l$ from the axis. Q in this case is the stress in the bar at the axis. The stress in a simple bar revolving about an axis at one end varies from Q at the axis to 0 at the outer end. If w be the weight of the bar per unit of volume and A its cross-sectional area, the stress Q' at any point distant x from the axis is

$$Q' = \frac{2\ wA\ (l^2 - x^2)\ \pi^2n^2}{g}$$

Giving Q' its proper value and making $x = 0$, the number of revolutions required to rupture a bar of any given size, length, and material may be found. Thus the number of revolutions required to rupture a steel bar one square inch in cross-section and 4 ft. long will be found by substituting in the formula $x = 0$, $l = 4$ ft., $A = 1$, $w = 3.4$ lb., $Q' = 60\,000$ lb. per sq. in., $g = 32.16$; then n equals about 42 revolutions per second.

Revolving Thin Hoop. A thin circular hoop having a thickness t and a radius r (ft.) revolving about its center generates a tension in the hoop due to the centrifugal force acting radially in a manner similar to the action of an internal pressure on a section of a thin cylinder. If S is the tensile unit stress (lb. per sq.-in.) in the hoop, W = weight of a piece of the material 1 in. square by 12 in. long, and the other symbols as above, then

$$S = \frac{4\pi^2 W n^2 r^2}{g} \quad \text{or} \quad S = \frac{4(\pi n r)^2 W}{g}$$

which applies only when t is very small compared with the radius r.

Revolving Thick Hoop or Solid Wheel. Let a thick hoop have an inside radius of r_1 and an outside radius of r_2. and let R be the radial unit stress and S the tangential unit stress, at a distance x from the axis, due to the angular velocity $2\pi n$. Other symbols remaining as above, then

$$R = \left(\frac{3}{2g}\right)(w\pi^2 n^2)\left(r_1^2 + r_2^2 - \frac{r_1^2 r_2^2}{x^2} - x^2\right)$$

$$S = \left(\frac{3}{2g}\right)(w\pi^2 n^2)\left(r_1^2 + r_2^2 + \frac{r_1^2 r_2^2}{x^2} - x^2\right)$$

For a solid wheel, such as a millstone or a grindstone, $r_1 = 0$ and $r_2 = r$. Whence, when $x=0$, the unit stresses at the center are $R=S=3/2(\pi nr)^2 w/g$. When $x = r$, then $R = 0$, and $S = (\pi nr)^2 w/g$ is the tangential stress at the circumference. The above formulas are deduced by supposing that the revolving body does not change its form, but they give the correct actual stresses at the circumferences. For the center of a solid wheel, the above formula for S does not take account of lateral deformation, but the equivalent simple radial and tangential stresses (the strain equivalents) corresponding to the actual deformation are $3/2\,(1 - p)\,(\pi nr)^2 w/g$. Here p is Poisson's Ratio, or the factor of lateral contraction. For a solid steel wheel, the simple unit stress corresponding to the deformation (the strain equivalent) at the axis is $9/8\,(\pi nr)^2 w/g$, but if there be a very small hole at the axis it is two times as great. All these formulas apply only when the elastic limit of the material is not exceeded.

Temperature Stresses. All structural materials undergo changes in length due to changes in temperature. The **Coefficient of Expansion** for any material is the factor which expresses the change per unit of length for each degree of temperature. A bar or other structural member of a length l under a change in temperature of t degrees will, if free to move, undergo a change in length of tln, n being the coefficient of expansion. If the bar or member is fixed so that the change in length cannot occur it is evident that there is generated in the bar or member a stress equal in amount to that required to produce a deformation of tln or a unit deformation tn. If E is the modulus of elasticity and S the unit stress produced, then

$$\frac{S}{E} = tn \quad \text{or} \quad S = tnE$$

from which it is seen that S is independent of the length of the member.

Average values for the coefficient of expansion based on one degree (Fahrenheit) are as follows: n = 0.000 006 2 for cast iron, 0.000 006 5 for steel, 0.000 006 7 for wrought iron, 0.000 005 0 for brick and stone, and 0.000 005 5 for concrete.

Shrinkage of Hoops. A hoop surrounding a cylinder such as a reinforcing band on a gun or a tire on a wagon wheel or locomotive driver, is usually held in place by turning the hoop to an inside diameter slightly smaller than that of the cylinder it is intended to surround, and then expanding it by heat until large enough to fit, the shrinkage in cooling holding it securely in place. In such a hoop a tangential stress is produced in the hoop and a radial pressure in the cylinder which it encloses. When the thickness of the hoop is small compared with its diameter, all the deformation produced may be considered as confined to the hoop. The tension in the hoop will be proportional to its change in diameter. Let d = the diameter of the cylinder to be enclosed, which is assumed to be the same after the hoop is in place. d_1 = the diameter to which the hoop has been turned. S = the tangential unit stress. E = the modulus of elasticity. R = the radial unit stress acting on the inside of the hoop, and t = the thickness of the hoop. Then

$$S = \frac{E(d - d_1)}{d_1} \quad \text{and} \quad R = \frac{2tS}{d_1}$$

The values found for S from this formula are somewhat too large since some change is made in the diameter d due to the radial pressure. For thick hoops such as the bands on heavy guns more exact formulas are usually employed.

Internal Friction. In 1893 the discovery was made by Hartmann that lines of stress became visible on polished metal specimens, these lines remaining after the removal of the load if the elastic limit had been exceeded. On a cylindrical specimen these lines are two sets of helices; on a flat specimen they are two sets of straight lines. In tension they make angles with the axis greater than 45°, in compression the angles are less than 45°. These lines indicate the direction of planes along which sliding or shearing is occurring. Internal friction occurs along these planes from the theory of which Merriman deduced the ratio between the shearing and compressive strength of brittle materials (Mechanics of Materials, 1910, p. 380) finding 0.13 for anthracite coal, 0.18 for sandstone, 0.23 for hard brick, 0.29 for concrete, and 0.35 for cast iron.

BEAMS, COLUMNS, SHAFTS

10. Moments and Shears

Flexure or Bending is the phenomenon which occurs when a straight bar is subjected to a force or a combination of forces so applied that the axis of the bar is caused to assume the form of a curve. The phenomenon of flexure is a combination of the three simple stresses of tension, compression, and shear. Thus a horizontal bar simply supported at the ends under the influence of its own weight assumes the form of a curve, concave upward, and is undergoing flexure. The fibers on the convex side of the bar are elongated and therefore are in tension, while the fibers on the concave side are shortened and are therefore in compression. Shear is taking place between each vertical plane of the bar and the one adjoining, between the middle of the bar and each support. The structural members which are ordinarily subject to flexure are called **beams** and are usually horizontal members, carrying loads acting vertically. Flexure, however, is not confined entirely to beams, since it may occur in any member of a structure under the influence of loads other than axial loads. Even in the case of a strut or column which is under compression, flexure may occur when the acting forces are eccentric with respect to the axis.

A Simple Beam is a horizontal member simply supported at the ends so that all parts have free movement in a vertical plane under the influence of vertical loads. Simple beams are the commonest structural members. Under ordinary loading the upper fibers are in compression and the lower fibers in tension. A **Cantilever Beam** is a member with one end projecting beyond the point of support, free to move in a vertical plane under the influence of vertical loads placed between the free end and the support. The fibers in the upper side of such a beam are in tension and those in the lower side in compression. A beam with one end rigidly fixed in a brick wall and the other end free is an example of a cantilever beam. Constrained beams are those rigidly fixed at one or both points of support. A beam with one or both ends rigidly built into brickwork is an example. A **Continuous Beam** is one having more than two points of support. More than two points of support cause a distribution of stress similar to that in a constrained beam.

Neutral Surface and Neutral Axis. Any beam under flexure takes the form of a curve. The fibers of the beam on the concave surface are subjected to compression, while those on the convex surface are subjected to tension. It is obvious that these stresses must decrease toward the middle of the depth of the beam; therefore at some point in the depth of the beam there is a surface where the fibers are neither in tension nor in compression and where no deformation is taking place. This surface is termed the **Neutral Surface.** The trace of this surface or plane on any cross-section of the beam is termed the **Neutral Axis** of that section.

From experiment it has been shown that where the elastic limit of the material has not been exceeded the deformation in any fiber and in consequence the stress in that fiber is proportional to its distance from the neutral surface. It may also be demonstrated analytically that the neutral surface passes through the center of gravity of the cross-section, and that for beams of **symmetrical cross-section** loaded in a plane of symmetry or at right angles thereto the neutral axis is perpendicular to the plane of the load.

End Reactions. In order that equilibrium may obtain in any vertical system of forces acting in one plane, as in the case of the loads and reactions of beams, it is known from analytical mechanics that, first, the algebraic sum of all vertical forces must equal zero, and second, the algebraic sum of all moments must equal zero. From the first of these laws it is apparent that the sum of the reactions must equal the sum of the loads. In a simple beam when the loads are systematically placed with reference to the supports, as in the case of loads uniformly distributed, such as the weight of the beam itself, or equal concentrated loads placed at equal distances from the supports or center of beam, each reaction will equal one-half the sum of the loads. When the loads are not systematically placed the reaction at each support may be ascertained from the second of the laws stated above. Thus consider the system of loads, Fig. 19. Taking moments about the left support $R_2 \times 20 - 4000 \times 5 - 5000 \times 8 - 10\,000 \times 10 = 0$, or $R_2 = 8000$ lb. Similarly moments taken at the right support give $R_1 = 11\,000$. It will be noted that the sum of the reactions equals 19 000, the sum of the loads thus fulfilling the first law of equilibrium. When some of the loads are uniformly distributed over a portion of or the whole of the beam the same method may be applied to find the reactions by considering the uniform loads as concentrated at their centers of gravity.

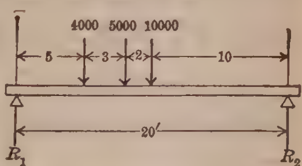


Fig. 19

Vertical Shear. At every section in a beam two equal and opposite forces, the one on the right and the other on the left of the section, tend to shear the beam at that point. This shearing tendency is greatest at the reactions where the shearing forces are each equal in amount to the reaction. The vertical shear at any section is the measure of the shearing tendency at that section and is equal to the algebraic sum of all the forces to the left of that section, upward forces or reactions being taken as positive and the downward forces or loads being taken as negative. The vertical shear may be either negative or positive, depending upon the relative values of the loads and reactions to the left of the section.

Figs. 20*a* and 20*b* illustrate diagrammatically the vertical shear. Fig. 20*a* shows a beam with a uniform load of 100 lb. per ft., the vertical shear at each support being equal to the reaction of 1000 lb. at that support and gradually decreasing to zero at the center. Fig. 20*b* shows diagrammatically the vertical shear under concentrated

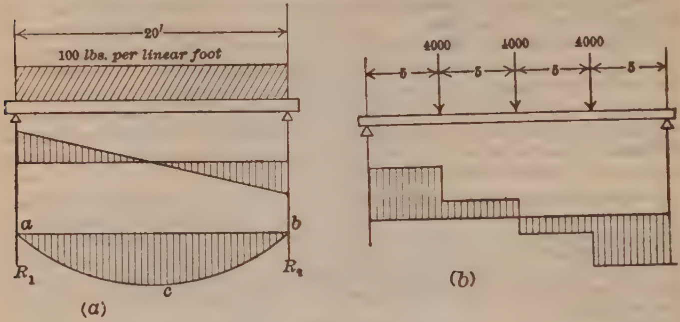


Fig. 20

loads, the vertical shear between the left reaction and first load being 6000 lb., between the first and second loads 2000 lb., at the center zero, between the second and third loads 2000, and between the third load and the right reaction 6000 lb.

The Bending Moment at any section is the algebraic sum of the moments of all forces on the left of that section, moments tending to cause rotation in the same direction as the hands of a clock being taken as positive, and those tending to cause rotation in the opposite direction being taken as negative. The bending moment at any section is the measure of the flexural stress at that section.

In Fig. 21*a* let the beam have a length l and be loaded with a uniform load of w lb. per lin. ft. The left reaction will then be $wl/2$, and if M be the bending moment at the distance x from the left support, then $M = 1/2\ wlx - 1/2\ wx^2$, whence it is seen that

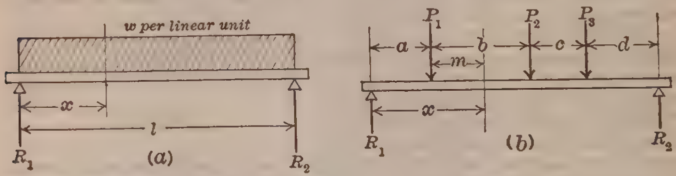


Fig. 21

$M = 0$ when $x = 0$, and $M = 1/8\ wl^2$ when $x = 1/2\ l$. In Fig. 21*b* let the beam be loaded with the loads P_1, P_2, P_3. The reactions R_1 and R_2 may be found by the principle of moments. The bending moment at any point between the first and second loads will be $M = R_1x - P_1m$.

Maximum Bending Moment. In a cantilever beam the maximum bending moment is at the support irrespective of the position of the loads. In a simple beam the point of maximum bending moment, or the dangerous section in a beam, is at the point where the shear passes through zero, that is, where the shear changes from positive to negative, or vice versa. This point may be located in any system of loading by beginning at one reaction and subtracting the loads in order from the reaction until a point is reached where the sum of the loads equals the reaction.

In Fig. 20*a* the bending moment under a uniformly distributed load is illustrated diagrammatically. From the expression for the bending moment at any point, $M = 1/2\ wlx - 1/2\ wx^2$, it will be noted that the bending moment at any section may be represented by the corresponding ordinate of the plane figure (Fig. 20*a*) formed by the straight line *ab* and the parabola *acb*. It will be noted that the maximum ordinate is at the center, coinciding with the axis of the parabola, and that at the supports the ordinates are zero.

11. Formulas for Flexure

Conditions of Equilibrium. The investigation of beams is governed by the following three conditions of equilibrium, which hold for any section:

Sum of all tensile stresses = sum of all compressive stresses.
Resisting shear = vertical shear.
Resisting moment = bending moment.

Resisting Shear. At any section in a beam the internal forces must equal the external forces in order that equilibrium may obtain. At any section, therefore, some internal force must oppose and be equal to the vertical shear at that section. This force is the shearing stress in the material and is called the **Resisting Shear.** Let V be the vertical shear at any section, S_s the maximum unit shearing stress at that section and A the area of the section. The **average** shearing stress at the section is

$$S'_s = \frac{V}{A}$$

The shearing stress is not, however, uniformly distributed over the cross-section and the maximum shearing stress on the section is

$$S_s = \frac{kV}{A} \qquad (1)$$

in which k is a constant depending on the shape of the section. For a rectangle k is 1.50, for a triangle k is 1.33, and for a circle k is 1.33. Values of factors for determining maximum shearing stress in I beams and channel beams are given on pp. 666–677.

In a beam the shearing stress is greatest at some point neat the center of depth and is 0 at the extreme fibers. At any point in a beam the horizontal shearing unit stress is equal to the vertical shearing unit stress at that point. Wooden beams which are weak in shear along the grain are in special danger of failure by horizontal shear.

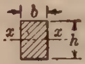
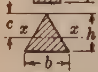
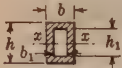
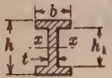
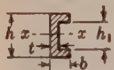
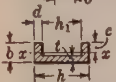
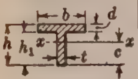
Resisting Moment. The bending moment at any section tends to cause rotation about that section. The tendency to rotate is resisted by the moment of tensile and compressive stresses in the material at that section, which act as an internal couple. This internal couple is called the resisting moment. Let S be the unit stress at any extreme fiber on the surface of the beam due to the bending moment and c the distance from that fiber to the neutral surface, M the resisting moment or its equal, the bending moment; then

$$M = \frac{SI}{c} \quad \text{or} \quad S = \frac{Mc}{I} \qquad (2)$$

in which I is the moment of inertia of the section about a gravity axis.

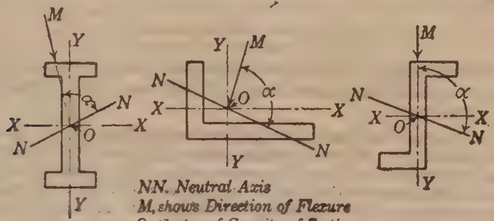
This is the common flexure formula. Strictly speaking it applies only to beams loaded in the plane of a principal axis of the cross-section (e.g., a plane of symmetry). However, in practice most beams are so loaded, and this formula may usually be used.

Properties of Common Sections of Beams

Sections of beams, Fig. 22	Distance from neutral axis to extreme fiber of section	Moment of inertia I	Section modulus I/c	Radius of gyration r $\sqrt{I/A}$
	1/2 h	$1/12\ bh^3$	$1/6\ bh^2$.	$\frac{h}{\sqrt{12}} = 0.289\ h$
	2/3 h	$1/36\ bh^3$	$\text{Min} = \frac{bh^2}{24}$	$\frac{h}{\sqrt{18}} = 0.236\ h$
	1/2 h	$1/12\ (bh^3 - b_1h_1^3)$	$\frac{bh^3 - b_1h_1^3}{6\ h}$	$\sqrt{\frac{bh^3 - b_1h_1^3}{12\ (bh - b_1h_1)}}$
	1/2 d	$\frac{\pi d^4}{64} = .0491\ d^4$	$\frac{\pi d^3}{32} = .0982\ d^3$	$\frac{d}{4}$
	1/2 d	$\frac{\pi(d^4 - d_1^4)}{64} = .0491(d^4 - d_1^4)$	$\frac{\pi(d^4 - d_1^4)}{32\ d} = .0982\frac{(d^4 - d_1^4)}{d}$	$\frac{\sqrt{d^2 + d_1^2}}{4}$
	1/2 h	$\frac{bh^3 - h_1^3(b-t)}{12}$	$\frac{bh^3 - h_1^3(b-t)}{6\ h}$	$\sqrt{\frac{bh^3 - h_1^3(b-t)}{12[bh - h_1(b-t)]}}$
	1/2 b	$\frac{2\ db^3 + h_1t^3}{12}$	$\frac{2\ db^3 + h_1t^3}{6\ b}$	$\sqrt{\frac{2\ db^3 + h_1t^3}{12[bh - h_1(b-t)]}}$
	1/2 h	$\frac{bh^3 - h_1^3(b-t)}{12}$	$\frac{bh^3 - h_1^3(b-t)}{6\ h}$	$\sqrt{\frac{bh^3 - h_1^3(b-t)}{12[bh - h_1(b-t)]}}$
	c	$\frac{h_1t}{12}\left[t^2 + 12\left(b - c - \frac{t}{2}\right)^2\right] + \frac{db}{6}\left[b^2 + 3\ (2\ c - b)^2\right]$	$\frac{I}{c}$	$\sqrt{\frac{I}{A}}$
	c	$\frac{bd}{12}\left[d^2 + 12\left(h - c - \frac{d}{2}\right)^2\right] + \frac{th_1}{12}\left[h_1^2 + 12\left(c - \frac{h_1}{2}\right)^2\right]$	$\frac{I}{c}$	$\sqrt{\frac{I}{A}}$

Oblique Flexure of Beams. If a beam is bent by a lateral force acting obliquely to an axis of symmetry of the cross-section, or if the beam section has no axis of symmetry, in general, the extreme unit stress is different from, and frequently greater than, the value given by formula (2). The neutral axis of a cross-section under oblique loading is not perpendicular to the plane of the load, but makes with it an angle α as illustrated in Fig. 23.

To determine the extreme unit stress set up by oblique flexure it is necessary to know three properties of the cross-section: I_y, the moment of inertia about some gravity axis, I_x, the moment of inertia about a gravity axis perpendicular to the axis for I_y, and J, the **product of inertia** about the axes for I_y and I_x. The product of intertia of an area about two axes at right angles is equal to the sum of the products of each elementary area and the x-coordinate and the y-coordinate of the elementary area, or $\mathbf{J = \Sigma\, xy\, dA}$. J may be either + or −, differing in that respect from the moment of inertia which is always +.



NN, Neutral Axis
M, shows Direction of Flexure
O, Center of Gravity of Section

Fig. 23

If the cross-section of a beam has an axis of symmetry the value of J for this axis and the one at right angles to it is 0. If this is not the case, J may be determined for the common structural shapes by the following graphical method. In Fig. 24 lay off $CA = I_x$ and $AB = I_y$. On CB as a diameter, with center at O, draw a circle, called the **dyadic circle.** In the tables of properties of sections (pp. 673–676 and 679) the minimum radius of gyration of the section $r_{\min}$ is given, then $I_{\min} = r^2_{\min} A$, A being the area of the cross-section. In Fig. 24 lay off $CD = I_{\min}$, and with radius OD draw circle DKF. Then AK perpendicular to CB gives, to scale, the value of J, which is measured in (inches)^4, as are the moments of inertia. Again note that J may be either + or −.

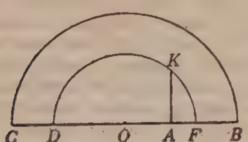
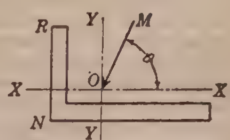


Fig. 24



Note that θ is not the same Angle as α in Fig. 23

Fig. 25

Note that θ is not the same angle as α in Fig. 23

If now a beam is subjected to oblique flexure, and the plane of the bending moment makes an angle θ with the X axis, as illustrated in Fig. 25, the extreme unit stress will be at one of the corners of the cross-section. It is usually possible to tell by inspection which corner will develop the maximum unit-stress, but in case of doubt try two or more corners. As an illustration, in Fig. 25 the extreme unit stress will be developed at either R or N, and it is to be noted that for R the x-coordinate is $-$ and the y-coordinate $+$, while for N both coordinates are $-$. The maximum unit stress is

$$S_{max} = \frac{M}{Q}, \text{ and } Q = \frac{I_x I_y - J}{y\,(I_y \sin\theta - J\cos\theta) + x\,(I_x \cos\theta - J\sin\theta)}$$

in which M denotes the bending moment in inch-pounds, and x and y are the coordinates of the critical corner.

For sections for which either the X axis or the Y axis is an axis of symmetry (I beams, H beams, and T beams), and which are obliquely loaded, J becomes zero, and

$$Q = \frac{I_x I_y}{y\,I_y \sin\theta + x\,I_x \cos\theta}$$

The above treatment of oblique flexure is based on the work of Otto Mohr and of Land. For a full discussion see L. J. Johnson, Trans. Am. Soc. C. E., Vol. LVI, p. 169 (1906).

For detail of drawing the dyadic circle is that used by Prof. H. M. Westergaard of the University of Illinois. Strictly speaking, where the term "axis of symmetry" is used the term "principal axis" would be more precise. The principal axes of an area are two axes at right angles for one of which the moment of inertia is a maximum, and for the other a minimum. For a pair of principal axes the product of inertia is zero. An axis of symmetry is always one of the principal axes of an area.

The Moment of Inertia of an area with reference to any axis may be defined as the sum of the products obtained by multiplying each elementary area of cross-section, da, by the square of the distance of that particular elementary area from the axis. The moment of inertia is represented by I, and

$$\mathbf{I = \Sigma z^2 da}$$

in which z is the distance of the elementary area da from the axis. The **Moment of Inertia** is a factor depending on the shape of the cross-section. From the above expression it will be found that the values of I will be greatest for sections having the largest area at the greatest distance from the axis of reference. Unless otherwise noted, the axis of reference is always a gravity axis of the section.

Section Modulus. This is the term $\mathbf{I/c}$ in the common flexure formula for the resisting moment. It is the measure of the resisting moment or the strength of a beam of given cross-section and is largely used as a basis of computation in the design and investigation of beams. In the table Fig. 22 are given the values of the moment of inertia I and of the section modulus I/c for the various sections most commonly used in structural design.

Investigation and Design. Formulas (1) and (2) are the formulas commonly used in the design and investigation of beams. Except in cases of short spans and heavy loads the question of resisting shear is not usually the controlling factor. Having obtained the bending moment from the conditions of loading in any particular case of design, and having assumed a proper working value for S, formula (2) is solved for I/c and the proper beam selected. In the investigation of beams, M is determined from the conditions of loading and from the kind of beam employed. The proper substitutions are then made in formula (2) and the equation solved for S, which may be wither tension or compression. A comparison of the value of S with the allowable working stress for the material will determine the degree of stability. See also the tabulated statement on p. 588 of the various ways in which beams may fail.

Effect of Combined Shearing and Bending Stress. In a beam of I-section or channel section both the shearing stress and the bending stress at the junction of web and flange may be high, and the combination of the two stresses sometimes causes stress on an inclined plane which is greater than the direct stress in the extreme fibers of the flange. This effect of combined stresses is rarely of importance except for deep beams with short spans. If S_t is the maximum stress on an inclined plane at the point under consideration, S' the direct bending stress at the junction of web and flange, and S'_s the shearing stress at the same point, which is slightly less than the shearing stress for the same cross-section at the neutral axis, then

$$S_t = 1/2\, S' + \sqrt{(S'_s)^2 + (1/2\, S')^2}$$

See also paragraph on Flexure and Tension, p. 598.

Elastic Curve. In a horizontal beam under a system of vertical loads the fibers in the upper surface are shortened, while those in the lower surface are elongated. This causes the beam to deflect downward and the neutral surface to assume the form of a curve whose radius of curvature at any section is dependent upon the bending moment at that section and the moment of inertia of the section.

Thus in Fig. 26 let mn represent the elementary section dl of a beam whose length is l and let $b'b$ and $c'c$ represent two sections separated by the distance dl. These sections, before bending, are assumed to be parallel; after bending, the sections produced intersect at some point o, the distance om being the radius of curvature R. Let c'_1c_1 be drawn parallel to $b'b$ through n. The distance c'_1c' represents the deformation of the upper fibers in the length dl, and the distance cc_1 represents the corresponding deformation in the lower fibers. Let $c'c'_1$ be represented by e; then $e = (S/E)dl$, in which S is the unit stress producing the deformation e, and E is the modulus of elasticity. Let c represent the distance from the neutral surface to the extreme fiber; it then follows that $R/dl = c/e$, whence $R = EI/M$, which expresses the value of the radius of curvature R at any section in terms of the modulus of elasticity, the moment of inertia and the bending moment at the section. When $M = 0$, R = infinity, or the curve at that point is a straight line; when M is a maximum, R is a minimum, or the curve is the sharpest. If the curve is referred to a system of coordinate axes in which x represents abscissas or horizontal distances and y ordinates or vertical distances, giving R its value as determined for very flat curves by the differential calculus in terms of x, y and l; then

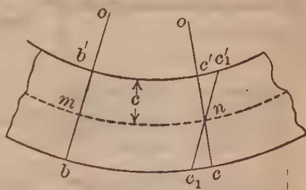


Fig. 26

$$\frac{EI}{R} = EI\frac{d^2y}{dx^2} = M \qquad (3)$$

which is the general differential equation of the elastic curve of any beam under any system of loading expressed in terms of the modulus of elasticity, the moment of inertia and the bending moment.

Slope of Beams. For a beam the integration

$$\int EI\frac{d^2y}{dx^2} = EI\frac{dy}{dx} + C$$

gives an expression involving $\frac{dy}{dx}$, the change of slope, or inclination of the elastic curve for any section of the beam. C is a constant of integration depending upon loading and upon end conditions. A beam so rigidly held that under bending action the slope remains horizontal at a support is said to be fixed at that support, and at the support $\frac{dy}{dx} = 0$.

Deflection. In formula (3), assuming the origin of coordinates at one support, where the deflection of a beam is zero, y becomes the deflection at any point at a distance x from the support or origin of coordinates. Substituting proper values for E, I, and M, integrating twice, and giving proper values to the constants of integration, the value of y, or the deflection, may be determined for any point in the beam.

12. Beams of Uniform Cross-section

Values for shear, V, and for moment, M, at any section; for maximum moment, M_{max}, for slope, dy/dx, and for deflection, y, at any section, and for maximum deflection, y_{max}, are given in the accompanying table for a number of common kinds of beams of uniform cross-section.

It is possible to combine values given in the table for different loadings to give values for loadings not given in the table. For example, by combining the values for a beam loaded with two symmetrical loads with the values for a beam loaded at one point in its span values for a beam with three loads may be obtained.

A beam with ends and loads overhanging the supports may be treated as a combination of a cantilever beam and an end-supported beam.

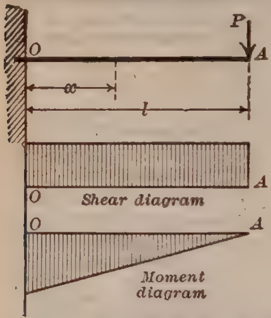


Fig. 27. Cantilever Beam, Concentrated Load at Outer End.

Shear at any section,

$$V = P \text{ (constant)};$$

Moment at any section,

$$M = -P(l - x);$$

$$M_{max} = -Pl \text{ at } O;$$

Equation of slope,

$$dy/dx = (-P/EI)(lx - x^2/2);$$

Equation of elastic curve,

$$y = (-P/EI)\left(\frac{lx^2}{2} - \frac{x^3}{6}\right);$$

Maximum deflection,

$$y_{max} = -Pl^3/3\ EI \text{ at } A.$$

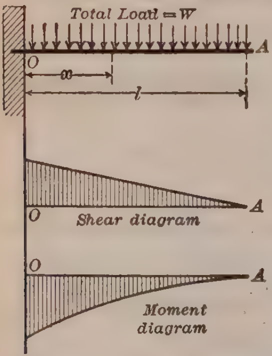


Fig. 28. Cantilever Beam, Uniform Load.

Shear at any section,

$$V = W - \frac{Wx}{l};$$

$$V_{max} = W \text{ at } O;$$

Moment at any section,

$$M = -W\left(l/2 - x + \frac{x^2}{2l}\right);$$

$$M_{max} = -Wl/2 \text{ at } O;$$

Equation of slope,

$$dy/dx = -(W/EI)\left(\frac{lx}{2} - \frac{x^2}{2} + \frac{x^3}{6}\right);$$

Equation of elastic curve,

$$y = -(W/EI)\left(\frac{lx^2}{4} - \frac{x^3}{6} + \frac{x^4}{24\,l}\right);$$

Maximum deflection,

$$y_{max} = -Wl^3/8\ EI \text{ at } A.$$

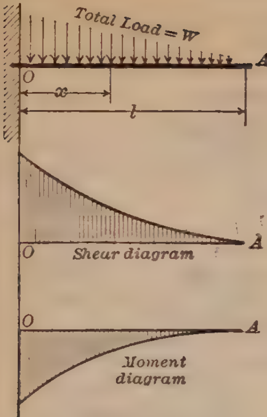


Fig. 29. Cantilever Beam, Load Increasing Uniformly from Free End to Support.

Shear at any section,

$$V = W\left(1 - 2\frac{x}{l} + \frac{x^2}{l^2}\right);$$

$$V_{max} = W \text{ at } O;$$

Moment at any section,

$$M = W\left(\frac{x^3}{3l^2} - \frac{x^2}{l} + x - \frac{l}{3}\right);$$

$$M_{max} = -Wl/3 \text{ at } O;$$

Equation of slope,

$$dy/dx = \frac{W}{EI}\left(\frac{x^4}{12\,l^2} - \frac{x^3}{3l} + \frac{x^2}{2} - \frac{lx}{3}\right);$$

Equation of elastic curve,

$$y = \frac{W}{EI}\left(\frac{x^3}{6} - \frac{x^4}{12\,l} + \frac{x^5}{60\,l^2} - \frac{lx^2}{6}\right);$$

Maximum deflection,

$$y_{max} = -Wl^3/15\,EI \text{ at } A.$$

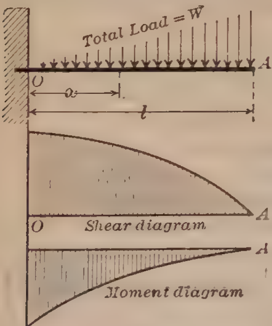


Fig. 30. Cantilever Beam, Load Increasing Uniformly from Support to Free End.

Shear at any section,

$$V = W\left(1 - \frac{x^2}{l^2}\right);$$

$$V_{max} = W \text{ at } O;$$

Moment at any section,

$$M = W\left(x - \frac{x^3}{3\,l^2} - \frac{2}{3}l\right);$$

$$M_{max} = -2/3\,Wl \text{ at } O;$$

Equation of Slope,

$$dy/dx = \frac{W}{EI}\left(\frac{x^2}{2} - \frac{x^4}{12\,l^2} - \frac{2}{3}lx\right);$$

Equation of elastic curve,

$$y = \frac{W}{EI}\left(\frac{x^3}{6} - \frac{x^5}{60\,l^2} - \frac{1}{3}lx^2\right);$$

Maximum deflection,

$$y_{max} = -\frac{11}{60}\frac{Wl^3}{EI} \text{ at } A.$$

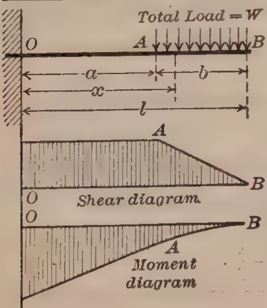


Fig. 31. Cantilever Beam. Uniform Load Over Part of Beam.

Shear at any section,

$$V = W \text{ for } OA;$$

$$V = W - \frac{W}{b}(x - a) \text{ for } AB;$$

Moment for any section,

$$M = -W\left(l - x - \frac{b}{2}\right) \text{ for } OA;$$

$$M = -\frac{W}{2b}(l - x)^2 \text{ for } AB;$$

$$M_{max} = -W\left(l - \frac{b}{2}\right) \text{ at } O.$$

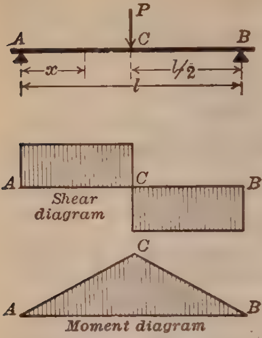


Fig. 32. Beam Supported at Ends, Concentrated Load at Middle of Span.

End reactions, $R_A = R_B = P/2$;

Shear at any section, $V = \pm P/2$;

Moment at any section,

$$M = Px/2 \text{ for } AC;$$

$$M_{max} = Pl/4 \text{ at } C;$$

Equation of slope,

$$\frac{dy}{dx} = \frac{P}{EI}\left(\frac{x^2}{4} - \frac{l^2}{16}\right);$$

Equation of elastic curve,

$$y = \frac{P}{EI}\left(\frac{x^3}{12} - \frac{l^2x}{16}\right);$$

Maximum deflection,

$$y_{max} = -\frac{Pl^3}{48\,EI} \text{ at mid span.}$$

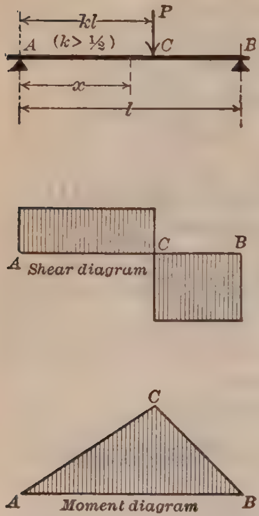


Fig. 33. Beam Supported at Ends, Concentrated Load at Any Point in Span.

End Reactions,

$$R_A = P(1-k);$$

$$R_B = Pk;$$

Shear at any section,

$$V = P(1-k) \text{ for } AC;$$

$$V = -Pk \text{ for } CB;$$

Moment at any section,

$$M = P(1-k)x \text{ for } AC;$$

$$M = Pk(l-x) \text{ for } CB;$$

$$M_{max} = Pkl(1-k) \text{ at } C;$$

Equation of slope,

$$dy/dx = \frac{P}{2EI}\left[(1-k)x^2 - \left(\frac{2}{3}k - k^2 + \frac{k^3}{3}\right)l^2\right]$$

for AC;

$$dy/dx = \frac{P}{EI}\left(lx - \frac{x^2}{2} - \frac{k^2l^2}{6} - \frac{l^2}{3}\right)k \text{ for } CB;$$

Equation of elastic curve,

$$y = \frac{P}{6EI}[(1-k)x^3 - (2k - 3k^2 + k^3)l^2x] \text{ for } AC;$$

$$y = \frac{P}{6EI}[-x^3 + 3lx^2 - k^2l^2x - 2l^2x + k^2l^3]k$$

for CB;

Maximum deflection,

$$y_{max} = -\frac{Pl^3}{3EI}(1-k)\left(\frac{2}{3}k - \frac{1}{3}k^2\right)^{3/2}$$

$$\text{at } x = l\sqrt{\frac{2}{3}k - \frac{k^2}{3}}.$$

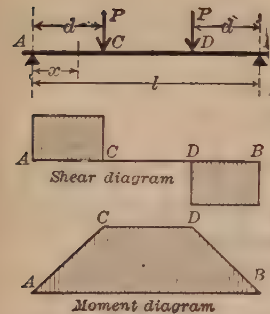


Fig. 34. Beam Supported at Ends with Two Equal, Symmetrical, Concentrated Loads.

End reactions,

$$R_A = R_B = P;$$

Shear,

$$V = P \text{ for } AC,\ O \text{ for } CD,\ -P \text{ for } DB;$$

Moment,

$$M = Px \text{ for } AC,\ Pd \text{ for } CD,\ P(l-x) \text{ for } DB;$$

$$M_{\max} = Pd \text{ at any section between } C \text{ and } D;$$

Equation of slope,

$$dy/dx = \frac{P}{2\,EI}(x^2 + d^2 - dl) \text{ for } AC;$$

$$dy/dx = \frac{Pd}{EI}\left(x - \frac{l}{2}\right) \text{ for } CD;$$

Slopes for DB symmetrical with those for AC.

Equation of elastic curve,

$$y = (P/EI)\left(\frac{x^3}{6} + \frac{d^2x}{2} - \frac{dlx}{2}\right) \text{ for } AC;$$

$$y = (Pd/EI)\left(\frac{x^2}{2} - \frac{lx}{2} + \frac{d^2}{6}\right) \text{ for } CD;$$

Elastic curve for DB symmetrical with that for AC.

Maximum deflection,

$$y_{\max} = \frac{Pd}{24\,EI}(4\,d^2 - 3\,l^2) \text{ at mid span.}$$

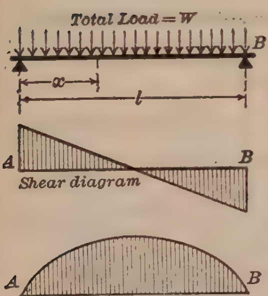


Fig. 35. Beam Supported at Ends, Uniform Load.

End reactions,

$$R_A = R_B = W/2;$$

Shear at any section,

$$V = W/2 - \frac{Wx}{l};$$

$$V_{\max} = \pm\frac{W}{2} \text{ at supports;}$$

Moment at any section,

$$M = \frac{Wx}{2} - \frac{Wx^2}{2\,l};$$

$$M_{\max} = \frac{Wl}{8} \text{ at mid span;}$$

Equation of slope,

$$\frac{dy}{dx} = \frac{W}{24\,EI}\left(6\,x^2 - \frac{4\,x^3}{l} - l^2\right);$$

Equation of elastic curve,

$$y = \frac{W}{24\,EI}\left(2\,x^3 - \frac{x^4}{l} - l^2x\right);$$

Maximum deflection,

$$y_{\max} = -\frac{5}{384}\frac{Wl^3}{EI} \text{ at mid span.}$$

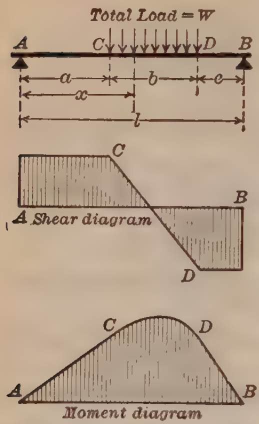


Fig. 36. Beam Supported at Ends, Uniform Load over Part of Span.

End reactions,

$$R_A = \frac{W\,(2\,c + b)}{2\,l};$$

$$R_B = \frac{W\,(2\,a + b)}{2\,l};$$

Shear at any section,

$$V = R_A \text{ for } AC;$$

$$V = R_A - \frac{W}{b}\,(x - a) \text{ for } CD;$$

$$V = -\,R_B \text{ for } DB;$$

Moment for any section,

$$M = R_A x \text{ for } AC;$$

$$M = R_A x - \frac{W}{2\,b}\,(x - a)^2 \text{ for } CD;$$

$$M = R_B\,(l - x) \text{ for } DB;$$

$$M_{\max} = R_A\left(\frac{a + R_A b}{2\,W}\right) \text{ at } x = a + \frac{R_A b}{W}.$$

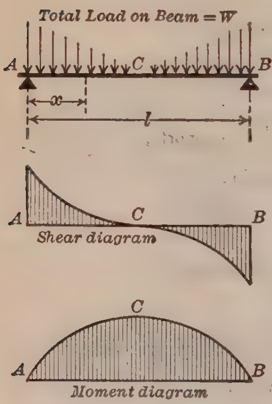


Fig. 37. Beam Supported at Ends, Load Increasing Uniformly from Center to Ends.

End reactions,

$$R_A = R_B = W/2;$$

Shear at any section,

$$V = -\,W\left(\frac{2\,x}{l} - \frac{2\,x^2}{l^2} - \frac{1}{2}\right) \text{ for } AC; \text{ shear}$$

diagram symmetrical about mid span;

$$V_{\max} = \pm\,W/2 \text{ at supports;}$$

Moment at any section,

$$M = Wx\left(\frac{1}{2} - \frac{x}{l} + \frac{2}{3}\,\frac{x^2}{l^2}\right) \text{ for } AC; \text{ moment}$$

diagram symmetrical about mid span;

$$M_{\max} = Wl/12 \text{ at } C;$$

Equation of slope,

$$\frac{dy}{dx} = \frac{W}{EI}\left(\frac{x^2}{4} - \frac{x^3}{3\,l} + \frac{x^4}{6\,l^2} - \frac{l^2}{32}\right);$$

Equation of elastic curve,

$$y = -\,\frac{Wx}{EI}\left(\frac{l^2}{32} - \frac{x^2}{12} + \frac{x^3}{12\,l} - \frac{x^4}{30\,l^2}\right);$$

Maximum deflection,

$$y_{\max} = -\,\frac{3}{320}\,\frac{Wl^3}{EI} \text{ at } C.$$

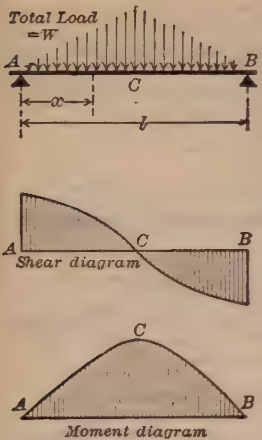


Fig. 38. Beam Supported at Ends, Load Increasing Uniformly from Ends to Center.

End Reactions,

$$R_A = R_B = W/2;$$

Shear at any section,

$$V = -W\left(2\frac{x^2}{l^2} - \frac{1}{2}\right) \text{ for } AC;$$

Shear diagram symmetrical about mid span;

$$V_{max} = \pm W/2 \text{ at supports;}$$

Moment of any section,

$$M = Wx\left(\frac{1}{2} - \frac{2}{3}\frac{x^2}{l^2}\right);$$

$$M_{max} = Wl/6 \text{ at } C;$$

Equation of slope,

$$\frac{dy}{dx} = -\frac{W}{EI}\left(\frac{x^4}{6\,l^2} - \frac{x^2}{4} + \frac{5}{96}l^2\right);$$

Equation of elastic curve,

$$y = -\frac{Wx}{EI}\left(\frac{x^4}{30\,l^2} - \frac{x^2}{12} + \frac{5}{96}l^2\right);$$

Maximum deflection,

$$y_{max} = \frac{Wl^3}{60\,EI} \text{ at mid span.}$$

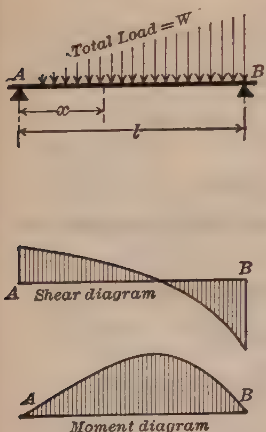


Fig. 39. Beam Supported at Ends, Load Varying Uniformly from One End to the Other.

End reactions,

$$R_A = W/3, \quad R_B = 2/3\,W;$$

Shear at any section,

$$V = -W\,(x^2/l^2 - 1/3);$$

$$V_{max} = -2/3\,W \text{ at } B;$$

Moment at any section,

$$M = -\frac{Wx}{3}\left(\frac{x^2}{l^2} - 1\right);$$

$$M_{max} = 0.128\,Wl \text{ at } x = 0.577\,l;$$

Equation of slope,

$$\frac{dy}{dx} = -\frac{W}{EI}\left(\frac{x^4}{12\,l^2} - \frac{x^2}{6} + \frac{7}{180}l^2\right);$$

Equation of elastic curve,

$$y = -\frac{Wx}{EI}\left(\frac{x^4}{60\,l^2} - \frac{x^2}{18} + \frac{7}{180}l^2\right);$$

Maximum deflection,

$$y_{max} = 0.0131\,\frac{Wl^3}{EI} \text{ at } x = 0.52\,l.$$

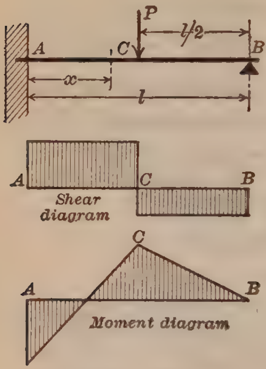


Fig. 40. Beam Fixed at One End, Supported at the Other, Concentrated Load at Mid Span.

End reaction at B,

$R_A = 11/16\ P$;

Shear at any section,

$V = +\ 11/16\ P$ for AC;

$V = -\ 5/16\ P$ for CB;

Moment at any section,

$M = P\ (11/16\ x - 3/16\ l)$ for AC;

$M = 5/16\ P\ (l - x)$ for CB;

Moment at mid span,

$M_C = 5/32\ Pl$;

Moment at fixed end,

$M_A = -\ 3/16\ Pl$;

Inflection point (M changes sign) at $x = 3/11 l$;

Equation of slope,

$$\frac{dy}{dx} = \frac{P}{EI}\left(\frac{11}{32}x^2 - \frac{3}{16}lx\right) \text{ for } AC;$$

$$\frac{dy}{dx} = \frac{P}{EI}\left(\frac{5}{16}lx - \frac{5}{32}x^2 - \frac{l^2}{8}\right) \text{ for } CB;$$

Equation of elastic curve,

$$y = \frac{Px^2}{EI}\left(\frac{11}{96}x - \frac{3}{32}l\right) \text{ for } AC;$$

$$y = \frac{P}{EI}\left(\frac{5}{32}lx^2 - \frac{5}{96}x^3 - \frac{l^2x}{8} + \frac{l^3}{48}\right) \text{ for } CB;$$

Maximum deflection,

$$y_{max} = -\ 0.0093\frac{Pl^3}{EI} \text{ at } x = 0.553\ l.$$

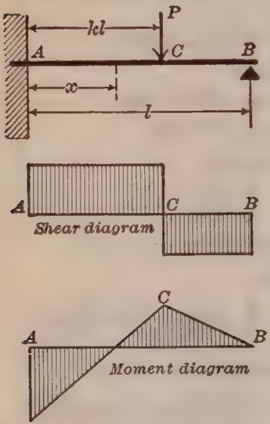


Fig. 41. Beam Fixed at One End, Supported at the Other, Concentrated Load at any Point.

Shear for AC,

$$V = (P/2)\ (2 - 3\ k^2 + k^3);$$

Reaction at supported end. Shear for CB,

$$R_B = V = \frac{P}{2}\ (3\ k^2 - k^3);$$

Moment for any section,

$$M = \frac{P}{2}\ [x\ (2 - 3\ k^2 + k^3) - l\ (2\ k - 3\ k^2 + k^3)] \text{ for } AC;$$

$$M = -\frac{P}{2}\ [x\ (3\ k^2 - k^3) + l\ (k^3 - 3\ k^2)] \text{ for } CB;$$

Moment at fixed end,

$$M_A = -\frac{Pl}{2}\ (2\ k - 3\ k^2 + k^3);$$

Moment under load,

$$M_C = \frac{Pl}{2}\ (3\ k^2 - 4\ k^3 + k^4).$$

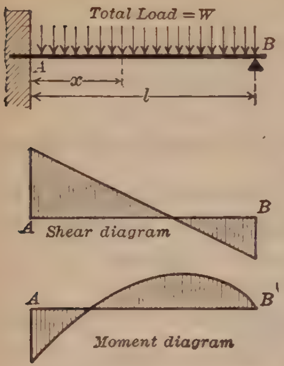


Fig. 42. Beam Fixed at One End, Supported at the Other, Uniform Load.

End reaction at B,

$$R_B = 3/8\,W;$$

Shear at any section,

$$V = -W\left(\frac{x}{l} - 5/8\right);$$

$$V_{\max} = 5/8\,W \text{ at fixed end;}$$

Moment at any section,

$$M = -W\left(\frac{x^2}{2\,l} - \frac{5}{8}x + \frac{l}{8}\right);$$

Moment at fixed end,

$$M_A = -Wl/8;$$

Maximum moment in span,

$$M_{\max} = 9/128\,Wl \text{ at } x = 5/8\,l;$$

Inflection point (M changes sign) at $x = l/4$;

Equation of slope,

$$\frac{dy}{dx} = -\frac{W}{EI}\left(\frac{x^3}{6\,l} - 5/16\,x^2 + \frac{lx}{8}\right);$$

Equation of elastic curve,

$$y = -\frac{Wx^2}{EI}\left(\frac{x^2}{24\,l} - \frac{5}{48}x + \frac{l}{16}\right);$$

Maximum deflection,

$$y_{\max} = 0.00543\,\frac{Wl^3}{EI} \text{ at } x = 0.578\,l.$$

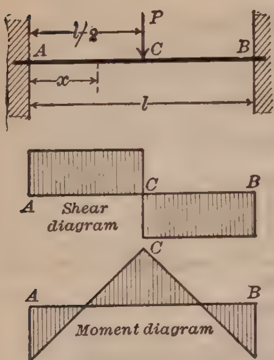


Fig. 43. Beam Fixed at Both Ends. Concentrated Load at Mid Span.

Shear at any section,

$$V = \pm\frac{P}{2};$$

Moment at any section,

$$M = P\left(\frac{x}{2} - \frac{l}{8}\right) \text{ for } AC;$$

Moment diagram symmetrical about mid span;

Moment at fixed ends,

$$M_A = M_B = -Pl/8;$$

Moment at mid span,

$$M_C = +Pl/8;$$

Inflection points (M changes sign) at $x = l/4$ and $x = 3/4\,l$;

Equation of slope,

$$\frac{dy}{dx} = \frac{Px}{EI}\left(\frac{x}{4} - \frac{l}{8}\right) \text{ for } AC;$$

Equation of elastic curve,

$$y = \frac{Px^2}{EI}\left(\frac{x}{12} - \frac{l}{16}\right) \text{ for } AC;$$

Slope curve and elastic curve symmetrical about mid span;

Maximum deflection,

$$y_{\max} = \frac{Pl^3}{192\,EI} \text{ at mid span.}$$

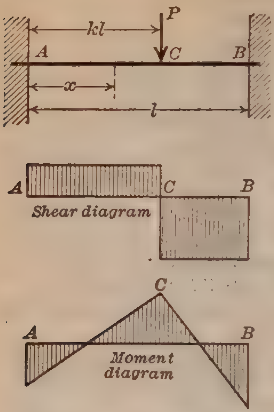


Fig. 44. Beam Fixed at Both Ends, Concentrated Load at any Point in Span.

Shear,

$$V = P\,(1 - 3\,k^2 + 2\,k^3) \text{ for } AC;$$
$$V = -\,Pk^2\,(3 - 2\,k) \text{ for } CB;$$

Moment,

$$M = P\,[(1 - 3\,k^2 + 2\,k^3)x - l\,(k - 2\,k^2 + k^3)] \text{ for } AC;$$
$$M = P\,[(2\,k^3 - 3\,k^2)\,x + (2\,k^2 - k^3)l] \text{ for } CB;$$

Moment at fixed ends,

$$M_A = -\,Plk\,(1 - 2\,k + k^2);$$
$$M_B = -\,Plk^2\,(1 - k);$$

Moment under load,

$$M_C = Plk^2\,(2 - 4\,k + 2\,k^2).$$

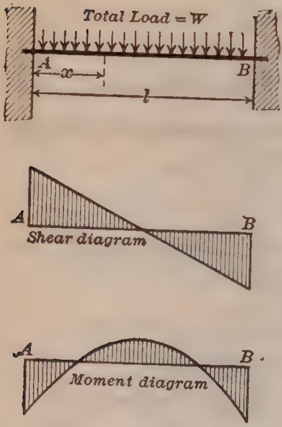


Fig. 45. Beam Fixed at Both Ends, Uniform Load.

Shear at any section,

$$V = -\,W\left(\frac{x}{l} - 1/2\right);$$

$V_{max} = \pm\,W/2$ at ends;

Moment at any section,

$$M = -\,W\left(\frac{x^2}{2\,l} - \frac{x}{2} + \frac{l}{12}\right);$$

Moment at fixed ends,

$$M_A = M_B = -\,\frac{Wl}{12};$$

Moment at mid span,

$$M_C = \frac{Wl}{24};$$

Inflection points (M changes sign) at $x = 0.211\,l$ and $x = 0.789\,l$;

Equation of slope,

$$\frac{dy}{dx} = -\,\frac{Wx}{EI}\left(\frac{x^2}{6\,l} - \frac{x}{4} + \frac{l}{12}\right);$$

Equation of elastic curve,

$$y = -\,\frac{Wx^2}{EI}\left(\frac{x^2}{24\,l} - \frac{x}{12} + \frac{l}{24}\right);$$

Maximum deflection,

$$y_{max} = -\,\frac{Wl^3}{384\,EI} \text{ at mid span.}$$

13. Beams of Uniform Strength

General Principles. The common flexure formula for the investigation and design of beams, $M = SI/c$, indicates that the stress S varies with the bending moment. In beams of uniform strength, S must be constant. Therefore from the flexure formula, since the bending moment varies, the quantity I/c, or section modulus, must be made to vary with the bending moment, in order to provide uniformity of strength for all sections under various conditions of loading. Only beams which are rectangular in cross-section will be considered. From the table in Fig. 22, $\mathbf{I/c = 1/6\ bh^2}$. Substituting in the flexure formula there results

$$\mathbf{M = 1/6\ Sbh^2} \qquad (1)$$

from which beams of uniform strength may be designed, giving proper values to the bending moment M, determined from the conditions of loading.

Deflection of Beams of Uniform Strength. The accompanying table gives values for dimensions, stresses, and deflections for the common kinds of uniform strength beams. (Figs. 46, 46*a*, 46*b*, 46*c*.)

Dimensions Stresses, and Deflections for Uniform Strength Beams with Rectangular Cross-section

At the ends of the beams where shear governs the design, the dimensions of the cross-section must be increased.

I. Cantilever beam, constant depth, load at outer end. Fig. 46.

Depth $= h$; width, $b = b_1 \dfrac{x}{l}$.

Fiber stress due to flexure, $S = \dfrac{6\ Pl}{b_1 h^2}$.

Maximum deflection, $y_{max} = \dfrac{6\ Pl^3}{Eb_1 h^3}$.

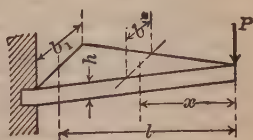


Fig. 46

Note. A flat spring made up of a number of leaves held together by a band acts approximately like a beam of varying width. b = the sum of the widths of the leaves piled together at any section.

II. Cantilever beam, constant width, load at outer end. Fig. 46*a*.

Depth, $h = h_1 \sqrt{\dfrac{x}{l}}$; width $= b$.

Fiber stress due to flexure, $S = \dfrac{6\ Pl}{bh_1^2}$.

Maximum deflection, $y_{max} = \dfrac{8\ Pl^3}{Ebh_1^3}$.

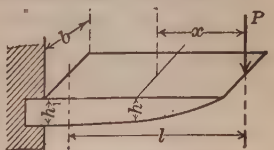


Fig. 46*a*

III. Beam supported at ends, constant depth, load at mid span. Fig. 46*b*.

Depth $= h$; width, $b = \dfrac{2\ b_1 x}{l}$.

Fiber stress due to flexure, $S = \dfrac{3}{2} \dfrac{Pl}{b_1 h^2}$.

Maximum deflection, $y_{max} = \dfrac{3}{8} \dfrac{Pl^3}{Eb_1 h^3}$.

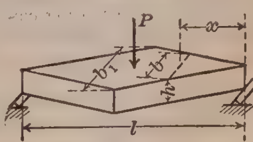


Fig. 46*b*

See note under I, above.

IV. Beam supported at ends, constant width, load at mid span. Fig. 46c.

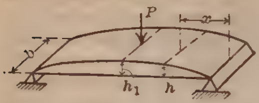


Fig. 46c

Depth, $h = h_1\sqrt{\frac{2x}{l}}$; width $= b$;

Fiber stress due to flexure, $S = \frac{3}{2}\frac{Pl}{bh_1^2}$.

Maximum deflection, $y_{max} = \frac{Pl^3}{2\,Ebh_1^3}$.

In the table of beams of uniform strength, the bending moment has been assumed to be the controlling factor. The shearing strength of the material, however, would modify a number of these forms near the ends, and this fact should be taken into account in actual practice. While the discussion of beams of uniform strength is of considerable interest, theoretically, it is of little practical value, except in the case of plate girders, which are designed in a manner somewhat different from that discussed above, and in the case of flat springs.

14. Continuous Beams

A Continuous Beam is one for which there are more than two supports. In the discussion here given it will be assumed that the supports are all on one level and that the beam is of constant cross-section. The expressions for the resisting shear, for the resisting moment SI/c, and the differential equation for the elastic curve $M = EI\,(d^2y/dx^2)$ apply to continuous beams with the same force as they do to simple, cantilever, or constrained beams. The elements which must be determined in order to apply these expressions are the vertical shears and the bending moments under various conditions of loading. Since the number of reactions will always be greater than two, it is obvious that they cannot be determined by the usual method of moments. It will be noted, moreover, that the several reactions of a continuous beam will depend for their values upon the elastic behavior of the beam under load. Therefore, instead of determining the bending moments from the reactions in the case of continuous beams, the process must be reversed and the bending moments must be determined from the elastic behavior of the beam under load and the reactions and vertical shears determined from the bending moments.

General Formulas. As in the case of simple beams, the vertical shear at any section is the algebraic sum of the reactions and the loads on the left of that section. Let V be the vertical shear at any section distant x to the right of any support. Let V' be the vertical shear at a section to the right of but immediately adjacent to the support. Let ΣP_1 denote the sum of the concentrated loads on the distance x, and let w be the uniform load per linear unit; then

$$\mathbf{V = V' - wx - \Sigma P_1} \qquad (1)$$

Assume any two supports of a continuous beam. The bending moment M at any section distant x from the left support being the algebraic sum of the moments of all forces to the left of that section, rotation in the direction of the hands of a clock being considered positive, and rotation in the opposite direction negative, assuming M' the moment at the left support, then

$$\mathbf{M = M' + V'x - 1/2\,wx^2 - \Sigma\,P_1\,(x - kl)} \qquad (2)$$

in which k is a fraction less than unity and l the distance between supports. If M'' be the moment at the right support, there results the relation

$$\mathbf{V'l = M'' - M' + 1/2\,wl^2 + \Sigma\,P_1\,(1 - kl)} \qquad (3)$$

From formulas (1), (2), and (3) it appears that V and M can always be determined, when M' and M'', or the bending moments at the supports, are known.

Theorem of Three Moments for Uniformly Distributed Load. For the purpose of determining the bending moments at the supports, which from the preceding paragraph it appears are necessary for the determination of the bending moment and the vertical shear at any section, the relation existing between the moment at any support in a continuous beam and the moments at the supports on either side is utilized. This relation between the moments at any three consecutive supports constitutes the theorem of three moments. In Fig. 47 let M', M'', and M''' be the moments at any three consecutive supports in a continuous beam, and let V' and V'' be the vertical shears on the right of the supports where the moments are respectively M' and M''. Let the span between the first and second supports be l' and between the second and third supports l'', and let the uniform load on the first span be w' per linear unit, and the uniform load on the second span w'' per linear unit. From the properties of the elastic curves, and by substituting for V' and V'' their values in terms of M', and M'', and M''' obtained from the use of formula (3), there follows the relation

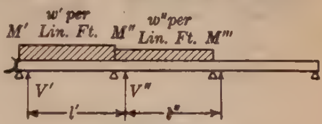


Fig. 47

$$M'l' + 2M''(l' + l'') + M'''l'' = -1/4\, w'l'^3 - 1/4\, w''l''^3 \qquad (4)$$

Formula (4) is the general expression for the theorem of three moments under uniform loads. When $w' = w''$ and when $l' = l''$, or, in other words, when the load is uniform throughout and equal to w per linear unit and the spans equal, formula (4) becomes

$$M' + 4M'' + M''' = -1/2\, wl^2 \qquad (5)$$

Formulas (4) and (5) are used as follows: In any continuous beam of n spans there will be $n + 1$ supports, and assuming the beam not to overhang the end supports, there will be $n - 1$ unknown moments, the moments at the end supports being each zero. An equation in the form of (4) or (5) is written for each of the unknown moments at the supports, and the solution of these equations will give the unknown quantities M', M'', M''', and the like. Substituting M' and M'' in formula (3), the vertical shear V' at a support may be determined, from which the bending moment at any point may be found by means of formula (2) The accompanying Fig. 48 gives reactions, shears, and bending moments for continuous beams with uniform load over from two to five equal spans.

Theorem of Three Moments for Concentrated Loads. If M', M'', and M''' are the moments at three successive supports of a continuous beam shown in Fig. 49, and P' and P'' are concentrated loads in the two spans considered, P' being distant an amount a from the support where the moment is M', and P'' being distant an amount b from the support, where the moment is M''', then the theorem of three moments becomes

$$M'l' + 2M''(l' + l'') + M'''l'' = -\frac{P'a\,(l'^2 - a^2)}{l'} - \frac{P''b\,(l''^2 - b^2)}{l''} \qquad (6)$$

In all cases load per unit length = w

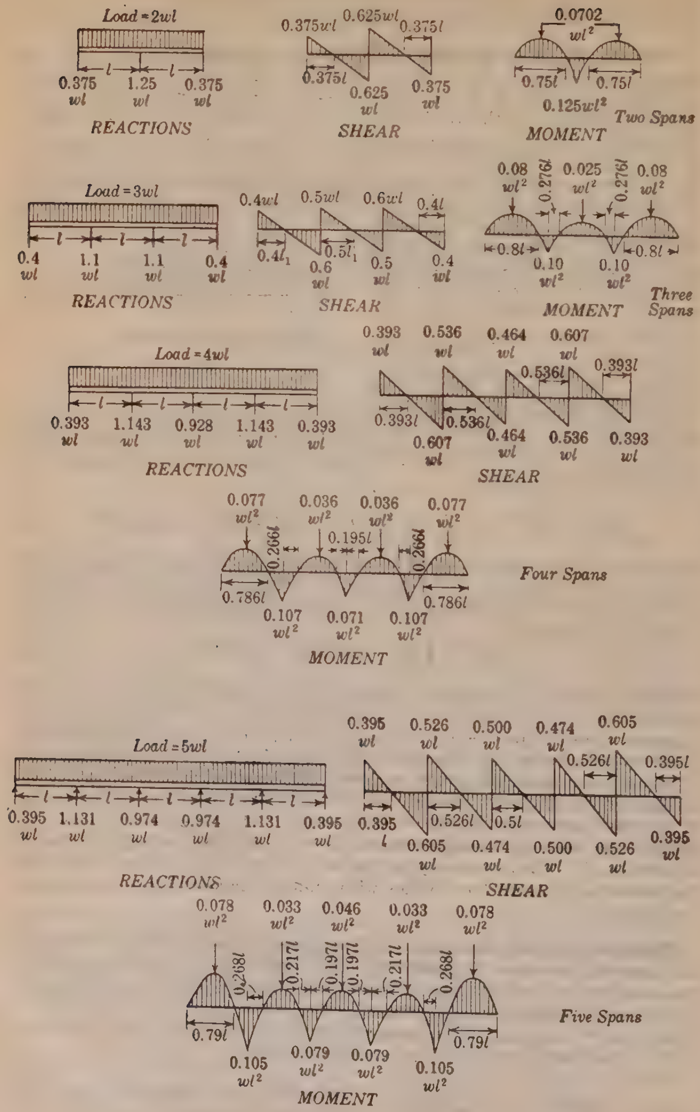


Fig. 48. Continuous Beams with Uniform Load and Equal Spans

If all the spans are the same length l, and all the loads are equal and located at the mid-points of the spans this reduces to

$$M' + 4\,M'' + M''' = -\,3/4\,Pl$$

For a combination of uniformly distributed load and concentrated load or for spans containing more than one concentrated load the moments and shears may be computed separately for uniformly distributed loading and for single concentrated loads in each span, and the resulting moments and shears superposed for the combined loading.

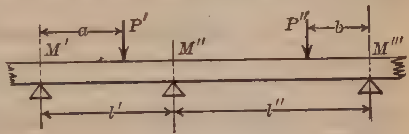


Fig. 49. . Continuous Beam Notation

15. Tests of Beams

Phenomena. In the discussion of beams, the general flexure formula $M = SI/c$ and the formulas for deflection are based upon the assumption that the values of E for stress and deformation below the neutral axis are the same as for stress and deformation above the neutral axis. While in most cases there is a slight difference between the value of E determined from compression tests and its value determined from tension tests, for the materials of which beams are usually constructed (steel, wrought iron, or timber), the difference is so slight that they may be assumed to be the same, as is usually done, without appreciable error. The formulas for the resisting moment and deflection are obviously valid only when the stresses produced are within the elastic limit. Beams under test loads, when the loads are small compared to the ultimate capacity and the deflections slight, exhibit phenomena which tend to confirm the assumption made regarding the distribution of stresses and the deflection. In tests on beams the measured deflections agree closely with the computed deflections and are proportional to the applied loads, as is indicated in the deflection formulas, up to a certain point, beyond which the deflections are found to increase more rapidly than the loads, indicating the elastic limit of the beam.

Modulus of Rupture. Since the expression $M = SI/c$ is no longer valid after the elastic limit of the material has been exceeded, it cannot be applied to determine actual conditions after that point has been reached. For purposes of comparison of brittle materials, however, it has been found convenient to determine the value of S from this formula when M equals the bending moment causing rupture. The value of S thus obtained for any material is termed the **modulus of rupture** and is usually found to lie between the ultimate tensile and the ultimate compressive strength. It is to be noted that the modulus of rupture does not express the actual stress in the extreme fiber of a beam, but is a quantity useful only as a basis of comparison.

Average values for the moduli of rupture of common structural materials are as follows, in pounds per square inch:

Brick	800	Timber	9 000
Stone	2000	Cast iron	40 000

It has been noted from tests that the shape of cross-section and the ratio of diameter to span affect to some extent the value of the modulus of rupture.

The manner of failure of beams under test depends upon the material of which they are constructed and the relative shape. Beams of brittle materials, such as plain concrete, stone or cast iron, fail by the fibers giving way in tension. In other cases where the material is less brittle and more plastic, such as timber, the beam fails by the crushing of the compression fibers. Beams of ductile materials, such as structural steel, bend when the yield point is reached and no value for the modulus of rupture can be obtained. Beams that are very deep in comparison to their length sometimes fail by the material shearing off near the supports. This manner of failure is, however, rare, except in the case of plain concrete or stone beams, where failure sometimes takes place along a diagonal line near a support.

16. Methods of Failure of Beams

The accompanying table gives a number of kinds of typical failures of beams with the general precaution to be observed in order to prevent such failures.

Typical Failures of Beams and General Methods of Prevention

Kind of beam	Danger of failure by	Precaution against failure
All beams.	Rupture or excessive deformation of extreme fibers.	Sufficiently large section modulus, I/c.
Long-span I beams and channel beams; long-span timber beams.	Sidewise buckling of compression fibers.	Additional material on compression side; sidewise bracing, low working stress in flexure formula, $S = Mc/I$.
Short-span beams.	Buckling of web of beam.	Sufficient thickness of web to resist diagonal column action; stiffening angles riveted to web.
Short-span beams with thin webs.	Excessive shearing stress in web.	Sufficient thickness of web.
Wooden beams.	Horizontal shear along neutral axis.	Sufficient breadth of beam to allow low working stress in shear.
Short beams, beams with short end-bearings, beams carrying concentrated loads.	Crushing of web adjacent to load or to end-bearing.	Sufficient thickness of web to carry compression under load or over bearing block; stiffening angles riveted to web and fitted against flange of beam under load or over bearing block.

17. Resilience and Impact on Beams

Resilience of a beam is the measure of its elastic resistance to external work. If in a simple beam a load P is gradually applied at the center until the beam comes to rest with a deflection of f, the load P has acted over the path f with a mean intensity of 1/2 P, so that the external work K producing the deflection f is $K = 1/2\ Pf$. This formula will also express the resilience of the beam provided the load P has been applied so that no energy has been lost in producing heat. For any beam of length l and under any conditions of loading, the work K may be expressed in terms of the volume of the beam by substituting for P its value obtained from the general flexure formula for the particular condition of loading, and substituting for f its value obtained from the proper deflection formula, thus:

$$\mathbf{K} = \frac{\mathbf{a}^2}{\mathbf{b}}\left(\frac{\mathbf{r}}{\mathbf{c}}\right)^2 \frac{1}{2}\frac{\mathbf{S}^2}{\mathbf{E}}\mathbf{A}l$$

in which a is a coefficient depending upon the kind of beam and the manner of loading being 1 for a cantilever loaded at the end, 2 for a cantilever with uniform load, and 4 for a simple beam loaded at the middle; b is a coefficient obtained from the formula for deflection, being 3 for a cantilever loaded at the end and 48 for a simple beam loaded at the middle and r is the radius of gyration of the section with respect to the neutral axis and is equal to $\sqrt{I/A}$, where A is the sectional area. The quantity Al is the volume of the beam.

Stresses Due to Impact. The effect of a weight falling upon a beam is to produce a deflection considerably greater than would be produced by the same weight being applied to the beam gradually or in small increments. Let P be a load which when falling from a height h on a horizontal beam produces a maximum deflection d_1. The work performed in producing this deflection is therefore $P\,(h + d_1)$. Let S_1 be the unit stress in the extreme fiber produced by the load P falling from the height h and S the unit stress produced by a static load P, and let d be the deflection which would be produced by the static load P. The elastic limit not being exceeded, the deflections are proportional to the unit stresses or $d_1/d = S_1/S$, or if Q be the static load which would produce the deflection d_1, then $d_1/d = Q/P$, or $Q/P = S_1/S$. If none of the energy is lost in producing heat or giving movement to the beam, the external work and the internal energy stored in the beam at the moment the deflection reaches d_1 may be expressed by $1/2\,Qd_1$, whence $1/2\,Qd_1 = P\,(h + d_1)$, whence by combination with the above

$$S_1 = S + S\left(1 + \frac{2\,h}{d}\right)^{1/2} \quad \text{and} \quad d_1 = d + d\left(1 + \frac{2\,h}{d}\right)^{1/2}$$

These formulas give the maximum unit fiber stress and the maximum deflection due to the dynamic load P. In place of h in the above discussion its value $V^2/2\,g$ may be substitured, in which V is the velocity at the moment of impact and g the acceleration due to gravity, usually taken at 32.16 feet per second per second. S may be obtained from the general formula for flexure, $M = SI/c$, and d from the proper formula for deflection under a static load.

If a force P moving horizontally with a velocity V strikes a beam the ends of which are secured against horizontal movement, producing a lateral deflection d_1, the work expended in producing this deflection may be expressed by Pd_1, whence

$$S_1 = S\,(2\,h/d)^{1/2} \quad \text{and} \quad d_1 = d\,(2\,h/d)^{1/2}$$

give values for the maximum unit stress in the extreme fiber under impact from a horizontal load moving with a velocity V, in which $h = V^2/2\,g$.

18. Columns

A Column is a structural member in which the length is such compared with its least lateral dimension that failure under compression is most likely to occur by the rupture of the material in the extreme fiber owing to flexural stresses accompanying lateral deflection. Any compression member in which the length exceeds 8 or 10 times its least lateral dimension is usually termed a column, strut, or post.

Columns differ from other compression members in that in a column it is assumed that an unequal distribution of stress is liable to take place, resulting in flexure and lateral deflection. While theoretically an axial load in a perfectly straight symmetrical member can produce only uniformly distributed stresses with consequently no flexure, actual experience teaches that when the length exceeds 8 or 10 times the least lateral dimension appreciable flexure will result owing to the fact that it is practically impossible to construct an absolutely straight and symmetrical member and to load it absolutely axially. It is evident, moreover, that under compression a lateral deflection once present, due to whatever cause, tends to increase with a corresponding increase in the flexural stress.

The Length of a Column is usually taken as the distance between the points at which it is rigidly secured against lateral deflection. A long compression member with a number of points of support dividing it into several sections may consist of several columns as in the case of the columns in a high steel building. In determining the unsupported length of each section, however, only such points of support are to be considered as will prevent deflection of the column in any direction.

Radius of Gyration. Since the strength of a column is so largely dependent upon its ability to resist flexural stress, the moment of inertia of its cross-section is an important factor in the determination of its carrying capacity. In place of the moment of inertia, however, for the purpose of comparison it has been found more convenient to use the **Radius of Gyration**, which is a linear dimension. If I is the moment of inertia of a section with reference to an axis, A the area of the section, and r the radius of gyration with reference to that axis, then the value of the radius of gyration in terms of the area and the moment of inertia may be found from the relation $r^2 = I/A$. Unless otherwise noted the radius of gyration is always taken with respect to the axis through the center of gravity of the cross-section.

The axis of a column is a line drawn through the centers of gravity of the cross-sections. If the column be perfectly symmetrical the moment of inertia and the radius of gyration will be the same for every axis through the center of gravity of every section. In many columns used in structural design the cross-sections are not perfectly symmetrical, and there are therefore several values for the radius of gyration, depending upon the position of the axis through the center of gravity. In the design of columns, therefore, it is necessary to know the value for the least radius of gyration in each particular case.

The radius of gyration for a circle is $d/4$, where d is the diameter. For a rectangle of height d and width b the radius of gyration for an axis parallel to d is $0.289\,b$ and that for an axis parallel to b is $0.289\,d$. For sections of rolled beams and shapes the values of the radius of gyration are given in Arts. 46–48.

The Slenderness Ratio of a column is the number obtained by dividing the length by the radius of gyration. The length and the radius of gyration both being linear dimensions, the slenderness ratio is an abstract number. The radius of gyration and the length must be expressed in the same linear unit, and since it is current practice to give the radius of gyration in inches, it is necessary also to reduce the length of the column to inches before the slenderness ratio can be obtained.

Condition of Ends of Columns. The strength of a column being dependent largely upon its ability to resist lateral deflection, the condition of the ends has a marked effect upon the carrying capacity. The various conditions which may exist at the ends of columns are usually divided into four classes. Columns with **round ends** are such that at the bearing at either end there is perfect freedom of motion, as would be the case were there a ball-and-socket joint at each end. Columns with **hinged ends** are such as have perfect freedom of motion at the ends in one plane, as would be the case of compression members in bridge trusses where the loads are transmitted through end pins. Columns with **flat ends** have the bearing surface normal to the axis of the column and of sufficient area to give at least partial fixity to the ends of the columns against lateral deflection. Columns with **fixed ends** have the ends rigidly secured so that under any load the tangent to the elastic curve at the ends will be parallel to the axis in its original position.

Experiments prove that columns with fixed ends are stronger than columns with either flat, hinged, or round ends, and that columns with round ends are weaker than any of the other types. Columns with hinged ends are equivalent to those with round ends in the plane in which they have free movement, while columns with flat ends have a value intermediate between columns with fixed ends and columns with round ends. It often happens that columns have one end fixed and one end hinged or have various other combinations. Their relative values may be taken as intermediate between those represented by the condition at either end.

Euler's Formula. Let A be the cross-sectional area of a column whose length is l and let P be an axial load under which the column is considered to assume a small lateral deflection f. Were the column absolutely straight and loaded absolutely axially and the material homogeneous it is evident that the load P could produce no lateral deflection. The deflection f, however, is assumed to be present, due to whatever cause, the load P being just sufficient to maintain the deflection f without the column returning to its original position or deflecting further. Assuming the origin of coordinates at one end with abscissas measured in the direction of the length and ordinates representing the deflections, the elastic curve assumed by the column would have the differential equation $EId^2y/dx^2 = -Pf$. From the integration of this equation and the determination of the constants of integration there results

$$\mathbf{P = n^2\pi^2EI}/l^2 \qquad (1)$$

in which E has its usual value and n is a number dependent upon the inclination of the tangent of the elastic curve at the ends. Giving to n proper values, formulas may be written for the several conditions of ends. For round ends n is taken as 1 and formula (1) becomes $P = \pi^2EI/l^2$, or, since $I = Ar^2$, in which r is the radius of gyration,

$$\mathbf{P/A = \pi^2E\,(r}/l)^2 \qquad (2)$$

For a column with fixed ends n is taken as 2, and formula (1) becomes

$$\mathbf{P = 4\,\pi^2\frac{EI}{\mathit{l}^2}} \quad \text{or} \quad \mathbf{\frac{P}{A} = 4\,\pi^2E\left(\frac{r}{\mathit{l}}\right)^2} \qquad (3)$$

Formula (1), of which formulas (2) and (3) are special cases, is the general form of Euler's formula for long columns. It is to be noted that the quantity f, or the deflection, does not appear. The load P in Euler's formula, therefore, is the load which would be required to maintain in the column any deflection which it might happen to have. A decrease in the load P would cause the column to return to its original position, while an increase in the load P would cause the deflection to increase indefinitely. The value of P in formula (1), therefore, may be taken as the ultimate strength of the column. For very long columns, where small loads produce marked lateral deflections, experimental results agree very closely with values computed from Euler's formula. It will be noted that Euler's formula does not involve the crushing strength of the material. For short columns failure occurs by crushing, not by flexure. For columns with values of l/r from 50 to 150 failure occurs by a combination of crushing and flexure. For short columns Euler's formula does not apply at all, the criterion being the ultimate in compression for brittle materials, or the yield point in compression for ductile materials. For columns with a value of l/r less than that about 150 Euler's formula gives results distinctly higher than those observed in tests. Euler's formula is rarely used in practice.

Rankine's Formula is widely used for the design and investigation of columns employed in engineering practice. It is a modification of the formula which for many years was in current use under the name of "Gordon's formula." Rankine's formula is based on the assumption that columns in engineering practice are intermediate between long columns and short prisms failing by a combination of direct compression and flexural stresses accompanying lateral deflection. While under an axial load a perfectly straight column will be stressed uniformly over its entire cross-section, any variation in structure will produce uneven distribution of stresses, resulting in lateral deflection, which must be taken into account in safe design. Thus in a column in which P is the direct load, A the cross-sectional area, and S_1 the compressive stress due to bending, the maximum unit stress in the column will be expressed by $P/A + S_1$. Determining the values of S_1 from the ordinary phenomena of flexure assuming that flexure action in columns is analogous to that in beams and taking into consideration the condition of ends of the column, Rankine's formula may be wirtten

$$\frac{P}{A} = \frac{S}{1 + \phi\,(l/r)^2}$$

in which S is the maximum stress in the column, P the axial load, A the area of cross-section, l the unsupported length of the column, r the radius of gyration of the cross-section, and ϕ a number depending upon the condition of the ends of the column and material of which it is constructed. The values of ϕ for various conditions are determined experimentally. Wide variations in the values are found in columns of different shapes and materials. Rankine's formula in the form above stated is applicable to the investigation of columns in which the factors P, A, l and r are known and ϕ is assumed for the given conditions. The value of S then determined when compared with the ultimate strength and yield point of the material will determine the factor of safety and the degree of stability of the column.

In Rankine's formula the average values for ϕ given in the accompanying table are to be used: see also Sect. 2, Arts. 2, 8 and 28, paragraphs on columns.

Material	Columns both ends fixed	Columns one end fixed, one end round	Columns both ends round
Timber	1/3000	1.95/3000	4/3000
Cast iron	1/5000	1.95/5000	4/5000
Wrought iron	1/36 000	1.95/36 000	4/36 000
Structural steel	1/25 000	1.95/25 000	4/25 000

The value of S to be used in the design of columns should be the allowable compressive unit stress, while for computing in cases of rupture it should be the ultimate compressive unit stress for brittle materials or the yield point in compression for ductile materials. Rankine's formula should be used between the limits of 20 and 150 for l/r.

Straight-line Formula. The plotted results of actual tests on columns show that the relation between ultimate load and l/r is fairly well represented by a straight line, for columns which fail by flexure of the whole column, and not by local collapse. For a value of $l/r = 0$ the average unit stress on the section, P/A, is equal to the ultimate in compression for brittle materials and to the yield point in compression for ductile materials. From the point thus determined the straight line representing the average results of tests is drawn tangent to the curve of Euler's formula for the material. The equation of the straight line is

$$P/A = S - C\,l/r$$

in which S is the unit stress at the ultimate in compression for a brittle material or at the yield point in compression for a ductile material, and C is a constant determined by experiment.

The accompanying table gives average values of S and C for structural steel, cast iron, and wood. The loads given by the straight-line formula using the constants in this table are ultimate loads:

Ultimate Strength Constants for the Straight-line Column Formula

$$P/A = S - Cl/r$$

Material and end condition	S	C	Limit of l/r
Structural steel:			
Round ends	35 000	150	160
Fixed ends	35 000	75	320
One end round, one fixed	35 000	100	240
Cast iron:			
Round ends	34 000*	175	90
Fixed ends	34 000*	88	160
One end round, one fixed	34 000*	116	115
Wood:			
Round ends	5 000*	40	75
Fixed ends	5 000*	20	150
One end round, one fixed	5 000*	30	112

* This is less than the ultimate in compression for small specimens of cast iron or wood, but from tests of full-size columns seems to be the value to be used for full-size castings or timbers which may contain defects.

For purposes of design the straight line formula is usually put in a form which gives working loads directly. Both the constant S and the constant C are divided by a factor of safety. Examples of such straight-line formulas for working loads on columns are given in Sect. 12, Art. 8, for structural steel columns; in the same article for cast-iron columns; and Sect. 9, Art. 3, for wooden columns. In a general way, in the straight-line formula P/A represents the average unit stress on the cross-section of the column at the middle of its length, and C^l/r represents the unit stress at the extreme fiber on the concave side due to flexure. S represents the stress at the extreme fiber on the concave side due to both direct compression and flexure.

Eccentric Loads. In many column designs the loading will not act along the axis of the column but at some distance from the axis; such loads are called eccentric loads, and the distance from the axis is called the eccentricity. It is evident that the greater the eccentricity the greater the flexural stresses, and that the unit compressive stress S on the most compressed side of the column will be greater than for axial loading. Let P be the load at distance z from axis of column; then $Pz = M$, the moment at axis. If the amount of unit stress along the side nearest the load is called S, then Rankine's formula may be written in the following form:

$$S = \frac{P}{A}\left(1 + \phi\left(\frac{l}{r}\right)^2 + \frac{cz}{r^2}\right)$$

where c = distance from axis of column to side of greatest compression. The straight-line formula may be notified for eccentrically loaded columns as follows:

$$(P/A)\ (1 + cz/r^2) = S - C\,l/r$$

Shear in Columns. For columns built up of two or more members held together by lattice bars the shear is of importance because the stress in the lattice bars is due mainly to shear in the column. The distribution of flexural stress in actual structural columns is too irregular to permit a satisfactory analysis for shear. Tests at the University of Illinois and general successful practice in the design of large columns indicate that the total shear on any cross-section of a column may be taken as not greater than 2.5% of the axial load, and that the lattice bars may be designed to take the stress corresponding to this shear.

If P is the axial load on the column, then the shear at any cross-section may be taken as 0.025 P. If there are lattice bars on each side of the column and θ is the angle which the lattice bars make with the axis of the column then the load carried by each bar is 0 025 $P/2 \cos \theta$ for single latticing and 0.025 $P/4 \cos \theta$ for double latticing. The average stress in the lattice bars should be very low as they themselves are often long, slender and eccentrically loaded columns. The average stress for a lattice bar should not be over 5000–6000 lb. per sq. in. Lattice bar design is further discussed in Sect. 12, Art. 29.

19. The Behavior of Columns under Load

Steel Columns may be either one-piece columns, such as H-section columns, or built-up columns, such as latticed channel columns. Fig. 50 shows various forms of cross-section for structural steel columns, and others are shown in Sect. 12, Art. 29. When loaded to failure well-designed steel columns fail by buckling of the whole column, showing points of inflection if the test column is fixed-ended. Columns made up of very thin parts may fail by local " wrinkling " of metal. The Manual of the Am. Ry. Eng. Assn. specifies that

Fig. 50. Sections of Steel Columns

the minimum thickness of web for a built-up steel column shall be 1/30 the width between the lines of rivets connecting it to the flanges, and that the minimum thickness of cover plate shall be 1/40 the distance between nearest rivet lines.

Several series of tests of full-size steel columns have been made, during recent years, at the U. S. Bureau of Standards (see Technologic Papers 101 and 328). Various types of structural steel column were tested and the values of l/r ranged from 15 to 90. The columns were tested " flat-ended." The yield point of the metal seemed to be the outstanding factor governing column strength. The effect of slenderness (l/r) was less than that indicated by the ordinary column formulas, which, in general, seem to give results on the safe side. The strength of structural steel in compression was found in some cases to be distinctly less than the strength in tension. No marked superiority was found for any particular type of column, but columns made up of channels connected by batten plates were found to be weaker than columns in which the flange members were connected by latticing.

Cast-iron Columns. The majority of designers at the present time have turned from the use of cast-iron columns to those of steel, but yet there seems to exist in the mind of some designers an impression that cast iron fulfills the necessary qualification as a material for columns. Tests of full-size cast-iron columns have been made at the Watertown Arsenal and at Phoenixville, Pa., and both series of tests showed that for short columns the strength in compression is much less than the strength of small test specimens of cast iron. This is probably due to the imperfections always present in iron castings. The cross-section of cast-iron columns is usually a hollow circle. For short columns failure of test columns takes place by shearing on an inclined plane; for long columns failure takes place by rupture on the convex side. Failure of cast-iron columns is always a sudden, shattering failure. Where the load is concentric and the stresses can be accurately determined, cast iron is used, by a good many designers, in certain places where steel would be more expensive. It is important that a very careful inspection be made of the cast iron and all pieces with flaws and blowholes be rejected.

Wood Columns are usually square in cross-section. Tests of full-size wood columns show lower strength for short columns than is shown by compression tests of small specimens of wood, the cause being, doubtless, the presence of defects and the non-homogeneity of the material. Wood columns fail less suddenly than cast-iron columns, but more suddenly and completely than steel columns. Wood columns are especially sensitive to effect of length on strength.

Brick and Terra Cotta Columns usually have low values of l/r so that length does not play much part in determining their strength. The care used in laying up the brick or the terra cotta tile, and the strength of the mortar used are important strength factors. If loaded to the danger point brick and terra cotta columns are liable to fail very suddenly and to collapse completely. See p. 645.

Concrete Columns are discussed in Sect. 11.

20. Torsion of Shafts

If a straight bar of material such as a round shaft be thoroughly gripped at one end and a twisting force applied at the other end, a state of stress is set up in the bar called **torsion**. The angle through which a fiber of unit's length is turned is called the **angle of torsion.** The twisting of the bar is produced by two couples, one consisting of the applied force with its lever arm, and one at the grip, with its lever arm. The conditions of equilibrium are that the moments of these couples must be equal.

Resisting Moment is the couple acting at the grip multiplied by its lever arm. If S is the unit shearing stress acting at a distance c from the axis of the bar, then S/c = the unit shearing stress at a unit's distance from axis of bar, and Sz/c would be the unit stress at any distance z from the axis. If this is considered to act over an elementary area da, then the moment of this stress, at distance z from axis, about axis would be $da.Sz^2/c$, and the summation of this between proper limits would be the resisting moment. The summation of da multiplied by z^2 is the sum of the products of each elementary area $\times$ square of its distance from the axis of bar. This expression is called the polar moment of inertia and is represented by the letter J.

The Polar Moment of Inertia of an area is the moment of inertia about an axis which is perpendicular to the plane of the area. It is equal to the sum of the moments of inertia of the area about two axes which are in the plane of the area, which intersect at the point where the axis of the polar moment of inertia pierces that plane, and which are at right angles to each other. For torsion the polar moment of inertia about an axis through the center of gravity of the area is of most importance. The polar moment of inertia of a circle is $\frac{\pi}{32} d^4$, twice its moment of inertia about a diameter.

Torsion Formula for Round Shafts. Considering a force P acting at one end of a shaft at a distance e from the axis, the moment of this force about the axis is equal to Pe. The resisting moment at the gripped end would be SJ/c, and from the conditions of equilibrium $\mathbf{SJ/c = Pe}$, or $\mathbf{S = Pec/J}$, which is the torsion formula for round shafts.

The twist of a circular shaft of length l subjected to a twisting moment Pe is $\theta = \mathbf{Pel/JF}$ in which F is the modulus of elasticity of the material in shear, and θ the angle of twist in radians. Values of F for various materials are approximately:

Steel (all grades)	12 000 000
Wrought iron	10 000 000
Cast iron	6 000 000

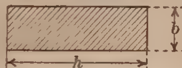
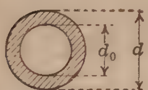
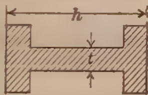
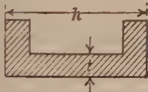
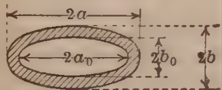
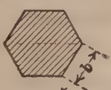
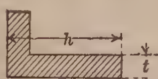
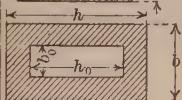
Solid and Hollow Shafts. In the design of a solid round shaft $J = \pi d^4/32$ and $c = 1/2\, d$. These values substituted in the preceding formula give $Pe = 1/16\, \pi d^3 S$. If the shaft is to be designed for the transmission of power, $d = 68.5\, (H/nS)1/3$, where H is the horsepower transmitted, n the number of revolutions per minute, S the allowable unit stress in pounds per square inch. On account of the greater strength of hollow round shafts over solid shafts of the same section area the former are often used for large sizes or where light weight is essential. In the design of hollow round shafts the value of J must be expressed in terms of the inner and outer diameters. Letting d_1 be the outer diameter, d_2 the inner diameter, $J = 1/32\, \pi\, (d_1^4 - d_2^4)$, and $c = 1/2\, d_1$, the torsion formula then becomes $S(d_1^4 - d_2^4)/d_1 = 321\,000\, H/n$. In hollow shafts as in hollow beams the material is removed from the axis where it is but little stressed and placed where it can resist the twisting moment more efficiently. It is obvious that the formulas for solid and for hollow shafts become the same when d^3 equals $(d_1^4 - d_2^4)/d_1$. When the section areas are equal then d^2 must equal $(d_1^2 - d_2^2)$. From these two equations the ratio of strength of a hollow shaft to a solid shaft of the same section area is $(2\, d_1^2 - d^2)/dd_1$.

Shafts with Non-circular Cross-section. The distribution of shearing stress in such shafts, while somewhat similar to the round shafts, is more complex. As an example, it has been found from experiments that no shearing stress exists at the corners of a rectangular shaft, while at the middle of the wide side the shearing unit stress is greater than at the middle of the long side. For any section with re-entrant angles, such as an I section, the stress at the apex of the angle is, theoretically, infinite, and practically there is a minute area of very high stress which may not be important under static loading, but forms a nucleus for cracks under repeated loading. The accompanying table gives values for the approximate stress set up under torsion in shafts of various cross-section. The values are those determined by Bach, and later modified by Kommers as the result of experiments on specimens of brittle material.

Shaft Couplings. When two lengths of shafting are to be joined together the connection is made with a coupling. The bolts used in such coupling transmit the torsion from one length of shafting to the other and therefore are subjected to shearing stress. This shearing stress is similar to that in the body of the shaft and is the greatest upon the side of the bolt most remote from the axis of shaft. If J_1 represents the polar moment of inertia of one bolt with respect to the axis of shaft, being equal to the polar moment of inertia of its cross-section about its axis plus the section area of the bolt into the square of the distance between the axis of shaft and axis of bolt, then in order that the strength of bolts and shaft be the same $\mathbf{J/c = J_1 n/c_1}$, c_1 being distance of the most remote fiber of bolt from axis of shaft, and n the number of bolts. Calling D the diameter of shaft, d the diameter of each bolt, and h the distance between axis of bolt and axis of shaft, and substituting the values of J and J_1 in the equation $J/c = J_1 n/c_1$, an expression for finding the proper diameter of the bolts would be formed. As this equation with proper substitutions is an awkward expression for finding the value of the diameter, it is general practice to assume the shearing stress uniformly distributed over the area of the bolts. On this assumption $c_1 = h$, $J_1 = 1/4\,\pi d^2h^2$, and $J/c = 1/16\,\pi D^3$. Equating the equation $J/c = J_1 n/c_1$, with the proper substitutions it reduces

Shearing Stress under Torsion in Shafts of Various Cross-sections.

S, maximum unit shearing stress, in pounds per square inch
Pe, twisting moment in inch pounds

Cross-section Fig. 51a	S	Cross-section Fig. 51b	S
	$\frac{16}{\pi}\frac{Pe}{d^3}$		$\frac{9}{2}\frac{Pe}{b^2h}$
	$\frac{16}{\pi}\frac{Ped}{d^4 - d_0^4}$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{2}{\pi}\frac{Pe}{ab^2}$ If $2a > 2b$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{2}{\pi}\frac{Peb}{ab^3 - a_0b_0^3}$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{1.09\,Pe}{b^3}$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{20\,Pe}{b^3}$		$\frac{9}{2}\frac{Peb}{b^3h - b_0^3h_0}$

For a shaft of any cross-section not having re-entrant angles St. Venant gives an approximate expression for the angle of twist measured in radians,

$$\theta = \frac{4\,\pi^2 PeJ}{FA^4}$$

in which A is the area of cross-section and the other symbols have the same significance as in the preceding paragraphs.

to $D^3 = 4\,nhd^2$ and $d = 1/2\ (D^3/nh)\ 1/2$, from which if n is known the diameter of bolt necessary can be found. The value found from the approximate formula is about 10% larger than the value found from the accurate formula.

21. Combined Stresses

Flexure and Compression. The ordinary manner of determining the amount of compression existing in a bar under an axial force is to consider the axial compression and the compression produced by flexure. Thus, calling

S the maximum unit stress due to the axial force and the flexure, S_a the axial compressive stress, S_f the unit compressive stress due to flexure, then $S = S_f + S_a$, where $S_f = Mc/I$ and $S_a = P/A$, I being the moment of inertia, c the distance of the extreme fiber, and A the area of cross-section. If the member is slender, a more exact method must be used in finding the total compression. Calling M the bending moment due to the transverse loads, P the axial load, e the deflection, then the total bending moment is $M + Pe$ and $S_f = (M + Pe)c/I$. The value of e may be assumed to equal $\alpha l^2 S_f/\beta Ec$ from the action of beams under transverse loads. Substituting this value of e, the maximum unit stress on the concave side of the column is

$$\mathbf{S} = \frac{\mathbf{P}}{\mathbf{A}} + \frac{\frac{\mathbf{Mc}}{\mathbf{I}}}{1 - \frac{\alpha \mathbf{Pl}^2}{\beta \mathbf{EI}}}$$

in which E is the modulus of elasticity, l the span, and α and β members depending upon the arrangement of ends and the kind of loading of the beam.

	α	β
Cantilever beam loaded at end	1	3
Cantilever beam uniformly loaded	2	8
Simple beam loaded at middle	4	48
Simple beam uniformly loaded	8	384/5
Beam fixed at one end, supported at other, uniform load	8	186
Beam fixed at both ends, loaded at middle	8	192
Beam fixed at both ends, uniformly loaded	12	384

In Flexure and Tension the method of finding the total amount of tension is similar to that for flexure and compression. Calling S the maximum unit stress due to the axial force and flexure, S_a the axial tensile unit stress, S_f the tensile unit stress due to flexure, then $S = S_a + S_f$, where $S_f = Mc/I$ and $S_a = P/A$. For a more exact formula, let M_1 equal the bending moment that produces the stress S, M the bending moment due to flexure, P the axial load or force, e the decreased deflection; then $M_1 = M - Pe$, and using the same reasoning as in the more exact method of flexure and compression, the maximum tensile unit stress is

$$\mathbf{S} = \frac{\mathbf{P}}{\mathbf{A}} + \frac{\frac{\mathbf{Mc}}{\mathbf{I}}}{\left(1 + \frac{\alpha \mathbf{Pl}^2}{\beta \mathbf{EI}}\right)}$$

the value of α and β being the same as given in the previous paragraph.

Flexure and Torsion result when a shaft is supported by hangers and loaded with pulleys for the transmission of the horsepower H. Find the flexural unit stress from $S = 32\ M/\pi d^3$ and the torsional unit stress from $S_s = 321\ 000\ H/nd^3$. Then the maximum tensile and compressive unit stresses S_t and S_c, and the maximum shearing unit stress S_{sh}, are given by

$$\mathbf{S_t} = \mathbf{S_c} = 1/2\ \mathbf{S} + \sqrt{\mathbf{S_s}^2 + (1/2\ \mathbf{S})^2} \qquad \mathbf{S_{sh}} = \sqrt{\mathbf{S_s}^2 + (1/2\ \mathbf{S})^2}$$

The above formulas may be used whether the shaft is solid or hollow by substituting the proper values for d. If shaft is round and hollow $(d_1^4 - d_2^4)/d_1$ should be substituted for d^3, where d_1 equals outer diameter and d_2 equals inner diameter.

Using the maximum strain theory the maximum "strain equivalent" ($E\times$ unit deformation) is

$$\mathbf{T_t} = 1/2\ (1 - \mathbf{p})\ \mathbf{S} + (1 + \mathbf{p})\sqrt{\mathbf{S_s}^2 + (1/2\ \mathbf{S})^2}$$

in which p is Poisson's ratio. For steel p may be taken as 1/4, and the above equation becomes

$$T_t = 3/8\,S + 5/4\,\sqrt{S_s^2 + (1/2\,S)^2}$$

22. Miscellaneous Cases

Eccentric Forces are sometimes applied longitudinally to the ends of a simple beam, instead of along the axis, in order that they may not increase the deflection of the beam. Let P be an eccentric tensile force acting at the distance e above the centers of gravity of the end sections, let f be the deflection and M the bending moment before the application of P. Then $P\,(e+f) = M$, whence the distance e should be $e = \dfrac{M}{P} - f$. When P is a compressive force it is applied below the axis at the distance $e = \dfrac{M}{P} + f$. As an approximation for both cases $e = \dfrac{M}{P}$.

Curved Beams. If the axis of a beam is curved the neutral axis no longer passes through the center of gravity of a cross-section, but is shifted towards the concave side of the beam. The maximum unit stress is increased by the curvature of the beam. The Winkler-Bach analysis for curved beams is the most widely used (see Appendix 1, Bulletin 18, Eng. Exp. Sta., Univ. of Ill.).

Wilson and Quereau, at the University of Illinois, from a study of the Winkler-Bach formula, have developed the following approximate formula, which, with an error not exceeding 15%, fits a large range of beams with cross-sections in the form of rectangles, ellipses, circles, trapezoids, T-sections, I-sections, and channel sections.

$$k = 1.0 + 0.5\,\frac{I}{bc^2}\left(\frac{1}{(R - c)} + \frac{1}{R}\right) \tag{1}$$

in which k = a correction factor by which the stress determined by the ordinary " straight beam " formula (formula (2), p. 569) is to be multiplied to give the extreme unit stress for the concave side of the curved beam.

I = the moment of inertia of cross-section.

c = the distance from the axis through the center of gravity of cross-section to the extreme fiber on the concave side of the curved beam.

b = the maximum breadth of the cross-section.

R = the radius of curvature of the centroidal line of the beam.

Helical Springs. A helical spring is formed by winding a wire closely around a cylinder which is then removed. Let d be the diameter of the wire, D the mean diameter of the helical coil, N the number of coils, P the londigitunal axial compressive load on the spring, and F the modulus of elasticity for shearing. The stress produced in the wire is one of shearing. The unit stress S_s and the shortening e of the spring are approximately

$$S_s = \frac{8\,DP}{\pi d^3} \qquad e = \frac{8\,ND^3P}{d^4F}$$

when P is a tensile load then e is the elongation of the spring. These formulas are valid only when S_s does not exceed the shearing elastic limit of the wire which is from 0.5 to 0.6 of the elastic limit in tension.

Circular Plates. A flat circular plate of radius r and thickness t supports a uniformly distributed load of p per square unit The flexural unit stress S of tension or compression at the middle of the plate is given by

$$S = m\,(r/t)^2 p$$

where m is a constant which varies with the kind of material and method of support.

	Wrought iron and steel and cast iron	Plain concrete
Plate supported around the circumference.	$m = 1.12$	$m = 1.27$
Plate fixed around the circumference.....	$m = 0.75$	$m = 0.85$

For a circular plate which carries at the middle a load of p lb. per sq. in. uniformly distributed over a circle of radius r_0, the value of S is, for a supported plate,

$$S = m\left(1 + 2\,\text{Nap}\log\frac{r}{r_0}\right)(r_0/t)^2 p$$

in which m has the same values as above. Example: when $r_0 = 1/4\,r$, then S is 0.26 $(r/t)^2p$ for cast iron and for steel.

Elliptical Plates. Let $2a$ and $2b$ be the axes, t the thickness and p be the uniform load per square unit. The plate tends to crack along the longer axis, and the greatest unit stress is

$$S = m\,\frac{a^2b^2}{a^2 + b^2}\,\frac{2p}{t^2}$$

in which the above values of m for circular plates may be used.

Rectangular Plates. Let 2 a be the shorter and 2 b the longer side; when the plate is fixed at the corners, then above formula for the elliptical plate may be used with above values of m for a circular supported plate. For a square plate of side 2 a and supported on edges $S = 9/8\,m\,(a/t)^2p$, while for one fixed on all edges $S = 3/4\,m\,(a/t)^2\,p$ in which m has above values for supported and fixed circular plates.

Sqaure Panels. Let an unlimited plate have many supports dividing the plate into many square panels, $2a$ being the side of each panel. The greatest unit stress S in the plate is $S = 8/9\,m\,(a/t)^2p$ in which m has the values given above for circular fixed plates. This applies to the flat stayed plates of steam boilers.

Fig. 52

Hooks. When a hook sustains a load P there exists in the bend of the hook combined tension and flexure. Let e be the distance from the line of action of P to the center of gravity of the cross-section at the bend, c the distance from that center of gravity to the inner edge of the bend, A the area and r the least radius of gyration of the cross-section then

$$S = k\frac{P}{A}\left(1 + \frac{ce}{r^2}\right)$$

gives the tensile unit stress at the inner edge of the bend in which k is the correction factor for curved beams given by formula (1), p. 599.

Circular Ring. When a circular ring of mean radius R is pulled in the direction of a diameter by two tensile forces each equal to P, the maximum bending moment is at the section, where P is applied, where it equals 0.318 PR and the unit stress is

$$S = \frac{0.318\ kPR}{0.098\ d^3}$$

in which k is the correction factor for curved beams given by formula (1), p. 599, and d is the diameter of the cross-section of the ring.

TESTING AND INSPECTION

23. Testing Apparatus

Testing Machines for determining the strength of materials of construction are of various types. A testing machine consists of a mechanism for applying a large force to a test specimen and a mechanism for determining the magnitude of the force applied.

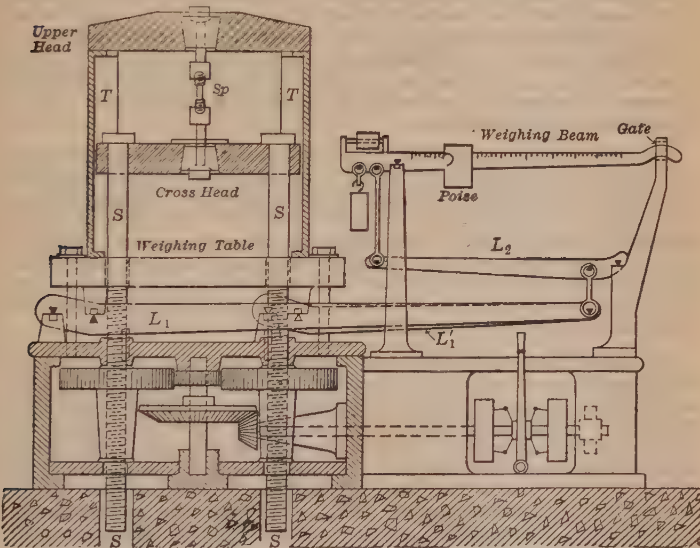


Fig. 53. Screw-power, Compound-lever Testing Machine

The Screw-power Compound-lever Testing Machine (Riehlé or Olsen **Machines**) is the type in most common use in the United States, and it is shown in diagram in Fig. 53. Power is supplied by a belt drive or by a direct-connected motor, and the power is transmitted through a series of gears to vertical screws which operate the crosshead of the machine. The force applied by the belt is multiplied many times by the gearing and the screws. This multiplied force is applied to the test specimen by the crosshead. If the specimen is a tension specimen it is placed between the crosshead and the upper head of the testing machine, if it is a compression specimen or a cross-bending specimen it is placed between the crosshead and the weighing table. In any event as the crosshead moves downward the specimen transmits downward pressure to the weighing table. The weighing table is supported on compound levers, $L_1L'_1$, Fig. 53, which are fitted with knife-edge bearings, and these compound levers transmit the force, reduced, to a simple lever, L_2, Fig. 53, which, in turn, transmits the force, still further reduced, to the weighing beam. As load is applied the beam is kept in balance by moving the poise, the position of which indicates the load on the specimen, when the beam is in balance.

For Flexure Tests, I-beams may be placed upon the weighing table, with suitable bearing blocks on their top flanges to support the beams to be tested, or the machines may be arranged with wings extending from the side and forming a part of the weighing table. The method of applying and registering the load is the same as in the compression test.

Hydraulic Pressure Testing Machines. The hydraulic press is a simple and comparatively inexpensive mechanism for applying the large forces required in testing machines. By using a pressure gage the total force on a ram of known size may be estimated with a fair degree of accuracy if the friction of packing against the ram is determined. The ordinary Bourdon pressure gage should be calibrated frequently, and very frequent adjustments made for zero reading.

The Amsler testing machine is made with rams so carefully ground to fit the cylinder that no packing is necessary, and the error due to friction is no larger than the error due to friction in knife-edge lever weighing devices. The pressure is measured by the load on a small piston which moves a pendulum so that the position of the pendulum bob indicates the force. This is a self-indicating weighing device, which eliminates much of the personal equation of balancing a weighing beam. The Amsler testing machine can be handled by a single observer, even when measurements of deformation are made, and is very rapid in its operation.

The Emery Testing Machine uses hydraulic pressure to apply the load to the specimen and transmits the load from the specimen to a hydraulic pressure cylinder, or pressure box, in which the plunger is replaced by a plate mounted on a thin metal diaphragm. This pressure box is filled with liquid and connected by means of a pipe to a smaller pressure box fitted with a plate and diaphragm, and the load on the smaller pressure box is weighed by means of a compound-lever balance. This hydraulic weighing device is extremely sensitive, and its accuracy is high. A recent model of the Emery machine uses a Bourdon pressure gage in place of the small pressure box and the compound-lever balance. The Bourdon gage is fitted with a special adjustment for zero load.

Calibration of Testing Machines. For small loads, testing machines may be calibrated by means of standard weights applied directly. For loads up to about 30 000 lb. testing machines may be calibrated by means of standard weights applied at the ends of a pair of simple levers of known leverage.

For larger loads the readings of the weighing mechanism of the testing machine may be checked against the elastic deformation of a standardized elastic member (a bar in tension, a block in compression, an elastic ring in flexure). The elastic member must have been previously calibrated by loading with standard weights, or with weights applied through simple levers of known leverage. At the U. S. Bureau of Standards is a dead weight calibrating apparatus with a capacity of 100 000 lb. Elastic calibrating devices up to 1 000 000 are on the market.

A fairly accurate calibration of a testing machine may be carried out by tests of a number of tension test pieces cut from a bar of mild steel. Half of these test pieces are tested to destruction in the machine to be calibrated, and half in a testing machine which has been standardized by one of the methods previously mentioned. The comparison of the average tensile strength of the two sets of specimens gives a measure of the accuracy of the testing machine under calibration.

Detailed specifications for calibrating testing machines are given in the 1927 Standards of the American Society for Testing Materials.

Accuracy and Sensitiveness of Testing Machines are two distinct characteristics. A machine is accurate if the readings of the machine agree closely with the actual loads applied by the machine. A testing machine is sensitive if a small change of load is indicated by a distinct movement of the beam or other weighing mechanism. A testing machine may be very sensitive, and yet very inaccurate.

Machines for Torsion Tests are of two types. In the Thurston type of machine one end of the specimen is held by a chuck in a horizontal spindle in the top of an A-frame. The other end is held in a similar manner in a parallel frame. One spindle is rotated by means of a worm gear and crank. The motion is transmitted through the specimen to the other spindle, to which is attached a weight on the end of a vertical bar. Any motion of this spindle will move the weight out of the vertical, and a torsional stress in the specimen results. The position of the weight will then be a measure of the stress, since its moment will be proportional to its deviation from the vertical.

Another form of machine consists of two parallel heads in which are chucks for holding the specimen, one head being operated by gears and the other attached to a weighing system, consisting of a compound lever. The moment in inch-pounds is read from a scale beam.

Abrasion Machines are used for determining the toughness or resistance of road materials to wear. The Deval machine is widely used. It consists of four cylinders 20 cm. in diameter and 34 cm. in depth, inside, mounted on a shaft at an angle of 30° with the axis of rotation. A charge of 5 kilograms of broken stone is placed in each cylinder, each charge consisting of 50 pieces. The cylinders are rotated 10 000 times at the rate of 30 to 33 per minute. The percentage of loss is computed from the amount of worn-off material that will pass through a 1/16-mesh screen.

For Abrasion Tests of paving brick, the cylinder or rattler adopted by the National Brick Manufacturers' Association is used. This consists of a barrel 20 in. in length, inside, and whose cross-section is a fourteen-sided polygon, inscribed in a circle 28-3/8 in. in diameter, supported on trunnions. The heads are made of cast iron, and the staves are of steel. The space between the staves must not exceed 5/16 in. The charge for bricks of "block-size" is ten bricks and with these bricks are placed in the rattler ten cast-iron spheres 3-3/4 in. in diameter and 300 lb. of cast-iron spheres 1-7/8 in. in diameter. These cast-iron spheres are weighed after every ten tests and when a large sphere is found to weigh less than 7 lb. or a small sphere less than 3/4 lb. new spheres are substituted.

The charge is rotated 1800 times at the rate of 29.5 to 30.5 per minute, and the percentage of loss is calculated in terms of the weight of the dry brick composing the charge. The average of five tests on separate charges of brick is considered an official test. (See p. 617.) For fuller details see 1927 Standards of the American Society for Testing Materials, Part II, p. 126.

Extensometers are used for measuring the small elastic deformations within the elastic limit, and for detecting the stress at which the elastic limit is exceeded. An extensometer consists essentially of two clamps which are fastened to the test specimen, and one or more micrometer devices by means of which a very small change of distance between the clamps can be measured. For all accurate measurement of elastic deformation of tension or compression specimens the extensometer should be attached so as to measure deformation along two or three symmetrically spaced axial lines on the specimen. Types of micrometer devices used in extensometers are: the screw micrometer, in which a very small axial motion of a screw is accompanied by a large circumferential motion of a dial attached to the screw; clockwork dial gages; multiplying levers; microscopes; and the "optical lever" in which a very small angular motion of a mirror changes the direction of a reflected ray of light. Extensometers for general use usually measure deformations to the nearest ten-thousandth of an inch.

The Strain Gage is a special form of extensometer which can be attached to a specimen or moved from one gage line to another between readings. It consists of a pair of trammel points PP', Fig. 54, attached to a frame F. One

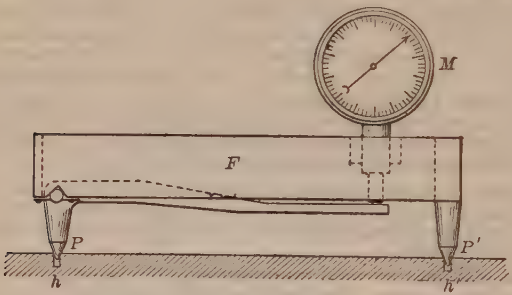


Fig. 54. Strain Gage

of the points, P, is movable and the motion of this point is measured by means of some micrometer device, M, usually a clockwork dial gage. The trammel points are conical in shape, and are made of hardened steel. On the specimen there are laid out gage lines along which deformation is to be measured, and at the ends of each gage line are drilled small holes hh' into which the trammel points fit. The strain gage may be clamped to a specimen, or the reading of the strain gage may be taken under zero load, the instrument removed, and after an increment of load has been applied another reading taken, the difference in the readings being the deformation. Corrections for temperature deformations are made by taking readings on a "standard bar" which is not subjected to stress, but is subjected to the same temperature changes as the specimen or structure under test. In skillful hands deformations can be measured with a strain gage to the nearest ten-thousandth of an inch The strain gage can be used to measure deformations in a structure or machine as well as in a test specimen. One instrument can be used for a large number of gage lines. In a test of stress determination in a reinforced concrete floor slab in the Soo Line Freight Terminal in Chicago measurements of deformation along over 1000 gage lines were made, and five strain gages were used. For detailed discussion of the use of the strain gage see Proceedings of the American Society for Testing Materials, Vol. XIII, p. 1019.

Recording Extensometers. One form of extensometer indicates changes of length by the change of electrical resistance of a pile of carbon disks when slightly compressed. This extensometer can be used as an indicating instrument, or, by the use of a mirror galvanometer recording the throw of its beam of light on a sensitized photographic film, as a recording extensometer.

It is especially useful for measurements in which the observer cannot conveniently be near the place where the deformation occurs.

See Proceedings of the American Society for Testing Materials, Vol. 23, Part II, p. 392 (1923), and Vol. 27, Part II, p. 522 (1927), articles by Peters and Johnston.

Another type of extensometer indicates changes in length by the change of impedance in an alternating-current electric circuit when the air gap between an electric magnet and its armature is slightly changed. This type may be used as a recording extensometer by the use of an oscillograph (a.c. galvanometer with very short period of vibration). It is also useful when it is convenient for the observer to be at a distance from the place where deformation occurs.

This type of instrument has been used by the Westinghouse Elec. & Mfg. Co., and by Prof. Earl Smith of Iowa State College.

A very simple form of recording extensometer is called the stremmatograph. In this form changes of length are recorded directly by the motion of a needle point over a plate of lightly smoked glass. This record is afterward examined under a microscope fitted with a micrometer eyepiece. The stremmatograph has been used successfully by P. H. Dudley and by Talbot to measure strains in railroad rails under moving trains.

See Trans. Am. Soc. C. E., Vol. LXXXII (1918), Progress Report of Committee on Stresses in Track.

For measuring the dimensions of cross-section of a test specimen micrometer calipers are usually used. For measuring the relatively large deformations beyond the yield point, and the elongation of tension test specimens after fracture measurements with dividers and steel scales are sufficiently accurate.

The Speed of Testing has an appreciable effect on the values determined for the mechanical properties of many structural materials. The accompanying table gives the requirements for speed in testing iron and steel as given in in 1928 Tentative Standards of the American Society for Testing Materials.

In testing steel and wrought iron in gage lengths of 2 and 8 in. in accordance with the specifications of the American Society for Testing Materials, the speed of the machine, by which is meant the speed of the crosshead when the machine is running idle, shall conform to the following requirements:

The crosshead speed of the testing machine shall be such that the beam of the machine can be kept balanced, but in no case shall the values given in the following table be exceeded:

Specified minimum tensile strength of material, lb. per sq. in.	Gage length, in.	Maximum crosshead speed for testing machine in determining, in. per minute	
		Yield point	Tensile strength
80 000 or under	2	0.50	2.0
	8	2.00	6.0
Over 80 000	2	0.25	1.0
	8	0.50	2.0

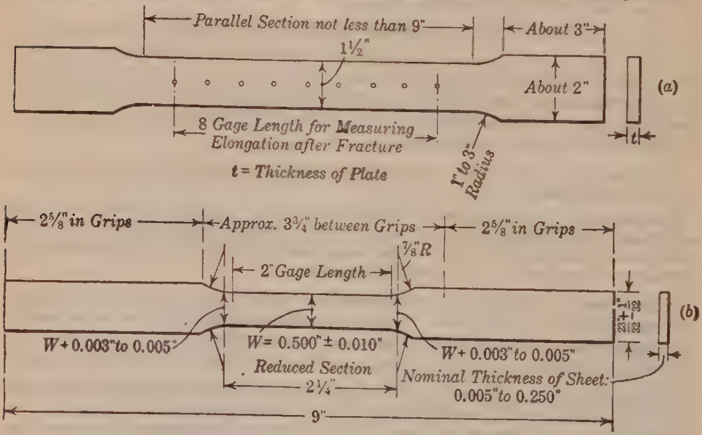
In determining the elastic limit by method I of the American Society for Testing Materials, described on p. 608, the crosshead speed for a specimen with a 2-in. gage length shall not exceed 0.125 in. per min. In determining the limit by method II (see p. 608), the crosshead speed shall not exceed 0.025 in. per in. of gage length per min.

Large Testing Machines in the United States. The U. S. Bureau of Standards has a compression testing machine of 10 000 000 lb. capacity, and another machine of 2 400 000 lb. capacity in compression and 1 200 000 lb. in tension. Several other government laboratories, college laboratories, and commercial laboratories have machines of 1 000 000 lb. capacity or more. The National Tube Co., Pittsburgh, Pa., has a torsion testing machine of 1 500 000 in-lb. capacity.

24. Tests and Test Specimens for Metals

This article is based largely on the Standards and the Tentative Standards of the American Society for Testing Materials.

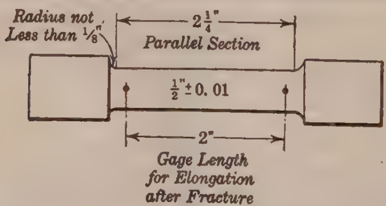
Tension Test Specimens for wire or for round bars usually consist of full-size pieces of the wire or the bar. Specimens of round bars are usually about



Note:- Reduced Section has Gradual Taper from Ends to Middle

Fig. 55

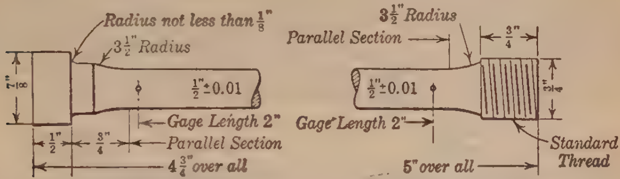
18 in. long, and the elongation after fracture is measured over a gage length of 8 in. Specimens for plate and shape material are usually of the form shown in Fig. 55*a* for material 1/4 in. thick and over. For material under 1/4 in. thick the specimen shown in Fig. 55*b* is recommended.



Note:- The Gage Length, Parallel Section, and Fillets shall be as Shown, but the Ends may be of any Shape to fit the Holders of the Testing Machine

Fig. 56

For thick plate material, forgings, large pieces, and general use in tension tests the specimen shown in Fig. 56 is recommended. Fig. 57 shows two designs of end for the specimen shown in Fig. 56, both of which have given satisfactory service. The gradual transition in cross-section from ends to middle of length is especially important in specimens of brittle metals.

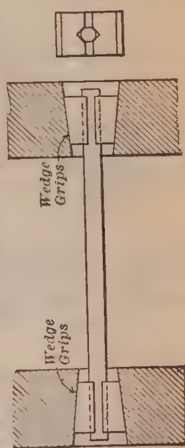


(a) SPECIMEN WITH SHOULDERED ENDS

(b) SPECIMEN WITH THREADED ENDS

Fig. 57

Grips for Tension Test Specimens. For long bars of ductile metal and for the flat specimens shown in Fig. 55 wedge grips with file-toothed faces are satisfactory (see Fig. 58). For short specimens and for specimens of brittle metals wedge grips are not satisfactory, usually causing fracture at the edge of a gripping wedge. For such specimens it is necessary to use the machined specimen (Figs. 56 and 57). Fig. 59*a* shows a satisfactory gripping device for machined specimens with screw ends, and Fig. 59*b* shows a device for machined specimens with shouldered ends. Both these devices transmit the applied force to the heads of the testing machine through spherical-seated bearings. For very brittle metals, such as cast iron, even the spherical-seated bearings shown in Fig. 59 are not always effective in avoiding bending stresses in the specimen. The gripping device shown in Fig. 60, known as the Robertson shackle, has proved satisfactory for testing specimens of brittle material, and it is an excellent device for gripping any machined tension test specimen.



Tension Specimen for Rod or Wire

Fig. 58

The gripping device is shown as used with threaded end specimens. A similar device fitted with split sockets would be used with shouldered specimens.

For specimens of thin sheet metal ordinary wedge grips are liable to cut the specimen near the edges of the grips, causing a tearing action rather than an axial pull. The self-aligning wedge grips shown in Fig. 61 (known as Templin grips) have proved satisfactory for tests of thin sheet metals.

Preparation of Tension Test Specimens. All machined surfaces on specimens should have a smooth finish. All shoulders should have generous fillets. If specimens of rolled metal are sheared from plates or shapes, or are roughed out with a cutting torch, at least 1/8 in. of the metal should be removed from the sheared or burnt edges of the specimen. For steel castings test coupons are usually attached to the casting, and the specimens machined from these coupons. The coupons should remain attached to the casting until the annealing or other heat treatment of the casting is completed.

Determination of the Elastic Limit in Tension Tests. The determination of elastic limit as defined on p. 547 logically involves the application and release of a series of increasing loads on the specimen until there is observed a set after the release of a load. This procedure is very slow, and since for many metals experience does not indicate any significant difference between the elastic limit so determined and the proportional limit, the determination of the proportional limit is regarded as a satisfactory determination of the elastic limit. It is obvious that the precise value obtained for elastic limit, or for proportional limit, depends on the delicacy of methods and instruments

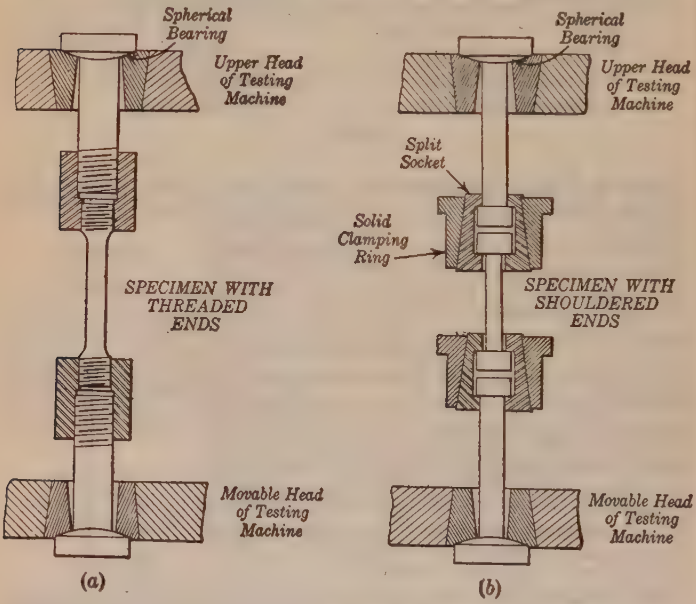


Fig. 59

used. It becomes necessary, therefore, that in any test the method used shall be clearly stated. The following methods are in common use for determining a value designated as the elastic limit:

Method I is suitable for metals of a fair degree of ductility. An extensometer, reading to 0.0002 in. (usually with a 2-in. gage length), shall be attached to the specimen. When the specimen is in the testing machine and the extensometer attached the testing machine shall be operated so as to increase the load on the specimen at a uniform rate. The observer shall watch the elongation of the specimen as shown by the extensometer and shall note the load for which the rate of elongation shows a sudden increase. The unit stress corresponding to this load shall be reported as the elastic limit.

Method II (the J. B. Johnson method) is somewhat more delicate than method I and is suitable for tests of metals of rather low ductility. The elastic limit is taken as that unit stress at which the rate of deformation is

50% greater than the initial rate of deformation. It is necessary to take a series of readings of load and stretch, or to use an autographic or a semi-autographic recording device to obtain a stress-strain diagram. Fig. 62 shows such a diagram. The initial rate of deformation is given by the ratio mn/Om; $nq = 0.5\ mn$, $mq = 1.5\ mn$, and the slope of Oq represents a rate of deformation 50% greater than the initial rate. $O'q'$ is drawn parallel to Oq and tangent to the stress-strain diagram. The point of tangency J locates the elastic limit. In using method II it is recommended that the extensometer

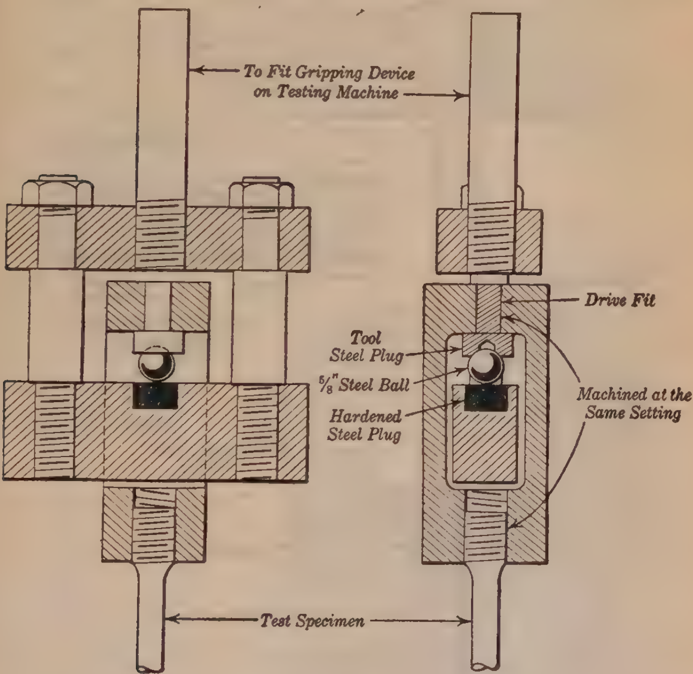


Fig. 60

be of such sensitiveness that it will indicate a change of one ten-thousandth of the gage length.

Determination of the Yield Point in Tension Tests. The yield point is the unit stress at which there occurs a marked increase of deformation without an increase in load. Two methods for determining yield point are in use.

Method I (" Drop of the Beam "). Load is applied to the specimen at a steady rate of increase and the operator keeps the beam in balance by running out the poise. At the yield point the increase of load ceases temporarily, but the operator, running out the poise at a steady rate, runs it a trifle beyond the balance position, and the beam of the machine drops for a brief but appreciable interval of time. In a machine fitted with a self-indicating weighing device there is a sudden halt in the motion of the load-indicating pointer, corresponding to the drop of the beam. The load at the " drop " or the " halt " is noted and the corresponding unit stress reported as the yield point.

Method II. An observer with a pair of dividers watches for visible elongation between two gage marks on the specimen. When visible stretch is

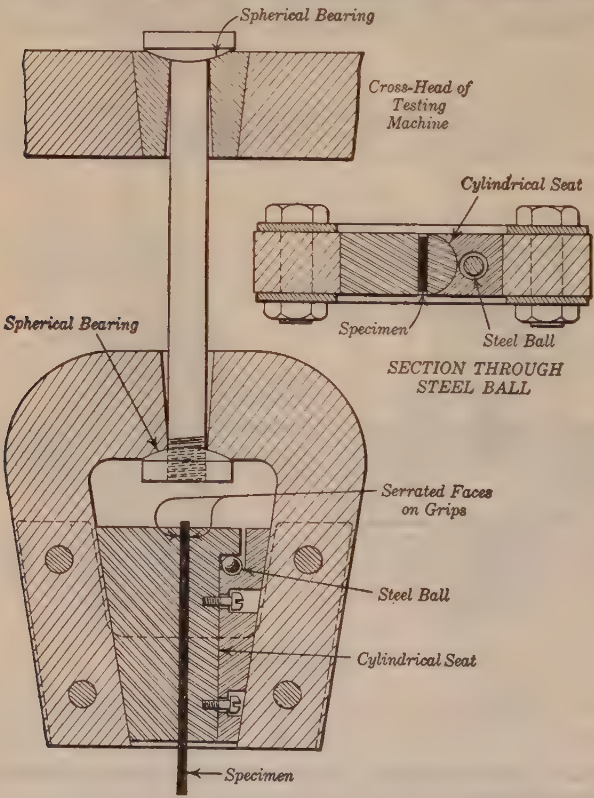


Fig. 61

observed the load is noted and the corresponding unit stress reported as the yield point. The gage length should not be greater than 2 in.

It should be noted that many metals do not have a well-defined yield point.

Significance of Elastic Limit and of Yield Point. Using careful methods of testing and especially avoiding high speeds of pulling, the yield point, if sharply defined, may be regarded as a satisfactory index of the elastic strength of a ductile metal. For metals of a good degree of ductility which do not show a clearly marked yield point method I for elastic limit gives a satisfactory index of elastic strength. For metals of fair ductility **method II gives a** satisfactory index of elastic limit (method II may also be used for metals of high ductility). For metals of low ductility (e.g., cast iron) none of the methods previously mentioned, nor any other as yet proposed, give a satisfactory index of elastic strength. It is doubtful whether any elastic limit should be reported in tests of such metals.

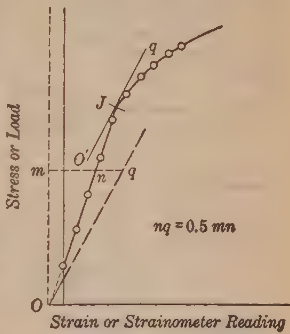


Fig. 62. Illustrating Method II for Determining Elastic Limit

Determination of Tensile Strength (Ultimate). The tensile strength of a material is determined by dividing the maximum load carried in a tension test by the original area of cross-section of the specimen. It is important that the speed of testing should not be so high as to render uncertain the balancing of the beam of the testing machine, or to cause "surging" of the indicating mechanism of a self-indicating load measuring device.

Determination of Elongation and Reduction of Area after Fracture. The measurement of elongation of a tension test specimen after fracture can be made with sufficient accuracy by means of a pair of dividers and a scale. In general, the elongation should not be measured on any specimen which breaks outside the middle third of the gage length. The measurement of reduction of the dimensions of specimen after fracture may be made by the direct measurement with a micrometer of the smallest cross-section of the specimen. For round specimens this measurement can best be made by holding the broken pieces together in a vise or between centers and then measuring the average diameter of the smallest section by means of a micrometer fitted with rather sharp points.

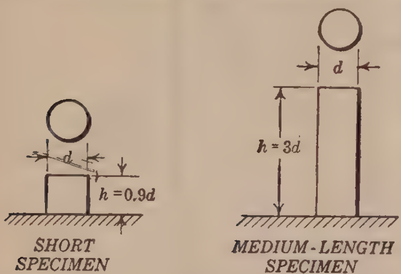


Fig. 63

Compression Test Specimens. Fig. 63 shows specimens for compression tests. The short specimen is used for tests of such metals as bearing metals, which in service are used in the form of a thin plate or shell. The medium-length specimen is useful for general use in compression tests. Compression specimens of metals should have all machined surfaces smooth and all ragged edges smoothed off.

Bearing Blocks for Compression Specimens. Both ends of a compression test specimen should bear on a carefully machined plane surface. The bearing blocks should be made of hardened steel or faced with hardened steel. One end of the specimen should bear on a spherical-seated bearing block. Fig. 64 shows a satisfactory arrangement of specimen and bearing block. The bearing block should be at the upper end of the specimen. It is important that the center of the spherical surface should be in the flat face which bears on the specimen; otherwise lack of parallelism between faces of the specimen sets up bending action in the specimen.

The object of using a spherical-seated bearing block is to give the specimen as even a distribution of initial load as possible. Owing to friction, the spherical-seated bearing cannot be relied on to adjust itself to bending action which may be set up after the test is started.

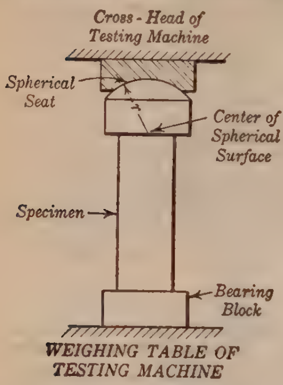


Fig. 64

Determination of Elastic Limit, Yield Point and Compressive Strength. The methods used for determining elastic limit and yield point in compression tests are, in general, the same as those used in tension tests. The compressive strength is determined by dividing the maximum load by the original area of cross-section of the specimen.

In the case of a material which fails in compression with a shattering fracture the compressive strength has a very definite value. In the case of materials which do not fail by a shattering fracture the value obtained for compressive strength is an arbitrary value depending on the degree of distortion which is regarded as indicating complete failure of the material.

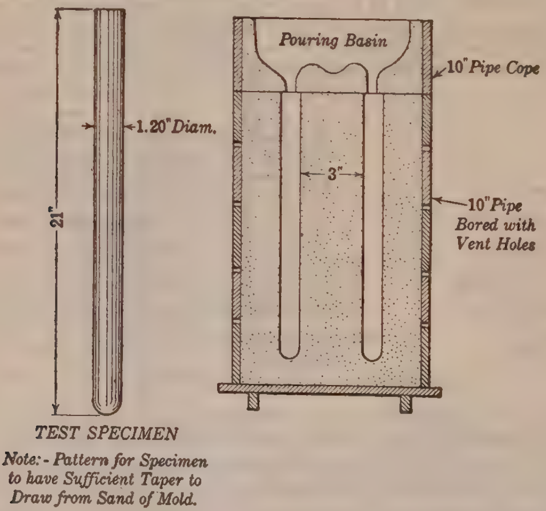


Fig. 65

Flexure Tests are used in determining the strength qualities of cast iron. For gray iron castings the standard test specimen is an "arbitration bar," the specifications for which are given by the 1927 Tentative Standards of the American Society for Testing Materials as follows:

"The mold for the bars is shown in Fig. 65. The bottom of the bar is 0.05 in. smaller in diameter than the top, to allow for draft and the strain of pouring. The pattern should not be rapped before withdrawing. The flask shall be rammed with green sand, a little damper than usual, well mixed and put through a No. 8 sieve, with a mixture of 1 to 12 bituminous facing. The mold shall be rammed evenly and fairly hard, thoroughly dried and not cast until it is cold. The test bar shall not be removed from the mold until cold enough to be handled. It shall not be rumbled or otherwise treated, being simply brushed off before testing."

The arbitration bar is loaded at the center of an 18-in. span, as shown in Fig. 66, and the load is applied at such a rate that 20 to 40 seconds are required to produce a deflection of 0.1 in.

For cast-iron pipes the standard test specimen is 26 in. long, 2 in. wide, and 1 in. thick, and is loaded at the middle of a 24-in. span.

Hardness Tests of Metals in common use are the Brinnell test, the Rockwell test and the Scleroscope test. In the Brinnell test a hardened steel ball of known diameter is pressed against the surface of the metal to be tested, the pressure exerted and the diameter (or depth) of the resulting permanent impression being noted. The smaller the indentation the harder the metal. The hardness is indicated by a "hardness number," which is the quotient obtained by dividing the pressure by the spherical surface of the indentation. The standard steel ball used is 10 mm. in diameter, and the standard pressure is 3000 kg. for the harder metals, and 500 kg. for the softer metals. A modification of the Brinnell test which is used to some extent is the Ludwik cone test, in which a hardened steel conical point is used in place of a steel ball.

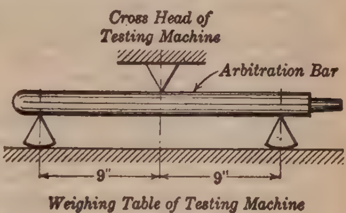


Fig. 66

In the Rockwell test a steel ball 1/16 in. in diameter (the Rockwell "B" test) or a conical diamond point (the Rockwell "C" test) is pressed against the surface of the specimen with a small but definite initial load. Then a load of 100 kg. ("B" test) or 150 kg. ("C" test) is applied by means of a weight acting at the end of a compound lever. This weight is removed leaving the small initial load still on the penetrating point and the depth of impression is measured by means of a micrometer dial gage attached to the indenting point. This gage is graduated to read directly an arbitrary system of hardness numbers corresponding (inversely) to the depth of impression. The Rockwell "B" numbers are different from the Rockwell "C" numbers.

The Scleroscope consists of a diamond-pointed plunger weighing 1 gram which drops vertically down the inside of a graduated glass tube to the surface of the metal to be tested, which is placed at the lower end of the tube. The height of rebound is measured, and is a measure of the hardness of the metal.

Brinnell tests, Rockwell tests, and scleroscope tests possess the great advantage that they may be made without damage to the material which is to be used, instead of being vicarious tests on samples chosen to represent the material. They are especially useful in testing the uniformity of different parts of the same piece, for example, the uniformity of hardness of the different teeth of a gear wheel.

In a general way there seems to be a fairly well defined relation between the Brinnell, Rockwell, and scleroscope hardness, and the tensile strength for the different grades of iron and steel. (See paper by R. R. Abbott, Proc. American Society for Testing Materials, Vol. XV, Part II, p. 42.) The relation between Rockwell number and Brinnell number is rather complicated and varies somewhat for different metals. (See Petrenko in Technologic Paper 334 of the U. S. Bureau of Standards.)

Impact Tests of Metals are sometimes made for the purpose of determining toughness, that is, the ability to resist without actual rupture a combination of high stress and great deformation. Tough metals resist heavy accidental overload without shattering failure. In impact tests the amount of energy required to rupture or permanently deform the test specimen is the quantity measured rather than the stress. In general, the results of an impact test may be predicted by measuring the area under the load-deformation diagram for a static test of the specimen, though there may be some variation introduced by the very rapid application of load in an impact test.

The Master Car Builders' Drop Testing Machine is an impact machine consisting of a framework fitted with a hoist and a weight which can be varied from 1640 lb. to 2000 lb. and which can be dropped from a height varying from 0 to 50 ft. The specimen is supported on an anvil weighing 20 000 lb. and the anvil is supported on heavy springs. With this machine tests are made of rails in flexure, of car couplers in tension, of car axles and locomotive axles in flexure. Impact tests of cast-iron car wheels are made on a machine somewhat similar to this. Drop tests are made by delivering on a test specimen one or more blows from a given height, and observing signs of failure in the test specimen.

The Charpy Impact Testing Machine is used for tests of small specimens of metal under impact. The specimen is held in a horizontal position and is in flexure over a short span. It rests against the vertical face of an anvil. The impact is furnished by a pendulum which is let fall from a predetermined height, striking the specimen at the bottom of its swing. The pendulum is of known weight, and is so hung that its center of percussion is at the point of impact against the specimen. After breaking the specimen the pendulum continues its swing, and an indicating apparatus shows the maximum height to which it rises. The energy supplied by the pendulum is the product of its weight and its height of fall; the energy left after the fracture of the specimen is the product of its weight and the height of rise; the difference is the energy required to break the specimen. Specimens for the Charpy machine are notched at the middle of their length to insure a sharp plane of fracture, and the uniformity of size of specimen and of size and shape of notch is very important.

The Izod Machine acts on a principle similar to that of the Charpy machine. The Izod specimen is a short notched cantilever beam held vertically and struck by a swinging pendulum. The energy required for fracture is measured in a manner similar to that used with the Charpy machine. The significance of the results of either Charpy or Izod tests is a matter of considerable uncertainty.

Repeated-stress Testing Machines. The commonest type of repeated-stress testing machine uses a specimen which acts as a rotating beam (either a simple beam or a cantilever beam). Known weights producing computable unit stresses are hung on the specimen, and for every revolution of the specimen there is a cycle of stress involving a complete reversal of flexural stress. A revolution counter records the number of revolutions necessary to cause fracture. For the general method of determining the endurance limit (fatigue limit) by the use of such a testing machine see p. 555.

The Haigh repeated-stress testing machine sets up cycles of repeated axial stress (tension-compression) by the use of low-frequency two-phase alternating current electromagnets. The force developed is measured by the current induced in coils of wire on the armature which is attached to the specimen, and which is caused to vibrate between the poles of the pair of magnets. The Haigh machine is a very expensive machine, capable of a high degree of accuracy.

For descriptions of other types of repeated-stress testing machine see

Bulletins 124, 136, 142, and 152 of the Engineering Experiment Station of the University of Illinois.

Tests of Magnetic Permeability of steel and iron seem to give some indication of the hardness, strength, and uniformity. They possess the great advantage that the test does not destroy the sample tested, and hence may be made on the actual parts to be used. Magnetic tests as an indication of hardness or strength are not standardized yet, but hold promise of future usefulness.

The variation of magnetic qualities in a bar or disk of metal has been used as a means of locating small invisible flaws. (See article by J. A. Capp, Proc. American Society for Testing Materials, Vol. 27, Part II, p. 268 (1927)).

Macroscopic and Microscopic Examination of Metals. The term **macroscopic** is applied to the examination of metals either by the unaided eye or by the use of a glass magnifying not more than 10 diameters. The term **microscopic** is applied to the examination by means of a magnifying lens or lenses yielding magnifications greater than 10 diameters. In both types of examination the surface of the metal is polished. Usually for macroscopic examination polishing with fine emery paper is sufficient; for microscopic examination emery paper polish is usually followed by a polish with jewelers' rouge or with very fine alumina. In most cases, after polishing, the surface of the metal is etched; nitric acid, picric acid, copper chloride, and ammonium persulfate are among the etching agents used. For inspection purposes the etched surfaces are examined by direct observation; where a permanent record is desired a micrograph of the surface is taken by means of a photographic camera attached to the microscope. For macrographs an ordinary camera may be used.

Macroscopic examination brings out the general features of the internal structure of the metal, but not the details. Macrographic examination has proved useful in showing the presence of segregation of ingredients in metals, the presence of small cracks and flaws, the " grain " or flow lines in forgings, and the slag streaks which distinguish wrought iron from steel.

Microscopic examination brings out the details of crystalline structure of metals. It shows the size and shape of the crystalline grains, the homogeneity of the internal structure, the presence of fine bands of segregated material, the presence of " slip bands " due to inelastic action, and the presence and growth of minute cracks. Magnifications commonly used for micrographs are 100, 500, 1000, and 1500 diameters. Micrographs with good detail have been made with a magnification of 3600 diameters.

X-ray Examination of Metals. X-ray apparatus has been used for the examination of metals with two distinct purposes in view: (1) to reveal internal defects, and (2) to determine the geometric pattern in which the atoms of a metal are arranged—the space lattice. X-ray examination for internal defects in metals is made with a very powerful X-ray apparatus. Defects have been shown in steel up to 4 in. thick. X-rays penetrate aluminum much more easily than they do steel, and lead much less easily. Defects shown by X-ray examination are not magnified, and the examination can detect no flaw smaller than one which could be detected by the unaided eye if the defect were on the surface of the metal. The technique of handling the powerful X-ray apparatus is very difficult, and unless elaborate precautions are taken to enclose the whole job in a thick lead chamber, there is great danger from X-ray burns and injury to the eyes of the attendants and persons in nearby rooms.

X-ray determination of the space lattice of a metal is as yet a problem for the physicist rather than the testing engineer The wave length of X-rays is much shorter than that of ordinary light rays, being of the order of magnitude of the distance between atoms. By the dispersion and reflection of X-rays as they pass through thin sheets of metal, or through finely powdered metal, it is possible to construct a diagram of the geometric arrangement of the atoms in a crystalline grain of metal—a diagram of the space lattice of the metal. This application of X-rays is known as X-ray spectroscopy, and it holds great promise for the more fundamental study of the atomic structure and the inherent strength of metals.

25. Tests of Stone and Brick

The Specific Gravity Test for stone shall be made on a specimen that has been dried to constant weight at a temperature of 212° to 230° Fahr. After removing all sharp corners and grains that are loose, the specimen shall be carefully weighed, then placed in water and put under the receiver of an air pump and all bubbles of air exhausted. The weight in water shall be noted and the specimen removed from the water and all surplus surface moisture removed either with a cloth or blotting paper. After again weighing, the specific gravity will be determined from the formula Sp. Gr. = Wt. of dry stone in air ÷ (Wt. of saturated stone in air − Wt. of saturated stone in water). Where an air pump is not available, the specimen may be weighed in the water after an immersion of 24 hours, all surface air bubbles being removed with a small brush or feather.

Tests for Crushing Strength of stone shall be made on cubes, the size of which will depend upon the capacity of the testing machine. Specimens shall be sawed out and not chiseled. They shall be tested on their natural beds, and the opposite faces shall be ground parallel and tested between steel plates in a machine having a spherical socket in the crushing head. At least three specimens shall be tested, and the load at which the specimen first cracks and the ultimate resistance shall be noted in the report.

Transverse Strength of stone shall be determined by placing a beam whose length is not less than 10 times its depth on slightly rounded knife-edges and applying a load at the center. The modulus of rupture shall be computed by the formula $\mathbf{R} = \frac{3\,Wl}{2\,bd^2}$, where W is the center load. l the span, b the width, and d the depth.

The Absorption Test shall be made by drying the stone to constant weight at a temperature of 212° to 230° Fahr., carefully weighing, then placing in water for 48 hours. At the end of this period the stone shall be removed from the water, all surplus water wiped off and the stone reweighed. The percentage of water absorbed shall be noted in terms of the original dry weight of the stone.

In the Freezing Test, five clean stone cubes of the same size shall be dried, weighed, and immersed in water for 24 hours. They shall then be alternately frozen and thawed twenty-five times. After the final thawing, they shall again be dried in an oven, and all loose particles removed. The loss of weight shall then be noted.

The Quenching Test shall consist of heating the stone specimens to 500° to 600° Fahr., and plunging them while hot in water of a temperature of about 70°. The loss due to spalling or disintegration shall be noted in terms of the weight of the original dry stone.

Toughness Test for rock. The following is from the 1927 Standards of the American Society for Testing Materials (Part II, p. 464):

"Test pieces shall be cylinders 25 mm. in diameter and 25 mm. in height, cut perpendicular to the cleavage of the rock. One set of three specimens shall be drilled perpendicular and another parallel to the plane of structural weakness of the rock, if such plane is apparent, otherwise one set of specimens shall be drilled at random.

"Any form of impact machine which will comply with the following essentials may be used in making the test:

"(a) A cast-iron anvil weighing not less than 50 kg. firmly fixed upon a solid foundation;

"(b) A hammer weighing 2 kg., arranged so as to fall freely between suitable guides;

"(c) A plunger made of hardened steel and weighing 1 kg., arranged to slide freely in a vertical direction in a sleeve, the lower end of the plunger being spherical in shape with a radius of 1 cm.;

"(d) Means for raising the hammer and for dropping it upon the plunger from any specified height from 1 to not less than 75 cm., and means for determining the height of fall to approximately 1 mm.;

"(e) Means for holding the cylindrical test specimen securely on the anvil without rigid lateral support, and under the plunger in such a way that the center of its upper surface shall, throughout the test, be tangent to the spherical end of the plunger at its lowest point."

Fig. 67 shows, in diagram, the testing machine.

The test consists of a 1-cm. fall of the hammer for the first blow, a 2-cm. fall for the second blow, and an increase of 1-cm. fall for each succeeding blow until failure of the test specimen occurs.

Paving Brick are tested for abrasion in the method described on p. 603.

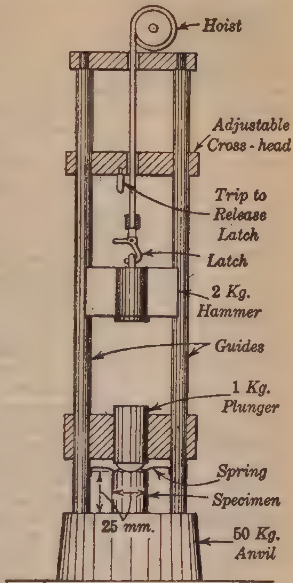


Fig. 67

26. Miscellaneous Tests

Tests of Strength of Wood have not been standardized to an extent which allows their inclusion in specifications for timber, but compression tests, flexure tests, shearing tests, and impact tests are frequently made. Compression tests are made both along the grain and across the grain. Compression specimens are usually rectangular blocks, and should be tested with spherical-seated bearing blocks. Wood has a rather poorly defined proportional limit in compression. In compression along the grain there is a well-defined ultimate. Shearing tests along the grain are made by the use of shearing blocks placed in the testing machine, or for larger specimens by testing short, deep beams. The shearing strength of wood along the grain is of great importance. Flexure tests both of small selected specimens and of large beams are common. In flexure tests the failure may be by longitudinal shear along the neutral axis, by compression along the upper side of the beam, or by tension along the lower side. Impact flexure tests show the shock-resisting qualities of wood. Hardness tests are made by using a steel ball 0.444 in. in diameter, and noting the load which causes the ball to penetrate to one-half its diameter.

Special Tests of Materials, Structures, and Machines

Structural part, machine part, structure, machine or material tested	Test	Measurements
Floor panel of building.	Proof test with dead load, not to destruction.	Deflections, tensile and compressive deformations in beams and columns.
Bridge	Proof test with dead load or with moving load.	Deflections, tensile and compressive deformations in various members.
Riveted joints	Tests of samples to destruction.	Ultimate load, slip of rivets.
Rivets, metal plates	Shearing test of samples to destruction using special shearing tools.	Ultimate load, results depend on hardness of shearing tools used.
Bolts	Tests of samples to destruction in tension or torsion.	Ultimate load.
Boilers	Tests with hydrostatic pressure. Proof tests with pressures somewhat above working pressures.	Observation of leaks, cracks, or permanent distortion of parts.
Car couplers and coupler yokes.	Proof tests, tests of samples to destruction under tension and impact tension.	Ultimate load, distortion under proof load.
Wire rope	Tests of samples to destruction in tension.	Ultimate load, observation of manner of fracture.
Brake beams for railway cars.	Tests of samples to destruction in flexure.	Ultimate load, deflections.
Chain	Proof tests of entire chain in tension, tests to destruction of sample sections.	Set after removal of proof load, ultimate strength.
Large pipe	Hydrostatic pressure proof test.	Observation of leaks, cracks, or other evidence of failure.
Gear teeth	Scleroscope or Rockwell test for uniform hardness.	Hardness for various teeth.
Engraver's plates	Brinnell, Rockwell, or scleroscope tests for uniform hardness of samples.	Hardness at various points.
Strain insulators for electric transmission lines.	Proof load in tension, tension test of samples to destruction.	Evidences of failure under proof load, ultimate strength.
Eyebars	Tension test of sample eyebars to destruction.	Yield point, ultimate load, stretch.
Columns	Compression tests of models or samples to destruction.	Yield point, ultimate load, deflection, axial compression.

Tests of concrete and of cement are discussed in Sect. 11 of this handbook. Further discussion or tests of road materials are given in Sect. 20.

The Turner-Hatt impact testing machine is in common use in the United States for impact testing of wood. This machine consists of an anvil on which the specimen is placed, and of a weight which can be dropped from various heights. Attached to the weight is a pencil which draws a record on a rotating drum. From this record the deflections of the specimen can be measured, as the weight is dropped from successively increasing heights until rupture occurs, or until the deflections increase abnormally, showing that the proportional limit has been passed. Another method of making an impact test consists in dropping the weight from such a height that the specimen is fractured by one blow. The pencil attached to the weight traces a curve on the rotating drum whose steepness is a measure of the velocity of the falling weight. Measuring the velocity of the weight before and after fracture of the specimen the energy absorbed in fracturing the specimen can be determined. For detailed description of tests of timber, see the 1927 Standards of the American Society for Testing Materials, Part II, p. 635.

Special Tests of materials, structures, and machines are often made. The table on p. 618 gives some such tests.

27. Inspection

A Shop Inspector has duties suggested largely by the specifications governing the work. Primarily his duties are to see that workmanship, method of fabricating material, and finished product are all in accordance with the plans and specifications, a complete set of which should be furnished him. The tools needed are inside and outside calipers, micrometer calipers, rule, steel tape, testing hammer, and report blanks. In structural work he should measure finished material, particularly with reference to field connections, examine abutting joints to see that they are full and square, test all rivets with a light hammer and have all loose or cock-headed ones cut out and replaced, should see that all members are free from twists, kinks, or bends, that pin holes are at right angles to the web of the member, countersunk rivets are chipped, that surfaces for rollers, splice plates, and bearing plates have been planed, that eyebars intended for the same pins should when piled all take the right pins at both ends at the same time, that threaded ends are wrapped with burlap, or otherwise protected against damage in transportation, etc. All accepted material should be marked with the inspector's private stamp.

The Cold Bend Test is of great importance as a shop test of ductility. It is made by bending cold a specimen of iron or steel flat on itself or round a mandrel of specified diameter. The bending may be accomplished either by pressure or by blows of a hammer. During and after bending the specimen must develop no cracks. For detailed requirements for cold bend tests of metals see the 1927 Standards of the American Society for Testing Materials, Part I.

Flattening or Upsetting Tests are made on rivets, on boiler tubes, and on pipes. Boiler tubes are also subjected to flanging tests in which a flange is formed on a cold specimen of tube. In flanging, upsetting, or flattening tests the distortion of the specimen must be accomplished without the development of cracks.

Hot Bend Tests and Quench Bend Tests are made on wrought iron, and quench bend tests are also made on material for boiler flues. Hot bend tests are made at a cherry-red heat, and quench bend tests are made after heating the specimen to redness and cooling in water. For both tests the specimen must bend round a mandrel of specified size without cracking.

Nick Bend Tests are made on wrought iron to show the fibrous structure and show the absence of scrap steel in the iron. A sharp nick is made across a test bar, which is then bent, opening out a cross-section. For round bars the specimen is nicked 25% around the circumference, for rectangular bars it is nicked along one side. The tool used to nick the bar has a 60-degree cutting edge, and the depth of the nick is not less than 8 nor more than 16% of the diameter or the thickness of the specimen. For the finest grades of iron the fracture at the nick shall be entirely free from bright crystalline spots; for cheaper grades not over 10% of the fractured surface shall show crystalline.

Wrought iron shows a gray, fibrous section, while the presence of scrap steel is indicated by bright, crystalline spots. This is a valuable shop test, but is not entirely conclusive.

IRON, STEEL, AND OTHER METALS

28. Cast Iron

Cast Iron is a saturated solution of carbon in iron, the amount of carbon varying ordinarily from a minimum of 1.7% to about 4%, depending upon the amount of silicon, sulfur, phosphorus, and manganese present in the solution. Other elements may also be present, but they are considered impurities.

Metallurgy. In the production of cast iron two steps may be necessary: First, the preparation of the ore for the blast furnace, involving the washing or dressing of the ore, and the calcination; and, second, the reduction in the blast furnace. In the case of the richer ores the first step is not always required, and this is the condition in the United States. The purpose of the roasting or calcination is the expulsion of volatile ingredients. The reduction or smelting of the ore is accomplished in a blast furnace, which, as the molten metal is drawn off at the bottom through the tapping-hole, is kept filled to the top or throat by adding metal in the form of iron ore, fuel in the form of coke, and flux, usually in the form of limestone, allowing the charge to work down. By means of tuyeres near the base of the furnace, a supply of air, or "blast," under a pressure of from 3 to 9 lb., is provided to maintain combustion at sufficiently high temperatures to reduce the ore. The molten iron is tapped from time to time near the bottom of the furnace. The flux combines, in the process of reduction, with the earthy matter of the ore and of the fuel, forming the slag. The slag, being very much lighter than the iron, floats on the surface of the molten metal and is tapped off near the top of the melting zone, through the cinder notch or cinder fall, into trucks called cinder tubs, by which it is transported to the cinder heap.

Pig Iron is the term applied to the form in which cast iron is obtained from the blast furnace. Pigs are usually cast in casting machines consisting of traveling metal molds, although some pigs are cast in open sand molds. A pig of iron weighs approximately 100 lb. and is approximately 3 ft. long. Machine-cast pigs are in the shape of rather thin slabs with rounded bottom side; sand-cast pigs have approximately equal width and depth with rounded bottom side. In its more restricted sense, cast iron is the form assumed after it has been again melted and cast into the finished form.

Composition. Commercial pig iron varies widely in chemical composition, depending on the uses to which it is to be put. The tendency of the present day is to purchase pig iron according to analysis instead of by grades. The constituents of good commercial pig iron vary as follows: Carbon from 2 to 4%; silicon from 0.5 to 5.0%; sulphur from 0.005 to 0.3%; phosphorus from 0.62 to 1.50%; and manganese from 0.10 to 1.75%. But these limits are by no means fixed.

Carbon. The amount in cast iron is largely dependent on the presence of other elements. While 4% is the ordinary maximum, the carbon may run as high as 7% if much manganese is present. The presence of silicon in large proportions, on the other hand, may reduce the solubility of the carbon to as low as 1%. The percentage of carbon present in cast iron in the combined form influences very largely the physical properties of the cast iron; thus, to get the maximum tensile strength, the combined carbon should be about 0.47%; for the maximum transverse strength it should be about 0.70%, and for the maximum crushing strength it should be over 1%. The hardness of cast iron increases regularly with the increase in the percentage of combined carbon. In Gray Iron the carbon exists almost wholly as graphite, having been precipitated as such in the process of solidifying. The graphite, known as "kish," gives the iron a somewhat spongy nature and a dark color. The condition is brought about in part by slow cooling, which tends to produce large crystals as well as graphite carbon. In White Iron the carbon is almost wholly combined and the iron has a more homogeneous texture, lighter appearance, and is composed of smaller crystals. Rapid cooling in solidifying tends to produce white iron. In Mottled Iron the proportions of combined and graphite iron are nearly equal, the fracture having, as the name indicates, a mottled appearance, due to the dark gray portions in the white matrix.

Silicon. The amount in cast iron determines, to some extent, the suitability of the material for various purposes. A certain amount is always desirable. Up to 0.8% it increases the hardness of the iron, but above that point it makes the iron soft and brittle. Silicon gives the iron a gray appearance, and if the proportion of silicon is large the iron is gray and highly crystalline. The shrinkage of cast iron is largely influenced by the presence of silicon, decreasing as the amount of silicon increases. A small proportion of silicon makes sound castings, free from blowholes.

Sulfur. The presence of sulfur is generally considered objectionable. A quantity greater than about 0.08% produces what is known as red shortness, that is, brittleness when in a heated condition. This condition makes the material unfit for use.

Phosphorus. The effect of phosphorus is to increase the fusibility and fluidity of the metal. Its presence, therefore, is desirable for light and ornamental castings, where well-defined impressions in the mold are wanted. At the same time, it tends to increase the brittleness of the iron. The maximum amount should not exceed 0.7% in good foundry pig for ordinary castings.

Manganese. This element is present in most pig iron and the amount varies greatly. For foundry use its presence is of no benefit when it exceeds about 1.0%. The value of the pig iron to the steel maker, however, increases in proportion to the amount of manganese present. Its presence prevents the absorption of sulfur in remelting. Hardness and closeness of grain are produced by the presence of manganese. **Spiegeleisen,** so called on account of its white glistening fracture, is a pig iron containing a large proportion of manganese, that is, from 5 to 20%. It is very hard, resisting cutting by cast-steel tools. When the proportion of manganese rises above 20% reaching sometimes as high as 80%, it is known as **ferro-manganese.**

The Grades of Pig Iron. Pig iron was formerly classified by grades, such as No. 1 soft, No. 2 soft, No. 1 foundry, No. 2 foundry, forge pig, etc., but to-day the practice is to buy pig iron by chemical analysis rather than by trade names of grades. Specifications for buying pig iron by analysis are given in the Standards of the American Society for Testing Materials. The accompanying table gives typical analyses of pig iron for various uses. All values are in per cent.

Use	Graphite carbon	Combined carbon	Silicon	Sulfur	Phosphorus	Manganese
Bessemer pig, for making steel by the acid Bessemer process or the acid open-hearth process.	3.50	0.05–0.10	1.00–2.00	Not more than 0.05	Not more than 0.10	0.25
Basic pig, for making steel by the basic open-hearth process.	3.50	0.05–0.10	Not more than 1.00	Not more than 0.05	0.30–1.00	0.25
Malleable pig, for making malleable cast iron	0.10	3.00	0.75–2.00	Not more than 0.05	Not more than 0.200	0.10
Foundry pig, for general casting.	3.15–3.90	0.05–0.10	0.75–4.00	0.02–0.08	0.10–1.00	0.20–0.25

Molding is the preparation of hollow molds to receive the molten metal. A wooden or metal pattern of the shape of the finished piece is prepared and imbedded in molding sand placed in wooden boxes so arranged in two parts, called the lower and upper flasks, that after the molding sand is thoroughly rammed and packed around the pattern, the upper flask may be taken off, the pattern withdrawn and the flask replaced and secured, leaving on the interior a hollow space of the size and shape of the finished piece. An opening is left through which the molten metal is poured into the mold. Smaller holes are also provided connecting with the hollow interior, to allow for the escape of the air and other gases that may be generated during pouring. When the finished casting is to be hollow a core is employed. Fig. 68 shows the cross-section of a mold ready to receive the molten iron. This particular mold is for a cast-iron pulley.

Wooden Patterns are generally made of thoroughly seasoned white pine or mahogany, the wood being carefully shellacked to keep the pattern from warping or being otherwise affected by moisture. If much used, patterns are often made of metal, in which case after being cast they are filed and scoured smooth, warmed and coated with wax. Aluminum patterns are coming into rather wide use. Patterns are sometimes also made of plaster of Paris, especially for highly ornamental castings in architectural iron work. The **shrinkage** of cast iron in cooling must be allowed for in the making of the patterns. The usual allowance is 1/8 in. per foot, but this cannot be laid down as a hard and fast rule, as the shrinkage varies with the relative dimensions of castings and with the character of the metal.

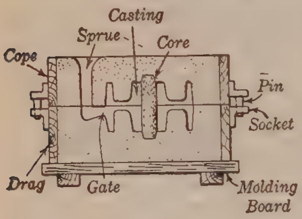


Fig. 68

Molding Sand consists chiefly of silica, 90 to 95%, which makes the sand sufficiently refractory; alumina and magnesia, 3 to 8%, which furnishes the necessary cohesion and plasticity to the sand; oxide of iron, about 1.5%; lime about 0.5%; and sometimes a small quantity of coal dust. Fine sand compacts too much, preventing the gases from escaping readily and causing blowholes in the castings, while coarser sand lacks cohesion and makes inferior castings.

Green Sand Molds are made of molding sand and are the molds most generally used, being most readily made and cheap. **Dry Sand Molds** are molds which have been baked in an oven to fix their shape and to give a hard surface. **Loam Molds** are generally used for large castings. They are built up of brickwork to the rough outline of the casting and are then finished off on the surface with loam laid on by a trowel.

Chills are metal molds used for certain castings, such as car wheels, where a hard surface is wanted, this being produced by the sudden cooling of the hot metal as it comes in contact with the comparatively cold surface of the mold.

Cores are used for the production of the hollow spaces in castings, and are made of baked sand and clay formed into shape and fixed in the molds. For large cylinders and other large castings, the cores are built up of brickwork to the approximate size and the surface finished off by facing with loam. For water pipes and similar castings the brickwork is often replaced by iron tubes.

The Cupola is the usual means for remelting pig iron to produce cast iron in its more restricted sense, namely, the finished castings. It consists of a cylindrical shaft provided with one or two rows of tuyeres near the base, through which air is forced at a pressure of about one-half pound per square inch. The charge consists of pig iron and coke in the proportion of about 200 lb. of coke for each ton of metal. A little limestone is usually also introduced as a flux. The molten metal drawn off at the bottom is poured into the molds by means of cup-shaped ladles fitted with long handles, one of which has a cross bar for tipping the ladle in pouring, or, for larger work, in ladles carried on wheels or by cranes and tipped by gears.

Malleable Castings are made in the same manner as ordinary castings, but are subjected to a further annealing process. Only small castings can be treated in this way. The castings are placed in cast-iron boxes called annealing pots, with decarbonizing material packed around them. For the decarbonizing agent an iron oxide in the form of hematite ore or forge iron scale is used. Only white iron low in sulfur can be used, and generally the best charcoal cast iron is selected. The castings, packed as stated, are placed in an oven in which the temperature is quickly raised and the castings kept at a cherry-red heat for three to five days, depending on their size, after which the furnace is allowed to cool slowly. The effect of this process is to make the castings " malleable " and to nearly double their strength. Castings of this kind may be bent cold, forged, or welded to a greater or lesser extent. They are used for pipe fittings, iron handles for tools, wheels, pinions, small parts of machinery, etc.

Semi-steel is the somewhat misleading trade name given to cast iron in the production of which 30 to 60% of steel scrap is used. With great care in foundry practice such " semi-steel " may be produced having higher strength than ordinary cast iron.

The Air Furnace which is sometimes used instead of a cupola to melt cast iron comprises a hearth on which fuel is burned and a separate chamber in which the pig iron and scrap are melted without direct contact with the fuel. Air furnace iron is freer from impurities than is cupola iron, but is much more expensive. Except for producing malleable cast iron the air furnace is not very extensively used.

The Fusibility of Cast Iron is dependent on the percentage of carbon and some of the other elements. The average fusion point is about 2200° Fahr. Its heat conductivity is 35.9, silver being 100.

The Specific Gravity of cast iron varies with its composition from about 6.9 to 7.5. It is usually taken at 7.22 (water at 2° F. being unity) corresponding to a weight of 450 lb. per cu. ft. In a general way the specific gravity increases with the strength of the metal and the number of remeltings.

The Coefficient of Expansion of cast iron may be taken at 0.000 0062 for 1° F. as an average. If exposed to continued heat, cast iron becomes permanently expanded 1-1/2 to 3%, a fact that must be remembered in installing grate bars or other castings exposed to heat.

The Modulus of Electricity varies from 12 000 000 to 14 000 000 lb. per sq. in. for ordinary commercial cast iron, and from 16 000 000 to 18 000 000 for special grades of strong cast iron, such as are used for ordnance.

The Elastic Limit. Cast iron has no clearly defined elastic limit either in tension or compression. For malleable iron the yield point in tension is about 30 000 lb. per sq. in.

The Ultimate Strength in Tension for ordinary castings can be taken at 15 000 to 18 000 lb. per sq. in., and for special grades of iron or for alloys of nickel with cast iron at 40 000 to 60 000. Malleable iron has a strength of about 50 000 lb. per sq. in.

Compressive Strength. 80 000 lb. per sq. in. may be taken as the average ultimate strength in compression. Tests show as low as 44 500 and as high as 215 000 lb. per sq. in.

Modulus of Rupture. The average value may be taken as 35 000 lb. per sq. in., the tests varying from a minimum of 9700 to a maximum of 80 000.

Shear and Torsion. Under torsion cast iron fails by tensile strain on an inclined section so that it is not safe to consider the shearing strength of cast iron as greater than the tensile strength.

Defects. The most common defects in cast iron are (1) blowholes, caused by the formation of steam when the hot metal comes in contact with the damp molding sand: (2) sand holes, and (3) roughness of surface, due to breaking down of the mold in spots; (4) cold-shuts or cold-shorts, which are seams caused by the too rapid congealing of the metal so that it does not completely fill the mold or join together solidly at points where two streams of iron meet, and (5) cracks resulting from uneven shrinkage in parts of the castings of unequal thickness. The last is sometimes not discoverable until the casting is put under load. Castings to be acceptable should present smooth, clean surfaces with all angles true and sharp, and should be soft enough to be dented on the edges by a hammer blow instead of breaking off.

For Cast-iron Pipe the metal shall be "of good quality and of such character as shall make the metal of the castings strong, tough and of even grain, and soft enough to satisfactorily admit of drilling and cutting." (Standard Specifications, American Society for Testing Materials.) For strength tests, see p. 613.

For Cast-iron Car Wheels the metal shall be soft, clean gray iron, closely approximating the following composition: Graphitic carbon 3.25%, combined carbon 0.60%, silicon 0.70%, manganese 0.60%, phosphorus 0.40%, sulfur 0.14%.

For Gray Iron Castings, that is, ordinary castings, the following standard specifications have been adopted by the American Society for Testing Materials (1927 Standards):

Unless furnace iron is specified, all gray castings are understood to be made by the cupola process.

The sulfur contents to be not over following percentages: Light castings, 0.10; medium castings, 0.10; heavy castings, 0.12.

In dividing castings into light, medium, and heavy classes, the following standards have been adopted: Castings having any section less than 1/2 in. thick shall be known as light castings. Casting in which no section is less than 2 in. thick shall be known as heavy castings. Medium castings are those not included in the above classification.

Transverse Test. The minimum breaking strength of the "Arbitration Bar" under transverse load shall be not under 1500 lb. for light castings, 1750 for medium castings, and 2000 for heavy castings. In no case shall the deflection be under 0.20 in. (See p. 613.)

Borings from the broken pieces of the "Arbitration Bar" shall be used for the sulfur determinations. One determination for each mold made shall be required. In case of dispute, the standards of the American Foundrymen's Association shall be used for comparison.

Castings shall be true to pattern, free from cracks, flaws, and excessive shrinkage. In other respects they shall conform to whatever points may be specially agreed upon.

The inspector shall have reasonable facilities afforded him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall, as far as possible, be made at the place of manufacture prior to shipment.

For **Malleable-iron Castings** the 1927 Standards of the American Society for Testing Materials specify tensile tests for acceptance.

The tension test specimen is a round specimen machined from a test lug on a casting 7-1/2 in. long, 5/8 in. in minimum diameter, and with a 2-in. gage length. Such tension tests shall show a yield point of not less than 30 000 lb. per sq. in., a tensile strength of not less than 50 000 lb. per sq. in., and an elongation in 2 in. of not less than 10%. Malleable castings shall be made from iron melted either in an air furnace, an open-hearth furnace, or an electric furnace.

Working Unit Stresses for cast iron depend upon the character of the applied loads and upon the grade of metal. Common average values for tension fibers of flexure members are 4000 lb. per sq. in. for steady loads and 3000 lb. per sq. in. for variable loads, while for compression values about four times as great may be used. In shearing the working unit stresses may be taken as about the same as those for tension. The properties of castings depend upon the quality of the ores and upon the method of manufacture. Cold-blast pig produces stronger iron than hot-blast pig, but it is more expensive. Drying the air before admitting it to the blast tends to secure uniformity of product. The darkest grades of foundry pig make the smoothest castings, but they are apt to be brittle; the lightest grades make tough castings, but they are apt to contain blowholes or imperfections.

Very low unit stresses should be used when cast iron is subject to repetitive stresses alternating from tension to compression; probably 2000 lb. per sq. in. is the highest value allowable. Cast iron is a brittle material which is unsuited to resist shocks, and since 1900 it has not been used in bridge construction. It is not generally used in direct tension.

29. Wrought Iron

Composition. Wrought iron is a product of the reverberatory furnace, and is composed principally of ferrite (pure iron) and slag (iron silicate); in these exist small amounts of impurities, an idea of the percentage of which can be obtained from the following analysis (Macfarlane):

	Common wrought iron	Best wrought iron
Carbon	0.05	0.06
Phosphorus	0.35	0.18
Sulfur	0.06	0.04
Silicon	0.23	0.20
Manganese	....	0.06
Slag ..... about	3.3	2.80

Metallurgy. Wrought iron is made in a reverbatory furnace from pig iron and less frequently from molten metal taken directly from the blast furnace. This method of manufacture is known as the puddling process. The furnace consists essentially of a firebox, puddling or working chamber, suitable draft openings and flue. The working chamber is provided with openings for the purpose of charging and working the metal, and for the removal of slag and the puddled metal. There are two distinct puddling processes, Wet Puddling and Dry Puddling.

In the **Wet Process** the hearth of the furnace is felted with high-grade iron ore or mill scale which acts as an oxidizing agent for reducing impurities. The reduction of impurities occurs in different stages, namely in the melting-down stage, during which most of the silicon and manganese and some of the phosphorus are removed; in the clearing stage, during which phosphorus and sulfur are removed, and in the boiling stage, during which carbon is removed and most of the remaining phosphorus and sulfur. Since pig iron melts at a much lower temperature than metallic iron, as the mass becomes purified it assumes a pasty condition. It is then collected in balls weighing about 80 lb., carried to the squeezers or a forge and most of the slag expelled. The resulting bars are then run through roughing rolls, the rolled bars trimmed, and thereafter known as " muck bar." The muck bars are then cut up into short lengths, and these short lengths are " box piled " in alternate layers with lengthwise and crosswise bars. These box piles are tied together with wire, placed in a furnace and heated to welding heat, and then rolled out into plates, rods, or other shapes. The piling and rerolling removes slag, and gives a metal with interlacing fibers of slag.

In the **Dry Process,** which is infrequently used, white pig iron is charged and subjected to the action of an oxidizing flame. The oxygen in this case is supplied by the furnace instead of from the fettling.

Appearance. A section of a wrought-iron rod or plate when polished shows more or less regular laminations of slag and iron. By notching one side of a bar and bending one end away from the notched side, the iron will break along the slag laminations and give a fracture known as "barking." If a bar is notched all around and then struck a blow heavy enough to cause it to break, the fracture will be coarsely crystalline; when broken in tension, the fractured section is generally irregular and fibrous.

To distinguish between Wrought Iron and Soft Steel (Iron Age, Dec. 23, 1909): The sample is cleaned from grease and scale and immersed in a solution of the following: water, 9 parts; sulphuric acid, 3 parts; muriatic acid, 1 part. The acids are poured into the water and the mixture allowed to cool. The specimen is allowed to remain in the solution for 15 or 20 minutes, when it is removed and rinsed in water. The fibers will now show plainly, or if not, the process is continued. Soft steel dissolves uniformly and without the fibrous structure found in wrought iron.

Grades. Wrought iron may be graded as follows: (1) Charcoal iron, the purest grade of wrought iron; (2) Puddled iron, classified according to quality into stay-bolt iron (grade A) and merchant iron, grades B and C; (3) Busheled scrap, a heterogeneous product made from iron scrap; steel is frequently mixed with the iron scrap, causing considerable irregularity in the resulting product.

Properties. Wrought iron possesses the important qualities of toughness, ductility, malleability, and weldability, but its properties are only slightly changed by tempering.

Coefficient of expansion..	0.00000648 per degree F. (Clarke)
Electrical conductivity...	0.16 (Cu = 100) (Lazare Weiler)
Melting temperature....	2732° to 2912° F. (Pouillet, Claudel, Wilson)
Specific heat...........	0.1138 (Röntgen)
Specific gravity.........	7.4 to 7.9 (Kent)

Tension. The average results of a great many tensile tests made at the Testing Laboratory of Columbia University on good wrought iron for general purposes give the following:

Yield point, lb. per sq. in..................	31 000
Ultimate strength, lb. per sq. in..............	51 000
Elongation in 8 in., per cent.................	21
Reduction of area, per cent..................	30
Modulus of elasticity, lb. per sq. in...........	28 200 000

Shear and Torsion. J. Platt and R. F. Hayward (Proceedings Inst. C. E., Vol. 90) give the following values for "crown" best wrought iron which had an ultimate tensile strength of 48 400 lb. per sq. in.:

Ultimate strength in single shear, lb. per sq. in.	42 050
Elastic limit in torsion, lb. per sq. in.........	20 530
Modulus of elasticity in torsion, lb. per sq. in..	12 800 000

Compression. The ultimate compressive strength of good wrought iron is not well defined. Practically, its yield point in compression should be considered as the ultimate for compression. This yield point is about the same as the yield point in tension.

The Strength of wrought iron is affected by its chemical composition, the mechanical work and heat treatment it has undergone, and also varies for different temperatures. Wrought iron has a well-defined yield point in both tension and compression which is from 2000 to 4000 lb. per sq. in. higher than the elastic limit. Beyond the yield point wrought iron is a plastic material which flows rapidly as the maximum strength is approached. The ultimate strength in tension increases with the amount of carbon, which, however, is rarely greater than 1/10 of 1 per cent. The strength of iron entirely free from carbon and phosphorus is probably between 39 000 and 40 000 lb. per sq. in.

Effect of Work of Rolling on Wrought-iron Plates (Holley)

Thickness in inches	Elastic limit, lb. per sq. in.	Ultimate strength, lb. per sq. in.	Elongation in 8 in., per cent	Reduction of area, per cent
1/4	32 400	51 800	11.2	18.4
1/2	31 180	49 760	14.2	22.0
5/8	30 775	50 200	15.5	22.5
3/4	30 400	49 050	16.0	22.4

The effect of the work of rolling on circular sections exhibits the same variation, that is, a material increase in the strength of the bar results from a reduction of the cross-section.

Wrought-iron Wire is manufactured by cold drawing through dies. Tensile tests on wire (U. S. Report on Tests of Metals, 1897) have given 110 000 lb. per sq. in. for wire 0.01 in. in diameter and 65 280 lb. per sq. in. for wire 0.2 in. in diameter. A rod 0.8 in. in diameter of the same kind of material as the wire gave 50 000 lb. per sq. in.

The Effect of Temperature upon the ultimate strength of wrought iron is shown by the following table (M. Rudeloff, Trans. Internat. Soc. Testing Materials, 1909).

Temperature, degrees C.	Ultimate strength, lb. per sq. in.	Elongation, per cent	Temperature, degrees C.	Ultimate strength, lb. per sq. in.	Elongation, per cent
20	49 300	30.5	250	70 700	23.0
50	51 400	25.5	300	68 600	30.0
100	54 300	16.0	350	57 100	35.0
150	60 700	14.0	400	46 100	40.0
200	67 100	17.5	....	....	....

Effect of Reheating and Rerolling. Within limits, puddled iron is much improved in quality by being cut up, piled, reheated, and rerolled or hammered. However, it is found that only in special cases is it advantageous to reheat puddled iron more than twice. The following figures (Johnson) show the effect of reheating and rerolling on the tensile strength: The original bar had a tensile strength of 43 900 lb. per sq. in.; after the second working this rose to 52 860 lb. per sq. in., and after the sixth working it became 61 820 lb. per sq. in.; the tensile strength then diminished with the number of workings until after the twelfth working it became the same as that of the original bar.

Uses. Wrought iron is used for spikes, nails, bolts and nuts, wire, chain rod, horseshoe bars, sheets and plates, stay bolts, pipes and tubing, third rails, armatures, electro-magnets, and in the manufacture of crucible steel. Wrought iron is sold as " merchant bar " for subsequent working into various wrought shapes.

Wrought iron was extensively used in bridge and building construction prior to 1890, but since 1900 structural steel has entirely taken its place on account of being about 20% stronger, as also lower in price. The permissible tension P on a wrought iron chain of the usual form is $\mathbf{P} = 0.4\ \mathbf{d}^2\ \mathbf{S}$ for open links and $\mathbf{P} = 0.5\ \mathbf{d}^2\ \mathbf{S}$ for stud links, where d is the diameter of the metal and S is the safe working unit stress (Goodenough and Moore, Bull. Univ. Ill., No. 18, 1907).

Behavior under Stress. Good wrought iron shows a fibrous structure when broken by tension or flexure. A stress exceeding the elastic limit causes a permanent set and raises the elastic limit higher than before. It is a fundamental rule that working unit stresses should not exceed the elastic limit. Under variable loads the allowable unit stress which is specified should seldom be greater than one-half of the elastic limit.

Defects. The principal defects in wrought iron as classified by Turner are rough edges, spilly places and blisters. To these might be added the presence of excess of slag. Rough edges are due to careless workmanship, imperfections in the rolls, and also to red shortness. Spilly places are spongy or irregular spotted parts, noticed particularly in sheets and occasionally in all kinds of wrought iron. They are generally attributed to imperfect puddling.

Specifications for wrought iron for various uses are given in detail in Part I of the 1927 Standards of the American Society for Testing Materials.

Working Unit Stresses for wrought iron will depend upon the kind of loading, the highest being for steady loads and the lowest for alternating stresses or shocks. The following values are for wrought iron of an average quality and these may be increased 30% for the very best grades. All values are in pounds per square inch.

	Steady Stresses or Stress Not Reversed	Alternating Stress
Tension..........	14 000	7000
Compression......	13 000	7000
Shear (torsion)....	8 000	5000

Wrought iron has a decided "fiber" due to the slag entrained in the process of rolling. The above values for tensile strength and working unit stresses apply to those along the fiber, and about 12% is to be deducted from them for cases where the tension acts across the fiber.

30. Classification of Steel

Steel and Iron. Steel was originally produced directly from pure iron ore by the action of a hot fire, which did not remove the carbon to a sufficient extent to form wrought iron. The modern processes, however, involve the fusion of the ore, and the definition of the United States law is that "steel is iron produced by fusion by any process, and which is malleable." Chemically, steel is a compound of iron and carbon generally intermediate in composition between cast and wrought iron, but having a higher specific gravity than either.

	Per Cent of Carbon	Specific Gravity	Properties
Cast iron......	5 to 2	7.2	Not malleable, not temperable
Steel.........	1.50 to 0.02	7.8	Malleable and temperable
Wrought iron..	0.30 to 0.05	7.7	Malleable, not temperable

It should be observed that the percentage of carbon alone is not sufficient to distinguish steel from wrought iron; also, that the mean values of specific gravity stated are in each case subject to considerable variation; further, only the hard steels are much affected by tempering, the softer grades resembling wrought iron.

Manufacture. The four principal methods are the crucible process, the electric-furnace process, the open-hearth process, and the Bessemer process. In the crucible process impure wrought iron, with carbon and a flux, is fused in a sealed vessel to which air cannot obtain access; some high grades of tool steel are thus made. In the electric-furnace process steel is refined out of contact with air by heat produced by means of an electric arc. In the open-hearth process pig iron is melted, scrap and iron ore being added until the proper degree of refining is secured. In the Bessemer process pig iron is completely decarbonized in a converter by an air blast and then recarbonized to the proper degree. The metal from the open-hearth furnace or from the Bessemer converter is cast into ingots which are rolled in mills to the required forms. The open-hearth process produces steel for machines, shafts, axles, springs, armor plates, rails, and for structural purposes. The Bessemer process mainly produces steel for railroad rails and for the cheaper grades of structural steel. A combination method known as the duplex process is sometimes used. In this process the refining action is started in a Bessemer converter, and after partial refining the molten steel is transferred to an open-hearth furnace, where the refining process is finished. The following description of these methods is taken mainly from Kent's Mechanical Engineers' Pocket Book:

Crucible Steel is commonly made in pots or crucibles holding about 80 lb. of metal. The raw material may be steel scrap; wrought iron with charcoal; cast iron with wrought iron or with iron ore; or any mixture that will produce a metal having the desired chemical constitution. Manganese in some form is usually added to prevent oxidation of the iron. Some silicon is usually absorbed from the crucible, and carbon also if the crucible is made of graphite and clay. The crucible being covered, the steel is not affected by the oxygen or sulfur in the flame. The quality of crucible steel depends on the freedom from objectionable elements, such as phosphorus, in the mixture, on the complete removal of oxide, slag, and blowholes by "dead-melting" or "killing" before pouring, and on the kind and quantity of different elements which are added in the mixture, or after melting, to give particular qualities to the steel, such as carbon, manganese, chromium, tungsten, and vanadium.

Bessemer Steel is made by blowing air through a bath of melted pig iron The oxygen of the air first burns away the silicon, then the carbon, and before the carbon is entirely burned away, begins to burn the iron. Spiegeleisen or ferro-manganese is then added to deoxidize the metal and to give it the amount of carbon desired in the finished steel In the ordinary or "acid" Bessemer process the lining of the converter is a siliceous material which has no effect on phosphorus, and all the phosphorus in the pig iron remains in the steel. In the "basic" or Thomas and Gilchrist process the lining is of magnesium limestone, and limestone additions are made to the bath, so as to keep the slag basic, and the phosphorus enters the slag. Basic Bessemer steel is not made in the United States.

Open-hearth Steel. Any mixture that may be used for making steel in a crucible may also be melted on the open hearth of a Siemens regenerative furnace, and may be desiliconized and decarbonized by the action of the flame and by additions of iron ore, deoxidized by the addition of spiegeleisen or ferro-manganese, and recarbonized by the same additions or by pig iron. In the most common form of the process pig iron and scrap steel are melted together on the hearth, and after the manganese has been added to the bath it is tapped into the ladle In the Talbot process a large bath of melted material is kept in the furnace, melted pig iron, taken from a blast furnace, is added to it, and iron ore is added which contributes its iron to the melted metal while its oxygen decarbonizes the pig iron. When the decarbonization has proceeded far enough, ferro-manganese is added to destroy iron oxide, and a portion of the metal is tapped out, leaving the remainder to receive another charge of pig iron, and thus the process is continued indefinitely. In the Duplex Process melted cast iron is desiliconized in a Bessemer converter, and then run into an open hearth, where the steel-making operation is finished. The open-hearth process may be either acid or basic, according to the character of the lining. The basic process is a dephosphorizing one, and is the one most generally available, as it can use pig irons that are either low or high in phosphorus.

Electric-furnace steel. Instead of using the gas flame of an open-hearth furnace the heat of an electric arc may be used to heat a charge of steel, which can be kept out of contact with air under a protecting blanket of molten slag. The purifying ingredients, iron oxide, manganese dioxide, etc., are in the slag, and the impurities are absorbed by it. Electric heat is very uneconomical for producing low temperatures, but much more economical for producing high temperatures, such as are used in the final refining stages of steel making. The electric furnace is sometimes used for the final stages of the steel making process, the earlier stages being carried out in an open-hearth furnace or a Bessemer converter.

Carburizing steel consists in heating it below its melting point in contact with material rich in carbon. Charcoal is sometimes used, also gas rich in carbon. There are a number of special carburizing compounds on the market. This gives the steel a " case " or skin of high-carbon steel with a " core " of lower-carbon steel.

Case Hardening consists in a carburizing process followed by suitable heat treatment. Case-hardened steel is used for service in which severe surface wear has to be resisted, and for giving a strong material at the surface of the metal, where strength is especially needed in parts subjected to bending or to torsion. A very thin skin of hardened steel may be given to soft steel by heating it in contact with a cyanide salt and subsequently heat-treating; this is called **cyaniding.**

The Physical Properties of steel depend upon method of manufacture, the heat treatment, and chemical composition, the carbon having a controlling influence upon strength. Phosphorus increases strength, but it promotes brittleness; manganese increases strength in a less degree, and it promotes malleability; sulfur causes red-shortness or a tendency of the steel to crumble while being rolled; and silicon increases hardness. Acid steel is regarded as of higher quality than basic steel having the same percentage of carbon. Since 1900 more than three-fourths of the open-hearth steel produced in the United States has been basic, the Bessemer product on the other hand being entirely acid.

The products described above are sometimes called carbon steel, because carbon is the controlling element in regard to strength. When the strength is largely governed by other elements the steels are called **alloy steels.**

The Methods of Manufacture greatly influence the strength of steel. Forged steel is stronger and more reliable than cast steel. Heat treatment by annealing and tempering has also a marked influence.

Forging and drawing greatly increase the strength of steel. Forging under a hammer or press renders the material more compact and increases both specific gravity and strength. The process of drawing steel bars into wire has a similar result, and wire has been made having a tensile strength of 300 000 lb. per sq. in., while the wire used for the cables of suspension bridges usually has a tensile strength of from 150 000 to 200 000 lb. per sq. in. By compressing steel while it is fluid, the strength may also be increased, and this process is used for the steel from which large guns and hollow shafts are made.

Annealing consists in raising cold steel to a light red heat and then allowing it to cool slowly. This process reduces the ultimate strength, but it increases the ductility. Quenching consists in plunging heated steel into a bath of water or oil, or by applying these fluids to its surface. The hardness of the steel and its ultimate strength are thereby much increased. Tempering consists in heating quenched steel to a temperature somewhat below that required for annealing.

Steel Castings have in recent years replaced steel forgings for many structural and machine parts. Their tensile strength is not widely different from that of steel forgings of the same chemical composition, but their elastic limits and their fatigue strengths average about 20% lower. Their strength and ductility are usually markedly improved by suitable heat treatment, usually an anneal to relieve internal strains. Detailed specifications for steel castings are given in Part I of the 1927 Standards of the American Society for Testing Materials. Steel castings are usually made by the open-hearth process, although some high grade castings are made by the electric-furnace process.

Carbon is the controlling element in regard to strength of plain carbon steels, and the same is the case with respect to ultimate elongation. The higher the percentage of carbon, within a reasonable limit, the greater is the strength and the less the ultimate elongation. The strength, especially for high-carbon steels, depends on the heat treatment used. Fully annealed high-carbon steel is not very much stronger than low-carbon steel.

General Classes. According to the percentage of carbon and its capacity for taking temper or being welded, steel may be classified as follows:

Soft, 0.05–0.20 C........	Not temperable, easily welded
Medium, 0.15–0.40 C.....	Slightly temperable, weldable
Hard, 0.30–0.70 C........	Temperable, welded with difficulty
Very hard, 0.60–1.00 C...	Highly temperable, not weldable

Uses. Plain carbon steel is frequently classified with reference to its uses, and the following classification gives a rough general idea of the elastic and ultimate strength and of the Brinell hardness of steels for various uses. Wide variations from these values occur for individual cases.

Steel	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.	Brinell hardness, number
Steel for rivets..........................	25 000	50 000	100
Steel for plates and shapes..................	35 000	60 000	120
Machinery steel as rolled....................	40 000	75 000	145
Machinery steel, oil quenched...............	65 000	100 000	200
Axle steel as forged.........................	45 000	90 000	180
Axle steel quenched and tempered...........	65 000	100 000	200
Spring steel, oil quenched...................	95 000	180 000	350
Iron or soft steel wire.......................	*	85 000	....
Crucible steel wire..........................	*	200 000	....

* Elastic limit poorly defined.

31. Structural Steel

Specifications. The following clauses give essential features of the specifications for structural steel which have been adopted by the American Society for Testing Materials and the American Railway Engineering Association:

1. Structural steel shall be made by the open-hearth process. (Structural steel for buildings may be made by the Bessemer process.)

2. Each of the three classes of structural steel shall conform to the following limits in chemical composition: Sulfur shall not exceed 0.05%; phosphorus shall not exceed 0.06% when the steel is made by the acid process and not exceed 0.04% when it is made by the basic process.

3. There shall be two classes of structural steel for bridges, buildings, locomotives, cars, and ships, namely, rivet steel and structural steel, which shall conform to the following physical requirements: Rivet steel shall range in tensile strength from

46 000 to 60 000 lb. per sq. in., have a yield point not less than 0.5 of the tensile strength, and the percentage of elongation in 8 in. shall be

$$\frac{1\,500\,000}{\text{tensile strength}}$$

Soft steel shall range in tensile strength from 55 000 to 68 000 lb. per sq. in., have a yield point not less than 0.5 of the tensile strength, and the elongation in 8 in. shall not be less than 25%.

4. For each increase of 1/32 in. in a flat specimen above a thickness of 3/4 in., a deduction of 1/4 shall be made from the above specified percentage of elongation. For each decrease of 1/32 in. below a thickness of 5/16 in., a deduction of 1-1/4 shall be made from the above-specified percentage of elongation.

5. For details of cold bend tests see the 1927 Standards of the American Society for Testing Materials, Part I.

6. The standard specimens are shown in Figs. 55*a* and 56. The flat specimen is used for plates and shapes, and the 2-in round specimen for pins, rollers, and bars. Rivet rounds and small bars are tested full size.

7. One tensile test specimen shall be taken from the finished material of each melt or blow, but in case this develops flaws, or breaks outside of the middle-third of its gage length, it may be discarded and another test specimen substituted therefor.

8. Material which is to be used without annealing or further treatment shall be tested for tensile strength in the condition in which it comes from the rolls. Where it is impracticable to secure a test specimen from material which has been annealed or otherwise treated, a full-size section of tensile-test specimen length shall be similarly treated before cutting the tensile-test specimen therefrom.

9. For the purpose of this specification, the yield point shall be determined by careful observation of the drop of the beam or halt in the gage of the testing machine.

10. In order to determine if the material conforms to the chemical limitations prescribed in paragraph No. 2 herein, analysis shall be made of drillings taken from a small test ingot.

11. The variation in cross-section or weight of more than 2-1/2% from that specified will be sufficient cause for rejection, except in the case of sheared plates, which will be covered by special allowances tabulated in the 1927 Standards of the American Society for Testing Materials.

When plates are ordered to gage, a variation of more than 1/100 in. below that specified for any dimension will be sufficient cause for rejection.

12. Finished material must be free from injurious seams, flaws, defective edges, or cracks, and have a workmanlike finish.

13. Every finished piece of steel shall be stamped with the melt number and name of the manufacturer and steel for pins shall have a melt number stamped on the ends. Rivets and lacing steel, and small pieces for pin plates and stiffeners, may be shipped in bundles, securely wired together, with the melt number on a metal tag attached.

14. The inspector representing the purchaser shall have all reasonable facilities afforded to him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall be made at the place of manufacture, prior to shipment.

Structural Nickel Steel. The following is a summary of the principal items in the specifications for structural nickel steel as given in the 1927 Standards of the American Society for Testing Materials:

1. The steel shall be made by the open-hearth process.

2. The phosphorus content shall not exceed: for acid structural steel, 0.05%; for basic structural steel, 0.04%; for acid rivet steel, 0.04%; for basic rivet steel, 0.03%. The sulfur content shall not exceed: for structural steel, 0.05%; for rivet steel, 0.45%. The nickel content shall not be less than 3.25%.

3. The following table summarizes the requirements as to tensile strength and ductility.

Properties considered	Rivet steel	Plates, shapes and bars	Eyebars, flats and rollers unannealed	Eyebars, flats and rollers annealed
Tensile strength, lb. per sq. in....	70 000–80 000	85 000–100 000	95 000–110 000	90 000–105 000
Yield point, min. lb. per sq. in...	45 000	50 000	55 000	52 000
Elongation, in 8 in., per cent., min.	1 500 000 / tensile str.	1 500 000 / tensile str.	1 500 000 / tensile str.	20*
Elongation, in 2 in., per cent., min.	....	18†	16†	20†
Reduction of area, per cent., min..	40	25	25	35

* Flat test specimens from plates and shapes.
† Round test specimens from pins and rollers.

4. For plates, shapes, and unannealed bars over 1 in. in thickness a deduction of 1/4 from the percentage of elongation specified in the above table shall be made for each increase of 1/32 in. thickness above 1 in., to a minimum of 14%.
5. The test specimens are similar in size and form to those used for structural carbon steel.
6. For details of cold bend tests see 1927 Standards of the American Society for Testing Materials, Part I.

Structural Silicon Steel. The following is a summary of the principal items of the specifications for structural silicon steel as given in the 1927 Standards of the American Society for Testing Materials.

1. The steel shall be made by the open-hearth process.
2. The phosphorus content shall not exceed: for acid steel, 0.06%; for basic steel, 0.04%. The sulfur content shall not exceed 0.05%. The silicon content shall not be under 0.20%.
3. The tensile strength shall be between 80 000 and 95 000 lb. per sq. in.; the yield point shall be not less than 45 000 lb. per sq. in.; the percentage of elongation in 8 in. shall not be less than 1 500 000/tensile strength; and the reduction of area shall not be less than 30%.
4. For material over 3/4 in. in thickness a deduction of 1/4 in the percentage of elongation and of 1/2 in the reduction of area shall be allowed for each increase of 1/16 in. in thickness, with a minimum value of 14% for elongation and 24% for reduction of area.

Items 5 and 6 for structural nickel steel apply to structural silicon steel.

Steel Castings should not contain over 0.07% of phosphorus, or over 0.06% of sulfur. The minimum physical qualities prescribed by the American Society for Testing Materials are:

	Soft Castings	Medium Castings	Hard Castings
Tensile strength, lb. per sq. in......	60 000	70 000	80 000
Yield point, lb. per sq. in..........	0.45 of the tensile strength		
Elongation, per cent in 2 in........	24	20	17
Reduction of area, per cent.......	35	30	25

Rivets. The ultimate strength of steel rivets as determined by various British authorities from experiments on riveted joints, assuming that the distribution of load was uniform on all rivets and that the friction of the plates is negligible, varied for iron from 42 582 to 62 362 lb. per sq. in., and for steel from 49 683 to 80 035 lb. per sq. in. Tests made at the University of Illinois gave average bearing strengths of 36 950 for boiler rivet steel, 40 950 for structural rivet steel, and 37 950 lb. per sq. in. for wrought iron. These values were, respectively, 76.4, 74.5, and 77.6% of the tensile strengths of the same material. The tests were made on 1/2, 3/4, and 1-in. rivets in single and double shear. More recent tests made by the American Railway Engineering Association on riveted steel joints gave for the rivets an average shearing strength of 48 580 lb. per sq. in.

Working Unit Stresses for members of roof and bridge trusses are given in Sect. 12. Working stresses under repeated loading may be taken as approximately one-third the endurance limit for the range of stress to which the metal is subjected. See p. 558. It is important to allow for localized stress in parts subjected to oft-repeated loading.

32. Alloy Steels

Three basic limitations of plain carbon steels are, according to Aitchison:

1. The impossibility of obtaining high strength with reasonably high toughness.
2. Irregularity in toughness, even with rather moderate strength.
3. A mass effect in hardening, making uniformity unlikely throughout parts of large masses.

By introducing alloying elements other than (usually in addition to) carbon, steels may be produced with tensile strength ranging from 80 000 to 250 000 lb. per sq. in. with a good degree of toughness and ductility. Such steels are capable of more effective heat treatment than plain carbon steels, especially in large masses. The accompanying table gives the general effects of various alloying elements; it is a summary of a table given by R. H. Aborn in the Iron Age for March 5, 1925.

Alloy steels give more trouble with shrinkage cavities (pipes) in ingots, and are more difficult to forge and stamp to shape than are low-carbon steels, and seem to be fitted for use where there are demanded properties which cannot be furnished by plain carbon steels.

In addition to nickel steel and silicon steel which are mentioned on p. 633, certain complex alloy steels are in common use, such as:

Chrome-nickel steels. Range of content, nickel, 1.5 to 3.5%; chromium, 0.7 to 1.5%; carbon, 0.2 to 0.4%. Similar in physical properties to nickel steel, but have higher elastic strength for same ductility, harder than nickel steel, more susceptible to heat treatment, more liable to embrittlement by improper heat treatment.

Chrome-vanadium steels. Content about 1.5% chromium, 0.2% vanadium. More easily machined than chrome-nickel steels, cleaner steel due to deoxidizing effect of vanadium, freer from scale and seams than steels containing nickel.

Chrome-molybdenum and chrome-nickel-molybdenum steels. Content, molybdenum, 1.0% minimum. Has advantages of chrome-vanadium steels over chrome-nickel, quenching and tempering ranges of temperature particularly large; disadvantages, loss of molybdenum by volatilization with resulting non-uniform composition, narrow range of temperature for forging.

Alloy steels have been widely used in the construction of long-span bridges. Owing to their increased strength over plain carbon structural steel lighter members can be used, thus diminishing the dead load on the bridge structure. It should be noted that the **stiffness** (as measured by modulus of elasticity) of alloy steels is not greater than that of ordinary structural steel, so that the elastic stiffness of such long-span structures is somewhat less than that of structures of plain carbon steel. In the case of long columns the strength is to some extent a function of the modulus of elasticity, and hence alloy steel loses some of its strength-increasing property when used in long columns.

Effect of Alloying Elements in Steels

Alloying element	General effect	Specific effect	Advantages and uses	Disadvantages
Manganese (Mn)	Deoxidizes; desulfurizes; stiffens atomic structure; gives finer crystalline grains.	Up to 2% acts mainly to deoxidize, desulfurize, and to increase strength.	Standard deoxidizer.	
		10–12% gives an amorphous structure, with a great resistance to abrasion.	Great resistance to abrasion. Used for mining and milling equipment, safes, frogs, switches, curved rails.	Extremely difficult to machine.
Nickel (Ni)	Stiffens atomic structure; gives finer crystalline structure.	Up to 4% increases tensile strength without great loss of ductility.	Makes heat treatment of large pieces possible with gain of strength. Used for structural and machine parts.	Scale formed in rolling; rough surface.
Silicon (Si)	Deoxidizes; stiffens atomic structure; gives finer crystalline structure.	Up to 1-3/4% increases elastic strength with little loss of ductility.	Standard deoxidizer. Used for structural steel, spring steel (with manganese).	Silico-manganese steel not very tough.
Aluminum (Al)	Deoxidizes.	Lessens harmful effect of gases in steel.	Standard degasifier. Used up to 0.05%.	Tendency to cause graphite formation if used in excess.
Chromium (Cr)	Forms carbide or double carbide; more stable than iron carbide. Intensifies hardening and embrittling effect.	Gives great hardness; makes quenching effective for considerable depth.	Very effective in adding strength. Used for cutting tools, ball bearings, cold rolls, 9–16% "stainless" irons and steels with nickel for general machine steels.	Danger of embrittlement with certain heat treatments.
Tungsten (W)	Similar to chromium. Forms extremely stable carbide.	Gives very fine crystalline structure; quenching effect not so deep as that of chromium.	Gives great cutting hardness, even at quite high temperatures. Used for cutting tools, permanent magnets.	Increased strength accompanied by brittleness, not suited for structural steel.
Molybdenum (Mo)	Similar to chromium and nickel.	Next to carbon most effective hardening element.	Allows very effective control of heat treatment. Used in combination with chromium, vanadium, or nickel.	Expensive; erratic in quality; volatilizes from surface layers on rolling. Tendency to seams and brittleness.

Effect of Alloying Elements in Steels — Continued

Alloying element	General effect	Specific effect	Advantages and uses	Disadvantages
Vanadium (V)	Deoxidizes; stiffens atomic structure; gives fine crystalline structure; forms carbide; intensifies hardening and embrittling effect.	Deoxidizing effect (0.1%); hardening agent; high elastic and tensile strength.	Forms fluid slag in deoxidizing, giving clean metal; inhibits growth of crystalline grains. Used for spring steel; with other elements for machine steel.	Expensive.

Alloy steels and heat-treated high-carbon steels are more susceptible to injury by grooves, holes, sharp shoulders and other causes of high localized stress than are the ordinary low-carbon structural and machine steels.

Invar steel is a special nickel steel with a nickel content of 36%. It has a coefficient of thermal expansion of 0.0000015 in. per inch of length per degree Fahrenheit as compared with 0.0000066 for ordinary steel. It is used for measuring tapes and scales.

33. Welding of Steel

Welds in steel may be divided into two classes: (a) welds made while the metal is heated to a plastic stage, and (b) welds made by actual fusion of the adjacent metal in the parts joined. In plastic welding the edges to be joined are pressed or hammered together. Wrought iron and soft steel can be thus welded without much difficulty, high-carbon steel can be welded by a skilled workman, while cast iron cannot be thus welded.

Plastic Welding is sometimes done in a forge fire, as is the case when the ordinary blacksmith shop welds rods or chain links, or the heat may be supplied by means of an electric current passing through the parts to be welded. Special welding machines are used for the welding of rods and the seams of vessels by this resistance heating method; such machines are to be distinguished from the machines for making electric welds by fusion of the metal, which are noted a little later.

Fusion Welding. If the edges of two adjacent pieces of metal are heated to fusion and then allowed to cool the process is known as **fusion** or **autogenous** welding. The high localized temperature required for such fusion may be obtained by the heat of a flame of acetylene gas burning in pure oxygen, by the heat of an electric arc, or by the heat from superheated molten iron produced by the burning of a mixture of iron oxide and aluminum.

Oxyacetylene Welding. In this process the oxygen and the acetylene are fed through a blowpipe and ignited at its tip. The flame has a temperature of approximately 5000° F., and a narrow strip of metal at the junction of the parts is fused. Usually additional metal is added to the joint by melting into it a small steel or iron rod. If an excess of oxygen is supplied to the torch it becomes an effective cutting, or more correctly burning, tool. The heated steel burns in the presence of the excess of pure oxygen, and a narrow gash in the iron or steel is burned. The oxyacetylene torch is widely used to cut up iron and steel scrap.

Electric Arc Process. In this process of fusion welding an arc is struck between the metal to be welded and an electrode, usually an iron or steel rod, held in an insulator and guided by the gloved hand of the operator. The heat of the arc melts the electrode and also melts metal at the surface to be joined. The strength and soundness of the weld depend to a considerable degree on the composition of the welding rod used. In England welding rod coated with some fluxing material is widely used. With coated rod there is some danger of slag inclusions in the weld unless the welding is very skillfully done. Oxidation at the weld is minimized by surrounding the arc with a jet of burning reducing gas, such as hydrogen; such a reducing atmosphere around the arc seems to serve the purpose of a coated rod without the danger of slag inclusions. Automatic welding machines are available for manufacturing purposes, and their use both cheapens the welding process and increases the uniformity of the welds produced.

Thermit Process. Fusion welding by means of the heat generated by the burning of iron oxide and aluminum is carried out in the thermit process, which is patented. A specially prepared mixture of aluminum and iron oxide is placed in a crucible and ignited. The heat of the resulting combustion raises the temperature of the mass to about 4800° F., and there is produced superheated molten iron which is poured around the joint to be melted, melting the edges and forming a solid casting at the joint.

Uses of Welding. Forge welding is confined to the welding of rods and small pieces. Plastic welding by the use of electric current is essentially a manufacturing process, and is used in quantity production of welded rods, tanks, etc. Thermit welding is used for welding heavy sections, such as rail joints, punch frames, and large shafts. Fusion welding either by the oxy-acetylene process or by the electric arc is very widely used for general repair work and for manufacturing processes. It is being used to some extent as a substitute for riveting in structural work, and its more general use in buildings and pressure drums is under consideration at the present time. A recent development of electric welding is the building up of machine frames of structural shapes welded together.

Strength of Welds. Plastic welds, if skillfully made and if hammered while cooling, form almost perfect joints, and the structure of the metal in such perfect welds is very similar to the structure in good forged steel. However, it is not safe to count on more than 75% of full strength for machine-made plastic welds, nor more than 50% of full strength for forge welds.

Fusion welds are steel castings at the joint, and the crystalline structure is that of a steel casting rather than that of rolled steel. Welds made under carefully controlled shop conditions involving automatic welding machinery are frequently produced which show strength **under static load** equal to that of the material welded. Under repeated load, available test data indicate that fusion welded joints cannot be counted on for more than 50% of the strength of the material welded. Welded joints made under field or repair shop conditions are not so strong as joints made by automatic machines under carefully controlled manufacturing conditions.

In using the fusion welding processes one of the marked sources of weakness is the presence of cooling strains set up by the unequal cooling of different parts. Where the welded parts can be heat-treated (usually normalized) or can be hammered while cooling such procedure should always be carried out.

34. Non-ferrous Metals

Aluminum is a white, malleable, and very light metal, its specific gravity being 2.75 when rolled and 2.55 when cast. It is almost non-corrodible, since even sulfuric acid has little effect upon it. Pure cast aluminum has a tensile strength of about 18 000 lb. per sq. in. When rolled into plates or drawn into wire this is raised to 30 000 or 40 000 lb. per sq. in. with an elastic limit one-half as great. On account of the softness of the pure metal it is usually alloyed with copper, iron, tin, or zinc. (Art. 35.)

Common impurities in aluminum are silicon and iron. It can, however, be obtained 99 or 99.5% pure. In making castings the great shrinkage must be taken into account. The melting point of pure aluminum 1150° F.

Its electric conductivity ranks next to that of copper among the commercial metals.

Copper is a reddish, malleable and ductile metal which can be drawn or rolled and also cast; its specific gravity varies from 8.9 in the first form to 8.6 in the second. It unites with oxygen at a red heat and melts at about 1900° F. It does not corrode in dry air. Its most important use is for electric conductors.

Copper wire has a tensile strength of about 50 000 lb. per sq. in. and a poorly defined elastic limit of about 37 000 lb. per sq. in. owing to the stiffening due to drawing; by annealing it may be softened and both ultimate strength and elastic limit thus lowered. Copper plates 1/4 to 3/4 in. thick have tensile strength of 32 000 lb. per sq. in. Cast copper has a tensile strength of about 25 000 lb. per sq. in. and a very poorly defined elastic limit of about one-third of this value. The strength of copper rods decreases as the temperature rises above 100°, the loss in strength being 16% at 500° F.

Small cylinders of annealed copper are used to measure the high pressures developed by the explosion of powder in guns, the pressure being inferred from the shortenings which the cylinders undergo. In this way pressures as high as 30 000 lb. per sq. in. have been noted.

Copper is alloyed with zinc, tin, and other metals to produce many useful alloys which are used in the arts and in engineering. (See Art. 35.)

Tin is a white, malleable metal of specific gravity 7.3. Commercial tin contains as impurities copper, iron, bismuth, and other metals. It melts at 450° F., and hence is often used for safety plugs in steam boilers. Its main commercial use is as a coating in the manufacture of the tin plates which are used for roofing, for household utensils, and for cans.

Roofing tin is thin sheet steel coated with an alloy of about 25% tin and 75% lead. A box of 112 sheets 14 by 20 in. in size will cover approximately 192 sq. ft. of roof with the flat-seam method of laying. For the standing-seam method a box of 112 sheets 20 by 28 in. in size will cover closely 370 sq. ft.

Lead has a low strength and is almost devoid of elasticity, but is very plastic, so that it flows readily under stress. The weights per square foot of sheet lead ordinarily rolled are 2-1/2, 3, 3-1/2, 4, 4-1/2, 5, 6, 8, 9, 10 lb. and upward. Small lead pipes are often lined with tin in order to prevent the lead from dissolving in the water; the common commercial sizes of these are 1/4 and 1/2 in. The use of lead pipes in bath rooms should usually be confined to the waste pipes. Lead melts at 625° F.

Zinc, called spelter when cast, has a tensile strength of about 9000 lb. per sq. in. and about 24 000 lb. per sq. in. rolled into thin plates. It is mainly used in coating iron and steel surfaces (galvanizing) in electric batteries, and for making brass and other alloys. Its melting point is 780° F.

Nickel is a ductile, hard, and tough metal. It is mainly used for plating and in alloys. Its melting point is about 3000°, so that it is very difficult to fuse. Not corrodible from the atmosphere. Its tensile strength, rolled and annealed, is about 70 000 lb. per sq. in. (See Art. 31.)

Mercury is a silver-white metal which is liquid at common temperatures. It boils at 680° F. and freezes at −38° F. When liquid its specific gravity ranges from 13.58 to 13.59. Its coefficient of cubical expansion for temperatures from 32° F. and 212° F. is 0.000101 per degree.

For other metals and for other properties of those above briefly described see Sect. 13, Art. 5. For atomic weights see Sect. 3, Art. 1.

35. Non-ferrous Alloys

An alloy is a mixture of two or more metals which is made by combining them when in a molten condition. The mixture is usually only mechanical, although some slight chemical union may take place in special cases. It is impossible to predict the properties of an alloy from the properties and proportions of its constituent metals.

Brasses are alloys of copper and zinc, the most valuable of which have from 60 to 80% of copper and from 40 to 20% of zinc. Brass can be cast or rolled. It is harder than copper. Tensile strength of castings is about 20 000 lb. per sq. in. The mean specific gravity of cast brass is 8.95.

Delta metal is brass with a small amount of iron. Its strength and ductility when rolled are equal to those of medium steel; when cast, its tensile strength is about 45 000 lb. per sq. in. Its resistance to corrosion is high.

Tobin bronze is an alloy of copper, tin, and zinc. It is of high tensile strength, is very non-corrodible, and can be obtained either in castings or in rolled sheets.

Strength of Copper-Zinc Alloys (Brasses)

Based on the report of the U. S. Board to Test Iron, Steel, and Other Metals, the report of Roberts Austen to the British Alloys Research Committee, and tests by J. M. Lohr.

Composition, per cent		Ultimate strength, lb. per sq. in		
		Tension		Compression cast
Copper	Zinc	Cast	Rolled	
0	100	9 000	24 000	5 000
20	80	9 000	10 000	55 000
40	60	7 000	7 000	......
50	50	30 000	65 000	115 000
55	45	47 000	77 000	90 000
60	40	48 000	61 000	75 000
65	35	38 000	53 000	65 000
70	30	37 000	50 000	55 000
75	25	35 000	51 000	45 000
80	20	33 000	50 000	39 000
85	15	32 000	45 000	37 000
90	10	31 000	41 000	30 000
100	0	25 000	40 000	40 000

Copper-Tin Alloys are called bronzes. The tin is added to harden the copper, and the alloy is denser, harder, and more fusible than copper. Bronze for making statues has about 87% copper, 7% tin, 3% lead, and 3% zinc. Bronze for medals has only 2% tin. Bronze is used to a slight extent for telegraph and telephone wires. Bronzes containing more than 24% of tin have insufficient strength for practical uses.

Strength of Various Non-ferrous Metals and Alloys

Average values for strength based on test data from various testing laboratories. Data are lacking for values of strength in compression. In the absence of such data a safe practice would be to consider the ultimate in compression as having a value equal to the elastic limit in tension.

Metal	Approximate composition, per cent	Weight, lb. per cu. in.	Strength in tension, lb. per sq. in.	
			Elastic limit	Ultimate
Copper, cast	Copper, 100	0.310	8 000	25 000
Hard drawn	Copper, 100	0.321	30 000	40 000
Zinc, cast	Zinc, 100	0.253	....	9 000
Rolled	Zinc, 100	0.253	4 000	24 000
Lead, rolled	....	0.411	800	* 2 700
Lead alloy, cast	Lead, 95.5; antimony, 4.5	0.380	4 000	6 400
Nickel, rolled and annealed	Nickel, 100	0.340	12 000	70 000
Cold-rolled	Nickel, 100	0.340	70 000	165 000
Aluminum, cast	Aluminum, 100	0.093	....	13 000
Rolled and annealed	Aluminum, 100	0.097	8 500	14 000
Cold-rolled	Aluminum, 100	0.097	20 000	30 000
Hard-drawn wire	Aluminum, 100	0.097	30 000	40 000
Duralumin, rolled and heat-treated	Aluminum, 96; magnesium, 1.5; copper, 2.5; iron and silicon, trace	0.102	19 000	55 000
60-40 Brass, cast	Copper, 60; zinc, 40	0.310	12 000	35 000
Rolled and annealed	Copper, 60; zinc, 40	0.310	16 000	60 000
Cold-drawn	Copper, 60; zinc, 40	0.310	43 000	96 000
Phosphor bronze, cast	Copper, 95; tin, 5; phosphorus, trace	0.320	16 000	32 000
Rolled and annealed	Copper, 95; tin, 5; phosphorus, trace	0.320	13 000	46 000
Cold-drawn	Copper, 95; tin, 5; phosphorus, trace	0.320	60 000	85 000
Aluminum bronze, cast	Copper, 90; aluminum, 10	0.270	25 000	60 000
Hot-rolled	Copper, 90; aluminum, 10	0.270	30 000	70 000
Cold-drawn	Copper, 90; aluminum, 10	0.270	80 000	90 000
Manganese bronze, cast	Copper, 60; zinc, 39; iron and manganese, 1	0.30	30 000	70 000
Rolled	Copper, 60; zinc, 39; iron and manganese, 1	0.30	45 000	70 000
Monel metal, hot-rolled	Nickel, 67; copper, 28; iron, silicon, carbon and manganese, 5	0.32	48 000	88 000
Cold-rolled and annealed	Nickel, 67; copper, 28; iron, silicon, carbon and manganese, 5	0.32	60 000	110 000

* Under long-continued static load lead will flow very slowly under stresses as low as 500 lb. per sq. in.

Phosphor Bronze is an alloy of copper and tin containing less than 1% of phosphorus. For hard castings of great strength from 4 to 10% of tin and from 0.5 to 1.0% of phosphorus is used. It is remarkable for its complete fluidity, so that most perfect castings can be made. It has been used for journal bearings, valve seats, and even for cannon. In the form of wire it is has been used for telephone service. It is hard and tough, and its ultimate tensile strength may range from 40 000 to 100 000 lb. per sq. in.

Copper-Zinc-Tin Alloys have been made in large numbers and are also called bronzes, or sometimes composition metals. The strongest of these was found by Thurston to be that which had 55% of copper, 44.5% of zinc, and 0.5% of tin, the tensile strength of a cast bar being 68 900 lb. per sq. in. With 55% copper, 40% zinc, and 5% tin the tensile strength was 28 000 lb. per sq. in.

Aluminum Bronze has from 5 to 12% aluminum with from 95 to 88% copper. These alloys have high ductility and great strength. With 6% aluminum it has a modulus of elasticity of about 18 000 000 lb. per sq. in. It has a high shrinkage in casting.

Composition of a Few Miscellaneous Alloys, in Per Cent

Name	Copper	Zinc	Tin	Lead	Bismuth	Other metals
Naval brass	62	37	1	....	....	
Bush metal	80	10	5	5	....	
Gold bronze	89.5	5.6	2.1	....	....	
Engine brass	76.5	11.7	11.8	2.8	....	
Spring brass	66	33	1	....	....	
German silver	55	25	....	....	....	nickel, 20
Britannia metal	1.9	81.9	....	....	....	antimony, 16.2
Fusible alloy *	....	....	25	25	50	
Fusible alloy †	....	....	13	27	50	cadmium, 10
Hard-type metal	....	75	....	....	....	antimony, 25

* Fuses at 93° C. † Fuses at 60° C.

Manganese Bronze is an alloy with a copper content of about 60%, a zinc content of about 38.5%, an iron content of about 1.5%, and a trace of manganese. The manganese " cleanses " the metal of oxide. Manganese bronze has a high strength and also a high ductility. Its elongation in 2 in. is about 25%. It resists corrosion remarkably well either in salt water or in fresh water, and is used for propeller blades and other submerged parts of ships.

Duralumin is an alloy having aluminum for its principal ingredient, with small percentages of magnesium, copper, iron, and silicon. By " aging " and heat-treating it can be given a tensile strength about that of structural steel while its weight per cubic inch is but little greater than that of aluminum. Its elastic strength and its fatigue strength are relatively low. Under the influence of salt water some heats of duralumin corrode badly, but, in general, it is highly resistant to corrosion. It is used in airplane construction and in other machines where extreme lightness combined with good strength is required.

Monel Metal is an alloy whose principal constituents are nickel and copper with small amounts of iron, silicon, carbon, and manganese. It is made from special ores containing these constituents. It is highly resistant to corrosion, and can be made about as strong as structural steel. It is used for steam turbine blades, tubing, and other parts exposed to corrosive agents.

Bearing Metals. A group of soft alloys of low strength is used mainly for forming the surfaces of bearings in machines. Steel rubbing on steel soon gives a rough torn surface. Steel rubbing on cast iron gives a smooth surface, but cast iron is so brittle that it is in danger of cracking in service. For heavy loads brasses or bronzes are used for bearing metals. They serve excellently, but are expensive. For bearings carrying light loads alloys of lead, antimony, and tin are used. These alloys have a low melting point and can be cast in place. One alloy in common use has the composition, lead 80%, antimony 20%. "Babbitt Metal," much used for high-grade bearings, has the composition, tin 89%, copper 4%, antimony 7%.

Platinite is an alloy of iron with 42 per cent of nickel which has the same coefficient of expansion as glass and hence may be used in glass to prevent cracking under heat.

36. Corrosion of Metals

Steel and Iron. The corrosion of structural steel and iron may be divided into five classes: (1) atmospheric, (2) water-immersion, (3) soil, (4) chemical, and (5) electrolysis. In atmospheric corrosion there is present a large excess of oxygen, and the quantity of moisture which reaches the surface of the metal, and the time of contact between metal and moisture are factors of prime importance. Structural steel with a copper content of 0.2% is more resistant to atmospheric corrosion than ordinary structural steel.

For steel immersed in water the amount of dissolved oxygen in the water and the acidity or alkalinity of the water are prime factors in producing corrosion. In general, acidity tends to increase corrosion, and alkalinity to decrease it.

In soil corrosion and chemical corrosion the chemical ingredients coming in contact with the steel are the prime factors, rather than the variations in composition of the ordinary structural steels and irons.

Disastrous corrosion by electrolysis by stray currents from power circuits sometimes occurs, but it can be guarded against in nearly all cases by suitable electrical precautions.

Corrosion of structural steel takes place rather slowly, and **careful inspection** of structural steel parts will give ample warning before corrosion has gone far enough to cause structural danger.

Mechanism of Corrosion. Iron when first brought into contact with moist air or with water containing dissolved oxygen goes into solution at a high initial rate. This initial rate is quickly retarded by an invisible film of hydrogen formed on the iron, and by films of the products of initial corrosion. If these films are sufficiently hard and dense they may stop the progress of corrosion altogether. Free oxygen, however, tends to combine with the film of nascent hydrogen formed, and to remove it, thus permitting the further progress of corrosion. Acidity also facilitates the progress of corrosion by the prevention of the formation of protecting films, and by the evolution of nascent hydrogen.

Corrosion of Wrought Iron, Steel and Cast Iron. The relative corrodibility of steel and wrought iron is a matter of dispute among chemists and engineers. It seems fairly certain that for either metal care in manufacture and homogeneity of metal are very vital factors in producing metal with a minimum of corrodibility. Ordinary cast iron seems to be rather more resistant to corrosion by water than either wrought iron or steel, and it is frequently used for the larger sizes of water pipes in spite of its brittleness.

Protection against Corrosion. Painting is the commonest means used for protecting structural steel against corrosion. Paint to be effective should be renewed every three or four years, so it is not suitable for protecting parts which cannot be reached for ordinary repainting. Clean metal is essential for the placing of an effective coat of paint. This may be secured by sand blasting and the use of a wire brush. Railway engineers report that the cost of cleaning and painting the structural steel of bridges averages about 50 cents per ton per year.

Asphalt or coal tar rubbed molten over a priming coat on the steel give good protection against water. A layer of rich concrete about 2 in. thick is perhaps the best protective coating for structural steel.

For further reference see Frank N. Speller, "Corrosion, Causes and Prevention" (McGraw-Hill, N. Y.)

Galvanizing iron or steel consists in coating with zinc. Zinc is resistant to atmospheric corrosion, and a coating of zinc blankets the iron or steel against attack. Moreover, zinc is electro-positive with respect to iron, and when two metals are in contact the one which is electro-positive with respect to the other tends to corrode, protecting the electro-negative metal against corrosion.

The common galvanizing process is the **hot dip process,** in which the metal to be coated is first cleaned (pickled) with acid, then washed, then dipped in, or drawn through, a bath of molten zinc. The thickness of coating of zinc depends on the temperature of the bath and the time the iron stays in the bath.

The approximate weight of zinc used for coating iron and steel is given by the following figures, which are based on the current Standards and Tentative Standards of the American Society for Testing Materials.

Sheets	1.75 to 2.75 oz. per sq. ft.
Wire	0.70 to 0.80 oz. per sq. ft.
Wire fencing:	
Wire less than 0.12 in.	0.35 to 0.70 oz. per sq. ft.
Wire, 0.12 in. or more	0.20 to 0.35 oz. per sq. ft.
Chain-link fence fabric:	
Galvanized before weaving	0.50 to 0.70 oz. per sq. ft.
Galvanized after weaving	0.12 oz. per sq. ft.

A heavy coating of zinc offers greater resistance to wear than does a light coating, but is more likely to crack and spall if the metal is bent than is a light coating. For galvanized plates or wire which have to be bent and shaped sometimes even lighter coatings of zinc are used than the minimum values given in the above table. Tests for the weight of zinc per square foot on wire or sheet are made in the laboratory by removing the zinc by chemical means and noting the loss of weight. Field tests sometimes employ chemical removal; sometimes the weight of zinc per square foot is estimated by the rise of temperature produced by attacking a standard sample of plate with a standard chemical reagent; and a third field test consists in determining the amount of hydrogen evolved from a given area of plate by the attack of a standard solution of hydrochloric acid and antimony chloride. See the 1927 Standards of the American Society for Testing Materials, Part I, p. 343.

Galvanizing is a fairly effective protection to iron and steel exposed to atmospheric conditions, unless the zinc coating is cracked or worn off mechanically. For iron or steel submerged in water galvanizing is less effective than it is for iron or steel in air.

Besides the hot dip method of galvanizing, iron or steel is cometimes coated with zinc by electroplating. **Sherardizing** is the process of coating iron or steel with zinc by the condensation of volatilized zinc dust. The bond between zinc and iron seems to be stronger for sherardized iron than for ordinary galvanized iron. Sherardizing is used on small pieces of hardware, parts of agricultural implements, wire clips, pipe fittings, etc.

Other Metal Coatings. Nickel, chromium, and cadmium plated on the surface of metal are used to protect against corrosion. Tin plate consists of sheet steel coated with molten tin. Any "pin hole" defects in the tin coating may lead to serious corrosion of the metal underneath.

Stress and Corrosion. Metals under stress, especially stress beyond the yield point, corrode more rapidly than do unstressed metals. Corrosion under repeated stress has already been discussed in Art. 7. Certain boiler waters seem to be particularly liable to cause dangerous corrosion at points of high localized stress around the riveting of boiler shells. This " caustic embrittlement," as it is called, can usually be prevented by adding suitable chemicals to the boiler feed water.

Stainless Iron and Steel. Iron and steel containing 13% or more of chromium are much more resistant to corrosion than are ordinary structural steels. Chromium in iron seems to cause the formation of a dense hard film of oxide on the surface of the metal, which acts as a protection against further corrosion. The name " stainless " iron or steel is given to such high-chromium metals. Stainless iron and steel present problems of manufacture and of fabrication of no small difficulty, and at present are rather too expensive for general structural use; their use, however, is rapidly growing.

The Non-ferrous Metals (Arts. 34 and 35) and their alloys are non-corrodible compared with iron or steel. Aluminum is one of the most non-corrodible. Lead dissolves in water to a slight extent, but carbonates or sulfates of lime deposited upon it from the water form a film on the surface which prevents further action. Nickel resists corrosion to a high extent. On zinc coatings a thin skin of zinc carbonate forms in the atmosphere which prevents corrosion.

NON-METALLIC MATERIALS

37. Stone

Data for Building Stones of Good Quality

Values based mainly on test data from the Watertown (Mass.) Arsenal

Kind of stone	Weight, lb. per cu. ft.	Compressive strength, lb. per sq. in.	Shearing strength, lb. per sq. in.	Modulus of rupture, lb. per sq. in.	Modulus of elasticity, lb. per sq. in.	Coefficient of expansion per degree F.	Absorption of water, per cent of weight of stone
Granite, range...	160 to 170	15 000 to 26 000	1800 to 2800	1 200 to 2 200	5 900 000 to 9 800 000		
Average......	165	20 200	2300	1 600	7 500 000	0.0000040	0.5
Sandstone, range	135 to 150	6 700 to 19 000	1200 to 2500	500 to 2 200	1 000 000 to 7 700 000		
Average......	140	12 500	1700	1 500	3 300 000	0.0000055	5.0
Limestone, range	140 to 180	3 200 to 20 000	1000 to 2200	250 to 2 700	4 000 000 to 14 700 000		
Average......	160	9 000	1400	1 200	8 400 000	0.0000045	7.7
Marble, range...	160 to 180	10 300 to 16 100	1000 to 1600	850 to 2300	4 000 000 to 12 600 000		
Average......	170	12 600	1300	1 500	8 000 000	0.0000045	0.4
Slate, range.....	170 to 180	14 000 to 30 000	....	7 000 to 11 000	13 900 000 to 16 200 000		
Average......	175	15 000	....	8 500	14 000 000	0.0000058	0.5
Trap, average...	185	20 000	....	....	....	....	....

Trap. This name is applied to a class of eruptive rocks, notably those of the Hudson River Palisades. Its most important properties are not among those in the table, for its most notable characteristics are resistance to wear and a high degree of binding power when powdered and moistened. Few tests have been made on its strength.

Artificial Building Stones. These as a rule are lower in compressive strength than the natural building stones, and without special test they may be rated with common building brick with a compressive strength of 3000 to 4000 lb. per sq. The Arsenal Tests for 1906 give for four varieties of "manufactured stone" values ranging from 2570 to 3390 lb. per sq. in.

38. Brick and Terra Cotta

Bricks are commonly made of clay, whose chief characteristics are a plasticity when wet and a rocklike hardness after being heated to a high temperature. Pure clay, or kaolin, is white, and is employed in the manufacture of china and porcelain ware, whereas the lower-grade clays are used in making building brick. Bricks are also made from pulverized shale, which is simply a clay that has been consolidated through geological processes. The more siliceous shales and clays are adapted to the manufacture of vitrified paving blocks. At the present time building bricks are also made from sand and lime.

Clay Bricks may be broadly classified as building and paving. The former include the common and the pressed bricks. Of more limited and yet important use are enamel brick, glazed brick and fire brick. The enamel brick are those that have a coating of enamel on one or two sides, while the body of the brick is usually of fire clay. Glazed bricks differ from enamel bricks in being coated with a transparent glass instead of an opaque enamel. Fire brick are ordinarily made of a mixture of flint clay and plastic clay. They are usually white, and are used in lining fire boxes and passages.

Defects. The finished brick may be defective because of errors in manufacture or because the clay contains harmful materials. Notable among these are limonite and pyrite. When the brick is fired, limonite concretions will cause fused blotches and weak spots, while pyrite burns away, leaving flaws in the brick.

Terra Cotta is a burnt clay product made in the same general way as brick. Hard terra-cotta blocks and tile are made by burning clay at a very high temperature. Porous or soft terra cotta, sometimes called terra-cotta lumber, is made by burning a mixture of clay and straw or sawdust. The straw or sawdust burns out, leaving a light, porous material. Nails and screws can be driven into porous terra cotta, and it can be cut with a wood saw. Terra-cotta lumber is weaker than hard terra cotta. Terra-cotta building blocks are made hollow with walls 3/4 in. to 1 in. thick.

Sand-Lime Brick are made from an intimate mixture of sand and lime in proportion of about 16 to 1, molded in a press, and hardened in a large cylinder

Properties of Brick of an Average Good Quality

Kind	Weight per cu. ft., lb.	Crushing strength, lb. per sq. in.	Shearing strength, lb. per sq. in.	Modulus of rupture, lb. per sq. in.	Modulus of elasticity, lb. per sq. in.	Absorption, per cent of weight
Common....	125	4 000	1000	600	2 000 000	15
Face........	130	6 000	1000	800	3 000 000	10
Paving.......	150	10 000	1400	2000	7 000 000	2
Sand-lime...	115	3 000	800	450	1 000 000	12
Terra cotta..	....	4 000	....	800	....	13

filled with steam at 125 lb. pressure. The bonding is a result of the union between the sand and lime which forms calcium silicate.

Average Compressive Strength of Brick Masonry

Brick or block used	Mortar	Ultimate stress in compression on test pier, lb. per sq. in.
Vitrified brick	1 : 3 Portland cement	2800
Face brick	1 : 3 Portland cement	2000
Face brick	1 : 3 lime mortar	1400
Common brick	1 : 3 Portland cement	1000
Common brick	1 : 3 lime mortar	700
Hard terra-cotta block	1 : 3 Portland cement	3000
Sand-lime brick *	1 : 3 Portland cement	750
Sand-lime brick *	1 : 3 lime mortar	500

* Estimated from the relative strength of individual sand-lime bricks and common bricks.

Strength of Brick Masonry. As in the case of stone masonry, brick masonry is always used in compression. The strength in compression of brick masonry is much less than the strength of individual bricks. The strength depends not only on the strength of the bricks, but to a large degree on the strength of the mortar joints, and on the skill used in laying the brick. The preceding table, based on test data from the Watertown Arsenal, Cornell University, the U. S. Bureau of Standards, and the University of Illinois, gives average values for ultimate strength in compression.

39. Timber

Classes of Timber Trees. Wood, as a building material, is produced by the spermatophyta, or seed-bearing trees, which may be divided into three groups, the conifers, the broad-leaved, and the tropical trees. The conifers and the broad-leaved trees produce the structurally valuable timbers and give rise to the general classification of timber in the lumber trade into **Soft Woods** (pines, spruce, cedar, cypress, larch, and fir) and **Hard Woods** (oak, walnut, maple, chestnut, hickory, ash, boxwood, whitewood, etc.). Of the tropical trees the products are the bamboos, palms, and rattans. Between the hard woods and the soft woods there is no sharply defined distinction in hardness, some of the hard woods, such as the basswood, poplar, and sycamore, being softer than the pines.

The conifers and the broad-leaved trees are known as the outward-growing or exogenous trees. The structure consists of three parts, the bark, the sapwood, and the heartwood. On the outside of the tree trunk is found from 1/4 to 2 in. or more of bark or protective tissue. As a structural material this is valueless and is always removed soon after the tree is felled, as it hastens the decay of the wood. Inside of the bark there is a soft portion made up of thin-walled cells which constitute the living portion of the tree, called the sapwood. Arranged in a circle inside of this soft tissue are many fibrous bundles making up the middle of the stem, giving it strength and stiffness and known as the heartwood. As the stem grows, new and branching bundles of these hollow fibers appear under the bark and form each season an annular ring. At the end of the season growth stops, to be resumed the following spring; and the rapid open growth of the spring against the slow and condensed growth of the summer gives rise to the peculiar marking in the bundles which indicates each year's increase. The last few rings formed constitute the sapwood, usually from 1/2 to 4 in. in thickness and light in color. The rings inside of the sapwood form the heartwood. The rings are interrupted by plates of tissue or radial cells communicating between the pith at the center of the tree and the soft tissue on the outside. These form the medullary rays. In the pine, the sapwood constitutes 40 to 60% of the cross-section of the tree, and the time required for sapwood to transform into heartwood varies from a few years in the case of the fir to many years in the oak tree. The approximate composition of all woods when dry is nearly uniform and consists by weight of the following elements: 49% of carbon, 6% of hydrogen, 44% of oxygen, and 1% of ash.

White Ash. Heavy, hard, very elastic, coarse-grained, and compact. Tendency to become decayed and brittle after a few years. Color, reddish-brown, with sapwood nearly white. Used for interior and cabinet work, but unfit for structural work.

Red Ash. Heavy, compact, and coarse-grained but brittle. Color, rich brown, with sapwood a light brown sometimes streaked with yellow. Used as a substitute for the more valuable white ash.

Green Ash. Heavy, hard and coarse-grained; brittle. Color, brown, with lighter sapwood. Used as a substitute for white ash.

Balsa. Extremely light, about half the strength of white pine. Appearance like poplar. Used for heat insulation and, when waterproofed, for life preservers.

White Cedar. Soft, light, fine-grained, and very durable in contact with the soil; lacks strength and toughness. Color, light brown, darkening with exposure. Sapwood very thin and nearly white. Used for water tanks, shingles, posts, fencing, cooperage, and boat building.

Red Cedar. Strong pungent odor repellent to insects. Very durable and compact, but easily worked and brittle. Color, dull brown tinged with red. Used as posts, sills, ties, fencing, shingles, and lining for chests, trunks, and closets.

Chestnut. Light, moderately soft, stiff, and of coarse texture. Shrinks and checks considerably in drying; works easily. Durable when exposed to weather. Color, heartwood dark and sapwood light brown. Used for cabinet work, cooperage, railway ties, telegraph poles, and exposed heavy construction.

Cypress. One of the most durable of woods, light, hard, close-grained but brittle. Easily worked, polishes highly and gives a satiny gloss. Color, bright clear yellow with nearly white sapwood. Used for interior finish and cabinet work, but used as extensively in the South as pine is in the North.

White Elm. Heavy, hard, strong and tough and very close-grained. Difficult to split and shape, but warps badly in drying. Capable of high polish. Color, light clear brown often tinged with red and gray, with broad whitish sapwood. Used for car, wagon, boat, and shipbuilding, bridge timbers, sills and ties, and furniture, also barrel staves.

Greenheart. Very heavy, strong, durable heartwood, dark green to dark chestnut color, free from knots. Used for shipbuilding, docks, implements, rollers.

Gum. Heavy, hard, tough, compact and close-grained. Tendency to shrink and warp badly in seasoning. Not durable if exposed. Takes high polish. Color, bright brown tinged with red. Used in the manufacture of furniture, wagon hubs, hat blocks.

Hickory. Medullary rays very numerous and distinct. Heaviest, hardest, toughest, and strongest of American woods. Very flexible. Color, brown, with very thin but valuable sapwood nearly white. Used for carriages, sleighs, handles, and bent-wood implements. Unfit for building material because of extreme hardness and liability to attack of boring insects.

Hemlock. Brittle, splits easily and likely to be shaky. Soft, light, not durable, with coarse and uneven grain. Color, light brown tinged with red and often nearly white. Used for cheap rough framing timber, crates and packing boxes.

Locust. Heavy, hard, strong, and close-grained. Very durable in contact with ground. Hardness increases with age. Color, brown and rarely light green, with yellow sapwood. Used for posts and turned ornaments.

Lignum Vitæ. Exceedingly heavy, hard, resinous, difficult to split and work, and has a soapy feeling. Color, rich yellow brown varying to almost black. Used for small turned articles, tool handles, and sheaves of block pulleys.

Hard Maple. Heavy, hard, strong, tough and close-grained. Medullary rays small but distinct. Curly and circular inflexion of fibers gives rise to "curly maple" and "bird's-eye maple." Susceptible of good polish. Color, very light brown to yellow. Used for flooring, interior finish, and furniture.

White Maple. Fine-grained, hard, strong, and heavy. Characteristics of grain the same as hard maple and more marked. Light colored. Used for flooring and furniture.

Mahogany. Strong, durable, and flexible when green, but brittle when dry. Free from shakes and less liable to attacks of dry rot or worms. Rapid seasoning causes deep shakes. Color, red-brown of various shades and degrees of brightness, often varied and mottled. Inferior qualities contain large numbers of gray specks. Used for interior finish, handrails, patterns, etc.

White Oak. Heavy, strong, hard, tough, and close-grained. Checks if not carefully seasoned. Well-known silver grain and capable of receiving high polish. Color, brown, with lighter sapwood. Used for framed structures, shipbuilding, interior finish, carriage, furniture making, and railway ties.

Chestnut Oak. Very durable in contact with soil. Color, dark brown. Used for railroad ties.

Live Oak. Very heavy, hard, tough, and strong. Difficult to work. Color, light brown or yellow, with sapwood nearly white.

Red and Black Oak. More porous than white oak and softer. Color, darker and redder than white oak. Used for interior finish and furniture, and railway ties.

Palmetto. Light but difficult to work when dry. Very durable under water and less subject to attacks of teredo. Color, light brown, with dark-colored fibers. Used for wharf piles, canes and handles.

White Pine. Light, soft and straight-grained and easily worked, but not very strong. Color, light yellowish brown often slightly tinged with red. Used for interior finish and pattern making.

Red Pine (**Norway Pine**). Light, hard, coarse-grained, compact, with few resin pockets. Color, light red, with a yellow or white sapwood. Used for all purposes of construction.

Yellow Pine (**Long-leaf**). Heavy, hard, strong, coarse-grained, and very durable when dry and well ventilated. Cells are dark colored and very resinous. Color, light yellowish red or orange. Cannot be used in contact with ground. Used for heavy framing timbers and floors. As house sills, sleepers, or posts it rapidly decays.

Yellow Pine (**Short-leaf Pine**). Varies greatly in amount of sap and quality. Cells broad and resinous, with numerous large resin ducts. Medullary rays well marked. Color, orange, with white sapwood. Used as a substitute for long-leaf pine.

Oregon Pine (**Douglas Fir**). Hard, strong, varying greatly with age, conditions of growth, and amount of sap. Durable but difficult to work. Of two varieties, red and yellow, of which yellow is the more valuable. Color, light red to yellow, with white sapwood. Used in all kinds of construction.

Poplar (**Whitewood**). Soft, very close and straight-grained, but brittle and shrinks excessively in drying. Warps and twists exceedingly, but when dry will not split. Easily worked. Color, light yellow to white. Used in carpentry and joinery.

Redwood (**California**). Light, soft, coarse-grained, and easily worked. Durable in contact with soil, but brittle. "Shrinks lengthwise as well as crosswise." Color, dull red, resembling pine. Used for railroad ties, fence posts, telegraph poles, and general building material.

Alaska Spruce (**Sitka Spruce**). Light, soft, medium strength. Heartwood, light, reddish brown; sapwood, white; trees very large. Used for general structural purposes; also for airplane frame-work.

Red Spruce. Light, soft, close and straight-grained and satiny. Color, light red and often nearly white. Used for piles, lumber, and framing timber, submerged cribs and cofferdams, as it well resists decay and the destructive action of crustacea.

White Spruce. Similar to black variety, but not so common. Color, light yellow, sapwood indistinct. Used as lumber for construction.

Tamarack (**Larch**). Wood like pine in appearance, quality and uses. Used for telegraph poles, railway ties, and in shipbuilding.

White Walnut (Butternut). Light, soft, coarse-grained, compact, and easily worked. Polishes well. Color, light brown, turning dark on exposure. Used for interior finish.

Black Walnut. Heavy, hard, strong, and checks if not carefully seasoned. Coarse-grained but easily worked. Color, rich dark brown, with light sapwood. Used for interior finish and cabinet work.

Standard Names for Structural Timber. The following classification of standard commercial names for structural timber is from the 1927 Standards of the American Society for Testing Materials:

1. Southern Yellow Pine. This term includes the species of yellow pine growing in the southern states from Virginia to Texas, that is, the pines hitherto known as long-leaf pine (*Pinus palustris*), short-leaf pine (*Pinus echinata*), loblolly pine (*Pinus taeda*), Cuban pine (*Pinus heterophylla*) and pond pine (*Pinus serotina*).

Under this heading, two classes of timber are designated: (*a*) dense southern yellow pine and (*b*) sound southern yellow pine. It is understood that these two terms are descriptive of quality rather than of botanical species.

(*a*) Dense southern yellow pine shall show on either end an average of at least six annual rings per inch and at least one-third summer wood, or else the greater number of the rings shall show at least one-third summer wood, all as measured over the third, fourth, and fifth inches on a radial line from the pith. Wide-ringed material excluded by this rule will be acceptable, provided that the amount of summer wood as above measured shall be at least one-half.

The contrast in color between summer wood and spring wood shall be sharp and the summer wood shall be dark in color, except in pieces having considerably above the minimum requirement for summer wood.

(*b*) Sound southern yellow pine shall include pieces of southern pine without any ring summer-wood requirement.

2. Douglas Fir. The term " Douglas Fir " is to cover the timber known likewise as yellow fir, red fir, western fir, Washington fir, Oregon or Puget Sound fir or pine, northwest and west coast fir.

3. Norway Pine, to cover what is known also as " Red Pine."

4. Hemlock, to cover Southern or Eastern hemlock; that is, hemlock from all states east of and including Minnesota.

5. Western Hemlock, to cover hemlock from the Pacific coast.

6. Spruce, to cover Eastern spruce; that is, the spruce timber coming from points east of and including Minnesota.

7. Western Spruce, to cover the spruce timber from the Pacific coast.

8. White Pine, to cover the timber which has hitherto been known as white pine, from Maine, Michigan, Wisconsin and Minnesota.

9. Idaho White Pine, the variety of white pine from western Montana, northern Idaho, and eastern Washington.

10. Western Pine, to cover the timber sold as white pine coming from Arizona, California, New Mexico, Colorado, Oregon and Washington. This is the timber sometimes known as "Western Yellow Pine," or " Ponderosa Pine," or " California White Pine," or "Western White Pine."

11. Western Larch, to cover the species of larch or tamarack from the Rocky Mountain and Pacific coast regions.

12. Tamarack, to cover the timber known as " Tamarack," or " Eastern Tamarack," from states east of and including Minnesota.

13. Redwood, to include the California wood usually known by that name.

Sizes of Timber. The following is summarized from the 1927 Standards of the American Society for Testing Materials.

Joist and Plank (joists, rafters, scaffold plank, factory flooring, etc.):

Nominal thickness	2 to 4 in.
Nominal width	4 in. and wider
Standard thickness	Nominal thickness less 3/8 in. finished on one or two sides
Standard width	2 to 7 in., nominal width less 3/8 in. finished on one or two sides 8 in. and wider, nominal width less 1/2 in. finished on one or two sides

Beams and Stringers (Beams, girders, stringers, etc.):

Nominal thickness .... 5 in. and wider

Nominal width .... 8 in. and wider

Posts and Timbers (posts, caps, sills, timbers, etc.);

Nominal size .... 6 by 6 in. and larger

Rough structural joist and plank shall not be thinner than the nominal dimension less 1/4 in., and not narrower than the nominal width less 1/4 in. for sizes 2 to 7 in. inclusive, and less 3/8 in. for sizes 8 in. and wider.

Rough structural beams and stringers, and posts and timbers shall not be smaller than the nominal size less 1/4 in. for sizes 7 in. and less, and less 3/8 in. for sizes 8 in. and over.

Sawing. The manner in which the stick of lumber is sawed from the tree has a remarkable influence upon its qualities and behavior, and the selection of cutting is determined by the character of the wood and the purpose for which it is destined. **Flat sawing** consists in cutting the timber tangential to the annular rings. **Rift sawing** is cutting the boards out of the log in such a manner that the annular rings are cut through as nearly as possible in a radial direction. Rift sawing and flat sawing give rise in the lumber trade to the terms edge grain and flat grain respectively. Rift sawing is done for the sake of the beauty of the grain thus obtained, as well as to expose the edge of the hard bands of summer wood. Edge-grain lumber shrinks and checks less, does not sliver, and wears more evenly and smoother than flat-grain lumber.

Air Seasoning. In the preparation of lumber for construction purposes, it is necessary to expel the moisture from the pores of the wood by the process of seasoning. The drier the timber, the less likely is it to shrink and decay. Natural seasoning consists in exposing the planks and boards, after sawing, to a free circulation of the air. The lumber is placed on skids in large square piles under shelter in a dry place, the layers being separated by three or four narrow strips or boards laid in the opposite direction. The lowest layer should be at least 2 ft. from the ground. At frequent intervals the decayed pieces are removed and the timber repiled. The time required for thorough seasoning varies from one to three years, depending upon the character of the wood, the purpose to which it is to be adapted, and the dimensions. Green wood contains from 30 to 35% of water. Air seasoning reduces this to 12 to 15%.

Artificial Seasoning or Kiln-drying hastens the evaporation of the moisture but at the same time tends to cause a rapid drying of the surface and ends of the material and a slow or imperfect drying of the interior. This may impair both the strength and elasticity of the wood. The timber is stacked in a drying kiln and exposed to a current of hot air, the temperature depending on the kind and the dimension of the stack. Sometimes the heat is supplemented by the employment of vacuum pumps. The best temperature to be employed depends on the kind and dimensions of the lumber, and varies from 100° F. for oak to 200° F. for pine. The time required depends on the thickness of the stack. About four days is necessary for one-inch pine, spruce, or cedar boards. Hard woods are usually dried in air from three to six months and then placed in the drying kiln from six to ten days. Kiln drying reduces the moisture content of wood to less than 10%.

Shrinkage. The concentric annular zones of wood are made up of pores or cells enclosed by walls of cellulose and are pierced at right angles by plates of similar fibers. As the average width of cell is 1/100 of the length, the greatest shrinkage will take place in the cross-section of the fibers or tangentially to the annular rings. This is known as the circumferential shrinkage. By rift sawing the medullary rays are cut across the length, in which direction shrinkage is least. Flat sawing produces lumber which checks and cracks to a greater extent in shrinking. The average values for shrinkage in width are:

Hard woods: radial, 3–8%; tangential, 7–14%
Soft woods: radial, 3–6%; tangential, 5–8%

The longitudinal shrinkage is usually less than 0.1%. The change in volume is therefore due to radial and tangential shrinkage, and expressed in percentage is approximately twice the figures given, as shrinkage takes place in two directions by approximately equal amounts.

The harder timbers are more compact in structure, with thicker cell walls, and therefore produce the greater shrinkage. The opposite effect to shrinkage is produced by the absorption of moisture, and protective checks must be resorted to in applying timber to construction; the expansion joints in wooden block pavement serve as checks. A roadway 40 ft. wide has been observed to expand 8 in.

Decay of Timber. The life of timber depends on its manner of felling, seasoning and working, and timber is subject both in its growing and converted states to decomposition and attack of animal and vegetable life. The proper time of year for cutting the tree is important. In the spring and late summer the sapwood contains an abundance of moisture with starches, sugars, and oils in solution, which tend to hasten its decay. In the drier summer months and in winter the growing and conducting cells are less active or altogether dormant, and the best wood is secured if the tree is cut during those seasons. Oak is claimed to be more durable if cut just after the leaves have fallen.

Trees are felled with the ax or saw, and hewed lumber is commonly accepted as more durable than sawed lumber, as the former process closes the cells at the cut and prevents the absorption of moisture. The agencies which produce the decay of wood may be classed as follows: alternate moisture and dryness, heat and confined air, bacteria and fungi, insects and worms. Well-seasoned wood in a uniform state of moisture or dryness, when well ventilated, should never decay. Timber kept constantly immersed may soften and weaken but does not decay; the elm, elder, oak, and birch in this condition possess great durability.

Dry Rot. This is caused by the fermentation and breaking down of the chemical compounds of wood when a certain fungus is introduced in the presence of moisture. These lower organisms excrete ferments which dissolve out parts of the cell wall and crumbling of the wood follows. Dry rot is prevalent in ill-ventilated places, such as the wall pockets at the ends of floor timbers, and is often prevalent in the core of timber columns in mill construction. The growth of the fungus is stimulated by moderate warmth, presence of damp, and lack of ventilation. In practice the decomposition is often accelerated by the use of unseasoned wood, the surfaces of such unprepared timber often being painted and tarred. Dry rot is indicated by the swelling of the timber, a change in color, the material gradually becoming covered with moldiness and emitting a musty odor. Sometimes reddish and yellowish spots appear on the timber and the fibers are gradually reduced to a powder. It is difficult to eradicate when once established, the only remedy being to remove all traces of the fungus and to disinfect the wood.

Wet Rot. Moisture, especially in the presence of warmth, will dissolve out the substance of the cell walls of sapwood and cause the decay of lumber. Wood felled between the months of April and October is especially subject to wet rot. **Common Rot** is manifested by the presence of external yellow spots on the ends of timber sticks and often by a yellowish dust in checks and cracks, especially where the pieces are in contact. The cause of common rot is improper seasoning in badly ventilated sheds.

Teredo. Of the worms most destructive to wood, the *Teredo navalis*, or shipworm, a species of mollusk, is the most active. It deposits its eggs on timber immersed in water, from which soon emerge small but rapidly developing worms. The head of the worm is equipped with a shell-like substance shaped like an auger, by means of which it bores its way into the timber in a direction generally parallel to the grain. As it progresses it lines the hole with a calcareous deposit and maintains communication with the outside through two small lids at the external opening. The teredo continually increases in size as its boring progresses, and specimens varying from 1/4 to 1/2 in. in diameter and 15 in. to 6 ft. in length have been found. Its habitat is the salt water of the warmer climates, though occasionally found in cold water. It avoids fresh water entirely, always prefers clear to muddy water, and is most active in the vicinity of calcareous shores. It may be found at work from half tide level down to the ground.

The Limnoria or gribble (*Limnoria terebrans*), is a crustacean about the size of a grain of rice and resembles the wood louse. It is able to swim, crawl, and jump, and requires both air and water for existence. In contradistinction to the teredo, it works only to a depth of 1/2 in., devouring the wood by means of claws or mandibles. The surface of the wood becomes undermined and brittle and is finally washed away, exposing a new surface to the action of the worm. The limnoria devours wood at the rate of 1 to 3 in. per annum, and is always found in large colonies, as many as 20 000 having been observed on a surface 12 in. square. It is found both in cold and in warm water, being capable of inhabiting colder climates than the teredo. It has a preference for siliceous shores, and confines its work to a space between high and low water marks. It is usually most active in brackish waters at low-water level.

The Lycoris fucata is a little worm with numerous legs, somewhat similar to the centipede. It lives in the mud and crawls up piles inhabited by the teredo, against which it is an active enemy. It enters the tunnel, devours the teredo, enlarges the entrance to the burrow and lives in it.

Impregnation of timbers with creosote is a rather uncertain protection for piles but seems to be somewhat more effective for piles in cold water than for piles in the warmer tropical waters. Creosote becomes ineffective if there exists or is formed in the pile a crack extending from the surface to the center where the creosote has not penetrated.

Various protective coatings, paints, asphalt, etc., have been applied to piling in the hope of preventing or retarding the attack of marine borers. Such protection is temporary at best. It becomes ineffective at any point where the surface is broken. No coating for which records are available has proved an effective protection over a period of years in water infested by active borers.

Tile pipe of burnt clay have been used for protective armor on piling. Tile is effective but is so brittle that it is in very great danger of being broken by the impact of vessels and floating logs against the piling. Copper sheathing makes an excellent non-corrosive armor for piles. It is, however, very expensive and is in danger of being stolen from piles in lonely locations. Timber piles have been successfully covered by concrete and not at very great cost but usually where permanent piles are required it is best to use precast concrete piles. They generally cost less per ton of load to be supported than wooden piles treated by a permanent cover of metal, tile or concrete.

Certain timbers from South American countries are found containing many minute grains of silica. These timbers give some promise of being somewhat resistant to rapid destruction by borers. They are under study at the present time. Hard wood resists the borers very much better than the softer woods; it is therefore recommended that oak or hickory instead of pine be used if the piles are not to be treated or protected. Piles from palm trees, on account of their structure, are not attacked by borers. Some of the hard tropical trees such as the Jucaro of Cuba are almost teredo proof.

Wooden hulls are often protected against the teredo by copper but sometimes by wood sheeting. This outer sheeting is quite effective until destroyed by borers because they will not usually cross a joint between timbers.

Under the general auspices of the National Research Council an extensive investigation of marine borers has been carried out in Los Angeles Harbor by Col. Wm. G. Atwood, U. S. Army. It was shown that marine borers could exist in water of lower salt content than had been previously supposed, and that creosote impregnation, properly carried out, and certain metallic armors, notably cast iron, were the most effective defence found against borers. (See The Military Engineer for May-June, 1923, and for Nov.-Dec., 1924.)

Classification of Lumber (Simplified Practice Recommendation No. 16, U. S. Department of Commerce). Lumber is the product of the saw and planing mill, not further manufactured than by sawing, resawing, and passing lengthwise through a standard planing machine, cross-cut to length and matched.

Use Classification

Yard Lumber. Lumber that is less than 6 in. in thickness, and is intended for general building purposes.

Structural Timbers. Lumber that is 6 in. or over in thickness and width.

Factory or Shop Lumber. Lumber intended to be cut up for further manufacture.

Size Classification

Strips. Yard lumber less than 2 in. thick and under 8 in. wide.

Boards. Yard lumber less than 2 in. thick, 8 in. or over in width.

Dimension Lumber. All yard lumber except boards, strips, and timbers; that is, yard lumber not less than 2 and under 7 in. thick, and of any width; this includes (1) Planks, yard lumber not less than 2 and under 4 in. thick and 8 in. and over wide; (2) Scantlings, yard lumber not less than 2 and under 6 in. thick and under 8 in. wide; (3) Heavy joists, yard lumber not less than 4 and under 6 in. thick and 8 in. or over wide.

Timbers. Lumber 6 in. or larger in least dimension.

Manufacturing Classification

Rough Lumber. Undressed as it comes from the saw.

Surfaced Lumber. Lumber that is dressed by running through a planer. It may be surfaced on one side (S1S), two sides (S2S), one edge (S1E), two edges (S2E), or a combination of sides and edges, (S1S1E), (S2S1E), (S1S2E), or (S4S).

Worked Lumber. Lumber which has been run through a matching machine, sticker, or molder. Worked lumber includes: (1) Matched lumber, which is edge dressed and shaped to make a close tongued and grooved joint at the edges or ends when laid edge to edge or end to end; (2) Shiplapped lumber, which is edge dressed to make a close rabbetted or lapped joint; (3) Patterned lumber, which is shaped to a patterned or molded form.

40. Data for Timber

Tests of Strength of Wood. The elastic limit of wood is not so clearly defined as is the elastic limit of structural steel. The values for elastic limit given in the table on p. 657 are values of the proportional limit as determined by tests with an extensometer. The tensile strength of wood is slightly higher than the compressive, but is rarely of practical importance, on account of the difficulty of attaching a piece of wood in tension without making the critical stress a shear along the grain. Wood is weak in shear along the grain, and in timber beams the shearing strength nearly always has to be considered.

Time Element in the Strength of Wood. Under long-continued steady load wood will fail under stresses much lower than those found at the ultimate strength in laboratory tests, which cover only a few moments. Under long-time loading test pieces of wood have broken under stresses only a little above 50% of the ultimate as given by short-time laboratory tests. This very pronounced time effect is one of the reasons why working stresses for timber are low as compared with ultimate strength.

Effect of Moisture on Strength of Wood. Up to a moisture content of about 25% for soft woods the fibers absorb water and are weakened and softened by it. For moisture content above this "fiber saturation point" water no longer affects the strength and stiffness of wood. The effect of moisture content is shown in the accompanying table, which is based on the results of tests by the U. S. Forest Service, principally by H. D. Tiemann.

Effect of Long Seasoning. Tests by A. C. Alvarez at the University of California on wood from the interior timbers of old buildings which had stood for some 37 years showed that Douglas fir and redwood did not deteriorate in strength with age when preserved from decay.

Effect of Slope of Grain (from 1927 Standards of the American Society for Testing Materials). "Slope of grain, resulting either from diagonal sawing or from spiral or twisted grain in the log, is limited in accordance with the recommendation of the Forest Products Laboratory based on the results of detailed study of the effect of cross and spiral grain upon strength, and the weakening of material by checks which invariably develop and, without exception, follow the grain. There is not much reduction in strength from cross grain until an angle of grain of 1 in 40 is reached. From that slope in a beam an angle of 1 in 20 reduces the strength about one-eighth; 1 in 15, about one-quarter; 1 in 11, about three-eighths; and 1 in 8, about one-half. In a post or column an angle of grain of 1 in 15 reduces the strength about one-eighth; 1 in 11, about one-quarter; 1 in 8, about three-eighths; and 1 in 6, about one-half."

Working Stresses for Timber are given in Sect. 9 on Timber Structures.

Inspection. The inspection of timber must be undertaken with two view-points, namely, the quality of the stock and the dimensions of the pieces; and all condemned pieces must be plainly marked with paint or branding iron. Strong and durable timber possesses the following characteristics: It is obtainable from the slowest-growing trees, as indicated by the narrowness of the annular rings. Wood containing the least amount of resin or sap in the pores is the strongest and most durable. The wood should be uniform in substance, straight in fiber, free from large and dead knots and all flaws, shakes, and blemishes. Freshly cut, sound timber smells sweet and shows a firm and bright surface with silky luster when planed. The surface should

Effect of Moisture on Strength of Wood

Based on test data from U. S. Forest Products Laboratory

Values are given in per cent of strength of oven-dry wood (moisture content negligible)

Moisture content, per cent of fiber saturation point	10	20	30	40	50	60	70	80	90	100
Modulus of rupture, flexure.....	91	82	73	66	60	55	50	46	43	39
Ultimate compression along grain	88	76	66	57	48	43	37	33	30	28
Ultimate shear along grain.....	93	85	77	70	64	58	54	51	48	46
Modulus of elasticity, flexure...	94	88	83	79	75	72	69	66	64	61
Modulus of elasticity, compression along grain............	93	85	77	71	66	63	59	57	54	52

never be woolly, and when cut should not clog the teeth of the saw. In highly colored woods, strength of color is generally indicative of strength and durability. Sound timber when struck lightly or scratched at one end transmits the sound to the ear placed at the other extremity through sticks of lumber 50 ft. in length. Sound material is sonorous when struck, while decaying timber gives forth a dull sound. Dull, chalky appearance and disagreeable odor are signs of bad timber. In the absence of the usual external signs, dry rot may be detected by boring test holes in the wood for the appearance and odor of the wood dust.

Relation of Strength of Wood to its Specific Gravity

Based on the work of Newlin and Wilson of the U. S. Forest Service. (See Bulletin 676 of the U. S. Department of Agriculture.)

G is the specific gravity of the wood, based on the weight oven-dry and the volume at the time of test and the condition when tested.

All values are in lb. per sq. in.

Strength property	Green wood	Air-dry wood
Flexure under steady load:		
Elastic limit	$10\,300\sqrt[4]{G^5}$	$19\,000\sqrt[4]{G^5}$
Modulus of rupture	$18\,500\sqrt[4]{G^5}$	$26\,200\sqrt[4]{G^5}$
Modulus of elasticity	$2\,500\,000\,G$	$3\,000\,000\,G$
Impact flexure with 50-lb. hammer:		
Elastic limit	$23\,500\sqrt[4]{G^5}$	$35\,000\sqrt[4]{G^5}$
Modulus of elasticity	$3\,000\,000\,G$	$3\,550\,000\,G$
Compression parallel to grain:		
Elastic limit	$6\,800\sqrt[4]{G^5}$	$11\,000\sqrt[4]{G^5}$
Ultimate strength	$6\,900\,G$	$12\,000\,G$
Modulus of elasticity	$2\,860\,000\,G$	$3\,500\,000\,G$
Compression perpendicular to grain:		
Elastic limit	$2\,900\sqrt[4]{G^9}$	$5\,200\sqrt[4]{G^9}$
Shear along grain		
Ultimate strength	$2\,650\sqrt[3]{G^4}$	$3\,800\sqrt[3]{G^4}$

Volume shrinkage of wood, per cent, 28 G.
Radial shrinkage of wood, per cent, 9.5 G.
Tangential shrinkage of wood, per cent, 17 G.

Defects. The following are adopted as standard by the 1927 Standards of the American Society for Testing Materials, Part II, p. 697. The diameters of knots and holes are mean or average diameters:

Knots. A **sound knot** is one which is solid across its face and which is as hard as the wood surrounding it; it may be either red or black, and is so fixed by growth or position that it will retain its place in the piece. A **loose knot** is one not firmly held in place by growth or position. A **pith knot** is a sound knot with a pith hole not more than 1/4 in. in diameter in the center. An **encased knot** is one whose growth rings are not intergrown and homogeneous with the growth rings of the piece it is in. The encasement may be partial or complete; if intergrown partially or so fixed by growth or position that it will retain its place in the piece, it shall be considered a sound knot; if completely intergrown on one face, it is a watertight knot. A **rotten knot** is one not as hard as the wood it is in. A **pin knot** is a sound knot not over 1/2 in. in diameter. A **standard knot** is a sound knot not over 1-1/2 in. in diameter. A **large knot** is a sound knot more than 1-1/2 in. in diameter. A **round knot** is one which is oval or circular in form. A **spike knot** is one sawn in a lengthwise direction.

Knots in timber do most damage when located near the faces or edges of structural pieces. Knots near the center of length of a piece are more injurious than those near the ends.

Detailed rules concerning the grading of timber with knots or other defects are given in the 1927 Standards of the American Society for Testing Materials, Part 11, pp. 581–603.

Pitch Pockets are openings between the grain of the wood containing more or less pitch or bark. These shall be classified as small, standard, and large pitch pockets. A small pitch pocket is one not over 1/8 in. wide. A standard pitch pocket is one not over 3/8 in. wide, or 3 in. in length. A large pitch pocket is one over 3/8 in. wide, or over 3 in. in length. A **pitch streak** is a well-defined accumulation of pitch at one point in the piece. When not sufficient to develop a well-defined streak, or where the fiber between grains, that is, the coarse-grained fiber, usually termed **Spring wood** is not saturated with pitch, it shall not be considered a defect. **Wane** is bark, or the lack of wood from any cause, on edges of timbers.

Shakes are splits or checks in timbers which usually cause a separation of the wood between annual rings. A **ring shake** is an opening between the annual rings. A **through shake** is one which extends between two faces of a timber.

Rot, dote, and red heart are forms of decay which may be evident either as a dark red discoloration not found in the sound wood, or as the presence of white or red rotten spots, and shall be considered as defects.

41. Preservation of Timber

Preservative Processes. The average life of timber used in the United States which is subject to decay, is about 8 years; its life can be extended to 12 years either by a chemical impregnation of the wood cells or by an exterior application of a preservative coating which will penetrate the fibers. Creosote oil is the preservative in most common use.

Preservation. The general method followed in the commercial treatment of timber with creosote involves the following steps: (1) seasoning the timber, (2) steaming the timber in a large cylinder to soften the wood fiber, (3) removal of air and moisture from the wood fibers by means of a vacuum pump, (4) admission of creosote oil to the evacuated chamber—the oil penetrates some distance into the timber, (5) the application of pressure forcing the preservative into the innermost fibers of the timber, and (6) the removal of pressure, after which the excess of creosote is allowed to drip off the timber and run into a tank.

A simpler process for treating small lots of timber consists of soaking the timber pieces in an open tank of hot creosote. This process does not impregnate timber so thoroughly as does the pressure-vacuum process, and it is wasteful of creosote.

Creosote is a rather expensive preservative, and other preservatives have been tried. Zinc chloride is violently poisonous to timber-destroying bacteria, can be forced into the inner fibers of wood, and is cheaper than creosote. However, zinc cloride is soluble in water, whereas creosote is not, and zinc chloride is an effective preservative only in extremely dry regions.

The following table gives approximate weights of creosote required per cubic foot of timber for various services.

Railroad ties:	
Hard wood	6–14 lb. per cu. ft.
Soft wood	13–20 lb. per cu. ft.
Dimension timbers	15–20 lb. per cu. ft.
Piles	20–30 lb. per cu. ft.
Poles and posts, treated by open tank method for a length of 7 ft:	
Chestnut poles	21 lb. per pole
Western pine poles	65 lb. per pole

Properties of Timber

Based mainly on Bulletin 556 of the U. S. Department of Agriculture (Forest Service) by Newlin and Wilson. Values are for wood tested green. Values for green timber are more reliable as a guide for structural use than values for dry timber, since in service the outer fibers of dry timber may readily reabsorb water. For working stresses see Sect. 7, Art. 1.

Kind of timber	Weight, lb. per cu. ft.	Strength in flexure, lb. per sq. in.			Strength in compression, lb. per sq. in.			Ultimate shearing strength, parallel to grain, lb. per sq. in.	Ultimate tensile strength, perpendicular to grain, lb. per sq. in.
		Elastic limit	Modulus of rupture	Modulus of elasticity in flexure	Elastic limit, parallel to grain	Ultimate, parallel to grain	Elastic limit, perpendicular to grain		
Ash	49	5500	9 900	1 490 000	3500	4200	800	1430	700
Cedar (white)	28	2600	4 200	640 000	1400	2000	290	620	240
Cedar (red)	27	3300	5 200	950 000	2500	2800	310	720	210
Chestnut	55	3100	5 600	930 000	2000	2500	380	800	430
Cypress	41	3800	6 500	1 070 000	2700	3200	440	820	270
Elm	52	3600	6 900	1 030 000	2300	2900	390	920	560
Gum (black)	45	4000	7 000	1 030 000	2400	3000	600	1100	570
Gum (blue)	70	7600	11 200	2 010 000	4900	5200	1020	1550	640
Hickory	64	5700	10 500	1 470 000	3500	4400	1270	930	680
Hemlock	45	3700	6 300	1 080 000	2500	3000	420	860	290
Locust	59	7200	12 000	1 570 000	4800	5600	1420	1710	850
Maple	67	4200	7 500	1 230 000	2500	3200	570	1150	630
Oak (white)	62	4700	8 300	1 250 000	3000	3600	830	1250	770
Oak (chestnut)	62	4600	8 000	1 370 000	2900	3500	660	1210	690
Oak (red and black)	64	3700	7 700	1 290 000	2300	3200	730	1120	740
Pine (white)	39	3400	5 300	1 070 000	2400	2700	310	640	260
Pine (western white)	39	3500	5 700	1 330 000	2800	3100	300	710	250
Pine (red or Norway)	42	3700	6 400	1 380 000	2500	3100	360	780	190
Pine (yellow long leaf)	50	5400	8 700	1 630 000	3800	4400	600	1070	290
Pine (yellow short leaf)	50	4500	8 000	1 450 000	3600	3800	480	890	330
Pine (western yellow)	46	3100	5 200	1 010 000	2100	2500	340	680	280
Pine (Oregon or Douglas fir)	38	5000	7 800	1 580 000	3400	3900	530	910	200
Spruce (red)	34	3400	5 700	1 180 000	2400	2700	350	770	220
Spruce (white)	33	3300	5 400	980 000	2300	2400	270	670	200
Tamarack (or larch)	47	4400	7 300	1 300 000	3100	3600	520	890	240
Walnut	58	5400	9 500	1 420 000	3600	4300	600	1220	570

Kyan's Process.—The timber is steeped in a solution of corrosive sublimate, 1 part of bichloride of mercury to 99 parts of water for a period (at least 5 to 10 days) sufficient to insure thorough penetration of the preservative. Sublimate is comparatively insoluble in water and remains in timber for a longer time than salts like zinc chloride. Great care must be exercised in handling corrosive sublimate as it is a deadly poison. It should not be used as a preservative where domestic animals can lick the treated parts.

Physical Properties.—Strength tests of the U. S. Forest Service in 1907 (H. F. Weiss, American Wood Preservers' Association, 1913) showed that both zinc chloride and creosote reduced the transverse strength of timber, though only to a slight extent in the case of creosote oil. Transverse and compression tests made by H. B. MacFarland (Vol. XIV, American Railway Engineers' Association, 1913) on long leaf pine showed greatly increased strength after one year for creosote treated wood. When tested immediately after treatment the strength was inferior. Thoroughly seasoned timber is stronger than green timber. Some processes tend to increase the moisture content and others to diminish it. If creosote is applied to green wood, the strengthening action of water evaporation is retarded. On the other hand, zinc chloride may cause a chemical dissolution of the wood fiber, thus weakening the structure. Zinc chloride is non-flammable, and wood so preserved is more fire-resisting than non-treated wood. (H. F. Weiss, American Wood Preservers' Association, 1913.) Zinc-treated wood ignited at a temperature of 500° C., 19% of the wood by weight being consumed. Freshly creosoted wood ignited at 225° F., 27% of the wood by weight being consumed. Creosoted wood is adapted for exterior work subject to moisture and is especially effective against marine borers. (See p. 652.)

42. Plywood

Manufacture. Plywood is made by gluing together thin layers of wood. The grain in one layer is at right angles to the grain in the next layer. An odd number of layers is used. It is important that the layers of wood are all split or sawed the same way from a stick of timber, and that the moisture content is approximately the same for all layers. If these precautions are not taken the plywood will tend to warp due to uneven stresses in the different layers.

Strength of Plywood. A limitation of the use of large boards is the difference in strength of wood in various directions. The tensile strength of wood parallel to the grain may be as much as twenty times the strength perpendicular to the grain, and the shearing strength parallel to the grain is much lower than the shearing strength across the grain. In plywood the strength is approximately equal in all directions and is equal, approximately, to the mean of the strength with and the strength across the grain. As plywood is built up of thin layers it becomes feasible to use a high grade of wood in its manufacture.

Woods Available for Plywood. Basswood, redwood, poplar, maple, birch, and red gum are domestic woods which can be cut into the thin sheets necessary for making plywood. Basswood, redwood, and poplar are not often used for the face layers.

Plywood is used mainly in the shape of thin sheets, and is of importance in manufacture of automobiles, street and railway cars, airplanes, and boats.

The strength of glue joints, such as are used in manufacturing plywood, are discussed on page 662.

Tensile Strength of Plywood

Based on test data from the U. S. Forest Products Laboratory, Madison, Wis.

Species	Weight of plywood, kiln-dry, lb. per	Moisture at test, per cent	Tensile strength of 3-ply plywood parallel to grain of faces, lb. per sq. in.	Tensile strength of single-ply veneer, parallel to grain, lb. per sq. in.
Basswood........	28	9.2	6 900	10 300
Yellow birch......	45	8.5	13 200	19 800
Maple, soft.......	38	8.9	8 200	12 300
hard......	46	8.0	10 200	15 300
Yellow poplar.....	34	3.4	7 400	11 100
Red gum.........	36	8.7	7 800	11 800
Redwood........	28	9.7	4 800	7 200

43. Cements and Plasters

For natural and portland cement see Sect. 11. For lime and lime mortar see Sect. 10, Art. 12. **Hydraulic lime** is obtained by burning limestone containing from 10 to 20% of clay. **Magnesian lime** is the term applied to limes containing more than 5% (usually 30% and over) of magnesia. It has the characteristics of being slow slaking or cool, but sets more rapidly and makes a stronger mortar than the high-calcium limes. Lime is never used alone as a binding material, as it shrinks very much in drying. One part of lime paste is combined with 3 or 4 parts of sand, the resulting mortar being about equal in volume to that of the sand.

Grappier Cements are made by grinding lumps of unburned limestone and overburned material which remains when a hydraulic lime is slaked. If lime silicate predominates in the mixture, grappier cement approximates to portland.

Lafarge Cement is a hydraulic grappier cement manufactured at Teil, France. It contains a small percentage of iron and soluble salts and does not stain porous stone masonry.

Plasters are classified by E. C. Eckel as (1) those produced by incomplete dehydration of gypsum ($CaSO_4$), the calcination being carried on at a temperature not exceeding 400° F., and (2) those produced by complete dehydration of calcium sulphate at a temperature exceeding 400° F. **Plaster of paris** is a product of the first class, no foreign material being added to the gypsum during or after calcination. **Cement plaster** is also of the first class, but is manufactured from an impure gypsum, or by the addition of certain impurities during manufacture to act as a " retarder " to the plaster. In the second class are included flooring plaster, commercially known as calcined plaster, being pure calcined gypsum, and also hard-finish plaster, obtained by adding alum or borax during manufacture. Structural gypsum weighs 80 lb. per cu. ft. and cement plaster about 115 lb. per cu. ft.

Common or Lime Plaster is made of quicklime and sand in which hair or fiber is beaten up and thoroughly incorporated with the lime paste. **Wall plasters** are made by the addition of lime and retarder to the calcined plaster.

The Strength of Plaster as determined from tests by Woolson is shown by the following tables (see Proceeding, American Society for Testing Materials, Vol. X, p. 328).

Tensile Strength of Plaster

Values given in pounds per square inch

Kind of lime	1 : 3 lime mortar				1 : 5 lime mortar			
Age	1 mo.	3 mo.	6 mo.	12 mo.	1 mo.	3 mo.	6 mo.	12 mo.
High-calcium (quicklime).....	27	34	47	60	25	36	38	39
High-calcium (hydrated lime).	80	103	113	122	46	62	73	79
Dolomitic (quicklime).......	56	60	74	125	51	72	93	124
Dolomitic (hydrated lime)....	88	90	96	130	27	56	84	122

The Compressive Strength of Gypsum varies from 70 lb. per sq. in. to 3000 lb. per sq. in., depending on the amount of water used in mixing the gypsum paste, the completeness of drying out, the foreign ingredients in the gypsum, and the process of calcination used. For highest strength the least possible amount of water should be used in mixing. From 33 to 38%

Compressive Strength of Plaster

Values given in pounds per square inch

Kind of lime	1 : 3 lime mortar				1 : 5 lime mortar			
Age	1 mo.	3 mo.	6 mo.	12 mo.	1 mo.	3 mo.	6 mo.	12 mo.
High-calcium (quicklime).....	160	120	130	230	90	95	125	250
High-calcium (hydrated lime).	180	750	740	750	175	320	405	485
Dolomitic (quicklime).......	370	350	340	355	170	260	350	530
Dolomitic (hydrated lime)....	270	530	710	1035	190	305	450	730

of water is necessary to make the gypsum paste sufficiently plastic to fill molds. From tests by W. A. Slater for the U. S. Gypsum Co. it was found possible to produce regularly gypsum having a compressive strength of 1400 lb. per sq. in. when a few days old. The modulus of elasticity was found to be about 1 000 000 lb. per sq. in.

44. Miscellaneous Materials

Leather as an engineering material is mostly applied in the manufacturing of belting for transmission trains. The best quality of well-tanned oxhide is cut into strips of from 4 to 6 ft. long and usually about 3/16 in. in thickness, which are scarfed, spliced, or cemented end to end to make the desired length of belt. According to the strength required, these are in turn cemented or riveted together in thickness to form " single " or " double " belts. Under light loads, the " single belt " gives the greater adhesion, but under heavy loads the "double belt " proves the more satisfactory. The " flesh side " or inside of the belt is customarily placed next to the pulley, as it gives the best wear, although when placed grain side to the pulley the belt is less liable to slip.

The weight of a hard well-tanned belt leather is about 62-1/2 lb. per cu. ft., whereas the tensile strength of a good quality is about 650 lb. per inch of width of single belt. When spliced or riveted, the tenacity is about one-half of the above figure, and when laced about one-third of the strength is developed. A safe working tension may be taken at about 50 lb. per inch of width.

Rawhide or untanned leather, finds many applications in textile machinery connections, looms, ships' tiller ropes, etc., and also in the manufacture of high speed gear wheels. When sound, it is much stronger than tanned leather and gives greater resistance to violent impact. Its tensile strength may be taken as one-half greater than that of tanned leather.

India Rubber is used for belting, sheets for packing, tires, electrical insulation, hose, etc. The rubber used by the engineer is generally vulcanized by heating and incorporating it with 7 to 30% of sulfur to render it less readily softened by heat and hardened by cold. When vulcanized with 30 to 40% of sulfur, it forms the various qualities of ebonite used in the manufacture of rules, scales, curves, etc. India-rubber belts are made by weaving cotton canvas of the required length and width, which is then coated with the vulcanized rubber. These are made 2, 3, or 4-ply, according as the power to be transmitted requires the strength of 2, 3, or 4 thicknesses of belt. Rubber belts are superior to leather belts in strength, usually run truer and more smoothly, are perfectly impervious to water, and give a higher coefficient of friction. However, when overloaded they are apt to be injured by the tendency to slipping.

The tensile strength of good quality soft rubber varies from 800 to 1200 lb. per sq. in., and the elongation before fracture is about 600%. Under com-

Tensile Tests of Rubber Belting

From Watertown Arsenal Reports

Width and kind of belt	Actual dimensions, in.			Weight per ft.	Sectional area	Tensile strength, lb.		Remarks on fracture
	Length	Width	Thickness	Lb. oz.	Sq. in.	Per sq. in.	Per in. of width	
4-ply:								
2 in.	60.17	2.02	0.26	0 4.4	0.525	3276	851	At face of jaws
6 in.	60.17	6.08	0.26	0 13.2	1.58	3227	839	3 in. from jaws
6 in.	60.12	6.13	0.26	0 13.6	1.59	3773	979	12 in from jaws
6 in.	60.17	6.05	0.26	0 13.0	1.57	2739	711	At face of jaws
12 in.	60.02	12.08	0.27	1 11.4	3.26	3037	819	10 in. inside gaged length of 30 in.
12 in.	60.14	12.24	0.26	1 11.5	3.18	2987	776	Near middle
6-ply:								
2 in.	60.17	2.14	0.36	0 6.0	0.77	3104	1116	4 in. from jaws
6 in.	59.98	6.26	0.37	1 3.1	2.32	2737	1014	At jaws
6 in.	60.08	6.27	0.36	1 2.9	2.26	3770	1358	Near middle
12 in.	60.15	12.04	0.36	2 3.7	4.33	3436	1236	At face of jaws
12 in.	60.17	12.16	0.34	2 3.7	4.13	3862	1311	At middle
24 in.	60.13	24.11	0.41	4 14.2	9.89	2381	977	At face of jaws
30 in.	60.04	30.18	0.40	6 2.3	12.07	2808	1123	16-3/4 in. from face of jaws

Tensile Tests of Leather Belting

From Watertown Arsenal Reports

Width and kind of belt	Actual dimensions, in.			Weight per ft.	Sectional area	Tensile strength, lb.		Remarks on fracture
	Length	Width	Thickness	Lb. oz.	Sq. in.	Per sq. in.	Per in. of width	
Single:								
2 in.	60.00	1.98	0.20	0 2.75	0.396	5045	1091	1 in. from jaws
6 in.	60.20	6.07	0.22	0 8.70	1.34	2537	560	At scarf joint
6 in.*	60.11	6.08	0.24	0 9.6	1.46	2219	533	At scarf joint
12 in.	60.11	12.05	0.18	0 14.9	2.17	3917	705	At scarf joint
12 in.	60.30	12.05	0.24	1 3.2	2.89	1557	373	At scarf joint
12 in.	....	....	....	....	....	3598	863	5 in. from jaws
Double:								
2 in.	60.00	2.07	0.42	0 5.35	0.869	4025	1690	In jaws of machine at end of scarf joint
4 in.	59.55	3.98	0.33	0 8.3	1.31	4931	1623	At scarf joint
6 in.	60.18	5.91	0.47	1 1.0	2.78	4309	2027	At scarf joint
6 in.*	59.93	6.00	0.40	1 0.6	2.40	5166	2066	At scarf joint
12 in.	59.90	11.90	0.39	1 12.8	4.64	4090	1595	At scarf joint
12 in.	60.06	11.93	0.36	1 14.0	4.29	4424	1591	4 in. from jaws
24 in.	60.00	23.90	0.47	4 8.0	11.23	2760	1297	At scarf joint
30 in.	59.90	29.95	0.43	4 12.7	12.88	2717	1169	At scarf joint

* Waterproofed.

Properties of Various Glues

Technical Note No. 207, U. S. Forest Products Laboratory, Madison, Wis.

Point of comparison	Animal glue	Casein glue	Vegetable glue	Blood glue	Liquid glue
Source of principal ingredient	Animal hides, bones, etc.	Casein from milk.	Starch, generally cassava.	Soluble dried blood.	Animal glue, or skins, bones, etc., of fish.
Spread * Extremes reported / Common range	20 to 50 / 25 to 35	30 to 80 / 35 to 55	35 to 70 / 35 to 55	30 to 100	No data.
Mixing	Soaked in water, then melted.	Mixed cold.	Mixed with alkali and water, with or without heat; can be made without alkali.	Mixed cold.	Requires no preparation.
Application	Applied warm with brush or mechanical spreader.	Applied cold with brush or mechanical spreader.	Applied cold with mechanical spreader.	Applied cold by hand or with mechanical spreader.	Applied cold or warm, usually by hand.
Temperature of press	Cold; hot cauls frequently used.	Cold.	Cold.	Hot or cold, depending on formula used.	Cold.
Strength (block shear test)	High grade; have greater shear strength than strongest American woods; medium grades slightly lower.	Similar to medium grade animal glue.	Similar to medium grade animal glue.	Similar to or slightly less than medium grade animal glue.	Good grades similar to medium grade animal glue; some brands very weak.
Water resistance	Naturally low, but can be increased by chemical treatment.	High or low, as required.	Low.	High.	Low.
Staining	Does not stain.	Stains wood of some species.	If mixed with caustic soda, stains wood of some species.	Does not stain, but the glue is very dark and may show through thin veneer.	Does not stain.
Uses in woodworking	High grade, where a strong joint is desired; low grade sometimes used for veneering, especially where it is desired to prevent staining.	Mainly where water resistance is desired in veneered or joint work.	To some extent for joint work, but mainly in veneered work where good strength at low cost is desired.	Almost entirely for water-resistant plywood for aircraft or automobiles and for articles to be molded after boiling in water.	Mainly for repair work and gluing small articles by hand.

* Expressed in square feet of single glue line per pound of dry glue for veneer work.

pressive load good soft rubber is shortened 50% under a stress of from 500 to 800 lb. per sq. in. The outstanding property of rubber, which makes it unrivaled as the material for automobile tires, is its capacity for absorbing energy. Stressed to a good working stress rubber under tension will absorb about ten times as much energy per cubic inch as will spring steel, and under compression about 3.75 times.

Glass. The tensile strength of common glass varies from 2000 to 3000 lb. per sq. in., and the compressive strength from 6000 to 12 000 lb. per sq. in. A series of transverse tests were undertaken on common glass at the Watertown Arsenal. Sheets of common window glass 2 ft. 6 in. long by about 4.95 in. wide and 0.121 in. thick were subjected to a central load with an unsupported length of 2 ft. The modulus of rupture varied from 3000 to 4000 lb. per sq. in and the modulus of elasticity from 10 000 000 to 11 000 000 lb. per sq. in.

Ice at 32° F. weighs 57.5 lb. per cu. ft., its specific gravity being 0.922 (water at 62° F. = 1). Its volume relative to water is 1.0855. Its melting point decreases from 32° F. at the rate of 0.0133° F. for each additional atmospheric pressure. Its specific heat is 0.504 (water = 1). Some German experiments made in 1885 gave a tensile strength of 142 to 223 lb. per sq. in. Tests made by the U. S. Engineer Corps on 6- and 12-in. cubes, gave crushing strengths varying from 100 to 1000 lb. per sq. in. depending on the structure of the ice and the purity of water from which it was formed. Before crushing, ice in cubes will compress from 6 to 30%. The sustaining capacity of ice is not definitely determined; 2-in. ice is considered safe for infantry, 4-in. ice for cavalry or light guns, 6-in. ice for heavy field guns, and 8-in. ice for loads not over 1000 lb. per sq. ft. on sledges. Railway trains have been run across ice which was 15-in. thick.

The expansive force of ice is given by Trautwine as probably not less than 30 000 lb. per sq. ft. The coefficient of expansion, as given by Ganot, is 0.000 052. By its expansion a sheet of ice 150 ft. in width has been known to tip a masonry bridge pier weighing 1000 tons 2 in. out of plumb, and in another instance to move masonry piers on pile foundations from 2 to 12 in. out of line. The expansive effect in river or lake ice, however, does not make itself felt until the ice is at least 5 in. thick.

Freshly fallen snow weighs from 5 to 12 lb. per cu. ft.; compacted or wet snow weighs from 15 to 50 lb. per cu. ft.

45. Paints and Oils

A Paint consists of a vehicle and a pigment; and many paints have a drier added. The whites, as white lead and zinc white, form the base upon which nearly all tints are made by the addition of colored pigments called stains. Red lead and oxide of iron may also be used as bases for paint without using a white base or they may be used as stains. The usual **vehicle** for the base is linseed oil. Its function is to enable the paint to be spread and also to form a binder for the paint materials after drying. Spirits of turpentine is often used as a paint **thinner.** Its function is to make the paint work more smoothly. The drying of the paint is often hastened by the addition of a **drier,** whose function is to hasten the solidification of the linseed oil by making its oxidation more rapid.

White Lead is a combination of lead carbonate and lead hydrate. The former gives the opacity or body, while the latter gives the saponifying and binding properties. White lead is supplied to the trade ground in linseed oil. It is a good drier of this oil, and very little artificial drier need be added to a white-lead paint.

Zinc White is the oxide of zinc. It is a permanent white pigment, and so is well adapted to interior decoration. Sulfureted hydrogen gases have no discoloring effect upon it, while such gases will darken a white-lead paint.

Red Lead is a double oxide of lead. It is used as a constituent of a priming paint for new woodwork, and its excellent anti-corrosive properties make it the best primer for coating ironwork. It excels all other pigments in withstanding abrasive wear. It is a strong drier of linseed oil, solidifying it in a short time, and it is for this reason that the pigment is sold in the dry powdered state. For the highest grades of red lead the content of lead monoxide (litharge) should not exceed 4%, for a lower grade 13% is allowable.

Raw Linseed Oil is produced by pressing flaxseed. Boiled linseed oil is prepared by heating the raw oil either alone or with a drier such as red lead. Boiled oil dries in about one-half the time that the raw oil does. For this reason the boiled oil is much used for exterior work. For interior work and for grinding up colors the raw oil is used. Linseed oil is subject to adulteration by the addition of cotton-seed, resin, hemp, mineral, and fish oils. As substitutes, fish oil, cotton-seed oil, and Chinese tung oil are used.

Spirits of turpentine is a solvent or diluent used with paints and varnishes. It is also employed for mixing pigments in making flatting colors for interior decoration. It is often adulterated with mineral oil. As substitutes, benzine and naphtha are used.

Staining Colors or Pigments are used for tinting paints which have a white base such as white lead or zinc white. The most common black pigments are the soot and charcoal blacks. The best known red pigments are Venetian red, which is an earth with ferric oxide as the coloring agent, and red lead. The brown pigments are burnt umber, and the raw and burnt siennas, which resemble umber in composition. The green most used is chrome green, a pigment compounded of Prussian blue and chrome yellow. The standard commercial mixture of chrome green contains three parts of base to one of chrome green. Prussian blue is the common blue pigment which has great strength of coloring matter. Another important pigment is the ultramarine blue, an artificial chemical product of complex composition. As this latter pigment has some sulfur, it should not be mixed with white lead but should be used with zinc white as a base.

Graphite Paint is prepared by mixing graphite with boiled linseed oil to which a small percentage of drier has been added. It is an excellent paint for iron. One objection to this paint is that it mars easily under slight abrasion.

Protection of Iron Work under Water. Refined coal tar dissolved in a vehicle makes the most durable paint coating known for iron under water. Asphalt paints made by dissolving asphaltum in some suitable vehicle such as naphtha or benzine is also used for this purpose. Coal tar has been especially effective as a protector for cast iron water pipe.

Varnish is made by dissolving gums or resins in oil, turpentine or alcohol. The primary purpose of varnish is to furnish a smooth, hard, transparent coating for a surface. Hardness and toughness are somewhat contradictory requirements for varnish both of which can be fairly well met in high-grade varnishes. Quick drying and durability are two contradictory requirements which have not been reconciled in any varnish.

For general reference on Paint and Varnish see Sabin, The Technology of Paint and Varnish, 3d Edition, New York, 1927.

STANDARD MARKET SIZES

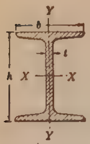
46. Rolled Structural Beams

Structural Beams and Shapes should be ordered by weight per foot or by the thickness wanted. A variation of 2-1/2% either way is allowed in the nominal weight of the shape. Unless otherwise arranged, structural shapes are cut to lengths as ordered with an extreme variation of 3/4 in. For cutting with a less variation, an extra charge is made.

The increase of weight in standard I beams and channels is secured by increasing the web uniformly in thickness for the depth of the beam. This results also in an increased width of flange equal to the increase in web thickness. Increase of weight of Bethlehem beams and girders is secured by increase of both web thickness and depth of beam. In angles the increase is made by increasing uniformly the thickness of the two legs. The effect of the spreading of the rolls to secure this additional thickness is to increase the length of the legs, amounting to about 1/16 in. for each 1/16 in. increase in thickness. As most sizes, however, are rolled in finishing grooves, the exact dimensions are generally maintained for different thicknesses. In Z bars the increase is made by increasing uniformly and by equal amounts the thickness of web and both legs. T shapes do not admit of any variation in weight, and are rolled only to fixed dimensions and weights.

Flanges of standard I beams and channels have a uniform slope of 2 in. per ft., or 16-2/3%. Flanges of Bethlehem I and girder beams have a uniform slope of 12-1/2%. On standard I beams and channels small fillets have a radius of 0.6 of the minimum web thickness, and large fillets a radius of the minimum web thickness plus 0.1.

The Section Modulus of the tables is the moment of inertia of the shape divided by the distance of the center of gravity from the top or bottom of the section. It is used to determine the fiber stress per square inch in a beam or other shape, by dividing it into the bending moment expressed in inch-pounds. It may also be used to guide in the selection of a beam, or other shape, required to sustain a girder load, by dividing the bending moment, in inch-pounds, by the allowable fiber stress per square inch, and finding the required shape in the tables. In those shapes which are not symmetrical about the neutral axis there are, for each case, two section moduli; and in the tables the smaller is always given. The fiber stress calculated from it will be the greater stress and would generally be the one sought.

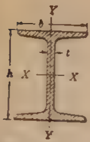


Properties of Standard I Beams

Dimensions are given in inch units

Depth of beam, h	Lb. per ft.	Area of section, A	Width of flange, b	Thickness of web, t	Axis XX: Moment of inertia, I	Axis XX: Radius of gyration, r	Axis XX: Section modulus I/c	Axis YY: Moment of inertia, I′	Axis YY: Radius of gyration, r′	Shearing stress factor *
24	120.0	35.13	8.048	0.798	3010.8	9.26	250.9	84.9	1.56	0.062
	115.0	33.67	7.987	0.737	2940.5	9.35	245.0	82.8	1.57	0.067
	110.0	32.18	7.925	0.675	2869.1	9.44	239.1	80.6	1.58	0.072
	105.9	30.98	7.875	0.625	2811.5	9.53	234.3	78.9	1.60	0.078
24	100.0	29.25	7.247	0.747	2371.8	9.05	197.6	48.4	1.29	0.067
	95.0	27.79	7.186	0.686	2301.5	9.08	191.8	47.0	1.30	0.073
	90.0	26.30	7.124	0.624	2230.1	9.21	185.8	45.5	1.32	0.079
	85.0	24.84	7.063	0.563	2159.8	9.33	180.0	44.2	1.33	0.087
	79.9	23.33	7.000	0.500	2087.2	9.46	173.9	42.9	1.36	0.097
20	100.0	29.20	7.273	0.873	1648.3	7.51	164.8	52.4	1.34	0.069
	95.0	27.74	7.200	0.800	1599.7	7.59	160.0	50.5	1.35	0.075
	90.0	26.26	7.126	0.726	1550.3	7.68	155.0	48.7	1.36	0.082
	85.0	24.80	7.053	0.653	1501.7	7.78	150.2	47.0	1.38	0.090
	81.4	23.74	7.000	0.600	1466.3	7.86	146.6	45.8	1.39	0.098
20	75.0	21.90	6.391	0.641	1263.5	7.60	126.3	30.1	1.17	0.093
	70.0	20.42	6.317	0.567	1214.2	7.71	121.4	28.9	1.19	0.105
	65.4	19.08	6.250	0.500	1169.5	7.83	116.9	27.9	1.21	0.117
18	90.0	26.29	7.236	0.796	1256.5	6.91	139.6	51.9	1.40	0.084
	85.0	24.81	7.154	0.714	1216.6	7.00	135.2	49.8	1.42	0.092
	80.0	23.34	7.072	0.632	1176.8	7.10	130.8	47.9	1.43	0.103
	75.6	22.04	7.000	0.560	1141.8	7.20	126.9	46.3	1.45	0.115
18	70.0	20.46	6.251	0.711	917.5	6.70	101.9	24.5	1.09	0.095
	65.0	18.98	6.169	0.629	877.7	6.80	97.5	23.4	1.11	0.106
	60.0	17.50	6.087	0.547	837.8	6.92	93.1	22.3	1.13	0.120
	54.7	15.94	6.000	0.460	795.5	7.07	88.4	21.2	1.15	0.141
15	75.0	21.85	6.278	0.868	687.2	5.61	91.6	30.6	1.18	0.094
	70.0	20.38	6.180	0.770	659.6	5.69	87.9	28.8	1.19	0.104
	65.0	18.91	6.082	0.672	632.1	5.78	84.3	27.2	1.20	0.118
	60.8	17.68	6.000	0.590	609.0	5.87	81.2	26.0	1.21	0.133
15	55.0	16.06	5.738	0.648	508.7	5.63	67.8	17.0	1.03	0.124
	50.0	14.59	5.640	0.550	481.1	5.74	64.2	16.0	1.05	0.144
	45.0	13.12	5.542	0.452	453.6	5.88	60.5	15.0	1.07	0.172
	42.9	12.49	5.500	0.410	441.8	5.95	58.9	14.6	1.08	0.188
12	55.0	16.04	5.600	0.810	319.3	4.46	53.2	17.3	1.04	0.126
	50.0	14.57	5.477	0.687	301.6	4.55	50.3	16.0	1.05	0.146
	45.0	13.10	5.355	0.565	284.1	4.66	47.3	14.8	1.06	0.174
	40.8	11.84	5.250	0.460	268.9	4.77	44.8	13.8	1.08	0.211
12	35.0	10.20	5.078	0.428	227.0	4.72	37.8	10.0	0.99	0.228
	31.8	9.26	5.000	0.350	215.8	4.83	36.0	9.5	1.01	0.275
10	40.0	11.69	5.091	0.741	158.0	3.68	31.6	9.4	0.90	0.165
	35.0	10.22	4.944	0.594	145.8	3.78	29.2	8.5	0.91	0.202
	30.0	8.75	4.797	0.447	133.5	3.91	26.7	7.6	0.93	0.262
	25.4	7.38	4.660	0.310	122.1	4.07	24.4	6.9	0.97	0.366
9	35.0	10.22	4.764	0.724	111.3	3.30	24.7	7.3	0.84	0.189
	30.0	8.76	4.601	0.561	101.4	3.40	22.5	6.4	0.85	0.238
	25.0	7.28	4.437	0.397	91.4	3.54	20.3	5.6	0.88	0.330
	21.8	6.32	4.330	0.290	84.9	3.67	18.9	5.2	0.90	0.441

* Shearing stress factor multiplied by shear at a section (in pounds) gives the maximum shearing unit stress in the web of the beam (in pounds per square inch).



Properties of Standard I Beams—Continued

Dimensions are given in inch units

Depth of beam, h	Lb. per ft.	Area of section, A	Width of flange, b	Thickness of web, t	Axis XX: Moment of inertia, I	Axis XX: Radius of gyration, r	Axis XX: Section modulus, I/c	Axis YY: Moment of inertia, I′	Axis YY: Radius of gyration, r′	Shearing stress factor *
8	25.5	7.43	4.262	0.532	68.1	3.03	17.0	4.7	0.80	0.284
	23.0	6.71	4.171	0.441	64.2	3.09	16.0	4.4	0.81	0.338
	20.5	5.97	4.079	0.349	60.2	3.18	15.1	4.0	0.82	0.420
	18.4	5.34	4.000	0.270	56.9	3.26	14.2	3.8	0.84	0.535
7	20.0	5.83	3.860	0.450	41.9	2.68	12.0	3.1	0.74	0.381
	17.5	5.09	3.755	0.345	38.9	2.77	11.1	2.9	0.76	0.486
	15.3	4.43	3.660	0.250	36.2	2.86	10.4	2.7	0.78	0.655
6	17.25	5.02	3.565	0.465	26.0	2.28	8.7	2.3	0.68	0.432
	14.75	4.29	3.443	0.343	23.8	2.36	7.9	2.1	0.69	0.625
	12.5	3.61	3.330	0.230	21.8	2.46	7.3	1.8	0.72	0.830
5	14.75	4.29	3.284	0.494	15.0	1.87	6.0	1.7	0.63	0.495
	12.25	3.56	3.137	0.347	13.5	1.95	5.4	1.4	0.63	0.684
	10.0	2.87	3.000	0.210	12.1	2.05	4.8	1.2	0.65	1.093
4	10.5	3.05	2.870	0.400	7.1	1.52	3.5	1.0	0.57	0.755
	9.5	2.76	2.796	0.326	6.7	1.56	3.3	0.91	0.58	0.912
	8.5	2.46	2.723	0.253	6.3	1.60	3.2	0.83	0.58	1.160
	7.7	2.21	2.660	0.190	6.0	1.64	3.0	0.77	0.59	1.510
3	7.5	2.17	2.509	0.349	2.9	1.15	1.9	0.59	0.52	1.147
	6.5	1.88	2.411	0.251	2.7	1.19	1.8	0.51	0.52	1.535
	5.7	1.64	2.330	0.170	2.5	1.23	1.7	0.46	0.53	2.250

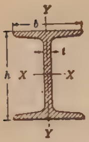
* Shearing stress factor multiplied by shear at a section (in pounds) gives the maximum shearing unit stress in the web of the beam (in pounds per square inch).

Comparative Tests on standard I beams and Bethlehem special I beams and girder beams, made at the University of Pennsylvania, gave following values of the modulu- of rupture in pounds per square inch as an average of three specimens in each case. The 15-in. beams were 15 ft. in span, the others were 20 ft. in span.

Standard I,	15-in., 42-lb.,	central loading,	S = 42 200
Standard I,	15-in., 42-lb.,	quarter point loading,	S = 34 700
Standard I,	24-in., 80-lb.,	quarter point loading,	S = 33 000
Bethlehem I,	15-in., 38-lb.,	central loading,	S = 46 100
Bethlehem I,	15-in., 38-lb.,	quarter point loading,	S = 37 900
Bethlehem I,	24-in., 72-lb.,	quarter point loading,	S = 34 600
Girder beam,	15-in., 73-lb.,	central loading,	S = 53 900
Girder beam,	15-in., 73-lb.,	quarter point loading,	S = 41 100
Girder beam,	24-in., 120-lb.,	quarter point loading,	S = 34 300
Girder beam,	30-in., 175-lb.,	quarter point loading,	S = 31 000

The modulus of elasticity for the 31 beams tested was nearly constant, its average value being 26 300 000 lb. per sq. in. This value, deduced from the full-size tests, was somewhat less than that obtained from the tensile tests on specimens cut from the flange, web and root of the several beams; this average was 28 700 000.

The Bethlehem Structural Beams are wide-flange I sections rolled by the Grey universal beam mill. Instead of the horizontal grooved rolls of the ordinary beam mill, the Grey mill has both horizontal and vertical rolls, by which the flanges and web of an I-beam shape are each produced by simultaneous combined rolling operations acting at right angles. This method of rolling makes it possible to obtain wider flanges than can be produced by the ordinary beam mill.

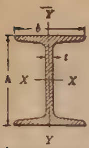


Properties of Bethlehem I Beams

Dimensions are given in inch units

Depth of beam, h	Lb. per ft.	Are of section, A	Width of flange, b	Thickness of web, t	Axis XX: Moment of inertia, I	Axis XX: Radius of gyration, r	Axis XX: Section modulus I/c	Axis YY: Moment of inertia, I′	Axis YY: Radius of gyration, r′	Shearing stress factor *
30-1/8	129.0	37.52	10.530	.580	5566.5	12.18	369.6	177.5	2.18	.066
30	121.0	35.36	10.500	.550	5213.6	12.14	347.6	164.3	2.16	.070
29-7/8	115.0	33.50	10.480	.530	4886.8	12.08	327.1	151.8	2.13	.072
28-1/8	104.0	32.98	10.030	.500	4285.5	11.40	304.8	142.3	2.08	.076
28	97.0	30.93	10.000	.470	3993.8	11.36	285.3	130.9	2.06	.080
27-7/8	91.0	26.86	9.980	.450	3723.4	11.30	267.1	120.2	2.03	.084
26-1/8	98.0	28.47	9.530	.500	3200.9	10.60	245.1	110.6	1.97	.087
26	91.0	26.55	9.500	.470	2962.8	10.56	227.9	100.9	1.95	.093
25-7/8	85.5	24.89	9.480	.450	2742.2	10.50	211.9	91.6	1.92	.098
24-3/32	104.5	30.63	9.775	.550	2967.7	9.84	246.4	132.9	2.08	.086
24	99.5	29.15	9.750	.525	2811.7	9.82	234.3	124.8	2.07	.090
23-29/32	95.5	27.79	9.730	.505	2663.1	9.79	222.8	117.1	2.05	.094
24	84.5	24.75	9.500	.460	2380.1	9.81	198.3	95.8	1.97	.099
24-3/32	79.5	23.17	9.035	.430	2245.3	9.84	186.4	81.2	1.87	.110
24	73.5	21.52	9.000	.395	2087.4	9.85	173.9	74.7	1.86	.120
22-1/8	67.5	19.84	8.520	.390	1637.5	9.08	148.1	61.8	1.70	.122
22	62.5	18.38	8.500	.370	1495.4	9.01	135.9	55.2	1.73	.134
20-3/32	78.0	22.77	8.905	.460	1568.3	8.30	156.1	84.6	1.93	.123
20	73.0	21.37	8.875	.430	1467.8	8.29	146.8	78.5	1.92	.132
20-1/8	64.5	18.79	8.025	.400	1283.2	8.26	127.6	54.3	1.70	.141
20-1/16	62.0	18.11	8.015	.390	1227.9	8.23	122.4	51.5	1.69	.146
20	59.5	17.33	8.000	.375	1169.7	8.22	117.0	48.6	1.68	.152
18-1/8	74.0	21.61	8.770	.440	1238.0	7.57	136.6	82.9	1.96	.142
18	69.0	20.20	8.750	.420	1142.5	7.52	126.9	75.6	1.93	.149
17-7/8	64.5	18.79	8.730	.400	1048.5	7.47	117.3	68.4	1.91	.158
18-1/8	54.5	15.95	7.540	.370	888.5	7.46	98.1	41.1	1.60	.169
18-1/16	52.0	15.22	7.525	.355	844.1	7.45	93.5	38.7	1.59	.177
18	49.0	14.32	7.500	.330	795.3	7.45	88.4	36.1	1.59	.190
15	71.5	20.79	7.500	.520	789.4	6.16	105.3	60.8	1.71	.144
15-1/8	59.5	17.32	7.040	.450	668.7	6.21	88.4	42.8	1.57	.168
15	54.5	15.87	7.000	.410	609.5	6.20	81.3	38.6	1.56	.185
14-7/8	50.5	14.66	6.975	.385	555.8	6.16	74.7	34.7	1.54	.198
15-3/32	42.5	12.39	6.785	.325	486.8	6.27	64.5	26.9	1.47	.230
15-1/32	40.0	11.68	6.765	.305	458.1	6.26	61.0	25.1	1.47	.246
15	38.5	11.26	6.750	.290	442.4	6.27	59.0	24.1	1.46	.260
12-1/8	44.0	12.97	6.780	.360	335.1	5.08	55.3	31.1	1.55	.304
12	40.0	11.80	6.750	.330	301.2	5.05	50.2	27.6	1.53	.346
12	28.0	8.28	6.500	.245	213.6	5.08	35.6	16.4	1.41	.373
10-3/32	26.0	7.61	5.770	.270	136.7	4.24	27.1	12.5	1.28	.412
10	23.5	6.89	5.750	.250	121.9	4.21	24.4	10.9	1.26	.449
9-1/16	22.0	6.45	5.510	.260	92.9	3.80	20.5	9.42	1.21	.478
9	20.5	6.02	5.500	.250	85.5	3.77	19.0	8.53	1.19	.500
8-1/16	19.0	5.62	5.270	.270	62.9	3.35	15.6	7.20	1.13	.522
8	17.5	5.14	5.250	.250	56.9	3.33	14.2	6.39	1.11	.565

* Shearing stress factor multiplied by shear at a section (in pounds) gives the maximum shearing unit stress in the web of the beam (pounds per square inch).

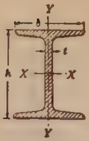


Properties of Bethlehem Girder Beams

Dimensions are given in inch units

Depth of beam, h	Lb. per ft.	Area of section, A	Width of flange, b	Thickness of web, t	Axis XX: Moment of inertia, I	Axis XX: Radius of gyration, r	Axis XX: Section modulus, I/c	Axis YY: Moment of inertia, I′	Axis YY: Radius of gyration, r′	Shearing stress factor *
30-1/4	200.0	58.52	15.04	.76	9148.8	12.50	607.5	628.5	3.28	.049
30-1/8	190.0	55.52	15.00	.72	8651.1	12.48	576.7	589.4	3.26	.052
30	181.0	52.82	14.97	.69	8181.0	12.45	547.6	552.0	3.23	.055
28-1/8	175.0	51.02	14.29	.70	6988.7	11.70	497.1	496.2	3.12	.057
28	165.0	48.19	14.25	.66	6577.9	11.68	469.9	462.8	3.10	.061
26-1/8	160.0	46.85	13.79	.67	5576.6	10.91	427.0	432.8	3.04	.064
26	151.0	44.16	13.75	.63	5237.1	10.89	402.9	402.7	3.02	.070
25-7/8	144.0	41.99	13.73	.61	4930.6	10.84	381.0	375.0	2.99	.071
24-1/8	149.0	43.57	13.29	.65	4451.1	10.11	369.1	383.3	2.97	.072
24	141.0	41.02	13.25	.61	4174.2	10.09	347.9	356.4	2.95	.076
23-7/8	133.0	38.71	13.22	.58	3912.4	10.05	327.7	330.7	2.92	.081
24-1/8	129.0	37.74	12.29	.58	3844.8	10.09	318.8	278.2	2.72	.080
24	121.0	35.30	12.25	.54	3585.3	10.08	298.8	256.9	2.70	.087
23-7/8	114.0	33.12	12.22	.51	3340.6	10.04	279.8	236.7	2.67	.092
20-1/8	149.0	43.44	12.78	.69	3106.6	8.46	308.8	384.5	2.97	.081
20	142.0	41.31	12.75	.66	2932.3	8.43	293.2	360.9	2.96	.085
19-7/8	135.0	39.18	12.72	.63	2760.6	8.39	277.7	337.6	2.94	.089
20-1/8	120.0	34.95	12.03	.59	2505.5	8.47	249.1	260.1	2.73	.095
20	113.0	32.90	12.00	.56	2340.2	8.43	234.0	240.8	2.71	.100
19-7/8	107.0	31.06	11.98	.54	2184.0	8.39	219.7	222.3	2.68	.104
18-1/4	100.0	29.25	11.79	.48	1725.7	7.68	190.5	211.3	2.69	.119
18-1/8	93.0	27.14	11.77	.86	1593.4	7.66	177.0	192.2	2.66	.129
18	87.5	25.40	11.75	.44	1472.8	7.61	164.7	174.9	2.53	.135
15-1/8	147.0	42.73	11.78	.83	1666.2	6.24	220.4	347.3	2.85	.092
15	141.0	40.86	11.75	.80	1577.7	6.21	210.4	328.3	2.83	.096
14-7/8	135.0	39.01	11.72	.77	1490.7	6.18	200.4	309.5	2.82	.100
15-1/8	111.0	32.40	11.29	.64	1306.3	6.35	172.8	231.2	2.67	.118
15	105.0	30.45	11.25	.60	1218.2	6.32	162.4	214.3	2.65	.126
14-7/8	99.0	28.65	11.22	.57	1134.7	6.29	152.5	198.4	2.63	.133
15-1/8	80.5	23.44	10.79	.48	968.5	6.43	128.1	143.0	2.47	.155
15	74.0	21.55	10.75	.44	883.8	6.40	117.8	128.9	2.45	.171
14-7/8	69.0	19.96	10.73	.42	806.4	6.36	108.4	115.8	2.41	.178
12-1/8	76.5	22.29	10.29	.51	589.0	5.14	97.2	132.1	2.43	.182
12	70.5	20.57	10.25	.47	538.4	5.12	89.7	119.7	2.41	.198
11-7/8	66.0	19.11	10.23	.45	491.7	5.07	82.8	108.3	2.38	.208
12-1/8	61.0	17.77	10.03	.41	479.9	5.20	79.2	95.8	2.32	.224
12	55.5	16.21	10.00	.38	431.8	5.16	72.0	84.9	2.29	.243
11-29/32	51.5	15.07	9.98	.36	396.9	5.13	66.6	76.9	2.26	.258
10-1/8	50.0	14.51	9.04	.36	275.5	4.36	54.4	66.4	2.14	.306
10	44.5	13.03	9.00	.32	244.7	4.33	48.9	58.2	2.11	.345
9-29/32	41.5	12.12	8.99	.31	223.8	4.30	45.2	52.6	2.08	.359
9-1/8	43.5	12.62	8.54	.35	193.8	3.92	42.5	51.3	2.02	.350
9	38.5	11.23	8.50	.31	170.3	3.89	37.9	44.4	1.99	.399
8-15/16	36.0	10.55	8.48	.29	158.9	3.88	35.5	41.0	1.97	.424
8-1/8	37.0	10.77	8.03	.33	131.1	3.49	32.3	38.7	1.90	.417
8	33.0	9.57	8.00	.30	114.2	3.45	28.6	33.2	1.86	.462
7-7/8	29.5	8.69	7.99	.29	100.7	3.41	25.6	28.4	1.81	.482

* Shearing stress factor multiplied by shear at a section (in pounds) gives the maximum shearing unit stress in the web of the beam (pounds per square inch).

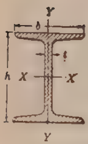


Properties of Standard H Beams

Dimensions are given in inch units

Depth of beam, h	Lb. per ft.	Area of section, A	Width of flange, b	Thickness of web, t	Axis XX: Moment of inertia, I	Axis XX: Radius of gyration, r	Axis XX: Section modulus, I/c	Axis YY: Moment of inertia, I′	Axis YY: Radius of gyration, r′	Shearing stress factor *
8	37.7	11.00	8.125	0.500	120.8	3.31	30.2	36.9	1.83	0.284
	34.3	10.00	8.000	0.375	115.5	3.40	28.9	35.1	1.87	0.372
	32.6	9.50	7.938	0.313	112.8	3.45	28.2	34.2	1.90	0.442
6	26.7	7.76	6.125	0.438	47.4	2.47	15.8	15.7	1.42	0.437
	24.1	7.01	6.000	0.313	45.1	2.54	15.0	14.7	1.45	0.604
	22.8	6.63	5.938	0.250	44.0	2.58	14.7	14.2	1.46	0.747
5	18.9	5.47	5.000	0.313	23.8	2.08	9.5	7.8	1.20	0.731
4	13.8	3.99	4.000	0.313	10.7	1.64	5.3	3.6	0.95	0.930

* Shearing stress factor multiplied by shear at a section (in pounds) gives the maximum shearing unit stress in the web of the beam (in pounds per square inch).



Properties of "Junior" I Beams

(Jones & Laughlin)

Dimensions are given in inch units

Depth of beam, h	Lb. per ft.	Area of section, A	Width of flange, b	Thickness of web, t	Axis XX: Moment of inertia, I	Axis XX: Radius of gyration, r	Axis XX: Section modulus I/c	Axis YY: Moment of inertia, I′	Axis YY: Radius of gyration, r′	Shearing stress factor *
12	11.13	3.27	2.99	0.17	67.19	4.531	11.20	0.84	0.508	0.584
11	9.74	2.86	2.80	0.16	49.83	4.171	9.06	0.67	0.483	0.677
10	8.42	2.48	2.61	0.15	35.95	3.809	7.19	0.52	0.457	0.792
9	7.23	2.13	2.42	0.14	25.31	3.449	5.62	0.40	0.431	0.943
8	6.12	1.80	2.23	0.13	17.13	3.086	4.28	0.29	0.405	1.135
7	5.10	1.50	2.04	0.12	11.10	2.722	3.17	0.22	0.378	1.400
6	4.16	1.22	1.85	0.11	6.77	2.353	2.26	0.15	0.352	1.790

* Shearing stress factor multiplied by shear at a section (in pounds) gives the maximum shearing unit stress in the web of the beam (in pounds per square inch).

47. Rolled Structural Shapes

Properties of Bethlehem Rolled Steel H Column Sections

Continued on page 672.

Dimensions are given in inch units

Section number	Weight lb. per ft.	Depth of section d	Width of flange b	Thickness of web t	Area of section A	For XX as neutral axis (see Fig. 69) Moment of inertia I	For XX Section modulus I/½d	For XX Radius of gyration r′	For YY as neutral axis (see Fig. 69) Moment of inertia I′	For YY Section modulus I/½d	For YY Radius of gyration r′
	32.0	7-7/8	8.00	.31	9.17	105.7	26.9	3.40	35.8	8.9	1.98
	34.5	8	8.00	.31	10.17	121.5	30.4	3.46	41.1	10.3	2.01
	39.0	8-1/8	8.04	.35	11.50	139.5	34.3	3.48	47.2	11.7	2.03
	43.5	8-1/4	8.08	.39	12.83	158.3	38.4	3.51	53.4	13.2	2.04
	48.0	8-3/8	8.12	.43	14.18	177.7	42.4	3.54	59.8	14.7	2.05
	53.0	8-1/2	8.16	.47	15.53	197.8	46.5	3.57	66.3	16.3	2.07
H 8	57.5	8-5/8	8.20	.51	16.90	218.6	50.7	3.60	73.1	17.8	2.08
	62.0	8-3/4	8.24	.55	18.27	240.2	54.9	3.63	80.2	19.4	2.09
	67.0	8-7/8	8.28	.59	19.66	262.5	59.2	3.65	87.1	21.0	2.11
	71.5	9	8.32	.63	21.05	285.6	63.5	3.68	94.4	22.7	2.12
	76.5	9-1/8	8.36	.67	22.46	309.5	67.8	3.71	101.9	24.4	2.13
	81.0	9-1/4	8.39	.70	23.78	333.5	72.1	3.75	109.2	26.0	2.14
	85.5	9-3/8	8.43	.74	25.20	359.0	76.6	3.77	117.2	27.8	2.16
	90.5	9-1/2	8.47	.78	26.64	385.3	81.1	3.80	125.1	29.6	2.17
	49.0	9-7/8	9.97	.36	14.37	263.5	53.4	4.28	89.1	17.9	2.49
	54.0	10	10.00	.39	15.91	296.8	59.4	4.32	100.4	20.1	2.51
	59.5	10-1/8	10.04	.43	17.57	331.9	65.6	4.35	112.2	22.3	2.53
	65.5	10-1/4	10.08	.47	19.23	368.0	71.8	4.37	124.2	24.6	2.54
	71.0	10-3/8	10.12	.51	20.91	405.2	78.1	4.40	136.5	27.0	2.56
	77.0	10-1/2	10.16	.55	22.59	443.6	84.5	4.43	149.1	29.4	2.57
H 10	82.5	10-5/8	10.20	.59	24.29	483.0	90.9	4.46	162.0	31.8	2.58
	88.5	10-3/4	10.24	.63	25.99	523.5	97.4	4.49	175.1	34.2	2.60
	94.0	10-7/8	10.28	.67	27.71	565.2	103.9	4.52	188.6	36.7	2.61
	99.5	11	10.31	.70	29.32	607.0	110.4	4.55	201.7	39.1	2.62
	105.5	11-1/8	10.35	.74	31.06	651.0	117.0	4.58	215.6	41.7	2.64
	111.5	11-1/4	10.39	.78	32.80	696.2	123.8	4.61	229.9	44.3	2.65
	117.5	11-3/8	10.43	.82	34.55	742.7	130.6	4.64	244.4	46.9	2.66
	123.5	11-1/2	10.47	.86	36.32	790.4	137.5	4.67	259.3	49.5	2.67

The clear distance between the flange fillets or the depth of the flat surface of the web available for connections, is 6.14 in. for the H 8 sections, 7.67 in. for the H 10 sections, 9.21 in. for the H 12 sections, and 11.06 for the H 14 sections.

All columns having the same section number are from the same rolls. Whenever possible it is advisable to confine the selection of column to the same section number.

The section modulus is needed when the sections are to be used as beams, and also when columns are subject to bending.

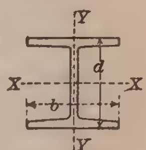
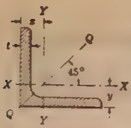


Fig. 69

Properties of Bethlehem Rolled Steel H Column Sections—Cont.

Dimensions are given in inch units

Section number	Weight lb. per ft.	Depth of section	Width of flange	Thickness of web	Area of section	For XX as neutral axis (see Fig. 69) Moment of inertia	Section modulus	Radius of gyration	For YY as neutral axis (see Fig. 69) Moment of inertia	Section modulus	Radius of gyration
		d	b	t	A	I	$\frac{I}{1/2\ d}$	r	I′	$\frac{I}{1/2\ b}$	r′
H 12	64.5	11-3/4	11.92	.39	19.00	499.0	84.9	5.13	168.6	28.3	2.98
	71.5	11-7/8	11.96	.43	20.96	556.6	93.7	5.15	188.2	31.5	3.00
	78.0	12	12.00	.47	22.94	615.6	102.6	5.18	208.1	34.7	3.01
	84.5	12-1/8	12.04	.51	24.92	676.1	111.5	5.21	228.5	37.9	3.03
	91.5	12-1/4	12.08	.55	26.92	738.1	120.5	5.24	289.2	41.3	3.04
	98.5	12-3/8	12.12	.59	28.92	801.7	129.6	5.27	270.1	44.6	3.06
	105.0	12-1/2	12.16	.63	30.94	866.8	138.6	5.30	291.7	48.0	3.07
	112.0	12-5/8	12.30	.67	32.96	933.4	147.9	5.33	313.6	51.4	3.08
	118.5	12-3/4	12.23	.70	34.87	1000.0	156.9	5.36	335.0	54.8	3.10
	125.5	12-7/8	12.27	.74	36.91	1069.8	166.2	5.38	357.7	58.3	3.11
	132.5	13	12.31	.78	38.97	1141.3	175.6	5.41	380.7	61.9	3.13
	139.5	13-1/8	12.35	.82	41.03	1214.5	185.0	5.44	404.1	65.4	3.16
	146.5	13-1/4	12.39	.86	43.10	1289.4	194.6	5.47	428.0	69.1	3.15
	153.5	13-3/8	12.43	.90	45.19	1366.0	204.3	5.50	452.2	72.8	3.16
	161.0	13-1/2	12.47	.94	47.28	1444.3	214.0	5.53	477.0	76.5	3.18
H 14	83.5	13-3/4	13.92	.43	24.46	884.9	128.7	6.01	294.5	42.3	3.47
	91.0	13-7/8	13.96	.47	26.76	976.8	140.8	6.04	325.4	46.6	3.49
	99.0	14	14.00	.51	29.06	1070.6	153.0	6.07	356.9	51.0	3.50
	106.5	14-1/8	14.04	.55	31.38	1166.6	165.2	6.10	387.8	55.2	3.52
	114.5	14-1/4	14.08	.59	33.70	1264.5	177.5	6.13	420.3	59.7	3.53
	122.5	14-3/8	14.12	.63	36.04	1364.6	189.9	6.16	453.4	64.2	3.55
	130.5	14-1/2	14.16	.67	38.38	1466.7	202.3	6.18	486.9	68.8	3.56
	138.0	14-5/8	14.19	.70	40.59	1568.4	214.5	6.21	519.7	73.3	3.58
	146.0	14-3/4	14.23	.74	42.95	1674.7	227.1	6.24	554.4	77.9	3.59
	154.0	14-7/8	14.27	.78	45.33	1783.3	239.8	6.27	589.5	82.6	3.61
	162.0	15	14.31	.82	47.71	1894.0	252.5	6.30	626.1	87.5	3.62
	170.5	15-1/8	14.35	.86	50.11	2007.0	265.4	6.33	662.3	92.3	3.64
	178.5	15-1/4	14.39	.90	52.51	2122.3	278.3	6.36	699.0	97.2	3.65
	186.5	15-3/8	14.43	.94	54.92	2239.8	291.4	6.39	736.3	102.1	3.66
	195.0	15-1/2	14.47	.98	57.35	2359.7	304.5	6.41	774.2	107.6	3.68
	203.5	15-5/8	14.51	1.02	59.78	2481.9	317.7	6.44	812.6	112.0	3.69
	211.0	15-3/4	14.54	1.05	62.07	2603.3	330.6	6.48	849.8	116.9	3.70
	219.5	15-7/8	14.58	1.09	64.52	2730.2	344.0	6.51	889.3	122.0	3.71
	227.5	16	14.62	1.13	66.98	2859.6	357.5	6.53	929.4	127.1	3.72
	236.0	16-1/8	14.66	1.17	69.45	2991.5	371.0	6.56	970.0	132.3	3.74
	244.5	16-1/4	14.70	1.21	71.94	3125.8	384.7	6.59	1011.3	137.6	3.75
	253.0	16-3/8	14.74	1.25	74.93	3262.7	398.5	6.62	1053.2	142.9	3.76
	261.5	16-1/2	14.78	1.29	76.93	3402.1	412.4	6.65	1095.6	148.3	3.77
	270.0	16-5/8	14.82	1.33	79.44	3544.1	426.4	6.68	1138.7	153.7	3.79
	278.5	16-3/4	14.86	1.37	81.97	3688.8	440.5	6.71	1182.4	159.1	3.80
	287.5	16-7/8	14.90	1.41	84.50	3836.1	454.7	6.74	1226.7	164.7	3.81

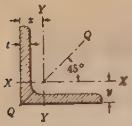


Properties of Standard Angles with Equal Legs

Dimensions are given in inch units

See paragraph on Unsymmetrical Loading of Beams, p. 570

Size	Thickness t	Lb. per ft.	Area of section A	Distance of center of gravity from back of flange x or y	Moment of inertia, axis XX or YY I	Radius of gyration, axis XX or XY r	Minimum radius of gyration, axis QQ r_{min}
8×8	1-1/8	56.9	16.73	2.41	98.0	2.42	1.55
8×8	1-1/16	54.0	15.87	2.39	93.5	2.43	1.56
8×8	1	51.0	15.00	2.37	89.0	2.44	1.56
8×8	15/16	48.1	14.12	2.34	84.3	2.44	1.56
8×8	7/8	45.0	13.23	2.32	79.6	2.45	1.56
8×8	13/16	42.0	12.34	2.30	74.7	2.46	1.57
8×8	3/4	38.9	11.44	2.28	69.7	2.47	1.57
8×8	11/16	35.8	10.53	2.25	64.6	2.48	1.58
8×8	5/8	32.7	9.61	2.23	59.4	2.49	1.58
8×8	9/16	29.6	8.68	2.21	54.1	2.50	1.58
8×8	1/2	26.4	7.75	2.19	48.6	2.51	1.58
6×6	1	37.4	11.00	1.86	35.5	1.80	1.16
6×6	15/16	35.3	10.37	1.84	33.7	1.80	1.16
6×6	7/8	33.1	9.73	1.82	31.9	1.81	1.17
6×6	13/16	31.0	9.09	1.80	30.1	1.82	1.17
6×6	3/4	28.7	8.44	1.78	28.2	1.83	1.17
6×6	11/16	26.5	7.78	1.75	26.2	1.83	1.17
6×6	5/8	24.2	7.11	1.73	24.2	1.84	1.17
6×6	9/16	21.9	6.43	1.71	22.1	1.85	1.18
6×6	1/2	19.6	5.75	1.68	19.9	1.86	1.18
6×6	7/16	17.2	5.06	1.66	17.7	1.87	1.19
6×6	3/8	14.9	4.36	1.64	15.4	1.88	1.19
5×5	1	30.6	9.00	1.61	19.6	1.48	0.96
5×5	15/16	28.9	8.50	1.59	18.7	1.48	0.96
5×5	7/8	27.2	7.98	1.57	17.8	1.49	0.96
5×5	13/16	25.4	7.47	1.55	16.8	1.50	0.97
5×5	3/4	23.6	6.94	1.52	15.7	1.50	0.97
5×5	11/16	21.8	6.40	1.50	14.7	1.51	0.97
5×5	5/8	20.0	5.86	1.48	13.6	1.52	0.97
5×5	9/16	18.1	5.31	1.46	12.4	1.53	0.98
5×5	1/2	16.2	4.75	1.43	11.3	1.54	0.98
5×5	7/16	14.3	4.18	1.41	10.0	1.55	0.98
5×5	3/8	12.3	3.61	1.39	8.7	1.56	0.99
4×4	13/16	19.9	5.84	1.29	8.1	1.18	0.77
4×4	3/4	18.5	5.44	1.27	7.7	1.19	0.77
4×4	11/16	17.1	5.03	1.25	7.2	1.19	0.77
4×4	5/8	15.7	4.61	1.23	6.7	1.20	0.77
4×4	9/16	14.3	4.18	1.21	6.1	1.21	0.78
4×4	1/2	12.8	3.75	1.18	5.6	1.22	0.78
4×4	7/16	11.3	3.31	1.16	5.0	1.23	0.78
4×4	3/8	9.8	2.86	1.14	4.4	1.23	0.79
4×4	5/16	8.2	2.40	1.12	3.7	1.24	0.79
4×4	1/4	6.6	1.94	1.09	3.0	1.25	0.79

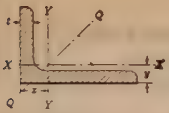


Properties of Standard Angles with Equal Legs—Cont.

Dimensions are given in inch units

See paragraph on Unsymmetrical Loading of Beams, p. 570

Size	Thickness t	Lb. per ft.	Area of section A	Distance of center of gravity from back of flange x or y	Moment of inertia, axis XX or YY I	Radius of gyration, axis XX or YY r	Minimum radius of gyration, axis QQ r_{min}
3-1/2×3-1/2	13/16	17.1	5.03	1.17	5.3	1.02	0.67
3-1/2×3-1/2	3/4	16.0	4.69	1.15	5.0	1.03	0.67
3-1/2×3-1/2	11/16	14.8	4.34	1.12	4.7	1.04	0.67
3-1/2×3-1/2	5/8	13.6	3.98	1.10	4.3	1.04	0.68
3-1/2×3-1/2	9/16	12.4	3.62	1.08	4.0	1.05	0.68
3-1/2×3-1/2	1/2	11.1	3.25	1.06	3.6	1.06	0.68
3-1/2×3-1/2	7/16	9.8	2.87	1.04	3.3	1.07	0.68
3-1/2×3-1/2	3/8	8.5	2.48	1.01	2.9	1.07	0.69
3-1/2×3-1/2	5/16	7.2	2.09	0.99	2.5	1.08	0.69
3-1/2×3-1/2	1/4	5.8	1.69	0.97	2.0	1.09	0.69
3×3	5/8	11.5	3.36	0.98	2.6	0.88	0.57
3×3	9/16	10.4	3.06	0.95	2.4	0.89	0.58
3×3	1/2	9.4	2.75	0.93	2.2	0.90	0.58
3×3	7/16	8.3	2.43	0.91	2.0	0.91	0.58
3×3	3/8	7.2	2.11	0.89	1.8	0.91	0.58
3×3	5/16	6.1	1.78	0.87	1.5	0.92	0.59
3×3	1/4	4.9	1.44	0.84	1.2	0.93	0.59
2-1/2×2-1/2	1/2	7.7	2.25	0.81	1.2	0.74	0.47
2-1/2×2-1/2	7/16	6.8	2.00	0.78	1.1	0.75	0.48
2-1/2×2-1/2	3/8	5.9	1.73	0.76	0.98	0.75	0.48
2-1/2×2-1/2	5/16	5.0	1.47	0.74	0.85	0.76	0.49
2-1/2×2-1/2	1/4	4.1	1.19	0.72	0.70	0.77	0.49
2-1/2×2-1/2	3/16	3.07	0.90	0.69	0.55	0.78	0.49
2-1/2×2-1/2	1/8	2.08	0.61	0.67	0.38	0.79	0.50
2×2	7/16	5.3	1.56	0.66	0.54	0.59	0.39
2×2	3/8	4.7	1.36	0.64	0.48	0.59	0.39
2×2	5/16	3.92	1.15	0.61	0.42	0.60	0.39
2×2	1/4	3.19	0.94	0.59	0.35	0.61	0.39
2×2	3/16	2.44	0.71	0.57	0.28	0.62	0.40
2×2	1/8	1.65	0.48	0.55	0.19	0.63	0.40
1-3/4×1-3/4	7/16	4.6	1.34	0.59	0.35	0.51	0.33
1-3/4×1-3/4	3/8	3.99	1.17	0.57	0.31	0.51	0.34
1-3/4×1-3/4	5/16	3.39	1.00	0.55	0.27	0.52	0.34
1-3/4×1-3/4	1/4	2.77	0.81	0.53	0.23	0.53	0.34
1-3/4×1-3/4	3/16	2.12	0.62	0.51	0.18	0.54	0.35
1-3/4×1-3/4	1/8	1.44	0.42	0.48	0.13	0.55	0.35
1-1/2×1-1/2	3/8	3.35	0.98	0.51	0.19	0.44	0.29
1-1/2×1-1/2	5/16	2.86	0.84	0.49	0.16	0.44	0.29
1-1/2×1-1/2	1/4	2.34	0.69	0.47	0.14	0.45	0.29
1-1/2×1-1/2	3/16	1.80	0.53	0.44	0.11	0.46	0.29
1-1/2×1-1/2	1/8	1.23	0.36	0.42	0.08	0.46	0.30
1-1/4×1-1/4	5/16	2.33	0.68	0.42	0.09	0.36	0.24
1-1/4×1-1/4	1/4	1.92	0.56	0.40	0.08	0.37	0.24
1-1/4×1-1/4	3/16	1.48	0.43	0.38	0.06	0.38	0.24
1-1/4×1-1/4	1/8	1.01	0.30	0.35	0.04	0.38	0.25
1×1	1/4	1.49	0.44	0.34	0.04	0.29	0.19
1×1	3/16	1.16	0.34	0.32	0.03	0.30	0.19
1×1	1/8	0.80	0.23	0.30	0.02	0.31	0.19



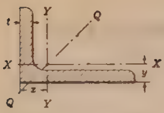
Properties of Angles with Unequal Legs

Dimensions are given in inch units

See paragraph on Unsymmetrical Loading of Beams, p. 570

Size	Thickness	Lb. per ft.	Area of section	Distance of center of gravity from backs of flanges		Moment of inertia		Radius of gyration		
						Axis XX	Axis YY	Axis XX	Axis YY	Min. Axis QQ
	t		A	y	x	I	I'	r	r'	r_{min}
*8×6	1	44.2	13.00	1.65	2.65	38.78	80.78	1.73	2.49	1.28
*8×6	15/16	41.7	12.25	1.63	2.63	36.85	76.59	1.73	2.50	1.28
*8×6	7/8	39.1	11.48	1.61	2.61	34.86	72.32	1.74	2.51	1.28
*8×6	13/16	36.5	10.72	1.59	2.59	32.82	67.92	1.75	2.52	1.28
*8×6	3/4	33.8	9.94	1.56	2.56	30.72	63.42	1.76	2.53	1.28
*8×6	11/16	31.2	9.15	1.54	2.54	28.56	58.82	1.77	2.54	1.29
*8×6	5/8	28.5	8.36	1.52	2.52	26.33	54.10	1.77	2.54	1.29
*8×6	9/16	25.7	7.56	1.50	2.50	24.04	49.26	1.78	2.55	1.30
*8×6	1/2	23.0	6.75	1.47	2.47	21.68	44.31	1.79	2.56	1.30
*7×3-1/2	1	32.3	9.50	0.96	2.71	7.53	45.37	0.89	2.19	0.74
*7×3-1/2	3/4	24.9	7.31	0.87	2.62	6.08	35.99	0.91	2.22	0.74
*7×3-1/2	1/2	17.0	5.00	0.78	2.53	4.41	25.41	0.94	2.25	0.75
6×4	1	30.6	9.00	1.17	2.17	10.75	30.75	1.09	1.85	0.85
6×4	15/16	28.9	8.50	1.14	2.14	10.26	29.26	1.10	1.86	0.85
6×4	7/8	27.2	7.99	1.12	2.12	9.75	27.73	1.11	1.86	0.86
6×4	13/16	25.4	7.47	1.10	2.10	9.23	26.15	1.11	1.87	0.86
6×4	3/4	23.6	6.94	1.08	2.08	8.68	24.51	1.12	1.88	0.86
6×4	11/16	21.8	6.41	1.06	2.06	8.11	22.82	1.13	1.89	0.86
6×4	5/8	20.0	5.86	1.03	2.03	7.52	21.07	1.13	1.90	0.86
6×4	9/16	18.1	5.31	1.01	2.01	6.91	19.26	1.14	1.90	0.87
6×4	1/2	16.2	4.75	0.99	1.99	6.27	17.40	1.15	1.91	0.87
6×4	7/16	14.3	4.18	0.96	1.96	5.60	15.46	1.16	1.92	0.87
6×4	3/8	12.3	3.61	0.94	1.94	4.90	13.47	1.17	1.93	0.88
6×3-1/2	1	28.9	8.50	1.01	2.26	7.21	29.24	0.92	1.85	0.74
6×3-1/2	15/16	27.3	8.03	0.99	2.24	6.88	27.84	0.93	1.86	0.74
6×3-1/2	7/8	25.7	7.55	0.97	2.22	6.55	26.38	0.93	1.87	0.75
6×3-1/2	13/16	24.0	7.06	0.95	2.20	6.20	24.89	0.94	1.88	0.75
6×3-1/2	3/4	22.4	6.56	0.93	2.18	5.84	23.34	0.94	1.89	0.75
6×3-1/2	11/16	20.6	6.06	0.90	2.15	5.47	21.74	0.95	1.89	0.75
6×3-1/2	5/8	18.9	5.55	0.88	2.13	5.08	20.08	0.96	1.90	0.75
6×3-1/2	9/16	17.1	5.03	0.86	2.11	4.67	18.37	0.96	1.91	0.75
6×3-1/2	1/2	15.3	4.50	0.83	2.08	4.25	16.59	0.97	1.92	0.76
6×3-1/2	7/16	13.5	3.97	0.81	2.06	3.81	14.76	0.98	1.93	0.76
6×3-1/2	3/8	11.7	3.42	0.79	2.04	3.34	12.86	0.99	1.94	0.77
5×3-1/2	7/8	22.7	6.67	1.04	1.79	6.21	15.67	0.96	1.53	0.75
5×3-1/2	13/16	21.3	6.25	1.02	1.77	5.89	14.81	0.97	1.54	0.75
5×3-1/2	3/4	19.8	5.81	1.00	1.75	5.55	13.92	0.98	1.55	0.75
5×3-1/2	11/16	18.3	5.37	0.97	1.72	5.20	12.99	0.98	1.56	0.75
5×3-1/2	5/8	16.8	4.92	0.95	1.70	4.83	12.03	0.99	1.56	0.75
5×3-1/2	9/16	15.2	4.47	0.93	1.68	4.45	11.03	1.00	1.57	0.75
5×3-1/2	1/2	13.6	4.00	0.91	1.66	4.05	9.99	1.01	1.58	0.75
5×3-1/2	7/16	12.0	3.53	0.88	1.63	3.63	8.90	1.01	1.59	0.76
5×3-1/2	3/8	10.4	3.05	0.86	1.61	3.18	7.78	1.02	1.60	0.76
5×3-1/2	5/16	8.7	2.56	0.84	1.59	2.72	6.60	1.03	1.61	0.76
5×3	13/16	19.9	5.84	0.86	1.86	3.71	13.98	0.80	1.55	0.64
5×3	3/4	18.5	5.44	0.84	1.84	3.51	13.15	0.80	1.55	0.64
5×3	11/16	17.1	5.03	0.82	1.82	3.29	12.28	0.81	1.56	0.64
5×3	5/8	15.7	4.61	0.80	1.80	3.06	11.37	0.82	1.57	0.64
5×3	9/16	14.3	4.18	0.77	1.77	2.83	10.43	0.82	1.58	0.65

* Angles marked with an * are listed by some steel makers as special angles, and command a slightly higher price per pound than standard angles, and, in small lots, are not always available to the purchaser.



Properties of Angles with Unequal Legs—Cont.

Dimensions are given in inch units

See paragraph on Unsymmetrical Loading of Beams, p. 570

Size	Thickness	Lb. per ft.	Area of section	Distance of center of gravity from backs of flanges		Moment of inertia		Radius of gyration		
						Axis XX	Axis YY	Axis XX	Axis YY	Min. Axis QQ
	t		A	y	x	I	I′	r	r′	r_{min}
5×3	1/2	12.8	3.75	0.75	1.75	2.58	9.45	0.83	1.59	0.65
5×3	7/16	11.3	3.31	0.73	1.73	2.32	8.43	0.84	1.60	0.65
5×3	3/8	9.8	2.86	0.70	1.70	2.04	7.37	0.84	1.61	0.65
5×3	5/16	8.2	2.40	0.68	1.68	1.75	6.26	0.85	1.61	0.66
*4×3-1/2	13/16	18.5	5.43	1.11	1.36	5.49	7.77	1.01	1.19	0.72
*4×3-1/2	3/4	17.3	5.06	1.09	1.34	5.18	7.32	1.01	1.20	0.72
*4×3-1/2	11/16	16.0	4.68	1.07	1.32	4.86	6.86	1.02	1.21	0.72
*4×3-1/2	5/8	14.7	4.30	1.04	1.29	4.52	6.37	1.03	1.22	0.72
*4×3-1/2	9/16	13.3	3.90	1.02	1.27	4.17	5.86	1.03	1.23	0.72
*4×3-1/2	1/2	11.9	3.50	1.00	1.25	3.79	5.32	1.04	1.23	0.72
*4×3-1/2	7/16	10.6	3.09	0.98	1.23	3.40	4.76	1.05	1.24	0.72
*4×3-1/2	3/8	9.1	2.67	0.96	1.21	2.99	4.18	1.06	1.25	0.73
*4×3-1/2	5/16	7.7	2.25	0.93	1.18	2.59	3.56	1.07	1.26	0.73
4×3	3/4	16.0	4.69	0.92	1.42	3.28	6.93	0.84	1.22	0.64
4×3	11/16	14.8	4.34	0.89	1.39	3.08	6.49	0.84	1.22	0.64
4×3	5/8	13.6	3.98	0.87	1.37	2.87	6.03	0.85	1.23	0.64
4×3	9/16	12.4	3.62	0.85	1.35	2.66	5.55	0.86	1.24	0.64
4×3	1/2	11.1	3.25	0.83	1.33	2.42	5.05	0.86	1.25	0.64
4×3	7/16	9.8	2.87	0.80	1.30	2.18	4.52	0.87	1.25	0.64
4×3	3/8	8.5	2.48	0.78	1.28	1.92	3.96	0.88	1.26	0.64
4×3	5/16	7.2	2.09	0.76	1.26	1.65	3.38	0.89	1.27	0.65
3-1/2×3	3/4	14.7	4.31	0.96	1.21	3.15	4.70	0.85	1.04	0.62
3-1/2×3	11/16	13.6	4.00	0.94	1.19	2.96	4.41	0.86	1.05	0.62
3-1/2×3	5/8	12.5	3.67	0.92	1.17	2.76	4.11	0.87	1.06	0.62
3-1/2×3	9/16	11.4	3.34	0.90	1.15	2.55	3.79	0.87	1.07	0.62
3-1/2×3	1/2	10.2	3.00	0.88	1.13	2.33	3.45	0.88	1.07	0.62
3-1/2×3	7/16	9.1	2.65	0.85	1.10	2.09	3.10	0.89	1.08	0.62
3-1/2×3	3/8	7.9	2.30	0.83	1.08	1.85	2.72	0.90	1.09	0.62
3-1/2×3	5/16	6.6	1.93	0.81	1.06	1.58	2.33	0.90	1.10	0.63
3-1/2×2-1/2	11/16	12.5	3.65	0.77	1.27	1.72	4.13	0.69	1.06	0.53
3-1/2×2-1/2	5/8	11.5	3.36	0.75	1.25	1.61	3.85	0.69	1.07	0.53
3-1/2×2-1/2	9/16	10.4	3.06	0.73	1.23	1.49	3.55	0.70	1.08	0.53
3-1/2×2-1/2	1/2	9.4	2.75	0.70	1.20	1.36	3.24	0.70	1.09	0.53
3-1/2×2-1/2	7/16	8.3	2.43	0.68	1.18	1.23	2.91	0.71	1.09	0.54
3-1/2×2-1/2	3/8	7.2	2.11	0.66	1.16	1.09	2.56	0.72	1.10	0.54
3-1/2×2-1/2	5/16	6.1	1.78	0.64	1.14	0.94	2.19	0.73	1.11	0.54
3-1/2×2-1/2	1/4	4.9	1.44	0.61	1.11	0.78	1.80	0.74	1.12	0.54
3×2-1/2	9/16	9.5	2.78	0.77	1.02	1.42	2.28	0.72	0.91	0.52
3×2-1/2	1/2	8.5	2.50	0.75	1.00	1.30	2.08	0.72	0.91	0.52
3×2-1/2	7/16	7.6	2.22	0.73	0.98	1.18	1.88	0.73	0.92	0.52
3×2-1/2	3/8	6.6	1.92	0.71	0.96	1.04	1.66	0.74	0.93	0.52
3×2-1/2	5/16	5.6	1.62	0.68	0.93	0.90	1.42	0.74	0.94	0.53
3×2-1/2	1/4	4.5	1.31	0.66	0.91	0.74	1.17	0.75	0.95	0.53
2-1/2×2	1/2	6.8	2.00	0.63	0.88	0.64	1.14	0.56	0.75	0.42
2-1/2×2	7/16	6.1	1.78	0.60	0.85	0.58	1.03	0.57	0.76	0.42
2-1/2×2	3/8	5.3	1.55	0.58	0.83	0.51	0.91	0.58	0.77	0.42
2-1/2×2	5/16	4.5	1.31	0.56	0.81	0.45	0.79	0.58	0.78	0.42
2-1/2×2	1/4	3.7	1.06	0.54	0.79	0.37	0.65	0.59	0.78	0.42
2-1/2×2	3/16	2.8	0.81	0.51	0.76	0.29	0.51	0.60	0.79	0.43

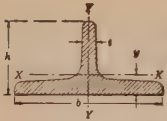
* Angles marked with an * are listed by some steel makers as special angles, and command a slightly higher price per pound than standard angles, and, in small lots, are not always available to the purchaser.

Properties of Standard Channels

Dimensions are given in inch units

Depth of channel	Lb. per ft.	Area of section	Width of flange	Thickness of web	Distance of center of gravity from back of web	Axis XX			Axis YY		Shearing stress factor *
						Moment of inertia	Radius of gyration	Section Modulus	Moment of inertia	Radius of gyration	
h		A	b	t	x	I	r	I/c	I′	r′	
15	55.0	16.11	3.814	0.814	0.82	429.0	5.16	57.2	12.1	0.87	0.106
	50.0	14.64	3.716	0.716	0.80	401.4	5.24	53.6	11.2	0.87	0.119
	45.0	13.17	3.618	0.618	0.79	373.9	5.33	49.8	10.3	0.88	0.136
	40.0	11.70	3.520	0.520	0.78	346.3	5.44	46.2	9.3	0.89	0.159
	35.0	10.23	3.422	0.422	0.79	318.7	5.58	42.5	8.4	0.91	0.192
	33.9	9.90	3.400	0.400	0.79	312.6	5.62	41.7	8.2	0.91	0.201
12	40.0	11.73	3.415	0.755	0.72	196.5	4.09	32.8	6.6	0.75	0.143
	35.0	10.26	3.292	0.632	0.69	178.8	4.18	29.8	5.9	0.76	0.169
	30.0	8.79	3.170	0.510	0.68	161.2	4.28	26.9	5.2	0.77	0.205
	25.0	7.32	3.047	0.387	0.68	143.5	4.43	23.9	4.5	0.79	0.264
	20.7	6.03	2.940	0.280	0.70	128.1	4.61	21.4	3.9	0.81	0.353
10	35.0	10.27	3.180	0.820	0.70	115.2	3.34	23.0	4.6	0.67	0.160
	30.0	8.80	3.033	0.673	0.65	103.0	3.42	20.6	4.0	0.67	0.191
	25.0	7.33	2.886	0.526	0.62	90.7	3.52	18.1	3.4	0.68	0.238
	20.0	5.86	2.739	0.379	0.61	78.5	3.66	15.7	2.8	0.70	0.323
	15.3	4.47	2.600	0.240	0.64	66.9	3.87	13.4	2.3	0.72	0.487
9	25.0	7.33	2.812	0.612	0.62	70.5	3.10	15.7	3.0	0.64	0.234
	20.0	5.86	2.648	0.448	0.59	60.6	3.22	13.5	2.4	0.65	0.311
	15.0	4.39	2.485	0.285	0.59	50.7	3.40	11.3	1.9	0.67	0.470
	13.4	3.89	2.430	0.230	0.61	47.3	3.49	10.5	1.8	0.67	0.573
8	21.25	6.23	2.619	0.579	0.59	47.6	2.77	11.9	2.2	0.60	0.278
	18.75	5.49	2.527	0.487	0.57	43.7	2.82	10.9	2.0	0.60	0.326
	16.25	4.76	2.435	0.395	0.56	39.8	2.89	9.9	1.8	0.61	0.394
	13.75	4.02	2.343	0.303	0.56	35.8	2.99	9.0	1.5	0.62	0.502
	11.5	3.36	2.260	0.220	0.58	32.3	3.10	8.1	1.3	0.63	0.675
7	19.75	5.79	2.509	0.629	0.58	33.1	2.39	9.4	1.8	0.56	0.295
	17.25	5.05	2.404	0.524	0.56	30.1	2.44	8.6	1.6	0.56	0.349
	14.75	4.32	2.299	0.419	0.54	27.1	2.51	7.7	1.4	0.57	0.427
	12.25	3.58	2.194	0.314	0.53	24.1	2.59	6.9	1.2	0.58	0.557
	9.8	2.85	2.090	0.210	0.55	21.1	2.72	6.0	0.98	0.59	0.805
6	15.5	4.54	2.279	0.559	0.55	19.5	2.07	6.5	1.3	0.53	0.384
	13.0	3.81	2.157	0.437	0.52	17.3	2.13	5.8	1.1	0.53	0.480
	10.5	3.07	2.034	0.314	0.50	15.1	2.22	5.0	0.87	0.53	0.648
	8.2	2.39	1.920	0.200	0.52	13.0	2.34	4.3	0.70	0.54	0.985
5	11.5	3.36	2.032	0.472	0.51	10.4	1.76	4.1	0.82	0.49	0.537
	9.0	2.63	1.885	0.325	0.48	8.8	1.83	3.5	0.64	0.49	0.763
	6.7	1.95	1.750	0.190	0.49	7.4	1.95	3.0	0.48	0.50	1.245
4	7.25	2.12	1.720	0.320	0.46	4.5	1.47	2.3	0.44	0.46	0.976
	6.25	1.82	1.647	0.247	0.46	4.1	1.50	2.1	0.38	0.45	1.240
	5.4	1.56	1.580	0.180	0.46	3.8	1.56	1.9	0.32	0.45	1.649
3	6.0	1.75	1.596	0.356	0.46	2.1	1.08	1.4	0.31	0.42	1.156
	5.0	1.46	1.498	0.258	0.44	1.8	1.12	1.2	0.25	0.41	1.620
	4.1	1.19	1.410	0.170	0.44	1.6	1.17	1.1	0.20	0.41	2.405

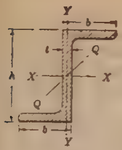
* Shearing stress factor multiplied by shear at a section (in pounds) gives the maximum shearing unit stress in the web of the beam (in pounds per square inch).



Properties of Standard T Shapes

Dimensions are given in inch units

Size flange by stem b × h	Lb. per ft.	Area of section A	Distance center of gravity from back of flange y	Axis XX			Axis YY		
				Moment of inertia I	Section modulus I/c	Radius of gyration r	Moment of inertia I′	Section modulus I′/c′	Radius of gyration r′
5×3	13.6	3.99	0.75	2.6	1.18	0.82	5.6	2.22	1.19
5×2-1/2	11.0	3.24	0.65	1.6	0.86	0.71	4.3	1.70	1.16
4-1/2×3-1/2	15.9	4.65	1.11	5.1	2.13	1.04	3.7	1.65	0.90
4-1/2×3	10.0	3.00	0.75	2.1	0.94	0.86	3.1	1.38	1.04
4-1/2×2-1/2	9.3	2.79	0.60	1.2	0.65	0.68	3.1	1.38	1.08
4×5	12.3	3.54	1.51	8.5	2.43	1.56	2.1	1.06	0.78
4×4-1/2	11.6	3.36	1.31	6.3	1.98	1.38	2.1	1.06	0.80
4×4	10.9	3.21	1.15	4.7	1.64	1.23	2.2	1.09	0.84
4×3	9.3	2.73	0.78	2.0	0.88	0.86	2.1	1.05	0.88
4×2-1/2	7.4	2.16	0.60	1.0	0.55	0.70	1.8	0.88	0.91
4×2	7.9	2.31	0.48	0.6	0.40	0.52	2.1	1.05	0.96
4×2	6.7	1.95	0.51	0.54	0.34	0.51	1.8	0.88	0.95
3-1/2×4	12.8	3.75	1.25	5.5	1.98	1.21	1.89	1.08	0.72
3-1/2×4	10.0	2.91	1.19	4.3	1.55	1.22	1.42	0.81	0.70
3-1/2×3-1/2	11.9	3.45	1.06	3.7	1.52	1.04	1.89	1.08	0.74
3-1/2×3-1/2	9.3	2.70	1.01	3.0	1.19	1.05	1.42	0.81	0.73
3-1/2×3	11.0	3.21	0.88	2.4	1.13	0.87	1.88	1.08	0.77
3-1/2×3	8.7	2.49	0.83	1.9	0.88	0.88	1.41	0.81	0.75
3-1/2×3	7.7	2.28	0.78	1.6	0.72	0.89	1.18	0.68	0.76
3×4	11.9	3.48	1.32	5.2	1.94	1.23	1.21	0.81	0.59
3×4	10.6	3.12	1.32	4.8	1.78	1.25	1.09	0.72	0.60
3×4	9.3	2.73	1.29	4.3	1.57	1.26	0.93	0.62	0.59
3×3-1/2	11.0	3.21	1.12	3.5	1.49	1.06	1.20	0.80	0.62
3×3-1/2	9.8	2.88	1.11	3.3	1.37	1.08	1.31	0.88	0.68
3×3-1/2	8.6	2.49	1.09	2.9	1.21	1.09	0.93	0.62	0.61
3×3	10.1	2.94	0.93	2.3	1.10	0.88	1.20	0.80	0.64
3×3	9.0	2.67	0.92	2.1	1.01	0.90	1.08	0.72	0.64
3×3	7.9	2.28	0.88	1.8	0.86	0.90	0.90	0.60	0.63
3×3	6.8	1.95	0.86	1.6	0.74	0.90	0.75	0.50	0.62
3×2-1/2	6.2	1.80	0.68	0.94	0.52	0.73	0.75	0.50	0.65
2-3/4×2	7.4	2.16	0.53	1.1	0.75	0.71	0.62	0.45	0.54
2-1/2×3	6.2	1.80	0.92	1.6	0.76	0.94	0.44	0.35	0.51
2-1/2×2-3/4	5.9	1.71	0.83	1.2	0.60	0.83	0.44	0.35	0.51
2-1/2×2-1/2	5.6	1.62	0.74	0.87	0.50	0.74	0.44	0.35	0.52
2-1/2×1-1/4	3.0	0.84	0.29	0.094	0.09	0.31	0.29	0.23	0.58
2-1/4×2-1/4	4.2	1.20	0.66	0.51	0.32	0.67	0.25	0.22	0.47
2×2	3.7	1.08	0.59	0.36	0.25	0.60	0.18	0.18	0.42
2×1-1/2	3.2	0.90	0.42	0.16	0.15	0.42	0.18	0.18	0.45
1-3/4×1-3/4	3.2	0.90	0.54	0.23	0.19	0.51	0.12	0.14	0.37
1-1/2×1-1/2	2.0	0.54	0.44	0.11	0.11	0.45	0.06	0.07	0.31
1-1/4×1-1/4	1.7	0.45	0.38	0.06	0.07	0.37	0.03	0.05	0.26



Properties of Standard Z Bars

Dimensions are given in inches

See paragraph on Unsymmetrical Loading of Beams, p. 570

Depth of bar h	Width of flange b	Thickness of web t	Lb. per ft.	Area of section A	Moment of inertia Axis XX I	Moment of inertia Axis YY I′	Radius of gyration Axis XX r	Radius of gyration Axis YY r′	Radius of gyration (Minimum) axis QQ r_{min}
6	3-1/2	3/8	15.6	4.59	25.32	9.11	2.35	1.41	0.83
6-1/16	3-9/16	7/16	18.3	5.39	29.80	10.95	2.35	1.43	0.84
6-1/8	3-5/8	1/2	21.0	6.19	34.36	12.87	2.36	1.44	0.84
6	3-1/2	9/16	22.7	6.68	34.64	12.59	2.28	1.37	0.81
6-1/16	3-9/16	5/8	25.4	7.46	38.86	14.42	2.28	1.39	0.82
6-1/8	3-5/8	11/16	28.0	8.25	43.18	16.34	2.29	1.41	0.84
6	3-1/2	3/4	29.3	8.63	42.12	15.44	2.21	1.34	0.81
6-1/16	3-9/16	13/16	31.9	9.40	46.13	17.27	2.22	1.36	0.82
6-1/8	3-5/8	7/8	34.6	10.17	50.22	19.18	2.22	1.37	0.83
5	3-1/4	5/16	11.6	3.40	13.36	6.18	1.98	1.35	0.75
5-1/16	3-5/16	3/8	13.9	4.10	16.18	7.65	1.99	1.37	0.76
5-1/8	3-3/8	7/16	16.4	4.81	19.07	9.20	1.99	1.38	0.77
5	3-1/4	1/2	17.9	5.25	19.19	9.05	1.91	1.31	0.74
5-1/16	3-5/16	9/16	20.2	5.94	21.83	10.51	1.91	1.33	0.75
5-1/8	3-3/8	5/8	22.6	6.64	24.53	12.06	1.92	1.35	0.76
5	3-1/4	11/16	23.7	6.96	23.68	11.37	1.84	1.28	0.73
5-1/16	3-5/16	3/4	26.0	7.64	26.16	12.83	1.85	1.30	0.75
5-1/8	3-3/8	13/16	28.3	8.33	28.70	14.36	1.86	1.31	0.76
4	3-1/16	1/4	8.2	2.41	6.28	4.23	1.62	1.33	0.67
4-1/16	3-1/8	5/16	10.3	3.03	7.94	5.46	1.62	1.34	0.68
4-1/8	3-3/16	3/8	12.4	3.66	9.63	6.77	1.62	1.36	0.69
4	3-1/16	7/16	13.8	4.05	9.66	6.73	1.55	1.29	0.66
4-1/16	3-1/8	1/2	15.8	4.66	11.18	7.96	1.55	1.31	0.67
4-1/8	3-3/16	9/16	17.9	5.27	12.74	9.26	1.55	1.33	0.69
4	3-1/16	5/8	18.9	5.55	12.11	8.73	1.48	1.25	0.66
4-1/16	3-1/8	11/16	20.9	6.14	13.52	9.95	1.48	1.27	0.67
4-1/8	3-3/16	3/4	23.0	6.75	14.97	11.24	1.49	1.29	0.69
3	2-11/16	1/4	6.7	1.97	2.87	2.81	1.21	1.19	0.55
3-1/16	2-3/4	5/16	8.4	2.48	3.64	3.64	1.21	1.21	0.56
3	2-11/16	3/8	9.7	2.86	3.85	3.92	1.16	1.17	0.55
3-1/16	2-3/4	7/16	11.4	3.36	4.57	4.75	1.17	1.19	0.56
3	2-11/16	1/2	12.5	3.69	4.59	4.85	1.12	1.15	0.55
3-1/16	2-3/4	9/16	14.2	14.2	5.26	5.70	1.12	1.17	0.56

48. Bars and Plates

Commercial Sizes of Steel Bars commonly kept in stock in warehouses are as follows (values in inches): **Rounds,** 3/16 to 1-1/8 advancing by 16ths; 1/14 to 4-3/4 advancing by 8ths; 5 to 8 advancing by 4ths. **Squares,** 1/4 to 5/8 advancing by 16ths; 3/4 to 2 advancing by 8ths; 2-1/4 to 3-1/2 advancing by 4ths; 4 to 5 advancing by halves. **Flats,** thickness, 3/16 to 1/2 advancing

Square and Round Steel Bars

Side or diameter, in.	Pounds per linear foot		Area in square inches		Circumference of round bar, in.	Side or diameter, in.
	Square	Round	Square	Round		
1/16	.013	.010	.0039	.0031	.1963	1/16
1/8	.053	.042	.0156	.0123	.3927	1/8
3/16	.119	.094	.0352	.0276	.5890	3/16
1/4	.212	.167	.0625	.0491	.7854	1/4
5/16	.333	.261	.0977	.0767	.9817	5/16
3/8	.478	.375	.1406	.1104	1.1781	3/8
7/16	.651	.511	.1914	.1503	1.3744	7/16
1/2	.850	.667	.2500	.1963	1.5708	1/2
9/16	1.076	.845	.3164	.2485	1.7671	9/16
5/8	1.328	1.043	.3906	.3068	1.9635	5/8
11/16	1.608	1.262	.4727	.3712	2.1593	11/16
3/4	1.913	1.502	.5625	.4418	2.3562	3/4
13/16	2.245	1.763	.6602	.5185	2.5525	13/16
7/8	2.603	2.044	.7656	.6013	2.7489	7/8
15/16	2.989	2.347	.8789	.6903	2.9452	15/16
1	3.400	2.670	1.0000	.7854	3.1416	1
1-1/16	3.833	3.014	1.1289	.8866	3.3379	1-1/16
1-1/8	4.303	3.379	1.2656	.9940	3.5343	1-1/8
1-3/16	4.795	3.766	1.4102	1.1075	3.7306	1-3/16
1-1/4	5.312	4.173	1.5625	1.2272	3.9270	1-1/4
1-5/16	5.857	4.600	1.7227	1.3530	4.1233	1-5/16
1-3/8	6.428	5.049	1.8906	1.4849	4.3197	1-3/8
1-7/16	7.026	5.518	2.0664	1.6230	4.5160	1-7/16
1-1/2	7.650	6.008	2.2500	1.7671	4.7124	1-1/2
1-9/16	8.301	6.520	2.4414	1.9175	4.9087	1-9/16
1-5/8	8.978	7.051	2.6406	2.0739	5.1051	1-5/8
1-11/16	9.682	7.604	2.8477	2.2365	5.3014	1-11/16
1-3/4	10.41	8.178	3.0625	2.4053	5.4978	1-3/4
1-13/16	11.17	8.773	3.2852	2.5802	5.6941	1-13/16
1-7/8	11.95	9.388	3.5156	2.7612	5.8905	1-7/8
1-15/16	12.76	10.02	3.7539	2.9483	6.0868	1-15/16
2	13.60	10.68	4.0000	3.1416	6.2832	2
2-1/8	15.35	12.06	4.5156	3.5466	6.6759	2-1/8
2-1/4	17.22	13.52	5.0625	3.9761	7.0686	2-1/4
2-3/8	19.18	15.07	5.6406	4.4301	7.4613	2-3/8
2-1/2	21.25	16.69	6.2500	4.9087	7.8540	2-1/2
2-5/8	23.43	18.40	6.8906	5.4119	8.2467	2-5/8
2-3/4	25.71	20.20	7.5625	5.9396	8.6394	2-3/4
2-7/8	28.10	22.07	8.2656	6.7771	9.2284	2-7/8
3	30.60	24.03	9.0000	7.0686	9.4248	3
3-1/4	35.92	28.20	10.563	8.2958	10.210	3-1/4
3-1/2	41.65	32.71	12.250	9.6211	10.996	3-1/2
3-3/4	47.82	37.56	14.063	11.045	11.781	3-3/4
4	54.40	42.73	16.000	12.566	12.566	4

by 16ths; 5/8 to 1-1/4 advancing by 8ths; 1-1/2 to 2 advancing by 4ths. 2-1/2 and 3; width, 3/8 to 6 advancing by 8ths. **Hexagons,** 1/2 to 9/16 advancing by 16ths; 5/8 to 2 advancing by 8ths.

Weight of Rolled Steel Plates in Pounds per Linear Foot

Width in inches	Thickness of plate in inches									
	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
1	0.638	0.850	1.06	1.28	1.49	1.70	1.92	2.12	2.34	2.55
1-1/4	0.797	1.06	1.33	1.59	1.86	2.12	2.39	2.65	2.92	3.19
1-1/2	0.957	1.28	1.59	1.92	2.23	2.55	2.87	3.19	3.51	3.83
1-3/4	1.11	1.49	1.86	2.23	2.60	2.98	3.35	3.72	4.09	4.47
2	1.28	1.70	2.12	2.55	2.98	3.40	3.83	4.25	4.67	5.10
2-1/4	1.44	1.91	2.39	2.87	3.35	3.83	4.30	4.78	5.26	5.75
2-1/2	1.59	2.12	2.65	3.19	3.72	4.25	4.78	5.31	5.84	6.38
2-3/4	1.75	2.34	2.92	3.51	4.09	4.67	5.26	5.84	6.43	7.02
3	1.91	2.55	3.19	3.83	4.46	5.10	5.74	6.38	7.02	7.65
3-1/4	2.07	2.76	3.45	4.15	4.83	5.53	6.22	6.91	7.60	8.29
3-1/2	2.23	2.98	3.72	4.47	5.20	5.95	6.70	7.44	8.18	8.93
3-3/4	2.39	3.19	3.99	4.78	5.58	6.38	7.17	7.97	8.76	9.57
4	2.55	3.40	4.25	5.10	5.95	6.80	7.65	8.50	9.35	10.20
4-1/4	2.71	3.61	4.52	5.42	6.32	7.22	8.13	9.03	9.93	10.84
4-1/2	2.87	3.83	4.78	5.74	6.70	7.65	8.61	9.57	10.52	11.48
4-3/4	3.03	4.04	5.05	6.06	7.07	8.08	9.09	10.10	11.11	12.12
5	3.19	4.25	5.31	6.38	7.44	8.50	9.57	10.63	11.69	12.75
5-1/4	3.35	4.46	5.58	6.69	7.81	8.93	10.04	11.16	12.27	13.39
5-1/2	3.51	4.67	5.84	7.02	8.18	9.35	10.52	11.69	12.85	14.03
5-3/4	3.67	4.89	6.11	7.34	8.36	9.77	11.00	12.22	13.44	14.67
6	3.83	5.10	6.38	7.65	8.93	10.20	11.48	12.75	14.03	15.30
6-1/4	3.99	5.31	6.64	7.97	9.29	10.63	11.95	13.28	14.61	15.94
6-1/2	4.14	5.53	6.90	8.29	9.67	11.05	12.43	13.81	15.20	16.58
6-3/4	4.30	5.74	7.17	8.61	10.04	11.48	12.91	14.34	15.78	17.22
7	4.46	5.95	7.44	8.93	10.41	11.90	13.39	14.87	16.36	17.85
7-1/4	4.62	6.16	7.70	9.25	10.78	12.32	13.86	15.40	16.94	18.49
7-1/2	4.78	6.36	7.97	9.57	11.16	12.75	14.34	15.94	17.53	19.13
7-3/4	4.94	6.58	8.23	9.88	11.53	13.18	14.82	16.47	18.12	19.77
8	5.10	6.80	8.50	10.20	11.90	13.60	15.30	17.00	18.70	20.40
8-1/4	5.26	7.01	8.76	10.52	12.27	14.03	15.78	17.53	19.28	21.04
8-1/2	5.42	7.22	9.03	10.84	12.64	14.44	16.26	18.06	19.86	21.68
8-3/4	5.58	7.43	9.29	11.16	13.02	14.87	16.74	18.59	20.45	22.32
9	5.74	7.65	9.56	11.48	13.40	15.30	17.22	19.13	21.04	22.94
9-1/4	5.90	7.86	9.83	11.80	13.76	15.73	17.69	19.65	21.62	23.59
9-1/2	6.06	8.08	10.10	12.12	14.14	16.16	18.18	20.19	22.21	24.23
9-3/4	6.22	8.29	10.36	12.44	14.51	16.58	18.65	20.72	22.79	24.86
10	6.38	8.50	10.62	12.75	14.88	17.00	19.14	21.25	23.38	25.50
10-1/4	6.54	8.71	10.89	13.07	15.25	17.42	19.61	21.78	23.96	26.14
10-1/2	6.70	8.92	11.16	13.39	15.62	17.85	20.08	22.32	24.54	26.78
10-3/4	6.86	9.14	11.42	13.71	15.99	18.28	20.56	22.85	25.13	27.42
11	7.02	9.34	11.68	14.03	16.36	18.70	21.02	23.38	25.70	28.05
11-1/4	7.17	9.57	11.95	14.35	16.74	19.13	21.51	23.91	26.30	28.68
11-1/2	7.32	9.78	12.22	14.68	17.12	19.55	22.00	24.44	26.88	29.33
11-3/4	7.49	10.00	12.49	14.99	17.49	19.97	22.48	24.97	27.47	29.97
12	7.65	10.20	12.75	15.30	17.85	20.40	22.95	25.50	28.05	30.60
12-1/4	7.82	10.42	13.01	15.62	18.23	20.82	23.43	26.03	28.64	31.25
12-1/2	7.98	10.63	13.28	15.94	18.60	21.25	23.96	26.56	29.22	31.88
12-3/4	8.13	10.84	13.55	16.26	18.97	21.67	24.39	27.09	29.80	32.52

Approximate rules for computing weights of bars, plates, and prisms: A wrought-iron bar one yard long and one square inch in section weighs 10 lb. Steel is about 2% heavier than wrought iron. Cast iron is about 6% lighter than wrought iron.

Weight of Rolled Steel Plates in Pounds per Linear Foot.—Continued

Width in inches	Thickness of plate in inches									
	13/16	7/8	15/16	1	1-1/16	1/1-8	1-3/16	1-1/4	1-5/16	1-3/8
1	2.76	2.98	3.19	3.40	3.61	3.83	4.04	4.25	4.46	4.67
1-1/4	3.45	3.72	3.99	4.25	4.52	4.78	5.05	5.31	5.58	5.84
1-1/2	4.14	4.47	4.78	5.10	5.42	5.74	6.06	6.38	6.69	7.02
1-3/4	4.84	5.20	5.58	5.95	6.32	6.70	7.07	7.44	7.81	8.18
2	5.53	5.95	6.38	6.80	7.22	7.65	8.08	8.50	8.93	9.35
2-1/4	6.21	6.69	7.18	7.65	8.13	8.61	9.09	9.57	10.04	10.52
2-1/2	6.90	7.44	7.97	8.50	9.03	9.57	10.10	10.63	11.16	11.69
2-3/4	7.60	8.18	8.77	9.35	9.93	10.52	11.11	11.69	12.27	12.85
3	8.29	8.93	9.57	10.20	10.84	11.48	12.12	12.75	13.39	14.03
3-1/4	8.98	9.67	10.36	11.05	11.74	12.43	13.12	13.81	14.50	15.20
3-1/2	9.67	10.41	11.16	11.90	12.65	13.39	14.13	14.87	15.62	16.36
3-3/4	10.36	11.16	11.95	12.75	13.55	14.34	15.14	15.94	16.74	17.53
4	11.05	11.90	12.75	13.60	14.45	15.30	16.15	17.00	17.85	18.70
4-1/4	11.74	12.65	13.55	14.45	15.35	16.26	17.16	18.06	18.96	19.87
4-1/2	12.43	13.39	14.34	15.30	16.26	17.22	18.17	19.13	20.08	21.04
4-3/4	13.12	14.13	15.14	16.15	17.16	18.17	19.18	20.19	21.20	22.21
5	13.81	14.87	15.94	17.00	18.06	19.13	20.19	21.25	22.32	23.38
5-1/4	14.50	15.62	16.74	17.85	18.96	20.08	21.20	22.32	23.43	24.54
5-1/2	15.19	16.36	17.53	18.70	19.87	21.04	22.21	23.38	24.54	25.71
5-3/4	15.88	17.10	18.33	19.55	20.77	21.99	23.22	24.44	25.66	26.88
6	16.58	17.85	19.13	21.40	21.68	22.95	24.23	25.50	26.78	28.05
6-1/4	17.27	18.60	19.92	21.25	22.58	23.91	25.23	26.56	27.90	29.22
6-1/2	17.95	19.34	20.72	22.10	23.48	24.87	26.24	27.62	29.01	30.39
6-3/4	18.65	20.08	21.51	22.95	24.39	25.82	27.25	28.69	30.12	31.56
7	19.34	20.83	22.32	23.80	25.29	26.78	28.26	29.75	31.23	32.72
7-1/4	20.03	21.57	23.11	24.65	26.19	27.73	29.27	30.81	32.35	33.89
7-1/2	20.72	22.32	23.91	25.50	27.10	28.68	30.28	31.88	33.48	35.06
7-3/4	21.41	23.05	24.70	26.35	28.00	29.64	31.29	32.94	34.59	36.23
8	22.10	23.80	25.50	27.20	28.90	30.60	32.30	34.00	35.70	37.40
8-1/4	22.79	24.55	26.30	28.05	29.80	31.56	33.31	35.06	36.81	38.57
8-1/2	23.48	25.30	27.10	28.90	30.70	32.52	34.32	36.12	37.93	39.74
8-3/4	24.17	26.04	27.89	29.75	31.61	33.47	35.33	37.20	39.05	40.91
9	24.86	26.78	28.69	30.60	32.52	34.43	36.34	38.26	40.16	42.08
9-1/4	25.55	27.52	29.49	31.45	33.41	35.38	37.35	39.31	41.28	43.25
9-1/2	26.24	28.26	30.28	32.30	34.32	36.34	38.36	40.37	42.40	44.41
9-3/4	26.94	29.01	31.08	33.15	35.22	37.29	39.37	41.44	43.52	45.58
10	27.62	29.75	31.88	34.00	36.12	38.25	40.38	42.50	44.64	46.75
10-1/4	28.32	30.50	32.67	34.85	37.03	39.21	41.39	43.56	45.75	47.92
10-1/2	29.00	31.24	33.48	35.70	37.92	40.17	42.40	44.63	46.86	49.08
10-3/4	29.69	31.98	34.28	36.55	38.83	41.12	43.40	45.69	47.97	50.25
11	30.40	32.72	35.06	37.40	39.74	42.08	44.42	46.76	49.08	51.42
11-1/4	31.08	33.47	35.86	38.25	40.64	43.04	45.42	47.82	50.20	52.59
11-1/2	31.76	34.21	36.66	39.10	41.54	44.00	46.44	48.88	51.32	53.76
11-3/4	32.46	34.95	37.46	39.95	42.45	44.94	47.45	49.94	52.44	54.93
12	33.15	35.70	38.25	40.80	43.35	45.90	48.45	51.00	53.55	56.10
12-1/4	33.83	36.44	39.05	41.65	44.25	46.86	49.46	52.06	54.67	57.27
12-1/2	34.53	37.19	39.84	42.50	45.16	47.82	50.46	53.12	55.78	58.44
12-3/4	35.22	37.93	40.64	43.35	46.06	48.77	51.48	54.19	56.90	59.60

Stone is about one-third the weight of wrought iron. Brick is about one-fourth the weight of wrought iron. Timber is about one-twelfth the weight of wrought iron. Concrete is slightly lighter than stone.

49. Bolts and Nuts

Standard Screw Threads

(A. S. M. E.)

National Coarse-thread Series (known also as U. S. Standard)

Recommended for general use in machines and structures.

Size	Threads per inch	Diameters, in.		Areas, sq. in.	
		Body	At root of thread	Body	At root of thread
No. 1	64	.073	.053	.0042	.0032
No. 2	56	.086	.063	.0058	.0032
No. 3	48	.099	.072	.0077	.0041
No. 4	40	.112	.080	.0099	.0050
No. 5	40	.125	.092	.0123	.0066
No. 6	32	.138	.097	.0150	.0074
No. 8	32	.164	.123	.0211	.0119
No. 10	24	.190	.136	.0284	.0145
No. 12	24	.216	.162	.0366	.0206
1/4 in.	20	.250	.185	.0491	.0269
5/16 in.	18	.312	.240	.0767	.0452
3/8 in.	16	.375	.294	.1105	.0679
7/16 in.	14	.438	.345	.1503	.0935
1/2 in.	13	.500	.400	.1964	.1257
9/16 in.	12	.562	.454	.2485	.1619
5/8 in.	11	.625	.507	.3068	.2019
3/4 in.	10	.750	.620	.4418	.3019
7/8 in.	9	.875	.731	.6013	.4197
1 in.	8	1.000	.838	.7854	.5515
1-1/8 in.	7	1.125	.939	.9940	.6925
1-1/4 in.	7	1.250	1.064	1.227	.8891
1-1/2 in.	6	1.500	1.284	1.767	1.295
1-3/4 in.	5	1.750	1.490	2.405	1.746
2 in.	4-1/2	2.000	1.711	3.142	2.302
2-1/4 in.	4-1/2	2.250	1.961	3.976	3.023
2-1/2 in.	4	2.500	2.175	4.909	3.719
2-3/4 in.	4	2.750	2.425	5.940	4.620
3 in.	4	3.000	2.675	7.069	5.428

National Fine-thread Series (above 1/4 in. known as S. A. E. threads)

Recommended for general use in automotive and aircraft work, and where special conditions require a fine thread.

Size	Threads per inch	Diameters, in.		Areas, sq. in.	
		Body	At root of thread	Body	At root of thread
No. 0	80	.060	.044	.0028	.0015
No. 1	72	.073	.055	.0042	.0023
No. 1	64	.086	.066	.0058	.0034
No. 3	56	.099	.076	.0077	.0045
No. 4	48	.112	.085	.0099	.0057
No. 5	44	.125	.096	.0123	.0072
No. 6	40	.138	.106	.0150	.0088
No. 8	36	.164	.128	.0211	.0129
No. 10	32	.190	.149	.0284	.0174
No. 12	28	.216	.170	.0366	.0227
1/4 in.	28	.250	.204	.0491	.0326
5/16 in.	24	.312	.258	.0767	.0524
3/8 in.	24	.375	.321	.1105	.0809
7/16 in.	20	.438	.372	.1503	.1090
1/2 in.	20	.500	.435	.1964	.1487
9/16 in.	18	.562	.490	.2485	.1888
5/8 in.	18	.625	.553	.3068	.2400
3/4 in.	16	.750	.669	.4418	.3513
7/8 in.	14	.875	.782	.6013	.4805
1 in.	14	1.000	.907	.7854	.6464
1-1/8 in.	12	1.125	1.017	.9940	.8119
1-1/4 in.	12	1.250	1.142	1.227	1.024
1-1/2 in.	12	1.500	1.392	1.767	1.521

Bolt Heads and Nuts

Rough and Semi-finished, Square and Hexagonal

(A. S. M. E.)

All dimensions are given in inches

Diameter of bolt	Bolt heads					Nuts				
	Width across flats		Minimum width across corners		Nominal height	Width across flats		Minimum width across corners		Nominal height
	Maximum	Minimum	Hexagonal	Square		Maximum	Minimum	Hexagonal	Square	
1/4	0.375	0.363	0.414	0.498	11/64	0.438	0.425	0.485	0.584	7/32
5/16	0.500	0.484	0.552	0.665	13/64	0.562	0.547	0.624	0.751	17/64
3/8	0.562	0.544	0.620	0.747	1/4	0.625	0.606	0.691	0.832	21/64
7/16	0.625	0.603	0.687	0.828	19/64	0.750	0.728	0.830	1.000	3/8
1/2	0.750	0.725	0.827	0.995	21/64	0.812	0.788	0.898	1.082	7/16
9/16	0.875	0.847	0.966	1.163	3/8	0.875	0.847	0.966	1.163	31/64
5/8	0.938	0.906	1.033	1.244	27/64	0.938	0.906	1.033	1.244	35/64
3/4	1.125	1.088	1.240	1.494	1/2	1.125	1.088	1.240	1.494	21/32
7/8	1.312	1.269	1.447	1.742	19/32	1.312	1.269	1.447	1.742	49/64
1	1.500	1.450	1.653	1.991	21/32	1.500	1.450	1.653	1.991	7/8
1-1/8	1.688	1.631	1.859	2.239	3/4	1.688	1.631	1.859	2.239	1
1-1/4	1.875	1.813	2.067	2.489	27/32	1.875	1.813	2.067	2.489	1-3/32
1-1/2	2.250	2.175	2.480	2.986	1	2.250	2.175	2.480	2.986	1-5/16
1-3/4	2.625	2.538	2.893	3.485	1-5/32	2.625	2.538	2.893	2.485	1-17/32
2	3.000	2.900	3.306	3.982	1-11/32	3.000	2.900	3.306	3.982	1-3/4
2-1/4	3.375	3.263	3.720	4.480	1-1/2	3.375	3.263	3.720	4.480	1-31/32
2-1/2	3.750	3.625	4.133	4.977	1-21/32	3.750	3.625	4.133	4.977	2-3/16
2-3/4	4.125	3.988	4.546	5.476	1-53/64	4.125	3.988	4.546	5.476	2-13/32
3	4.500	4.350	4.959	5.973	2	4.500	4.350	4.959	5.973	2-5/8

Upsetting reduces the strength of iron, so that bars having the same diameter at root of thread as that of the bar, invariably break in the screw end, when tested to destruction, without developing the full strength of the bar. It is therefore necessary to make up for this loss in strength by an excess of metal in the upset screw ends over that in the bar. Lengths of upset ends for use with turnbuckles of standard length (6 in. between heads) and with clevises should be 1 in. longer than specified in table, and additions for upset should be correspondingly increased.

Upsets on Square and Round Steel Bars

All dimensions are given in inches

Round bars				Size of upset						Square bars			
Diameter	Area, sq. in.	Addition for upset	Excess of area at root of thread, per cent	Diameter	Length	Diameter at root of thread	Area at root of thread	Number of threads per inch	Weight of one turnbuckle lb.	Side of square	Area, sq. in.	Addition for upset	Excess of area at root of thread, per cent
5/8	0.307	4-1/2	36.8	7/8	4	0.731	0.420	9	2-1/2	.....	.....	.....	....
3/4	0.442	3-7/8	24.4	1	4	0.837	0.550	8	3-1/2	.....	.....	.....	....
.....	.....	.....	....	1-1/8	4	0.940	0.694	7	4	3/4	0.563	3-1/2	20.6
7/8	0.601	5	48.3	1-1/4	4	1.065	0.891	7	5-1/4	7/8	0.766	4	16.3
1	0.785	4-3/8	34.7	1-3/8	4	1.160	1.057	6	6	.....	.....	.....	....
1-1/8	0.994	3-7/8	30.3	1-1/2	4	1.284	1.295	6	7-1/2	1	1.000	4	29.5
1-1/4	1.227	3-7/8	23.5	1-5/8	4-1/2	1.389	1.515	5-1/2	8-1/2	1-7/8	1.266	4-1/2	19.7
1-3/8	1.485	3-1/2	17.4	1-3/4	4-1/2	1.490	1.744	5	10	.....	.....	.....	....
.....	.....	.....	....	1-7/8	4-1/2	1.615	2.049	5	11-1/2	1-1/4	1.563	4-1/2	31.1
1-1/2	1.767	4-5/8	30.3	2	5	1.712	2.302	4-1/2	13	1-3/8	1.891	4-1/8	21.7
1-5/8	2.074	4-1/4	27.8	2-1/8	5	1.837	2.651	4-1/2	15	.....	.....	.....	....
1-3/4	2.405	4	25.7	2-1/4	5	1.962	3.023	4-1/2	18	1-1/2	2.250	4-3/4	34.0
1-7/8	2.761	4-1/8	23.9	2-3/8	5-1/2	2.087	3.410	4	20	1-5/8	2.641	4-5/8	29.6
2	3.142	3-7/8	18.3	2-1/2	5-1/2	2.175	3.716	4	24	1-3/4	3.063	4-1/4	21.3
2-1/8	3.547	3-5/8	17.1	2-5/8	5-1/2	2.300	4.155	4	28	.....	.....	.....	....
.....	.....	.....	....	2-3/4	6	2.425	4.619	4	30	1-7/8	3.516	5-1/8	31.4
2-1/4	3.976	4-5/8	28.5	2-7/8	6	2.550	5.107	3-1/2	34	2	4.000	4-3/4	27.7
2-3/8	4.430	4-3/8	22.6	3	6	2.629	5.430	3-1/2	38	2-1/8	4.516	4-3/8	20.2
2-1/2	4.909	4-3/8	21.3	3-1/8	6-1/2	2.754	5.957	3-1/2	50	.....	.....	.....	....
2-5/8	5.412	4-1/4	20.3	3-1/4	6-1/2	2.879	6.510	3-1/2	50	2-1/4	5.063	5-1/8	28.6
2-3/4	5.940	4-1/4	19.3	3-3/8	7	3.004	7.088	3-1/4	65	.....	.....	.....	....
.....	.....	.....	....	3-1/2	7	3.100	7.548	3-1/4	65	2-3/8	5.641	6-1/8	33.8
2-7/8	6.492	5-1/2	25.9	3-5/8	8	3.225	8.170	3-1/4	......	2-1/2	6.250	6-1/4	30.7
3	7.069	5-1/4	22.2	3-3/4	8	3.317	8.641	3-1/4	......	.....	.....	.....	....
3-1/8	7.670	5-1/8	21.3	3-7/8	8	3.442	9.305	3	......	2-5/8	6.891	6-3/4	35.0
3-1/4	8.296	4-7/8	20.7	4	8	3.567	9.994	3	......	2-3/4	7.563	6	25.1
3-3/8	8.946	6	26.6	4-1/4	9	3.798	11.33	2-7/8	......	2-7/8	8.266	8	37.0
3-5/8	10.32	4-1/2	23.6	4-1/2	9	4.028	12.75	2-3/4	......	3	9.000	7-1/2	41.7

Standard Cast Washers are proportioned to the bolt as follows: The diameter of the larger or bearing surface is 4 times the diameter of the bolt plus 1/4 in.; the diameter of the smaller surface (against which the bolt head or nut bears) is twice the diameter of the bolt plus 1/4 in.; the thickness is equal to the diameter of the bolt; and the bolt hole is 1/8 in. larger than the diameter of the bolt.

Weight in Pounds of 100 Bolts with Square Heads and Nuts

Length under head to point, in.	Diameter of bolts, inches												
	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1-1/4	1-1/2	1-3/4	2
1-1/2	4.0	7.0	10.5	15.2	22.5	39.5	63.0	.....	....	.....	.....	.....	....
1-3/4	4.4	7.5	11.3	16.3	23.8	41.6	66.0	.....	....	.....	.....	.....	....
2	4.8	8.0	12.0	17.4	25.2	43.8	69.0	109.0	163	.....	.....	.....	....
2-1/4	5.2	8.5	12.8	18.5	26.5	45.8	72.0	113.3	169	.....	.....	.....	....
2-1/2	5.5	9.0	13.5	19.6	27.8	48.0	75.0	117.5	174	.....	.....	.....	....
2-3/4	5.8	9.5	14.3	20.7	29.1	50.1	78.0	121.8	180	.....	.....	.....	....
3	6.3	10.0	15.0	21.8	30.5	52.3	81.0	126.0	185	358	589	900	1312
3-1/2	7.0	11.0	16.5	24.0	33.1	56.5	87.0	134.3	196	375	613	934	1355
4	7.8	12.0	18.0	26.2	35.8	60.8	93.1	142.5	207	392	638	967	1399
4-1/2	8.5	13.0	19.5	28.4	38.4	65.0	99.1	151.0	218	409	662	1001	1442
5	9.3	14.0	21.0	30.6	41.1	69.3	105.2	159.6	229	426	687	1034	1486
5-1/2	10.0	15.0	22.5	32.8	43.7	73.5	111.3	168.0	240	443	711	1068	1529
6	10.8	16.0	24.0	35.0	46.4	77.8	117.3	176.6	251	460	736	1101	1573
6-1/2	....	....	25.5	37.2	49.0	82.0	123.4	185.0	262	477	760	1135	1616
7	....	....	27.0	39.4	51.7	86.3	129.4	193.7	273	494	785	1168	1660
7-1/2	....	....	28.5	41.6	54.3	90.5	135.0	202.0	284	511	809	1202	1703
8	....	....	30.0	43.8	59.6	94.8	141.5	210.7	295	528	834	1235	1747
9	....	....	....	46.0	64.9	103.3	153.6	227.8	317	562	883	1301	1835
10	....	....	....	48.2	70.2	111.8	165.7	244.8	339	596	932	1368	1922
11	....	....	....	50.4	75.5	120.3	177.8	261.9	360	630	982	1435	2009
12	....	....	....	52.6	80.8	128.8	189.9	278.9	382	665	1031	1502	2096
Per in. additional	1.4	2.1	3.1	4.2	5.5	8.5	12.3	16.7	21.8	34.1	49.1	66.8	87.2

Permissible Variations in the Sizes of Hot-rolled Steel Bars

(1927 Standards of the American Society for Testing Materials)

Rounds, squares, and hexagons: sizes specified	Variations in size, in.	
	Under	Over
Up to 1/2 in., inclusive	0.007	0.007
Over 1/2 in. to 1 in., inclusive	0.010	0.010
Over 1 in. to 2 in., inclusive	1/64	1/32
Over 2 in. to 3 in., inclusive	1/32	3/64
Over 3 in. to 5 in., inclusive	1/32	3/32
Over 5 in. to 8 in., inclusive	1/16	1/8

Flats: widths specified	Variations in width, in.		Variations in thickness, under or over, in., for thicknesses specified			
	Under	Over	3/16 in. or under	Over 3/16 to 1/2 in. inclusive	Over 1/2 to 1 in. inclusive	Over 1 to 2 in. inclusive
Up to 1 in., inclusive	1/64	1/32	0.006	0.008	0.010	....
Over 1 to 2 in., inclusive	1/32	3/64	0.008	0.012	0.015	1/32
Over 2 to 4 in., inclusive	3/64	1/16	0.010	0.015	0.020	1/32
Over 4 to 6 in., inclusive	1/16	3/32	0.010	0.015	0.020	1/32

50. Sheet Metal, Board Measure

Dimensions of Sheets of Corrugated Iron

Width of corrugation, w	Depth of corrugation, D	Number of corrugations to sheet	Covering width with a lap of one corrugation	Width of sheet after corrugation	Length of longest sheets
2-1/2	5/8 in.	10	24 in.	26 in.	10 ft.
1-1/4	1/2 in.	19-1/2	24 in.	26 in.	8 ft.
3/4	1/4 in.	34-1/2	25 in.	26 in.	8 ft.

Weight of Corrugated Iron

Weight calculated for sheets 30-1/2 in. wide before corrugating

Number by U. S. std. gage	Thickness, in.	Weight per sq. ft., flat, lb.	Weight per sq. ft., corrugated, lb.	Weight per square of 100 sq. ft. when laid, allowing 6 in. lap in length and 2-1/2 in. or one corrugation lap in width for sheet lengths of						Galvanized, weight per sq. ft., flat, lb.
				5 ft.	6 ft.	7 ft.	8 ft.	9 ft.	10 ft.	
16	.062	2.61	3.28	365	358	353	350	348	346	2.95
18	.050	1.97	2.48	275	270	267	264	262	261	2.31
20	.038	1.40	1.76	196	192	190	188	186	185	1.74
22	.031	1.12	1.41	156	154	152	150	149	148	1.46
24	.025	.88	1.11	123	121	119	118	117	117	1.22
26	.018	.72	.91	101	99	97	97	96	95	1.06

Number of Square Feet of 2-1/2-in. Corrugated Iron, Required to Lay One Square of 100 Sq. Ft. with Side Lap of One Corrugation

Length of sheet, ft.	Length of end lap					
	1 in.	2 in.	3 in.	4 in.	5 in.	6 in.
5	110	112	114	116	118	120
6	110	111	113	115	117	118
7	110	110	112	114	115	117
8	109	110	112	113	114	115
9	109	110	112	113	114	115
10	108	109	110	111	112	113

The United States standard gage, adopted by act of Congress in 1893, is in general use by manufacturers of sheet steel, and the above table gives the thickness and weight of corrugated iron in accordance with that standard. For weight per square, 5% should be added to figures given in the table of weights when sheets are laid with one and one-half laps, and 10% when laid with two laps. If the corrugated iron is to be painted, 2 lb. per sq. ft. should be added to weights given in this table. The 2-1/2-in. corrugation is the one generally employed for roofing and siding, and the regular lengths of sheets are 6, 7, 8, 9, and 10 ft.

The transverse strength of a sheet of corrugated iron is found by the formula $\mathbf{W = 90\,000\ tbd/l}$, in which W = the uniformly distributed load in pounds that will produce failure, t is the thickness of the sheet in inches, b is the width of the sheet in inches, d is the depth of corrugations in inches, and l is the unsupported length of the sheet in inches.

For Roofing and Siding of buildings, corrugated iron is applied directly upon steel purlins or studding by means of clips of hoop iron, placed not more than 12 in. apart,

United States Standard Gage for Sheet Iron and Steel

Adopted as standard by Amer. Rail. Mas. Mechan. Asso. and the Asso. of Amer. Steel Manufacturers

Number of gage	Approximate thickness in fractions of an inch	Approximate thickness in decimal parts of an inch	Approximate thickness in millimeters	Weight per sq. ft. in pounds avoirdupois of iron	Weight per sq. ft. in pounds avoirdupois of steel	Weight per sq. meter in kilograms of steel	Number of gage
0000000	1/2	.5	12.70	20.	20.4	99.601	0000000
000000	15/32	.46875	11.91	18.75	19.125	93.376	000000
00000	7/16	.4375	11.11	17.50	17.85	87.151	00000
0000	13/32	.40625	10.32	16.25	16.575	80.926	0000
000	3/8	.375	9.53	15.0	15.3	74.701	000
00	11/32	.34375	8.73	13.75	14.025	68.476	00
0	5/16	.3125	7.94	12.50	12.75	62.251	0
1	9/32	.28125	7.14	11.25	11.475	56.026	1
2	17/64	.265625	6.75	10.625	10.8375	52.913	2
3	1/4	.25	6.35	10.	10.2	49.800	3
4	15/64	.234375	5.95	9.375	9.5625	46.688	4
5	7/32	.21875	5.56	8.75	8.925	43.575	5
6	13/64	.203125	5.16	8.125	8.2875	40.463	6
7	3/16	.1875	4.76	7.5	7.65	37.350	7
8	11/64	.171875	4.37	6.875	7.0125	34.238	8
9	5/32	.15625	3.97	6.25	6.375	31.125	9
10	9/64	.140625	3.57	5.625	5.7375	28.013	10
11	1/8	.125	3.18	5.	5.1	24.900	11
12	7/64	.109375	2.78	4.375	4.4625	21.788	12
13	3/32	.09375	2.38	3.75	3.825	18.675	13
14	5/64	.078125	1.98	3.125	3.1875	15.563	14
15	9/128	.0703125	1.79	2.8125	2.86875	14.006	15
16	1/16	.0625	1.59	2.5	2.55	12.450	16
17	9/160	.05625	1.43	2.25	2.295	11.205	17
18	1/20	.05	1.27	2.	2.04	9.960	18
19	7/160	.04375	1.11	1.75	1.785	8.715	19
20	3/80	.0375	0.953	1.50	1.53	7.470	20
21	11/320	.034375	0.873	1.375	1.4025	6.848	21
22	1/32	.03125	0.794	1.25	1.275	6.225	22
23	9/320	.028125	0.714	1.125	1.1475	5.603	23
24	1/40	.025	0.635	1.	1.02	4.980	24
25	7/320	.021875	0.556	0.875	0.8925	4.358	25
26	3/160	.01875	0.476	.75	0.765	3.735	26
27	11/640	.0171875	0.437	0.6875	0.70125	3.424	27
28	1/64	.015625	0.397	0.625	0.6375	3.113	28
29	9/640	.0140625	0.357	0.5625	0.57375	2.801	29
30	1/80	.0125	0.318	0.5	0.51	2.490	30
31	7/640	.0109375	0.278	0.4375	0.44625	2.179	31
32	13/1280	.01015625	0.258	0.40625	0.414375	2.023	32
33	3/320	.009375	0.238	0.375	0.3825	1.868	33
34	11/1280	.00859375	0.218	0.34375	0.350625	1.712	34
35	5/640	.0078125	0.198	0.3125	0.31875	1.556	35
36	9/1280	.00703125	0.179	0.28125	0.286875	1.401	36
37	17/2560	.006640625	0.169	0.265625	0.2709375	1.323	37
38	1/160	.00625	0.159	0.25	0.255	1.245	38

Section Areas of Rectangular Plates, in Square Inches

Width of plate, in.	Thickness of plate, in inches									
	1/8	1/4	3/8	1/2	5/8	3/4	7/8	1	1-1/4	1-1/2
1	.1250	.2500	.3750	.500	.6250	.9375	.8750	1.00	1.250	1.500
1-1/2	.1875	.3750	.5625	.750	.9375	1.125	1.312	1.50	1.875	2.500
2	.2500	.5000	.7500	1.00	1.250	1.500	1.750	2.00	2.500	3.000
2-1/2	.3125	.6250	.9375	1.25	1.562	1.875	2.188	2.50	3.125	3.750
3	.3750	.7500	1.125	1.50	1.875	2.250	2.625	3.00	3.750	4.500
3-1/2	.4375	.8750	1.312	1.75	2.188	2.625	3.062	3.50	4.375	5.250
4	.5000	1.000	1.500	2.00	2.500	3.000	3.500	4.00	5.000	6.000
4-1/2	.5625	1.125	1.687	2.25	2.812	3.375	3.937	4.50	5.125	6.750
5	.6250	1.250	1.875	2.50	3.125	3.750	4.375	5.00	6.250	7.500
5-1/2	.6875	1.375	2.062	2.75	3.438	4.125	4.812	5.50	6.875	8.250
6	.7500	1.500	2.250	3.00	3.750	4.500	5.250	6.00	7.500	9.000
6-1/2	.8125	1.625	2.438	3.25	4.062	4.875	5.688	6.50	8.125	9.750
7	.8750	1.750	2.625	3.50	4.375	5.250	6.125	7.00	8.750	10.50
7-1/2	.9375	1.875	2.812	3.75	4.688	5.625	6.562	7.50	9.375	11.25
8	1.000	2.000	3.000	4.00	5.000	6.000	7.000	8.00	10.00	12.00
8-1/2	1.062	2.225	3.188	4.25	5.312	6.375	7.438	8.50	10.62	12.75
9	1.125	2.250	3.375	4.50	5.625	6.750	7.875	9.00	11.25	13.50
9-1/2	1.188	2.375	3.563	4.75	5.938	7.125	8.312	9.50	11.88	14.25
10	1.250	2.500	3.750	5.00	6.250	7.500	8.750	10.0	12.50	15.00
10-1/2	1.313	2.625	3.938	5.25	6.563	7.875	9.188	10.5	13.12	15.75
11	1.375	2.750	4.125	5.50	6.875	8.250	9.625	11.0	13.75	16.50
11-1/2	1.438	2.875	4.312	5.75	7.188	8.625	10.06	11.5	14.38	17.25
12	1.500	3.000	4.500	6.00	7.500	9.000	10.50	12.0	15.00	18.00
12-1/2	1.563	3.225	4.688	6.25	7.812	9.375	10.94	12.5	15.62	18.75
13	1.625	3.250	4.875	6.50	8.125	9.750	11.38	13.0	16.25	19.50
14	1.750	3.500	5.250	7.00	8.750	10.50	12.25	14.0	17.50	21.00
15	1.875	3.750	5.625	7.50	9.375	11.25	13.13	15.0	18.75	22.50
16	2.000	4.000	6.000	8.00	10.00	12.00	14.00	16.0	20.00	24.00
17	2.125	4.250	6.375	8.50	10.62	12.75	14.88	17.0	21.25	25.50
18	2.250	4.500	6.750	9.00	11.25	13.50	15.75	18.0	22.50	27.00
19	2.375	4.750	7.125	9.50	11.88	14.25	16.63	19.0	23.75	28.50
20	2.500	5.000	7.500	10.0	12.50	15.00	17.50	20.0	25.00	30.00
21	2.625	5.250	7.875	10.5	13.12	15.75	18.38	21.0	26.25	31.50
22	2.750	5.500	8.250	11.0	13.75	16.50	19.25	22.0	27.50	33.00
23	2.875	5.750	8.625	11.5	14.38	17.25	20.13	23.0	28.75	34.50
24	3.000	6.000	9.000	12.0	15.00	18.00	21.00	24.0	30.00	36.00
26	3.250	6.500	9.750	13.0	16.25	19.50	22.75	26.0	32.50	39.00
28	3.500	7.000	10.50	14.0	17.50	21.00	24.50	28.0	35.00	42.00
30	3.750	7.500	11.25	15.0	18.75	22.50	26.25	30.0	37.50	45.00
32	4.000	8.000	12.00	16.0	20.00	24.00	28.00	32.0	40.00	48.00
34	4.250	8.500	12.75	17.0	21.25	25.50	29.75	34.0	42.50	51.00
36	4.500	9.000	13.50	18.0	22.50	27.00	31.50	36.0	45.00	54.00
38	4.750	9.500	14.25	19.0	23.75	28.50	33.25	38.0	47.50	57.00
40	5.000	10.00	15.00	20.0	25.00	30.00	35.00	40.0	50.00	60.00

This multiplication table is applicable to any kind of material, but it is especially useful in computations on the plates which are used in the beams and members of steel structures for roofs and bridges.

which encircle the purlin or stud. Numbers 20 and 22 are the gages most frequently used for roofs, and numbers 22 and 24 for siding. The sheets are either painted or galvanized, preferably the latter.

Feet Board Measure, for Stuff 1 In. Thick

Width, in.	Length, in feet 1	2	3	4	5	6	7	8	9	12
1	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	1.00
1-1/4	.1042	.2083	.3125	.4167	.5208	.6250	.7292	.8333	.9375	1.25
1-1/2	.1250	.2500	.3750	.5000	.6250	.7500	.8750	1.000	1.125	1.50
1-3/4	.1458	.2917	.4375	.5833	.7292	.8750	1.021	1.167	1.313	1.75
2	.1667	.3333	.5000	.6667	.8333	1.000	1.167	1.333	1.500	2.00
2-1/4	.1875	.3750	.5625	.7500	.9375	1.125	1.313	1.500	1.688	2.25
2-1/2	.2083	.4167	.6250	.8333	1.042	1.250	1.458	1.667	1.875	2.50
2-3/4	.2292	.4583	.6875	.9167	1.146	1.375	1.604	1.833	2.063	2.75
3	.2500	.5000	.7500	1.000	1.250	1.500	1.750	2.000	2.250	3.00
3-1/4	.2708	.5417	.8125	1.083	1.354	1.625	1.896	2.167	2.437	3.25
3-1/2	.2917	.5833	.8750	1.167	1.458	1.750	2.042	2.333	2.625	3.50
3-3/4	.3125	.6250	.9375	1.250	1.563	1.875	2.188	2.500	2.812	3.75
4	.3333	.6667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	4.00
41-/4	.3541	.7083	1.062	1.416	1.771	2.125	2.479	2.833	3.187	4.25
4-1/2	.3750	.7500	1.125	1.500	1.875	2.250	2.625	3.000	3.375	4.50
4-3/4	.3953	.7917	1.188	1.584	1.979	2.375	2.776	3.167	3.562	4.75
5	.4167	.8333	1.250	1.667	2.083	2.500	2.917	3.333	3.750	5.00
5-1/2	.4583	.9167	1.375	1.833	2.292	2.750	3.208	3.667	4.125	5.50
6	.5000	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	6.00
6-1/2	.5417	1.083	1.625	2.167	2.708	3.250	3.792	4.333	4.875	6.50
7	.5833	1.167	1.750	2.333	2.917	3.500	4.083	4.667	5.250	7.00
7-1/2	.6250	1.250	1.875	2.500	3.125	3.750	4.375	5.000	5.625	7.50
8	.6667	1.333	2.000	2.667	3.333	4.000	4.667	5.333	6.000	8.00
8-1/2	.7083	1.417	2.125	2.833	3.542	4.250	4.958	5.667	6.375	8.50
9	.7500	1.500	2.250	3.000	3.750	4.500	5.250	6.000	6.750	9.00
9-1/2	.7917	1.583	2.375	3.167	3.908	4.750	5.542	6.333	7.125	9.50
10	.8333	1.667	2.500	3.333	4.167	5.000	5.833	6.667	7.500	10.0
10-1/2	.8750	1.750	2.625	3.500	4.375	5.250	6.125	7.000	7.875	10.5
11	.9167	1.833	2.750	3.667	4.583	5.500	6.417	7.333	8.250	11.0
11-1/2	.9583	1.917	2.875	3.833	4.792	5.750	6.708	7.667	8.625	11.5
12	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	12.0
12-1/2	1.042	2.083	3.125	4.167	5.280	6.250	7.292	8.333	9.375	12.5
13	1.083	2.167	3.250	4.333	5.417	6.500	7.583	8.666	9.750	13.0
14	1.167	2.333	2.500	4.667	5.833	7.000	8.167	9.333	10.50	14.0
15	1.250	2.250	3.750	5.000	6.250	7.500	8.750	10.00	11.25	15.0
16	1.333	2.667	4.000	5.333	6.667	8.000	9.333	10 67	12.00	16.0
17	1.417	2.833	4.250	5.667	7.083	8.500	9.917	11.33	12.75	17.0
18	1.500	3.000	4.500	6.000	7.500	9.000	10.50	12.00	13.50	18.0
19	1.583	3.167	4.750	6.333	7.917	9.500	11.08	12.67	14.25	19.0
20	1.667	3.333	5.000	6.667	8.333	10.00	11.67	13.33	15.00	20.0

A board 1 in. thick, 1 ft. wide, and 1 ft. long contains 1 foot board measure (1.00 ft. b.m.). This table gives feet board measure for lumber of different widths and lengths. For stuff more than 1 in. thick, multiply tabular values by the thickness in inches; thus, for a plank 3 in. thick, 10-1/2 in. wide, and 1 ft. long, $3 \times .875 = 2.625$ ft. b.m. Also, for 418 lin. ft. of such plank, $418 \times 2.625 = 1097$ ft. b.m.

The number of feet board measure in a lot of lumber may also be found by taking 12 times its contents in cubic feet. Or, conversely, the number of cubic feet is one-twelfth of the number of feet board measure.

51. Pipes and Fittings

Standard Welded Pipe

Diameter: Nominal internal, in.	Diameter: Actual external, in.	Diameter: Actual internal, in.	Thickness, in.	Transverse areas: External, sq. in.	Transverse areas: Internal, sq. in.	Transverse areas: Metal, sq. in.	Length of pipe containing 1 cubic foot, ft.	Nominal weight per foot, lb.	Number of threads per inch of screw
1/8	0.405	0.27	.068	.129	.0573	.0717	2513.	.241	27
1/4	0.54	0.364	.088	.229	.1041	.1249	1383.3	.42	18
3/8	0.675	0.494	.091	.358	.1917	.1663	751.2	.559	18
1/2	0.84	0.623	.109	.554	.3048	.2492	472.4	.837	14
3/4	1.05	0.824	.113	.866	.5333	.3327	270.	1.115	14
1	1.315	1.048	.134	1.358	.8626	.4954	166.9	1.668	11-1/2
1-1/4	1.66	1.38	.14	2.164	1.496	.668	96.25	2.244	11-1/2
1-1/2	1.9	1.611	.145	2.835	2.038	.797	70.66	2.678	11-1/2
2	2.375	2.067	.154	4.43	3.356	1.074	42.91	3.609	11-1/2
2-1/2	2.875	2.468	.204	6.492	4.784	1.708	30.1	5.739	8
3	3.5	3.067	.217	9.621	7.388	2.243	19.5	7.536	8
3-1/2	4.0	3.548	.226	12.566	9.887	2.679	14.57	9.001	8
4	4.5	4.026	.237	15.904	12.73	3.174	11.31	10.665	8
4-1/2	5.0	4.508	.246	19.635	15.961	3.674	9.02	12.34	8
5	5.563	5.045	.259	24.306	19.99	4.316	7.2	14.502	8
6	6.625	6.065	.28	34.472	28.888	5.584	4.98	18.762	8
7	7.625	7.023	.301	45.664	38.738	6.926	3.72	23.271	8
8	8.625	7.982	.322	58.426	50.04	8.386	2.88	28.177	8
9	9.625	8.937	.344	72.76	62.73	10.03	2.29	33.701	8
10	10.75	10.019	.366	90.763	78.839	11.924	1.82	40.065	8
11	11.75	11.000	.375	108.4	95.03	13.37	1.51	45.0	8
12	12.75	12.000	.375	127.6	113.0	14.6	1.27	49.0	8

Boiler Tubes (National Tube Works), of charcoal-iron, lap-welded, come in standard sizes of 1-3/4 in. to 4 in. external diameter, advancing by quarter inches. There is also a 4-1/2-in. size. Above that, beginning at 5 in., they advance by inches to 16 in.

For estimating the effective steam-heating or boiler surface of tubes, the surface in contact with air or gases of combustion (whether internal or external) is to be taken. For heating liquids by steam, superheating steam, or transferring heat from one liquid or gas to another, the mean surface is to be taken. The square feet of surface, S, in a tube of a given L feet long and d inches in diameter may be obtained by the formula $\mathbf{S = 0.2618\ dL}$.

The Standard Pipe Fittings given in the tables were adopted by the American Society of Mechanical Engineers in 1914 after conferences between a committee of that society, a committee representing the manufacturers of pipe fittings, and a committee of the National Association of Master Steam and Hot Water Fitters.

Extra Strong Welded Pipe

Diameter: Nominal internal, in.	Diameter: Actual external, in.	Diameter: Actual internal, in.	Thickness, in.	Transverse areas: External, sq. in.	Transverse areas: Internal, sq. in.	Transverse areas: Metal, sq. in.	Nominal weight per foot, lb.	Number of threads per inch of screw
1/8	.405	.205	.100	.129	.033	.096	.29	27
1/4	.540	.294	.123	.229	.068	.161	.54	18
3/8	.675	.421	.127	.358	.139	.219	.74	18
1/2	.840	.542	.149	.554	.231	.323	1.09	14
3/4	1.050	.736	.157	.866	.425	.441	1.39	14
1	1.315	.951	.182	1.358	.710	.648	2.17	11-1/2
1-1/4	1.660	1.272	.194	2.164	1.271	1.893	3.00	11-1/2
1-1/2	1.900	1.494	.203	2.835	1.753	1.082	3.63	11-1/2
2	2.375	1.933	.221	4.430	2.935	1.495	5.02	11-1/2
2-1/2	2.875	2.315	.280	6.492	4.209	2.283	7.67	8
3	3.500	2.892	.304	9.621	6.569	3.052	10.25	8
3-1/2	4.000	3.358	.321	12.566	8.856	3.710	12.47	8
4	4.500	3.818	.341	15.904	11.449	4.455	14.97	8
4-1/2	5.000	4.280	.360	19.635	14.387	5.248	18.22	8
5	5.563	4.813	.375	24.306	18.193	6.113	20.54	8
6	6.625	5.751	.437	34.472	25.976	8.496	28.58	8
7	7.625	6.625	.500	45.664	34.472	11.192	37.67	8
8	8.625	7.625	.500	58.426	45.664	12.762	43.00	8
9	9.625	8.625	.500	72.760	58.426	14.334	48.25	8
10	10.750	9.750	.500	90.763	74.662	16.101	54.25	8
12	12.750	11.750	.500	127.68	108.43	19.25	65.00	8

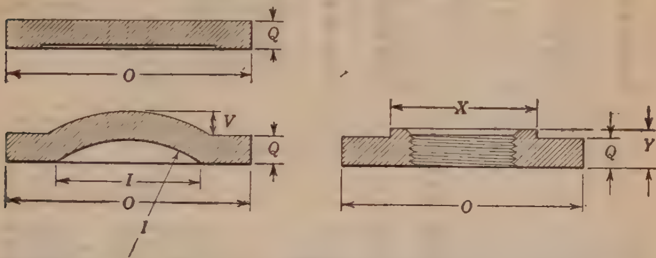
Double Extra Strong Welded Pipe

Diameter: Nominal internal, in.	Diameter: Actual external, in.	Diameter: Actual internal, in.	Thickness, in.	Transverse areas: External, sq. in.	Transverse areas: Internal, sq. in.	Transverse areas: Metal sq. in.	Length of pipe containing 1 cubic foot, ft.	Nominal weight per foot, lb.	Number of threads per inch of screw
1/2	0.840	0.25	0.294	0.55	0.05	0.51	2280	1.70	14
3/4	1.050	0.43	0.308	0.87	0.14	0.73	1030	2.44	14
1	1.315	0.60	0.358	1.36	0.27	1.04	533	3.65	11-1/2
1-1/4	1.660	0.89	0.382	2.16	0.62	1.55	232	5.20	11-1/2
1-1/2	1.900	1.10	0.400	2.84	0.93	1.91	155	6.40	11-1/2
2	2.375	1.50	0.436	4.43	1.74	2.69	82.8	9.02	11-1/2
2-1/2	2.875	1.77	0.562	6.49	2.42	4.07	59.5	13.70	8
3	3.500	2.30	0.600	9.62	4.10	5.52	35.2	18.60	8
3-1/2	4.000	2.73	0.636	12.57	5.79	6.77	24.9	22.80	8
4	4.500	3.15	0.674	15.90	7.72	8.18	18.6	27.50	8
4-1/2	5.000	3.58	0.710	19.64	9.98	9.66	14.4	32.50	8
5	5.563	4.06	0.750	24.31	12.97	11.34	11.1	38.10	8
6	6.625	4.89	0.864	34.47	18.67	15.81	7.70	53.10	8
7	7.625	5.87	0.875	45.66	27.11	18.56	5.30	63.08	8
8	8.625	6.87	0.875	58.43	37.12	21.31	3.88	72.42	8

Standard Pipe Flanges for 125 lb. per sq. in. Pressure

Approved by American Engineering Standards Committee, February, 1928. Sponsor organizations: Heating and Piping Contractors National Association, Manufacturers' Standardization Society of Valve and Fittings Industry, American Society of Mechanical Engineers.

All dimensions are given in inches



Dimensions and Theoretical Weights (Pounds) of Screwed Companion and Blind Flanges *

Nominal pipe size †‡ I	Diameter of flange O	Thickness of flange (min.) Q	Metal thickness‡ (min.) V	Diameter hub (min.) X	Length of hub and threads§ (min.) Y	Theoretical weights ‖ Companion flanges	Blind flanges
1	4-1/4	7/16	....	1-15/16	0.68	2	2
1-1/4	4-5/8	1/2	....	2-5/16	0.76	2	3
1-1/2	5	9/16	....	2-9/16	0.87	3	3
2	6	5/8	....	3-1/16	1.00	5	5
2-1/2	7	11/16	....	3-9/16	1.14	7	7
3	7-1/2	3/4	....	4-1/4	1.20	8	9
3-1/2	8-1/2	1-3/16	....	4-13/16	1.25	11	12
4	9	15/16	....	5-5/16	1.30	14	16
5	10	15/16	....	6-7/16	1.41	17	20
6	11	1	....	7-9/16	1.51	22	25
8	13-1/2	1-1/8	....	9-11/16	1.71	31	42
10	16	1-3/16	....	11-15/16	1.93	45	63
12	19	1-1/4	13/16	14-1/16	2.13	63	88
14 O.D.	21	1-3/8	7/8	15-3/8	2.25	82	115
16 O.D.	23-1/2	1-7/16	1	17-1/2	2.45	105	160
18 O.D.	25	1-9/16	1-1/16	19-5/8	2.65	120	190
20 O.D.	27-1/2	1-11/16	1-1/8	21-3/4	2.85	150	250
24 O.D.	32	1-7/8	1-1/4	26	3.25	220	370
30 O.D.	38-3/4	2-1/8	1-7/16	....	....	....	620
36 O.D.	46	2-3/8	1-5/8	....	....	....	990
42 O.D.	53	2-5/8	1-13/16	....	....	....	1470
48 O.D.	59-1/2	2-3/4	2	....	....	....	2000

* All 125-lb. flanges have a plain face.

† Sizes 14 in. and larger are to be used with O.D. pipe of the same sizes.

‡ All blind flanges for sizes 12 in. (19 in. O.D.) and larger must be dished, with inside radius equal to the port diameter.

§ This column is the same as column for effective thread (E) in Table I of the American Pipe Thread Standard, published by the American Engineering Standards Committee, except in sizes 1-1/4, 1-1/2, and 2 in.

‖ All weights listed are for flanges faced only, based upon minimum thicknesses given in the table above without allowances for variation. Cast iron is considered to weigh 0.26 lb. per cu. in.

Templets for Drilling 125-lb. Pipe Flanges

All dimensions are given in inches

Nominal pipe size	Diameter of bolt circle	Number of bolts *	Diameter of bolts	Diameter of drilled bolt holes *	Length of bolts † ‡	Length of bolt-studs with two nuts §	Total effective area bolt metal	Stress lb. per sq. in. bolt metal ‖	Size of ring gasket
1	3-1/8	4	1/2	5/8	1-1/2	......	0.504	1340	1×2-5/8
1-1/4	3-1/2	4	1/2	5/8	1-1/2	......	0.504	1755	1-1/4×3
1-1/2	3-7/8	4	1/2	5/8	1-3/4	......	0.504	2215	1-1/2×3-3/8
2	4-3/4	4	5/8	3/4	2	......	0.808	2065	2×4-1/8
2-1/2	5-1/2	4	5/8	3/4	2-1/4	......	0.808	2885	2-1/2×4-7/8
3	6	4	5/8	3/4	2-1/4	......	0.808	3510	3×5-3/8
3-1/2	7	8	5/8	3/4	2-1/2	......	1.616	2410	3-1/2×6-3/8
4	7-1/2	8	5/8	3/4	2-3/4	......	1.616	2870	4×6-7/8
5	8-1/2	8	3/4	7/8	2-3/4	......	2.416	2440	5×7-3/4
6	9-1/2	8	3/4	7/8	3	......	2.416	3110	6×8-3/4
8	11-3/4	8	3/4	7/8	3-1/4	......	2.416	4915	8×11
10	14-1/4	12	7/8	1	3-1/2	......	5.04	3485	10×13-3/8
12	17	12	7/8	1	3-1/2	......	5.04	5065	12×16-1/8
14 O.D.	18-3/4	12	1	1-1/8	4	......	6.60	4685	14×17-3/4
16 O.D.	21-1/4	16	1	1-1/8	4-1/4	......	8.80	4575	16×20-1/4
18 O.D.	22-3/4	16	1-1/8	1-1/4	4-1/2	......	11.10	4135	18×21-5/8
20 O.D.	25	20	1-1/8	1-1/4	4-3/4	......	13.88	4030	20×23-7/8
24 O.D.	29-1/2	20	1-1/4	1-3/8	5-1/4	......	17.86	4385	24×28-1/4
30 O.D.	36	28	1-1/4	1-3/8	5-3/4	......	25.00	4700	30×34-5/8
36 O.D.	42-3/4	32	1-1/2	1-5/8	6-1/2	......	41.41	4035	36×41-1/4
42 O.D.	49-1/2	36	1-1/2	1-5/8	7-1/4	9-1/2	46.57	4810	42×47-7/8
48 O.D.	56	44	1-1/2	1-5/8	7-1/2	9-1/2	56.93	5200	48×54-3/8
54 O.D.	62-3/4	44	1-3/4	2	8-1/4	10-1/2	76.82	4755	54×61
60 O.D.	69-1/4	52	1-3/4	2	8-1/2	11	90.79	4925	60×67-1/2
72 O.D.	82-1/2	60	1-3/4	2	9-1/2	12	104.70	6150	72×80-5/8
84 O.D.	95-1/2	64	2	2-1/4	10-1/2	13	147.33	5825	84×93-1/2
96 O.D.	108-1/2	68	2-1/4	2-1/2	11-1/2	14-1/2	205.56	5415	96×106-1/4

* Drilling templets are in multiples of four, so that fittings may be made to face in any quarter, and bolt holes straddle the center line. For bolts smaller than 1-3/4-in. the bolt holes shall be drilled 1/8 in. larger in diameter than the nominal diameter of the bolt. Holes for bolts 1-3/4 in. and larger shall be drilled 1/4 in. larger than nominal diameter of bolts.

† The bolt holes on cast-iron flanged fittings are not spot-faced for ordinary service. When required, the fittings and flanges in sizes 36 in. and larger can be spot-faced or back-faced, so that standard length bolts can be used.

‡ All 125-lb. cast-iron standard flanges have a plain face.

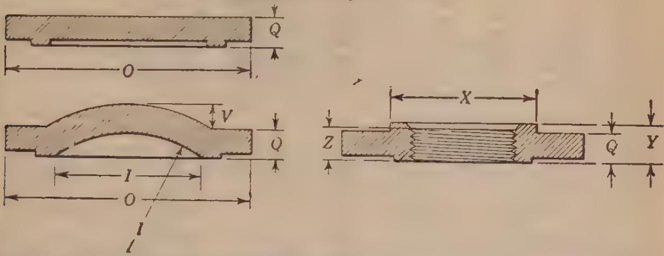
§ Bolts shall be of steel with standard "Rough Square Heads" and the nuts shall be of steel with standard "Rough Hexagonal" dimensions; all as given in the "Tentative American Standard on Wrench Head Bolts and Nuts, and Wrench Openings." For bolts 1-3/4 in. in diameter and larger, bolt-studs with a nut on each end are recommended. Hexagonal nuts for pipe sizes 1 in. to 48 in. can be conveniently pulled up with open wrenches of minimum design of heads. Hexagonal nuts for pipe sizes 48 in. to 96 in. can be conveniently pulled up with box wrenches.

‖ The stress shown is that of internal pressure only, assumed to act on a circular area equal in diameter to the outside diameter of a ring gasket covering the flange to the inside of bolts.

Standard Pipe Flanges for 250 lb. per sq. in. pressure

Approved by the American Engineering Standards Committee, February, 1928. Sponsor organizations: Heating and Piping Contractors National Association, Manufacturers' Standardization Society of Valve and Fittings Industry, American Society of Mechanical Engineers.

All dimensions are given in inches



Dimensions and Theoretical Weights (Pounds) of Screwed Companion and Blind Flanges *

Nominal pipe size	Diameter of port †‡ I	Diameter of flange O	Thickness of flange * (min.) Q	Metal thickness (min.) V	Diameter hub (min.) X	Length through hub (min.) Y	Length of threads § (min.) Z	Diameter of raised face * W	Theoretical Weights‖ Companion flanges	Theoretical Weights‖ Blind flanges
1	1	4-7/8	11/16	......	2-1/16	7/8	0.68	2-11/16	3	3
1-1/4	1-1/4	5-1/4	3/4	......	2-1/2	1	0.76	3-1/16	4	4
1-1/2	1-1/2	6-1/8	13/16	......	2-3/4	1-1/8	0.87	3-9/16	6	6
2	2	6-1/2	7/8	......	3-5/16	1-1/4	1.00	4-3/16	7	8
2-1/2	2-1/2	7-1/2	1	......	3-15/16	1-7/16	1.14	4-15/16	11	12
3	3	8-1/4	1-1/8	......	4-5/8	1-9/16	1.20	5-11/16	14	16
3-1/2	3-1/2	9	1-3/16	......	5-1/4	1-5/8	1.25	6-5/16	18	20
4	4	10	1-1/4	......	5-3/4	1-3/4	1.30	6-1/516	23	26
5	5	11	1-3/8	......	7	1-7/8	1.41	8-5/16	29	34
									37	46
6	6	12-1/2	1-7/16	......	8-1/8	1-15/16	1.51	9-11/16	56	75
8	8	15	1-5/8	......	10-1/4	2-3/16	1.71	11-15/16	81	120
10	10	17-1/2	1-7/8	15/16	12-5/8	2-3/8	1.92	14-1/16	115	155
12	12	20-1/2	2	1	14-3/4	2-9/16	2.12	16-7/16	155	210
14 O.D.	13-1/4	23	2-1/8	1-1/8	16-1/4	2-11/16	2.25	18-15/16	195	270
16 O.D.	15-1/4	25-1/2	2-1/4	1-1/4	18-3/8	2-7/8	2.45	21-1/16	240	350
18 O.D.	17	28	2-3/8	1-3/8	20-5/8	3-1/8	2.65	23-5/16	300	440
20 O.D.	19	30-1/2	2-1/2	1-1/2	22-3/4	3-5/16	2.85	25-9/16	450	670
24 O.D.	23	36	2-3/4	1-5/8	27-3/8	3-11/16	3.25	30-5/16	....	1070
30 O.D.	29	43	3	2	....	....	....	37-3/16		

* All 250-lb. cast-iron standard flanges have a 1/16 in. raised face. This raised face is included in the minimum thickness of flange dimensions.

† Sizes 14 in. and larger are to be used with O.D. pipe of the same sizes.

‡ All blind flanges for sizes 10 in. (17-1/2 in. O.D.) and larger must be dished, with inside radius equal to the port diameter.

§ This column is the same as column for effective thread (E) in Table 1 of the American Pipe Thread Standard, published by the American Engineering Standards Committee, except in sizes 1-1/4, 1-1/2 and 2 in.

‖ All weights listed are for flanges faced only, based upon minimum thicknesses given in the table above without allowances for variation. Cast iron is considered to weigh 0.26 lb. per cu. in.

Templets for Drilling 250-lb. Pipe Flanges

All dimensions are given in inches

Nominal pipe size	Diameter of raised face	Diameter of bolt circle	Number of bolts *	Diameter of bolts	Diameter of drilled bolt holes *	Length of bolts † ‡ §	Length of bolt-studs with two nuts §	Total effective area bolt metal	Stress lb. per sq. in. bolt metal ‖	Size of ring gasket
1	2-11/16	3-1/2	4	5/8	3/4	2-1/4	.....	0.808	970	1×2-7/8
1-1/4	3-1/16	3-7/8	4	5/8	3/4	2-1/2	.....	0.808	1520	1-1/4×3-1/4
1-1/2	3-9/16	4-1/2	4	3/4	7/8	2-1/2	.....	1.208	1345	1-1/2×3-3/4
2	4-3/16	5	8	5/8	3/4	2-1/2	.....	1.616	1595	2×4-3/8
2-1/2	4-15/16	5-7/8	8	3/4	7/8	3	.....	2.416	2090	2-1/2×5-1/8
3	5-11/16	6-5/8	8	3/4	7/8	3-1/4	.....	2.416	2030	3×5-7/8
3-1/2	6-5/16	7-1/4	8	3/4	7/8	3-1/4	.....	2.416	2460	3-1/2×6-1/2
4	6-15/16	7-7/8	8	3/4	7/8	3-1/2	.....	2.416	3120	4×7-1/8
5	8-5/16	9-1/4	8	3/4	7/8	3-3/4	.....	2.416	4385	5×8-1/2
6	9-11/16	10-5/8	12	3/4	7/8	3-3/4	.....	3.624	3915	6×9-7/8
8	11-15/16	13	12	7/8	1	4-1/4	.....	5.04	4400	8×12-1/8
10	14-1/16	15-1/4	16	1	1-1/8	5	.....	8.80	3625	10×14-1/4
12	16-7/16	17-3/4	16	1-1/8	1-1/4	5-1/2	.....	11.10	3975	12×16-5/8
14 O.D.	18-15/16	20-1/4	20	1-1/8	1-1/4	5-3/4	.....	13.88	3735	13-1/4×19-1/8
16 O.D	21-1/16	22-1/2	20	1-1/4	1-3/8	6	.....	17.86	2255	15-1/4×21-1/4
18 O.D.	23-5/16	24-3/4	24	1-1/4	1-3/8	6-1/4	.....	21.43	4505	17×23-1/2
20 O.D.	25-9/16	27	24	1-1/4	1-3/8	6-1/2	.....	21.43	4845	19×25-3/4
24 O.D.	30-5/16	32	24	1-1/2	1-5/8	7-1/2	9-1/2	31.06	4500	23×30-1/2
30 O.D	37 3/16	39-1/4	28	1-3/4	2	8-1/4	10-1/2	48.89	5590	29×37-1/2
36 O.D.	43-11/16	46	32	2	2-1/4	9-1/4	11-1/2	73.70	5355	34-1/2×44
42 O.D.	50-7/16	52-3/4	36	2	2-1/4	9-3/4	12	82.90	5945	40-1/4×50-3/4
48 O.D.	58-7/16	60-3/4	40	2	2-1/4	10-1/2	13	92.08	7315	46×58-3/4

* Drilling templets are in multiples of four, so that fittings may be made to face in any quarter, and bolt holes straddle the center line. For bolts smaller than 1-3/4 in. the bolt holes shall be drilled 1/8 in. larger in diameter than the nominal size of the bolt. Holes for bolts 1-3/4 in. and larger shall be drilled 1/4 in. larger than nominal diameter of bolts.

† The bolt holes on cast iron flanged fittings are not spot-faced for ordinary service. When required, the fittings and flanges in sizes 36 in. and larger can be spot-faced or back-faced, so that standard length bolts can be used.

‡ All 250-lb. cast-iron standard flanges have a 1/16 in. raised face. This raised face is included in the the face to face center to face and the minimum thickness of flange dimensions.

§ Bolts shall be of steel with standard "Rough Square Heads" and the nuts shall be of steel with standard "Rough Hexagonal" dimensions; all as given in the "Tentative American Standard on Wrench Head Bolts and Nuts, and Wrench Openings." For bolts 1-3/4 in. in diameter and larger, bolt-studs with a nut on each end are recommended. Hexagonal nuts for pipe sizes 1 in. to 16 in. can be conveniently pulled up with open wrenches of minimum design of heads. Hexagonal nuts for pipe sizes 18 in. to 48 in. can be conveniently pulled up with box wrenches.

‖ The stress shown is that of internal pressure only assumed to act on a circular area equal in diameter to the outside diameter of the raised face.

Note. For tongue-groove and male-female facings it is recommended that the dimensions given in Table 5 of the Tentative American Standards for Steel Pipe Flanges and Flanged Fittings be used.

52. Ropes and Chains

Weight and Strength of New Manila Rope

Eng. News, Dec. 6, 1890

Diameter, in.	Circumference, in.	Weight of 100 ft. of rope, lb.	Ultimate tensile strength of rope, lb., calculated by the formulas of	
			Hunt	Miller
3/16	9/16	2	230	280
5/16	1	4	630	790
3/8	1-1/8	5	900	1 140
1/2	1-1/2	7-2/3	1 620	2 020
5/8	2	13-1/3	2 880	3 380
13/16	2-1/2	20	4 500	5 030
1	3	28-1/3	6 480	7 020
1-1/8	3-1/2	38	8 820	9 370
1-5/16	4	52	11 500	12 000
1-1/2	4-1/2	65	14 600	14 900
1-5/8	5	80	18 000	18 100
2	6	113	25 900	25 200
2-1/4	7	153	35 300	34 300
2-5/8	8	211	46 100	44 800
3	9	262	58 300	56 700
3-1/4	10	325	72 000	70 000

"Stevedore" Rope is made by lubricating the fibers with plumbago mixed with sufficient tallow to hold it in position, so as to prevent the internal chafing and wear due to the friction between the several strands and the yarns of these strands in passing over sheaves. After running awhile the exterior of the rope becomes compressed and coated with the lubricant. The breaking strength (according to Hunt), in pounds, may be taken at 720 times the square of the circumference in inches; and the weight in pounds per foot at 0.032 times square of circumference in inches.

Tests of Cordage

Tests of Metals, Watertown Arsenal

Circumference		Actual diameter, in.	Number of strands	Yarns per strand	Lay, one turn, in.	Tensile strength, lb.
Nominal	Actual, in.					
		Manila rope				
6 threads	0.99	.29	3	2	.90	620
9	1.22	.37	3	3	1.02	1 250
9	1.30	.42	3	3	1.25	1 260
15	1.55	.49	3	5	1.42	1 860
15	1.64	.52	3	5	1.60	1 620
21	1.80	.57	3	7	1.90	2 640
1 inch	1.10	.34	3	3	1.02	820
1-1/2	1.64	.56	3	6	2.04	2 550
2	2.53	.84	4	8	2.72	4 800
2-1/2	2.85	.92	4	11	3.16	5 790
3	3.52	1.10	4	16	3.54	8 050
3-1/2	4.30	1.38	4	23	4.18	11 800
4	4.60	1.52	4	30	4.24	14 700
4-1/2	5.02	1.66	4	35	4.40	17 050
5	5.32	1.80	4	40	4.70	18 100
5-1/2	6.02	2.00	4	50	5.40	18 750
6	7.10	2.33	4	62	5.80	25 300
7	8.02	2.63	4	90	6.60	29 400
		Hemp rope				
2-1/2	3.00	.96	3	15	2.80	3 800
3	3.20	1.02	4	19	2.70	5 140
3	3.33	....	4	17	....	5 960
3-1/2	3.87	1.26	4	26	3.16	6 100
3-1/2	3.93	....	4	23	....	7 850

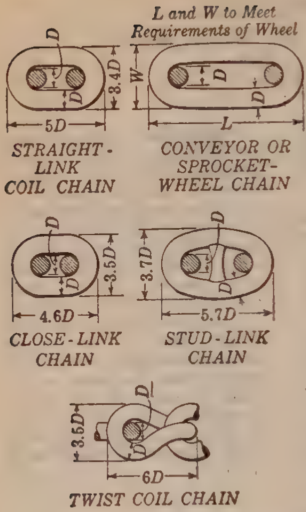


Fig. 69

Chains are classified as follows in the 1927 Standards of the American Society for Testing Materials:

Class AA. Wrought iron chain made from iron free from any admixture of scrap or of steel, for slings, cranes, hoists, steam shovels, and marine uses.

Class A (**Dredge Chain**). Made from puddled iron or a mixture of puddled iron and reworked scrap for the same uses as noted for Class AA.

Class BBB. Wrought iron or open-hearth steel chain for ordinary slings and hoists.

Class BB. Wrought iron or open-hearth steel chain for railroads and construction work.

Class Proof Coil. Wrought iron or open-hearth steel chain for railroad cars, construction, and forestry work.

The tensile strength of chain as determined by tests of lengths not less than 2 ft., cut from the finished chain shall not be less than the values given in the accompanying table. Under a proof load of half the required tensile stength the chain shall not show any defects.

Required Tensile Strength of Chain

1927 Standards of the American Society for Testing Materials. Values are in pounds

Nominal size of chain bar, in.	Approximate weight of 100 ft. of chain	Class AA, wrought iron and Class A (dredge) iron	Class BBB, iron or steel	Class BB, iron or steel	Class proof coil, iron or steel
1/4	75	....	4 500	4,050	3 400
5/16	112	....	6 800	6 100	5 300
3/8	162	8 700	9 300	8 350	7 700
7/16	215	11 900	12 500	11 250	10 500
1/2	270	15 600	16 500	14 850	13 700
9/16	345	19 800	20 500	18 450	17 300
5/8	425	24 300	25 000	22 500	21 400
3/4	605	35 000	35 400	31 800	30 700
7/8	815	47 600	48 000	43 200	41 800
1	1030	62 200	62 000	55 800	54 700
1-1/8	1300	78 700	78 000	70 200	....
1-1/4	1585	97 200	96 000	86 400	....
1-3/8	....	117 600	....	....	....
1-1/2	....	140 000	....	....	....
1-5/8	....	163 000	....	....	....
1-3/4	....	187 800	....	....	....
1-7/8	....	214 200	....	....	....
2	....	242 300	....	....	....

Fig. 69 shows the approximate proportions for common types of welded-link chain. Up to about 2 in. diameter chain is made of wrought iron or mild steel with the links forge-welded. Large anchor chains made as steel castings have come into successful use during the past few years, although many large anchor chains are still forged and welded.

53. Wire and Wire Gages

Sizes and Weights of Steel Wire

(American Steel and Wire Co.'s Gage)

Number of gage	Diameters: Fractions of inch	Diameters: Decimals of inch	Diameters: Millimeters	Sectional area, sq. in.	Weight: Pounds per foot	Weight: Pounds per mile	Number of feet per pound
....	1/2	.5000	12.70	.19635	.6668	3521.	1.500
0000000	....	.4900	12.45	.18857	.6404	3381.	1.562
....	15/32	.46875	11.91	.17257	.5861	3094.	1.706
000000	....	.4615	11.72	.16728	.5681	2999.	1.76
....	7/16	.4375	11.11	.15033	.5105	2696.	1.959
00000	....	.4305	10.93	.14556	.4943	2610.	2.023
....	13/32	.40625	10.32	.12962	.4402	2324.	2.272
0000	....	.3938	10.00	.12180	.4136	2184.	2.418
....	3/8	.3750	9.525	.11045	.3751	1980.	2.666
000	....	.3625	9.2075	.10321	.3505	1851.	2.853
....	11/32	.34375	8.731	.092806	.3152	1664.	3.173
00	....	.3310	8.407	.086049	.2922	1543.	3.422
....	5/16	.3125	7.938	.076699	.2605	1375.	3.839
0	....	.3065	7.785	.073782	.2506	1323.	3.991
1	....	.2830	7.188	.062902	.2136	1128.	4.681
....	9/32	.28125	7.144	.062126	.2110	1114.	4.74
2	....	.2625	6.668	.054119	.1838	970.4	5.441
....	1/4	.2500	6.350	.049087	.1667	880.2	5.999
3	....	.2437	6.190	.046645	.1584	836.4	6.313
4	....	.2253	5.723	.039867	.1354	714.8	7.386
....	7/32	.21875	5.556	.037583	.1276	673.9	7.835
5	....	.2070	5.258	.033654	.1143	603.4	8.750
6	....	.1920	4.877	.028953	.09832	519.2	10.17
....	3/16	.1875	4.763	.027612	.09377	495.1	10.66
7	....	.1770	4.496	.024606	.08356	441.2	11.97
8	....	.1620	4.115	.020612	.07000	369.6	14.29
....	5/32	.15625	3.969	.019175	.06512	343.8	15.36
9	....	.1483	3.767	.017273	.05866	309.7	17.05
10	....	.1350	3.429	.014314	.04861	256.7	20.57
....	1/8	.125	3.175	.012272	.04168	220.0	24.00
11	....	.1205	3.061	.011404	.03873	204.5	25.82
12	....	.1055	2.68	.0087417	.02969	156.7	33.69
....	3/32	.09375	2.381	.0069029	.02344	123.8	42.66
13	....	.0915	2.324	.0065755	.02233	117.9	44.78
14	....	.0800	2.032	.0050266	.01707	90.13	58.58
15	....	.0720	1.829	.0040715	.01383	73.01	72.32
16	1/16	.0625	1.588	.0030680	.01042	55.01	95.98
17	....	.0540	1.372	.0022902	.007778	41.07	128.60
18	....	.0475	1.207	.0017721	.006018	31.77	166.20
19	....	.0410	1.041	.0013203	.004484	23.67	223.00
20	....	.0348	.8839	.00095115	.003230	17.05	309.60
21	....	.0317	.8052	.00078924	.002680	14.15	373.10
....	1/32	.03125	.7938	.00076699	.002605	13.75	383.90
22	....	.0286	.7264	.00064242	.002182	11.52	458.40
23	....	.0258	.6553	.00052279	.001775	9.374	563.30
24	....	.0230	.5842	.00041548	.001411	7.45	708.70

For iron wire multiply columns 6 and 7 by 0.98
For copper wire multiply columns 6 and 7 by 1.12.
For other wire gages see next page.

Comparison of Standard Gages

Number of gage	Thickness in decimals of an inch: Birmingham	Browne & Sharpe	United States Standard Plate Iron and Steel	British Imperial	American Steel & Wire Co.	Stubs Steel Wire	Number of gage
0000000	....	....	.500	.500	.4900	....	0000000
000000	....	.58	.46875	.464	.4615	....	000000
00000	.500	.5165	.4375	.432	.4305	....	00000
0000	.454	.46	.40625	.400	.3938	....	0000
000	.425	.40964	.375	.372	.3625	....	000
00	.380	.3648	.34375	.348	.3310	....	00
0	.340	.32486	.3125	.324	.3065	....	0
1	.300	.2893	.28125	.300	.2830	.227	1
2	.284	.25763	.265625	.276	.2625	.219	2
3	.259	.22942	.25	.252	.2437	.212	3
4	.238	.20431	.234375	.232	.2253	.207	4
5	.220	.18194	.21875	.212	.2070	.204	5
6	.203	.16202	.203125	.192	.1920	.201	6
7	.180	.14428	.1875	.176	.1770	.199	7
8	.165	.12849	.171875	.160	.1620	.197	8
9	.148	.11443	.15625	.144	.1483	.194	9
10	.134	.10189	.140625	.128	.1350	.191	10
11	.120	.090742	.125	.116	.1205	.188	11
12	.109	.080808	.109375	.104	.1055	.185	12
13	.095	.071961	.09375	.092	.0915	.182	13
14	.083	.064084	.078125	.080	.0800	.180	14
15	.072	.057068	.0703125	.072	.0720	.178	15
16	.065	.05082	.0625	.064	.0625	.175	16
17	.058	.045257	.05625	.056	.0540	.172	17
18	.049	.040303	.05	.048	.0475	.168	18
19	.042	.03589	.04375	.040	.0410	.164	19
20	.035	.031961	.0375	.036	.0348	.161	20
21	.032	.028462	.034375	.032	.03175	.157	21
22	.028	.025347	.03125	.028	.0286	.155	22
23	.025	.022571	.028125	.024	.0258	.153	23
24	.022	.0201	.025	.022	.0230	.151	24
25	.020	.0179	.021875	.020	.0204	.148	25
26	.018	.01594	.01875	.018	.0181	.146	26
27	.016	.014195	.0171875	.0164	.0173	.143	27
28	.014	.012641	.015625	.0148	.0162	.139	28
29	.013	.011257	.0140625	.0136	.0150	.134	29
30	.012	.010025	.0125	.0124	.0140	.127	30
31	.010	.008928	.0109375	.0116	.0132	.120	31
32	.009	.00795	.01015625	.0108	.0128	.115	32
33	.008	.00708	.009375	.0100	.0118	.112	33
34	.007	.006304	.00859375	.0092	.0104	.110	34
35	.005	.005614	.0078125	.0084	.0095	.108	35
36	.004	.005	.00703125	.0076	.0090	.106	36
37	....	.004453	.006640625	.0068	.0085	.103	37
38	....	.003965	.00625	.0060	.0080	.101	38
39	....	.003531	....	.0052	.0075	.099	39
40	....	.003144	....	.0048	.0070	.097	40

In the U. S. Standard, the numbers correspond to the weight in ounces per square foot and an equal number of 640ths of an inch in thickness.

See p. 688 for fuller data of U. S. standard gage.

Tensile Strength of Wire Rope

(American Steel & Wire Company)

Approximate breaking load in tons of 2000 lb.

Diameter of rope, in.	Wire transmission rope; one hemp core surrounded by 6 strands of 7 wires each				Wire hoisting rope; one hemp core surrounded by 6 strands of 19 wires each			
	Iron	Crucible cast steel	Extra strong crucible cast steel	Plow steel	Iron	Crucible cast steel	Extra strong crucible cast steel	Plow steel
2-3/4	....	....	....	....	111	211	243	275
2-1/2	....	....	....	....	92	170	200	229
2-1/4	....	....	....	....	72	133	160	186
2	....	....	....	....	55	106	123	140
1-3/4	....	....	....	....	44	85	99	112
1-5/8	....	....	....	....	38	72	83	94
1-1/2	32	63	73	82	33	64	73	82
1-3/8	28	53	63	72	28	56	64	72
1-1/4	23	46	54	60	22.8	47	53	58
1-1/8	19	37	43	47	18.6	38	43	47
1	15	31	35	38	14.5	30	34	38
7/8	12	24	28	31	11.8	23	26	29
3/4	8.8	18.6	21	23	8.5	17.5	20.2	23
5/8	6	13	14.5	16	6	12.5	14	15.5
9/16	4.8	10	11	12	4.7	10	11.2	12.3
1/2	3.7	7.7	8.85	10	3.9	8.4	9.2	10
7/16	2.6	5.5	6.25	7	2.9	6.5	7.25	8
3/8	2.2	4.6	5.25	5.9	2.4	4.8	5.30	5.75
5/16	1.7	3.5	3.95	4.4	1.5	3.1	3.50	3.8
9/32	1.2	2.3	2.95	3.4	....	....	....	....
1/4	....	....	....	....	1.1	2.2	2.43	2.65

Wire Rope is made of iron, open-hearth steel, crucible steel, or plow steel wires, and accordingly has an ultimate strength per square inch of 45 000 to 100 000 lb. for iron, 50 000 to 130 000 lb. for open-hearth steel, 130 000 to 190 000 lb. for crucible steel, and 190 000 lb. for plow steel.

The wires are either laid parallel to each other as in suspension bridge cables, or twisted together into strands as in smokestack guys or cases where only moderate flexibility is needed. Wire strands consist of 4, 7, 12, 19, or 37 wires, depending on the work intended. The most pliable rope, for hoisting and transmission, contains 19 wires to the strand, while ropes of 12 or 7 wires to the strand are better adapted for use as standing ropes, guys, or rigging.

Wire rope must not be coiled or uncoiled like hemp rope. When mounted on a reel the latter should be so mounted that the rope may be paid off. When furnished in a coil it should be rolled over the ground like a wheel, and the rope run off in that way. All untwisting and kinking must be avoided.

The Flexibility of Wire Rope depends largely on the size of the individual wires used in making up the strands of the rope, the smaller these wires (and consequently the more numerous) the more flexible the rope. For guy wires and other wire ropes not bent around sheaves wire rope is usually made up of 6 strands of 7 wires each. For hoisting rope which is to pass around sheaves the rope in commonest use has 6 strands of 19 wires each. The makeup of two extra flexible hoisting ropes and the approximate ratio of their strength to that of 6 strand 19 wire rope of the same material and the same nominal size is:

8 strand 19 wires	0.87
6 strand 37 wires	0.95

Bending Stresses in Wire Ropes which pass around sheave wheels are sometimes of considerable magnitude. If a straight rod or wire of diameter d is bent into a circle of diameter D there is set up a bending stress S.

$$S = E\,d/D$$

E is the modulus of elasticity of the material.

In a wire rope bent around a sheave each individual wire is bent into the arc of a circle and in the outer fibers of each wire there are set up bending stresses. The extreme fibers on the outer side of each wire must withstand the combined tensile stress due to bending and the direct tensile stress due to the pull on the wire. If a wire breaks, its share of the direct tensile stress is transferred to the remaining wires, but the bending stress in the outer fibers of the other wires is not increased. Owing to the twisting of the strands the length of wire subjected to bending in a wire rope is greater than that of a straight rod bent to the same size circle. This increased length decreases the stiffness of the wire rope and has the same effect on the bending stress as reducing the value of E in the above formula. For 6-strand ropes tests by the American Steel and Wire Co. give for bending stresses in pounds per square inch:

$$S = 12\,000\,000\,D/d$$

D is the diameter in inches of the sheave wheel to the center of the rope, d is the diameter of the largest individual wires in the rope. In determining allowable tensile load on a wire rope passing around a sheave the stress due to bending should be deducted from the allowable working stress for a straight rope, so that no fiber of any wire shall be over-stressed.

The table on p. 703 gives values of tensile load equivalent to bending wire rope around sheaves of various diameters for two common types of hoisting rope. The values are based on tables given in the wire rope handbook of the American Steel & Wire Co., to which reference may be made for a fuller discussion of bending stresses.

Working Loads on Wire Ropes. Since wire ropes are used only in tension the ultimate tensile strength is a good basis for estimating working strength. After making allowance for direct load stresses, bending stresses, and other known stresses the working load should be taken at from 10 to 25% of the estimated breaking load. Factors which tend to reduce the allowable working load are: disastrous results of failure, especially danger to life and limb; high speed of rope travel; difficulty of inspection, length of time between inspection; presence of corroding agencies. A good general rule is to keep the working load below 1/5 of the estimated breaking load. If reasonably effective inspection service is available for a rope the use of a low factor of safety usually means short life of the rope rather than a disastrous failure.

Strength of Wire Rope Fastenings. The best fastenings for a wire rope is a steel socket with the wire rope fastened into it by means of zinc cast around the wire. When such sockets are fitted by skilled workmen under shop conditions their strength may be made equal to the strength of wire rope itself. Made under field conditions the strength of such sockets will hardly be greater than 75% of the values given for wire rope if the work is carefully done, and the strength may fall far below this if the fitting of the rope to sockets is not carefully done. Temporary fastenings for wire rope in the form of clips or clamps are often used. To develop the maximum strength of such a fastening four clips should be used for small-sized ropes and six clips for the larger sizes. The holding power of clips or clamps depends to a large extent on the care with which all the clips are tightened and kept tight. In any event clips or clamps weaken the rope by "crimping" it at the point of application. Carefully made clip or clamp fastenings may develop 60 to 75% of the full strength of a wire rope.

Effect of Bending Wire Ropes Around Sheaves and Drums

(American Steel and Wire Company)

The effect of bending a wire rope around a sheave or drum is equivalent to a pull on the rope of the value (in tons of 2000 lb.) given below.

Diameter of wire rope, in.	Diameter of sheave or drum, in feet								
	20	15	10	8	6	4	3	2	1
	Ropes of 6 strands 7 wires to a strand								
1-1/2	......	5.04	7.56	9.45	12.60	18.90	......	......	......
1-3/8	......	3.88	5.82	7.27	9.70	14.54	......	......	......
1-1/4	......	2.91	4.37	5.46	7.28	10.92	......	......	......
1-1/8	......	2.09	3.14	3.92	5.24	7.84	......	......	......
1	......	1.49	2.24	2.80	3.74	5.60	7.48	......	......
7/8	......	1.03	1.55	1.94	2.58	3.88	5.16	......	......
3/4	......	0.63	0.94	1.18	1.56	2.36	3.12	4.72	......
5/8	......	0.37	0.55	0.69	0.92	1.38	1.84	2.76	......
9/16	......	0.26	0.39	0.49	0.66	0.98	1.32	1.96	......
1/2	......	0.19	0.28	0.35	0.46	0.70	0.92	1.40	......
7/16	......	0.13	0.19	0.24	0.32	0.48	0.64	0.96	......
3/8	......	0.08	0.12	0.15	0.20	0.30	0.40	0.59	......
5/16	......	0.04	0.07	0.09	0.12	0.18	0.23	0.35	......
9/32	......	0.03	0.05	0.06	0.08	0.12	0.16	0.24	......
	Ropes of 6 strands 19 wires to a strand								
2-3/4	11.63	15.51	23.26	29.08	38.78	......	......	......	......
2-1/2	8.74	11.65	17.48	21.84	29.14	......	......	......	......
2-1/4	6.37	8.49	12.74	15.92	21.22	31.84	......	......	......
2	4.48	5.97	8.96	11.20	14.94	22.40	......	......	......
1-3/4	3.00	3.99	5.99	7.48	9.98	14.96	19.96	......	......
1-5/8	2.40	3.20	4.80	6.00	8.00	12.00	16.00	......	......
1-1/2	1.88	2.51	3.77	4.72	6.28	9.44	12.56	......	......
1-3/8	1.46	1.94	2.91	3.64	4.84	7.28	9.68	14.56	......
1-1/4	1.09	1.45	2.18	2.72	3.64	5.44	7.28	10.88	......
1-1/8	0.80	1.06	1.59	1.98	2.66	3.96	5.32	7.92	......
1	0.56	0.75	1.12	1.40	1.86	2.80	3.72	5.60	......
7/8	0.37	0.50	0.75	0.94	1.26	1.88	2.52	3.76	7.52
3/4	......	......	0.47	0.59	0.80	1.18	1.60	2.36	4.72
5/8	......	......	0.27	0.34	0.46	0.68	0.91	1.36	2.72
9/16	......	......	0.20	0.24	0.33	0.48	0.66	0.96	1.92
1/2	......	......	0.14	0.17	0.23	0.34	0.46	0.68	1.36
7/16	......	......	......	0.12	0.15	0.24	0.30	0.47	0.94
3/8	......	......	......	......	0.10	0.15	0.20	0.30	0.60
5/16	......	......	......	......	0.06	0.09	0.12	0.17	0.34
1/4	......	......	......	......	0.03	0.05	0.06	0.09	0.17

54. Terra Cotta

Terra Cotta is either dense, semi-porous, or porous. The dense tile is very hard and is used for floor arches because of its high crushing strength. Porous and semi-porous terra cotta is made by mixing sawdust with the clay, the sawdust being destroyed during the burning operation. In the porous tile the proportion of sawdust is about 25 to 35%; in the semi-porous tile it is about 20%. Porous terra cotta can be cut with a saw or edge tools, and nails may be easily driven into it.

In a series of tests at Columbia University for the New York Building Department, terra-cotta blocks secured in the open market developed a net crushing stress for dense tile of 5820 lb. per sq. in., and for semi-porous terra-cotta of 3292 lb. per sq. in., the figures in each case being the average results of ten tests. See also p. 645.

Hollow Terra-Cotta Blocks

(National Fireproofing Co.)

Thickness, in.	Partition blocks		Wall-furring			Roof blocks		
	Weight, lb. per sq. ft.	Size of block, in.	Thickness, in.	Weight, lb. per sq. ft.	Size of block, in.	Thickness, in.	Weight, lb. per sq. ft.	Size of block, in.
2	14	6×12	1-1/2	9	12×12	3	20	12×18
2	14	8×12	2	10	12×12	3	20	12×20
2	14	12×12	Hollow brick Haverstraw size			3	20	12×24
3	17	6×12				4	22	12×24
3	17	8×12						
3	17	12×12	Brick	Size, in.	Weight, lb.	Ceiling blocks		
4	18	6×12						
4	18	8×12						
4	18	12×12						
5	20	8×12	Stretcher.	2-1/8×3-1/2×8	3	2	12	12×16
5	20	12×12	Header. . .	2-1/8×3-1/2×7-1/4	2-1/2	2	12	12×18
6	26	8×12	Porous. . .	2-1/4×3-3/4×8	2-1/2	2	12	12×20
6	26	12×12	stretcher			3	20	12×16
7	29	12×12	Solid	2-1/4×3-3/4×8	3-1/3	3	20	12×18
7	29	12×12	porous			3	20	12×20
8	32	12×12	stretcher			3	20	12×24
9	36	12×12						
10	38	12×12						
12	42	12×12						

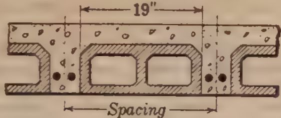
55. Gypsum

Sizes and Weights of Gypsum Tile

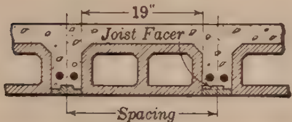
(U. S. Gypsum Co.)

Size of gypsum tile, in.	For ceiling heights up to	Weight, tile, lb. per sq. ft.	Weight, mortar, lb. per sq. ft.	Weight, plaster, one side, lb. per sq. ft.	Total weight plastered one side, lb. per sq. ft.	Weight, plaster, two sides, lb. per sq. ft.	Total weight plastered two sides, lb. per sq. ft.
1-1/2-in. split (1-1/2×12×30)	Furring	4.9	1.00	3	7.9	6	10.9
2-in. split (2×12×30)	Furring	6.4	1.00	3	9.4	6	12.4
2-in. solid (2×12×30)	10 ft.	9.4	1.00	3	12.4	6	15.4
3-in. hollow (3×12×30)	13 ft.	9.9	1.2	3	12.09	6	15.9
3-in. solid (3×12×30)	15 ft.	12.4	1.2	3	15.4	6	18.4
4-in. hollow (4×12×30)	17 ft.	13.0	1.63	3	16.00	6	19.0
5-in. hollow (5×12×30)	25 ft.	15.6	2.04	3	18.60	6	21.6
6-in. hollow (6×12×30)	28 ft.	16.6	2.45	3	19.60	6	22.6
8-in. hollow (8×12×30)	40 ft.	22.4	3.26	3	25.40	6	28.4

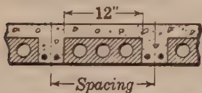
Fig 70 shows, in diagram the application of gypsum floor voids to a beam and girder concrete floor.



19" FLOOR VOID, WITHOUT JOIST FACERS



19" FLOOR VOID WITH JOIST FACERS



12" FLOOR VOID WITHOUT JOIST FACERS

Fig. 70

Gypsum Floor Voids

(U. S. Gypsum Co.)

Size each dimension in inches	Weight, per linear foot, lb.	Size each dimensions in inches	Weight, per linear foot, lb.
3×12×30	11	6×19×18	26
4×12×30	15	8×19×18	29
5×12×30	20	10×19×18	35
6×12×30	23	12×19×18	41

56. Slates and Shingles

Number of Slates and Pounds of Nails per Square of 100 Sq. Ft.

Size of slate, in.	Number of inches exposed when laid	Number per square	Weight of galvanized nails per square, lb. oz.	Size of slate, in.	Number of inches exposed when laid	Number per square	Weight of galvanized nails per square lb. oz.
14×24	10-1/2	98	4d. 1 6	12×16	6-1/2	185	3d. 2 2
12×24	10-1/2	115	1 10	10×16	6-1/2	222	2 8
12×22	9-1/2	126	1 12	9×16	6-1/2	247	3 0
11×22	9-1/2	138	1 15	8×16	6-1/2	277	3 2
12×20	8-1/2	142	2 0	10×14	5-1/2	262	3 0
10×20	8-1/2	170	2 6	8×14	5-1/2	328	3 12
12×18	7-1/2	160	3d. 1 13	7×14	5-1/2	374	4 4
10×18	7-1/2	192	2 3	8×12	4-1/2	400	4 9
9×18	7-1/2	214	2 7	7×12	4-1/2	458	5 3
				6×12	4-1/2	533	6 1

Number and Weight of Pine Shingles per Square of 100 Sq. Ft.

Length, in.	Width, in.	Number of inches exposed to weather	Number of shingles per square	Weight of shingles per square, lb.	Pounds of nails per 1000 shingles
16	4	4	900	188	5 (approx.)
16	4	4-1/2	800	172	5
16	4	5	720	156	5
16	4	5-1/2	655	140	5
16	4	6	600	124	5

Roofing Slate is supplied commercially in thicknesses of 1/8, 3/16, 1/4, increasing by 8ths to 1 in. Slate roofing as laid weighs about 6.5 lb. per sq. ft. for 3/16-in. thickness, and 8.75 lb. for 1/4-in. thickness, the smaller sizes weighing more on account of lap.

Wood Shingles are made from cypress, redwood, cedar, pine, and spruce, this being the order of their durability. Redwood is much less inflammable than any of the others. For hip roofs, 5% should be added to the quantities in the table to allow for cutting, and for irregular roofs with dormer windows 10% should be added.

Common shingles are of random widths, varying from 2 to 14 in. They come in bundles, four of which contain a " thousand " or the equivalent of 1000 shingles 4 in. in width. The above table makes no allowance for waste.

SECTION 8

FOUNDATIONS AND EARTHWORK

BY

WALTER J. DOUGLAS *

MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

FOUNDATIONS ON LAND †

FOUNDATIONS UNDER WATER†

EARTHWORK † ‡

* This section takes the place of part of Section 6 of previous editions, which was written by the late Prof. Ira O. Baker. All changes and revisions of these chapters have been prepared by Mr. Douglas.

† H. A. Foster assisted in writing these chapters.

‡ W. H. Correale assisted in writing this chapter.

FOUNDATIONS ON LAND

1. Examining the Site

If the nature of the soil has not already been revealed to a considerable depth by excavation it will be necessary to make an examination of the subsoil preparatory to deciding upon the details of the method of constructing the foundation. In the case of simple structures such as dwellings, it is usually sufficient in advance of design and estimates to dig one or two test pits to the depth of the proposed foundation and examine the soil for 4 or 5 ft. below by means of a post auger or iron bar. Generally, however, it is necessary to examine the soil to greater depths, as hereafter described. The examination of the foundation soil other than ledge rock should be extended to a depth at least equal to the width of the largest footing.

Test Pits. The simplest method of examining the soil is by digging open test pits. Test pits, however, are expensive if carried over 10 or 12 ft. in depth, especially if there is much ground-water. The best practice is to sink one or two test pits for each foundation area of a building, dam, or bridge foundation (if the foundation is to be built on land) and then check the remaining area with probings or borings. Test pits have the advantage of permitting the direct examination of the material in place and its degree of compactness and of permitting test loads on the foundation bed.

Soundings can be made in soft soil to a depth of 20 or 30 ft. by driving a rod or sections of gas-pipe with a hammer or maul from a temporary scaffold, the height of which will of course depend upon the length of the rod or of the sections of the pipe. Good judgment is required in interpreting the results of such tests, particularly if the structure is to be a heavy one or a bridge abutment or pier in a stream liable to scour. A layer of compact sand or cemented gravel, which may be scoured away, may be mistaken for a ledge of rock; but the difference can usually be detected by striking the rod or pipe with a hammer, since rock will give a decided rebound, while gravel or sand will not. A boulder may be mistaken for bedrock; but the difference can usually be detected by making one or more additional tests, and accurately noting the depths at which rock is struck. If samples of the soil are desired, use a 2-in. pipe open at the lower end. If much of this kind of work is to be done, it is advisable to fit up a hand piledriving machine, using a block of wood for the dropping weight.

Borings 50 to 100 ft. deep can be made very expeditiously in common soil or clay with a common wood-auger turned by men with levers 3 or 4 ft. long. Or the boring may be made with any one of several earth augers having a spoon-like form for bringing up samples of the soil. An auger will bring up samples sufficient to determine the nature of the soil, but not its compactness, since it will probably be compressed somewhat in being cut off. When the testing must be made through sand or loose soil, it may be necessary to drive down a steel tube to prevent the soil from falling into the hole. The sand may be removed from the inside of this tube with an auger, or with the "sand-pump" used in digging artesian wells.

Water Jet and Wash Borings. In soft soil or clay that can be washed with a stream of water, a hole can be sunk rapidly by driving a pipe, inserting a smaller pipe inside of it, and forcing water down the inside pipe, the débris and water flowing up between the two pipes. See also Sect. 15, Art. 29.

Caution should be used in interpreting the results of wash borings unless suitable undisturbed samples of the soil are secured from the bore hole at intervals. Such samples have generally been secured by driving a pipe into the undisturbed soil beneath the bore hole and withdrawing a sample with this pipe, but this method tends to compact the soil or to alter its condition. No entirely satisfactory means have been developed for securing samples in an undisturbed condition.

Drilling. When the subsoil is composed of various strata, particularly if there are strata of hard soil or rock, it is necessary to use a percussion drill in connection with some form of core drill; and in extreme cases the diamond drill is employed. Great care is needed in interpreting the results of such borings. In using the percussion drill, care must be taken that a stratum sufficiently hard to serve as a foundation is not passed by unnoticed. This can be prevented by taking dry cores at frequent intervals. In using a core drill care must be taken to discriminate between erratic boulders and native ledge rock. See also Sect. 15, Art. 30.

Cost of Explorations. Engineers usually do not spend nearly enough money upon the investigation of foundation conditions in advance of design, making of contracts and actual construction. This often results in (1) change of design after binding estimates of costs or contracts have been entered into, resulting in embarrassing added costs for which funds are not available; (2) lawsuits between the client and contractor; (3) disaster. The engineer usually has difficulty in getting his client or company to provide sufficient funds for such investigation; nevertheless he will be blamed for any added cost resulting from lack of advance explorations. One per cent of the estimated cost of construction may not be excessive to secure in advance sound data upon which to predicate designs, estimates and contracts.

If thorough explorations are not made before preparation of designs, estimates and contracts, the actual cost of foundations may prove to be five or six times as great as the engineer and his client or company were led to expect.

2. Bearing Capacity of Soils

The rational design of the foundation of any structure must be based on an assumed value for the safe bearing power of the underlying soil. For unimportant structures, the bearing power may be estimated from the table (p. 711) after determining the character of the soil by observation. For heavier loads, the soil should be carefully examined and classified, and its bearing power determined by direct test, as explained in Art. 3. If a soil of the same character in some other location has been tested or loaded, and its behavior is already known, this may be used as a guide in selecting a suitable value for the working load.

Ultimate bearing capacity is the maximum load in short tons per square foot which can be applied to a given area of ground without causing sudden or rapid settlement. The **bearing power** of a soil is the load in tons per square foot which may be applied to a given area of ground without causing a total settlement of more than a given amount. The **allowable soil pressure** is the load in tons per square foot which is considered as applied to the ground in designing the foundation, when the maximum assumed loading is transmitted by the structure.

Practical considerations will generally control the allowable soil pressure with reference to the bearing power or ultimate bearing capacity. Inequalities of bearing power cause unequal settlement under a uniform load. Where this would endanger the structure, as in the case of a tall chimney, the foundation pressures should be kept low. In a heavy foundation over a large area, the pressures may be somewhat greater, as unequal settlement would not have such serious results. The pressure per unit of area should be less for a light structure subject to the passage of heavy loads than for a heavy structure subject only to a quiescent load, since the shock and jar of the moving load are far more serious than the heavier quiescent load.

Classification of Soils. In determining the safe bearing power of a soil by comparison with similar soils in other locations, it is essential that the material be accurately classified. As different soils are frequently given the same name in different localities, the following definitions are given to eliminate some of the resulting confusion (see Report of Committee on Soils, Proc. Am. Soc. C. E., Feb., 1921):

Adobe is a sandy, calcareous clay occurring in the southwestern part of the United States.

Alluvium is the finer deposit of earth, sand, gravel and other transported material which has been washed away and deposited by rivers, floods, or other causes on land not permanently submerged.

Bog is a quagmire covered with grass or other plants.

Clay is a general name for cohesive soils, which are firmly coherent, weighty, compact, and hard when dry, but stiff, viscid and ductile when moist, and smooth to the touch. Clay absorbs water greedily but not readily, is diffusible in water and, when mixed, does not readily subside in it. Other characteristics of clay are discussed in Art. 3.

Drift or **glacial drift** is a deposit of loose detrital material, fragments of rocks, boulders, sand, gravel, or clay, or other soil driven together, or a mixture of two or more of these deposits, resting on the surface of the bedrock.

Gravel consists of small stones or fragments of stone or very small pebbles larger than the particles of sand, but often mixed with them.

Grit consists of angular, rough, hard particles of sand or gravel in a loose form.

Gumbo is a dark-colored, very sticky, highly plastic clay, occurring abundantly in the central and southern parts of the United States.

Hardpan is a rather loosely used term, but is most commonly applied to a very dense heterogeneous mass of clay, sand and gravel of glacial drift origin; it is also applied to the hard stratum of consolidated soil underlying the surface soil.

Loam is a mixture of sand and clay with oxide of iron, and generally a varying amount of organic matter.

Peat is a brown soil of vegetable origin consisting of partly decomposed roots and fibers, more or less saturated with water.

Quicksand is a condition rather than a type of soil; it is generally a fine granular soil, supersaturated with water temporarily and when under pressure acting as a fluid.

Sand is any mass or collection of fine particles of stone, particularly of siliceous stone, but not strictly reduced to powder or dust.

Silt is a fluvial sediment of mud or fine soil deposited from running or standing water.

"In classifying soils, it is a decided advantage to the investigator to be able to check his estimates by rubbing samples of the soil between his fingers. As a rule sand feels gritty; silts are velvety, smooth and floury; and clays, which are smooth and plastic, offer considerable resistance to pressure, roll into balls when moist and adhere to the fingers. By the degrees in which they combine these salient characteristics of the primary constituents, soils other than pure sands, silts and clays may be distinguished as sandy loams, silt-loams, sandy clays, clay-loams, silty clay-loams and silty clays." (From "Public Roads," Vol. 6, No. 5, July 1925.)

Allowable Soil Pressure. Where the unit load on the foundation of an unimportant structure is to be assumed, without making any field tests of the soil, the values of allowable soil pressure given in the following table may be used for design. The writer, however, wishes to emphasize that, for any important structure, a thorough examination of the soil is essential, together with suitable loading tests. The accuracy of the allowable pressures given in this table depends entirely upon the proper classification of the soil. Unfortunately soil classification is still too indefinite for accurate use with tabulated allowable soil pressures. In classifying a soil, it should be noted that a dry soil may become wet due to variations in the ground-water level caused by floods, or by leaking or broken water mains.

Allowable Soil Pressure in Short Tons per Square Foot

Kind of material	Minimum	Maximum
1. Quicksand; alluvial soil	0.5	1
2. Soft clay	1	2
3. Wet clay; soft wet sand	1	2
4. Moderately dry sand; fine sand, clean and dry	2	3
5. Clay and sand in alternate layers	2	3
6. Firm and dry loam or clay; hard dry clay or fine sand	2	5
7. Compact coarse sand; stiff gravel	3	6
8. Coarse gravel; stratified stone and clay; rock inferior to best brick masonry	5	8
9. Gravel and sand well cemented	6	10
10. Good hardpan or hard shale	6	10
11. Good hardpan or hard shale unexposed to air, frost or water	10	15
12. Very hard native bedrock	15	25
13. Very hard native bedrock, in thick layers, under caisson	30	

In many cities, the local building codes specify the maximum allowable soil pressures which may be used in foundation design and these must govern.

Rock. Solid rock provides the best foundation. The ultimate compressive strength of good stone, as determined by crushing cubes, is greatly in excess of the strength of stone masonry. The crushing strength of slabs is greater than that of cubes; and the crushing strength of slabs when the pressure is concentrated on only a portion of the upper surface is much greater than for a load uniformly distributed over the entire upper surface. Therefore it is safe to say that any ordinary rock (except the weaker shales) in its native bed will bear any load that can be brought upon it by an artificial structure.

Great care must be taken in limestone areas where the rock may be dissolved by ground waters, and caves may occur close to the surface. In such cases, the engineer should seek advice from persons familiar with local geological conditions and with due appreciation of the engineering problems involved, in deciding on the extent and location of testing or drilling necessary to determine the allowable bearing pressure. Similar precautions should, of course, be taken in mining regions, especially where there is a possibility of abandoned workings extending under the site of the proposed structure.

Even when the engineer has every reason to believe that visible rock is solid ledge, he should sink drill or bore holes from 10 to 30 feet into the rock to verify its soundness.

In preparing a rock foundation, the loose and decayed portions of the rock should be removed, and the surface of the rock should be cut down or stepped if necessary in order to eliminate any possibility of the structure slipping on it. This precaution is particularly important where the rock strata dip at an angle with the horizontal, or the structure is subject to lateral thrust as in a retaining wall or dam.

Engineers who are not experienced geologists should consult a practical geologist concerning the broad foundation problem of important structures such as high dams. The geologist should be able to throw some additional light upon (a) the location of any pre-glacial river beds with relation to ledge rock; (b) the probable dip of the rock at various points; (c) the probable tightness of the reservoir banks; (d) probability of sulfur or other gases being encountered in the excavation.

3. Foundations on Soft Ground

Sand and Gravel. These soils may vary from coarse gravel to fine sand or a mixture of various sizes and proportions of sand and gravel. Coarse gravel when of sufficient thickness forms one of the firmest and best foundations, whereas fine sand when saturated with water is practically a liquid. Sand when dry, or wet sand when prevented from spreading laterally, forms one of the best beds for a foundation. Porous, sandy soils are, as a rule, unaffected by water that is not in motion, but are easily removed by running water, as with water seeping under a dam; in the former case they present no difficulty, but in the latter they require extreme care in the hands of the designer and constructor. Sand and gravel are but little affected by frost unless the water table is near the ground surface. Compact sands and gravels have the advantage of being only slightly compressed under load, this compression occurring when the load is first applied and not being extended over a period of time as with clay soils. Loose gravels and sands may have high ultimate bearing capacity but are compressed under load, causing settlement.

Compact sand or gravel, in beds of considerable thickness and protected against flowing water, and from lateral flow of the sand under pressure, may be loaded with 4 to 10 tons per sq. ft. This statement is given with a warning against the use of such high unit pressures without most thorough study and verification by test loads. Examples of loads on sand and gravel foundations are given below.

	Load, tons per sq. ft.	
Cincinnati, Suspension Bridge:		
Coarse gravel, 12 feet below water.........	4	Disregarding friction on sides of pier.
Brooklyn Bridge, Manhattan pier:		
2 ft. of sand (artificial cushion to reduce injurious vibration) over bedrock, 44 ft. below river bed.	6-3/4	Sand confined within a cup excavated in rock.
Chicago:		
Sand and gravel about 15 ft. below surface.	2 to 2-1/2	
Berlin:		
Sandy soil..................................	2 to 2-1/2	
Washington Monument:		
Gravel.....................................	5.25 to 6	

Vibrations may cause settlement of sand foundations. The same foundation which will sustain 2 tons per sq. ft. static load without any measurable settlement may settle irregularly several inches if the same pressure is exerted by machinery producing violent vibrations. One-half the allowable static pressure is suggested for machinery foundations and two-thirds for railroad bridge and retaining wall foundations.

Clay Soils. The Committee on Soils of the American Society of Civil Engineers (Proc., March, 1922) defines clay as follows: "Clay is an adventitious mixture of inert materials such as sand and a complex compound possessing colloidal properties such as silicates of aluminum, iron, the alkalies and the alkaline soils. Some clays are not wholly colloidal but may resemble a mass of mineral particles partly covered with a colloidal substance. An excessive proportion of sand will produce a sandy clay, and if the colloidol substance is in excess the clay is plastic."

Clay soils vary from slate or shale, which generally will safely support most masonry loads, to a soft wet clay which will squeeze out in every direction when a moderately heavy pressure is brought upon it. Some shale though hard when first exposed becomes soft when exposed to air or water. Whether or not this will occur can usually be determined by observations of short duration and by immersion in water. Foundations on clay should be placed at such depths as to be unaffected by the weather, since clay, at even considerable depths, will gain and lose an appreciable amount of water as the seasons change. The bearing capacity of clay soils can be very much improved by preventing the penetration of water. The drainage of clays is usually a difficult matter and sometimes impossible. The writer has, however, noted in the Province of Ontario clays containing holes or natural pipes which carry water readily and where drainage would be helpful. If the foundation is laid upon wet and undrained clay, care must be taken that excavations made in the immediate vicinity do not allow the clay under pressure to escape by oozing away from under the structure. When the clay occurs in strata which dip toward the excavation, great care is necessary to prevent flow of the soil. There are few foundation problems more difficult than those in which a heavy foundation is placed upon wet or moist layers of clay dipping toward the lower ground. One must consider the possibility of the added pressure causing the upper foundation layers of clay to slide upon the lower ones. A heavy structure built upon moist or wet clay (Fig. 1) might cause, first, the failure of the retaining wall and, second, disaster to the main structure. It also requires an abnormally heavy retaining wall to withstand a wet clay bank. Even a masonry wall with base equal to its height may not suffice.

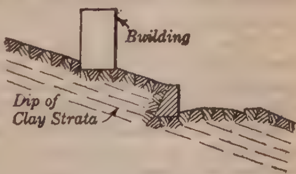


Fig. 1

When coarse sand or gravel is mixed with the clay, its supporting capacity is greatly increased, being greater in proportion as the quantity of these materials is greater. When they are present to such an extent that the clay is just sufficient to bind them together, the combination will bear nearly as heavy loads as the softer rocks. The bearing capacity of soft clays may be increased by the use of "sand piles" (Art. 5).

A proper understanding of the action of clay under load requires a knowledge of its physical properties. Considerable research has been carried out on this subject in recent years. Many investigators have claimed that the properties of clay can be traced to the presence of colloidal substances in the clay. Although the theory of colloids has not been fully developed, perhaps engineers may get some light upon the action of soils, particularly clays, by trying to understand the chemist's idea of "colloidal properties."

A Colloid is a "substance of gelatinous nature, permeable by crystalloid solutions, diffusing not at all or very slowly through animal or vegetable membranes" (Webster's New International Dictionary). A colloidal solution is regarded as a suspension of very fine particles in some liquid. The particles are so small that their total superficial area is very large compared with their mass. There is no sharply defined limit to the size of colloidal particles, but it is generally assumed that particles which are smaller in average diameter than one micron (.001 mm.) are colloidal. A colloidal solution differs from a true solution in that the particles are much larger, though there is no definite line of distinction between the two—a true solution containing particles under 0.001 micron in diameter.

Many of the properties of clay have been explained by assuming the presence of colloidal substances. However, it seems to be quite impossible, with present knowledge, to be certain of the substances to which clays owe their chief characteristics. Many of the properties of clays are closely connected with the colloidal matter present, such matter being in the form of a film of colloidal gel surrounding particles which are of a non-colloidal or a colloidally inert nature. The particular kind and quantity of gelatinous matter present, the size and shape of the grains of non-colloidal material, and the relative proportions of large and small grains are important factors in determining the various physical properties of clays, particularly their binding power, compressive strength, tensile strength and air shrinkage. (For discussion of colloids, see report of Committee on Soils, Proc. Am. Soc. C. E., March, 1922; May, 1925.)

The Physical Characteristics of Clay which are most important in relation to its use as a foundation material are swelling and shrinking with change of water content, compressibility, plasticity, and the relation of these to compressive strength.

(1) Clay will absorb large quantities of water, by direct contact or indirectly from the atmosphere. The amount of water that can be absorbed is apparently controlled by the amount of colloids present in the clay. As the water is absorbed, the clay expands; and if the wet clay is allowed to dry out it will shrink to a smaller volume, and eventually cracks will develop throughout the mass. A clay soil which is likely to have a variable water content may therefore be undesirable as a foundation material.

(2) When a wet clay is subjected to pressure, it is compressed. If the loading is continued for a considerable length of time some of the water in the clay may be forced out and a settlement may occur. If the load is released soon after being applied there will be a "rebound," but this will not be equal to the original compression. The rate of settlement under constant load depends on the rate at which the water can be squeezed out, and this in turn depends on the permeability of the soil. A heavily loaded, impermeable clay may require years to develop its full compressibility, whereas in a more permeable clay the water will be squeezed out more quickly, resulting in a more rapid rate of settlement.

Experiments with mixtures of sand and mica seem to show characteristics under load very similar to clay. It is known that some clays contain a considerable amount of finely divided mica, or similar flat-grained particles, and their presence may have material bearing upon the compressibility of such clays. (See "Compressibility of Sand-Mica Mixtures" by G. Gilboy, Proc. Am. Soc. C. E., Feb., 1928, p. 555.)

(3) An important characteristic of clay is its plasticity, which is supposed to be closely related to the colloidal content. The plasticity increases with the water content. A clay bed which appears hard and sound when examined in a dry state may become highly plastic when saturated with water.

(4) The compressive strength of a clay is closely related to its water content and plasticity. For moderate loads, the clay may undergo elastic deformation, with a "rebound" after the load is released. The limit to which such loading can be applied will be reduced as the water content (and consequently the plasticity) is increased. For a given degree of saturation, this limit will depend on the character of the clay, and apparently is related to the amount of colloids present.

For an exhaustive study of clay soils, see articles by Charles Terzaghi, Eng. News-Rec., Vol. 95, pp. 742, 796, 832, 874, 912, 987, 1026, 1064.

Settlement of Loads on Soft Foundations. Any structure built on a soft foundation will settle. The length of time over which settlement occurs will depend on the nature of the soil. This settlement is in no way serious, provided it is fairly uniform in amount over the entire area of the foundation and is not excessive. The total settlement may be kept within reasonable limits by using moderate unit-pressures in design of the footings, as suggested in the table of Art. 2. It is of vital importance to keep the settlement uniform. This requires careful estimates of the loads that will come upon the footings, and proper proportioning of the footing areas in order to maintain uniform unit-pressure on the soil under all footings. (See Arts. 6 and 7.) If the soil is not uniform under different portions of the structure, particular care will be required in order to secure a reasonably uniform settlement of all parts of the foundation.

In a large flat-slab concrete building upon gravel (column spacing 20 ft. on centers) the writer has observed a difference in settlement in adjacent footings of more than 1 in. without cracks obvious to the eye. If the footings are independent of each other, and one footing settles more than an adjacent one, the floors of the structure will tend to bend as shown in Fig. 2, with maximum bending moments somewhere between the edges of the column and the quarter points. The bending in the floor caused by the direct loads on the floor will produce maximum bending moments at the column and at the middle of the span, with practically no bending at the quarter points. If the floor is adequately designed to carry its dead and live loads, it should also be able to sustain the bending stresses caused by a moderate amount of inequality of settlement of the adjacent footings.

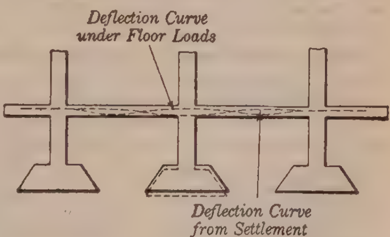


Fig. 2

The question as to whether unequal settlement will be injurious to the structure can be determined only by examination and study of its structural design. It may not be feasible to secure absolute equality of settlement but it should be closely approximated. Special methods may be adopted in the design of the foundations and of the structure itself in order to prevent unequal settlement from causing serious injury to the structure. (See Article by Charles Terzaghi, Trans. Am. Soc. C. E., Vol. 93, p. 270.)

At times uniform pressure upon separate footings based upon assumed dead and live loads is attainable but the bearing "capacity" of the soil is so radically variable that approximate uniform settlement can be obtained only by exhaustive study. Attention is also called to the fact that theoretical live loads and actual ones vary. Certain bays may be heavily loaded at the times others are without live load. The foregoing discussion indicates the wisdom of using conservative "safe bearing capacities" of soils and of studying the subject with more than usual care.

The trained eye may more accurately determine the safe bearing capacity of the soil than the probable settlement under load. Test loads are important for both, particularly the latter; but even test loads over a limited area may be deceptive.

In computing the total load upon a footing, the weight of the backfilled earth should be considered (Fig. 4).

Relation of Settlement to Area of Foundation. For a given soil, the settlement produced by the weight of the structure will not be the same if the footings are spread over a large area as for smaller footings, even if the unit loading on the soil is the same in each case. This is due to the fact that, in soils possessing cohesion, the load of the footing is transmitted to the subsoil both by direct pressure on the loaded area and by friction and shearing stresses into the adjacent soil surrounding the loaded area, and the influence of these shearing forces relative to the direct pressure becomes less as the area increases. Moreover, as shown below, the pressure under the footing is not uniform, and the maximum unit pressure under a large footing is greater than under a smaller area for the same average pressure, causing greater elastic deflection under the larger area. Although no precise formula can be given to describe this feature, it is known that the settlement under a given unit load increases somewhat in proportion to the square root of the loaded area, for soils possessing cohesion. If the soil has little or no cohesive strength the settlement is practically independent of the loaded area.

Relation of Settlement to Depth below Surface. The capacity of a soil to support a load increases with the depth below the surface, probably because the material is more compact due to the weight of covering material that has been removed from it. In general, the settlement produced by a given unit load decreases with the depth. The influence of the depth upon the settlement is less for cohesive soils than for soils without cohesion.

Lateral Flow of Plastic Soils. If the supporting soil possesses considerable plasticity it is liable to be pushed out laterally from under the loaded area. Fig. 3 shows that with such a soil the settlement will be due to two causes: (1) The settlement from aa to a_1a_1 is due to the bulging out or flow of the underlying material, from the straight line ab to the curved line acb; (2) The settlement from a_1a_1 to a_2a_2 is due to direct compression of the soil under the loaded area. The lateral flow may be prevented in whole or in part by driving steel sheet piling around the foundation area. It may also be prevented by the weight of the material surrounding the foundation area. If the footing is placed at some depth below the surface, lateral flow cannot take place without lifting the overlying material surrounding the footing. (See Article by Charles Terzaghi, Trans. Am. Soc. C. E., Vol. 93, 280.)

If the soil contains beds of fine sand, settlement may also be caused by the removal of the sand by flowing water. In New York City one of the large buildings settled because of the pumping of fine sand from an artesian well

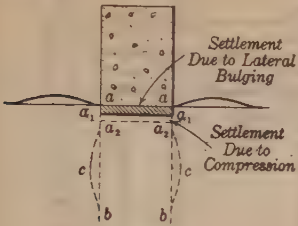


Fig. 3

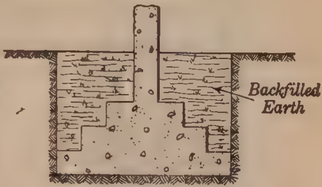
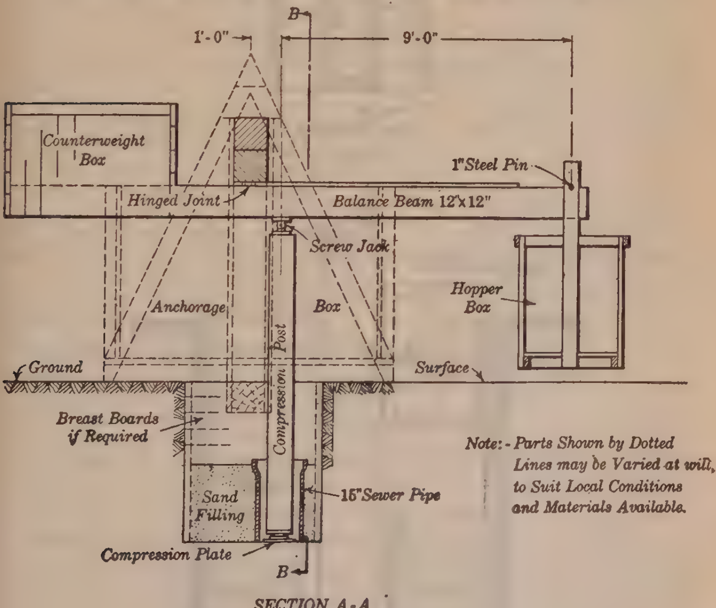


Fig. 4

on the site in getting water for the boilers of the building. The removal of large volumes of ground-water may cause settlement.

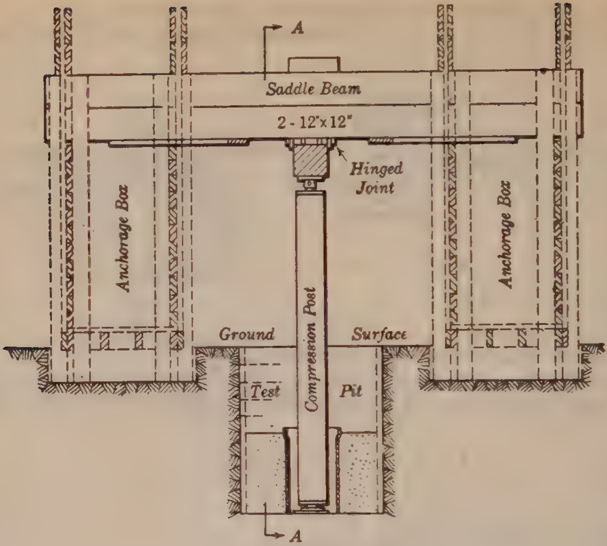
Loading Tests. If the laws of soil mechanics were fully known, it would be possible to compute the bearing power and probable settlement of a soil



SECTION A-A

Fig. 5a

under load. This would require the previous determination of certain physical constants, such as permeability, cohesive strength, etc. The present knowledge of soil mechanics, however, is not sufficient to make such computations of



SECTION B-B

Fig. 5*b*

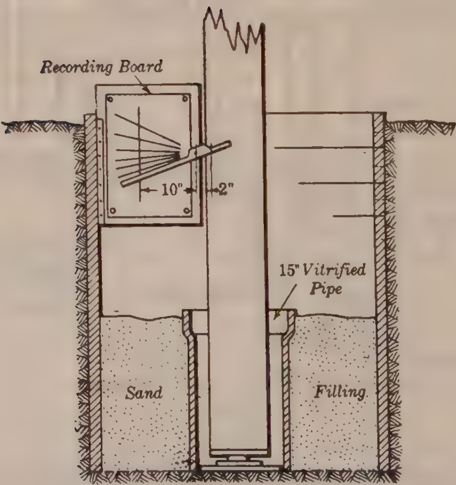


Fig. 5*c*

value. Therefore, the bearing capacity of soils must be determined by test loads. Such tests should be made for the design of important foundations and over as large an area and for as long a period of time as possible.

The Committee on Soils of the American Society of Civil Engineers (Proc. Aug., 1920, p. 907) recommends the apparatus shown in Fig. 5. The load is applied by filling the hopper box with earth or water, this weight being transferred to the compression post by means of the lever or balance beam. The balance beam reacts against the saddle beam which is held down by the anchorage boxes, the latter being also weighted down by earth or other material. The actual compression of the soil is measured by the recording board (Fig. 5-*c*) "on which is tacked a sheet of paper, with a straight-edge of thin material fastened to the board at a certain fixed distance from a nail on the compression post, this nail being against the upper edge of the straight-edge. By drawing a vertical line on the paper, at a definite multiple of the distance between the hinge and nail in the post, a multiplied diagram of the compression for different loads is obtained, reading from a zero line drawn on the paper across the straight-edge before starting the test. At various pressures and compressions, similar lines can be drawn and the time and load noted on the paper. The capacity of this apparatus is limited to about 10 tons, which is sufficient to cover all cases of ordinary soils, and gives a compression diagram exceeding the safe load. In the event that soils of greater or less bearing value are encountered, it would be necessary to substitute other bearing plates under the compression post, of smaller or larger areas."

If this apparatus cannot be made available, use a platform as in Fig. 6, and weight with pig iron or steel, being careful to prevent tipping by wedges marked *w*. If the platform tends to tip adjust the iron or steel and continue loading. In lieu of such apparatus one may get a rough idea of the soil's bearing capacity by standing upon a small block of wood and noting its settlement.

Load tests made by makeshift apparatus, over small areas or of short duration, must be conservatively interpreted.

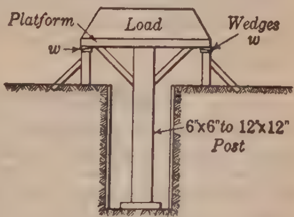


Fig. 6

Existing Structures a Guide. If the designer has not had experience with the terrain under consideration, the opinions of experienced engineers and builders, and perhaps geologists who have had such experience, should be obtained and existing structures and their foundations should be studied. One should at least avoid the obvious mistakes of others. Structures with settlement cracks are very instructive.

Semi-Liquid Soils like mud, silt, alluvium, or quicksand, have little or no supporting power; and with any of these soils it usually is necessary (1) to remove it entirely, or (2) to sink piles, tubes or caissons through it to a solid substratum, or (3) to consolidate the soil by adding earth, sand, stone, etc. The method of performing these operations will be described later. It is impossible to give useful results of the bearing power of soils of this class.

Effect of Frost on Foundation. If foundations are not carried down below the frost line in soils which retain moisture, which is the case with most soils, they may be lifted several inches during the winter. The writer's knowledge of such action is limited to concrete roadways, sidewalks and small building piers. Although the weight of heavy structures may be too great to be "lifted by frost," all foundations should be carried below the frost line.

The following discussion is taken from the paper by Charles Terzaghi, Trans. Am. Soc. C. E., Vol 93, p. 291. The freezing starts at the surface of the ground and gradually proceeds downward. At the same time the soil freezes to the masonry, the adhesion being practically equal to the shearing strength of the frozen soil. Suppose that at some intermediate state freezing has proceeded from the surface, *s–s*, (Fig. 7), to the level, *aa*, the frozen soil being cemented to the pier along the faces *as*. If the freezing process continues still farther, causing expansion of the soil within the space *aabb*, there are then three possibilities:

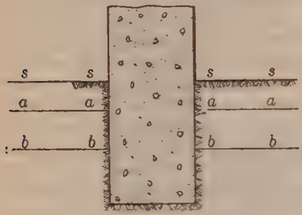


Fig. 7

(1) The ground beneath the freezing layer is only slightly compressible, and the adhesion between the faces, *as*, and the soil may be greater than the weight of the pier. In this case the pier may be lifted bodily as freezing continues, leaving an empty space beneath the pier. (2) The ground is easily compressible, in which case the expanding soil in the area *aabb* may compress the soil below *bb* without raising the pier. Then when the ground thaws, the pier will settle. (3) The ground is slightly compressible but the weight of the pier is greater than the adhesion. In this case the expansion of the soil, *aabb*, will cause a slow upward creep of the soil, *ssaa*, along the outside of the pier.

Reference has already been made to the paper by Charles Terzaghi, "The Science of Foundations," Trans. Am. Soc. C. E., Vol. 93, p. 270. See also Eng. News-Rec., Vol. 100, pp. 520 and 629. The reports of the Committee on Soils of the American Society of Civil Engineers also contain much valuable material; see Proc., Am. Soc. C. E., Aug., 1920, p. 905; Feb., 1921, p. 11; March, 1922, p. 523; May, 1925, p. 884.

4. Distribution of Pressure in Soils

In the design of foundations, it has generally been assumed that the pressure of the soil against the bottom of the footing is uniformly distributed over the foundation area, when the resultant load on the footing coincides with the center of gravity of the loaded area. This assumption is probably closely correct for loads on hard ground or on rock but it has been proved incorrect for compressible soils. It is thought that further study along the lines of Terzaghi and Paaswell will develop information that will enable more correct footing designs. In the interim it is suggested to assume a uniform distribution and conservative soil bearing capacities.

Experiments made by the U. S. Bureau of Public Roads show that the soil unit pressure is a maximum under the center of the load, and is less than the average under the edges. The maximum unit pressure under the center of the load also increases as the bearing area increases for the same average unit loading. Fig. 8 shows the pressure distribution measured at a depth of 54 in. in damp sand, loaded with circular bearing blocks of various sizes and with 5,000 lb. per sq. ft. average load. (See Proc. Am. Soc. C. E., May, 1925, p. 896; also paper by A. T. Goldbeck, Trans. Am. Soc. C. E., Vol. 88, p. 264.)

So far as the computed stresses in an isolated footing are concerned, the assumption of uniform distribution of the load is on the safe side. The stresses computed under this assumption will be greater than if a non-uniform distribution of the loading were adopted. With a footing carrying several concentrated loads, the assumption of uniform distribution may result in lower computed stresses than would result from a non-uniform distribution. For this condition more than ordinary conservatism must be used or it may be best to assume a parabolic distribution of the loading upon the soil, having a maximum value under the center of the footing-area of 1.5 times the average unit-pressure, and zero unit-pressure at the edges of the footing area. (See paper by Charles Terzaghi, Trans. Am. Soc. C. E., Vol. 93, p. 277.)

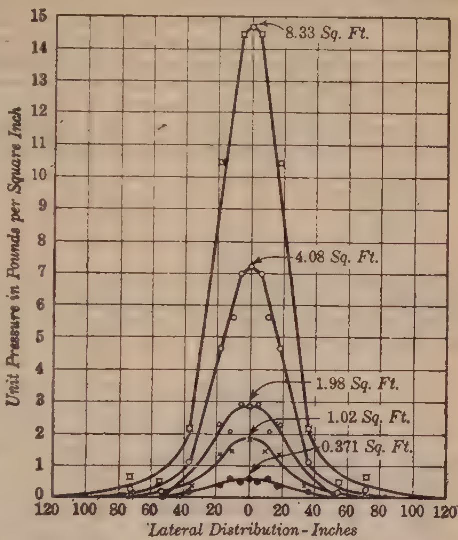


Fig. 8

The distribution of the pressure into the subsoil has been discussed in an interesting and valuable way by George Paaswell (Trans. Am. Soc. C. E., Vol. 85, 1922, p. 1563). He shows that, in any type of foundation material, the pressure from a load is transmitted both downward and outward through the soil, in the manner of a "bulb of pressure." At any given depth below the footing, the load is spread out to an indefinite distance in all directions. However, the limiting zone of pressure influence is contained within a slope of 1 vertical to 2 horizontal from the bottom edge of the footing (see Fig. 9). There is such great variation in the character of soil that further scientific soil classification and further experiments and study are needed before discarding the obvious approximate and probably inaccurate assumption of 1 to 1 distribution. In other words, although the actual distribution may more nearly follow the curve marked "pressure distribution" in Fig. 9, the average pressure based on a 1 : 1 slope limit is recommended. This gives the average distribution shown by the rectangle of "average pressure."

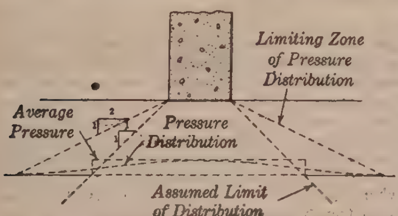


Fig. 9

Soil Pressure on Trench Sheeting. The pressure of earth placed back of a retaining wall is discussed in Sect. 10, Art. 19. An entirely different problem is involved in the calculation of the pressure that must be sustained by trench sheeting. Soils which possess a large amount of cohesion may not require sheeting for trenches of moderate depth, as such material will stand up with vertical faces for some length of time. If sheeting is not used the engineer must watch the banks with care to guard against ultimate sloughing. If the cohesiveness of the soil is not great, sheeting is of course necessary. Experiments have shown that the unit pressure on the sheeting generally reaches a maximum at or slightly above a point half way between the top of the trench and the bottom, and the total pressure in the upper half of the cut is greater than the total pressure in the lower half. Measurements at the Flatbush Avenue subway cut in Brooklyn (1916) showed the following results:

Depth in feet	Actual pressure, lb. per sq. ft.
15	900
45	900
55	300

The actual line of rupture of an earth bank *in situ* is approximately a curved line, roughly described by a quarter-circle meeting the bottom of the cut and tangent to a vertical line distant $d/2$ from the top edge of the cut (Fig. 10). (See Trans. Am. Soc. C. E., Vol. 60, p. 27.) This rule applies to cohesive materials. Therefore no pressure should be exerted upon the trench sheeting from any material lying behind the line of rupture. Experience has shown that when earth or sand is left in place over the top of a timbered excavation, the overlying material arches over, relieving the roof timbering of a large portion of the superimposed load. In a similar way, it is probable that the pressures in a sheeted trench are produced by the arching action of the soil lying between the sheeting and the line of rupture. The material at the bottom of the trench may exert little pressure but it must be prevented from running into the trench, which would destroy the arch action and permit the full pressure of the soil to come upon the lower portion of the sheeting.

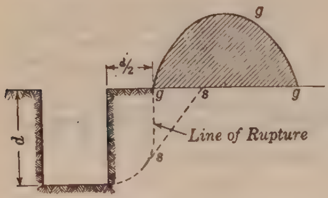


Fig. 10

Experience in subway excavation has shown that the line of break of the soil reaches the surface at a distance from the trench equal to about one-half the depth of the cut. It has been contended that some buildings adjacent to subway work would probably have been safe without underpinning, whereas they have been badly cracked and injured after underpinning because the work incident to underpinning weakened the bank near its critical line. This is possible; but attention is called to the fact that buildings or excavated material adjacent to a trench (*ggg*, Fig. 10) and within a slope *ss*, which varies with different soils and with the extent of soil disturbances as a result of excavating, may exert great pressures upon the sheeting. This is especially true in moist or wet clays where heaving of the bottom may also be caused by such surcharge. (For pressures on trench sheeting, see "Earth and Rock Pressures" by H. G. Moulton, Trans. Am. Inst. Mining and Metallurgical Engineers, Vol. 68, p. 327; also discussion by E. G. Haines and J. C. Meem.)

5. Improvement of Bearing Capacity

Increasing the Depth. The simplest method of increasing the bearing capacity is to dig deeper. Ordinary soils will bear more weight the greater the depth reached, owing to their becoming more condensed from the superincumbent weight. Depth is especially important with clay and saturated fine sands, since it is then less liable to be displaced laterally owing to other excavations in the immediate vicinity, and also because at greater depths the amount of moisture in it will not vary so much. However, occasionally the soil grows more moist as the depth increases beyond a moderate distance, in which case increasing the depth is undesirable. For example, in Chicago the clay grows softer after a depth of about 12 to 14 ft. below the sidewalk is reached.

Drainage. Another simple method is to drain the soil. The water may find its way to the bed of the foundation down the side of the wall, or by percolation through the soil, or through a seam of sand. In most cases the bed can be sufficiently drained by surrounding the building with a tile drain laid a little below the foundation. This drain must have an outlet to permit accumulated water to flow off. In more difficult cases, the expedient is employed of covering the site with a layer of gravel, the thickness depending upon the plasticity of the soil, the gravel serving the double purpose of distributing the concentrated loads of the footings to a larger area of the native soil and of improving the drainage of the bed of the foundation. In extreme cases, it is necessary to enclose the entire site with a puddle-wall to cut off drainage water from a higher area.

Adding Sand. A simple method of improving the bearing capacity of a compressible soil is to spread sand or gravel or broken stone over the bed of the foundation, and pound it into the soil, thus forming a comparatively compact stratum upon which to found the structure. This method is not very effective, since at best the effect of the blow cannot extend very deep, whereas the heavy masses of the masonry make themselves felt at great depths. A more efficient way is to make an excavation a little larger than the proposed structure and cover the bed of the foundation with a layer of sand or gravel. The sand should be deposited in successive layers, each of which should be thoroughly tamped before laying the next. The sand should be moist, so it will pack well. Sand when used in this way possesses the valuable property of assuming a new position of equilibrium and stability should the soil on which it is laid yield at any of its points; and not only does this take place along the base of the sand bed, but also along its edges or sides. The bed of sand or gravel must be thick enough to distribute the pressure on its upper surface over the entire base of the trench.

Short Piles. If the soil is very soft, it can be consolidated to a considerable depth by driving short wood piles, for which purpose many small ones are preferable to fewer but larger ones. It is customary to employ wood piles about 6 ft. long and about 6 in. in diameter, since this size can be driven with a hand maul or by dropping a heavy block of wood with a tackle attached to any simple frame, or by a hand piledriver. They may be driven as close together as necessary, although 2 to 4 ft. in the clear is usually sufficient. Clay is compressible, whereas sand is not; hence this method of consolidating soils is not applicable to sand, and is not very efficient in soils largely composed of sand.

When the piles are driven primarily to compact the soil, it is customary to load them and also the soil between them, by depositing a bed of concrete between and over the heads of the piles. For further discussion of piles, see Articles 9 and 10.

Sand Piles. Experiments show that in compacting the soil by driving wood piles it is better to withdraw them and immediately fill the holes with sand, than to allow the wooden piles to remain. This advantage is independent of the question of the durability of the wood. When the wooden pile is driven, it compresses the soil an amount nearly or quite equal to the volume of the pile, and when the latter is withdrawn this consolidation remains, at least temporarily. If the hole is immediately filled with sand this compression is retained permanently, and the consolidation may be still further increased by ramming in the sand in thin layers, owing to the ability of the latter to transmit pressure laterally. Further, the sand pile will support a greater load than the wooden pile; for, since the sand acts like innumerable small arches reaching from one side of the hole to the other, more of the load is transmitted to the soil on the sides of the hole. To secure the best results, the sand should be fine, sharp, clean, and of uniform size.

Sand piles are not suitable where liable to be disturbed by nearby excavations, which may not be possible to foresee.

6. Loads for Designing Foundations

Spread Foundations are those in which the width is increased until the load can be safely carried, and for soft or compressible soils the loads must be accurately determined. The structures requiring the most careful consideration are buildings, since usually they give a greater unit pressure upon the soil and are more likely to settle unevenly and crack. The load consists of three parts, that of the building itself, the movable loads on the floors, and the part of the load that may be transferred from one part of the foundation to the other by the wind.

Dead Load. The weight of the building is ascertained by calculating the cubical contents of all the various materials in the structure. If the weight is not equally distributed, care must be taken to ascertain the proportion to be carried by each part of the foundation. For example, if one vertical section of the wall is to contain a number of large windows while another will consist entirely of solid masonry, it is evident that the pressure on the foundation under the first section will be less than that under the second. In this connection it must be borne in mind that concentrated pressures are not transmitted, undiminished, through a solid mass of masonry in the line of application, but spread out in successively radiating lines; and hence, if any considerable distance intervenes between the foundation and the point of application of this concentrated load, the pressure will be nearly or quite uniformly distributed over the entire area of the base.

For final design, the actual weights of the various parts of the structure as designed should be computed. Weights of partitions are given in Sect. 12, Art. 6, and weights of fireproof floors in the same section, Art. 5. Weights of different kinds of masonry are given in Sect. 10, Art. 17. For preliminary design the following approximate loads may be assumed: For ordinary dwellings, weight of floor including framing is about 15 lb. per sq. ft. exclusive of plastered ceiling; fireproof floors including framing should be taken at not less than 75 lb. per sq. ft. for large public buildings, and will be from 135 to 200 lb. per sq. ft. for reinforced concrete warehouses.

Live Load. The movable load on the floors depends on the nature of the building. For dwellings it is usually taken at 40 lb. per sq. ft. but is probably considerably less; for office buildings it is usually taken at 50 lb. per sq. ft. For theaters and other public places of congregation the assumed live load is usually 100 lb. per sq. ft. Actually to approximate this load people must be crowded like sardines within a closed area. However, in public places the possibility of rhythmic stamping justifies conservatism. For stores, warehouses and factories, the live load should be from 100 to 400 lb. per sq. ft., according to the purposes for which they are used. (See also Sect. 12, Art. 6.) In designing a structure to be built in a city, the minimum live loads specified in the local building code must be considered.

The above live loads are generally reduced in the designing of columns and foundations. The New York City code states that, in computing the column loads "in buildings more than five stories in height, the live load on the floor next below the top floor may be assumed at 95% of the allowable live load, on the next lower floor 90%, and at each succeeding lower floor at correspondingly decreasing percentages, provided that in no case shall less than 50% of the allowable live loads be assumed" at any one floor. The footing loads are taken the same as the reduced column loads. Wind loads are generally not considered in the design of footings except for high, narrow buildings; and where used, they are usually reduced to 50% of their theoretical value.

On a compressible soil it is very important that the live load assumed as reaching the footings be neither greatly over- nor under-estimated. The dead load can be estimated with accuracy, and as the load on the footings under the walls is mostly dead load, it can be computed readily and with accuracy. But the possible maximum load on the footings of interior columns may be made up largely of live load (as in the case of a warehouse). The assumed live loads upon the interior footings may be a large percentage of the dead load or may even equal or be three times the dead load. Further, the actual live loads on adjacent footings may vary so radically that one must anticipate unequal settlement. This being an operating necessity, one should be conservative in the unit pressures allowed upon compressible soils.

D. E. Moran suggests the following procedure: The footing receiving the largest ratio of live to dead load is proportioned for the combined live and dead loads. The dead plus one-half probable live load on this footing is then divided by the area of the footing, and this value is then divided into the dead plus one-half probable live load of the other footings to get their areas. (See Eng. News, Vol. 69, p. 463; Eng. News-Record, Vol. 85, p. 219.)

Experience in Chicago in founding on a compressible soil shows that the settlements of the columns and walls of eight- and ten-story office buildings, hotels and retail stores are almost exactly the same when designed on the assumption that no live load reaches the footings. In other words, the live load apparently caused no settlement. This, however, is believed to be the exception and not the rule.

Attention must be given to the manner in which the weight of the roof and floors is transferred to the walls. For example, if the floor joists of a warehouse run from back to front, it is evident that the back and front walls alone will carry the weight of the floors and the goods placed upon them, and this will make the pressure upon the foundation under them considerably greater than under the other walls. Again, if a stone-front is to be carried on an arch or on a girder having its bearings on piers at each side of the building, it is manifest that the weight of the whole superincumbent structure, instead of being distributed equally on the foundation under the front, will be concentrated on that part of the foundation immediately under the piers.

7. Unit Pressure on Foundations

Area of Foundation. This is determined by dividing the load on the foundation by the pressure which may safely be brought upon the soil. Then having found the area of foundation, the base of the structure must be extended by footings of masonry, concrete or timber, so as to cover that area and distribute the pressure uniformly over it. The object is not so much to secure an absolutely unyielding base as to secure one that will settle as little as possible, and uniformly. All soils will yield somewhat under the pressure of any building, and even masonry itself is compressed by the weight of the load above it. If the foundation bed is uniform, the pressure per square foot should be the same for all parts of the building, so that the settlement may be uniform. This can be secured only when the axis of the load (a vertical line through the center of gravity of the weight) passes through the center of the area of the foundation. If the axis of pressure does not coincide exactly with the axis of the base, the ground will yield most on the side which is pressed most; and, as the ground yields, the base assumes an inclined position, and carries the lower part of the structure with it, thus producing unsightly cracks, if nothing more.

The coincidence of the axis of pressure with the axis of resistance is of the greatest importance. The principle is almost self-evident, and yet the neglect to observe it is the most frequent cause of failure in the foundations of buildings. Fig. 11 is an ex-

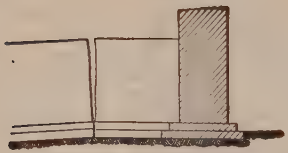


Fig. 11

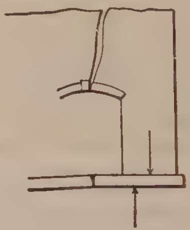


Fig. 12

ample of one way in which this principle is violated. The shaded portion represents a heavily loaded exterior wall, and the unshaded portion a lightly loaded interior wall. The foundations of the two walls are rigidly connected at their intersection. The center of the load is under the shaded section, and the center of the supporting area is at some point farther to the left; consequently the exterior wall is caused to incline outward producing cracks at or near the corners of the building. The two foundations are connected in the belief that an increase of the bearing surface is of advantage; but the true principle is that the coincidence of the axis of pressure with the axis of resistance is of more importance. Fig. 12 is another illustration of the same principle. The foundation is continuous under the opening, and hence the center of the foundation is to the left of the center of pressure; consequently the wall inclines to the right, producing cracks, usually over the opening.

One conclusion to be drawn from the above examples is that the foundation of a wall should never be connected with that of another wall either much heavier or much lighter than itself, as both are equally objectionable. Large brick or concrete chimneys should not be firmly connected to a building. A second conclusion is that the axis of the load should strike a little inside of the center of the area of the base, to make sure that it will not be outside. Any inward inclination of the wall is rendered impossible by the interior walls of the building, the floor beams, etc.; whereas an outward inclination can be counteracted only by the bond of the masonry and by anchors. A slight deviation of the axis of the load outward from the center of the base has a marked effect, and is not easily counteracted by anchors.

The center of the load can be made to fall inside of the center of foundation by extending the footings outward, or by curtailing the foundations on the inside. The latter finds exemplification in the properly constructed foundation of a wall containing a number of openings. For example, in Fig. 13, if the foundation is uniform under the entire front, the center of pressure must be outside of the center of the base; and consequently the two side walls will incline outward, and show cracks over the openings. If the width of the foundation under the openings is decreased, or if this part of the foundation is omitted entirely, the center of pressure will fall inside of the center of base and the walls will tend to incline inward, and hence be stable.

The bearing capacity of the soil may vary greatly over the area of a single building. In a glacial district a small area may contain clay, fine sand, coarse sand and gravel, with mixtures of these, and a bearing capacity varying between 2 and 5 tons. In this event each footing must be especially designed with an effort to keep the settlements uniform.

Where horizontal loads must be considered with vertical loads, as in bearing walls subject to earth or water pressure, the distribution of pressure upon the soil involves the principles contained in Sect. 10, Arts. 19 and 26.

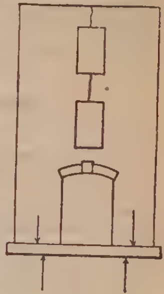


Fig. 13

An Important Principle derived from the above conclusions is the following: Foundations should be so constructed as to compress the ground slightly concave upward, rather than convex upward. On even slightly compressible soils, a small difference in the pressure on the foundation will be sufficient to cause the bed to become convex upward. At Chicago, in buildings founded upon soft clay an omission of 1 to 2 per cent of the weight (by leaving openings) usually causes sufficient convexity to produce unsightly cracks. With very slight differences of pressure on the foundation, it is sufficient to tie the building together by careful bonding, by hoop-iron built in over openings, and by heavy bars built in where one wall joins another.

The art of constructing foundations on a compressible soil was brought to a high degree of development by the architects of Chicago between 1870 and 1890, when the principal buildings were founded upon a bed of soft clay. The special feature of the practice in that city, is what is called " the method of independent piers "; that is, each tier of columns, each pier, and each wall has its own independent foundation, the area of which is proportioned to the load on that part. The interior walls are fastened to the exterior ones by anchors which slide in slots.

The opposite extreme is to rest the structure upon a platform of concrete, timber, or steel beams so strong as to resist local settlement. This method has not always been successful; and when it is successful, it is usually expensive.

The post-office building erected in Chicago in 1875 rested upon a bed of concrete 3 feet thick over the entire site; but the concrete was insufficient to resist the unequal loading, and the building settled so badly and so unevenly that it was necessary to demolish it after it had stood only seventeen years. It is said that some noted buildings in Europe rest upon beds of concrete 8 or 10 feet thick. With a properly designed reinforced-concrete slab, it is thought that similar difficulties may be overcome. In fact, in soft ground this may be the best solution.

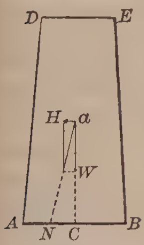


Fig. 14

Effect of Wind. On chimneys and other tall narrow structures the action of wind increases the pressure on one side and decreases it on the other. The maximum horizontal pressure of the wind is usually taken as 40 lb. per sq. ft. on a flat surface perpendicular to the wind (in regions of recorded cyclones, 50 lb. should be used) and on a cylinder at about 25 lb. per sq. ft. of the projection of the surface. In Fig. 14 let $ABED$ represent a vertical section of the chimney; a is a point horizontally opposite the center of the surface exposed to the pressure of the wind and vertically above the center of gravity of the chimney; C is the position of the center of pressure when there is no wind; N is the center when the wind is acting. Let $H =$ the total pressure of the wind against the exposed surface,

W = the weight of the structure above the section considered, L = the distance AB, h = the distance aC, d = the distance NC found from $d = hH/W$. When there is no horizontal force acting, the load on AB is uniform; but when the wind blows from the right, the pressure is greatest near A and decreases towards B. The maximum pressure at A will be that due to the weight of the chimney plus the compression due to overturning moment and the pressure at B will be the compression due to the weight minus the tension due to the overturning moment. For a rectangular base with length L normal to the width b the average pressure is W/Lb, and

$$\textbf{Maximum unit-pressure at A} = \frac{W}{Lb}\left(1 + 6\,\frac{d}{L}\right)$$

$$\textbf{Minimum unit-pressure at B} = \frac{W}{Lb}\left(1 - 6\,\frac{d}{L}\right)$$

These equations show that when $d = NC = 1/6\ L$, the maximum pressure at A is twice the average, and that the pressure at B is zero. This is equivalent to what is known, in the theory of arches, as the principle of the middle-third. It shows that as long as the center of pressure lies in the middle-third, the maximum pressure is not more than twice the average pressure, and that there is no tendency to produce tension at B. The average pressure per unit on AB is supposed to have been adjusted to the safe bearing power of the soil, and if the maximum pressure at A does not exceed the ultimate bearing power, the occasional maximum pressure due to the wind will do no harm; but if this maximum exceeds or is dangerously near the ultimate strength of the soil, the base must be widened. (See Sect. 10, Art. 22.)

8. Footings

The footing is the bottom course or courses of masonry, timber, or steel beams employed to increase the area of the foundation. Whatever the character of the soil, footings should extend beyond the face of the wall (1) to add to the stability of the structure and lessen the danger of the work being thrown out of plumb, and (2) to distribute the weight of the structure over a larger area and thus decrease the settlement due to the compression of the ground. To serve the first purpose, footings must be securely bonded to the body of the wall; and to produce the second effect, they must have sufficient strength to resist the transverse strain to which they are exposed.

In Masonry Footings that part of the footing that projects beyond the portion above it acts as a cantilever beam uniformly loaded by the reaction of the soil or of the footing below. To determine the safe offset, let P equal the pressure in pounds per square foot at the bottom of the footing course under consideration, S = the working tensile stress of the material in pounds per square inch (Sect. 10, Art. 16), o = the greatest possible offset or projection of the footing course in inches, t = thickness of the footing course in inches. Then with sufficient accuracy $\mathbf{o = 7\,t\,\sqrt{S/P}}$. The accompanying table gives values of o/t for three values of **P**. For example, if **P** is 4000 lb. per sq. ft., then for granite $o/t = 1.4$, and hence if t is 12 in. the offset o would be 16.8 inches.

The table is strictly correct only for the lower offset, because flexure of the short projecting cantilevers brings indeterminate concentrations upon the edges of the upper courses. Attention is called to the necessity of keeping the outside vertical joints well within the outer edges of the upper stones. (See Fig. 15). The values in the table are applicable to any of the several projecting courses, provided no stone projects more than half its length beyond

Safe Offset for Masonry Footings

Kind of stone	S in lb. per sq. in.	Ratio of offset to thickness P = 1000	P = 2000	P = 4000
North River bluestone	200	3.1	2.2	1.6
Granite	150	2.7	1.9	1.4
Limestone	125	2.5	1.8	1.2
Sandstone	75	1.9	1.4	1.0
Good quality clay brick, 1 : 3 portland cement mortar age 90 days	25	1.1	0.8	0.6

the end of the next course above. The proper projection for rubble masonry lies somewhere between the values for stone and for concrete. If the rubble consists of large stones well bedded in good strong portland-cement mortar, then the values for this class of masonry will be but little less than those given for stone; but if the rubble consists of small irregular stones laid with portland-cement mortar, the projection should not exceed that given for concrete. Footing courses should not be laid of small stones in either natural-cement or lime mortar.

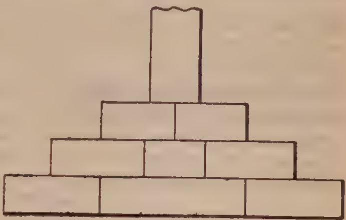


Fig. 15

After the safe length of the offset of the footing has been determined, it should be examined to see if it is safe against failure by shearing. Since footings are subject to heavy loads and consequently to great shearing stress, and since the allowable shearing stress for stone and brick is much less than the crushing strength, the allowable shearing stress should be considered, as given in Sect. 10, Art. 16.

Timber Footings were formerly much used but are not as efficient as reinforced-concrete footings. They should be used only in places where the ground is always wet and where it will not subsequently become dry by the lowering of the ground-water. They may be desirable: (1) for temporary construction; (2) where disintegration of concrete might be probable due to the chemical condition of the water; (3) since timber has much greater flexural strength than masonry, the timber footing requires less thickness, with less depth of excavation and lower cost; (4) where concrete materials are not readily available.

The safe projection, as computed by the formula under masonry footings, is given in the table on page 730. On account of the flexure of the timbers, the pressures on such footings will not be uniformly distributed. The soil pressure under the footing will be greater near the center, so that the stress in the timber may be less than assumed in the table.

Steel-Beam Footings are specially useful in the foundations (particularly of the columns) of a heavy building supported upon a compressive soil. The footing consists of a row of steel beams placed side by side and embedded in rich concrete; on top of this and at right angles to it is a shorter row; and above this, one and sometimes two other rows. The advantages of such footings are that they decrease the area of the foundation and at the same time decrease the weight of the footing and increase the usable space in the cellar.

Safe Offset for Timber Footings

Kind of timber	S in lb. per sq. in.	Ratio of offset to thickness		
		P = 1000	P = 2000	P = 4000
Chestnut	760	6.1	4.3	3.1
Southern yellow pine	1200	7.7	5.4	3.8
Oak	1120	7.4	5.2	3.7
Douglas fir—Coast region	1200	7.7	5.4	3.8
Rocky Mountain	800	6.6	4.6	3.3
Hemlock—West Coast	1040	7.1	5.0	3.6
Eastern	880	6.6	4.6	3.3
Spruce—red, white	880	6.6	4.6	3.3

These values are for best-grade timber. For common grade, use two-thirds of the projections given in the table.

Steel-beam footings were first used in Chicago in 1878. For design of steel beam grillages, see "Foundations of Bridges and Buildings," by Jacoby and Davis, 2nd Ed., p. 510.

Reinforced-Concrete Footings are sometimes more economical than steel-beam footings. See Sect. 11, Art. 40.

Inverted Arches were formerly sometimes built under and between the bases of piers, as shown in Fig. 16. Employed in this way, the arch simply distributes the pressure over a greater area; but it is not well adapted to this use, for it is nearly impossible to prevent the end piers of a series from being pushed outward by the thrust of the arch, and it is generally impossible, with inverted arches, to make the areas of the different parts of the foundation proportional to the load to be supported. The only advantage the inverted arch has over masonry footings is in the shallower foundations obtained. This construction is used now only in reinforced concrete.

Fig. 16

9. Piles and Piledriving

Timber Piles are widely used for foundations, the softer varieties for easy driving and the harder varieties for difficult driving. Piles should never be less than 6 in. and preferably not less than 8 in. in diameter at the small end; never more than 18 in. and preferably not more than 14 in. at the large end. To prevent bruising and splitting in driving, 2 or 3 in. of the head is usually chamfered off. As an additional means of preventing splitting, the head is often hooped with a strong iron band, 2 to 3 in. wide and 1/2 to 1 in. thick. The expense of removing these bands and of replacing the broken ones, and the consequent delays, led to the introduction of a hood or cap for the protection of the head of the pile. The hood consists of a cast-iron block with a tapered recess above and below, the chamfered head of the pile fitting into the lower recess and a cushion piece of hard wood, upon which the hammer falls, fitting into the upper one. The hood preserves the head of the pile, adds to the effectiveness of the blows, and keeps the pile head in place to receive the blows of the hammer.

A further advantage of the pile hood is that it saves the piles by preventing brooming and consequently the necessity of successively cutting off the pile when the driving is hard. Sometimes, particularly in stony ground, the point is protected by an iron shoe.

The shoe may be only two V-shaped loops of bar iron placed over the point, in planes at right angles to each other, and spiked to the piles; or it may be a wrought- or cast-iron socket.

Piles for permanent construction should be cut from sound trees, should be close-grained and solid and free from defects and sharp bends. A line drawn from the center of the butt to the center of the tip should lie within the body of the pile. Piles should be peeled soon after cutting, especially when they are cut in the summer, in order to avoid decay due to fungi and the ravages of worms; if not done at this time, the bark must be removed before driving.

The bearing power of wood piles in soft ground has sometimes been increased by bolting lagging pieces on the sides of the piles. This increases the area of cross-section and also the surface area, thereby providing a greater surface for supporting the pile by friction on the surrounding ground. (See Trans. Am. Soc. C. E., Vol. 54, F, pp. 8, 27.)

Concrete Piles are of two types, precast piles which are molded and cured before driving, and various types of cast-in-place piles. Precast piles are generally required in docks, wharves or piers, where a portion of the pile is left standing above the surface of the firm supporting ground, and is subject to bending stresses from transverse loads on the structure. In certain soils, such as plastic clay, or fine wet sand, the soil may react against the soft concrete of a cast-in-place pile and cause great damage. Under such conditions, the precast pile is preferable. Cast-in-place piles are well suited to compressible soils which have sufficient stability to prevent damage to the soft concrete when it is first placed in the pile form.

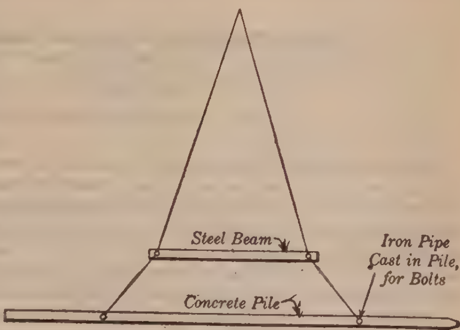


Fig. 17

A Precast Pile is a reinforced-concrete pile which is molded in a form, and after hardening and curing is handled and driven like a timber pile. All precast piles are reinforced, since appreciable bending stresses are developed in the pile during handling. Before the pile is designed, the method of handling to and from the storage piles and piledriver must be determined in order that the pile may be properly reinforced for such handling. Fig. 17 shows a common method of handling. The pile may need extra reinforcement to take care of the resulting bending strains. The head of the pile is often provided with additional reinforcement to withstand the impact of the piledriver. The general design of premolded piles is similar to that of concrete columns. (Sect. 11, Art. 39.) The reinforcing steel is generally made up in units. The foot of the pile is sometimes provided with a metal cap, for protection during driving; more frequently the foot is strengthened by additional reinforcing steel. The foot of the pile is pointed. Several types of precast piles in use are unpatented, though there are a number of patented designs. Precast piles are constructed either with taper or without it. Tapering adds to the cost of manufacture but probably increases the carrying capacity. If the piles are to be driven with a water jet, they are constructed with a hole in the center, either by casting a jet pipe in the pile, or by leaving a tapered wooden core or collapsible form in the center of the pile. The reinforcing steel should have a cover of concrete of at least 3 in. to protect against rusting. Precast piles are generally molded in horizontal forms, but sometimes vertically. They must be thoroughly cured after removal from the forms, and therefore they should be molded at least thirty days before driving. As soon as they are driven, they are ready to receive their load, which is an advantage that they hold over the cast-in-place piles. Fig. 18 shows precast piles used in a pier at Havana, the maximum length of these piles being 85 ft.

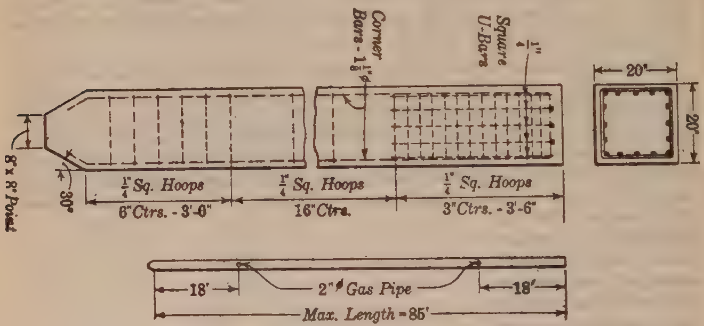


Fig. 18. Concrete Piles in Havana Piers

Cast-in-Place Piles are generally patented. There are a number of types available which may be grouped in two classes: (1) piles in which a permanent shell or protection is allowed to remain in the ground to protect the newly placed and unset concrete from the action of the surrounding soil and water and to prevent distortion of the shaft by natural earth pressure or from the pressure caused by driving adjacent piles; (2) piles in which no permanent protection remains around the concrete after it has been placed in the ground.

The best-known pile of the first class is the **Raymond** pile, which is made by driving a tapering reinforced steel shell into the ground to refusal, by means of a collapsible steel core, withdrawing the core and then filling the shell with concrete. The concrete within the shell may be reinforced or not, as may be desired. Raymond piles are 8 in. in diameter at the point and the diameter increases at the rate of 0.4 in. per ft. The shell is made of sheet steel of a thickness suited to local conditions, and is reinforced by means of steel wire spirally wound on the interior of the shell on a 3-in. pitch and held in place by grooving the shell around the wire. No. 24 gage steel in the shell and No. 3 wire are most commonly used. A " boot " or point of pressed steel is placed on the end of the core or mandrel, and completely encloses the bottom of the shell. Due to the weight of the mandrel, a strongly constructed piledriver is required. The Raymond piles have the following advantages: (1) The form can be inspected before placing the concrete; (2) the bearing power of every pile is tested by the average penetration of the steel core under the final blows of the hammer; (3) the steel core permits driving through very hard material; (4) due to the large taper, the surface friction is increased, which enables them to support under ordinary conditions about twice the working load which would be used on wood piles of the same length.

Simplex and MacArthur piles are of the class which has no permanent shell. The **Simplex** pile is made by driving a cylindrical steel pipe, usually 16 in. in diameter and about 3/4 in. thick, which is sealed at the bottom by a conical cast-iron point. After the pipe has been driven to the desired depth it is filled with concrete and withdrawn, leaving the cast-iron point at the bottom of the concrete shaft. A ram is often used to force each batch of concrete into place against the surrounding soil as the pipe is withdrawn. Occasionally a precast pile is inserted in the pipe after it has been driven, the space around this pile is filled with grout, and the pipe is then withdrawn. The Simplex type of pile is adapted to stiff, non-water-bearing, or clay soils, which are not plastic.

The **MacArthur** or "pedestal pile" is placed by driving a steel pipe through which extends a steel ram. After the pipe has been driven to the required depth, the ram is withdrawn and is then used to force concrete out of the bottom of the pipe to form an enlarged base. When this operation has been completed, the ram is removed, the pipe is filled with concrete, and finally the pipe itself is pulled out, leaving the concrete shaft superimposed on the base. This pile permits taking advantage of the larger bearing power of a lower stratum of soil, by means of the pedestal base. The ramming process also compacts and strengthens the concrete. If for any reason the earth at the base of the pile should present unequal resistance to pressure, the pedestal is likely to assume an eccentric shape.

The Pretest Pile is a patented method, well adapted to underpinning of existing foundations on sand or clay soil. It consists in forcing a hollow steel pipe or tube, 12 in. to 16 in. in diameter, into the soil by means of jacks which react against the bottom of the existing footing. The pipe is in sections of relatively short length, and as the sinking progresses additional sections are added, being joined together by special collars. As the pipe is forced down, the material inside is removed, generally by a water jet and pump. And finally the pipe is filled with concrete.

Precautions for Cast-in-Place Piles. All such piles may be subject to injury to the green concrete by driving forms for adjacent piles, either by displacement of the concrete by the earth pressure, or by vibration of the fresh concrete after the initial set of the cement has taken place. This difficulty may be avoided by providing that no pile shall be driven within a distance of 5 to 10 ft. from a freshly placed pile until 3 to 7 days after the latter has been driven, in order to permit the green concrete in the pile to attain a strength adequate to withstand injury. The construction of cast-in-place piles often requires more careful supervision than is needed for driving precast concrete piles.

Composite Piles have been used successfully in water-bearing ground, to eliminate the great cost of placing extremely long concrete piles. A long wood pile is driven to such a depth that its top is below the permanent water level. A concrete pile is then placed on top of the wood pile, and properly doweled to it, the concrete extending up above the water level to the required elevation. A similar construction is by driving a hollow reinforced-concrete pile over the wood pile, removing the water and mud from the hollow pile and filling the latter with concrete. In clay care should be taken to be sure that the concrete "tops" first built are not shoved off the wooden piles by the displacement of the clay by the piles subsequently driven.

Concrete vs. Wood Piles. Advantages of concrete piles are: (1) Can extend above permanent water level without decaying and can be used in water infested with borers. (2) Are not subject to decay through unforeseen changes in ground-water level. (3) Greater bearing capacity; timber piles are generally restricted to 20 tons, whereas concrete piles (depending upon their size and the soil through which and to which they are driven) may be safely loaded up to 50 tons. (4) As foundation may be above ground-water level, there is a saving in excavation and in foundation masonry, with a possible reduction in time of construction. (5) They can be readily bonded into a grillage or capping of concrete. (6) They may save time where it is necessary to wait for delivery of timber piles. (7) When properly designed they have great flexural strength.

Advantages of wood piles are: (1) Less expensive than concrete. (2) They can be driven more quickly and with lighter equipment and often can be obtained in less time. Precast concrete piles usually require thirty days to cure. (3) Where the length of pile cannot be accurately determined in advance the wood pile has a decided advantage. Five or six feet of additional pile may be purchased at relatively small cost and if necessary cut off with little additional cost.

The Drop-hammer Piledriver consists of a heavy block of iron, called a ram, monkey, or hammer, which is carried by a rope or cable passing over a pulley fixed at the top of an upright frame and allowed to fall freely on the head of the pile. The machine is generally placed upon rollers or on a car or scow. The frame consists of two uprights, called leaders or leads, from 10 to 60 ft. long, placed about 2 ft. apart, which guide the falling weight in its descent. The leaders are either wooden beams or steel channel-beams, usually the former. The hammer is generally a mass of iron weighing from 500 to 7600 lb. (usually about 2000), with grooves in its sides to fit the guides and a staple or pin in the top by which it is raised. The rope employed in raising the hammer is usually wound up by a steam engine placed on the end of the scow or car, opposite the leaders.

The hammer is operated by attaching the rope permanently to the staple in the top of the hammer, and dropping the hammer by setting free the winding drum by the use of a friction clutch. In this way the hammer can be dropped from any height, thus securing a light or heavy blow at pleasure.

Steam-Hammer Piledrivers are of two types—single-acting and double-acting. The **single-acting steam-hammer** consists essentially of a steam cylinder (stroke about 3 ft.) the piston-rod of which carries a striking weight of 3000 to 5000 lb. The steam cylinder is fastened to and between the tops of two I-beams 8 to 10 ft. long, the beams being united at the bottom by a piece of iron in the shape of a frustum of a cone, which has a hole through it. The under side of this connecting piece is cut out so as to fit the top of the pile. The striking weight, which works up and down between the two I-beams as guides, has a cylindrical projection on the bottom which passes through the hole in the piece connecting the feet of the guides and strikes the pile. The steam to operate the hammer is conveyed from the boiler through a flexible tube. In principle the single-acting steam hammer does not differ from the drop-hammer.

The whole mechanism can be raised and lowered by a rope passing over a pulley in the top of the leaders. After a pile has been placed in position for driving, the machine is lowered upon the top of it and entirely let go, the pile being its only support. When steam is admitted below the piston, it rises, carrying the striking weight with it, until it strikes a trip, which cuts off the steam, and the hammer falls. At the end of the down stroke the valves are again automatically reversed, and the stroke repeated. By altering the adjustment of the trip-piece, the length of stroke (and thus the force of the blows) can be increased or diminished. The admission and escape of steam to and from the cylinder can also be controlled directly by the attendant, and the number of blows per minute is increased or diminished by regulating the supply of steam. The machine can give 60 to 80 blows per minute.

The Double-Acting Steam-hammer not only lifts the ram by steam pressure, but also uses the steam to drive the ram down. This action increases the energy of the blow exerted by the hammer, and permits the use of a lighter hammer. The double-acting hammers are made in various sizes, with piston-stroke ranging from 2 1/4 in. to 20 in., and weight of ram varying from 5 1/2 lb. to 2 tons, and running at speeds of from approximately 1500 to 120 strokes per minute. The lighter hammers are used for driving wood sheeting, and the heavier machines can be used to drive all sizes of wood or concrete piles. The advantages of the double-acting over the single-acting or the drop-hammer are: (1) The lighter weight permits use of lighter handling equipment; (2) The shorter hammer lessens the headroom required in the leads. (3) The increased frequency of blows gives easier driving, as the motion of the pile never actually ceases between blows. This also permits driving with less injury to the pile, especially in the case of large concrete piles.

When a pile is to be driven in water, and the top of the pile must be below the water surface, a "follower" is commonly used, especially with the drop-hammer. This is a round stick of hard timber about 20 ft. long, banded with steel at either end, and placed upon the top of the pile as the latter is driven into the water. The need of a follower may be avoided by using a special type of double-acting steam-hammer which is adapted to driving under water. For this purpose, telescopic leads are used, the inner leads being lowered through the water to the bottom, then the pile with the hammer is let down within the lower leads and driving begins. Other advantages of the submarine driving are: shorter piles, full force of blow delivered to the pile, bearing power determined in the same way as for driving at the surface, piles can be driven straight and accurately spaced, no delays on account of high water, divers not required except for inspection and occasional sawing. An important use of submarine driving is for driving piles within a cofferdam. If the cofferdam is pumped out before the piles are driven, the driving of the piles may open up "sand boils," with the result that some of the piles may be lifted up after they are driven. By driving the piles before unwatering the cofferdam, such troubles are avoided. (See article by E. R. Evans in "The Florida Engineer and Contractor," Aug., 1927.)

Another special use for steam hammers is in pulling piles, and particularly steel sheet piling. For this purpose, the hammer is suspended from a derrick in an inverted position and attached to the pile by wire rope. The derrick pulls on the hammer, which transmits this pull to the pile; in addition the blows of the hammer, acting upward, are transmitted to the pile.

The size of hammer required will depend on the weight of the pile, and on the difficulty of driving. The McKiernan–Terry Drill Co. recommends for concrete piles that a double-acting steam hammer be used exerting an energy of not less than 5500 ft-lb. per cubic yard of concrete in the pile, and that in no case should the total energy per blow be less than 6000 ft-lb. Any of the steam hammers may be operated with compressed air.

Cost of Driving Piles. It is impossible to give any general costs that will be applicable to most conditions. The actual cost is a function of locality, varying soil conditions, length of pile, the number of piles that are to be driven and the time in which they are to be driven. In small work the cost of getting the piledriving equipment to and from the job may be the major item of cost, whereas in the larger work this is a negligible factor. The cost of wooden piles varies from 20 to 40 cents per lin. ft., depending upon the locality and the length. The cost of driving varies between

15 and 30 cents per lin. ft., depending upon the above-mentioned conditions. Precast concrete piles cost from $1.50 to $4.00 per lin. ft. in place, including the cost of driving. The reasons for variations are given above for wood piles. In addition, some of the longer precast piles may be as large in cross-section as 24 in. × 24 in. and are therefore expensive. The cost of manufacturing piles having a cross-section of 12 × 12 in. to 16 × 16 in., including all materials, is about $1.00 per lin. ft. The remaining portion of the above cost is the cost of handling and driving, including the rental of plant. Raymond Concrete Pile Company states that the cost of their cast-in-place pile is approximately $2.00 per ft. in place.

In Water-jet Piledriving a jet of water is discharged below the point of the pile, thus loosening the soil and allowing the pile to sink by its own weight or with very light blows of a drop-hammer. The water may be conveyed to the point through a hose or pipe loosely tied or stapled to the pile, or, if the pile is a concrete one, through a hole molded in the center for that purpose. It makes very little difference, either in the rapidity of the sinking or in the accuracy with which the pile preserves its position, whether the nozzle is exactly under the middle of the pile or not. The efficiency of the jet is greatest

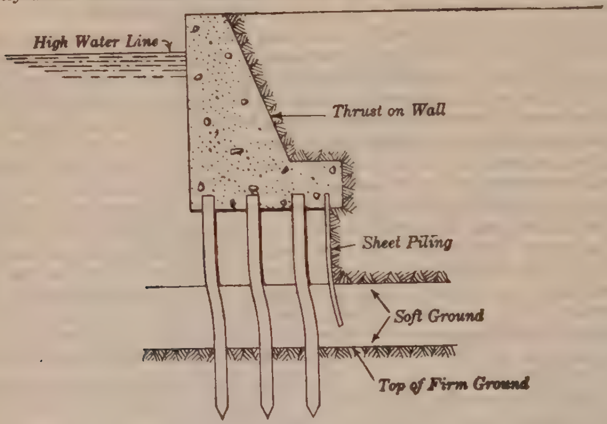


Fig. 19

in clear sand, mud or soft clay; and it is almost useless in gravel, or in sand containing a large percentage of gravel, or in hard clay. The use of the water jet is practically indispensable for driving large piles in sand. The rate of jetting piles in sand that does not contain clay or silt may be much increased by raising the water pressure of the jet, and without decreasing the bearing capacity of the pile.

Lateral Stiffness of Piles. The lack of stiffness of a pile supporting a structure subject to horizontal or lateral pressures is not generally understood. If a pile is placed in a horizontal position, the unsupported projecting ends will bend from 3 in. to 1 ft., depending upon the length unsupported. The same pile in a vertical position will be no stiffer. This may be observed, in the case of piles extending 20 to 40 ft. above firm ground, by a person standing on the pile and working it back and forth by swaying the body. Such piles can often be pushed apart with the hand. When they are framed together with a top platform the piles will work together, but the flexure under lateral or horizontal thrust will be approximately the same.

Attention is also called to the fact that the point of flexure in soft ground may be far below the top surface of the ground (see Fig. 19). The lateral stiffness of vertical piles should be considered only where the piles are driven in firm ground such as coarse sand, and do not extend much above the surface

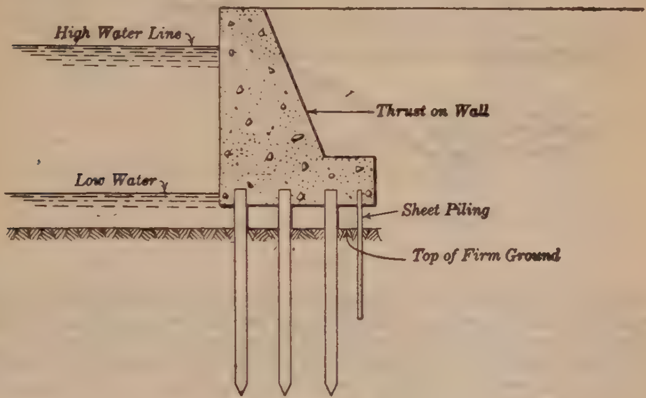


Fig. 20

of this ground, as in Fig. 20. To prevent a retaining wall from moving, as in Fig. 19, either the wall must be carried down into firm ground or riprap, shore anchors or batter piles must be used, as in Fig. 21.

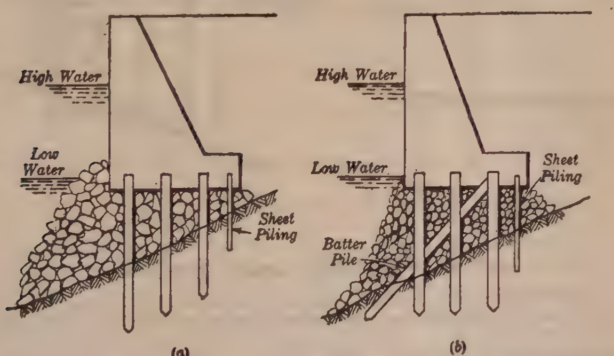


Fig. 21

Attention is called to the fact that a pile bent with a vertical and a batter pile framed as in Fig. 22 is not stable unless there is sufficient length of vertical pile, *a*, below the ground level to act as an anchor through skin friction. If this is not the case a vertical load such as the earth fill above a platform, Fig. 23, must be placed on the vertical piles to give the necessary weight to hold the structure down. This is one reason for the so-called "platform type" of wall shown in Fig. 23.

Attention is also called to the fact that sometimes the stresses at the connection of the vertical and batter pile of Fig. 23 are not given sufficient con-

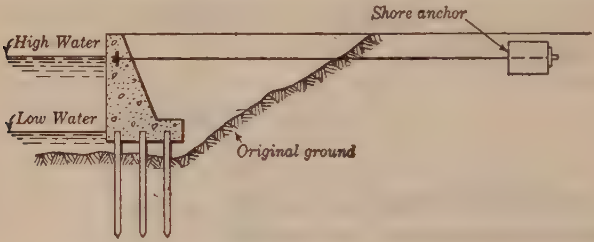


Fig. 21c

sideration. The writer knows of one wall of this type 1000 or more feet in length which gives serious trouble because one bolt was used at this joint instead of three, although the rest of this structure was well designed.

Further comments on the design of such connections are given in Sect. 19, Art. 13. The detail of the joint should be in accordance with the principles of timber detailing given in Sect. 9.

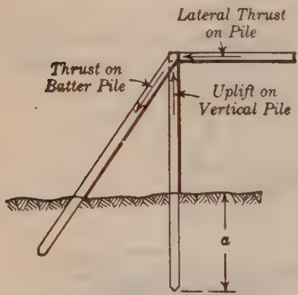


Fig. 22

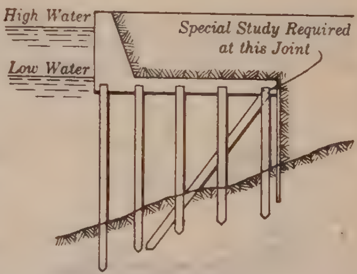


Fig. 23

10. Safe Bearing Loads for Piles

(The following discussion of safe bearing loads for piles is based largely on the paper on "The Science of Foundations" by Charles Terzaghi, published in Trans. Am. Soc. C. E., Vol. 93, p. 288.)

Bearing Capacity of Individual Piles. Piles can be grouped in two general classes: (1) The lower end of the pile rests upon a hard stratum, and the body of the pile receives little or no support from the earth above. In this case the pile must be designed as a column, and its bearing capacity will depend upon its strength as a column rather than upon the bearing capacity of the supporting ground. (2) The pile transmits its load largely by friction along its sides into the adjacent ground. This is the condition most frequently encountered, and to which the following discussion mainly applies.

In attempting to determine the bearing capacity of a pile as based on the effect of the blows of the piledriver, the soil into which the pile is driven must be considered under two general classes:

Class I. In certain materials (particularly in sand, gravel, and permeable artificial fills) the resistances acting while the pile is being driven are practically identical with those acting on the pile under static load. Under such conditions the piledriving formulas can be expected to furnish accurate results. The formulas most commonly in use are the "Engineering News formulas," which are as follows:

For a pile driven with a drop-hammer, $$\mathbf{P} = \frac{2\,\mathbf{Wh}}{\mathbf{s}+1}$$

For a pile driven with a single-acting steam-hammer, $$\mathbf{P} = \frac{2\,\mathbf{Wh}}{\mathbf{s}+0.1}$$

For a pile driven with a double-acting steam-hammer, $$\mathbf{P} = \frac{2\,\mathbf{h}\,(\mathbf{W}+\mathbf{ap})}{\mathbf{s}+0.1}$$

In these, **P** is the safe load in pounds, **W** is the weight of the hammer in pounds, **h** is the free fall of the hammer in feet, **a** is the effective area of the piston in square inches, **p** is the mean effective steam pressure in pounds per square inch, and **s** is the penetration of sinking in inches. **s** is generally measured as the average penetration under the last 5 or 10 blows under a drop-hammer or as the average of the last 20 blows under a steam-hammer, and is assumed to be at an approximately uniform rate; it should not be less than 1 in. under a drop-hammer or 1/4 in. under a steam-hammer, for consistent results with this formula. These formulas are based on an assumed factor of safety of 6.

The Engineering News formula was developed for use with wood piles; it is not as satisfactory for concrete piles, since their great weight influences the amount of penetration under the hammer. For concrete piles, Eytelwein's formula is frequently used:

For a concrete pile driven with a drop-hammer, $$\mathbf{P} = \frac{2\,\mathbf{Wh}}{\mathbf{s}\,(1+\mathbf{W_p}/\mathbf{W})}$$

For a pile driven with a single-acting steam-hammer, $$\mathbf{P} = \frac{2\,\mathbf{Wh}}{\mathbf{s}+0.1\,\mathbf{W_p}/\mathbf{W}}$$

For a pile driven with a double-acting steam-hammer, $$\mathbf{P} = \frac{2\,(\mathbf{W}+\mathbf{aP})}{\mathbf{s}+0.1\,\mathbf{W_p}/\mathbf{W}}$$

in which W_p is weight of pile in pounds and the other symbols are as given above.

For piles sunk with a water jet or those cast in place, the formulas do not apply, except when the pile is tested by driving some time after it has been placed. For these conditions, a static loading test is more satisfactory.

Class II. In very fine-grained silts, soft clays and saturated fine sands and similar fine water-bearing materials, the friction acting on the pile during the driving (hydrodynamic pile friction) is very much less than that which develops after a couple of days' rest (static pile friction), whereas the resistance of the point of the pile under impact (dynamic point resistance) is very much greater than its resistance under static load (static point resistance). The difference between the dynamic and the static resistance is thought to be caused by the squeezing of a certain quantity of water out of the soil beneath the point of the pile in the driving. The water escapes toward the surface through the space between the pile and the ground and forms a film acting as a lubricant, which greatly reduces the friction along the sides of the pile. The force required to squeeze the water rapidly out of the soil beneath the pile is greater than that required to compress the soil slowly. During a period of rest, the film of water is gradually absorbed by the soil and the full static pile friction develops. When piledriving is resumed, both the static friction and the dynamic point resistance must be overcome. For such materials, the pile formulas based on resistance under the hammer give only an approximate indication of the static bearing power of the pile. The safe load is probably far in excess of that given by the formula.

The best way to distinguish whether a material belongs to Class I or Class II is by comparing the penetration per blow immediately before and after a period of rest of at least 24 hours. If these two penetrations are approximately the same, it is evident that the material belongs in the first class, and that the piledriving formulas can be expected to furnish reliable results. If after 24 hours the penetration is two-thirds or less of that previously noted, the material belongs to the second class, and the only satisfactory way to determine the bearing capacity of the pile is by a static load test.

Test Piles. Before designing a structure or ordering piles for a structure, test piles should be driven over the area to determine the total number and requisite lengths. If the behavior of the pile in driving definitely indicates the safe applicability of the foregoing formulas, test loads may be omitted. If in doubt, test several of the piles, framing a platform upon three adjacent piles, and load with pig iron or other heavy material.

In making a driving test there should be little or no bouncing of the hammer. If the hammer bounces to any considerable extent, the fall is too great, or the pile has struck a solid obstacle, or the hammer is too light. Frequently, decreasing the fall will decrease the bouncing and also increase the effectiveness of the blow. If the pile has struck an impenetrable stratum, and the driving is continued, it is probable that there will be a small and continuous apparent penetration due to the mashing of the foot of the pile, which may be followed by a sudden drop of several inches, showing that the pile is broken. Not infrequently when a pile is pulled the bottom is found badly bruised, and sometimes the body of the pile is shattered.

Unless the observed penetration is uniform or at a uniformly decreasing rate, it should not be used in the formulas. If the penetration is practically zero, it is probable that the pile is against an impenetrable stratum and further driving will crush the point. When the penetration has reached as small an amount as 1/4 or 1/2 in. per blow and the hammer rebounds, it is safe to conclude that the limit of safe driving of that pile has been reached. Care should be taken that the test blow is struck on sound wood, otherwise the observed penetration may be much too small and the computed supporting power much too great.

Static Load Tests should be made on piles driven in soils belonging to Class II described above. In such a test, the pile is allowed to stand at least 24 hours after driving. It is then loaded by placing weights on a platform constructed upon a pile (or three piles), keeping the load symmetrical. Careful records of the settlement should be taken, as the load is increased. After a load of 20 or 30 tons per pile has been reached, the pile should be allowed to stand for one or two days, after which the load should be still further increased. The allowable load must not exceed one-half the ultimate load and preferably should not exceed one-third. A record should also be made of the distance which the top of the pile rises after the load is removed, to serve as an indication of the elasticity of the supporting ground.

Bearing Capacity of Pile Foundation. Tests of a few piles in advance of design may be misleading as to the number of piles that are needed. If the piles are driven through soft ground to very hard ground or rock, practically to refusal, it is safe to assume that the bearing capacity of the entire foundation is equal to the safe load as determined by the test pile multiplied by the number of piles used. On the other hand, the test load on a few piles may be misleading when (a) the test pile is driven into spongy ground such as clay which is more disturbed than compacted by the driving of piles, or (b) short piles are driven into ground which is uniform for the full length of pile. These tests may indicate a larger safe bearing capacity than will be true for a group of piles. In the case of (a) the single pile might not disturb the ground sufficiently to injure the skin friction, whereas a group of piles in the same ground may make the skin friction negligible. The writer has observed clay which rose from 5 in. to 5 ft. when piles were driven on 3-ft. centers. The disturbance of the ground was so great that large numbers of the piles first driven rose several feet as a result of subsequent driving of piles in the adjacent area. In the case of (b) the load will be merely transferred from the bottom of the foundation slab to a point a short distance below it, say 25 or 30 ft.; and unless the soil at or near the bottom of the piles is much firmer than the upper layers it is probable that the total resistance of the piles will be materially less than would be indicated by a test load upon a single pile. See discussion of this subject by C. Terzaghi, Trans. Am. Soc. C. E., Vol. 93, p. 290.

11. Deep Foundations*

When an earth or rock stratum suitable for the foundation of a structure can be reached by excavation, by unwatering the site, or by unwatering and subsequent excavation, a number of methods are available, the choice of which must be determined by the particular conditions which may exist.

Open Excavation. When the excavation will be above water level and the ground is cohesive as well as dry, it may be practical to excavate to the desired depth without supporting the sides of the excavation. Usually such excavation does not safely exceed 10 or 15 ft. in depth but, in very dry materials, such pits can be carried down to considerable depths. The foundations for the Nebraska State Capitol, at Lincoln, are concrete piers, belled out at the bottom and resting upon Dakota sandstone, at depths of 25 to 45 ft. below the surface. The excavations for these pits were made through the dry loess and cemented sand and gravel, without sheeting or bracing.

Wood Sheet Piles. The ground below the surface is generally moist, water-bearing, or it may be non-cohesive, and the sides of the excavations must then be supported. For this purpose braced planks are used in ordinary excavations. In all except dry and cohesive ground, the planks are driven vertically and close together, and are called sheet piles. The plank should be 2 to 12 in. thick, depending upon the earth pressures to be resisted and the spacing of the bracing. Except in dry materials the plank should be tongued and grooved, or splined; or Wakefield sheet piling should be used. Tongued-and-grooved sheeting is made or milled so that on one edge of each plank there is a projecting tongue and on the opposite edge a groove into which the tongue of the adjoining plank engages when driven. Splined sheeting has a groove cut into both edges of each plank. A narrow wood strip is then nailed in one of these grooves, projecting beyond the edge of the plank, so that it

* Most of this article was written by Ralph H. Chambers.

will engage with the groove in the adjoining plank in the same manner as tongued-and-grooved sheeting. The connection between the planks is stronger when splined sheeting is used as the splines can be made wider and thicker than the tongues of tongued-and-grooved sheeting. Also, the width of splined sheeting is not reduced by the milling as is the case with tongued-and-grooved sheeting. The tongues and grooves are sometimes made by spiking strips of wood to ordinary dimension lumber.

Wakefield sheeting (Fig. 24) is made of three layers of plank which are nailed or bolted together to form each sheet pile, so that the middle plank projects beyond the edges of the planks on each side of it, thus forming a tongue on one edge of each sheet pile and a groove on the other edge. This form of sheeting is not as strong as solid sheeting of equal total thickness of either of the other two forms mentioned. On the whole splined sheeting is preferable.

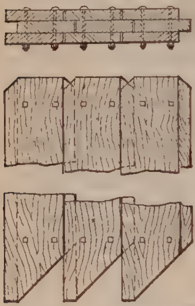


Fig. 24. Wakefield Sheet Pile

Wood sheet piles should be beveled at the bottom on one side and one edge to facilitate driving and to cause each succeeding sheet pile to drive close against the adjacent sheet pile and tight against the bracing. In general, wood sheeting should be driven as the excavation proceeds and the bottom of the sheeting should never at any time be driven more than a few feet below the bottom of the excavation. Wood sheeting may be driven by wooden mauls, the tops of the sheet piles being protected by an iron cap, or may be driven by steam- or air-hammers.

In large open excavations the sheeting is braced by one or more sets of horizontal timber rangers or wales supported by braces which slope down to the bottom of the excavation, or by horizontal braces across the excavation. Rangers, wales or sets are usually spaced 6 or 7 ft. apart vertically at the top, and closer together near the bottom. In deep excavation, especially if there is a large hydrostatic head, it may be necessary to place them almost touching each other near the bottom. Both sheeting and bracing must be proportioned to resist the lateral pressure of the earth and water (see Sect. 10, Art. 19). In pits of small horizontal dimensions, the computed pressure should be modified in accordance with Art. 4. When the ground is very unstable or is water-bearing, hydrostatic pressure should be used as a measure of the forces to be resisted.

The excavation may be made by hand tools, casting the excavated material to the top of the excavation, or to successive benches and so to the top; or the excavation may be loaded into buckets and hoisted by windlass or derrick. Clamshell or orangepeel buckets, operated by derrick or crane, may also be used Water in the excavation must be removed, as will be later described. When the headroom over the site of a foundation is restricted, as under an existing building, the sheeting may be placed horizontally. It is then put in, plank by plank, in horizontal sets, notched and butted together at the corners. After four or five of these sets have been placed, vertical breast pieces and horizontal braces must be placed, if needed.

In comparatively dry ground, excavation by the use of wood sheet piles may be carried to a depth of 60 or 70 ft. When the depth is more than 20 ft., it is usual to make the horizontal dimensions of the first pit (i.e., the upper 20 ft.) sufficient so that, when a depth of 18 to 20 ft. is reached, a second set of sheeting can be started inside the rangers of the upper pit, and thus a pit of smaller dimensions can be carried another 20 ft. If the excavation is to be carried still deeper a third set of sheeting may be driven inside the second set.

In order to obviate the necessity of enlarging the excavation in the upper section, the so-called "poling board" method may be used. Fig. 25 shows the details of this method, which is similar to that already described except that the sheeting, in each successive drive, is only 4 or 5 ft. long and is driven slanting outward, so that the average horizontal dimensions of the pit are kept constant. It is obvious that this method cannot be used in wet or very unstable materials. This method is more generally used when the great depth to which the excavation must be carried has not been anticipated. When the material to be excavated is very soft or is water-bearing to the extent that the ground rises inside the excavation, or is carried up into the excavation by water, wood sheeting cannot be used advantageously and steel sheet piling driven at all times 10 or 15 ft. below the bottom of the pit excavation works better (Art. 12).

Fig. 25. Poling Board Method

Chicago Method. When the material to be excavated is a fairly stiff clay, as that which underlies the city of Chicago, the excavation may be carried down to comparatively great depths by the Chicago Method. In this method the excavation is carried down in a circular well 3 to 8 ft. in diameter in successive sections of 5 ft. each. The sides of these sections are lagged with 6-in. tongued-and-grooved staves, usually maple, 2 or 3 in. thick and 5 ft. long Each 5-ft. section of lagging is held in place by two steel rings 3 × 3/4 in., each ring consisting of two semicircles flanged at the ends and connected by bolts. The rings are usually removed as the wells are filled with concrete but the lagging remains in place. It is usual to bell out the bottoms of these wells to a frustum of a cone, having a bottom diameter twice the diameter of the well. These wells are excavated to rock to depths of 100 ft. or more.

Gow Method. In material similar to the Chicago clay, or even in a softer and more water-bearing material, a method somewhat similar to the Chicago method is sometimes used. In this method a single cylindrical concrete shaft of sufficient cross-sectional area (generally not less than 3 ft. in diameter) is sunk through any unsuitable upper layers of soil to a satisfactory bearing material. The method consists of sinking a series of short steel cylinders varying slightly in their several diameters so as to telescope into one another, the largest size being used as a starter and the others inserted successively through those already in place. When the cylindrical excavation has reached the desired depth the bottom of the opening is enlarged in the form of a truncated cone having a base of sufficient area upon which to impose the entire load carried by the particular column without danger of overloading the underlying soil. Concrete is then deposited in the opening thus prepared so as to fill it entirely, the several steel cylinders being successively withdrawn as the concreting work progresses. The excavation is done by hand tools.

Drop Shafts. Another method which can be used in ground which is not too soft or too water-bearing is similar to the dredging caisson method (Art 14). This consists of the sinking of a "well curb" which is a hollow cylinder built of reinforced concrete or brick, often sheathed on the outside with wood plank, and provided with a steel cutting edge at the bottom and which is constructed to a convenient height at the site and sunk by excavating inside and under it. The well curb will sink of its own weight until the friction of the ground against the sides becomes sufficient to support the weight. The well shaft must then be weighted until the friction is overcome. The intensity of the friction on the sides of the shaft is equivalent to that on dredging caissons (Art. 15). Sometimes the shafts are built in two sections, the lower telescoping within the upper. This method was used in the construction of a mine shaft for the St. Albert Collieries, near Calgary, Alberta, Canada. The object of using two sections of shaft is the reduction of the amount of weight which must be used at one time. It will be obvious, however, that the use of two sections greatly increases the quantity of excavation and concrete.

Well-Drilling Method. Recently, large pipes or steel cylinders have been sunk to rock by well-drilling methods. It is believed that cylinders up to 48 in. in diameter can be sunk by such means. This cannot, however, be done when many boulders are encountered, because the boulders force the drilling apparatus and the cylinders out of line. The method consists of driving a steel pipe into a hole in the ground which is drilled out ahead of the pipe. The drilling is done by a heavy "bit" which is lowered into the pipe on the end of a cable, and is churned up and down in the soil below the pipe. The loosened soil is removed by a "bailing" bucket, which is lowered into the pipe after the bit is hoisted out. After the hole is drilled out for some distance below the end of the pipe, the latter is driven down to the bottom of the hole by a special drop-hammer.

This method was used to construct the foundations for the west ventilating shaft of the Holland Vehicular Tunnel, under the Hudson River, between New York and Jersey City, N. J. Two groups of 24-in. diameter pipes, forty-two in each group, were driven to support two rectangular foundations. The pipes were spaced around the peripheries of the two proposed foundations. The pipes were driven to rock from a temporary platform of piles and timber through 30 ft. of water and 220 ft. of silt. They were cleaned out by pumping water into the pipe, then reinforced with hooped reinforcing bars, concreted to a level about 100 ft. below water level and cut off (from inside the pipes) at that point, 70 ft. below the surface of the silt. Two pneumatic caissons were then sunk until they rested upon the two groups of pipes. Each of these pipes supports a load of 97 tons which may become 126 tons under wind pressure. (See Eng. News-Rec., Vol. 90, p. 242.)

The Well-Point Method consists of sinking a row of pipes vertically into the ground around the excavation, at intervals of a few feet, to a depth somewhat greater than that of the proposed excavation. The lower end of each pipe is provided with a "well-point," which is a metal tube with a solid pointed end, the sides of the tube containing a large number of small holes. After the pipes are placed, their tops are connected with mains leading to the pumps, which draw the ground-water through the well-points and thus lower the ground-water level. The length of time required to lower the water to the desired level will depend upon the degree of fineness of the sand. This method has been much used in the construction of the New York City subways and was used to excavate portions of the Cape Cod Canal by steam shovels working "in the dry."

An example of its use on a large scale is given in the report of the construction of the lock for the inner Harbor Navigation Canal at New Orleans, La. The subsurface materials were alternate strata of clay and quicksand. The unwatering and excavation were accomplished by using driven pipe wells and sheet piling to cut off the quicksand strata. The site (1625 ft. by 650 ft.) was surrounded by an enclosure of wood sheet piles, 48 ft. long, within which the excavation was done by suction dredges. Then a second enclosure of wood sheet piles 55 ft. long was driven within the first enclosure. This second enclosure was 1285 ft. long and 280 ft. wide. Inside the second enclosure a third enclosure of 50-ft.-long Lackawanna (Bethlehem) steel sheet piles was driven. Thirty-two pipe wells were driven just outside of the second sheet pile enclosure and 24 pipe wells were driven outside and close to the innermost enclosure of steel sheet piling. Both of these lines of wells were driven to a level about 40 ft. below original ground surface. Just inside the enclosure of steel sheet piling 126 pipe wells were driven and 5 pipe wells were driven in a line at the middle of the proposed excavation. The two latter groups were driven to a level about 60 ft. below the original ground surface. All pipe wells were 10 in. in diameter. The inner enclosure of sheet piling and the several lines of pipe wells were driven successively as the water level was lowered. The innermost enclosure of steel sheet piling was braced by horizontal timbers running across the enclosure. After months of pumping the water level within the innermost enclosure was lowered to a depth of 47 ft. below the level of the original ground surface and the lock floor was built.

See also account of this method as used at Lynn, Mass., by Charles Terzaghi, Journal Boston Soc. Civ. Eng., Sept., 1927.

Freezing Method. This method, invented by F. H. Poetsch of Prussia, in 1883, has been much used in Germany and has also been used in Belgium and England but has been used in this country in only a few instances. It was used in the construction of the Chapin mine shaft, Iron Mountain, Mich., and it was also used for stopping a leak in a cofferdam for the Detroit–Superior Bridge at Cleveland, Ohio. It is used where the subsurface materials are fine sands and quicksands which cannot be successfully handled by ordinary methods. The extensive installation necessary makes this method expensive, but it is invaluable in very deep shafts and it will doubtless be increasingly used for such shafts. Shafts more than 2000 ft. deep have been so sunk. The site was surrounded by pipes driven to a depth somewhat greater than that of the proposed excavation. These pipes were closed at the bottom and inclosed smaller pipes which were open at the bottom. A brine freezing mixture was pumped through the inside pipes and, issuing from the bottoms of these pipes, rose through the larger driven pipes. After many days' operation of the freezing apparatus, the ground within the circle of driven pipes was frozen and was excavated as solid material.

Costs. The cost per cubic yard of excavation for foundations must include not only the cost of the excavation itself but also the cost of supporting the sides of the excavation, as well as the costs of pumping, an allowance for plant and the cost of operating the plant. By the Chicago method, at present prices and labor rates, the cost per cubic yard, of an excavation 6 or 7 ft. in diameter, including lagging and pumping, as well as overhead charges, will be about $15.00. The cost of excavation by the Gow method will be about the same.

FOUNDATIONS UNDER WATER

12. The Cofferdam Process *

When the site of a proposed foundation is covered by water it must first be unwatered, unless caissons are used, before the excavation work can be done and the foundation built. Recourse may then be had to one of the following methods.

* Most of this article was written by Ralph H. Chambers.

Earth Dikes. When the site is covered by comparatively quiet water 4 or 5 ft. deep, or even more, and the bottom is rock or firm and impervious material, the unwatering can be done by constructing an enclosing dike composed of sand- or clay-filled bags, or a bank consisting of two walls of sand-bags with a clay filling between. The dike should be at least as thick as its height and usually the clay puddle between the sand-bags should be 3 ft. or more in thickness.

Puddle Cofferdams. When the bottom is a stiff impervious material into which sheeting can be driven, puddle cofferdams are built by driving two lines of sheeting to retain the clay puddle (Fig. 26). Temporary timber piles, at intervals of 10 ft. or more, are first driven in two lines around the site. To these, timber wales, *w* and *g*, are bolted above water level. Within these timber rangers are driven an inner and an outer enclosure of wood sheet piling, of one of the forms that have been described. The space between the two lines of sheet piling is filled with clay puddle. The innermost sheet piling and the supporting rangers or wales should be designed to support the clay puddle. The outer ring of sheet piling, which will always have the water pressure against it, may be comparatively thin. The whole cofferdam may depend for its support, after the site has been unwatered, upon the temporary timber piles, or the inside wales may be braced across the site. The puddle in a cofferdam of this form should never be less than 3 ft. thick and should be at least one-half as thick as the height of the cofferdam.

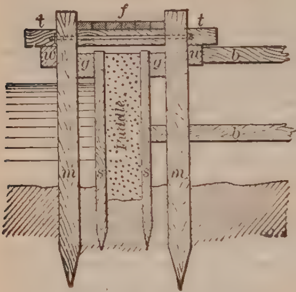


Fig. 26. Puddle-Wall Cofferdams

Steel Sheet-Pile Cofferdams. Except when the bottom is bare rock, it is now usual to build a cofferdam consisting of a single wall of steel sheet piling rather than any of the foregoing types. (Rock-filled cribs with sheet piling, as in Figs. 27 or 28, are used if the width of excavation is too great to be braced.) Even on bare rock if the rock is soft enough to get a toe-hold steel piling may be used. On some of the piers of the Arlington Memorial Bridge at Washington, D. C., steel piling about 50 ft. long was used successfully, the rock being usually a hard gneiss and the overlying material of little value either for a toe-hold or as a water stop (Fig. 29). Such cofferdams consisting of wood sheet piles have already been described. When the water is more than 20 ft. deep or when it is necessary to drive the sheet piles to a depth below the bottom to cut off the water, steel sheet piling is used. There have been many sections of steel sheet piling designed but only a few of these have been found satisfactory and some of the forms formerly used are now obsolete. The sections now used in this country for cofferdams are Bethlehem sections D P 165 and SPE 14, and the Larssen section II which is imported from

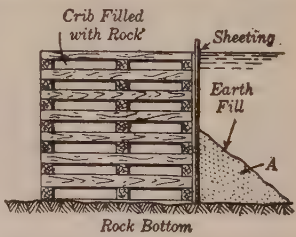
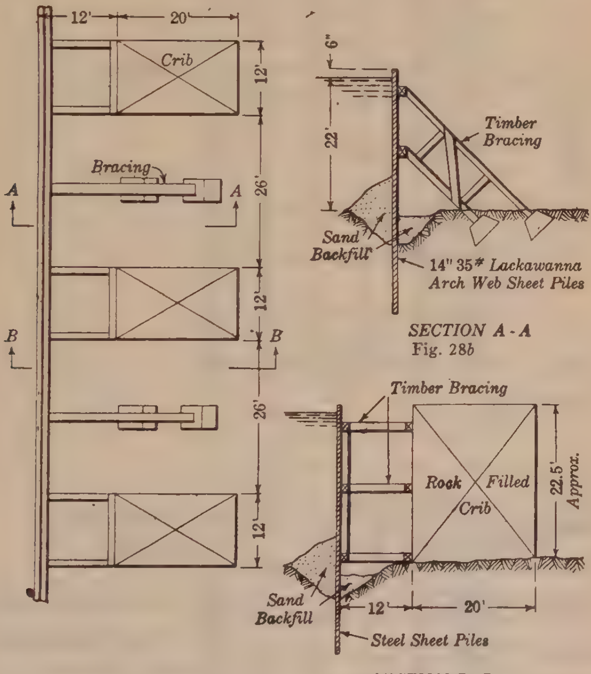


Fig. 27

Germany. These sections are shown in Fig. 30. Fig. 30a shows a new section introduced in 1928 by Jones & Laughlin Co., which is also suitable for cofferdams and deep foundations. Steel sheet piling is also often used in excavations for foundations where the site is not covered by water. The advantage in using steel sheet piling is that it can be driven far below the bottom of the excavation, to cut off a flow of water or to retain soft and unstable materials. It can be pulled after the work is done and can be used again. In



SECTION A - A
Fig. 28b

SECTION B - B

Fig. 28a Fig. 28c

Cofferdam Sherman Island Dam

excavations of small dimensions, on land, the sections most used are the Carnegie section M 104 and the Bethlehem sections SPB 12 and SPE 14.

When steel sheet piling is used for a cofferdam, at a site covered by water, a guide frame should first be constructed around which the piling is driven. When the water is not over 12 ft. deep, such a frame may be constructed by driving a line of timber piles around the site and bolting timber wales to these piles, usually outside and inside of the proposed line of sheet piling. When water is deeper it is usual to build a framework which will serve to brace the sheet piling after the cofferdam is pumped out. This frame is built at the site and lowered by weighting into an excavation which has been dredged for it;

or the frame may be built elsewhere, towed to the site and sunk into place. Timber piles are usually driven to hold the bracing frame in place. The wales may be braced across the cofferdam, above the water.

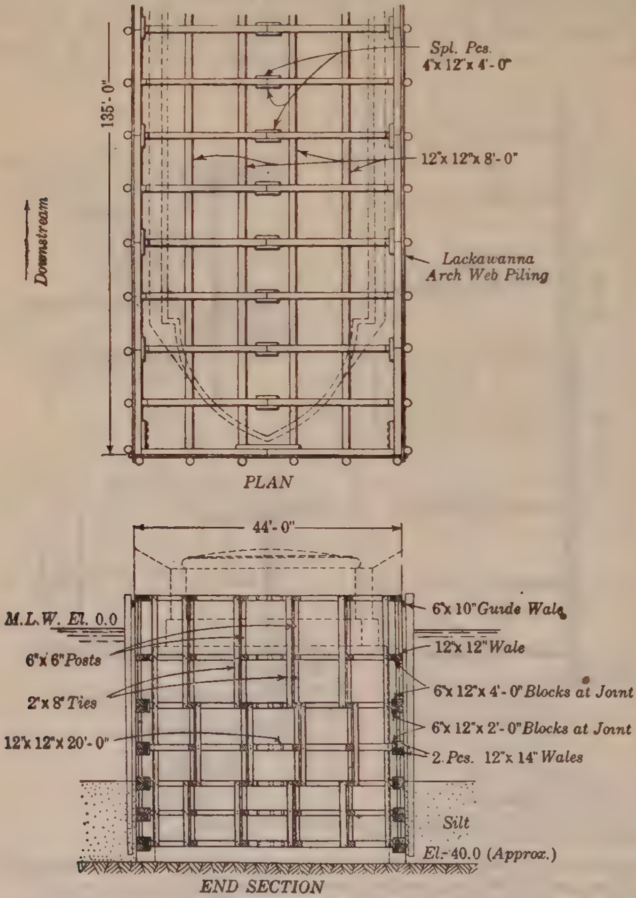
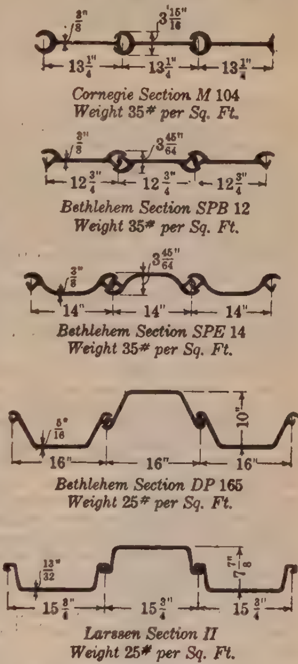


Fig. 29. Cofferdam for Arlington Memorial Bridge Piers

Steel sheet piling is usually driven by a double-acting steam- or air-hammer. Such a hammer, suspended from a derrick or frame, is also used to pull the sheeting by reversing the direction of the blows (Art. 9). Steel sheet piling will usually leak when first driven. After the pumping in the cofferdam has been started, cinders dumped into the water outside of the sheet piling will be drawn into the interlocks of the piling by the currents induced by the pumping and will effectually calk these joints. The vertical spacing of the wales and the size and spacing of the bracing should be determined by computation, considering the water and earth pressure upon the piling and the strength and stiffness of the piling. Excavation inside cofferdams is usually done by clamshell or orangepeel buckets, operated by derrick or crane, or by hand labor and dump buckets.

Steel sheet-pile cofferdams have been built of very large horizontal dimensions and it is common to build such cofferdams to depths of 45 ft. or more.



SECTIONS OF STEEL SHEET PILING

Fig. 30

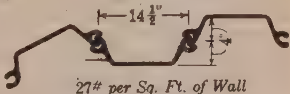


Fig. 30

No discussion of this subject would now be complete without mention of the cofferdams which were used for the construction of the foundations of the western tower of the Hudson River Bridge from 178th Street, New York City, to Fort Lee, New Jersey, which is now in course of construction (1928). Two cofferdams, each about 100 ft. square, were built for this foundation near the west bank of the Hudson River. The rock surface, at the west side of these cofferdams, was about 45 ft. below the water surface. From the west side to the middle of the cofferdams the rock sloped gradually down and from the middle the slope was steep down to the east side where the depth below surface was about 85 ft. The site was first dredged down almost to the level of the rock at the west side. Bracing frames, consisting of 30-in. Bethlehem beam rangers and timber braces and posts, were built in place and sunk. Larssen steel sheet piling, Section II, was then driven around the frame, down to rock. On the west side and about half-way on the north and south sides, a single wall of sheet piling was driven. Along the balance of the north and south sides and along the east side two lines of sheet piling, about 8 ft. apart, were driven and connected together by steel sheet piling at intervals to form pockets. These pockets were dredged out to rock and were filled with concrete, placed under water, up to a level about 50 ft. below the water surface. The balance of the height of the pockets was then filled with sand. These cofferdams were unwatered without difficulty and the excavation to the full depth was made without mishap, except for one accident at the northeast corner of the north cofferdam where the rock berm under the sheet piling failed suddenly, causing the flooding of this cofferdam. The sheet piling at this point was pulled and replaced and the work was successfully completed.

These are, by far, the deepest cofferdams that have been constructed Their successful completion was undoubtedly due to the use of concrete-filled double walls on the deep sides.

The essential feature for the success of any cofferdam is that the stratum into which the sheet piling is driven shall be practically impervious to water for a considerable depth below the proposed bottom of the cofferdam. If there appears to be any doubt of this, the method described in Art. 14 should be adopted.

Driving Sheet Piles. The Bethlehem Steel Company gives the following recommendations for driving steel sheet piling: "A drop-hammer or steam-hammer mounted on a suitable rig are the more common. The latter is preferable as it puts a constant weight on the pile, delivers quick, short blows which keep the pile moving, is less apt to batter the piling tops unless the hammer selected is too light, and as a rule does the most satisfactory work. The use of a cap over the end of the pile while being driven is not essential, but is recommended wherever hard driving is encountered, to save the top of the pile from being battered. Piles which are to be pulled and re-used should invariably be protected in this way, and as occasional piles must usually be withdrawn and re-driven in any work, the use of the driving cap is advisable. Good types of caps, each consisting of a steel casting, slotted to fit the piledriver leads and the top of the pile, and containing a recess in which a wooden block on end can be placed to cushion the blows of the hammer, can be secured from the manufacturers of pile-driving hammers.

"Driving piling under water or below the piledriver leads is facilitated by using a follower, consisting of another pile fitted with projecting plates that conform with the section below. (See also under Double-Acting Steam Hammers, Art. 9.)

"Care should be observed in driving to keep each pile plumb. If the first pile driven is vertical, maintaining a wall straight or in true form and making closures will not prove difficult. Piling that deflects badly or refuses to penetrate on account of serious buried obstructions should never be forced, otherwise it may separate at the interlock and be twisted out of usable shape. Where the decreased movement of the piling under the hammer blows indicates serious obstruction, it is often advisable to continue the line, leaving the obstinate piles projecting above the rest until after excavation, when the obstruction may be removed and all parts of the piling wall properly set.

"The cost of driving steel sheet piling must necessarily depend so much upon the size, length and type of section used, the nature of the soil, the amount of piling used, local labor conditions, the method of driving, etc., that general figures are not practical."

In driving the sheet piles it is usually best to place them around the entire cofferdam and then work them down gradually (this is called "gang driving") rather than to drive each one down as it is placed. The latter method is more liable to result in getting them out of plumb. In this case it is necessary to use tapered piles which have to be specially made, are expensive, and require time to manufacture. In the construction of the cofferdams of the Sherman Island hydroelectric development and in driving the permanent sheet steel piling cutoff, much trouble was found in keeping the piling plumb when each pile was driven down as placed. These piles were driven into sand containing occasional small boulders. In some cases additional sheet piling was driven back of and in contact with the original sheeting because the interlock opened up when the piles encountered boulders.

Reinforced-Concrete Sheet Piles are used where the piles are to form a permanent part of the structure, as in piers and wharves. These are designed as beams to carry the lateral pressure of the material which they support, though they may also carry vertical loads. The thickness is generally 8 in. or more. They may be tongue-and-grooved; or a semicircular groove may be cast in each edge, leaving a cylindrical space where the adjacent pile is driven, this space being later cleaned out and filled with mortar. Combination concrete and steel sheet piles have also been used, in which the steel pile is cut longitudinally along its web, and the two pieces are cast into a concrete pile on opposite edges, leaving the interlocking portions of the steel pile exposed.

Cost. A statement of the cost of construction work, unless all the conditions are taken into account, is likely to be misleading. However, to give some idea of the cost of such work as has been described the following are the approximate labor costs of the cofferdam for the screen well for the Narraganset Electric Light Co. built at Providence, Rhode Island, in 1925:

Driving steel sheet piling, 24 cents per square foot.
Pulling steel sheet piling, 14 cents per square foot.
Timber bracing, placing and removing, $97.00 per M. b. m.
Excavation, $2.00 per cubic yard.
Placing concrete under water, $2.65 per cubic yard.
These costs do not include plant, power, insurance and general overhead expenses.

Miscellaneous Types of Cofferdams. It is sometimes expedient to construct the foundation for a structure upon a base of concrete, deposited under water. To confine this concrete a crib with sheeted sides but open at the top and bottom is sunk at the site, upon a bottom which has previously been dredged out and leveled. Such a method was used for the construction of the river wall of the lock at the Scotia dam, on the Mohawk River near Schenectady, New York, for the New York State Barge Canal. Eight cribs, 26 ft. wide and 44 to 51 ft. long, were sunk in a continuous line upon a previously dredged gravel bottom about 30 ft. below the water surface. Concrete to a depth of 12 ft. was placed in the bottom of these cribs by bottom-dump buckets. After the concrete had set and the sides of the cribs had been connected by divers, the cribs were pumped out and the concrete wall was constructed in the cofferdam.

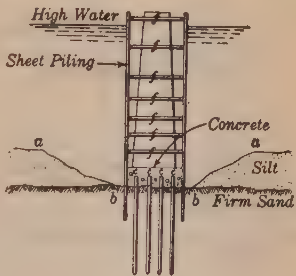


Fig. 31

This method was also successfully used in the construction of three piers in the Niagara River between Buffalo, New York, and Fort Erie, Ontario, Canada. Each of the cribs used was built of timber in skeleton form. The current in the river has a velocity of 6 to 8 miles per hour and it was impossible to tow the cribs to site by ordinary means. Therefore heavy anchors were set in the river about 1500 ft. above the site and each of the cribs was towed to site by a winding drum, mounted upon a barge and pulling against the anchors previously set above the site. After the cribs had been anchored in place and sunk in 16 to 19 ft. of water, steel sheet piling was set around the frame and driven tight into the rock. The joint between the piling and the rock was then calked by concrete in bags, placed by divers. In the quiet water within the sheet piling, concrete was placed through the water to a depth of 8 ft. When this concrete had set the space above was pumped out and the masonry of the piers was completed.

In soft ground the method used is as follows (Fig. 31): (1) Excavate to line *a b b a*. (2) Drive wooden piles and cut off as at *c c c c*. (3) Drive wood or steel sheet piling to form cofferdam. (4) Cap piles with concrete to seal against water coming up through bottom when unwatered. (5) Float in bracing frames and force them into positions *f f f*, by weighting or otherwise. (6) Unwater the cofferdam. (7) Build pier on concrete cap in the dry. (8) Pull sheet piling.

Rock-filled cribs are adapted to either hard or soft bottoms, using wood sheeting if the cribs rest on rock, or wood or steel sheeting if built to act as a water stop upon a soft bottom. Cribs generally should have a width at any point equal to the probable height of the water above the point. These crib coffers may be used for dams or piers and obviously without a sealed bottom.

On rock it may be desirable to fill in an earth cut-off as at *A*, Fig. 27. Fig. 28 shows the cribs used for the cofferdam of Sherman Island dam on the Hudson River.

13. Leakage and Pumping

Leakage. A serious objection to the use of a cofferdam is the difficulty of preventing leakage through or under it. It is nearly impossible to prevent considerable leakage, unless the bottom of the crib rests upon an impervious stratum or the sheet piles are driven into such stratum. Water will find its way through nearly any depth or distance of gravelly or sandy bottom, and seams of sand are very troublesome. Logs or stones under the edge of the dam are also a cause of considerable annoyance. The object of a cofferdam is not to prevent all infiltration, but only to reduce it to the extent that a moderate amount of pumping will keep the water out of the way.

Large leaks in a sand bottom often carry fine particles of soil. Sometimes this water appears clear to the eye, but if a glassful of such water is held up to the light, particles may be seen in it. If particles are being transported,

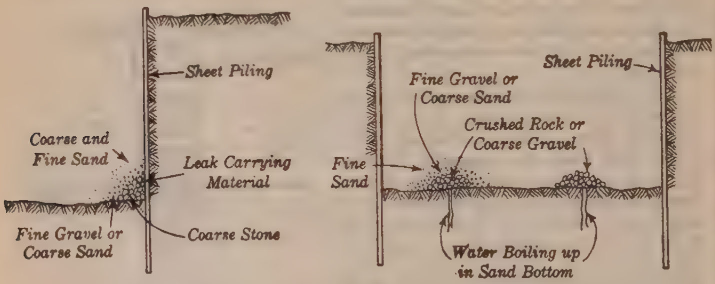


Fig. 32

settlement may take place in the structures outside of and adjacent to the excavation and subsequently it may take place in the foundation which is being built, but to a less degree. The following methods may be used to decrease the flow of water into the cofferdam and to reduce the amount of material that is being carried by the water coming into the bottom of the excavation by reducing its velocity:

(1) Leaks or boils at the bottom of the sheet piling may be prevented from carrying material by checking the velocity of flow, as in Fig. 32.

(2) Similarly, the flow of material carried by water " backing up " in the foundation bottom may often be prevented, as in Fig. 32.

(3) If there are bubbling springs all over the foundation area with sufficient velocity either to carry material or to wash out the cement or mortar of the concrete, cover the whole area with a thin and preferably a porous fabric which will stand the weight of a light layer of stone or gravel (5 to 8 in. thick) and then place the concrete. A sump outside the concrete forms is desirable

to permit the concrete above the first 2 or 3 ft. to be placed in the dry (see Fig. 33).

(4) With steel or wood sheet piling driven through water, leaks in the sheeting may be stopped by depositing, outside of the sheeting, cinders, sawdust, manure, or fine sand. This fine material will be drawn into the joints of the piling and stop a surprisingly large quantity of water.

The method to be employed in removing the water will vary greatly with the amount present, the depth of the excavation, the appliances at hand, etc. If the excavation is shallow and the amount of water small, it can be bailed out; but usually some form of pump must be employed. The pumps generally used for this kind of work are the direct hand-lift foundation-pump, the diaphragm pump, the steam siphon, the pulsometer, and the centrifugal pump.

The Direct Hand-lift Foundation-pump consists of a straight tube at the bottom of which is fixed a common flap valve, and in which works a piston carrying another flap valve. The tube is either a square wooden box or a sheet-iron cylinder, usually the latter, since it is lighter and more durable. The pump is operated by applying the power directly to the upper end of the piston-rod, the pump being held in position by wooden stays or ropes. The only advantage of the wood-box hand-lift pump is that it may be improvised on the job; and the disadvantage for foundation work of all pumps having flap valves is the danger that straw, sticks, mud, etc., will interfere with the action of the valves.

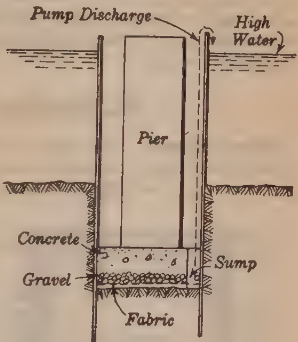


Fig. 33

The Diaphragm Pump is the usual form of hand pump for foundation work. This pump consists of a short cast-iron cylinder having a rubber hose connected to its lower end, and being divided about midway of its height by a flexible horizontal rubber diaphragm. The central portion of the diaphragm is connected to a bent-lever handle, and there is a valve in the center of the rubber disk. The rise and fall of the center of the disk acts as a piston. A pump of this form throws a large amount of water, allows sand and gravel to pass without choking, is not easily clogged by straw, leaves, etc., and is easily unclogged. Various forms of diaphragm pumps are also driven by a gasoline engine and are much superior to the hand pump.

The Steam Siphon is the simplest of all pumps, since it has no movable parts whatever. It consists essentially of a discharge pipe, open at both ends, through the side of which enters a smaller pipe having its end bent up. The lower end of the discharge pipe dips into the water; and the small pipe connects with a steam boiler. The steam, in rushing out of the small pipe, carries with it the air in the upper end of the discharge pipe, thus tending to form a vacuum in the lower end of that pipe; the water then rises in the discharge pipe and is carried out with the steam. The steam siphon is limited practically to lifting water only a few feet; its cheapness and simplicity are recommendations in its favor, and its efficiency is not much less than that of other forms of pumps. One of the advantages of the steam siphon is that frequently it can be improvised on the work from ordinary pipe and fittings. Several forms and sizes are upon the market, ranging in capacity from 5 to 200 gal. per minute, and are much better than one made from pipe. They are usually called jet-pumps by the manufacturers.

The Pulsometer is an improved form of the steam siphon. It may be properly called a steam pump which dispenses with all movable parts except the valves. The height to which it can lift water is practically unlimited, and it is in very common use for pumping out cofferdams.

A Centrifugal Pump must be used when very large quantities of water must be handled. The distance from the water to the pump is limited by the height to which the ordinary pressure of the air will raise the water and is generally not over 18 or 20 ft. vertically; but the height to which a centrifugal pump can lift the water is limited only by the velocity of the outer ends of the revolving blades. Since there are no valves in action while the pump is at work, the centrifugal pump will allow sand and large gravel to pass. Pumps having a 6-in. to 10-in. discharge pipe are the sizes most frequently used in foundation work.

14. Floating Caisson Process *

Open Caissons. This method, which is very old, is still used and, under some conditions, is very reliable and practical. It consists of the sinking of a floating box in which the masonry is built as the box sinks (Fig. 34). The

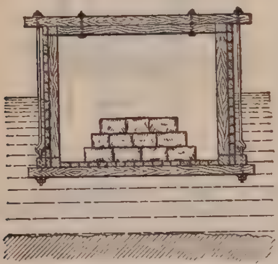


Fig. 34 Open Caisson and Pier

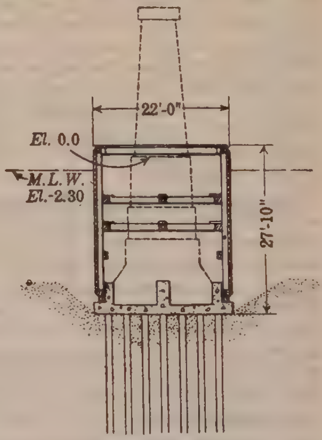


Fig. 35. Pier Caissons, Newark Bay Bridge, C.R.R. of N. J.

sides are then detached, leaving the masonry resting upon the bottom of the box which in turn rests upon a previously prepared foundation. In its modern adaptation the bottom of the box is usually made of reinforced concrete and the sides of braced timber.

The subway and highway bridge over Flushing Creek at Flushing, Long Island, is an example of its use (1927). Two piers, one on each side of the creek, were built for a large bascule span. Each of these piers was 93 ft. wide and 118 ft. long and the bottom of the masonry was placed at an elevation 36 ft. 9 in. below mean high water on a foundation of timber piles which had been previously driven and

* In part by Ralph H. Chambers.

cut off under water. An enclosure of 12-in.-thick wood sheet piles was built around the site, before the piles were driven. The purpose of this sheeting, which was only braced above water level and was never pumped out, was simply to keep out the mud which had previously been dredged down only part way to the level of the top of the proposed pile foundation. After the balance of the mud was dredged out by clamshell bucket and the piles had been driven, which was done by a steam-hammer working under water, the tops of the piles were cut off to an exact level. A comparatively thin wooden floor was built in sections, assembled and connected together within the area enclosed by the sheet piling. To this floor wooden sides were connected and braced. A thick slab of concrete was then placed on the floor, the box formed by the floor and sides sinking as the concrete was placed. This concrete slab or base was reinforced by steel, longitudinally and transversely. Upon the concrete base the concrete and masonry of the pier above were built up as the box sank, until finally it rested upon the pile foundation. The sides of the box were then detached and used again on the second pier.

The same method was used in the construction of a large number of piers for the Central Railway of New Jersey Bridge over Newark Bay, New Jersey (Fig. 35).

15. Dredging through Wells*

Dredging Caissons. When it is necessary to excavate to great depths to obtain a satisfactory foundation, particularly when the site is under water, dredging caissons are used. This method has the advantage that it can be carried to great depths, far below the limit that is possible with pneumatic caissons (Art. 17). A dredging caisson is simply a large box or shaft, built of timber or steel and filled with concrete, or it may be built of reinforced concrete. It is provided with a well, or wells, through which the material below may be dredged. The caisson sinks as the material below is dredged and as the compartments of the caisson are filled with concrete. It may be of the form of a hollow cylinder or may be rectangular in section. In the circular form there is only one central dredging well; in the rectangular form there are usually several wells. The bottom edge of the caisson, called the "cutting edge," is usually V-shaped in section and shod with steel to prevent distortion or damage in sinking. The caisson is built up, as it sinks, and the dimensions are usually such that the weight of the caisson will overcome its buoyancy and the friction of the sides against the surrounding materials, without the use of the additional weight which is usually necessary in sinking drop shafts and many pneumatic caissons.

The friction to be overcome in sinking these caissons is usually of greater intensity than that encountered in sinking pneumatic caissons, as the air escaping from the latter somewhat relieves the friction. This friction may be 400 to 1500 lb. or more per square foot of surface. Probably the average friction encountered is 500 to 700 lb. per sq. ft. Dredging caissons are often provided with jetting pipes, running from the top down to the cutting edge and discharging around it, to relieve the friction. Dynamite sunk down into the bottom at the cutting edge and exploded will often start the movement of a caisson. Another difficulty encountered is the proper guiding of the caisson, particularly when the caisson has a number of dredging wells and when obstacles, such as sunken logs or boulders, are encountered. This

* By Ralph H. Chambers.

method is not as sure and definite as sinking by the pneumatic method, but has been successful in the many years of its use, though only after the surmounting of many difficulties.

This method was used to sink the piers of the Poughkeepsie Bridge, New York, to a depth of 134 ft. below mean high water (Eng. News, Oct. 29, 1887); of the Hawks-

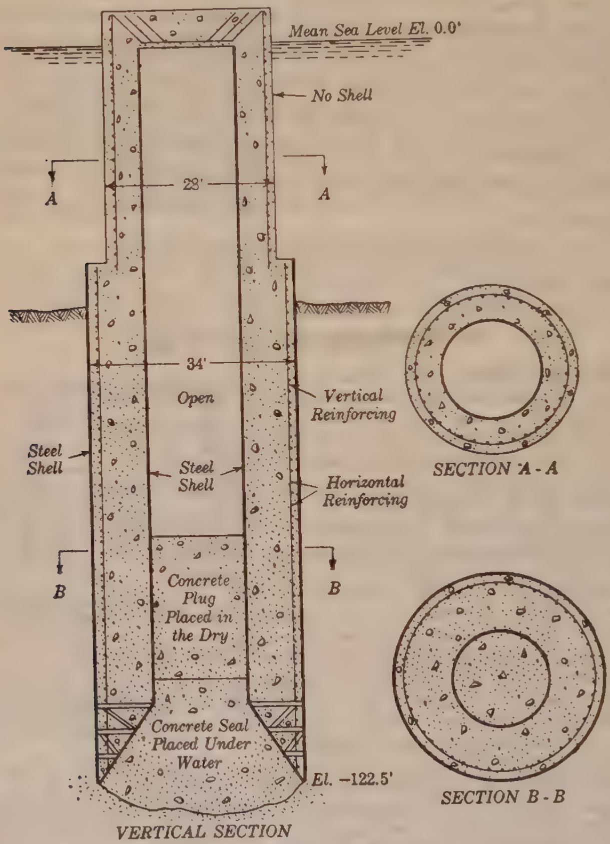


Fig. 36

bury River Bridge, Australia, to a depth of 162 ft. below high water; and of the Morgan City Bridge over the Atchafalaya River in Louisiana, to a depth of 120 ft. The greatest depth ever attained was at the bridge over the Ganges River at Sara, India, where one pier was dredged to 185 ft. below high water. The bridges over Chef Menteur and Rigolets Passes between Lake Borgne and Lake Pontchartrain, in Louisiana, for the Louisville and Nashville Railroad, are recent illustrations (1925) of this method. One of the piers at Chef Menteur reached a depth of 122.5 ft. below mean sea level.

The piers for these bridges were sunk as hollow concrete cylinders surrounded, inside and out, by a steel-plate shell. The diameter of the piers was 34 ft. These were sunk through gray sand. The maximum friction amounted to 765 lb. per sq. ft. Fig. 36 shows some of the features of these piers.

16. Divers and Diving

In laying foundations under water it is sometimes necessary to employ a diver to examine the site, to prepare the bed, or to lay the masonry. A diver is also sometimes required in other engineering operations, as in tunneling to erect or remove a bulkhead; in water-supply engineering to inspect and repair a conduit, clear an inlet, or take out pumps; in wrecking operations, etc. With the best modern apparatus it is possible for any able-bodied man to prosecute any ordinary kind of work under a moderate head of water without serious risk of personal injury. No man suffering from chronic disease, alcoholic excess, ear or heart troubles, having a sluggish circulation or a great excess of fat, should attempt the work of a diver; and if the work is to be carried on at a greater depth than 35 to 40 ft. the man should undergo a thorough examination by a physician before attempting diving.

A Diver's Outfit consists of a metal helmet or head-covering, a breast-plate which rests upon the chest and back, and an airtight flexible diving suit which envelops the body from the breast down. The helmet is provided with at least one window in front, and usually also one on each side, and sometimes one above the face; and is also provided with a valve in either the head-piece or the breast-plate for receiving air, and also a safety valve and a regulating valve. The diving suit is closed at the feet, but is open at the hands, which are provided with rubber bands that fit closely around the wrists. The helmet is connected by a hose to an air pump above the water. A rope, called the life line, passing around the diver's waist and reaching to the surface, is used in raising and lowering him. To overcome buoyancy weights are hung to the diver's waist, and his shoes have lead or iron soles. The weights upon the feet are more particularly intended to prevent the diver from losing his upright position by undue buoyancy of the lower portion of his diving suit, caused by an excessive air pressure on the inside. The air pressure in the diving dress may be controlled by the diver by means of the regulating valve. To prevent the possibility of the diver being blown to the surface by the inflation of the lower part of his diving dress, a new type of suit has been developed which has the legs tightly laced up, thus preventing the air from getting into the legs when the diver suddenly stoops over or if he falls down; and consequently if he loses his upright position, he is able to right himself. The air pump is always operated by hand power, and may be any one of several forms, according to the amount of work or the depth: one, two, or three cylinders; single- or double-acting; lever or fly-wheel type; with or without a water-jacketed cylinder. The diver communicates with his attendant by jerks upon the life line through a code of signals, or by a telephone the wires of which are embedded in the life line or in the air hose. Except in very clear water, the surface light does not penetrate very deep, and hence the diver usually works in the dark, although sometimes he is provided with an electric light.

To Prepare for Diving the diver takes off all his own clothing, puts on a heavy flannel shirt, a pair of drawers which must be carefully adjusted outside of the shirt and be well secured to prevent slipping down, and then a pair of heavy stockings. If the water is cold, he may put on two or more of each of these articles. He also puts on a woolen cap, drawing it well down over his ears. Usually a shoulder pad is put on and tied under the arms. The diver then gets into the diving suit and draws it well up to the waist; he next puts his arms into the sleeves, and an assistant opens the cuffs, by means of the cuff expander provided with the apparatus, so that the diver can push his hand through the cuff. Sometimes the diver wears rubber mittens; and if so, they are next put on by first inserting a ring into the cuffs, and then each mitten is fastened in place by a clamp. Canvas overalls are usually next put on over the diving suit to preserve it from injury. The diver now sits down, and the shoes are put on. Next the inner cape is drawn up around his neck and tied loosely with a yarn string. The breast-plate is then put on, and the rubber collar of the diving suit is put over the projecting screws of the breast-plate; and the sectional clamping piece is put over the projecting screws and fastened with the thumb nuts. The helmet, with the front plate or window removed, is next put on and screwed fast to the breast-plate; but before putting the helmet over the diver's head, the attendant should place it over his own head and place his mouth over the place where the air escapes and blow, to see whether the safety valve is in working order. The life line is looped around the diver's waist, brought up in front of the man's body, and secured with a small rope passing around his neck or is fastened to the stud on the helmet. The waist belt is then buckled on with the knife pocket at the left. The end of the air hose is passed through the ring on the belt on the diver's left and up to the inlet valve of the helmet, and secured to it, and then the upper part of the hose is made fast by lashing it to the loop on the helmet. A ladder or a rope must be provided which reaches from the surface to the bottom, for the use of the diver in descending and ascending; and must be heavily weighted at the bottom. The ladder, usually a rope one, is the better for the inexperienced man although an expert diver prefers a single rope. The diver steps on the ladder, and weights are hung to his waist and fastened in place. Two men take their places at the pump. When the attendant is sure that everything is right and the diver understands the code of signals, he gives orders for the pump to start, screws the front plate into place, takes hold of the life line, and gives the signal for the diver to descend. While the diver is under water, an attendant should give constant attention to air hose and life line, keeping the former free and clear of kinks and the latter just taut enough that any signals by the diver can be easily felt; no talking, laughing or distracting noise should be allowed on the surface.

The diver should descend slowly, and as soon as his head is under water should stop to see that everything is satisfactory, particularly the escape valve. The valve is kept closed by a spring, and the farther the valve is screwed up the less air escapes. If too much air escapes, breathing is difficult; and if too little escapes, the diving dress may become inflated to such an extent that the diver will rise. The proper adjustment is that which just takes the pressure of the weights from the diver's shoulders. As he descends, if he feels any pain in his ears, he should shut his mouth and inflate his cheeks, or blow through his nose, or swallow several times, to equalize the pressure on the inside of the drum of his ear. He should continue his descent slowly, and not demand too much air. If the pain is not relieved or if he feels oppressed, he should slowly rise a yard or two, and remain at that point a minute or two, and inflate his cheeks or swallow or do both; and if the oppression, or the singing in the ears, or the headache should continue, he should slowly return to the surface. However, unless there is some serious physical defect a man with a cool head and good judgment is not likely to have any trouble in diving. Arriving at the bottom, the diver signals his attendant, usually by one jerk on the life line to indicate "all-right." The diver takes down in his hand a coil of small rope, one end of which is fastened to his waist, and the other end of which he attaches to the bottom of the ladder or rope down which he descended, so that he can always find his way back to the ladder when he wishes to return to the surface.

If the diver has been down some time, particularly in deep water, the liquids and tissues of his body have gradually become saturated with compressed air and other gases, and consequently when he ascends it should be slowly to allow time for desaturation. Remaining down too long or ascending too rapidly is likely to cause a form of paralysis, which in pneumatic caisson work (Art. 18) is called by the physicians caisson disease and by the workmen "bends." It occurs only after returning to the normal atmosphere; and in ordinary cases in diving can be prevented by care in ascending. The desaturation is more rapid and less harmful if the diver ascends say halfway and then stops for a time (the exact time depending upon the depth and the time under pressure), and then continues the ascent, again stopping when halfway up. This is what is known in the British navy as stage decompression, and an elaborate series of tables has been established for that service which give the time to be consumed in the ascent for different depths and for various times down. For a depth of 36 ft. and unlimited time down, the time of ascent is 1 minute; for a depth of 60 ft. the time of ascent after being down 20 minutes is 2 minutes, and after being down 2 or 3 hours it is 32 minutes; and for a depth of 120 ft. after being down 15 minutes the time of ascent is 15 minutes, and after being down 30 minutes it is 33 minutes.

The Nominal Maximum Depth for a diver is 100 ft. and the ordinary maximum for a strong and expert diver is 150 ft., although the maximum reached is over 300 ft. Strong and experienced divers can stay under 2 hours at 100 ft. and 5 or 6 hours at 90 ft., but the maximum depth at which a diver can do any considerable work is 60 to 80 ft. It must not be forgotten that the work of a diver is much less efficient than that of a man above water, owing to restraint of his diving apparatus, the resistance of the water, the seeming tendency of his tools to rise, etc. The catalog price of a diving outfit varies from $350 to $1300, the former being for an outfit for a brief examination in shallow water and the latter for the best apparatus suitable for the deepest work.

Skin Divers. In emergency work, naked expert divers, working in water not over 25 ft. deep, can do certain kinds of simple work. Hartley Rowe successfully repaired a sea wall in Colon, C. Z., during a storm, by underpinning with concrete in bags placed by skin divers.

17. The Pneumatic Method*

When Expedient. The pneumatic method offers a certain and dependable solution when a foundation cannot be carried down to a satisfactory stratum by means of piles or by open methods. Such a situation arises (1) when the intensity of load upon a foundation is too great to permit the use of piles; (2) when subsurface conditions, such as boulders or buried obstructions, would prevent the penetration of piles to the desired stratum; (3) when the depth of the proposed bearing stratum and the hydrostatic head would make the driving of sheet piling and the bracing of an open cofferdam difficult and expensive; (4) when it is desirable or necessary to build a deep foundation adjacent to an existing structure and such a foundation must be carried down through unstable and water-bearing materials, the pneumatic method provides a means which will reduce to a minimum the possible settlement and hazard to the adjoining structure; (5) even when an open dredging caisson might be used, it may be desirable to unwater and explore the proposed foundation stratum and to avoid the placing of concrete through water. These considerations have led to the wide use of this method for the construction of the foundations of bridges, dams, and piers, and for the foundations of heavy buildings, especially in congested districts.

Referring to the use of this method for building foundations it should be understood that, even by its use, the settlement of nearby wall footings and piers cannot be entirely prevented. The friction of the sides of caissons upon the surrounding materials induces some settlement; and the excavation of material from space outside of the limits of the caissons, which cannot always be prevented, leaves voids which are later filled by a movement of the surrounding ground with some consequent settlement. Nevertheless, this method does furnish the safest means for constructing deep foundations under such conditions, and possible settlement of adjacent structures must be guarded against by preliminary shoring and underpinning of these structures.

On the other hand, on account of its usually greater cost, the pneumatic method should not be used when some other method is possible, provided the

* Most of this article was written by Ralph H. Chambers.

work could be done by such other method with certainty and without hazard to adjoining structures or to the success of the undertaking.

Limits. Foundations may be constructed by the pneumatic method to a depth of about 110 ft. below water or ground-water level. Even this depth has been exceeded; but usually, because of the rapidly increasing costs at the greater depths it is not advisable to use this method to a depth of more than 90 ft.

Essentials of the Method. The essential feature of the pneumatic method is the use of compressed air to drive the water out of the space in which the

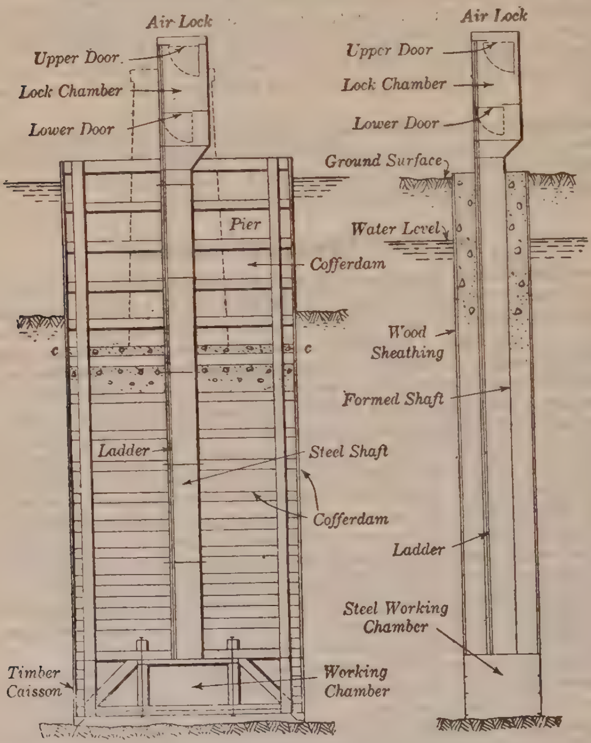


Fig. 37

men are working and out of the voids in the material which is being excavated, so that this material becomes comparatively dry and easy to handle, will not rise in the bottom of the excavation, and will stand to a considerable height without other lateral support. Excavation is done within a suitable box or chamber (Fig. 37) closed on the sides and top but open on the bottom, into which compressed air is fed. Men working upon the ground in this chamber remove the material under the chamber and cause the chamber and the construction above it to sink as the excavation proceeds. When the bottom of the working chamber reaches the desired stratum, the chamber is filled with concrete. It is evident that the excavators working in a dry, well-aired and well-lighted space can work advantageously in the removal of the excavation, including sunken logs or boulders, in leveling or stepping up the foundation, and in concretng the foundations within the chamber. This method is therefore dependable for use in foundation work.

It is necessary to provide means of access to the working chamber, from the surface of the ground or water above, so that men may go into and out of the working chamber and so that excavated materials may be removed and concrete and other materials may be lowered into it. This is accomplished by means of a cylindrical well or shaft (Fig. 37) leading from the working chamber to a point above water level. The shaft is provided with a valve or air lock which permits ingress and egress to and from the shaft, without appreciable loss of air pressure in the working chamber.

Working-Chamber Details. The form and horizontal dimensions of the working chamber are usually determined by the form of foundation required and by the required bearing area upon the foundation stratum, although in very small caissons the size is sometimes determined by the minimum space required in the working chamber. The smallest working chamber now used is about 3 ft. in diameter and 6 ft. high. This is used in the underpinning of heavy buildings. A caisson of this form and size was called a "pneumatic pile" in the older treatises upon the subject but this term is no longer used. When it is possible to use them, caissons of circular cross-section can be sunk with greater ease and less disturbance to the surroundings than other forms.

The exterior sides of the working chamber are now invariably made plumb. Formerly it was customary to batter the sides so that the working chamber at the bottom would be larger than at the top. This was done with the idea that a working chamber of this form would, in sinking, leave a space all around and thus the friction of the sides of the caisson upon the surrounding ground would be relieved. Experience has shown that the result hoped for was not obtained and that such caissons were more difficult to keep in alignment. Further, when it is necessary to guard against the settlement of adjacent structures, it is necessary to prevent the removal of any material which will not be replaced by the sinking caisson.

The bottom edges of the working chamber, known as the "cutting edge," were formerly made sharp or V-shaped in section. This is no longer regarded as necessary and the cutting edge is now often 12 in. or more in width. It is usual to protect the bottom edges of the working chamber by a steel or hardwood curb to prevent damage due to blasting or to the landing of a portion of the cutting edge upon a boulder or other hard obstruction. The inside height of the working chamber should be 6-1/2 to 7 ft. The inside faces of the side walls are often battered so that the thickness of the walls increases above the cutting edge.

Formerly, working chambers and a portion of the pier above, known as the "crib," were always made of timber, although some of the largest caissons such as those used in the Forth Bridge (Scotland) were built of steel. Timber is still used, especially when the working chamber must be built on shore and launched; but, when it is possible, reinforced concrete is a much better material except in small caissons where lack of space requires the use of steel. The risk of fire in the working chamber which, before the now invariable use of electric lights, was a constant danger, is eliminated by the use of incombustible materials. These fires were especially bad on account of the large amount of oxygen due to air compression. Timber working chambers must be thoroughly calked, inside and out. Fig. 38 shows a large timber pneumatic caisson used for the new Memphis Bridge (1915).

The sides of the working chamber should be designed for a load equal to the hydrostatic pressure, at the greatest depth to which the caisson is to be sunk, so that, in case of a blow-out and the sudden loss of pressure in the working

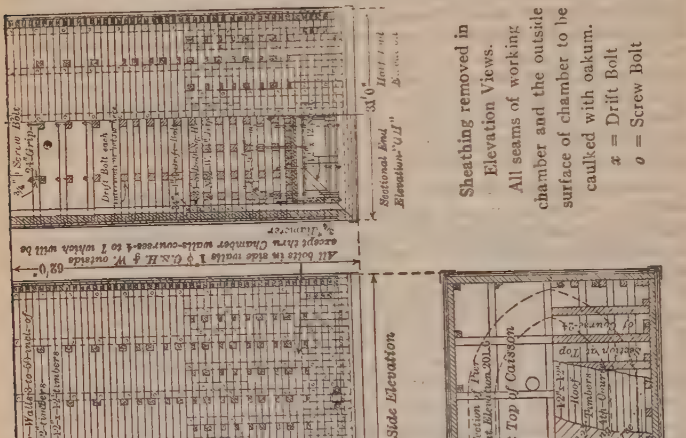


Fig. 38. Pneumatic Caisson and Crib for Second Memphis Bridge

chamber, the walls will not collapse. It is also usual to design the walls to resist a bursting pressure of about one-half the hydrostatic pressure. The roof of the working chamber must be made sufficiently strong to support the load of the pier upon it. This, however, does not mean that it must support the whole weight of the pier above, but only so much of it as is not carried directly to the side walls of the working chamber, due allowance being also made for the supporting effect of the friction of the sides of the pier against the adjacent soil.

Cofferdam or Pier Shaft. In large caissons where the top of the masonry built upon the roof of the working chamber may, at times, be below the water surface or when the pier shaft above the working chamber will cover less area than the top of the working chamber, a cofferdam is constructed above this working chamber (Fig. 37). This is usually built of timber and calked. It is built up as the caisson sinks. This cofferdam is filled with concrete or the pier masonry is built inside of it, as shown above $c-c$, Fig. 37. The amount of concrete or masonry in the cofferdam while the caisson is sinking should be regulated so that its weight will only slightly overbalance the buoyancy of the caisson and the friction on the sides. Provision must be made to prevent the pulling apart of the cofferdam or its separation from the working chamber because of the friction.

When caissons are built upon dry ground, as in the case of building foundations, the foundation above the working chamber is built up of concrete as the caisson sinks and without a cofferdam. The method of operation when building on land is to build the working chamber and then build upon it a small amount of concrete masonry, sinking the working chamber into the ground always to such a depth that the masonry pier above it will not be in danger of tipping. The preliminary sinking may be done without air if there is no ground-water. When the top of the concrete sinks to about the level of the ground the sinking is interrupted until another concrete section can be built up 10 or 15 ft., the air pressure being maintained in the meanwhile.

Caisson Shafts. Access to the working chamber is provided by one or more vertical shafts, depending upon the size of the caisson, extending up through the masonry in the cofferdam from the top of the working chamber to a level above the water surface. These shafts may be of steel or may be formed in the concrete. They are usually 3 to 4 ft. in diameter, but may be as small as 30 in. in diameter in small caissons. Steel caisson shafts are usually built in 10-ft. lengths with outside angle flanges bolted together and having yarn or rubber gaskets. The caisson shafts are provided with iron ladders and furnish a passage for men and for buckets. In large caissons, separate shafts for men and for materials are provided. In smaller caissons only one shaft is used, this being for both men and materials. In some cases a single shaft has been used with separate passages for men and for materials. Upon the completion of the caisson the shafts are filled with concrete. If made collapsible they can be removed and the space filled with concrete.

Air Locks. The air lock (Fig. 37) is a device used to allow the men to enter or leave a working chamber without appreciable loss of air pressure through the shaft and to allow the introduction of materials into the working chamber and the removal of excavated materials from it. The air lock is simply a chamber, usually 4 to 6 ft. in diameter and 7 ft. high, having doors at each end. These doors both open in a direction against the air pressure. By closing the lower door and opening the upper, men can enter the lock chamber. The upper door being then closed and the air pressure in the lock chamber being gradually made equal to that in the working chamber, the lower door can be opened and the men can proceed down the caisson shaft into the working chamber. The air lock should always be placed above water level so that in case of a blow-out, or sudden loss of pressure in the working chamber, the men may take refuge in the lock and then be passed out. Fig. 37 shows the general air lock, diagrammatically.

In order that excavated materials may be removed from the working chamber by buckets, the cable carrying the bucket must pass through the top of the air lock or through the upper door. In the first locks the upper door was located in the side of the lock chamber and the lower door in the bottom. The bucket cable then passed through the top of the lock. This arrangement is still used in one form of lock but it necessitates the dumping of the contents of the bucket from the side of the lock chamber and its rehandling, or the releasing of the cable from the bail of the bucket in the lock, and the connection of another cable, outside the lock, by which the bucket can be hoisted and dumped. This disadvantage led to an arrangement of details of the upper door which would permit a bucket to pass out of the top of the lock without disconnecting the cable. This is accomplished by making the upper door in two sections which will close around the cable, or by entering the bucket into the lock and then moving it sidewise so that the cable enters a slot in the edge of the upper door as it is closed. The upper doors, of locks now most used, are simply hinged or hinged and provided with a connecting lever so that they will swing laterally as they swing open. The lower doors are simply hinged. Both doors are usually counterbalanced and provided with outside operating levers.

Air locks must be provided with piping and valves to permit the equalization of the air pressure in the lock and the discharge of air from it, and the discharge should be located so that it can be operated by men in the lock.

Air Connections. Connections for the supply of compressed air must be made to the air lock and to the caisson shaft below the lower door of the lock. A connection must be carried down to the working chamber itself in order to supply fresh air direct to the working chamber. This is done by a pipe leading down through the caisson shaft or by a pipe buried in the concrete and leading down to the working chamber. All these connections should be of 3-in. pipe except in very large caissons when they are made of 4-in. To the top of the connection to the working chamber and to the connection to the air lock, two lines of hose lead to a manifold which in turn is connected to the main air line from the receivers and compressors. At the manifold, each line leading to the hose is provided with a gage and these gages are attended by a gage tender who regulates the pressure and supply of air to the caisson.

Connections are made for electric light, high-pressure air for drilling, and for a signal whistle to the working chamber, through the shaft or by pipes buried in the concrete. Ordinary signals are made by the men in the working chamber rapping upon air pipes or caisson shafting, answered by men rapping upon the air lock above.

Compressors and Air Mains. To maintain a constant supply of compressed air, suitable low-pressure compressors must be provided. High-pressure compressors might be used on small jobs by dropping the pressure the required amount at the caissons, but this would be uneconomical. Usually a high-pressure compressor with a separate air line is also installed to supply air for drilling. Compressors may be driven by steam, oil, gasoline or electricity, but when electric power is available, at equivalent cost, it is the most convenient. Air receivers, of adequate capacity, are required to equalize the pressure. When the work is done in warm weather, provision for cooling the air should be made just before it reaches the caissons as the air is likely to be highly heated by long supply mains lying in the sun. The mains should be 4 to 8 in. in diameter, depending upon the amount of air used and the lengths of mains. The quantity of air which will be required will depend upon the size of the caissons, the number being sunk simultaneously and the character of the ground through which the caissons are being sunk. A supply of 1500 cu. ft. of free air per minute may be enough for a small number of small caissons sinking in tight ground whereas large caissons and porous ground may require several times as much. Usually 2500 to 3000 cu. ft. per minute will be sufficient for fair-sized works in ordinary ground. At least two compressors should be provided or, better, a stand-by compressor should be installed, in case of trouble with the compressors.

Launching and Placing. The working chambers of caissons which are to be sunk through open water before reaching ground are usually built on land, launched on sliding ways similar to shipbuilding ways, and towed to site. Or the working chambers may be built on a submersible barge. False bottoms in working chambers are sometimes necessary when the depth of water is otherwise insufficient for launching. These false bottoms are removed after air pressure has been applied, or by weighting them off in deep water with sand. Provision is usually made for anchoring, holding and guiding such caissons, by driving clusters of piles around the site and by current breakers and anchors. For caissons to be sunk from the ground surface, the working chambers are built upon the site.

Sinking. Caissons are usually sunk a little below water level before compressed air is applied, except those which are placed in open water. When air is applied, the organization for each caisson consists of a working gang foreman and as many men as can conveniently work in the working chamber, two lock tenders on the air lock and a gage tender. A general caisson foreman is in charge of all the caissons. Besides this there are required sufficient derricks with their operators to handle the excavating buckets, and a complement of men to build up the caissons and to supply mixed concrete to the air locks, also operators for the compressors and a general works organization.

The material excavated in the working chambers is handled into buckets and hoisted through the air locks. When the excavated material is sand, gravel, or silt, it may be advantageously handled out of the caissons by blow pipes, except in foundation work in city districts. These consist of pipes, usually 4 in. in diameter, running up through the roof of the working chamber to the top of the caisson and terminating in an elbow. The lower end of the pipe extends down into a shallow, water-filled pit just below the level of the cutting edge. At a point about 4 ft. above the cutting-edge level, the blow pipe is fitted with a full-way valve. Excavated material is shoveled into the pit at the bottom of the blow-pipe and from time to time the valve is opened for short periods. The air pressure in the working chamber blows the material through and out of the pipe at the top of the caisson. It is necessary to protect the bend in the elbow at the top of the pipe, by chilled cast iron, to prevent rapid wear. This apparatus is an expeditious means of raising excavated materials, when used with proper material and when the air pressure exceeds 10 lb.

Men working under compressed air are exhilarated and usually work energetically. Each man in the working chamber should handle 2 or 3 cu. yd. per eight hours of working time. Boulders or rocks may be drilled and blasted in the working chamber in the ordinary manner but the men must be taken out of the working chamber at each blast.

In sinking a caisson care must be taken to keep it in correct position and plumb. Improper handling of caissons has caused them to turn over. In order to keep the caisson plumb and in alinement it may be necessary to excavate ahead of whichever side is lagging behind in the sinking, or to use shores above the ground if the caisson is sunk from the ground or in shallow water, or by using shores within the working chamber and also sliding blocks. Sliding blocks are heavy timbers placed under the cutting edge and sloping in the direction in which it is desired to shift the caisson.

As the sinking proceeds and the pressure increases, the length of time during which men can safely work in the caisson rapidly decreases and the time required for decompression in the air lock increases. These periods are sometimes regulated, as in New York State, by law (see Art. 18) and by union rules.

It will be noted in Art. 18 that the higher the air pressure and therefore the deeper the caisson the shorter the day's work. The following tabulation shows also that the higher the air pressure the higher the per diem wages: For pressures up to and including 17 lb., $12.00 per shift (two working periods).

	Per shift
For pressures of 18 to 25 lb.	$12.50
For pressures of 26 to 32 lb.	13.00
For pressures of 33 to 37 lb.	13.50
For pressures of 38 to 42 lb.	14.00
For pressures of 43 to 48 lb.	14.50

From the foregoing it will be seen that under pressures of less than 18 lb. the cost of labor is $1.50 per hour of actual working time, whereas, at 34 lb. pressure the labor cost per hour is $3.38 and at 48 lb. pressure the cost per hour is $9.67.

Friction and Weighting. Only very large caissons will sink of their own weight as the material under them is excavated, overcoming the friction of the surrounding ground against the sides of the caissons. This friction varies with the character of the ground materials, from 150 to 1500 lb. per sq. ft. of surface or even more. The usual friction is 350 to 500 lb. per sq. ft. Loam, silt and sand cause the least friction, clay often causes more, and strata of boulders and loose rock sometimes raise the friction to an amount which cannot be overcome. The air escaping from the working chamber tends to reduce the friction. Water jets around the sides are sometimes effective. Dynamite exploded in the ground below the working chamber will sometimes start the movement of a caisson. When it can be done without risk of settlement of adjacent structures, the sudden release of air pressure in the working chamber will often cause a sinking of the caisson. Caissons of steel or timber will sink with less friction than concrete caissons. In fact it is advisable to sheath concrete caissons with 2- or 3-in. wood plank to reduce the friction.

To overcome the friction and the buoyancy of the air pressure, which is equivalent to the hydrostatic pressure at the bottom, it is usually necessary to add to the weight of the caisson by loading it with sand or gravel, concrete blocks, or iron. The weight usually used is pig iron, loose or in boxes, or cast-iron blocks (about 2 tons each). A load of 300 to 1000 tons of added weight on a caisson is not unusual.

Concreting Working Chambers. When a caisson has reached the desired foundation stratum and the bottom has been cleaned, the working chamber is filled with concrete. The concrete is either lowered in buckets through the air lock, or is dumped into the lock chamber and dropped by opening the lower door, either free or through a pipe. The concrete should be spaded and packed tight against the roof of the working chamber and should then be grouted, through the air pipes, or through the caisson shaft. The workmen usually receive additional pay for concreting under air pressure.

Rate of Sinking. When the sinking of a pneumatic caisson has been begun it is usual to continue the sinking night and day without interruption, although this is not absolutely necessary provided air pressure is constantly maintained. The rate at which a caisson can be sunk depends upon its size, upon the character of the material through which it is sinking, and the number of men working in the working chamber. Small caissons, 5 to 7 ft. in diameter, sinking through silt, sand or light clay, may be sunk 5 to 7 ft. per eight hours of working time. Larger caissons, 20 by 50 ft. or more in horizontal dimensions, may be sunk 1.5 ft. or more in eight hours.

Costs. From the foregoing it will be evident that there are so many variable factors entering into the cost of pneumatic caisson work that any statement of costs would probably be misleading. The cost of any particular work, per cubic yard of caisson, might vary within limits as wide as $35 to $100, depending largely upon the magnitude of the work. Before such work is undertaken, careful borings should be made, also a thorough study of the costs of materials and labor, power, equipment and liability insurance. This having been done, it is generally possible to estimate very closely the cost of pneumatic caisson work and, as has already been stated, the results which can be obtained by the use of this method can usually be more definitely forecast than can be done with any other method.

Other Hazards. The danger of fire in the working chamber has already been mentioned. This in the past has been the cause of a number of serious accidents, but by the use of electric lighting and the increasing use of incombustible materials this danger has been largely eliminated. There is always, in ground of certain character, a possibility of a blow-out or sudden loss of air pressure in the working chamber, the subsequent inrush of water and mud drowning the men before they have had a chance to escape. Similar accidents have been caused by the dropping of a loaded bucket through the lower door of the lock, causing it to open and suddenly releasing all the pressure in the working chamber. This can be guarded against only by constant vigilance on the part of the experienced men who are usually employed in this work. Marsh gas and other inflammable, explosive, or suffocating gases may be encountered in some localities and means must be adopted to recognize immediately the presence of such gases.

18. Physiological Effect of Compressed Air

After entering the air lock, as the pressure increases, the first sensation experienced is one of great heat. As the pressure is still further increased a pain is felt in the ear, arising from the abnormal pressure upon the ear-drum. The tubes extending from the back of the mouth to the bony cavities over which this membrane is stretched are so very minute that compressed air cannot pass through them with a rapidity sufficient to keep up the equilibrium of pressure on both sides of the drum (for which purpose the tubes were designed by nature), and the excess of pressure on the outside causes the pain. These tubes can be distended, thus relieving the pain, (1) by the act of swallowing, or (2) by closing the nostrils with the thumb and finger, shutting the lips tightly, and inflating the cheeks. Either action facilitates the passage of the air through these tubes, and establishes the equilibrium desired. The relief is only momentary, and the act must be repeated from time to time as the pressure in the air lock increases. This pain is felt only while the air in the lock is being "equalized," and is most severe the first time compressed air is encountered, a little experience generally removing all unpleasant sensations.

The Rate of Entering the compressed air, or of "locking in," should be as rapid as possible, the only limitation being the ability of the individual to equalize the air pressure in his head cavities. For this reason the novice should go in slowly, but an experienced workman may enter as quickly as the valve will admit the air.

When the lungs and whole system are filled thoroughly with the denser air, the general effect is rather bracing and exhilarating. The increased amount of oxygen breathed in compressed air very much accelerates the organic functions of the body, and hence labor in the caisson is more exhaustive than in the open air; and on getting outside again a reaction with a general feeling of prostration sets in. At moderate depths, however, the laborers in the caisson, after a little experience, feel no bad effects from the compressed air, either while at work or afterward. In passing through the air lock on leaving the air chamber, the workman experiences a great loss of heat owing (1) to the expansion of the atmosphere in the lock, (2) to the expansion of the free gases in the cavities of the body, and (3) to the liberation of the gases held in solution by the liquids of the body. Hence, on coming out the men should be protected from currents of air, should drink a cup of hot strong coffee, or preferably hot beef extract, take a warm shower, dress warmly, and lie down for a short time.

The Cause of Compressed Air Sickness, which is known as caisson disease or the "bends," is the release of free gas bubbles in the blood. The blood dissolves both oxygen and nitrogen in passing through the lungs, and if exposed to compressed air the amount of gas dissolved will increase proportionally to the pressure. The oxygen gas thus dissolved will generally be absorbed by the bodily functions, but the nitrogen remains in solution. If the air pressure is reduced too rapidly when "locking out," this dissolved nitrogen is not able to escape through the lungs, and some of it will form bubbles in the blood which will be carried to the various tissues. In most cases this will cause pain in the joints of the limbs, which is the form of the disease known as the "bends," this symptom being generally of a non-dangerous type. More dangerous forms are the "chokes" caused by the pressure of gas bubbles in the lungs, and the "staggers" or vertigo caused by bubbles in the brain or the middle ear. Very severe cases may result in collapse, followed by death. Caisson disease is more likely to occur where the men remain too long under heavy pressure, and when the decompression is too rapid.

Precautions to be taken to limit the occurrence of the disease are as follows:

(1) Each workman should be examined by a physician before being employed and at intervals not exceeding two months. The men should be young, of spare build, and with low blood pressure.

(2) The hours of work should be limited, and should be less at the higher pressures. The New York State regulations (1921) specify hours of labor per man in any 24 hours as follows:

Gage pressure lb. per sq. in.	1st period of work in compressed air—hours	Period of rest—normal air—hours	2nd period of work in compressed air—hours
0–20	4.0	0.5	4.0
21–29	3.0	1.0	3.0
30–34	2.0	2.0	2.0
35–39	1.5	3.0	1.5
40–44	1.0	4.0	1.0
45–49	0.75	5.0	0.75

(3) Decompression should be at a slow rate, and the time allowed for decompression should be increased with higher pressures. The time required may be reduced by adopting "stage-decompression," in which the pressure is first reduced to half its original amount at a rate not exceeding 5 lb. per

minute and then reduced to normal at a much slower rate. For pressures under 20 lb. per sq. in. the rate of decompression may be fairly rapid. For higher pressures, the total time required should be about one minute for each pound of gage pressure.

(4) Care should be taken to avoid chill during and after decompression as noted above.

(5) The men should be required to remain near the work for at least one hour after decompression, as it has been found that a large proportion of cases of the disease occur within that time. The men should also be advised not to go to work with an empty stomach.

The Cure for Caisson Disease is recompression to the same pressure as that at which the man had been working, followed by decompression at a much slower rate than prescribed for actual work. For this purpose, a "hospital lock" should be required on all compressed-air work. This is a chamber arranged to receive the required air pressure, and provided with necessary facilities for the comfort and treatment of the patient. The treatment is generally effective after one recompression, but in more serious cases it may have to be repeated several times to obtain permanent relief.

The comfort and safety of the workmen will be greatly increased by keeping the air supplied to the compressed-air chambers as clean as possible, and properly cooled. The air lock should also be heated during decompression in cold weather. If proper precautions are taken, the pneumatic process can be applied at depths of 80 to 90 ft. without serious consequences, but it is generally considered to be limited to depths of 110 ft.

References: "Caisson Disease and Its Prevention," Trans. Am. Soc. C. E., Vol. 65, p. 1, "Compressed Air Illness and Its Engineering Importance," by Edward Levy, U. S. Bureau of Mines Technical Paper 285 (1922).

EARTHWORK

19. Loosening and Shoveling

Loosening. If the earth is not to be handled by steam power, it must first be loosened, unless it is sand or sandy loam. The loosening is done with a pick, a mattock, or a plow. The **pick** is used in trenches and other confined positions, and the plow elsewhere. The pick is much more economical than a mattock; and the latter should be used only in trimming a surface. The amount that a man can loosen in a given time with a pick varies greatly with the man, the supervision, the character of the soil, the depth of breast, etc.; but is usually about as follows: hardpan or cemented gravel, 0.5 to 1.0 cu. yd. per hour, and ordinary loam 3.0 to 5.0. The **plow** is usually drawn by two horses or mules in ordinary loam, four in stiff clay and gravel, and six in hardpan. In each case there is a driver and a plowman, and in the last also one or two men to ride the plow-beam to keep the plow in the ground. Gasoline tractors may often be more economical than horses or mules. The amount loosened is in ordinary loam 40 to 50 cu. yd. per hour, in stiff clay 25 to 30, and in hardpan 15 to 20. If the plow worked continuously and straight along, it could loosen a great deal more than this; but the plowing must usually be done in short sections, and for various reasons much time is lost.

Shoveling. The shovel employed may be either square-end or round-end, and may have either a long or a short handle. Pointed shovels are generally used except for handling loose material from a platform or other smooth surface. The long-handled shovel is usually more economical, unless the shovelers are crowded closely together. A man will shovel well-loosened earth into a wagon at the rate of 1.2 to 1.5 cu. yd. per hour; if the wagon is comparatively low the larger amount can be realized, and if the wagon is high the smaller amount is a good output. If the man shovels earth from a platform he can handle 2 or 2.5 cu. yd. per hour. In soil that can be spaded easily, a man can dig and load more solid earth with a spade than of loose earth with a shovel. Ordinary men can spade and load as task work 20 cu. yd. of brick clay per day; and experienced and skilful men have spaded and loaded month after month 40 cu. yd. in 8 to 9 hours, and occasionally 56 cu. yd. A man can pick and load about 1 cu. yd. of loam per hour, about 3/4 cu. yd. of stiff clay, and 1/3 to 1/2 cu. yd. of hardpan.*

The Pneumatic Spade, operating on the same principle as the percussion drill (Sect. 10, Art. 4), is efficient for excavating stiff clay.

20. Scrapers

Scrapers are useful on small jobs which do not warrant the use of extensive mechanical equipment. There are three kinds of drag scrapers commonly in use: the drag, the Fresno, and the flat-bottomed tongue scraper. They are efficient in sand or gravel with or without clay which is not indurated and does not contain a large percentage of heavy cobbles or boulders. If the soil is very fine the output is decreased by the miring of the horses. Large cobbles are hard on the horses and upset or block the scrapers.

The Drag Scraper, the form of scraper most frequently used, consists of a solid steel bowl with two handles by which to load and dump it, and of a bail to which to hitch the team. It is made in three sizes, which vary a little with the maker. The smallest, for one horse, has a rated capacity of about 3 cu. ft., and the larger sizes, for two horses, have nominal capacities of about 5 and 7 cu. ft. respectively. Under working conditions they will carry only about 70% of these capacities. The drag scraper is admirable for borrowing at the sides of embankments and for wasting from cuts or ditches, and also for opening the mouths of large cuts; but is not economical except for short distances. There is no danger of the scraper getting out of order until it is worn out and unfit for use, and the manner of using it is quickly learned by any one.

Drag scrapers, in the usual short hauls, are operated in gangs of three, each with a man to load the scrapers; and the teams, even on the shortest hauls, travel in a circuit of about 150 ft. Usually the driver dumps the scraper; but when an embankment is being built, there is sometimes a man to keep the dump level, who also aids in dumping the scrapers. If the earth is sand or sandy loam and unobstructed with grass or tree roots, it may be scraped without plowing; but usually it is economical to plow before scraping, since then the scrapers are filled more nearly full. Sometimes a man is required to grub roots, etc. The " drag " is not economical to move earth more than about 200 ft. and generally at more than 100 ft. the wheel scraper is more economical than the drag.

The output on a drag scraper job depends very much upon the experience and constant supervision of the foreman. Rough average outputs for various lengths of haul are as follows:

* Although the rates given in this paragraph are possible, under average conditions they should be reduced 50%.

Haul distance, feet	Cu. yd. per 10-hr. day
25	70
50	60
100	50

These figures are for loose material.

The Fresno Scraper differs from the ordinary drag scraper in the form of the bowl and in having adjustable runners upon which the bowl is carried in dumping and returning and which permit the scraper to distribute its load in dumping and to level off the fill. The Fresno scraper is made in three sizes, the cutting edge being 3 1/2, 4 and 5 ft., and having nominal capacities of 10, 12 and 15 cu. ft. respectively. The working loads are about 75% of these capacities, except on downhill work. The distance from the cutting edge to the rear of the scoop is comparatively small, which enables it to be easily loaded to its full capacity; and the runners enable it to deposit its load in layers. It is usually drawn by three or four horses abreast.

The Fresno scraper is very efficient in easily dug material, especially on downhill work, where large loads can be pushed along in front of it. The greatest loss in Fresno work is found in short loading, which under proper supervision can be largely eliminated. Hauls may be made economically up to 300 ft. The following table gives the output for a standard 4-ft. Fresno for various lengths of haul and for average management. Good management will increase these outputs about 30%. This table was compiled by J. L. Harrison of the U. S. Bureau of Public Roads.

Haul distance, feet	Loads per Fresno per 10-hr. day, cu. yd.
50	380
100	270
150	220
200	190
250	160
300	140
350	130
400	115

The Flat-Bottomed Tongue scraper is an iron-shod wood or solid-metal scoop employed in filling ditches and leveling off the bottom of an excavation, particularly in pavement and macadam road construction, and is made in two sizes, 36 and 48 in. wide.

The Power Scraper, described in detail in Art. 26, is found useful in all types of earth-moving operations. It is particularly useful in removing overburden and in making the smaller cuts and fills.

The Wheel Scraper consists of a steel box mounted on wheels and furnished with levers for raising, lowering, and dumping. All of the movements may be made without stopping the team. There are two forms, the two-wheel and the four-wheel. The former has long been in use; the latter was first made in 1909 but is not used to any extent at present.

The **two-wheel scraper** is made in five sizes, Nos. 0, 1, 2, 2 1/2 and 3, having a rated capacity of 7, 9, 12, 14 and 16 cu. ft., respectively. Some manufacturers make an automatic front end-gate which adds materially to the load the scraper will carry, particularly on a rough or downhill road, and is useful in preventing sand and gravel loads from spilling. The two-wheel scraper is drawn by two horses; but it is usually necessary with the larger sizes to hitch another team, called a snatch team, ahead to aid in loading. One man, and in tough soil two, in addition to the driver is required in loading the largest scraper.

Except under the most favorable conditions the wheel scraper is not entirely filled owing to the difficulty of forcing the earth to the back of the bowl, hence it is not usually safe to count upon the scraper placing in the dump more than 60 to 75% of its nominal capacity. Sometimes when the haul is long, the bowl is filled heaping full by men with shovels as the scraper leaves the pit. The smallest wheel scraper is used when the haul is short and the rise is steep. It is usually more economical than the drag scraper, but where there are many stones the latter is the better.

The wheel scraper is used largely on excavation jobs where the hauls are from 200 to 800 ft. and in conjunction with Fresno or drag scrapers, the change from these to the wheelers being made at about 300 ft. J. L. Harrison of the U. S. Bureau of Public Roads found that there cannot be any generalization made as to the point at which one should change from Fresnos to wheelers. The wheeler generally requires a snatch team which is not required by the Fresno. For a short haul the Fresno output is large, but it decreases with the increase in length. For a short haul, the wheeler uses a large percentage of time loading with the snatch team but becomes increasingly efficient as the length of haul increases. It is therefore of greatest importance that the change point be considered carefully in order to save losses which would be caused by an error of even 50 ft. in the slection of the change point.

The wheelers can carry 8 cu. yd. per wheeler per hour for a 100-ft. haul, and about 4 cu. yd. per wheeler per hour for a 500-ft. haul. It has also been found that in some cases a tractor can be used instead of teams with savings in labor of more than 25%.

In constructing the Chicago Sanitary Canal, the average output for 300 000 cu. yd. having a cut ranging from 4 to 8 ft. deep, a vertical lift of 10 to 20 ft. and a horizontal haul of 400 ft., ranged from 39 to 50 cu. yd. per scraper per 10 hours, and from 27 to 35 cu. yd. per team; on another section the average output per 10 hours was 46 cu. yd. per scraper, and 30 cu. yd. per team.

The Four-Wheel Scraper is a steel box or scoop suspended from a frame supported on four wheels. It is made in two sizes, 3/4 yd. and 1 1/4 yd. capacity. It is economical because it is self-loading and self-dumping, and on long hauls also because of the larger load carried. It is usually tractor-drawn, in trains of 2 to 6, and with this type of operation will move 350 to 500 cu. yd. a day.

In excavating a channel 800 ft. long, 200 ft. wide, and up to 4 ft. deep for a temporary dam on Schoharie Creek for the Board of Water Supply of New York City, two men operated one 5-ton tractor pulling 3 scrapers of 1 cu. yd. rated capacity and excavated 8000 cu. yd. in two months, working 8 hours per day (1924). The material was mostly light loam, with gravel for about one foot at the bottom that required plowing.

See also article in "Contractors' and Engineers' Monthly," June, 1928, p. 395.

21. Grading Machines

The Scraping Grader is a machine primarily for smoothing a roadway, and incidentally for moving small quantities of earthtoward the crown. It consists of a cutting blade sliding upon the ground or of a cutting blade, adjustable in height and direction, supported upon two wheels or suspended from a frame carried by four wheels. In addition to its use in smoothing the trackway and restoring the crown of an earth road, the scraping grader is used in pavement and macadam-road construction in smoothing up the surface of an excavation, and also in backfilling trenches.

The Elevating Grader is a combined digging and loading machine and consists of a frame resting upon four wheels, from which are suspended the plow and a frame carrying a wide traveling belt. The plow loosens the soil and throws it upon the traveling inclined belt, which delivers it upon the embankment direct or into wagons. The carrier is built in sections and its height is adjustable. The machine can be had in three sizes, small, standard and large. The standard is the size best adapted for general use. The small size is recommended for use where space is limited. The large size is used on embankment work and will deliver earth 30 ft. from the plow. The machine has loaded an average of 650 to 1000 cu. yd. per 10-hour day.

The elevating grader is an effective machine for grading roads and building open ditches and earth embankment, or filling wagons. It works best in light loam, and when hauled by a tractor. Large wagons or trucks should be used to remove the excavated material. The elevating grader cannot be used in coarse sand or gravel, since the plow will not throw this material upon the elevator; and it is not suitable for use where there are roots or stones enough to interfere with the work of the plow. On an average road-grading job the usual output for the grader is about 700 cu. yd. per 10-hour day. Theoretically this should be much greater, but losses on the job cut down the output. The elevating grader is most efficient in cutting out shallow road ditches. It can be operated by one man. (See " Public Roads," 1925, for much detailed information on the elevating grader.)

The Shuart Grader consists of a scraping steel blade attached to a frame borne upon four low wheels, having at each side a guard that enables the blade to push considerable earth along in front of it. The height of the blade may be adjusted by levers. It is used in the West in preparing the ground for irrigation. If the guards are removed, this machine can be used as a grader, that is, to smooth off a surface and to push small quantities of earth horizontally sidewise.

22. Loading and Hauling

Wheelbarrows are never economical where teams can be used. A man will pick and load into a wheelbarrow 1 cu. yd. of ordinary loam per hour. A wheelbarrow load will make about 1/15 cu. yd. (or about 2 cu. ft.) in the settled embankment. The time consumed in going and returning is about 1 1/3 minutes per 100 ft., since the load is usually taken up a rather steep grade; or say 20 minutes per cu. yd. per 100 ft. For each load the time lost in dumping, fixing runway, and changing the position of the barrow is 1/2 minute per load, or 7 1/2 minutes per cu. yd.

Carts drawn by one horse are used in some parts of the country for transporting earth. They are not economical except where the haul is so short that a driver can tend two carts by taking one to the dump while the other is being loaded. The loading is usually done by four men casting in at the back end of the cart. The chief advantage of the cart is the ease with which it is dumped, especially into hoppers or scows; but usually either the wheeled scraper or the wagon is preferred. A load is 1/2 cu. yd. for level hauls, and 1/4 for steep ascents, or say 1/3 cu. yd. per load on the average; and the speed is 200 ft. per minute, or 3 minutes to transport 1 cu. yd. 100 ft. It requires about 3 minutes for 4 men to load a cart, and about 1 minute to dump it; or the lost time is 4 minutes per load, equivalent to 12 minutes per cu. yd.

Wagons. There are two general forms of wagons made especially to transport earth: the bottom-dump and the end-dump wagon. The ordinary farm wagon is unsuitable for the purpose, because the box or bed is so light as soon to be knocked to pieces by being struck with the shovels, and also because the load must be shoveled out of it.

Of the **Bottom-Dump Wagons** there are two forms: a box which is placed upon the running gears of an ordinary wagon, and a wagon made especially for hauling earth. In each the bottom of the box consists of two doors, usually hinged at the sides, which drop down to discharge the load. The doors are kept in place by chains which are wound around a spool by means of a lever operated by the driver; and when the chains are released the load drops. The load can be dumped almost instantly by the driver without stopping the team, and the bottom can be closed while the wagon is returning for another load. The drop-bottom box holds 1-1/2 to 2 cu. yd. of loose earth, and the bottom-dump wagon 1 to 5 cu. yd., although 1-1/2 to 2-1/2 are the most common. The larger sizes of bottom-dump wagons are made of steel, and are drawn by three horses and sometimes by four. The bottom-dump wagon is better than the drop-bottom box on ordinary running gears, since (1) it has larger capacity, (2) there is no coupling pole to interfere with the dumping device, (3) the front wheels go under the bed, thus greatly facilitating the turning of the wagon, and (4) it has wider tires and hence draws easier.

The **end-dump** wagon is used only where it is necessary to dump into hoppers, or onto barges, or into railroad cars. This form of wagon is too heavy and too high for use in ordinary earthwork construction.

Loading Wagons by Hand. If the haul is comparatively short, a wagon is left to be loaded while the team takes another to dump, whereby the team and driver are kept busy, and besides this gives an opportunity to fix the responsibility for the amount of output. The number of shovelers and extra wagons will depend upon the length of haul. If no extra wagons are used, and the haul is short, it is important that the team be kept on the road as much as possible, which requires that as many men as possible should be employed in loading. If the haul is extremely long, it is not important that the wagon be loaded quickly, since the team then needs a little rest. Ordinarily ten men, four on each side and two at the rear, are as many as can profitably be used. The height of the wagon materially affects the cost of loading by hand. The top of the box of a dump wagon is about 4 1/2 or 5 ft. high, and at this height a man can load 15 cu. yd. per day; but for greater heights the amount will be decreased about 10% for each 6 in. of additional height. The loading of wagons can be greatly facilitated by first excavating narrow cuts 5 or 6 ft. deep, and then placing the wagons in these trenches while being loaded. The earth at the sides of the trenches can be shoveled into the wagons much more rapidly than if the men stood on the same level as the wagon. As the sides are shoveled off, the trench can be deepened. The slopes of the material at the sides of the wagon should not be steeper than about 2 to 1, or it will be difficult to operate the plow on them or for the men to stand on them. If the wagon must be loaded from only one side, and particularly if the haul is long, an extra side-board should be placed upon the opposite side of the box to increase the size of the load.

Wagons are also loaded with a steam shovel, in which case the earth should preferably be first dumped into a hopper under which the wagon is driven to load, as otherwise there is time lost in centering the wagon under the shovel and there is much dribbling around the wagon, and besides the shovel is likely to strike and damage the wagon. The hopper should rest upon a trestle which is mounted upon wheels so that it may move forward as the shovel advances. Wagons are also loaded with an orange-peel excavator or grab bucket swung from a derrick.

Motor Trucks are now being used extensively on earthwork jobs, particularly on large jobs and on work in the large cities. For small jobs and in locations where the truck would be hard to handle, the wagon is still used. The motor truck has greater power and capacity than team-drawn wagons and makes better speed, but it cannot get into some places that are accessible to wagons. Waits for loading and dumping are more expensive for trucks than for wagons, so that trucks are most economical for long-haul work. Trucks are obtainable with power dump and hoist equipment. Trucks are available of the sizes and capacities following:

Tons	Cubic yards
1-1/4	1-1/2
1-1/2	1-1/2
2	2
2-1/2	2-1/2 to 3
3-1/2	4 to 5
5 to 7-1/2	5

The capacities of course are nominal and vary with conditions. In some cases they may require under-loading, and in others would permit over-loading.

Belt Conveyors have been used with success on a few large jobs for the transportation of excavated materials. Due to the cost of installation, they are limited to cases where the amount of earth to be moved is very great, and where mechanical means are available for loading the excavated earth. They are not practical unless the material is of fairly uniform character, and does not contain any large stones. They can be successfully used on grades as high as 25 or 30%. For description of belt conveyors in construction of the earth dam at Wanaque, N. J., see Eng. News-Rec., Vol. 95, p. 252.

23. Power Shovel Work

Where very large quantities of earth are to be moved the power shovel is generally economical. The dragline excavator competes with the power shovel in excavating large volumes of material, but is not effective in hard ground.

There are two types of power shovel: the boom-swinging or railroad type, and the revolving. The railroad type has a boom with its main bearing and swing circle at its base. The total swing of the boom is from 180 to 200 deg. The dipper handle passes through the struts of the boom and is operated by a rack and pinion. The boom and dipper are at one end of the machine and the engines and boiler at the other. All are mounted on a two-truck car body and generally operate on standard-gage railroad tracks. The size of this type ranges from 3/4- to 6-yd. dipper, the largest size having a rated capacity of 300 to 500 cu. yd. per hour. The maximum depth of through cut which this type will make is about 16 to 18 ft. It can dig from 4 to 6 ft. below track level. For loading into dump cars it will lift to a height of 14 to 18 ft.

The revolving shovel has its boom fixed to a revolving frame which forms the base for the engines, boilers, and other machinery. The frame rotates on several large rollers. This type is well balanced and does not need jack arms or struts to keep it from overturning, as with the railroad type. The revolving shovels are made with dipper sizes of 1/2 to 8 cu. yd. They can rotate in a full circle in either direction. The smaller shovels can lift 11 to 18 ft. for loading. The larger shovels can lift from 20 to 70 ft.

Some of the newer boom-swinging type shovels are mounted on caterpillar or crawler tractors. The ease and rapidity with which this type can be moved around the excavation make it a very efficient machine. It not only saves in time but also in the number of laborers needed whenever it is shifted in the process of excavating. The small and medium sized revolving shovels are mounted on wide flat-tread wheels, or on caterpillar traction. Economy of operation is causing the wheel types to be abandoned in favor of the caterpillar type. Caterpillar traction is now generally used for all but the very large shovels and the tendency is to use this type on the larger sizes. Power is furnished the shovels by either steam or electric motors, the smaller sizes sometimes using gasoline engines. Gasoline shovels are more compact and can be used in close quarters. Diesel oil engines are used on some of the large machines.

Besides the operator, who is the only man required on the smaller revolving shovels, there is sometimes a fireman. From one to three groundmen are needed to do the labor jobs around the machine. On the railroad type it is usual to have a man to operate the boom as well as a dipper operator.

The revolving type has some advantages over the boom-swinging in that it can revolve completely and can dig at any point in its swing, is more compact and can work in closer quarters, and can be converted very easily and quickly into dragline, trenching machine or crane. Caterpillar mounted, and steam- or gasoline-driven revolving shovels of 1/2-yd. to 1-yd. capacity are best for small contracting jobs. The large revolving shovel is used on large-scale mining operations.

The boom-swinging type is still best for large capacity requirements where standard railroad equipment is available. However, because of its limited digging range, considerable benching is required for it in deep cuts.

The power shovel will excavate almost any material, except solid rock, without blasting. The output of the smaller shovels varies but they will do from 400 to 600 cu. yd. per 10-hour day. Working time is usually from 50 to 65% of total time. The larger shovels (such as 70 tons, with 2 1/2-yd. dipper) will do from 1000 to 1600 cu. yd. per 10-hour shift, depending upon their actual size and the materials handled.

In general, the commonest sizes are 60- to 80-ton shovels for the usual railroad work. For light grading, up to 25,000 cu. yd. per mile, where a shovel can be used economically, a light revolving shovel is desirable. For 25,000 to 40,000 yd. per mile, a shovel of 50 tons is a good size. For 40,000 to 60,000 cu. yd. per mile, a shovel of 60 to 80 tons is well suited. For anything over 60,000 cu. yd. per mile, the shovel may run up to well over 100 tons economically if its transportation is not too expensive, and if the ground is fit to carry its weight during excavation.

The greatest cause of delay in shovel work is in the removal of the excavated material. Too great attention cannot be given to securing proper and ample equipment in the matter of cars and locomotives, transportation and disposal. A shovel or a dredge can theoretically dig three or four times as much material as can be transported and disposed of, for the following reasons.

1. Shovels are usually not adapted in size and character to the material in which they are used. Contractors buy shovels for specific work for which they are suitable and then use them for all kinds of work almost regardless of conditions. On large work it would often pay them to sell their existing equipment and buy new plant of size and kind best adapted to the work.

2. Old shovels and poorly maintained equipment cause daily losses. It pays to have only the best equipment, thoroughly maintained. The master mechanic is an important factor in output. Second-hand plant is rarely economical. When the shovel is broken down the contractor loses the time not only of the shovel but of all collateral equipment, and the efficiency and enthusiasm of the whole organization suffer.

3. Excavation is usually more of a transportation problem than one of digging; therefore:

(a) Lay out the plan of excavation and your track plan and dump locations months in advance of the work. Be sure the tracks are not only properly located but ample for the transportation requirements.

(b) Use heavy and good rails and plenty of ties, and keep the trackwork in repair.

(c) On large work use large-size cars and locomotives and keep them in good condition. Assume that at least 15% of the cars and one locomotive will always be in the shop.

(d) Select dump sites requiring a minimum haul. A high dump saves delays in "jacking dump track."

(e) Note that in northern latitudes winter work is slow and expensive.

As the plant charge against shovel work is always an important item, either three 8-hour shifts or two 10-hour shifts are recommended. Continuous work for a long period is not practicable without spare plant because of the impossibility of adequate maintenance.

An excellent study of power-shovel operation is given in "Public Roads," Feb., March and April, 1928. See also report of Committee on Roadway of the American Railway Engineering Association, 1921 Manual.

24. Shrinkage and Settlement

Shrinkage and Settlement. In all operations involving the moving of earth, it should not be forgotten that the act of excavation so breaks up the earth that it occupies more space after excavation than before; but when the material has been placed in an embankment, it will usually occupy less space than in its original position. The expansion due to excavation is usually 8 to 12% of the volume, and in extreme cases may be 40%; but in placing the material in the embankment, it is compacted by the weight of the embankment itself, by the pounding of the hoofs, and by the action of the wheels, until usually the final volume is less than the original. Ordinary earth in its original position is more or less porous owing to its soluble portions having been carried away by the percolating water, to the penetration of vegetable roots which subsequently decay, and to the continued action of frost. There is usually also more or less earth lost in transporting it from cut to fill. The amount of shrinkage depends chiefly upon the character of the material and the means by which it is put into the embankment, and somewhat upon the moisture of the soil, the rainfall conditions while the work is in progress and soon afterwards, and the depth to which frost usually penetrates. If the soil is moist when placed in the bank, it will become more compact than if it were dry. Rain greatly affects the shrinkage; and embankments put up during a rainy season will be more compact than those built during a dry time. Soil from above the usual frost line is more porous than that not subject to the heaving effect of alternating freezing and thawing, and consequently shrinks more when put into an embankment. The shrinkage of the ordinary soils is in the following order: (1) sand and sandy gravel least, (2) clay and clayey soil intermediate; and (3) loams most. The shrinkage according to the method of handling is in the following order, beginning with the greatest: (1) wheelbarrows, (2) cars, (3) wagons, (4) wheel scrapers, (5) drag scrapers, and (6) caterpillar traction equipment. The usual allowance for shrinkage for drag scraper work is as follows: gravel 8%, gravel and sand 9%, clay and clayey earth 10%, loam and light sandy earth 12%, loose vegetable surface soil 15%. The above results are for ordinary earth, and do not apply to such unusual materials as "buck-shot," gumbo, very fibrous soil, etc., which have a much greater shrinkage.

Broken rock placed in an embankment will occupy a volume 40 to 50% greater than in its original solid state.

The shrinkage of earthwork referred to above takes place chiefly during construction; but the continued action of the weight of the embankment and the effect of rain and traffic will usually cause a comparatively small settlement after completion. Sand or gravel embankments built with wheel scrapers will usually settle 1 to 2% after completion, and clay or loam embankments about 2 to 3%. With drag scrapers the settlement will usually be a little less than the above; and with dump carts or wagons a little more. With wheelbarrows the settlement is usually about 10%, but may be as much as 25%. A sand and gravel embankment on which caterpillar traction equipment was used was found to have practically no settlement. The settlement of steam-shovel work depends upon the method of dumping, the length of time the work is in progress, the season and the soil.

The Manual of the American Railway Engineering Association (1921) states that because it is easier to add to the height of a fill than to decrease it, little or no allowance should be made for vertical shrinkage after construction. Allowance in width should be from about 5% to 20% of the height of the fill depending on the materials and conditions. For estimating, figure a shrinkage of 10% measured in excavation, on earth removed from excavation to embankment.

Subsidence is due to compression or displacement of the strata of earth under the embankment. Some subsidence occurs under all embankments, built on any ground but rock. It must always be anticipated in bogs or swamps, or any land on which there is standing water. Serious subsidence is local and it is impossible to fix any rule as a guide in estimating it.

Fig. 39

For the above reason, culverts in compressible soil should be built with a camber; and if of concrete, additional reinforcement should be used to prevent cracks. Fig. 39 shows the effect of building a culvert with camber.

25. Levee Construction

A Levee is a bank of earth thrown up along the side of a stream to prevent the flooding of the adjoining land. The cross-section of a levee depends upon the material of which it is constructed and the length of time it will be subjected to the flood. To prevent the levee from being overtopped by waves, the crown should be 3 ft. above the highest water for the smaller levees and 5 ft. for the larger. A levee usually has a top width of 3 to 12 ft., a slope on the water side of 1 to 3 or 4, and a slope on the land side of 1 to 2 or 3. A 1 : 4 slope on the water side is more desirable because it facilitates maintenance and increases resistance to wave action. A 1 : 3 slope is satisfactory if the surface is paved soon after construction. If the material of the levee is very light, the slopes should be flatter than the above. The above slopes are easily constructed, are readily kept clear of weeds and brush by the use of a mowing machine, do not slide when tramped over by stock when wet, are not difficult to get set in grass, and resist wave action reasonably well. It is important to avoid sharp corners, and flat curves should be used even if arable land is thereby lost; and on the curves the levee should be thicker and have flatter slopes than on the straight portions.

The slopes are determined also by the lines of seepage of water through the levee, and it is desirable to keep the line of seepage below the toe of the levee. Some seepage is unobjectionable when the levee is built on good foundation and is of good construction. H. St. L. Coppee in a paper in the Transactions of the American Society of Civil Engineers, Vol. 39, p. 229, shows as a result of tests made that the slope of the line of seepage for small sections is about 8 : 1 and large sections 5 : 1. He also estimated the relative strength of various levees to resist deformation due to seepage in the following order: (1) Clay and gravel tamped in shallow layers, (2) clay mixed with sharp sand in shallow layers, (3) clay, (4) heavy strong soils, (5) coarse sharp sand, (6) light soils, (7) fine sandy rounded particles. The Mississippi River Commission has adopted a minimum line of seepage of 7 : 1 in the landside slope for its new sections. (See Eng. News-Rec., Vol. 100, p. 398). Figs. 40*a* and 40*b* show the old and the proposed new levee sections of the Mississippi River Commission. In the old section, a banquette (reinforcing bank) was constructed where the foundation was weak and the height more than 10 or 12

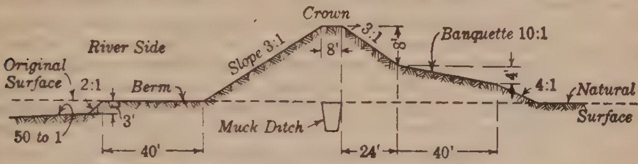


Fig. 40*a*. Standard Levee of Mississippi River Commission

ft. The chief object of the banquette is to keep the levee from sloughing off when saturated with water, which it does by keeping the surface of saturation well within the embankment. In the proposed new section, the banquette has been eliminated.

In order to help maintain equilibrium of the bank during high water, the Mississippi river levees are provided with iron pipe siphons, 10 or 12 in. in diameter, running up the face of the levee, over the top and down the back.

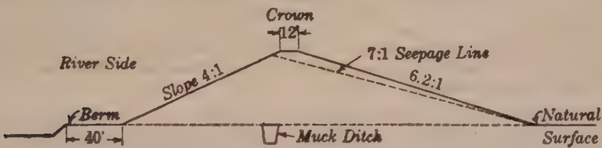


Fig. 40*b*. Proposed New Levee Section

The siphons are started by exhausting the air from the pipe with a hand pump, and are used to flood the borrow pits at the rear of the levee.

After the alignment is fixed, the surface should be freed from trees, logs, brush, and débris; and all buried logs, stumps, and roots of any considerable size should be removed. For the Mississippi river levee all logs, stumps, and roots are removed down to a diameter of 4 in. and for a width extending 5 ft. each side of the base of the levee. After clearing the foundation the ground is plowed deeply, and any unsuitable soil is removed. If the soil is not reasonably uniform, the foundation is tested by sinking pits at various points, and any defects are remedied as far as practicable. A muck ditch is usually cut on the river side of the center line, to explore the foundation and to aid in preventing the seepage of water along the plane of the base of the levee. It should extend through the porous surface soil; and should be filled with the best soil available, preferably clay or clayed gravel, which is thoroughly tamped into place. The standard dimensions of the muck ditch of the Mississippi river levees are 12 ft. wide on top, 8 ft. wide at the bottom, and 8 ft. deep, these dimensions being selected so that the muck ditch can be dug and filled with wheel scrapers. The width of the muck ditch may vary with the quality of the soil used for filling; if pure clay is employed, the width need not be more than 1 or 2 ft. except for the highest levees. Where the muck ditch will not efficiently cut off seepage, sheet piling is used. This is very expensive, but is sometimes necessary.

The smaller levees are constructed in layers not more than 2 ft. thick extending over the entire base, usually with drag or wheel scrapers. For the very large levees, draglines, steam shovels with long booms, and tower machines are used. Only unfrozen earth free from sticks, leaves, and straw should be used. The earth is taken when practicable from borrow pits on the stream side of the levee. For the Mississippi river levees the borrow pits on the river side must be at least 40 ft. from the toe of the levee, the side slope of the pit must not be steeper than 1 on 2, the depth next to the levee must not exceed 3 ft., the bottom of the pit must have a uniform slope toward the river of about 1 : 50, the depth on the farther side must not exceed 6 ft., and traverses (tongues of undisturbed earth) must be left at intervals of 300 to 400 ft. The traverses are to prevent destructive currents along the base of the levee; and should be pierced at the river end by drainage ditches, since standing water affords refuge for crawfish, muskrats, etc., which endanger the levee. The berm between the borrow pit and the levee should be kept intact, and any depressions in it should be filled to the level of the natural surface. Ditches and borrow pits on the inside are very objectionable; and, if the earth is borrowed from the land side, the pit should be well back from the levee. On the Mississippi the distance from the toe of the levee to the edge of a borrow pit on the land side must be at least 100 ft.

When the levee has been carried to the proper height and the crown and side slopes have been dressed to planes, tufts of Bermuda grass 2 to 4 in. square are planted at intervals of about 2 ft., which soon spread over the entire embankment and effectually protect it from the wash of the rain and to a considerable extent from the wash of waves and the current. Weeds and bushes should not be permitted to grow on levees, since weeds die and blow over, thereby loosening the soil, and bushes by shading the ground kill the grass and their roots loosen the earth. The injurious effect of wave action is greatly increased by the presence of weeds and bushes, as their roots break the surface and cause erosion to begin. Large trees should not be permitted to grow near the levee, as their roots will penetrate the base of the levee and give an opportunity for seepage, and when the tree is cut or dies its roots will decay and leave large openings which may cause a crevasse.

The smaller levees are built with drag or wheel scrapers, elevating graders and wagons, and the large ones with power shovels, dragline excavators and cableway excavators. The use of the elevating grader discharging directly on the embankment is unsatisfactory because it brings the pit too close to the levee. The same is true for the power shovel and dragline. They all are very useful for loading cars and trucks where the material has to be hauled. The grapple-dredge with an orangepeel bucket and a long boom is recommended by C. W. Okey of the U. S. Bureau of Public Roads for use in wet or marshy lands. It is superior to the dipper dredge for this use. The hydraulic dredges are used where the site is favorable for them, namely, where large quantities of material are present suitable for hydraulic dredging and where there are facilities for carrying off excess water, etc. They have been used on the Mississippi to transfer materials at high water to points within range of other type of equipment.

The Tower Machine has been used with very much success on the new larger sections of Mississippi levee. It is essentially a dragline or cableway excavator with the main tower on the land side of the levee, and the tail tower on the farther or river side of the borrow pit, usually up to 1000 ft. from the main tower. The two are connected by a cable which carries a 5-cu. ft. bucket. Both towers are traction mounted and can be moved parallel to the direction of the work. They eliminate the need of a haul, but the cost is considerable for moving the plant.

Drains are sometimes constructed through levees to carry away storm water during a low stage of the stream, but such openings are always a menace to the stability of the levee. If used at all, they should have substantial and adequate walls of concrete at both ends and the barrel should be of iron or vitrified pipe so laid that the joints will not leak. The lower end should be provided with a substantial automatic gate, usually a flap valve, for closing at the time of high water in the stream; and the inner end should be provided with a sliding gate to be moved by hand, for use in case the outer valve fails to work at the time of high water.

Roads along or across the top of levees are objectionable since ruts and holes result, and the edges of the crown are cut off by the wheels. Further, water collects in ruts and flows down the slope where it does considerable damage. If the crown is to be used as a roadway, the levee should be given an additional top width of at least 12 ft., and the completed structure should be carefully and frequently inspected. As better practice, it is recommended that the road be put on a shoulder on the landside slope, about 5 ft. below the crown.

26. Dredges and Dredging

Classification. Dredges may be classified according to the method of handling the material, as (1) dipper dredges, (2) elevator or ladder dredges, (3) hydraulic or suction dredges, (4) the grapple or grab-bucket dredges, and (5) scraper or dragline bucket dredges. Dredges may be classified according to the way in which they are moved forward while working, as (1) floating dredges, (2) traction dredges, (3) roller dredges, (4) drag dredges, and (5) walking dredges, of which the last four are used chiefly in ditch work. In harbor work the dredge usually discharges into a scow moored alongside; but sometimes the barge carrying the dredging machinery is provided with pockets large enough to hold several hours' output, and when these are filled the dredge steams out to sea and dumps its load. The latter type of dredge is called a **sea-going** or **hopper dredge**; and is used where the water is too rough to fasten scows alongside of the dredge. The hopper dredge is advantageous to use wherever the material is suitable and the excavation is deep enough to permit its use, and where the dump is not too remote. It is well adapted to sand and gravel when the latter is not too coarse, but cannot operate in mud because this will not settle in the bins. The Ambrose Channel at New York Harbor is maintained by such dredges, the material being all sand and the distance to the dump about 15 miles. Excavation by this method costs about 9 cents per cu. yd., bin measurement.

A Dipper Dredge is simply a steam shovel of the railroad type (see Art. 23) mounted upon a scow; and differs from an ordinary steam shovel chiefly in having a longer dipper-handle and a longer boom. In extreme cases, for ditch and levee work, the dredge can dig as deep as 60 ft. at low water. The dipper ranges in size from 1/3 to 15 cu. yd. It is better adapted to all kinds of work than any of the other types, since it can handle any material any other type can, and it can excavate material too tough for any of the other forms; but when large quantities of earth are to be moved and when other conditions are favorable, some of the other types are more economical.

The dipper dredge is excellent for hard digging, in which it has no competitor excepting the ladder dredge and that is rarely a serious competitor. A great deal of soft or stratified rock is dug by dipper dredges without blasting. They are the only machine adapted to removing old cribs and docks. In operation, the dipper dredge is not afloat, but is hoisted or " pinned up " on the " spuds." In soft ground these spuds have wide shoes to spread the pressure upon the bottom. These are adjustable steel posts or legs which are forced down into the bottom and hold the dredge rigidly in place. A modern

15-yd. dipper dredge costs from $700,000 to $1,000,000. Dipper dredges are much used for harbor and ditch work. (See Bull. 300, Bureau of Public Roads, U. S. Dept. of Agriculture.)

The Elevator Dredge, also called a ladder dredge and a chain-and-bucket dredge, consists of a series of scraper buckets attached to a chain, which scoop up the material and deliver it at the top of the ladder, where it is discharged into a chute or onto a belt conveyor. The tower which carries the upper end of the ladder is usually 20 to 25 ft. high. One " high tower " type has towers 75 to 80 ft. in height. The buckets have capacities of 1/2 to 15 cu. ft., are spaced from 3 to 6 ft. apart, and travel at a speed of 40 to 60 ft. per minute. The first machine of this type was made in France in 1859 for use on the Suez Canal. This type is more common in Europe than in the United States. For certain types of work where the material is free from roots and stones, it is rapid and economical of power. It can work in deeper water than the dipper dredge, and has continuous operation and large capacity; but it is more complicated and more likely to get out of repair. It is used in placer mining and for sand and gravel digging. On canal work in New Orleans a small ladder dredge having 32 buckets of 6-ft. capacity and working in swampy muck averaged 24,000 cu. yd. per month (Eng. News, Aug. 14, 1913).

Hydraulic Dredge. The essential feature of the hydraulic or suction dredge is a centrifugal pump which draws water and suspended earth through a suction pipe and discharges them through a line of pipe floated on buoys or pontoons. With light alluvium or fine sand, a reasonable amount of solid material is drawn up with the water, if the lower end of the suction pipe is nearly in contact with the soil; but in any other soil some form of agitator must be employed to loosen and stir up the material to be excavated. With medium soils the material can be stirred up sufficiently with one or more water jets; but in tough material a mechanical agitator, usually in the form of a hollow rotary cutter, is required. The agitator and the suction head are mounted on a ladder which, in the forward-feeding type of dredge, swings in a vertical plane. For lateral motion the whole dredge must be moved. In the radial feeding type, the ladder is designed to permit horizontal as well as vertical movement. The dredge may be either steam-, electric-, or Diesel-driven. Some smaller ones are gasoline-driven. The hydraulic dredge is used principally for construction and maintenance of rivers and harbors, for filling in and reclaiming land, and for canals and ditches, provided the material to be handled is of a nature which lends itself to hydraulic dredge handling. There must be a large spoil area available for dumping and it must not be too far away, although there are dredges which will transport material economically 10,000 ft. The following table taken from Bulletin 300, Bureau of Public

Diameter of suction and discharge pipes, in.	Normal capacity, gal. per min.	Cubic yards solids pumped per hr. Per cent of solids			Approximate horsepower required for each foot of total head
		10%	15%	20%	
4	450	12	18	24	0.4
6	1 000	30	45	60	0.8
8	1 800	50	75	100	1.5
10	2 800	90	135	180	2.5
12	4 000	130	195	260	3.0
18	9 000	300	450	600	7.0
24	17 000	500	750	1000	10.0

Roads, U. S. Department of Agriculture, gives an indication of what these dredges can do; but it should be considered as giving only a very approximate idea of the output, which is a function of the kind of material, the length of discharge pipe, and the height to which the material is lifted.

Material pumped by a hydraulic dredge and used for fill will take slopes approximately as follows: Sand pumped into place under water, 1 : 8 to 1 : 15; sand pumped into fill on land, 1 : 20 to 1 : 60; clay pumped into place on land, 1 : 4 or 1 : 5 if the clay forms balls in passing through the delivery pipe, and practically horizontal if the material comes out as liquid mud.

A Grapple or Grab-Bucket Dredge consists of a self-filling bucket suspended from a swinging or rotary boom; and is often called a **clamshell dredge** from the former form of the two halves of the bucket. At present there are only two forms of bucket in use, the orangepeel and the grab-bucket. The former consists of three or more curved triangular spades which when closed form a hemisphere; and the latter consists of two quadrants of a cylinder, which when closed form half of a short cylinder. These buckets are suspended by two chains or wire cables, one to close the bucket in loading it and one to open it in discharging. Both forms of buckets are made of rated capacities varying from 1/2 to 12 cu. yd. loose measurement. The great advantage of the grapple dredge over the dipper form is the greater length of boom permissible and the fact that there is practically no limit to its digging depth. A grapple dredge has done excellent work with a boom 240 ft. long. (Eng. Rec., Vol. 74, p. 360.) A long boom is sometimes of great advantage in ditch or levee work. The orangepeel bucket fills itself only in comparatively soft ground; and consequently the grab-bucket must be employed in hard or tough ground, for which work it is sometimes provided with teeth on the cutting edge and with extra power for closing. This form of dredge is peculiarly adapted to very deep dredging or to work in confined places like the inside of a cofferdam or even a pier cylinder. The grapple excavator is sometimes economical in excavating trenches, and can be used to load wagons or cars in dry excavation. It is particularly advantageous in cellar excavation, since the wagons can remain on the natural surface. In good material it can deliver a bucket load every minute. A 10-yd, 15-ton 2-part bucket working in water 65 ft. deep with 10 men and 5 tons of coal averaged 4000 cu. yd. in 10 hours, and in another case 2300 cu. yd. in 10 hours (Eng. News, 1899, vol. xli, p. 66).

Grapple dredges can be operated very cheaply due to their rapidity. Where the material is favorable, such as mud and sand or soft clay, it can be put into a scow lying alongside by a grapple dredge more cheaply than by any other method.

A Dragline Bucket Dredge is a form in which material is handled with a scoop roughly resembling the ordinary drag scraper suspended from a swinging boom, the scoop being drawn toward the machine by a line attached to the front and a second line at the rear holding it at the proper angle to slice the earth away as it is moved forward. When the scoop is filled, it is lifted to the point of the boom, both lines being kept taut, and is then swung around; when on slacking the drag or hauling line, the scoop dumps automatically. The bucket or scoop is lighter than an orangepeel or grab-bucket of equal size, and hence the whole machine may be lighter. It will excavate either soft or fairly hard material. There are several slightly different forms of bucket on the market. An important advantage of the dragline excavator is the wide reach possible by the use of a long boom. The dragline excavator is at present the favorite machine for widening and deepening small streams and ditches and for building levees and earth dams.

The dragline machine may be used for excavating either under water or in the dry. It is best adapted for jobs where the excavated material does not have to be removed from the site, but may be piled up temporarily by the dragline and later backfilled or spread out on the "dump" by the same machine. In cut-and-fill work, it is sometimes economical to handle the material as many as four times, the dragline being used first to scrape the soil from the cut to the beginning of the fill, and afterwards being moved to the fill and used again to scrape the excavated material still farther onto the fill.

The engineers of the Miami Conservancy District, Dayton, Ohio, under the direction of Arthur E. Morgan, Chief Engineer, made an elaborate investigation of different earth-excavating machines, and adopted the dragline excavator for that mammoth work, and had extensive experience in the use of this machine in excavating channels, and in building levees and earth dams. The District had nineteen dragline excavators at work, ranging from a 40-ft. boom and a 1 1/2-yd. bucket to a 160-ft. boom and a 5-yd. bucket. The following are the advantages, as stated by C. A. Bock, Division Engineer, which led to the adoption of the dragline excavator on this work almost to the exclusion of other devices.

Advantages. 1. The shape of the bucket and the method of loading make it possible to excavate boulders and also to pull stumps and large roots. The capacity of the buckets in use ranges from 3/4 to 5 cu. yd. Clamshell and orangepeel buckets may also be used, in which case the machine operates like an ordinary grab-bucket dredge. 2. The dragline has greater reach than steam shovels or any type of dredge. The horizontal radius for excavating or dumping may be 20 to 30 ft. beyond the end of the boom; and the vertical reach may be as low as 50 ft. below the machine bed and as high as 30 ft. above. 3. While the dragline ordinarily digs toward itself, with comparatively simple rigging it may be arranged to dig away from itself. 4. It is efficient in loading cars, in some cases even surpassing the steam shovel. 5. The dragline excavator may often be used to advantage as a movable revolving derrick for lifting track and equipment in conjunction with its own operation, which frees it from many interruptions to which other types are subject. 6. The lighter dragline machines may be mounted on caterpillar traction, or may be provided with walking devices, which give greater speed and more freedom of movement than any other type of excavating machine. 7. The heavier types are usually mounted on skids and rollers or on wheels and track, in which case they have greater range of action and require less track shifting than steam shovels or other types of dredges. 8. The dragline picks up, moves ahead, and relays, its own track.

Disadvantages. The disadvantages of the dragline excavator are: 1. In narrow cuts there is not sufficient dragging distance to fill the bucket properly and hence under this condition the dragline is not as efficient as the steam shovel or the dipper dredge. 2. The dragline ordinarily leaves the bottom of cuts and borrow pits quite rough, although a skillful operator can secure fairly satisfactory results.

The Cableway Excavator consists of an excavating self-loading bucket carried on a cable which is supported by towers at its two ends. One tower is high and usually is the one at which the operating machinery is located. The other tower is lower and is at the other side of the work. The commonest type of cableway excavator is the slackline cableway. The track cable in the slackline type can be loosened and tightened to permit raising and lowering of the bucket. The tightline type has a fixed track cable and the bucket is raised and lowered by mechanical attachments. The slackline can excavate up to one-third its span, which may be from 200 to 1200 ft. It can lift excavated material about 150 ft. It is used mainly for gravel pit operations, but has been found very useful on Mississippi river levee construction. The towers may be fixed or may be mounted to travel along the work. The power may be steam, electric, or gasoline. In gravel pit operation the material is dumped usually into a screening and handling structure. In levee work the material is dumped into the embankment. The accompanying table indicates roughly what may be expected from this type of excavator. The variations are due to differences in materials and other local conditions.

Capacities in Cubic Yards per Hour on Average Hauls for Various Lengths of Span

(Data furnished by Sauerman Bros., Inc.)

Size of bucket, cu. yd.	300-ft. span, 200-ft. av. haul	500-ft. span, 300-ft. av. haul	700-ft. span, 400-ft. av. haul	1000-ft. span, 550-ft. av. haul
1/2	19 to 26	18 to 24	....	....
1	38 to 52	36 to 48	30 to 40	....
2	78 to 104	72 to 96	60 to 80	50 to 68
3	114 to 156	108 to 144	90 to 120	75 to 102

The Power Scraper is a bottomless drag scraper which is pulled back and forth across the job by cables by means of a steam-, gasoline-, or electric-driven hoist. It may be crescent-shaped or V-shaped and the bottom or cutting edge may be equipped with teeth. The scraper is dragged through the material by the front cable, gets its load, and deposits it at the load point. It is then dragged by the rear cable back to the digging point. Towers are needed at the head and tail points to carry the sheaves over which the cables operate.

The scrapers come in 1/3- to 6-cu. yd. capacity sizes. They are used for stripping operations, for moving large quantities of materials in storage operations, and for highway construction. They are often used in conjunction with slackline cable operations. The power drag scraper is found most economical on hauls of 200 to 400 ft.

Estimated Hourly Capacities in Cubic Yards

(Data from Sauerman Bros., Inc.)

Length of haul, ft.	Size of scraper, cu. yd.							
	1/3	1/2	3/4	1	1-1/2	2	4	6
100	22	34	51	80	120	160	320	480
200	12	18	27	42	63	84	168	254
300	8	12	19	28	42	56	112	168
400	6	10	14	21	32	43	86	126
500	5	7	11	17	26	34	68	104

27. Ditching Machinery

It has been estimated that there are still in this country something like 100,000,000 acres, or over 150,000 square miles, of swamps and wet lands. This area is about two and a half times as large as all the New England States. A considerable part of the redemption of this vast tract will consist of building levees and dredging main drainage channels. The building of levees and the construction and operation of dredges have been described in Arts. 25 and 26, and the principles of drainage are discussed in Sect. 16. It is proposed to briefly consider here some of the machines employed in constructing medium-sized drainage and irrigation ditches.

The **dipper dredge** (see Art. 26) is an important machine in the drainage of swamp or overflowed land. There are four forms of this machine specially adapted to ditch work, namely, the floating type, the land type, the walking dredge, and the drag boat.

The **floating type** is a steam shovel mounted upon a wooden or steel hull. It may work either up or down stream, but the latter is much the more common, since then dams are not necessary behind the dredge to retain water in which to float it. The capacity of the dipper varies from 0.5 to 5 cu. yd. The maximum size of ditch that can be dug economically with this type varies from 100 to 1200 sq. ft. and the minimum is 10 ft. wide and 5 ft. deep.

The **land type** of dipper dredge consists of a boom, a dipper, and the necessary machinery mounted upon a frame which straddles the ditch and runs upon wheels resting directly on the banks or upon a track. Some machines have caterpillar traction. The capacity of the dipper varies from 1/2 to 1-1/2 cu. yd., and the span from 15 to 50 ft. The land type can be dismantled and moved more easily than the floating form; but in stumpy ground it is not as good as the latter, since it lacks power to uproot large stumps. The land type is objectionable unless the ditch is narrow and the banks of the ditch are firm. With soft banks the caterpillar traction is better than either broad wheels or a track. The chief advantage of the land dredge is that all of the excavation is in sight, and hence a better ditch can be dug than when the excavation is done under water as with the floating type.

The **walking dredge** straddles the ditch. Its distinguishing characteristic is that each corner of the frame rests upon a built-up foot of which the horizontal cross-section is about 6 × 8 ft., and in the middle of each side of the frame is a similar foot about 6 × 12 ft. When the machine is to be moved forward, it raises itself and also the four corner feet until it is entirely supported on the two center feet; and then the machine slides itself about 6 ft. forward on the middle supports. The corner feet are then lowered, and the weight of the frame thrown upon them; and next the center supports are raised, and brought forward ready for another move. "Walking Speed" is about 500 to 800 ft. per hour. The chief advantage of the walking device is that it enables the whole machine to move itself across country without dismantling, thus saving much time and also the cost of dismantling and rebuilding.

The **drag-boat ditching machine** is mounted on a hull whose transverse cross-section is somewhat like that of the ditch to be dug. The whole machine is moved forward by winding up a cable anchored ahead. This type has booms from 25 to 30 ft. long, and dippers of 1/2- to 3/4-cu. yd. capacity; and will excavate a ditch having a bottom width of 4 to 6 ft. It cannot be used if the banks of the ditch will not stand without caving, since that may wedge the hull fast.

The **drag-line excavator** (see Art. 26) is in many ways the most developed type of machine for ditch work. It may follow the center line of the ditch, backing away from the completed work, or it may travel on one side of the ditch. The first method is usual for small ditches, and the second for large ditches; and sometimes for large ditches the machine travels down one side and back on the other, excavating half of the channel each trip. Dragline excavators have been made with booms 40 to 150 ft. long and buckets holding from 1 to 5 cu. yd.; but the longer booms and larger buckets are not very common. The comparatively wide reach of the dragline excavator fits it for excavating channels wider than can be dug economically with the floating dipper dredge, but the cost of ditch work with a dragline excavator is usually more than with a dipper dredge. The dragline can cut the side slopes of the ditch more accurately and more economically than the dipper dredge and also can leave a wider berm, both of which are important advantages.

When arranged specially for ditch work, the dragline excavator may be mounted upon traction wheels, or on skids and rollers, or on car wheels on a track; or it may be mounted as a walking machine like the walking dipper dredge previously described. It may also be mounted on a tractor, and is then specially adapted to ditches of small cross-section.

A form of dragline scraper is called the **templet excavator.** It has two buckets, back to back, working on a frame transverse to the ditch in such a manner that one bucket shaves off a thin slice down one slope and across the bottom and up the other slope, and then dumps. One scraper does the excavating one way and the other in the opposite direction. The frame carrying the buckets is lowered automatically as the ditch increases in depth, so that the slopes and the bottom are cut accurately to specifications. When the desired depth is reached, the frame is raised and the machine moved forward or backward, as the case may be, for a new cut.

28. Hydraulic Sluicing

Hydraulic Sluicing has been used successfully for the excavation of large volumes of material, and notably on the Panama Canal and by the Miami Conservancy District in Ohio. The method consists in breaking down the material in the " borrow pit " by means of a powerful water jet which is discharged from a large nozzle or monitor commonly called a " giant." The nozzle diameter varies from 2 1/2 to 5 in., and the jet or " muzzle " velocity of the stream varies up to 150 ft. per sec. The working pressure for a 5-in. nozzle may be as high as 150 lb. per sq. in.

The water is supplied by centrifugal pumps, which must have the required water capacity and must exert a head sufficient to overcome the suction and the friction losses in the pipe line and to develop the necessary head at the monitor. The pipe line from the pump to the monitor should be 12 to 15 in. in diameter in order to reduce friction losses, but is reduced in size near the monitor where the pipe requires frequent handling.

In operation, the water jet is directed against the base of the bank which is being excavated, and washes out the material. This causes caving of the bank and furnishes loosened material which is readily washed away by the jet. The water, with its burden of excavated soil, flows out of the borrow pit and is conducted to the site of the fill in a " sluiceway " which generally is merely the natural channel formed by the flowing water. As a rule, the supply of water from the jet is not sufficient to carry off the excavated soil properly, and an additional stream of water is provided for this purpose by low-pressure pumps. The sluice will generally require a slope of about 4 per cent. Where the ground is particularly hard, especially with some types of clay, it may be necessary to loosen the bank partially by blasting. Gravel and sand are the easiest material to excavate by this method, if there are not too many cobblestones or small boulders scattered through the bank. Clay is more difficult to handle; some kinds of clay will not entirely break down, but will pass through the sluice in round balls several inches in diameter.

The monitor should be placed as close to the pit face as possible in order to take advantage of the full force of the jet, but this distance will be limited by the danger of caving of the bank. The slope of the sluice should be as flat as possible in order to keep the elevation of the monitor at a minimum, as this will give a higher pit face and increase the quantity of material available for excavation.

The hydraulic sluicing method is well adapted to excavating jobs which have suitable material for breaking down, where there is an ample water supply, and where the borrow pit is at a high enough elevation so that a sufficient grade may be obtained for the sluiceway. The method is particularly adapted to the construction of hydraulic-fill earth dams. It has also been used on large street grading work in Seattle, Wash., in 1912 (Eng. Rec., Vol. 65, p. 480), and in highway grading near Vancouver, Wash., in 1926 (Eng. News-Rec., Vol. 97, p. 776). In the Miami Conservancy District large quantities of material were handled by this method, at costs ranging from 21 to 65 cents per cu. yd., the lower costs being for gravel handled under favorable conditions. On this work, however, the cost was higher than should be expected on an ordinary excavation job, due to the necessity of securing the requisite amount of fine material for the core of the dam. The output ranged from 12,000 to 50,000 cu. yd. per month, the latter being accomplished with two giants having nozzles of 5-in. and 2 1/2-in. diameters. The contract price for the work at Seattle was 10 cents per cu. yd. for excavation and 15 cents per cu. yd. for embankment.

See "Methods and Plant for Excavation and Embankment" by C. H. Paul and C. S. Bennett; also "Construction Plant, Methods and Cost," Technical Reports, Part X, Miami Conservancy District.

For sluicing of hydraulic fill dams, see Sect. 15, Art. 6.

29. Excavation Classification

There is probably no single problem in the interpretation of contracts that is more difficult than the classification of excavation into rock, loose rock, and earth. No specifications have yet been evolved that will be interpreted by two men precisely alike. Wide variations of interpretation are possible by those of opposing interest. It is the duty of the engineer to write them as clearly as he can and to interpret them as broadly and definitely as he can to prospective bidders, especially to the contractor to whom the contract is to be awarded. The following specification is in part from the Reading Railroad company.

Excavation is classified as: solid rock, loose rock, and earth—dry or in water.

(1) **"Solid Rock"** shall include all rock which is found in ledges or detached masses, exceeding 1 cu. yd. each, and which may be best removed by blasting.

(2) **"Loose Rock"** shall include all other rock and cemented gravel which may be removed without blasting, although blasting may occasionally be resorted to; also detached stones less than 1 cu. yd. and more than 1 cu. ft. in volume.

(3) **"Earth"** shall include all materials of whatever name or character not unquestionably rock, as above defined.

(4) **Excavation in Water**: (a) This term shall apply to the removal of all materials in water, or materials thoroughly saturated or impregnated with water, except surface water which can be drained away.

(b) It shall include drainage, bailing, pumping and all labor and other expenses connected with such excavation.

(c) In changing, deepening, or making water courses, all materials above the water level and materials that can be removed without becoming saturated with water shall be classified as dry.

(5) **Payment for Excavation.** Excavations in all the several classes thereof shall be estimated and paid for by the cubic yard of material removed, measured in its original undisturbed state. Estimates shall be based on cross-sections, made by the engineer, of the undisturbed surface before the work is started, of the uncovered surfaces as the work progresses (when different classes of material are uncovered), and of the completed surfaces. Specifications for dredging should state whether or not excavated material is to be paid for in its original undisturbed state or on basis of scow measurement.

(6) **Payment for Embankment.** Embankments shall not be estimated or paid for when made from excavations; but in special cases when made from material furnished by the owner to the contractor they shall be estimated and paid for by the cubic yard of material placed in the embankment measured in its finished state. Estimates shall be based on cross-sections, made by the engineer, of the surface before the filling is started and of the finished surfaces.

(7) **Haul.** The prices paid for the several classes of excavation shall be understood to cover the entire cost of removal and disposal in the manner and at the location prescribed, it being understood that no allowance will be made for over-haul.

Comments on Specifications: There often arises a question whether or not shale, soft sandstone, or hardpan belong under solid rock or loose rock. In justice to the contractor and in the interests of all parties to the contract, an interpretation of this should be made in advance of the signing of the contract. Even then the final line of demarkation is usually a matter of judgment.

It is extremely difficult at times to determine with justice the difference between earth and loose rock. For instance, if the work consists of ordinary sand and gravel and it is understood that it is to be removed by scrapers, nests of small boulders of 1 or 2 cubic feet make the operation of scrapers impossible. If such nests could not be foreseen by observation or from the borings that were taken in advance of the contract, the contractor has some equity in his contention that such material is not earth.

It is also difficult to specify accurately what is not rock. It has been classified by some as that which cannot be plowed and by others that which cannot be loosened with an ordinary pick. However, a very heavy plow and tractor can loosen ground which would not normally be classified as earth unless this was clearly explained to the contractor in advance of signing the contract. In other words, standard specifications without a specific interpretation, applicable to the particular work to be undertaken, are dangerous.

Referring to paragraphs 5 and 6, the common expression is that "excavation shall be measured and paid for in cut and not in fill."

If item 7 is included in the specification the contractor must be careful to determine, in advance of signing the contract, where the fill is to be spoiled; or in some way the haul must be predetermined.

In some specifications slate, coal, shale and soft friable sandstone and soft soapstone are not classified as solid rock but as loose rock. It should also usually be stated that the prices agreed upon for excavation include the cost of all clearing and grubbing and for the removal and disposing of all materials covered by the contract; if not so stated, separate prices should be given for clearing and for grubbing. These terms should be defined by the contract substantially as follows: **Clearing** is the disposal of all trees, brush and other perishable material, which shall be removed or destroyed by burning. Large trees shall be cut not more than 2-1/2 ft. from the ground; and in embankments less than 4 ft. and more than 2-1/2 ft. high, they shall be cut even with the ground. **Grubbing** is the removal of stumps and roots. Clearing and grubbing are paid for on the basis of the area involved.

Overcutting or excess excavation is not usually paid for unless due to causes beyond the control of the contractor. Rock excavation is generally carried 6 in. below subgrade.

SECTION 9

TIMBER STRUCTURES

BY

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SIMPLE TIMBER CONSTRUCTION

1. Working Unit Stresses

The recommended unit stresses for material complying with structural grades of the American Society for Testing Materials, as determined by Forest Products Laboratory, U. S. Forest Service, 1926, are given in the following table:

Safe Allowable Working Unit Stresses in Pounds per Square Inch

Dry Location—Common Grade Timber

Kind of timber	Allowable stress: Bending: In extreme fiber (tension with grain)	Allowable stress: Bending: Horizontal shear (with grain)	Allowable stress: Compression: Perpendicular to grain	Allowable stress: Compression: Parallel to grain; short columns	Intermediate columns, value of K: Select grade	Intermediate columns, value of K: Common grade	Modulus of elasticity, E
Cedar, western red	720	64	200	560	24.2	27.1	1 000 000
Northern and southern white	600	56	175	440	24.5	27.3	800 000
Port Orford	880	72	250	720	23.4	26.2	1 200 C00
Alaska	880	72	250	640	24.8	27.8	1 200 000
Chestnut	760	72	300	640	22.7	25.3	1 000 000
Cypress, southern	1040	80	350	880	21.2	23.7	1 200 000
Douglas fir, Coast region	1200	72	325	880	23.7	27.3	1 600 000
Rocky Mountain region	880	68	275	640	24.8	27.8	1 200 000
Fir, balsam	720	56	150	560	24.2	27.1	1 000 000
Golden, noble, silver, white	880	56	300	560	25.4	28.4	1 100 000
Hemlock, West Coast	1040	60	300	720	25.3	28.3	1 400 000
Eastern	880	56	300	560	25.4	28.4	1 100 000
Larch, western	960	80	325	880	22.0	24.6	1 300 000
Oak, red and white	1120	100	500	800	24.8	27.8	1 500 000
Pine, southern yellow	1200	88	325	880	23.7	27.3	1 600 000
White, sugar, western white, western yellow	720	68	250	600	23.4	26.2	1 000 000
Norway	880	68	300	640	24.8	27.8	1 200 000
Redwood	960	56	250	800	22.2	24.8	1 200 000
Spruce, red, white, Sitka	880	68	250	640	24.8	27.8	1 200 000
Englemann	600	56	175	480	23.4	26.2	800 000
Tamarack, eastern	960	76	300	800	23.1	25.8	1 300 000

(See A. S. T. M. Standard Specifications D 245–27; Dept. of Commerce Report on "Recommended Building Code Requirements for Working Stresses in Building Materials," 1926; also Trans. Am. Soc. C. E., Vol. 91, p. 400.)

The following comments on the use of this table are based on the authorities quoted.

The allowable stresses given are for **Common Grade** timber, which contains many knots and blemishes but which is suitable for general utility and construction purposes. For **Select Grade** timber, which is generally clear and limited in defects both in size and number, these stresses may be increased 33-1/3% for Douglas fir (Coast) and Southern pine, and 25% for all others, excepting compression perpendicular to the grain and modulus of elasticity which are the same for both grades.

The stresses also are given for dry locations. In locations where the timber will be exposed to the weather, the allowable stresses shall be reduced 25%, excepting horizontal shear and modulus of elasticity which, in designing, are assumed not to vary with conditions of exposure.

In joints, the shearing stress with the grain may be taken as twice the value given in the table, for both grades of timber.

Stress due to dead and live loads acting singly or in combination, without wind load, shall not exceed the allowable stress given above. For stresses produced by wind loads or by a combination of wind loads and dead and live loads, the working unit stresses may be increased by 50% provided the resulting sections are not less than those required for dead and live loads alone. The recommended working stresses may be used without any increase of live loads to allow for impact.

Stress in compression perpendicular to the grain may be increased by 50% above that given in the table in the case of joists supported on a ribbon board and thoroughly spiked to the studding rather than resting upon or in masonry.

The unit stresses for columns whose ratio of unbraced length to least width l/d is 11 or under are those given in the table for short columns. The unit stresses for columns whose ratio l/d is 12 or greater must be reduced by column formulas (Art. 3). The values for modulus of elasticity are average for the several kinds of timber and not safe working values. They are for use in computing average deflections of beams. When it is desired to prevent or limit sag in a beam, one-half the values given in the table should be used. For instance, in a building, beams supporting an ornate plastered ceiling should be figured using the value $E/2$. E cannot be used to determine the shortening of timber when the pressure is applied to side grain, such as the pressure of a post upon a cap or sill.

For continuous heavy loading, or loading causing reversal of stress in a member, 80% of the table stresses is suggested. The strength of timbers in old exposed structures should not be assumed higher than 80% of the table stresses, after deducting for timber exposed to weather as noted above, and only the section exclusive of rotted or partly rotted portions should be considered as effective. Old timbers should be bored at numerous points, in order to determine the effective sections. For timber to be immersed in water deduct 33%, except for side grain impression for which deduct 50%.

In the design of structures to be built in cities, the unit stresses permitted by the building codes must be considered.

In designing, only the net section of dressed timber should be considered. Undressed seasoned timber is generally a little scant of the listed or nominal cross-sectional dimensions. Ordinary differences between nominal and actual standard dimensions of dressed timbers are:

Timber	Nominal	Standard
Joists, rafters, plank, flooring.	Thickness, 2 to 4 in.	3/8 in. less
	Depth, 4 to 7 in.	3/8 in. less
	Depth, 8 in. and over	1/2 in. less
Beams, girders, stringers.	Thickness, 6 in. and over	1/2 in. less
	Depth, 8 in. and over	1/2 in. less
Posts, caps, sills, timbers.	Size 6 × 6 in. and over	1/2 in. less

Standard lengths are in multiples of 2 ft.

The constructor should inspect important timbers to be sure that the actual dimensions equal those assumed in the design.

2. Properties of Timber

Unit Weights of various kinds of timber are given in the table herewith. In designing, an average value of 40 lb. per cu. ft. or 3.3 lb. per ft. board measure may be used; this is closely the actual weight of commercially seasoned southern pine.

Kind of timber	Weight per cu. ft. air-dry	Kind of timber	Weight per cu. ft. air-dry
Cedar, Western red	23	Hemlock, West Coast	28
Northern and Southern white	23	Eastern	28
Port Orford	29	Larch, Western	36
Alaska	29	Oak, red	44
Chestnut	30	White	48
Cypress, Southern	32	Pine, Southern	41
Douglas, fir, Coast region	34	White, sugar, Western white, western yellow	27
Rocky Mountain	30	Redwood	30
Fir, Balsam	26	Spruce, red, white, Sitka	27
Golden, noble, silver, white	27	Engelmann	23
		Tamarack, Eastern	37

These are average values, as determined by the U. S. Forest Products Laboratory.

Sizes and Specifications. In designing small structures, standard sizes and lengths should be used, as special sizes and long lengths are expensive. Standard lengths are in multiples of two feet, and usually do not exceed 20 ft. In many cases, considerable saving can be made by using "short-length lumber," less than 8 ft. long. (See "Marketing of Short-Length Lumber," published by U. S. Dept. of Commerce, 1926.) Nominal cross-sectional sizes of undressed timber are in multiples of 1 or 2 in.

"Simplified Practice Recommendation No. 16," on Lumber, published in 1926 by the U. S. Department of Commerce through the Bureau of Standards, gives a recommended set of standards and specifications for sizing, classifying or grading timbers which was generally accepted by the lumber interests throughout the country and was adopted by the American Railway Engineering Association and the American Society for Testing Materials.

Indentation under Side Grain Compression. The safety of structures is seldom endangered by crushing across the grain. The indentation made, however, is important in some structures and allowance for it should be made in design. It is suggested that in computing the amount of side grain indentation or compression, 0.05 in. be used for all exposed timbers under the safe load unit pressures and 0.10 in. for double these unit pressures. If timber is to be immersed in water, double the above allowances. In rough carpentry work where joints are not well made, double the above allowances; if such joints are to be saturated, use four times the above values. In the construction of the Arlington Memorial Bridge, Washington, D. C., white and red oak wedges, which carried the arch load although subjected to the weather, failed when saturated for three days by flood waters. It is thought that if these wedges had been thoroughly oiled or greased (which is good practice for such wedges subjected to weather or flood) failure would not have occurred.

For thoroughly seasoned timber in a housed structure and for first-class carpentry work, it is suggested that the indentation be computed as 0.01 in. for the allowable compressive stress given in Art. 1.

Non-Uniform Distribution of Pressure on timber requires special consideration, and the excess pressure caused by such distribution should be considered in designing. Such a condition commonly occurs at the end of a roof-truss or arch centering, where the center of the support is not directly in line with the resultant load of the structure. Fig. 1 shows this condition under arch-

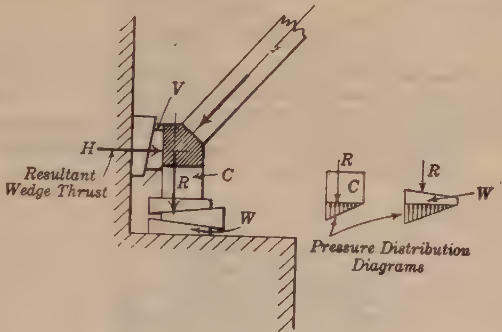


Fig. 1

centering, where the resultant thrust from the timber framing and the wedges which take the horizontal thrust falls outside the center of the supporting timber. This causes non-uniform stress distribution in the latter as indicated by the pressure diagram.

Attention is also called to the fact that the point of application of *H* depends upon the care with which the wedges are made and driven. It may vary by one-fourth the length of the wedge *V*. Therefore the unit side-grain compression should be conservative in both the vertical and horizontal wedges and in the cap *C*.

In a study made in 1917 by the Forest Service of the U. S. Department of Agriculture, hardness of air-dried timbers was tested by measuring the load required to embed a 0.444-in.-diameter ball to one-half its diameter in the wood. Hardness across the grain may then be determined from the accompanying table.

Kind of timber	Load required to imbed ball, lb.	Relative hardness	Kind of timber	Load required to imbed ball, lb.	Relative hardness
Cedar, Western red..	380	.28	Hemlock, Eastern...	490	.36
Northern white....	340	.25	Larch, Western.....	870	.65
Port Orford.......	700	.52	Oak, white, red.....	1350	1.00
Alaska...........	580	.43	Pine, Southern yellow...........	1020	.76
Chestnut..........	580	.43	Western yellow, white, sugar....	460	.34
Cypress, Southern...	550	.41	Norway (red).....	600	.44
Douglas Fir, Coast region.........	810	.60	Spruce, red Sitka...	520	.39
Rocky Mountain..	660	.49	Engelmann......	290	.21
Fir, Balsam.........	500	.37	White..........	560	.42
White............	460	.34	Tamarack.........	640	.47
Hemlock, Western...	620	.46			

Compressive Stress on Surfaces Inclined to the Fibers. Many formulas exist for determining the compressive strength of wood on surfaces at an angle to the fiber. One of the more recently developed ones, known as the Hankinson formula, is recommended by the U. S. Forest Products Laboratory for general use in timber framing. The equation for the formula is

$$N = \frac{PQ}{P \sin^2 \theta + Q \cos^2 \theta}$$

in which N = allowable unit stress on the inclined surface;
P = unit stress in compression parallel to the grain;
Q = unit stress in compression across the grain;
θ = angle between the direction of the load and the direction of the grain. It is equal to 90° minus the angle between the inclined surface and the direction of the grain. (See Fig. 1*a*.)

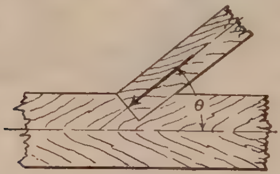


Fig. 1*a*

The table given here is based on this equation

Safe Loads in Pounds per Square Inch

Common Grade Timber—Dry Location

Kind of timber	Angle θ, degrees						
	7-1/2	15	30	45	60	75	82-1/2
Cedar, Western red	540	500	390	295	240	210	200
Cypress, Southern	860	800	640	500	410	370	355
Douglas fir, Coast	860	790	615	470	390	340	330
Fir, Balsam	535	470	330	240	180	160	150
Hemlock, Western	705	660	535	425	355	315	305
Oak, red and white	790	770	695	615	550	515	505
Pine, Southern yellow	855	790	620	470	390	340	330
Redwood	770	695	515	380	300	260	255

Select Grade Timber—Dry Location

Kind of timber	Angle θ, degrees						
	7-1/2	15	30	45	60	75	82-1/2
Cedar, Western red	670	600	430	310	245	210	205
Cypress, Southern	1060	965	715	530	420	365	355
Douglas fir, Coast	1125	1000	710	510	395	340	330
Fir, Balsam	660	560	365	250	185	160	155
Hemlock, Western	870	790	600	450	360	315	305
Oak, red and white	985	940	800	665	570	515	505
Pine, Southern yellow	1125	1000	710	510	395	340	330
Redwood	950	830	570	400	310	265	255

For timber in locations exposed to the weather, the safe loads in these tables should be reduced by 25%. If timber is to be saturated, reduce by 50%.

Coefficients and Angles of Friction

Kind of surface. Position of fibers with reference to direction of sliding	Coefficient of friction	Angle of friction
Wood on wood, undressed, side grain, fibers parallel	0.45	24° 10′
Wood on wood, undressed, side grain, fibers at right angles	0.35	19 20
Wood on wood, undressed, end grain on side grain	0.30	16 40
Wood, undressed, on rough metal	0.45	24 10
Wood, undressed, on smooth metal	0.30	16 40
Wood on wood, dressed smooth	0.35	19 20
Wood dressed smooth on smooth metal	0.25	14 00
Wood on masonry. See Sect. 10, Art. 18	....	....

These coefficients are average values for dry seasoned wood as determined by comparison of published data. "Fundamental Ideas of Mechanics" by Morin contains good data on this subject. Soft, wet or unseasoned timber usually gives higher coefficients, although very green timber under small loads gives lower values.

When very heavy loads are applied, the fibers interlock and give higher values. End grain timber on side grain and metal on wood under heavy loads bite into the wood, tearing the fibers (Fig. 2) and causing mechanical restraint against sliding.

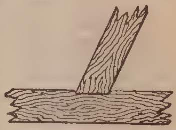


Fig. 2

If lubricants are used and the pressures are small, the coefficients of friction may be reduced 50 or 70% of the tabulated ones. If the pressures are as great as those given in Art. 1, the lubricant is squeezed out, the fibers bite into each other and the coefficient of friction will not be reduced.

Friction should not generally be regarded in the design of framed structures except as an added factor of safety. For temporary structures or simple members of permanent ones, such as battered posts of a highway trestle, it may properly be considered, but it is suggested that not more than 50% of the tabulated coefficients be used.

3. Beams and Columns

Wooden Beams are generally of rectangular section and of uniform depth, and only such beams are considered in what follows. A beam should be designed so that the extreme fiber stress due to bending, the maximum horizontal shear, and the compression across the grain, at the end bearings, do not exceed the allowed unit stresses, and so that the maximum deflection does not exceed a limit which is fixed by the use to which the structure is to be put. For highway bridges and trestles this limit is generally 1/200 of the span, for railway bridges and trestles 1/300, for plastered ceilings 1/360 and for ceilings supporting shafting 1/700. When the ratio of length to width in a beam exceeds 20, it should be laterally braced, cross-bridging being commonly used. If not braced, the safe extreme fiber stress in Art. 1 should be reduced as follows:

Ratio of length to width	20 to 30	30 to 40	40 to 50	50 to 60
Percentage of reduction	25	34	42	50

In the case of a beam used without a floor (which acts partly as lateral bracing) the ratio of length to least width should not exceed 30.

Wooden beams are often continuous over several supports and may be so figured, but most designers compute all timber beams as simple beams.

To design a beam for any loading, the depth d should be assumed, consistent with the general governing conditions and the fact that beams over 12 in. deep cost more per board foot than shallower ones. Having selected d, the width of beam can be determined by the proper formulas, selecting values for the unit stresses from Art. 1. For long and shallow beams the flexure formula will generally govern, for short and deep beams the shear formula. The designer must be careful to consider only the actual or standard cross-sectional dimensions of lumber and not the nominal ones. The safe loads given in the tables are based upon the standard or actual depths, although the depths given are nominal ones.

If the amount of deflection is important, formulas for depth in terms of deflection and unit stress should be used. The end-bearing area A should not be less than V/S_a, in which S_a is one-half of the allowed unit stress across the grain; this is recommended because flexure of the beam makes the pressure upon the front of the bearing greater than that at the back.

Safe Loads in Pounds Uniformly Distributed for Rectangular Beams One Inch Wide

For an allowable fiber stress of 1000 lb. per sq. in.

Span, ft.	Nominal depth of beam, in.										
	4	6	8	10	12	14	16	18	20	22	24
4	370	880	1560	2510	3670	5060	......	......	......	......	......
6	240	590	1040	1670	2450	3380	......	......	......	......	......
8	180	440	780	1260	1840	2530	......	......	......	......	......
10	150	350	630	1000	1470	2020	2670	3400	4220	5140	6140
12	120	290	520	840	1220	1690	2220	2840	3520	4280	5110
14	100	250	450	720	1050	1450	1900	2430	3020	3660	4380
16	90	220	390	630	920	1260	1670	2120	2640	3220	3840
18	80	200	350	560	820	1120	1480	1890	2340	2860	3400
20	70	180	310	500	730	1010	1330	1700	2110	2570	3060
22	.....	160	280	460	670	920	1210	1540	1920	2340	2790
24	.....	150	260	420	610	840	1110	1420	1760	2140	2560
26	.....	140	240	390	560	780	1030	1310	1620	1980	2360
28	.....	130	220	360	530	720	950	1210	1510	1830	2190
30	.....	120	210	330	490	670	890	1130	1410	1710	2040
32	.....	110	200	310	460	630	830	1060	1320	1610	1920
34	.....	100	180	290	430	590	780	1000	1240	1510	1810

The weight of the beam need be considered only when the ratio of span to depth of beam is large. For concentrated loads at the middle of a beam, divide table safe loads by 2. For allowable fiber stresses other than 1000 lb., multiply safe loads of table by the (allowable fiber stress) ÷ 1000.

Safe Loads Based on Horizontal Shear. Safe loads uniformly distributed, for rectangular beams one inch wide, with assumed allowable horizontal shearing stress along the grain = 100 lb. per sq. in., are given in the following table:

Nominal depth, in.	4	6	8	10	12	14	16	18	20	22	24
Safe load, lb.	480	750	1000	1260	1530	1800	2060	2330	2600	2870	3130

The weight of the beam need not be considered, because when the shearing strength of a beam governs the design the weight of the beam is negligible. For short and deep beams these values will generally govern rather than those in the preceding table.

For allowable horizontal shearing stress other than 100 lb., multiply safe loads of table by (allowable shearing stress) ÷ 100.

Depths given in these tables are nominal (i.e., they are the sizes used for ordering timbers in the lumber trade), but the safe loads are computed for the corresponding standard depths (i.e., the probable actual depths). The total safe load is found by multiplying the load for a beam one inch wide by the standard or actual width in inches.

Notched Beams. Beams are frequently notched at their ends in order to cut down clearance and to bring the top surfaces of adjacent beams to the same level. They are also occasionally notched at intermediate points to clear other parts of a structure, such as the top lateral rods of a highway deck bridge. When notched at the ends, the strength of the beam is materially decreased if the depth of the notch is a large percentage of the depth. For beams whose depth is great compared to their length, the effect of notching is greatest because such beams fail in horizontal shear. It is suggested that in designing beams with end notches the actual end depth be used to determine the horizontal shearing stresses, or the safe load based upon horizontal shear, and that only 80% of the allowed horizontal shearing stresses given in Art. 1 be used.

When the notches are at or near the middle of the beam, the depth of the beam, in determining the extreme fiber stresses or the safe loads based on same, should be regarded as the net depth. It is further recommended that only 80% of the allowed fiber stresses and the safe loads based thereon be used. Report of the Chief of Engineers, U. S. A., 1883, p. 1496, contains valuable tests upon the subject of notches.

Knots. Beams containing large or loose knots should not be used, unless an allowance is made for the knots. Beams containing knots should be placed with the knots in the compression side of the beam, that is, for simple beams the knots should be at the top.

The Deflection formula for a beam 1 in. wide under uniform load may be written $f = 270\ (l/d)^3W/E$. The following table gives values of $C = 270\ (l/d)^3$ for various spans and widths. In this formula, l is expressed in feet, f and d in inches, W in pounds of total load on beam divided by standard or actual width in inches.

Values of C in the Formula, Deflection = CW/E for Beams One Inch Wide and Uniformly Loaded

Span, ft.	Nominal depth of beam, in.										
	4	6	8	10	12	14	16	18	20	22	24
6	1 220	330	140	68	38	24	16	.....	.....	.....	....
8	2 900	780	330	160	91	56	37	.....	.....	.....	....
10	5 670	1 520	640	320	180	110	73	50	36	27	21
12	9 790	2 620	1100	540	310	190	130	87	63	47	36
14	15 550	4 160	1760	870	490	300	200	140	100	75	57
16	23 200	6 210	2620	1290	730	450	300	210	150	110	85
18	......	8 840	3720	1840	1040	640	420	290	210	160	120
20	......	12 130	5110	2520	1420	880	580	400	290	220	170
22	......	......	6800	3360	1890	1170	770	540	390	290	220
24	......	......	8840	4360	2460	1520	1000	700	500	380	290
26	......	......	......	5540	3120	1930	1280	880	640	480	370
28	......	......	......	6910	3890	2410	1590	1100	800	600	460
30	......	......	......	8500	4790	2960	1960	1360	980	730	560
32	......	......	......	......	......	......	2380	1650	1190	890	680
34	......	......	......	......	......	......	2850	1980	1430	1070	820
36	......	......	......	......	......	......	3380	2350	1700	1270	970

The depths given in this table are nominal, but the tabulated values are computed for the corresponding standard sizes.

For a concentrated load at the middle of a beam, multiply the tabulated value of C by 1.6.

To Design a Beam for a span of 15 ft. under a uniform load of 4000 lb., deflection not to exceed 1/200 × span, the beam to be of southern pine, common grade. Art. 1 gives an extreme allowable fiber stress of 1200 lb. per sq. in. for southern pine. Using the first table of this article, assuming a nominal depth of 12 in., the safe load is 980 lb., but as this table is for a fiber stress of 1000 lb. per sq. in. the safe load for a pine beam 1 in. wide = 980 × 1200/1000 = 1180 lb. As beams come in widths which are generally in multiples of 1 in., select a beam 4 in. wide, nominal, or 3-5/8 in. standard.

The allowable horizontal shearing stress, with the grain, for southern pine is 88 lb. per sq. in. The preceding tabulation, based on shear, gives for a pine beam 1 in. wide and 12 in. nominal depth 1530 × 88/100 = 1350 lb. For a beam 4 in. wide, nominal size, the safe load = 3-5/8 × 1350 = 4900 lb. Therefore the dimensions of the beam are governed by flexure rather than by shear.

From Art. 1, $E = 1\,600\,000$. For the given span and depth $C = 600$, $W = 4000/3.625 = 1100$, and $f = 600 \times 1100/1\,600\,000 = 0.41$ in., which is less than 1/200 × span, and hence satisfactory. The safe compression across the grain is 325 lb. per sq. in. and using one-half of 325 lb. the area of the end bearing must be at least 4000/(2 × 163) = 12.3 sq. in. Since the beam is 4 in. wide nominal, or 3-5/8 in. actual, the length of bearing required is 3-1/2 in.

If the ratio of length of unbraced span to width exceeds 20, the safe loads of the table must be reduced by the percentages given in the first part of this article. For example, the ratio = 180/3.625 = 50. Therefore, if there is no lateral support, the safe load = 1180 × 0.50 × 3-5/8 = 2140. If this beam is detached so that it has no lateral support in its length, the nominal width should be increased to 8 in. (actual width 7-1/2 in.). If the beam supports joists or stringers or a floor fastened to it, it may be regarded as braced laterally and the 4-in. width may be used. A timber beam which is not braced at one or more points very seldom occurs in practice.

Columns. The following formula adopted by the American Railway Engineering Association, 1926, is recommended for columns whose ratio of unsupported length to least dimension is greater than 11 and less than K:

$$\frac{P}{A} = S\left[1 - \frac{1}{3}\left(\frac{l}{Kd}\right)^4\right]$$

in which P/A = safe working stress for the column, S = safe compressive stress parallel to the grain (Art. 1), l = unsupported length of column in inches, d = least side of column in inches. K is a constant which varies with the variety of timber, and is given in the table of Art. 1.

The above formula is used for intermediate columns whose l/d is greater than 11. When l/d is 11 or less, the safe load may be found by multiplying the column area by the allowable stress for short columns given in Art. 1. When l/d is greater than K, the following formula (based on Euler's formula for long columns) is used:

$$\frac{P}{A} = \frac{0.274\,E}{(l/d)^2}$$

Column **slenderness** should be limited to $l/d = 50$.

Columns abutting against side-grain lumber, at top or bottom, must usually be provided with caps or shoes of hardwood or metal, otherwise the pressure upon the side-grain will be excessive.

Columns may be designed by the above formulas, or by the following table which is based on them.

Safe Stresses for Square and Rectangular Columns

Select Grade Timber

(Working stresses in pounds per square inch)

Kind of timber	Ratio of length to least dimension										
	11	12	14	16	18	20	25	30	35	40	50
Cedar, Western red	700	686	674	652	625	590	438	304	224	171	110
White	550	539	531	515	495	467	350	244	179	137	88
Port Orford	900	882	859	831	790	740	525	365	268	206	132
Alaska	800	785	775	750	725	685	525	365	268	206	132
Chestnut	800	782	760	734	691	638	438	304	224	171	110
Cypress, Southern	1100	1060	1025	984	910	811	525	365	268	206	132
Douglas fir, Coast region	1175	1150	1125	1090	1038	976	700	487	358	274	175
Rocky Mountain	800	785	775	750	725	685	525	365	268	206	132
Fir, Balsam	700	686	674	652	625	590	438	304	224	171	110
Golden, white, silver	700	690	680	660	640	610	483	335	246	188	121
Hemlock, Western	900	885	873	848	822	781	616	426	313	240	153
Eastern	700	690	680	660	640	610	483	335	246	188	121
Larch, Western	1100	1070	1045	1000	935	847	568	395	291	233	142
Oak, red and white	1000	980	968	938	906	856	656	457	336	257	164
Pine, Southern yellow	1175	1150	1125	1090	1038	976	700	487	358	274	175
White, sugar, Western white, Western yellow	750	735	715	692	658	617	438	304	224	171	110
Norway	800	785	775	750	725	685	525	365	268	206	132
Redwood	1000	975	950	912	854	778	525	365	268	206	132
Spruce, red, white, Sitka	800	785	775	750	725	685	525	365	268	206	132
Englemann	600	588	572	554	526	494	350	244	179	137	88
Tamarack, Eastern	1000	980	950	920	872	813	568	395	291	233	142

Common Grade Timber

Kind of timber	Ratio of length to least dimension										
	11	12	14	16	18	20	25	30	35	40	50
Cedar, Western red	560	554	548	538	522	505	417	304	224	175	110
White	440	435	430	423	412	398	332	244	179	137	88
Port Orford	720	713	700	685	665	642	521	365	268	206	132
Alaska	640	632	627	615	600	582	500	365	268	206	132
Chestnut	640	629	620	603	585	556	438	304	224	171	110
Cypress, Southern	880	861	843	816	780	732	525	365	268	206	132
Douglas fir, Coast region	880	870	862	845	821	795	665	487	358	274	175
Rocky Mountain	640	632	627	615	600	582	500	365	268	206	132
Fir, Balsam	560	554	548	538	522	505	417	304	224	171	110
Golden, silver, white	560	554	549	540	528	512	449	335	246	188	121
Hemlock, Western	720	713	706	693	680	657	575	426	313	240	153
Eastern	560	554	549	540	528	512	449	335	246	188	121
Larch, Western	880	861	850	825	793	750	568	395	291	233	142
Oak, red and white	800	790	783	768	750	726	625	457	336	257	164
Pine, Southern yellow	880	870	862	845	821	795	665	487	358	274	175
White, sugar, Western white, Western yellow	600	594	583	572	553	535	434	304	224	171	110
Norway	640	632	627	615	600	582	500	365	268	206	132
Redwood	800	785	775	750	725	685	525	365	268	206	132
Spruce, red, white, Sitka	640	632	627	615	600	582	500	365	268	206	132
Englemann	480	475	466	457	443	428	347	244	179	137	88
Tamarack, Eastern	800	790	777	758	735	706	568	395	291	233	142

The heavy line marks the division between intermediate and long columns.

The table can be used for other than rectangular columns, the l/d being equivalent to 0.289 l/r, where r is the least radius of gyration of the section.

The stresses are given for dry locations. In locations where the timbers will be exposed to the weather, these stresses should be reduced 25%.

For columns with l/d intermediate between those given in the table the safe load may be found by interpolation.

Example. Design a Douglas fir (Coast region) column (common grade) 13 ft. long to support a load of 50 000 lb. To get an approximate size, assume that the column is a short one. The allowable stress from Art. 1 is 880 lb. per sq. in. for common grade, dry location, so that the section needed is 50 000/880 = 56.8 sq. in., or a 7.5 × 7.5-in. square column. The l/d for this trial section is 13 × 12/7.5 = 20.8 which, being greater than 11, indicates that the reduced stresses of the column table must be used. From the above table, we find for Douglas fir (Coast) and an l/d = 20.8, a safe stress allowed of 775 lb. per sq. in. The area of the section needed is then 50 000/775 = 65 sq. in. The nearest commercial size which will satisfy this requirement is an 8 × 10-in. nominal or 7-1/2 × 9-1/2 actual, giving 71 sq. in. of section.

4. Fastenings

Nails for ordinary structural work are of two general types, cut nails of rectangular cross-section tapering from head to point and wire nails of circular cross-section without taper. Both types are usually of steel. Cut and wire nails of larger cross-section than common nails are called spikes. Boat spikes, used for very heavy timber work, have larger cross-sections than ordinary spikes. A clinch nail is similar to a cut nail, but is so made that it may be clinched or bent down, so as better to withstand shock or vibration.

Screws are preferable to nails for permanent work and work subject to vibration or shock, when a joint or connection of great strength is desired, and for lumber which splits readily under the driving of nails.

The common sizes of cut and wire nails and spikes and the approximate number per pound are given in the following tables. The numbers per pound are approximate, as different manufacturers use slightly different standards of cross-section, taper, and head. Nails are usually designated in size by the "penny" system. For instance, a 3-in. nail is called a ten-penny nail (written 10d.), probably because such nails once were sold at ten pence per 100. A keg of nails or spikes weighs 100 lb.

Square Steel Boat Spikes

Approximate number in a keg of 200 lb.

Size, in.	Length of spikes, in.											
	3	4	5	6	7	8	9	10	11	12	13	14
5/8	.....	.....	.....	330	292	260	234	210	194	178	166	152
1/2	.....	648	512	434	384	338	294	262	236	212	198	186
7/16	1082	812	692	572	484	414	380	348	318	292	268	246
3/8	1476	1110	920	748	634	554	500	458	416	378	348	320
5/16	2176	1646	1386	1138	974	858	778	708	648	592	.....	.....
1/4	3400	2600	2040	1748	1456	1294	.....	.....	.....	.....	.....	.....

Screws for structural work are divided into two general types, wood screws and lag screws. Lag screws (Fig. 10q, p. 808) are used for heavy timber work. Lag screws are often used instead of bolts where it is difficult or impossible to place through bolts. They are also used for trestles, especially to attach guard-rails to ties, as they require less maintenance than bolts and render dapping of the guard-rails unnecessary.

For a wood screw, a hole should be bored smaller than the diameter of the shank and about one-half its depth, before inserting the screw. For a lag screw, a hole should be bored slightly smaller than the diameter of the shank. Wood screws and lag screws should not be hammer-driven.

Strength of Nails. Nails are seldom used in permanent structural work where the strain is parallel to the length of the nail, but for scaffolding such a fastening is not uncommon. The following formulas for safe loads on a nail

Nails and Spikes

Size	Steel cut nails and spikes: Length, in.	Approximate number per pound: Common nails	Spikes	Clinch nails	Size	Steel wire nails and spikes: Length, in.	Approximate number per pound: Common nails	Spikes	Clinch nails
2d.	1	740	....	400	2d.	1	876	....	710
3d.	1-1/4	460	....	260	3d.	1-1/4	568	....	429
4d.	1-1/2	280	....	180	4d.	1-1/2	316	....	274
5d.	1-3/4	210	....	125	5d.	1-3/4	271	....	235
6d.	2	160	....	100	6d.	2	181	....	157
7d.	2-1/4	120	....	80	7d.	2-1/4	161	....	139
8d.	2-1/2	88	....	68	8d.	2-1/2	106	....	99
9d.	2-3/4	73	....	52	9d.	2-3/4	96	....	90
10d.	3	60	....	48	10d.	3	69	41	69
12d.	3-1/4	46	....	40	12d.	3-1/4	63	38	62
16d.	3-1/2	33	17	34	16d.	3-1/2	49	30	49
20d.	4	23	14	24	20d.	4	31	23	37
25d.	4-1/4	20	12	....	30d.	4-1/2	24	17	....
30d.	4-1/2	16	10	....	40d.	5	18	13	....
40d.	5	12	9	....	50d.	5-1/2	14	10	....
50d.	5-1/2	10	8	....	60d.	6	11	9	....
60d.	6	8	7	....	....	7	....	7	....
....	6-1/2	....	6	....	....	8	....	4	....
....	7	....	5	....	....	9	....	3-1/2	....
....	....	....	....	....	....	10	....	3	....
....	....	....	....	....	....	12	....	2-1/2	....

Wood Screws*

Length, in.	Gage numbers
3	10 to 26
3-1/2	10 to 26
4	12 to 26
4-1/2	16 to 26
5	18 to 28
6	20 to 30

Approximate Screw Gages†

Number of gage	Diameter, in.
6	0.14
10	0.19
15	0.26
20	0.32
25	0.39
30	0.45

Lag Screws‡

Length, in.	Diameter, in.
3	5/15 to 7/8
3-1/2	5/16 to 1
4	5/16 to 1
4-1/2	5/16 to 1
5	5/16 to 1-1/8
5-1/2	5/16 to 1-1/8
6	5/16 to 1-1/4
6-1/2	7/16 to 1-1/4
7	7/16 to 1-1/4
7-1/2	7/16 to 1-1/4
8	7/16 to 1-1/2
9	1/2 to 1-1/2
10	1/2 to 1-1/2
11	9/16 to 1-1/2
12	9/16 to 1-1/2

* Flat heads are commonly used.

† Other gages may be approximated by proportion.

‡ Lag screws may be obtained with hexagonal or square heads.

were determined from a comparison of tests on the resistance to pulling out. When one piece of timber is nailed to another, the penetration of the nail into the second timber should not be less than one-half the length of the nail. The formulas are given for a penetration of one-half the length of the nail, when driven across the grain.

	Cut nails	Wire nails
In Southern pine............	$P = 6\text{-}1/2 \times$ d.	$P = 4 \times$ d.
In Douglas fir...............	$P = 6\text{-}1/2 \times$ d.	$P = 4 \times$ d.
In White pine...............	$P = 4 \times$ d.	$P = 2\text{-}1/2 \times$ d.
In White oak...............	$P = 13 \times$ d.	$P = 8 \times$ d.

In this table, d. is the number of penny weight of nail, and P is safe load in pounds. For example, a 60d. wire nail in Southern pine, if driven perpendicular to the grain, has a safe resistance to pulling out of $4 \times 60 = 240$ lb.

If driven parallel to the grain use 60% of the above values. If subject to shock, the resistance of the nail to pulling out is very low as was shown by W. C. Riddick in 1890 and as may be observed when a nailed plank is struck by a sledge in the demolition of timber structures. Nails should never be depended upon for important joints, in either temporary or permanent structures.

The Safe Lateral Resistance of Nails driven across the grain may be approximated by the following formulas, in which P is in pounds when d. is the penny weight.

Cut nails in oak, $P = 12$ d.
Wire nails in oak, $P = 10\text{-}1/2$ d.
Cut nails in Southern pine and Douglas fir, $P = 7\text{-}1/2$ d.
Wire nails in Southern pine and Douglas fir, $P = 7$ d.

Nails should be driven at right angles to the contact surfaces. When nails are driven parallel to the grain, use 60% of the tabulated values. For shock see preceding paragraph. The nails should not be driven closer to each other than about one-half their length nor closer to the edge of the timber than about one-quarter their length. It is generally impractical to develop the full strength of timber by the use of nails.

Safe Resistance to Pulling Out, in Pounds per Linear Inch of Threaded Length of Wood Screws when Inserted across the Grain

Kind of wood	Gage number								
	4	8	10	12	14	16	18	20	24
Cypress................	40	50	60	70	80	90	100	100	100
Southern pine...........	90	120	150	165	180	195	210	220	220
White oak..............	100	140	180	210	240	250	260	260	260

For wood screws inserted with the grain, use 75% of the values in the table.

Large diameters should be avoided for screws 3 in. and under in length. The screw with the greater length and smaller diameter should be chosen when practicable. For wood screws holes should be bored about 70% of the core diameter of the screw in soft woods and 90% in hard woods. Lubricants may be used with screws without any great loss in holding power.

The Lateral Resistance of Screws is much higher than that of nails. The accompanying table gives safe working values for lateral resistance of common wood screws, assuming that the screw is imbedded in the main piece of timber

Safe Lateral Resistance of Wood Screws in Southern Pine and Douglas Fir

Gage of screw	Safe lateral resistance in pounds	Gage of screw	Safe lateral resistance in pounds	Gage of screw	Safe lateral resistance in pounds
6	82	14	256	22	527
8	116	16	314	24	612
10	156	18	381	26	700
12	204	20	451		

approximately 6/10 the length of the screw. (See Cornell Civil Engineer, Nov., 1913.)

Safe Resistance to Pulling Out of Lag Screws when Inserted across the Grain

In pounds per linear inch of thread

Kind of wood	Diameter of lag screw in inches				
	1/2	5/8	3/4	7/8	1
Southern pine	315	390	470	550	630
Douglas fir	315	390	470	550	630
White pine	195	245	295	345	390
White oak	470	590	700	825	940

The Lateral Resistance of Lag Screws was determined in a series of tests by H. D. Dewell, which are described in *Engineering News*, Vol. 76. He recommends an allowance for a 7/8 × 5-in. lag screw of 1050 lb. for all-timber joints and 1200 lb. for a joint in which a metal plate is fastened to timber, and 900 lb. and 1030 lb. for a 3/4 × 4-1/2-in. lag screw under similar conditions.

A Dowel or dowel pin is a pin of wood or steel extending into adjacent parts of a structure (Figs. 3 and 4) to keep them from being displaced. Wood

Fig. 3

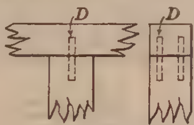


Fig. 4

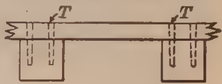


Fig. 5

dowels are also called tree nails. Steel dowels are troublesome in renewals. Dowel pins are usually not computed to transmit stress. They should be made a tight fit and extend at least 4 diameters into side-grain and 6 diameters into end-grain timbers. It is better to use two rather than one to a joint. Tree nails are occasionally computed to transmit stress, the strength of the connection being regarded as 1/2 the shearing strength of the cross-section of the nail. They are often substituted for bolts in lumber work below high water. In Figs. 3 and 4 the dowels or tree nails are marked *D*. In Fig. 5, the tree nails fastening grillage planks are marked *T*.

Drift Pins or drift bolts are long pins of steel or wood, usually the former, driven through the timber and into an adjacent timber to hold them together

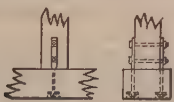


Fig. 6

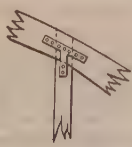


Fig. 7

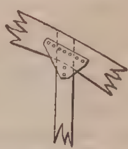


Fig. 8

and to transmit stress. Drift pins are made with or without heads. Round pins are preferable and whether round or square, a hole should be driven having a diameter from 1/16 to 1/8 in. less than the diameter of the bolt and for the full length of the bolt. Boat spikes may be used for drift pins.

Safe Resistance to Pulling Out, in Pounds per Linear Inch, of Drift Bolts when Driven across the Grain

Kind of wood	Diameter in inches				
	1/2	5/8	3/4	7/8	1
Southern pine	160	200	240	275	315
Douglas fir	160	200	240	275	315
White pine	95	120	140	170	195
White oak	240	300	350	410	470

For drift bolts driven with the grain, use 50% of the tabular values.

Common Bolts (see Sect. 7, Art. 49) are generally preferable to nails, screws or drift pins. For holding power of bolts see Art. 6. **Strap Bolts** are bolts with a strap at one end. They are used for the purpose of fastening the end of one timber to the side of another timber, Fig. 6. Common bolts are often used with steel straps as in Fig. 7, and steel gussets or plates as in Fig. 8.

Washers are metal disks placed under the heads and nuts of bolts to distribute the pressure upon the wood over such an area that the unit compression will not exceed a safe limit. Five types of washers are used in timber construction: (1) cast-iron ogee (Fig. 9*a*), (2) cast-iron ribbed (Fig. 9*d*), (3) malleable iron (Fig. 9*g*), (4) circular pressed steel (Fig. 9*b*), and (5) square plate washers (Fig. 9*c*). Washers of standard sizes and shapes may be bought

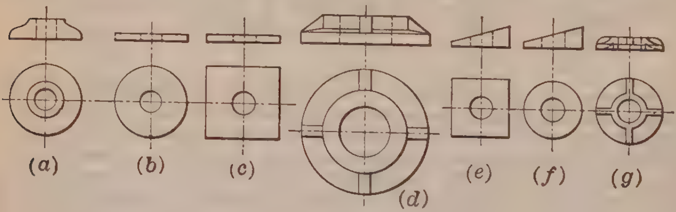


Fig. 9. Types of Metal Washers

from the manufacturers. For bolts over 2 in. in diameter, special cast-iron washers (Fig. 9*d*) are often used, though this type is very expensive unless a large number of the same size are required. Beveled cast-iron washers (Fig. 9*e*, *f*), for cases in which the axis of the bolt makes an angle with the bearing surface, may also be purchased from the manufacturers.

When the tension in a bolt is at right angles to the base of the washer, the washer should be designed so that the compression upon the wood, due to tension in the bolt, will not exceed by more than 50% the allowable compressive unit stresses of Art. 1. The malleable iron washer does not have its full area of base available for bearing; therefore when used with a bolt in tension 60% of its nominal area should be taken. In joints where the bolt acts only in shear and bending, smaller washers, such as the malleable washer, may be used. However, if there is any chance that the bolt may be brought into tension by washer tightening in maintenance work, square plate washers or special cast-iron washers should be used. The circular cut or pressed steel washer should not generally be used except between metal and metal.

Wrought-iron or Steel-plate Round Washers

Diameter in inches	Size of hole in inches	Approximate number in a pound	Area of washer, sq. in.
1-3/8	9/16	27.0	1.3
1-1/2	5/8	22.0	1.5
1-3/4	11/16	13.0	2.0
2	13/16	10.0	2.6
2-1/4	15/16	8.6	3.3
2-1/2	1-1/16	6.2	4.0
2-3/4	1-1/4	5.2	4.5
3	1-3/8	4.0	5.6
3-1/4	1-1/2	2.8	6.5
3-1/2	1-5/8	2.5	7.6
3-3/4	1-3/4	2.4	8.6
4	1-7/8	2.2	9.8
4-1/4	2	1.9	11.0
4-1/2	2-1/8	1.7	12.4
4-3/4	2-3/8	1.0	13.3
5	2-5/8	1.0	14.2

The size of hole in washer is 1/16 in. larger than the diameter of the bolt up to and including bolts 1 in. in diameter. For larger bolts the hole is 1/8 in. larger.

Wrought-iron or Steel-plate Square Washers

Width in inches	Size of hole in inches	Approximate number in a pound	Area of washer, sq. in.
2	9/16	5.0	3.8
2-1/4	3/4	3.2	4.6
2-1/2	7/8	2.5	5.7
3	1	1.7	8.2
3-1/2	1-1/8	.9	11.3
4	1-1/8	.7	15.0
4-1/2	1-1/4	.5	19.0
5	1-1/2	.4	23.2
6	1-5/8	.3	34.0

For size of bolts see round plate washers.

Cast-iron Washers

Diameter in inches	Size of hole in inches	Approximate number in a pound	Area of washer, sq. in.
2-1/4	5/8	3.3	3.8
2-3/4	3/4	2.5	5.5
3-1/4	7/8	1.4	7.9
3-3/4	1	1.0	10.1
4	1-1/8	.5	11.6
4-1/2	1-1/4	.4	14.7
5	1-3/8	.3	18.2
6	1-3/4	.2	25.9

The size of hole in washer is 1/8 in. larger than the diameter of the bolt up to and including bolts 1-1/4 in. in diameter. For larger bolts the hole is 1/4 in. larger.

Where the bolt makes an angle with the bearing surface, thetimber may be notched instead of using a beveled washer, provided the notch does not reduce the timber section beyond a safe limit. Bearing in this case would be inclined to the direction of the fiber, and greater unit stresses can be taken advantage of (see Art. 2). Where notching is impracticable, especially if large washers are required, a combination of a large flat washer and a small beveled washer may be economical.

To determine the size of washer required, divide the tension in the bolt by the safe unit compression of the wood and add the area of the hole of the washer, which gives the gross area required. From this the outside dimensions are computed. Its thickness, if of wrought iron or steel, should be at least 1/8 its width or diameter. If of cast iron (Fig. 9*a*), the thickness should be not less than about 1/4 its diameter. The square plate washer should have a thickness of 1/2 the diameter of the bolt plus 1/16 in. If of the type shown in Fig. 9*d*, it may be designed similar to cast-iron shoes, the unit pressure upon the bottom of the washer being the allowable compression upon the wood. If the tension in the bolt is small use standard washers.

If the washer is bevelled (Fig. 10*n*), the pull on the washer should be resolved into the components shown by the arrows and the pressure upon the wood in either direction P or N should not exceed the allowed safe limits of Art. 1. The washer should be so designed that the line of pull and the line of resistance P and N meet in a point.

Common Joints are shown in Fig. 10. *a* is a butt joint. *b* is a bevel joint. *c* is housing. *d* is single notching. *e* is double notching. *f* is halving. *g* is beveled halving. *h* is cogging. *i* is dovetail halving. *j* is a dapped joint. *k* is a double-step dapped joint. *l* is a head block. *m* is a lap joint. *p* is a bird's-mouth joint. *q* is a lag screw.

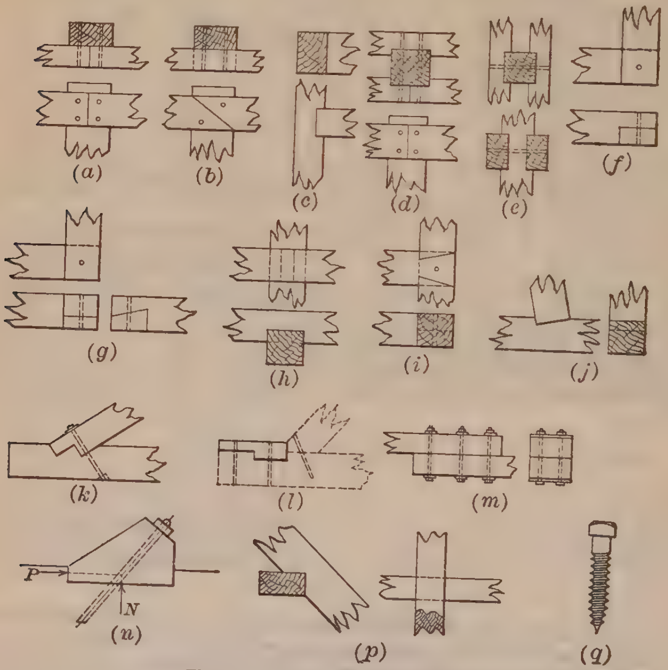


Fig. 10. Common Timber Joints

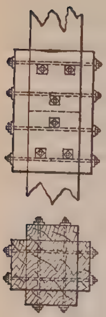


Fig. 11

5. Compression Joints

The Butt Joint, shown in Fig. 11, is used to splice columns and struts. The ends of adjacent sections should be sawed at right angles, and in important work the ends should be dressed to an even surface. If the contact surfaces or ends are not flush and true, the column is loaded eccentrically. This joint need not be computed, but braces should hold the lower or upper section of the joint to correct alinement, so as to prevent buckling of the column at the joint.

A Column with beams framed into its sides (Fig. 12*a*) must not be reduced at section *A*, to such an extent that the unit compression exceeds the safe end-bearing compression. The beveled dap may not give sufficient end bearing for the beams, in which case the beam bearings may be increased as in Fig.

12a, by use of angles, by bolted side blocks (Fig. 12b) or by use of side and bent plates (Fig. 12c).

Example. (Fig. 12a) Girder 10 × 10 in., columns 10 × 12 in., both Southern pine, common grade. Location is dry. Load on upper column = 70 000 lb. End-beam reactions each = 9500 lb. The allowed compression across the grain = 325 lb. per sq. in. It is suggested that only 50% of the allowed unit compression be used, as the flexure of the beam causes greater compression at the front of the support than at the back. The minimum bearing area of the beams = 9500/163 = 58.2 sq. in. As the beam is 10 in. wide, the length of bearing should be 6 in. As the load on the upper

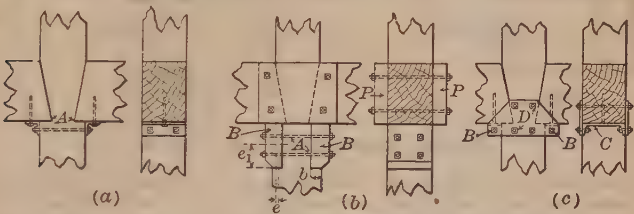


Fig. 12. Struts Framed into Beams

section of column is 70 000 lb. and the safe bearing on end grain for short columns is 880 lb. per sq. in., the minimum reduced area of the column = 70 000/880 = 80 sq. in. Therefore the length of bearing on the column for each beam may be 2 in. The additional length of end bearing needed for each beam = 6 − 2 = 4 in. This may be provided by using 4 × 4 × 3/8-in. angles as in Fig. 12a. The load on each angle will be 9500 × 4/6 = 6350 lb. The bolts are in single shear.

Bolts should be selected of such diameter that the shear will not exceed safe limits and so that the compression brought upon the wood by the bolts will not exceed the allowed stresses of Art. 1. At least two bolts should be used. The shear on the ends of each bolt = 6350/2 = 3175 lb. 3/4-in. bolts will be safe against shearing and bearing of the angles on the bolts. As the bolts bear on the end grain of the wood, the safe bearing for the two bolts = 2 × 3/4 × 12 × 880 = 15 850 lb., which is more than the total load on the two angles.

The joint of Fig. 12b should be designed similarly to Fig. 12a. The load on block B is transmitted to its end b. The lines of action of the pressure at the top and bottom of the block B are not coincident. The eccentricity e causes a couple whose moment equals the load multiplied by e. This couple is resisted by a couple consisting of the bolts acting in tension and the adjacent wood acting in compression parallel to the length of the bolts. The arm of this resisting couple may be considered as equal to 2/3 e_1 in which e_1 equals the distance from a point halfway between the bolts to the shoulder b. The washers should be designed to develop the tension in the bolts without stressing the timber higher than allowed in Art. 1.

In the joint of Fig. 12c, the end-bearing area of the beams is increased by bent plates or channels marked C. The load on these bent plates is transmitted to the side plates through bolts B, and the load on the side plates is transmitted to the post by bolts D.

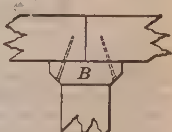


Fig. 13

Joints at Top and Bottom of Columns. When beams rest upon the top of a column, a hardwood bolster marked B, Fig. 13, may be used to decrease the side-grain compression upon the ends of the beams. In this case the unit side-grain compression may be excessive upon the bottom of the bolster. To decrease this compression, the tops of the columns may be widened as in Fig. 12b. Cast-iron caps of the type shown in Fig. 14, a, b, are often used at the tops of columns of buildings. The compression upon the side grain of caps and sills, at the top and bottom of columns, will generally be excessive if the columns be designed to take the maximum safe loads and the top and bottom of the columns are not

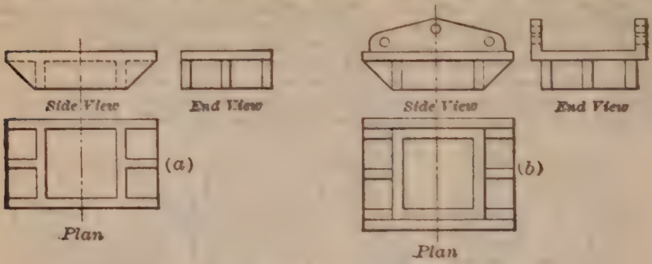


Fig. 14. Metal Caps for Timber Columns

widened similar to Figs. 12, 13. In trestles it is often cheaper to design the columns for loads which, without widening the top and bottom, do not produce excessive side-grain compression upon the caps and sills. Joints like Fig. 12*b* exposed to the elements hold water and rot quickly.

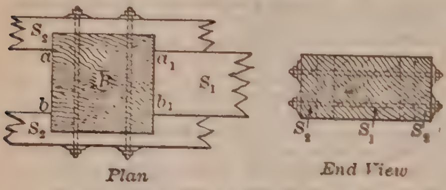


Fig. 15. Dapped Compression Joint

Dapped Compression Joints (Fig. 15) may be designed as follows: Assume that a load of 80 000 lb. is to be transferred from the middle 12-in. × 12-in. Douglas fir (common grade) timber, marked S_1, to two outside Douglas fir timbers, marked S_2. The length of the block *B* should be such that it will not shear along the lines aa_1 and bb_1. Using a white oak block which has a shearing value with the grain of 200 lb. per sq. in. (for joints), and assuming that the depth of the side pieces S_2 will be 12 in. to correspond with the middle piece S_1, the length of the block must be at least $80\,000/(2 \times 12 \times 200) = 17$ in. The length of bearing in each side timber, with the safe end-bearing compression on Douglas fir = 880 lb. per sq. in., must be at least

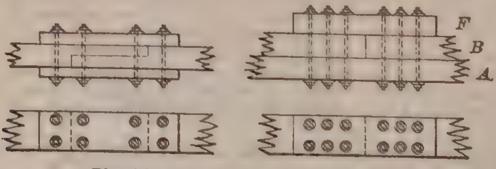


Fig. 16 Fig. 17

$80\,000/(2 \times 12 \times 880) = 3.8$ in. A sufficient number of bolts with standard washers should be used to clamp the wood closely together and prevent displacement of the block. The outside timbers S_2 may have the same combined cross-section as the middle timber S_1, unless l/d for S_2 is greater than l/d for S_1, in which l/d is the ratio of the length of the timbers to their least width.

Fig. 16 shows a half-lap compression joint with side or fishplates. Fig. 17 shows a compression joint in which the timber *A* is continuous and timber *B* is spliced. *F* is a fishplate and *A* acts as a fishplate in splicing *B*.

6. Tension Joints

Plain Fishplate Joints (Figs. 19 and 20) can be economically used for small timbers or large timbers under small unit stresses. The fishplates marked *F* may be of wood or steel. The design of a joint with metal plates (Fig. 20) is similar to that for wood plates, excepting that the strength of the bolts in shear and bearing on the metal must be carefully considered. Round bolts are generally used and are the only ones considered in what follows. When planks less than 3 in. thick are joined by bolts they may be designed without considering the bending of the bolt.

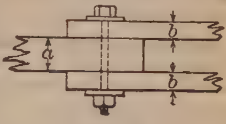


Fig. 18

The following table may be used in determining the safe loads upon bolts of plain timber fishplated joints. In this class of joints, the bolts are not in tension, and therefore standard washers, preferably of cast iron, may be used. The bolts should be taut so as to bring the pieces tightly together, to prevent

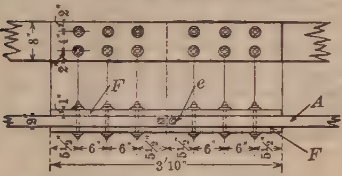


Fig. 19

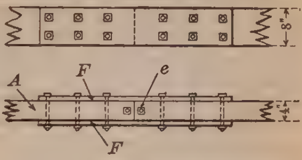


Fig 20

buckling of the side plates and to exclude moisture. Both values given by the table should be considered and the lesser safe load be used.

Safe Bearings and Uniform Loads for Bolts

Diameter of bolt in inches	Safe bearing load, lb. per lin. in.		Safe load in pounds on bolt for dimension in inches as shown in Fig. 18					
	End grain	Side grain	a = 2 b = 1	a = 3 b = 1-1/2	a = 4 b = 2	a = 6 b = 3	a = 8 b = 4	a = 10 b = 5
1/2	500	50	1 100	740	550	370	280	220
5/8	625	63	2 160	1 440	1080	720	540	430
3/4	750	75	3 720	2 490	1860	1250	930	750
7/8	875	88	5 920	3 940	2960	1970	1470	1180
1	1000	100	8 820	5 880	4410	2940	2210	1760
1-1/8	1125	113	12 580	8 390	6290	4190	3140	2510
1-1/4	1250	125	17 250	11 500	8640	5750	4300	3450

The safe loads per linear inch of bolt shown in the second and third columns are based on a safe compressive stress parallel to grain of 1000 lb. per sq. in., and a safe compressive stress across the grain of 100 lb. per sq. in. For other allowed unit stresses the safe loads may be obtained by proportion.

In a tension joint such as Fig. 18, it is often assumed that the pressure of the bolt against the timber is uniformly distributed along the length of the bolt. However, this is not correct, as the unit pressure will be greatest at the point of contact between the center timber and the outside timbers. It is therefore recommended for design that the safe compressive stress of the bolt against the timber be taken as 75% of the working stresses given in table of Art. 1, where the pressure of the bolt on the timber is assumed as uniformly distributed.

The safe loads on the bolt in the table are for the entire bolt, based upon an extreme fiber stress in bending in the bolt of 22 500 lb. per sq. in. For higher or lower assumed safe fiber stresses the safe load can be obtained by proportion.

Design of a Plain Fishplate Joint with wooden fishplates (Fig. 19). The timber *A* to be spliced is a 2-in. × 8-in. southern pine (common grade) tension member, stressed to 1200 lb. per sq. in. of net section (see Art. 1). It will be assumed, in order to get the approximate net section, that two bolts will be necessary in the same plane at right angles to the length of the member and that the diameter of each hole equals 1 in. Therefore the approximate net section equals 16 − 4 = 12 sq. in. The total tension in the member = 1200 × 12 = 14 400 lb. Using the same kind of wood for the fishplates, as for timber, *A*, their gross area must be 16 sq. in. Therefore use two 1-in. × 8-in. fishplates.

The Number of Bolts required may be obtained by assuming any convenient diameter of bolt and finding the safe load on the bolt as a restrained beam, taking the length of the beam as the distance between the center lines of fishplates, and finding also the safe load upon the end grain of the timbers against which the bolts bear. The smaller safe load should be used. Dividing the smaller safe load upon the bolt into the total stress, 14 400 lb., the number of bolts necessary may be obtained. This work is much simplified by using the table on p. 811.

Assuming that a 3/4-in. bolt will be used, the table shows that the safe bearing value is 750 lb. per lin. in., with 1000 lb. per sq. in. working stress. Two inches of the length of the bolt will bear upon the end grain of the tension member, and also of the fishplates. The safe compression with the grain for Southern pine is 880 lb. per sq. in., which should be reduced to 660 lb. per sq. in. as recommended above. Therefore the safe load upon a 3/4-in. bolt, determined by bearing on the timber, is 2 × 750 × 660/1000 = 990 lb.

From the table, the safe load on the bolt as determined by bending is 3720 lb. The strength of the bolt is, therefore, limited by bearing on the timber, and the number of bolts required = 14 400/990 = 14-1/2. As the fishplates are wide, there should be two lines of bolts; if one line were used the splice plates might buckle and the splice would be very long. In order to cut down the number of bolts, use 12 1-in. bolts, which are safe in bearing for a load of 12 × 2 × 660 = 15 800 lb.

The distance between adjacent holes of the same line from each other, and from the end of the tension member and the fishplates, must be such that the unit shearing stress with the grain will not exceed 80% of that allowed in Art. 1. The reduction of 20% is made on account of a splitting action transverse to the line of tension when round bolts are used; for square bolts this reduction need not be made. Bolts tend to shear the wood in two planes tangent to the top and bottom of the bolt. There are 12 1-in. bolts on either side of the joint and the length of the bolt in the tension member and the fishplates is 2 in. The safe shear with the grain is 176 lb. per sq. in. (twice the value in table of allowable stress). Therefore the distance between the bolts must be not less than 14 400/(12 × 2 × 2 × 176 × 0.8) = 2.1 in. Add 1-1/8 in. for the bolt hole, which is assumed as 1/8 in. larger than the bolt, and the spacing of bolts will be not less than 3-1/2 in. The distance from the ends may be 1/2 in. less than between bolt holes, as only half of the bolt hole need be added. (The splice as here computed differs in details from that shown in Fig. 19.)

The bolts marked *e* in Figs. 19 and 20 are inserted in the ends of the main timbers for the purpose of overcoming the tendency of the wood to split under the pressure of the round bolts.

Tabled fishplate joints are commonly used for heavy timber work. They offer the best type of tension joints and should be used in permanent work where practicable.

Design of a tabled plate joint with wooden fishplates (Fig. 21). Southern pine (common grade) will be used in both the tension member, *A*, which is to be spliced and the fishplates. From Art. 1, the safe stresses per square inch for Southern pine are: for tension with the grain, 1200 lb.; for compression with the grain, 880 lb.; for compression across the grain, 325 lb.; and for shear with the grain, 176 lb. The fishplates *F* are to transmit the full tension of the member across the joint, which in this example is 56 000 lb. The size of the tension member is 8 × 10 in.

The tension to be transferred to and carried by one fishplate = 56 000/2 = 28 000 lb. The net area therefore must be 28 000/1200 = 23.3 sq. in. A bolt should be placed at *h*, the end of each timber *A*, to hold it firmly and to keep the fishplates from buckling. Assume all of the bolts in the joint to be 3/4 in. and their holes 7/8 in. The net width of the fishplate is then 10 − 7/8 = 9-1/8 in. Dividing the net area by this net width gives the net depth, thus 23.3/9.125 = 2 56 in. This should be made the same at *f* and *g*. The fishplate receives its tension through the bearing of the shoulders of the tables *TTTT*, which bear against the end grain of the tables of the main tension member *A*. Assume 4 tables in each fishplate, 2 either side of the middle of the joint. Two tables should be used only for small timbers. The net area of each shoulder should therefore be 28 000/(2 × 880) = 15.9 sq. in. Since the tables are the same width as the depth of timber *A*, or 10 in., the depth of the tables should be 15.9/10 = 1.59 in. The total depth of the fishplates must therefore be 2.56 + 1.59 = 4.15 in. The nearest standard size timber is 5 in., which should be used.

The length of the tables on the fishplates is determined by the necessary shearing strength, with the grain, along lines *bb*. Since there are two tables on either side of the

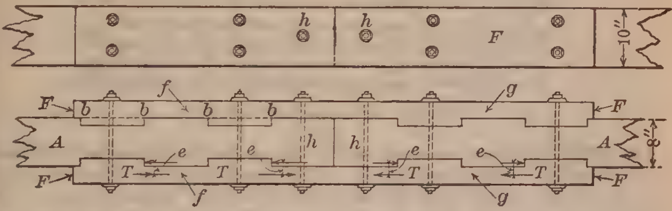


Fig. 21. Tabled Fishplate Joint

joint, the required area for each table is 28 000/(2 × 176) = 79.6 sq. in. If *x* is the length of the table, then 10 *x* is the gross area, from which must be deducted 2 × 0.60, the area of two bolt holes, since two lines of bolts are necessary because of the great width of the fishplates, which might buckle if only one line were used. Then 10 *x* − (2 × 0.60) = 79.6 from which *x* = 8.08 in. The tables of the main timbers *A* are generally made the same length as those of the fishplates. When no bolts go through the tables of the timber, they may be made shorter than those of the fishplates.

The bolts do not transmit the stresses by bending as in the plain fishplate joint, but by tension. The tension in the fishplate and the compression upon the shoulders, shown by arrows in Fig. 21, do not act in the same plane, thus causing a couple, = (28 000/2) *e* for each table, where *e* = one-half the net thickness of the fishplate plus one-half the depth of the shoulder, or 1-3/4 + 3/4 = 2.5 in, using nominal thickness of timber. The moment of the couple for each table = (28 000/2) × 25 = 35 000 in-lb. This moment must be resisted by a couple consisting of tension in the adjacent bolts and compression in the adjacent portion of the fishplate and timber *A* acting in a direction parallel to the bolt. The arm of this couple may be taken at one-half the length of the table, or 4.04 in. The stress in each bolt hence = 35 000/(2 × 4.04) = 4330 lb. A 3/4-in. bolt has a net area of 0.302 sq in., and therefore, based on the allowed tension of 16 000 lb. per sq. in., the strength of the bolt = 4830 lb.; hence 3/4-in. bolts can be used. The bolts should be tightened to their full working strength when constructed, and as this is difficult to determine in practice, an excess of strength is desirable. For the same reason it is suggested that for this type of joint the unit compression across the grain under the washers should not exceed that allowed in Art. 1.

The washers must have a net area of 4330/325 = 13.3 sq. in. A standard cast-iron washer may therefore be used. If a standard washer cannot be used, the gross area should be determined by adding the area of the hole in the washer, and from this gross area the diameter or width of the washer computed.

Fig. 22 shows a modified type of Fig. 21. This joint is costly, but as the bolts are not located at the minimum section of the main timber or fishplates, the area of the bolts need not be deducted in figuring its strength. Fig. 23 shows a common eccentric tabled splice. Timber *A* is continuous in this joint, while *B* is spliced by *C*.

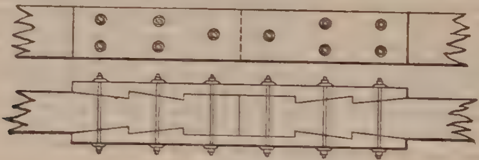


Fig. 22. Scarf Tabled Joint

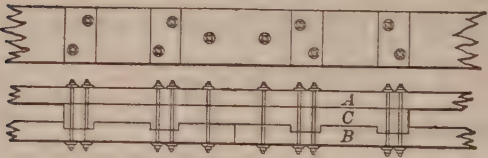


Fig. 23. Eccentric Tabled Joint

Lap Joints (Fig. 24*a*) are often used in temporary work; they are inefficient but quickly and inexpensively made. To compute this joint either in tension or compression assume that the bolts take a bending moment equal to the tension or compression times the distance *e*. The shear in the bolts and the compression on the timber brought by the bolts must also be computed. The table given herewith is convenient in finding the safe resisting moment and

Safe Stresses for Bolts when Used in Timber Connections

Bolt	Area in gross inches	Net area in inches	For tension	For single shear	For resisting moment
1/2	0.196	0.126	2 020	1 960	275
5/8	0.307	0.202	3 230	3 073	540
3/4	0.442	0.302	4 830	4 420	930
7/8	0.601	0.420	6 720	6 010	1 480
1	0.785	0.550	8 800	7 850	2 205
1-1/8	0.994	0.694	11 100	9 940	3 145
1-1/4	1.227	0.893	14 290	12 300	4 310
1-3/8	1.485	1.057	16 910	14 800	5 745
1-1/2	1.767	1.295	20 720	17 700	7 460
1-3/4	2.405	1.746	27 940	24 000	11 835
2	3.142	2.302	36 830	31 400	17 670
2-1/4	3.976	3.023	48 370	39 800	25 155
2-1/2	4.909	3.719	59 510	49 100	34 475
2-3/4	5.940	4.620	73 920	59 400	45 935
3	7.069	5.428	86 850	70 700	59 635

Unit stresses used in this table are: for tension 16 000, for single shear 10 000, and for resisting moment 22 500 lb. per sq. in.

Values for tension are for net area; values for shear for gross area.

safe shear in the bolt, while the safe bearing on the bolts may be obtained from the table on p. 811. The shearing resistance may be increased by the use of hardwood or iron keys, shown in the figure.

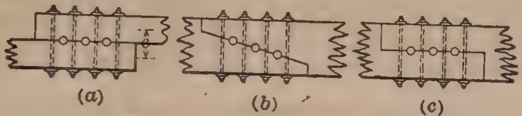


Fig. 24. Scarf Joints

Scarf Joints (Figs. 24, *b*, *c*) are used to transmit tension or compression and sometimes flexure. They are less efficient for tension than fishplated joints, but may be used when the tension is small. They are more generally used to transmit compression stresses with possible small reversal of stress.

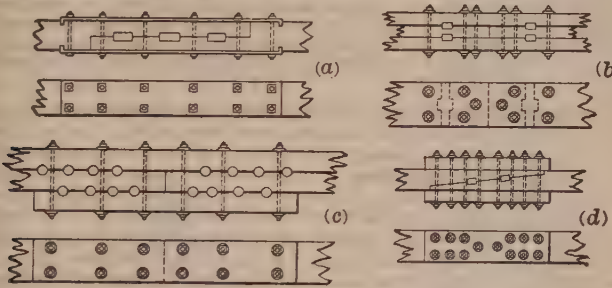


Fig. 25. Four Types of Tension Joints

7. Joints in Flexure

Built-Up Truss Members are occasionally designed for both compression and flexure. Cases of this kind can usually be avoided and should be whenever possible. Compound beams built of two pieces of timber, one above the other (Figs. 26, 27), have been used, but in present practice trussed beams are generally employed if beams of single timbers cannot be obtained of large enough dimensions. The general method of designing a compound beam is given below. When such a beam is in compression, as the top chord of a Howe truss, directly supporting floor beams intermediate between panel points, the direct compression stresses must be considered in determining the cross-sections.

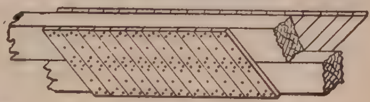


Fig. 26

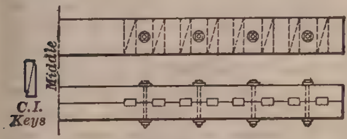


Fig. 27

A Compound Beam should be designed so that the horizontal shear between the adjacent timbers is properly taken care of, so that when the beams bend under the applied loads they will not slide upon each other. If no such provision were made, the strength of the beam would be equal only to the sum of the strengths of each timber acting independently. Compound beams cannot in practice be made as strong as those of single timbers of equal dimensions, and the deflection of the former exceeds the deflection of the latter. For properly designed compound beams of well-seasoned timber carefully framed together and keyed with metal wedges the difference will not be material.

For temporary structures the two timbers may be fastened together by a continuous sheeting of planks, well spiked to the main timbers as in Fig. 26. The thickness of each side piece should be at least 1/8 of the width of the main timbers and the nails should extend at least 1/2 their length into the timbers. On the basis of Kidwell's tests (Trans. Am. Inst. M. E., vol. 27) it is recommended that the strength of this type of compound beam be taken at 70% of a solid beam of equal dimensions.

For permanent work, keys of hardwood or metal should be used and the pieces fastened together by bolts as in Fig. 27. The efficiency of keyed beams on the basis of Kidwell's test will be taken at 75% for hardwood keys and 80% for metal keys. The keys are generally made of wedges or tapered pieces so that they may be made a driving fit; they are also made of flat plates or cast-iron blocks without taper. It is thought that a stronger and stiffer beam will result by the use of wedged or tapered keys.

To Design a Compound Beam with iron keys, since the efficiency is taken at 80% of that of a solid beam of the same dimensions, multiply the load upon the beam by 1.25, adding the weight of the beam, determined by approximate design, and use this increased load to determine the stresses and necessary dimensions. From the bending moment determine the necessary dimensions of the beam, making the width of beam approximately 4/10 its depth. Compute the vertical external shears at governing points and draw the vertical shear diagram. Determine the total horizontal shear between the point of zero shear and the ends of the beam. If d is the total depth of the beam, and V is the total vertical shear between the point of zero shear and the end of the beam, the total horizontal shear will be $3V/2d$. The number of keys in each half of the beam may be assumed, and five is a desirable number. Making all keys of the same size, the compression to be resisted by each key equals the total horizontal shear in the left or right portion of the beam divided by the number of keys. As the horizontal shear will be different to the right and left of the point of zero shear, in an eccentrically loaded beam, the compression to be resisted, per key, or the number of keys, may be different in the right and left portions of the beam.

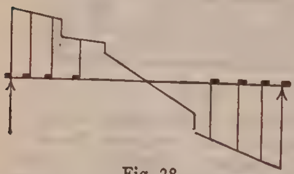


Fig. 28

To find the positions of the keys, divide the right and left areas of the shear diagram by vertical lines into as many equal areas as there are keys to the right and left of the point of zero shear. This may be done graphically or by trial. Fig. 28 shows a shear diagram for an eccentrically loaded beam divided by vertical lines into equal areas. The position of each key is fixed by the vertical lines of division. The keys should be placed back toward the ends, and not in front of the vertical lines. They are shown on the horizontal axis of the diagram. The dimensions of the keys must be such that the safe end compression on the wood is not exceeded. The distance between keys must be such that the safe horizontal shearing stresses are not exceeded. The end keys may be moved back of the point of support to decrease the unit horizontal shear between the first and second keys from the end.

Stresses in the bolts and their sizes may be determined as in the design of a tabled fishplate joint. Additional bolts should be used to bind the timbers together. The friction resulting from the tightening of the bolts increases the strength of the beam, but to an indeterminate amount, and should not be depended upon in design.

If the keys are of wood, the bearing of the end grain of the timbers against the side grain of the keys will govern their cross-sectional dimensions. If the keys are inclined the stress in the bolts and also the available shearing area between keys will be increased. For inclined keys, the shearing area of the timber for any key may be regarded as extending from its outer end to the middle of the next key. The breadth of a key should not be less than twice and preferably not less than 2-1/2 times its thickness.

8. Trussed Beams

For Spans Exceeding 20 ft. or when the loads are unusually large and where head room permits, trussed beams may economically be used. For economy the trusses should be generally as deep as head room permits. Trussed beams are of two types (Fig. 29). Type 1 is used when available head room is below the beam seats and type 2 when the head room is above. The member *aba* of both types may be of a single timber or of two or more timbers placed side by side. Usually each timber *aba* is a single unspliced piece extending from end support to end support. When it is of a single timber, the tie rod or rods of type 1 pass through oblique holes bored through the ends

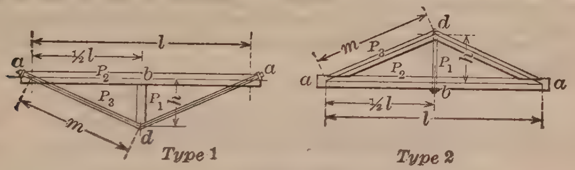


Fig. 29. King Types of Trussed Beams

of the piece. If it is of two or more pieces, they are separated so as to allow the tie rod or rods to pass between them. Type 1 should be designed with ends of tie rods made adjustable, or with an intermediate portion of rods, made adjustable by turn-buckles, or preferably with both adjustments. The adjustable tie rods should be made taut when erected, which causes initial stresses in the beam, and if the tightening is not carefully done, the strut *bd* may be pulled out of the vertical. The base of *bd* in type 1 should be made wide to better withstand unequal tightening of adjacent tie rods.

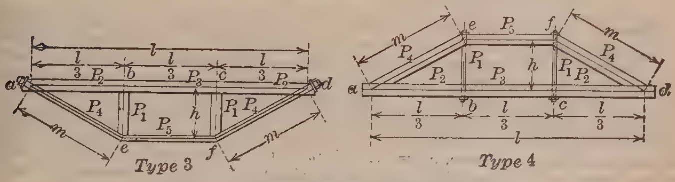


Fig. 30. Queen Types of Trussed Beams

For Spans over 25 ft., or when the head room is small or when loads are concentrated at two intermediate points, types 3 and 4 (Fig. 30) are preferable to types 1 and 2. The struts *be* and *cf* are generally placed vertical in these types, but they would be more efficient if placed so as to bisect the angle between the horizontal and inclined portions of the rods. In the design of all beams, the line of action of all pieces intersecting in a joint should meet in a point, or secondary stresses must be considered.

Type 1 (Fig. 29) may be designed by the following approximate formulas, in which W in pounds = total load on beam. (1) When **W is uniformly distributed** over the length of beam, $\mathbf{P_1}$ = compression in bd = $\mathbf{5/8\,W}$; $\mathbf{P_2}$ = compression in ab = $\mathbf{5\,Wl/32\,h}$; $\mathbf{P_3}$ = tension in ad = $\mathbf{5\,Wm/16\,h}$; and $\mathbf{S_2 = P_2/bd}$ = unit compression in ab, due to direct compression. To find the maximum unit compression in ab, add to S_2 the flexural or bending unit extreme fiber stress, due to the load acting upon ab as a beam. The extreme fiber stress due to bending $\mathbf{S_3 = 2.25\,Wl/bd^2}$. To find the maximum unit fiber stress add S_2 to S_3, or use the formula $\mathbf{S = Wl\,(5/h+72/d)/32\,bd}$. All lengths are expressed in feet, except that b and d, the width and depth of the chord aba, are expressed in inches. P_1, P_2 and P_3 are expressed in pounds, and S_2, S_3 and S in pounds per square inch. (2) When **W is concentrated** at the middle of the beam, $\mathbf{P_1}$ = compression in bd = $\mathbf{W}$; $\mathbf{P_2}$ = compression in ab = $\mathbf{Wl/4\,h}$; $\mathbf{P_3}$ = tension in ad = $\mathbf{Wm/2\,h}$. For this loading there are no flexural stresses except those due to dead load of the beam, which, excepting in long spans, under small loads, may usually be ignored.

Type 2. The stresses will be of the same intensity as in type 1, but the character of the stresses will be reversed. Members bd and ab will be in tension, and ad in compression. If the load is uniformly distributed, the foregoing flexural formulas may be used, but the maximum unit stress in ab will be tension.

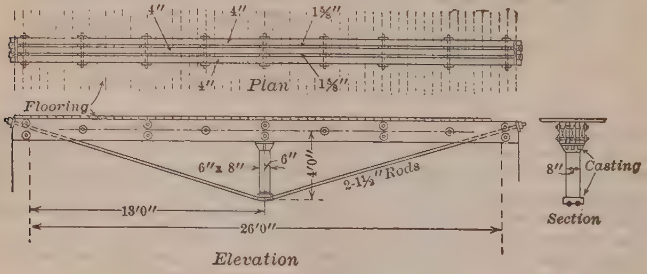


Fig. 31. Beam Trussed with Steel Rods.

Example, Fig. 31. To design a trussed beam, type 1, span 26 ft., depth of truss 4 ft.; load, exclusive of weight of truss, 1500 lb. per lin. ft., using Southern yellow pine, common grade, for top chord and vertical post. The compression in $bd = 5/8 \times 1500 \times 26 = 5/8 \times 39\,000 = 24\,375$ lb. Assuming that the ratio of length of bd to its least width will be less than 11, the cross-sectional area of $bd = 24\,375/880 = 27.7$ sq. in. The direct compression in $ab = 5\,Wl/32\,h = 39\,600$ lb. The necessary cross-sectional area of ab therefore $= 39\,600/880 = 45.0$ sq. in. The extreme fiber stress in ab acting as a beam, $S_3 = 2.25\,Wl/bd^2$. Use a safe extreme fiber stress 80% of that given in Art. 1 in order to allow approximately for direct compression. From the above formula $d = \sqrt{2.25\,Wl/bS_3}$. Assume $b = 12$ in., and for S_3 substitute eight-tenths of the allowed extreme fiber stress or $1200 \times 0.8 = 960$ lb. per sq. in. Then $d = \sqrt{2.25 \times 39\,000 \times 26/12 \times 960} = 14.1$ in. The maximum fiber stress per square inch, due to direct compression and flexure, is $S = Wl\,(5/h + 72/d)/32\,bd = 1190$ lb. per sq. in. The weight of the truss per lineal foot = approximately 1.3 of the weight per lineal foot of the top chord aba or $1.3 \times 12 \times 14/12 \times 3.3 = 60$ lb., where

12 × 14/12 equals the number of board feet, and 3.3 equals weight of a board foot of timber. As this weight is only 4% of the load, it may be ignored. Instead of using 80% of the allowed extreme fiber stress, the necessary size of *ab* due to bending may be found by formula, using the allowed extreme fiber stress and adding the necessary area for direct compression.

For the top chord 3 planks 4 × 14 in. will be used, separated so as to allow two steel rods to pass between them. This chord will be assumed to be stiffened laterally by a plank floor. Allowing 16 000 lb. per sq. in. in the tie rods, the net section A in square inches for $S = 16\ 000$ and $P_2 = 5\ Wm/16\ h = 41\ 450$, is $A = 2.6$ sq. in., or the two rods having a diameter of 1-1/2 in. may be used if the ends are not upset. If the ends are upset, two rods of 1-3/8-in. diameter may be used, as they have a combined area slightly in excess of 2.6 sq. in. The pieces of the top chord should be separated, so as to permit the insertion of the rods and their upset ends.

As the necessary cross-sectional area of *bd* based on end compression is 27.7 sq. in. a timber 6 × 6 in. might be used if the columnar reduction is not too great. The safe stress for a 6 × 6 Southern pine column 4 ft. long with $l/d = 8$ is 880 lb. per sq. in. Therefore the safe load for a 6 × 6 column 4 ft. long = 5-1/2 × 5-1/2 × 880 = 26 600 lb. For stiffness and simplicity of end connections, 6 × 8-in. timber will be used.

A cast-iron cap may be advantageously used at the top of the vertical strut to transmit the pressure to the side grain of the top chord. The width of the top must be sufficient to engage the top timbers, and its length must be sufficient to reduce the unit compression across the grain to that allowed in Art. 1. A metal foot of cast iron or steel should be used to reduce the unit compression with the grain of *bd* within a safe limit. The width of *bd* was, in this case, increased to 8 in. in order to receive the rods, which, on account of the top chord construction, are 5-1/2 in. apart on centers. The washer at the ends of the tie rods must be of such size that the compression on the surfaces inclined to the grain will not exceed the assigned safe limits. (Art. 2.)

The Design of Type 3 (Fig. 30), when the three panel lengths are all equal, may be made as follows. W = total load on beam in pounds. (1) **W uniformly distributed.** The end reactions brought by *ab* and *cd* acting as beams = $(4/30)W$. The stresses in the various members acting as a truss may be determined graphically or analytically. To the stresses so determined the flexural beam stresses in the top chord of type 3 and in the bottom chord of type 4 should be added. The following approximate formulas are easily applied when the panels are of equal lengths. They are sufficiently accurate for practical purposes. $\mathbf{P_1}$ = compression in *be* or *cf* = $\mathbf{1/3\ W}$; $\mathbf{P_2 = P_3}$ = compression in *ad* = $\mathbf{P_5}$ = tension in *ef* = $\mathbf{W\mathit{l}/9\ h}$; $\mathbf{P_4}$ = tension in *ae* and *fd* = $\mathbf{Wm/3\ h}$. The extreme fiber stress per square inch in the top chord panels acting as beams = $S_3 = \mathbf{W\mathit{l}/bd^2}$. The maximum extreme fiber stress $S = \mathbf{W\mathit{l}(h/9 + 1/d)/bd}$. All lengths are expressed in feet except that b and d are expressed in inches. (2) For **1/2 W on each vertical strut.** Let P_1 = compression in *be* or *cf* = $\mathbf{1/2\ W}$; $\mathbf{P_2 = P_3}$ = compression in *ad* = $\mathbf{P_5}$ = tension in *ef* = $\mathbf{W\mathit{l}/6\ h}$; $\mathbf{P_4}$ = tension in *ae* and *fd* = $\mathbf{Wm/2\ h}$. For this loading there are no flexural stresses except those due to dead load of beam, which may be usually ignored.

Type 4. The stresses will be of the same intensity as in type 3, but the character of the stresses will be reversed. *be*, *cf* and *ad* will be in tension and *ae*, *ef* and *fd* in compression. If the load is uniformly distributed, the flexural formulas of type 3 may be used, but in this case the maximum stress in *ad* will be tension.

9. Grillages and Wharves

Grillages of Timber are used on the tops of piles or upon foundation beds to support masonry. Fig. 32 shows a design of a grillage supporting a masonry retaining wall. The top of the grillage blocks B should not be higher than about 18 in. above low tide and the bottom timbers should not be so low that they must be laid by divers. The sizes of timbers may be computed by means of the beam formulas and tables. As the timber is under water the unit side-grain compression should be taken at 50% of that given in the table of Art. 1. The caps *C* and blocks *B*, which help prevent the masonry from sliding on top of the grillage, should be drift-bolted or lag-screwed, using long special screws, to the top of the piles. The grillage plank *A* should be fastened to the caps with boat spikes.

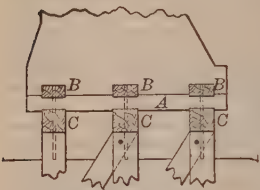


Fig. 32.

Grillages Supporting Building Bearing Walls are of the type shown in Fig. 33. These grillages should not be used except where they will be permanently wet. The bottom course of plank should be at least 2 in. thick and the top course at least 3 in. thick, provided that the clear spacing of the middle timber does not exceed 18 in. This middle timber must be designed so that the extreme fiber stresses do not exceed those specified in Art. 1. The bearing of the top plank upon the middle timber will not need to be investigated, except for soft wood. As in the preceding design, only 50% of the compression across the grain specified in the table of Art. 1 should be used.

The middle timber should be figured as a cantilever beam whose length is the portion extending beyond the outer edge of the wall. The load upon this cantilever beam equals the allowed safe load on the foundation bed $\times\, l_1 \times c$ (Fig. 33). If the depth of the middle timber is assumed, its width may be determined by

$$\mathbf{b_1 = 36\, w\mathit{l}_1^2/Sd^2}$$

and

$$\mathbf{b = 3\, w\mathit{l}_1c/2\, S_h d}$$

where b = width of timber in inches, based on extreme fiber stress, b_1 = width of timber in inches, based on horizontal shearing stress; l_1 = portion of middle timber extending beyond the edge of wall in feet; d = depth of beam in inches; c = distance between consecutive middle timbers in feet; w = safe load on foundation in pounds per square foot; S = safe extreme fiber stress in pounds per square inch; S_h = safe horizontal shearing stress in pounds per square inch. For shallow beams the width b will generally govern, and for deep beams the width b_1; the greater width should be used.

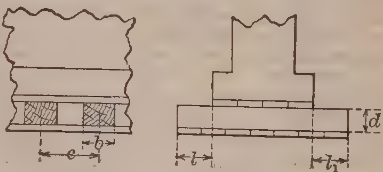


Fig. 33. Grillage for Wall

Wharves. The width of a wharf or pier is usually governed by local and commercial conditions, and its length by conditions and regulations of the United States government. The direction of the length of a wharf, as a rule, should be perpendicular to the bulkhead wall or shore line. In narrow rivers this direction may be made oblique to the shore line, the outer end being farther downstream than the shore end. This arrangement facilitates the entrance of vessels into slips between adjacent wharves. Timber wharves are of two general types: (1) having the floor system or deck supported on piles; (2) having the floor system or deck supported upon timber cribs. The first type is generally the better and cheaper and should be built wherever conditions permit. It cannot be built where the bottom of the river is too hard for pile penetration. When a small penetration, from 4 to 10 ft., may be gotten, the requisite stiffness and holding power of the piles may be obtained by using riprap. The riprap, if of small stones, may be placed, in whole or part, in advance of the piledriving. This is unusual practice and, if adopted, the pile points should be shod with iron.

The top of the deck of a wharf must be above high water, but low enough to permit the economical loading of vessels at low tide. The wharf should be designed with the greatest practical free waterway, so as not to obstruct the flow of water, sewage or ice. Pile piers are usually built with piles in transverse rows 8 to 12 ft. on centers, the piles in the rows being 3 to 10 ft. on centers. The close spacing is for heavy loads or where the pile resistance is small. When timber cribs are used they extend for the transverse width of the wharf and are usually spaced 15 to 35 ft. on centers. Where cost of cribs and their foundations is high, trussed spans may be found economical for distances much over 25 ft. A wharf need not, as a rule, be braced longitudinally, as the impact of vessels in this direction may be taken care of by the bulkhead wall or the earth at the shore end; and as the length of a wharf is generally much greater than its width, the force of impact is distributed over a greater number of piles than when the force acts in a transverse direction.

The lines of transverse piles should be placed as nearly as possible parallel to the thread of the current. They should be strongly sway-braced, the braces being carried down at least as low as low tide. If the piles are to be driven into soft bottom or deep water or in a rapid current, riprap or batter piles should be used to stiffen the wharf against the impact of vessels. A horizontal stick of 12×12-in. timber, 20 ft. long, neglecting its weight, acting as a cantilever, will deflect 1/4 in. under a load of 100 lb. applied at its end; if 40 ft. long, the deflection will be about 2 in. Wharf piles, if unbraced and under the impact of vessels, are cantilever beams whose length equals the distance from their top to firm river bottom. If the upper portion of the piles is sway-braced sufficiently, the length of pile, acting as a cantilever under vessel impact, is the portion below the bottom of the sway bracing. As the deflection under a given load varies as the cube of the length, it is evident that the sway bracing should be carried down as low as possible.

Guard or fender piles should be used to protect the structure from abrasion of vessels lying alongside, and clusters of piles should be placed near corners to protect them. In deep water, or where the river mud is deep and for large vessels, the clusters should consist of from 10 to 50 piles. The corners of wharves for large vessels should be rounded and strongly reinforced. The rounded corner makes it easier for boats to enter slips between the adjacent wharves.

A timber wharf in fresh water, or in salt water where marine wood borers do not exist, lasts about 30 years, with minor repairs and floor renewals. To increase the life of a wharf all joints and bearings should be treated with a preservative; drift bolts should be driven below the tops of the timber, and the hole thus formed filled with tar or asphaltic cement. Large washers should be used under all bolt heads and nuts, and the bolts should not be tightened so as to break the outer wood fibers. When wood pieces are in contact and it is possible to force them into closer contact with bolts in order to exclude water, this should generally be done. Beams built up of two or more pieces and all joints should be packed at least 1 in. apart, using metal packing pieces or separators. The timber should, in general, be so designed as to permit as much circulation of air around each piece as is possible. Wooden floors should be built so as to drain the rain water to outlets. Leaving open joints between adjacent floor planks in order to get rid of rain water decreases the life of a floor subject to vehicular travel, due to the pounding of the wheels across the joints and the abrasion of the edges. Exposed timbers will last longer if beveled so as to shed water freely. Creosoting lumber, if well done, increases the life of the structure. (See Sect. 7, Art. 41.)

In fresh water the portion of the pile between about 1 ft. above low water and high water usually rots before the rest of the structure. By making the caps continuous over several piles, the rotted pile tops may be removed and replaced without removing the top timbers. In fresh water, piles generally last longer with bark removed above low tide. In salt water, piles in warm climates are attacked by marine borers which generally enter the piles between high tide and a point about 4 ft. below low tide.

They may continue their destructive work above high tide and many feet below low tide. They never enter the wood below the mud line. Wood borers are classified by the Forest Service as Teredo, Xylotrya and Limnoria, Chelura and Sphaeroma. The first two are commonly called Teredo and the latter three Limnoria. These borers do not live in fresh water nor in salt water containing much sewage, but may exist in brackish water. The average length of time in which the Teredo and Xylotrya have destroyed barked and unprotected piles in the Gulf of Mexico ranges from 1 to 5 years, the minimum time being sometimes only a few months. Circular 128, U. S. Forest Service, gives methods of protection. (Ref. Sect. 7, Art. 41.)

Live Loads for Wharves range between 75 and 1200 lb. per sq. ft. for concentrated loads of wagons, unloading apparatus, trains, and posts of wharf buildings. The lighter loads are used for excursion boat landings. For ordinary commercial wharves 600 lb. per sq. ft. should be used. The necessary sizes of timbers are determined from the preceding beam formulas and tables. All the parts of the structure should be well bolted, drift-bolted, spiked or lag-screwed together, so that the wharf will act as a unit under the impact of vessels. As the piles between high and low water decay before the other portions, except the floor plank, the unit stresses in the piles should be taken at about 50% of those recommended in Art. 1. This will usually be taken care of by the proper application of pile formulas.

The cost of pile wharves varies, depending upon the depth of water, nature of foundation, necessity of batter piles or riprap, and the loads to be carried. Wharves of the crib bridge type cost one and one-half to twice as much as the pile wharves, depending upon similar conditions.

For additional data on wharves, see Sect. 19.

FALSEWORK AND TRUSSES

10. Trestles

Timber Trestles are generally used for semi-permanent work when their first cost is less than an earth fill or an earth fill and culvert, if a stream of water is to be taken care of. When permanent work is desirable, there is usually no sound reason for timber trestle construction, other than the lack of funds, because, due to the high cost of lumber and timber maintenance, an earth fill or steel structure will be cheaper in the end. Trestles usually consist of a series of bents spanned by beams or stringers. Each bent generally consists of two or more vertical posts, depending upon the width of roadway and the load to be carried, two outside battered posts, a cap and sill, and sway bracing. The outside posts are given a batter of from 1 1/2 to 3 in. in 12 in. and the bent is sway-braced to resist lateral force such as wind or rack of a locomotive. The roadway or track is carried upon stringers, which should be continuous over two spans. Fig. 34 shows a common form of trestle less than 30 ft. in height as used in 1906 on the Baltimore & Ohio R.R. Trestles over 30 ft. high are generally built in stories of from 15 to 30 ft. in height, as in Fig. 35, which shows the 1908 standard of the New York, New Haven & Hartford R.R. In this design the bent is framed into two separate stories, the second story being erected upon the cap of the completed story below. This is the more common design and is easy to erect, but occasionally the vertical and batter posts are made continuous from top to bottom of trestle. The latter method reduces to a minimum the number of joints where end grain bears upon side grain. This is desirable, as the side grain is often injured by such compression, and in the case of sills dampness is retained by the broken fibers and rot takes place rapidly. Trestles over 50 ft. high are sometimes built with pairs of bents framed into towers at intervals of 20 to 30 ft., the stringers being compound or trussed beams.

Longitudinal bracing for railway bridges must be designed to take the thrust of a braking train, which may amount to 75 000 to 100 000 lb. It is generally assumed as 20% of the live load. The longitudinal bracing should not only consist of horizontal struts between bents to take compression, but the bents should also be X-braced as shown in Fig. 34. For highway trestles the bracing may be of smaller cross-section and the X-bracing at less frequent intervals. Where the trestle is of short length and there are abutments of earth, timber or masonry at either end, the X-bracing may be safely reduced in cross-section.

Trestle Design. The sizes of the stringers and posts can be obtained from the timber tables and formulas, using the specified loads given in Sect. 12, but it is advisable to increase the dimensions so determined, to allow for decay. Stringers rot quickest where the ties or flooring rest upon them and where they rest upon the caps, as dampness lodges in such joints. The stringers of railway bridges should be packed 1 to 2 in. apart, to permit circulation of air, using metal separators or packing spools, as in Fig. 34. Bolts should pass through the spools, having large washers at their ends to permit the bolts to be tightened without injury to the fibers. The sizes of the posts depend upon the safe allowable compression upon the side grain of the caps and sills, see Art. 1. It is therefore desirable to use hardwood caps and sills when possible. The caps and sills should have about the same dimensions as the posts. As a rule the posts can be so placed with reference to the stringers that there will be practically no bending stress in the caps, and similarly there will be none in the sills, the function of caps and sills being to hold the posts in position and transmit shear.

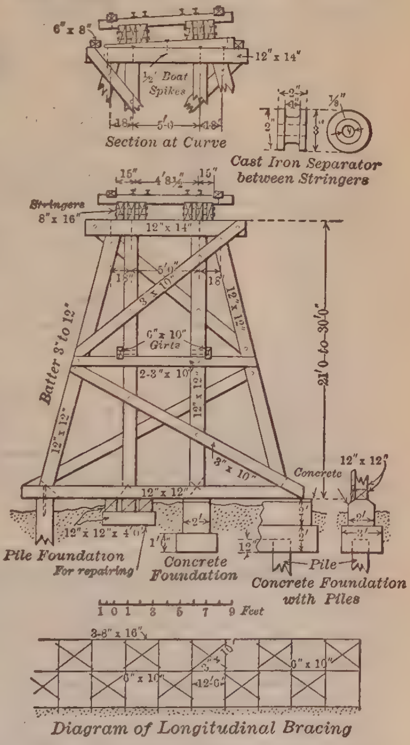


Fig. 34. Trestle, Baltimore & Ohio R.R.

The posts may be drift-bolted or mortised to the caps and sills.

The bottoms of sills should be supported by masonry so as to be at least 12 in. above the ground. If the trestle consists of piles driven into dry ground, the piles at ground level will rot badly in 5 or 6 years, and as the life of a trestle is from 10 to 14 years, it is advisable to double, or nearly double, the number of piles per bent, in order to allow for decay, or cap the piles with masonry, as in the extreme right footing of Fig. 34.

Bracing. Good examples of sway bracing are given in Figs. 34 and 35. For highway bridges the sways may be lighter. Use 3/4- or preferably 1-in bolts for fastening sways. The design of this bracing for trestles under 30 ft. high is generally made on the basis of precedent, the effect of wind being small and the other stresses indeterminate. For higher trestles the wind

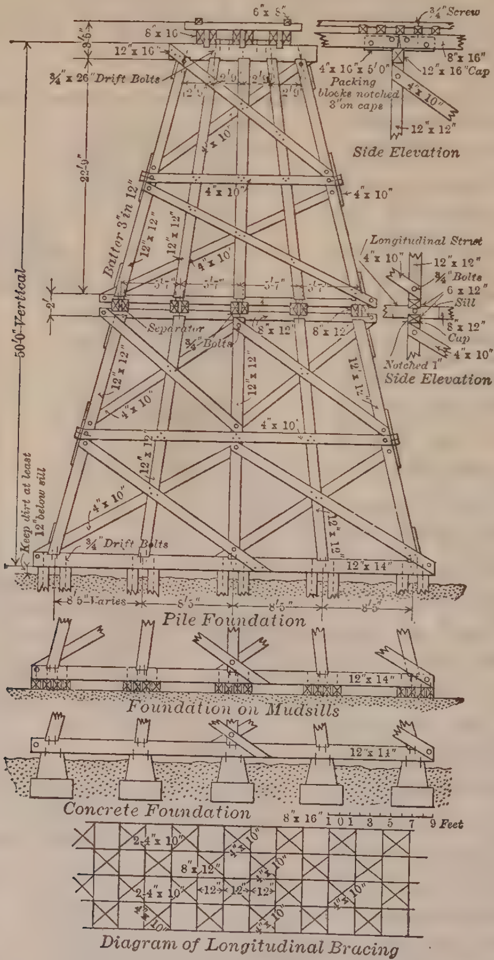


Fig. 35. High Trestle, New York, New Haven & Hartford R. R.

stresses should be computed as guide in the selection of proper size and in the design of joints. The horizontal struts or girts of the longitudinal bracing should not, for railway work, depend upon a single bolt for the transmittal of stress, but such stress should be transmitted by end bearing. For railway trestles the struts should be from 6 by 8 to 8 by 12 in. For highway trestles, the span being longer, the size should not generally be less than 6 by 6 in. The longitudinal X-bracing need not be used in every panel, except for railway trestles on curves over 7°. In high railway trestles on curves over 8°, the centrifugal force of the moving train should be further guarded against by additional braces on the convex side of the trestle. The sizes of the X-braces cannot be determined by computation, but should be selected from existing designs. The details of all trestle work should be simple. When pieces are in contact they should be drawn tight, to exclude water, and where possible they should be separated at least 1 in. and preferably 2 in. to allow circulation of air. In the design of high trestles, in order to decrease the number of bents the span of the beams is reduced by means of corbels and A-frames as shown in Fig. 36, thereby permitting a greater distance center to center of columns.

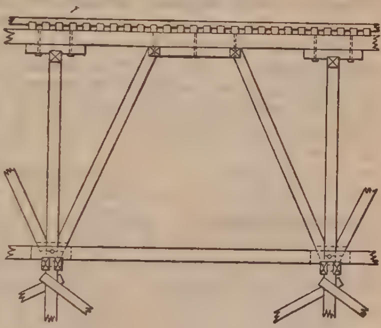


Fig. 36

Life of Trestles may be increased by the use of wood preservatives, such as creosote, and all surfaces in contact or which have been disturbed by ax or adze, all screw and bolt holes and the tops of all drift bolts should be thoroughly coated with similar preservatives. The tops of caps and stringers are often covered with sheet zinc or iron to protect them from water and hot cinders. These coverings are unquestionably of value as fire protection, but as they decrease the circulation of air, it is doubtful if the life of the caps and stringers is prolonged thereby.

Construction Trestles are for temporary use, and therefore in selecting the sizes for posts the compression across the grain may be taken 50% higher than allowed in Art. 1, and no allowance need be made for rot. Trestles for light industrial tracks and narrow-gage construction tracks require only light transverse and longitudinal bracing. In making earth fills from temporary trestles constructed therefor and from old trestles constructed for semi-permanent use, as railway trestles, the lateral and longitudinal bracing, and in high fills even the caps and sills, may be broken by the fall or weight of the material or its settlement. Such fills should be brought up evenly on either side of the axis of the trestle to diminish the possibility of large and sudden movements of the fill, which occur most frequently in fills made upon side hills. It is often advantageous to remove the bracing when it is reached by the fill to prevent the pull in the bracing, under the load of the fill, from distorting the posts and even breaking them. This is a matter which can be determined only after the character of the fill and contours of the ground are known. If the bracing is old and rotted it is often best to put on a new and substantial bracing before starting the fill.

When eath fills are made from trestles of piles driven into 15 to 20 ft. of mud, sudden movements of the mud and newly made fill frequently not only break the bracing but snap off the piles and demolish the entire structure. Such trestles should be built of piles of large diameter, driven to hardpan and heavily sway-braced, and the sway bracing removed when reached by the fill. But even with these precautions sudden failure may take place.

11. Floors for Trestles and Bridges

Floors for Steel Highway Bridges are illustrated in Fig. 27 of Sect. 12. In designing the roadway joists concentrated wheel loads may be regarded as distributed over two joists, when the distance center to center of joists does not exceed 2 1/2 ft. and the flooring is at least 3 in. thick. In maintenance work, when the center distances do not exceed 2 ft. and the wheel load is not applied near the outer line of stringers, it may be regarded as distributed over three joists, provided the flooring is at least 3 in. thick. The front roller of a steam roller may be computed as equally distributed over all joists which it can cover or partially cover. The wheel loads upon cross-ties of electric railways may be regarded as distributed over three ties.

Roadway Flooring may consist of a single thickness of 3- or 4-in. plank, the distance between centers of stringers not exceeding 9 times this thickness; 3-in. flooring should be used for light and infrequent travel and 4-in. where it is heavy and continuous. Roadway plank should be laid with a transverse grade of from 1 to 1 3/4 in. in 10 ft., to drain the water to the sides of the roadway. In bridges over 16 ft. wide, the plank may be cut in two lengths and the level of the roadway raised at the middle. **Floor plank** should be laid snug, as the planks at open joints are easily torn, and the floor is made very rough and wears quickly. They may be laid transverse to the bridge or oblique; the former method is slightly cheaper and appears equally satisfactory. Planks need be dressed on only one side, but all joints should be adzed smooth when the floor is laid. Roadway plank should have a uniform width of 12 in., although if ordered in variable widths of 8, 10 and 12 in., a substantial saving results. When variable widths are used it is necessary in making repairs to select particular widths of plank for each defective one replaced, which is troublesome. For bridges having little travel the life of the flooring depends upon the rotting of the timber, and therefore timber which rots slowly, such as Southern pine, is desirable. For bridges carrying heavy and continuous travel the floor does not rot but is worn out, and therefore the harder and tougher woods such as white oak or chestnut oak are best.

Three-inch flooring should be fastened to the joists with 6-in. nails and four-inch with 7-in. nails. Flat-head wire nails are more easily driven than round-head nails. Well-seasoned hardwood floors often require holes to be bored in advance of driving nails, but most commercially seasoned oak and all pine flooring may be nailed down without such holes. All flooring should have a one-heart face. The heart face should be laid down for light travel and up for heavy. The general rule is to put the heart face down, but experience with bridges in Washington, D. C., shows that as a rule it should be up.

The life of the flooring under light travel is from 4 to 5 years, and under heavy travel 3-1/2 to 4 years. In the latter case probably 10% of the plank will have to be replaced before a general replacement is made, using a low grade of timber.

Double floors consisting of a 3- or 4-in. creosoted subfloor, laid with 1/2- to 2-in. open joints and having a wearing floor of from 1-1/2 to 2 in., are sometimes used. They should never be used with untreated plank in the subfloor. A creosoted subfloor lasts 12 or 15 years. The lower planks are laid transverse or oblique to the bridge and the upper ones at an angle of from 45° to 60° with the ones below. The subfloors should be nailed to the joists with 60d. nails, using two per joist, and the top planks nailed to the lower floor with two 40d. nails at intervals of 2 ft. This type of floor, while resulting in higher first cost, has the advantage of cheap resurfacing at frequent intervals, and therefore usually gives a smoother roadway than single planking.

A Roadway Joist should be at least 3 in. thick and should not deflect more than 1/400 of the span under the full load. The general sizes of joists used are 3 by 12, 3 by 14, 4 by 14 and 4 by 16 in. When they rest upon the tops of floor beams they may lap or butt, if there is sufficient bearing. When lapped, an air space of 1/2 in. should be left between the lapping ends to permit circulation of air. Neither bridging nor fastening of the joist to the floor beams is necessary, unless planks thinner than 3 in. are used. The floor will be too heavy to be disturbed by wind.

Southern pine and oak joists last from 12 to 15 years, provided they are selected of such depth that they may be turned over once. Joists rot where the planks rest upon them, and when this depth reaches an average of one inch they should be turned and used until the top of the joist, in the new position, rots about the same amount. Southern pine joists are better than oak joists, as they do not warp, and pine rots from the outside and oak often from the inside. The painting of the top of joists with a timber preservative is recommended.

Sidewalk Flooring should be 2 in. thick, dressed one side, 6 or 8 in. wide and supported by joists not over 2 ft. on centers. They should be laid snug, as open joints make rough floors and the public object to seeing through the open cracks of floors of high bridges. Pine floor planks may be spiked with two 20d. nails per joist and oak ones with two 30d. nails. Floors should be given a transverse grade of 3/8 in. per ft. **Sidewalk joists** are usually 3 by 12 or 3 by 14 in., placed 2 ft. on centers. These joists should be fastened to the floor beams, as sidewalks are usually narrow and of light weight per square foot, and if the joists are not fastened the floor may be disturbed by wind, although cases of this kind are extremely rare.

When steel stringers are used the planks in small country bridges are often fastened by clinching the nails under the flanges. The best arrangement is to bolt spiking strips 4 to 5 in. thick to the flanges of the beams, countersinking the bolt heads.

Wheel Guards should be provided on either side of the roadway to protect the adjacent trusses or to act as a curb protection for the sidewalks. They should consist of 4 by 6 or 6 by 6-in. string pieces supported upon 2- or 3-in. shims, 6 to 12 in. long placed at intervals of 4 to 6 ft. on centers. The roadway water should drain through the openings left between the shims. Guard rails should be bolted to the joists or floor beams, the bolts passing through the shims. Joints in the string pieces should be made at the shims and with halved laps. These rails should be protected with steel angle facings when laid upon a steep grade, as they are often used by teamsters as brakes.

Electric Railway Tracks may be supported upon cross-ties or directly upon longitudinal stringers, the latter method being used when the bridge also carries vehicles. Cross-ties should not be less than 6 by 6 in. when the stringers are 6 1/2 ft. on centers. For a greater distance they should be designed to carry the maximum wheel load, assuming a distribution of this load over three ties, the fiber stress not to exceed allowable limits. The openings between ties should not exceed 6 in. Alternate ties should be drift-bolted to the stringers. No notching is necessary. Guard rails 6 by 8 in. should be placed at either side of the track, 3 ft. 7 in. from the track, and lag-screwed to every tie. No notching over the ties is necessary with the use of lag screws, which should be staggered 2 in. in the guard rails.

Solid Timber Floors covered with ballast have been used for trestles and have given satisfactory results. The timber is creosoted and not afterwards cut if it can be avoided. When cut the end surfaces are painted with creosote. This type of floor is expensive to build and maintain and is somewhat difficult to inspect, but is regarded as offering a good protection against fire, decreases noise and gives a better track than the common type of floor. Arguments for and against this type of floor are given with interesting detail in Proc. of Assoc. of Railway Superintendents of Bridges and Buildings, Oct., 1906. Fig. 37 shows a ballast trestle floor on the El Paso and South Western R.R.

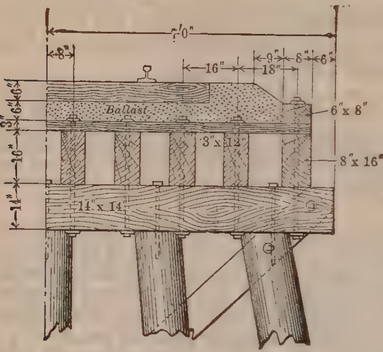


Fig. 37

The 1921 Manual of the American Railway Engineering Association gives the following relative advantages of solid timber ballast deck and reinforced-concrete trestles: **Concrete trestles** have advantage in fire resistance, and permanence; and a slight advantage in bridging waterways subject to flood currents or wide fluctuations of water surface. **Creosoted timber trestles** are more economical than concrete except when the cost of the concrete structure is less than one and one-half times the cost of the wooden structure. Neither type has the advantage in the matter of appearance, nor in general service qualities.

Railway Trestle and Bridge Floors. Figs. 34 and 35 show typical trestle timber floors and Fig. 38 shows floors for Howe bridges.

The following clauses are taken from the 1921 Manual of the American Railway Engineering Association:

It is recommended as good practice to use guard timbers on all open-floor bridges and they should be so constructed as to space the ties properly and hold them securely in their places.

It is recommended as good practice to use inner guard rails on all open-floor and on the outside tracks of all solid-floor bridges, and similar structures longer than 20 ft. in main-line tracks and on similar bridges and structures in branch line tracks in which the speed of trains is 20 miles per hour or more.

The inner guard rails should be of some form of metal section and extend beyond the ends of the bridges for not less than 50 ft. They should be spiked to every tie.

Ties and guard timbers should be sized one dimension. Omit dapping of guard timbers and ties. Use lag screws in every tie. Holes should be bored for lag screws 1 in. deeper than penetration of screw. Holes should be bored 1/16 in. smaller than diameter of lag in guard timber and 1/4 in. smaller than diameter of lag in ties. Lag screws must not be driven but screwed to position. Fasten alternate ties to stringers. Lag screws should be stagged 2 in. in guard timbers.

12. Falsework for Bridges

Truss Bridges are generally erected upon timber trestles or scaffolds consisting of vertical timber posts and outside batter posts, supporting caps upon which rest longitudinal stringers (Fig. 39). The bottom chord of the bridge is laid upon the falsework and blocked several inches above its correct position to allow for settlement of the falsework under the load of the bridge.

Bents are usually placed at panel points of the bridge, but where clearance makes it necessary the panel loads may be carried at any point of the stringers.

Small and light bridges may be erected by gin poles, which are timber masts guyed in a vertical position and carrying a block and tackle at their tops, by means of which the separate truss members are hoisted into their positions. Most bridges were formerly erected by timber travelers, consisting

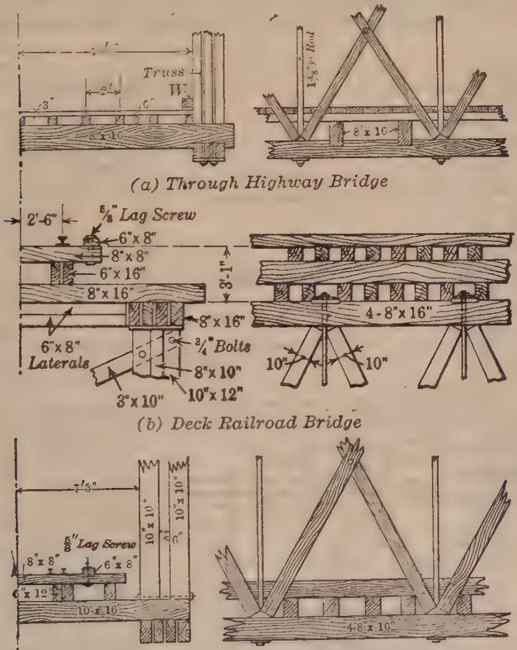


Fig. 38. Floors for Howe Bridges

of two or more overhead bents braced together to form a tower (Fig. 39). Such travelers are no longer used, and modern practice is to erect all but the largest bridges by locomotive cranes or derrick cars. For important large bridges, special steel travelers may be constructed. The falsework must be designed to carry the weight of the bridge and traveler or crane. Where there is danger from injurious freshets or where there must be large clearances maintained as for navigation, trussed falsework, generally of the Howe bridge type, is used. This falsework may be erected upon temporary timber trestles or upon pontoons. Fig. 39 shows the falsework and traveler used on the Baltimore and Ohio R.R. in erection of a small bridge.

Design of Falsework. The loads to be carried by the falsework consist of its weight, which, except for trussed falsework, is negligible; the weight of the bridge, including the girders, but not necessarily the stringers and floor resting upon the girders, and the weight of the traveler or crane. In the design, consideration must be given to wind pressure exerted upon the bridge and the traveler; where a stream is to be bridged the effect of ice and débris carried by freshets must be regarded and preferably the erection completed within a period of time when ice gorges or serious freshets are least probable. Where trussed spans are used, slightly higher unit stresses may generally be used than are allowed for permanent work, as the period of erection is not sufficient for a material deterioration of the timber. The trusses should be

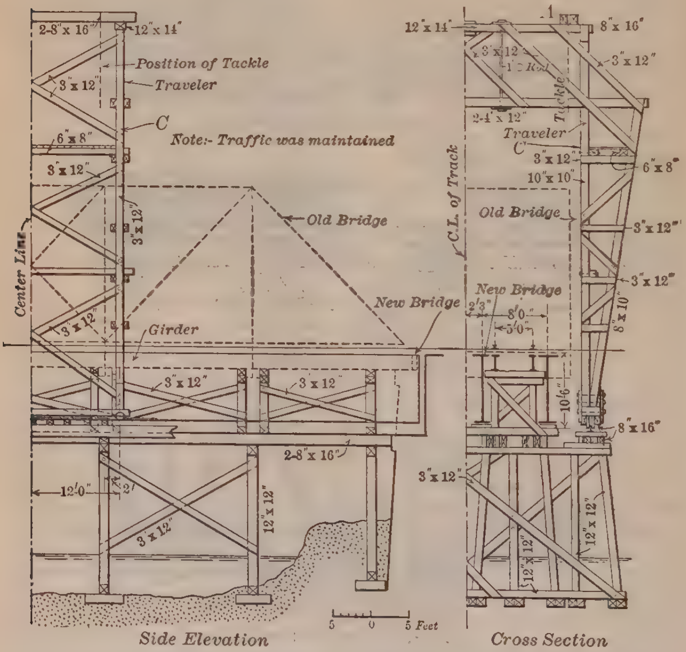


Fig. 39. Bridge Erection, Baltimore & Ohio R. R.

of first-class carpentry work to reduce the truss deflection to a minimum and should be strongly sway-braced.

When viaducts or bridges over land are erected, the falsework may consist of a series of ordinary trestle bents resting upon either mud sills or spread timber footings. The trestles for water work are generally formed, in whole or part, of piles. When the trestles are over 30 ft. high they are generally built in stories of from 15 to 30 ft. high, the timbers of adjacent stories being tied together by fishplates, to transmit tension resulting from wind pressure upon the bridge and traveler.

For light bridges the trestle sills, posts and caps may be of 6 by 6 or 8 by 8-in. timbers, the sway and longitudinal bracing of 3 by 8-in. timbers and the stringers of 6 by 12-in. timbers. For most bridges the sills, posts and caps should be 12 by 12-in., the sway and longitudinal bracing 4 by 10 in., and the stringers 8 by 16 in. The approximate dimensions of the timbers should be determined by computation, but as stiffness and sometimes weight are governing factors the sizes should generally be larger than theoretically required.

All falsework should be designed so as to permit economical erection and demolition and so that the timbers will not be materially injured for future use. Bolts should be used instead of spikes, and beveled, mortised or other expensive joints should be avoided.

The Settlement of the Falsework under its full load should be a minimum consistent with economy. A practically unyielding foundation should be constructed; the number of horizontal joints and particularly the number of joints where end grain bears upon side grain should be reduced to a minimum and the unit compression across the grain should not exceed that allowed in Art. 1. The probable settlement of the falsework, exclusive of foundations, may be approximated as follows: (1) By figuring the columnar shortening, c, in inches, by means of the following formula, $c = 12\ Sl\ /\ E$, in which S is the unit compression per square inch, l = length of column in feet, and E the modulus of elasticity. When pile foundations are used the length of piles should be added to obtain the total columnar length to be used in the formula. (2) By allowing 1/16-in. settlement for each horizontal joint for " taking up" and an additional 1/16 in. at each horizontal joint where end grain bears upon side grain for " biting" of the grain into the side grain, provided the unit stresses of Art. 1 are not exceeded. The total settlement will be the total of that obtained from (1) and (2).

In railway bridge renewals or other bridge erection where the falsework carries heavy locomotive or vehicular loads the locomotives or heavy vehicles may cause settlements of the falsework foundation so much greater than the deformation of the timber that the latter is negligible. Frequently, even in highway bridge construction, falsework foundation settlements of from 6 to 12 in. occur without serious mishap. The panel points are wedged up as the falsework settles, but in design such settlements should be guarded against.

13. Falsework for Arches

Masonry Arches are built upon temporary falsework, called centers or centering, usually of timber consisting of parallel frames, bents or ribs with top members whose upper surface is cut to the curve of the arch soffit. The top members may be joists; or bows, also called back pieces, may be used. Upon the top members plank called lagging are laid which support the stone voussoirs or concrete sheeting of the arch. Wedges or jacks are placed at convenient points of the falsework to lower sections of it after the arch is built. The wedges may be located at any point between the soffit of the arch and the falsework foundation. All arches should be built up symmetrically and the centering should be designed for such loading.

The Centering for Short Span Arches, when the span is under 10 ft. and the arch is flat, may be built of single planks on edge with tops cut to curve of soffit, supporting lagging dressed on one side and supported at the ends by posts resting upon wedges, as at *a* in Fig. 40. If the centering is built for a stone or brick arch, the lagging may be laid with open joints not wider than 1/3 the width of voussoirs or bricks; if for concrete, tongued and grooved lagging should be used. For spans over 6 ft. the top member or bow may be in two pieces tied by small battens (*b* in Fig. 40), the curve of the arch being cut out of the top piece. To save labor of cutting bows when the barrel of the arch is long enough, the bows may be placed 4 or 5 ft. on centers and 2- or 3-in. lagging used. The lagging and bows are computed as beams and the posts as columns under the full load of the arch. In buildings 1 in. lagging is generally used, supported on two or three bows about 12 in. on centers. When the arch is flat and the span is over 10 ft., intermediate props or posts are generally used to shorten the beam length of the bow; 2- to 4-in. lagging may be used, the bows being placed 4 to 6 ft. on centers.

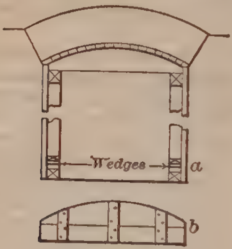


Fig. 40

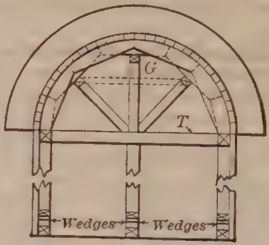


Fig. 41

Centers for Short Span Arches

When the arch has a rise of over 1/6 the span, centering of the form of Fig. 41 may be used. The bow may be made up of two or three planks breaking joints and nailed together. For economy the bows should be as far apart as possible, preferably 4 to 6 ft., using from 2- to 4-in. lagging. For the larger spans, the thicker lagging is the more economical. A center post should be used for spans over 10 ft., and when this cannot be done the bottom piece, *T*, should be stiff and strong enough to carry the load brought by the radial braces. This load is indeterminate, but in design it may be assumed at 1/4 of the total load of the arch, the bow also carrying 1/4 of the load as an arch and the balance being carried by the masonry arch acting as a compression member.

This type of centering may be used up to spans of 30 ft., but for spans over 15 ft. one or two lines of horizontal girts should be used for each frame to take care of the horizontal thrust of the arch load. A girt, marked *G*, is shown in dotted lines in Fig. 41.

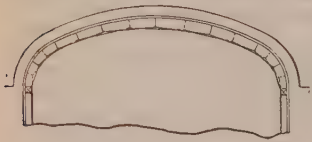


Fig. 42. Bow Centering

Unbraced arch centering of the type shown in Fig. 42 should never be used, except where other forms are impossible, and then only for spans under 20 ft., as it deforms badly, rising at the crown when loaded at the haunches and sinking at the crown under the crown loads, and is at best risky. By placing a temporary load at the crown, the change in the form of the curve can be lessened. If used at all, the carpentry work should be first-class, the abutting sections of the bows in intimate contact, wedges of shingles being used just prior to loading if the shrinkage of the timber has caused the joints to open.

Centering for Arches Over 30 Ft. in span may be of four types: (1) Arches, (2) Howe trusses, (3) Bowstring trusses, (4) Bents. The first is expensive to build, gives small salvage for lumber, deforms badly under loading and therefore requires temporary crown loading and causes cracks at the haunches during the arch construction unless loaded in alternate transverse sections. Where a deep gorge is to be spanned it may be economical. The second type is objectionable for general use for the foregoing reasons, except that the crown will not rise when the haunches are loaded. Maximum stresses should be determined for all web members under various stages of loadings. There is no reason for using this type except to utilize existing available trusses.

The curve of the arch must be built up by additional timbers resting upon the top chord at panel points. For the design of this type see Art. 16.

Bowstring Centering shown in Fig. 43 has been used extensively for spans up to 100 ft., and may be necessary for arches over streams, but should never be used where intermediate supports can be gotten, as in Fig. 44, or where vertical posts may be carried to intermediate foundations. It has all the objections given for type 1, but is cheaper to build and deforms less. This type is preferable to the Howe truss bridge type. Bowstring centering should be built with Pratt bracing, as in Fig. 43. The stresses in the various

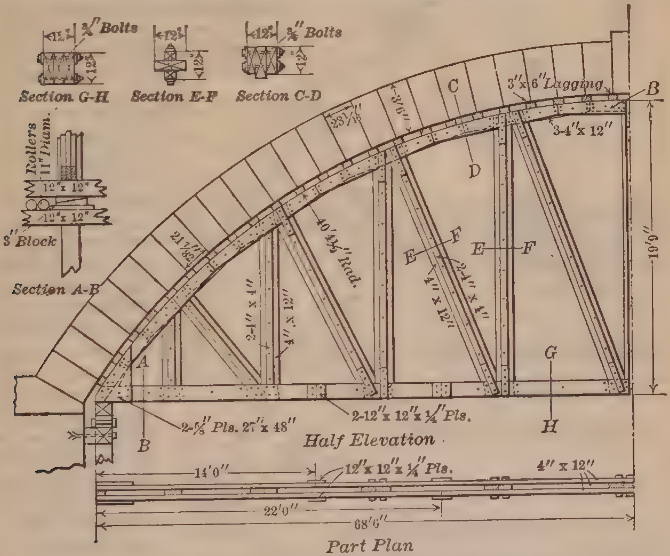


Fig. 43. Bowstring Centering

members should be computed for the centering, one-third, one-half, two-thirds, and fully loaded, and each member designed for the maximum stress. Reversal of stress will take place in the bracing, and all joints of this bracing should be made with straps or gussets to transmit tension as in Fig. 43. The crown of this centering should always be loaded with a temporary load of about 1/6 the total arch load, but even with this precaution hair cracks will generally develop at the haunches, showing that the tensional value of the mortar has been destroyed.

Fig. 44 shows a modification of the bowstring centering which was used in the construction of the Massachusetts Ave. culvert, Washington, D. C. The bowstring truss was designed to be moved ahead in sections upon the bottom bents or A-frames. Eight bows were struck at one time and skidded upon pipe rollers resting upon a stringer track. One additional A-frame was erected in advance in order to permit this movement.

The Bent Type of Centering is recommended for the construction of all arches where clearance does not prohibit its use. Its first cost is generally least, it deforms less than the other types, and gives the greatest salvage of lumber. Its design should not be begun until the method of building the arch has been determined. If the arch is to be built up continuously from the springing line to the crown, it should deform or settle less than if built in either longitudinal rings or alternate transverse sections. But however strongly built, if the arch is built up continuously cracks will develop at the haunches when the crown is built. This of course will not take place if the arch is small enough to be built complete in a day. The stiffer the center, the smaller the cracks. These cracks are not serious except that in the case of a reinforced-concrete arch they may permit the infiltration of water and the rusting of the steel, and they are the cause of the abnormal sinking of the crown of the arch when the centering is lowered. Where the cracks are larger than hair cracks, the masonry above the extrados of the arch should not be built until the centering has been lowered sufficiently to close up the cracks, otherwise the closing up of the cracks in the arch may crack the upper masonry.

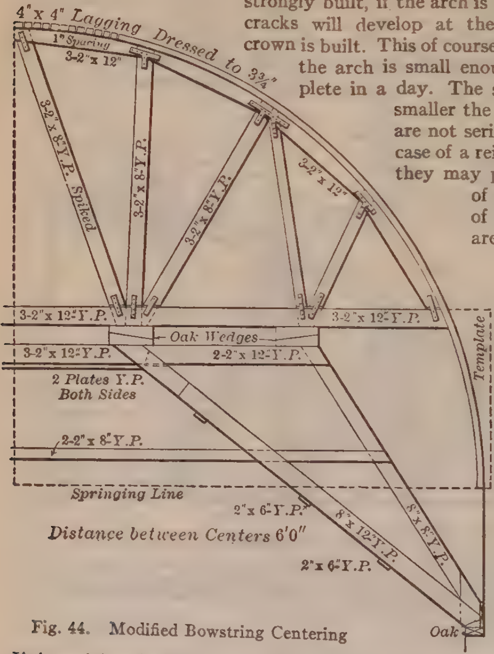


Fig. 44. Modified Bowstring Centering

If the arch is to be built in longitudinal rings three or four feet wide, each ring to be completed in a day, which is common practice in the construction of reinforced-concrete arches, the centering should deform or settle less than if built in transverse blocks, but may deform more than if built continuously for the full width of the bridge. When built in alternate transverse blocks, a settlement equal to 1-1/4 in. per 100 ft., and proportionally for larger or smaller span, is good practice, but a less settlement is desirable. For arches in longitudinal rings, the settlement should be less than half this amount, and for arches built up continuously from the springling line to the crown a smaller settlement should be gotten if possible. Since the settlement of the falsework varies almost directly as the distance from the foundation to the crown, arches of large span and rise and those over deep gorges or valleys should be built in alternate sections, whereas arches of small span and rise over flat ground may be built according to the other methods.

Two Types of Centerings of Bents. The first type (Fig. 47), which is used for concrete arches, generally is designed so that the load upon the lagging is transmitted to joists spaced from 10 to 24 inches on centers, supported by transverse caps resting upon vertical or inclined posts. In the second type, Fig. 45, which is generally used for stone arches, the lagging rests directly upon bows spaced from 3 to 6 ft. on centers, the bows being supported by the posts. In Fig. 45 the voussoirs are shown supported by wedges, but usually lagging is used in lieu of wedges. The first type is preferable for concrete bridges, because, upon the completion of the arches and the striking of the centering, lagging, studs, and wales of proper sizes are available for the balance of the concrete construction. The second type is better for stone arches, because by laying the lagging with open joints the bottom or soffit joints of the voussoirs are accessible and can be " packed with rope or rags " for grouting, the packing removed and the work pointed prior to the removal of the lagging. Further, this second method permits the placing of wedges under each voussoir, allowing small errors in the top curve of the centering or the top curve may be omitted and the curve of the soffit made by shimming pieces and wedges of variable heights, as in Fig. 45.

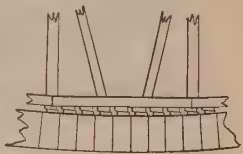


Fig. 45

Loads and Stresses. The centering should be designed to carry the weight of the arch ring alone, except in the case hereafter noted. In the case of concrete the nature of the load varies, the concrete being deposited as a semi-liquid. For unit weights see Sect. 10, Art. 17. The weight of the falsework is negligible, except in very high falsework. It is suggested that, in determining the load upon the centering, friction be neglected, as this is on the safe side and other assumptions reduce to an absurdity. Fig. 46 shows a voussoir or section of stone or concrete, *abcd*, supported by the lagging and by the masonry or strut *SS* below it. The weight *W* is resolved parallel and perpendicular to a tangent to the lagging at its middle point, giving a normal component *N*, which must be carried by the centering. Assuming that there is friction along *ab*, which reduces the intensity of *N*, it is evident that the reduction cannot equal *T* times the coefficient of friction, because, if such were the case, the arch ring from the joint *ab* to the springing line would have to be capable of supporting itself. Concrete when deposited being in a semi-liquid form would cause little friction along the joint *ab*, and the setting of the concrete could not modify the initial stresses.

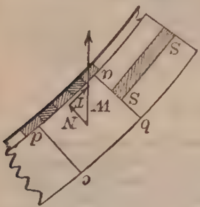


Fig. 46

In addition to the loads of the arch ring, the effect of wind pressure upon the falsework, prior to the keying of the arch, should be considered, and where the arch is over a stream, masonry footings should extend to above the high-water line, or other precautions taken to protect the falsework from injury, due to the pressure of ice or débris.

Information upon the Design of Centerings. The lowest 30° of a semicircular arch and of other arches of less rise exert little vertical pressure upon the falsework and may be built upon a templet or light falsework. In applying this general rule to an arch which is not semicircular, the 30° point is one at which a perpendicular to the tangent to the intrados makes an angle with the horizontal equal to 30°. Fig. 44, which is a design used in the construction of a brick arch, shows that a template was used in the lower portion of the arch.

The settlement of the centering may be computed approximately by using the rules given in Art. 12, but as arch centering should be, and generally is, of a better grade of carpentry work, an allowance is suggested of 1/32 inch for " taking up " and 1/32 inch additional for each joint, where end grain bears upon side grain, provided the unit stresses of Art. 1 are not exceeded. If particular care is not taken with the joints, such as adzing off rough places, the settlement may more nearly equal 1/16 in. for "taking up" and 1/32 in. for "biting."

At the Piney Branch Bridge, Washington, D. C., there were used six joints between the soffit and foundation, two of which were in end bearing (Fig. 47). Assuming first-class carpentry work, which was obtained, the settlement due to "taking up" and "biting" would be 6 × 1/32 + 2 × 1/32 = 1/4 in. To this must be added the columnar shortening which, on the basis of the formula in Art. 12, gives a total of 9/16 in., agreeing closely with the actual measurements. Measurements of other centerings show that the limits lie between 1/16 and 1/32 in., depending upon the grade of carpentry work. However, when the unit compression upon side grain exceeds the allowable unit stresses, the settlement of the centering is greater. In some parts of the Connecticut Ave. and the Edmondson Ave. bridge centerings where the stresses were more than 50% higher than allowed in Art. 1, the "bite" was 1/8 in. per joint, due in part to defective framing.

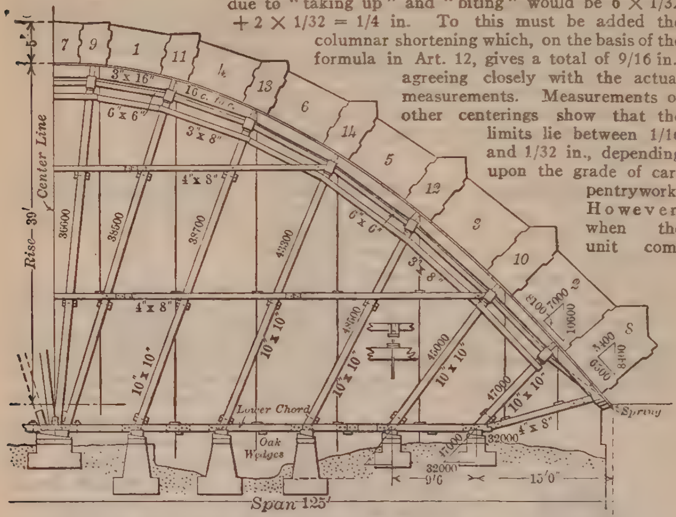


Fig. 47. Centering for Voussoir Arch

When the posts are placed vertically, and not radially or approximately radially, there is a lateral deformation of the centering as well as the vertical. When posts are vertical, struts must bear against the transverse caps, to prevent displacement under the radial pressure, or the joists resting upon the caps must be notched out and shimmed tight against the caps. Centerings are frequently designed with only one-half as much load on the outside posts of the same bent as upon the middle and intermediate ones and the cross-section of all timbers is made the same. As a result the center settles and the face forms, if the arch is of concrete, go out of plumb often as much as 1/2 in. in 5 ft., and if the arch is wide a crack may be formed. When the arch is to be built of concrete, the caps should extend about 3 ft. beyond the face of the arch ring to support the forms for the face of the arch and also to provide a walkway.

Sway-Bracing need not be computed for wind pressure, except for high arches. It should, as a rule, be heavier than such computations indicate to be necessary. A study of successful designs is suggested. Very little longitudinal bracing is required, except such girts as are necessary to take the horizontal component of the load of the arch when vertical posts are used. The use of steel, except for sway rods and in the form of plates or channels to increase end bearing, is not generally recommended. Steel is greatly affected by temperature and should not generally be used. However, in centerings of high arches the posts for 1/3 the height of the centering may be of steel, as their expansion or contraction will have comparatively little effect upon the total height of the falsework.

For a description of the centering of a large arch which collapsed due to insufficient bracing, see Eng. News, Vol. 77, p. 314.

Since centerings are designed for temporary work, the less permanent woods, such as hemlock and short-leaf pine, may be used for posts, lagging, and sway-bracing. Only hard wood should, as a rule, be used for sills, caps, and wedges, although Southern yellow pine is sufficiently hard for caps. Southern yellow pine is best for joists.

Layout. Having determined upon a centering of bents, the types of foundation, span and curve of the arch govern the layout. The distance between the posts at the top of the centering should be, in general, as great as the available depths of joist or, in case of a stone arch centering, the available depths of timber for the bows will permit. Joists cannot generally be obtained of larger dimensions than 6 × 16 or 4 × 16 and sometimes only 4 × 14 in. If rock or other suitable foundation can be obtained at shallow depths, Fig. 47 gives a good layout. If suitable foundation is at considerable depth and foundations are expensive, Fig. 48, which shows the center for the main arch of the Edmondson Avenue bridge at Baltimore, gives a good layout. In this design the railway track near the center of the arch was cleared, which somewhat modified the layout.

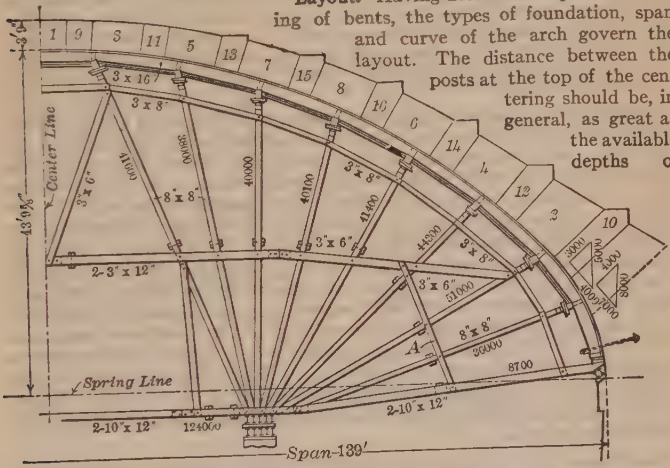


Fig. 48. Centering for Voussoir Concrete Arch

Fig. 44 shows a good way of supporting a centering upon an offset in the foundation masonry. Small arches upon high piers may be supported in a similar manner upon a projecting coping or upon projecting blocks of concrete or masonry.

Details for a Concrete Arch Centering. The **lagging** should be of tongued and grooved flooring, such as second-class Southern pine flooring, dressed one side and having a thickness of from 25/32 to 1 5/8 in., depending upon the distance center to center of joists. The deflection of the lagging under full load should not exceed 1/8 in. per ft. and preferably should be less. **Joists** may have their tops sawed to the curve of the soffit, or a spiking strip 2 in. wide may be used. This spiking strip may be of a low grade of lumber, and its use will result in a saving, particularly where the radius of the curve of the arch is less than 150 ft. **Posts,** when inclined, as in the case of the lower posts of Fig. 48, should be supported at one or more points by timber marked *A* in figure, otherwise they will sag materially and be inefficient as columns. Where end grain bears upon side grain, the end-bearing area may be increased by angles, as in Fig. 47, or by steel plates or channels. The number of joints in side-grain compression should be reduced to a minimum, and therefore for high centering continuous posts are preferred to independent stories.

It is expensive to cut bevels and miters at the ends of timbers and notches at intermediate points, and since the salvage of the timber is less when so cut, they should be omitted when possible. The difference in the cost of framing may be materially increased by failure to consider this point in design. Caps should extend over three or more posts. They should be drawn tight upon the posts by means of bolts or other fastenings and be tied in place by fishplates and bolts, by notches in the joists above them, or by continuous struts, as in Fig. 47.

Wedges may be placed at the tops of posts, as in Fig. 48, or at the bottom, as in Fig. 47. When placed at the top, they permit easier adjustment of the top before concreting, as there is less load to lift. When at the top they should be placed at right angles to the length of the cap, as in this position they are more easily driven than when placed parallel to the cap. Wedges at or near the top should not be loaded with more than 15 short tons per wedge. At the bottom they may be loaded to 20 short tons. With this loading they may be loosed with a 12-lb. sledge hammer. Wedges should be of hard oak with a bevel of 1 : 5 to 1 : 10, preverably the former. They need not be spiked in order to hold them in correct position relative to each other. Lubricants do not appear to facilitate striking, as they are forced out under the pressure of the arch load, and the friction to be overcome is due largely to the mechanical interlocking of the fibers. At Piney Branch Bridge the coefficient of friction of white-oak wedges was found to be about 0.4. Where wooden wedges are liable to become watersoaked, they may be oiled or otherwise weatherproofed.

Sand Boxes have been used extensively in Europe, but only in a few cases in America. They permit the centering to be lowered without jar and quickly if desired. Their first cost is high, but in large arches the cost of striking wooden wedges is so great that it is thought sand boxes result in economy. They cannot be used to raise the centering before it is loaded, in order to adjust the curve, and great care must be taken to keep the sand dry during the entire construction. The sand should be thoroughly washed and dried before being placed in the boxes. Fig. 49 shows details of the sand boxes used in the main arch of the Edmondson Avenue Bridge, Baltimore.

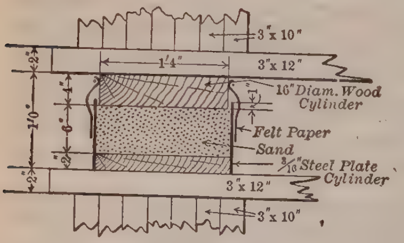


Fig. 49. Sand Box

This sand box consisted of a cylindrical shell built of 3/16-in. sheet steel inclosing, at the bottom of the cylinder, an oak bottom, which fitted snugly against the steel. The box was then filled with dry clean sand, upon which rested a circular oak plunger, which supported the bottom of the posts of the centering. Near the bottom of the box was a 1-in. circular hole, closed by a wooden plug, which was removed when it was necessary to lower the centering, permitting the sand to run out of the hole. The top of the cylinder was covered with several thicknesses of tar paper, in order to exclude rain. The maximum pressure upon the sand was 620 lb. per sq. in. The stress in the steel shell was figured on the basis of this pressure applied to each square inch of the shell, or on the basis of water pressure. (For comments on design and use of sand boxes, see Eng. Rec., Vol. 74, p. 531.)

Erection. A full-size drawing of the centering should be laid out upon a level floor, using transit and steel tape. Templets of thick paper or preferably of white pine should be made of all curved pieces and of the ends of all members having beveled or mitered ends, and the full length of each piece measured, intermediate notches being accurately located. The separate pieces of the centering should be laid out, using these templets and measurements. In making the layout drawing, the true curve should be first laid out and then a second curve drawn, adding sufficient camber to allow for the probable settlement of the falsework.

All carpentry work should be the best class of heavy timber work. Care should be taken to see that all saw cuts are true, and surfaces of abutting timber should be adzed off to make a snug fit. Just before the centering is loaded it should be inspected and all joints which have opened up should be shimmed tight, using shingles or thin wedges. During the construction of the arch similar wedging should be done, as the settlement of the falsework may open joints.

As large centerings are expensive and if destroyed by fire not only the centering but the arch also is lost, they are occasionally insured and given fire protection. At Connecticut Ave. Bridge, the centering for which cost about $50 000, a 4-in. water main was laid along the top of the construction trestle and a hose connection with hose attached provided opposite each large arch. The centering was watered every evening as an additional precaution. The wetting of the centering, however, tends to cause softening of the wood fiber of the side grain, and when used, somewhat lower stresses upon side grain should be used than are allowed in Art. 1.

For concrete arches the tongued and grooved lagging should be laid with a board omitted every 10 ft., to allow for the swelling of the flooring when it rains, otherwise much of the flooring will buckle and have to be relaid. If continuous forms are built to form the face of the arch ring, they should be sawed through with radial saw cuts at intervals of about every 25 ft.; otherwise when the centering settles these forms will twist and warp and an unsightly face results.

It is customary to speak of lowering the centering, but when the wedges at the top of the falsework are loosened, the centering actually rises. It has been held down by the weight of the arch, and when the wedges are driven out, the centering rises due to the elastic resilience of the wood. If the wedges are at the bottom of the falsework, the general movement of the falsework is downward, but the falsework is not really lowered until the timber is no longer under restraint.

It seems to be the common opinion that when wedges are moved and still remain tight the arch is lowering and following the centering. This usually is not true. Arches of concrete, built of alternate blocks, do settle very slightly, due to the fact that the concrete of each block shrinks in setting up, but in arches from 60 to 150 ft., of moderate rise, the settlement is seldom over 1/8 in. Arches built in longitudinal rings settle less.

If the temperature rises after the arch is keyed, the expansion of the concrete may be sufficient to develop arch action while the arch is still on the centering, and if so, probably no settlement will take place. Or if the falsework is weak, the arch will key itself tight, due to its own load. If the falsework is very wet when keyed, the arch may also develop arch action by the shrinkage of the timber due to drying out. If the temperature of the air falls 15° or 20° for any length of time after keying the arch and the centering is stiff, the contraction of the arch masonry may cause cracks to appear in the arch, while still upon the centering, unless the arch is reinforced with steel.

If an arch is built in alternate transverse sections, the falsework may have to carry loads in excess of the weight of the arch, due to the fact that on account of the shrinkage of the concrete in the several sections the arch ring consists of a number of separate blocks which are not quite in contact. A load, A, if placed upon the arch before striking the wedges, may have to be carried by the centering. At both the Connecticut Ave. arch and the Edmondson Ave. arch a settlement of about 1/4 in. took place in the crown when the load *A* was placed. After this there was no appreciable lowering of the crown, except that upon striking the settlement may have been as much as 1/16 in.

Details of a Stone Arch Centering. All the foregoing data are generally applicable to centering for stone arches. The lagging for stone arches may consist of timber from 3 to 6 in. thick and laid with open joints, as in Figs. 43–45. The wedges are placed under the lagging or at lower points. These figures also show typical bows which may be used for the trussed or the bent type of centering. As a considerable portion of the arch stones may be placed temporarily upon the falsework to decrease deformation without inconvenience to the contractor or added cost, the bowstring centering is somewhat better adapted to stone than to concrete arches.

14. Roof Trusses

Definitions and Description. The top chords of roof trusses, sometimes called the main rafters, usually support timbers called purlins, laid at right angles to the planes of the trusses. These purlins support timbers called common rafters, laid at right angles to the purlins or parallel to the main rafters. Upon the common rafters are laid boards called sheathing, and upon the sheathing a roof covering, such as tin, shingles, or terra cotta. The purlins may be laid close together and the common rafters omitted, or heavy planking may be used to span between main rafters and both purlins and common rafters omitted. This latter method is uneconomical and seldom used.

Trusses are usually placed from 12 to 16 ft. on centers, purlins 8 to 10 ft., and common rafters 1 1/2 to 2 1/3 ft. Purlins should be placed at or near panel points so as not to cause flexure in the top chord members. When the common rafters are omitted, the top chord members should always be computed for both compression and flexure. Purlins may be placed with their sides either vertical or at right angles to the main rafters. The latter method is slightly more expensive, except where the purlins are supported by steel hangers.

Roof trusses are designed to support the roof and snow load and withstand wind pressure and occasionally to support a ceiling or lower floors, which are hung from the trusses by means of hanger rods. For loads and computation of stresses see Sect. 12.

Weights of Trusses. The following formula, recommended by H. S. Jacoby (Structural Details, 1910), is based upon 121 designs of English and Belgium roof trusses of spans ranging from 48 to 196 ft., with a rise of one-sixth to one-third of the span, spacing of trusses 8 to 12 ft., a snow load of 25 lb. per sq. ft. and a normal wind load corresponding to a pressure of 40 lb. per vertical sq. ft.

$$W = 1/2\, al\, (1 + 0.15\, l)$$

in which W is the weight of a truss in pounds, a the distance between centers of trusses, and l the span, both a and l being expressed in feet. The above approximate weight of truss is suggested for use in design, but the weight of the truss, when designed, should be estimated and compared with the approximate one used.

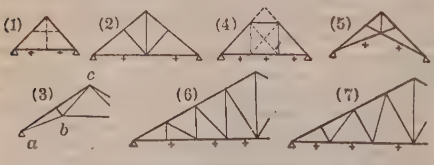


Fig. 50. Types of Roof Trusses

Common Types of Roof Trusses are shown in Fig. 50. In these diagrams the heavy lines indicate timbers, the lighter ones iron or steel ties, the light dotted lines counterbracing and the heavy dotted lines auxiliary timbers. When timbers are used as tension members they are marked with a plus sign. The ends of timber ties should be detailed to transmit tension, using steel straps or gusset plates and bolts for this purpose. Trusses with all tension members of iron or steel are called combination trusses.

Type (1) is used for temporary buildings such as sheds, the spans not exceeding 24 ft. The horizontal tie is generally supported at its middle by a tie rod or plank, shown by the dotted vertical line. An intermediate horizontal plank, shown by the dotted horizontal line, spiked to the main rafters, may be used to stiffen the truss. Type (2) is a king-rod truss, which is a modification of type (1), the rafters being supported by timber struts at their middle points. This type is used for 24- to 36-ft. spans. Type (3) is a combination truss with main rafters supported by struts at middle points, which may be used for spans of 25 to 45 ft. Joint *b* should be pin-connected, and ties *ab* and *bc* should not be in one length. Type (4) is a queen-rod truss suitable for spans between 24 and 36 ft. The middle panel should be counterbraced, as shown by light dotted lines, to take care of the wind pressure upon the roof and eccentric loading. When this type is used with a pitch roof, rafters are placed above the top chord, as shown by heavy dotted lines. For spans between 36 and 45 ft. struts should be placed to support the main rafters at their middle points. Type (5), called a scissor truss, is suitable for spans of 24 to 30 ft., and where a high ceiling is desired. Modifications of this are sometimes used for slightly larger spans. Type (6), called an English roof trusses, is used for spans from 40 to 60 ft., or for larger spans with an increased number of panels. For the shorter lengths it is built in 6 or 8 panels. Type (7), called the Belgian roof truss, may be used for the same spans as Type (6). In this truss the struts are perpendicular to the rafters and there are two less panels in the lower chord than in the upper one.

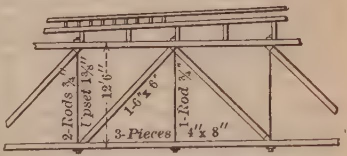


Fig. 51. Howe Roof Truss

Howe trusses are generally used for flat roofs, for spans from 40 to 130 ft. They are built with depths equal to from 1/7 to 1/10 of the span. Fig. 51 shows the middle portion of one or these trusses. The upper chord is horizontal, but the roof is made to have a low pitch by using short vertical struts of different lengths. There are no counterbraces, since there is no rolling load. This figure will apply to roofs of from 85- to 125-ft. span.

15. Design of a Roof Truss

Data of Design. (Fig. 52.) The truss will have a span of 48 ft., pitch 30°, distance center to center of trusses 16 ft. The roof covering will be of 3/16-in. slate laid upon wooden sheathing. The sheathing will be supported by common rafters resting upon purlins at or near panel points of the top chord. Slate roofing is selected in preference to tin, corrugated iron, or wooden shingles because of its permanence and low cost of maintenance, and in preference to clay tile because appearance is not desirable and slate is the cheaper. For slate roofs the pitch should not be less than 1 : 2 or 26° 30′. This rule for pitch may be applied to the other kinds of covering mentioned. The trusses were spaced 16 ft. apart on centers, as this is generally the maximum limit of span for untrussed purlins. This spacing therefore is the economical one. The roof has been designed to carry the dead load, a snow load of 10 lb. per square foot of horizontal area and a wind pressure of 30 lb. per square root of vertical area, which for a 30° pitch gives a pressure normal to the roof of 24 lb. per sq. ft. (see Sect. 12). The weight of the slate covering and sheathing was taken at 11 lb. per square foot of roof.

Unit Stresses. Southern pine, select grade, was used throughout. The allowable unit stresses for dry location are given in Art. 1 as follows: Tension with the grain = 1600; compression with the grain = 1175; compression across the grain = 325, for washers 50% greater or 490; extreme fiber stress in bending = 1600; shearing with the grain for joints = 176; horizontal shear in beams = 117.

Common Rafters. The rafters will be placed 2 ft. on centers, so that 7/8-in. tongued and grooved sheathing dressed one side may be used. Occasionally the sheathing may have to be figured as a beam to carry the load upon it, but usually it may be selected without computation. The vertical load upon the rafter consists of the weight of roof covering and sheathing, snow load and the weight of rafter assumed at 3.5 lb. per lin. ft., or $2 \times 9.25 \times 11 + 2 \times 8 \times 10 + 3.5 \times 9.25 = 396$ lb. The wind load acting perpendicular to the top of the rafter is $2 \times 9.25 \times 24 = 444$ lb.

The total load normal to the top of the rafter equals $444 + 396 \times \cos 30° = 787$ lb. The load parallel to the length of the rafter $= 396 \times \sin 30° = 198$ lb. (Fig. 52*e*). The latter component is transmitted directly to the purlin, although a part may be carried up through the common rafters to the apex of the roof. In this design it will be assumed that the entire component is taken by the purlins. To select the size of rafter, enter the beam table (Art. 3) with a load of $787 \times 1000/1600 = 492$ lb. since the table is based on a fiber stress of 1000 lb. For a span of 9 ft. a 1×6-in. rafter would be not quite strong enough, so that a 2×6-in. should be used. This will be wide enough for nailing the sheathing. If the rafter is investigated for bearing at the ends, horizontal shear and deflection, it will be found large enough.

Purlins. Since the load upon the purlins (Fig. 52*f*) is inclined, it is necessary to find the inclination of the neutral axis in order to find the true stresses. The following formulas from Jacoby's "Structural Details" are recommended:

$$\mathrm{Tan}\,\beta = (d^2/b^2)\tan\alpha \qquad I = 1/12\, bd\,[(d\cos\beta)^2 + (b\sin\beta)^2]$$

$$c = 1/2\,(d\cos\beta + b\sin\beta) \qquad S = Mc/I$$

in which α is the angle made by the load with the longer side of the beam and β is the angle between the neutral axis and the shorter side of the beam; **d** and **b** are the depth and width of the beam in inches; **I** is the moment of inertia; **c** is the distance from the neutral axis to the most remote fiber; **M** equals the bending moment in inch-pounds and **S** equals the maximum fiber stress per square inch.

The load normal to the top of a purlin exclusive of its weight, since there are 8 rafters in each length of purlin, is $787 \times 8 = 6296$ lb.; the load normal to the side of the purlin is $198 \times 8 = 1584$ lb. The weight of the purlin is approximately 22 lb. per ft., or 350 lb. Resolving this weight normal to the top and side of the purlin gives respectively 304 lb. and 175 lb. to be added to the foregoing loads, or the total load normal to the top = 6600 lb. and that normal to the side = 1760 lb. The total load on the purlin equals 6830 lb. and then $M = 6830 \times 16 \times 12/8 = 164\,000$ in.-lb., the load being assumed as uniformly distributed. Tan $\alpha = 1760/6600$. Assuming the dimensions of the purlin to be 8×10 in. and the 10-in. side normal to the main rafter, $\beta = 22° - 35'$, $I = 630$, $c = 6.15$, $M = 164\,000$ in.-lb. and $S = 1600$ lb. per sq. in. which is within the allowable unit stress.

Allowable Compression on Inclined Surfaces. Hankinson's formula for compressive stress on surfaces inclined to the fibers is

$$N = \frac{PQ}{P\sin^2\theta + Q\cos^2\theta} \text{ (see Art. 2),}$$

where θ is angle between the direction of the load and the direction of the grain. For designing joints in the present problem $P = 1175$, $Q = 325$, so that the equation may be written

$$N = 450/(1.382 - \cos^2\theta)$$

This is a convenient form for designing. Either this formula or the table in Art. 2 may be used in designing the timber joints. To facilitate design, the following table is computed from this formula:

$\theta°$	N lb. per sq. in.	$\theta°$	N lb. per sq. in.
0	1175	40	570
10	1090	50	465
15	1000	60	395
20	900	75	340
30	710	90	325

Truss Members. The maximum stresses are given in Fig. 52*c*. For the **top chord** the maximum stress is 36 000 lb. compression. The length of the chord per panel is 9 ft. 3 in. The allowed stress per square inch for columns under 11 diameters is 1175 lb. The approximate net area hence is 36 000/1175 = 31 sq. in. Use an 8 × 8-in. timber in order to allow for the cutting of daps and notches, holes for bolts and tie rods. Although theoretically smaller-sized timbers may be used in the upper panels, it will be cheaper to make the main rafter or top chord of one length, and even where a single splice is necessary, due to the length of rafter, the use of the same size of timber throughout the length is economical. For the **bottom chord** the maximum stress is 31 300 lb. tension and the allowed stress per square inch 1600 lb. To allow for the cutting of daps and bolt holes, particularly at the end joint, *a*, use an 8 × 8-in. timber. Care must be taken to consider actual and not nominal dimensions of timber.

The **strut** *Bc* has a maximum compressive stress of 7200 lb. Its length is 9 ft. 3 in. Since the column length is small, the side-grain compression of the chord by this member will determine its size. For the purpose of selecting the requisite size take the allowable bearing stress from the table above, the angle between the top chord and *Bc* being 60°. The requisite size is 7200/325 = 22 sq. in. Since this member will sag sightly, due to its inclination, use a 6 × 6 in. timber.

The **strut** *Cd* has a maximum compressive stress of 9500 lb. Its length is 12.0 ft. This strut, on account of its length, will be first computed as a column. Assume a 6 × 6-in. section. The l/d would be 12 × 12/6 = 24. The allowable stress as given in the table in Art. 3 is 755 lb. per sq. in. or an allowable load for the strut of 5-1/2 × 5-1/2 × 755 = 22 800 lb. For bearing across the grain of the top chord an area is required of 9500/325 = 29.2 sq. in., which is less than provided.

The **tie rod** *Bb*, if there is no ceiling to be supported as in this design, carries very little stress, its function being to prevent any sag in *ac*; a 5/8-in. rod will be used. The tie rod *Cc* is stressed 3600 lb. To select the proper steel rod, enter table in Art. 6 for recommended stresses in bolts. A 5/8-in. rod is too small, therefore use a 3/4-in. rod which is good for 4830 lb. It should be noted, in this table, that the gross area of a 5/8-in. rod is the same as the net area of a 3/4-in rod; therefore if the ends of a 5/8-in. rod are upset it will be strong enough. A 3/4-in. rod without upset ends will be used, because in small trusses where only a few rods are required they are often made in small local shops and the welds are not first-class. The tie rod *Dd* may be selected in a similar manner, a 1-1/4-in. rod being used; if the ends are upset a 1-1/8-in. rod will answer.

Joint C. The washer for the tie rod should be a beveled one (Art. 4). The end of the strut should be cut as shown in the figure so that the short bevel bisects the angle between the top chord and strut *Cd*. If so cut this bevel will make the same angle with the fiber of both the top chord and *Cd*, and the allowable unit stress will be the same for both the top chord and strut. If cut at any other angle the allowable stress will be less for one member and greater for the other and the lesser will govern. The angle made by the surface of the small bevel with the fiber of both chord and *Cd* equals 50°, the angle between the line of the load and the fiber is 40°, and the safe allowable pressure, from table above, is 570 lb. per sq. in. The pressure on the bevels is found by resolving the stress in the strut, 9500 lb., into two components, one perpendicular to the short bevel and the other perpendicular to the long bevel. This gives a pressure on the short bevel of 4700 lb. and on the long bevel of 6600 lb. Therefore the length of the short bevel must be 4700/570 × 6 = 1-3/8 in. Make the bevel 1-1/2 in. to allow for inaccurate framing. The surface of the long bevel makes an angle with the top chord of 7° 30′ and the safe compression from the table is about 330 lb. per sq. in. The length of the bevel is 5-1/2 in., and the pressure per square inch therefore equals 6600/6 × 5-1/2 = 200 lb., which is safe.

Joint D. The stress in the vertical rod is 14 400 lb., and the washer makes an angle of 30° with the fibers of the chord. The safe bearing from above table, and allowing an increase of 50% for washers, = 600 lb. per sq. in. The net area of washers required = 14 400/600 = 24 sq. in. A standard cast-iron washer can therefore be used, see Art. 4. The horizontal pressure at the joint is 18 700 lb., and the angle of the joint surface with the fibers is 60°. From the table the allowable compression is 710 lb. per sq. in. The net area of contact, deducting the area cut out by the bolt hole, must be at least 18 700/710 = 26.3 sq. in. The actual area is greater than this.

Joint d. The stress in the vertical rod is 14 400 lb. The washer presses against the side grain of the bottom chord, and therefore its net area must be at least 14 400/490 = 29.4 sq. in. Use a 6 × 6 × 3/4 washer (Art. 4). The bevel surface of the angle block makes an angle of 40° with the fiber of the block. The allowable compression from the table is therefore 465 lb. per sq. in., and the stress in *Cd* being 9500 lb., the necessary area is 9500/465 = 20.4 sq. in. The actual area is 36 sq. in. The bearing of the bottom of the block upon the side grain of the bottom chord is evidently sufficient. The dap in which the angle block is seated must be deep enough so that the difference between the horizontal components of the stress in *Cd* and in *C′d* can be transmitted when the wind blows on either side of the truss. The horizontal component equals 2700 lb. Now since both the block and chord bear on end grains, the depth of the dap must be at least 2700/1175 × 8 = 1/4 in. Use 1/2-in dap to allow for the inacccurate framing. In this design it is not necessary to compute the actual horizontal shearing stress in the block, due to the pressure of 2700 lb., as it is evidently very low.

Joint c. The washer was designed similarly to those of joints *D* and *d*. The short bevel at the end of the timber is on the line bisecting the angle between the lower chord and *Bc*, see joint *C*. The angle made by the surface of the short bevel with both the chord and *Bc* is 75° and the allowable compression for both chord and strut is 1000 lb. per sq. in. Therefore since the stress *Bc* is 7200 lb., the length of the short bevel must be 7200/1000 × 6 = 1-1/4 in., but 1-1/2 in. will be used to allow for inaccurate framing.

Joint a. The stress in the tie equals 31 300 lb. The allowable unit tension equals 1600 lb. per sq. in. Therefore the net cross-section of the tie must be at least 31 300/1600 = 20 sq. in. which requires a net depth of timber of at least 2-3/4 in. Using 3 in., the length of the heel bevel of the top chord will be 4.5/cos 30°, or 5-1/4″. The toe bevel in this design will be made 2 in. The total area of end bearing therefore equals (5-1/4 + 2) 7-1/2 = 54.4 sq. in. The angle made by the bevels with the fiber of the bottom chord equals 60°. Therefore, from table in this article, the allowable compression is 710 lb. per sq. in. Therefore the total allowable pressure on the bearing is 710 × 54.4 = 38 600 lb., which is greater than the thrust of the top chord. In order to take care of the horizontal component of the thrust of the top chord, the distance from the heel to the end of the lower timber must be such as to develop the necessary horizontal shear, or the length must be at least 31 300/6 × 176 = 30 in.

In this design the bolt tying the chords together is not figured to transmit stress, but is designed to hold the members together. The bolt at the end of the timber is introduced to increase the resistance against horizontal shear.

The bearing area on the post must be able to take the total reaction of the truss, or 21 600 lb. Since side grain bears on side grain, this area must be at least 21 600/325, or 67 sq. in. The block should therefore be at least 9 in. wide. However, since the truss will deflect under its load, the pressure on the inside of the block will be greater than on the outside. The pressure is not uniform, but the exact difference is unknown. An area of 25% in excess of that theoretically required is recommended, or 67 × 1.25

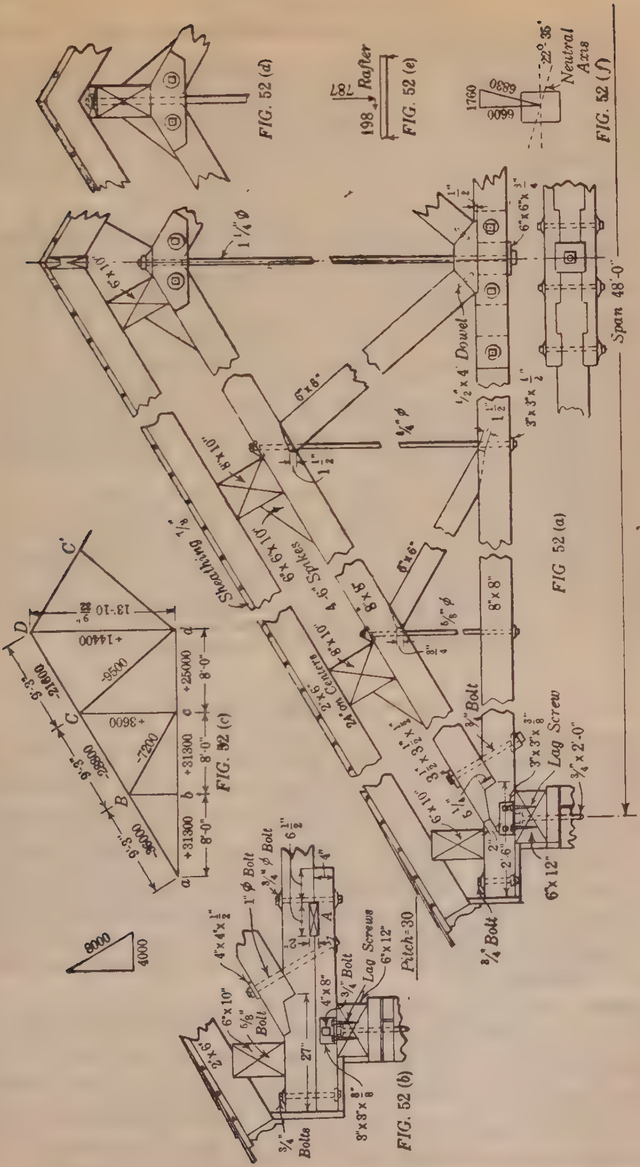


FIG. 52 (a)
FIG. 52 (b)
FIG. 52 (c)
FIG. 52 (d)
FIG. 52 (e)
FIG. 52 (f)
Span 48'-0"
Sheathing
Neutral Axis
Rafter
Dowel
Lag Screw
Lag Screws
Bolt
Bolts
Spikes
Pitch=30
24" on Centers
6"x10"
8"x10"
6"x6"
8"x8"
6"x12"
4"x8"
2"x6"
6"x6"x10"
8000
4000
+31300
+25000
+3600
+14000
-9500
-7200
-21600
-28800
-36000
8'-0"
9'-3"
13'-10"
1760
6880
6600
22°-35'

Fig. 52

= 84 sq. in. Since the chord is 8 in. wide, the block should be at least 11-1/2 in. long and a 6 × 12-in. block should be used.

If it is impracticable to get a length of 30 in. to take care of the horizontal shear, a bolt or bolts may be used to take part of the shear as at Fig. 52*b*. Assume that the length is 27 in., then the shear to be taken by the bolt equals 3 × 7-1/2 × 176 = 4000 lb. Resolving this horizontal stress parallel to the bolt and perpendicular to the tie member, as in Fig. 52*b*, gives a stress in the bolt of 8000 lb. Enter the table of recommended stresses for bolts (Art. 6), selecting a 1-in. bolt, which is a little larger than necessary. The top washer, which bears on side grain, should have a net area of 8000/490 = 16.4 sq. in. Use a 4 × 4 × 1 2 in. square plate washer.

A bolster will be used under the end of the truss so that it will not be necessary to cut into the bottom chord timber in order to get a proper bearing for the lower washer. This bolster will stiffen the truss and is particularly valuable when the support cannot be placed directly under the intersection of the neutral axes of the top and bottom chord timbers. The bolster will be made of a 4 × 8-in. timber. It must transmit to the lower chord not only the horizontal component of the pull of the bolts, but must also take the horizontal component of the wind pressure.

The design of the bottom washer is similar to those in joints *C* and *D*. A key of Southern pine will be provided at *A* to transmit the horizontal pressure which amounts to 4000 + 4200 = 8200 lb. The area of bearing of the key in both the bolster and lower chord members, since both the key and adjacent timbers bear on end grain, must be at least 8200/1175 = 7.0 sq. in., and the depth of the key must be 2 × 7/7.5 = 1.87 in. A depth of 2 in. will be used. The length of the key should be 8200/176 × 7.5 = 6.2 in., but 6-1/2 in. will be used. The distance of the end of the key from the end of the bolster will also be 6-1/2 in.

The key tends to rotate, the moment being equal to 8200 in.-lb., the arm of the horizontal forces acting on the top and lower halves of the key being 1 in. This moment compresses the top and bottom of the key and the adjacent chord and bolster timbers. The maximum compression $S = 6\,M/bl^2$ in which M is the rotating moment in in.-lb., b is the width of the key and l its length in inches, or $S = 6 \times 8200/(7.5 \times 6.5 \times 6.5) = 156$ lb. per sq. in., which is safe. The rotating moment also causes tension in the adjacent bolt. This bolt acts with a leverage of at least 1/2 the length of the key, or 3-1/4 in. The stress in the bolt will not exceed 8200/3.25 = 2520 lb. A 3/4-in. bolt, although stronger than theoretically necessary, will be used.

16. Howe Bridge Trusses

General Data. Timber bridges are generally of the Howe type, shown in Fig. 53. All members of this type are of timber except the vertical tension members and the lateral and sway tension members. The depth of the trusses of through bridges is generally governed by the necessary overhead clearance, which is 14 ft. for highways and from 17 to 25 ft. for railways. The weights of Howe trusses and of Howe bridges are about 1/3 larger than those of steel bridges of the same span, character of floor and class of loading. (See Sect. 12, Art. 9.) The stresses in the Howe truss are similar to those in the Pratt type, except that the diagonals carry compression instead of tension. The panels of the Howe truss are seldom over 10 or 12 ft. long and sometimes less. The truss is not suited for long spans, though it has been used for spans up to 250 ft.

Chords. The top and bottom chord sections of the trusses are built of from two to four pieces of timber varying from 3 by 8 in. to 8 by 16 in. in section. The greatest available length of these sections should be used in order to decrease the number of splices. Timbers 65 ft. long have been used for chord timbers. Fig. 53 shows a common method of packing the top and bottom chord timbers. Not more than one timber should be spliced in the same panel. Splices should be made at packing blocks. The simplest lower chord joint is the tabled fishplate type, and other styles are shown in Art. 6.

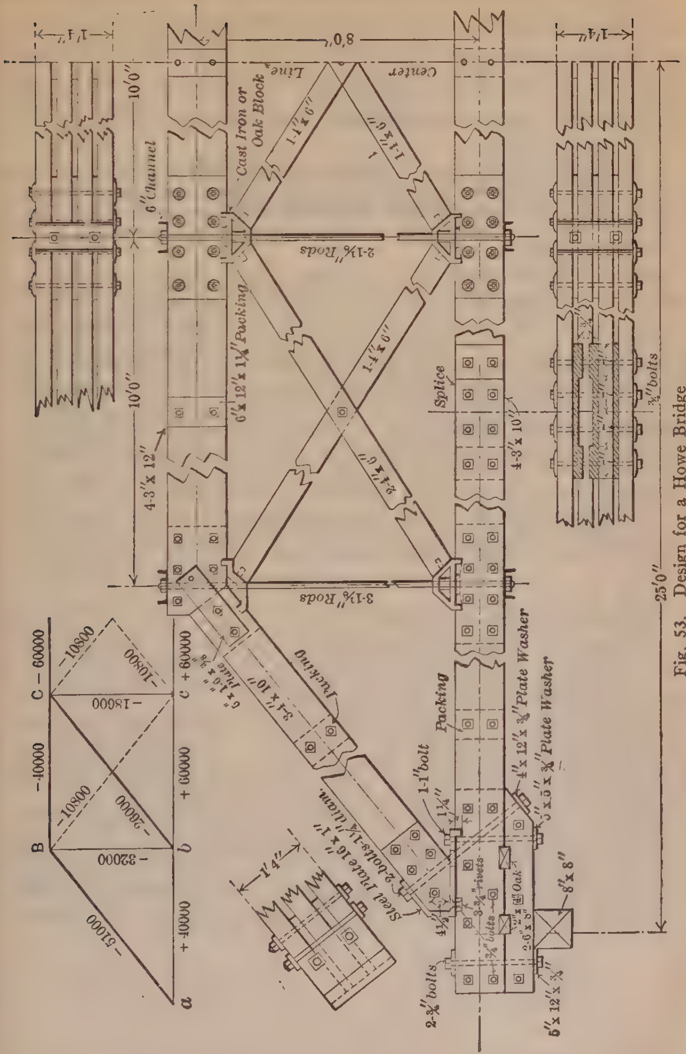


Fig. 53. Design for a Howe Bridge

Typical floors for timber bridges are shown in Fig. 38. For long bridges with heavy loads, more than one floor beam can be used per panel, and for greater strength they are made trussed beams. More than one rod can be used for the vertical tension members of a bridge of this size. The struts are usually made up of two sticks, with counterbracing in the panels made of one stick passing between the two of the strut. The counterbracing is needed to give the truss camber, and to take care of reversal of stress in the diagonal due to the moving loads.

Angle Blocks. Hollow or solid cast-iron or oak angle blocks are used to distribute the compression brought by the web members upon the side grain of the chords, and channel washers extending across the width of the chord as in Fig. 53 are often used in lieu of plate washers to transmit to the chords the pull of the vertical tie rods.

Design of a Howe Truss. The truss in Fig. 53 was designed for a highway bridge having a span of 50 ft. and a width of 16 ft. The bridge was designed for a live load of 100 lb. per sq. ft. and a 15-ton road roller. Southern pine, select grade, was used throughout. The sizes of the members were determined in a similar way to those of the roof truss (Art. 15). The same sizes of timbers are used in all panels of the top chord and all panels of the bottom chord, although theoretically uneconomical. In using the column tables or applying the column formula to the top chord or other built-up compression members, the separate pieces should be regarded as detached columns. When the separate pieces are well packed and bolted, they may act together as a single column, but on account of the shrinkage of the lumber it is not safe to depend upon such action. That chord which supports the floor beams, unless these are placed at panel points, should be computed for both flexure and direct stress. In this design, the floor beams are placed at panel points only.

Joint C. The stress on the vertical tie rod is 18 600 lb. The allowable bearing on the side grain of the top chord from the table of Art. 1 is 490 lb. per sq. in., allowing 50% increase under washers. The necessary net area of washer is 18 600/490 = 38 sq. in. Allowing for the bolt holes, the necessary gross area is 41 sq. in. A 6-in. channel the full width of the top chord, or 16 in. long, is used, as this size washer is needed at joint *B* and the same size will be used throughout.

A cast-iron angle block is shown in the figure, but in this design oak angle blocks might be used. When cast-iron blocks are used, their webs should be designed to transmit the thrust of the main brace and counterbrace. Counterbraces are shown by dotted lines in the diagram of Fig. 53. Each lug should be designed to be able to transmit the entire horizontal component of the main brace to the top chord.

The vertical component of the stress in the brace is 18 600 lb.; therefore the area of the base of the angle block should be at least 18 600/325 = 57 sq. in. In order to provide bearing for the brace and counterbrace the area of the brace will be made much larger. The horizontal component of the stress in the brace being 21 000 lb., the depth of the lugs, assuming that the packing pieces take their part of the horizontal component of the brace must be at least 21 000/16 $\times$ 1175 = 1-1/8 in. Ignoring the packing pieces, the depth should be 1-1/2 in., which depth will be used.

SECTION 10

MASONRY AND MASONRY STRUCTURES

BY

WALTER J. DOUGLAS

MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

* The late Prof. Ira O. Baker was associate editor of Section 6 of previous editions. The greater portion of the articles marked with an * has been retained as written by Professor Baker.

† H. A. Foster assisted in writing these chapters.

‡ Thomas C. J. Bailey, Jr., assisted in writing these articles.

§ E. E. Halmos assisted in writing these articles.

STONE QUARRYING AND CUTTING

1. Building Stone

Selection of Building Stone. In passing upon the suitability of unquarried rock for construction purposes, the engineer must decide the following points: (1) Whether or not the rock can be quarried in blocks of sufficient size for his purposes. This is dependent upon the frequency and character of the seams or joints. (2) Whether or not the stone will weather. This can best be determined by observing exposed faces, to some extent by determining the relative porosity of the stone and by obtaining advice of experienced geologists. The national government in Washington and practically all state governments have experts whose advice is readily obtainable. Similar help can be secured at most colleges and universities. (3) Whether or not the stone can be economically shaped for ashlar. This is dependent upon the hardness of the stone and the extent to which it may contain dry seams. The writer recalls a pier that was constructed from local stone which was so hard and so full of dry seams that the pier would have cost probably less than one-half had the stone been procured at a good quarry a thousand miles away. It was difficult in this case to keep stonecutters at work continuously over a week; their wrists would not stand the work. (4) Whether or not the stone can be effectively carved. Carving requires a fine-grain stone and one that is not very hard or brittle. Some granites can be carved almost as effectively as limestone.

In passing upon a quarry for crushed stone one should determine the following: (1) Its weathering qualities, hardness and durability. (2) The readiness with which it can be quarried. The inexperienced engineer should solicit the opinion of expert quarry foremen or obtain the academic advice of geologists. In any event, he should note the following: (*a*) Whether the rock can be readily drilled. It will be found that some stone contains hard material such as garnets, iron and flint which make drilling very expensive. (*b*) If the rock is fissured it may be difficult to blast. (*c*) If garnets, iron, flint, etc., are found in large quantities the crushing will be difficult and the crusher maintenance very high. (*d*) It should also be noted that some laminated stones break into flat slabby pieces which do not make first-class concrete or road aggregate.

Classification.* Building stones are classified according to origin (igneous, sedimentary, and metamorphic), and subdivided according to mineral and chemical composition, texture and structure, and geologic age. The most common granular crystalline-igneous rocks are granites, diorites, and gabbros or diabases; the more common dense varieties of volcanic origin are rhyolite, andesite, and basalt. Igneous rocks that have conspicuous crystals disseminated through a fine-grained to dense ground mass are called porphyries (e.g., granite-porphyry, rhyolite-porphyry). Sedimentary rocks include conglomerates (consolidated gravel), sandstones and many quartzites, shales (consolidated clays), limestones and dolomites. The more common metamorphic rocks are gneisses, schists, slates, some quartzites, and marbles. Varieties of any one of these rocks are distinguished by certain characteristic minerals (e.g., coarse-grained gneissoid granite, fine-grained thin bedded limestone).

Igneous Rocks.* The most abundant of the granular crystalline rocks is **granite** which consists mainly of feldspars and quartz with minor quantities of mica or hornblende, and minute quantities of certain other minerals.

* Largely from G. F. Loughlin.

Commercially, however, other granular igneous rocks, like **diorite** and **gabbro,** which consist mainly of soda-lime feldspar and hornblende or pyroxene, are referred to as "black granite." These rocks are commonly massive, although closely spaced fractures or joints parallel to the surfaces of some quarries resemble a bedded or stratified structure. **Gneisses** (granular metamorphic rocks similar to granites in mineral composition but distinguished by a foliated arrangement of mineral grains) are also called granite commercially. Granites as a whole are composed mainly of hard minerals that are extremely resistant to chemical weathering and are so firmly welded together by crystallization under great pressure that they make by far the strongest stone in common use. Granites that contain unusually large percentages of mica are comparatively soft, but are resistant to weathering if free from alteration. Gray and light-red granites are generally more durable, as they consist of minerals which are more resistant to weathering. True granite generally has the advantage of breaking with regularity and is readily formed into simple shapes, but is so much harder and tougher than most sedimentary rocks that the elaborate finishing of blocks is comparatively expensive; however, in structures of monumental character and those subjected to severe conditions of physical and chemical weathering it may be ultimate economy to use granite.

Sedimentary Rocks.* **Limestone** is widely distributed and is more widely used for building and crushing than any other kind of stone. It includes many varieties, which differ in color, composition, and adaptability for engineering and building purposes. Many limestones are dull gray or bluish gray and unattractive from an architectural standpoint. Some of these are massive and suitable for crushing, and sufficiently free from impurities to be used in metallurgical and chemical industries; but others contain so many shaly streaks and are so impure that they are of no value unless situated where they can be used with other material in the manufacture of Portland cement. The famous Indiana oölitic limestone, which contributes more than half of the building stone quarried annually in the United States, is a light buff to gray, porous, granular stone, similar to sandstone in mode of origin, although its "sand" grains are practically all minute shells or shell fragments. It is very easily quarried and worked, and is adapted for architectural and engineering uses where chemical weathering is not too severe.

Most limestones and marbles consist mainly of the mineral calcite which is slightly soluble in rainwater and in other waters or vapors containing acids and certain salts, especially ferric sulphate. Dolomite stone, or high magnesium limestone, which consists mainly of the mineral dolomite, is similarly affected but much more slowly. The effect of this corrosion is to roughen the surfaces of unevenly grained stones and gradually to remove small details of finish; but the rate of corrosion on the most exposed parts of buildings in the larger eastern cities of the United States is only about 1 inch in 600 to 800 years. It is much faster where attack by acid waters is more concentrated and continuous. The texture of limestones and dolomites varies from very porous to impervious, and their strength and resistance to weathering vary accordingly; but resistance to freezing depends not so much upon the degree of porosity as upon the size and distribution of the pores and the presence of impurities, especially certain varieties of clay minerals, that readily absorb and give off water with changing climatic conditions.

Sandstones * also vary greatly in color, texture, and usefulness. Their more common colors include light gray, buff, pink, red, and brown. Siliceous sand-

* **Largely from G. F. Loughlin.**

stones consist almost entirely of quartz grains cemented by silica, and the impervious variety of this composition, called quartzite, is extremely hard and difficult to work; but it is used especially in the manufacture of silica brick, and occasionally as crushed stone and building stone. Calcareous sandstones contain calcite (calcium carbonate) as the principal cementing material, and like limestones are noticeably affected by severe or prolonged chemical weathering. Argillaceous sandstone contains clayey material as its chief cementing constituent, and is subject to relatively rapid disintegration, especially if certain of the clay minerals are concentrated along layers. Ferruginous sandstone contains considerable red, brown, or yellow iron oxide in its matrix. Its durability depends mainly upon whether the stone is otherwise siliceous, calcareous, or argillaceous.

Metamorphic Rocks.* Gneiss, marble, quartzite, and slate have already been mentioned in connection with closely related igneous and sedimentary rocks. Mica and hornblende schists if too thinly foliated are of little use for anything but low-class rubble and riprap. The less foliated varieties, however, are locally crushed and used in concrete masonry.

2. Tests for Building Stone

Weight or Density is an important property since upon it depends to a large extent the strength and durability of the stone. Methods of determining the specific gravity are described in Sect. 7, Art. 25; weights of various classes of stone are given in Sect. 7, Art. 37.

Crushing Strength † may be determined as described in Sect. 7, Art. 25. In making tests of crushing strength, the position of the test specimen with reference to bedding planes or other planes of weakness should be carefully noted. As the crushing strength is usually highest when pressure is applied at right angles to the bedding (or to the rift), care should be taken to mark the direction of bedding, or of rift and grain, on the test specimens. (See Art. 4.)

If test specimens are dressed by hand, the concussion of the tool has a shattering effect, especially on small specimens. Tool-dressed 2-in. cubes usually show only about 60% of the strength of sawed and rubbed cubes of the same size, which most nearly represent the conditions of actual practice.

Values of crushing strengths of various classes of stones are given in Sect. 7, Art. 37. These show that granites and slates far exceed any requirements placed upon them in structures; that most marbles do so also; that limestones and sandstones have a great range in strength, even the weakest far exceeding the immediate pressure of the load placed upon them, but subject to some concern as they age. The strongest limestones may not be attractive architecturally, and the strongest sandstones (or quartzites) may be unattractive as well as very expensive to work; but both are quarried to a greater or less extent for crushed stone.

3. Durability of Building Stone †

Methods of Determining Durability. There are three methods of determining the durability of stone under different conditions: examination of old structures, mineralogical examination, and physical and chemical tests. The

* Largely from G. F. Loughlin.
† In part from G. F. Loughlin.

first, when feasible, is the best as it shows exactly how the stone has withstood conditions of natural weathering, pressure of heavy loads, abrasion, corrosion by acid waters, and any other special agencies that affect durability. The stones most resistant to weathering retain sharp edges and corners and delicate tool marks for many years, but there are exceptions to the general rule.

This uncertainty, however, can be cleared away to a considerable degree by mineralogical examination, which shows the minerals present, the manner in which they are held together, and the kind and degree of alteration undergone by the different minerals whereby their strength and durability may be affected. Such an examination will aid in the selection of physical and chemical tests that are necessary to determine those properties of the stone that are of critical importance.

The use of physical and chemical tests is based upon the assumption that the relative durability of stones is proportional to their crushing strength, their absorptive power, their resistance to freezing, and their solubility in acids; but in the making of the tests each element acts by itself, whereas in the structure the stone is exposed to the combined action of all these methods of attack, and their action may be, and frequently is, different from that when acting separately. The principal tests made to determine the durability of a stone are the absorptive power, the solubility in sulfuric and hydrochloric acids separately and combined, the resistance to fire, the loss due to freezing and thawing, and the artificial freezing test, which consists in soaking the sample in a solution of sulfate of soda and then hanging it up to dry, the crystallizing of the salt in the pores of the stone having an effect somewhat similar to that of freezing of water. None of these tests gives wholly satisfactory results. (For description of tests see Sect. 7, Art. 25.)

Increase in Durability may be secured by proper seasoning and by finishing the surface in a suitable manner. All stones, and especially limestones and sandstones, when first quarried contain considerable quarry sap; and when full of sap the stone works somewhat more easily under the tool than when well seasoned. If a stone freezes while full of quarry sap, it is nearly certain to crack; but if it is first allowed to season it is not likely to be appreciably damaged by a single freezing. The hardening by seasoning also adds materially to the weathering qualities of the stone. The increased strength and durability caused by seasoning are due to the fact that the sap holds in solution a small amount of calcareous or siliceous matter, and by the process of seasoning this material is drawn to the surface and is deposited in the pores of the stone by the evaporation of the sap, the matter in solution thus becoming an additional cementing material and binding the grains more firmly together. It is surprising that so small an amount of liquid can produce so marked an effect; but it has been in the stone for such an extremely long time that it has formed around each mineral grain a "colloidal" film of mutually dissolved water and mineral, thus accounting for the softness of the newly quarried stone. When this film has become thoroughly dried, its mineral matter adheres firmly to the grains and matrix, thus increasing the cohesive strength of the stone. When this mineral matter is once deposited it is not readily dissolved again and the stone will not regain its original softness or tendency to injury by frost except when in contact with ground-water for a very long time, unless it contains a large percentage of extremely fine pores. Certain minerals of the clay group are exceptions to this statement, as they both give up and take on water very readily. It is the presence of these minerals that renders argillaceous limestones and sandstones and certain altered igneous rocks subject to rapid disintegration.

The method of dressing a stone may have a marked influence upon the details of weathering. If the surface is finished with a tool similar to the bush hammer, a too severe application of the tool may produce minute cracks that render the tooled surfaces subject to injury by frost; or by crystallization in them of salts leached from mortar or concrete backing. This is especially true of some of the softer granites that contain considerable mica, both as distinct flakes and as microscopic inclusions in feldspar. If the granite is otherwise satisfactory, disintegration due to such causes does not penetrate beyond the limits of these minute cracks, and granites from which the finely hammered surfaces have crumbled in a short time have continued in use as much as 25 years without showing any further effects of weathering. The choice of finished surfaces should be made with a view toward the preservation of details of appearance rather than the durability of the stone as a whole.

Many methods have been devised for preventing or checking the action of the weather upon building stones; but none of them is entirely satisfactory. These preservatives consist of some liquid into which the stone may be dipped or which may be applied with a brush to its outer surface, to fill the pores and prevent the access of moisture. Paint, coal tar, linseed oil, paraffin, and numerous chemical preparations have been used. With porous stone a 10% solution of paraffin in gasoline has been found effective. (See "Exposure Tests on Colorless Waterproofing Materials" by D. W. Kessler, Bureau of Standards Technologic Paper No. 248, 1924.)

Distinctly laminated stone should be placed on its natural bed to obtain its maximum resistance to pressure and to the disintegrating effects of frost action. The advantages gained by placing granite and other strong stone so as to obtain greatest resistance to pressure is too slight to be of importance. Although the Indiana oölitic limestone has a distinct bedding it is not highly laminated and in most structures may be used indiscriminately on bed or on edge. Sandstone should usually be placed on its bed to prevent disintegration.

4. Methods of Quarrying

Hand Tools. When the stone is thin-bedded, it may be quarried by hand tools alone. The principal tools are pick, crowbar, drill, hammer, wedge, and plug and feathers (Fig. 15). The **plug** is a narrow wedge with plane faces, and the **feathers** are wedges flat on one side and rounded on the other. When a plug is placed between two feathers, the three will slip into a cylindrical hole; and if the plug is then driven, it exerts a great force. If these plugs and feathers are placed a few inches apart in a row, and all driven at the same time, the stone will be cracked along the line of the holes, even though it is comparatively thick. The **drill** ordinarily used to cut the holes for the plug and feathers is a bar of steel furnished with a wide edge sharpened to a blunt angle and hardened. It is operated by one man, who holds the drill with one hand and drives it with a hammer in the other, rotating the drill between blows. The holes are usually from 3/8 to 3/4 in. in diameter. Sandstones and limestones occurring in layers thin enough to be quarried as above are usually of inferior quality, suitable only for slope walls, paving, riprap, and concrete.

When the stone is hard a **pneumatic drill** is generally used to make the holes for the plug and feathers. It is handled by one man, and the power may be furnished by a fixed compressed-air plant or a portable compressor with gasoline or electric drive.

Channeling consists in cutting long narrow channels in the rock to free the sides of large blocks of stone. A channeling machine consists of an engine running on a track on the floor of the quarry and operating a drill bit which cuts the channel as the machine is run back and forth along the track. The drill is sometimes a rotary steel or diamond bit, but is usually a percussion bit. The percussion channeler is ordinarily employed for marble, massive limestone, and thick-bedded sandstones; and the diamond drill for the harder stones. With the percussion channeler, the channels are 1 to 3 in. wide according to depth, and may be 10 to 14 ft. deep, different lengths of drill rods being used as the depth of the channel increases. The diamond channeler first bores a series of holes close together, and then bores or channels out the partitions between the holes. Some channeling machines can operate at any angle with the vertical, even horizontal. The channeler is generally operated by steam, and in that case each machine has its own boiler. Compressed air is also used. A special type of operation is by the "electric-air" machine, in which a "pulsator," operated by electric motor, forces air back and forth to the two ends of the drill cylinder.

Quarrying with Explosives. In this method drill-holes are put down to the depth to which the rock is to be split, and the requisite amount of powder or other explosive put in, covered with sand, and fired by a fuse. Sometimes numerous charges in a line of drill-holes are fired simultaneously by means of electricity. Coarse gunpowder is the explosive ordinarily used, since the quick-acting explosives, like nitroglycerine and dynamite, have a tendency to shatter the stone and break it in many directions. For quarrying each class of rock there is a characteristic method employed, which is, however, varied in detail in different quarries. The minor details of quarry methods are as various as the differences existing in the textures, structures, and modes of occurrence of the rocks quarried. Even such an apparently unimportant matter as the form of the bottom of the drill-hole into which the explosive is put has a very marked effect. If bored with a hand-drill, the hole is generally triangular at the bottom, and a blast in such a hole will break the rock in three directions. In some quarries the lines of fracture are made to follow predetermined directions by putting the charge of powder into canisters of special forms.

Drills are of three forms. The **jumper** is a drill similar to that used for drilling holes for plugs and feathers, except that it is larger and longer. It is usually held by one man, who rotates it between the alternating blows from hammers in the hands of two other men. **Churn drills** are long, heavy drills, usually 6 to 8 ft. in length. They are raised by the workmen, let fall, caught on the rebound, raised and rotated a little, and then dropped again, thus cutting a hole without being driven by the hammer. They are more economical than jumpers, especially for deep holes, as they cut faster and make larger holes than hand-drills. **Machine rock drills** bore much more rapidly than hand-drills, and also more economically, provided the work is of sufficient magnitude to justify the preliminary outlay. They drill in any direction, and can often be used in boring holes so located that they could not be bored by hand. They are worked either by steam directly, or usually by compressed air.

Rock-drilling machines use either percussion or **rotary** drills. The method of action of the **percussion drill** is the same as that of the churn drill already described. The usual form is that of a cylinder, in which a piston is moved by steam or compressed air, and the drill is attached to this piston so as to make a stroke with every complete movement of the piston. An automatic device causes it to rotate slightly at each stroke. A well-known type of percussion drill is the "**jackhammer,**" which is obtainable in weights of 27 to 75 lb. This is a one-man drill, and can be used for all types of work. Larger drills (known as "drifters") are generally mounted on tripods. When holes are to be drilled close together, as in channeling, the "quarry-bar" is used. This is a tubular bar, mounted on adjustable legs at each end, the drifters being supported on the bar in such a way that they can be moved back and forth into position, thus keeping all holes in a straight line.

Of the **rotary drills** there are two forms, one in which the cutting is done by teeth on the end of a hardened steel tube and the other by diamonds. In either case the drill-rod is a long tube, revolving about its axis, kept in contact with the rock, and by its rotation cuts in it a cylindrical hole, generally with a solid core in the center. The drill-rod is fed forward, or into the hole, as the drilling proceeds. The débris is removed from the hole by a constant stream of water which is forced to the bottom of the hole through the hollow drill-rod, and which carries the debris up through the narrow space between the outside of the drill-rod and the sides of the hole. The **diamond drill** is the only form of rotary rock-drill extensively used in America. The tube has a head at its lower end, in which are set a number of carbons or black diamonds. The diamonds usually project slightly beyond the circumference of the head, which is perforated to permit the ingress and egress of the water used in removing the débris from the hole and at the same time prevent the head from binding in the hole. When it is desirable to know the precise nature and stratification of the rock penetrated, the cutting points are so arranged as to cut an annular groove in the rock, leaving a solid core, which is broken off and lifted out whenever the head is brought up. Where it is not desired to preserve the core intact, a solid boring-bit is used instead of the core bit. They are made of any size up to 15 inches in diameter.

Dynamite is the name given to any explosive which contains nitroglycerine mixed with a granular absorbent. If the absorbent is inert, the mixture is called true dynamite; if the absorbent itself contains explosive substances, the mixture is called false dynamite. Dynamite is exploded by means of sharp percussion, which is applied by means of a cap and fuse. The cap is a hollow copper cylinder, about 1/4 in. in diameter and an inch or two in length containing a cement composed of fulminate of mercury and some inert substance. The cap is called single-force, double-force, etc., according to the amount of explosive it contains. **True dynamites** must contain at least 50% of nitroglycerine, otherwise the latter will be too completely cushioned by the absorbent, and the powder will be too difficult to explode. **False dynamites,** on the contrary, may contain as small a percentage of nitroglycerine as may be desired, some containing as little as 15%. The added explosive substances in the false dynamites generally contain large quantities of oxygen, which are liberated upon explosion, and aid in effecting the complete combustion of any noxious gases arising from the nitroglycerine. The false are generally inferior to the true dynamites, since the bulk of the former is increased in a higher ratio than its power; and as the cost of the work is largely dependent upon the size of the drill-holes, there is no economic gain. High-power dynamites are required for hard and refractory rock, but for the softer rock a low-power is better. In quarrying, 40% dynamite is ordinarily used.

Gunpowder is sold in kegs of 25 lb. Dynamite is sold in cylindrical, paper-covered cartridges, from 7/8 in. to 2 in. in diameter and 6 to 8 in. long, or longer, which are packed in boxes containing 25 or 50 lb. each. They are furnished, to order, of any required size.

Methods of Quarrying are generally quite different for granites than for the softer limestones and sandstones, being determined largely by the existence or non-existence of joint planes or, in other words, the facility with which the rock can be split. Granites generally have no natural bedding planes, but they usually show a tendency to split more readily in one direction (called the "rift") than in any other. There will also be a plane at right angles to the rift along which splitting is not quite as easy, which is called the "grain" or "run." The direction in which resistance to splitting is greatest is called the "head" or "hardway" (at right angles to rift and grain). In many granite quarries the rift is horizontal or nearly parallel to the surface; in others it may be nearly vertical. Granites that are divided by seams or joints into nearly horizontal layers or sheets which are successively thicker with increasing depth are quarried by cutting each layer into blocks of desired size by wedging or channeling.

In some granite quarries where the ledge is dome-shaped and seams are very scarce, artificial sheets are separated from the mass, just as an onion is peeled, by the successive use of powder and compressed air (Fig. 1). "In the center of the sheet or area to be 'lifted' a drill hole 3 or 4 in. in diameter is sunk to a depth of 5 to 8 ft., depending on the greatest thickness of stone required and the contour of the surface at that particular point. The bottom of the hole is then enlarged into a pocket by exploding half a stick of dynamite, as shown in Fig. 1. A small charge of powder, about a handful, is then exploded in the pocket, thus starting a horizontal crack or cleavage across its greater diameter. Charges increasing in size are then exploded in the cavity, the drill hole being plugged at each blast to confine the powder gases and thus exert a more or less constant force upon the stone.

"After the cleavage has extended to a radius of 75 or 100 ft. in all directions from the lift hole, a pipe is inserted into the hole, tamped tight and connected by means of a globe valve to the pipe line. Compressed air at 70 to 80 lb. pressure is gradually admitted and the cleavage rapidly extended until it comes out upon the sloping face of the quarry in a thin edge, as indicated in Fig. 1. A sheet of granite several acres in extent may be raised in this manner, affording a bed plane approximately level to which the quarrymen can work, thus securing stone of any required thickness." (From pamphlet of North Carolina Granite Corp.)

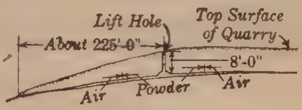


Fig. 1

The sizes of stones that may be quarried by this method are limited only by the handling and transporting equipment. Where horizontal sheeting is absent and the granite is divided into natural blocks by steeply inclined joints, quarrying consists of removing the smaller blocks and cutting the larger blocks into smaller sizes, taking all possible advantage of rift and grain.

In Sedimentary Formations * the beds or strata may lie nearly horizontal or may be inclined at any angle. In some quarries single beds are distinctly separated from one another by open bedding planes or partings, and in others there are few or no such separations and the formation may for practical purposes be regarded as one bed. Marble, limestone, and sandstone are commonly quarried by cutting a bed or layer over a considerable area into rectangular blocks by channeling, and by loosening these blocks, if necessary, from the underlying rock by wedging. Where the dip of the beds is at a considerable angle the channeling may be done on the inclined surface of the beds; but in many quarries it is conducted on a horizontal plane and the dimensions of the quarry blocks are quite independent of bedding. Bedding is usually distinct in the ledge and in large quarry blocks, but may be quite inconspicuous in small specimens.

In channeling the softer stones, care must be taken not to operate the machine so rapidly as to injure the stone. If the blows of the channeler are too violent, the adjacent rock may become overstressed, with the formation of minute seams which may open up when exposed to the weather or may cause trouble when the stone is being carved. This effect is called " stunning."

For references on quarrying, see "Practical Stone Quarrying" by Greenwell and Elsden; "Sandstone Quarrying in the United States," by Oliver Bowles, Bull. No. 124, U. S. Bureau of Mines.

For Reducing the Large Quarry Blocks to proper sizes for the shop, the quarry foreman is furnished a list of pieces required, with the dimensions of each piece, these dimensions including necessary allowance for finishing the surfaces. Each piece on the list is numbered for identification. The quarry blocks are then cut to the sizes given on the list, this subdivision being done by drilling holes along the required lines, and driving wedges or plugs-and-feathers in the holes. The small stones are marked with the same numbers as on the foreman's list, and are then sent to the finishing shop.

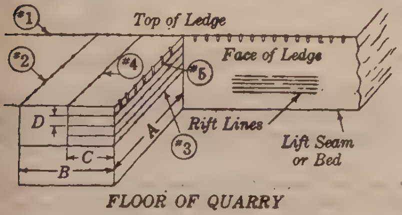


Fig. 1a

Where many small pieces of the same size are required, as for curb stones or paving blocks, the large quarry blocks are cut out originally to sizes which are multiples of

* In part from G. F. Loughlin.

the rough dimensions required for the small pieces. Fig. 1*a* (from J. D. Sargent) illustrates a method used in one of the large granite quarries for getting out curb-stones. Dimension *A* is the rough length of a single curb-stone, *B* is twice the rough height of one curb ($B = 2C$) and *D* is the thickness of the curb before it is finally dressed. The stone is broken successively along the planes numbered 1-5, by drilling and wedging.

When the quarry blocks are not too thick, the sub-dividing may be done as illustrated in Fig. 1*b* (from J. D. Sargent). This shows the method used in preparing granite paving-blocks. A flat slab is first broken out of the quarry, with a thickness of 8 in. (twice the thickness of a rough block) and with a length and width as shown. This slab is then marked on the flat side across its center (transversely) with a sharp chisel, and is broken cleanly along the chisel line by striking a hard blow on the back with a 30-lb. hammer directly opposite the chisel line. This method of breaking the stone is called "knapping." The final subdivision into the small blocks may be done by splitting the stone along the rift or grain with a sharp chisel.

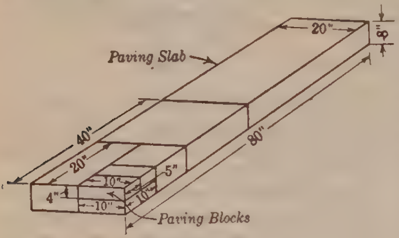


Fig. 1*b*

5. Tools for Stonecutting

A knowledge of the tools used in stonecutting is necessary for an understanding of the methods of preparing stones and is also necessary for a recognition of the names of the various kinds of dressed surfaces. The following names and description of tools were first proposed in 1877 by a committee of the American Society of Civil Engineers and have been widely adopted and used:

The **Double-Face Hammer** (Fig. 2) is a heavy tool weighing from 20 to 30 lb., used for roughly shaping stones as they come from the quarry and for knocking off projections. This is used only for the roughest work.

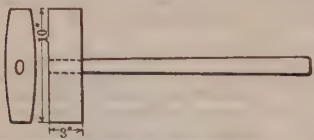


Fig. 2. Double-Face Hammer

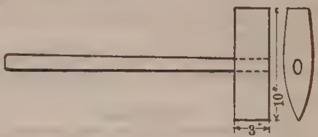


Fig. 3. Face Hammer

The **Face Hammer** (Fig. 3) has one blunt and one cutting end, and is used for the same purpose as the double-face hammer where less weight is required. The cutting end is used for roughly squaring stones, preparatory to the use of finer tools, and also for splitting stones.

The **Cavil** (Fig. 4) has one blunt and one pyramidal, or pointed, end, and weighs from 15 to 20 lb. It is used in quarries for roughly shaping stone for transportation.

The **Pick** (Fig. 5) somewhat resembles the pick used in digging, and is employed for rough dressing, mostly on limestone and sandstone. Its length varies from 15 to 24 in., the thickness at the eye being about 2 in.

The **Ax** or **Peen Hammer** (Fig. 6) has two opposite cutting edges. It is used for splitting and for making drafts around the arris, or edge, of stones, and in reducing faces, and sometimes joints, to a level. Its length is about 10 in. and the cutting edge about 4 in. It is used after the point and before the patent hammer.

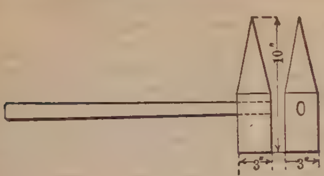


Fig. 4. Cavil

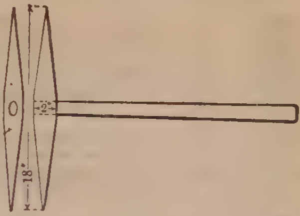


Fig. 5. Pick

The Tooth Ax (Fig. 7) is like the ax, except that its cutting edges are divided into teeth, the number of which varies with the kind of work required. This tool is not used on granite and gneiss.

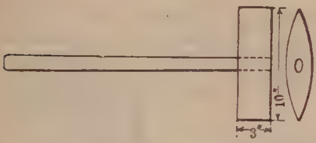


Fig. 6. Ax or Peen Hammer

Fig. 7. Tooth Ax

The **Point Bush Hammer** is a square prism of steel with ends cut into a number of pyramidal points. The length of the hammer is from 4 to 8 in., and the cutting face from 2 to 4 in. square. The points vary in number and in size with the work to be done. One end is sometimes made with a cutting edge like that of the ax.

The **Crandall** (Fig. 8) is a malleable-iron bar about 2 ft. long, slightly flattened at one end. In this end is a slot 3 in. long and 3/8 in. wide. Through this slot are passed ten double-headed points of 1/4-in.-square steel, 9 in. long, which are held in place by a key.

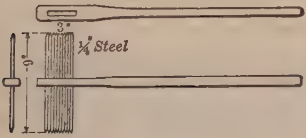


Fig. 8. Crandall.

Fig. 9. Patent Hammer

The **Patent Hammer** or **Bush Hammer** (Fig. 9) is a double-headed tool so formed as to hold at each end a set of wide thin chisels. The tool is in two parts, separated by about 7/8 in., which are held together by the bolts which hold the chisels. Lateral motion is prevented by four guards on one of the pieces. The tool without the teeth is 5-1/2 by 2-3/4 by 1-1/2 in. The teeth are 2-3/4 in. wide. Their thickness varies from 1/12 to 1/6 in. This tool is used for giving a finish to the surface of stones.

The **Bush Chisel** has its end similar to one-half of the patent hammer, and may be used by hand or with the pneumatic surfacer.

The **Pitching Chisel** or **Chipper** (Fig. 10) is usually of 1-1/8-in. octagonal steel, spread on the cutting edge to a rectangle of 1/8 by 2-1/2 in. It is used to make a well-defined edge to the face of a stone, a line being marked on the joint surface to which the chisel is applied and the portion of the stone outside of the line broken off by a blow with the hand hammer on the head of the chisel.

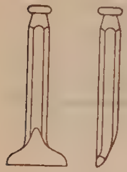


Fig. 10. Pitching Chisel

Fig. 11. Point

Fig. 12. Chisel

The **Point** (Fig. 11) is made of round or octagonal rods of steel, from 1/4 in. to 1 in. in diameter. It is made about 12 in. long, with one end brought to a point. It is used until its length is reduced to about 5 in. It is employed for dressing off the irregular surface of stones, either for a permanent finish or preparatory to the use of the ax. According to the hardness of the stone, either the hand hammer or the mallet is used with it.

The **Chisel** (Fig. 12) of round steel 1/4 to 3/4 in. in diameter and about 10 in. long, with one end brought to a cutting edge from 1/4 in. to 2 in. wide, is used for cutting drafts or margins on the face of stones.

The **Tooth Chisel** (Fig. 13) is the same as the chisel, except that the cutting edge is divided into teeth. It is used only on marbles and sandstones.

Fig. 13. Tooth Chisel

The **Splitting Chisel** (Fig. 14) is used chiefly on the softer stratified stones, and sometimes on fine architectural carvings in granite.

Fig. 14. Splitting Chisel

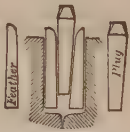


Fig. 15. Plug and Feather

The **Plug**, a truncated wedge of steel, and the **Feathers** of half-round malleable iron (Fig. 15), are used for splitting unstratified stone. A row of holes is made with the **Drill** (Fig. 16) on the line on which the fracture is to be made, in each of these holes two feathers are inserted,

Fig. 16. Drill

and the plugs are lightly driven in between them. The plugs are then gradually driven home by light blows of the hand hammer on each in succession until the stone splits.

Machinery is used to a large extent in all modern stonecutting plants, and consists of pneumatic drills and hand tools, pneumatic surfacing machines, saws, polishing machines, lathes, carborundum machines, and sand-blast equipment. Saws are used in cutting practically all kinds of stone. The reciprocating saw is a thin blade of soft steel, either flat or with vertical corrugations, which cuts by abrasives applied with water between the blade and the stone. Circular saws generally have teeth, either diamonds or carborundum blocks, the diamond saw being used largely for limestone. For thinner stones, a solid carborundum wheel or carborundum steel-center wheel may be used. For round columns, either plain or fluted, and balusters for stone railings, the lathe is used. Carborundum cutting-wheels are used in cutting moldings and for rabbetting, especially in marble and limestone. Hand carving is done largely with pneumatic tools. Machinery has replaced handwork to a very large extent, especially in marble and limestone; but handword is still required for the more intricate details, and especially in finishing architectural granite.

The Sand-blast Process has been used for a number of years in the granite memorial industry, but is equally applicable to the finishing and carving of granite for general structural and architectural purposes. It consists essentially of an intaglio or etching process, whereby a design is engraved or cut into the flat surface of the stone. The face of the stone is first dressed by any of the methods described in Art. 6. The finished surface is then covered with a plastic protective coating, in the form of sheets or melted and poured on, which when hard has a rubber-like consistency. The design to be etched in the stone is drawn or traced on the coating. The coating is then cut out so as to leave the granite accessible to the blast in accord with the design. A blast of white silica sand is blown against the coated surface, and quickly cuts into the area from which the rubber-like coating has been removed, while the portion of the surface which was protected by the coating is left untouched. After the blasting has been carried far enough, the composition coating is peeled off the stone, leaving the ornament as a clean-cut etching. Usually the design is finished with hand tools. If it is desired to have lettering or ornamentation raised above the general surface of the stone, the coating is cut and the background of the design is removed, leaving the design that was protected by the coating. This work is generally done at the quarry shop, though portable equipment is available.

6. Dressing the Surface

Building stones are divided into three classes according to the finish of the surface: unsquared stone, squared stone, and cut stone.

Unsquared Stones include all stones which are used as they come from the quarry without other preparation than the removal of very acute angles and excessive projections from the general figure. The term " backing," which is frequently applied to this class of stone, is inappropriate, as it properly designates material used in a certain relative position in a wall, whereas stones of this kind may be used in any position.

Squared Stones include all stones that are roughly dressed on beds and joints. The dressing is usually done with the face hammer or ax, or in soft stones with the tooth hammer. In gneiss it may sometimes be necessary to use the point. The distinction between this class and the third lies in the degree of closeness of the joints. Where the dressing on the joints is such that the distance between the general planes of the surfaces of adjoining stones is 1/2 in. or more, the stones properly belong to this class. Three subdivisions of this class may be made, depending on the character of the face of the stones: **Quarry-faced Stones** are those whose faces are left untouched as they come from the quarry. **Pitch-faced Stones** are those on which the arris is clearly defined by a line beyond which the rock is cut away by the pitching chisel, so as to give edges that are approximately true. **Drafted Stones** are those on which the face is surrounded by a chisel draft, the space inside the draft being left rough. Ordinarily, however, this is done only on stones in which the cutting of the joints is such as to exclude them from this class. In ordering stones of this class the specifications should state the width of the bed and end joints which are expected, and also how far the surface of the face may project beyond the plane of the edge. In practice, the projection varies between 1 in. and 6 in. It should also be specified whether or not the faces are to be drafted.

Cut Stones include those with smoothly dressed beds and joints. As a rule, all the edges of cut stones are drafted, and between the drafts the stone is smoothly dressed. The face, however, is often left rough where the construction is massive.

The Surface Finishes * usually met in engineering work, arranged in the approximate order of relative cost are: Rock-faced, pointed, peen-hammered, 4-cut, 6-cut, 8-cut, rubbed, honed and polished. **Rock-faced** may vary from ordinary split or quarry-faced stone with split or roughly squared beds and joints, to carefully quarried faces having practically uniform projection, sometimes with rusticated or drafted margins. **Pointed Work** may be rough, medium, fine or machine-pointed. Rough-pointing is used when it is necessary to remove an inch or more from the face of the stone, and is done with the pick or heavy point (Fig. 17). In dressing limestone and granite this operation precedes all others. If a smoother finish is desired, rough pointing

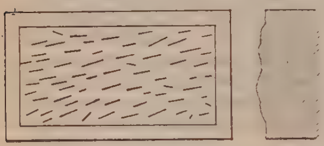


Fig. 17. **Rough-Pointed**

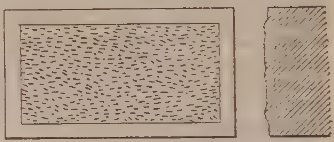


Fig. 18. **Fine-Pointed**

is followed by fine-pointing (Fig. 18), which is used only where this finish is to be final and never as a preparation for a final finish by another tool. Machine pointing is generally more uniform than hand-pointing. The depressions in fine-pointing will be approximately 3/8 in. apart, in medium and machine-pointing 5/8 in. to 3/4 in. apart, and coarse pointing 1 to 1-1/4 in. apart. **Crandalling** is a speedy method of pointing, the effect being the same as fine-pointing, except that the dots on the stone are more regular. The tooth-ax is practically a number of points, and gives the same effect as fine-pointing. It is usually, however, only a preparation for bush-hammering, and the work is then done without regard to appearance so long as the surface of the stone is sufficiently leveled.

Peen-Hammered Finishing (Fig. 19) covers the surface with chisel marks, which are made parallel as far as practicable, and is a final finish. It is adapted to rougher work such as steps, curbings, or to portions of high-class work not exposed to the eye. Peen-hammering is coarser and less regular than 4-cut, and the point marks are not entirely eliminated by the axing.

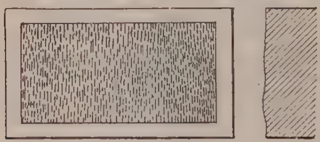


Fig. 19. Peen-Hammered

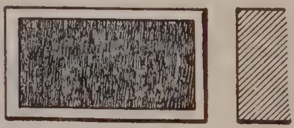


Fig. 20. Bush-Hammered

Four-Cut, Six-Cut and **Eight-Cut** finishes are produced by pounding off the roughness of a stone with the bush hammer and the stone is then said to be "bushed" (Fig. 20). In 4-cut work, the patent hammer contains four blades in each pair of jaws;

* Taken in part from "Architectural Granite," published (1926) by the National Building Granite Quarries Association, Inc.

and, as the jaw opening is about 7/8 in., there are actually about five cuts to the inch, so that the specification of "four-cuts per inch" is incorrect. Similarly 6-cut and 8-cut finishes are sometimes incorrectly specified as 6 or 8 cuts to the inch. In bush-hammered finish the resultant surface has the appearance of being uniformly corrugated to the fineness determined by the number of cuts used, but while the bushing is kept parallel and in the required direction the bush marks are not necessarily continuous or mechanically precise. Six-cut and eight-cut finishes are continuations of the four-cut process, each finer hammer being used consecutively until the required finish is reached. Pneumatic surfacing machines are also used for bushing, especially on all larger surfaces. Sawed surfaces are also bushed, in which case much of the preliminary work is avoided. In sandstone and limestone, bush-hammering should not be used, as experience has shown that soft stone thus treated is likely to scale off (see Art. 3). **Ten-Cut** and **Twelve-Cut** finishes are also produced, but less frequently used on regular building work.

Rubbed and **Honed** finishes are produced by grinding a pointed or sawed surface under the polishing wheel. They vary from coarse-rubbed, with small surface scratches and adapted to work requiring fine smooth finish but not close to the eye, to honed finish with velvet smooth surface practically free from scratches. **Polished** finish is produced by glossing a honed surface under a heavy, felt-coated wheel. Small surfaces and moldings generally have to be rubbed, honed or polished by hand. Polishing usually produces a darker tone, and brings out the color of each crystal, whereas a hammered finish shows up much lighter and softens the colors of the contrasting minerals.

In dressing sandstone, limestone and marble, it is very common to give the stone a plane surface at once by the use of the stone-saw. Any roughnesses left by the saw are removed by rubbing, generally with the carborundum machine. Such stones, therefore, have no margins. They are frequently used in architecture for string courses, lintels, door jambs, etc.; and they are also well adapted for use in facing the walls of lock chambers and in other localities where a stone surface is liable to be rubbed by vessels or other moving bodies. Sometimes the space between the margins is sunk immediately adjoining them and then rises gradually until the four planes form an apex at the middle of the pannel. In general, such panels are called diamond panels, and the one just described is called a sunk diamond panel. When the surface of the stone rises gradually from the inner lines of the margins to the middle of the panel, it is called a raised diamond panel. Both kinds of finish are common on bridge quoins and similar work. The details of this method should be given in the specifications.

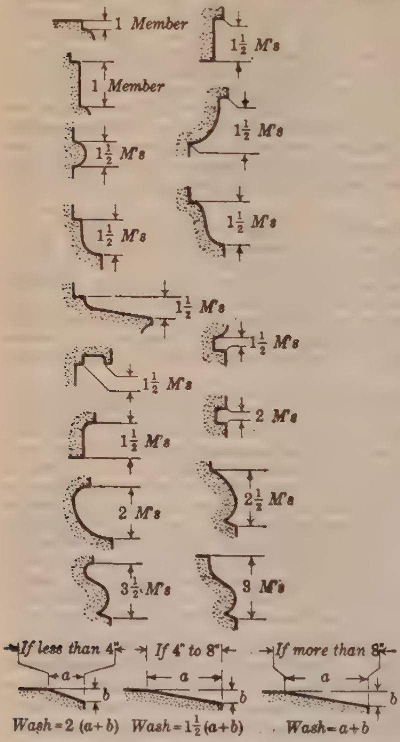
Granite Moldings.* The cutting and finishing of moldings on granite work is almost exclusively a hand process. (Some quarries are equipped with saws and sand-blasts, thus reducing hand cutting.) No practical machine has yet been devised which will do this work and eliminate the more expensive hand-work. In limestone and marble, machine processes are almost universally used on molded work, and here lies one of the most essential differences between the cutting of granite and of the softer stone. The pneumatic tool has materially aided the hand-cutting process, making it possible to produce the most delicate and intricate detail of molding or carving at materially reduced costs. But the cuttings of moldings will continue to be a hand process and its cost one of the principal factors in the total cost of granite work.

In estimating the cost of cutting granite moldings, the so-called "member system" is generally used, by which each molded section is divided into "members." A molded "member" is a more or less arbitrary unit in which the edges, contour and changes in direction of the surface between edges, and the width between edges are taken into consideration. In general, every 4 in. or fraction thereof across the molding is counted as one member. In estimating cost of work, one lineal foot of one molded member equals one

* From "Architectural Granite," published by National Building Granite Quarries Association (1926).

square foot of plain work. The "member chart," Fig. 21, represents the combined judgment of several experienced granite estimators. It is designed for approximate comparisons of relative cost in the study of molded granite details, and is not intended as an exact basis for estimating cost in any particular granite.

For rough estimates, use $2.00 per sq. ft. for plain work or per lin. ft. of a member; thus the molding marked "3 M's" costs 3 × $2 or $6 per lin. ft.



Relative Cost of Wash or Chamfer in Terms of Plane Surface

Fig. 21. Member Chart

Cost of Cut Stone will depend on the character of the rock and conditions at the quarry, as well as on the type of finish required, and can be determined accurately only by obtaining actual bids for any given structure. The cost of transportation is also a large factor. To illustrate the relative cost of various finishes, the following prices for a large **granite-faced** bridge (1926) are given; these prices are in dollars per cubic foot delivered, but do not include placing.

Forming the Surface. The surfaces most frequently required in stone-cutting are plane and cylindrical; but sometimes warped, conical, spherical, and irregular surfaces are required.

To secure a **Plane Surface** the stonecutter chisels draft spots on the three low corners of the face, and cuts down the high corner until it is in the same plane as the other three. Chalk lines are drawn around the

	Rubbed	10-Cut	8-Cut	6-Cut	4-Cut
Facing of piers and abutments below springing line	....	$3.54	$3.34	$3.22	$3.08
Arch voussoirs and spandrel facing	$5.93	4.83	4.70	4.55	4.48
Coping	9.95	6.33	6.01	....	....
Balustrades	19.80	13.08	12.28	....	....

four edges of the stone joining these draft spots, and draft lines are chiseled in the face from corner spot to corner spot to serve as guide lines. To tell when the drafts are in the same plane, he uses two straight-edges having parallel sides and equal widths (Fig. 22). The stone is then sent to the surface cutter, which is a pneumatic machine in which may be used either a point tool or a bush hammer. With this machine the rough surface of the stone between the draft lines is quickly cut down to a true plane surface.

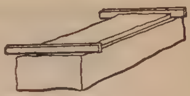


Fig. 22. Plane Surfaces

To form a second plane surface at right angles to the first one, the workman draws a line on the cut face to form the intersection of the two planes; he also draws a line on the ends of the stone approximately in the required plane. With the ax or the chisel he then cuts a draft at each end of the stone until a steel square fits the angle. He next joins these drafts by two others at right angles to them, and brings the whole surface to the same plane. The other faces may be formed in the same way. If the surfaces are not at right angles to each other, a bevel is used instead of a square, the same general method being pursued.

To form a **cylindrical surface,** the stone is first reduced to a parallelopipedon, after which the curved surface is produced in either of two ways: (1) by cutting a circular draft on the two ends and applying a straight-edge along the rectilinear elements (Fig. 23); or (2) by cutting a draft along the line of intersection of the plane and cylindrical surface, and applying a curved templet perpendicular to the axis (Fig. 24).

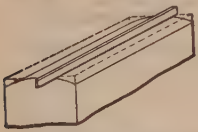


Fig. 23. Cylindrical Surfaces

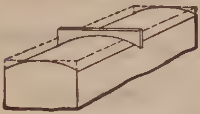


Fig. 24. Cylindrical Surfaces

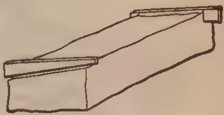


Fig. 25. Warped Surfaces

Warped Surfaces include what the stonecutter calls twisted surfaces, as well as the general warped surface. The former is seldom used in masonry, and the latter almost never. The method of forming a surface equally twisted right and left is shown in Fig. 25. Two twist rules are required, the angle between the upper and lower edges being half of the required twist. Drafts are then cut in the ends of the stone until the tops of the twist rules, when applied as in Fig. 25, are in plane. The remainder of the projecting face is removed, until a straight-edge, when applied parallel to the edge of the stone, will just touch the end drafts and the intermediate surface. If the surface is to be twisted at only one end, a parallel rule and a twist rule are used.

Stereotomy is the art of making the plans and drawings by which the stonecutter is enabled to prepare the patterns, bevels and templets which are required in shaping the blocks of stone for the structure. The engineer should prepare drawings in sufficient detail so that these directing-instruments may be prepared, but fully detailed drawings are not generally necessary.

The general drawings of the stonework should be at a scale of not less than 1/4 in. to one foot, and details should be at least 3/4 in. to one foot. General drawings should show all the dimensions of the structure. Detail drawings may be required for individual stones, and for complicated work the separate stones should be shown in isometric projection. Plane structures (such as retaining walls, wing walls, piers, etc.) may have dimensions of the individual stones directly on the general drawings. Structures with curved or warped surfaces must have the dimensions of the stones properly indicated, these being determined by descriptive geometry. In heavy walls, particularly, the stonecutters should be allowed as much freedom as possible in determining the sizes of individual stones.

Fig. 26 shows an example of the stonework drawing for a small skew-arch culvert, constructed as a "false-skew arch" (Art. 39). This example is rather extreme, as a "false-skew" arch should be used only where the angle of skew is small. The faces

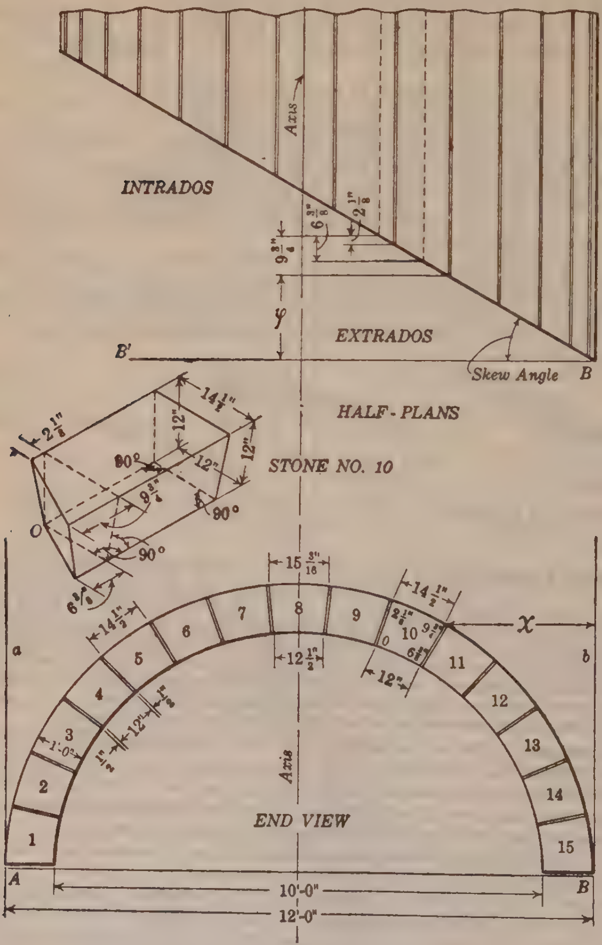


Fig. 26

of the ring-stones are dimensioned in detail. In the end view, figures should be given at the corners of each stone to indicate the distances of these corners from a vertical plane (a different plane for each stone) passing through the corner marked O and at right angles to the axis of the arch. This is indicated on the figure for voussoir 10 only. These dimensions should be computed, and may be checked by scaling from a large-scale plan or preferably from a full-scale drawing laid out upon a floor of a building or upon one especially constructed for this purpose.

The computation is carried out by simple trigonometry, by determining the horizontal distance of each stone corner from a vertical line (Aa or Bb) through A, or B, in the end view. This distance (in this case measured from Bb) is multiplied by the tangent of the skew-angle to obtain the horizontal distance of the given corner measured parallel to the arch axis, from a vertical plane BB', perpendicular to the arch axis. The proper dimensions to be noted on each stone are then readily derived from these computed distances. An isometric view of stone No. 10 is shown, to clarify the meaning of these dimensions.

For examples of more complicated problems, see "Stereotomy" by A. W. French and H. C. Ives, 1902; "Modern Stone Çutting and Masonry" by J. S. Siebert and F. C. Piggin, 1896.

In important or complicated work (right or skew arches), templets should be made for each arch stone and the templets should be checked by laying out the structure at full scale on a large wooden or concrete platform, using steel tape and transit. The proper fitting of the stones can thus be accurately verified, thereby avoiding the necessity of expensive re-cutting.

The joints of the stones back of the face or ring-stones are generally dependent upon the size of the available stones and the judgment of the stonecutter.

STONE AND BRICK MASONRY

7. Kinds of Stone Masonry

Classifications of stone masonry are: (1) according to the finish of the face of the stones, (2) according to whether the horizontal joints are more or less continuous, (3) according to the care which is employed in dressing the beds and joints.

(1) **Quarry-faced Masonry** is that in which the face of the stone is left as it comes from the quarry (Fig. 27). **Pitch-faced Masonry** is that in which the face edges of the beds are pitched to a right line (Fig. 28). The outer edge of a horizontal joint of pitch-faced masonry is straight, while in quarry-faced it is not. **Cut-stone Masonry** is that in which the face of the stone is finished by any one of the methods described in Art. 6, as rough-pointed, fine-pointed, crandalled, axed, bush-hammered, rubbed.

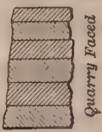


Fig. 27

Fig. 28

(2) **Range Masonry** is that in which a course is the same thickness throughout its length (Fig. 29). **Broken Range** is that in which the course is of uniform thickness for only parts of its length. (Fig. 30). **Random Masonry** is that which is not laid in courses at all (Fig. 31); it is sometimes designated

Fig. 29. Range

Fig. 30. Broken Range

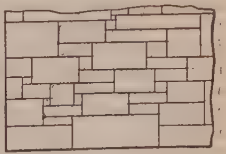


Fig. 31. Random

as one-against-two or two-against-three, the first term indicating that there is one stone on one side of a vertical joint and two on the other, and similarly for the second term.

(3) **Ashlar Masonry** is composed of any of the kinds of cut-stones (Art. 6). It is usually held that when the dressing of the joints is such that the distance between the general planes of the surfaces of adjoining stones is one-half inch or less, the masonry belongs to this class. From its derivation ashlar apparently means large, square blocks; but practice seems to have made it synonymous with cut-stone, and this secondary meaning has been retained for convenience. The coursing of ashlar is described by prefixing range, broken range, or random; and the finish of the face is described by prefixing a name to designate the finish of the face of the stone of which the masonry is composed, as for example fine-pointed ashlar, rubbed ashlar. **Dimension Stone Masonry** is that composed of cut-stones all of whose dimensions have been fixed in advance. Ordinarily the specifications of ashlar are so written as to prescribe the dimensions of the stones to be used, and hence it is seldom or never necessary to make a new class of masonry composed of dimension stones.

Squared-Stone Masonry is that in which the stones are roughly squared and roughly dressed on beds and joints. The distinction between squared-stone masonry and ashlar lies in the degree of closeness of the joints. "When the dressing on the joints is such that the distance between the general planes of the surface of adjoining stones is one-half inch or more, the stones properly belong to this class."

Rubble Masonry is composed of unsquared stones, and it may be coursed or uncoursed. **Coursed Rubble** is masonry composed of unsquared stone which is leveled off at specified heights to an approximately horizontal surface (Fig. 33). **Uncoursed Rubble** is masonry composed of unsquared stones laid without any attempt at regular courses (Fig. 32). The specifications for rubble may require that the stones shall be roughly shaped with the hammer. Rubble is sometimes designated as one-man or two-man rubble, according to the number of men required to handle a stone.

Fig. 32. Uncoursed Rubble

Fig. 33. Coursed Rubble

General Rules. The following general principles apply to all classes of stone masonry: (1) The largest stones should be used in the foundation to give the greatest strength and lessen the danger of unequal settlement. (2) A stone should be laid upon its broadest face, since then there is better opportunity to fill the spaces between the stones. Where stone is used as a veneer covering for steel or concrete, the stones are laid with the broadest face vertical. (3) For the sake of appearance, the thickness of the courses should decrease gradually toward the top of the wall. (4) Stratified stones should be laid upon their natural bed, i.e., with the strata perpendicular to the pressure, since they are then stronger and more durable. (5) The masonry should be built in courses perpendicular to the pressure it is to bear. (6) To bind the wall together laterally, a stone in any course should break joints with or overlap the stone in the course below; that is, the joints parallel to the pressure in two adjoining courses should not be too nearly in the same line. This is briefly comprehended by saying that the wall should have sufficient lateral bond. (7) To bind the wall together transversely, there should be a considerable number of headers extending from the front to the back of thin walls or from the outside to the interior of thick walls; that is, the wall should have sufficient transverse bond. (8) The surface of all porous stones should be moistened before being bedded, to prevent the stone from absorbing the moisture from the mortar and thereby causing it to become a friable mass. (9) The spaces between the back ends of adjoining stones should be as small as possible, and these spaces and the joints between the stones should be filled with mortar. (10) If it is necessary to move a stone after it has been placed upon the mortar bed, it should be lifted clear and be reset, as attempting to slide it is likely to loosen stones already laid and destroy the adhesion, and thereby injure the strength of the wall.

Specifications should be as simple as possible, clear, and not unnecessarily rigid. Care should be taken to keep the terminology consistent. For ashlar work, give depth of bed and end joints. State allowable projection of the face beyond plane of the edge of any stone; whether face is drafted, and width of draft; surface finish between drafts. If rubble is specified, do not call for joints that will require stonecutting. Hollows in the bed face of a stone may not be harmful, especially in hard stone, unless too close to an edge.

For heavy masonry, headers should be not less than 4 ft. long, or else should extend all the way through the wall. Stretchers should be not less than 4 ft. long, with depth of bed 1-1/4 times the thickness. Length of stretchers should be from 2-1/2 to 5 times the thickness.

If full dimensions are not given, specify the limiting dimensions of headers and the proportion of headers (generally not less than 25% of the wall face area), thickness of joints and allowable depressions. Beds of headers need not be dressed to a greater depth than that of the stretchers. Headers may taper horizontally back of the stretchers. Vertical joints need not be dressed to full depth of the stretchers. Insistence on uniform color and texture may add unnecessary expense. The finer finishes are often specified where coarser finishes would be as good. Treads of steps and platforms should never be finer than 4-cut, for safety purposes.

If the stone is paid for at the quarry, the volume of measurement is the smallest rectangular block from which the required stone can be cut. If the masonry is paid for in the structure, the volume should be cubic yards in place, rectangular measure, including joints.

The National Building Granite Quarries Association makes the following suggestions:

Allow at least 2 in. between top of concrete and bottom of steps to obviate bedding off steps. Avoid projections on courses which might otherwise be sawed or machine surfaced. Detail rustication, rabbets and sinkages to take fullest advantage of carborundum saw cutting. Avoid raised seats on balustrade base and substitute round for square plinths on the balusters.

(See also " Graphical Statics and Masonry " by G. F. Swain.)

8. Ashlar Masonry

Ashlar is the best quality of stone masonry, and is employed in all important structures. It is used for piers, abutments, arches, and parapets of bridges; for hydraulic works; for facing quoins, and string courses; for the coping of inferior kinds of masonry and of brickwork; and, in general, for work in which great strength and stability are required. The dimensions of the blocks should vary with the character of the stone employed. With the weaker sandstones and granular limestones the length of any stone should not be greater than three times its depth, as otherwise it is likely to be broken across; but with the stronger stones the length may be four or five times the depth. With the weaker stones the breadth may range from one and a half to two times the depth; and for the stronger stones it may range from three to four times the depth.

Dressing. The dressing consists in cutting the side and bed joints to plane surfaces, usually at right angles to each other. The accurate dressing of the bed joints to a plane surface is exceedingly important. If any part of the surface projects beyond the plane of the chisel draft, that projecting part will have to bear an undue share of the pressure, the joint will open at the edges, and the whole will be wanting in stability. On the other hand, if the surface of the bed is concave, having been dressed down below the plane of the chisel draft, the pressure is concentrated on the edges of the stone, to the risk of splitting them off. Such joints are said to be flushed. They are more difficult of detection, after the masonry has been built, than open joints; and are often executed by design, in order to give a neat appearance to the face of the building. Their occurrence must therefore be guarded against by careful inspection during the progress of the stonecutting.

Great smoothness is not desirable in the joints of ashlar masonry intended for strength and stability, for a moderate degree of roughness adds at once to the resistance to displacement by sliding and to the adhesion of the mortar. When the stone has been dressed so that all the small ridges and projecting points on its surface are reduced nearly to a plane, the pressure is distributed nearly uniformly, for the mortar serves to transmit the pressure to the small depressions. Each stone should first be fitted into its place dry, in order that any inaccuracy of figures may be discovered and corrected by the stonecutter before it is finally laid in mortar and settled in its bed.

The entire bed area of a stone should be dressed to a plane; but, unless the wall is so thin that the stones extend clear through, it is not necessary to dress the entire area of the ends of the stones; and it is not necessary to dress any portion of the back side of the stones. The specifications should state the distance back from the face of the stone that the end is to be dressed to a plane surface. This distance is sometimes stated in inches and sometimes as a fractional part of the thickness of the course. Sometimes specifications permit the vertical joints to be wider than the bed joints. This decreases the cost of cutting, and may not materially reduce the strength of the masonry; but may slightly affect the durability and the architectural appearance.

The thickness of mortar in the joints of the very best ashlar masonry for architectural work is about 1/8 in.; in first-class railroad masonry the joints are from 1/4 to 1/2 in. A chisel draft 1-1/2 or 2 in. wide is usually cut at each exterior corner. In the best work, as fine cut-stone buildings, all projecting courses, as window sills, water tables, cornices, etc., have grooves or " drips," cut in the under surface a little way back from the face, so as to cause rainwater to drop from the outer edge instead of running down over the face of the wall and disfiguring it.

Bond. The bond is the arrangement or overlapping of the stones to tie the wall together longitudinally and transversely, and is of great importance to the strength of the wall. No joint of any course should be directly above a joint in the course below; but the stones should overlap, or break joints from one to one and one-half times the depth of the course, both along the face of the wall and also from the front to the back. The effect is that each stone is supported by at least two stones of the course below, and assists in supporting at least two stones of the course above. The object is twofold: first, to distribute the pressure, so that inequalities of load on the upper part of the structure (or of resistance at the foundation) may be transmitted to and spread over an increasing area of bed in proceeding downward (or upward); and second, to tie the building together, both lengthwise and from face to back.

The strongest bond is that in which each course at the face of the structure contains a header and a stretcher alternately, the outer end of each header resting on the middle of a stretcher of the course below, so that rather more than one-third of the area of the face consists of ends of headers. This proportion may be deviated from when circumstances require it, but in every case it is advisable that the ends of headers should not form less than one-fourth of the whole area of the face of the structure. A header should be over the middle of the stretcher in the course below. **In a thin wall a header should extend entirely through the wall.**

Where very great resistance to displacement of the masonry is required (as in the upper courses of bridge piers, or over openings, or when new masonry is joined to old, or where there is danger of unequal settlement), the bond is strengthened by dowels or by cramp irons of, say 1-1/4-in. round iron set with cement mortar. Stone veneer should be thoroughly anchored to the backing or supporting structure.

Backing. Ashlar is usually backed with rubble masonry which in such cases is specified as coursed rubble. Special care should be taken to secure a good bond between the rubble backing and the ashlar facing. Two stretchers of the ashlar facing having the same width should not be placed one immediately above the others. The proportion and the length of the headers in the rubble backing should be the same as in the ashlar facing. The " tails " of the headers, or the parts which extend into the rubble backing, may be left rough at the back and sides; but their upper and lower beds should be dressed to the general plane of the bed of the course. These tails may taper slightly in breadth, but should not taper in depth. The backing should be carried up at the same time with the facework, and in courses of the same depth; and the bed of each course should be carefully built to the same plane with that of the ashlar facing. The rear face of the backing should be lined to a fair surface. If the backing is concrete, similar care should be given to the bond, and in important structures the stone should be secured to the concrete by metal anchors.

Pointing. In laying masonry of any character, whether with lime or cement mortar, the exposed edges of the joints will naturally be deficient in density and hardness. The mortar in the joints near the surface is especially subject to dislodgment, since the contraction and expansion of the masonry are liable either to separate the stone from the masonry or to crack the mortar in the joint, thus permitting the entrance of rain-water, which upon freezing forces the mortar from the joint. Therefore it is usual, after the masonry is laid, to refill the joints as compactly as possible, to the depth of at least an inch, with mortar prepared especially for this purpose. This operation is called pointing.

The very best cement mortar should be used for pointing, as the best becomes dislodged all too soon. Neat portland-cement mortar is the best, although 1 volume of cement to 1 of sand is frequently used in first-class work. The mortar, when ready for use, should be rather incoherent and quite deficient in plasticity. In the best work, it may be advisable to use a special non-staining cement in the mortar for pointing.

Before applying the pointing, all mortar in the joint should be dug out to a depth of at least 1 in. or better; in setting the stones, the mortar should be kept back an inch or more from the face, and thus save the labor of digging out the joints preparatory to pointing. For the bed joints this may be accomplished by keeping the mortar back from the face of the wall about 3 in., and then when the stone is put into place the mortar will probably be forced out to about 1 or 1-1/2 in. from the face of the joint, and consequently little or no labor will be required to dig out the mortar. Frequently in laying a stone the mortar is spread to the very edge of the joint; and then when the pointing is done, it is so difficult to dig out the mortar that the joint is cleared only about half an inch deep, which depth does not give the pointing sufficient hold, and consequently it soon drops out. The difficulty of digging out the mortar from the vertical joints may be obviated by bending a strip of tin or thin steel to the form of a U having one leg considerably longer than the other, and nailing the long leg to the side of a light strip of wood so that the closed end of the U will project beyond the edge of the wood a distance equal to the depth of the pointing, and then inserting the closed end of the U in the vertical joint before it is filled with mortar.

When the surplus mortar has been removed, the joint should be cleansed by scraping and brushing out all loose material and then it should be well moistened. The mortar is applied with a mason's trowel, and should be well " set in " with a calking iron and hammer. The joint should be rubbed smooth and finished even with the pitch line or with the face of the stone. In the very best work, the joint is also rubbed smooth with a steel polishing tool. Walls should not be allowed to dry too rapidly after pointing; and therefore pointing in hot weather should be avoided.

There are four general forms of finishing the edges of the horizontal joints of cut-stone masonry, whether or not thay are formally pointed as above described. Fig. 34 shows these four forms. When the horizontal joints are finished as in either of the first two examples in Fig. 34 it is customary to finish the vertical joints by the first method; but when either of the last two methods is employed, it is used for both the vertical and the horizontal joints. Occasionally in cut-stone masonry, and frequently in brick masonry, the weather joint is improperly made to slope in the opposite direction, due to the fact that the mason stands at the back of the wall and "strikes" the joint by reaching down and resting the edge of the trowel on the stone below the joint. If the mason stands behind the wall, it is not comfortable to make the weather joint as shown in Fig. 34 at the time the masonry is laid. The grooved joint is frequently called a tuck-pointed joint, and is sometimes made with a V-like face. The beaded joint is not very durable, since the projecting portion soon becomes detached. In making the beaded joint, the beading tool is sometimes guided by a straight-edge, called a "rod," and the joint is then said to be "rodded."

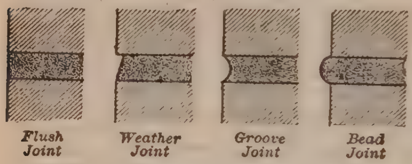


Fig. 34. Finish of Horizontal Joints

The Amount of Mortar required for ashlar masonry varies with the size of the blocks, and also with the closeness of the dressing. With 3/8- to 1/2-inch joints and 12- to 20-in. courses, there will be about 2 cubic feet of mortar per cubic yard; with larger blocks and closer joints, i.e., in the best masonry, there will be about 1 cubic foot of mortar per yard of masonry. For quantities of materials in mortar, see Art. 13.

9. Squared-Stone and Rubble Masonry

Squared-Stone Masonry. Squared-stone masonry is distinguished from ashlar in having less accurately dressed beds and joints; and from rubble in being more carefully constructed. In ordinary practice, the field covered by this class is not very definite. The specifications for "second-class masonry" as used on some railroads usually conform to the above description of quarry-faced range squared-stone masonry; but sometimes this grade of masonry is designated "superior rubble." Squared-stone masonry is employed for the piers and abutments of highway bridges, for small arches, for box culverts, for basement walls, etc. The quoins and the sides of openings are usually reduced to a rough-smooth surface with the face-hammer, the ordinary ax or the tooth-ax. This work is a necessity where door or window frames are inserted; and it greatly improves the general effect of the wall, if used wherever a corner is turned.

The remarks concerning size of stones and backing under ashlar above apply substantially also to squared-stone masonry. As the joints of squared-stone masonry are thicker than those of ashlar, the pointing should be done proportionally more carefully, whereas as a rule it is done much more carelessly. The mortar is often thrown into the joint with a trowel, and then trimmed top and bottom to give the appearance of a thinner joint. Such work is called **ribbon pointing.** Trimming the pointing adds to the appearance but not to the durability. When the pointing is not trimmed, it is called **dash pointing.**

Rubble Masonry. This is the lowest grade laid with mortar; and is built of stones as they come from the quarry without other preparation than the removal of very acute angles and excessive projections from the general figure. The stone used for rubble masonry is prepared by simply knocking off all the weak angles of the block. It should be cleaned from dust, etc., and moistened, before being placed on its bed. This bed is prepared by spreading over the top of the lower course an ample quantity of good mortar of ordinary consistency in which the stone is firmly embedded. The vertical joints should be carefully filled with mortar. The interstices between the larger masses of stone are filled by thrusting small fragments or spalls of stone into the mortar. Careful attention should be given to bonding the wall laterally and transversely. It is frequently specified that one-fourth or one-fifth of the mass shall be headers. The corners and jambs should be laid with hammer-dressed or cut stones.

A very stable wall can be built of rubble masonry without any dressing, except a draft on the quoins by which to plumb the corners and carry them up neatly, and a few strokes of the hammer to spall off any projections or surplus stone. This style of work is not generally advisable, as very few mechanics can be relied upon to take the proper amount of care in leveling up the beds and filling the joints; and as a consequence, one small stone may jar loose and fall out, resulting probably in the downfall of a considerable part of the wall. Some of the naturally bedded stones are so smooth and uniform as to need no dressing; a wall of such stones is very economical, since there is no expense of cutting and no time is lost in hunting for the right stone, and yet strong massive work is assured. However, many of the naturally bedded stones have inequalities on their surfaces, and in order to keep them level in the course it becomes necessary to raise one corner by placing spalls or chips of stone under the bed, and to fill the vacant spaces well and full with mortar. Here the disadvantage of this style of work becomes apparent.

When carefully executed with good mortar, rubble possesses all the strength and durability required in structures of an ordinary character, and is much less expensive than either ashlar or squared-stone masonry. But it is difficult to get rubble well executed. The most common defects are (1) not bringing the stones to an even bearing; (2) leaving large unfilled vertical openings between the several stones; (3) laying up a considerable height of the wall dry, with only a little mortar on the face and back, and then pouring mortar on the top of the wall; (4) using insufficient cement, or that of a poor quality. The only way to prevent the first defect is to have an inspector on the job all the time. The second and third defects can be detected by probing the wall with a small pointed steel rod. To prevent the fourth defect it is customary for the owner to furnish the cement to the contractor. Apparently it is commonly believed that the rougher the stones and the poorer the grade of masonry, the poorer the cement or the leaner the mortar should be. The principal object of the mortar is to equalize the pressure; and the more nearly the stones are reduced to closely fitting surfaces, the less important is the mortar. Consequently, when a substantial rubble is required, it would not be amiss to use a first-class cement mortar, particularly if the stones are comparatively small.

The Amount of Mortar required for rubble masonry varies greatly with the character of the surfaces with which the stone quarries out. If the stone is stratified sandstone or limestone yielding flat-bedded stones with good end surfaces, the rubble may not require much if any more mortar than ashlar built of the more refractory stones; but if the rubble is built of stone that quarries out in irregular chunks and is difficult to dress, a very large per cent of mortar may be required. The amount of mortar required can be considerably reduced by packing spalls into the vertical spaces between the stones, a proceeding that is always economical since spalls are always much cheaper than cement mortar. However, when the cement is furnished by the owner, the mason is apt to fill the joints entirely with mortar since it requires less time.

If rubble masonry is composed of small and irregularly shaped stones, about one-third of the mass will consist of mortar. If the stones are large and regular in form, one-fifth to one-quarter of the mass will be mortar. For quantities of materials in mortar, see Art. 13.

Rubble Concrete is ordinary concrete in which large irregular stones, sometimes called plums, are embedded. This form of masonry is adapted to moderately massive construction. The plums decrease the cost of crushing the stone and also decrease the amount of cement required, and increase the density of the mass. The plums are usually limited to about 40% of the entire volume, to insure that they shall be surrounded by concrete. It is usually difficult and expensive to place more than 25% of plums. If the concrete is wet, there is little or no trouble in getting the large stones thoroughly bedded, and consequently this form of masonry is as good as or better than ordinary concrete.

Cyclopean Masonry. Sometimes in building large structures, as dams, the rubble is made of very large irregular rocks and wet concrete is used instead of mortar. The large stones are placed in the wall by means of a derrick, and concrete is deposited from a bottom-dump pocket. This form of construction was first used about 1900, and is specially applicable in building a dam in which the faces are laid in ashlar, squared-stone, or rubble, and serve as forms in which to place the rock and concrete. The term concrete-rubble is sometimes used for this kind of filling.

10. Dry Stonework

Riprap is rough stone of various sizes placed compactly or irregularly to prevent scour by water. Often riprap is dumped in promiscuously, the size of the stone depending upon the material at hand and the velocity of the current, in extreme cases stone of 15 to 20 cu. ft. being used. Such riprap is sometimes called " cyclopean riprap."

Hand-placed Riprap (sometimes called " slope wall ") is usually made of thin-bedded stones 6 to 12 in. wide set on edge. If the bank on which the riprap is placed consists of material which may be washed out by the water, a layer of sand or gravel should be placed under the riprap, a thickness of 4 to 8 in. usually being sufficient. The stones should break joints horizontally, so that if any stone is displaced those above will bridge the opening and prevent the raveling of the whole bank. The joints should be made reasonably close to prevent the washing out of the bank material, but ordinarily a width of 1 or 1-1/2 in. is sufficient. The length and width of the face of the stone are unimportant, and will depend upon the way the stone quarries out. Ordinarily such riprap can be built more cheaply and better of stones that one man can lift readily, than of two-men stones or larger. The paving should be laid approximately to a plane, and in fairly regular courses. Sometimes the riprap is made of cobblestones or rounded boulders, but such stones are more troublesome to place, and do not make as desirable or stable a protection.

Dry Rubble Masonry is sometimes used for low retaining walls, or at the foot of exposed banks of excavations to prevent the loose material of the bank from flowing into the cut. It is constructed like ordinary rubble masonry but without using any mortar. Walls of dry masonry should be about 60% wider than would be required for rubble work laid up in mortar.

11. Qualities of Brick

Good Bricks have the following qualities to recommend them as a building material: (1) Bricks are practically indestructible, since they are not acted upon by fire, the weather, or the acids in the atmosphere. (2) Bricks may be had in most localities of almost any shape, size, or color. (3) Bricks are comparatively easy to put into place in the wall. (4) In most localities brick masonry is cheaper than stone masonry, even rubble, and under some conditions is a competitor with concrete. The disadvantages of brick as a building material are: (1) Owing to the smallness of the unit, bricks are comparatively expensive to lay, and require considerable skill to secure a strong and good-appearing wall. (2) Ordinarily brick masonry is not as durable as ashlar, since a considerable part of the face of the wall is mortar, which is not as durable as the brick; but by making thin joints or using superior mortar in the exterior edges of the joints, a durable wall may be constructed.

Clay Brick. Until about 1900 the word brick always meant a prism of burned clay; but at about that date bricks composed of sand and lime were put upon the market, and at present many such brick are used annually, although their number is very small in comparison with that of ordinary clay brick. Ordinarily the word brick means a burned-clay brick, and a brick composed of sand and lime is called a sand-lime brick. Clay brick is made by submitting clay which has been prepared properly and molded into shape to a temperature which converts it into a semi-vitrified mass. Building brick are usually made from surface clay, and paving brick from shale (a fine-grained and indurated clay), since the latter gives a tougher, denser, and stronger brick.

The method of molding gives rise to the following terms.

Soft-mud Brick: One molded from clay which has been reduced to a soft mud by adding water. It may be either hand-molded or machine-molded. **Stiff-mud Brick**: One molded from clay in the condition of stiff mud. It is always machine-molded. **Pressed Brick**: One molded from dry or semi-dry clay. **Re-pressed Brick**: Usually a stiff-mud brick which has been subjected to an enormous pressure to render the form more regular and to increase its strength and density. It is doubtful whether the re-pressing increases either the strength or the density. Occasionally in the East, and more formerly than at present, a soft-mud brick, after being partially dried, is re-pressed, which process greatly improves the form and also the strength and the density. A re-pressed brick is sometimes, but inappropriately, called a pressed brick. **Slop Brick**: In molding brick by hand, the molds are sometimes dipped into water just before being filled with clay, to prevent the mud from sticking to them. Brick molded by this process is known as slop brick. It is deficient in color, and has a comparatively smooth surface, with rounded edges and corners. This kind of brick is now seldom made. **Sanded Brick**: Ordinarily, in making soft-mud brick, sand is sprinkled into the molds to prevent the clay from sticking; the brick is then called sanded brick. The sand on the surface is of no serious advantage or disadvantage. In hand-molding when sand is used for this purpose, it is certain to become mixed with the clay and occurs in streaks in the finished brick, which is very undesirable; and owing to details of the process, which it is here unnecessary to explain, every third brick is especially bad. **Machine-made Brick**: Brick is frequently described as "machine-made" but this is very indefinite, since all grades and kinds are made by machinery.

When bricks were usually burned in the old-style up-draft kiln, the classification according to position was important; but with the new styles of kilns and improved methods of burning, the quality is so nearly uniform throughout the kiln that the classification is less important. Three grades of brick are taken from the old-style kiln: arch brick, body brick, and salmon brick. **Arch** or **Clinker Bricks**: Those which form the tops and sides of the arches in which the fire is built. Being overburned and partially vitrified, they are hard, brittle, and weak. **Body, Cherry,** or **Hard Bricks**: Those taken from the interior of the pile. The best bricks in the kiln. **Salmon, Pale,** or **Soft Bricks**: Those which form the exterior of the mass. Being underburned, they are too soft for ordinary work, unless it is for filling. The terms "salmon" and "pale" refer to the color of the brick, and hence are not applicable to a brick made of a clay that does not burn red. Although nearly all brick clays burn red, yet the localities where the contrary is true are sufficiently numerous to make it desirable to use a different term in designating the quality.

The form of the brick gives rise to the following terms. **Compass Brick**: One having one edge shorter than the other. **Feather-edge Brick**: One having one edge thinner than the other. Used in arches; and more properly, but less frequently, called voussoir brick. **Face Brick**: Those which, owing to uniformity of size and color, are suitable for the face of the wall of buildings. Sometimes face bricks are simply the best ordinary brick; but generally the term is applied only to re-pressed or pressed brick made specially for this purpose. **Sewer Brick**: Ordinary hard brick, smooth, and regular in form. **Paving Brick**: Very hard, ordinary brick. A vitrified clay block, very much larger than ordinary brick, is sometimes used for paving, and is called a paving brick, but more often a brick paving-block. **Vitrified Brick**: The introduction of brick for street pavements about 1890 led to a new grade, one burned to the point of vitrifaction and then annealed or toughened by slowly cooling. Vitrified brick and paving blocks, though originally made for paving purposes, are now much used in building and engineering structures.

The Size of common brick varies widely with the locality and also with the maker, and with the same maker the brick are likely to be larger as the working season advances, owing to the wear of the molds or the die. Hard-burned bricks are smaller than soft-burned ones, owing to the greater shrinkage in burning; and this difference varies with the different kinds of clays. The standard sizes accepted by a number of the brick manufacturers' associations in 1923 are: 8 × 3-3/4 × 2-1/4 in. for common and rough-face brick, and 8 × 3-7/8 × 2-1/4 in. for smooth-face brick. For sizes of paving brick, see Sect. 20, Art. 18.

Form. A good brick should have plane faces, parallel sides, and sharp edges and angles. In regularity of form re-pressed brick ranks first, dry-clay brick next, then stiff-mud brick, and soft-mud brick last. Regularity of form depends largely upon the quality of the clay and the method of burning. A good brick should not have depressions or kiln marks on its edges caused by the pressure of the brick above it in the kiln.

Texture. A good brick should have a fine, compact, uniform texture; and should contain no fissures, air bubbles, pebbles, or lumps of lime. It should give a clear ringing sound when struck a sharp blow with a hammer or another brick. A brick which gives a clear ringing sound is strong and durable enough for any ordinary work.

The compactness and uniformity of texture, which greatly influence the durability of brick, depend mainly upon the method of molding. As a general rule, hand-molded bricks are best in this respect, since the clay in them is more uniformly tempered before being molded; but this advantage is partially neutralized by the presence of sand seams. Machine-molded soft-mud bricks rank next in compactness and uniformity of texture. Then come machine-molded stiff-mud bricks, which vary greatly in durability with the kind of machine used in their manufacture. By some of the machines, the brick is molded in layers (parallel to any face, according to the kind of machine), which are not thoroughly cemented, and which separate under the action of frost. In compactness the dry-clay brick comes last. However, the relative value of the products made by the different processes varies with the nature of the clay used.

Formerly it was believed that the capacity of a building brick to absorb water had an important effect upon its ability to resist destruction by frost; but experiments and a more careful study of experience has shown that this has little or nothing to do with its durability. The absorptive capacity varies with the chemical composition of the clay, and there seems to be no close relation between the absorptive power and the strength of a brick or the loss of strength by freezing.

The Crushing Strength is valuable only in comparing different brands, and gives little idea of the strength of walls built of such bricks. The crushing strength of brick is of relatively less importance than that of stone, since owing to the relatively smaller size of the brick and consequently the relatively larger proportion of mortar, the strength of brick masonry is more dependent upon the strength of the mortar than is stone masonry. The strength of the brick is of relatively small importance unless the mortar is nearly as strong as the brick.

Soaking a brick in water decreases its compressive strength, apparently because the water acts as a lubricant on the plane of rupture. In a series of experiments with the United States testing machine at Watertown, of thirty tests upon ordinary building brick from ten localities, all but two showed a loss of strength due to immersion in water for one week; and the wet half of a brick gave an average crushing strength of only 85% of the strength of the dry half.

Experiments indicate that a brick will show maximum strength when tested flatwise, less strength if tested edgewise, and will be weakest when tested endwise. The crushing strength of brick is quite variable, depending on the method of molding, the character of the clay, and the manner of burning. The tentative specifications (1927) for Building Brick of the American Society for Testing Materials contain the following classification of brick according to strength:

Name of grade	Compressive strength flatwise		Modulus of rupture flatwise	
	Mean of 5 tests, lb. per sq. in.	Individual minimum, lb. per sq. in.	Mean of 5 tests, lb. per sq. in.	Individual minimum, lb. per sq. in.
Grade " A "	4500+	3500	600+	400
Grade " B "	2500+	2000	450+	300
Grade " C "	1250+	1000	300+	200

Grades " A " and " B " are usually hard or well-burned. Bricks of grade " C " are usually underburned. The actual strength of well-made brick may be considerably higher than the specified minimum.

Sand-lime Brick consist of a mass of sand cemented together with lime. There are two classes of sand-lime brick: one in which the binding material is carbonate of lime, and the other in which it is silicate of lime.

The first is virtually a brick made of ordinary lime mortar, molded as are soft-mud clay brick, and hardened in the open air or in an atmosphere rich in carbon dioxide (CO_2), either with or without pressure. This form may properly be called a lime-mortar brick. It is the older form of sand-lime brick, and was formerly made in a small way where sand and lime were cheap and clay and fuel were expensive; but the brick is so weak and friable that it has not given satisfaction, and needs no further consideration here.

The second kind is made from a mixture of sand and lime which is molded in a press and hardened by being subjected to steam under pressure. In this case the binding material consists chiefly of hydrosilicate of lime. Probably part of the lime is converted into carbonate by absorbing carbon dioxide; but the most of the lime combines with the silica of the sand and forms hydrosilicate of lime, a stable and comparatively strong cementing material. This form is the only one to which the term sand-lime brick is now applied; but in consulting the past literature on the subject, a careful distinction should be made between the two forms of so-called sand-lime brick. This form of sand-lime brick was first manufactured in Germany about 1880, and was introduced into America about 1900. There are localities where this form of brick is an important factor in building operations.

Sand-lime brick are made which in appearance and quality are the equal of dry-clay (pressed) brick. The average sand-lime brick will be equivalent in strength to medium clay-brick. When sand-lime bricks are manufactured under standard specifications they may be used as a substitute for clay-brick in masonry construction.

12. Lime and Lime Mortar

Lime Mortar, a mixture of lime paste and sand, is generally used for brick masonry because of its cheapness and the comparative ease with which it is used. There are two classes of lime on the market, high-calcium lime and magnesian or dolomitic lime. The former is made of nearly pure limestone, and the latter of a limestone containing a considerable quantity of magnesia; the latter slakes more slowly, evolves less heat, expands less, sets more slowly, and makes the stronger mortar. The former is known as hot or quick lime, and the latter as cool or slow-setting lime.

The lime must be slaked before being mixed with the sand. Lime is usually slaked by placing the lumps in a layer 6 or 8 in. deep in either a water-tight box or a basin formed in the sand to be used in mixing the mortar, and pouring upon the lumps a quantity of water 2-1/2 to 3 times the volume of the lime. If the quantity of water added is just right, the lime will be reduced to a thick paste; but if too much water is used, the lime will be reduced to a semi-liquid condition and a considerable part of its binding quality will be destroyed.

With a High-Calcium or quick-slaking lime the best results are obtained when all the water is added at once; but with a magnesian or slow-slaking lime only a little water should be added at first, and then after the lime and water are hot, more water may be added gradually so as not to chill the mixture and retard the slaking. The slaking proceeds more rapidly and is more complete if the mass is hot. The lime absorbs the water, and the chemical action generates heat enough to change part of the absorbed water into steam which bursts the lumps of lime apart and thus exposes new surfaces to the action of the water; but if cold water is added after the slaking has begun, it chills the mass, prevents the formation of steam and the consequent bursting of the lumps, and hence the slaking is not complete, and the amount of paste formed is less than it should be. Further, when the slaking has been thus retarded, a thin paste forms on the outside of the fragments of the unslaked lime, which excludes the water from the interior or unslaked portion of the lump; and hence it is difficult, if not impossible, to slake lime thoroughly if it has ever been chilled in the slaking. Partial air slaking is harmful in much the same way, since the slaked lime on the outside of a lump prevents the free access of the water to its interior.

Stirring the lime while slaking chills the mass and thereby retards the slaking; but on the other hand, stirring breaks up the friable lumps and thereby aids the slaking. Therefore if the mass is stirred at all, the stirring should be done in such a manner as to cool the mass as little as possible. The swelling of the lime in the lower portion of the mass frequently lifts some of the lumps out of the water, the heat in the lump causes a column of steam to rise from it, and the lump is said to "burn." This burning is detrimental, since a film of slaked lime is formed on the surface of the unslaked portion which tends to prevent complete slaking. Therefore it is important that lumps which are burning should be pushed back into contact with the water. Burning can be prevented by covering the box with boards or a tarpaulin to retain the heat and the moisture.

Lime slakes spontaneously when exposed to the air by absorbing moisture from the atmosphere; and lime that is thoroughly aid-slaked is as good as, or even better than that slaked in the usual way, a popular prejudice to the contrary notwithstanding. However, lime that is only partially air-slaked is undesirable, since it is more difficult to slake by the ordinary process than lime that is not partially air-slaked.

Slaked lime is sold under the name of hydrated lime. It is a dry powder, and is usually packed in paper sacks. In certain classes of work, hydrated lime is of decided advantage, since it is ready for immediate use without waiting to slake it. For quantities of materials in mortar, see Art. 13.

Mortar. Sand is added to the lime paste for four reasons: (1) to divide the paste into thin films and make the mortar more porous, thus allowing the penetration of air and facilitating the absorption of the carbonic acid which causes the setting of the mortar; (2) to prevent excessive cracking of the mortar owing to shrinkage due to the evaporation of the water in the lime paste;

(3) to give greater strength to the mortar against crushing (practically the only stress that comes upon mortar), since sand has a greater resistance to compression than lime paste either before or after it has set; and (4) to reduce the amount of lime necessary to make a given bulk of mortar, thus decreasing the cost. See Sect. 11, Appendix 4, for the requisites of good sand, and Sect. 11, Appendix 2, for cement mortar which is used in brickwork of the highest class.

After the lime is slaked, the sand is spread evenly over the paste, and the ingredients are mixed with a shovel or hoe, a little water being added occasionally if the mortar is too stiff. The mixing should be thorough, i.e., it should be continued until the mortar is of a uniform color.

To determine whether the proportion of sand is right, hold the hoe handle nearly horizontal and lift up a hoeful of mortar. If the mortar will not of itself slide from the hoe, it does not contain enough sand; and if a hoeful of mortar cannot be thus lifted up, it contains too much sand. The brick-mason on the wall by a somewhat similar process checks the proportions of the mortar by the way in which it slips from the trowel. If there is an excess of sand, the mortar will be "brash" or "short," and will drop from the trowel so abruptly as to make it impossible to "string out the mortar," namely, to spread the mortar over several bricks by simply allowing it to flow from the trowel as the latter is drawn along. On the other hand, if there is an excess of paste, the mortar will not flow from the trowel, at least in sufficient quantity to make the joint. This method of proportioning gives a mortar that works well under the trowel, and with reasonably clean sand also a mortar of practically maximum strength.

If the sand is very fine and contains a good deal of finely pulverized clay, the above test may be satisfied when the mortar contains too little lime; but lime paste is so cheap, and lime mortar is so weak, that a sand with any considerable amount of clay should not be used in lime mortar; since the clay is a source of weakness.

Certain forms of diatomaceous silica, originally developed for use in concrete, are claimed to be beneficial in lime-cement mortars. When this material is added to the mortar, it is said to increase the plasticity and also the strength.

13. Laying Brick

Wetting the Brick. Since most bricks have a great avidity for water, it is best to dampen them before laying. If the mortar is stiff and the bricks are dry, the latter absorb the water so rapidly that the mortar does not set properly, and will crumble in the fingers when dry. Neglect in this particular is the cause of most of the failures of brickwork. Since an excess of water in the brick can do no harm, it is best to drench them thoroughly with water before laying. Lime mortar is sometimes made very thin, so that the brick will not absorb all the water. This process interferes with the adhesion of the mortar to the brick. Watery mortar also contracts excessively in drying, which causes undue settlement and, possibly, cracks or distortion. Wetting the brick before laying will also remove the dust from the surface, which otherwise would prevent perfect adhesion. When the very strongest work is desired, as in brick sewers, it is customary to require that the brick shall be immersed in water for 3 to 5 minutes before being laid. Wetting in the pile is not as effective as immersion, since in the pile the water is not likely to reach all the surfaces of all of the bricks. Masons very much dislike to lay wet brick, since the water softens the skin on their fingers and causes it to wear away rapidly. The softer the bricks the more necessary that they should be thoroughly wet when laid. In freezing weather, care should be taken that the water does not form a film of ice on the brick.

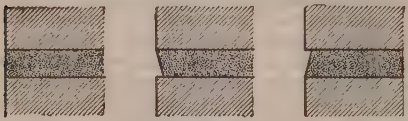
Laying. The bricks should not be merely laid, but every one should be pressed down in such a manner as to force the mortar into the pores of the brick and produce the maximum adhesion. This is more important and also more difficult to accomplish with cement than with lime mortar. The increased value of the cement mortar can be attained only by bringing the brick and the mortar into close contact; and this is more difficult to do, since cement mortar is not as plastic as that made with lime. The mason is apt either (1) to butter the edges of the brick, and thus secure a joint that looks well after the brick is laid; or (2) to place insufficient mortar to make a full bed joint of the required thickness, run the point of his trowel through the middle of the mass, making an open channel with a sharp ridge of mortar on each side, and then lay the brick upon the top of these two ridges, thus leaving the center of the brick unsupported. The first method is the one employed with thin joints, which is a reason why they should not be required; the second method is popular because it requires less exertion and is more rapid than fully bedding the brick.

If strength is a matter of any moment, care should be taken to see that the vertical joints are filled solidly full of mortar; this is called slushing the joints. Unless slushing is insisted upon, masons are apt to butter the end joints, lightly bed the brick, throw a little mortar into the top of the vertical joints, and scrape off the excess above the top of the brick, thus leaving the major portion of the vertical joints open; and sometimes little or no attempt is made to fill the vertical joint between adjacent tiers of stretchers, thus leaving also long and high unfilled vertical spaces. Where resistance to dampness is more important than compressive strength, no particular attempt to fill interior vertical joints is necessary.

For the best work it is specified that the brick shall be laid with a " shove joint"; that is, that the brick shall first be laid so as to project over the one below, both at the end and the side, and be pressed into the mortar, and then be shoved into its final position. Masons are very reluctant to lay brick with a shove joint, partly because it is hard work and partly because many of them have not acquired the art. If brick are not laid with a shove joint, it is highly improbable that the lower part of the vertical joints will be filled with mortar, and consequently the wall will not be as strong or as impervious to water, air and heat as it would otherwise be.

Pointing. In laying inside walls that are to be plastered, the mortar that is forced out when the brick is pressed into position is merely cut off with the trowel; but for outside walls and also for inside walls that are to be left exposed, the joints should be more carefully finished. In laying common brick the mortar in the vertical joints is simply pressed back with the flat face of the trowel; but there are three methods of pointing or finishing the bed joints: (1) flush joints, (2) struck joints, and (3) weather joints.

Flush pointing consists in pressing the mortar flat with the trowel, thus making the edge of the joint flush with the face of the wall (Fig. 35). The struck joint is formed by resting the lower edge of the blade of the trowel upon the edge of the brick below the joint and drawing the trowel along the joint, which smooths the face of the joint and slightly consolidates the mortar, and leaves the joint as shown in the center of Fig. 35. The weather joint is formed as shown in right-hand side of Fig. 35 by pressing the mortar back with the upper edge of the trowel. This form of finish is much more durable than the struck joint, since water will not lodge in the joint and soak into the mortar and on freezing dislodge the mortar; but this form of joint is much more difficult to make, since the mason stands above and back of the brick he is laying. If the weather joint is desired, it must be distinctly specified and the inspector must be watchful to see that it is secured. Brick masonry is usually laid with lime mortar or with lime-cement mortar, the lime giving cohesive strength to the mortar so that enough mortar stays in the joint to permit it to be successfully struck; but when cement mortar or mortar containing but little lime is used, the mortar is so lacking in cohesion that enough does not remain in the joint to permit striking it, and hence with cement mortar it is necessary to formally point the masonry.



Flush Joint *Struck Joint* *Weather Joint*

Fig. 35. Pointing Bed Joints of Common Brickwork

Pressed brick are usually laid with a mortar made of one volume of stiff lime paste (called lime putty) and one volume of fine sand; and when this mortar is used, the brick is buttered, i.e., a little mortar is spread upon only the edges of the brick before it is laid. If the above mortar were spread over the entire surface of the brick, the joint could not be made as thin as is usually specified; but some of the better architects specify thicker joints for pressed work so that the bricks can be laid otherwise than by being buttered. If the mortar is to be spread in the usual way, it should consist of 1 volume of lime paste to about 2 volumes of rather fine sand. Some architects specify 1 volume lime paste, 1 volume portland cement, and 2 volumes of fine sand. Some contractors prefer to substitute at their own expense a rich portland cement mortar and lay thicker joints rather than lay thin buttered joints, since the brick-mason can lay more brick with the former than with the latter.

The joints of pressed-brick work are finished by grooving or beading (Art. 8, Fig. 34), the former being more common. The grooved joint is preferred to the flush joint, because of the variation in light and shade that the former gives to the face of a wall.

Bond is the arrangement of the bricks in successive courses to tie the wall together both longitudinally and transversely. The primary purpose of bond is to give strength to the masonry, but architects employ various longitudinal bonds to improve the appearance of the wall. Although numerous bonds are employed for artistic effect, in the construction of ordinary brick masonry only three bonds are used, the common, the English, and the Flemish. The common bond consists of four to seven (for better work not over six) courses of stretchers to one of headers. The proportionate numbers of the courses of headers and stretchers should depend on the relative importance of transverse and longitudinal strength. The proportion of one course of headers to two of stretchers is that which gives equal tenacity to the wall lengthwise and crosswise. English bond consists of alternate courses of stretchers and headers, and Flemish bond consists of a header and stretcher alternately in each course so placed that the outer end of each header lies on the middle of a stretcher in the course below.

If the wall is more than one brick thick, it should be bonded transversely as well as longitudinally. The exact arrangement of the transverse bond varies with the thickness of the wall, but is easily worked out if a little attention is given to it. The face bond is likely to receive more attention than the transverse bond, and it can be readily inspected after the completion of the wall; but the transverse bond cannot be examined after a course is laid on top of it, and therefore it should be carefully looked after as the work progresses.

The Report of the Building Code Committee of the Dept. of Commerce (June 26, 1924) on " Recommended Minimum Requirements for Masonry Wall Construction " contains many valuable suggestions.

Estimates. The following tables are from "Brick—How to Build and Estimate," published by The Common Brick Manufacturers' Association of America * (1926).

The time required to perform work is based on the average performance of experienced bricklayers and laborers. In some districts where bricklaying is done by men who are not so familiar with the work, the time allowance should be correspondingly increased. Laborer's time includes time for making and handling mortar, handling brick from pile on the ground, waiting on bricklayer, moving scaffold, etc. Laborer's time for cleaning brickwork is not included. Bricklayer's time is given for the average number of brick laid on a small job in a country town. This time may be decreased through good management. On large work and in cities, bricklayers will lay 1500 brick per day (8 hours) on a solid wall, including facing and backing. Under good management, the time for bricklayers given in the table should be considerably reduced. In case the work has many special features, such as pilasters or special patterns, the bricklayer's time should be increased.

* This book also contains other useful information on brick construction.

In figuring the wall area, deduct all openings, using the exact masonry opening. To the cost computed from the table add cost of foreman, rental of hoisting machinery, engineers' time, fuel, special scaffolding, cleaning down, overhead and profit. An allowance should be made in quantity of brick for breakage and waste; in massive work this may be 1 or 2%, for buildings 3 to 5%.

Tables for Estimating Brickwork

	Number of bricks	Cubic feet of mortar	Laborer's time, hours	Bricklayer's time, hours	
				Lime or cement-lime mortar	Cement mortar
I. Piers (1/2-in. joints. All joints filled with mortar).					
8 × 12-*in. solid*—Brick laid flat. Height = 10 ft.	124	2.25	1.	....	1.75
12 × 12-*in. solid*—Brick laid flat. Height = 10 ft.	185	3.25	1.50	....	2.50
12 × 16-*in. solid*—Brick laid flat. Height = 10 ft.	247	4.50	2.	....	3.25
II. Solid Basement Exterior Walls (1/2-in. joints; exterior 4-in. thickness of wall laid with all joints filled. Remaining brick laid on full bed of mortar, but brick touching end to end. Vertical space between each 4-in. thickness filled with mortar. Every fifth course a header course.)					
8-*in. wall*—Surface area = 1000 sq. ft.	12 706	195.	97.	73.	93.
12-*in. wall*—Surface area = 1000 sq. ft.	19 252	314.	149.	110.	140.
16-*in. wall*—Surface area = 1000 sq. ft.	25 797	433.	200.	129.	159.
III. Solid Exterior Walls above Grade (1/2-in. joints, common bond; mortar in joints as per Table II.)					
8-*in. wall*—Surface area = 1000 sq. ft.	12 706	135.	93.	84.	93.
12-*in. wall*—Surface area = 1000 sq. ft.	19 252	195.	140.	128.	140.
16-*in. wall*—Surface area = 1000 sq. ft.	25 797	255.	187.	137.	172.

Tables for Estimating Brickwork—Continued

	Number of bricks	Cubic feet of mortar	Laborer's time, hours	Bricklayer's time, hours: Common bond: Lime or cement-lime mortar	Common bond: Cement mortar	Other bonds: Lime or cement-lime mortar	Other bonds: Cement mortar
IV. Solid Walls. All bonds (1/2-in. joints, all joints filled with mortar. "Other bonds" includes Flemish and English).							
8-*in. wall*—Surface area = 1000 sq. ft.	12 321	195.	95.	90.	99.	104.	110.
12-*in. wall*—Surface area = 1000 sq. ft.	18 482	314.	144.	135.	148.	156.	164.
16-*in. wall*—Surface area = 1000 sq. ft.	24 642	433.	192.	152.	179.	179.	197.

Number of Bricks Laid by One Bricklayer per 8-Hour Day

Class of masonry *	Common bond		Other bonds	
	Lime or cement-lime mortar	Cement mortar	Lime or cement-lime mortar	Cement mortar
I. Piers:				
8 × 12-in.	....	570	....	....
12 × 12-in.	....	590	....	....
12 × 16-in.	....	610	....	....
II. Solid Basement Exterior Walls:				
8 in. thick	1390	1090	....	....
12 in. thick	1400	1100	....	....
16 in. thick	1600	1300	....	....
III. Solid Exterior Walls above Grade:				
8 in. thick	1210	1090	....	....
12 in. thick	1200	1100	....	....
16 in. thick	1507	1200	....	....
IV. Solid Walls:				
8 in. thick	1100	1000	950	900
12 in. thick	1100	1000	950	900
16 in. thick	1300	1100	1100	1000

* See preceding table for joint details.

Quantities of Material for 1000 Cu. Ft. of Mortar

(Based on use of good quality lime. Lime quantities are approximate and will vary with the grade of lime and the size of particles composing the sand. In cement mortars, 1/10 of the cement by weight is replaced by dry hydrated lime or its equivalent in lump-lime paste.)

Mortar	Lump lime, 180-lb. bbl., number of bbl.	Hydrated lime, 50-lb. sacks, number of sacks	Cement, 94-lb. sacks, number of sacks	Sand, cu. yd.
Lime mortar:				
1 : 2-1/2	57	350	....	37
1 : 3	47	292	....	37
Cement-lime mortar:				
1 : 1 : 6	24	146	130	37
Cement mortar:				
1 : 2	16	92	442	34
1 : 3	12	69	331	39
1 : 4	10	55	264	41

In using the above tables, the following points should be considered: (a) Locality has much influence upon the number of brick laid per man. In the small communities, especially in the South, the number of brick laid may be 50 to 60% higher than those laid per man in the large cities of the East. This is influenced also by the available supply of bricklayers. (b) The quality of the brick has a material bearing. Rough, poorly shaped brick are laid at a slower rate than uniform, well shaped brick. (c) The proper and commonly employed method of laying brick is to " butter and shove." In some places only the outside brick are so laid; the others are simply placed and slushed. The latter method results in laying about 50% more brick, but produces a weak wall. (d) The table does not cover special work such as small window arches, small offsets, etc. Engineers generally over-estimate the rate of laying such work. It is thought that in general the rate of doing this special work is about one-third of that given in the above tables. (e) The time of year in which brick are laid seriously affects the rate of laying. When the temperature is below 20° F. all materials should be heated and the work protected against frost. Men wearing gloves and mitts lay only about 50% the usual amount.

14. Efflorescence *

Efflorescence is a white deposit which may occur on the surface of any masonry wall, and which is caused by the crystallization of soluble salts brought to the surface by moisture in the masonry. These salts may have been in any or all of the masonry materials in the wall—in facing, backing, or mortar. The moisture may have entered the wall during construction through exposure to rain or snow; or it may have entered after the wall was completed, from the earth at the base, from improperly flashed parapet walls at the roof, from leaking gutters and downspouts, from lack of drips on cornices and sills, or from poorly filled or cracked mortar joints in the wall.

" Once in the wall, the moisture dissolves some of the salts present, and later passes to the surface when conditions become favorable. When evaporation takes place, the salts are left behind on the wall as efflorescence. No masonry material is always exempt from contributing to the development of noticeable efflorescence, but some materials contain much larger quantities of soluble salts than others, and consequently are much more important sources of trouble."

* This article is based largely on " Wet Walls and Efflorescence," by L. A. Palmer, Research Associate, National Bureau of Standards, and published by American Face Brick Assn., 1928.

"Prevention of efflorescence must begin with the design of the building, follow through the selection of materials and the methods of work during the construction period, and continue with maintenance of the structure after its completion. The neglect of certain simple precautions may bring very unsatisfactory results."

The most important precaution is to prevent moisture from entering the wall, either during construction or later. This precaution will also aid in preventing disintegration of the masonry through the freezing and thawing of the moisture, and the undesirable effects of damp interiors.

"The presence of soluble salts in clay products can be practically eliminated by the manufacturer by certain procedures in the drying and burning of the ware, together with the judicious use of barium compounds in the clay." The resultant increase of cost renders this procedure impracticable in the manufacture of the less expensive building products, though it has been generally used in the face brick industry. "From the standpoint of efflorescence it is therefore very essential that the backing-up materials be protected from excessive water penetration."

One of the important sources of efflorescence is free lime, which occurs in the lime mortar and also in cement. The development of efflorescence from the mortar may be largely prevented by the use of water-repellent materials, especially calcium or ammonium stearate (2% by weight of the cement and lime used in the mortar).

"Mortars so treated are effective in preventing the passage of moisture where only capillary forces obtain. Calcium stearate is a powder which is mixed dry with the cement before adding sand and water, while ammonium stearate is a paste which is usually added to the mixing water. On the ordinary building job the powder has proved the more practical and accurate to use with the equipment commonly available. Although best practice would call for the use of stearate in mortar for backing as well as facing materials, it is especially important that it be employed in laying up the facing where the joints will be subjected to unusual exposure to driving rains or to water running down the surface of the wall."

Joints of copings, window sills and parapet walls are all in need of special care, as well as the repointing of joints that have cracked.

"Specific recommendations that will help in preventing water from entering the wall: Use impervious copings, and point joints carefully with water-repellent mortar. Flash parapet walls, carrying the flashing completely through the wall one or two courses above the roof level, and waterproof the entire inner side of the wall. Always provide adequate drips on all copings, cornices and window sills, making the grooves at least 5/8 in. wide and 3/8 in. deep; and provide projections of 2 in. or more so as to keep the drip away from the wall. Where practical avoid all joints in sills. If brick sills are used, flash under them with metal flashing provided with drips. Protect walls under construction to prevent rain or melting snow from entering. Keep gutters and downspouts in good repair. Point up cracked joints promptly. Coat the backs of retaining walls with an impervious layer, and keep brick building walls away from contact with the earth."

In concrete construction, give particular care to grading of the aggregate, proper consistency, using a minimum amount of water, compacting to get density without too much cement, and careful curing. The use of chemicals may be helpful but cannot be considered a substitute for these precautions.

"Any efflorescence appearing on the wall of a building that is suitably designed, constructed and kept in repair in order to prevent excessive water penetration, will be due to rare and abnormal conditions. It should in time gradually diminish and finally disappear. Prolonged and heavy wind-driven rains will bring out efflorescence on almost any wall, but this is generally of short duration."

Efflorescence may be partially removed by water, and sometimes is easily brushed off. Where special treatment is necessary, the wall should first be washed down with water, then treated with a dilute solution of hydrochloric acid (3 parts to 100 parts of water), and finally washed thoroughly with water.

No great success has been met with the use of washes in waterproofing exposed surfaces; but any wash that will prevent the saturation of the brick and mortar from driving rains should be helpful in decreasing the amount of efflorescence. Such a moisture-proof coating should be applied after the mortar and bricks have dried out.

The writer wishes to emphasize the fact that great care should be taken in the use of calcareous (shell) sand for mortar or concrete. Some calcareous sands give a weaker mortar and one that is more porous and therefore more liable to effloresce. If in doubt use a thicker covering over reinforcing steel for concrete subject to salt water or salt air, or protect the finished concrete by the use of waterproofing materials.

DATA FOR MASONRY DESIGN

15. General Discussion

Stone Masonry. The strength depends upon the strength of the stone, accuracy of dressing and bedding, proportion of headers to stretchers and upon the thickness of the mortar joints and the strength of the mortar. However, the strength of the stones need be given little consideration if they are sound and durable unless the masonry is of the best grade of ashlar and is subjected to high unit stresses. The relative strength of mortar has greater effect especially in rubble masonry, where the joints are usually large and the amount of mortar relative to the total volume of masonry is high.

Unfortunately, the bulk of existing masonry structures throughout the world were built prior to the time when analytical study was possible and when ultra-conservatism was necessary. Therefore the older structures give little indication of safe unit stresses.

Few tests have been made upon stone masonry which give conclusive evidence of the ultimate strength of masonry in actual structures. We are therefore more or less dependent upon actual or computed stresses in actual structures of record. Considerable light, however, is given by experiments made upon brick piers with lime and cement mortar which indicate that the strength of the masonry is much less than the strength of the individual bricks. An increase of 50% in the strength of the brick produces no appreciable effect upon the strength of the masonry; but the substitution of cement mortar for lime mortar increases the strength of the masonry as much as 70%. In stone masonry, and usually in brick, the mortar is the weaker material and therefore the less used the stronger the masonry. Therefore masonry of large dressed blocks with thin joints (ashlar) is stronger than rubble.

The following table shows how difficult it is to predicate design upon existing structures:

Name and location	Short tons per sq. ft.	Pounds per sq. in.
Church of All Saints, Angers, France	43	600
Pont-y-Prydd, stone arch bridge, Wales (1750), hard limestone rubble in lime mortar	72	1000
Saltash Bridge, granite piers	9	125
Brooklyn Bridge, granite masonry	28.5	400
Brooklyn Bridge, limestone masonry	10	140
Washington Monument, dead load	19.8	275
Washington Monument, with wind load	25.4	350

The Washington Monument is faced with marble, the backing being granite. Joints in the marble were about 1/4 in. The joints in the backing were probably thicker and the greater amount of strain in these joints in the lower part of the shaft caused an excessive load to come on the facing. As a result, the facing joints have been almost squeezed tight together, and the pressure tends to chip off the edges of the marble blocks.

Brick Masonry. The allowable pressure upon brick masonry depends upon:

1. The quality of the materials employed.
2. The thickness of the mortar joints and the degree of care with which the work is executed.
3. The accuracy with which the load upon the masonry could be or was estimated.

Referring to the second statement, in some sections of the country bricklayers carefully "butter" each brick and "shove" the joint "hard." In other sections they lay the brick in the wall and "slush" them. The writer cannot state from actual test the relative value of two walls or piers so built, but he is much afraid that the latter method decreases the strength materially. Partly as a result of this, bricklayers at some localities lay only 500 brick per day in a straight wall and in other points they will lay nearly 2000. The amount of brick laid, however, per man day is equally affected by local labor conditions.

Examples of Pressures in Brick Structures

	Short tons per sq. ft.	Pounds per sq. in.
American Surety Building, N. Y., brick piers........	7.25	100
Chimney at Glasgow, 468 ft. high, dead load.........	9	125
Chimney at Glasgow, 468 ft. high, with wind.........	15	210

The U. S. Bureau of Standards made tests on brick piers, 2.5 ft. square by 10 ft. high (Technologic Paper No. 111, 1918), which led to the following conclusions regarding brick masonry:

1. Primary cause of failure is transverse failure of individual bricks.
2. Ultimate strength may be increased by any method of construction which will increase the depth of the component parts of the pier, as by laying the bricks on edge instead of flat, breaking joints every few courses instead of every course, or using bricks of more than ordinary thickness.
3. Strength may be increased by the introduction of wire mesh in all horizontal joints.
4. Varying the number of header courses does not appreciably affect the ultimate strength.
5. Mortar joints should be made as thin as possible and of uniform thickness, and regularity in shape of the bricks is essential.
6. Ultimate strength of brick masonry is proportional to the compressive and transverse strength of the bricks used. (The writer is of the opinion that this is true only for bricks of medium strength, because with the harder and stronger varieties of brick it has been demonstrated that the strength of the masonry is more dependent upon the strength of the mortar and the thickness of the joints.)
7. The kind of mortar used has an important effect on the strength of brick masonry. Pure lime mortar is inefficient under high stress. In 1 : 3 portland cement mortar, 25% by volume of the cement may be replaced by hydrated lime without appreciably affecting the strength of the masonry.

16. Working Unit Stresses

Maximum Safe Unit Stresses for masonry design are given in the following tables. The values are based upon good material and workmanship in accordance with the best practice. It is assumed that the masonry is to be properly bonded and the mortar ingredients suitably tested, also that the foundation is properly designed to carry all the forces which the masonry transmits to it. The masonry is assumed to be laid in mortar of the following proportions:

Cement mortar—1 part portland cement to 3 parts sand.
Lime and cement mortar—1 part portland cement, 1 part lime and 6 parts sand.
Lime mortar—1 part lime paste to 4 parts sand (hydrated lime may be added to cement mortar, up to 15% of the cement by volume).

The unit stresses given are for masonry which is at least two months old if laid in warm weather and six months old if laid when the temperature is below or near freezing.

For large structures, such as dams over 70 ft. high and arches over 150 ft. in span, or in a series of arches where the amount of masonry is great and where each unit of masonry is dependent for its stability upon each and every other unit, it is advisable to use lower working unit stresses than those in the tables. For such cases it is recommended that 75% of the tabulated values be employed.

If, for special reasons, it is desirable to consider masonry laid in warm weather one month old, or that laid when the temperature is below or near freezing, two months old, it is best to use 60% of the values given in the tables.

Ashlar Masonry. The following are safe working compressive stresses recommended to be used in the design of structures of ashlar masonry laid in 1 : 3 portland cement mortar with joints not exceeding 1/2 in. in width. With lime-cement mortar, use 75% of these stresses.

Kind of ashlar	Pounds per sq. in.	Short tons per sq. ft.
Granite, syenite, gneiss	700	50.4
Limestone, hard	600	43.2
Limestone, medium; marble	500	36.0
Limestone, soft; sandstone	400	28.8

For ashlar laid with 1 : 3 portland cement mortar, and having joints greater than 1/2 in. and less than 1 in., it is best to use 450 lb. per sq. in. (32.4 tons per sq. ft.) for all kinds of sound building stone.

For ashlar laid in lime mortar with joints not exceeding 1/2 in., it is best to use 225 lb. per sq. in. (16.2 tons per sq. ft.) for all kinds of sound building stone.

For ashlar laid in lime mortar with joints greater than 1/2 in. and less than 1 in. it is best to use 150 lb. per sq. in. (10.8 tons per sq. ft.) for all kinds of sound building stone.

For Brick Masonry in portland cement mortar, with joints not exceeding 3/8 in., the following working unit stresses are recommended:

	Pounds per sq. in.	Short tons per sq. ft.
For hard, best quality clay brick	300	21.6
For hard to medium clay brick and for sand-lime and concrete brick	250	18.0

For brick masonry in lime-cement mortar, use 75% of these values. For brick masonry in lime mortar use 50% of these values. If the bricks have been tested and show a uniform ultimate compressive strength of at least 7000 lb. per sq. in., and they are laid with great care in portland cement mortar, the safe working compressive strength may be taken at 400 lb. per sq. in. (28.8 tons per sq. ft.). Masonry of soft brick should not be used except for abnormally light loads, its safe working compressive strength being taken at 40 lb. per sq. in. (2.9 tons per sq. ft.).

For Hollow Tile, Concrete Block or Concrete Tile Masonry, the following working compressive stresses are recommended: With portland cement mortar, 80 lb. per sq. in. gross sectional area; with cement-lime mortar (1 : 1 : 4), 70 lb. per sq. in. gross area.

For Rubble Masonry laid in portland cement mortar, the following working compressive unit stresses are recommended:

Kind of rubble	Pounds per sq. in.	Short tons per sq. ft.
Flat or scabbled stones	250	18.0
Irregular stones, not scabbled	200	14.4
Very irregular stones (field stones)	100	7.2

For rubble masonry in lime mortar about 50% of the preceding values should be used as working unit stresses.

For Plain Concrete Masonry made with portland cement the following safe working compressive stresses in pounds per square inch are recommended: 450 for 1 : 2 : 4 concrete, 350 for 1 : 3 : 6, and 250 for 1 : 4 : 8.

The above unit stresses apply to compression from static loads, and do not apply to masonry where only a small portion of the area is under direct compression as under the footings of steel columns. (See Art. 22, Fig. 59*a*.) For shocks lower values should be used. Under bed plates of steel bridges use 50% of above values.

The Working Shearing Unit Stress for all stone masonry should be taken as one-quarter of the working compressive unit stresses above given. For concrete masonry experiments have shown that it may be taken as 30% of the compressive working stress provided diagonal tension does not accompany the shear or is properly provided for by steel reinforcement.

For Stone Slabs or single blocks of stone, the following working unit stresses are recommended, all being in pounds per square inch:

Kinds of stone	Compression	Tension*	Shear	Weight, lbs. per cu. ft.
Granite, syenite, or gneiss		150	200	170
Hard	1500			
Medium	1200			
Soft	1000			
Limestone		125	150	165
Hard	1000			
Medium	800			
Soft	700			
Marble		125	150	165
Hard	900			
Soft	700			
Sandstone	700	75	150	150
Blue stone flagging	1500	200		

* Values in this column apply to both direct and flexural tension.

If the shearing stress occurs without diagonal tension, the working shearing unit stress may be taken at 4/10 of the working compressive unit stress, but with detached blocks and slabs of stone shear is almost always accompanied by diagonal tension.

Tension, whether direct or flexural, should not occur in stone masonry, and working tensile unit stress should be taken as zero in the computations of designing. In the analysis of existing structures, where the mortar is found to be strong and adhesive, a tensile unit stress of 15 lb. per sq. in. may be allowed for masonry laid in portland cement mortar, 10 lb. per sq. in. for that in natural cement mortar and 5 lb. per sq. in. for that in lime mortar.

The resistance of masonry joints against tension is often wholly or partially destroyed by erection stresses, by shrinkage of the mortar in setting, and by expansion and contraction of the mass under changes of temperature. The mortar in the vertical joints, as a rule, wholly loses its adhesion from these causes. The mortar of the bed joints, however, sets under pressure and hence is more or less available to transmit tension. Since the shrinkage of mortar is less below ground, due to ground moisture and a uniform temperature, and since expansion and contraction are also less, masonry below ground is stronger in tension than that in air.

When it is necessary to build masonry to take tension, either direct or flexural, special care should be given to the bond. In the design of concrete footings a tensile unit stress not to exceed 8% of the safe compressive unit stresses may be allowed.

Allowed Working Compressive Stresses in Pounds per Square Inch, for Masonry, According to Building Laws

Kind of masonry	New York 1925	Philadelphia 1927	Baltimore 1924	Washington 1925	Buffalo 1926	Detroit 1926	Kansas City 1927	New Orleans 1927	Denver 1926	San Francisco 1926
Ashlar (portland cement mortar):										
Granite	600	.....	1000	800	.....	.....	600	600	800	389
Limestone	600	.....	600	500	.....	.....	600	600	500	.....
Sandstone	300	.....	400	500	.....	.....	300	300	400	.....
Rubble masonry:										
Portland cement mortar	140	139	125	.....	125	139	170	140	140	.....
Natural cement mortar	110	.....	.....	.....	.....	.....	.....	.....	100	.....
Lime-cement mortar	100	111	100	.....	97	97	120	105	100	.....
Lime mortar	.....	70	60	.....	70	70	70	70	70	.....
Brickwork:										
Portland cement mortar	250	208	150	170	250	208	225	175–450	170	208
Natural cement mortar	210	.....	.....	.....	.....	.....	175	.....	130	.....
Lime-cement mortar	160	167	.....	130	195	153	175	130–340	.....	139
Lime mortar	110	111	110	.....	125	97	120	90–225	90	97
Plain concrete:										
(1 : 2 : 4; portland cement)	500	208	500	400	348	111	500	By test	400	278

In design of structures to be built in cities, the unit stresses permitted by the building codes must be considered.

17. Unit Weights and Other Constants

Unit Weights of Masonry are slightly less than those of the materials of which it is composed. Average values are given in the accompanying table; the third column also gives the approximate moduli of elasticity and the fourth the coefficients of expansion for one degree Fahrenheit (based on using cement or cement-lime mortar).

Physical Properties of Masonry

Kind of masonry	Weight, lb. per cu. ft.	Modulus E, lb. per sq. in.	Coefficient *e* of expansion
Ashlar: granite, syenite, gneiss	165	4 000 000	0.000 0035
Limestone, marble	160	4 000 000	0.000 0035
Sandstone	140	4 000 000	0.000 0035
Mortar rubble: Granite, syenite, gneiss	155	2 000 000	0.000 0035
Limestone	150	2 000 000	0.000 0035
Sandstone	130	2 000 000	0.000 0035
Dry rubble: Granite, syenite, gneiss	130	....	....
Limestone	125	....	....
Sandstone	110	....	....
Brick: Pressed, thin joints	140	2 000 000	0.000 0030
Common, 3/8-in. joints	120	2 000 000	0.000 0030
Soft, 3/8-in. joints	100	....	....
Concrete: Broken stone, 1 : 2 : 4	145	2 500 000	0.000 0060
Broken stone, 1 : 3 : 6	145	2 000 000	0.000 0060
Cinder	110	....	....
Cyclopean: Masonry with maximum volume of stone	155	....	....

Slabs or detached block stone have values of E much higher than those in the table. Approximate values are 7 000 000 for granite, syenite and gneiss, and the harder limestones; 8 000 000 for hard marble; 5 500 000 for soft limestone; 2 800 000 for sandstone. Coefficients of expansion are 0.000 0040 for the granitic rocks, 0.000 0037 for limestone and 0.000 0050 for sandstone.

Values in the last two columns are used in investigating the temperature stresses which may come upon masonry arches (Art. 40) and for computing their deformations.

Weights of Other Materials which may bring pressure upon masonry walls and arches are given in the next table, in pounds per cubic foot.

Weights of Miscellaneous Materials

Sand, dry clean	90	Cinders, bituminous, dry compact	45
Sand, wet	115	Ashes, anthracite, dry compact	30
Gravel, clean	100	Paving in place:	
Broken stone	100	Asphalt top and binder	107
Clay, dry, compact	100	Asphalt block	145
Clay, plastic	100	Granite block	155
Sand, gravel, and clay, mixed:		Wooden block	50
Dry, compact	100	Brick	140
Wet	115	Macadam	105
Mud	110	Water, fresh	62.5
Rock, rotten, soft compact	110	Water, salt	64
Rock, hard, loose	100	Snow, fresh	8

The weight of snow to be used in designing is generally assigned by specification, as is also the lateral wind pressure, the latter being usually 30 lb. per sq. ft. of vertical surface.

The Slope of Repose of a bank of loose earth, in its natural state, is a factor which governs the lateral pressure which the earth may bring to exert against a retaining wall. In the next table the slope is the ratio of horizontal to vertical projection. It will be noted that the weight per cubic foot of filling material generally varies between narrow limits when the slope of repose varies between wide limits.

Slopes of Repose and Weights for Loose Earth

Kind of earth	Slope of repose	Angle of repose	Weight, lb. per cu. ft.
Sand, clean	1.5 to 1	33° 41′	90
Sand and clay	1.33 to 1	36 53	100
Clay, dry	1.75 to 1	29 44	100
Clay, damp, plastic	3 to 1	18 24	100
Gravel, clean	1.33 to 1	36 53	100
Gravel and clay	1.33 to 1	36 53	100
Gravel, sand and clay	1.33 to 1	36 53	100
Soil	1.5 to 1	33 41	100
Soft rotten rock	1.33 to 1	36 53	110
Hard rotten rock	1 to 1	45 00	100
Bituminous cinders	1 to 1	45 00	45
Anthracite ashes	1 to 1	45 00	30

The Angle of Repose given in the third column of this table is the angle ϕ which the sloping face of a bank of loose earth makes with the horizontal (Fig. 36). The cotangent of this angle is the slope ratio given in the second column; thus, 1.5 is the cotangent of 33° 41′. In general, if s is the slope of repose, or the ratio of horizontal to vertical projection, and ϕ the angle of repose, then $s = \cot \phi$. The term "natural slope" is sometimes used as synonymous with "slope of repose." The tangent of this angle is the coefficient of friction for earth upon earth, or $f = \tan \phi$ (Art. 18).

In the theory of retaining walls (Art. 19), the coefficient of friction along planes not on the surface but within the mass of the backing material is of interest. The angle corresponding to this coefficient is termed the angle of internal friction.

It has been found by experiment, that, for non-cohesive materials, the angle of internal friction is generally larger than the angle of repose given for loose earth in the above table, but, in determining the theoretic pressure upon retaining walls, the values of ϕ given in this table are recommended and are on the safe side. Non-cohesive materials, such as dry sand, gravel, broken stone, cinders, etc., consist of granular particles and resist shearing forces through friction only. Attention is called to the fact that the experiments of Jacob Feld show the angle of repose is the greater.

In the case of cohesive backing materials, such as clays which possess a degree of shearing (cohesive) strength, the angle of internal friction is considerably smaller than the angle of repose given in the above table and varies from 0° to 16°. The following table (used in Art. 19) was taken from A. L. Bell's paper (Proc. Inst. C. E., 1915):

Cohesive Strength and Angle of Internal Friction in Clays

Description of material	Cohesive strength lb. per sq. ft. c	Angle of internal friction ϕ_1
Puddle clay, wet	400–900	0°–2-1/2°
Stiff clay puddle	1200	2°
Sandy clay, wet	1200	2-1/2°
Stiff sandy clay	1000	10°
Moderately firm boulder clay	1400	6-1/2°
Very stiff, boulder clay fairly dry	3800	16°

For sand, the cohesive strength is very small, probably not more than 10 lb. per sq. ft., and should be entirely disregarded in practical design.

Material Excavated by a wet or a dry process, and dumped into water, as at the back of a sea wall, has submerged weights and slopes approximated as follows:

Material	Slope of repose	Angle of repose	Weight, lb. per cu. ft.
Sand, clean	2 to 1	26° 34′	60
Sand and clay	3 to 1	18 26	65
Clay	3-1/2 to 1	15 57	80
Gravel, clean	2 to 1	26 34	60
Gravel and clay	3 to 1	18 26	65
Gravel, sand and clay	3 to 1	18 26	65
Soil	3-1/2 to 1	15 57	70
Soft rotten rock	1 to 1	45 00	65
Hard rock, riprap	1 to 1	45 00	65

When the material is excavated by suction dredging and pumped back of a retaining wall which has inefficient drains to carry off the water, the weight per cubic foot may be taken at 110 lb. and the slope of repose as 2 to 1 for sand and clay, clay and gravel, or clay, gravel, and sand combined. River mud in situ may be taken at 100 lb. per cu. ft. with a slope of 3 to 1. If pumped back of wall, use 100 lb., with slope of 10 to 1. For fills placed by hydraulic dredge, see Sect. 8, Art. 26.

18. Data Regarding Friction

The Law of Friction for one stone beginning to move on another along their plane of contact is $F = fN$, in which F is the force parallel to the plane, N is the normal pressure on the surface of contact, and f is the coefficient of friction. Fig. 37 shows a stone about to move along a surface inclined to the horizontal at the angle ϕ, the conditions being exactly the same as those seen in Fig. 36 for a particle of earth on the face of a sloping bank. In both cases $F = fN$ and $f = \tan\phi$, where ϕ is the angle of repose, which in the case of stone surfaces is often called the "angle of friction." When the plane of contact is horizontal, as in Fig. 38, motion is about to begin when the resultant force R against that plane makes with the normal an angle ϕ such that $\tan\phi = f$. The angle of repose ϕ and the coefficient of friction f depend upon the roughness of the surface of contact.

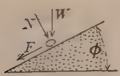


Fig. 36

Fig. 37

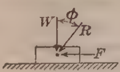


Fig. 38

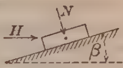


Fig. 39

The following table assumes that the surfaces of the stones are rougher than rubbed work, that is, they are for fine-pointed, bush-hammered, crandalled or sawed surfaces, while the wood surfaces are those of undressed yellow pine. For rubbed surfaces of stone the angles of repose are about 5° less than those given, and for polished stone about 10° less.

Moisture usually increases the angle ϕ, so that wet wood has a larger value than dry wood. For stone surfaces with fresh wet mortar between them, the angle ϕ is about 5° less than that in the table, the value given corresponding to the consistency generally used.

Arch Masonry built upon timber centering has usually a greater angle of friction than given in the table, due to mortar setting between adjacent pieces of lagging; in the case of concrete there is also a mechanical bond with the lagging, since when the lagging is removed the imprint of the grain can always be seen upon the concrete surface.

When a plane makes with the horizontal an angle β less than ϕ, the horizontal force H (Fig. 39) which is required to slide a body up the plane is $H = fN(\cos\beta - f\sin\beta)$, in which N is the pressure normal to the surface. This formula gives also the horizontal force required to slide the body down the plane, if the minus sign be changed to plus.

Coefficients and Angles of Friction

Kind of surface	Coefficient of friction f	Angle of friction ϕ
Granite, Limestone, Marble:		
Soft dressed upon soft dressed	0.70	35° 00′
Hard dressed upon hard dressed	0.55	28 50
Hard dressed upon soft dressed	0.65	33 00
Stone, brick or concrete:		
Masonry upon masonry	0.65	33 00
Masonry upon wood (with the grain)	0.60	31 00
Masonry upon wood (across the grain)	0.50	26 40
Masonry upon dry clay	0.50	26 40
Masonry upon wet or moist clay	0.33	18 20
Masonry upon sand	0.40	21 50
Masonry upon gravel	0.60	31 00
Soft stone upon steel or iron	0.40	21 50
Hard stone upon steel or iron	0.30	16 40

Values of the coefficient of friction given in this table are approximate, and in any actual case may vary within ± 30% of the figures given.

The angle of friction is materially lessened by jar, shock, or vibration caused by blasts, the passage of trains, or motion of attached machinery. For such cases it is recommended that in masonry design the angles of friction be taken as 5° less than those in the table and that the coefficients of friction be likewise modified by the relation $f = \tan\phi$.

For clay foundations it is best to use the values given for wet clay, unless it is reasonably certain that water cannot reach the bed.

19. Lateral Pressure of Earth

Conditions Affecting Earth Pressure. A wall backed with earth is subject to a lateral pressure which tends to overturn it as well as to cause it to slide. The amount of the pressure depends on the density of the earth, and on the internal friction and cohesion that may be developed in the earth and therefore to some extent on the water content.

Many attempts have been made by experiment and theory to determine the direction, magnitude and point of application of the resultant lateral pressure upon the back of a retaining wall so that the design of such walls may be made with precision. The materials used in filling behind walls and the methods and conditions of deposit are variable and it is difficult to determine with any degree of precision the characteristics which would permit the correct use of theory. However, even though only approximate results are obtainable no retaining wall should be designed without the application of theory to determine the distribution of pressures upon the foundation. The failure of retaining walls has generally been due to insufficient foundations, insufficient drainage permitting accumulation of water behind walls or neglect to consider surcharge. (See Art. 26.)

The Pressure on Trench Sheeting follows different laws from those applying to retaining walls. These laws are discussed in Sect. 8, Art. 4.

Earth Pressure Theories. Two principal theories have been proposed for the computation of earth pressure on retaining walls: (1) the "wedge theory," developed chiefly by Coulomb, and (2) the elastic or Rankine theory. All the theories assume that the backing material is of uniform granular consistency without cohesion, that the angle of internal friction is the same at every point in the mass, and that the pressure intensities increase uniformly with the depth (as shown by the arrows in Fig. 40).

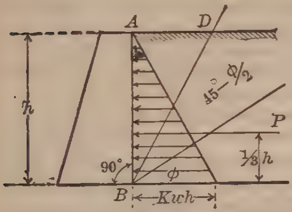


Fig. 40

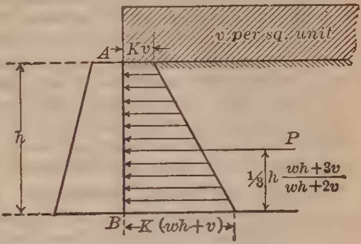


Fig. 41

Rankine's theory of earth pressure upon the back of retaining walls has been most commonly used. It is based on principles of elasticity applied to granular materials, the pressure being determined by means of the ellipse of stress. Coulomb's theory assumes that the maximum thrust on the wall is exerted by a wedge of the backing materials, this wedge being bounded on one side by the back of the wall and on the other side by a plane called the "plane of rupture" (Fig. 42) upon which the wedge tends to slide as the retaining wall is strained under the pressure. Experiments have shown that there is no actual motion on a plane of rupture, the concept of such a plane being useful only for purposes of analysis. However, it has been found that loads on the fill beyond the wedge have practically no effect on the lateral pressure. Actually, if the wall should fall over, the soil would break down with a concave surface rather than on a plane, the upper end of this surface of rupture approaching a vertical direction in many materials.

If cohesion is acting, the true earth pressure will be less than indicated by the theory. The extent to which the actual pressure will correspond to that derived from these theories depends on how homogeneous the filling material is, and upon its compactness and cohesion. As these factors are very uncertain, and quite variable in any given location, no attempt will be made here to include the influence of cohesion in the theoretical analysis. (For theories including cohesion, see "Stresses, Graphical Statics and Masonry," by G. F. Swain, 1927.)

Engineers interested in the pressure of earth on sheet piling should consult "Erddruck, Erdwiderstand," by H. Krey, published by Wilhelm Ernst & Son, Berlin, 1926. So far as the writer knows, this book has not been translated. It is one of the few books giving any information upon the pressures existing below the bottom of a retaining wall or sea wall that will enable one to determine the proper length of sheet piling to be driven below the bottom of the wall, or the weight of riprap to be placed in front of the wall, to prevent any blow-out of the material under the wall.

Direction of Earth Pressure. Different authorities make different assumptions as to the direction of the earth pressure acting on the back of retaining walls. Rankine makes the direction dependent on the slope of the retained bank and on the inclination of the back of the wall. Coulomb's theory gives the direction of the earth pressure upon the back of the wall as between the horizontal and a downward pointing line making an angle with the normal to the back of the wall, this angle not exceeding the angle of friction of earth on masonry.

Experiments made to determine the direction of earth pressure have shown considerable divergence in results. The most recent tests, however, indicate that the resultant pressure may be inclined downward to the wall, deviating from the normal by an angle equal to the angle of friction between the fill and the wall, thus agreeing with Coulomb. The amount of this possible friction is likely to be reduced, however, due to seepage of water behind the wall. For conservative design, therefore, the effect of friction on the back of the wall should be neglected; and the following assumptions are recommended:

(1) In all walls, regardless of whether or not the back of the wall is vertical or inclined, the pressure is assumed to be horizontal.

(2) In walls with backs inclined away from the fill (Fig. 44) the weight of the fill *ABL* resting upon the back should be regarded as part of the wall.

Magnitude and Point of Application. Formulas have been deduced giving the magnitude of the resultant earth pressure for all inclinations of the back of the wall and slope of the bank. Rankine and Coulomb agree as to the formula given below for the magnitude of the lateral pressure of level banks upon walls with vertical backs. Formulas for other conditions are complicated and for these, Rebhann's graphical method, based on Coulomb's theory, is hereafter given and is recommended. It gives results practically in accord with Rankine and is more simple. In the following a slice of wall of unit length will be considered.

Wall with Vertical Back. Level Bank. For conditions shown on Fig. 40, let w be the weight of the earth per cubic unit; ϕ its angle of repose, and h the height of the wall. The magnitude of the resultant earth pressure is

$$\mathbf{P} = 1/2\ wh^2 \frac{1 - \sin\phi}{1 + \sin\phi} = 1/2\ wh^2 \tan^2 (45° - \phi/2)$$

For a similar wall, but with the bank uniformly loaded with a weight v per unit of surface (Fig. 41)

$$\mathbf{P} = (1/2\ wh^2 + vh) \tan^2 (45° - \phi/2)$$

For a backing of clay, denoting the value of the cohesional resistance per unit surface by c and the angle of internal friction by ϕ_1 (Art. 17), Bell proposes the following formula for the resultant lateral pressure.

$$\mathbf{P} = 1/2\ wh^2 \tan^2 (45° - \phi_1/2) - 2ch \tan (45° - \phi_1/2)$$

In these formulas P is expressed in the same unit of weight as w and v.

ϕ	$\text{Tan}^2\left(45° - \frac{\phi}{2}\right)$	ϕ	$\text{Tan}^2\left(45° - \frac{\phi}{2}\right)$	ϕ	$\text{Tan}^2\left(45° - \frac{\phi}{2}\right)$
0°	1.000	20°	0.490	40°	0.218
5	0.840	25	0.406	45	0.172
10	0.703	30	0.333	50	0.132
15	0.589	35	0.271		

Intensity and Point of Pressure Application. In the value of P, $1/2\ wh^2$ and $(1/2\ wh^2 + vh)$ represent the magnitude equivalent to a hydrostatic pressure resulting from a head h of a fluid weighing w per unit volume and loaded with v if any; **tan**^{2} ($45° - \phi/2$) is the ratio of the true lateral pressure of earth to the hydrostatic pressure and is a constant for the same material. It is a fraction which increases when ϕ decreases; in other words, the flatter the angle of repose the greater the thrust. Denoting this ratio by K, the earth pressure P in the case of an unloaded bank is $1/2\ wh^2\ K$ which is equal to the area of a triangle of height h and base Kwh (Fig. 40); in the case of a loaded bank $P = (1/2\ wh^2 + vh)\ K$ which is equal to the area of a trapezoid of height h, the upper side equal Kv and the base equal $K\ (wh + v)$ (Fig. 41). Horizontal lines drawn in these diagrams will represent the intensity of the earth pressure at any point of the back of the wall. The resultant earth pressure will pass through the center of gravity of the intensity diagrams, its distance from the base, therefore, being **h/3** for the unloaded bank and **h/3 (wh + 3v)/(wh + 2v)** for the loaded bank. This latter value is greater than **h/3**, but the most excessive load cannot raise it as high as **h/2**. Jacob Feld has shown (Trans. A. S. C. E. Vol. 86, pg. 1448) that the resultant pressure generally acts at a distance from the base greater than $h/3$, but not more than $.4h$ for an unloaded fill. The ratio of this distance to the height of the wall tends to increase as the wall becomes higher.

Graphic Determination of Earth Pressure. In an embankment made of granulous material possessing friction but no cohesion and supported by a

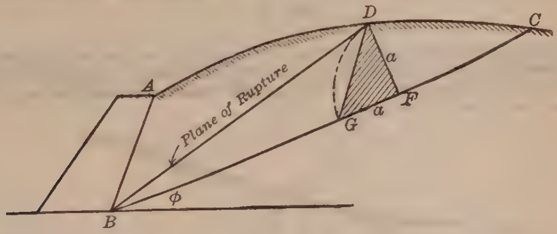


Fig. 42

retaining wall, the plane of maximum shear (plane of rupture) is located as follows: If, in Fig. 42, AB represents the back of the wall, BC the slope of repose and ADC the surface of the bank, then BD is the plane of rupture if area ABD is equal to the area of triangle BDF, DF being perpendicular to BC. Make $FG = DF$. The magnitude of the earth pressure is given by the product of the area, $DFG = 1/2\ DF = 1/2a^2$, and the unit weight of the earth, w. In the case of vertical back and level bank (Fig. 40) the plane of rupture bisects the angle $90° - \phi$ and this graphic method leads to the same formula as given above or $P = 1/2\ wh^2\ \tan^2\ (45° - \phi/2)$.

In the case of a wall with inclined back or supporting a sloping bank, or both, the plane of rupture and the magnitude of the earth pressure can be obtained by a cut and try method, making area $ABD = BDF$ on either side of the plane of rupture BD, or by the following construction. In Fig. 43 a wall with the back slightly inclined toward the fill supports a bank with a positive surcharge. Draw BC to make angle ϕ with the horizontal, C being the intersection of the slope of repose with the surface of the ground, and draw a semicircle with diameter BC. Draw AH perpendicular to BC and make $BF = BH$. Then draw $DF = a$, at right angles to BC and make $FG = DF$. The areas BAD and BDF are then equal and therefore BD is the plane of rupture. The area of triangle DFG, $= 1/2\ a^2$, multiplied by w will give the magnitude of the resultant earth pressure acting upon AB in a horizontal direction. $P = 1/2\ a^2w$. The intensity diagram will be a triangle the base of which is $BM = 2\ P/h$. This solution is perfectly general and applies for all slopes of the bank and inclinations of the back of the walls. In the case of a wall leaning away from the bank, however, the weight W resting upon the back of the wall (ABL in Fig. 44) should be regarded as part of the wall and the earth pressure upon the vertical BL determined. Combine P graphically

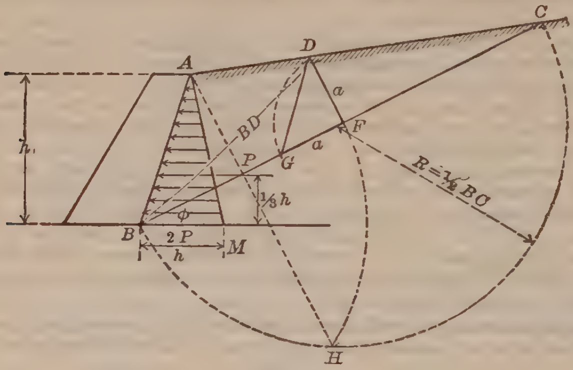


Fig. 43

with W to get the pressure P' upon the back of the wall. P' will be inclined downward.

If the surcharge (Fig. 45) is parallel to the slope of repose of the retained bank, the above (Fig. 43) construction is not necessary, the distance between the lines AD and BC being equal to a in $P = 1/2\ a^2w$.

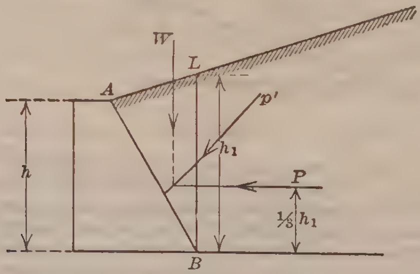


Fig. 44

If the bank is loaded with a load of v per unit surface, draw an imaginary slope, parallel to the true slope and at distance $h_1 = v/w$ above that slope (Fig. 46); find P_1 for the extended wall A_1B by the construction shown in Fig. 43; and construct the intensity triangle for wall A_1B, the base of the triangle being $2\ P_1/(h + h_1)$. Then the area of trapezoid $ANMB$ will represent the pressure P on wall AB, being applied at the center of gravity of the trapezoid.

If the surface of the retained bank is broken, as in Fig. 47, locate the plane of rupture BD by trial and successive approximation so that area $BAMND$ shall equal area BDF. Then $DF = a$ and $P = 1/2\ a^2w$. In this case the point of application of the resultant earth pressure is not at $1/3\ h$ from the base, the intensities not being a linear function of the depth. The actual distribution of the pressure on the back of wall AB and consequently the distance of the resultant pressure from the base cannot be determined graphically in any simple way but for this distance the use of $0.4\ h$ is recommended.

If the bank has a plane top surface, but is only partially loaded, apply the same method as shown above (Fig. 47), the loading being transformed into fill resulting in a broken surface.

In computations h is taken in feet, w in pounds per cubic foot, and v in pounds per square foot, so that values of P are in pounds for one linear foot of wall. For

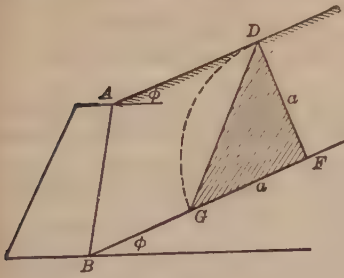


Fig. 45

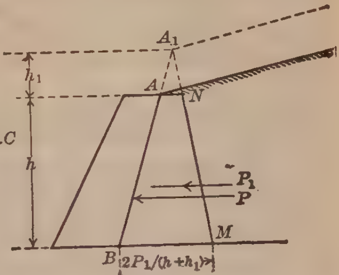


Fig. 46

example, let h = 18 ft., w = 100 lb. per cu. ft., ϕ = 34°. For an unloaded level bank retained by a wall with vertical back (Fig. 40) P is 4580 lb. and its point of application is 6 ft. above the base. If this bank is loaded (Fig. 46) so that v = 120 lb. per sq. ft., then P is 5190 lb. and its point of application is 6.35 ft. above the base.

Experiments on Earth Pressure. Many retaining walls which have stood for years, if investigated by means of the foregoing theories would be found

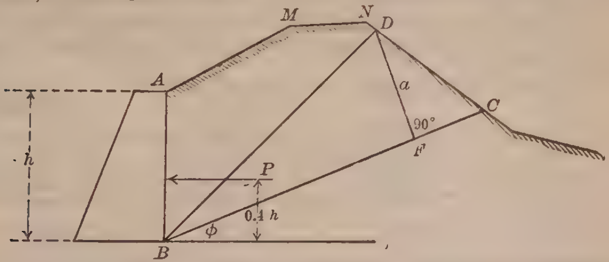


Fig. 47

inadequate. This fact shows that the theories furnish excessive values for the lateral pressure of earth, due possibly to the neglect of cohesion, which to a greater or less amount exists in the backhill, unless the latter is granulous. During the last 40 years many experiments were made to obtain data upon which more accurate theories than those of Rankine and Coulomb might be based. The most important recent experiments were made by Jacob Feld at the University of Cincinnati, 1919-1921 (Trans. Am. Soc. C. E. Vol. 86, p. 1448; this paper also gives a summary of all the important publ shed tests on earth pressure). Many of the earlier experiments were made on small-scale models and the results have not always been confirmed by large-scale experiments. Some of the more important experiments are summarized below:

M. M. L. Leygue (Annales des Ponts et Chaussées, 1885) showed that the amount of cohesion existing in the dry sand is negligible and that the surface of rupture is not plane.

Professor Mueller-Breslau (Erddruck auf Stützmauern, 1906) experimented on a wall 29 in. high. He measured both horizontal and vertical components of the lateral pressure. His results agree closely with the Coulomb theory. He investigated the wedge of rupture photographically, and found that the surface of rupture was slightly curved down at the lower end.

P. M. Crosthwaite (Proceedings Inst. C. E., Vol. 103 and Vol. 109) in experiments made between 1916–1920 found no visible plane of rupture. His results agreed with the Coulomb theory for walls without surcharge, if the angle of internal friction was used. The horizontal component of the earth thrust was independent of the wall friction.

A. R. Fulton (Proceedings Inst. C E., Vol. 109) made tests on a wall 7 ft. high, and obtained results approximating Coulomb's theory, with resultant pressure at one-third the height of wall and making an angle equal to the angle of repose with the normal to the wall.

Dr. Charles Terzaghi has done extensive experimental work on the physical properties of soils and the laws of soil mechanics. His results, however, are more applicable to the bearing power of soils under foundations. (See Sect. 8, Art. 3.)

Measurements of pressure on actual retaining walls 30 ft. high were made by the U. S. Bureau of Public Roads (Public Roads, July, 1925). Pressures were measured by cells placed on the back face of the wall. The results showed pressures varying somewhat from a triangular distribution, with resultant higher than the one-third point. The pressures were found to increase with moisture content in the fill, indicating the importance of suitable provisions for draining the fill.

20. Pressure of Water

Hydrostatic Pressure against a surface immersed in water is wAd, where w = weight of a cubic unit of water, A = area of surface, and d = depth of the center of gravity of the surface below the water level. When dimensions are in feet, the total pressure in pounds is 62.5 Ad for fresh water and 64 Ad for salt water. For an immersed plane the direction of this pressure is always normal to the surface.

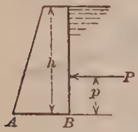


Fig. 48

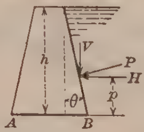


Fig. 49

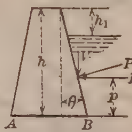


Fig. 50

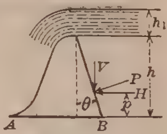


Fig. 51

Pressures on a Dam. In Figs. 48 and 49 the water level is at the top of the dam, in Fig. 50 it is below and in Fig. 51 it is above the top. The horizontal pressure H and the height of its point of application above the base are given by

For Figs. 48 and 49 $\mathbf{H} = 1/2\ \mathbf{wh}^2$ $\quad \mathbf{p} = 1/3\ \mathbf{h}$

For Fig. 50 $\mathbf{H} = 1/2\ \mathbf{w}\ (\mathbf{h} - \mathbf{h}_1)^2$ $\quad \mathbf{p} = 1/3\ (\mathbf{h} - \mathbf{h}_1)$

For Fig. 51 $\mathbf{H} = 1/2\ \mathbf{wh}\ (\mathbf{h} + 2\mathbf{h}_1)$ $\quad \mathbf{p} = 1/3\ \mathbf{h}\dfrac{\mathbf{h} + 3\mathbf{h}_1}{\mathbf{h} + 2\mathbf{h}_1}$

Also the normal pressure and its vertical component are

$$\mathbf{P} = \mathbf{H} \sec \theta \qquad \mathbf{V} = \mathbf{H} \tan \theta$$

The notation is shown on the figures and P is for unit length of dam. The distribution of unit pressures on the back of dams is exactly that for earth illustrated in Figs. 40 and 41. Thus, at the base the unit pressure for Fig. 48 and 49 is $\mathbf{wh}$, for Fig. 50 it is $\mathbf{w}\ (\mathbf{h} - \mathbf{h}_1)$ and for Fig. 51 it is $\mathbf{w}\ (\mathbf{h} + \mathbf{h}_1)$.

An Overflow Dam, or spillway dam, is one where the water flows over the top, as in Fig. 51. The effect of the water above the top is both to increase the pressure P and to raise its point of application. The following are the percentages of increase in P and p over those for the case of Fig. 49, due to different ratios of h_1 to h;

h_1/h........	0.05	0.10	0.15	0.20	0.30	0.40	0.50
For P......	10	20	30	40	60	80	100
For p.......	4.5	8.3	11.5	14.3	18.8	22.2	25.0

The flow of water over a spillway dam may cause a partial vacuum beneath the falling water sheet or nappe if it is not in contact with the masonry. If the atmospheric pressure between the masonry and the falling sheet of water is thus reduced from 14.7 lb. per sq. in. to a value q over the whole front face, the practical effect is to add a unit pressure $14.7 - q$ lb. per sq. in. to the back of the dam and thus to cause an additional normal pressure $144\ (14.7 - q)\ h$ lb., h being expressed in feet, whose point of application may be as high as $1/2\ h$. The possibility of decrease of atmospheric pressure between the dam and the falling water sheet should be kept in mind by a designer, but it is obvious that only a small percentage of the theoretically possible total pressure need be used in design. (Art. 32.)

For a Dam Backed with Earth, the hydrostatic pressure upon the back of the dam may be increased or decreased by the backfilling material depending upon whether or not the material is porous or so non-porous that it will not permit the transmission of water pressure. A puddled bank of sand, clay and gravel in which the clay fills the voids of the harder materials would materially reduce the pressure when compacted. Silt and detritus deposited behind a dam may increase the pressure. Sand, gravel and broken stone (riprap) backing increase the pressure due to water alone. For this case, in addition to the full hydrostatic pressure, the pressure of the backfill must be calculated by the methods given in Art. 19, using the values of w and ϕ as given in Art. 17 for material excavated by a wet process and dumped into water.

Water below the Base exerts an upward pressure upon the base which has the effect of diminishing the weight of the masonry, thereby reducing its resisting value against both overturning and sliding. This is one reason why it is very important to prevent water from entering below the base of a dam as far as it is practicable. To exclude such water entirely and completely prevent uplift is generally difficult, expensive, requires experience and a thorough investigation of the foundation bed and underlying strata. Where the dam rests upon solid unfissured rock the problem is fairly simple but in the average case of a dam resting on ordinary rock, hardpan or more pervious material the problem is difficult. One must allow for uplift of an intensity dependent upon the character of the foundation bed and upon the cost of the special construction necessary to prevent or diminish it. The magnitude of this upward pressure depends upon the head, the character of the foundation, design and construction, and no general rule can be given for its intensity and distribution (Art. 30). Specifications often call for full upward pressure acting at the heel B (Fig. 49) and diminishing to zero at the toe A. This is the extreme case if there is no apron as in Fig. 49. For this case the unit upward pressure at the heel is wh, the **total uplift 1/2 whb,** where b is the base width, and the force is applied at a distance $1/3\ b$ from the heel B. If there is an apron, the unit upward pressure at the toe is greater than zero as in Fig. 87. When uplift is considered as acting upon the base, no diminution of the weight of the masonry is to be made in the computations.

21. Internal Stresses in Walls and Dams

The external forces which act on a masonry structure are its weight and the loads that it carries, the lateral pressures of earth, water and wind, if such exists, and the reactions of the foundation. The deformation caused by these external forces induces internal stresses of compression, shear and sometimes tension within the structure. For the purpose of determining the internal stresses, the wall will be considered as having a uniform cross-section, uniformly acted upon at every unit of its length by the same vertical or vertical and lateral (horizontal or inclined) forces, so that to determine the actual stress in the wall any unit of its length may be considered. It will be assumed in computing stresses that the masonry of the wall is homogeneous and obeys (within the limits of working stresses) Hooke's law for elastic solids, that is, stress is proportional to strain. A horizontal joint of a wall, AB in Figs. 52 to 55 will be investigated. It forms a parallelogram of width $AB = b$, length unity and area b.

Vertical Loads, Normal Stresses on Horizontal Joints. Let W be the resultant of the vertical loads acting on Joint $AB = b$, c the middle point of AB, e the eccentricity of W or the distance of the point of application of W from c, and x the distance of any point of the joint from c, positive if measured in the direction of W and negative for the other half of the joint. Then the normal stress per unit area at any point x, expressed in the same units as W and b, is

$$\mathbf{S} = \frac{\mathbf{W}}{\mathbf{b}}\left(1 + 12\,\frac{\mathbf{ex}}{\mathbf{b}^2}\right)$$

which formula is called the **law of the trapezoid.** At the middle point c, where $x = o$, $\mathbf{S}_c = \mathbf{W/b}$ for all values of e; in other words, at the middle of the joint the unit stress is always equal to the average stress along the joint.

At the edges of the joint where $x = +b/2$ or $-b/2$, the unit normal stress is a maximum (S_1) or minimum (S_2) for any value of e and

$$\mathbf{S}_1 = \frac{\mathbf{W}}{\mathbf{b}}\left(1 + \frac{6\mathbf{e}}{\mathbf{b}}\right); \quad \mathbf{S}_2 = \frac{\mathbf{W}}{\mathbf{b}}\left(1 - \frac{6\mathbf{e}}{\mathbf{b}}\right)$$

S_1 is always positive (compression) for any value of e. S_2 is positive (compression) as long as $e < b/6$ (Fig. 52). It becomes zero if $e = b/6$ (Fig. 53). If the resultant falls outside the middle third, $e > b/6$, the formula furnishes a negative value (tension) for S_2 (Fig. 54) and this formula does not apply unless the masonry is able to develop tension. This case is discussed below. Usually no dependence should be placed upon masonry to take tension.

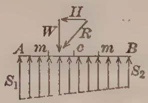


Fig. 52

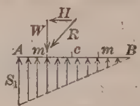


Fig. 53

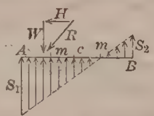


Fig. 54

Since in design the maximum and minimum stresses largely govern, these formulas for the stresses are more generally used and will be further discussed. If $e = o$, the resultant load is applied at the middle point of the joint and $S_1 = S_2 = W/b$ (compression), i.e., the stress is uniform along the joint. If $e = b/6$, the resultant load is applied at the middle third point and $S_1 = 2W/b$ (compression) or twice the average value of the stress, and $S_2 = 0$.

If $e > b/6$ and the joint cannot take tension, part of it will not be stressed at all, while the other part will transmit compressive stresses only. This condition of stress distribution is shown in Fig. 55, where the distance e of the point of application g of the resultant load is greater than $cm = b/6$. Let r be the distance of W from the nearest edge of the joint A in Fig. 55; then the point of zero stress O will lie at a distance $3r$ from this edge and $S_1 = 2W/3r$; for $r = 0$, S_1 becomes infinity. Substituting the safe unit stress s for S_1 we get $r =$

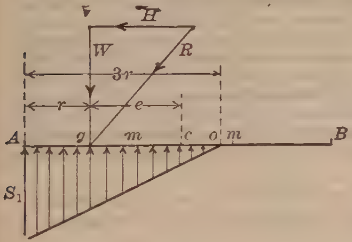


Fig. 55

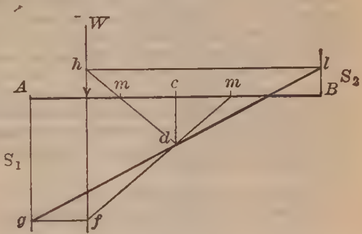


Fig. 56

$2W/3s$. This gives the minimum distance r from the edge of the joint to the resultant permissible with a given safe unit stress.

If tensile stresses are considered in a masonry wall the general formulas given above for S_1 and S_2 should be used even though the resultant falls outside the middle third. For such a case, if $e = 1/2\ b$, the resultant is applied at the edge of the joint, $S_1 = 4W/b$ (compression) or four times the average stress, and $S_2 = 2W/b$ (tension) or twice the average stress.

Graphically the stresses along AB may be found by the following construction. Lay off (Fig. 56) $cd = W/b$ the average stress, connect the middle third points m and m with d and produce lines md to intersect with W in f and h. Draw horizontal fg and hl to intersect verticals A and B in g and l. Then Ag and Bl will be the maximum and minimum normal stresses and the stress at intermediate points will be obtained by scaling the vertical intercept between AB and gl, ordinates above AB indicating tension.

Example. Let $W = 30\,000$ lb. and $b = 6$ ft. Then for W applied at the middle of the joint, $e = 0$, $S_1 = S_2 = 5000$ lb. per sq. ft. For W at the middle third point, $e = 1$ ft., $S_1 = 2\ W/b = 10\,000$ lb. per sq. ft., $S_2 = 0$. For W at $1/3\ b$ from the center, $e = 2$ ft., $S_1 = 3\ W/b = 15\,000$ lb. per sq. ft., $S_2 = -Wb = -5000$ lb. per sq. ft. If the joint cannot take tension, $r = 1$ ft., $S_1 = 2W/3\ r = 20\,000$ lb. per sq. ft. If the masonry is rubble, with a safe compressive stress of 18 tons per sq. ft., how near in safety can the resultant come to the edge of the joint? $r_{min.} = 2W/3\ S = 2 \times 30\,000/3 \times 36\,000 = 5/9$ ft.

Vertical and Lateral Loads. Normal Stresses on Horizontal Joints. If lateral forces act upon a wall, the resultant of these and the vertical loads will be inclined to the joint as R in Figs. 52 to 55. In order to find the normal stresses on the joint, resolve R into a vertical and a horizontal component, W and H at the point of application of R and treat W exactly as shown in the preceding discussion. H, the horizontal component of R, will produce compression on vertical planes and shearing stressses along the joint, if it is capable of taking shear. If not, the stability of the structure is dependent upon the friction developed along the joint. Designers should not add the safe shearing value to the safe frictional resistance, because the latter will not come into action unless the shearing resistance has been overcome.

Shearing Stresses in Walls. Careful consideration should be given to the shearing stresses in the design of dams over 50 ft. in height but they need rarely be considered in the design of other walls. The magnitude and distribution of shear in walls cannot be determined by the customary formulas for beams since they generally are not prismatic and are always subject to axial loads. The only case in which shearing stresses do not exist in horizontal and vertical planes is when the wall has a vertical face and back and carries an axial or an evenly distributed vertical load, but in this case there are shearing stresses along every other plane.

Let a prismatic wall (Fig. 57) be subjected to an axial compressive force P, then the intensity of the compressive stress on joint AB normal to the axis will be $= P/a$, if a is the area of joint AB. On a plane CD whose normal is inclined to the axis at an angle ϕ there is a stress in the direction of the axis and therefore oblique to the plane CD, of intensity P/a_1 where a_1 is the area of joint CD, that is, $a_1 = a/\cos\phi$. The whole stress P on CD may be resolved into two components, one normal to CD and the other a shearing stress tangential to CD. The normal component is $P\cos\phi$, the tangential component is $P\sin\phi$. The unit intensity of normal compression on CD, therefore, is $(P/a)\cos^2\phi$ and the intensity of shearing stress is $(P/a)\sin\phi\cos\phi$. This expression makes the shearing stress a maximum when $\phi = 45°$; surfaces inclined at 45° to the axis are called surfaces of maximum shearing stress, the intensity of shearing stress on them is $P/2a$.

In the case of retaining walls and dams which are subjected to horizontal loads besides vertical ones and generally have a wedge-shaped cross-section, it

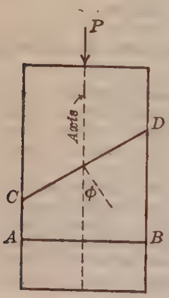


Fig. 57

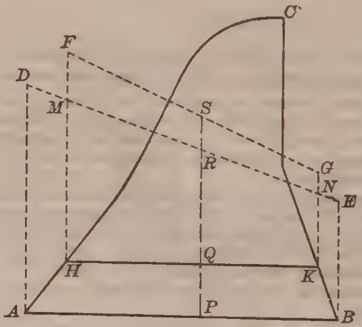


Fig. 58

is obvious that shearing stresses exist on every horizontal joint, the total of such stresses being equal to the resultant horizontal force above the plane of the joint. The intensity of the horizontal shear will vary from point to point along the joint. Its distribution can be determined by applying the principle that at any point of the structure the horizontal and vertical shear intensities must be equal. Let Fig. 58 represent the cross-section of a dam, a slice of unit thickness being considered. According to the **law of trapezium,** the distribution of the normal pressure on joint AB may be represented by $ADEB$; and if forces are expressed in masonry units (the unit of weight being the weight of one cubic unit of masonry) and the same scale used for dimensions and forces, the area $ADEB$ will be equal to the area $AHCKB$. Similarly the distribution of thrust on joint HK will be represented by $HFGK$, which is equal to HCK. The weight of the layer between AB and HK will be represented to the same scale by $AHKB$. Consequently, $ADMH + KNEB = MFGN$. Take any vertical PS. The difference between the vertical forces to the right of PS and acting on AB and HK respectively $= RSGN - KNEB$, and that to the left of $PS = ADMH - MFSR$. These two resultant forces are equal but of opposite sign, and either of these differences represents the resultant of the vertical forces on either side of the layer $AHKB$, to the right or left of PQ, and therefore, the shear on PQ. Hence the intensity of shearing stress at P (horizontal and vertical) is approximately either $(RSGN - KNEB)\ PQ$ or $(ADMH - MFSR)/PQ$. The approximation will be closer the smaller is PQ (Unwin, Engineering, Vol. 79).

Applying the foregoing method to a triangular dam with vertical back, the distribution of the shearing stresses is found to be uniformly varying from zero at the back to a maximum at the face, the shear diagram being a triangle. In the case of a rectangular dam the shear diagram is a parabola with vertical axis; the shear intensity is zero at either face and maximum at the center where its value is $q_{max} = 3\,h^2/4\gamma b$, h being the depth of water to the joint in question, b the width of the dam and γ the density of the masonry relative to water.

For the important practical case of a dam with vertical back and inclined face, E. P. Hill (Proc. Inst. Civ. Eng., 1908, Vol. 172) gives the following formula for the **intensity of shear on horizontal or vertical planes** at any point of a horizontal joint:

$$q = \left[4\,Wm(b - 3d) + \frac{h^2(3b - 2\,mh)}{\gamma}\right]\frac{x}{b^3} + \left[6\,Wm(3d - b) - \frac{3h^2(b - mh)}{\gamma}\right]\frac{x^2}{b^4}$$

In this formula W is the total weight above the joint in masonry units (i.e., if b, h, d and x are expressed in feet, W must be multiplied by the weight in pounds per cubic foot of masonry and q then will be expressed in pounds per square foot), b the width of the joint and h the depth of the water above it, d the distance of the center of gravity of W from the water edge of the joint, x the distance from the water edge of the point where q is to be determined, m the tangent of the angle between the face of the dam and the vertical and γ the specific gravity of the masonry. The formula shows that the shear intensity on horizontal (or vertical) planes is practically proportional to the distance from the back of the dam at which point it is zero. The experiments of Ottley, Brightmore, Wilson and Gore (Proc. Inst. Civ. Eng., 1908, Vol. 172) have verified this with the exception of the joints at and near the base, where, on account of the rigidity of the fixing of the dam, the shear intensities are practically uniformly distributed along the joint. The shearing stresses to be considered in the design are not those at the base but higher up and near the face. For practical purposes, except in the case of a very high dam (for which the closer analysis is necessary), the shearing stress along horizontal joints more than $1/5\ h$ above the base can be taken as uniformly varying from zero at the back to a maximum at the face at which point it equals $2\ H/b$, H being the magnitude of the resultant of the horizontal components of the lateral forces above the joint considered and equal to the total shear along the joint. For the joints near to and at the base, the shearing stress should be taken as uniformly distributed having an average value of H/b. (See example, Art. 31.)

Horizontal Pressures on Vertical Planes. The lateral external forces (water or earth pressure) also generate normal stresses on vertical planes. For a dam with vertical back and inclined face, Hill gives the following formula to determine the intensity p' of these stresses, the notation being the same as in the preceding paragraph:

$$p' = \frac{h}{\gamma} + \left[\frac{2\,Wm^2}{b^3}\left(2 - \frac{9d}{b}\right) - \frac{3h(b - mh)^2}{\gamma b^4} + \frac{m}{b}\right]x^2 + \left[\frac{6\,Wm^2}{b^4}\left(\frac{4d}{b} - 1\right) + \frac{2h(b - mh)\,(b - 2\,mh)}{\gamma b^5} - \frac{m}{b^2}\right]x^3$$

p' is in masonry units, similarly to q in the preceding paragraph. The horizontal stress intensity on vertical planes, therefore, increases from the back. For $x = o$, $p' = h/\gamma$. But since p' is expressed in masonry units, h/γ must be multiplied by $62.5 \times \gamma$ and becomes $62.5\ h$, or in other words, at the back of the dam p' equals the unit water pressure For all walls, except very high structures, where a closer investigation is necessary, the normal stress intensity on vertical planes, for every point of a horizontal joint, can be taken as constant and equal to the intensity of the lateral forces at the elevation of the joint under investigation. (See example, Art. 31.)

Maximum and Minimum Stresses. Both normal and shearing stresses exist at every plane passing through any point of an elastic solid which is in equilibrium under the influence of external forces. These stresses vary in magnitude for the same point according to the direction of the plane and there invariably exists one plane on which the normal stress intensity is a maximum, and another plane perpendicular to the first, on which the normal stress intensity is a minimum. These limiting values of the normal stresses are termed principal stresses and they can be determined both as to magnitude and direction if the normal and shearing stresses at a certain point are known on two planes at right angles to each other. At any point of the horizontal joint investigated, let S and p' be the intensities of the normal stresses on horizontal and vertical planes and let q be the intensity of the shear on these two planes, both kinds of stresses being determined by the methods given above (compressions taken as positive and tensions as negative), then the magnitude of the principal stresses at the same point is given by the formula:

$$f = \frac{S + p' \pm \sqrt{(S + p')^2 - 4(Sp' - q^2)}}{2}$$

the positive sign before the radical furnishing the maximum and the negative sign the minimum principal stress.

Let θ be the inclination of the principal stress to the horizontal, then

$$\cot \theta = (f - p')/q$$

These formulas show that even in the case when the resultant cuts the horizontal joint within the middle third, in other words when S is compression along the whole joint, tensile stresses will occur at some points in a certain direction, if $Sp' < q^2$. According to Levy, in order that tensile stresses should entirely be eliminated, the design should be such that not only shall the resultant lie within the middle third of the joint, but that S_2 (minimum normal stress) shall be not less than the intensity of the horizontal components of the lateral forces at the elevation of the joint investigated (or the hydrostatic pressure in the case of a dam). It can also be shown that Sp' is always greater than q^2 at the toe of retaining walls and dams designed for the requirement that the resultant shall fall within the middle third of the horizontal joints and, therefore, no tension can exist at and near the face of such walls. The maximum shearing stresses in walls with vertical face and back and vertical loading only are inclined 45° to the vertical and equal to $S/2$ for every point of the horizontal joint, S being the intensity of the constant normal stress upon the joint. For other methods of loading, according to Wilson and Gore, the maximum shearing stress can be approximately expressed by the formula $q_{max} = S/2 \cos^2 \phi$, where S is the normal stress on the horizontal plane at the point considered and ϕ the angle between the resultant R and the vertical. According to Bouvier, the maximum stresses in dams of the usual cross-section are about 13/9 times the stresses on horizontal joints. This rule gives good approximate values also for other types of walls for preliminary design.

G. F. Swain shows that the maximum compressive stress at the downstream face is on a plane normal to the face, and equals $S \sec^2 \beta$, where S is normal unit stress on a horizontal plane at the downstream face and β is angle between downstream face and the vertical. (Stresses, Graphical Statics and Masonry (1927), p. 468.)

The above discussion of stresses and the formulas do not take in account the factor of lateral deformation of the masonry which is probably very small. (See example, Art. 31.)

Joints in Arches. The above discussion has been developed for the horizontal joints of walls and dams, but it applies equally well to the radial joints between the voussoirs of stone arches. Against such a joint there acts a resultant force R, and the component of this parallel to the joint gives the total shear H; the component normal to the joint may be called N. Then the preceding formulas apply to such a radial joint if W is replaced by N, and the distributions of the normal stresses are exactly like those shown in Figs. 52-55.

22. Local and Eccentric Loads

A Bridge Pier which sustains only vertical loads whose resultant coincides with the axis of the pier has the compressive unit stresses uniformly distributed on the base. The least area for the base is $A = W/S$, where W is the total vertical load on that base and S is the allowable unit stress from Art. 16. Practical considerations, however, usually necessitate a larger section. Small piers in buildings should have this least area, unless the r length is such that columnar action exists, and then S must be taken smaller (Art. 29).

Under a Concentrated Load on the top of a pier, the compressive unit stress may be taken higher than for a section of the pier lower down. If the area under local compression is 0.3 of the total area of masonry over which the compression is distributed, the values in Art. 16 may be increased 60%; if that area is 0.1 of the masonry area, those values may be increased 200%. For other values of the ratio r the increase may be made in direct proportion: for r between 0.1 and 0.3, per cent increase $= 270 - 700\ r$; for r between 0.3 and 1.0, per cent increase $= 600\ (1 - r)/7$. These percentages apply when the concentrated load is at the middle of the top of the pier as at (a) in Fig. 59; thus, for a steel column having a base 15 by 15 in. and a masonry pier 24 by 24 in. the value of r is 0.4, and the allowable S of Art. 16 may be increased 51% under the base of the column.

An Eccentric Load is one placed so that its line of action does not coincide with the gravity axis of the pier as at (b) in Fig. 59. Here the distributions of unit stresses on the base are like those shown in Art. 21, and when the eccentricity e is large enough the stresses on one side may be tensile. For this case the working unit stress S cannot be increased to the full extent indicated in the last paragraph, and when the load is close to the edge of the pier top no increase should be made. A specially unfavorable condition is that in the first diagram of Fig. 61a, where there is an eccentricity in both horizontal directions, so that the opposite corner of the pier should be anchored down in order to prevent it from rising or from cracking by tension.

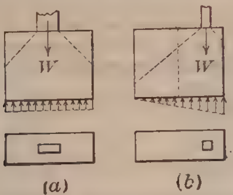


Fig. 59

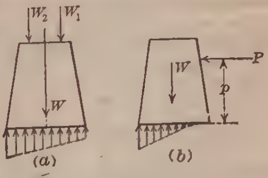


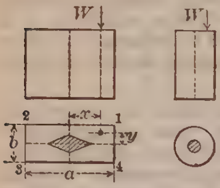
Fig. 60

The inclined lines in Fig. 59a show the manner in which the local compression spreads out until it finally becomes uniformly distributed over the horizontal cross-section; the inclination of these lines is about 45°. For the eccentric load in Fig. 59b, one of the inclined lines quickly reaches the edge of the pier, indicating that the unit stress under the column must be taken lower for an eccentric load than for a central one.

A Bridge Pier under Vertical Loads (Fig. 60a) is subject to non-uniform compression on the base when the two adjacent spans of the bridge differ in length or when only one of two equal spans is covered with live load. Thus in Fig. 60*a* let W be the weight of the pier itself above the base, and W_1 and W_2 the weights brought upon the top of the pier from the two spans of the bridge, these being applied at the distance c from the middle of the top. Here the resultant vertical pressure cuts the base at a distance e from the center which is found from $e = c\,(W_1 - W_2)/(W + W_1 + W_2)$, and the distribution of compressive unit stresses on the base is the same as in Art. 21. When $W_1 > W_2$ in Fig. 60*a*, this point is at the right of the center; when $W_1 < W_2$, it is at the left of the center. The proper formula of Art. 21 enables the maximum compressive unit stress to be found.

Wind blowing on a tower or chimney, as shown in Fig. 60*b*, causes a similar eccentricity of the resultant on the base; for this case, the value of $e = Pp/W$. Wind blowing at right angles to the line of a bridge causes a similar longitudinal eccentricity on the base of a pier, whereas the lateral eccentricity due to live loads is the same as given in last paragraph. Two horizontal wind forces may act on a bridge pier, P_1 for wind blowing against the bridge spans and P_2 for wind against the pier itself; here the above equation also holds if Pp is replaced by $P_1p_1 + P_2p_2$.

The Kern is an area on the base of the pier within which the resultant force that acts on the base must lie in order that no part of the base may be subject to tension. To prevent tension on any corner or side of the base of a masonry pier, wall, or dam, the line of action of the resultant must cut the base within the kern. For a rectangular section with sides a and b, the kern is a rhombus which has the diagonals 1/3 a and 1/3 b, the corners of this rhombus pointing toward the middle of the sides of the rectangle as shown in the plan of Fig. 61*a*. For a square with side d, the kern is a smaller square which has the diagonal 1/3 d and the sides of which are parallel to the diagonals of the given square. For a circular section of diameter d the kern is a smaller concentric circle of diameter 1/4 d as in Fig. 61*b*. The resultant is the weight W above the base in case no lateral force acts upon the structure, otherwise it is the R of Art. 21, which is found by combining W and H.



(a) (b)

Fig. 61

A Rectangle with sides a and b (Fig. 61*a*) is the most common case. Let x and y be the eccentricities with respect to axes of the rectangle parallel to the sides a and b. Then the unit stresses at the four corners are

$$S_1 = \frac{W}{ab}\left(1 + 6\frac{x}{a} + 6\frac{y}{b}\right) \qquad S_3 = \frac{W}{ab}\left(1 - 6\frac{x}{a} - 6\frac{y}{b}\right)$$

$$S_2 = \frac{W}{ab}\left(1 - 6\frac{x}{a} + 6\frac{y}{b}\right) \qquad S_4 = \frac{W}{ab}\left(1 + 6\frac{x}{a} - 6\frac{y}{b}\right)$$

When W lies within the kern all of these are compression; the formulas hold, however, even if W is without the kern, provided the masonry can take tension near the corners and edges.

For a Circular Cross-Section of diameter d, the maximum and minimum unit stresses, due to a load having the eccentricity e, are

$$S_1 = \frac{W}{A}\left(1 + 8\frac{e}{d}\right) \qquad S_2 = \frac{W}{A}\left(1 - 8\frac{e}{d}\right)$$

in which A is the area of the circle. The unit stress S_2 is tensile if the load is without the kern, but the formula gives it correctly when the masonry can resist tension. The radius of the kern is 1/8 d (Fig. 61b).

If the material is unable to develop tensile stresses and W acts outside the kern in one of the main axes of the figure comprising the horizontal joint of the pier, the following methods to determine the distribution of normal stresses are recommended.

For a pier rectangular in plan as in Fig. 61a and W being applied on the axis parallel to side a at a distance r from the nearest short side of the joint, $S_1 = 2\,W/3\,rb$.

For a horizontal joint having any symmetrical figure (as, for instance, a chimney section), according to Mohr the following construction will give the compressive stress distribution, Fig. 62. A is the point of application of W lying on the symmetry axis BMB_1 of figure F. Draw an equilibrium polygon for area F with an arbitrary pole distance H. Draw a perpendicular to axis BB_1 at A to intersect the tangent $B'O$ in A'. Draw $B'N$ so that the area of triangle $B'A'N$ shall be equal to area $B'CN$, in other words, that the two shaded areas shall be equal. Then the line NN drawn at right angles to BB_1 is the zero line from which the compressive stresses increase in proportion to the distances. Make $B'D = W/H$ = weight ÷ area (H being measured in the same unit as F); draw DE parallel to $B'N$. Then for the point B, farthest from NN, the unit stress $S_1 = EE_1$ which is drawn parallel to BB_1 from point E. EE_1 is measured in the same unit as W/H.

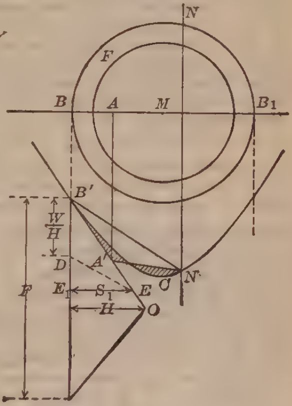
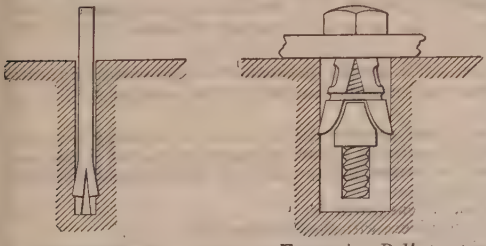


Fig. 62

If the point of application of the resultant does not lie on an axis of symmetry of the joint, or if the joint has an unsymmetrical figure, the determination of the purely compressive stresses becomes very complicated. The following method is recommended: Connect the point of application with the center of gravity of the joint and find in this direction the conjugate axis of the ellipse of inertia of the figure. Then the line of zero stress will be parallel to the direction thus found and its location should be determined by trial, remembering that the stresses will be proportional to the distance from the zero line and that the center of gravity of the intensity wedge (the volume of which is equal to the vertical component of the resultant) must fall in the same vertical with the point at which the resultant cuts the joint.


Fox Bolt *Expansion Bolt*
Fig. 63

Anchor Bolts. Masonry structures sometimes are required to be anchored to a rock foundation, especially where tension may occur under eccentricity of loading. A fox bolt (Fig. 63) is a straight iron bar, split for about an inch at the lower end, the split allowing an iron wedge to enter as the bolt is driven down; the end of the bolt then expands and is held in the hole by friction aided by cementing material. The expansion bolt (Fig. 63) is the best form of anchor bolt for fastening timber or steel members to masonry or rock.

Ordinary straight round rods, which should be "hacked" or otherwise roughened, may be cemented in place with lead, babbitt metal or cement grout, the last being most common.

M. Ferret found, for plain round rods in 1 : 2 : 4 concrete, that the holding capacity was 237 lb. per sq. in. of embedded surface. A. N. Talbot found 424, and W. O. Withey 452 lb. per sq. in. Plain square rods give slightly higher values. A working unit stress of 75 lb. per sq. in. is recommended for plain round rods in 1 : 2 : 4 concrete, or 100 lb. per sq. in. if rods are roughened.

For fox and expansion bolts E. S. Wheeler found 264 lb. per sq. in. when embedded in 1 : 2 portland mortar, 843 lb. per sq. in. when in sulphur and 485 lb. per sq. in. when in lead. A working unit stress of about one-fifth of these values should be used. Sulphur should not be used where exposed to rain or other water, as acid will be formed which rusts the steel and often disintegrates the stone near the top of the hole.

Brick Chimneys. The overturning effect of wind on a tall isolated chimney will generally result in some tensile stress in the brick masonry. The amount of this tensile stress may be determined by analyzing the chimney as a vertical cantilever, and computing the bending stresses at various heights. The difference between the compressive unit stress due to dead load and the maximum unit stress from the bending will give the maximum unit tensile stress in the brickwork; the sum of the dead load and bending stresses will give the maximum compressive stress. As the strength of the mortar during construction is greater at the base of the chimney than higher up, due to the greater age of the mortar at the base, the allowable masonry stresses are sometimes given higher values for the lower portions of the chimney. G. Lang (Eng. Rec., July 20, 1901) gives the following formulas for maximum allowable tensile and compressive stresses in brick chimneys:

Tension: $S = 18.5 + 0.056\,L$, for single shell chimneys
$S = 21.3 + 0.056\,L$, for chimneys with complete lining

Compression: $S = 71 + 0.65\,L$, for single shell chimneys
$S = 85 + 0.65\,L$, for chimneys with complete lining,

where S = stress in pounds per square inch, and L is the distance in feet from the top of the chimney to the section considered.

If the computed maximum tensile stress is greater than the tensile strength of the mortar, the joints will tend to open up slightly on the windward side. However, the chimney may be stable even in this case, provided the compressive stress on the other side of the chimney does not exceed the allowable compressive stress for brick masonry. For this condition, Mohr's method (page 909) may be used to compute the maximum compressive stress. An approximate method is also given for concrete chimneys in Sect. 11, Art. 60.

Building codes do not, as a rule, specify methods for design of isolated brick chimneys. The following requirements are given in the code of the City of Toronto:

(1) Based upon a horizontal wind pressure of 25 lb. per sq. ft. as acting over their vertical projection, the circular portions of isolated chimneys constructed of approved radial brick may be designed for stresses, due to weight and wind moment combined, not exceeding the following amounts:

(a) Maximum compression, 17 tons per sq. ft.

(b) Maximum tension in accordance with the following table:

Maximum height of chimney in feet	Tension in tons per sq. ft.	Maximum height of chimney in feet	Tension in tons per sq. ft.
100	2-1/4	225	1
125	2	250	3/4
150	1-3/4	275	1/2
175	1-1/2	300	1/4
200	1-1/4	over 300	0

(2) Radial brick shall be hard burned, also weather and acid proof. The brickwork when laid in place shall weigh at least 120 lb. per cu. ft.

(3) Circular steel reinforcing hoops or rings, not less in cross-sectional area than 3/8 × 2-1/2 in., shall be built into the brickwork at each change of wall thickness and just above and below the flue opening. The spaces between the inside of the hoops and the brickwork shall be well filled with mortar.

It will be noted that the Toronto code specifies a decreasing maximum tensile stress as the height of the chimney increases, which is contrary to Lang's formulas. The writer considers the allowable stresses in the Toronto code to be too conservative. It is suggested for design, based on a wind load of 25 lb. per sq. ft. of vertical projection, that the maximum compressive stress in the brickwork of the chimney be not over 210 lb. per sq. in. (15 tons per sq. ft.) and that the maximum tensile stress at any point in the chimney be not over 28 lb. per sq. in. (2 tons per sq. ft.).

MASONRY WALLS AND DAMS

23. Principles of Stability

Definitions. A **Bearing Wall** is a vertical wall which supports a building and is subject mainly to vertical pressures. A **Retaining Wall** is subject to its own weight and to the lateral pressure of earth which is deposited behind it after it is built. A **Brest Wall,** or face wall, is one built to prevent the fall of an undisturbed bank of earth. A **Buttress** is a projection of masonry built into the front of the wall to strengthen it, and a **Counterfort** is a projection of masonry built into the back of the wall.

A **dam** is a structure to obstruct the flow of a stream or to impound water, thus creating a storage of water, or head, or both. Masonry dams are, as a rule, of the gravity type, resisting the lateral water pressure by virtue of their weight. Arched masonry dams are dams curved in plan and for their resistance against water pressure rely partly or entirely upon arch action.

The front of a wall or dam is the face seen after it is finished, while the back is the face which is acted upon by earth or water. The lowest point of the front, at the top of the foundation, is called the toe, and and the lowest point of the back is called the heel.

The Batter of the face of a pier, wall or dam is its inclination to the vertical, measured by the ratio of horizontal to vertical projection, which is usually expressed in inches per foot of height. The following table gives the angle θ (Fig. 49) between the inclined face and the vertical for various batters and also trigonometric functions of those angles:

Angles and Functions for Batters

Batter, in. per ft.	Angle θ	Sin θ	Cos θ	Sec θ	Tan θ
0	0° 00′	0.0000	1.0000	1.0000	0.0000
1/2	2 23	0.0416	0.9991	1.0009	0.0417
1	4 46	0.0831	0.9965	1.0035	0.0833
1-1/2	7 08	0.1242	0.9923	1.0078	0.1250
2	9 28	0.1645	0.9864	1.0138	0.1667
2-1/2	11 46	0.2039	0.9790	1.0210	0.2083
3	14 02	0.2425	0.9702	1.0308	0.2500
3-1/2	16 15	0.2798	0.9601	1.0416	0.2915
4	18 26	0.3162	0.9487	1.0541	0.3333
4-1/2	20 33	0.3510	0.9364	1.0680	0.3750
5	22 37	0.3846	0.9231	1.0825	0.4166
5-1/2	24 37	0.4166	0.9091	1.1000	0.4582
6	26 34	0.4472	0.8944	1.1181	0.5000

The last column in this table gives the batter expressed as a decimal, that is, the horizontal projection for a vertical projection of unity. The column next to the last gives the length of the inclined face for a vertical projection of unity.

Failure of a masonry wall or dam usually occurs in three ways: (1) by sliding along the joint at the base, (2) by overturning or rotating, (3) by crushing of the masonry. In order that failure may not occur, the design should be so made that the structure may have such size and weight that sliding, rotation or crushing cannot occur.

Failure of retaining walls by tension seldom, if ever, occurs, but tension has been the cause of some of the most disastrous failures of dams. The failure of the Habra dam in Algiers and that of the Bouzey dam in France were due primarily to tension fracture and subsequent shearing action. (Unwin, Min. Proc. Inst. C. E., Vol. 172, p. 160.)

The most common cause of failure of walls and dams is a defective foundation. A washout around the foundation of a wall due to excessive rainfall may cause its destruction. Water entering beneath the base of a masonry dam or around its ends may impair the foundation as well as exert an upward pressure. In this article it is assumed that the foundation is securely built and that precautions have been taken to prevent the action of water beneath it.

Sliding Stability is secured by giving the structure a sufficient weight so that there is no danger of motion along a joint at the base or any other horizontal joint. For the horizontal joint in Fig. 64*a* let H be the horizontal pressure and W the sum of all the vertical weights above the joint. Motion occurs when $H = fW$ where f is the coefficient of friction. In order that there may be no motion, let n be a number greater than unity, called the **safety factor,** then if W is sufficiently large so that $nH = fW$, stability is secured. A common value of n for use in designing is 2, so that the formula $2H = fW$ produces a proper degree of security, and then $W = 2H/f$. For an existing wall or dam the safety factor is $n = fW/H$.

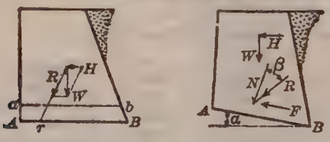
For an inclined joint (Fig. 64*b*) let α be its angle with the horizontal, N the normal pressure on the joint and F the force acting parallel to it; then for motion about to begin $F = fN$, and for stability $nF = fN$. For this case

$$\mathbf{W = H\ (n - f \tan\alpha)/(n \tan\alpha + f)}$$

gives sufficient security if the safety factor n is taken as 2.

Another method of consideration is by means of the angle which the resultant R makes with the normal to the joint. In Fig. 64*b*, motion will occur when the angle β equals the angle of repose ϕ, for masonry upon masonry (Art. 18). For stability the angle β should be less than ϕ, and proper security requires that $n \tan\beta = \tan\phi$, or $\tan\beta = f/n$.

Stability against Rotation is secured by giving the wall or dam such size and weight that R, the resultant of all the external forces above the joint, cuts the base AB well within the joint (Fig. 64*a*). When R passes through A



(*a*) (*b*)

Fig. 64

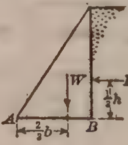


Fig. 65

failure occurs by rotation, and if R passes near A the material may begin to crush so that rotation is imminent. Some writers require that R shall cut AB at the edge of the middle-third or even within it; others require that R shall cut the base at A when the acting horizontal force H is doubled. The first method is used for dams and the second for walls. The **safety factor** against rotation is defined as the number by which the horizontal pressure must be multiplied in order that R shall pass through the toe. Let l_1 and l_2 be lever arms of H and W with respect to A in Fig. 64b; then the safety factor is $n = Wl_2/Hl_1$ for an existing wall or dam. For designing, the value of W must be such that $W = nHl_1/l_2$ and n may be taken as 2 for walls and between 2 and 3 for dams.

For a triangular wall or dam with vertical back (Fig. 65) a safety factor of 2 causes R to cut the base at the edge of the middle third. For a trapezoidal section of top width a and base width b and a vertical back, the safety factor required in order that R may cut the edge of the middle third is $n = 2 + a^2/(b^2 + ab - a^2)$, which shows that n should be 3 for a rectangular dam and between 2 and 3 for a trapezoidal one.

Stability against Crushing is secured by making the greatest compressive unit stress S_1 much less than the crushing strength of the masonry. For S_1 the safe working value given in Art. 16 should be used in design, the wall or dam being given such size that this shall not be exceeded.

The Resistance Line is a line drawn within the cross-section of a wall or dam which shows for every horizontal joint the point where the resultant of the external forces above it cuts the joint. Let ab in Fig. 64a be any horizontal joint in a masonry wall, H the resultant of the horizontal forces above ab, and W the weight of the masonry above ab; then the resultant R of H and W cuts ab at a point r. If ab is conceived to move up and down, the point r describes the line called the resistance line. Figs. 66 to 69 show four forms of walls in each of which two broken lines are drawn dividing each horizontal joint into three equal parts, the middle part being the "middle-third." (See Art. 21.) Fig. 66 is a rectangular pier or wall subject only to its own weight, and the vertical line rr is the resistance line. Fig. 67 is a vertical rectangular wall subject to the horizontal pressure of earth or water, and the curved line rr is the resistance line. Fig. 68 is a triangular wall with vertical back under horizontal pressure, and the straight line rr which cuts the base at the middle is the resistance line. Fig. 69 is a trapezoidal wall with vertical back under horizontal pressure, and the resistance line rr is curved.

Fig. 66

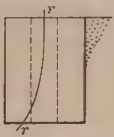


Fig. 67

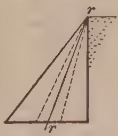


Fig. 68

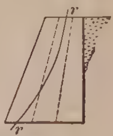


Fig. 69

The conditions of stability discussed in this article apply not only to the base joint but also to every horizontal joint of the cross-section of a wall or dam. In the case of stone and brick structures, the shearing strength of the joints is disregarded, and friction alone is assumed to resist sliding. In the design of concrete walls, including dams, the tendency to slide is assumed to be resisted by the shearing strength of the masonry. Both friction and shear should not be assumed to be simultaneously available to resist sliding because frictional resistance will not come into action until the section has failed in shear.

The conditions of stability (against sliding, rotating and crushing) here discussed apply also to radial joints of masonry arches, where a normal pressure N acts upon the joint and a shearing force H along the joint.

24. Moments for Trapezoidal Sections

The Overturning Moment M for a wall with vertical back is $\mathbf{P \times 1/3\ h}$, where P = horizontal pressure and $1/3\ h$ is lever arm with respect to the toe.

For a level bank of earth Art. 4 gives for this overturning moment per unit length of wall,

$$M = 1/6\, wh^3 \tan^2 (45° - \phi/2)$$

and values of M are given in Table A, below, for earth having w = 100 lb. per cu. ft. and for various values of ϕ. These values of M are in thousands of foot-pounds. This table is useful in making approximate computations and estimates for walls with backs nearly vertical. It does not apply to banks which have upper surfaces inclined.

The table may be used to find M for earth less than 100 lb. per cu. ft. by multiplying the tabular values by the ratio of the unit-weight to 100; for example, for cinders weighing 45 lb. per cu. ft. multiply by 0.45. Interpolation may be made for earths having slopes different from those given. For a height of wall intermediate between the heights given in the table, take the overturning moment for a wall 100 ft. high and multiply it by the ratio of the cube of the height of the desired wall to the cube of 100. For example, for a wall 55 ft. high and a slope of repose 1.33 : 1, the overturning moment = 4 167 000 × $(55/100)^3$ = 692 000 ft.-lb.

For a Loaded Level Bank of Earth (Art. 19, Fig. 41), the overturning moment M may be found by multiplying the tabular values by $1 + 3v/100\,h$, where v is the weight of the load per square foot of level surface and h equals height of wall in feet. When the load consists of merchandise or building materials back of a wall, v may be taken as from 200 to 400, the higher the wall the greater to be the value of v. For walls retaining railroad embankments v should be taken at 750 lb. The tables are not directly applicable to a dock wall or to any wall in which the retained earth is saturated with water.

(A) Overturning Moments for Retaining Walls in Units of 1000 Foot-Pounds

Angle of repose of earth	Slope of repose of earth	Height of wall in feet									
		10	20	30	40	50	60	70	80	90	100
45° 00′	1 : 1	2.9	22.9	77.1	182.8	357.1	617.0	980	1463	2083	2 857
39 50	1.2 : 1	3.7	29.2	98.6	233.6	456.3	788.4	1252	1869	2661	3 650
37 30′	1.3 : 1	4.1	32.4	109.4	259.2	506.3	874.8	1389	2074	2953	4 050
36 50	1.33 : 1	4.2	33.3	112.5	266.7	520.8	900.0	1429	2133	3038	4 167
35 30	1.4 : 1	4.4	35.4	119.3	282.9	552.5	954.7	1516	2263	3222	4 420
33 40	1.5 : 1	4.8	38.2	128.8	305.3	596.3	1030	1636	2442	3477	4 770
26 40	2 : 1	6.4	50.8	171.3	406.1	793.1	1371	2176	3249	4626	6 345
18 30	3 : 1	8.6	69.1	233.3	553.0	1080	1866	2964	4424	6299	8 640
For water.........		10.4	83.3	281.3	666.7	1302	2250	3573	5334	7594	10 417

(B) Resisting Moments for Walls of Type B in Units of 1000 Foot-Pounds

Ratio of base to height	Height of wall in feet											
	10	15	20	25	30	40	50	60	70	80	90	100
0.33	8.2	26.5	65.3	127.6	220.5	522.7	1021	1762	2802	4182	5954	8 168
0.34	8.7	28.1	69.1	135.5	234.1	554.9	1084	1873	2974	4439	6320	8 670
0.35	9.2	29.8	73.5	143.5	248.1	588.0	1149	1985	3151	4704	6697	9 188
0.36	9.7	31.5	77.8	151.9	262.4	622.1	1215	2100	3334	4977	7086	9 720
0.37	10.3	33.3	82.1	160.1	277.2	657.1	1283	2218	3522	5257	7485	10 268
0.38	10.8	35.1	86.6	169.2	292.4	693.1	1354	2339	3715	5545	7895	10 830
0.39	11.4	37.0	91.3	178.2	308.0	730.1	1426	2464	3913	5841	8316	11 408
0.40	12.0	38.9	96.0	187.5	324.0	768.0	1500	2592	4116	6144	8748	12 000
0.41	12.6	40.0	100.4	197.0	340.4	806.9	1576	2723	4324	6455	9191	12 608

(C) Resisting Moments for Retaining Walls of Type C in Units of 1000 Foot-Pounds

Ratio of base to height	Height of wall in feet									
	10	20	30	40	50	60	70	80	90	100
0.35	8.1	60.2	199.0	469	909	1567	2482	3 692	5 374	7 208
0.40	10.3	78.0	259.0	609	1185	2042	3236	4 824	6 861	9 403
0.50	15.8	120.7	402.2	933	1846	3113	5045	7 527	10 759	14 670
0.60	22.3	172.7	577.0	1361	2652	4574	7254	10 819	15 393	21 103
0.70	30.1	205.1	781.9	1849	3618	......	......	......	......	......

(D) Resisting Moments for Walls of Type D in Units of 1000 Foot-Pounds

Ratio of base to height	Height of wall in feet									
	10	20	30	40	50	60	70	80	90	100
0.35	8.6	61.0	193.9	444.0	848	1443	2265	3352	4 740	6 465
0.40	11.0	78.0	249.0	572.0	1095	1866	2933	4344	6 147	8 390
0.50	16.5	118.0	379.5	876.0	1683	2874	4526	6712	9 509	12 990
0.60	23.0	166.0	537.0	1244	2395	4098	6461	9592	13 599	18 590
0.70	30.5	222.0	721.5	1676	3233	......	......	......	......	......

(E) Resisting Moments for Walls of Type E in Units of 1000 Foot-Pounds

Ratio of base to height	Height of wall in feet									
	10	20	30	40	50	60	70	80	90	100
0.35	8.4	62.7	205.6	480.0	929	1595	2521	3 751	5 326	7 290
0.40	10.8	81.0	266.5	623.3	1208	2075	3282	4 883	6 915	9 498
0.50	16.5	124.7	412.0	966.0	1874	3224	5103	7 599	10 799	14 790
0.60	23.3	177.7	589.0	1383	2687	4625	7324	10 911	15 510	21 248
0.70	33.3	240.0	797.5	1875	3645	......	......	......	......	......

Interpolation is not satisfactorily made except approximately by an arithmetic method in tables (B), (C), (D), (E), for heights intermediate between those given. For a case of this kind, it is recommended that interpolation be made by help of a curve drawn through plotted points whose ordinates represent values taken from the table.

The Resisting Moment of a wall or dam is defined as its weight multiplied by the lever arm of that weight with respect to the toe. Tables (B), (C), (D), (E) give resisting moments in thousands of foot-pounds for masonry walls 1 ft. long of four types, and for various ratios of height to base and for top width between 2 and 3 ft. These types, also called B, C, D, E, are shown in Figs. 70-73. Walls B and C may have a vertical face or one with a batter not exceeding 1 in 3. The weight of the masonry was taken at 150 lb. per cu. ft. in computing these tables. If the unit weight w of the masonry is mater-

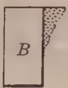


Fig. 70

Fig. 71

Fig. 72

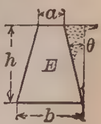


Fig. 73

ially less, multiply the tabulated resisting moments by $w/150$. The resisting moment will be called M_1. The weight of earth upon the back of walls, types C and E, was taken as effective. Assumed weight of earth 100 lb.

The Unit Stress S_1 at the toe of a trapezoidal wall or dam is given approximately in the following table. It applies to each of the four types (Figs. 70-73) when the resisting moment M_1 is twice as great as the overturning moment M. The values are in pounds per square foot when h is taken in feet.

(F) Unit Stress S_1 for Toe of Wall

Type of wall	Height of wall in feet		
	5 to 10	10 to 20	20 to 100
B	400 h	400 h	400 h
C	392 h	392 h to 378 h	378 h to 362 h
D	263 h	263 h to 198 h	195 h to 158 h
E	319 h	319 h to 272 h	270 h to 237 h

Example. To find the compressive unit stress at the toe of a wall of type B, the height being 20 ft. The table gives $S_1 = 400 \times 20 = 8000$ lb. per sq. ft. = 56 lb. per sq. in. When the value of S_1 approaches the working value (Art. 16), the foundation must be widened by an offset on the front side.

Formulas for Use with the Tables. Let a = width of top, b = width of base, h = height, e = distance from middle of base to point where resistance line cuts it; all in feet; f = coefficient of friction. M = overturning moment; M_1 = resisting moment.

Horizontal pressure without surcharge = $P = 3\,M/h$.
Weight of wall, types B and D = $W = 75\,h\,(a + b)$.
Weight of wall and earth on back, type C = $W = 25\,h\,(a + 5\,b)$.
Weight of wall and earth on back, type E = $W = 50\,h\,(a + 2\,b)$ approx.
Eccentricity e of resultant for type B = M/W.
Eccentricity e of resultant for type C = M/W approximately.
Eccentricity e of resultant for type D = $M/W - b/6$ approximately.
Eccentricity e of resultant for type E = M/W approximately.
Unit compression at toe = $S_1 = 2\,W/3\,(1/2\,b - e)$.
Safety factor against rotation = $n = M_1/M$.
Safety factor against sliding = $n = fW/P$.

Centers of Gravity. Let a be the top width, b the base width and h the height of any trapezoid whose back face makes the angle θ with the vertical, this face being inclined forward as in Fig. 73. Then the distance t from the toe to the vertical passing through the center of gravity of the trapezoid is given by

$$\mathbf{t = 2/3\,b - 1/3\,h\tan\theta - \frac{a\,(a + h\tan\theta)}{3\,(a + b)}}$$

When the back is vertical, tan θ is zero. For a rectangular section, $b = a$ and $t = 1/2\,a$. For a triangular section, $a = 0$ and $t = 2/3\,b$. When the back inclines backward, tan θ is to be taken negative. For a wall with vertical back, top width a and base width b, the distance from the toe to the vertical line through the center of gravity is $(2\,b^2 + 2\,ab - a^2)/3\,(a + b)$.

25. Investigation of Retaining Walls

To Investigate a wall is to determine its degree of stability against sliding, rotation, and crushing. Often this may be done by inspection in the case of an existing wall, for if it has stood for many years and no signs of failure are seen, it is to be regarded as safe. The problem, however, generally occurs in the case of a proposed design, where all dimensions and data are given and it is required to ascertain whether these satisfy the conditions of stability. The process of investigation should determine at least these three things: (1) the factor of safety against sliding on the base, (2) the point where the resultant cuts the base, (3) the compressive unit stress S_1 which exists at the toe.

The height of retaining walls seldom exceeds 30 ft. and a height of 60 ft. is about the limit. The condition of stability against rotation, with a proper safety factor, will usually insure against excessive stresses; further, in retaining walls it is not imperative that the whole joint should be under compression. It is therefore not necessary to apply for retaining walls the investigation discussed in the latter part of Art. 21. The vitally important theoretic determination is the maximum unit pressure upon the foundation bed at the toe.

A Graphic Analysis is often convenient and satisfactory. First, the pressure P acting on the back of the wall is to be determined by Art. 19, using for the earth which is to be put behind the wall values of w and ϕ which are most unfavorable to stability. Second, the weight W of the wall and the weight of earth resting upon the back, if any, are computed and the line of action determined, this line passing through the center of gravity of the combined cross-section of the wall and earth upon it. Then a drawing of this cross-section is made to scale and the resultant R of P and W determined, Fig. 64a; this gives the point r where the resultant cuts the base or the lowest point of the resistance line. The ratio W/P may be computed, or may be scaled from the drawing by taking W as unity. Then $n = fW/P$ is the factor of safety against sliding. The base b being divided into three equal parts, it is at once seen if the resultant falls within the middle-third, in which case there is ample security against rotation. Lastly, the distance between the center of the base and the point r is measured, and this gives e (Art. 21), from which S_1 is computed, and if S_1 does not exceed the allowable unit stress there is full security against crushing.

A Better Graphic Method is that shown in Fig. 74, where the force and equilibrium polygons are used. W_1 is the weight of the rectangular part of the wall and W_2 that of the triangular part. The force polygon is seen on the right, R_1 being the resultant of P and W_2, and R the resultant of P and $W_1 + W_2$. The equilibrium polygon is seen within the wall, P being produced to m_2, through which m_2m_1 is drawn parallel to R_1; then through m_1 the line m_1r is drawn parallel to R, thus determining r, the point in the line of resistance where the resultant cuts the base. Finally e is found by measuring the distance from r to the middle of the base. This method, although here illustrated for only two weights, is a general one readily applied to Fig. 75 or to any number of weights (Section 2). It obviates the necessity of computing the line of action of the resultant of the given weights or of finding it by a geometric construction.

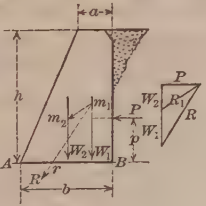


Fig. 74

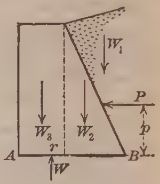


Fig. 75

The Analytic Method computes e by the principles of mechanics, the cross-section being divided into rectangles and triangles whose centers of gravity are known. For example, take the wall with vertical back in Fig. 74 which is held in equilibrium by the forces shown, W_1 being the weight of the rectangle, W_2 that of the triangle, and $W_1 + W_2 = W$ the vertical reaction due to the total weight, its point of application being at a distance e from the center of the base or at $1/2\ b - e$ from the toe A. Here the equation of moments with respect to the toe is

$$Pp - W_1(b - 1/2\ a) - W_2\, 2/3\ (b - a) + W\ (1/2\ b - e) = 0.$$

Inserting for W_1 and W_2 their values vah and $1/2\ v\ (b - a)\ h$, where v is the weight of a cubic unit of the masonry, there results

$$\mathbf{e = 1/2\ b - [vh\ (2\ ab + 2\ b^2 - a^2) - 6\ Pp]/3\ vh\ (a + b)}$$

which is a general formula for computing e for a trapezoidal wall or dam with vertical back. The weight W and the safety factor against rotation are

$$\mathbf{W = 1/2\ vh\ (a + b) \qquad n = vh\ (2\ ab + 2\ b^2 - a^2)/6\ Pp.}$$

The analytic method is more readily subject to detailed check than the graphic method.

Example. A wall of good rubble with a vertical back retains a level bank of earth, the data being w = 100 lb. per cu. ft., $\phi = 34°$, h = 18 ft., a = 2 ft., b = 5 ft., v = 140 lb. per cu. ft., f = 0.65. From Art. 19, P = 4580 lb. per lin. ft. of wall, and p = 6 ft. Then the formula gives e = 2.47 ft., so that the resultant comes outside the middle-third. The safety factor against rotation is n = 1.02, which is too small. For sliding, W = 8820 lb. and $n = 0.65 \times 8820/4580 = 1.25$, which is too low a degree of security, since n should be at least 2 (Art. 23). The thickness of this wall is too small to give proper security against either sliding or rotation.

For the Trapezoidal Wall in Fig. 75, let W_1 be the weight of the triangle of earth above the inclined back, W_2 and W_3 the weights of the masonry, and let p_1, p_2, p_3 be their lever arms with respect to the toe A, while p is the lever arm of the horizontal pressure P. Then the equation of moments is

$$Pp - W_1p_1 - W_2p_2 - W_3p_3 + W\ (1/2\ b - e) = 0$$

the solution of which gives the value of e. Here W is the total vertical force $W_1 + W_2 + W_3$. The factor of safety against sliding is $\mathbf{n = fW/P}$, and that against rotation is $\mathbf{(W_1p_1 + W_2p_2 + W_3p_3)/Pp}$.

After e is found, the compressive unit stress S_1 at the toe is ascertained by the proper formula of Art. 21. For the example given above, where the resultant comes without the middle-third, $S_1 = 2\ W/3\ (b/2 - e)$ = 196 000 lb. per sq. ft. = 1360 lb. per sq. in., which is much greater than the allowable.

Approximate investigations may be quickly made by the help of the tables in Art. 24 if interpolation is not required. Example: Let h = 60 ft., b = 24 ft., a = 3 ft for a wall of type C which retains earth having a slope of 1.3 to 1. Table (A) gives M = 874 800 ft-lb., and table (C) gives M_1 = 2 042 000 ft-lb., so that the safety factor against rotation is $n = M_1/M_2 = 2.3$. Then P = 874 800/20 = 43 740 lb., $W = 25 \times 60\ (3 + 5 \times 24)$ = 184 500 lb. The factor of safety against sliding if f = 0.65 is $n = 184\ 500 \times 0.65/43\ 740 = 2.8$. Lastly, the compressive unit stress at the toe, from table (F), is approximately 370×60 = 22 230 lb. per sq. ft. Table (F) was made for a factor of safety against rotation of 2, and should not be applied, except for design or roughly approximate pressures, when factor of safety exceeds or is less than 2.

To find the resistance against sliding upon the foundation bed, by the help of the tables of Art. 24, use $F = f\ (W_1 + M_1/Kb)$, in which F = resistance in pounds, M_1 = resisting moment taken from the proper table, b = width in feet of base of wall as previously determined, W_1 = weight of foundation in pounds, f = coefficient of friction of masonry upon the material of which the foundation bed is composed; also K = 0.50 for wall B, K = 0.47 for wall C, K = 0.66 for wall D, K = 0.57 for wall E.

26. Design of Retaining Walls

General Data. The top width of a retaining wall should not be less than 2 ft. where frost penetrates into the ground more than 3 ft., and not less than 1.5 ft. for smaller penetrations. For railway walls these widths should be increased to 3 ft. and 2.5 ft. The front face of the wall should have a batter of at least 1 in 24, preferably 1 in 12. The foundation masonry should be offset at least 6 in. both at heel and toe. Offsets to reduce the unit compression on the foundation bed should always be at the toe (Fig. 76), never at both heel and toe, since the compression at the heel will be small under the horizontal pressure of the earth. A factor of safety of 2 against rotation is recommended, although this always brings some tension in the back joints over the heel.

Fig. 76

Convenient Approximate Data on retaining walls of type C for preparation of estimates and use in the field will now be given. Let h = the height of a wall in feet; P = the horizontal pressure in pounds applied at $h/3$ above the heel, for a wall one foot long; W = the weight of the wall including earth upon its back in pounds for a wall one foot long; W_1 = weight of foundation masonry in pounds for a wall one foot long; b = width in feet of the wall at top of foundation; b_1 = width in feet of bottom of foundation masonry; S_1 = the pressure upon the toe of the wall in tons per square foot; S_0 = the safe load upon the foundation base in tons per square foot; f_1 = the coefficient of friction of masonry upon rock = 0.66: f_2 = the coefficient of friction of masonry upon sand and gravel = 0.50; and f_3 = the coefficient of friction of masonry upon clay = 0.33.

For approximate design, the width of the base may be taken as $b = 0.45\ h$ for ordinary walls retaining level banks; $b = 0.66\ h$ for railway walls; $b = 0.70\ h$ for surcharged walls, that is, walls retaining an inclined bank. The batter of the foundation masonry with a vertical line may be taken as 1 : 0.9. Also $b_1 = 1/2\ b\ (1 + 0.18\ h/S_0)$, and $P = 13\ h^2$ for ordinary unsurcharged walls, $P = 20\ h^2$ for railway walls, $P = 22\ h^2$ for surcharged walls, $W = 50\ h^2$ for ordinary unsurcharged walls, $W = 80\ h^2$ for surcharged walls, $W = 85\ h^2$ for railway walls, and $S_1 = 0.2\ h$. The resistance to sliding $= f(W + W_1)$, which must be greater than P.

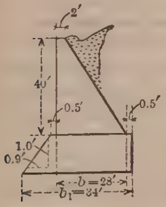


Fig. 77

To determine, in the field, the necessary width of base and of foundation, proceed as in the following example of a design made for estimating purposes (Fig. 77). Let h = 40 ft. for a surcharged wall which is to be built upon a foundation having a safe bearing capacity of 5 tons per sq. ft. The above approximate formulas then give $b = 0.70\ h$ = 28.0 ft., and b_1 = 14.0 (1 + 1.44) = 34.2 ft.

A retaining wall is probably slightly stronger if the back is built in steps. This is the most convenient form of construction for ashlar and rubble walls, but when the wall is to be built of concrete, either the stepped or unstepped back may properly be used. In calculating the stability of a wall with a step back, it may be regarded as having a uniform batter. The placing of earth back of a retaining wall in layers, and ramming it, may or may not reduce the horizontal pressure against the wall, but the calculated width of wall should not be reduced if the earth is to be so compacted. The ramming of soft rotten rock or earth which is a combination of clay, sand and gravel, may reduce the earth pressure.

Designing by Tables. As an example take a retaining wall 30 ft. high for a highway bridge approach which for architectural reasons must have a vertical face. The masonry will be taken at 150 lb. per cu. ft. and the earth at 100 lb. per cu. ft. The slope of repose of the earth will be taken at 1.33 : 1. A factor of safety 2 will be used for rotation and hence the overturning moment in Table (A) must be doubled. Entering table (A) for a slope of repose of 1.33 : 1 and a height of 30 ft., M is found to be 112 500 ft-lb.; hence the overturning moment for the proper factor of safety is 225 000 ft-lb. As a vertical faced wall is desired, either type B or C must be used. Type B is a very uneconomical type and should be used only for dry walls. Therefore type C will be adopted in the design.

Entering table (C) with the multiplied overturning moment, the nearest resisting moment is 259 000 ft-lb. for a wall 30 ft. high, and the required ratio of the base of the wall to its height is found to be 4/10 and therefore a base 12 ft. wide is needed. Entering table (F), the compression at the toe of the wall is found to be intermediate between that for a 20-ft. and a 100-ft. wall. By proportion the expression for the compression upon the toe of a 30-ft. wall is found to be 376 h or 11 280 lb. per sq. ft. or 5.6 + tons per sq. ft. If the foundation bed is rock or hard rotten rock and the bed is rough so that the wall will not slide, the design is completed. Offsets of 6 in. should be made at the top of foundation for reasons heretofore given. The stability of the design may now be investigated in detail by the methods of Art. 23.

The capacity of a granite ashlar wall to resist overturning, as compared with a concrete wall, is generally overestimated. Since both kinds of walls resist the overturning thrust of the earth by gravity alone, the granite ashlar wall is little stronger than the concrete one. Calling the resistance of a concrete wall 1, the resistance of a granite ashlar wall for types B, C and D would be 1.07, 1.05 and 1.07 respectively. A brick wall is only 0.85 as strong as a concrete one. A sandstone ashlar wall has about the same strength as a concrete. On the basis of the recommended factor of safety 2 against overturning, the maximum unit compression stresses are so low that, except for masonry laid in lime mortar, they need not be considered. The maximum compression in a retaining wall 100 ft. high is given in table (F) as 400 h = 20 tons per sq. ft.

Walls with Vertical Backs. The top width a is assumed and the horizontal earth pressure P computed by Art. 19. Then for the case where the resultant cuts the base at the edge of the middle-third, so that the compression at the heel is zero, the width of base is

$$\mathbf{b = -1/2\,a + 1/2\,\sqrt{5\,a^2 + 8\,P/v}}$$

where v = unit weight of the masonry. When the condition is inferred that the resultant cuts the toe when P is doubled, the formula is

$$\mathbf{b = -1/2\,a + 1/2\,\sqrt{3\,a^2 + 8\,P/v}.}$$

The first formula gives the greater value of b, and it should be used when it is important that no tension should exist at the heel. See Art. 27 for a comparison of designs made by these two methods. For rectangular sections, where $b = a$, these formulas become $b = \sqrt{2\,P/v}$ and $b = 2\,\sqrt{P/3\,v}$.

In Construction a trench is excavated in which to lay the foundation of the wall. Fig. 78 shows a wall whose foundation $aebd$ is at a considerable depth below the ground surface sg, while m_1m_2 and n_1n_2 are the sides of the trench which will be excavated in order to lay the foundation masonry $aebd$. It is important that upon the completion of the retaining wall the remaining trench spaces m_1aem_2 and $bn_1n_2\,d$ be filled with well selected earth thoroughly rammed into place. This gives an additional factor of safety against overturning. If the wall leans forward it will meet resistance at the front, which is shown by P_1 applied at $1/3\ ae$ measured down from a and also at P_2 applied at $1/3\ bd$ measured up from d. The values of P_1 and P_2 cannot usually be approximated, but the most extreme case is that where $P_1 = 0$, and then the height of the wall would be that of the top above the foundation bed. In construction, however, it is important that backfilling the space between the masonry and the sides of the trench should be well done. In very deep foundations the trench sheeting should be left in place so as not to disturb the earth back and front of the foundation masonry.

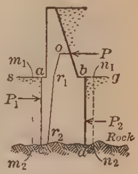


Fig. 78

Deep foundations for retaining walls, however, are very seldom used, because it is more economical, as a rule, to use piles to transmit the loads to soil of sufficient bearing capacity. In pile foundations it is important in the design to take care of the horizontal thrust by land ties or battered piles.

In the case of clay foundations also the proper backfilling of the trench is important, as it tends to keep water away from the foundation.

Sliding on the Base is to be prevented by making the base as rough as possible. For a wall on a timber grillage, some of the top timbers well bolted to the grillage may project above the level of the top of the grillage, or bolts well fastened to the timbers may project into the vertical masonry joints. When this is not possible the base should be placed on an inclined grillage. Fig. 64*b* of Art. 23 applies to this case and there will be no force F parallel to the incline AB if $\tan \alpha = H/W$, and it is usually easy to secure such an inclination in construction. Bonding the stones together will usually give ample security against sliding of a portion of a wall, on a lower portion. The designer should provide such methods of preventing sliding rather than to widen the base and thereby increase the weight in order to meet the theoretic condition of stability against failure by this cause.

The Principal Cause for the Failure of retaining walls is inadequate foundations. The application of theory is recommended in the design of all retaining wall foundations to guard against excessive toe pressures and sliding upon the foundation bed. The foundation bed, if of compressible material, should be investigated by borings to determine the character of the underlying strata. A wall built on a side-hill may require a special study against sliding and against movement of the side of the bank on account of the foundation pressure. Springs in side-hills not only endanger the foundations but may bring excessive pressure upon the wall itself if there is not adequate drainage provided.

The manner of placing the backfill may have serious effect upon the stability of a retaining wall. Less pressure is brought upon the wall if the fill is placed in layers, especially if these layers are compacted as they are placed. On the other hand, with low walls, the additional surcharge of a roller may have serious effect. If the layers are placed so as to dip away from the back of the wall somewhat less pressure is brought upon the wall and the drainage of water away from the wall is accelerated. The placing of a crushed rock or gravel drain over the whole area of the back of the wall is beneficial, provided that adequate drains are provided at the base of the wall to allow a free discharge of accumulated water.

The following references to failures should be read: Engineering News-Record, 1920, Vol. 85, p. 1058, p. 1076; Engineering News-Record, 1927, Vol. 98, p. 146; Vol. 99, p. 681.

Waterproofing and Drainage. Fig. 79 shows a design of a retaining wall to be built of concrete or of concrete faced with ashlar. If it is important that the face of the wall shall be dry, the treads of the steps of the back must have a batter with the horizontal, as shown in Fig. 79, of at least one inch.

Fig. 79

The back of the wall should be given two coats of coal tar pitch or one of the standard brands of waterproofing material; and immediately adjacent to the back a stone drain, marked *d-d* in the figure, should be built. Near the bottom of the wall, but at such height above the surface of the ground as will permit free discharge of the water back of the wall, drain pipes not less than 3 in. in diameter should be laid in the masonry. These drains should be at every 25 ft. of the length of the wall, and at their inside ends piles of broken stone, from 1/4 cu. yd. to 1 cu. yd. in volume, should be placed, depending upon the height of the wall. This broken stone prevents the pipes from becoming stopped up with earth. The ground in front of the wall should also be properly drained.

In order to guard against seepage of water through cracks which may result from temperature changes, a concrete wall may be built with vertical expansion joints at intervals of 50 ft., which extend from the foundation bed through the coping. This method of construction causes the temperature cracks to occur at known vertical lines. If expansion joints are provided, the sections should be properly keyed together to maintain the wall in alignment, as at *c*, Fig. 80. The wall may also be built without such expansion joints, provided it is reinforced with sufficient steel to take care of the tension in the concrete resulting from the shortening of the wall under fall of temperature. Water may be prevented from seeping through these joints by forming a rectangular vertical recess, as the wall is built up, which is filled with plastic clay well rammed in place or with asphalt. Fig. 80 is a plan of a portion of the length of a wall showing a horizontal section of the vertical expansion joint and the recess *a a a a* into which the clay is packed. Even though there should be a slightly uneven settlement of the adjacent sections, the clay will be effective in stopping the seepage of water.

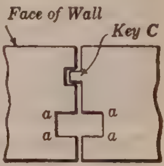


Fig. 80

A Frost Batter is a forward inclination of the back face near the top of the wall, as shown in Fig. 74. The object of this is to allow the earth to lift upward under the action of frost, and thus prevent an additional horizontal pressure at the top of the wall. A drainage ditch, parallel to the wall, a few feet away from the top, is often provided to carry off rain water before it percolates into the earth.

Pre-cast Concrete Crib Walls. These walls are built of pre-cast reinforced concrete, each member approximating in cross-section and length a railroad tie. The crib is then built up like a timber crib or one constructed of cross ties. This type may be advantageously used for temporary work and, if the concrete "ties" are held in stock, for emergency work. Where installed for temporary use they have much salvage value (see Engineering News-Record, Vol. 91, p. 783; Vol. 96, p. 654).

27. Comparison of Retaining Walls

The Profile of a wall or dam means the shape of its cross-section as shown by the bounding lines. An economical profile is one which has the least material consistent with proper stability and with the local conditions where it is built. Retaining walls are usually of trapezoidal form, but the batter of the faces has much influence upon economy of material. A sea wall may have its front face more or less curved, the curve being introduced to deflect upward the waves that strike it and thus to lessen the lateral pressure which may be caused by their impact.

The Relative Economy of retaining walls of different types may be approximately ascertained by comparing their resisting moments given in the tables of Art. 24. In type C the weight of the earth upon the back of the wall adds to the resistance. The efficiency of earth above the back in resisting overturning of the wall has been demonstrated by the successful construction of reinforced-concrete walls with counterforts where stability against overturning depends almost entirely upon the weight of the earth resting upon the wall. It should be noted in table (F) that the pressures upon the toe of walls of type D are much less than for C or in fact any other type, and therefore type D is a desirable one when the carrying capacity of the foundation bed is low. This type, however, cannot often be used in cities, since its front batter encroaches upon valuable land. Type B should rarely be used except for dry rubble walls, since it is a very uneconomical type. It has one advantage, however, that, due to the greater weight, it offers the most resistance to sliding, of all the types. Even in this respect, however, type B has little advantage over type C. Type D offers the least resistance to sliding. Type E is really a modification of C, and when the front batter of E is small compared with its back batter, type E becomes type C. Type E need be given little consideration in practice.

Counterforts on the Back are advisable only when built of reinforced concrete, because of the practical impossibility of taking care of the tension at the junctions of the back face and base with the counterforts, especially in walls of rubble or ashlar. Buttresses should rarely be used except when needed to prevent failure of a wall which is seen to be weak. A buttressed wall has no place in engineering except for architectural effect.

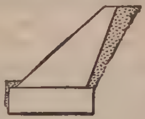


Fig. 81

Fig. 82

A Wall with Back Inclined Backward (Fig. 81) is not usually an economical one, owing to the added cost of construction, although theoretically such a form tends toward economy of material. Fig. 82 shows an excellent form of wall with a short and steep batter *aa* at the front a little above the surface of the ground, while the greater part of the front is vertical. In walls of types B and C the batter of the front face should be at least 1 : 24.

Theoretic Comparison of Walls. The following is a comparison of the width of base computed for several retaining walls with vertical backs all being 18 ft. high, masonry 150 lb. per cu. ft., and retaining a level bank of earth weighing 100 lb. per cu. ft. and having a slope of repose of 34°. The top width a in feet in the first line being assumed, the base width b in feet in the second line is computed under the condition that the distance e (Art. 21) shall be 1/6 b, thus giving a high degree of stability against rotation.

$a =$	0	1	2	3	4	5	6	7	7.8 ft.
$b =$	7.8	7.4	7.1	7.0	7.0	7.1	7.3	7.5	7.8 ft.
% =	50	53	58	65	71	78	85	93	100

The last line shows the relative amounts of materials in these walls as percentages of the rectangular wall in the last column. The triangular wall ($a = 0$) has 50% of the material in the rectangular one.

The following shows a comparison for the condition that there shall be a factor of safety of 2 against rotation, that is, that the resultant cuts the toe of the wall when the pressure P is doubled:

$a =$	0	1	2	3	4	5	6	6.4 ft.
$b =$	7.8	7.4	7.0	6.7	6.6	6.5	6.4	6.4 ft.
% =	60	66	70	76	83	90	97	100

The width of base is here less than for the more rigid requirement, that for the rectangular wall being 1.4 ft. less. The triangular wall has here only 60% of the material of the rectangular wall. In practice 1.5 ft. is the least allowable width of the top of a retaining wall (Art. 26).

28. Brest Walls and Dock Walls

A Brest Wall, often called a **face wall,** is one built against a bank of earth or rock to prevent it from falling. Since theory has so far proved useless in determining pressures from earth in situ, experience and practice are the only guides. The following rules may serve as a basis for thickness needed for different materials.

Hard rock and hard firm rotten rock will usually stand with a vertical face or with a slight batter 1 : 10 to 1 : 4 for an indefinite length of time. If the dip of the rock is away from the face, a wall is seldom necessary. If the dip of the rock is toward the face, a base of wall greater than 4/10 its height is seldom necessary, provided, however, that drains are built into the wall so as to prevent the wall being forced out by the formation of ice between the face of the rock and the back of the wall. Brest walls for all classes of material should have drains not only at the bottom but also at frequent intervals between the bottom and the top. Soft and seamy rotten rock, the dip of which is away from the face, seldom requires a wall with base greater than 1/5 its height. Soft and seamy rotten rock, the dip of which is towards the face, seldom requires a wall whose base is greater than 1/4 its height.

Firm sand and combinations of clay, sand and gravel seldom require a wall with base greater than 1/4 its height. If the dip of the stratum, however, is steep and toward the face, a base at least 4/10 the height of the wall should be used. If the walls are surcharged, these ratios should be increased to 4/10 and 5/10 respectively.

Clay and other materials resting upon a moist stratum of clay back of the wall, that dips toward the face, may exert enormous pressures, particularly when the wall is surcharged. If, for this case, the earth back of the wall is level with its top, a base larger than 6/10 its height is seldom required, and for a level top, if the dip is away from the face, a base larger than 5/10 its height is seldom required. If the wall is surcharged and the dip of the moist clay stratum is away from the face, a base 6/10 its height is usually sufficient. If the wall is surcharged and the dip of the moist clay stratum is towards the face and making an angle with the horizontal not exceeding 15°, a base 6/10 its height is usually sufficient. If the wall is surcharged and the dip of the moist clay is toward the face and over 15°, but under 30°, a base equal its height is usually sufficient. If the dip exceeds 30°, the ratio should be increased to 3/2. If any such walls rest on a clay foundation exceptional precautions may be necessary to prevent sliding, especially if the foundation is wet.

A Dock Wall differs from an ordinary retaining wall in that water pressure is exerted against its front face in the lower portion of the wall, while water, and earth submerged in water, exerts pressure upon its back in the lower portion of the wall; also the weight of the masonry below the water line is reduced by the weight of the volume of water which the wall displaces.

Assumption for Design. The water pressure upon the face of the wall should be regarded as balancing the water pressure upon its back. When the wall is built in tidal water, particularly when the range of tide is large, there is probably a lag in the tide back of the wall, and therefore it is advisable above low water to increase the width of the wall arbitrarily above the average back batter line, as shown in Fig. 83 by the dotted lines marked *rs*,

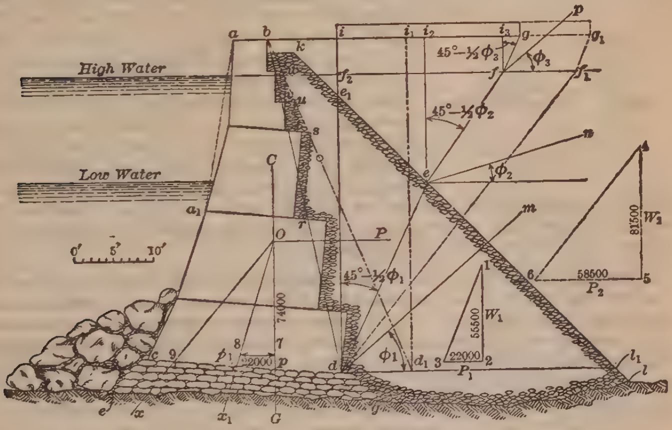


Fig. 83. Concrete Dock Wall on Rock

tu and *vw*. The weight of the earth on the inside of the wall and below high water is decreased by the weight of the volume of water displaced. The angles of repose of submerged earth will generally be much less than for earth above high water. The angle of repose of riprap or large cobble stone is not decreased by submergence and this fact makes both materials desirable as filling, back of a sea wall. Art. 17 gives the weights and angles of repose of submerged earth. It should be noted that the weight of river mud is given as 100 lb. per cu. ft. and its slope of repose as 10: 1. River mud should therefore be regarded as a liquid whose weight is 100 lb. per cu. ft. exerting a horizontal pressure against the back of the wall equal to 50 h_1^2, in which h_1 equals the vertical distance from the heel of the wall to high water. This horizontal pressure is applied at 1/3 h_1 above the heel. To get the net or effective pressure against the back of the wall, due to river mud, subtract from the liquid mud pressure the pressure of the water against the front, which gives 18.8 h_1^2 for fresh water and 18 h_1^2 for salt water. The pressure of liquid mud against the back of a wall need seldom be considered, but in the case of high walls, and where the earth filling is dumped from the shore toward the wall (Fig. 84), the river mud becomes banked against the back of the wall and exerts great pressure. The wedge b_1 b_2 a will be a mixture of river mud and earth with angle of repose of practically 0°. Due to the filling from the shore, a wave of mud, caused by the sudden settlement of the fill ab_3b_4, may add impact to the river mud and exert enormous pressure. Therefore when the depth of river mud is great, the filling should be made from the top of the wall, but even if this is done, it is advisable to regard the slope of repose of the submerged earth filling as having its base increased 30%; because the mud mixes with earth filling and the result is a flattening of the angle of repose. For example, riprap should be regarded as having a slope of repose of 1.3 : 1, and clay, sand and gravel as having a slope of 2 : 1.

The pressure at the back of a dock wall is considered, in what follows, to be applied at 4/10 the height of the wall above the heel. The point of application of the horizontal pressure is raised above that recommended in the design of ordinary retaining walls because the submerged portion of the earth back of the wall is generally semi-liquid in character, that is, it has a flat angle of repose. If the back fill were a liquid, under an abnormally large live load, the point of application of the horizontal pressure would never be higher than 1/2 the height of the wall above the heel. The effective weight of a masonry dock wall, below high water, equals its weight in air less the weight of the volume of water displaced by the wall, or the weight per cubic foot of masonry below high water should be taken as its weight in air less 62-1/2 lb. for fresh water and 64 lb. for salt water.

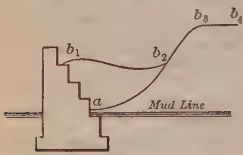


Fig. 84

Masonry Docks have walls of three general types: (1) A gravity masonry wall resting upon rock or other hard foundation or upon piles in firm material extending to the pile tops. Fig. 83 shows this type of wall founded upon rock. (2) A masonry wall resting upon a timber or concrete platform, called a relieving platform, the top of which is generally from 12 to 18 in. above low tide, resting upon a pile foundation, Fig. 85. (3) A rubble wall resting upon riprap; this is seldom used for dock walls, but may be so used by the construction of a timber wharf in front. It is more generally used in the reclamation of land from the sea.

Dock Wall of Type 1. Fig. 83 shows a type of dock wall founded upon rock. The rock should be cleaned off by dredge and divers, and the foundation of concrete in bags be laid by derrick boat and divers. Assume the safe unit compressive stress of well-bonded portland cement bag concrete as 10 tons per sq. ft. If the wall is built in salt water where the temperature of the air falls below 32° F. the face of the dock wall between the top and 2 ft. below low tide should be faced with ashlar masonry, which will withstand the action of salt water and frost. Type 1 is generally built of large blocks of portland cement concrete from the top of the foundation concrete to several feet above low tide. These blocks weigh from 15 to 80 tons, depending upon the size of the wall, and are generally cast on the shore, hauled by boat to the site of the wall and lowered by a derrick boat. The blocks are made with their beds beveled approximately 10 : 1, so that the bed joints are approximately normal to the resultant pressure. Grooves or recesses are molded in the vertical joints of each block and these grooves or recesses are connected by a hole passing through the block. By means of chains passing through these grooves and holes, the blocks are lowered into place, and when in place the chains are removed. The vertical grooves are made to match, and the square recesses thus formed by adjacent blocks are packed with concrete. Above low water the concrete is molded in place. The top of the wall should be at least 3 ft. wide to withstand shock from vessels. The front of the wall may be built with an increasing batter forming a curve as in Fig. 83. The back of the wall should be built in steps to agree with the heights of the concrete blocks, or the back of each block may be divided into several steps. If the wall rests upon a foundation bed of material that may be scoured out if unprotected, a bank of riprap should be placed in front of the wall to protect its foundation.

The resultant pressure P for a section one foot long in Fig. 83 is found as follows: The wall is to be built of concrete weighing 150 lb. per cu. ft., and in sea water weighing 64 lb. per cu. ft. It will be backed with riprap, and will have a live load back of it equal to 750 lb. per sq. ft. The filling back and above the riprap will be a combination of gravel, sand and clay. All material above high water will be regarded as dry and below high water will be regarded as submerged. The following data are here applicable:

Kind of material	Slope	Angle	Lb. per cu. ft.
Hard rock or riprap, submerged	1 : 1	45° 00′	65
Hard rock or riprap, dry	1 : 1	45 00	100
Gravel, sand and clay, submerged	3 : 1	18 30	65
Gravel, sand and clay, dry	1.33 : 1	36 50	100

Draw an inclined line bd, which is an average line for the stepped back. This line should pass through the heel d always; through d draw a vertical line di and a horizontal line dl_1 to intersect with kl, the line which represents the limiting surface of the riprap which is placed back of the wall to reduce the intensity of the earth pressure; through d draw de making an angle $45° - \phi_1/2$ with the vertical, in which ϕ_1 is for riprap and e is a point in kl; at e draw ef making an angle $45° - \phi_2/2$ with the vertical, ϕ_2 being for submerged gravel, sand and clay, while f is a point in the line of high water; at f draw fg making an angle $45° - \phi_3/2$ with the vertical, ϕ_3 being for dry gravel, sand and clay. Then d-i-g-f-e is the wedge which tends to slide down the lower plane de and cause a horizontal pressure P against the back of the wall, this being applied at a distance 4/10 di measured from d. To the weight of the wedge must be added the live load of 750 lb. per sq. ft., or, since a wall one foot long is being considered, a load of 750 lb. per lin. ft. The weight of the sliding wedge in pounds equals area $de_1e \times 65$ + area $e_1f_2fe \times 65$ + area $f_2igf \times 100 + 750\,ig = 55\,500$ lb.

To find the horizontal pressure P lay off to any scale of intensity 55 500 on the line 1–2; at 1 draw line 1–3 parallel to de and at 2 draw horizontal line 2–3 to intersect 3. Then length of line 3–2 gives 22 000 lb. = horizontal pressure P. The weight of the material between the back of the wall and di will be regarded as helping the wall to resist the overturning moment of the horizontal pressure. The center of gravity of this material can be found analytically or graphically. It should be remembered in getting the weight of this material and its center of gravity that submerged and unsubmerged materials have different weights. In finding the center of gravity of the masonry and its weight also, it should be remembered that the masonry above high water weighs 150 lb. per cu. ft. and below high water $150 - 64 = 86$ lb. per cu. ft. CG is a vertical passing through the combined center of gravity of the wall and the earth bid resting upon it. The total weight acting through CG is 74 000 lb.

To find the resultant pressure P, lay off on any scale the line 0–7 equal to 74 000 lb., and from 7 lay off line 7–8 = P = 22 000 lb. Then line 0–8 gives the resultant in direction and magnitude, and this intersects the base at p_1. Dock wall designs need not be investigated for pressures at the toe, because their weight is materially reduced by the buoyancy of the water. Their foundation pressures and their tendency to slide upon their foundations should be investigated as in retaining walls and all designs of dock walls should be investigated for overturning. The factor of safety against overturning for this example equals the ratio of the distances pc to pp_1 or 3.4. For dock walls the factor of safety against overturning should be at least 2.5.

If no riprap is placed back of the wall, df_1 is the plane of rupture down which the wedge dig_1f_1 tends to slide and the live load is applied to ig_1. The line 4–5, to the same scale as line 1–2, represents the weight of wedge dig_1f_1 and its live load, while line 5–6 equals the horizontal pressure of this wedge. The line Ox gives the position and direction of the resultant pressure for the wall without rip-rap. The factor of safety against rotation of the wall without riprap equals the ratio of the distances pc to p_9, or 1.3 : 1. The wall is therefore not safe, and either riprap should be placed back of it or its base should be increased to cd_1 for a factor of safety of 2.5 against overturning. The foregoing analysis shows the value of riprap in decreasing the pressure upon the back.

Dock Wall of Type 2. (Fig. 85.) This wall is built where rock or other hard foundation bed lies at considerable depth, and for permanent work, but not where the piles are subject to the action of the teredo. It is also used where a hard foundation cannot be reached by piles, that is, where the wall is floated in mud; in this case, no analysis is of much value in making or checking a design, but the wall should be designed so as to permit settlement and forward displacement without demolition, and land ties should be used to keep the wall from excessive forward movement. Fig. 85 shows the method

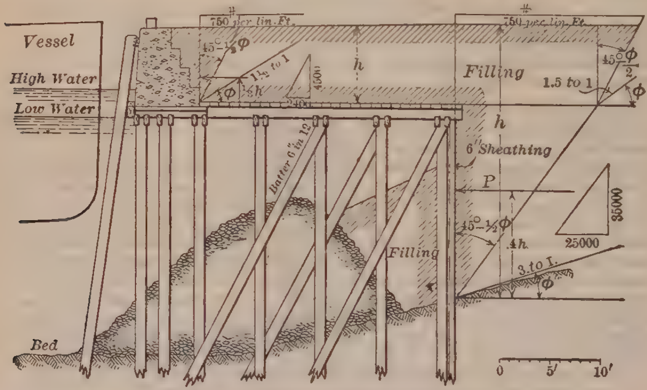


Fig. 85. Relieving Platform Dock Wall

of finding the horizontal pressure against the back of the masonry wall and also that against the timber sheeting; these methods are similar to those for type 1. For discussion of the timber substructure see Sect. 9, Art. 9. This type may also be constructed with a concrete platform on piles. If the water is infested with the teredo, concrete piles can be used instead of wood piles.

Engineers interested in the pressure of earth on sheet piling should consult "Erddruck, Erdwiderstand," by H. Krey, published by Wilhelm Ernst & Son, Berlin, 1926. So far as the writer knows, this book has not been translated. It is one of the few books giving any information upon the pressures existing below the bottom of a retaining wall or sea wall that will enable one to determine the proper length of sheet piling to be driven below the bottom of the wall, or the weight of riprap to be placed in front of the wall, to prevent any blow-out of the material under the wall.

Dock Wall of Type 3. This wall may be used for any class of foundation bed, and is well adapted to mud flotation. The riprap walls settle and go out of line, but cannot be overturned, and, if possible, the construction of the masonry wall should be delayed for a year or two after the riprap is placed. The riprap extends up to the low water line, its slope on the land side being 1 to 1 and on the water side 1-1/2 to 1. The rubble wall on top of the riprap is about 6 ft. in width. This wall is generally the cheapest type of permanent sea wall, but is unsightly, and vessels cannot approach so close to it as to the other types. When floated on mud, dock walls cannot be subjected to computation, but they require extraordinary precautions in design and construction. See paper by S. W. Hoag in Proc. Munic. Engrs. of City of New York, 1905. For examples of quay and dock walls, see Sect. 19.

29. Walls and Piers in Buildings

A Building Bearing Wall may fail (1) by the crushing of the masonry, (2) by tension of the masonry over openings, (3) by overturning about any horizontal joint due to the thrust of arches adjacent to corners, (4) by overturning of an exterior wall about any horizontal joint, due to wind, or (5) by failure of foundation. For (1) the same methods of preventing failure are to be used as for retaining walls (Art. 23). For (2) arches of masonry or lintels of stone, reinforced concrete, or metal must be provided to support the load of the masonry over openings. For (3) tie rods or buttresses must be provided to take care of the horizontal pressure when the width of wall outside of the arch is insufficient to act as an abutment. For (4) the walls must have a thickness not less than hereinafter recommended.

Failures are liable to occur during construction, due to the application of concentrated loads, as a rule eccentrically applied and when the mortar is not well set. Many cracked stone lintels are to be found over openings, but these seldom cause failure except of a local nature. Eccentric loads resulting from placing the bearing plates of beams and lintels very close to the inside edges of the masonry walls may cause failure by shearing the masonry or by overturning the walls. The flexure of beams and lintels causes an inclined reaction which tends to overturn the walls. Failures may also be caused by the lack of sufficient anchorage of the floor-beams or joists to the wall.

Thickness in Inches of Exterior Walls of Buildings

(Recommended for design)

	Residence buildings	Public and business buildings
Walls over 75 ft.:	In.	In.
Uppermost 25 ft. of wall.........	12	16
Next lower 35 ft. of wall.........	16	20
Next lower 40 ft. of wall.........	20	24
Next lower 40 ft. of wall.........	24	28
Etc.		
Walls under 75 ft. high:		
Uppermost 25 ft. of wall.........	12	16
Next lower 30 ft. of wall.........	12	16
Remainder of wall...............	16	20

Walls not over 60 ft. high may have uppermost 20 ft. with thickness of 12 in.

Walls not over 40 ft. high may be 12 in. thick throughout.

Basement walls should be at least 4 in. thicker than the walls immediately above.

The values given apply to masonry walls of brick, stone, plain concrete or hollow building blocks. If built of rubble stone, increase the specified thickness by at least 4 in., with a minimum thickness of 18 in.

Bearing walls with face brick bonded with clip courses or ties, and walls faced with ashlar should have a thickness increased by at least 4 in. over that given above. However, if the ashlar is at least 8 in. thick in alternate courses and is bonded to the wall, the thickness specified need not be increased.

When clear span between bearing walls is over 26 ft., increase the thickness by 4 in. for every 13 ft. that span is over 26 ft.

All walls over 100 ft. long between cross-walls, or suitable piers or buttresses, should be increased in thickness by at least 4 in. for every 100 ft. that their length exceeds 100 ft.

Reinforced-concrete walls may have a minimum thickness two-thirds of that specified above.

The height of stories for all given thicknesses of walls must not exceed 11 ft. in the clear for basements, 18 ft. in the clear for the first story, 15 ft. in the clear for the second story, and 14 ft. in the clear average height for any upper stories, unless the walls of such story and all the walls below that story shall be increased 4 in. in thickness in addition to the thickness already mentioned.

In case any wall is increased in thickness in accordance with one of the above requirements, it need not be further increased to meet other requirements.

The writer considers the above requirements reasonable. All engineers and architects, in designing structures to be built in cities, must conform to the requirements of the local building code.

The following specifications are quoted from "Minimum Requirements for Masonry Wall Construction," report of Building Code Committee of U. S. Department of Commerce (1925):

"The minimum thickness for solid brick bearing or party walls shall be 12 in. for the uppermost 35 ft. of their height, and shall be increased 4 in. for each successive 35 ft. or fraction thereof, measured downward from the top of the wall; except that the top story exterior bearing wall of a building not exceeding three stories or 40 ft. in height, or the wall of a one-story commercial or industrial building, may be 8 in. thick, provided that such 8-in. wall does not exceed a 12-ft. unsupported height and that the roof beams are horizontal.

"Where solid brick exterior bearing or party walls are stiffened at distances not greater than 12 ft. apart by cross-walls, or by internal or external offsets or returns at least 2 ft. deep, they may be 12 in. thick for the uppermost 70 ft. measured downward from the top of the wall, and shall be increased in thickness 4 in. for each successive 70 ft. or fraction thereof.

"The minimum thickness of solid brick exterior non-bearing walls shall be 12 in. for the uppermost 70 ft. of their height, and shall be increased 4 in. for each successive 35 ft. or fraction thereof measured downward from the top of the wall; in other respects, minimum thickness of non-bearing walls is the same as for bearing walls.

"Solid brick walls shall be supported at right angles to the wall face at intervals not exceeding 18 times the wall thickness in the top story or 20 times the wall thickness elsewhere.* Such lateral support may be obtained by cross-walls, piers or buttresses when the limiting distance is measured horizontally, or by floors when the limiting distance is measured vertically. Sufficient bonding or anchorage shall be provided between the wall and the supports to resist the assumed wind force acting in an outward direction."

When girders are supported at the ends by masonry walls the center of the bearing plates shall be practically concentric with the center of the wall. Small joists of steel or wood need not have more bearing in the wall than is necessary to reduce the unit stresses of the bearing within safe limits.

Stone Beams may be designed by $\mathbf{w} = \mathbf{Sbd^2/9}l^2$, where w = safe total uniform load in pounds per linear foot, b = width and d = depth of beam in inches, l = clear span in feet, S = working tensile strength (Art. 16) in pounds per square inch; for a concentrated load at middle use $1/2\ w$. For depth of a stone lintel in inches use $d = 4 + a\sqrt{l^3}$, where l = span in feet, and $a = 0.65$ for granite, 0.75 for limestone and marble and 0.85 for sandstone.

In Designing Piers with the unit stresses given in Art. 16, the piers should not have a greater ratio of unbraced height to least width than 12 for monoliths of stone and portland cement concrete and ashlar piers or blocks having the full horizontal dimensions of the piers, 10 for brick and well-bonded ashlar piers laid in portland cement mortar, 8 for brick and well-bonded ashlar piers laid in lime mortar, 8 for rubble piers of flat or scabbled stones laid in

* The writer thinks these requirements are too conservative.

portland cement mortar, 6 for rubble piers of unscabbled stones laid in portland cement mortar, and 4 for rubble piers of unscabbled stones laid in lime mortar. The use of bond stones at intervals in brick piers is not advocated, as they are apt to split under fire exposure.

The Unit Stresses of Art. 16 should be reduced for piers when the ratio of the unbraced height to least width exceeds 10 for monoliths of stone and portland cement concrete and ashlar blocks having the full horizontal dimensions of the piers, 8 for ashlar masonry, 6 for brick masonry, 4 for rubble masonry. This may be done by multiplying the S of Art. 16 by $(1 + 0.1c/d - 0.1h/d^2)$, in which h is height of pier in feet, d is the least width of pier in feet, h/d is the ratio of unbraced height to least width, and $c = 10$ when S is from 700 to 500, $c = 8$ when S is from 500 to 300, $c = 6$ when S is from 300 to 200, $c = 5$ when S is from 200 to 100, $c = 4$ when S is below 100 lb. per sq. in. When brick piers are faced with pressed brick, thin ashlar or terra-cotta, the facing should be neglected in figuring the area of the piers.

Example. Find the safe compressive unit stress for a brick pier of hard brick laid in portland cement mortar, the unbraced height being 16 ft. and least width 16 in. From Art. 16, $S_1 = 300$ lb. per sq. in., $c = 8$, $h/d = 12$, $h = 16$, and $d = 1.33$. Therefore the pier may be built of the stated dimensions, but the unit stress must be reduced by the above method to 210 lb. per sq. in.

30. General Data for Masonry Dams

Engineers interested in the flow of water over dams, or in the design of earth dams, siphon or shaft spillways, should consult "Hydraulic Laboratory Practice," edited by John R. Freeman and published in 1929 by the Am. Soc. Mech. Engineers, New York.

Introduction to Dams. The design of a gravity dam is a comparatively simple matter until one reaches the foundation. There the engineer is confronted with greater difficulties and responsibilities than in the design of the foundations of most engineering structures. In all cases, the design of a dam should be predicated upon extensive and thorough borings and test pits. The engineer may find it difficult to induce the owner of the dam to provide adequate funds for this purpose, but it is incumbent upon the engineer to insist that the necessary information be obtained in advance of the design and construction. Even though the results of extensive subsoil explorations are available, difficulties will arise during construction which will require changes in design to be determined by sound judgment.

The foundations must be designed not only to take the horizontal or lateral thrust of the water, but also to be as nearly as possible watertight. Twenty-five years ago the highest masonry dam in this country was the New Croton Dam, 297 ft. high above bottom of excavation. The highest existing masonry dam (1928) is tbe Pacoima Dam in California, about 400 ft. high, and engineers are now discussing dams up to 800 ft. in height. The amount of horizontal thrust varies with the square of the height. The tendency to leakage increases in a generally unknown proportion with the height of the dam, and in dams of over 200 ft. the problem of watertightness is an extraordinary one to solve. One may be reasonably content, for dams 50 or 60 ft. high, with borings and general personal study of the geology of the dam and reservoir site; but in the higher dams (100 ft. or more) it is suggested that a report of a practical geologist should also be obtained unless the terrain is thoroughly well known to the designer and the constructor. The engineer must study the watertightness not only at the dam site itself but throughout the reservoir to make sure that it will hold water. In this investigation the aid of a practical geologist should be of great value.

If the engineer is not a geologist it may be well to have his judgment confirmed as to the probability of any soft foundation rock becoming softer or slippery when the reservoir is filled and when the dam and its foundation are subjected to the pressure of water. Such changes in consistency are found in the case of certain hardpans, soft rocks and fissured rocks in which there has been an infiltration of clay into the joints or fissures. As infiltration of clay is usually of shallow depth, the fissured rock should be removed to a point where the clay has disappeared or is negligible.

Further, there remains the problem of properly tying the dam into the banks on either side of the valley, because lack of care in this respect is probably the most frequent cause of worry and occasional failure. The rock on the banks may be weaker than in the bottom of the valley. The end of the dam may abut against a ridge projecting from the side of the valley, so that there is less rock in front of the toe at that part of the dam; or the rock in the bank may dip downstream from the structure. Under such conditions, the ends of the dam should be very thoroughly anchored into the banks.

The writer does not wish to criticise unduly the tremendous amount of mathematical talent that for a generation has been expended in the determination of economical dam profiles with the view of saving a few per cent of masonry. He does, however, wish to emphasize that almost any engineer can with intelligent study of the standard rules determine a safe and reasonably economical cross-section of a dam; but, when this is done, little of the real problem of safe design has been accomplished. The foundations are the real problem. This fact is too little understood.

Due to the high cost of construction and the alleged commercial necessities, the profession has been contented to work with lower factors of safety than are customary in the design of most other engineering structures. This is properly justified in the construction of the dam above its foundations because it is a simple structure with definite loads. When the foundation is reached, however, the hazards are greater and the factors of safety there should be at least as great as in any other statically determinate structure.

Gravity dams are usually designed with the line of pressure at the outer edge of the middle-third, which gives a factor of safety against rotation of slightly over 2. The writer recommends that the line of pressure lie within rather than at the edge of, the middle-third. The factor of safety against sliding within the body of the dam is much less if friction alone is considered. The writer recommends that in every plane of the dam the resultant horizontal pressure be resisted by shearing in the masonry with a factor of safety greater than 2, ignoring friction—this being accomplished by proper bonding of the masonry. In the design of foundations resting upon hardpan or rock, it is recommended that the dam be stepped or sunk into the foundation to such a depth that the factor of safety against sliding or shearing of the foundation shall be at least 4. The base of the dam should be sunk into the foundation a sufficient distance to carry the entire horizontal pressure by the bearing of the vertical face of the toe or steps against the foundation, omitting the consideration of frictional resistance on the foundation.

Attention is called to Fig. 85*a*, showing a dam 185 ft. high from base of dam to water surface, and with the base sunk into the rock foundation to a depth of 10 ft. at the toe. The horizontal pressure on the back of this dam amounts to approximately 580 tons per lin. ft. Ignoring friction and assuming a unit bearing pressure of 40 tons per sq. ft. at the vertical face of the toe, such a dam would have to be sunk into the rock a depth of 14 ft. For most ordinary rock with the usual seams, the safe load on the rock in front of the dam would probably not be in excess of 20 tons per sq. ft., in which event a dam of this height should be sunk into the rock at the toe approximately 28 ft., and in soft rock approximately 30 or 35 ft. If the rock surface is roughened or stepped as shown by the dashed line of Fig. 85*a*, the sum of the heights of the vertical shoulders may be considered as the height of the rock face at the toe for resisting horizontal thrusts.

At the banks, the dam should be sunk into the foundation to a greater percentage of the height than at the middle of the valley. If the dam is a spillway and the rock is soft or fissured it should be protected at and below the toe against erosion of the falling water, otherwise one cannot safely count upon the rock immediately in front of the toe being permanently available to take the horizontal thrust of the water.

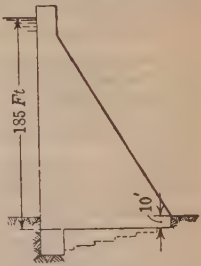


Fig. 85*a*

Extreme care must be taken to prevent a dam from sliding should the rock slope downward from the heel of the dam toward the toe. This is especially true in high dams on laminated rock. In that case there is a possibility of the rock bulging in front of the dam or shearing along lamination planes.

Thorough borings and test pits at the dam site are recommended not only for design but also to assure against excessive cost, because otherwise upon excavation the foundation rock may be found at a greater depth than assumed. The cost of many dams has been far in excess of the money provided due to the fact that the borings made were not sufficient to locate accurately the rock and overlying material in advance of construction.

If the foundation rock is at all fissured and requires grouting, this should be thoroughly done under expert supervision. The amount of grouting that may be found necessary in such rock cannot well be determined in advance unless the explorations have been extensive, by core borings and test pits. The engineer in his estimates, therefore, must allow a considerable amount of money for grouting if the explorations indicate that it may be needed.

Dams are often constructed at a great distance from the operating engineer's headquarters, and daily or weekly inspection of the finished structure is left to laborers or farmers living in the vicinity. Just how frequent expert inspection will be made cannot usually be foreseen by the designers and constructors. The writer recommends four engineering inspections a year and that at least a weekly record be kept of all leaks through the dam or adjacent banks. Practically all dams leak and this is true of the adjacent banks, and the amount of leakage varies from week to week and year to year. If there is no undue or irregular settlement and no cracks in the masonry and if these leaks do not increase materially and the issuing water is not discolored, it may usually be assumed with reasonable safety that the dam foundation is all right. The writer wishes, however, to accentuate the fact that the matter of discoloration or the carrying of materials from the dam foundation is not a matter that should be left to inspection by laymen. Water which is apparently clear if put in a clean glass and held up to the sunlight may indicate the carrying of minute particles even though there is no discoloration which can be determined by observing the issuance of the water.

Needless to say, if the amount of water is increasing from time to time it should be most carefully watched under expert supervision. In order that one may not be unduly apprehensive in the matter of leaks below the dam or through the banks, a careful observation should be made of all springs in the adjacent banks and below the dam prior to the filling of the pond. Otherwise it may be difficult to determine after the pond is filled whether or not these are old springs or new leaks.

Theory of Design. **Dams** built up to the middle of the 19th century apparently were designed without theoretical analysis. The principles upon which safe and economical dams should be designed were laid down first by Sazilly in 1853 and after him by such authorities as Graeff, Delocre, and Rankine. According to these principles the requirements of a safe dam are:

(a) At all horizontal sections, the resultant of the acting forces shall strike at or within the middle-third for all conditions of the water elevation behind the dam. This principle implies the condition that no tensional stresses are permissible in a masonry structure of this character, the assumption being made that the pressure intensities along the section or joint vary uniformly, according to the law of trapezium. (Art. 21.)

(b) At no point along any horizontal joint or the base shall the unit normal stress upon the plane be more than a specified safe limit. (Art. 16.)

Investigations of the old Spanish dams, standing for over three hundred years, show a maximum compression stress on horizontal joints of from 11 to 14 tons per sq. ft.

(c) The friction developed by the weight of the structure at any horizontal joint or at the base shall be greater than the resultant horizontal force acting upon the joint tending to slide the part of the dam above it. (Art. 18.) Frictional resistance on the base or at horizontal sections above the base may be reduced by the entrance of water beneath or within the dam.

No exact mathematical expression has been found as yet which would determine the outline of the dam section in accordance with the three requirements enumerated and with given data as for permissible stress and specified factor of safety against sliding and overturning. If only condition *a* is considered, the economical section will be a right triangle with back vertical and base width $= h/\sqrt{\gamma}$, where h is the height of water above the base and γ the specific weight of the masonry. The authorities mentioned and a number of others have endeavored to find the best modification of this triangle form to comply with requirements *b* and *c*, the result being a series of proposed forms of more or less complex outline, but none having an unimpeached general value and most being entirely impracticable. A very complete record of these forms and of the dams built according to the different theories can be found in E. Wegmann's "The Design and Construction of Dams." The "practical profiles" recommended by him (Art. 32) represent a valuable addition to the profiles mentioned.

In 1895 M. Levy and following him in 1904 Karl Pearson and Atcherley made thorough analytical studies of stresses in gravity dams of masonry and came to the conclusion that in such dams the planes subject to maximum compressive and tensional stresses are not horizontal as a rule; that the direction of the maximum pressure intensity is parallel to the face of the dam and the pressure is greatest near the outer profile; and that, in every case, in a dam designed according to the heretofore accepted principles tensional stresses occur on inclined planes near the heel. These analytical deductions have been verified by the famous experiments of Ottley and Brightmore on plasticine models and by the experiments of Wilson and Gore on India rubber models. To eliminate tension at or near the heel, Levy proposes to design dams so that "the normal pressure on every joint at the upstream face should be greater than the hydrostatic pressure at that point." Recently built French dams are all designed on this principle. German engineers use a clay or loam fill at the back of high dams, to assist in the plastering of the masonry on the waterside and to fill every crack possibly developed by tensional stresses. Wilson and Gore pointed out the advantage of a rounded or projecting inner heel to decrease tensions. In American practice this method is extensively used.

It is recommended that for the lower one-fifth of the height of dams, the safe working stresses of Art. 16 shall be reduced 30% if the dam is designed without considering the maximum stresses resulting from the combination of normal and shearing stresses. (Art. 21.)

Horizontal Pressure on Dams. Whereas the physical properties of water are well known (Art. 20) it takes a good deal of judgment to determine the horizontal forces exerted by water upon a dam. As a rule the stratum upon which the dam is built is covered by more or less pervious or impervious layers of sand, gravel, loam or clay. The dam generally extends through this overlying material and the question arises, how far down and to what extent shall the horizontal water pressure be considered. In the case of spillway dams the back pressure from the water downstream (tailwater) may decrease the net pressure upon the back and this pressure may also be exerted through the material overlying the foundation.

How much numerical value should be given to this underground pressure is entirely dependent on the kind of overlying material. Clean gravel will practically not reduce the hydrostatic pressure at all, fine sand will reduce it to a great extent, while puddled alluvial soil may not transmit any water pressure at all, but such soil quickly disintegrates with a running leak. The intensity of this horizontal underground water pressure cannot be accurately determined but its possibility should not be overlooked. The overlying material also exerts a horizontal thrust upon the back of the dam which should always be considered in the design (Art. 20), although usually this pressure is practically negligible. The active thrust and passive resistance of the overlying material at the downstream side of the dam on the other hand should generally be disregarded.

Uplifting Pressure on Dams. Masonry dams of the gravity type are preferably built upon rock, but low dams of a gravity type have been successfully constructed upon gravel and sand. These materials may afford a safe foundation if compact (of the nature of hardpan), or if the free percolation of water under the dam is prevented by carrying down a cutoff at the heel to a depth of 1/2 to 3/4 of the height of the dam, by concrete, steel or wood piling. (See also Dams on Sand Foundation.)

In the design of all dams, except those whose foundations will rest on solid, impervious and unfissured rock, careful consideration must be given to the possibility of uplift or upward water pressure upon the base due to the water being forced under the base of the dam through veins, fissures or interstices. Neglect to consider this great force may result in disaster. The designer must allow for uplift or upward pressure on the base of the dam on all but the best rock foundations or he must provide an absolutely impervious cutoff that will prevent uplift entirely or make it negligible. In "Professional Memoirs," Jan.-Feb. 1915, Capt. W. A. Mitchell describes many experiments and observations made on the uplift on dams, especially those made on the Ohio River dams built by the federal government.

The amount of uplift which should be assumed either on rock or on sand and gravel foundations is a matter of judgment and can only be subject to broad general rules. The observations that have been stated relative to the uplift in existing structures and laboratory experiments are indicative only. The wide variation in the character of rock, sand and gravel leaves the matter to judgment, and experience in the construction of dams makes one feel that ultra-conservatism is necessary. The writer therefore recommends that, except where the dam rests on impervious rock or where an impervious concrete cutoff is carried to such rock, full uplift on the whole area shall be considered at the heel and zero uplift at the toe; or if there is any probability of water in the lower pond, the uplift at the toe should be assumed equal to the full head of the lower pond water. In designing the dam the center of pressure should fall within the middle-third considering uplift and a reasonable amount of ice pressure (p. 941).

Experiments by D. M. Francis, F. R. Shunk and J. P. Jervey show that water percolates through joints between concrete and rock and travels in small veins in these joints. If allowed free exit, the pressure varies irregularly between upper and lower pools; if the exit is closed the pressure quickly becomes that of the upper pool as soon as enough water has passed through the veins to fill the test holes. The amount of space of these small veins, that is, the area of upward water pressure, varies from nearly zero in excellent granite foundations to 50% or more in rotten shale. The final height of the rise of water in the test holes when free exit is provided below the dam varies roughly as the distance from the lower pool relative to the distance between pools. If efficient drains are introduced downstream of the cutoff, the upward pressure beyond such drains is practically negligible in good rock. The drains must be effective, must not be too small and must not be stopped. With rock foundations the Ohio River Board in a general case used the following rules:

(a) Upward water pressure varying at a constant ratio from upper pool at upstream edge to lower pool at drains.

(b) Drains to be located in the rock on the downstream side of the key or cutoff.

(c) Pressure under 50% of base for firm, solid rock and under 100% of base for weak rock.

(d) Neglecting upward pressure, the center of pressure to fall within the middle-third of the base; including uplift, the center of pressure to fall far enough inside the toe so that the maximum compression shall not exceed 400 lb. per sq. in. for good rock and 70 lb. per sq. in. for poor or doubtful rock.*

Tests were made by the Bureau of Reclamation at the Willwood Dam of the Shoshone Irrigation project in northern Wyoming (Eng. News-Rec., Vol. 99, p. 660; Trans. Am. Soc. C. E., Vol. 93, p. 1538). This is a concrete gravity overflow dam, about 55 ft. high, built upon horizontally stratified and seamy shale and sandstone. The foundation was grouted and observation pipes were conducted from the dam foundation to the inspection gallery built within the dam. The uplift at the heel was about one-half the reservoir head and at the toe full tailwater head. These measurements were not sufficiently extensive to give indication as to what proportion of base was acted upon by water pressure. It will be helpful to the designer and the profession at large, and no great expense to the owner, if uplift observation pipes are provided as in the case of the Willwood Dam. These will also be useful in the inspection of the structure from time to time.

Uplift may also occur between concrete or masonry joints of the dam above the foundation bed, and in constructing the dam special precaution should be made to decrease this uplift to a minimum. The writer recommends the following rules, regardless of any attempts that may be made to eliminate such uplift; at all sections of a gravity dam uplift equal to 25% of the reservoir head above any joint shall be considered at the back of the dam and zero at the front; without uplift the resultant should lie well within the middle-third, and considering such uplift the line of pressure should not go outside the middle-third. (See also Art. 20.)

Waterproofing the back of the dam will decrease uplift and also disintegration of the concrete where subjected to water and the atmosphere alternately. Waterproofing may be applied as an exterior asphaltic or wash coating or by "gunite" as at the Elephant Butte Dam, or by enriching the cement content for a space of one or two feet from the back of the dam as was done at Arrowrock and Hetch Hetchy dams.

Cutoff Walls and Drains. It is customary and proper to build a cutoff of concrete as shown in Fig. 86, the concrete being made of the best quality,

* The writer believes that the center of pressure should fall within the middle-third for reasonable assumptions of uplift.

most carefully placed so as to prevent the water getting under the dam or at least to increase the travel of the water by the length *abcd*. The higher the dam and the more pervious or fissured the strata the deeper this cutoff should be. Sometimes the cutoff wall is made quite deep, the rock being channeled out by machine. It is good practice to provide at *d* (Fig. 86) a continuous drain with proper openings *p* extending to the face of the dam, so that any water that passes the cutoff may not exert uplift except of little intensity, but will flow out at *e*. On account of the possibility of these small drains and discharge pipes getting stopped up it is recommended in large dams to build instead of a small drain, as in Fig. 86, a gallery (Fig. 94) of such height that a man may walk in it from one end to the other to inspect and clean the drains so that the real uplifting pressure will be known. This gallery should be reached through manholes located at or near the top of the dam.

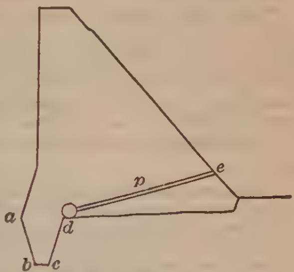


Fig. 86

Experiments made on the Gatun dam and the Ohio River at Dam No. 18 show that water pressure is transmitted through concrete and is transmitted through joints between old and new concrete more freely than through solid concrete. If allowed free exit, the pressure is practically negligible. Mitchell suggests that in order to utilize the full value of the weight of the masonry, drainage should be provided from near the back of the dam at other levels than the base only; or if such drainage is not provided for in the design, allowance should be made for the upward pressure at every horizontal joint investigated, but a smaller relative intensity can be assumed than that considered at the base. Uplift on joints above the base or drains above it have rarely been considered. Certainly if entirely disregarded, the line of pressure above the base should lie well within the middle third.

Dams Built on Sand. In the first paragraphs of this article, the writer has sounded a word of caution to designers and constructors because of foundation difficulties. Should the engineer be confronted with the problem of building a dam upon sand he will in all probability find the foundation problem exceedingly difficult.

Fig. 103 (p.960) shows the cross-section of the Sherman Island Dam which is one of the best cross-sections developed for a reasonably high dam upon a soft bottom. While this dam was designed for sand, it was built upon a mat of interlocked glacial boulders 5 or 6 ft. deep overlying saturated fine sand. This interlocked mat of boulders gave a great deal of resistance against sliding. There are two lines of sheet piling at the heel of the dam, one line 55 ft. deep and the other 45 ft. deep; and at the middle of the base of the dam a large mass of concrete was provided to add resistance to sliding. There is also a concrete apron below the dam, from *F* to *G*, to increase the percolation factor; and the dam base and apron were weighted with sand and gravel to give greater weight against sliding. Upstream a sand and gravel backfill was placed to further increase the percolation factor. Before there can be leakage under the dam the water must travel from *B′* to *B*, *B* to *C*, *C* to *D*, *D* to *E*, *E* to *F*, *F* to *G*. This gives a percolation factor of about 5.

It is thought that this dam could not have been built without steel sheet piling, but the mere use of steel sheet piling was not in itself a guarantee against serious percolation. It was only intended to drive one line of sheet piling 55 ft. in length. After this was driven it was discovered that at three points the sheet piling did not interlock at the top, and it was therefore highly probable that it did not interlock at the bottom. It was then decided to drive an additional lapping piling at these three points so as to provide a certain barrier. It was also decided to protect the dam further by driving a second line of steel sheet piling (as shown on Fig. 103) back of the original 55 ft. of piling. There is a record of steel sheet piling having been driven into sand containing boulders, as a cutoff, where one or two sheets when driven "curved up" and came to the surface of the stream bed. Steel sheet piling makes high dams on sand possible, but only if driven with great care.

There is no satisfactory way of grouting sand, and test borings give only approximate information as to the condition of the sand into which the piling is to be driven. The hazard of water cutoff for a dam on sand is so great that it is doubtful if any masonry dam should be built upon sand to a greater height than 75 or 80 ft. In such high dams upon sand the writer recommends increasing the percolation factor by placing a backfill, as shown by Fig. 103, and also recommends increasing the length of the apron as shown in this sketch.

The Sherman Island Dam presented a particularly difficult problem because the ends of the dam rest on rock while 90% of the length of the dam is on a mat of boulders overlying fine sand. It was necessary to make expansion joints in the foundation mat at a point where the rock ended and sand began, and under one buttress the rock was excavated and the base built upon a layer of sand 2 ft. thick resting directly upon the rock. This permitted adjustment without cracking. The writer is of the opinion that had there been no boulder overlay at this site the resistance against sliding should have been further increased either by flattening the back slope of the dam or otherwise spreading the dam base, and possibly also by extending the apron and placing additional earth fill on it.

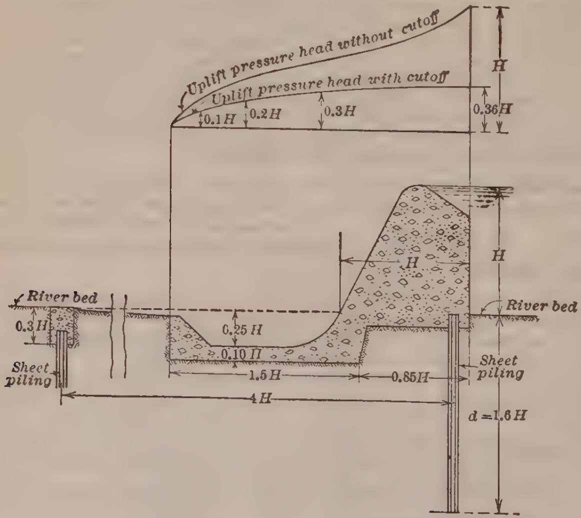


Fig. 87. Spillway Dam on Sand Foundation

The uplifting pressure of water below the base is most serious in the case of dams built on sand or gravel foundation. Water pressure is transmitted more or less freely through washed sand, gravel aggregate and washed river gravel. Sand offers the greatest, clean gravel the least resistance to uplifting action. Besides being dependent on the physical properties of the foundation strata, the magnitude and the method of distribution of the upward pressure are functions of the depth of the cutoff or sheet piling, of the location of sheet piling and of the width of the base and apron. A watertight sheet piling or cutoff wall is most effective if located at or near the heel of the dam. Another line of sheet piling placed near the toe will be effective in confining the material below the base but will increase the uplifting pressure if made watertight. The results of a thorough analytical investigation of the matter are published in Trans. Am. Soc. C. E., 1911, vol. 73, by G. E. Smith, in discussing A. C. Koenig's paper on dams on sand foundation. Figs. 87 and 88 are taken from this article, illustrating the influence of the depth of the cutoff wall and the width of the base of the dam and the length of the apron upon the distribution of upward water pressure. The diagram above the dam section, Fig. 87, shows that for the relative dimensions of this particular instance the total of uplift is reduced by the sheet piling to about one-half of that which would prevail without cutoff. For other dimensions the uplift can be estimated with the help of the diagram, Fig. 88. Koenig, in the paper above referred to, proposes the following formula for the depth d of the necessary penetration of the sheet piles:

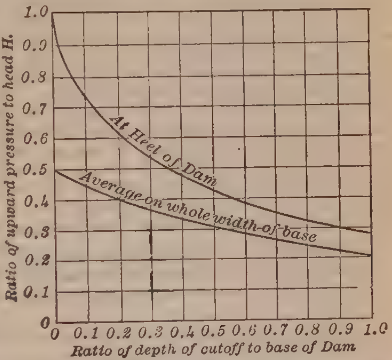


Fig. 88. Variation of Upward Pressure with Change in Depth of Cutoff

$d = h/sx$ where h = the maximum head of water on the upstream side above the bed of the stream, s = the specific gravity of the material penetrated and x = the proportion of solids in the strata. In sand with $s = 2.65$ and $x = 0.6$, $d = 0.629\ h$, which is the absolute minimum depth needed. For low heads, Koenig suggests $d = 2\ h$ and for medium heads up to 30 ft. $d = h$.

J. D. Justin proposes a somewhat different method for determining the depth of sheet piling, based on assumed factors defining the velocity of flow of water through the soil. (Trans. Am. Soc. C. E., Vol. 87, p. 54.)

A number of failures of dams occurred due to the uplifting action of water upon the aprons of dams. The floor of the Lost River dam, California, and that of the Narrora weir on the Ganges were destroyed by being lifted up by upward water pressure.

Percolation. The flow of water under dams on porous foundations can not be entirely stopped and this is the origin of a frequent cause of failure of such dams, the " blow out " at the toe, which is due directly to a higher rate of percolation than permissible for the material in question. If the percolating water has a greater velocity than a certain limit, it will carry with it parts of the material lining the vein of its passage, thus increasing the size of the vein. This action being progressive, the size of the vein continues to increase until it becomes a large hole and the dam collapses from lack of support. The blowout always starts with the appearance of a boil at the toe of the dam, that is, a strong spring carrying sediment. The time between the first appearance of the boil and the blowout varies from a few hours to several days.

The only way to prevent a blowout in pervious foundations is to cut down the velocity of the percolating water below that necessary to disturb the particles of the foundation bed, by artificially making the underground travel of the water from headwater to tailwater as long as is necessary for this purpose. The ratio of this length of underground travel to the head of water is called the **percolation factor**. The most efficient way to create a large percolation factor is by a cutoff or sheet piling.

W. G. Bligh (Eng. News, 1910) proposes the following percolation factors: Fine sand and silt 18, fine micaceous sand, 15; ordinary coarse sand, 12; gravel and sand, 9; boulders, gravel and sand, 4–6. The percolation factor may also be increased by a substantial impervious apron downstream or an impervious mat upstream of the dam, which latter may be an upstream concrete apron or a protected clay fill or the dam may be given a large inclination on the upstream face. These types have been mostly developed for dams across streams in India, where many failures occurred before the importance of the use of a proper percolation factor was recognized. The knowledge there acquired was used to good advantage on some dams in the United States. The Laguna weir across the Colorado River, with a head of 15 ft., has a total base width of 244 ft., whereas the Madaya weir in Burmah, India, for a head of 11 ft. is 288 ft. wide. In the example shown on Fig. 87 the sheet piling, the base of the dam and the apron take care of a percolation factor of about 6, corresponding to a foundation stratum of gravel and sand. If steel sheet piling is used, it must be driven with care to insure proper tightness, or else " piping " may result in spite of the cutoff piling.

Grouting of sand foundations, especially if saturated with water, is of questionable value. Primarily successful grouting can be accomplished only where there is a vent, as in the case of seamy rock, to permit the displacement of water or air with grout. It is thought that the forcing of material under pressure into sand disturbs the sand and that the grout may not restore it to the same degree of compactness that prevailed before grouting. The writer has tried it without success. Some success may be expected in loose gravel.

Grouting of soft or hard rock foundation should decrease the percolation of water under the dam if such grouting is so done as to permit free discharge of the water or air in the crevices or fissures. It is recommended in all except solid rock without fissures or open joints. If the fissures in the rock are very fine, neat cement grout must be used. Where the fissures or openings in the rock are sufficiently large, grout consisting of one part cement and one part fine sand may be used.

An excellent description of the methods used in grouting the dams of the Catskill reservoirs is given in Trans. Am. Soc. C. E., Vol. 83, p. 1042.

Usually the best method is to drill holes into the rock to a depth of from 1/10 to 4/10 the height of the dam, depending on the character of the rock, these holes to be placed from 5 to 20 ft. apart, also depending on the character of the rock. Grouting should then be started at one side of the valley and the grout forced in at only one hole at a time, noting whether or not it rises in the other holes. The same method should be carried on across the dam slowly so as to permit the grout to set up in each set of holes before grouting goes on in the near vicinity. There are records of grout having traveled several thousand feet in very seamy rock.

At the Mazoe River Dam in South Africa the shale foundation was solidified by starting the grouting at the top because the shale was too weak to sustain high grout pressures. To achieve this the grout holes were filled with clay to within 3 ft. of the top and then grouted. The hole was then cleaned of grout and clay to a further depth of 3 ft., and grout again injected. This process was repeated until the entire shale formation, about 25 ft. deep, was thoroughly grouted.

In the Hales Bar Dam on the Tennessee River, the Dix River Dam in Kentucky, and the Wilson Dam in Alabama, asphalt grouting proved successful. These were cases where there were large fissures in the rock through which water flowed with appreciable velocity. This grouting work was accomplished by installing a pipe in a grout hole, the lower end of the pipe being perforated. An electrical resistance wire was placed inside of the pipe, fastened at its bottom and insulated from the pipe. The hot asphalt was pumped into the pipe, flowing through the perforations and being kept in fluid condition by the electrically heated wire. When the asphalt reached the open fissures it cooled and hardened and was not washed away by the water, which would have been the case had cement grout been used. (See Eng. News-Rec., Vol. 96, p. 798; Vol. 100, p. 627.)

Ice Pressure at the top of the dam must sometimes be taken into account in the design and use of the dam. The under surface of a sheet of ice, being in contact with water, remains practically at the freezing point, while the temperature of the top surface may be as low as that of the atmosphere with resulting shrinkage cracks. These cracks subsequently fill with water and freeze. When the temperature rises the sheet tends to expand, and if the ice is confined it will develop thrusts. If the area of the reservoir is very large the thrust action will be lessened due to buckling of the ice sheet, and if the reservoir banks have flat slopes it is thought the pressure will be negligible.

Some designers disregard ice pressure where reservoirs are designed to be filled during the flood season and drawn down in whole or part during the winter to a level where the ice pressure would have a negligible effect. The writer feels that it is unwise to design and build a dam on the assumption that the reservoir is to be operated in any definite manner that will protect the dam structure. Conditions of operation change and operators have little regard for theory.

After a dam has been built (this should not be considered in its design) ice thrust is often made negligible where there is frequent change in the reservoir level as in the case of a forebay dam. The rise in the water level will cause the ice to break 10 or 15 ft. back of the dam. The writer has observed four thicknesses of ice adjacent to the back of a forebay dam, each about a foot thick, separated by about a foot of water and so broken as to be harmless. The damage to an existing dam by ice may be prevented by maintaining a narrow channel of water back of the dam by cutting, steam or compressed air. The following statements indicate the great difference among designers as to assumed ice pressures. Investigations in Norway indicated that the maximum thrust per lineal foot would not exceed 12 000 lb. for a 30-in. sheet and 18 000 lb. for a 40-in. sheet of ice.

The table below gives the ice pressure assumed in the design of several large dams:

	Lb. per lin.ft.
Wachusett, Mass.	47 000
Olive Bridge, N. Y.	47 000
Kensico, N. Y.	47 000
Croton Falls, N. Y.	30 000
Cross River, N. Y.	24 000
New Croton, N. Y.	0
Scioto, Ohio	34 000
Wanaque Reservoir, N. J.	20 000

The writer thinks that except where the back of the dam is approximately vertical and where also the reservoir shore is steep and the distance from the dam to the shore does not exceed 200 times the assumed thickness of the ice, these tabulated pressures are far in excess of actual ones, otherwise low dams which have been built would have failed on account of ice pressure, whereas the only records which have come to the writer's attention of the failure of low dams have occurred where the back of the dam and the banks were practically vertical and the distance between the dam and the banks small.

The writer computed some years ago that, based on such assumed ice pressures, a dam 10 ft. high should have a base 40 ft. wide, which is absurd. (See Trans. Am. Soc. C. E., Vol. 75, p. 219; Eng. News-Rec., Vol. 99, p. 742.)

Effect of Wave Action. The height of the top of the dam and the thickness of the top should be sufficient to prevent overtopping and to withstand the force of the waves.

Temperature Changes often seriously affect dams, and expansion joints should be provided in gravity dams perpendicular to the axis of the dams at intervals of about 30 to 60 ft. If expansion joints are not provided at about these intervals, cracks may form which will be unsightly and will leak. (See Sect. 11, Art. 1.)

Comparative Cross-sections of several high masonry dams are shown in Fig. 89, and data regarding them and a large number of other dams are given in Sect. 15.

Cost of Masonry for Retaining Walls and Dams. The cost of masonry for retaining walls and dams varies with the locality, size of wall, availability of local materials, the difficulties of foundation construction and, in the case of dams, with the difficulties of handling the water during construction. The average cost per cubic yard is about as follows: Ashlar facing, first class masonry \$40.00 to \$50.00 per cu. yd.; second class and backing \$30.00 to \$35.00 per cu. yd., rubble \$15.00 to \$20.00, concrete \$8.00 to \$12.00 depending on location and quantity involved.

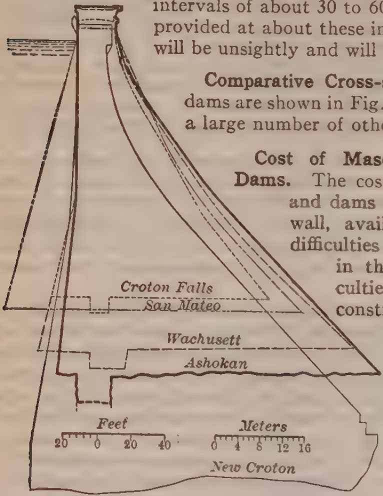


Fig. 89. Five American Dams

31. Investigation of Masonry Dams

An Investigation of a Proposed Section must be made for two cases: (1) when the reservoir is full and the water level is at its maximum height; (2) when the reservoir is empty. In the first case, the resultant should always be within the middle-third, and then the compressive unit stress is greatest at the toe and least at the heel. (Art. 21.) In the second case, the resultant may be allowed to come at the edge of the middle-third nearest the heel, and then the compressive unit stress is greatest at the heel and least at the toe. Maximum economy of material will result when the resistance line for reservoir full lies just inside the limit of the middle-third and when the resistance line for reservoir empty coincides with the limit of the middle-third nearest the back. For other heights of water the resistance line will lie well within the middle-third.

The following formula gives the safety factor against rotation

$$n = \frac{Wt}{Hp}$$

where W = weight above any horizontal plane considered;
t = distance from toe to point where resistance line cuts that plane;
H = horizontal water pressure above the plane;
p = height of H above the plane.

A Trapezoidal Section, such as is generally used for low dams, can be investigated by the methods given for retaining walls in Art. 25. The resultant must cut the base within the middle-third for all cases. This will always occur when the reservoir is empty, so that the entire base is then under compression. For reservoir full to the top of the dam, the resultant cuts the base on the other side of the middle, and it should also lie within the middle-third in order that the entire base may be under compression.

Example. Fig. 74 may represent a section of a trapezoidal ashlar masonry dam of 30-ft. height, 5-ft. top, and 18-ft. base, the back face plumb and the foundation sound, impervious rock. The weight of the masonry will be taken at 160 lb. per cu. ft. Let a be the top width, b the base width, and h the height, all in feet; then the weight of a section one foot long is $80\,h\,(a + b)$ = 55 200 lb. The line of action of this weight passes through the center of gravity of the trapezoid whose horizontal distance from the toe A is

$$t = (2\,b^2 + 2\,ab - a^2)/3\,(a + b) = 11.64 \text{ ft.}$$

Therefore the resisting moment about the toe is M_1 = 55 200 × 11.64 = 642 530 ft-lb. For reservoir full the water surface is taken as level with the top of the dam, and its horizontal pressure P is 31.25×30^2 = 28 125 lb. The overturning moment about the toe is $M = P \times 1/3\,h$ = 281 250 ft-lb. Hence the factor of safety against rotation is $M_1/M = 2.3$, which is ample provided the resultant cuts the base within the middle-third. That this is the case is found by using the formula

$$e = 1/2\,b - t + 1/3\,Ph/W = 2.45 \text{ ft.},$$

where t is the distance 11.64; the value of e is less than $1/6\,b$, so that the resultant lies within the middle-third. The ratio $6\,e/b$ is $6 \times 2.45/18 = 0.817$, and $W/b = 3070$; hence (Art. 21), the maximum and minimum pressures on the base are $S_1 = 3070\,(1 + 0.817) = 5580$ lb. per sq. ft., and $S_2 = 3070\,(1 - 0.817) = 560$ lb. per sq. ft. The coefficient of friction required to prevent sliding on the base with a safety factor of 2 is $f = 2\,P/W = 1.02$, so that the rock should be made very rough. For reservoir empty, $e = 9.0 - 11.64 = 2.64$ ft. on the other side of the middle, so that here $6\,e/b = 0.88$, and hence the pressure at the heel is slightly greater than at the toe for reservoir full, and the pressure at the toe is slightly less than at the heel for revervoir full, but the line of resistance here also falls within the middle-third. For ice pressure see Art. 30. The formulas for walls with a vertical back (Art. 25) could have been used in this example.

Fig. 74 shows a simple graphic analysis for this case by the use of the force and equilibrium polygons, the trapezoid being divided into a rectangle and a triangle whose centers of gravity are known without computation and the lines of action of the weights W_1 and W_2 being drawn through these centers.

A Dam with Curved Front (Fig. 90) is 60 ft. high, 40 ft. wide on the base, top width 10 ft., built of granite ashlar weighing 160 lb. per cu. ft., and is to be founded on impervious rock. The first step in its investigation is to divide the cross-section into subdivisions A, B, . . . G by horizontal lines, these being sufficiently near together so that the dubdivisions may be regarded as trapezoids. The areas and centers of gravity of the subdivisions are next found, the latter marked on the drawing by small circles. The horizontal lines at a, b, . . . g are, for brevity, called joints, although no joints may really exist. P_a, P_b . . . P_g denote the horizontal water pressure acting above the joints a, b, . . . g, each of these being applied at a distance above the joint equal to one-third of the depth of the joint below the water level at O.

Resistance Line for Reservoir Full. To determine the point where the resistance line cuts any joint as b in Fig. 90a, the weight of $A + B$ is to be laid off vertically through its center of gravity and P_b produced until it intersects this vertical at the point marked by the small square, then the resultant of the weight $A + B$ and the pressure P_b is determined, and this, being produced until it intersects the joint, locates a point in the line of resistance. The steps of the graphic analysis are as follows:

(1) In Fig. 90*b* the resultant forces acting against the joints are found. The load line O_1g_1 is first laid off to scale to give the different weights, O_1a_1 being that of A, a_1 b_1 that of B, and so on. On the horizontal line at the top the distances O_1P_a, O_1P_b, etc., are laid off to represent the horizontal forces

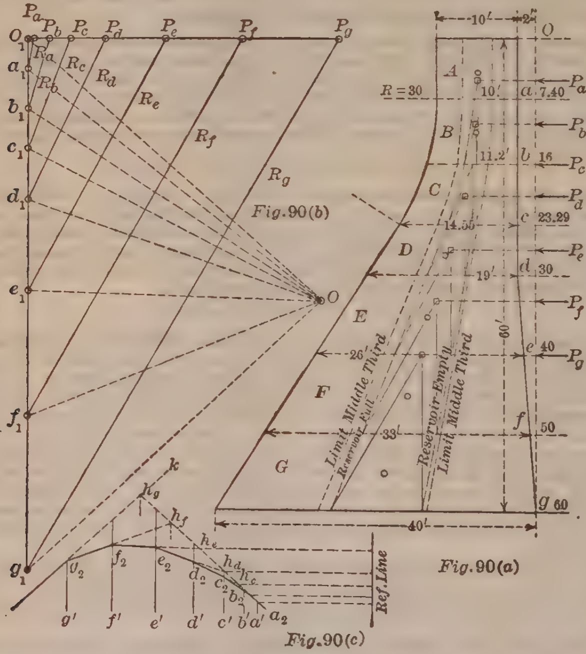


Fig. 90. Graphic Analysis of a Masonry Dam

P_a, P_b, etc. Joining corresponding points R_a, R_b, etc., are the resultants in direction and magnitude which act on the joints a, b, etc.

(2) In Fig. 90*b* a force polygon is constructed with pole at any point O, as shown by the broken lines, this to be used for locating lines through the centers of gravity of the areas $A + B$, $A + B + C$, etc., the points where these lines intersect the horizontal forces P_b, P_c, etc., being indicated by small squares.

(3) To find these vertical lines, Fig. 90*c* is used where verticals through a', b', etc., are first drawn at the same horizontal distances apart as the small circles on Fig. 90*a*. Usually a larger scale is used for Fig. 90*c* and the horizontal distances are laid off to this scale from a reference line, these corresponding to the horizontal distances from a vertical through the heel g to the small circles. Then an equilibrium polygon a_2, b_2, . . . g_2 is constructed

and its sides produced to cut the first side in the points h_a, h_b, h_c, . . . h_g. The distances from these points to the reference line are the distances to be laid off on Fig. 90*a* from the vertical through the heel, proper consideration being given to the different scales, in order to find the verticals through the centers of gravity of $A + B$, $A + B + C$, etc.

(4) Producing the horizontal directions of P_a, P_b, . . . P_g to these last vertical lines, the points of intersection are marked by small squares. Through these points draw lines parallel to the resultants R_a, R_b, . . . R_g in the force polygon, and the intersection of these with the joints a, b, . . . g will give points on the resistance line for reservoir full. A curve is then drawn through these points, and it is seen that the resistance line for reservoir full lies well within the middle-third, so that there is full security against rotation and against any ocurrence of vertical tension at the heel of the dam.

Resistance Line for Reservoir Empty. Through the small squares draw verticals to cut the corresponding joints and pass a curve through the points thus determined. The resistance line thus determined is also within the middle-third but nearer to its limit than for the preceding case.

Stresses at the Base Joint. The weight of the dam, equal to the full load line O_1g_1 in Fig. 90*b* is 201 050 lb. and hence the average normal pressure on the base is 5026 lb. per sq. ft. The resistance lines for reservoir full and empty cut the base at distances of 5.5 and 6.0 ft. from the middle, so that the ratios for $6e/b$ are 0.825 and 0.9. Then for reservoir full, using formula $\mathbf{S = W/b \times (1 \pm 6e/b)}$, (Art. 21), the maximum vertical pressure intensity is at the toe and is equal to 5026 (1 + 0.825) = 9170 lb. per sq. ft., whereas for reservoir empty the maximum vertical pressure intensity is at the heel and is equal to 5026 (1 + 0.9) = 9550 lb. per sq. ft. Both these are very low for both the masonry and the rock foundation. The value of the principal stresses will now be found for the case of reservoir full. The following data are necessary:

Vertical pressure intensity at the toe	S_1 = 9170 lb. per sq. ft.
Vertical pressure intensity at the heel	S_2 = 880 lb. per sq. ft.
Intensity of water pressure	$p' = 60 \times 62.5$ = 3750 lb. per sq. ft.
Average intensity of shear	$q = 1/2 \times 60^2 \times 62.5/40$ = 2810 lb. per sq. ft.

Then the principal stresses at the toe will be (Art. 21)

$$S_{max} = 1/2\ \left(9170 + 3750 + \sqrt{(9170 + 3750)^2 - 4\ (9170 \times 3750 - 2810^2)}\right) = +\ 10\,365 \text{ lb. per sq. ft.}$$

$$S_{min} = 1/2\ \left(9170 + 3750 - \sqrt{(9170 + 3750)^2 - 4\ (9170 \times 3750 - 2810^2)}\right) = +\ 2555 \text{ lb. per sq. ft.}$$

Both principal stresses being positive at the toe, no tension exists there on any plane.

The principal stresses at the heel will be

$$S_{max} = 1/2\ \left(880 + 3750 + \sqrt{(880 + 3750)^2 - 4\ (880 \times 3750 - 2810^2)}\right) = +\ 5470 \text{ lb. per sq. ft.}$$

$$S_{min} = 1/2\ \left(880 + 3750 - \sqrt{(880 + 3750)^2 - 4\ (880 \times 3750 - 2810^2)}\right) = -\ 840 \text{ lb. per sq. ft.}$$

The latter value being negative a tensile stress of about 5.8 lb. per sq. in., which is so small as to be negligible, exists on a plane at the heel of the dam, which plane is inclined to the horizontal at an angle θ determined by the relation

$$\cot\theta = -\frac{840 - 3750}{2810} = -\ 1.63 \text{ from which } \theta = 121°\ 30'$$

Security against Sliding. The base of masonry dams on rock foundation is usally placed several feet below the surface of the rock (Fig. 94). If the base line of the dam here investigated (Fig. 90*a*) represents the joint at the level of the original rock surface, sliding of the dam is prevented by the shearing resistance of the masonry. If the dam were placed with its base upon the surface of the rock it would slide on the base if the tangent of the angle between the resultant pressure on the base and a vertical line is greater than the coefficient of friction between the materials of the dam and the foundation, or if the coefficient of friction were less than $P/W = 0.56$. For full security the base should be roughened so that the frictional resistance will be greatly augmented. In high dams " stepped shoulders " should be cut in the rock in sufficient depth and number so that independent of the frictional resistance at the base the entire horizontal thrust might be taken by these shoulders without too high stresses on the rock shoulders or the abutting concrete, thus assuring at least a factor of safety of 2 against sliding. If the rockbed slopes downward from the heel of the dam, these steps are especially necessary. In high dams or poor laminated rock the possibility of the rock bulging up in front of the dam should be considered. Make doubly sure that the dam will not slide. Serious dam failures have been quite usually due in whole or part to failure to provide adequately for the transfer of the horizontal thrust to the foundation bed.

32. Details of Design of Masonry Dams

The Back of the Dam is often vertical or has a uniform batter, this being always the case for a low dam. For a high section, however, the profile of the back as well as the front of the section is often formed by straight lines of different inclinations (Fig. 93), but more often the front is curved. The water pressure is normal against a battered back, but its horizontal and vertical components may be used instead. (Art. 20.) It is customary to neglect the vertical component V (Art. 20), since such neglect is on the side of safety, and since the percolation of water beneath the heel would entirely annul it. For a uniform inclination above an assumed joint the normal water pressure $\mathbf{P} = 1/2\ \mathbf{wh_1^2 \cos\theta}$, where h_1 = depth of joint below water level and w = weight of water per cubic unit. For further security it is best to consider θ as 0, and then $\mathbf{H} = 1/2\ \mathbf{wh_1^2}$, which is strictly true for a vertical back.

Above the assumed joint there are hence only two forces acting, the horizontal water pressure and the weight of the masonry. The resultant of these should cut the base within the middle-third. The point where the resultant cuts the planes of several joints gives points through which the resistance line can be drawn. This may be done either graphically (Art. 31) or analytically, the latter leading to complicated formulas.

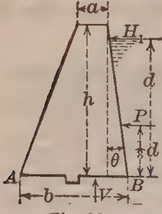


Fig. 91

Trapezoidal Dams with vertical backs are designed by the same methods as those given in Art. 25 for retaining walls, the water pressure being taken horizontal and equal to $\mathbf{1/2\ wd^2}$ when no overflow occurs, or in pounds $\mathbf{P = 31.25\ d^2}$, if d is in feet; d is height from water line to base, this being usually less than the height h of the dam. The resultant should cut the base at or inside the edge of the middle-third for reservoir full, and for the resultant at the edge required width of base is $\mathbf{b = -0.5\,a + \sqrt{1.25\,a^2 + d^2/s}}$, where s is the specific gravity of the masonry. This formula applies for water pressure only and takes no account of the action of ice or waves.

For any Trapezoid (Fig. 91) let a = top width, h = height, d = height of water line above base, θ = inclination of back to vertical, w = unit weight of water, v = unit weight of masonry, H_1 = ice or wave pressure acting at water line on one unit of length of dam. Then proper base width so that resistance line cuts base at edge of the middle-third is $\mathbf{b} = (\mathbf{A}^2 + \mathbf{B})^{1/2} - \mathbf{A}$, in which

$$A = 1/2\ (a - h \tan\theta) \qquad B = a^2 + 2\,ah \tan\theta + (6\,H_1 + wd^2)\,d/vh$$

Example. For Fig. 74, let $a = 5$, $h = d = 30$ ft., $\theta = 0°$, $w = 62.5$, $v = 160$, $H_1 = 0$; then $A = 2.5$, $B = 376.5$, and the formula gives $b = 17.1$ ft. If, however, $H_1 = 43\,000$ lb. for ice pressure, then $B = 537.8$ and the formula gives $b = 20.8$ ft. The amount of material in this dam is increased over 30% by taking ice pressure into account.

A trapezoidal dam with vertical back requires less material than one with inclined back if vertical water pressure on the inclined back is ignored. For example: $h = 60$ ft., $d = 57$ ft., $a = 9$ ft., $v = 150$ lb. per cu. ft., $H_1 = 0$. Then

for $\tan\theta = 0$	$b = 32.7$ ft.	area = 1251 sq. ft.	per cent = 100
for $\tan\theta = 1/12$	$b = 36.2$ ft.	area = 1356 sq. ft.	per cent = 108
for $\tan\theta = 1/6$	$b = 39.8$ ft.	area = 1664 sq. ft.	per cent = 117

When a trapezoid is used for a high dam, the compression at the toe may become greater than the allowable value even though the resultant cuts the base at the edge of the middle-third. When this is found to be the case by computation or graphic analysis, the section is too small and the base must be widened. Let S be the safe working compressive stress, and the other quantities as above; then the required width of the base is $\mathbf{b} = (\mathbf{C} + \sqrt{\mathbf{C}^2 + \mathbf{D}})$, in which $C = 1/2\ vh^2 \tan\theta/S$; and $D = (wd^3 + vha^2 + 2\,vah^2 \tan\theta)/S$.

A Pentagonal Section (Fig. 92) is a modification of a triangular one by giving to the top a width of 5 ft. or more and then making the front face fg vertical until it reaches the inclined line dc. The top width may be 10 to 17% of the average height, though this width may be governed by ice pressure, or the desirability of a roadway on top of the dam. For a triangular section the necessary width of the base is $b = 7.91\ h\sqrt{1/v}$. For the pentagonal section the width of base should be $b_1 = b/\sqrt{1 + 2\,r^2 - 2\,r^3}$, where r is the ratio of the width of the top to the computed b. A short back batter of height 1/4 h above the heel, and base width of 1/12 b_1 may here, as in other cases, be advantageously used to diminish any tendency to tension at the heel. This formula does not include the effect of ice or wave pressure at the water line.

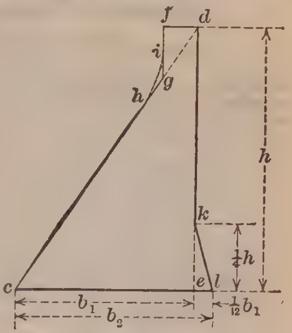


Fig. 92

Water under the Base reduces the unit weight of masonry by 62.5 lb. per cu. ft. if this produces a full uniform pressure. It is best to keep the unit weight of masonry at its full value, however, and to introduce beneath the base such upward pressures as may be specified. For discussion of the uplift see Art. 30.

Dams over 60 Feet High may be designed by means of Fig. 93, known as Wegmann's "Practical Type No. 2," and with the help of the following table, these being taken from Wegmann's "Design and Construction of Dams" (1927) and modified in columns 4 and 5 in order to diminish the tendency to tension at the heel (Art. 21). The figure gives this profile for a 200-ft. dam, the masonry of which was assumed to weigh 145.8 lb. per cu. ft. This profile is drawn for a top width of 20 ft. or one-tenth the height of the dam, and the table is made for the same ratio of top width to base. For a dam of the same top width cut off the section at any desired height. For a dam of less top width, for example 12 ft. and 100 ft. high, draw a dam 12 ft. × 10 or 120 ft. in height, by reducing the given profile in the proportion of 120/200 and cut off a height of 100 ft. If the width of the top of the dam is less or greater than 20 ft., the pressures in columns 6 and 7 must be reduced or increased proportionately.

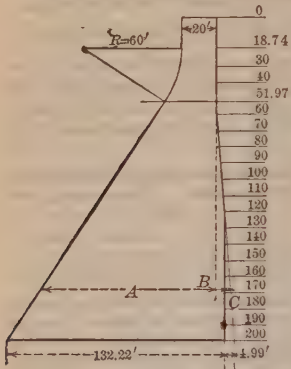


Fig. 93

For a dam having a top width of 12 ft. and a height of 120 ft., the maximum pressure on the base for reservoir full is 14.32 × 0.6 = 8.59 tons per sq. ft., and for reservoir empty 14.65 × 0.6 = 8.79 tons per sq. ft. The minimum width of the top of a dam over 60 ft. should be at least 8 ft., and this width must often be arbitrarily increased by the designer on account of the pressure of ice. The height of the dam above high-water level varies from 2 ft. in low dams to 10 ft. in high dams. The tables and profile recommended are for dams upon impervious

Elements of Wegmann's Practical Profile No. 2

Slightly modified in fourth and fifth columns

Depth of water below top of dam, ft.	Length of joint in feet				Maximum pressure, tons per sq. ft.		Tangent of resultant with vertical
	A	B	C	Total	Reservoir full	Reservoir empty	
18.74	20.00	0	0	20.00	1.89	1.36	0.20
30	21.07	0	0	21.07	3.68	2.37	0.31
40	23.89	0	0	23.89	5.03	3.53	0.41
51.97	30.04	0	0	30.04	5.53	4.91	0.50
60	35.38	0	0	35.38	5.59	5.63	0.54
70	42.03	0.62	0	42.65	5.94	6.11	0.58
80	48.68	1.25	0	49.93	6.45	6.59	0.61
90	55.33	1.87	0	57.20	7.02	7.09	0.62
100	61.98	2.50	0	64.48	7.62	7.61	0.63
110	68.63	3.12	0	71.75	8.26	8.15	0.63
120	75.28	3.74	0	79.02	8.90	8.69	0.63
130	81.93	3.74	0.62	86.29	9.55	9.46	0.64
140	88.58	3.74	1.25	93.57	10.22	10.22	0.64
150	95.23	3.74	1.87	100.84	10.89	10.96	0.64
160	101.88	3.74	2.49	108.11	11.56	11.71	0.64
170	108.53	3.74	3.12	115.39	12.25	12.44	0.64
180	115.18	3.74	3.74	122.66	12.95	13.18	0.64
190	121.83	3.74	4.37	129.94	13.63	13.91	0.64
200	128.48	3.74	4.99	137.21	14.32	14.65	0.64

rock, no upward water pressure being considered. For dams upon pervious rock proceed as in trapezoidal and pentagonal dams. The profile recommended is made without regard to the effect of ice pressure in tending to overturn the dam, except that the upper section has been arbitrarily increased so as to provide for this pressure. For heavy ice pressures the specified pressures per linear foot at the water line can be taken into account in a graphic analysis.

The Ashokan Dam, completed in 1912, in the lower Catskill region, for impounding the additional water supply of the city of Greater New York, has some novel features in design which are shown on Fig. 94. The length of the central and masonry section of the dam is 1000 ft., height 220 ft., width of base 190.2 ft., width of top 23 ft., which is corbeled to 26.3 ft. The dam is built of cyclopean concrete faced with solid concrete blocks laid up as ashlar. The interior of the masonry is drained into galleries approximately level, one at the water level and the other about 40 ft. above the base. The galleries are connected at intervals by vertical porous concrete drains. This drainage system is designed to intercept water so as to prevent seepage through the dam.

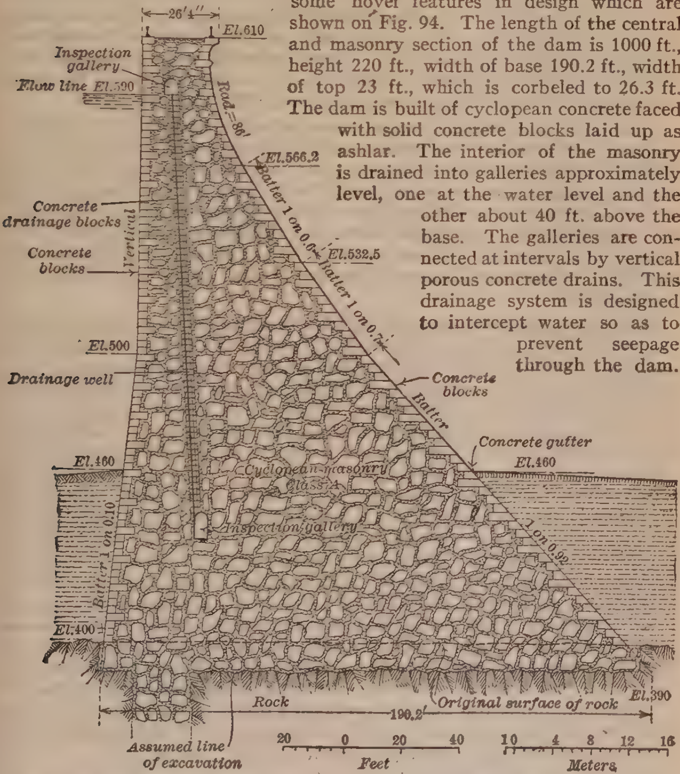


Fig. 94. Maximum Section of Ashokan Dam

A Spillway Dam is one built to discharge water over its crest. The design of the spillway dam embodies all the essential features of the design of masonry dams in general and besides must provide for the thrust and discharge of the overtopping water. The spillway dam is subjected to all the forces of a reservoir dam and in addition to the pressure of the overtopping water and, if not properly designed, to a pressure resulting from the formation of a partial vacuum between the face of the dam and the overflowing water (Art. 20). To prevent the formation of such a vacuum the face of the dam must have a sufficiently flat slope so that the falling sheet of water will not separate from it in transit from the top to the bottom. Special care must be taken to so design the bottom of the dam that the water in leaving the dam will cause a minimum amount of erosion. This is accomplished by so designing the base of the dam that the delivery of the water is tangential to the stream bed below the dam. If it is not of the hardest rock, the lower stream bed adjacent to the dam should be paved with concrete or masonry and in some cases a pool is created at the toe of the dam to absorb the shock of the falling water. If the designer is not experienced in the erosive action of falling water he should, before designing the spillway section, make a thorough study of such effect by observing the erosion at other existing dams. Often the paving is made too light or is not carried far enough below the dam, or the downstream end of the paved apron is not provided with a proper cutoff trench filled with masonry.

Unless it is made with the greatest care and of the best grade of materials, concrete has not usually been found satisfactory for the discharge surface of spillways, especially of high dams and those built in northern latitudes. The writer recommends concrete for the discharge surface of spillways only in southern latitudes, or in northern latitudes for spillway dams having a height not exceeding 50 ft. For higher dams and those in northern latitudes it is recommended that the discharge surface be paved with either hard brick thoroughly bonded to the concrete backing or with ashlar built of hard and durable stone.

The Form of the Crest, as shown by the Cornell experiments on weirs (Sect. 13, Art. 14), has an important bearing on the discharging capacity and on the economy of spillway dams. It is often difficult, in narrow valleys of large drainage area, to get sufficient length of spillway to discharge freshets without a great depth of water passing over the dam. Every effort should be made to keep this depth a minimum because it increases the pressure on the dam and the scour at the bottom and increases back water and thereby floods more land.

Flat-topped rectangular dams up to 1 ft. top-width discharge practically the same volume of water as the standard sharp-crested weir. With wider crests, the discharge diminishes rapidly, a 6-ft. horizontal crest passing only 80% of the standard discharge. So-called compound crests (Fig. 96), with a sloping top (dotted line in figure), discharge more than standard weirs, the volume of discharge being about 20% more for an inclination of 45 deg. of the sloping top, which is connected by a circularly or parabolically rounded portion to the down-stream face. This face should be inclined about 30 deg. to the vertical (Fig. 96) so that the nappe of the overflowing water will adhere. For any dam the precise profile to insure an " adherent nappe " may be computed by the ordinary principles of hydraulics, assuming the depth of the overflow.

The sloping top is also useful in preventing injury of the upper part of the dam, due to ice and logs, and also reduces ice pressure of the reservoir if full, since the ice will slide up the slope without exerting pressure.

If to the compound crest and sloping face a smooth transition to the river bed or to an apron protecting same is added (*ik* in Fig. 96) the familiar ogee section results. This section, which, in masonry spillway dams, is of general use, not only approximates the theoretical economical section, but possesses several practical advantages. The lower curved part helps to keep the eddies and back waves at a distance from the toe of the dam and thereby protects the dam against the impact of logs and ice revolving with great velocity in these eddies and waves. The inclined face and the rounding of the downstream top make the falling sheet of water adhere to the dam, eliminating the possibility of a vacuum below the nappe, and help somewhat to destroy by friction a part of the energy of the falling water. This energy may be very large and is the cause of the greatest danger to spillway dams by the scouring action upon the bed of the stream at and near the toe. The destruction of this energy is sometimes attempted by building the downstream face in steps, as for instance at the Pedlar River dam in Virginia (Fig. 97). The Gilboa Dam of the New York City water supply was built with a stepped spillway, the action of the steps in dissipating the energy of the overflowing water being increased by the addition of baffle vanes on the top step. (See Trans. Am. Soc. C. E., Vol. 86, p. 280.) The use of a stepped spillway may not be advisable with a concrete dam in that it means the pounding out of the energy of the overflow upon the dam itself. This may result in rapid wear of the concrete steps, requiring numerous expensive repairs; and the rounding off of the steps may destroy their effectiveness in breaking up the sheet of overflowing water. The face of the Bassano dam (height of spillway 38 ft., 13 ft. of water over the crest) is equipped with two staggered rows of baffle piers built like snow plows and pointed upstream. The baffle piers are not designed to take the shock of the falling water, but to split and divert the stream into interfering jets and so create eddies and disturbances within the body of the downstream pool, which is 50 ft. long. The Gatun dam (height of spillway 59 ft., 18 ft. of water over the crest) has two staggered rows of baffle piers on the apron close to the toe. These piers have vertical upstream faces, the upper row consisting of piers 14 ft. wide and 9 ft. high and 12 ft. long. They are designed to take the shock and are faced with heavy cast-iron protection. The destruction of the kinetic energy of the falling water is almost complete. The cast-iron protection was later extended around the sides of the piers to prevent erosion of the concrete; and the spillway floor for a distance of 3 ft. in front of and around the sides of the piers was protected with 3/4-in. steel plates, for the same reason. The wear on the concrete was due only to the impact of the water, as there is no silt carried over the spillway. The metal protection has been quite effective.

Baffle piers were also used on the spillway aprons of Pitt River Dams Nos. 3 and 4, in California, the baffles being staggered in two rows and placed just beyond the toe and in a pool about 15 ft. deep. (Trans. Am. Soc. C. E., Vol. 93, p. 451.)

The necessity of adequate protection of a dam with free overfall is shown at natural waterfalls where at the foot a deep pool is formed in the stream bed. The erosion of the bed stops as soon as the pool becomes deep enough for the contained water cushion to take up and dissipate the shock of the falling water. A similar artificial pool, the bottom and sides well protected by heavy concrete lining, should therefore be built at the toe of free overfall dams, except in case of low dams on hard rock, the dimensions of the pool and its lining being a function of the material of the river bed, the height of the fall and the volume of the discharged water.

In the case of a spillway dam with an ogee face or any other form with adherent nappe, it is customary to protect the bed of the stream with a concrete or masonry apron for a considerable distance below the toe of the dam. The design of this apron on any but the hardest rock foundation is a matter of the greatest importance. Besides its function of protecting the bed from scouring, it also increases the percolation factor of dams built on sand, gravel or other pervious foundations. (Art. 30.) The apron should extend far enough downstream so that the river bed would be protected at least to the point where the velocity of the flow becomes normal, in other words where the velocity of the flow becomes the same as it was before the dam was built. The dissipation of the energy of the water flowing over a dam apron may be conveniently and economically accomplished by the creation of the " hydraulic jump " close to the toe of the dam. (Sect. 13, Art. 17.) If this energy is not dissipated and the stream bed is readily erodible it may be highly important to create an hydraulic jump near the toe of the dam to save the expense of extensive paving down the stream.

Professor Sherman M. Woodward (Eng. News-Rec., Vol. 99, p. 974) states in part: " There is some lack of agreement among authorities as to just what phenomena should be considered to be the typical hydraulic jump, but probably all will agree that the jump occurs in its most definite, striking and conspicuous form when it is produced on a practically level floor by a stream of water moving with a high velocity striking against a higher wall of relatively quiet water. In this typical jump the kinetic energy of the moving stream is transformed . . . into heat so that the out-flowing water moves away in a quiet, placid stream with a uniform low velocity . . . . In order that the jump may form, however, there must be a definite mathematical relation between depth and velocity of the high velocity stream and the depth of the tail water . . . . The easiest way to insure the formation of the jump is to provide a sloping floor so that by moving up or down the slope the jump may automatically adjust itself to such position as will provide the needed depth of tail water. Although a sloping floor provides a convenient automatic regulation of the jump, it has the disadvantage that if the floor is given much slope the nature of the slope is modified. If the slope is made greater than 1 on 3, experiments indicate that the certainty and effectiveness of the jump in destroying energy are perceptibly reduced. If the slope is made as steep as 45 deg. the usefulness of the jump is mostly lost."

See also "Hydro-Electric Handbook," by W. P. Creager and J. D. Justin, p. 142;

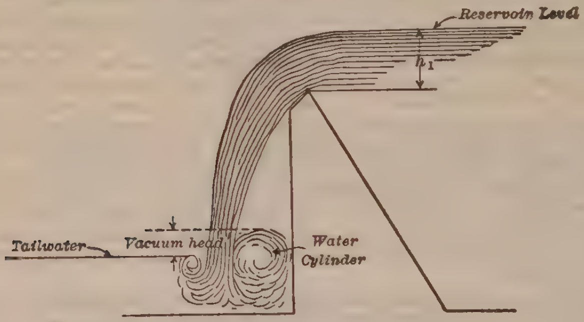


Fig. 95

"The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," by J. Hinds, Eng. News-Rec., Vol. 85, p. 1034.

The thickness of the apron must be so proportioned that it will more than equalize, by its weight, the uplifting force from below, or it will "blow up." Weep-holes can be used if the foundation is seamy rock, but they should be avoided in material which easily erodes. It is best to give the apron a form similar to Fig. 87, where an artificial pool of moderate depth is formed. Such an apron is very efficient in dissipating the kinetic energy of the falling water and brings the location of the jump to the point desired, i.e., to the lower end of the pool. Baffle walls on the ogee or on the apron will be found necessary only in the case of very high dams or if a very large volume of water is discharged.

Rarefied air space forms between the nappe and the face of the dam if the nappe is not adherent and the space between the falling sheet of water and the dam is not connected with the outside atmosphere. Air ducts are occasionally provided for to equalize the pressure of the air, as shown in Fig. 97 illustrating the Pedlar River dam. There is, however, a tendency among engineers to attribute too much importance to the vacuum which may possibly occur behind the nappe (Art. 20), some authors stating that the hydrostatic pressure may thereby increase by as much as the full atmospherical pressure, equivalent to 34 ft. of additional head. Observations do not admit the possibility of such a condition. When the air is gradually exhausted from behind the nappe by action of the falling water, a water cylinder is pushed up into the space between the nappe and the dam from the foot of the overfall. The height to which this cylinder (Fig. 95) rises is a measure of the rarefaction of the air and of the additional pressure upon the dam. The cylinder, however, reaches only a certain limiting height which has been found to depend upon the quantity of the discharge over the spillway. When this height is reached, the nappe is pierced very soon and air admitted through it so that the pressure beneath the nappe again becomes atmospheric. In one case, where the water stood about 2-1/2 ft. over the crest, the nappe was pierced every 10 seconds with a thunderlike report, at which time the pressure of the air in the enclosed space under the nappe was diminished by about 1/20 atmosphere. The experiments were repeated with different amounts of the overflow and showed that the limit of the rarefaction is proportional to the thickness of the nappe. It is recommended, for a dam with non-adherent nappe and the space between nappe and the face not ventilated, that an increase of pressure on the back equivalent to 60% of the height of the water above the crest be considered.

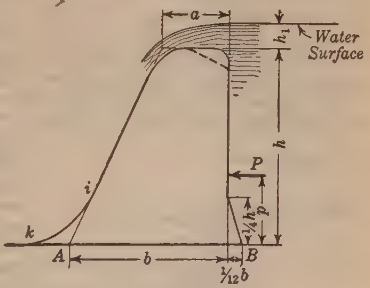


Fig. 96

The spillway dam is usually treated as a masonry dam with trapezoidal cross-section, the curved portion *Aik* (Fig. 96) being disregarded in design and investigation. For the magnitude of the acting forces and their point of application, see Art. 20 and Art. 21.

The spillway dam at Austin, Tex., which failed in 1900, was 60 ft. high and had an effective width at the base of 44 ft., the height of the water above the crest being 11 ft. at the time of the failure. Sliding occurred on the defective rock foundation and also shearing on four vertical cross-sections so that two pieces of the dam, each about 250 ft. long, were pushed out of the main portion and moved downstream without overturning. For full details, see Eng. News, April 14, 1910.

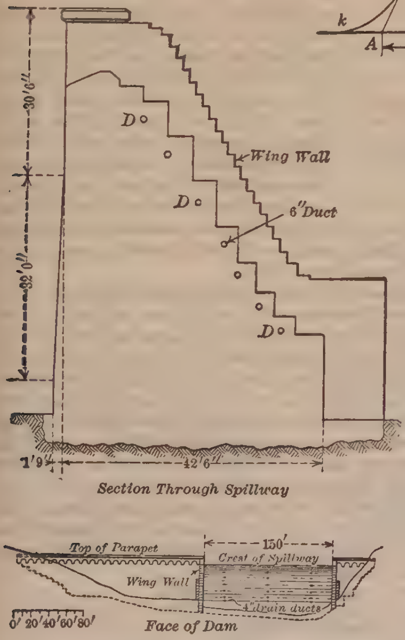


Fig. 97. Pedlar River Dam

The Siphon Spillway, a simple form of which is shown in Fig. 98, is adapted to cases where close regulation of reservoir level is desirable. The common overflow spillway requires considerable rise of water surface above crest level before the discharge reaches any large amount unless the length of spillway is very great. The only way in which the discharge can be increased, without lengthening the spillway, is by increasing the head on the spillway. This may be accomplished without raising the pond level by using flashboards or crest gates, or by constructing a conduit within the dam with its discharge end at an elevation lower than maximum pond level.

Flashboards may be used to limit backwater rise during floods, but do not give a close regulation of pond levels because the water must rise considerably above top of flashboards before the boards will go out and the boards cannot be replaced until the flood has subsided. Ordinary crest gates may be used to give close regulation of pond levels, even during floods, but they are not automatic like flashboards, and involve considerable expense for operation. Automatic crest gates may be used, but they are expensive, and their operation may be uncertain due to complicated machinery, especially if they are designed to maintain close regulation of pond level.

The siphon spillway is automatic and involves no operating cost and it permits close regulation of the pond level. While more expensive than an overflow spillway with flashboards, it should compare in cost with crest gates, for the same discharge capacity. It may be used in conjunction with an overflow spillway, the siphon serving to regulate the pond level for ordinary rates of flow, while floods are discharged in part over the main spillway. As the water entrance is below freshet water level, this type of spillway is useful where it would be undesirable to discharge débris or ice below the dam. On the other hand, if close regulation of pond level is not essential, the siphon spillway may not be desirable, for it prevents storage of water above ordinary pond level, as is possible with flashboards, and it may involve considerable loss of water in case of a slight rise of pond level.

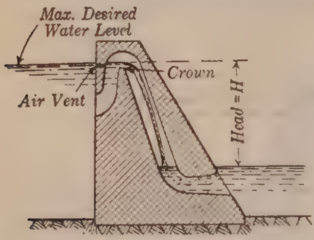


Fig. 98

The essential feature of a siphon spillway is that it takes advantage of a considerable portion of the head between pond water and tailwater level. The siphon consists of a conduit constructed within the dam, as shown in Fig. 98, the upper entrance being below the desired maximum water level in the reservoir. The lower end of the conduit may discharge into the air or under a pool of water at the downstream side of the dam. The discharge end is placed lower than maximum water surface of the reservoir, but this difference should not be more than the atmospheric head unless the cross-sectional area of the conduit is reduced toward the lower end. For efficient operation the siphon at all points should lie below the hydraulic gradient (Sect. 9, Art. 8). The crown of the siphon is placed somewhat below maximum desired water level. An air vent must be provided from the top of the siphon to the upstream face of the dam, its upstream entrance lying between maximum desired water level and the level of the siphon crown.

The discharge of the siphon in cubic feet per second is:

$$Q = C A \sqrt{2 gH}$$

where C is a coefficient depending on the detail of the conduit and the curves of entrance and exit and varying between 0.6 and 0.8, A is minimum cross-sectional area in square feet, H is head in feet and g = acceleration of gravity. As the velocity of flow varies with the square root of the head, the discharge does not change much for considerable variations in the head. If the total head from the highest point of the siphon to the outlet minus frictional losses is greater than atmospheric head, the lower end of the conduit must be reduced in area in order to avoid breaking the water column. The capacity of the siphon, however, will not be increased by making the gross head greater than atmospheric head.

In operation, as the water in the reservoir rises above the crown of the siphon, the air vent is cut off and water begins to flow through the conduit. The air trapped in the siphon is exhausted by the flowing water and siphonic action is quickly started. When the pond level starts to fall again, the air vent is exposed and admission of air to the conduit breaks the siphonic action. The water will then merely flow over the crown of the siphon until the pond drops below this level. If an air valve is placed at the top of the siphon, the siphonic action can be stopped or prevented by opening the valve.

Where several siphons are used, their crowns may be placed at different elevations, so as to cause the siphons to come into action successively as the pond level rises, thereby preventing all the siphons from acting under a slight rise of pond level. At the Conklingville Dam of the Hudson River Regulating District, siphon spillways are provided in order to prevent breaking up the ice-sheet on the reservoir and canal due to a sudden rise of water level above the overflow spillway. In some cases, the siphons have been constructed at a convenient location near the dam, but separately from the dam itself. Siphons have also been used in place of sluice gates in canal locks. (See paper on "Siphon Spillways" by G. F. Stickney, Trans. Am. Soc. C. E., Vol. 85, p. 1098; also Eng. Rec., Vol. 67, p. 488.)

A Shaft Spillway consists of a vertical shaft usually built in rock connected with a tunnel for the purpose of discharging water from the upper to lower ponds in lieu of the normal spillway. It may be economically used in connection with an earth-fill or rock-fill dam and where the cost of a separate spillway built adjacent to the earth-fill or rock-fill dam would be prohibitively expensive, or in a narrow and deep gorge where the handling of freshet water during and after construction cannot be satisfactorily worked out without it.

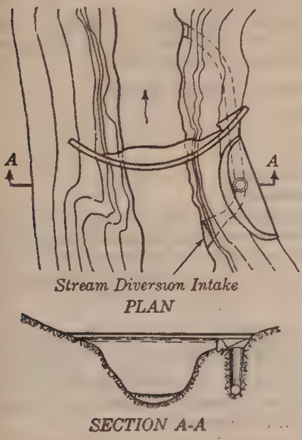


Fig. 99

The writer has had occasion to design two such spillways where the shaft type was necessary because of the narrowness and depth of the gorge, the available width being hardly more than was required for the power house, and this type affording the further advantage of using the tunnel portion of the spillway during construction. Fig. 99 indicates a shaft spillway and tunnel planned for an arch dam in a narrow gorge. A somewhat similar spillway was built at an earth-fill dam on the North Platte River, Wyoming. (Eng. News-Rec., Vol. 100, p. 264.)

A shaft spillway was used at Davis Bridge Dam (see article by Ford Kurtz, Trans. Am. Soc. C. E., Vol. 88, p. 1). This type of spillway was built in connection with an earth-fill dam, the tunnel being used during the construction period and the shaft in connection with the tunnel after completion. The article referred to goes carefully into the hydraulics of the design, and is followed by most interesting criticism.

Special care is required in the design to insure that the actual discharge capacity of the shaft for a given head will be equal to that assumed, or the reservoir level may rise to dangerous height in case of flood. The mouth of the shaft must be protected against inflow of logs and ice in order to avoid damage to the lining of the shaft and tunnel, and if high velocities are used in the shaft and tunnel the lining must be given careful consideration. Where large trees may be carried by the stream as a result of freshet in storm, care must be taken to protect the tunnel against stoppage in whole or part.

33. Arched Dams

Single Arch Dams. When a masonry dam is to be built in a gorge with firm rock sides it may be economical to curve it in plan so that it will act as an arch, although the masonry near the rock foundation will act partly as a cantilever beam. Even in a high, straight gravity dam in a narrow gorge, it is obvious that arch action must take place to some degree, although this may reduce to a wedging effect in extreme cases. Consideration of arch action will permit the use of less masonry than in the gravity type. This saving will be increased where the costs of materials and labor are high as in the case of dams built at inaccessible places.

Although there are obvious economies in the volume of masonry required in an arch dam, there are certain objections:

1. The internal stresses are more indeterminate than for a gravity dam, and therefore low unit stresses must be used in the design.
2. In the upper portions where the masonry is thin, greater care must be taken to build a watertight structure, seeping through the masonry being detrimental on account of leeching out of soluble material and injury by freezing.
3. The rock foundation, being subject to high pressures, must be of good quality.

An arched dam may be considered as consisting of two systems of structural elements:

1. Horizontal arches cut by parallel horizontal planes.
2. Vertical elements contained between radial planes which transfer part of the water load directly to the foundation through beam action. If these elements are fixed to the foundation they will act as cantilevers.

To determine the approximate thickness, use the "cylinder formula,"

$$T = PR/s,$$

where T = thickness of the arch ring in feet;
R = radius of upstream face in feet;
P = water pressure in pounds per square foot;
s = allowable stress in pounds per square foot.

It will be noted from this formula that the thickness increases with the height, that is, with the water pressure, and decreases with the assumed allowable stress. It may be proven theoretically that the volume of concrete will be a minimum when R is such that the subtended angle between the abutments is about 133 deg. This principle forms the basis of design of the "constant angle" form of dam. (Trans. Am. Soc. C. E., Vol. 78, p. 685).

A few years ago all arched dams were designed by the cylinder formula, this being possible because of the low unit stresses assumed. In designing high dams, however, and for economy of material, a more accurate method of computation is desirable. In such designs the influence of cantilever action is considered. The problem then is divided into two parts:

1. Division of the water load between the arches and the cantilevers.
2. Computation of stresses in the arches and in the cantilevers.

The problem is complicated by the fact that, although the arches may be symmetrical, the cantilevers are not all the same length. As the subdivision of the load between arches and cantilevers depends upon relative stiffness, in any given horizontal arch the portion of the water load carried by the arch will be much greater in the middle than near the abutments. This factor, however, is disregarded in most methods of analysis.

The methods that have been recently developed for analyzing arch dams are summarized below, the details being omitted as it is evident that more correct methods of analysis are in the process of early development with the help of models.

1. Method of F. A. Noetzli (Trans. Am. Soc. C. E., Vol. 84, p. 1). The distribution of water load is computed at the center of the dam (crown of the arches) and is assumed to be the same throughout the length of the dam. The relative amount of load carried by the arches is fixed by the condition that the deflection of any arch at the crown must be equal to the deflection of the cantilever at the same elevation. A trial division of the load is assumed and the deflections computed, the load division being adjusted until the deflections of the arches and cantilever are approximately the same. Arch deflections are computed approximately for temperature and load, the arch being assumed to have hinged ends. Noetzli's method neglects the following:

(a) Influence of shear.

(b) Influence of Poisson's ratio (the fact that an elastic material that is under stress in one direction will develop strains in a plane at right angles to that direction).

(c) Unsymmetric profile of dam.

(d) Temperature and shrinkage cracks.

2. Method of B. H. Smith (Trans. Am. Soc. C. E., Vol. 83, p. 2027). This method is a mathematical analysis of

(a) A cylindrical dam of uniform thickness.

(b) A cylindrical dam with a trapezoidal cross-section, the thickness therefore varying uniformly from crest to base. This method involves most of the approximations of Noetzli's method, but is possibly more precise mathematically, though its application is complicated.

3. Trial Load Method. Described by C. H. Howell and A. C. Jaquith (Trans. Am. Soc. C. E., Vol. 93, p. 1191). This method is similar to Noetzli's but takes into consideration the variable heights of the cantilevers and the unsymmetric dam profile. The arch deflections are computed by the elastic theory, taking into consideration temperature and rib shortening. This method seems more laborious than Noetzli's but appears to be somewhat more accurate.

In all the foregoing methods the computation of the arch stress is dependent on the assumptions as to distribution of water load over the arches, that is, the division of the water load into that carried by the masonry as an arch and as a cantilever. If the loading on any given arch is assumed to be uniform throughout its length (as in Noetzli's and Smith's methods) the computation of arch stresses may be simplified by the use of formulas that have been developed by Professor Wm. Cain (Trans. Am. Soc. C. E., Vol. 85, p. 233; Vol. 90, p. 522). These formulas have been plotted diagrammatically by F. H. Fowler (Trans. Am. Soc. C. E., Vol. 92, p. 1512). The Cain formulas assume horizontal circular arches of constant thickness, fixed or hinged, and subjected to uniform water pressure. These formulas include the influence of shear on arch stresses, and take into account that the neutral axis of a thick arch is closer to the intrados than the extrados. If the arch load is not assumed as uniform, the elastic method of arch analysis must be used. (See also Trans. Am. Soc. C. E., Vol. 93, p. 1191.)*

The theory of arch dams involves so many indeterminate factors and is so difficult of mathematical analysis that attempts have recently been made to check the available theoretical analysis or enable the development of better methods by means of experiments on models. Engineering Foundation made observations of an experimental arch dam at Stevenson Creek, California. This dam was 60 ft. high, 2 ft. thick at the crest, 7-1/2 ft. thick at base, and had a constant upstream radius of 100 ft. It was built in a narrow canyon. Facilities were provided for measuring deflections, strains and temperature changes.

* An excellent paper on "Load Distribution in High Arch Dams," by R. A. Sutherland (Trans. Am. Soc. C. E., Vol. 93, p. 1623), gives a summary of various methods of determining load distribution on arch dams, and develops formulas and diagrams to assist in computing loads and stresses.

The Stevenson Creek experiments have also been checked by tests upon a celluloid model made by Prof. G. E. Beggs. A model was constructed to a scale of 1 to 40 and loaded with mercury in order to magnify the liquid load. The results of the model experiments compared with those at Stevenson Creek are indicated in Figs. 100, 101 and 102.

The laboratory experiment furnishes an excellent check upon the field tests. It is thought that the reason for certain disagreements was the cracking of the concrete in the Stevenson dam, and that results from the celluloid model would check the Stevenson Creek Dam even more closely if the model were grooved deeply to simulate cracks in areas of high tensile strain. Both experiments indicate combined arch and cantilever action; and they show that an appreciable portion of the water load at the intermediate levels is transferred to the upper arches through the cantilevers, as indicated by the fact that the curvature of the cantilevers is reversed near their upper end. Fig. 100 shows the peculiar effect of an appreciable upstream deflection near the quarter points of the upper arches. The field and laboratory experiments indicate that the theoretical methods should give approximately correct values for stresses in the dam, provided proper assumptions are made as to the degree of fixity of the dam at the base and at the abutments. The report of the Committee on Arch Dam Investigation of Engineering Foundation is published in Proc. Am. Soc. C. E., May, 1928. Every engineer engaged on design of arched dams should study this report carefully.

It is believed that the use of laboratory models is worth while in the design of large arch dams and as a check on stresses computed by theoretical methods.

The writer, though convinced of the economy resulting from the use of arch dams in narrow rock gorges, thinks that the design should be conservative in the unit stresses used because of indeterminate stresses. Based on a 1 : 2 : 4 portland cement concrete made according to the best practice and with first-class aggregates, the maximum unit stress computed by the more accurate methods should not exceed 400 lb. per sq. in. The following stresses computed by the cylinder formula were used in design:

		Lb. per sq. in.	Trans. Am. Soc. C. E.
Barren Jack Dam, New South Wales.......	1907	264	Vol. 78, p. 603
Huacal Dam, Sonora, Mexico..............	1912	225	Vol. 78, p. 603
Goodman Dam, California................	1913	300	Vol. 78, p. 603
Lake Spaulding, California................	1913	330	Vol. 78, p. 716
Salmon Creek, Alaska....................	1914	330	Vol. 78, p. 708
Kerckhoff Dam, California................	1921	225	Vol. 84, p. 106

Multiple-Arch Dams. The multiple-arch dam will be found economical in many locations. This type requires less masonry than the gravity dam but involves more labor. If of concrete, the formwork is increased and the narrowness of the forms results in a higher cost of placing. Usually reinforcing steel is also required. Further, on account of the thinness of the masonry the dam must be built more carefully to insure strength and watertightness and is therefore more expensive per unit.

In approximate terms, the multiple-arch dam on a rock foundation will require from one-fourth to one-half as much concrete as a solid gravity dam and will cost from one-half to two-thirds as much as the gravity type. It is more advantageous where the cost of materials is high and the cost of skilled labor is relatively low. In a spillway dam the saving in material and cost is smaller, due to the necessity of providing a slab upon the downstream side of the buttresses for the overflow water.

This type of dam, if an inclined back is used, has the further advantage of making use of the weight of the water upon the inclined back to increase the

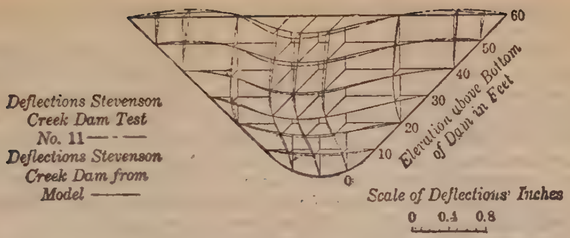


Fig. 100. Comparison of Deflections of Stevenson Creck Dam with Those Computed from Celluloid Model

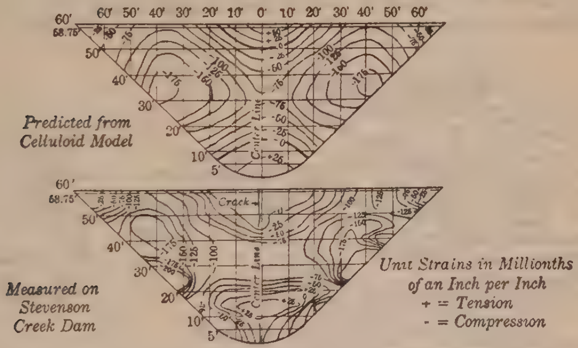


Fig. 101. Measured and Predicted Values for Equal Horizontal Strain, Down-stream Face, 60 ft. Head

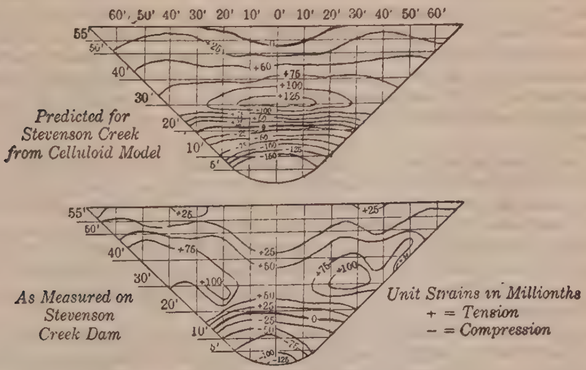


Fig. 102. Measured and Predicted Values for Equal Vertical Strain, Down-Stream Face, 60 ft. Head

stability against sliding and overturning and to obtain a lower pressure per square foot upon the foundations.

On a rock foundation the only advantage a multiple-arch dam usually offers is a saving in cost. On a soft foundation it may be the only feasible masonry type, the designer having to choose between a multiple-arch dam or an earth-filled dam due to the fact that a gravity section brings excessive pressures upon the soft foundation. Further, the wide base of the multiple-arch type increases the distance the water must travel under the dam and therefore the safety of the structure. With a multiple-arch dam, by flattening the slope of the back and by steepening the face of the buttress as in Fig. 103, the pressure upon the foundation may be distributed over a very large area.

When constructed upon a soft or porous foundation special provision must be made to prevent percolation of the water under the dam and the washing out of the sub-soil (for discussion of cutoff walls see Art. 30). A recent multiple-arch dam upon a soft foundation was constructed at Sherman Island in 1922, which is shown in Fig. 103. (Trans. Am. Soc. C. E. Vol. 88, p. 1257.)

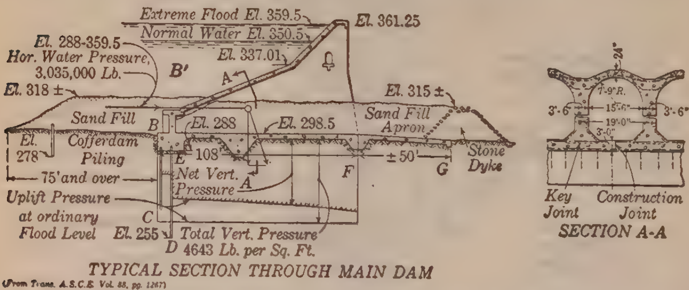


Fig. 103. Sherman Island Dam

The early designs of multiple-arch dams were constructed with buttresses which were braced together with tie struts to prevent buckling and to take the unbalanced arch thrust during construction. With this type of buttress the economical limit of height is about 125 ft. More recent designs use hollow buttresses or solid buttresses with counterforts which serve the same purpose as tie struts. With buttresses of this type it has been possible to construct economically multiple-arch dams, more than 200 ft. in height, on rock foundations.

On soft foundations, sand or gravel, the possible safe height of the dam is limited by the practicability of getting a water cutoff and also by the low supporting capacity of the foundation bed. Assuming the maximum safe depth of the steel sheet pile cutoff in sand or gravel at 75 ft., the maximum safe height of a multiple-arch dam is thought to be less than 100 ft. Even if a concrete caisson cutoff is extended to rock, the practical safe height is probably not greater than this due to the resulting high pressures upon the foundation bed.

In order that the resultant thrust of adjacent arches may lie parallel to and substantially coincident with the middle line of the buttresses, all arches should have the same span in any multiple-arch dam. If this is not possible the buttresses must be designed to take the unbalanced thrusts.

The economical arch span will depend somewhat upon the height, being about 25 ft. for a dam 30 ft. high, and increasing to 50 ft. for a dam 100 ft. high. This theoretically economical span, however, must be reduced for soft foundations in order to avoid excessive bending in the foundation slab between buttresses.

If the dam is built with an inclined back the arches should preferably be circular in any horizontal plane, resulting in an ellipse on a plane normal to the axis of the arch barrel. The horizontal radius of the upstream face should be constant throughout, and for a minimum of material the angle subtended between the abutments should be between 120 and 145 deg. The amount of material saved by this limitation of angle is, however, small unless the angle is smaller than 100 deg. or greater than 160 deg. The economical slope of water face will usually vary between 50 and 60 deg. with the horizontal, but with a soft foundation the slope should be flatter in order to spread the load over a greater foundation area. For convenience and economy in construction of inclined arches, it may be advisable to design the extrados and intrados on a section normal to the axis in the form of a three-centered curve approximating an ellipse. In this way the horizontal sections can be maintained approximately circular. The form work will be greatly simplified if the same intradosal curve is used throughout the height of the dam and if the increase in the thickness of the arch is attained by offsetting the interior or exterior forms at intervals.

After a preliminary design is made a check should be made of the arch stresses, considering the water pressure and the weight of the concrete. The arch should be analyzed in slices perpendicular to the axis, the arch being divided into voussoirs with lengths such that I/s is constant (see Art. 39). The weight of each voussoir should be resolved into two components, one parallel to the sloping face of the arch and one normal to the slope, that is, in a direction parallel to the plane of the abutment. The water load upon each voussoir should be added to the second component of the weight and the arch analyzed by the usual methods. For preliminary design the static method may be used (Art. 36). The final design should be checked by the elastic method (Art. 38) and the effect of temperature and rib shortening considered (Art. 40). It may be necessary in large arches to consider combined temperature and dead load with the reservoir empty. The arches should not be made unnecessarily heavy as this will cause high stresses from temperature changes when the reservoir is empty. If the computations indicate that tension will exist in the concrete, reinforcing steel must be introduced in the design and construction.

The upper portions of the arches should not be made very thin, because the difficulty of placing the concrete, especially if reinforcing steel is used, would be increased and the watertightness jeopardized. If the arches are vertical the thickness should be further increased where ice pressure is expected.

The buttresses should be analyzed as gravity walls, considering the computed reactions of the arches and the weight of the buttress and bracing. If, however, the arches are vertical the weight of the arch concrete cannot be assumed to be of value against overturning unless there is sufficient shear value to transmit the concrete arch weight to the buttress. The method of analyzing the buttresses will be similar to that given in Art. 31 for masonry dams. If the dam is high it may be necessary to brace the buttresses by struts or counterforts or by using the hollow reinforced type. (Trans. Am. Soc. C. E. Vol. 87, p. 342.)

If the buttresses rest directly upon the foundations (rock or hardpan) it will be unnecessary to consider hydraulic uplift; if the buttresses rest on spread footings (soft ground) or a continuous foundation slab, uplift must be considered.

A multiple-arch dam with a steep face is likely to have a small factor of safety against sliding upon the foundations, requiring special provision against sliding by roughening the foundation or providing keyways (see Fig. 103, showing Sherman Island Dam).

In the higher dams of reasonably thick cross-section, considerable saving in cost may be obtained by the use of large-sized aggregate; but watertightness is jeopardized unless the arch cross-section is relatively large.

It is of special importance that the foundation of a multiple-arch dam upon soft ground shall be uniform, to guard against cracks and leakage of water. In the case of Sherman Island dam 90% of the buttresses rest on a concrete slab built upon sand; the buttresses at either end, however, rest upon rock. In order to prevent non-uniform settlement, it was necessary on one side of the river to excavate the rock and cover it with 2 ft. of sand upon which the buttress slab was constructed.

In constructing this type of dam it is probable that the arch action may be developed by the impounded water back of the dam before it is complete. This assumes that the dam will first be built for one-half or two-thirds the river width, the freshet water discharging in the remaining portion of the river. If the remaining width of the river is not adequate, water will be impounded against the back of the arches of the incompleted dam and therefore provision must be made to take this unbalanced thrust, either by having an anchor buttress at the end of the incomplete series of arches or by provision of tie struts. Such enlarged buttresses or tie struts are recommended in any large series of multiple arches to facilitate construction.

Among recent multiple-arch dams constructed on soft foundations may be mentioned:

Sherman Island Dam (Fig. 103), constructed on Hudson River in 1921–23. 80 ft. high, 590 ft. long; buttresses 19 ft. on centers, standing on concrete base 108 ft. by 3 ft. Upper portion of arch barrel 18 in., lower portion 24 in. thick. Built on sand, with steel sheet-pile cutoff. Interior of bays filled with sand to increase resistance to sliding. Apron slab extended 50 ft. downstream to increase length of water travel under dam; sand fill on upstream side for same purpose. (See also Art. 30, p. 937, Dams Built on Sand.)

Cave Creek Dam, near Phoenix, Arizona (1922). 60 ft. high above river bed, 120 ft. to foundation of cemented gravel. Buttresses 44 ft. on centers. Total length 1692 ft. Upstream face vertical at top and curved to 56 deg. slope 80 ft. below crest; below this point, arches are vertical. (Trans. Am. Soc. C. E., Vol. 87, p. 402).

The following are some of the high multiple-arch dams constructed in recent years:

Palmdale Dam, Little Rock Creek, California (1923–24). Maximum height 175 ft.; 690 ft. long. Buttresses 24 ft. on centers, braced with tie struts and counterforts. Arch slope 45 deg., top 10 ft. vertical.

Lake Pleasant Dam, Arizona (1927). Maximum height 256 ft., 170 ft. above river bed; 1850 ft. long. Double-wall buttresses, 16 ft. out to out, with cross walls. Buttresses 60 ft. on centers; no tie struts. Arch slope 47 deg. (Eng. News-Rec., Vol. 100, p. 180; Vol. 102, p. 275.)

Sutherland Dam, Santa Ysabel Creek, near San Diego (started Feb., 1927). Maximum height 180 ft.; length 780 ft. Buttresses 60 ft. on centers, 3 ft. thick at crest, and 10 ft. thick at 180-ft. level.

Tirso Dam, Sardinia (1919–23). Maximum height 239 ft.; length 930 ft. Buttresses of cut-stone masonry, 49.2 ft. on centers, 8.2 ft. thick at top, 23.8 ft. thick at 200-ft. level; with tie struts. Arches of concrete. Arch slope 57 deg. (Trans. Am. Soc. C. E., Vol. 87, p. 397.)

Pavana Dam, Italy (1924–25). Combination of multiple-arch and hollow gravity types. Maximum height of multiple-arch section 177 ft.; spans 54.1 ft. Arch slope 60 deg.

Tidone Dam, Italy (1924–25). Maximum height 171 ft.; length 900 ft. Buttresses 32.8 ft. on centers, 2.3 ft. thick at top, 7.2 ft. at base. Arch slope 45 deg.

Gleno Dam, Italy. As built, about 160 ft. high. This dam failed, but the failure was due to improper construction and not to its design as a multiple-arch structure. This is the only recorded instance of the failure of an arch dam.

The Flat-Slab Reinforced-Concrete Dam is closely related to the multiple-arch dam. This type is known as the "Ambursen Dam." It is a hollow structure, with buttresses similar to those of the multiple-arch dam, but the upstream face is built of flat reinforced-concrete slabs instead of masonry arches. The action of the structure as a whole is similar to that of the multiple-arch dam. Therefore all the statements under the heading Multiple-Arch Dams, except those referring to the arches, will apply to the Reinforced-Concrete, or Ambursen Dam. This is true as to economical location, stability and safety, precautions to be taken with soft or porous foundations, slope of water face, design of buttresses, safe height, uplift, safety against sliding, and importance of uniform foundation.

The Flat-Slab Dam has certain **advantages** over the Multiple-Arch Dam: (1) It is less subject to injury due to slight inequalities in settlement of the buttresses, on soft foundations. (2) The formwork is less expensive. (3) There are no unbalanced arch thrusts on the buttresses during construction. The Flat-Slab Dam has the **disadvantage** that the economic span (distance between buttresses) is less than for a multiple-arch dam. This spacing is generally between 15 and 20 ft. Expansion and contraction under temperature variations are arranged in the flat-slab dam by providing a sliding joint on both sides of each deck slab. Expansion and contraction of the multiple-arch dam are taken up by the flexibility of the arches, which must be adequately reinforced for the resulting stresses.

The details of design will be similar to those of other reinforced concrete structures (Sect. 11).

MASONRY ARCHES AND PIERS

34. General Data for Arches

Definitions. Fig. 104 shows most of the technical terms relating to the construction of stone arches. The **span** is the horizontal distance between abutments. The **soffit** is the under or concave surface of an arch. The **back** is the upper or convex surface of an arch, marked *BBB* in figure. The **rise** is the vertical distance between the lowest and highest points of the soffit. The **crown** is the highest part of the arch ring. The **skewback** is the inclined surface or joint upon which the end of an arch rests. The **springing line** is the inner edge of the skewback. The **axis** is an imaginary horizontal line, parallel to the abutments, passing through the middle point of a line joining the springing lines; in an arch built to withstand horizontal pressure only, as for a shaft, the axis is vertical. The **intrados** is the line of intersection of the soffit with a vertical plane perpendicular to the axis. The **extrados** is the line of intersection of the back with a vertical plane perpendicular to the axis. The **arch ring** is the entire arch included between the skewbacks, the soffit and the back. The **haunch** is the portion of the arch ring between the springing line and the crown; **haunching** or **backing** is masonry, with bed joints nearly horizontal, which is placed above the haunch. The **spandrel**

is the space between the extrados and the roadway, marked *SSS* in figure. **Spandrel filling** is earth deposited between the back and roadway. A **voussoir** is one of the wedge-shaped blocks of stone or concrete of which the arch is composed; in design the voussoir is regarded as having a depth equal to the depth of the arch ring, but in construction the voussoir may be of less depth; the **keystone** is the highest voussoir. **Ring stones** are voussoirs which show at the ends or faces of the arch ring. **Arch sheeting** comprises all voussoirs ex-

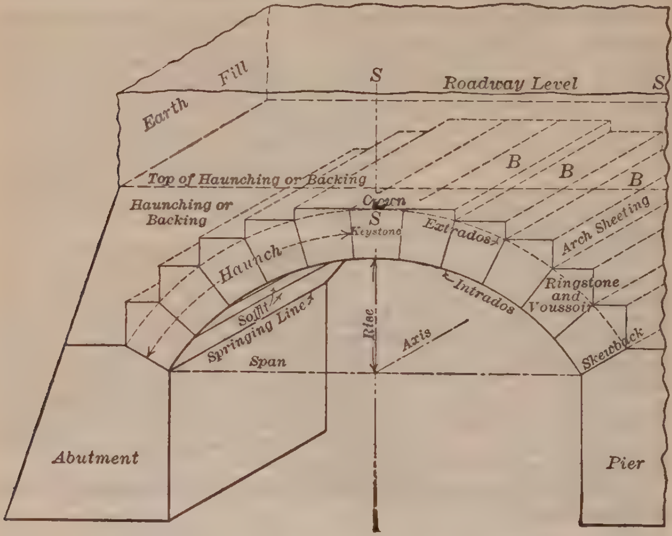


Fig. 104. Technical Terms for Masonry Arches

cept the ring stones; in concrete arches, which are not faced with stone voussoirs, the term arch sheeting includes the entire arch ring.

Kind of Arches. **Hinged arches** are those with stone, steel or lead hinges at crown and springing line; hinged masonry arches are always built with three hinges. **Fixed arches** are those constructed without hinges. A **full-centered arch** is one whose intrados is a semicircle. A **segmental arch** is one whose intrados is less than a semicircle. A **pointed arch** is one in which the intrados

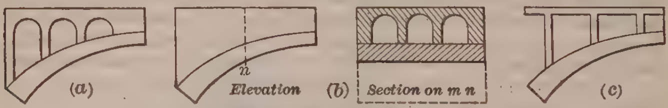


Fig. 105. Spandrel Arches and Columns

consists of two arcs of radii greater than the half-span, these arcs intersecting at the crown. A **right arch** is one terminated by vertical planes perpendicular to its axis. A **skew arch** is one terminated by vertical planes oblique to its axis. **Spandrel arches** are those which rest upon the back of the larger or main arch; they are called transverse when their axes are parallel to that of the main arch (Fig. 105*a*) and longitudinal when their axes are at right angles to that of the main arch (Fig. 105*b*). **Spandrel columns** are shown in Fig. 105*c*.

Loads. Dead load includes the weight of the masonry itself, the weight of the spandrel filling and that of the roadway. The live load to be used in investigating a masonry arch is the greatest that comes or is liable to come upon its roadway, while for design that given in the specifications is to be employed. Since the spandrel filling distributes the load to a large extent, uniform live loads are often used, about 150 lb. per sq. ft. for heavy highway traffic and about 700 lb. per. sq. ft. for railroad traffic for spans over 80 ft., and 1000 lb. per sq. ft. for shorter spans. Culverts and small arches are designed for a uniform live load over the entire span. Larger arches are to be analyzed not only for full uniform load but also for a live load extending over certain portions.

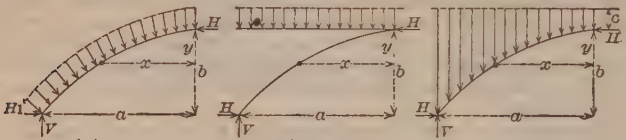
Earth-filled arches should be designed for uniform live load over the span and half-span. Arches with the roadway supported on spandrel arches or spandrel columns should be designed for uniform live load over the span, half-span, middle-third of the span, and the outer thirds of the span. Narrow highway arches with spans under 100 ft., and all railway arches should be designed for concentrated loads. Impact of live loads need not be generally considered except for short span arches with arched or column spandrels. In northern latitudes a snow load of 25 lb. per sq. ft. over the entire span should be considered for a highway bridge if a uniform live load of less than 100 lb. per sq. ft. is provided for. Arches for aqueducts must be figured for the live load of the water. Wind loads are rarely taken into account for the arch, but may be needed for a pier.

In computations an arch one unit in width parallel to the axis is considered. The total live load over the portion considered being divided by the width of arch gives the weight per unit of width, and this may be reduced, for convenience of representation on the drawings, to a rectangle whose height is that of an equivalent weight of masonry.

For an Arch with Horizontal Axis under water pressure, the loading at any point of the extrados depends on the depth of that point below the water surface, and its direction is normal to a tangent to the extrados at that point. If the arch is circular, bending moments as well as thrusts are developed in the arch. For symmetrical arches, the loading (both dead and live) will be symmetrical. The arch may be analyzed by the usual methods, taking into consideration both the vertical and the horizontal components of the water pressure.

A Horizontal Thrust H (Fig. 109, Art. 35) is produced at the crown when the arch is loaded symmetrically. For non-symmetrical loading an inclined pressure P acts at the crown, and its horizontal component H is also called the horizontal thrust for that loading. The semi-arch is held in equilibrium by the system of forces consisting of the horizontal crown thrust, the vertical loads and the reaction of the skewback. For any joint 2-2 the part of the arch on the right is held in equilibrium by the horizontal thrust H, the vertical loads W_1 and W_2 and the reaction of the arch below the joint. This reaction is equal and opposite to the resultant of H and the loads W_1 and W_2, and the point r_2 where this resultant cuts the joint is a point in the resistance line, the definition of this term being the same as in Art. 23, except that in the arch the joints are not horizontal. The resistance line itself is a curve joining the points $o\ r_1r_2r_3r_4r_5$.

A Linear Arch is a curve which is in equilibrium under the action of a given system of loads; it is an equilibrium polygon with an infinite number of sides. In an actual arch the resistance line is a linear arch for the actual loading. Each kind of loading has a linear arch which corresponds to it and which holds that load in equilibrium. The three simplest cases are: (1) The linear arch for water pressure is a circle if the axis of the arch is vertical or if the depth of water over the crown is very large compared to the rise of arch (Fig. 106*a*). (2) The linear arch for uniform horizontal load is the common parabola (Fig. 106*b*). (3) The linear arch for a load of uniform density between a horizontal roadway and the curve itself is the transformed catenary (Fig. 106*c*). Let a be the half-span and b the rise of a linear arch. Let x and y be abscissa and ordinate of any point in the curve with respect to the crown as an origin. Let



(*a*) *Circle* (*b*) *Parabola* (*c*) *Trans. Catenary*

Fig. 106. Loadings for Linear Arches

c be the depth of the load at the crown, and $n = (b + c)/c$. Then the equations of these linear arches are:

for circle, $y = r - \sqrt{r^2 - x^2}$, where $r = 1/2\, b + a^2/2\, b$.
for parabola, $y = bx^2/a^2$.
for transformed catenary, $y = c\,(\cosh\,(\beta x/a) - 1)$

in which β is the Napierian logarithm of $n + \sqrt{n^2 - 1}$. Let w = load per linear unit at the crown. Then the horizontal thrust at the crown and the vertical reaction at the skewback are:

for circle, $H_1 = wr - wb$ $H = wr$ and $V = wa$
for parabola, $H = wa^2/2\,b$ and $V = wa$
for transformed catenary, $H = wa^2/c\beta^2$ and $V = (wa/\beta)\sinh\beta$

The resultant thrust at the skewback is the square root of the sum of the squares of H and V. (For tables of Napierian logarithms and hyperbolic functions see Sect. 1.)

Curves for Arches. The central curve of an arch, that is, the curve lying half-way between intrados and extrados, may be the circle, the parabola, the transformed catenary, the ellipse, or various combinations of these. In Fig. 107 are shown the first three curves for four different ratios of rise to span, these having been plotted from the above equations. The transformed catenary curves are for a height of roadway above the crown equal to 1/20 of the span, or $c = 1/10\,a$. When the ratio of rise to span is 1/10 or less, the three curves are practically the same. **Economic curves,** or those giving a minimum amount of masonry for various loadings, have been determined to be as follows: (1) A circle should be used for water or other fluid material under a head which is large compared with the span. (2) A parabola should be used for a uniform load on the horizontal projection, which is closely the case for an arch having spandrel columns or spandrel arches. (3) A transformed catenary should be used for vertical loads like those of Fig. 106*c* or for an earth-filled arch having a ratio of rise to span less than 1/4. (4) An ellipse should be used for an earth-filled arch having a ratio of rise to span greater than 1/4.

The transformed catenary is difficult to lay out and is unsightly, but an approximate catenarian curve consisting of a compound curve made up of circles of different radii may be used. The parabola may be modified for the sake of appearance by short circular curves at its ends, made tangent to the parabola and to the vertical side of the pier.

When arches are built of cut-stone voussoirs, the cost of cutting each stone to a different curve more than offsets the saving in material resulting from using the theoretically correct curves. Many designers for this reason prefer the segmental arch, and for flat arches use the same depth of voussoirs from the crown to the springing line. The economic curves may be used advantageously for concrete arches, because falsework may be laid out as easily for a parabolic, elliptical or multi-centered arch as for a segmental arch.

When the maximum waterway for the given span and height of crown is desired the ellipse must be used. The flatter the ellipse the greater the area of waterway. The flatter the arch, however, the greater the thrust and the larger and more expensive the requisite abutments. For economic abutment design the ratio of the rise of the arch to its span should be as large as possible.

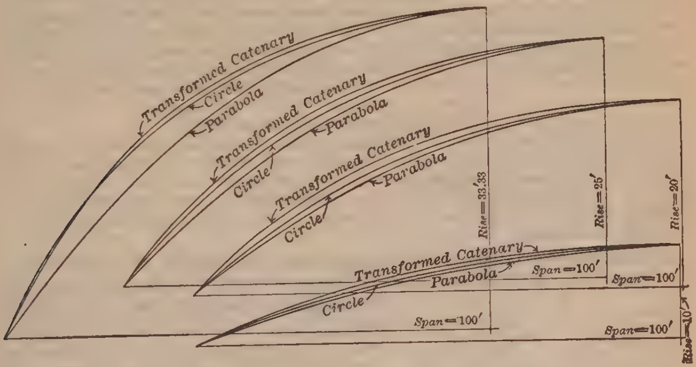


Fig. 107. Comparison of Curves for Arches

The Span and Rise are, as a rule, determined by physical conditions. A creek or river must be spanned and ample waterway provided. Roadways must pass under the bridge in fixed positions. Railway rights-of-way must be left unobstructed. Piers and abutments must be located, if possible, where the cost of foundation is a minimum. Where the cost of foundation is uniform from end to end of bridge, a series of arches should for economy be of equal spans. When unequal spans are mandatory, the spans should be made as nearly equal as possible. The rise of the soffit of the arch is always governed by the necessary clearance under the arches or the necessary level of the roadway or by both.

The length of span and the curve of an arch are often determined by the necessities of river travel. The position of the piers should be made parallel to the thread of the current, and this condition may necessitate a skew arch. Passage of vessels may require maximum clearance over certain portions of the river, or a maximum waterway may be necessary on account of freshets or ice gorges.

Appearance is sometimes the governing factor in selecting the curve of an arch. The semicircular arch best satisfies this demand, and next in sequence are the segmental, modified parabolic and basket-handle arches. A series of arches should, for appearance, always be of approximately equal spans, unless the profile of the ground demands unequal spans; if of unequal spans, a symmetrical arrangement of spans is desirable. The number of spans should, for appearance, always be odd unless the number exceeds seven, in which case it may be odd or even. For the most satisfactory appearance, the spans should increase slightly in length from the banks toward the center of the bridge, and the profile of the roadway should be cambered. If the spans are all of the same length, the middle span will appear shorter, due to an optical illusion; and if the profile is straight, the roadway will appear to sag at the middle of the bridge. If the spans are unequal, the piers must be large enough to withstand the unbalanced arch thrusts.

Economic Types. If the arches are of low rise or of high rise and under 100-ft. span, the earth-fill bridge is generally the cheapest type. However, if the foundations are poor, regardless of span, the three-hinged ribbed arch (Fig. 110) is the safer one and therefore the better type to use. If the foundations are neither poor nor first-class, the arches should be heavily reinforced with steel, to provide better for possible deformation of the arches due to settlement, however small.

If the arches are of high rise and spans over 100 ft., the arch with arched or column spandrels is generally cheaper. This is due to the great cost of the retaining walls of earth-filled bridges. However, for very wide bridges where the cost of the retaining walls per foot width of bridge is small, comparative designs should be made before selecting the type of design. The three-hinged ribbed type of bridge is generally expensive, due to the cost of the hinges and the added cost of two-faced masonry, or, if concrete, due to the added cost of formwork. The column spandrel is used only for reinforced concrete, but it is cheaper than the arched spandrel.

Masonry arches may be built of cut stone, or plain or reinforced concrete. Concrete is generally more economical if suitable materials are available, as it does not require skilled labor and permits a lighter structure, especially if reinforced. A stone arch, however, is more permanent, as it is less subject to attack by weathering, and the workmanship is liable to be better. A stone arch is generally a more beautiful structure, but more costly.

Dimensions for Designs. The approximate thickness at the crown of an arch built of masonry, with portland cement mortar, is given in the table below in feet, l being the span of the arch in feet. The thickness at the springing line may be approximated by the following empirical rules in which the percentages are to be added to the crown thickness found from the table: (1) Add 50% for circular, parabolic and catenarian arches having a ratio of rise to span less than 1/4. (2) Add 100% for circular, parabolic, catenarian and three-centered arches having a ratio of rise to span greater than 1/4. (3) Add 150% for elliptical, five-centered, and seven-centered arches. These thicknesses should be measured along radial joint as in Fig. 108, namely at *aa* for cases (1) and (2) and at *bb* for case (3).

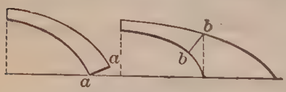


Fig. 108

Thickness in Feet at Crown for Highway Arches (Original)

Kind of masonry	Span in feet			
	Under 20 *	20 to 50 *	50 to 150 †	Over 150 ‡
First-class ashlar....	$0.04\,(6+l)$	$0.020\,(30+l)$	$0.00012\,(11\,000+l^2)$	$0.018\,(75+l)$
Second-class ashlar or brick...........	$0.06\,(6+l)$	$0.025\,(30+l)$	$0.00016\,(11\,000+l^2)$	$0.025\,(75+l)$
Plain concrete......	$0.04\,(6+l)$	$0.020\,(30+l)$	$0.00014\,(11\,000+l^2)$	$0.020\,(75+l)$
Reinforced concrete..	$0.03\,(6+l)$	$0.015\,(30+l)$	$0.00010\,(11\,000+l^2)$	$0.016\,(75+l)$

* For culverts under a high fill, add 60%; for railroad arches, add 25%. † For railroad arches add 20%. ‡ For railroad arches, add 15%.

The Top Thickness of a Pier capable of supporting adjacent arches of equal span may be taken as 3-1/2 times the crown thickness. In a series of arches of more than five spans, abutment piers capable of resisting the entire thrust of one adjacent arch should be introduced at every third or fifth span, so that in case of failure of the foundations of one arch the entire series would not fail. The top width of abutment piers may be assumed at five times the crown thickness.

Cost of an Arch Bridge in 1928 was approximately given in dollars as follows:

For a bridge of concrete, plain or reinforced, Cost $= 2.0\, bl \sqrt{d}$.

For a bridge of ashlar, Cost $= 3.5\, bl \sqrt{d}$,

in which b = the width in feet of the bridge, l = its length in feet, and d = the average depth in feet of the bed of the stream below the roadway level. These formulas cannot be used when foundation work is expensive.

Before laying out the spans and the curves of the arches, a contour map and profile of the site of the bridge should be made. On the profile the depth and character of the foundation and its overlying material should be indicated. The position of the piers should be located with reference to economic foundation, clearance of waterway, roadway and right of way; the spans to be made equal unless impossible or, on account of the cost of foundation, manifestly uneconomical to do so. The most economical type of bridge should then be selected and also the curves of the arches requiring a minimum amount of material, due consideration being given, however, to cost of labor of cutting stones, clearances and appearance.

Long Span Stone Masonry Arches

Name	Location	Span, ft.	Rise, ft.	Crown thickness, ft.	Date of completion
Bellefield	Pittsburgh, Pa.	150.0	36.7	4.0	1900
Main St.	Wheeling, W. Va.	159.0	28.3	4.5	1892
Grosvenor	Chester, England	200.0	42.0	4.0	1834
Gour Noir	Near Uzerche, France	203.4	52.8	5.6	1889
Krummenau	Krummenau, Switzerland	207.5	45.4	5.9	1911
Jaremcze	Jaremcze, Poland	213.3	58.7	6.9	1894
Cabin John	Washington, D. C.	220.0	57.3	4.2	1864
Sidi Rached	Algiers	225.6	82.0	4.9	1912
Morbegno *	Near Morbegno, Italy	229.7	32.8	4.9	1903
Steyrling	Near Steyrling, Austria	229.7	51.5	6.6	1905
Trezzo †	Near Trezzo, Italy	236.2	69.2	6.7	1377
Montanges	France	263.4	67.2	4.9	1909
Luxemburg	Luxemburg	277.7	101.7	4.7	1903
Salcano	Near Salcano, Austria	278.9	71.5	6.9	1906
Plauen	Plauen, Saxony	295.3	59.1	4.9	1905

* Three-hinged during construction; afterwards fixed.
† Destroyed 1416.

Long Span Plain Concrete Arches

Name	Location	Span, ft.	Rise, ft.	Crown thickness, ft.	Date of completion
Connecticut Ave.	Washington, D. C.	150.0	75.0	5.0	1907
Almandares *	Havana, Cuba	190.9	32.5	5.5	....
Neckarhausen †	Neckarhausen, Germany	194.9	41.3	2.8	1900
Walnut Lane	Philadelphia, Pa.	233.0	70.3	5.5	1908
Rocky River	Cleveland, Ohio	280.0	80.8	6.0	1910
Monroe St.	Spokane, Wash.	281.0	113.7	6.8	1911
Villeneuve	Villeneuve, France	315.8	50.7	4.8	1916

* Built on pile foundation.
† Three-hinged arch.

Long Span Reinforced-Concrete Arches

Fixed Arches

Name of bridge	Location	Span, ft.	Rise, ft.	Crown thickness, ft.	Date of completion	Reference
Washington Memorial *	Wilmington, Del.	250.0	40.0	6.0	1923	Eng. N.-R., 91 : 96.
Monongahela	Fairmont, W. Va.	250.0	52.0	5.0	1921	Eng. N.-R., 87 : 472.
Beechwood Ave.	Pittsburgh, Pa.	267.4	57.0†	6.5	1923	Eng. N.-R., 89 : 655.
Monroe Street	Spokane, Wash.	281.0	113.7	6.75	1911	Eng. News, 62 : 241; 65 : 546.
Ft. Snelling–Mendota	Ft. Snelling, Minn.	283.3	80.0	....	1926	Eng. N.-R., 97 : 621; 98 : 514.
Intercity	St. Paul-Minneapolis, Minn.	300.0	87.4	5.0	1927	Eng. N.-R., 99 : 754.
Larimer Ave.	Pittsburgh, Pa.	300.4	67.0	7.5	1912	Eng. Rec., 66 : 698.
La Balme ‡	Yenne, France	312.0	29.5	2.0	1916	Cornell Civil Engr., 32 : 17.
Langweis Viaduct	Langweis, Switzerland	315.0	137.8	6.9	1914	Genie Civil, 64 : 287.
Risorgimento	Rome, Italy	328.0	32.8	0.66	1911	Genie Civil, 60 : 21.
Cappelen	Minneapolis, Minn.	400.0	88.0	8.0	1923	Eng. N.-R., 84 : 335; 90 : 148.
St. Pierre-du-Vauvray	France	432.4	82.0	8.2	1923	Eng. N.-R., 92 : 476.
Elorn Viaduct	Brest, France	590.6	108.0	....	in constr. 1927	Eng. N.-R., 92 : 860.

Arches with Temporary Hinges

Name of bridge	Location	Span, ft.	Rise, ft.	Crown thickness, ft.	Date of completion	Reference
Vermilion River §	Wakeman, Ohio	145.0	32.6	....	1909	Eng. News, Nov. 18, 1909.
Latah Creek ‖	Spokane, Wash.	150.0	....	3.25	1913	Eng. N.-R., 67 : 312.
Pasteur ‖	Lyons, France	216.0	23.5	4.25	1923	Genie Civ., 84 : 53.
Vesubie River ‖	Var, France	315.0	51.0	6.3	1922	Eng. N.-R., 91 : 917.

* Skew 30 deg. † Theoretical rise. ‡ Open spandrel superstructure integral with rib designed to assist arch rib in carrying loads.
§ Cast steel hinges with 1/4-in. lead plate, later encased in concrete. ‖ Hinges: flexible reinforced concrete.

Long Span Reinforced-Concrete Arches

Arches with Permanent Hinges

Name of bridge	Location	Span, ft.	Rise, ft.	Crown thickness, ft.	Date of completion	Reference	Hinge description
Alexandra Ave.	Portland, Ore.	150	35.0	1.7	1924	Eng. N.-R., 93 : 328	3-hinged for construction, 2-hinged for live load. Rods fitted tight against hinge castings.
Connecticut River	Springfield, Mass.	176	29.7	4.75	1922	Eng. N. R., 84 : 817; 88 : 514	3-hinged steel arch used as centering, made permanent 2-hinged before pouring concrete. Pins in steel hinge-plates.
Maudit River	Nantes, France	180	31.0	6.5	1923	Eng. N.-R., 91 : 917	Bowstring arch with horizontal tie; 2 hinged concrete; 1 rocker bearing, 1 roller bearing.
Tarn River	Montauban, France	184	30.6	1.34	1915	Genie Civ., 68 : 65	3 flexible reinforced-concrete hinges.
Salmon River	Pulaski, N. Y.	200	26.5	3.0	1923	Eng. N.-R., 92 : 320	Old wrought-iron 2-hinged arch used for reinforcement.
Candelier	Charleroi, Belgium	210	21.3	5.1	1922	Genie Civ., 83 : 514	2 reinforced-concrete rolling hinges.
Boutiron	Vichy, France	223	....	....	1912	Genie Civ., 78 : 50	3 hinges.
Veurdre	Vichy, France	238	17	....	1911	Genie Civ., 78 : 50	3 hinges.
Aar River	Olten, Switzerland	269	30.4	3.9	1914	Schweiz. Bauzeitung, 2–1–25	3 hinges of soft lead between steel plate and granite block.
Bagneux	Paris, France	286.5	49	7.8	1927	Eng. N.-R., 99 : 272	Bowstring arch with horizontal tie; 2 hinges.
Öre River	Sweden	295.6	95.1	4.26	1919	Handbuch für Eisenbetonbau, 3d Ed., V., 7, p. 588	3 hinges; structural steel and pin.
Grafton	Auckland, New Zealand	320	84	5.5	1910	Eng. N.-R., 62 : 12	3 hinges.

35. Equilibrium and Stability

Methods of Failure. A masonry arch may fail in the following ways. (1) By crushing of the masonry. (2) By sliding of one voussoir upon another. (3) By one voussoir or section of masonry overturning about an adjacent voussoir or section. (4) By shearing in a horizontal or vertical plane, this applying to solid concrete arches and not to voussoirs. (5) As a column when the ratio of the unsupported length of an arch to its least width is greater than 12. (6) From striking the centering before the mortar is hard or when the arch, though stable under the full load, is not stable under its weight alone. (7) By striking the centering or loading the arch during construction unsymmetrically. (8) By settlement of the foundations. (9) By sliding upon the foundations. (10) By overturning about any point in the pier or abutment. Methods (8) and (9) are the most common ways of failure. All methods of failure, however, must be guarded against in design.

Just before an arch fails the forces acting upon it are in equilibrium, but there is no stability. For any degree of stability, however, the forces acting on the arch are in equilibrium. The conditions of stability are, in general, the same as those explained in Art. 23 for walls and dams.

Conditions of Stability corresponding to the above methods of failure are as follows: (1) The unit compression must not be greater than the safe unit compressive stress given in Art. 16. (2) The angle between the resistance line at any joint and a normal to the joint should be less than the angle of friction of masonry upon masonry. (3) The resistance line should lie within the middle-third of the arch ring; in reinforced-concrete arches it may depart a small distance outside the middle-third, but there should be sufficient steel to take care of the tension which will develop upon the portion of such joints farthest from the resistance line. (4) The greatest tendency to shear is in a horizontal plane for a high arch, and in a vertical plane for a low arch; it is sufficient to calculate the shear for points near the springing line, and the frictional resistance in the vertical or horizontal plane should be regarded as an additional factor of safety against failure by shear. (5) When the span is more than twelve times the width of the arch at the crown, either the crown width must be widened or the faces of the arch ring must be battered so as to reduce the ratio; for an arch formed of independent ribs, the ribs must be well bonded together by transverse walls. (6) The exposed mortar in the joints should always be inspected before striking the center, and in case of large concrete arches it is best to drill holes through the last concrete placed, to make sure the concrete is set up within the interior of the arch ring; an analysis should be made to find whether the arch ring is stable under its own weight, and if not, the centering should not be struck until the total dead load rests upon the arch. (7) If the masonry above the arch is to be built after the center is struck, it should be built symmetrically; if for any reason this appears impracticable, the arch must be analyzed for the desired unsymmetrical load. (8) Foundations for fixed arches must be practically unyielding, and the allowed unit load should not, as a rule, exceed 50% of that for ordinary foundations. (9) The horizontal thrust on the foundation should be less than the vertical load upon the foundation multiplied by the coefficient of friction of masonry upon the material of which the foundation bed is composed, or in case of hard or rotten rock, the foundation bed should be roughened; when piers and abutments rest upon piles, the piles should be driven parallel to the line of the resultant thrust or they should be braced by means of horizontal struts to unyielding foundation. (10) The line of thrust must lie within the middle-third of the piers and abutments, including the foundation masonry.

The External Forces (Fig. 109) holding a semi-arch in equilibrium are the vertical loads W_1, W_2, etc., the horizontal thrust H, a vertical reaction V_1 at the skewback, and a vertical shear V_0 at the crown which is due to the action of the other semi-arch. When both semi-arches are loaded equally and symmetrically, then $V_0 = 0$. For the case where the position of the resistance line within the arch ring is known, so that the half-span a and the rise b of the linear arch (Art. 34) are also known, the forces V_1, V_0, H are found as follows: first, the vertical reaction V_1 is computed in exactly the same way as for vertical loads on a simple beam, by taking the center of moments at the right end of the arch; second, the shear V_0 is equal to the sum of all the loads on the semi-arch under consideration minus V_1; third, the horizontal thrust H is found by taking moments about r_5 at the left end, or $\mathbf{Hb + V_0a = \Sigma W\mathit{l}}$, in which the last term is the sum of the moments of the loads.

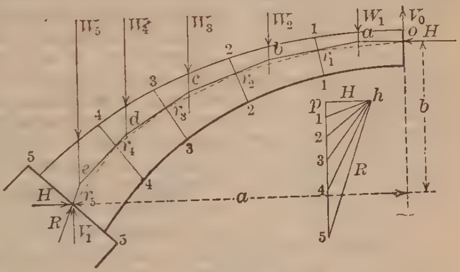


Fig. 109

Example. For arch loaded symmetrically, let $a = 30$ and $b = 21$ ft.; $W_1 = 3$, $W_2 = 4$, $W_3 = 7$, $W_4 = 10$, $W_5 = 12$ tons, their lever arms being $l_1 = 28$, $l_2 = 16$, $l_3 = 10$, $l_4 = 4$, $l_5 = 1.5$ ft.; here $V_1 = 36$ tons and $V_0 = 0$; $H \times 21 = 3 \times 28 + 4 \times 16 + \text{etc}, = 276$ ton-ft., whence $H = 13.1$ tons. For arch with no load on right-hand span and above loads on left-hand span: $V_1 \times 60 = 3 \times 32 + 4 \times 44 + \text{etc.} = 1882$ ton-ft., whence $V_1 = 31.4$ tons; $V_0 = 36.0 - 31.4 = 4.6$ tons, which acts downward upon the right half-span or upward upon the left half-span; $H \times 21 + 4.6 \times 30 = 276$ ton-ft., whence $H = 6.6$ tons.

For a Three-Hinged Arch (Fig. 110) the above method gives the correct values for V_1, V_0 and H, since a is the half-span between end and middle

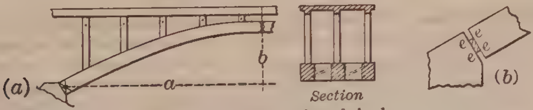


Fig. 110. Three-Hinged Arch

hinge and b the rise of the middle hinge. For a common unhinged arch it is usual to take a as the horizontal and b as the vertical distance between the middle of skewback and middle of crown, but the values found are approximate only, since the assumption that the resistance line passes through the middle points of crown and skewback joints is rarely correct, and since the method of computing V_1 is not exact.

The three-hinged arch in Fig. 110*a* consists of three ribs, which support the roadway on spandrel columns. Each rib has hinges at both ends and one at the crown, the hinge being a steel pin set in a pedestal and upon which abuts the steel shoe which holds the end of the concrete arch. Another form is that in Fig. 110*b*, where a lead plate or a steel plate covered with lead is placed between two abutting concrete surfaces; this form limits the resistance line to a smaller area, but can scarcely be called a real hinge. (See Art. 37.)

Stability against Sliding along any joint will be secured when the resultant of all the forces on each side makes an angle β with the normal to the joint such that 2 tan β is less than tan ϕ where ϕ is the angle of friction of masonry upon masonry (Arts. 18 and 23). Or, let F and N be the components of the resultant parallel and normal to a joint (Fig. 111) and f the coefficient of friction; then fN/F should not be less than 2. After V_0 and H have been found, F and N for any joint are computed by $F = H \sin \theta - (W - V_0) \cos \theta$, $N = H \cos \theta + (W - V_0) \sin \theta$, in which W is the sum of all the loads between the crown and the joint, and θ is the angle which the joint makes with the vertical. When F is positive it acts upward; when negative it acts downward.

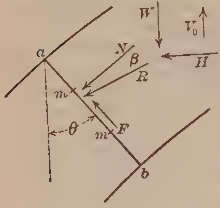


Fig. 111

Stability against Rotation at any joint is secured when the resultant cuts the joint ab within the middle-third mm (Fig. 111), for, the entire joint being then under compression, no tension or opening of the edges of the voussoirs will occur (Arts. 21 and 23).

Stability against Crushing of the material is secured if the compressive unit stress S_a at the edge nearest the resultant is less than the allowable value given in Art. 16. The average unit stress on the joint is N/A, where A is the area of the joint, and the unit stress at a is found by multiplying N/A by $1 + 6\,e/b$, where b is the length of the joint and e is the distance from its middle to the point where the resultant R cuts it (Fig. 111). When R cuts the joint without the middle-third, this rule must be modified unless the joint can take tension (Art. 21).

At each and every joint throughout the arch the above conditions must be satisfied in order that security may prevail throughout. The points cut by the resultants at all joints being connected by a curved line, the resistance line is constructed. The most important of the above conditions is that the resistance line must lie within the middle-third. Although the word joint has been used in this discussion, all the conclusions apply to the imaginary radial joints of a plain concrete arch.

36. Static Analysis of an Arch

The Static Method of investigating an arch is that outlined in a general way in Art. 35, and is so called because only the principles of statics are used. To determine the horizontal pressure H by that method it is necessary to assume its point of application at the crown joint and also the point of application of the resultant at the skewback joint. Then H can be computed and the resistance line be drawn within the arch ring, but there is no assurance that this is the true resistance line. It is, however, generally accepted that an arch has proper stability against rotation if a resistance line can be constructed which will lie within the middle-third of the arch ring at all radial joints. Hence, different assumptions as to points of application are to be made and different resistance lines to be constructed, and the design should be pronounced deficient in stability if a resistance line cannot be found which lies within the middle-third. (See " Stresses, Graphical Statics and Masonry," by G. F. Swain, 1927, p. 411.)

The static method was used in America almost exclusively prior to 1900. Although the elastic method (Art. 38) has the advantage that it determines both H and its point of application, yet the static method is still of great value in preliminary investigations.

The Resistance Line can be constructed, after H has been computed, by help of the force polygon. In Fig. 109 the thrust H is laid off horizontally and the loads vertically in succession; then from h as a pole the rays are drawn to the points of division between the loads, the last ray thus giving the magnitude and direction of the resultant R on the joint 5-5. An equilibrium polygon is then constructed, the first side being in the line of H produced, the second parallel to the first ray, and so on until the last side through e gives the position of R. The points r_1, r_2, r_3, r_4, r_5, where these sides cut the joints, are points in the resistance line, and the resistance line itself is the curve joining them.

This force polygon is for the the case of symmetrical loading on the arch so that the crown shear V_0 is zero. The method, however, is perfectly general and may be used to give closely approximate results for unsymmetrical loading by drawing V_0 as the first vertical force in the force polygon, laying off its value upward if the same is positive and downward if it is negative.

A Graphic Method of finding the horizontal thrust H is shown in Fig. 112 for a simple case where only three loads are used for the sake of clearness. The arch has the live load on the half-span shown, while the other half-span has only dead load. The horizontal thrust is assumed to act at the crown at the upper middle-third limit, and the resultant at the skewback is assumed to act at the lower middle-third limit. The correct horizontal thrust H on the foregoing assumptions is that which will, with the three loads W_1, W_2, W_3, give an equilibrium polygon passing through the assumed point a at the crown and the assumed point r at the skewback. The procedure is as follows:

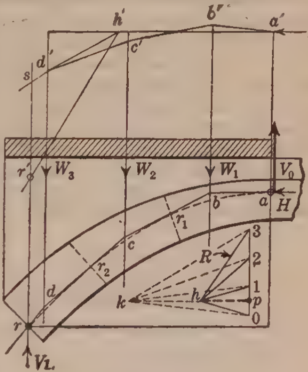


Fig. 112

(1) The reaction V_1 is computed in exactly the same way as for a simple beam. The crown shear V_0 is found by subtracting V_1 from the sum of the three loads. The load line for the force polygon is laid off from p, the distances 3–2, 2–1, 1–0, op being made equal on a proper scale to the values of W_3, W_2, W_1, V_0, respectively.

(2) Assume pk in the force polygon to be the value of H; with k as a pole draw the rays $k0$, $k1$, $k2$, $k3$, the last of which should be the resultant at the skewback if the assumed H is correct. Above the arch, in order not to confuse the drawing, extend upward the lines of action of the loads and place the points a' and r' above a and r, and at the same vertical distance apart. Then construct the equilibrium polygon $a'b'c'd's$, each side being parallel to the corresponding ray in the force polygon. The last side $d's$ of this trial equilibrium polygon does not pass through r'. Extending its line of direction upward until it meets that of H', there is found at h' the point through which the resultant of the vertical forces acts. Then a line joining r' and h' gives the direction of the true resultant.

(3) In the force polygon, draw R parallel to $r'h'$ then ph is the value of the horizontal thrust H on the scale employed. From h draw a new set of rays, and in the arch ring itself construct the equilibrium polygon $abcdr$, which must now pass through the point r on the skewback joint.

(4) This equilibrium polygon cuts the two radial joints at the points r_1 and r_2, these being points in the resistance line, which is a curve joining a, r_1, r_2, and r. This curve lies everywhere within the middle-third of the arch ring, and hence there is full stability against rotation. The direction and magnitude of the resultants acting on these joints are given by the rays $k1$ and $k2$ in the force polygon.

An Actual Investigation of a proposed design by the static method involves no principles not explained above. The practical procedure is as follows: (1) Vertical lines are drawn dividing the arch structure into an even number of parts, about 16 parts being used for a span of 80 ft. (2) The weight of the masonry and earth filling for each of these parts is found for an arch one unit in width, as also the line of action of that weight; earth is usually reduced to an equivalent height of masonry, and then the center of gravity of an area is the point through which its weight acts. (3) For loads W_1, W_2, etc., which act through these centers of gravity the horizontal thrust may be found and the resistance line be located as above explained. (4) The points assumed at crown and skewback may be the middle points for the first investigation, but the upper limit of the middle-third at one joint and the lower limit at the other joint may be used for other resistance lines. (5) One set of lines should be found for full dead and live load over the whole span, another for the case when the live load is on the left half-span only. Lines for other loadings may be sometimes necessary (Art. 34).

To Design an Arch by this static method, a drawing is made which seems to give good proportions and dimensions for the given local conditions and loads, the thickness of the arch ring being assumed from the data in Art. 34. Then several resistance lines are constructed, and, if none can be found which lies within the middle-third, the proportions or thicknesses are unsafe ones and must be changed. Another drawing with different dimensions is then made and resistance lines constructed for it. When several drawings or designs all furnish proper stability for the given loads, then that one should be chosen which can be built for the minimum cost.

Cain's Method of finding the crown thrust H and crown shear V_0 for a case of unsymmetric loading is as follows (Fig. 113). P_1 = weight of left half of arch and the load upon it, P_2 = weight of right half of arch and the load upon it, P_3 = weight of the portion of the arch and load between a joint rr_1 and the crown, other notations in the figure. It is necessary to assume the points of application of the resultants on the lowest joints mm_1 and nn_1 and the point of application of the resultant upon some other intermediate joint as at L, while I, L, and K are points through which it is desired to pass the resistance line. All of the points should be taken within the middle-third of their respective joints. If I and L are on the more heavily loaded half of the span and K therefore on the more lightly loaded half, I should be assumed a little below the middle of its joint and L and K a little above the middle of their joints. If I and L are on the lighter half and K on the heavier half, I should be taken a little above the middle of its joint and L and K a little below the middle of their joints. The following formulas (Cain's Voussoir Arches, 1904) give the horizontal and vertical components of the inclined crown thrust and its point of application:

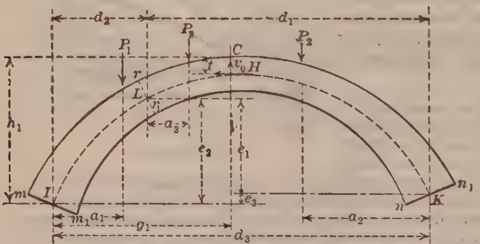


Fig. 113. Unsymmetric Loading

$$V_0 = \frac{a_1e_1P_1 - a_2e_2P_2 + a_3e_3P_3}{e_2d_3 - e_3d_2} \qquad H_0 = \frac{a_1d_1P_1 + a_2d_2P_2 - a_3d_3P_3}{e_2d_3 - e_3d_2}$$

$$t = h_1 - (a_1P_1 - g_1V_0)/H$$

Limitations of Static Method. The static method of design gives curves of resistance which agree closely with those determined by the elastic method. The maximum unit stresses determined by the static method agree with those found by the elastic method within less than 10%, and usually the difference is less than 5%. It is therefore a safe method of design, and may, in general, be used for the design of an arch. The elastic method, however, should be used in the final design of all important arches. The static method ignores temperature stresses (Art. 40). Inclined loads, resulting from pressure of earth, may usually be treated more simply by the elastic method.

Longitudinal Walls and masonry haunching resting upon the back of the arch do not exert their full weight upon the back of the arch, but, for safety, the full weight of walls and haunching should be used in determining the line of resistance.

A Retaining Wall resting upon the back of the arch exerts greater pressure upon the arch at the toe of the wall than at the heel, and the pressure varies between the heel and toe, but it is exact enough to regard the pressure upon the base of the wall as uniformly distributed upon the back of the arch. The ring stones should be tied by iron cramps to the sheeting back of the ring stones, otherwise the horizontal thrust upon the back of the retaining wall may separate them from the arch sheeting.

Change in Spandrel Loading. The line of resistance may be shifted in position so as to lie closer to the neutral axis, by changing the spandrel loading. For instance, in Fig. 114, which is an arch of an earth-filled bridge, the resistance line will be changed if transverse hollow arches are formed through the haunches on each side of the arch; such hollow tunnels are used in the longest masonry arch in the world, that at Plauen in Saxony. Changes in the spacing of spandrel columns will also modify the resistance line; such changes, however, should be mainly confined to cases where errors in design are detected after the arch is erected and before the spandrel columns are built. In an earth-filled arch it is manifestly impracticable to change the line of resistance by changing the unit weight of the earth fill at selected points.

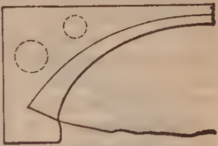


Fig. 114

Transverse Spandrel Arches (Fig. 115). The spandrel columns or walls of transverse arches should not generally be spaced parallel to the span at

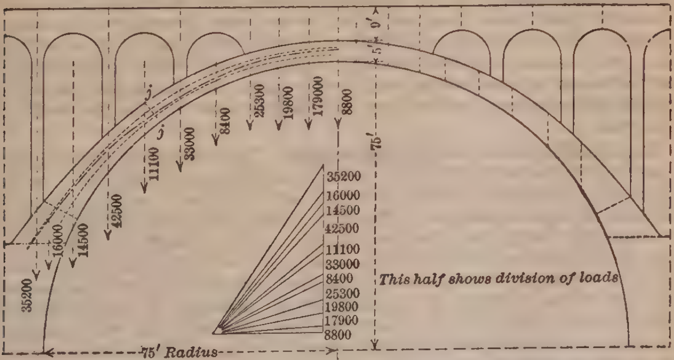


Fig. 115. Connecticut Ave. Arch, Washington, D. C.

intervals exceeding about 1/7 of the span. The spandrel columns should not generally be spaced parallel to the axis of the arch, transverse to the span, at intervals exceeding about two and one-half times the thickness of the arch ring upon which the columns rest. When the columns are so placed, the load brought upon the back of the arch may be regarded as distributed uniformly across the transverse section of the arch. In the case of a reinforced-concrete arch this transverse spacing of the columns may exceed two and one-half times the thickness of the arch.

Fig. 115 shows the method of determining the resistance line for transverse spandrel arches, this being shown by a heavy dot and dash line, while the limits of the middle-third are shown by light dotted lines. This arch is one of the main spans of the Connecticut Avenue bridge in Washington, D. C. The 5-ft. arch ring is of concrete blocks and the arch sheeting of plain monolithic concrete. The arch is a full-centered one of 150-ft. span, and the span of transverse arches is 14 ft. The analysis shown is for a full uniform load over the entire span. The resistance line comes nearest the edge of the middle-third at the joint *jj*, and the maximum compression there is 350 lb. per sq. in.

37. Three-Hinged Arches

Advantages. (1) If the foundations are compressible, such as pile or clay foundations, the three-hinged arch offers the best type, as it permits considerable settlement without injury to the arch. (2) If the arch has a rise of less than 1/4 the span, on account of the high temperature stresses in a solid arch, the three-hinged arch is the better. (3) Since the temperature stresses are practically nothing, and as the arch is statically determinate, higher unit stresses may be used than in the solid arch.

Disadvantages. (1) High cost of the hinges, which may be offset somewhat by the saving in concrete for long-span arches. (2) Difficult maintenance and possible short life of the hinges, if of iron or steel, as compared with the life of the masonry. (3) The three-hinged arch usually lacks the graceful appearance of most fixed arches, due to the increased thickness that is necessary at the haunches.

Approximate Design. The crown thickness t_0 may be taken a little less than those given by empirical rules for the solid arch. The thickness at the springing line may be approximately 1.25 t_0, and that at the middle of the haunch approximately 1.5 t_0. Having selected these dimensions, the arch should be drawn to a large scale and be analyzed according to the static method. The cheapest type of the three-hinged arch is one of detached ribs supporting spandrel columns, although there appears to be no reason why an arch with continuous sheeting should not be used. In what follows the lower hinges will be assumed at the same level and the crown hinge at the middle of the arch.

Dead and Live Load over the Entire Span. The weights and lines of application of the loads may be found as in the static method (Art. 36). As the loads are symmetrical, only the left-hand half of the arch need be considered. The crown shear is here zero. The crown thrust H, which is horizontal for symmetrical loading, may be computed from $\mathbf{H = \Sigma Wz/b}$, in which W = any load applied upon the left half of the arch, z = horizontal distance from it to the left hinge, b = rise of arch measured from the horizontal line passing through the lower hinges to the crown hinge. Knowing H, lay off the load line, construct the force and equilibrium polygons as in the static method, applying H at the center of the crown hinge. If the graphical work is correct, the last line of thrust will pass through the center of the lower hinge.

For dead load on entire span and live load on the left half, let W = any dead load of the left half-span, W' any live load, and z and z' be the horizontal distances from them to the left end hinge. Then

Thrust $\mathbf{H = (\Sigma Wz + 1/2\ \Sigma W'z')/b}$ Crown shear $\mathbf{V_0 = +\ \Sigma W'z'/2a}$

in which a = half-span. For dead load over entire span and live load on the right half the same formulas apply, except that V_0 is negative. The force polygon for either case is then constructed as in Fig. 112, and the equilibrium polygon drawn, taking H and V_0 as applied at the middle hinge; the last line giving the direction of the resultant thrust at the skewback should pass through the end hinge.

Special Method for Thickness of Arch Ring. The hinges may be drawn at the given points and the approximate loads of the spandrels and arch be applied as shown in Fig. 116. These loads and their point of application may be determined by a practically complete design of the spandrels made prior to the arch design. The weight of the arch ring may be approximated, using the dimensions recommended, but the arch ring itself need not be drawn. After computing H and V_0 for each case construct the equilibrium polygons or thrust lines for (1) dead and uniform live load over the entire span, (2) for dead and uniform live load over the right half-span, (3) for dead and uniform live load over the left half-span. The first line is shown in Fig. 116 as a full

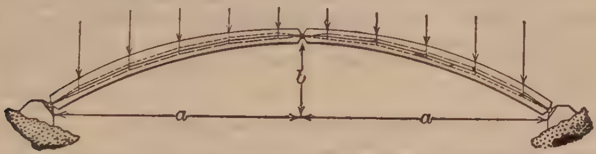


Fig. 116. Three-Hinged Arches

line, the second by a line of dashes and the third by a line of dots. After the lines are drawn, knowing, from the force polygon, the intensity of the resultant thrust at any point, the correct thickness of the arch ring at any point may be determined by keeping all thrust lines within the middle-third and the unit stresses within safe limits. If the assumed thicknesses are found to be wrong, a second analysis may be necessary.

This method, considering only dead load, or dead and uniform load over the entire span, may be used to determine the curve of the neutral axis of a solid arch, which will closely approximate the linear arch. This approximate linear arch would be the one requiring a minimum amount of material for any given loading.

Hinges have been used for stone masonry arches in many European bridges, but they are almost unknown in America. Recent European practice has been to construct concrete arches also with hinges. The lack of experience with such construction has been the chief cause for not using hinges with concrete arches in America. Hinges may be of cut stone, structural or cast steel pins (Fig. 110*a*), lead plates (Fig. 110*b*), or reinforced concrete. The steel hinge requires painting, but locates the line of thrust more precisely than the others. The lead hinge requires no maintenance; it should not carry a greater pressure than 1250 lb. per sq. in. for a plate 1/2 in. thick, 1000 lb. per sq. in. for 3/4 in. and 750 lb. per sq. in. for 1 in. in thickness. Adjacent to the hinges, blocks of hard stone should be placed to withstand the high unit pressure; or if concrete is used, it should be rich and heavily hooped. Stone hinges have not been used to an extent to warrant their general adoption.

A recent development in French practice is the use of reinforced-concrete hinges. These are not true hinges, but consist of a decided reduction in thickness of the arch at the crown and springing lines, with the addition of reinforcing steel, so that the flexibility of the structure at the reduced section is greatly increased. This type of hinge has given satisfactory service, and requires no maintenance. (See Hool's "Reinforced Concrete Construction," Vol. III; also article by J. F. Brett, Journal Western Soc. of Engineers, Sept., Oct., 1926.) Examples of long-span reinforced-concrete arches with flexible concrete hinges are given in Art. 34.

In some cases, arches have been made "semi-hinged." These are constructed as 3-hinged arches in order to keep the resistance line for dead-load closer to the center of the masonry. After the centering is removed, and the arch has attained its dead-load deflection, the hinges are filled in with concrete or mortar, and the arch is then fixed under live load and temperature stresses. The stone masonry arch at Morbegno, Italy (see Table in Art. 34), was constructed by this method. The Knox Bridge over the Aqueduct Canal, near Montreal, was constructed with flexible concrete temporary hinges (Canadian Engineer, Vol. 52, p. 521, May 17, 1927). In reinforced-concrete arches, the main arch reinforcement may be carried past the hinges, or overlapped at those points, and both hinges and main reinforcement are then concreted in solid to complete the structure. The Candelier Bridge near Charleroi, Belgium, was constructed with two permanent concrete rolling hinges at the abutments. This arch was actually 3-hinged for dead load, as jacks were provided at the crown to introduce a horizontal thrust into the arch (see Method of Rib-Compensation, Art. 43), but is a 2-hinge arch under live-load and temperature.

38. Elastic Method of Analysis

The True Resistance Line for an arch ring is found in the static method by a series of approximations which start with assumed points on the crown and skewback joints. In the elastic method this true line is found without such approximations in its exact location. This can be done entirely by computation, but graphic work may also be used to draw the equilibrium polygon within the arch ring after H and its point of application have been found. An arch one unit in length is considered, as in the static method, and the **neutral axis** of the flexural forces is represented by a central line drawn half-way between intrados and extrados. The thickness of the arch at any joint being called t, the moment of inertia of the surface of that joint for an arch of a unit length about the neutral axis is $I = 1/12\ t^3$. The **bending moment** for any point on the neutral axis is the algebraic sum of the moments of all the forces on one side of that point. A moment is positive when it tends to increase the compression on the back of the arch, this being the same convention as for beams (Sect. 7). The **resisting moment** at any joint is the product of the normal pressure N acting upon the joint and its distance e from the middle of that joint. The word "moment" when used without qualification applies to the bending moment.

Notation (Fig. 117). At the crown: V_0 = vertical shear, H = horizontal thrust. M_0 = bending moment, e_0 = distance of H from crown of the neutral axis, He_0 = resisting moment. For any point: x = horizontal distance from it to crown, y = vertical distance of it below crown, V = vertical shear, M = bending moment, R = resultant thrust on joint, N = component of R normal to joint, e = distance of N from middle point of joint or from the neutral axis, Ne = resisting moment.

Preliminary Steps. (1) The arch ring is to be drawn to scale and be divided into an even number of parts by the broken radial lines shown in Fig. 117, while half-way between these are drawn solid radial lines to represent joints. When the arch ring is of constant thickness, the arch divisions are equal in length; when it increases in thickness from crown to skewback, the lengths of the divisions are to be determined so that the ratio I/s shall be the same for all, s being the length of any division and I the average of the moments of inertia of the two limiting joints. Let the middle points of the joints be marked 1, 2, 3, etc., and the coordinates x and y be found for each point by computation or measurement. (2) Let Σx = sum of the values of x for these points, Σx^2 = sum of the squares of the abscissas, Σy = sum of their ordinates, Σy^2 = sum of the squares of these ordinates. (3) For a load W, placed at one of these points, let z denote the distance from it, toward the nearest skewback, to another middle point; also let Σz = sum of all these distances, that is, the sum of the distance of W from each of the points nearer the nearest skewback, Σzx = sum of the products of all values of z by the corresponding x, and Σzy = sum of all products of z by the corresponding y; that is, each z

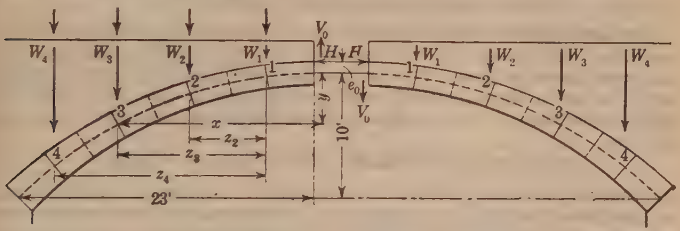


Fig. 117. Data for the Elastic Method

in the last two summations is multiplied by the x or y of the point back of W which corresponds to Z.

For a Single Load W on the left semi-arch of Fig. 117, the elastic theory of the arch furnishes the following formulas, n being the number of parts into which the semi-arch is divided.

For horizontal thrust, $$\mathbf{H = 1/2\,W\frac{n\Sigma zy - \Sigma y.\Sigma z}{n\Sigma y^2 - (\Sigma y)^2}} \qquad \text{(A)}$$

For moment at crown, $$\mathbf{M_0 = (1/2\,W\Sigma z - H\Sigma y)/n} \qquad \text{(B)}$$

For shear at crown, $$\mathbf{V_0 = 1/2\,W\Sigma zx/\Sigma x^2} \qquad \text{(C)}$$

Summations are for the half arch only. The value of H is always positive; that of M_0 may be either positive or negative, depending upon the position of W. The formula for V_0 gives positive values only, this force acting upward with respect to the left semi-arch, the load being on the left of the crown, but downward with respect to the right one.

When W is on the right semi-arch the same formulas apply if z is measured from W toward the right-hand skewback. For this case, however, the value of V_0 is to be taken as negative with respect to the left semi-arch, and positive with respect to the right one.

For Two Symmetric and Equal Loads, such as W_2 on the left and W_2 on the right in Fig. 117, let W be the weight of each; then the horizontal thrust and crown moment due to both loads are double those found by the above formulas, while the crown shear V_0 is zero.

For Several Loads, the formulas are to be applied to each in succession and the results added. Thus if one load produces a crown moment of 640 ft-lb. and another − 870 ft-lb., then for both loads M = − 230 ft-lb.

The denominator in formula (A) depends only on the coordinates of the curve and is independent of the span. Hence, when H is to be computed for several loads, the different numerators should be added and then their sum be divided by the constant denominator. Art. 39 gives an example.

For any Joint whose middle point is at horizontal distance x from the crown, values of the moment and shear may be computed, after H, M_0, V_0 are known, from the formulas

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{H}\mathbf{y} + \mathbf{V}_0\mathbf{x} - \Sigma\mathbf{W}\mathbf{z} \qquad \mathbf{V} = \mathbf{V}_0 - \Sigma\mathbf{W} \tag{D}$$

where ΣW is sum of all loads between that joint and the crown, and ΣWz is the sum of the moments of those loads with respect to the middle of the joint. The components of the resultant thrust normal and parallel to the joints are

$$\mathbf{N} = \mathbf{H} \cos \theta - \mathbf{V} \sin \theta \qquad \mathbf{F} = \mathbf{H} \sin \theta + \mathbf{V} \cos \theta \tag{E}$$

where θ is the angle which the plane of the joint makes with the vertical (Fig. 111). The resultant thrust itself is $\mathbf{R} = \sqrt{\mathbf{H}^2 + \mathbf{V}^2}$.

Resistance Lines should be determined, in the design of an important arch, for at least three cases: (1) For dead load only. (2) For dead load plus live load over the entire span. (3) For dead load plus live load on left of crown only. In each of the cases let M_0 and H be found for the crown, and M and N for any other joint. Then the distances from the neutral axis to the resistance line are

$$\text{At the crown, } \mathbf{e}_0 = \mathbf{M}_0/\mathbf{H} \qquad \text{At any joint, } \mathbf{e} = \mathbf{M}/\mathbf{N} \tag{F}$$

and thus that line may be located at every joint. Or, after having found e_0, an equilibrium polygon can be drawn as in the static method, but here no approximation is needed, since H and e_0 being correctly found for the crown, the force and equilibrium polygons are immediately drawn in correct magnitude and position.

The coefficient of friction which is necessary in order that there may be full security against sliding along the joint is $f = nF/N$, in which F and N are to be computed from (E) and n should be 2 or greater.

A common masonry arch is a statically interderminate structure like a continuous beam or like a beam fixed at its ends. The elastic theory, by which the above formulas are deduced, makes the following assumptions: (1) that the material is elastic and obeys Hooke's law; (2) that the material is homogeneous so that modulus E is the same in all parts of the arch; (3) that the arch is fixed at its ends so that a tangent there remains unchanged under the loading; (4) that the loads cause no change in the length of the span; (5) that the ends remain in the same horizontal plane under all loadings. Although it is often difficult in practice to secure the complete observance of these assumptions, the theory gives correct results if they are fulfilled. When an arch is built upon piles or compressible soil the conditions (3), (4), (5) may be fulfilled only partially, and for such cases the three-hinged arch (Art. 37) may be preferable. For rock foundations the elastic theory is entirely satisfactory, and in an important case the old static method of trial and approximation ought not to be used, except as a check.

In concrete arches with reinforced-concrete floor and spandrels, the use of models will be helpful to indicate errors resulting from the application of the elastic theory and the difficulty of properly considering the effect of the stiff superstructure upon the actual stresses. (See Art. 42.)

39. Example of the Elastic Method

Data. It is required to design a plain concrete arch for a double-track railroad which shall have a width of 24 ft., a span of 46 ft. between centers of skewback joints, and a rise of 10 ft. from the springing line to the center of the crown joint. The railroad track is to be 3.6 ft. above the center of the crown joint, and the filling above the back of the arch will be earth. The live load per linear foot of track is specified as 4000 lb. Assume the thickness of the arch as 1.42 ft. at the crown and 2.14 ft. at the skewback. Through the given end and middle points draw a curve approximating a parabola, and lay off these two radial joints. Then draw the extrados and intrados curves so that the radial distance between them shall increase uniformly from middle to end, that is, so that the thickness of the arch at horizontal distance x from crown is $1.42 + 0.0313\ x$. Following is an investigation of this proposed design (Fig. 117).

(1) Divide the central curve of one semi-arch into four parts, of unequal length, those nearest the crown being the shortest. Let the lengths of these divisions, measured on the curve, be represented by s_1, s_2, s_3, s_4. Let radial lines be drawn through the middle points of these divisions and their lengths t_1, t_2, t_3, t_4 be found. Compute the ratios t_1^3/s_1, t_2^3/s_2, t_3^3/s_3, t_4^3/s_4; if these are equal, the division is correctly made; if not, the process must be repeated until four points 1, 2, 3, 4 are found for which these ratios t^3/s have approximately the same value.

A convenient graphical method for making this sub-division is shown in Fig. 118. A curve is plotted showing the variation of I $(= t^3/12)$ with respect to the distance, S, along the arch axis, starting from the crown. A series of isosceles triangles is drawn under this curve (as shown in the figure) with parallel sides but variable heights, the slope of the sides being determined by trial, so that the base of the last triangle will just reach the skewback. The bases of these triangles will then represent the proper lengths of the arch subdivisions, measured along the arch axis.

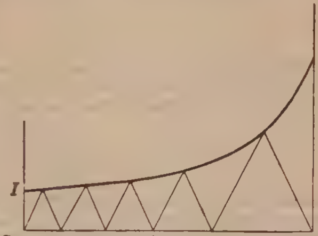


Fig. 118

(2) Let the final results of this work be that the thicknesses of the four joints are $t_1 = 1.52$, $t_2 = 1.72$, $t_3 = 1.90$, $t_4 = 2.06$ ft.; that the abscissas of their middle points are 3.22, 9.47, 15.20, 20.50 ft.; and that the corresponding ordinates are 0.21, 1.84, 4.62, 8.31 ft. These coordinates are entered in the second and third columns of the table below.

(3) Drawing vertical lines at the limits of the four divisions, the loads for the points 1, 2, 3 are, for an arch one foot in length,

For dead load, $W_1 = 4500$ $\quad W_2 = 5400$ $\quad W_3 = 7000$ lb.
For live load, $W_1 = 2100$ $\quad W_2 = 2000$ $\quad W_3 = 1800$ lb.

The last load, W_4 in this case, is not used, because by this method of division and computation it does not produce stresses in the arch; in reality, it does, of course, cause stresses, but they are small on account of its nearness to the skewback. In Fig. 117 the dead loads are shown for both halves of the span, but the live loads are shown only for the left-hand half.

(4) The summations and quantities involving only x and y, which are required for use in the formulas of Art. 38, are computed and entered in the following table; the denominator in the formula for H has the constant value 150.9 for all loads in all positions.

Point	x	y	y^2	x^2
1	3.22	0.21	0.04	10.4
2	9.47	1.84	3.39	89.7
3	15.20	4.62	21.34	231.0
4	20.50	8.31	69.06	420.2
$(\Sigma y)^2 = 224.4$ $\quad n = 4$ $n\Sigma y^2 - (\Sigma y)^2 = 150.9$		$\Sigma y = 14.98$	$\Sigma y^2 = 94.87$	$\Sigma x^2 = 751.3$

(5) The products and summations involving z, which depend upon both position of load and the other points of division, are computed and entered in a second table.

Since the load W_1 is taken as applied at point 1, the values of z for it are found by subtracting the first value of x from each of the following ones; also for W_2 the second value of x is subtracted from the following ones. The distances z_2, z_3, z_4, shown in Fig. 117, are the values of z for the load W_1.

Point	Values of z			Values of zy			Values of zx		
	W_1	W_2	W_3	W_1	W_2	W_3	W_1	W_2	W_3
1	0	......	......	0	......	......	0	......	......
2	6.25	0	......	11.5	0	......	59.2	0	......
3	11.98	5.73	0	55.3	26.5	0	182.1	87.1	0
4	17.28	11.03	5.30	143.6	91.7	44.0	354.2	226.1	108.6
Σz =	35.5	16.8	5.3	......	......	......	......	......	......
$\Sigma y.\ \Sigma z$ =	531.8	251.7	79.4	......	......	......	......	......	......
Σzy =	......	......	......	210.4	118.2	44.0	......	......	......
Σzx =	......	......	......	......	......	......	595.5	313.2	108.6

(6) Formula (A) of Art. 38 is now used to obtain the horizontal thrust caused by each load, and then Formulas (B) and (C) to find the crown moment and crown shear. For example, the work for the first load is

$$H = 1/2\ W_1\ (4 \times 210.4 - 531.8)/150.9 = 1.026\ W_1$$
$$M_0 = (1/2\ W_1 \times 35.5 - 1.026\ W_1 \times 14.98)/4 = 0.595\ W_1$$
$$V_0 = 1/2\ W_1 \times 595.5/751.3 = 0.396\ W_1$$

This value of V_0 is positive if W_1 is on the left and negative if it is on the right of the crown, because it is understood that the left semi-arch is the one for which the analysis is to be made. For each load in Fig. 117 the computed results for the crown now are

for W_1, $H = 1.026\ W_1$ $\quad M_0 = 0.595\ W_1$ $\quad V_0 = 0.396\ W_1$
for W_2, $H = 0.733\ W_2$ $\quad M_0 = -\ 0.645\ W_2$ $\quad V_0 = 0.208\ W_2$
for W_3, $H = 0.320\ W_3$ $\quad M_0 = -\ 0.535\ W_3$ $\quad V_0 = 0.072\ W_3$

(7) Only one case of loading will be here investigated, namely, that when the live load covers the left semi-arch only, as in Fig. 117. For dead load over the whole span, the values $W_1 = 4500, W_2 = 5400$, $W_3 = 7000$ lb. are to be inserted above, the products added and the sums doubled for H and M, while those for V cancel each other on account of the double sign. For live load over left semi-arch only, the values $W_1 = 2100$, $W_2 = 2000$, $W_3 = 1800$ lb. are to be inserted and the products added. Thus, for

dead load, $\quad H = 21\ 600 \quad M_0 = -\ 9100 \quad V_0 = 0$
for half live load, $\quad H = 4\ 200 \quad M_0 = -\ 1000 \quad V_0 = +\ 1400$

Lastly, the addition of these gives the final values for the case of loading shown in Fig. 117, namely, $H = 25\ 800$ lb., $M_0 = -\ 10\ 100$ ft-lb., $V_0 = 1400$ lb. The negative sign of M_0 shows that it tends to produce tension on the back of the arch.

(8) These final crown moments and shears reduce formula (D) of Art. 38 to

$$M = -\ 10\ 100 + 25\ 800\ y + 1400\ x - \Sigma Wz, \qquad V = 1400 - \Sigma W$$

and the values of the moment and shear for each joint are placed in the table below. Also the first formula (E) takes the form $N = 25\ 800 \cos\theta - V \sin\theta$, and the computed normal pressures for each joint are given in the last column.

Joint	x, ft.	y, ft.	cos θ	sin θ	M, ft-lb.	V, lb.	N, lb.
Crown	0.0	0.0	1.00	0.00	− 10 100	+ 1 400	25 800
1	3.2	0.2	0.99	0.12	− 460	+ 1 400	25 370
2	9.5	1.8	0.94	0.34	+ 8 390	− 5 200	26 020
3	15.2	4.6	0.86	0.50	+ 8 390	− 12 600	28 490
4	20.5	8.3	0.79	0.61	− 9 570	− 21 400	33 440
Skewback	23.0	10.0	0.77	0.64	− 19 210	− 21 400	33 560

(9) Finally, the eccentricity e, or the departure of the resistance line from the neutral axis, is obtained for each joint by formula (F) of Art. 38, namely $e = M/N$. The following table shows that the resistance line is within the middle-third of the arch ring except at the crown and at joint 2 and skewback; the values of $1/6\ t$ are distances from neutral axis to limits of middle-third. The maximum compression S_1 and the maximum tension S_2, using the formulas of Art. 21, and using only the normal component N of the thrust, are computed for the joints under the assumption that no rupture of the material occurs under the tensile stress; if such occurs, then the compression on the other side is increased to the value shown in the last column.

Joint	Thickness, t, ft.	Middle-third, 1/6 t, ft.	Eccentricity, e, ft	Maximum unit stress, lb. per sq. in.		
				Compression, S_1	Tension, S_2	S_1
Crown	1.42	0.237	−0.391	330	80	370
1	1.52	0.253	−0.018	120	....	....
2	1.72	0.287	+0.322	220	13	230
3	1.90	0.317	+0.294	200	....	....
4	2.06	0.343	−0.286	210	....	....
Skewback	2.14	0.357	−0.572	280	65	310

Other cases of loading cannot here be investigated, but a live load over the entire span, or perhaps live loads W_1 and W_2 acting on both sides of the crown, may produce a greater moment at the crown and raise the resistance line higher above the neutral surface. The result of the investigations thus far made shows that the resistance line passes outside the middle-third at three points, and hence the design must be modified in order to be satisfactory. This may be done by increasing the rise, by increasing the thickness of the arch ring at the crown, or by introducing steel reinforcement.

The above method, in its essential features, was first given in America by Howe (Arches, 1890), the formulas being materially simplified by Turneaure and Maurer (Reinforced Concrete, 1907); both formulas and tabulations are here given in somewhat different form. The way of obtaining the loads and applying them above the centers of the joints is regarded as not perfectly satisfactory. There is, however, nothing in the theory which requires the loads to be applied at the points 1, 2, 3 in Fig. 117; they may be applied at any other positions, but the x's for any W must be measured from the position of W to all the points between it and the skewback.

Multiple-Arch Bridges on high, elastic piers are difficult to analyze since the elastic deformation of the piers affects the stresses in the arches. Elaborate analytical methods have been proposed for this purpose. (Trans. Am. Soc. C. E., Vol. 90, p. 1094; Vol. 88, p. 1142; Vol. 91, p. 459.) A simpler method of deriving stresses for such structures is by the use of models (see Art. 42).

Skew Arches should be avoided if possible because they are more expensive and involve many complications in design and construction. An approximate method of analysis is outlined in G. F. Swain's "Stresses, Graphical Statics and Masonry" (1927), p. 426. (Also see Eng. News-Rec., Vol. 88, p. 638.) An elaborate analytical method is given by J. C. Rathbun, Trans. Am. Soc. C. E., Vol. 87, p. 611. See also article by S. C. Hollister in Proceedings, Am. Concrete Inst., Vol. 24 (1928).

A skew stone arch may be one of three types: (1) If courses are laid parallel to arch axis, it is called a "false skew arch." (See Stereotomy, Art. 6.) This method is suitable only where the angle of skew is small (not over 20 deg.) unless the span is very short. (2) A ribbed skew arch (Fig. 134) in which the soffit is offset at regular intervals, each rib being constructed as a right arch. The ribs should be tied together by cramps at the extrados. (3) The real skew arch is constructed with the courses laid out in spiral curves, so as to maintain the joints perpendicular to the pressure line. (Books on Stereotomy referred to in Art. 6 contain descriptive matter.)

40. Temperature and Deformation

General Effect of Temperature. If a ring of any elastic material, as in Fig. 119, is rigidly fastened to fixed abutments, the expansion of the ring under a rise of temperature exerts a horizontal thrust against the abutments. The abutments being immovable, the ring distorts, and the points *dd* retreat from the horizontal diameter *ab*. If the temperature falls, the ring is also distorted, but the points *dd* approach *ab*. Since the ring is still in equilibrium, the stresses at all sections will have undergone a change. The removal of the lower half of the ring would evidently not affect the distortion of the upper half under the change of temperature, provided the upper half is rigidly fastened to the immovable abutments. The condition of the upper portion of the ring is approximately that of a solid masonry arch built upon a rock foundation. If the upper portion of the ring is of loose voussoirs (Fig. 119*b*),

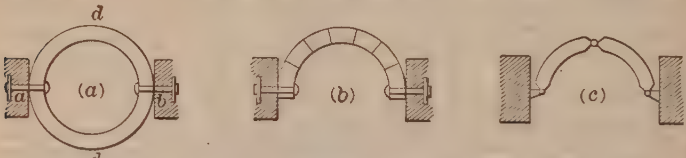


Fig. 119. Influence of Temperature

that is, without mortar in the joints, it is evident that the arch would adjust itself with but little strain. In practice, however, the arch ring is held down by the loads upon it so that the stresses due to the loads are increased or diminished by a change in temperature. Fig. 119*c* shows a three-hinged arch which, disregarding friction of the hinge, is free to move under change of temperature without developing temperature stresses.

Changes in Crown Thrust and Moment occur when the temperature varies from that prevailing at the time of closing the arch. When the temperature rises above the standard, the horizontal thrust is increased; when it falls below the standard, the horizontal thrust is decreased. Let H' = the change in H due to a change of T degrees; let e = the coefficient of expansion and E = modulus of elasticity of the material (Art. 17); let n = the number of divisions into which the half arch ring is divided and m = the constant ratio I/s for each division (Art. 38); let Σy and Σy^2 be the sum of the ordinates and the sum of their squares for the middle points of the several joints; let a = half-span of the arch. Then, for a rise of T degrees,

$$\text{Horizontal thrust} = \mathbf{H'} = \frac{\mathbf{ETenma}}{\mathbf{n\Sigma y^2 - (\Sigma y)^2}}$$

$$\text{Crown Moment} = \mathbf{M'_0} = -\mathbf{H'\Sigma y/n} \qquad \text{Crown shear } \mathbf{V_0} = \mathbf{0}$$

The bending moment at any point due to this rise in temperature is $\mathbf{M'} = \mathbf{M'_0} + \mathbf{H'y}$. The computed values of H', M_0' and M' are to be combined with those found for H, M_0 and M in Art. 39 for the case of loading under consideration. For a fall of temperature T is to be taken negative.

The standard temperature is usually assumed in the design at 50° Fahr. and a range of from 20° to 45° above and below it is allowed, or $T = \pm 20°$ for a spandrel filled arch and $T = \pm 45°$ for an isolated arch ring. Instead of using a standard temperature, it is best to compute the design with reference to the temperature conditions that will probably be met. These will be not only a function of latitude and altitude, but also of the time of year in which the arch will be built and, to some extent, of the method of building. For instance, if the arch is built as a continuous monolith the heat resulting from chemical action will add to the normal temperature. If, however, keyways are provided and these keyways are not concreted until a week or so after the balance of the arch has been built, there will be little added temperature as a result of chemical action.

There are few actual temperature observations of arch concrete. Some tests, however, indicate that the variation of the temperature in the arch will be about 75% of the mean seasonal atmospheric variation for a spandrel filled arch; and for an exposed arch ring the temperature of the arch will vary nearly as much as in the atmosphere. (See Bulletin 30, Eng. Exp. Sta., Iowa State College, 1913.)

The values of M due to temperature are most conveniently taken from a diagram, being the ordinates between the neutral axis and a straight horizontal line drawn at a distance $(\Sigma y)/n$ below the crown if the ordinate below the crown represents on a certain scale the value of M_0', or these ordinates in feet multiplied by H'.

Example. For the arch discussed in Art. 39, the value of m is 0.45. Then for T = 45° and E = 2 500 000 × 144 lb. per sq. ft., the horizontal thrust due to temperature is $H' = \pm$ 27 000 lb. Hence under dead load the horizontal thrust ranges from 21 600 + 27 000 = 48 600 lb. at 95° Fahr. to 21 600 − 27 000 = − 5400 lb. at 5° Fahr.

Temperature changes will often, on the basis of the formulas, cause the line of thrust to pass outside of the middle-third of the arch ring. It does not, however, seem necessary, when temperature is considered, that the line of thrust at all joints, based upon the maximum moments, shall lie inside the middle-third. The following tabulation shows the stresses, in pounds per square inch, as given by the formulas, for an arch of 150-ft. span with a uniform thickness of 4 ft., the range of temperature being 40° Fahr. from the normal.

Ratio of rise to span	Crown			Skewback		
	Concrete *	Ashlar †	Ashlar ‡	Concrete *	Ashlar †	Ashlar ‡
1/2	40	35	65	70	65	110
1/4	110	105	180	210	190	340
1/5	150	140	240	270	250	445
1/10	340	320	550	625	580	1010
1/20	770	720	1260	1375	1280	2245

* E = 2 500 000 lb. per sq. in. and e = 0.0000060. † For average joints, E = 4 000 000 and e = 0.0000035. ‡ For very thin joints, E = 7 000 000 and e = 0.0000035.

Shortening of the Arch Ring under the compressive stresses would produce the same effect as a decrease in temperature if the compression were uniform throughout. If S = average compressive unit stress in the arch ring, this being found by taking the average of several values, or known in advance by specification, then the shortening per unit of length is S/E instead of eT, and accordingly

$$\text{Horizontal thrust } \mathbf{H'} = -\frac{\mathbf{Snma}}{\mathbf{n\Sigma y^2 - (\Sigma y)^2}}$$

but the expression for crown moment is same as before and crown shear is zero. For the numerical example of Art. 39, this value of H' is − 5300 lb., or about 25% of the thrust due to dead load.

If keyways are not provided the shrinkage of the concrete after setting will have the same effect as shortening of the arch ring under compression. There is no accurate way of computing the horizontal thrust resulting from shrinkage, but the following formula indicates the elements that enter into it.

$$\mathbf{H''} = -\frac{\mathbf{EC_snma}}{\mathbf{n\Sigma y^2 - (\Sigma y)^2}},$$

where C_s = shrinkage coefficient (Sect. 11, Art. 2).

The writer has given this formula to indicate that stresses result from concrete shrinkage, but suggests that in design it is best to use conservative allowable unit stresses rather than to attempt to apply any formula.

The Deflection of the Crown under load may be closely found by the following formula. For dead load, or for full live load, it is

$$f = -(M_0\Sigma x + H\Sigma xy - \Sigma Wzx)/mE$$

in which the summations are for the half span only, as in Art. 39. For any loading on the left of the crown the deflection of the crown is

$$f = -(M_0\Sigma x + H\Sigma xy + V_0\Sigma x^2 - \Sigma Wzx)/mE$$

and this expression applies also to any loading on the right of the crown if the negative sign be used before V_0. With these formulas, a positive result indicates downward deflection, and a negative result shows upward deflection.

The stiffness of the arch will be considerably increased if the roadway is part of a concrete superstructure which is integral with the arch. This will be the case with an open spandrel bridge, if the spandrel columns are fastened rigidly to the arch by reinforcing steel; or with a closed spandrel bridge with hollow superstructure, where the interior walls or columns are similarly attached to the arch. The resulting increase in stiffness will generally reduce the stresses in the arch due to dead and live loads, but may cause a decided increase in temperature stresses. The actual effect of the superstructure on the arch stresses is difficult to determine analytically, but may be investigated by the use of models. (See Art. 42.)

For Temperature Changes the deflection of the crown is much greater than that due to the loads. If the rise of the arch is b, and the half-span a, the deflection may be computed from

$$f_1 = -eTb - \frac{eTa(n\Sigma xy - \Sigma x\Sigma y)}{n\Sigma y^2 - (\Sigma y)^2}$$

the deflection being negative (or upward) for an increase in temperature.

In this formula, the first term gives the change in the rise of the arch which would occur if the ends were entirely free to move under temperature variations, and the second term gives the change in rise due to the restraint of the abutments. This is also shown in Fig. 120, in which SCS is the arch curve under normal temperature, $S''C'S''$ is the shape which the arch would assume after a temperature increase if the abutments were entirely free, and $SC''S$ is the shape which the arch must take due to the restraint at the ends. Most engineers have been accustomed to compute temperature deflection according to the second term of the above formula, neglecting the influence of the first term, but this results in a computed deflection less than the writer believes to be correct.

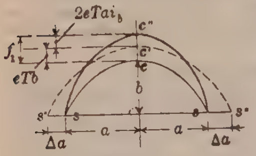


Fig. 120

The above equation can also be written

$$f_1 = -eT(b + 2ai_b)$$

where i_b is the thrust at the crown due to a unit vertical load on the arch at the crown. (See article by Prof. Hardy Cross, Eng. News-Rec., Vol. 96, p. 190.)

For important arches the deflection from load and temperature should be computed and allowance for this deflection should be made in the cambering of the centering.

In computations for deflection, either from direct loads or from temperature, it is essential that the sub-divisions of the arch be made relatively small in order to avoid errors due to the summation process involved in the formulas. This is particularly true in the case of an arch which has a considerable variation in thickness from crown to skewback. Where the skewback is exceptionally heavy, it may be better to consider the lower portion of the arch as part of the abutment, basing the computations on an arch of reduced span and rise.

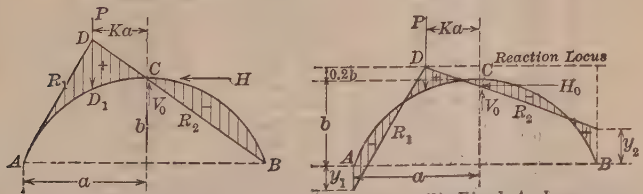
Tests made by the Austrian Society of Engineers on masonry arches ranging in span from 6 to 75 ft. have confirmed the correctness of the above formulas for deflection under load, and hence also the validity of the elastic theory (see Eng. News, Nov. 21, 1895, and April 9, 1896). Measurements of temperature deflections were made on a 145-ft. arch of the 6-span concrete bridge at Danville, Ill., in 1927, by the Engineering Experiment Station of the University of Illinois, the results confirming the above formula for temperature deflection. (Bulletin 174, Eng. Exp. Sta., Univ. of Illinois.)

Flow of Concrete. The elastic method of arch design assumes that the deformation of the masonry under a sustained load is the same as under a load applied for only a short time. Experiments and tests, however, have shown that when concrete is subjected to continuously acting stress, the deformation does not stop after the application of the load, but gradually increases during a considerable period of time, and the concrete is subject to additional deformation without increase of stress. (See paper by R. E. Davis, Proceedings Am. Concrete Inst., Vol. 24, 1928.) This yielding or "flow" of the concrete is variable, depending on the proportions of the concrete aggregate, conditions of mixing and curing of the concrete, and the intensity of the stress. In arches, temperature changes are generally gradual and extended over a considerable length of time, and the stresses due to temperature change will cause a gradual flow of the concrete, which tends to neutralize the temperature effect to a considerable extent. It is suggested that in reinforced concrete arches with reinforced spandrels and floor, if temperature stresses are computed as described hereinbefore, the unit allowed stresses given in Art. 16 may be increased 25%.

41. Diagrams and Influence Lines

Formulas for a Three-hinged Arch (Fig. 121*a*). For a single load on left of the crown,

$$\mathbf{H = Pa\,(1 - k)/2\,b} \qquad \mathbf{V_0 = P\,(1 - k)/2} \qquad \mathbf{M_0 = 0}$$



(*a*) *Three-Hinged Arch* (*b*) *Fixed Arch*

Fig. 121. Moments due to Load *P*

The moments are shown by the vertical lines above and below the arch curve. The moments at all hinges are 0. If the ordinate DD_1, to some scale equals the moments at D_1, then the moment at any other point equals the length of the vertical line at that point included between the neutral axis ACB and the resultant R_1 or R_2, measured to the same scale. Or the moment at any point equals H multiplied by the vertical line or ordinate included between ACB and R_1 or R_2, measured to the same scale as the arch. For uniform load over the entire arch, $H = wa^2/2\,b$, while $V_0 = 0$ and $M_0 = 0$. The resistance line is a curve passing through the hinges; it may coincide with the neutral axis or may lie above or below this axis.

Formulas for a Fixed Parabolic Arch (Fig. 121*b*). For a single load P at a distance ka from the crown,

$$y_1 = 2\,b\,(1 - 5\,k)/15\,(1 - k) \qquad y_2 = 2\,b\,(1 + 5\,k)/15\,(1 + k)$$

$$H = \frac{15\,Pa\,(1 - k^2)^2}{32\,b} \qquad V_0 = \frac{P\,(2 - 3\,k + k^3)}{4}$$

$$M_0 = \frac{-\,Pa\,(3 - 16\,k + 18\,k^2 - 5\,k^4)}{32}$$

y_1 and y_2 are laid off above AB when positive and below AB when negative. The reaction locus, or the locus of the intersection of all loads with their resultants, is a horizontal line 0.2 b above the crown of the neutral axis. The moments are shown graphically at all points of the arch. This may be drawn for any load by laying off the reaction locus and y_1 and y_2 computed by formulas. This moment diagram should be interpreted in the same manner as explained for the three-hinged arch. For uniform load over the entire arch, $H = wa^2/2\,b$, while $V_0 = 0$ and $M_0 = 0$.

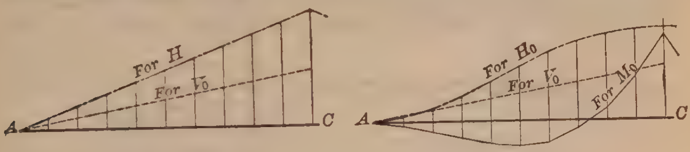
The following Table for Fixed Parabolic Arches is useful in obtaining H, V_0, and M_0 for a single load P at a distance ka to the left of the crown. The numbers of each column should be multiplied by the factors shown in the first line. This table may be used for the design of earth-filled bridges or for arches with spandrel arches or columns, if the loads are applied at tenth points. By use of formulas this table may be extended to cover all classes of parabolic arch design. The table may be used as a close approximate check for all positions of loading and for non-parabolic arches in which b/a is less than 0.5.

Thrust, Crown Shear, and Moment for Fixed Parabolic Arches

k	H	V_0	M_0
0	0.4687 Pa/b	+0.5000 P	+0.0938 Pa
0.1	0.4593	+0.4252	+0.0494
0.2	0.4320	+0.3520	+0.0161
0.3	0.3881	+0.2818	−0.0069
0.4	0.3308	+0.2160	−0.0203
0.5	0.2636	+0.1562	−0.0254
0.6	0.1920	+0.1040	−0.0240
0.7	0.1219	+0.0608	−0.0181
0.8	0.0607	+0.0280	−0.0103
0.9	0.0169	+0.0072	−0.0031
1.0	0.0000	+0.0000	−0.0000

An Influence Line is a line whose ordinates represent the values of a function as a single unit load travels over the span, the ordinates being drawn at the positions of the load. Thus in Fig. 122*a* let a load P travel from the left end A of the span to the middle C. Then the influence line for the horizontal thrust H is constructed by laying off from AC an ordinate at each position of the load to represent the value of H for that position. Since H varies as the first power of k in a three-hinged arch, the influence line is straight and it is only necessary to determine the ordinates for $k = 0$ and $k = 1$. For the three-hinged arch both H and V_0 are 0 when the load is at the springing line, and reach their maximum values at the crown, while M_0 is always 0. For the arch with fixed ends. Fig. 122*b* shows influence lines for H_0, V_0 and M_0 which have been constructed by the help of the preceding table. For both cases the ordinates for the other half span have the same values as for CA, except that those for V_0 are negative, as the load travels from C to B. The influence lines clearly show that loads near the crown produce the greatest thrusts, shears, and moments.

Where a moving live-load must be considered, the influence lines are most useful in determining the distribution of the loading which will cause maximum moments, shears and thrusts. Thus, from Fig. 122*b*, the maximum positive



(*a*) *Three-Hinged Arch* (*b*) *Fixed Arch*

Fig. 122. **Influence Lines**

value of M_0 will be found by multiplying the several positive ordinates of the influence diagram by the loads which may be applied at those points and taking their sum; and similarly, to obtain the maximum negative value of M_0.

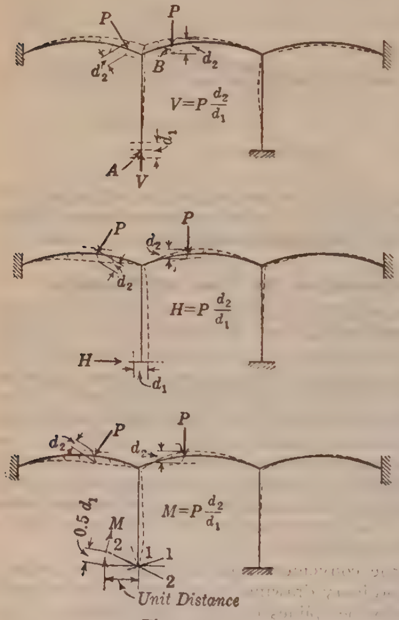
42. Analysis of Structures by Use of Models

Many masonry structures, especially bridge arches and arched dams, are statically indeterminate, i.e., their stresses can be computed only by taking into consideration the elastic deformation of the structure. As pointed out in Art. 38, the use of the elastic theory for this purpose involves certain assumptions and approximations which render the results approximate. Moreover, the correct use of this theory for the analysis of any complicated structure is accompanied with serious mathematical difficulties. To eliminate these troubles, various methods have been developed by which it is possible to find the stresses in a statically indeterminate structure by producing known deflections at selected sections of a model of the structure, and observing deflections at points of assumed loading. The writer recommends the use of models for checking stresses in indeterminate structures.

In order to make possible the convenient application of these methods within the limited space of an ordinary drawing board, use is made of small-scale elastic models. These may be splines of steel, hard rubber or other elastic material for the case of continuous beams or simple frames of uniform section. For more complicated structures, the models may be cut from celluloid sheets or fine quality cardboard. The widths of the model members are varied so that the moment of inertia of any section of the model will be proportional to that of the corresponding section of the actual structure.

An elementary application of the model method, for analyzing a continuous beam, is described by R. Fleming, Eng. News-Rec. Vol. 83, p. 428 (Aug. 28, 1919). This method is credited to Prof. George E. Beggs of Princeton University, and was first used by him to check the reaction influence lines for the Bessemer and Lake Erie Bridge near Pittsburgh. The extended development of this method for the practical solution of continuous structures of various kinds was described by Professor Beggs in Proceedings Am. Concrete Inst., Vol. 18, p. 58, (1922) and Vol. 19, p. 53 (1923).

Others employ steel splines or metal rods of suitable thickness, clamped or soldered together to form models of proportionate stiffness. Reference is made to the "Continostat" developed by Otto Gottschalk, Buenos Aires (described in Journal of Franklin Institute, July 1926, p. 61; and in Journal of Boston Society Civil Engineers, Vol. 14, p. 495); and to the "Nupubest" of Christian Rieckhof, Darmstadt, Germany (see Bauingenieur, Heft 7, 1925). A home-made apparatus applicable to continuous beams and simple frames employs hardened brass wires soldered together at joints by special details (see description by Anders Bull, Eng. News-Rec., Vol. 99, p. 920; also Vol. 100, p. 370). In the methods used by Gottschalk, Rieckhof and Bull, appreciably large deflections are applied to the model so that the desired deflections at the load points may be read without magnification. Although such devices are in special cases more convenient, they cannot easily be applied to structures with rapidly varying sections. They are not so generally applicable as Professor Beggs's apparatus. A detailed comparison of existing devices for use with structural models is given by Rudolph Bernhard in Bauingenieur, Heft 8, 1928, pp. 127-132.



Figs. 123–125

The above methods of analysis by models are based in general on Maxwell's theorem of reciprocal deflections. As a corollary to this theorem, it follows that for any elastic structure (Fig. 123) the reaction V at a point A due to a load P at B is given by $\mathbf{V = Pd_2/d_1}$, provided the deflections d_1 and d_2 are so minute that the structure is not appreciably deformed from its geometrical shape. The value of the ratio d_2/d_1 may be obtained by an experiment with an elastic model. It is only necessary to introduce a relatively small known deformation d_1 at the point A in the direction of the desired reaction component (see Figs. 123–125), and measure accurately the corresponding deflection component d_2 at B in the direction of the assumed applied load. Then for any assigned value of the load, the reaction component is computed by the formula $P\,d_2/d_1$. It is evident that this deflection ratio is independent of the value of the coefficient of elasticity E, provided E is constant in the range of stress.

Professor Beggs has developed the Deformeter Gage, shown in Fig. 126, for the accurate production of the deformations noted as d_1, in Figs. 123, 124 and 125. The essential feature of this instrument is the pair of clamps at Y. The upper bar is screwed to the drawing board; the lower bar is clamped tightly to the model, and is held to the upper bar by spring bolts. By changing the size or shape of the pins in the V-notches (marked g) a definite deflection d_1 of known amount may be applied to the model, either as a vertical or horizontal translation, or as an angular rotation. Thus, by changing from plugs h to i, a vertical upward deflection is produced in the model; plugs j will cause a counter-clockwise rotation; plugs k will produce a horizontal deflection to the left and plugs l a horizontal deflection to the right. By means of a microscope, the reciprocal deflection d_2 of the model at the assumed load point P is measured. The reaction at V is then given by the relation: $V = Pd_2/d_1$. The Beggs apparatus can be used to study practically any type of structure for which a model of suitable material has been made. (See description in

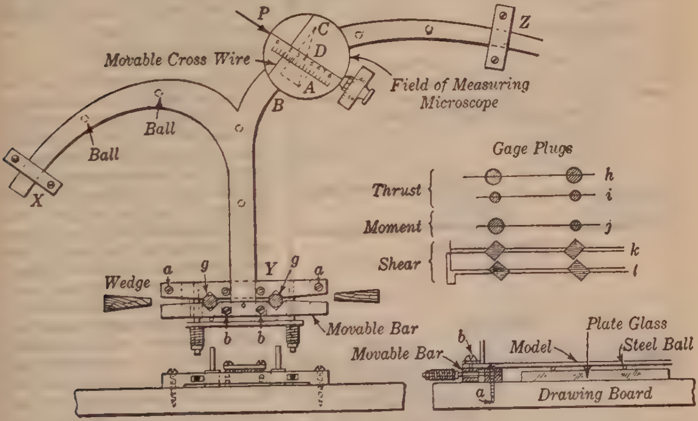


Fig. 126

Trans. Am. Soc. C. E., Vol. 88, p. 1208; also Journal Franklin Institute, March, 1927.)

The advantages of using models in analyzing indeterminate structures are:

1. This method makes unnecessary some of the assumptions required in the application of the elastic theory, particularly the omission of the influence of shear, and of the unknown distribution of stresses around corners in frames.

2. It gives an independent check upon analyses made by the elastic theory and permits further easy and rapid analyses which, on account of the time involved, would not be practicable with the elastic theory.

3. It eliminates elaborate mathematical formulas and laborious computations.

4. The method is largely self-checking because the components of the reactions obtained by model deflections may be readily checked by using the three equations of static equilibrium, which have not been previously used in the solution of the problem.

5. Difficult problems may often be more readily solved by models.

6. The use of models permits greater freedom in the choice of structural forms which one might hesitate to use because of the difficulty or impossibility of analysis by usual methods.

An important use of these methods in analyzing open spandrel arches is to determine the influence of the superstructure on the strength of the arch-barrel. This effect has generally been disregarded in design, although elaborate studies were made in connection with the La Balme Bridge (see Table, Art. 34) by the use of a plate-glass model and polarized light.' (Cornell Civ. Eng., Vol. 32, p. 17.) In general, the study of such structures by models shows that the superstructure acts to strengthen the arch-barrel in supporting direct loads, which may be taken advantage of by saving material in the arch itself. However, the increased stiffness produced by the integral superstructure may result in greatly increased temperature and rib-shortening stresses, both in the arch-barrel and in the superstructure itself. (See Art. 40.) Fig. 127 shows the effect of a continuous superstructure upon the moments and thrusts of the arch, and also illustrates the stresses that may be produced in the superstructure itself due to its construction integral with the arch.

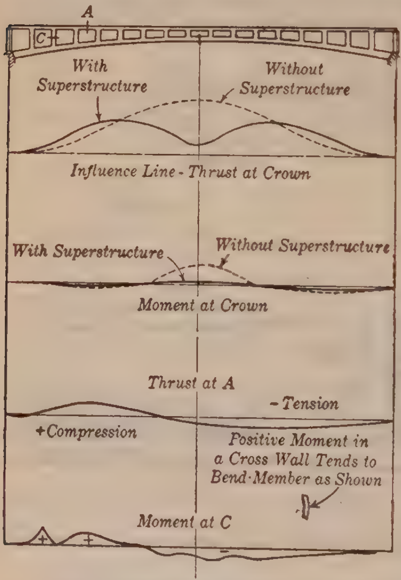


Fig. 127

In the design of the Arlington Memorial Bridge at Washington, D. C. (Fig. 136) the stresses in the arch and in the superstructure were studied by the use of models, the superstructure being designed to act integrally with the arch. In this bridge, the deck is supported by transverse walls resting on the arch. The ring stones are of granite ashlar, and the spandrel walls are concrete faced with granite. The weight of this heavy spandrel wall is transferred to the concrete arch-sheeting by the transverse walls, and the effectiveness of these walls for this purpose was also studied by means of a three-dimensional model. (See description by John Nagle, Military Engineer, Vol. 20, p. 154.)

The use of models is not limited to masonry structures. This method may be applied effectively to the study of indeterminate structures of many types and materials. (See also Art. 33, "Arched Dams.") For an application to reinforced concrete frames, see article by A. G. Hayden, Eng. News-Rec., Vol. 96, p. 686.

43. Construction and Erection

Materials of Construction. In selecting the material of which the arch is to be built, the designer must be governed by the available materials in the local market, due consideration being given to labor conditions. If cement, sand, and broken stone are cheaper than cut stone, the arch should be built of concrete, provided, however, architectural conditions do not demand stone. For arches over 150 ft. in span the stone arch may be cheaper than the concrete arch, but usually this will not be so. Occasionally when labor rates of cutting and setting stone are low and stone can be quarried at abnormally low rates, the stone arch may be cheaper for all spans

If the foundations are compressible the reinforced-concrete arch with 1% of steel is preferable, as it can better withstand deformation without failure. It is not correct to figure the reinforcement as carrying the total bending moment at all joints as is occasionally done, while the concrete is assumed to take the thrusts only. Such assumptions result in an absurdly high percentage of steel. However, when the cost of falsework is abnormally high, as for an arch of large span over a deep gorge, the percentage of reinforcement may be made high so as to reduce the amount of concrete and therefore the load upon the falsework. In such cases the cost of the falsework governs the design. As steel can never be stressed higher than 15 times the maximum compression in the concrete, or about 7500 lb. per sq. in., a large percentage of steel is not generally economical nor is a steel of high strength advantageous.

In a large work where the amount of arch construction is small, it is often better to build arches of stone, as they may not get the inspection care necessary for concrete. When stonecutters and setters are first-class and concrete labor poor, the stone arch is the safer arch to build, although if first-class inspection can be had, concrete work can be well done under nearly all conditions.

In order to decrease the cost of work at the quarries, it is common practice to make the voussoirs of a stone bridge of the same depth from the crown to the springing line, and to haunch the arch with concrete or rubble. When this is done the arch should generally be designed regarding the haunching as load and indirectly as a factor of safety. By bonding this haunching with stone voussoirs, however, they may be regarded as acting together and the combined masonry may be analyzed as an arch, in which case it is suggested that the safe working unit compressive stresses be taken as 70% of those recommended in the tables of Art. 16 for the weaker material.

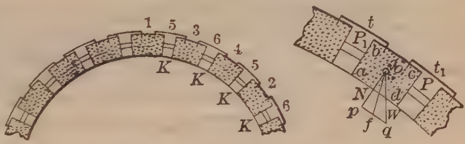


Fig. 128. Concrete Voussoirs

For arches over 100 ft. in span it is not good practice to use cut ring stones with the arch sheeting of rubble, concrete or brick, due to the great difference in the moduli of elasticity. Where large concrete arches, which should be built in alternate sections, are faced with stone voussoirs, the difficulties of erection are much increased.

The Architectural Details for all masonry bridges should be simple and logical. The accentuation of the roadway by an ornamental but simple coping and parapet, the accentuation of the ring stones and springing blocks by projection and simple ornamentation, the apparent strengthening of the abutment adjacent to the arch by projection, all tend to enhance the appearance of the bridge. The paneling and ornamentation of the spandrels, more commonly seen in concrete bridges, injure the appearance of the structure and lower the magnitude of its scale. No amount of ornamentation will relieve the awkward appearance of a bridge where spans and arch curves have been illogically selected. The ornamentation of a bridge depends upon its location. Where appearance is of paramount importance, a bridge should be designed so as to be in harmony with the surrounding landscape, present and future, whether it be rural or formal.

Drainage. The roadway of a masonry bridge should have drains at either side of the roadway at intervals of 30 to 40 ft., for a level roadway, and 100 ft. when the roadway is on a grade. These drains should have a minimum diameter of 2 in. and preferably 3 in. The minimum area of a drain in square inches should be $a = A/200$, where A = area of the surface drained in square feet. If a drain is placed at every 40 ft. of each gutter for a bridge 50 ft. wide, $A = 50/2 \times 40 = 1000$ sq. ft. and $a = 1000/200 = 5$ sq. in. If the drains are at 100-ft. intervals, $A = 50/2 \times 100 = 2500$ and $a = 2500/200 =$

12.5 sq. in. The drains should be designed with an intake trap, and the entire drainage system carrying the water to the ground should be so designed as to permit of easy cleaning.

In an arch with solid spandrels, the extrados of the arch sheeting should be waterproofed so as to prevent seepage of water through the masonry or concrete, and resultant discoloration of the soffit. This precaution may be advisable even where the roadway is properly drained, as there is always a possibility of rain water leaking from the roadway to the interior of the bridge.

Falsework. Arches are generally built upon temporary falsework called centers or centering, which may be of timber or steel. Timber centering is generally more economical and less subject to deformation from temperature changes, but it deforms due to changes in moisture content. The deflection of steel centering due to loads is about the same as for timber centering. The actual deflection of any type of centering does no harm, however, provided the amount of settlement can be estimated in advance and occurs prior to final closure of the arch. (For the design and construction of timber centering, see Sect. 9, Art. 13.)

Steel centers are generally more economical than timber when they can be used over again about four times, either by moving them transverse to the bridge or to several different arches of the same span length. They may be desirable where a clear opening is required under the arch during construction, for traffic, or to leave adequate water way. If the necessary foundations are provided the cost of steel centers will be approximately as follows:

For	Cost per cubic yard of arch concrete
40-foot span	\$4.00
80-foot span	6.00
150-foot span	8.00

Steel centering, however, is not usually feasible unless the steel sections can be handled by means of derricks.

Steel centers are generally constructed as 3-hinged trussed arches. They may be designed for unit stresses 10 to 15% higher than are customary for permanent structures, the loads being nearly static. It is advisable to make the general designs of the centering before the bridge piers are built, because usually the reactions of the centering can best be carried by masonry offsets near the top of the piers. This means of support saves the cost of building additional foundations to take the centering loads. (See Fig. 136.) The design of the offset must be predicated upon the design of the centering.

Provision also must be made to take the horizontal thrust of the centers at their supports. If the span is short, a horizontal tie rod can be used for this purpose, with turnbuckles for adjustment. If the arch is flat and of long span, the horizontal thrust of the centers must be taken directly by the pier, by the use of wood or steel wedges. Steel wedges are better than wood, and should be planed. The horizontal tie rod should also be provided, however, in order to facilitate lowering and moving the centering.

Provision must be made to enable lowering the centering after the arch is closed. This may be done by sand boxes (Sect. 9, Art. 13) or by wedges. Sand boxes are excellent but require more expert care, and generally steel wedges are best for this purpose. As steel centers are affected by temperature changes, the concrete should generally be placed in alternate sections, with keyways. The final closure should not be made at a time when a sudden drop in temperature is expected, for the resulting contraction of the centering might cause the fresh concrete to carry practically the entire dead load of the arch ring. (For comments on steel centers, see Eng. Rec., Vol. 65, p. 478.) Fig. 128a shows steel centers for the Arlington Memorial Bridge.

Concrete arches have been built using structural steel ribs as the reinforcing arch steel and to support the forms and green concrete during construction. This may be economical for a single narrow and short span, but not where the steel centers may be used several times.

Methods of Construction. All arches must be built symmetrically. That is, the centering must be loaded equally on either side of its middle. Stone arches under 75 ft. in span may be built continuously from springing line to crown, but unless the centering is abnormally unyielding for spans of more than 40 ft. hair-line cracks will occur at one or more joints, showing that the tensional value of the mortar is destroyed. Where the span is over 75 ft., these cracks may amount to 1/8 in. As a result, when the centering is struck

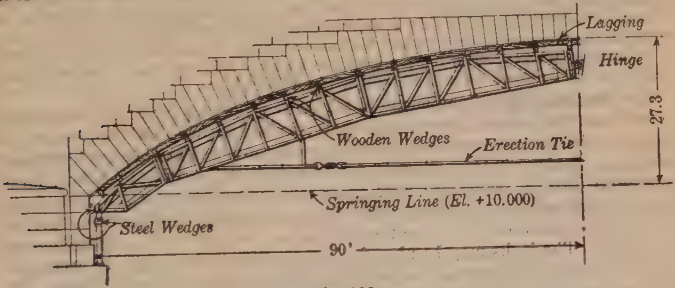


Fig. 128a

there will be a settlement of the crown of at least one inch. Therefore for spans of more than 75 ft., the masonry work should be laid up in alternate sections as shown in Fig. 128. The sections are numbered in the general order in which they should be built. The sections last built, called the keying sections and marked K, should be as small as will permit the men to work. The length of the larger sections should not exceed about 15 ft. In arches built of voussoirs each of which is composed of more than one stone, the masonry blocks should be "racked back." This is not shown in the figure.

Plain and reinforced-concrete arches under 40 ft. in span should be built as continuous work between umbrella points if possible. Arches of plain concrete over 40 ft. and under 75 ft. may be built continuously from the springing line to the crown; if over 75 ft. the alternate block system should be used.

Reinforced-concrete arches over 40 ft. and under 100 ft. in span may be built in longitudinal ribs 3 or 4 ft. wide, so arranged as to incorporate two or more ribs or lines of reinforcing steel. If so built the centering should be made very stiff, so that having one rib complete, the adjacent rib will not so deform the center as practically to strike the center under the first rib before the concrete of that rib is hardened. If built in ribs, transverse reinforcement should be used to tie the ribs together. Reinforced-concrete arches over 100 ft. in span would better be built in alternate blocks extending transversely across the bridge because of the danger of an adjacent longitudinal rib striking the centering under the completed ribs. The longitudinal rib method, however, has been successfully used in spans over 100 ft. As all centers settle and deform, the steel reinforcement becomes more or less buckled in construction and therefore will not be found in its theoretical position. In the construction of large arches it appears better to have the reinforcing steel in lengths not exceeding 30 ft., the length to be spliced after the large blocks have been placed. The splice may be an overlap, a bolted or a riveted joint. In concrete arches the concrete should be laid in horizontal courses or, where the inclination of the soffit is not too great, in courses parallel to the soffit. If alternate concrete blocks are used, the size of each block should be such that it can be completed as continuous work. Several keying blocks can usually be made in a day. The use of construction keyways also permits shrinkage due to settling and drying to take place without developing additional stresses in the concrete.

When arches are built in alternate sections the large blocks must be supported by stone, concrete or steel piers or columns until the key block masonry is in place (Fig. 128), or tie rods may be used as shown. The stresses in these piers or tie rods may be computed by resolving the weight of the blocks normal and tangent to the soffit (Fig. 128) and deducting the friction. $W = oq$ = weight of block. $N = op$ = normal thrust against the centering. $T = pq$ = tangential component. of is a line making an angle with N = the angle of friction of masonry upon wood. Then fq is the total stress in the pier P, due to the weight of block $abcd$. When the angle poq = or $<$ pof, the block exerts no pressure upon P. The upper strut P_1 may bring additional load upon P, due to the tendency of the upper blocks to slide down the arch centering. If tie rods are used, the stresses are found in a similar manner, but in that case the accumulated stress in the tie rod t is due to the lower blocks. As the arch centering settles or deforms, the block above the pier P rotates about the foot of the pier P and in consequence the pressure at the top and bottom of the pier is applied close to the edge. It is therefore suggested that when stone or concrete piers are used the unit compressive stress be taken at 40% of those given in Art. 16.

The Method of Rib-Compensation has been developed by French engineers to facilitate the construction of long-span concrete arches. In this method, a keyway is left at the crown until after the arch concrete has attained its proper strength. Jacks are then placed at intervals in this keyway, and a large horizontal force is applied to each half-arch through the jacks. This force may be made sufficiently large to raise the arch slightly, at the crown, thereby relieving the centering of its load and avoiding the difficulties of striking the centering. The jacks may be placed in any position relative to the arch axis, so as to bring the resistance line under dead load to the desired location. By this method it is possible to compensate for stresses due to rib-shortening under dead load. After the desired pressure has been applied, the spaces between the jacks are filled in with concrete, and subsequently the jacks are removed. Special provision must be made to properly strengthen the concrete of the arch at the crown in order to sustain the high unit pressures developed by the jacks. (See article by J. F. Brett, Jour. West. Soc. Engineers, Sept. and Oct. 1296; also Eng. News-Rec., Vol. 93, p. 463.) This method has been used on the St. Pierre du Vauvray, Villeneuve, Veurdre, Elorn and Boutiron bridges (see table, Art. 34).

Striking or Lowering of the Centering should be done gradually. The wedges or sand boxes should be lowered symmetrically, beginning at the crown and working to the springing lines. In a series of arches the centering between abutments or abutment piers should be struck simultaneously. The centering, which should be designed to carry the load of the arch, only, may be struck before the portion of the bridge above it is built, if the arch is stable under its own weight. If this is not done, expansion joints should be provided in the upper masonry, otherwise this masonry will crack, should there be any settlement of the crown of the arch, upon striking the center. Fig. 129a shows such joints for an earth-filled bridge, the expansion joints being marked jj, and 129b is a cross-section of one of these joints. Fig. 130 shows expansion joints in arched spandrels; they need only be left open, the spandrel arch centering remaining in place, until the large arch is struck. As the rise and fall of an arch, due to temperature, has a similar tendency to crack the masonry above the arch, such expansion joints are desirable to prevent temperature cracks. The joints of Fig. 130, when made permanent, should be about 1 in. wide, and the spandrel arches should be reinforced with steel so as to act as beams and cantilever beams. The spandrel expansion joints of the Walnut Lane Bridge, described in Trans. Am. Soc. C. E., Vol. 65, p. 423, are of excellent design. Expansion joints should extend from the back of the arch to the top of the parapet.

In an open spandrel bridge, if the crown of the arch is close to the roadway, the deck structure and arch will be joined together for a short distance on each side of the crown, forming a "saddle." In this case, no expansion joints need be provided adjacent to the saddle, the only joints required in the deck being near the ends of the arch above the skewbacks. Measurements of the action of expansion joints were made on a multiple-span open spandrel bridge at Danville, Ill. (Bulletin No. 174, Eng. Exp. Sta. Univ. of Illinois), which lead to

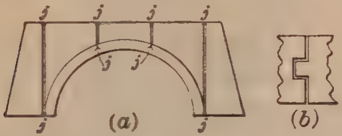


Fig. 129

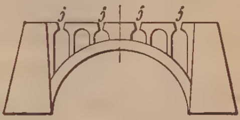


Fig. 130

the following conclusions: " There was practically no movement at the expansion joints adjacent to the saddles. These expansion joints are not needed to relieve the temperature stress in the deck, and they seem to weaken the structure by localizing the temperature deformation in the rib at the ends of the saddle."

In a three-hinged arch with permanent hinges, an expansion joint must be provided in the spandrels and deck over the crown hinge, as well as over the end-hinges.

The permanent paving of earth-filled bridges should not be laid for two years after the earth fill is placed, as the settlement will crack and warp the pavement. The filling should be spread in layers and thoroughly rammed to decrease the amount of settlement.

44. Large American Masonry Arches

Cabin John Bridge at Washington, D. C., completed in 1864, is the longest stone arch and the one having the largest ratio of span to width of bridge in America. It is further notable as the first American example of the use of hollow or cellular abutments. The arch was built up continuously from springing line to crown. The clear span is 220 ft. and the rise is 57.3 ft.

Walnut Lane Bridge at Philadelphia, Pa. (Fig. 131), completed in 1908, is the largest masonry arch in America. The main span consists of two detached parallel plain concrete arch ribs each supporting spandrel arches upon which rest low spandrel walls. These walls support steel I beams encased in concrete and the concrete jack arches of the floor. The main arch was built in alternate blocks.

The Edmondson Avenue Bridge at Baltimore, Md. (Fig. 132), completed in 1909, has piers and arches of plain concrete. The spandrels and floor system are of reinforced concrete. The order of building the large arch in alternate transverse blocks is shown in the sectional elevation.

The Bellefield Bridge at Pittsburgh, Pa. (Fig. 133), is a stone arch completed in 1900. It is the only large masonry bridge in America built with longitudinal spandrel arches. The necessary great width of the outside walls which must resist the spandrel arch thrust limits this type of construction to special

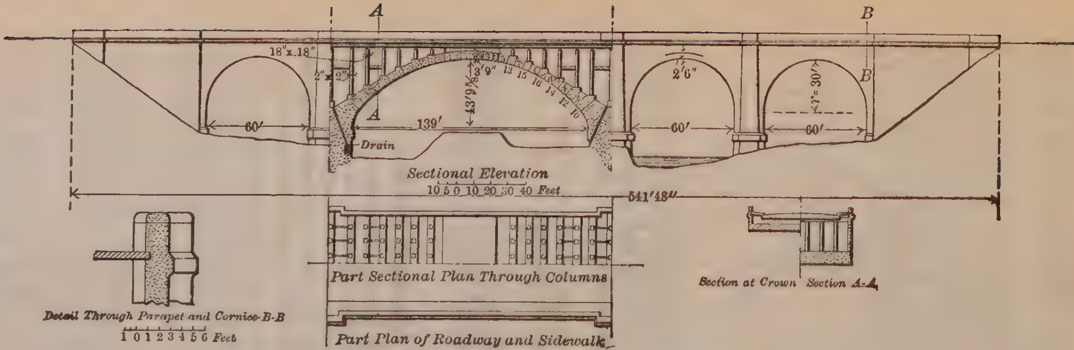


Fig. 132. Edmondson Ave. Bridge, Baltimore, Md.

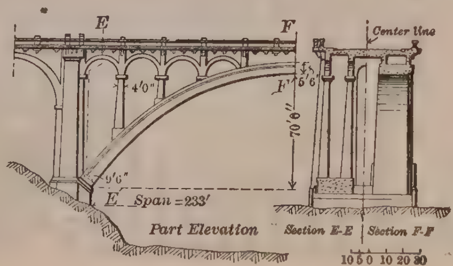


Fig. 131. Walnut Lane Bridge, Philadelphia, Pa.

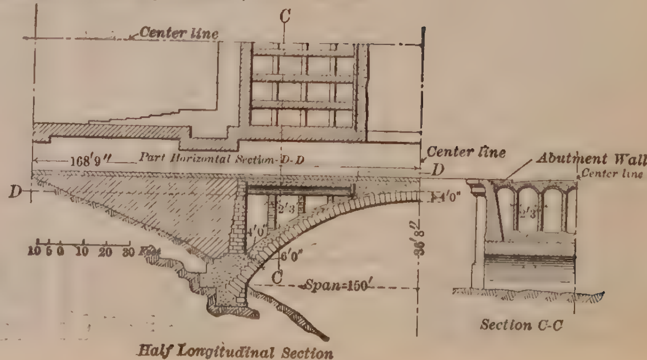


Fig. 133. Bellefield Bridge, Pittsburgh, Pa.

cases. The face wall in section *C-C* is marked abutment wall. The Bellefield arch was built continuously from springing line to crown.

The Pennsylvania R. R. Bridge at New Brunswick, N. J. (Fig. 134), consists of a series of stone arches having spans between 51 ft. and 72 ft. Two of these arches are shown in the figure.

Connecticut Avenue Bridge, Washington, D. C. (Fig. 135), was completed 1907. It consists of five 152-ft. spans and two 82-ft. spans. It is built of

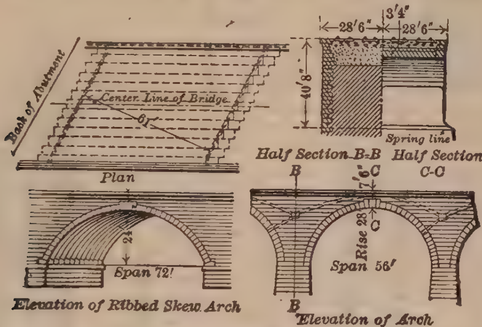


Fig. 134. Railroad Bridge, New Brunswick, N.J.

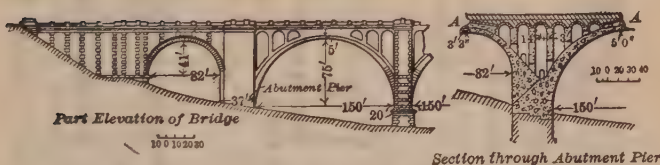


Fig. 135. Connecticut Ave. Bridge, Washington, D. C.

plain concrete. The coigns, belt courses, copings and other ornamental details were cast upon the ground, dressed and set in place as ashlar blocks. These details are indicated, in part, in the figure. Each spandrel arch nearest a pier and nearest the crown of the main arch has an expansion joint at its crown. These spandrel arches are therefore not true arches, and were reinforced with steel so as to act as simple and cantilever beams.

The Arlington Memorial Bridge, Washington, D. C. (Figs. 128*a* and 136), was under construction in 1928. This bridge, which is of monumental character, crosses the Potomac River, and consists of eight reinforced-concrete fixed arches and one steel-truss double-leaf bascule span. The clear arch spans vary from 166 ft. to 180 ft., and rises vary from 22.56 ft. to 27.38 ft.; crown thickness is 2.25 ft. The ring stones are of granite ashlar, and support a solid concrete spandrel wall faced with granite. The deck is supported by transverse walls spaced about 10-ft. centers, both deck and supports being reinforced concrete.

The Cappelen Memorial Bridge, Minneapolis, Minn. (Fig. 137), was completed in 1923. This bridge has the largest span of any concrete bridge in the United States (400 ft.) and with two side spans of 199 ft. The arches are fixed, and consist of two ribs, with open spandrels and beam-and-slab deck construction. The main arch reinforcement consists of latticed steel trusses.

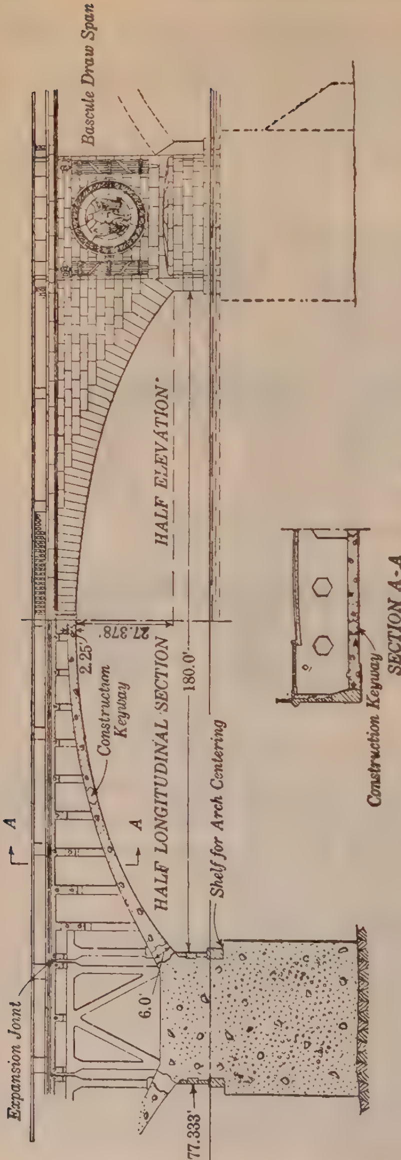


Fig. 136. Arlington Memorial Bridge

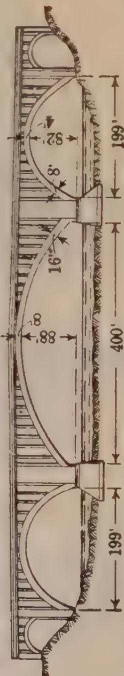


Fig. 137. Cappelen Memorial Bridge

45. Piers and Abutments

Engineers interested in design of bridge piers should consult "Hydraulic Laboratory Practice," edited by John R. Freeman, and published in 1929 by the Am. Soc. Mech. Engineers, New York.

Location of Bridge Piers should be based on (a) keeping the combined cost of superstructure and substructure a minimum, (b) securing a stable foundation, and (c) causing a minimum disturbance to stream flow and navigation. The cost of a pier does not vary greatly with the length of span. Topographical conditions, however, or the depth of water and the vertical distance to rock at various points of the river materially affect the cost of the piers and therefore the economical span length. The minimum cost for a steel bridge may be approximately attained if the cost of one river pier equals the cost of the main and lateral trusses of one span. (See " Roofs and Bridges," by Merriman and Jacoby, Part III, Art. 9.)

Piers should be located approximately parallel to the direction of the current so as not to cause (1) a shift in the river channel, (2) currents that may interfere with navigation, (3) erosion of the foundation bed, or (4) unnecessary obstruction to the flow of the stream resulting in increased backwater upstream in times of flood and possible damage and lawsuits. In order that the axes of piers may be approximately parallel to the direction of the current, it may be necessary to build skew spans. As skew spans result in higher cost of design and bridge fabrication (both of steel and masonry), they should not be adopted unless freshet and ice conditions and the direction of the current make them mandatory.

In navigable rivers the location of piers must be approved by the Secretary of War. The most important considerations in this case are: (a) Location of crossing relative to current and to river traffic requirements, (b) type of span over navigable channel, and (c) clear width of channel opening. (See paper by C. E. Smith, Proceedings, Am. Railway Eng. Assoc., Vol. 14, p. 185 of appendix.) If a movable span is undesirable because of its cost or the operating expense, or if the river banks are high, it may be desirable to build a high bridge. The War Department will generally approve a height that will clear shipping, which height on important streams may be 150 ft. or more above high water.

Dimensions of Piers. Height is usually determined by design of superstructure and required clearances above high-water level. With non-navigable streams, 10 ft. clearance above high water is usually sufficient to pass drift and ice. The pier top should be wide enough to allow a clearance of 1 to 1-1/2 ft. outside of bridge pedestals. Width under coping should be not less than 4 ft., and at least 1 ft. wider than the pedestals. Length under coping should be not less than distance out-to-out of pedestals plus 1-1/4 times the width of the pier. The coping thickness usually varies from 1 to 2-1/2 ft. The sides of the pier are usually battered, for stability and to improve the appearance. For tall piers, on rock or solid foundations, use a minimum batter of 1 : 48 or 1/4 in. per foot; for piers of ordinary height the usual batter is 1 : 24 or 1 : 12.

Forces Acting on a river pier may include:

(1) **End loads** of two adjacent bridge spans, including all dead load, and live load on either one or both spans. Impact of live load is generally omitted. Eccentricity of pier loading must not be overlooked where the two adjacent spans differ widely in length.

(2) **Wind Load,** generally assumed as 300 lb. per sq. ft. of exposed vertical surface of the pier and of both trusses and train, or 150 lb. per linear foot of bridge for both the upper and lower lateral systems applied at the panel points, and 300 lb. per linear foot of train applied 7 ft. above the base of rail. If end of pier is rounded, use 20 lb. per sq. ft. for wind on end of pier.

(3) **Traction Force of Train,** for which is used from 10 to 20% of the live load on one track applied at top of bearing. In determining the stability of a pier this force must be used with a good deal of discretion because of the indeterminate factors resulting from the continuity of the roadbed (the ballast and rails) and the friction in the roller bearings, all of which tend to transmit this force over more than one pier if there are a series of piers. Moreover, with a long train it is improbable that all the brakes will act simultaneously.

(4) **Centrifugal Force,** which occurs only when the track is on a curve, and which equals $0.00001167\ WV^2D$, where W = weight of train on track supported by the pier, V = velocity of train in miles per hour and D = degree of curvature of track. V may be taken as $60 - 2\text{-}1/2\ D$. Centrifugal force will be applied about 6-1/2 ft. above the rail and acts horizontally and in the direction of the radius of the curve away from the center of curvature.

(5) **Impact of Floating Debris and Ice,** which must be considered, as it may occur, but its value can only be conjectured. Unless it is liable to be considerable, it may generally be neglected. For small river piers it would be well to add to the normal pier thickness to guard against floating ice if records show probable large floes of ice. There are a few records of piers having been moved by floating ice.

(6) **Pressure of Flowing Water and of Solid Ice.** Ice pressure may usually be neglected. There are records, however, of piers having been moved by solid sheets of ice. This was believed to be partly due to the uplift of rising water and ice expansion. Therefore in designing small river piers in cold climates make them somewhat heavier than normal and have smooth masonry between high and low water. (See Art. 30, for pressure of ice against masonry dams.) Pressure of flowing water may be computed by the formula: $P = KWV^2/64.4$, where P = pressure in pounds per square foot, V = velocity of current in feet per second, W = weight of a cubic foot of water, and K is a constant. Greiner gives value for $KW/64.4$ as 1.5 for flat surfaces and 0.75 for rounded surfaces and minimum value for P as 150 lb. per sq. ft. for flat surfaces subjected to freshets and 50 lb. per sq. ft. for flat surfaces in tidal streams.

(7) **Forces Exerted by Expansion or Contraction of the Superstructure.** If the expansion support is a sliding plate, this force may be the reaction of the superstructure multiplied by the coefficient of friction; where suitable roller or rocker supports are used, the force will be much less than this maximum.

(8) **Weight of the Pier Itself.** If the foundation is pervious, the weight of that portion of the pier below water level should be reduced by 62.5 lb. per cu. ft.

Stability. A pier should be stable (based on the forces given in the preceding paragraphs) as to: (1) sliding downstream, (2) sliding parallel to the axis of the bridge, (3) overturning about the downstream toe, (4) overturning in the direction of the axis of the bridge. The maximum unit pressure at the downstream toe and at the side should not exceed the allowable masonry or foundation pressure (Art. 16). The stability should be examined under various combinations of track, wind and water loads.

Determination of stability consists of computing the point of action of the resultant reaction of the foundation, and the maximum unit pressure exerted on the foundation. In a plane through the pier at right angles to the bridge axis (see Fig. 138, sect. AA), let M_x = sum of the moments of all forces (vertical and those horizontal and transverse to the bridge) about the center line of the footing, O_1. In a plane through the pier parallel to the bridge axis (sect. BB), let M_y = sum of moments of all forces (vertical and those horizontal and parallel to the bridge) about the center O_2. W = sum of all vertical loads, that is, the reaction or the magnitude of the resultant; x = longitudinal eccentricity; and y = transverse eccentricity of the reaction. Then $\mathbf{x} = \mathbf{M_x/W}$ and $\mathbf{y = M_y/W}$. These coordinates therefore locate the point where the resultant intersects the pier base. With W and x and y known, the maximum and minimum unit stresses on the rectangle base are computed by the formulas of Art. 22. The stability against sliding of the pier on its foundation should also be considered, using the frictional coefficients given in Art. 16. Except upon firm and rough rock, the ratio of the sum of the horizontal loads to the vertical, $\frac{\Sigma P}{W}$, should not be greater than 0.36, and the departure of the resultant from the vertical should not exceed 20 deg. if friction alone is to be depended on. On rock the horizontal loads or pressures may be entirely taken by cutting the rock into steps, or by cutting depressions in the rock into which the masonry may be keyed.

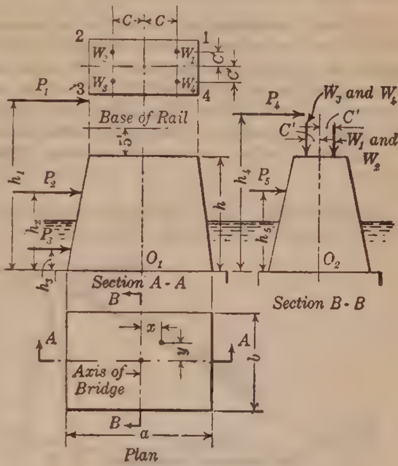


Fig. 138

For example, let the vertical loads W_1 and W_2 of the larger span (Fig. 138) include live load, W_3 and W_4 of the smaller span being dead load only. Assume two double-track spans, one 400 and the other 300 ft. long, dead load being 2000 lb. per linear foot of track and live load 4000 lb. per linear foot of track. Then $W_1 = W_2 = 600$ and $W_3 = W_4 = 150$ short tons. Lateral wind pressure is 300 lb. per lin. ft. (150 lb. per linear foot of top and bottom lateral systems) on truss and 300 lb. per linear foot of train. Wind reaction on pier from long span = 30 tons on truss applied 25 ft. above top of pier (the distance of the middle of the truss above the top of pier) and 30 tons on train applied 12 ft. above top of pier; wind reaction on pier from short span = 22.5 tons applied 20 ft. above top of pier. Hence resultant wind reaction is P_1 = 82.5 tons, and is applied 18.9 ft. above top of pier.

Dimensions of pier are a = 26 ft., b = 18 ft., h = 21 ft. Weight of pier (approximated) = 360 tons, after subtracting weight of displaced water; c = 6 ft., c' = 2.5 ft. P_2 = wind on end of pier (approximated) = 2 tons. P_3 = pressure of current (150 lb. per sq. ft.) = 10 tons. P_4 = traction force of train (taken as 10% of live load reaction of one track only) = 0.1 × 400 = 40 tons. P_5 = wind on side of pier = 0. (For very high piers a check computation should be made with the wind 30 lb. per sq. ft. in this direction and 0 parallel to the length of the pier.) h_1 = 39.9 ft.; h_2 = 14 ft.; h_3 = 4 ft.; h_4 = 21 ft. (All h's are approximate in this example.)

Then $\mathbf{M_x = P_1h_1 + P_2h_2 + P_3h_3 = 3360}$ **ton-ft.**

$\mathbf{M_y = P_4h_4 + (W_1 + W_2)c' - (W_3 + W_4)c' = 3090}$ **ton-ft.**

$\mathbf{W}$ **= sum of vertical loads =** $\mathbf{1500 + 360 = 1860}$ **tons**

Then $\mathbf{x = M_x/W = 1.81}$ **ft.**

$\mathbf{y = M_y/W = 1.66}$ **ft.**

By the formulas of Art. 22, the stress at corner 1 of the base of the pier $= S_1 = \frac{W}{ab}\left(1 + 6\frac{x}{a} + 6\frac{y}{b}\right) = 7.84$ tons per sq. ft., compression; and at the opposite corner $S_3 = 0.12$ ton per sq. ft., compression. The resultant lies within the kern of the base, hence compression everywhere prevails.

Tendency to slide or rotate may be diminished, when the pier is on a rock foundation, by the use of anchor bolts (Art. 22). If the current is very strong, or if it is probable that the river may become choked with ice, round dowel rods, extending for at least 40 diameters into both rock and pier, may be used, as seen in pier F of Fig. 139. This figure illustrates the forces which act on the piers of a bascule bridge, which may include wind on the open bascule leaf in the direction of the bridge axis in addition to the forces discussed above.

Shape of Pier. The rectangular shape is the cheapest, but causes greater interference with the current. Experiments by F. A. Nagler at the University of Michigan led to following conclusions (Trans. Am. Soc. C. E., Vol. 82, p. 334): (1) Best practical form of nose is either the half-round or the semi-elliptical, these being better than the pointed nose. (2) Backwater may be appreciably reduced by using an efficient tail. Best practical form of tail is either half-round or pointed. (3) The 90-deg. nose is not satisfactory; best angle for a pointed nose is 45 deg. or less. (4) For piers of same design upstream and downstream, the half-round shape gives least backwater. Where the nose is pointed and circular segments are used, the curves should be tangent to sides of the pier and the radius should be greater than half the pier thickness.

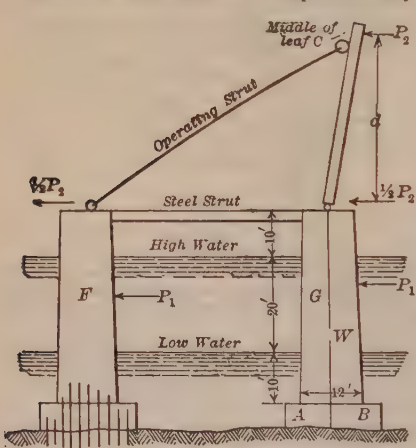
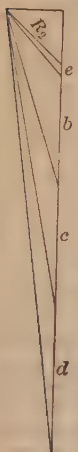


Fig. 139. Piers of Bascule Bridge

Piers of Arch Bridges are subject to horizontal thrust from the skew-backs of the adjacent spans, and the difference of these acts at the top and causes an overturning tendency. The ordinary arch pier should be analyzed for one adjacent arch without live load and the other adjacent arch with live load over the whole span. It is not necessary to consider thrust due to temperature or rib shortening except when two reinforced concrete arches of widely different spans rest upon the same pier and the larger arch has a rise of less than 1/6 the span; even then the consideration of these thrusts may be an unwarranted refinement. If the pier is an abutment pier it should be analyzed for either arch standing and the other arch removed and for both arches standing. Every third or fourth pier in a series of arches should be built wider than the others so as to act as an abutment in case of failure of one or more of the arches.

Fig. 140 shows graphic analysis for an abutment pier for the case of the left arch standing and the right arch removed. The pier is arbitrarily divided into horizontal courses for the purpose of analysis. The resultant pressures are shown for each joint, and the resistance line passes through the points where the resultants cut the joints. For each joint the distance *e* from the middle to the resistance line is measured and then the maximum stresses may be found from the formulas of Art. 21. The resistance line should lie within the middle-third of all joints. The inclination of the resultant thrust at each joint and at the foundation should be noted and the safety factor against sliding be determined.

Abutments for Arches. The method of analyzing the portion of an arch abutment which takes the arch thrust is similar to that used for piers, except that for an earth-filled bridge the earth pressure against the abutment should be combined with the other forces. In the case of a railway bridge the pressure produced by the earth fill behind an abutment should be increased by a surcharge of 800 lb. per sq. ft. For a single span, the cost of the abutments often exceeds the cost of the arch. The U abutment, Fig. 141, is uneconomical except for a very wide or very low bridge. If built upon a compressible foundation a crack *cc* will usually form at the junction of the transverse and longitudinal walls. This may be prevented by steel rods *rr* or by providing an expansion joint at *ee*.



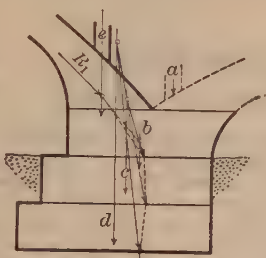


Fig. 140

In the design of U abutments the earth pressure against the back of the longitudinal walls may be less than against an ordinary retaining wall, since the wedge of earth which exerts pressure against the back of the wall is smaller. At the approach end of the abutment, the earth at the outside of the wall is effective in resisting the earth pressure at the back of the wall. It is not believed advisable, however, to decrease materially the masonry at the approach end on this account, as the outside fill may be disturbed by future excavation. As an additional factor of safety against rotation, the longitudinal walls may be tied together by steel rods.

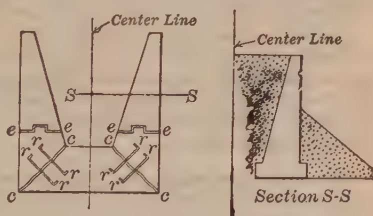


Fig. 141

A good type of abutment is shown in Fig. 142 for the case where the earth pressure is against the outside of the walls. The interior is cellular as shown in the plan or horizontal section *bb*. The earth thrust is taken by the interior walls acting as buttresses, *ss* being a line at the top of the outside earth-filled slope. The cells below *ss* may be partially filled with earth to help resist the pressure of the outside earth. Above this line the masonry walls may be changed to columns, as they carry only vertical loads of the roadway. The abutment may be built of stone, plain concrete, or reinforced concrete. If built of stone or plain concrete, the masonry should be arched as shown by dotted lines in the sections *aa* and *bb*; if built of reinforced concrete the arches may be changed to flat slabs as shown in full lines.

Owing to lack of knowledge regarding the material that may be deposited behind abutments by contractors, no very definite computations regarding earth thrust can generally be made. Probably a horizontal pressure equal to one-third of that of water will be sufficient for most cases.

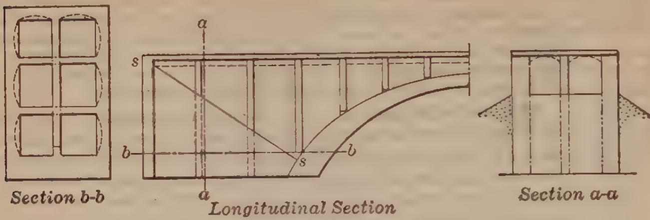


Fig. 142. Cellular Type of Abutment

DOMES OF MASONRY

46. General Data for Domes

Definitions. A dome (Fig. 143) is a spherical or spheroidal vault, a solid of revolution with vertical axis. The section of the inner surface of the dome (soffit) may be **semicircular** (Fig. 144), **pointed** (Fig. 145) or **segmental** (Fig. 146); in the last case the dome is also termed a " cupola." Domes may be closed, as that shown in Fig. 149, or open on the summit (Fig. 143), the opening being termed the " eye " of the dome. The rim of the eye frequently supports a structure called a lantern (Fig. 147). The section of the outer surface (extrados) is usually similar to that of the soffit, but may have a different curve. The solid bounded by the soffit and extrados is the **shell** of the dome. The bed joints are conical. Annular parts of the shell between bed joints (true or imaginary) are called **crowns.** In Fig. 143 the points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, inclose one-fourth of a crown. **Voussoirs** are parts of the crown included between meridian joints, that is, vertical planes passing through the axis of the dome.

The plan of the dome is always a circle; it may be supported on a circular wall (drum) or carried over a square or polygonal area, resting upon walls or arches whose plan circumscribes the plan of the dome. The vaulted areas between the arches or walls and the base of the dome are called **pendentives** (Fig. 148). The joint between the shell of the dome and the supporting walls is the springing joint.

Data and Precedents. The dome is a remarkably stable form of structure; it can be shown analytically to be stable with a uniform thickness of 23/1000 of its diameter, which is less than 1/3 of the necessary thickness of a semicircular masonry arch of uniform thickness, carrying only its own weight. If the thickness of the shell is tapered from the springing joint toward the summit, it need only have a volume of 9/16 of the thinnest uniform dome of the same span. If enough steel bands are provided at the springing joint, the upper portion of the shell extending down 26 deg. from the summit can be made abnormally thin and yet be stable. The dome of the Cathedral of St. John The Divine, in New York City, built by Gustavino, is an excllent example of this. It has a diameter of 110 ft. and is built of tile, having a thickness of 4 in. for 29 deg. from the summit or 3/1000 of the span. For the next 19 deg. it is 6 in., and from there to the springing line 12 in. thick.

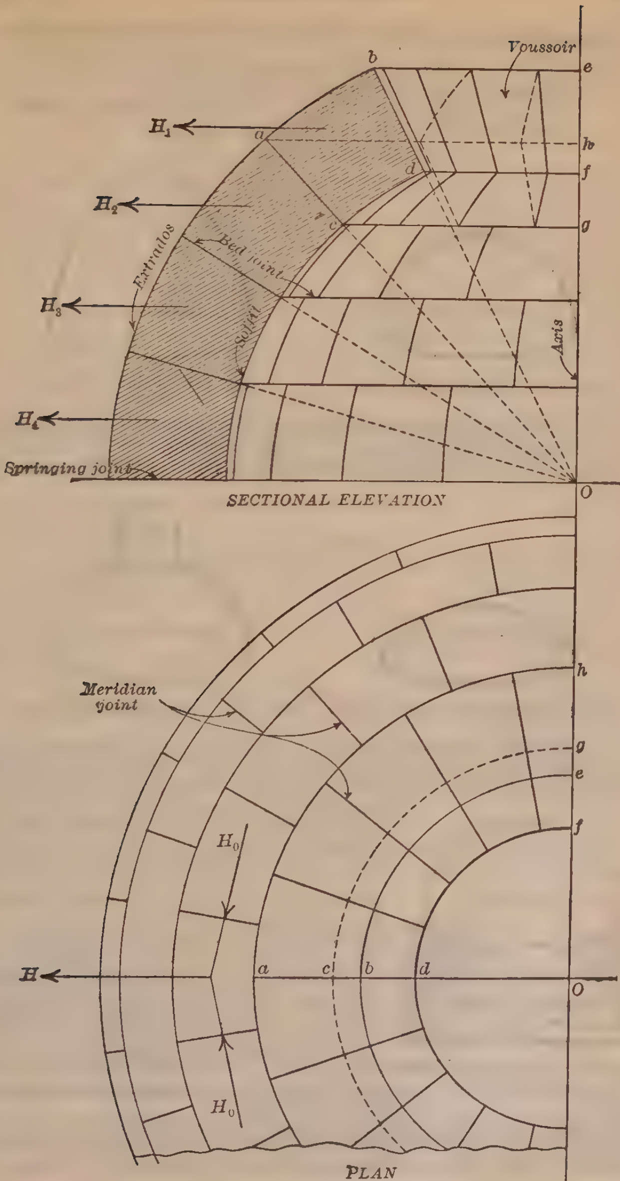


Fig. 143. Plan and Elevation of a Dome

According to Rondelet the following thicknesses are usually given to domes built of brick:

For spans up to 13 ft., a uniform thickness of 4 in.
For spans from 13 to 20 ft., a uniform thickness of 8 in.
For spans from 20 to 25 ft., 12 in. at the springing, 8 in. at the summit.
For spans from 25 to 35 ft., 16 in. at springing, 8 in. at the summit.

Fig. 144. Hemispherical Dome

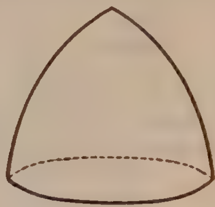


Fig. 145. Pointed Dome

Pointed domes are generally stronger than hemispheres; a dome generated by the revolution of an equilateral arc (60 deg.) requires, to be self-supporting, a thickness of only 5/8 of that of a hemispherical dome of the same span.

Fig. 146. Segmental Dome (Cupola)

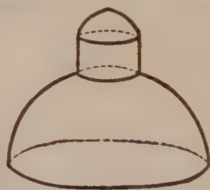


Fig. 147. Open Dome with Lantern

The dome of the Pantheon in Rome, built about A.D. 112, is the largest existing hemispherical dome of masonry, its diameter being 142 ft. It rests on a circular wall nearly 20 ft. thick which, however, is interrupted by 8 large and 8 small niches. The dome is open on the top, the "eye" being about 30 ft. in diameter. The lower courses of the dome, built of Roman tile, were all laid horizontal up to about 40 deg. from the springing line. Above this the construction is not known but is said to consist of tile arches and ribs of concrete. The outside of the dome shows a strong masonry backing beginning at the springing line and extending about half-way up.

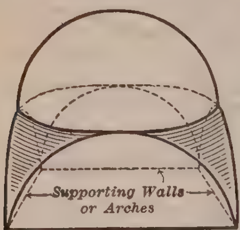


Fig. 148.—Hemispherical Dome and Pendentives

The dome of Santa Sofia in Constantinople, built by Justinian, A.D. 532–537, from the designs of Anthemius of Tralles and Isidorus of Miletus, has a diameter of 104 ft. and is carried on pendentives over a square area. The shell consists of 40 stone ribs and the space between same (about 6 ft. at the springing line) is filled with brick masonry. The dome is closed at the summit and has 40 windows between the ribs.

The dome of St. Peter's Cathedral in Rome, built by Michelangelo, has an inside diameter of 140 ft. and is built of brick. The lower quarter of the shell is of solid construction 9 ft. thick. The upper three-quarters of the height consists of a double shell.

The outer shell, which carries a lantern, is raised higher than the inner one and the two shells are connected by 16 stone ribs. St. Paul's Cathedral in London, designed by Sir Christopher Wren, has three shells; the innermost is hemispherical, open at the summit, the middle one is conical and carries the lantern, the outer dome is framed in timber and covered with lead

47. Conditions of Stability

The dome is usually built without centering, by constructing the crowns in succession, beginning at the springing joint. Every complete crown is self-supporting; consequently open domes may be stable. This stability is the result of tangential pressures acting normally on the meridian joints (H_0 in Fig. 143). Pressures of this character do not exist in simple arches. In Fig. 149, two meridian planes are shown in plan making a small angle ϕ with each other. They cut from the dome two lunes having contact only in a line at O, or, if the dome were open at the summit, the lunes would have no common point at all. The difference between the stability of the arch and the dome is manifest. The arch for its stability depends upon the balancing of the horizontal thrust of each half on either side of the crown or summit. In case of the dome the contact of symmetrically opposite sections (lunes) is in a line without area and incapable, therefore, of transmitting horizontal thrust which would counteract the tendency of the opposing sections to fall into the space below. Therefore the stability of the lunes must be the result of tangential forces or forces acting upon their sides (that is, upon the meridian joints), and normal to them.

These forces, called crown thrusts, are horizontal and act upon both sides of each voussoir (H_0 in Fig. 143). They result in the forces H or H_1, H_2, etc., which act radially outward in a horizontal direction. The forces H are termed resultant crown thrusts. Consider first the uppermost crown of a dome. To insure stability, in other words to prevent the voussoir from rotating or sliding upon its bed joint, the magnitude of H_1 must be such that if it is combined with the weight of the voussoir W_1, the resultant force P_1, called the meridian thrust, shall cut the bed joint ac and its direction shall not be inclined to the normal to the bed joint by more than the angle of friction of masonry upon masonry, say 30 deg. The meridian thrust acting upon the bed joint ac of the first voussoir will be transmitted to the crowns below, and at each voussoir new forces H_2, H_3, etc., must exist to maintain stability. In the case of domes having a high rise there will be found a bed joint called the joint of rupture below which no additional forces H occur, the above requirements of stability (as to the resultant) being fulfilled without the action of such additional horizontal forces. From this joint downward, the dome can be treated as a simple arch.

48. The Line of Pressure

Crown Thrust. In the analysis of stresses in domes of masonry it is assumed that the least horizontal force which is sufficient to keep the voussoirs from rotating and sliding will be the resultant crown thrust H (Fig. 143) if its combination with the weight of the voussoir, the meridian thrust, does not cut the bed joint so close to the edge as to crush the masonry or, due to the elasticity of the masonry, cause the joint to open on the opposite side.

The Meridian Thrusts, P_1, P_2, etc., in Fig. 149, result from combining weight W of the voussoir and the resultant crown thrust H. In order to make the latter a minimum, H must be applied as near to the extrados as possible and P must cut the bed joint as near the soffit as possible, consistent with the foregoing requirements. This may readily be proven graphically by assuming any other point of application of these forces. Theoretically, in order that no

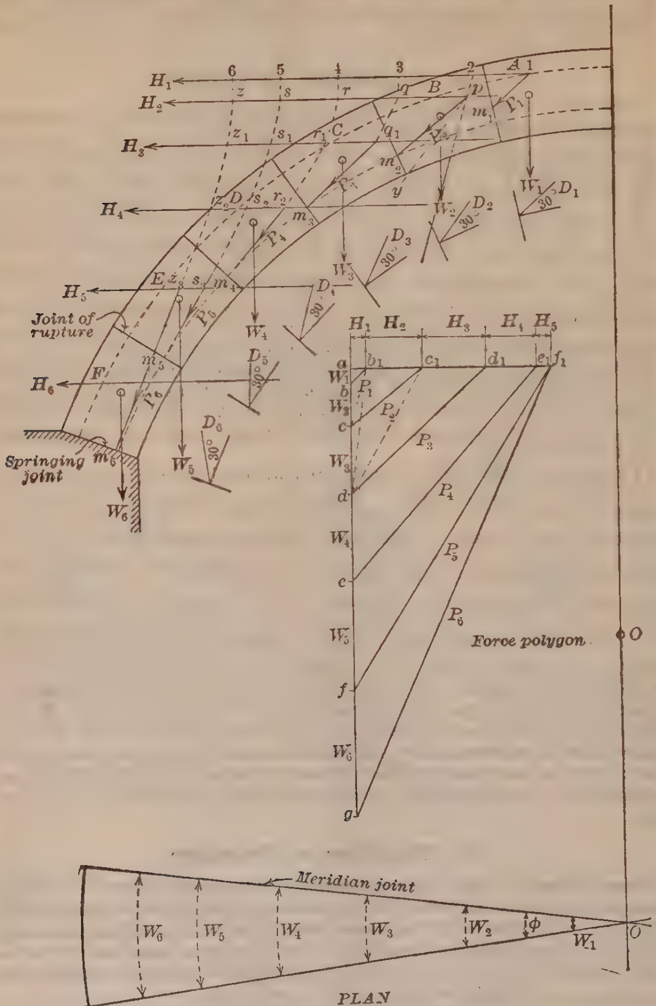


Fig. 149. Graphic Analysis of a Dome

joint may open, the resultant force upon any joint shall not lie outside the kern or the middle-third (Art. 21 and 22). In the case of domes, however, investigation of domes which have been standing for centuries shows that the limits of the middle half of the meridian and bed joints can be safely used as points of application for both crown thrust and meridian thrust. The use of these limits furnishes smaller dimensions than the use of the middle-third points and is here recommended for practical design. Therefore the point of application of the minimum force H should be taken at the upper middle half limit of the shell and the meridian thrust should be applied at the lower middle half point of the bed joint, provided that the condition of stability against sliding permits this location of the meridian thrust. If with the help of the so-found forces H a line of pressure can be drawn that will be within the middle half of the shell, the dome will be stable, provided that the unit stresses on both the meridian and the bed joints are not excessive.

49. Graphic Analysis

In Fig. 149 are shown a plan of a lune and a meridian half section of a cl sed dome. The soffit of the dome is segmental, the radius of the soffit being 8 ft. and its rise 5 ft. The central angle ϕ of the lune is 15 deg. The shell of the dome is 1 ft. 3 in. thick throughout. The section is divided by full radial lines into voussoirs 1 ft. 9 in. long on the center line. These radial lines form the bed or conical joints. The middle half limits of the shell are shown by the dotted curves and their intersections with the bed joints give the middle half limits of these joints. The weight of each voussoir acts at its center of gravity, which may be taken, with sufficient accuracy, as coincident with the geometrical center of the voussoir, except at the topmost wedge-shaped one in which the center of gravity is taken at the lower third. The weights of the voussoirs, including the superimposed loads if any, may be determined graphically or analytically. In the example given (Fig. 149) the weights of the voussoirs, W_1, W_2, W_3, etc., are proportional to the lengths of the arcs W_1, W_2, etc., of the plan which pass through the geometrical centers of the voussoirs. The lengths of these arcs are used directly in the force polygon to represent the weights of the voussoirs. The true magnitude of all the forces shown on the force polygon will therefore be found by measuring their lengths to the same scale as that of the plan of the dome, and multiplying these scaled lengths by the length of the voussoir $\times$ the thickness $\times$ the weight per unit volume of the masonry. W_1, W_2, to W_6 show on the section the lines of action of the vertical force.

The lines of action of the horizontal resultant crown thrusts H_1, H_2, etc., are assumed to be and are shown as applied at points A, B, C, D, E, and F, which points are located at the upper limit of the middle half of the shell and on the same radii as the centers of gravity of the corresponding voussoirs. The lines of action of the combined weights $W_1 + W_2$, $W_1 + W_2 + W_3$, etc., may be determined graphically or analytically and are shown in Fig. 149 as passing through points 2 to 6, inclusive. For example, the line of action of combined loads $W_1 + W_2 + W_3 + W_4$ passes through point 4. At D_1, D_2, etc., the limiting directions of the meridian thrusts (which must not be inclined to the normal of any bed joint by more than 30 deg.) are shown for every conical joint.

Point 1 is the intersection of the weight W_1 of the first voussoir and of the resultant crown thrust H_1. The resultant of these two forces must pass through this point and its direction, if the dome is to be stable, must be such that it will intersect the first bed joint between the middle half points, and it must not be steeper than the limiting direction D_1. Draw $1m_1$ (in section) and also bb_1 (force polygon) parallel to D_1; bb_1 is the probable magnitude of the meridian thrust P_1, because if its direction were steeper it would not fulfill the condition necessary to prevent sliding; if it were less inclined, it would require a greater resultant crown thrust than the minimum. Therefore, m_1 is a point of the probable line of pressure and ab_1 (force polygon) is the magnitude of the resultant crown thrust H_1 acting on the first voussoir.

The combined weight of voussoirs 1 and 2 passes through point 2. If no additional crown thrust were present the meridian thrust on the second bed joint would be obtained by combining H_1 and $W_1 + W_2$, in other words by drawing line $2\,x$ through 2 parallel to b_1c (force polygon). This line $2\,x$ falls outside the second bed joint and is too steep for stability against sliding. Therefore, at point p (the intersection of $2\,x$ and H_2) an additional horizontal force acts, if the dome is stable. If through point p, a line py parallel to the limiting direction D_2, is drawn, this line is also seen to fall outside the joint. The minimum resultant crown thrust H_2 acting at the second voussoir and consistent with the requirements of stability, will therefore be such as to make the meridian thrust pass through the lower middle half limit m_2 of the second bed joint.

Draw pm_2, in which m_2 is a seond point of the line of pressure; and draw cc_1 (force polygon) parallel to pm_2. Then cc_1 is the magnitude of the meridian thrust P_2 and b_1c_1 the magnitude of the resultant crown thrust H_2.

A similar construction is carried out for the other voussoirs to determine meridian thrusts P_3, P_4, P_5, and the resultant crown thrusts H_3, H_4, H_5. At voussoir 6 it is found that the sum of the horizontal forces H_1 to H_5 is sufficient to insure the stability of the voussoir and therefore the value of H_6 reduces to zero. Bed joint 5 then is the joint of rupture of this dome.

The magnitude of the crown thrust H_0 (the normal force acting on the meridian joints (Fig. 143) can be calculated by the formula: $H_0 = H/(2\sin \phi/2)$ or if ϕ is small $H_0 = H/\phi$, ϕ being measured in radians and H being the value of H_1, H_2, etc., for the voussoir in question. After the magnitudes of both the crown thrusts and meridian thrusts have been determined, the maximum unit stresses upon the bed joints and the meridian joints can be obtained by the methods given in Art. 21 and Art. 22.

It is sometimes necessary, in order to insure the stability of a dome or its supporting walls, to eliminate the transmission of the resultant crown thrust either at the joint of rupture or at the springing joint. This may be done by hoops of iron or steel. The magnitude of the tension for which these hoops must be figured can be determined by the above equation, where, in this case, H_0 denotes the tension in the band (hoop tension) and H denotes the sum of the values H_1, H_2, etc., at and above the joint in question.

Numerical Example. How large is the crown thrust H_0 acting on the meridian joint of voussoir 3 (Fig. 149) and what is the maximum compressive stress on this joint? By scaling (force polygon) H_3 is found to be 1.0 ft., the length of the voussoir is 1.75 ft., its thickness is 1.25 ft., and the weight of the masonry is 165 lb. per cu. ft. Then

$$H_3 = 1 \times 1.75 \times 1.25 \times 165 = 360 \text{ lb.}$$

ϕ being 15 deg., $H_0 = 360/0.262 = 1385$ lb.

By Art. 22, $$S_{max} = 2\,H_0/3\,rb,$$

where $r = 1/4 \times 1.25$ and $b = 1.75$ ft. for the case here discussed.

$$S_{max} = 8 \times 1385/3 \times 1.25 \times 1.75 = 1740 \text{ lb. per sq. ft.}$$

What is the required section of iron bands necessary to take the hoop tension at the springing joint so as to make the thrust at the springing vertical? $\Sigma H = 3.15$ (force polygon) $\times 1.75 \times 1.25 \times 165 = 1135$ lb. $H_0 = 1135/0.262 = 4335$ lb. If the allowable unit stress for iron is 8000 lb. per sq. in., the iron section required $= 4335/8000 = 0.54$ sq. in.

In a Conical Dome, if the base is kept from spreading by friction on a firm abutment or by means of a hoop, tension is nowhere exerted from the summit down.

References on analytical solution of domes: Trans. Am. Soc. C. E., Vol. 52 (1904), p. 262; Vol. 55 (1905), p. 201; Vol. 88 (1925), p. 102.

SECTION 11

CONCRETE AND REINFORCED CONCRETE

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PLAIN CONCRETE

1. Factors Controlling Quality of Concrete

Concrete is a comparatively new major structural material and the technique of its manufacture is subject to constant revision as we learn more concerning it by experience and scientific research. Present-day opinions regarding such effects as that of curing temperature, of prolonged mixing (such as several hours in a mixer-conveyor), of re-tempering and of other conditions may later be modified. Some recent investigations seem to indicate this. Caution should be exercised, however, in accepting the results of practical experimenting. Deviation from the rules of established practice may be permissible under one given set of conditions and not under another set of conditions. Practical experiments give results that are sometimes apparently contradictory. Established practice should then be discarded only after research by persons accustomed to separate and recombine effects and deduce general principles thereby, and not on the strength of ordinary observation. The rules set down in this section are intended to be in accordance with what is now considered good practice.

The essential factors in proper control of concrete manufacture are: Selection of good materials, correct proportioning of all ingredients, and proper technique in manipulation. The neglect of any one factor will offset to a great degree the good results anticipated by observance of the other factors. Adequate discussion cannot be given here; only the high points will be touched upon, and the reader is referred to standard treatises for detailed discussion.

Concrete is a physico-chemical mixture of cement, water and aggregate (sand and stone). The cement and water combine by a process known as hydration of the cement which binds the particles together. Insufficient cement paste in concrete to fill the voids in the aggregate will weaken the product. More water in the cement paste than is necessary to hydrate the cement remains uncombined and leaves minute water voids in the concrete, which weakens it structurally and leaves it pervious to the passage of water. Only about 2-1/2 gal. of water is necessary to hydrate a bag of cement and anything in excess of this produces water voids. With this minimum amount of water, however, it is not possible to produce a mixture that can be manipulated. A considerable amount of water in excess of this minimum is required to give mixtures suitable for placing. "Workability" is an essential requirement. Fortunately workable mixes can be made, with quantities of water so limited that the water voids are negligible in effect and the concrete strong, durable and impervious.

It is obvious that another essential requirement for a good concrete is structural uniformity throughout the mass. The water and cement must be intimately incorporated so that all the cement will be hydrated and unavoidable water voids uniformly distributed; the cement paste must be uniformly distributed throughout the mass of aggregates so that no portion of the mass will be impoverished; the fine and coarse aggregates must also be uniformly mixed and thoroughly coated with the cement paste. Thorough mixing is therefore essential.

After mixing, the material must be transported and deposited in the forms so that the results of good mixing will not be vitiated by subsequent segregation before the concrete has set. All these requirements may be classified under the general head of proper manipulation.

The chemical changes incident to setting and hardening do not take place at once but, after hydration of the cement, continue over a considerable period of time in the presence of water. This process is called "curing" and therefore concrete should be kept wet, after it has set, for as long a period as is practicable. Curing not only increases the strength of concrete and therefore its structural value, but is a necessity for the production of watertight and durable concrete. Excess mixing water damages concrete by producing water voids in the concrete but a plentiful supply of curing water is essential.

The essential requirements for making good concrete may therefore be summarized as follows, each item being treated in detail later in this section:

(1) Proper selection of materials
- (a) Cement
- (b) Water
- (c) Aggregate

(2) Correct proportioning
- (a) For strength and durability
- (b) For workability

(3) Proper manipulation
- (a) Mixing
- (b) Transporting
- (c) Depositing

(4) Continued curing to increase strength and durability.

2. Physical Properties of Concrete

The physical properties of concrete vary widely depending upon the quality and proportion of the ingredients and the technique of manufacture. Concrete is characterized by a relatively high compressive strength, increasing with age, a relatively low tensile strength (about 10% to 12% of the compressive strength) and a true shearing strength (as distinguished from the combined shear and tension action in the webs of beams) of about one-half the compressive strength. (It is on the basis of the combined action that web shear is sometimes given as 1-1/2 to 2 times the tensile strength.) The modulus of rupture as determined by transverse bending tests on plain concrete beams is about twice the tensile strength.

From the above general comparisons it is seen that plain concrete is suitable only for structures not subject to tension, either direct or in bending; such as foundations for structures, piers and abutments, dams, gravity retaining walls and small arches in which the pressure line can be kept within the middle third of the section for all conditions of loading and all ranges of temperature For structures subject to tension in transverse bending, such as floor slabs, beams, large arches, etc., the deficiency in tensile strength is supplied by reinforcing steel. Reinforced concrete will be discussed later.

In the discussion immediately following, the classes of concrete are designated by the proportions of the dry ingredients — cement : fine aggregate : coarse aggregate, as for example 1 : 2 : 4, 1:2-1/2 : 5 and so on.

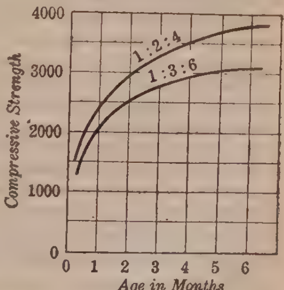


Fig. 1

The principal element affecting the strength of the concrete is the amount of mixing water relative to cement. The approximate variation of compressive strength with the water-cement ratio $\frac{W}{C}$ is shown in Fig. 5, Article 6. The strength of concrete increases with age. Fig. 1 shows typical curves for a particular set of laboratory conditions.

The ratios of compressive strength, tensile and shearing strength and modulus of rupture to each other vary considerably; the values depending upon the kind and age of the concrete and upon the methods used in their determination The ratios given at the beginning of the article are approximate.

Elastic Properties. Concrete does not possess such perfect elasticity as steel and many other metals. In a **compression** test, for example, small permanent deformations or set occur at comparatively low loads and the stress-strain curve is slightly curved almost from the beginning. If the load is not excessive, however, repeated loading causes the material to become more perfectly elastic and the stress-strain curve will approach a straight line up to the amount of stress which is repeated, as illustrated in Fig. 2. The elastic limit

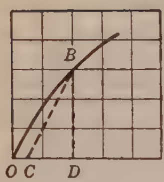


Fig. 2

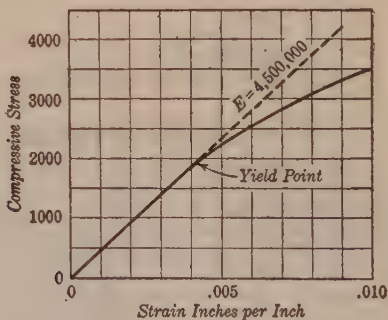


Fig. 3

is, for the reasons stated above, not well defined. There appears, however to be a limit to the stress which can be repeated indefinitely without continuing to add to the deformation and this limit may for all practical purposes be taken as the elastic limit. It is about one-half the ultimate strength; above this point the rate of deformation increases rapidly as the stress increases as indicated in Fig. 3. Since concrete is not perfectly elastic, the true modulus of elasticity is a variable depending upon the degree of stress existing at the time the increments of stress and strain are assumed to be measured. The stress-strain curve being nearly straight, however, within working limits of stress the modulus may be calculated on the basis of the working stress and the corresponding total deformation. In this regard there is some disagreement in method of experimenters; some calculating E as the ratio of total stress to corresponding total deformation (B D ÷ O D in Fig. 2); others calculating it as the ratio of total stress to the total deformation minus the permanent set occurring when the specimen experimented upon is first stressed; that is $\frac{BD}{OD - OC}$. The modulus also varies with the different kinds of concrete and with the age. Typical values are given in the table at top of next page.

The moduli for tension and compression are about equal.

Coefficient of Expansion. Tests show little variation in the coefficient of

Name of concrete	Modulus of elasticity, lb. per sq. in.		
	Based on elastic deformation		Based on total deformation
	At 114 lb. per sq. in.	At 570 lb. per sq. in.	At 570 lb. per sq. in.
1 : 2-1/2 : 5 (broken stone)........	4 660 000	3 590 000	3 440 000
1 : 2-1/2 : 5 (gravel)............	3 170 000	2 520 000	2 200 000
1 : 3 : 6 (broken stone)..........	3 870 000	2 990 000	2 570 000
1 : 3 : 6 (gravel)................	3 000 000	2 240 000	2 110 000

Kind of concrete	Modulus of elasticity between loads of 100 and 600 lb. per sq. in. based on elastic deformation		
	7 to 10 days	1 month	2 months
1 : 2 : 4......................	2 090 000	2 620 000	3 650 000
1 : 3 : 6......................	1 970 000	2 590 000	3 210 000

expansion, which may be assumed as 0.0000065 per degree Fahrenheit or about equal to that of steel.

Contraction after Hardening. The shrinkage of concrete after setting and hardening and drying may be assumed as 0.02% to 0.04% for 1 : 2 : 4 concrete.

Weight of Concrete. Average concrete weighs about 145 lb. per cubic foot. Reinforcing steel may be assumed to add 5 lb. per cubic foot, giving the usual figure of 150 lb. used in calculating dead loads.

3. Cement, Cement Testing

Natural Cement is the finely pulverized product resulting from the calcination (dehydration) of an argillaceous limestone at a temperature below fusion, which, when mixed with water, hydrates again into a solid. The proportions of lime and clay in the raw material vary considerably, depending upon the source, and therefore natural cement is not so uniform in quality as the artificial product known as **Portland cement.** It sets quicker, hardens more slowly, attains less strength—approximately 1/3 that of portland cement, and varies more in strength. For "Early Strength Cements" see page 158, Section 20.

Portland cement, which has almost entirely displaced natural cement for all structures, is an artificial mixture of calcined argillaceous and calcareous materials having more definite chemical and structural qualities. The proper control of the chemical composition in the process of manufacture determines its structural value.

The composition and method of manufacture of cement are discussed briefly in Appendix 1.

Testing. Cement being the active ingredient of concrete should be tested if it is to be used on important work, although there is little danger in using a well-known brand without testing, particularly if the engineer has used it in previous work within a reasonable period of time and knows its qualities. The results of tests depend largely upon the methods employed. Tests should therefore be made by experts who specialize in this work and in accordance with the methods standardized by the American Society for Testing Materials.

The usual tests made for portland cement are: (1) For fineness, (2) for soundness, (3) time of setting, (4) tensile strength.

Chemical tests are sometimes required on important work. The methods of testing are described in Appendix 2 and the requirements under tests are as given in Appendix 3.

Sampling. The above tests are to be made on samples selected, in accordance with A. S. T. M. standard methods, by the engineer or under his supervision as follows:

Sampling. 1. Tests may be made on individual or composite samples as may be ordered. Each test sample should weigh at least 4 lb.

2. (a) *Individual Sample.* If sampled in cars one test sample shall be taken from each 50 bbl. or fraction thereof. If sampled in bins one sample shall be taken from each 200 bbl.

(b) *Composite Sample.* If sampled in cars one sample shall be taken from one sack in each 40 sacks (or 1 bbl. in each 10 bbl.) and combined to form one test sample. If sampled in bins or warehouses one test sample shall represent not more than 200 bbl.

3. Cement may be sampled at the mill by any of the following methods that may be practicable, as ordered:

(a) **From the Conveyor Delivering to the Bin.** At least 4 lb. of cement shall be taken from approximately each 100 bbl. passing over the conveyor.

(b) **From Filled Bins by Means of Proper Sampling Tubes.** Tubes inserted vertically may be used for sampling cement to a maximum depth of 10 ft. Tubes inserted horizontally may be used where the construction of the bin permits. Samples shall be taken from points well distributed over the face of the bin.

(c) **From Filled Bins at Points of Discharge.** Sufficient cement shall be drawn from the discharge openings to obtain samples representative of the cement contained in the bin, as determined by the appearance at the discharge openings of indicators placed on the surface of the cement directly above these openings before drawing of the cement is started.

4. Samples preferably shall be shipped and stored in airtight containers. Samples shall be passed through a sieve having 20 meshes per linear inch in order to mix the sample thoroughly, break up lumps and remove foreign materials.

4. Gaging Water

Certain impurities in mixing water will seriously impair the quality of concrete. If it is necessary to use water from a stream liable to pollution from sewage or manufacturing wastes, tests should be made. It is customary to specify that mixing water shall be free from oil, acids, alkalis and organic impurities.

Oil may be detected by the iridescent film which it forms on quiet water. Oils of organic origin are very injurious to concrete. Mineral oils in small quantity lower the compressive strength but only to a slight degree.

Acids or alkalis may be detected by litmus paper, which may be procured at a drug store. Blue litmus paper changing to red when dipped in the water indicates the presence of acid. Red litmus paper changing to blue indicates the presence of alkali. If the color changes are rapid and decided, dangerous amounts of acid or alkali are indicated. If the color changes are slow and the resulting color faint, the water is safe to use.

Turbidity of the water indicates impurities, that may or may not be injurious, held in suspension. Investigation should be made to determine whether such matters comes from sewage or manufacturing waste or whether it is natural silt.

The use of sea water for mixing is generally prohibited, though very limited tests indicate that it is not injurious to concrete. Further investigation is needed to determine its effects definitely, especially its effects upon reinforcing steel.

The action of sea water on finished concrete structures seems to be physical rather than chemical. Marine structures of a dense concrete, manufactured under rigorous control, are durable. The destructive effect of sea water other than direct physical damage seems to be the disruptive effect of crystallization within the pores of concrete, in which excess mixing water has been used, or the action of frost within the voids of concrete which has been improperly transported or deposited; or similar causes.

5. Aggregates. Test for Aggregates

The aggregates — stone or gravel and sand—are the inert ingredients of concrete but their quality and grading affects the quality of the concrete. All aggregates should be structurally sound; the particles should not be soft nor friable. Aggregates consisting of flat or elongated particles do not make good concrete. Trap, granite and gravel are generally the best aggregate; some limestones are their equal; in particular, those having a high coefficient of wear. Soft limestones and, in general, sandstones and shale are weak. Blast furnace slag used with care makes a strong concrete, but sulphur should have been removed by two or three years' weathering to prevent subsequent deterioration of the concrete. Also in proportioning mixing water the absorption of slag should be measured, otherwise the mix is likely to be harsh.

Aggregates containing mica should be rejected. Even small amounts of mica in the aggregate will seriously reduce the strength of the concrete.

Aggregates can usually be obtained from sources supplying materials of known quality; so that tests are unnecessary. If there is doubt as to the quality of the aggregates offered, compression tests should be made on concrete cylinders as described in Appendix 6, to determine their suitability. The aggregates may be provisionally accepted on the 7-day test subject to satisfactorily passing the 28-day test also.

Pre-mixed Aggregates. Aggregates are often placed upon the market as "pre-mixed," containing about the correct proportions of fine and coarse materials for making good concrete. The fine and coarse materials tend, however, to segregate in the cars, or barges, so that earlier batches of concrete may vary considerably from later batches. It is considered better practice by some engineers to store fine and coarse aggregates separately on the job so that, although the finer particles in each may settle to a degree, correction may be made as the work progresses by varying the proportions of fine to coarse so that uniformity of the mixed materials may be maintained. The use of pre-mixed aggregate, however, decreases the cost of the concrete. If properly graded in the first instance and if segregation is not invited by long transportation in cars or in rehandling, there is no disadvantage in its use.

Size Classification. Fine aggregate (sand, fine gravel or stone screenings) is defined as material that will pass a No. 4 screen (4 meshes to the inch, or 3/16-in. square spaces between wires). Coarse aggregate (gravel, broken stone or slag) is the material retained on the No. 4 screen. Coarse aggregates are designated by the largest sized particles as measured by the clear space between the sides of the smallest opening through which 95% by weight of the material can be passed; thus we have 3-in. coarse aggregate, 2-1/2-in., 2-in., 1-1/2-in., 1-in., 3/4-in., 1/2-in.

Grading of Aggregates. Fine aggregate should be graded from fine to coarse within the following limits:

Passing 3/8-in. sieve (3/8-in. square openings)........	100%
Passing No. 4 sieve (4 meshes to the inch)...........	85% to 100%
Passing No. 16 sieve (16 meshes to the inch)..........	45% to 80%
Passing No. 50 sieve (50 meshes to the inch), not more than.....	30%
Passing No. 100 sieve (100 meshes to the inch), not more than...	5%

Coarse aggregate should be graded from fine to coarse; from 40% to 75% passing a sieve with square openings the side dimension of which is one-half the size classification of the aggregate. Not more than 10% should pass the No. 4 sieve.

Sampling Aggregates. The engineer should select stock samples of both fine and coarse aggregate from which to obtain the smaller samples for making the various tests to determine structural soundness, proper grading, presence or absence of organic impurities, amount of clay and silt, etc., as described elsewhere. A 50-lb. sample of fine aggregate and 100-lb. sample of coarse aggregate should be obtained, selection being from various locations in the supply. Bear in mind the tendency of the larger particles to roll to the bottom and outside of stock piles.

The small samples for the various tests to be made should be selected from the stock sample as follows in order that they too shall be representative:

Place the stock sample on a square of canvas or oilcloth and mix thoroughly. Spread the mixed material in the form of a circle 3 or 4 in. thick. Quarter the circle and discard opposite quarters. Re-mix the remainder and repeat the operation until the sample is of the required size for the test that is to be made.

Natural Sand. The selection of a natural sand for the fine aggregate requires particular attention since many sand pits have become impregnated with organic impurities of a humus nature from the overlying soil. The presence of silt or clay is not a necessary indication of organic impurity in the sand; neither is its absence an indication that there are no organic impurities. A clean-looking sand may contain organic impurities in the nature of a thin coating on the grains that will result in a very inferior concrete. Only 10% of organic loam may decrease the strength of concrete 50% or more. On the other hand, the presence of as much as 5% weight of clay or silt is not seriously detrimental if it is free from organic matter. The effect of organic matter on the strength of concrete is illustrated in Fig. 4.

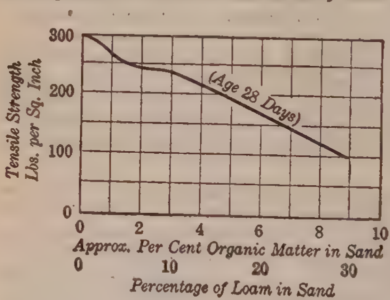


Fig. 4

Colorimetric Test for Organic Impurities. The detection of organic impurities in sand may be accomplished by the colorimetric test devised by Professor Duff A. Abrams and Oscar Harder, formerly of Lewis Institute, Chicago. Obtain a 12-oz. graduated prescription bottle and a stock of sodium hydroxide from a drug store. Make a 3% solution of the hydroxide by dissolving 1 oz. of it in 32 oz. of water. Fill the bottle to the 4-1/2 oz. mark with a representative sample of the sand to be tested, and add enough of the sodium hydroxide solution to bring the mixture to the 7-oz. mark after shaking thoroughly. If after 24 hours the liquid above the sand is colorless or of a light yellowish color, the sand may be considered safe in so far as organic impurities are concerned.

A standard color solution with which to compare the color of the test sample may be made as follows: Make a 2% solution of tannic acid in 10% alcohol. Add 2.5 c.c. of this to 97.5 c.c. of a 3% solution of sodium hydroxide, place in a 12-oz. prescription bottle, stopper it, shake and allow to stand 24 hours. This will give the standard light yellowish color. Any sand which by the colorimetric test produces a color darker than this should be subjected to strength tests in mortar or concrete to decide its suitability for making concrete. Washing sand will remove organic impurities but re-tests should be made afterward.

Structural Qualities of Sand. Tests to determine the structural value of sand are also of importance when the sand is obtained from a source where the quality of the product is unknown. The grains of some sands are soft and friable, which renders the sand unsuitable for making concrete. The structural value of a sand may be determined by making the standard tensile or compressive tests on specimens made with the sand and comparing with similar tests of specimens made with standard Ottawa sand. The standard tests are given in Appendices 2 and 6. The standard sand generally used for test comparisons is that obtained from a natural sand bank in Ottawa, Ill.

Clay in sand is sometimes beneficial and sometimes slightly detrimental. Lean concrete—a mixture of aggregate and lean mortar—is improved up to the point where the voids in the sand are completely filled by the cement and clay. The strength of richer concretes, in which the voids of the aggregate are filled due to careful proportioning and the addition of sufficient cement, is somewhat reduced by the presence of clay which reduces cohesion without increasing density. The effect of 5% of clay is not particularly harmful if it is finely divided and well distributed in the mass, although the American Society for Testing Materials' standard specification for fine aggregate limits the amount to 3%. The determination of the amount of clay and silt in sand or fine aggregate may be made by the Decantation test devised by the American Society for Testing Materials or by the approximate Sedimentation test as follows:

Fill a 32-oz. graduated prescription bottle to the 14-oz. mark with the sand or fine aggregate to be tested. Add clear water to the 28-oz. mark, shake vigorously and allow to settle one hour or more. If more than one ounce of sediment appears above the aggregate, the aggregate should be rejected or else it will require washing. Re-tests should always be made after washing.

6. Proportioning of Concrete

Theoretically the best concrete as to both quality and economy would be that in which the mixed aggregate is so graded that the minimum percentage of voids results and in which the voids in the aggregate are completely filled with a cement paste containing just enough water to hydrate the cement completely so that absolutely no water voids would be formed in the concrete to weaken it structurally. Practically, ideal conditions cannot be realized, for reasons stated at the beginning of the section. Some sacrifice of strength and density must be made by the use of more water than is necessary merely to hydrate the cement, in order to obtain workable mixes economically; but beyond a certain range the use of excess water is extremely damaging to concrete even as a practical structural material.

Evidences of this are on every hand in many structures. Unnecessary excess water is the cause of "**day's work**" **planes**—planes indicating the interruption of the depositing of concrete. This is due to laitance carried

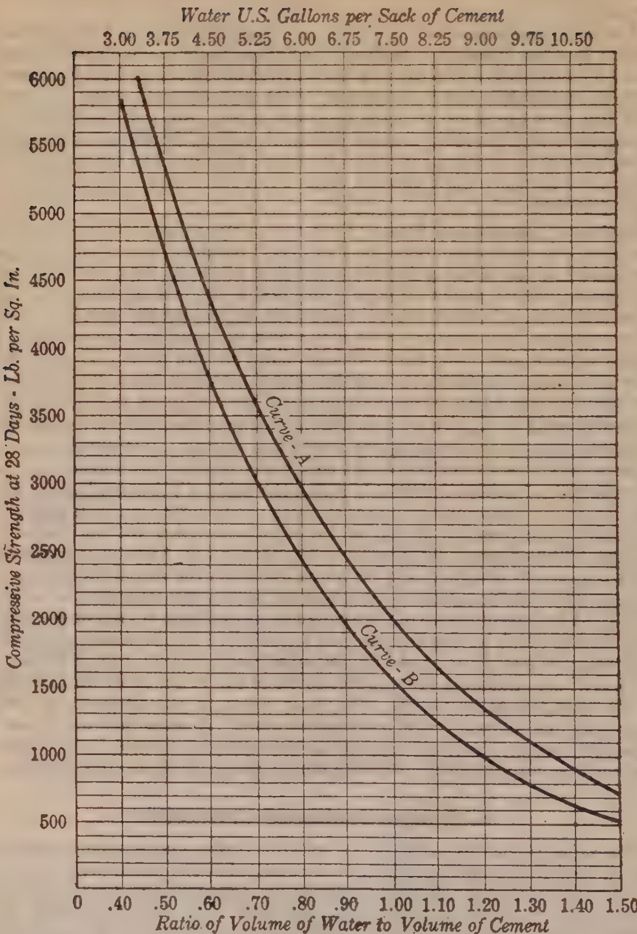


Fig. 5

Effect of quantity of mixing water on the strength of concrete. Curves based on average values from nine series of tests made over a period of four years. Curve *A* to be used for design where the water-cement ratio is carefully controlled by accurate measurement of quantities of water, cement and aggregate, with proper correction for water carried by the aggregate. Curve *B* to be used for design where the water-cement ratio is indifferently controlled and where only rough methods are used for measuring quantities of materials. These are curves developed by Duff A. Abrams.

by the excess water, which forms a structurally weak deposit as soon as concreting ceases and prevents the new concrete from bonding to the old. A pervious stratum is formed through which moisture percolates carrying along the solubles and forming a disfiguring chalky efflorescence. Within the body of the concrete, too, the minute water voids cause structural weakness not at first evident to the eye. The formation of minute frost particles due to the percolation of water or the physico-chemical formation of crystals in the minute voids exerts a tremendous disruptive force which causes the disintegration of many concrete structures. Such a condition is very unfortunate because entirely avoidable.

A prime requisite for making good concrete is therefore the strict limitation of the amount of water in the mix relative to the cement. Strength and durability are a direct function of the water-cement ratio $\left(\frac{W}{C}\right)$, other factors being constant, as shown in Fig. 5. For work requiring concrete that will develop a given strength in a given time as determined from Fig. 5, the relative amounts of water and cement must be kept constant. If it is found necessary

Classes of Concrete for Different Degrees of Exposure

From "Design and Control of Concrete Mixtures," Portland Cement Association.

Class of concrete (expected strength at 28 days lb. per sq. in.)	Maximum quantity * mixing water U. S. gallon per sack of cement (94 lb.)	Type of structure or degree of exposure
3000	6	Roadways, piles, pressure pipe and tanks. Thin structural members in severe exposure. Walls, dams, piers, etc., where exposed to severe action of water and frost.
2500	6-3/4	Sewers, bridges, walls, dams, piers, etc., for all weather conditions and moderate action of water and frost.
2000	7-1/2	Ordinary enclosed reinforced-concrete buildings. Bridges and retaining walls of heavy sections in moderate exposure.
1500	8-1/4	Mass concrete, basement walls, etc., protected from water or severe weather conditions.

* Free water or moisture carried by the aggregates must be included as part of mixing water.

to increase workability so that the concrete can be properly manipulated in the forms and around reinforcing steel, the proportion of cement paste to aggregate may be increased but **relative amounts of water and cement must not change.** The unit of field measurement of cement being the bag (94 lb., or about one cubic foot), the proportions of cement paste and aggregate may best be varied by changing the amount of aggregate per batch.

From the above it is seen that classification of concrete by arbitrary proportions has no real significance with respect to quality and is only a general indication (subject to modification) of the workability required for particular classes of work, hence, the approximate amount of cement paste relative to aggregate. A relatively stiff mix can be properly placed and compacted en masse as in abutments or gravity retaining walls and the mix may approach

1 : 3 : 5 or 1 : 3 : 6. The placing of concrete around steel reinforcement in the bottoms of deep beams or in thin vertical walls requires more workability and the concrete may approach 1 : 1-1/2 : 2 or 1 : 1-1/2 : 3. Above the reinforcement in the beams where the concrete can be more effectively manipulated, it may approach 1 : 2 : 3 or 1 : 2 : 4. In any case, however, the water must not exceed the amount that will produce the required strength.

Aggregates stored in the field, particularly the fine aggregates, usually contain a considerable amount of moisture and this must be taken into account in preparing the mix. Fine aggregates in storage piles under ordinary conditions will contain 1/4 to 1/2 gallon of water per cubic foot, and after a heavy protracted rain may contain as much as a gallon. Coarse aggregate retains less—from 1/8 to 1/4 gal. Account of this in determining the quantity of water to be added to the mix may be taken by weighing samples of the aggregate before and after drying and converting the loss in weight into gallons of water per cubic foot of aggregate. Moisture content of storage piles will remain constant for two or three days under settled weather conditions Determination of the content may therefore be made at such intervals. After a rain a re-determination is necessary.

METHODS OF PROPORTIONING

Proportioning by Determination of Voids in the Aggregates.

Proportioning by Trial Method for Maximum Density.

These "rational" methods require considerable modification of the proportions determined by them, in order to obtain smooth workable mixtures and, in fact, resolve themselves in the final analysis into a method of judgment.

Proportioning by Fineness Modulus, as developed by Professor Duff A. Abrams, is based upon careful research and the results of numerous laboratory experiments, and is the most scientific and accurate method given uniform aggregates. Even this method requires some modification in the field, and further study is needed to adapt its rules to field conditions. The necessary procedure is complicated and has been the main obstacle to its general adoption. The presentation of the technique will therefore not be given here.

Proportioning by Arbitrary Proportions. The customary arbitrary proportions (1 : 1-1/2 : 3 or 1 : 2 : 4 for richer mixes, and 1 : 2-1/2 : 5 or 1 : 3 : 6 for leaner mixes) give fairly satisfactory results with well graded aggregates proportioned dry. Coarse aggregate measured loose contains from 30% to 55% voids, depending upon grading; dry sand from 30% to 40%. In a 1 : 2 mixture of fine and coarse aggregates, both reasonably well graded from coarse grains to fine, the fine aggregate would presumably fill the voids in the coarse aggregate with a little to spare as allowance for bulking of the coarse aggregate in the matrix. Aggregate under job conditions, however, contains moisture which causes it to bulk. This bulking is considerable for fine aggregate. Coarse aggregate bulks little from surface moisture. A very wet sand may bulk so that a measured quantity will contain only about 80% of the solid material measured dry. Therefore a 1 : 2 mix of fine and coarse aggregates measured under job conditions without allowance for bulking will sometimes be "undersanded" and result in a harsh mix. Modification of the standard proportions, according to the nature of the work and conditions on the job, is therefore necessary.

Proportioning by Judgment Based on Experience. On all ordinary work satisfactory results may be obtained by modifying arbitrary proportions as the work proceeds, according to the trained judgment of the engineer.

The following observations will be an aid to judgment:

(a) The larger sized aggregates, reasonably graded down from coarse to fine particles, will produce the more economical concrete, that is, one which will require less cement paste to make a concrete of given consistency or degree of workability. Thus a concrete of 1 part cement and 5 parts of mixed fine and coarse aggregate well graded down from particles 2 in. in diameter will have about the same consistency or degree of workability as a concrete of 1 part cement and 4-1/2 parts of mixed fine and coarse well graded down from particles 1 in. in diameter. In this connection it may be noted that the volume of mixed fine and coarse aggregates will be about 85% of the sum of the volumes of the separated aggregates.

(b) Larger ratios of coarse to fine aggregate may be used with the larger size aggregates; thus with a 2-in. coarse aggregate the fine and coarse may well be mixed in the proportion of 2 : 3-1/2 whereas a 1-in. coarse aggregate will require a larger proportion of the fine to make a smooth working mixture, say 2 : 3. Too little fine aggregate tends toward harshness and leads to the danger of honeycombing. Too much, on the other hand, will produce porous concrete of low density. The yield in cubic yards of concrete for a given quantity of cement is also less for mixes containing larger proportions of fine aggregate relative to coarse aggregate. The best proportion depends upon the grading and other characteristics of the aggregates.

Summary of Procedure. (1) Select coarse aggregate of the maximum size appropriate for the work, say 1/1-2-in. to 3-in. for exclusively mass work; 1-1/2-in. for heavy reinforced-concrete work (heavy reinforcing bars with at least 2-in. clear spacing); 1-in. for light reinforced-concrete work. (2) From Fig. 6 determine the water-cement ratio for the strength concrete required. For example, for a 2000-lb. concrete, and using curve B (if moisture contents of aggregates are estimated from tables instead of being determined by tests) we find the required water-cement ratio to be 0.9. Quantity of water per bag of cement (about 1 cu. ft.) $= 0.9 \times 7.5 = 6.75$ gal.

(3) Determine the water content of the aggregate by test as previously explained, or, approximate from the following table:

Sand, very wet	3/4 to 1 gal. per cu. ft.
moderately moist	about 1/2 gal. per cu. ft.
moist	about 1/4 gal. per cu. ft.
Coarse aggregate, moist	about 1/4 gal. per cu. ft.

Deduct such amount from the quantity determined above in (2) to obtain the quantity of added mixing water.

(4) Adopt tentative proportions of cements: fine aggregate: coarse aggregate as follows:

1 : 2 : 3-1/2 for maximum size aggregate 2 in. or over.

1 : 2 : 3 for maximum size aggregate 1 in.

and proportionately for intermediate or smaller sizes.

If it so happens that the aggregates are well graded and have been measured dry (stock piles containing little moisture) the proportion of coarse aggregate to fine may be increased without resulting in harshness. The yield in concrete per unit volume of cement will be slightly increased thereby.

If, however, the first trial mix proves, for some aggregates, to be harsh, the proportion of coarse aggregate to fine may be decreased until smoothness is reached. Avoid, however, excessive proportions of the fines, for reasons before stated. Smoothness may also be secured by increasing the cement content.

Vary the consistency or degree of workability required for various portions of the work by varying the quantity of total aggregate per unit volume of cement; but always maintain the predetermined ratio of water and cement.

An idea of the relationship between consistency, size of aggregate and water-cement ratio may be gained from the following table furnished by the Portland Cement Association:

Consistency	Trial mix, dry compact volumes for maximum size of aggregate indicated	
	1 in	2 in
Water-Cement Ratio, 5-1/2 Gals. per Sack		
Stiff	1 : 2 : 3	1 : 2 : 3-1/2
Plastic	1 : 1-3/4 : 2-1/2	1 : 1-3/4 : 3
Soft	1 : 1-1/2 : 2	1 : 1-1/2 : 2-1/2
Water-Cement Ratio, 6 Gal. per Sack		
Stiff	1 : 2-1/4 : 3-1/4	1 : 2-1/4 : 4
Plastic	1 : 2 : 3	1 : 2 : 3-1/2
Soft	1 : 1-3/4 : 2-1/2	1 : 1-3/4 : 3
Water-Cement Ratio, 6-3/4 Gal. per Sack		
Stiff	1 : 2-1/2 : 3-1/2	1 : 2-1/2 : 4
Plastic	1 : 2-1/4 : 3-1/4	1 : 2-1/4 : 3-3/4
Soft	1 : 2 : 3	1 : 2 : 3-1/2
Water-Cement Ratio, 7-1/2 Gal. per Sack		
Stiff	1 : 3 : 4	1 : 3 : 4-3/4
Plastic	1 : 2-1/2 : 3-3/4	1 : 2-1/2 : 4-1/4
Soft	1 : 2-1/4 : 3-1/2	1 : 2-1/4 : 3-3/4

Water-cement ratios indicated include moisture contained in the aggregate.

Proportions are given by **volume, aggregate dry** and **compact.** Thus 1 : 2 : 3-1/2 indicates 1 volume of cement, 2 volumes of sand and 3-1/2 volumes of coarse aggregate.

If the aggregates are to be measured in the **damp** and **loose** condition they will occupy greater volume than when dry and compact. Amount should be determined by test. Approximate average value for sand 20%, for coarse aggregate 6%.

For approximate proportions **by weight** add 15% to proportions of aggregate shown in the table.

Absorptive Aggregate. The particles of light and porous aggregates will absorb an appreciable amount of water and for such condition a corrective addition to the quantity of mixing water may be made. This correction may be determined by means of the absorption tests recommended by the American Society for Testing Materials (1920 Proceedings, Part 1, Appendix 2, Report of Committee C-9). A rough test may be made by thoroughly drying samples in an oven to expel all absorbed water, weighing the sample, immersing in water for about half an hour, then removing surface water by spreading out for a period in the air so that the surface water is evaporated, and then weighing. The difference between the two weighings will indicate the amount of absorbed water. Very light and porous aggregate may absorb as much as 25% by weight of water; porous sandstone 7%. Average sand, pebbles and crushed limestone particles will absorb about 1% by weight; trap rock and granite 1/2%, and for these no correction need be made.

Yield. The yield in concrete of a given mix may be easily determined due to the fact that the volume of concrete is equal to the sum of the absolute volume of its ingredients, which may be calculated from the weights per unit volume.

One sack of cement (assumed 1 cu. ft.) weight 94 lb. The specific gravity of cement being 3.1, the "absolute weight" of a cubic foot of solid particles would be $3.1 \times 62.5 = 194$ lb.

The specific gravity of the aggregates may be assumed as 2.65; hence the "absolute weight" of the material of which it is composed is $2.65 \times 62.5 =$ 165 lb.

For a concrete mixture of 1 sack cement, 2 cu. ft. fine aggregate, 3-1/2 cu. ft. coarse aggregate, and 7 gal. (0.93 cu. ft.) water, the yield in concrete would then be calculated as follows:

First $\begin{cases} \text{weigh 1 cu. ft. fine aggregate (say 110 lb.)} \\ \text{weigh 1 cu. ft. coarse aggregate (say 100 lb.)} \end{cases}$

Calculate:

From cement $\frac{94}{194}$ = 0.49 cu. ft.

From fine aggregate $\frac{110 \times 2}{165}$ = 1.33 cu. ft.

From coarse aggregate $\frac{100 \times 3\text{-}1/2}{165}$ = 2.11 cu. ft.

From water 0.93 cu. ft.

Yield in concrete.............. 4.86 cu. ft.

7. Field Measurement of Materials

The application of accurate proportioning necessitates accurate measurement of materials for the mixer. The measurement of the cement is easily done by counting the bags. On important work close measurement of the aggregates by means of calibrated bottomless boxes should be required. Attempts to measure the aggregates by calibrating the barrows, carts or buckets used to convey the materials to the mixer are not effective. Unless measuring boxes are used it is better to adopt arbitrary proportions (say 2 to 3 or 2 to 4) of fine to coarse aggregate and rely on struck loads.

Accurate measurement of water should be demanded on all structural work and rigid inspection at the mixer should be provided. Reliable water measuring devices are on the market and installation of an approved device tested by experience should be required of the contractor.

Of equal importance with correct proportioning are proper mixing, transporting, depositing and curing. Neglect of proper observance of any one element will vitiate the good results of observance of all the others.

8. Mixing

Mixing of the ingredients must be thorough, so that fine and coarse particles of aggregate will be uniformly distributed, so that the water and cement will be intimately incorporated to cause complete hydration of cement, so that the water voids caused by the unavoidable excess will be well distributed in the mass, and so that the aggregate particles will be completely coated with the cement paste. A minimum of one minute actual mixing, after all ingredients including the water, are in the mixer, should invariably be demanded.

A batch a minute from a batch mixer means a net time of mixing after all the materials are in, of little more than half a minute. Fig. 6 shows the effect of **time of mixing** upon the **strength of the concrete.** Thorough mixing also **increases the workability** of the mix and makes it easier to place in the forms. It is also essential for watertight construction. Speeding up a rotary mixer by increasing the revolutions per minute has little effect in reducing the time required to produce the desired results; it is the time element rather than speed element that is important in mixing. Time of mixing also affects **uniformity of product.** Test specimens made of concrete mixed 15 seconds may show a mean variation of 30% in strength whereas those mixed 2 minutes should vary less than 10%.

Machine Mixers are of two general types, (1) the batch mixer and (2) the continuous mixer. In the batch mixer the proper amounts of material for a single "batch" of concrete are placed in the mixer and the contents are then mixed either by means of moving paddles or blades, or by the rotation of the receptacle itself, in which are generally placed deflectors to aid in the mixing. In the continuous mixer the operation is continuous, more or less adequate provision being made for maintaining the proper proportions of materials.

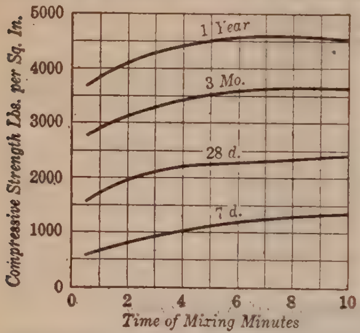


Fig. 6

The gravity mixer is a type of continuous mixer into which the material is introduced at the top and is mixed by striking various obstructions or deflectors in its descent. Continuous power mixers utilize some form of screw or paddle blades.

Although good results may be secured by either type, the correct proportions are more readily secured in the batch mixer and it is generally to be preferred, especially for small plants where the supervision is likely to be inadequate.

9. Transporting

The effect of methods of transporting concrete from the mixer to the forms upon the quality of the set concrete is of prime importance.

(1) Concrete should be transported quickly enough so that it will not get its initial set before its manipulation in the form has ceased. Since initial set should not occur within 45 minutes after mixing there is little danger from a violation of this principle except where there is a central mixing plant for several jobs and transportation is by truck. In such case avoidable delays by drivers may be prevented by a time checking system.

(2) The mode of transportation should prevent segregation or loss of materials. Buckets, chutes, belts of conveyors, etc., should be watertight so that the cement will not be lost from the mix. Chutes should be at such an angle (not less than 30 deg. with the horizontal) that the coarser materials will not be left behind. Continuous delivery from the chute will help to prevent segregation.

10. Depositing

The results of all previous care may be lost to a great degree by lack of care in depositing.

(1) Concrete must be deposited evenly in the forms in uniform layers and not allowed to pile up. Localized delivery causes the more liquid mortar to flow away from the aggregates and not only impoverishes part of the concrete but promotes the formation of laitance from the segregated cement and water, which has been discussed in Art. 6. In beam and slab floors the beams should be completely filled first; the concrete should not be dumped on the slab floor and allowed to flow into the beams thus causing segregation. The temptation to localize delivery from chutes, particularly in high walls, is very great.

(2) Concrete should be deposited so as to prevent as far as possible the entraining of air. Concrete should not be dropped from a height of more

than 5 ft. without the aid of a pipe or chute. It should be puddled or tamped so as to expel entrained air as well as to insure complete filling of the forms and close contact with reinforcing steel. In inaccessible forms as for thin walls, the outside of the forms should be tapped vigorously with automatic air hammers or with sledge hammers or mauls.

(3) Concreting should proceed without interruption so far as possible in order to prevent stoppage planes by the formation of laitance under initial set. Stoppage planes are only less pronounced than days' work planes.

(4) Concrete should not be deposited during freezing weather without special precautions, since it hardens very slowly at low temperatures. Freezing of the water while setting exerts a tremendous disruptive force which disintegrates the concrete and invariably causes failure although concrete frozen **before** setting will sometimes set up after thawing, with apparently little harmful effects.

If it is necessary to deposit concrete in cold weather, the aggregates throughout their mass and the water should be heated, but the water must not be so hot as to cause flash-set of the cement. The concrete should be delivered into place at a temperature above 50° F. but not over 100° F., and all materials should be heated to within these limits. A sheet-iron pipe or inverted trough buried in the aggregates and fired with wood is the usual method of heating small quantities of aggregates. Large quantities are sometimes heated by a network of steam pipes underlying the storage piles.

(5) Fresh concrete must be protected from the action of flowing water. In ordinary construction excavations should be unwatered. When ground-water is encountered, it is advisable to enlarge the excavations one or two feet in all dimensions, build the forms to dimension and surround them with a channel to lead the ground-water to a sump whence it may be removed without disturbance to the fresh concrete until it has set. Where springs are encountered, the water should be piped and led away from the area upon which concrete is to be placed.

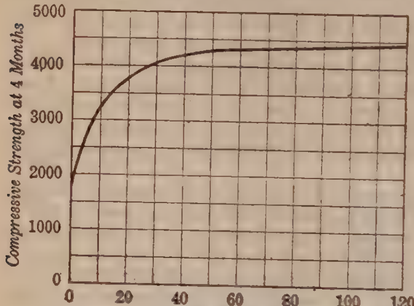
Under-water Concrete. In marine work proper, concrete may be lowered through the water by means of tightly closed buckets which may be opened at the bottom and discharged when in contact with the concrete already deposited; or, it may be deposited through a pipe or tremie. The latter method is usually to be preferred both on account of the better results obtained and on account of economy of operations.

Depositing by Tremie. Concrete may be placed under water through a tremie or watertight pipe 12 in. to 24 in. in diameter, having a conical shaped top the better to receive the charge of concrete. At the start of operations the mouth of the pipe is set on the bottom and the pipe filled with concrete. The pipe is then slowly raised, allowing the concrete to flow out as more concrete is poured in at the top, always keeping the pipe fully charged. It is essential, once the concreting operations are started, that the lower end of the pipe be kept buried in the concrete until the entire operation is completed. In this manner, washing out of the cement is prevented. The upper layer of concrete displaces the water, and laitance is kept at the surface of the concrete whence it may be finally and effectively removed. If at any time during the operations the bottom of the pipe loses contact with the concrete, the charge will be lost; therefore particular care must be taken in continuing operations not to stir up the fresh concrete in place. Tremie operations should be continuous until the structural unit has been completed.

Tremie concrete must be of a consistency that will permit it to flow readily and it therefore requires more cement and water than concrete placed in the dry.

11. Curing

Proper curing of concrete after setting is an important factor for successful work, as indicated in Art. 1. Excess mixing water is an evil. There is no danger from an excess of curing water. Fig. 7 shows graphically the value of proper curing. Keeping concrete wet for the first ten days after setting increases its strength and durability 75% over that which might be expected from its ordinary aging. Three weeks' curing shows 115% gain and four weeks' curing shows 145% gain. These results were observed from compression cylinder tests as follows: Specimens A were allowed to harden in air for four months and then tested — 1800 lb. per sq. in. Specimens B were cured in damp sand 3 days, kept in air the remaining 117 days and tested — 2100 lb. per sq. in. Specimens C were stored in damp sand 21 days, kept in air the remaining 99 days and then tested — 3900 lb. per sq. in. Specimens D were stored in damp sand the full 120 days and then tested — 4400 lb. per sq. in.



Storage - Days in Damp Sand Followed by Air Storage until Tested. Total Age 4 Months

Fig. 7

Concrete structures should be cured by adequate means. Drenching the forms with water before placing concrete will reduce absorption. Horizontal surfaces such as floors and pavements can be covered with damp sand the day after they are laid when they have hardened sufficiently to prevent pitting the surface, and kept damp by frequent sprinkling. Sometimes small dikes of clay are built around a section of floor or pavement, which is then flooded by water. Vertical surfaces can be kept damp by frequent sprinkling of the forms or exposed concrete. Sometimes walls are coverd with canvas or burlap which is drenched with water several times a day. In road work continuous drenching from sprinkler heads is something practical.

For like moisture-curing conditions, the time required for concrete structures to set and develop a given strength depends upon the external temperature maintained during the period of curing and also upon the rate of dissipation of internal heat generated by the chemical reactions of setting, as affected by massiveness. Fig. (8) shows graphically the effect of temperature upon the rate of strength increment of small test specimens for which the dissipation of internally generated heat would be rapid.

Measurements of the internal heat generated within massive structures have been made by embedding thermometers and making observations. Fig. 9 shows the results observed during the construction of the Arrowrock Dam by the U. S. Reclamation Service near Boise, Idaho (" Temperature Changes in Mass Concrete," by Chas. H. Paul and A. B. Mayhew, Trans. Am. Soc. C. E. 1915).

The resultant effect of external temperature, generation of internal heat and rate of radiation upon the curing of structures in general is largely a matter of guess. Massive structures will not require elaborate cold-weather protection to develop early strength. Structures having relatively thin sections, such as reinforced-concrete buildings and bridges, do require careful **cold-weather protection** to develop sufficient strength to permit early removal of supporting falsework. It is probable that many so-called inexplicable cold-

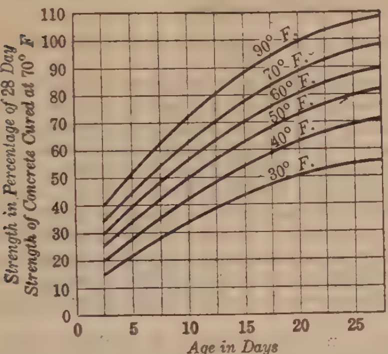


Fig. 8

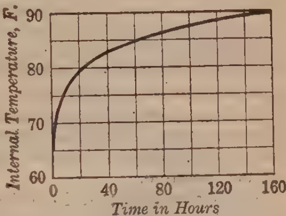


Fig. 9

weather failures are due to slow setting at low temperature or to actual freezing. In such construction cold-weather protection by enclosure and heating by salamanders may be necessary but this does not warrant dispensing with preheating the materials since the transference of heat through forms is slow.

12. Miscellaneous Notes

12a. Effect of Cold Weather Concreting

Although the rate of strength increment is slow at low temperatures, cold weather concreting need not be detrimental to the final quality of the product. If actual freezing is prevented during the setting process, and supports are left in place sufficiently long to permit the concrete to develop self-sustaining strength, there need be no concern as to the effects of low temperatures upon final quality. Supports for structures should not be removed, however, without making due time allowance for delayed rate of setting or until a sufficient period of warm weather has intervened.

12b. Construction Joints

Proper treatment of construction joints, stoppage planes or days' work planes is of vital importance and is too frequently neglected. Even in structures not required to carry a head of water, improperly treated joints permit percolation and disfigurement by efflorescence, besides being an element of structural weakness. An almost perfect joint can be made if mixing water is properly regulated to prevent excess, if the more liquid portions of the concrete are not allowed to segregate in transporting and depositing, if the surface of the concrete in place is thoroughly scrubbed with stiff wire brushes to remove laitance completely so that clean aggregates are exposed; and if the surfaces are again drenched before concreting proceeds and coated with a rich thin mortar, just before the fresh concrete is placed. The procedure may seem to be "fussy," but it is easy of execution and the results will pay both in avoiding unsightliness and planes of weakness. The formation of efflorescence on the surfaces of completed structures in which the concrete was not properly placed will appear sometimes promptly, sometimes after the lapse of considerable time.

12c. Payment for Concrete

Payment for concrete is usually by the cubic yard measured in place in the completed work.

Complete control of the proportions, however, is predicated on the use of sufficient cement to keep the water-cement ratio constant and at the same time give the required degree of workability. Since the contractor has necessarily based his bid upon the assumption of definite proportions, orders to increase cement content instead of merely adding water to increase workability will invariably lead to disputes, and as a matter of fact equity is on the side of the contractor, despite the technique in the matter. Specifications calling for strength concrete do not solve the question. No court would uphold orders to replace, extensively, structures for which the field tests of concrete indicated inferior quality. The real responsibility rests with the engineer, and the contractor is entitled to payment for the materials entering the construction in so far as is possible.

A separate payment item for the number of barrels of portland cement ordered to go into the concrete should be provided and payment for this amount should be made in addition to the number of cubic yards of concrete in the work. Cause for argument will be removed and responsibility for the quality of the product will be indisputably in the hands of the engineer where it belongs.

13. Admixtures

Various admixtures are sometimes used to increase workability; to increase impermeability or watertightness; to hasten setting; to facilitate curing; etc. In general the judicious use of such admixtures is not harmful and may be beneficial to the leaner mixtures; they are as a rule, however, palliatives and unnecessary in high-grade work.

Anti-freezing Admixtures. Admixtures to prevent freezing may be used with judgment in moderately cold weather to avoid expensive housing, but should not be relied upon when extremely cold weather is likely to follow.

Common salt (NaCl) is sometimes used to prevent freezing, though it retards the setting process and somewhat reduces the strength of the concrete. About 1% by weight to that of the mixing water is required to lower the freezing point each degree below 32° F. Up to 10% it is not particularly harmful to concrete if the tendency to reduction of strength is overcome by the addition of a little excess cement.

Calcium Chloride ($CaCl_2$) is used as an anti-freezing compound and also to accelerate setting and assist the curing process. It may be safely used up to 4% by weight to that of mixing water. With concrete cured at low temperatures a mixture of 2% calcium chloride and 9% common salt is more effective as an anti-freezing compound than common salt alone. 2% calcium chloride is effective as an accelerator for road concrete and permits opening the road to traffic in less time than ordinarily required.

Workability Admixtures. Neutral admixtures such as the diatomaceous earths (marketed under such trade names as diatomite and celite), kaolin and hydrated lime, are designed to increase workability and act as a void-filler to increase imperviousness. The amount of such admixtures that can be used beneficially depends upon the quality of the concrete and the character of the aggregate. It is more beneficial to the leaner mixes. Beyond the point of maximum of density these admixtures begin to decrease the strength by reducing cohesion. A rule for the " judicious use " of such admixtures as given by Pearson and Hitchcock, Proc. American Concrete Institute, Vol. 20, 1924, is as follows: percentages being by weight relative to the cement: For concretes leaner than 1 : 1-1/2 : 3 the maximum percentage of celite used should be equal to the index number for the coarse aggregate; of kaolin, to twice the same index number; of hydrated lime, three times the same index number. For example, for 1 : 3 : 6 concrete the maximum percentages would be 6 for celite, 12 for kaolin, 18 for hydrated lime. Some engineer-chemists claim that siliceous admixtures may be used to advantage in sea-water concrete to combine with the free lime of the cement into a calcium silicate and prevent the combination of the free lime with the magnesium sulphate and chloride of the sea water into a calcium sulphate which is expansive, soluble and destructive to concrete. Two pounds of a siliceous admixture per bag of cement is the usual proportion.

Clay. Clay acts much the same as lime. Finely divided and evenly distributed, it fills the voids of lean mixtures and improves quality, while reducing the cohesion of rich mixes.

Oils. Mineral oils are sometimes used in gaging concrete to increase workability and imperviousness. Up to 5% or 10% by weight of cement they lower the strength of ordinary concrete but little. In excess they are injurious. Some commercial oils for concrete contain oils of organic origin in addition to the mineral oils and are very detrimental. Rich mixes designed for imperviousness require no waterproofing compounds.

14. Watertight Concrete

The best concrete is watertight. Strict observance of the rules outlined in the preceding text will result in a strong, durable, impervious material without the use of waterproofing compounds. To obtain the required density for watertightness the water content of the mix should be kept low, say 6 gal. per sack of cement, which will require possibly a 1 : 2 : 3 or 1 : 1-1/2 : 2 mix for workability so that it will compact properly. Mix thoroughly, prevent segregation in transporting and depositing, spade or tamp 6 or 8-in. layers in the forms to break up stone pockets and expel entrained air, cure thoroughly. Tanks and other structures to be subjected to hydrostatic pressure should be thoroughly cured before the pressure is applied. If construction joints are unavoidable observe the rules for bonding new concrete to old, strictly. Abundant experience has shown that care will produce the desired results.

In tests conducted by the U. S. Bureau of Standards, thin slabs of a lean (1 : 6) portland cement mortar and 1 : 1-1/2 : 2 concrete were subjected to a water pressure of 60 lb. per sq. in. This pressure is equivalent to a 138-ft. head of water. Although water penetrated through 1-5/8-in. limestone slabs in periods ranging from 20 seconds to 20 minutes, it took 3-1/2 hours for water to penetrate through a 2-in. slab of 1 : 6 mortar, while at the end of 24 hours, when the test was terminated, the 2-in. slab of 1 : 1-1/2 : 2 concrete was still dry.

Hundreds of concrete tanks are being used for the storage of fuel oil, which is lighter than water, and these tanks are oiltight, and of course watertight. Concrete basements, pits, bridges, and tanks will also be watertight if proper care is taken in their construction. Experience and tests have shown that proper practice will make watertight concrete.

15. Follow-up Tests of Concrete

Field tests of concrete afford a means of determining the effectiveness of the field control of concrete. It is not to be expected that the tests on the standard compression-test cylinders sampled from mixes in the field would agree very closely with tests on blocks cut out from the completed structure, when such tests are made, or that they prove the exact quality of concrete in the work. Such factors as differences in curing conditions, differences in degree of compactness due to static head and a variety of other factors would be reflected in the results. Such tests are, however, an approximate indication of the quality of the concrete if the material in the form is representatively sampled and if a sufficient number of test specimens are made to reflect daily variation.

Since the test specimens are small, they should be made with strict observance of a rather elaborate technique, and should be stored and cured under standard conditions and tested in an exact manner in order to be of any value at all.

Appendix 6 describes the proper method of making and storing concrete test pieces.

If test cylinders are tested at 7 days' age, the 28-day strength may be approximately estimated by the following formula $S_{28} = S_7 + 30\sqrt{S_7}$.

Test cylinders should be cast in pairs; at least one pair for each 100 cu. yd. of concrete placed, or a minimum of two pairs for each day's work. The samples of concrete should be taken at the point of delivery and be as representative of the product as possible.

16. Forms for Concrete

General Requirements. Forms for concrete should be strong and rigid and should be tight enough to prevent leaking of the mortar. The material should be planed on one side and be of even thickness if a smooth face is desired. This is generally advisable for convenience in handling. Material which warps readily, such as hemlock or oak, is not desirable. Pine or spruce is commonly the best available material. For repeated use in built-up sections, tongued and grooved material is to be preferred.

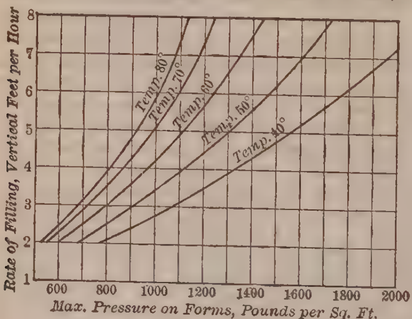


Fig. 10. Pressure of Concrete

Pressure of Concrete. The pressure of wet concrete, such as is frequently used, will be the full hydraulic pressure of a liquid weighing about 150 lb. per cu. ft., until initial setting begins. If the forms are to be filled before this occurs they must be designed for the full pressure. If setting begins while the work is in progress, then the maximum pressure will be less than the full hydrostatic pressure, this maximum limit depending upon the rapidity of filling and rate of setting of the concrete. It will therefore depend largely upon the existing temperature. Results of experiments made by F. R. Shunk on the maximum pressure of concrete are shown in Fig. 10. This diagram gives the maximum pressure which occurred for various rates of filling and at various temperatures. The tests were made by means of a pressure board 9.23 in. in diameter, set into the side of a form supporting a large mass of concrete. The concrete was made of 1 : 3 : 5-1/2 proportions and was very wet, the workmen sinking into it about 18 in. (Eng. News, Sept. 9, 1909.)

Design of Forms. Either 1-in. or 2-in. stuff is suitable for lagging but, excepting for small parts, 2-in. is preferable. One-inch stuff requires supports spaced 18 to 24 in. apart and 2-in. stuff a spacing of 4 to 5 ft. Studding or joists should be 2 in. by 4 in. to 2 in. by 5 in. for 1-in. lagging, and 4 in. by 6 in. up to 4 in. by 10 in. for 2-in. lagging. Forms for opposite faces of walls of ordinary thickness should be connected together by ties designed to take the full pressure. Thus connected, the outside bracing need be only sufficient to steady the forms as a whole while the concrete is being placed. Forms are conveniently connected by wire twisted into a sort of turn-buckle for adjustment. Iron rods fitted with nuts are more readily adjusted. These may be passed through small pipes cut slightly shorter than the thickness of the wall, which are afterwards filled with cement.

Where no cross ties are used the entire pressure must be borne by outside props placed, generally, as diagonal struts. Great pains must be taken to secure the ends of such struts from settlement. Rigidity is as important as strength, and deflections of forms should be limited to very small amounts.

Column Forms should be made of 2-in. plank and well clamped together to resist the heavy pressures involved. They are conveniently held together by wooden blocks connected by bolts and adjusted by wedges as shown in Fig. 11. Care must be taken to see that the bottoms of column forms are thoroughly clean before pouring concrete. To render this easy and certain it is advantageous to arrange a small removable panel at the base of the form.

Beam Forms are preferably made of 2-in. stuff. The bottoms should be well supported and the sides held together by wedges or clamps. Wedges are also conveniently used at top or bottom of the supporting struts.

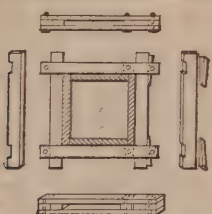


Fig. 11. Form for Column

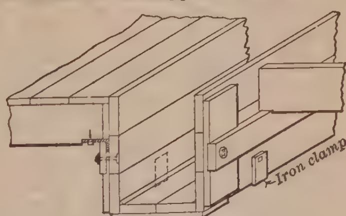


Fig. 12. Form for Beam

Slab Forms are made of 1-in. or 2-in. stuff. If the former is used it should be supported on joists spaced not more than 2 ft. apart. Joists are conveniently supported on 2-in. strips fastened to the sides of the beam forms or may be independently supported. Fig. 12 shows a typical arrangement of beam and slab forms.

Adhesion of Concrete to the forms can be prevented by oiling the forms with crude oil, soap or other greasy material. This aids also in preventing warping. If not oiled the forms should be thoroughly wet before placing the concrete. This is especially necessary in hot dry weather.

Wall Forms. Forms for thin walls are generally made to be supported by the concrete already in place. Fig. 13 shows a common arrangement. The bolts and slotted holes enable the sections to be raised conveniently as the work proceeds.

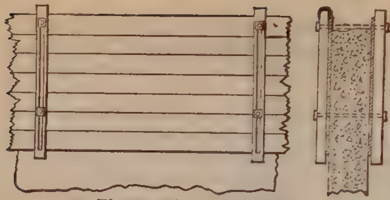


Fig. 13. Form for Wall

Removal of Forms should be done only under expert supervision. In warm weather column and wall forms can be removed in 2 to 4 days, slab forms in 10 to 15 days, and beam forms in 2 to 3 weeks. Low temperatures require much more time for hardening. Forms should be removed very carefully and gradually and the concrete examined before general removal is ordered.

17. Quantities of Material for One Cubic Yard of Compacted Concrete

(Without allowance for waste)

Based on 3.8 cu. ft. per bbl. of cement. Sand and stone measured loose

Proportions by volume			Ratio: Mortar / Stone	Quantities of material		
Cement	Sand	Stone		Cement, bbl.	Sand, cu. yd.	Stone, cu. yd.
1	1	1-1/2	0.98	3.10	0.44	0.65
		2	0.72	2.75	0.39	0.78
		2-1/2	0.58	2.48	0.35	0.88
		3	0.48	2.25	0.32	0.95
		3-1/2	0.42	2.05	0.29	1.01
1	1-1/2	2	0.92	2.40	0.51	0.68
		2-1/2	0.73	2.20	0.47	0.78
		3	0.61	2.00	0.42	0.85
		3-1/2	0.53	1.85	0.39	0.91
		4	0.46	1.72	0.36	0.97
1	2	3	0.74	1.85	0.52	0.78
		3-1/2	0.63	1.72	0.49	0.85
		4	0.55	1.60	0.45	0.90
		4-1/2	0.49	1.48	0.42	0.94
		5	0.44	1.39	0.39	0.98
1	2-1/2	3-1/2	0.75	1.60	0.56	0.79
		4	0.66	1.48	0.52	0.83
		4-1/2	0.58	1.38	0.49	0.87
		5	0.52	1.30	0.46	0.91
		5-1/2	0.47	1.22	0.43	0.95
		6	0.44	1.17	0.41	0.99
1	3	4	0.75	1.40	0.59	0.79
		4-1/2	0.67	1.30	0.55	0.82
		5	0.60	1.22	0.52	0.86
		5-1/2	0.55	1.16	0.49	0.90
		6	0.50	1.10	0.46	0.93
		6-1/2	0.46	1.04	0.44	0.95
		7	0.43	1.00	0.42	0.99
1	4	5	0.78	1.10	0.62	0.77
		6	0.65	1.00	0.56	0.84
		7	0.55	0.92	0.52	0.91
		8	0.48	0.85	0.48	0.96
		9	0.43	0.80	0.45	1.01
1	5	9	0.52	0.73	0.51	0.93
		10	0.47	0.68	0.48	0.96
		11	0.42	0.64	0.45	1.00
1	6	10	0.55	0.63	0.53	0.89
		12	0.45	0.58	0.49	0.98

REINFORCED CONCRETE

18. Reinforcement for Tension

Concrete having very little tensile strength must be reinforced with steel when subjected to bending stresses. In beams and slabs the usual amount of main tension steel is about 3/4% to 1% of the cross-sectional area of the beam; the size of the rods or bars depending upon the size (as to cross-section) of the beam; say 1/2-in. for thin floor slabs; from 3/4-in. to 1-1/4-in. for rectangular beams; structural shapes for very heavy girders. Plain round rods are sometimes used though the use of deformed rods (having depressions or projections) is general practice in order to increase the grip or bond between the concrete and steel. Four types of deformed bars are illustrated in Fig. 14.

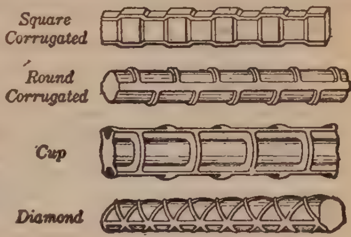


Fig. 14

Standard specification for reinforcing steel is given in Appendix 5.

Adhesion of Concrete to Steel. The grip or bond between concrete and plain rods (clean but not too smooth) depends somewhat upon the quality of the concrete — in general the stronger the concrete the higher the bond.

Bond stress is determined in two ways: (1) by direct pulling out of a rod embedded in a concrete block; (2) by embedding the two ends of a rod in a concrete beam (the middle of the rod being exposed by a recess in the beam) subjecting the beam to bending by loads and measuring the stress in the exposed middle portion of the rod by means of strain-gage readings. The latter method gives somewhat lower results than the former. Under test, initial slip of plain bars, indicating the destruction of pure adhesion, occurs at about 2/3 the maximum bond stress, after which frictional resistance comes into play until the maximum is reached. Maximum bond stress is in plain bars about 20% of the compressive strength of the concrete.

Deformed bars behave under test as do the plain rods up to the point of maximum bond of the plain rods. Beyond this point the bond of plain rods falls off rapidly up to failure, whereas the bond of the deformed rods continues to increase due to the abutting action between the concrete and the bar projections. Maximum bond of deformed bars at a slip of 0.05 in. is about 50% greater than the maximum bond of plain bars at a slip of 0.01 in.

Ordinary rust increases the bond. Mill or rust scale, however, destroys it.

Bond of flats is much less per square inch than the bond of square or round rods.

Right-angle bent ends do not affect initial slip but increase ultimate bond near the hook about 50%.

Hooked ends — 180° semicircular bends with a short length of straight beyond the bend — are more effective, increasing ultimate bond near the hook about 100%.

19. General Principles of Design

Use and Advantages of Reinforced Concrete. Steel is a material especially well suited to resist tensile stresses, and for such purposes the most economical form, the solid compact bar, is well adapted. A disadvantage in the use of steel in many locations is its lack of durability, thus rendering it necessary to add a protective covering to prevent corrosion and injury from fire. Concrete is characterized by low tensile strength, relatively high compressive strength, and great durability. It is a good fire-proof material and therefore serves both as a good inhibit of rust and as a good fire-proof covering for steel. A combination of steel and concrete constitutes a form of construction possessing in a large degree the advantages of both materials without their disadvantages.

For those structural members carrying purely tensile stresses steel must be employed, but it may be surrounded by concrete as a protection against corrosion and fire, or for the sake of appearance. For large and compact compression members plain concrete may be used. For more slender members, however, such as long columns, plain concrete is too brittle a material, and therefore too much affected by secondary and unknown stresses to be satisfactory; and for such members steel alone, or the two materials in combination, will preferably be used. For those structural forms in which both tension and compression exist, that is to say, in all forms of beams, the combination of the two materials is particularly advantageous. In this form the tensile stresses are carried by the steel and compressive stresses by the concrete.

Some of the most important types of construction in which reinforced concrete can be advantageously employed are the following: In buildings, for floors or for the complete structure; in foundations, especially where broad footings are required; in culverts and small beam bridges; in retaining walls, dams and abutments; in arch bridges; in bins and tanks for coal, grain and other material; in conduits and pipe lines subjected to low pressures; in chimneys and towers; and in separate structural forms such as piles, railroad ties, poles, etc.

General Assumptions in Calculations. In calculations of stresses and sections of reinforced-concrete structures the following general principles are usually followed:

(1) Calculations are made with reference to working stresses and safe loads rather than with reference to ultimate strength and ultimate loads, although some prefer to calculate ultimate loads and then apply a safety factor.

(2) Perfect adhesion is assumed between concrete and reinforcement. Under compressive stresses the two materials are therefore stressed in proportion to their moduli of elasticity. Under tensile stresses this relation also holds up to the ultimate strength of the concrete.

(3) Inasmuch as the extensibility of concrete is small, its tensile resistance is usually neglected and the entire tensile stress is assumed to be taken by the reinforcement. Where the stresses are very small, as frequently occurs in arches, the resistance of the concrete is sometimes considered. Certain beam formulas also take account of the tensile resistance of the concrete.

(4) Under working stresses the modulus of elasticity of concrete in compression is constant and the variation of compressive stress on a section of a beam is therefore rectilineal. The variation of tensile stress is also rectilineal and at the same rate. For ultimate loads a curvilinear law is usually assumed, the parabola being a convenient approximate curve to use.

(5) In beams, a plane section before bending remains plane after bending.

(6) Initial stress in the reinforcement due to contraction or expansion in the concrete is neglected.

20. Theory of Reinforced Beams

General Arrangement of the Reinforcement. The purpose of steel reinforcement is to carry the principal tensile stresses, the concrete being depended upon for the compressive and direct shearing stresses. If no steel were present the concrete would tend to rupture on lines perpendicular to the direction of maximum tension, as shown in Fig. 15, and hence we may conclude that the ideal tension reinforcement would require the steel to be distributed in the beam along the lines of maximum tension.

Fig. 15

Fig. 16

Fig. 16 shows by the dotted lines the lines along which failure of the concrete in a reinforced beam tends to occur. The inclined full lines show how the beam may be reinforced against such failures.

Where the shearing stresses exceed about 50 lb. per sq. in. some form of shear reinforcement is necessary. Fig. 17 shows various methods of arranging

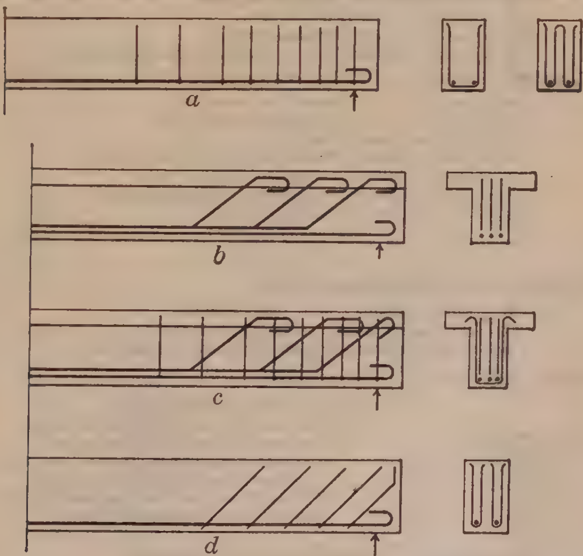


Fig. 17. Methods of Shear Reinforcement

such reinforcements. Figs. (*a*) and (*c*) show the use of vertical stirrups and Fig. (*d*) inclined stirrups. Inclined stirrups must be firmly fastened to the horizontal rods. For maximum strength form (*c*) or (*d*) should be used.

Methods of Failure. A reinforced-concrete beam tested to destruction will usually fail in one of three ways: (a) By the yielding of the steel at or near the section of maximum bending moment. (b) By the crushing of the concrete at the same place. (c) By a diagonal tension failure of the concrete at a place where the shear is large.

General Results of Tests. From the results of tests made by various experimenters the following general conclusions may be drawn:

(1) The elastic limit or, more strictly speaking, the yield point of the steel, may safely be taken as its ultimate strength in reinforced beams.

(2) The crushing strength of concrete as determined by tests on cylinders hardened under similar conditions will be fully realized in the beam.

(3) The usual shearing strength for beams reinforced with straight rods only is from 100 to 150 lb. per sq. in., but this can readily be increased by the use of proper web reinforcement to 300 or 400 lb. per sq. in.

(4) Steel used on the compressive side of beams is stressed in accordance with the usual assumptions.

Flexure Formulas for Working Loads. Assumptions: Rectilineal law of stress variation is assumed and the tension in the concrete is neglected. The notation used is as follows:

For Rectangular Beams:

$\mathbf{f_s}$ = tensile unit stress in steel.
$\mathbf{f_c}$ = compressive unit stress in concrete.
$\mathbf{E_s}$ = modulus of elasticity of steel.
$\mathbf{E_c}$ = modulus of elasticity of concrete.
M = moment of resistance, or bending moment in general.
$\mathbf{M_c}$ and $\mathbf{M_s}$ = moments of resistance with respect to the concrete and the steel.
A = steel area. b = breadth of beam.
d = depth of beam to center of tension steel.
t = total depth of beam.
k = ratio of depth of neutral axis to effective depth d.
z = depth of resultant compression below top.
j = ratio of lever arm of resisting couple to depth d.
jd = $d - z$ = arm of resisting couple.
$\mathbf{n} = Es/Ec$. $m = \dfrac{fs}{fc}$. $p = A/bd$.

For T Beams (in addition to preceding):

b = width of flange. b' = width of stem.
t = thickness of flange (see also t above).
p = steel ratio based on circumscribing rectangle bd.

For Beams Reinforced for Compression:

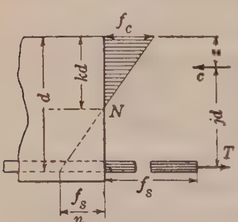


Fig. 18

$\mathbf{A'_s}$ = area compressive steel.
$\mathbf{p'}$ = steel ratio for compressive steel.
$\mathbf{f'_s}$ = compressive unit stress in steel.
C = total compressive stress in concrete.
C'' = total compressive stress in steel.
d' = depth to center of compressive steel.
z = depth to resultant of C and C'.

From Fig. 18 by proportion

$$\frac{k}{1-k} = \frac{f_cE_s}{E_cf_s} = \frac{n}{m} \qquad (1)$$

By composition

$$\frac{k}{k+(1-k)} = \frac{n}{m+n}\,nk = \frac{n}{m+n} \qquad (2)$$

From (2)

$$j = 1 - \frac{n}{3\,(m+n)} \qquad (3)$$

Since the total compression in the beam must equal total tension ($\Sigma H = 0$);

$$Af_s = \frac{bkdf_c}{2} = \frac{n}{m+n} \times \frac{bdf_c}{2}$$

from which
$$\frac{A}{bd} = p = \frac{n}{2\,m\,(m+n)} \qquad (4)$$

Assuming ratios for $m = \frac{f_s}{f_c}$ and $n = \frac{E_s}{E_c}$ equations (3) and (4) give all necessary solutions of a reinforced concrete beam. For $n = 15$ and $m = 24.5$ $\left(\text{as for example } \frac{16\,000}{650}\right)$ we have

$$j = 1 - \frac{5}{15 + 24.5} = \frac{7}{8}. \qquad p = \frac{15}{49 \times 39.5} = 0.0077$$

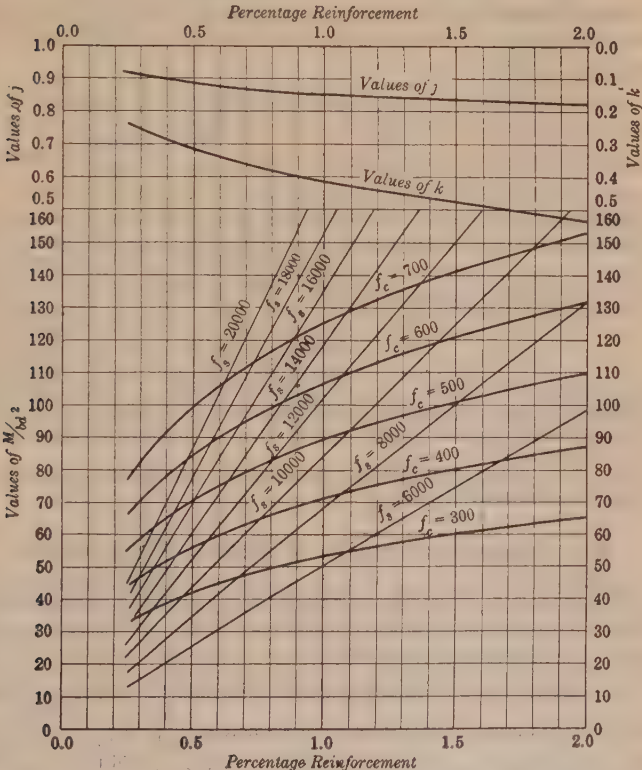


Fig. 19. Diagram for Reinforced Rectangular Beams. ($n = 15$)

The above values for j, p, m, n are usual values, hence for design calculations the moment of resistance of a beam

$$= 0.0077\,bdf_s \cdot \frac{7}{8}d = M \quad \text{or} \quad bd^2 = \frac{148\,M}{f_s}$$

Also (for n = 15), $bd^2 = \frac{10\,M}{f_c}$.

To design a beam for a bending moment of 416 000 in-lb. at 17 000 lb. in the steel and 700 lb. in the concrete $\left(\frac{17\,000}{700} = 24.5\right)$

$$bd^2 = \frac{148 \times 416\,000}{17\,000} = 3630. \quad \text{If } b = 12 \text{ in., } d^2 = 303 \text{ or } d = 17\text{-}1/2 \text{ in.}$$

$$\text{Cover} = \underline{\quad 2 \quad}$$
$$19\text{-}1/2 \text{ in.}$$

Use of Diagram Fig. 19. Fig. 19 gives values of k and j for various percentages of tensile reinforcement, also steel and concrete stresses for various values of $\frac{M}{bd^2}$ (for $n = 15$).

Assume a beam under bending moment 416 000 in-lb. to design for 16 000 lb. in the steel and 650 lb. in the concrete. At intersection of proper curves for unit stresses read to left $\frac{M}{bd^2} = 108$ or $bd^2 = \frac{M}{108} = \frac{416\,000}{108} = 3860$ and read to bottom $p = 0.77\%$. Various values of b and d will suit the case.

T-Beams. A T-beam may be treated as a rectangular beam $h\,w$ in Fig. 20 if the neutral axis as for a rectangular beam lies in the flange (at or above the bottom of the floor slab). In such case the T-beam is theoretically a rectangular beam with part of the neglected tension area of the concrete omitted, leaving sufficient stem to contain the reinforcing steel and resist longitudinal shear between the tension and compression flanges. If any considerable compression area of the enclosing rectangular unit is missing in the T-beam the structure will require special consideration. Derivation of formulas will not be given here.

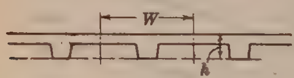


Fig. 20

Fig. 21 is a diagram for designing T-beams, the use of which will be illustrated in an example later on.

21. Over-reinforcement to Reduce Compression

From Fig. 19 it is seen that if the dimensions of a concrete section are for any reason limited, its moment of resistance may be increased (within limits) without exceeding allowable stresses by increasing the percentage of tension steel. The resulting steel stress will be low in order that the stress in the concrete may be kept within limits. For example, assume a beam 15 in. wide, 20 in. deep to carry a bending moment of 845 000 lb. without exceeding 16 000 lb. in the steel and 650 lb. in the concrete. $\frac{M}{bd^2} = \frac{845\,000}{15 \times 20 \times 20} = 141$. In diagram Fig. 19 find $p = 2\%$ for $f_c = 650$. f_s will then be a little less than 9000. Steel area $0.02 \times 15 \times 20 = 6$ sq. in.

It is usually considered better practice to keep the concrete stress within proper limits by over-reinforcement with tension steel as shown above, than by the use of compression steel. If, however, the compression steel is well tied into the concrete by inverted stirrups so as to make the compression flange in effect like the hooped column, it will be effective. The following discussion of beams reinforced for compression is therefore given:

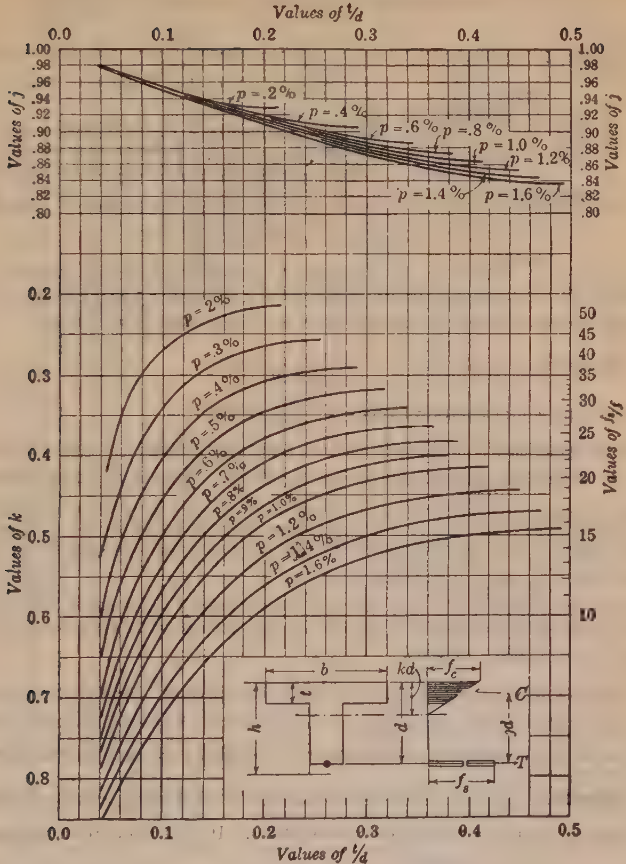


Fig. 21. Diagram for Reinforced T Beams. (n = 15)

22. Beams Reinforced for Compression

The normal capacity of a 15-in. × 20-in. beam for 16 000 lb. in the steel and 650 lb. in the concrete would be as follows: From Fig. 19 $\frac{M}{bd^2} = 108$; moment of resistance $= 15 \times 20 \times 108 = 648\,000$ in-lb.; $p. = 0.77\%$; $A_s = 15 \times 20 \times 0.0077 = 1.3$ sq. in. As in the above example a bending moment of 845 000 in-lb. is to be carried. 845 000 − 648 000 = 197 000 in-lb. to be carried by additional steel reinforcement. Compressive steel will be placed 2 in. from the upper face. From Fig. 19, $k = 0.38$; $kd = 7.6$ in.; therefore from Fig. 22 unit stress in compressive steel (for $n = 15$) =

$\frac{650 \times 15 \times 5.6}{7.6} = 7200$ lb. per sq. in. Required compressive steel $\frac{197\ 000}{18 \times 7200} = 1.5$ sq. in.

To maintain equilibrium of tension and compression forces, add tension steel $\frac{197\ 000}{18 \times 16\ 000} = 0.7$ sq. in. Total tension steel $2.3 + 0.7 = 3$ sq. in.

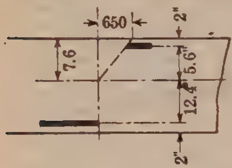


Fig. 22

Fig. 23 is a diagram for obtaining the arm z of the resisting couple and the ratio between the unit stress in tension steel and that in the extreme fiber of the compression flange. (Unit stress in the compressive steel will always be small.) The diagram is based on $n = \frac{E_s}{E_c} = 15$.

Example 1. Required the carrying capacity of a beam 15 in. wide, effective depth 20 in., having 3 sq. in. tensile steel and 1.5 sq. in. compressive steel placed 2 in from the top of the beam so that $\frac{d'}{d} = \frac{2}{20} = 0.1$.

Steel ratio p for tensile steel $= \frac{3}{15 \times 20} = 0.01$.

Steel ratio p' for compressive steel $= \frac{1.5}{15 \times 20} = 0.005$.

In the upper part of the diagram pass a straight-edge through $p = 0.01$ at left and $p' = 0.005$ at right and at its intersection with line for $\frac{d'}{d}$ read off $\frac{fs}{fc} = 24.5$. In lower part of diagram find in like manner $j = 0.88$. The moment value of the beam as determined by the tensile steel at a working stress of 16 000 lb. per sq. in. $= jdAsfs = 0.88 \times 20 \times 3 \times 16\ 000 = 845\ 000$ in-lb. The corresponding unit stress in the concrete $= fc = \frac{fs}{m} = \frac{16\ 000}{24.5} = 650$ lb. per sq. in.

If the allowable stress in the concrete were to be limited to say 600 or 1/27 of the stress in the steel, the moment value of the beam as determined by the concrete would be $\frac{24.5}{27} \times 845\ 000 = 766\ 000$.

Value of k may be found from the upper part of the diagram. From the intersection of the straight edge with the proper line for $\frac{d'}{d}$ follow the direction of the radial $\frac{fs}{fc}$ lines to the left hand ordinate, then horizontally to the scale for k. In the above example $k = 0.38$.

Example 2. A floor slab carries a moment of 12 000 in-lb. per inch of width. Design for unit stresses 16 000 lb. and 600 lb. If $\frac{fs}{fc}$ as found from the upper diagram is less than $\frac{16\ 000}{600} = 27$ the strength of the slab will be limited by the tension steel. Upper steel is to be 1-1/2 in. from the upper face. Assume $p = 0.01$ and $p' = 0.005$. Assume tentatively $\frac{d'}{d} = 0.15$. Find from upper part of diagram $\frac{fs}{fc} = 24$; then unit stress in steel must not exceed $\frac{24}{27} \times 16\ 000$ to give 600 in the concrete. From lower part of diagram find $j = 0.866$.

$$\frac{24}{27} \times 16\ 000 \times 0.01d \times 0.866d = 12\ 000$$

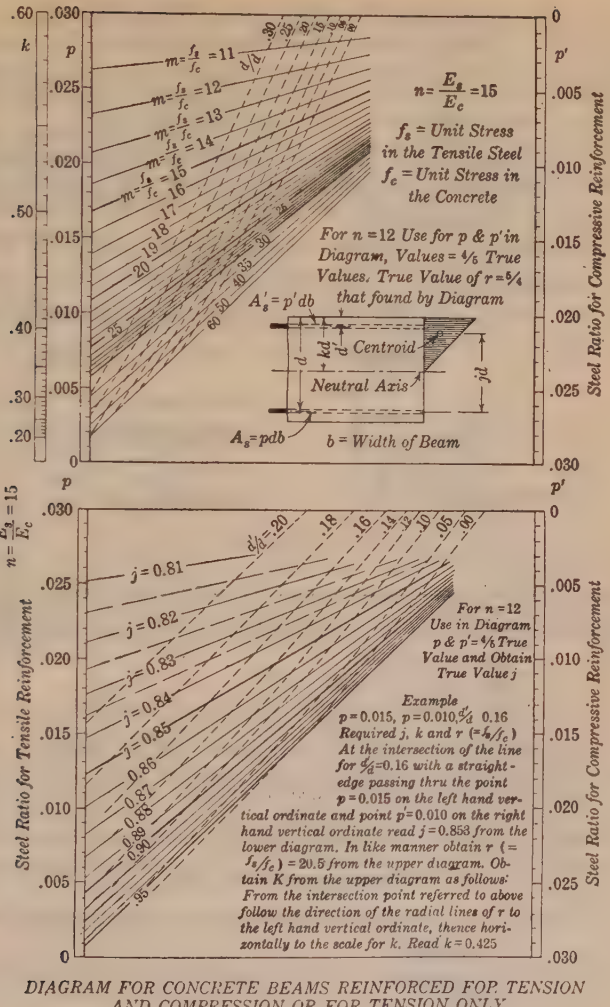


DIAGRAM FOR CONCRETE BEAMS REINFORCED FOR TENSION AND COMPRESSION OR FOR TENSION ONLY

Fig. 23

Solve for $d = \sqrt{97}$ = say 10 in.

Actual $\frac{d'}{d} = 0.15$, as assumed; hence no correction to the calculation is needed.

23. Combined Bending and Direct Stress

Case 1. Tension Over Part of the Section

As in the case of beam action alone, the compressive stress in the concrete may in case of necessity be kept within proper limits by increasing the percentage of tensile reinforcement or by adding compressive steel.

Assume that on a concrete section 1 ft. wide and 34 in. effective depth there has been calculated a bending moment of 113 000 ft-lb. and a direct thrust of 20 000 lb. acting along the axis of the structure or at the middle of the section. To avoid confusion reduce all figures to inch and pound units. $M = 113\,000 \times 12 = 1\,360\,000$ in-lb.

The bending moment and thrust are equivalent to a single force 20 000 acting a distance $\frac{1\,360\,000}{20\,000} = 68$ in. from the axis $= 84$ in. from the steel (Fig. 24). This is equivalent to a pure bending moment 1 680 000 in-lb.

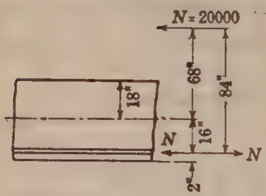


Fig. 24

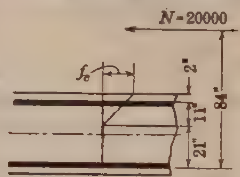


Fig. 25

acting on the section and a direct force $N = 20\,000$ lb. acting directly on the steel which will reduce the area of steel required for bending alone by $\frac{20\,000}{16\,000}$ $= 1.25$ sq. in. $\frac{M}{bd^2} = \frac{1\,680\,000}{12 \times 34 \times 34} = 121.$

From Fig. 19 for 16 000 lb. or 650 lb. limiting find $f_c = 650$; $f_s = 13\,000$; $p = 1.1\%$. Required $A_s = 0.011 \times 12 \times 34 - 1.25 = 3.25$ sq. in.

Since the concrete stress is limited, in the above example, compressive steel might have been used. Moment of resistance of normal beam 12 in. $\times$ 34 in. is $108 \times 12 \times 34 \times 34 = 1\,500\,000$ in-lb. with $f_s = 16\,000$ lb.; $f_c = 650$ lb.; $k = 0.38$; $kd = 13$ in.; $p = 0.77\%$; $A_s = 3.15$ sq. in. Balance of moment $1\,680\,000 - 1\,500\,000 = 180\,000$ in-lb. may be provided by additional compressive steel and tensile steel. (See Fig. 25.) Unit stress in compressive steel $= 650 \times 15 - 11/13 = 8250$ lb. per sq. in. Required compressive steel area $\frac{180\,000}{8250 \times 32} = 0.68$ sq. in. Required additional tensile steel (neglecting effect of direct stress) $\frac{180\,000}{32 \times 16\,000} = 0.35$ sq. in. Total tensile steel required $3.15 + 0.35 - \frac{20\,000}{16\,000} = 2.25$ sq. in.

Case 2. Compression Over Entire Section

The following construction is by E. H. Harder as published in the December 1927 number of "Concrete."

For a plain concrete section no tension exists if the direct force is within the middle-third as limited by the " core points," which are then situated a distance 1/6 t from the center of the section. If the section (as for a concrete arch, for example) is reinforced, the effect of the reinforcement is to move the " core points " outward. Ten per cent reinforcement may move the core points to a distance 1/3 t from the axis. An assumption of 1/4 t will in general lead to sufficiently accurate results without further approximation. Without generalizing a concrete example will illustrate the method of determining the stresses in the concrete and the steel for an eccentric direct thrust. Assume a section 12 in. × 30 in. acted upon by a thrust of 150 000 lb. and a bending moment of 840 000 in-lb. Eccentricity $e = \frac{840\,000}{150\,000} = 5.6$ in. Find the necessary reinforcement to keep compressive stress in the concrete within 600 lb. Refer to Fig. 26. Assume core points 30/4 = 7-1/2 in. from the center line, as indicated. Lay off $bf_c = 12 \times 600 = 7200$ to scale at the extreme compressive fiber and carry over to the direct force 150 000 lb. as indicated at a. Connect a with core points c_1 and c_2.

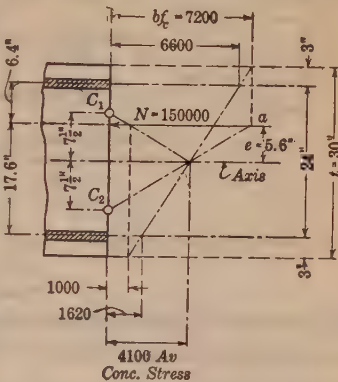


Fig. 26

Where line aC_2 cuts the axis, scale the average concrete stress = 4100. Load carried by concrete = N_c = 4100 × 30 = 123 000 lb. Remainder to be carried by steel = 150 000 − 123 000 = 27 000 lb.

$$\text{Carried by upper steel } 27\,000 \times \frac{17.6}{24} = 19\,800 \text{ lb.}$$

$$\text{Carried by lower steel } 27\,000 \times \frac{6.4}{24} = 7\,200 \text{ lb.}$$

At these steel areas the concrete stresses may be found by scale as follows:

$$\text{At upper steel } \frac{6600}{12} = 550 \text{ lb.}$$

$$\text{At lower steel } \frac{1620}{12} = 135 \text{ lb.}$$

$$\text{Required area of upper steel } (n = 15)\ \frac{19\,800}{15 \times 550} = 2.4 \text{ sq. in.}$$

$$\text{Required area of lower steel } \frac{7200}{15 \times 135} = 3.5 \text{ sq. in.}$$

24. Shear and Bond Stresses

Horizontal shear at any point in a beam is assumed as a measure of the diagonal tension for which secondary reinforcement may be required.

In Fig. 27 is indicated a small length x of beam with opposing compressive stresses on opposite sides indicated as C and C', opposing tensile stresses T

and T' and shear $= V$, no loads being assumed in the length x. The total horizontal shear above the steel and below the neutral axis is then $T' - T$ and **unit** horizontal shear

$$v = \frac{T' - T}{bx} \quad (1)$$

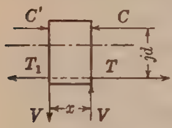


Fig. 27

in which b = width of beam. Taking moments with CC' as center,

$$(T' - T) = \frac{Vx}{jd} \quad (2)$$

From (1) and (2) find **unit** horizontal shear above the steel and below the neutral axis

$$= v = \frac{8\,V}{7\,bd} \quad (3)$$

Total horizontal shear per unit of length for the full width of beam

$$= \frac{8\,Vb}{7\,bd} = \frac{8\,V}{7\,d} \quad (4)$$

In that portion of the beam containing no other tension reinforcement than the horizontal rods, the horizontal shear $\frac{8\,V}{7\,d}$ per inch of length must be developed by the grip of the concrete on the rods. If $\Sigma 0$ = the sum of the perimeters of all the rods, the **bond** per square inch $= \frac{8\,V}{7d\,\Sigma 0}$.

In the above the approximate value of 7/8 for j was used and is correct within 1% or 2% for rectangular beams. For T-beams j may possibly vary from 0.85 to 0.95 and since the proportioning of T-beams usually necessitates the use of a diagram giving correct theoretic values of j, the correct value might as well be used.

25. Web Reinforcement for Shear

Fig. 28 represents a portion of a beam having some of the main rods bent up to resist diagonal tension which tends to open up cracks as indicated.

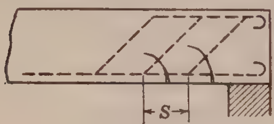


Fig. 28

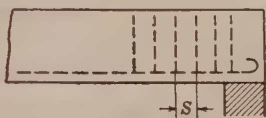


Fig. 29

Horizontal shear is taken as a measure of the diagonal tension. S = distance between bend-up of rods, and n rods each of cross-section A are bent up at any one point.

From equation (4) total horizontal shear in length $\mathbf{S} = \frac{Vbs}{bjd} = \frac{Vs}{jd}$ carried by concrete and bent-up rods. At 40 lb. per sq. in. allowable in the concrete, the concrete will carry 40 **bs** lbs. At 16 000 lb. per sq. in. in the steel, the steel will carry $16\,000\ \sqrt{2}nA$, assuming the rods bent up at 45 deg. Then

$40bs + 16\,000\,\sqrt{2}\,nA = \frac{Vs}{jd}$, from which the proper spacing s may be determined.

Fig. 29 represents a portion of beam having straight rods and **vertical** stirrups. If A' = area of stirrup rod and n = number of legs (2 for U-stirrup, 4 for W-stirrup, etc.) $40bs + 16\,000\,nA' = \frac{Vs}{jd}$.

26. Example: Complete Design of T-Beam for a Floor

Figs. 30 and 31. Span 40 ft. Dead load 1200 lb. per lin. ft. Moving live load concentration 9000 lb.

Assume floor slab to have been designed and required total depth $t = 8$ in. Working stresses $\left\{ \begin{array}{l} f_s = 17\,000 \\ f_c = 700 \end{array} \right\}$

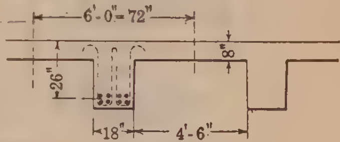


Fig. 30

Maximum dead load M at center $= 1/8 \times 1200 \times 40 \times 40 \times 12 = 2\,880\,000$ in-lb.
Maximum live load M at center $= 1/4 \times 9000 \times 40 \times 12 = 1\,080\,000$ in-lb.

3 960 000 in-lb.

Uniform dead and moving live loads give parabolic variation of maximum

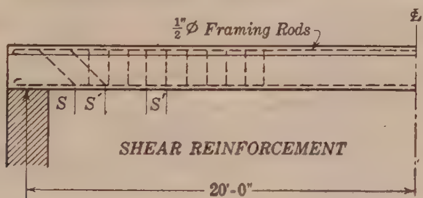


Fig. 31

moments from point to point as shown to scale on Fig. 32. Eight rods **1-1/4** in. round would give 10 sq. in. of steel.

For this area, approximate $d = \frac{3\,960\,000}{10 \times 17\,000 \times 7/8} = 26 \pm$ in.

For $d = 26$, $\frac{t}{d} = \frac{8}{26} = 0.31$ $\quad p = \frac{10}{26 \times 72} = 0.53\%$

From diagram Fig. 21 $j = 0.90$ $\quad \frac{f_s}{f_c} = 30$

Stress in steel $= \frac{3\,960\,000}{0.90 \times 26 \times 10} = 16\,900$ lb. per sq. in.

Stress in concrete $= \frac{16\,900}{30} = 540$ lb. per sq. in.

Moment value of each bar $= 1.23 \times 17\,000 \times 0.90 \times 26 = 495\,000$ in-lb. Plotting **twice** this moment value accumulatively on diagram for bending moment in the beam, Fig. 32, indicates where each pair of rods may be bent up for shear reinforcement.

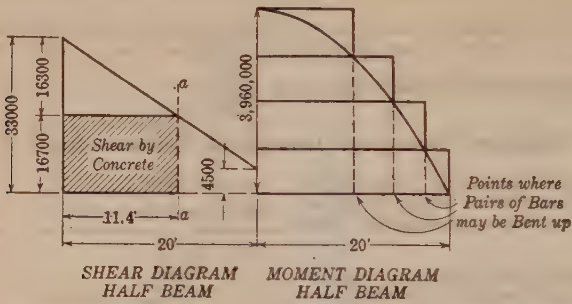


Fig. 32

Shear and Bond Stresses.

End shear. Dead load 1200 × 20 = 24 000 lb.
Live load 9 000 lb.

33 000 lb.

Center shear = 4500 lb. Plot shear diagram as shown.

Horizontal shear acts just above the rods in the stem of the beam; hence for shear and bond calculations b = 18 in. (not 72 in.).

Unit horizontal (longitudinal) shear above rods $\dfrac{33\,000}{18 \times 0.90 \times 26} = 79$ lb.

Allowable without shear reinforcement or special anchorage, as explained in Art. 29 = 40 lb. per sq. in.; hence shear reinforcement is required.

Circumference of 1-1/4-in round rod = 4 in. Four rods of upper layer will be bent up for shear reinforcement; hence at the end of the beam the maximum bond will act on 4 lower rods.

Bond $\dfrac{33\,000}{0.90 \times 26 \times 4 \times 4} = 88$ lb. per sq. in.

Shear capacity of concrete alone 18 × 0.90 × 26 × 40 = 16 700 lb.
Shear capacity of one rod bent up at 45 deg.

$$\frac{1.23 \times 17\,000 \times \sqrt{2} \times 0.90 \times 26}{S} = \frac{693\,000}{S}$$

Shear capacity of one 3/8-in. round vertical W-stirrup (4 legs):

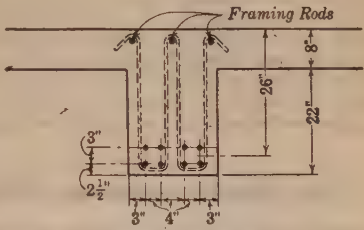
$$\frac{4 \times 0.11 \times 17\,000 \times 0.90 \times 26}{S} = \frac{175\,000}{S}$$

On the shear diagram plot shear capacity of concrete alone = 16 700 lb. Shear in excess of this must be supplied by bent-up rods and stirrups. To right of line $a - a$ shear is less than shear capacity of concrete and no shear reinforcement is needed. Shear carried by steel at first bend-up point is scaled 14 500. Hence S is calculated at $\dfrac{693\,000}{14\,500} = 48$ in., but should not exceed about 3/4 d = 20 in.

Shear just to right of bent-up rods is scaled 11 000. Hence S' at this point $= \dfrac{175\,000}{11\,000} = 16$ in. but should not exceed $1/2\,d = 13$ in. Bond stress in stirrups is assumed to be developed by grip of the concrete through half the depth of the beam.

$$\text{Bond} = \frac{17\,000\,A}{1/2 d \Sigma 0} = \frac{17\,000 \times 4 \times 0.11}{1/2 \times 26 \times 4 \times 1.2} =$$

120 lb. per sq. in.



STIRRUP DETAIL

Fig. 33

Space stirrups on 11-in. centers, reducing bond to 11/13 × 120 = 100 lb. per sq. in., unless reinforcement is provided with special anchorage as described in Art. 29.

27. T-Beams in Beam and Slab Construction

Total width of flange to be considered as effective width in design calculations not to exceed 1/4 span length of beam. Overhanging width on either side of stem to be considered as effective, not to exceed 1/2 clear distance to next beam nor 8 times the flange thickness. (See Fig. 34.)

For beams having flange on one side only, effective width in design not to exceed 1/12 span length of beam, and overhanging width not to exceed 1/2 clear distance to next beam nor 6 times the flange thickness.

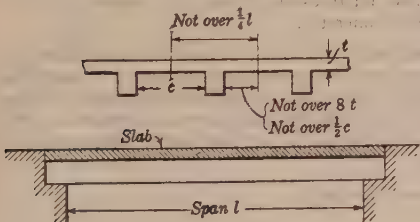


Fig. 34

Isolated T-Beams (designed to provide additional compressive area):

Flange thickness to be not less than one-half width of stem.

Total flange width to be not more than four times width of stem.

In all cases adequate shear and bond resistance must be provided between flange and stem.

The rules governing design as given in the following, Articles 28 to 33, are in agreement with the Joint Code of the American Concrete Institute and Concrete Reinforcing Steel Institute. Unit stresses mentioned in the text are for 2000-lb. concrete (that is, concrete that will develop an ultimate compressive strength of 2000 lb. per sq. in. as determined by tests in accordance with A. S. T. M. standards at age of 28 days.) For concrete of different strength the allowable unit stresses are in proportion.

28. Ordinary Anchorage of Main Reinforcement

In simple beams or free ends of cantilever beams, at least one-fourth of the positive reinforcement should extend along the tension face and beyond the face of the support a distance equal to at least 10 diameters of the bar.

In any span of a continuous, restrained or cantilever beam, the tensile negative reinforcement should have a length of anchorage beyond the face of support sufficient to develop full maximum tension at an average bond stress of 80 lb. per sq. in. for plain bars and 100 lb. per sq. in. for deformed bars (2000-lb. concrete assumed). At least one-third of the negative reinforcement should extend straight to or beyond the point of inflection and the remainder should be bent down not sharper than 45 deg. and overlapped with the positive reinforcement or anchored in a region of compression. Not less than one-fourth the positive reinforcement should extend along the same face into the support and be embedded at least 10 bar diameters beyond face of support.

29. Special Anchorage of Main Reinforcement

Tests show that beams in which the main bars and shear bars or web reinforcement (when required) are more securely anchored than as specified above have greater resistance against ultimate failure. In such beams, higher unit stresses in shear and bond may be allowed. The designer must determine whether it is more economical, in a particular case involving heavy shear and bond stresses, to use special anchorage to justify higher unit shear and bond, or to proportion the beam for the lower unit stresses with ordinary anchorage.

In simple beams or free end of continuous beams, at least one-half of the tensile reinforcement should extend along the tension face to provide anchorage beyond face of support for one-third the allowable working stress in tension at an average bond stress not to exceed 80 for plain bars nor 100 for deformed bars (based on 2000-lb. concrete).

In continuous and restrained beams, anchorage beyond points of inflection of at least one-third of the negative reinforcement and anchorage beyond face of support of at least one-third of the positive reinforcement should be provided to develop one-third of the allowable working stress in tension at average bond stresses not to exceed 80 for plain bars nor 100 for deformed bars (based on 2000-lb. concrete).

Figs. 35 to 38 illustrate types of anchorage.

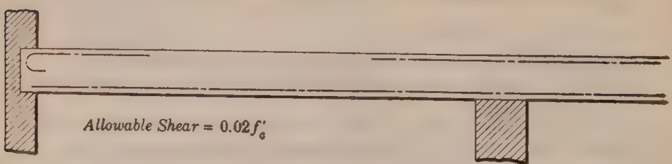


Fig. 35

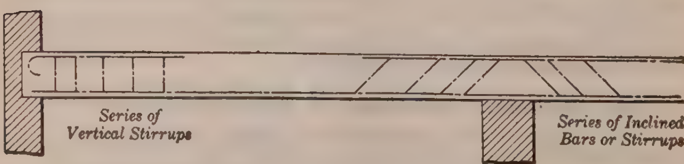


Fig. 36

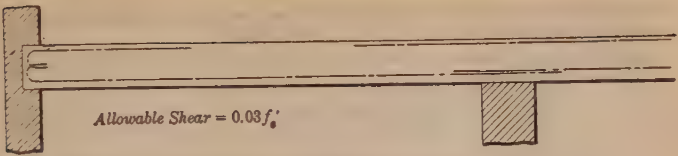


Fig. 37

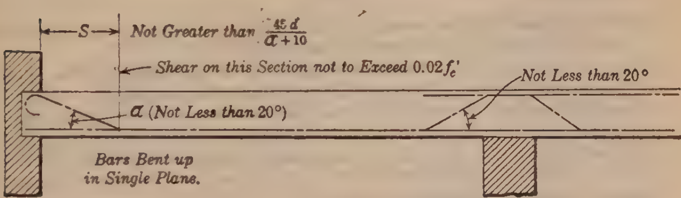


Fig. 38

30. Anchorage of Web Reinforcement

Web reinforcement shall be anchored at both ends by one of the following methods or a combination thereof, but only anchorage meeting the requirements of (a), (b) and (c) shall be used for shearing unit stresses in excess of 160 lb. per sq. in. (for 2000-lb. concrete).

(a) Providing continuity with the main reinforcement.

(b) Bending around the longitudinal reinforcement.

(c) A hook which has a radius at least 4 times the diameter of the web bar.

(d) A length of embedment in the compressive half (upper or lower) of the beam sufficient to develop the stress in the stirrup at bond stresses of 80 for plain bars or 100 for deformed bars (based on 2000-lb. concrete) and also provided that the other end of the stirrup is anchored as specified in (a).

The free end anchorage of a web member not bearing on the longitudinal reinforcement shall be such as to engage an amount of concrete sufficient to prevent the bar from pulling out. In all cases the stirrup should be carried close to the upper and lower surfaces.

31. Unit Stresses in Beams and Slabs

Allowable unit stresses for 2000-lb. concrete are, expressed in pounds per square inch:

Flexure. Extreme fiber stress in compression, 800 lb., excepting adjacent to the supports of continuous beams, 900 lb.

Shearing Stress. (a) Beams without web reinforcement and with ordinary anchorage of longitudinal bars, 40 lb.

(b) Beams without web reinforcement, and with special anchorage of longitudinal bars, 60 lb.

(c) Beams with properly designed web reinforcement and ordinary anchorage of longitudinal bars 120 lb.; the concrete being assumed to carry its share at 40 and the balance being carried by the web steel.

(d) Beams with properly designed web reinforcement and special anchorage of longitudinal rods 180 lb.; the concrete being assumed to carry its share at 60 and the balance being carried by the web steel.

Bond Stresses. Allowable bond stress between concrete and reinforcement for ordinary anchorage of main rods:

80 for plain rods.
100 for deformed rods.

In beams provided with special anchorage of main rods, the above values may be doubled.

Tension in Steel Reinforcement. Structural steel grade, 16 000 lb. per sq. in. Intermediate grade, hard grade or rail steel, 18 000.

Note. The Joint Code recommends 20 000 for the **intermediate** grade as the single standard for billet steel reinforcement conforming to the "Standard Specification for Billet-Steel Concrete Reinforcement Bars," (Serial Designation A15-14, of the American Society for Testing Materials) or for "Rail Steel Concrete Reinforcement Bars," conforming to the Standard Specification A. S. T. M. (Serial Designation A16-14.) See Appendix 5 for Specifications.

Ratios of Moduli of Elasticity, to be used as a basis for design calculations, are recommended as follows:

$$\frac{E_s}{E_c} = \begin{cases} 15 \text{ for 2000-lb. concrete at 28 days.} \\ 10 \text{ for 3000-lb. concrete at 28 days.} \end{cases}$$

and proportionately for other values; but not less than 10.

32. Calculation of Web Reinforcement

The calculation of web shear reinforcement consisting of bent-up main rods at 45 deg. and of vertical stirrups separately in different portions of the beam has been illustrated in the example of T-beam design. In more extreme cases a combination of bent-up rods and stirrups in the same region may be required. In such cases the total shearing resistance of the beam may be assumed as the sum of the shearing resistance of the concrete (at its proper value, 40 or 60 lb. per sq. in., see Art. 31) and the shearing resistances of each type of reinforcement, providing the total calculated resistance does not exceed the values given above (Art. 31). The effectiveness of an inclined bar or stirrup depends upon the angle of inclination (α) with the horizontal, expressed in degrees, and is assumed to be as follows:

When α is less than 45 deg.,

$$\frac{\text{Allowable total direct stress in bar}}{\sin \alpha}.$$

When α is between 45 and 90 deg.,

$$\text{Allowable total stress in bar} \times (\sin \alpha + \cos \alpha).$$

(See Fig. 39.)

Main bars bent up for shear should have an inclination not less than 15 deg.

Inclined stirrups should have a minimum inclination of 30 deg. (from 30 to 90 deg.).

When the total unit shearing stress is less than 120 lb. sq. in. (for 2000-lb. concrete) the distance from edge of support to the first stirrup or first bend-up point of main rod or between stirrups or bend-up rods shall not exceed $S = \frac{45\,d}{\alpha + 10}$, in which d = effective depth of beam.

When the total unit shearing stress exceeds 120 lb. sq. in. (for 2000-lb. concrete) S shall not exceed $\frac{30\,d}{\alpha + 10}$.

When the web reinforcement consists of bars bent-up at a single plane (see Fig. 38) the point of bending and shear at point of bending must not exceed the values indicated, and the portion of the total unit shear on the concrete assumed to be carried by the steel should not exceed 75 lb. per sq. in.

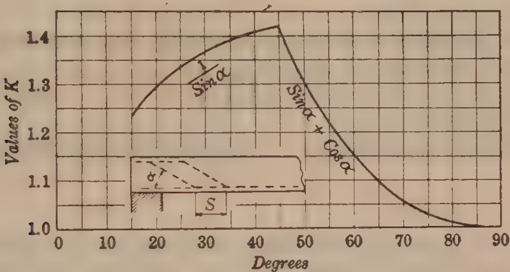


Fig. 39

33. Practical Points of Design

Minimum clear spacing between reinforcing bars:

(a) Unanchored bars: { 1-1/2 times diameter of round bars. / 1-1/2 times diagonal of square bars.

(b) Anchored bars (see Sect. 29) { One diameter of round bars. / One diagonal of square bars.

(c) Never less than 1 in. or 1-1/3 times maximum size of aggregate.

Maximum spacing of **principal slab reinforcement** in other than flat slabs and ribbed floor construction should be 3 times the slab thickness.

Minimum clear distance between bar and side of beam or other member should be equal to bar diameter but not less than 1 in.

Minimum protective cover of reinforcement in fire-resistive construction; 1 in. for slabs and walls, 1-1/2 in. for beams, girders and columns; in ordinary construction 3/4 in. for slabs and walls, 1 in. for beams, girders and columns.

When concrete is deposited directly on the ground, reinforcement near the ground should have 3-in. protection. At other surfaces of concrete exposed to the ground or weather, 2-in. protection should be provided.

Splices. Splicing by over-lap of bars:

(a) Preferably at other points than those of maximum stress.
(b) Splices of adjacent bars to be staggered.
(c) Minimum spacing between laps, as given for separate bars.
(d) Amount of overlap to be sufficient to transfer stress between bars by shear and bond; in general a minimum of 40 diameters of the bar.

Span Length (assumed for design purposes):

Simple beams; center to center distance of supports, but not more than clear span plus depth of beam.

Continuous or restrained beams built to act integrally with supports; clear distance between supports.

Unsupported Length of compression flange of a beam should not exceed 24 times width of flange.

Reinforcement against Shrinkage and Temperature Stresses. Reinforcement normal to the principal reinforcement in roof and floor slabs reinforced in one direction only should be from 0.2% to 0.3%; but in no case should bars be spaced more than 5 times the slab thickness nor more than 18 in.

34. General Notes on Building Construction

Floor Slabs. Reinforced-concrete floors are well adapted to use where the framework is of steel columns and beams, as well as where the entire structure is of concrete.

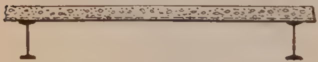


Fig. 40

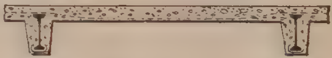


Fig. 41

Figs. 40 to 43 illustrate various designs of floors supported on steel I-beams. In Figs. 40 and 41 the reinforcement may be of small rods or a metal fabric of some sort, the latter being convenient for short spans. In Figs. 42 and 43

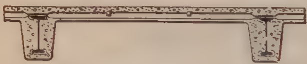


Fig. 42

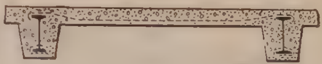


Fig. 43

the reinforcing bars are hooked around the flange of the beam, thus providing anchorage.

Fig. 44 illustrates floor arch construction in the case of a very heavy floor to support a load of 1500 lb. per sq. ft. In this case the shearing stresses were

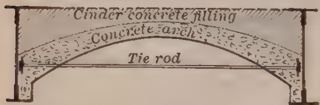


Fig. 44

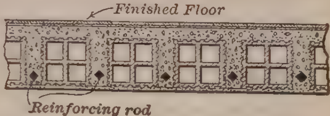


Fig. 45

so great that a sufficient bond strength could not readily be obtained without anchorage. When such anchorage is used it changes the beam to an arch, and enables the thickness of the concrete to be greatly reduced near the center of the span. Fig. 45 shows a common form of floor construction in which terra-cotta tile is used with concrete. The ribs of concrete form essentially small T-beams.

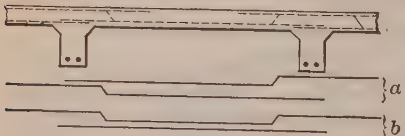


Fig. 46

Where concrete beams are used the slab and beam are built simultaneously. For short spans a metal fabric is convenient, as in Fig. 41. For longer spans, Fig. 46 shows common arrangement of reinforcement, the negative moments being provided for by bending up a part of the rods as in (*a*) or (*b*). The result may also be arrived at by using separate short straight rods over the support.

T-Beams. Beams and girders are usually designed as T-beams, utilizing a portion of the floor slab as a part of the beam. The proportions of the beam below the slab will be determined by considerations of strength, economy, requirement of head-room, and space for reinforcing material. Generally the ratio of depth to width will vary from 1 to 3 as extreme values, the larger ratio being suitable for very large beams. Deep beams are economical of concrete, but cost more for forms than shallow beams. An effective bond should be provided at the junction of the beam and slab. When the principal slab reinforcement is parallel to the beam, transverse reinforcement should be used extending over the beam and well into the slab.

If designed as continuous, the beam will be a rectangular beam at the support and will require strengthening at that point by compressive reinforcement, or by increase of depth. An increase of compressive stress at the support, amounting to 10 or 15%, is often permitted. Fig. 47 illustrates the main features of beam and girder design.

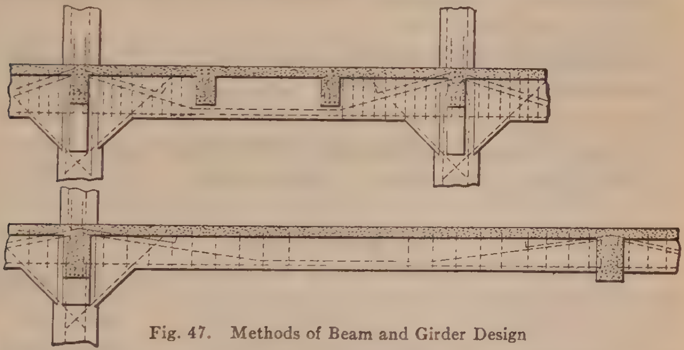


Fig. 47. Methods of Beam and Girder Design

Walls and Partitions

Concrete or Reinforced Concrete is well adapted to the construction of all walls where considerable strength is desired. For walls supporting light loads, such as curtain walls between the framework of a reinforced-concrete building, or partitions, the requirements of strength are met by the use of very thin walls, but such walls become relatively expensive on account of the cost of forms and do not make a dry or warm construction.

Walls of Double Thickness. Concrete walls are often made of double thickness with an air space enclosed, each part being 3 to 4 in. thick. The air spaces are formed by core boxes which are drawn up with the outside forms as the work proceeds. Cheap pipe of thin metal may also be used and left in place.

Brick Facing. Walls of single thickness are often finished with a single layer of face brick, with satisfactory results both in appearance and dryness.

Reinforcement of walls should be sufficient to prevent cracks. unless the wall is to be faced. A small amount of reinforcement is desirable in any case to insure adequate strength.

Partitions may be made as thin as 2 or 3 in., if the concrete is slightly reinforced. Hollow concrete tile or metal lath and plaster will, however, generally be more economical for partitions than poured concrete.

Roofs

General Design. Reinforced-concrete roofs are designed and constructed in the same manner as floors. For steep slopes the concrete in the slabs must be laid quite dry, or else top forms must be used to retain the concrete. The latter method is slow and expensive. Cinder concrete may frequently be used to advantage as the loads are relatively light.

Imperviousness. Concrete roofs may be designed merely as the supporting area for a roof covering of other material, or may be designed to be imvious and complete without such covering. To secure absolute imperviousness requires special precautions, and a maximum amount of reinforcement against shrinkage cracks (0.5 to 1.0%). If designed to support a separate roof-covering, the construction is much simplified and the design can be made very economical. To support the roof covering nailing strips of wood are embedded at suitable intervals.

Adaptability. Reinforced concrete is well adapted to roof construction where a fireproof construction is desired. To support the roof, beams may be employed for spans up to 40 or 50 ft. and trusses for still longer spans. The arch type of construction can also be readily carried out in reinforced concrete. Domes have been built of reinforced concrete in several instances. The stresses are resisted chiefly by circumferential tension rods and by the compressive stress in the concrete in a radial direction.

Girder Bridges, Trestles

Beam or Girder Bridges are designed in the same manner as other heavy concrete floors. Spans up to 20 or 30 ft. may well be made in the form of a simple slab of uniform thickness spanning the opening. For railroad structures the loads are relatively so large that shearing stresses will usually require careful attention. For longer spans a gain in economy will result by the use of main horizontal girders of relatively great depth, with a floor supported by the girders and reinforced transversely. The bridge may be made either a "through" or "deck" girder, according to the requirements of the case, the latter being the more economical. The details are arranged in a variety of ways, but the calculation and design of the reinforcement to meet the given conditions require no special consideration. The proper allowance for impact is an important point in this connection. Durability is an important factor favorable to the use of reinforced concrete for bridge floors.

Trestles. Where several short spans are required and concrete is used for both the girders and the piers, the latter may usually be made of comparatively small cross-section—much smaller than possible if ordinary masonry is used. The structure then approaches the ordinary floor and column construction in the relation of its parts. The piers may generally consist of two or more columns connected by a suitable portal. Where the piers are small, as here assumed, they must be built rigidly in connection with the girders of one or more spans, as are the columns in a building. The girders must be designed with proper reference to their continuity, and the piers must be reinforced somewhat near the top to enable them to resist a part of the bending moment due to the continuity of beams and piers.

Expansion joints should be provided at intervals not exceeding 75 to 100 ft., otherwise the movement due to contraction and expansion will be concentrated at the ends of the structure, resulting in severe stresses in the end columns. Fig. 48 illustrates the general arrangement of a large concrete trestle. Low trestles may be made without longitudinal bracing, the bending resistance of the columns giving the necessary stability. The end piers or abutment must be designed to act also as retaining walls.

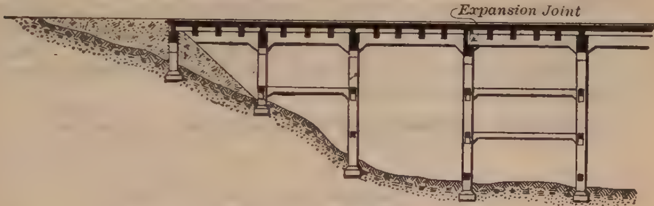


Fig. 48. Concrete Trestle

Pile Trestles, in which reinforced-concrete piles are used, are advantageous in many locations. They are somewhat more expensive than wooden trestles, but are durable and are well adapted to ballast floor construction. In their construction the concrete piles are driven into place and then caps of reinforced concrete are molded about the heads of the piles. On the caps are then placed slabs, forming the supporting floor for ballast.

35. Two-Way Reinforcement

Reinforcement of Slabs in Two Directions. If the length of a slab supported on all 4 sides exceeds 1.5 times its width the entire load should be carried in the short direction. Square slabs may well be reinforced in both directions. The exact distribution of load on square and rectangular slabs, supported on four sides and reinforced in both directions, cannot readily be determined. The following method of calculation will give results on the safe side. The distribution of load is to be determined by the formula

$$r = \left(\frac{l}{b} - \frac{1}{2}\right)$$

in which r = proportion of load carried in the short direction, l = length, and b = breadth of slab. For various ratios of l/b the values of r are:

l/b = 1.0	1.1	1.2	1.3	1.4	1.5
r = 0.50	0.6	0.7	0.8	0.9	1.0

Using the values above specified each set of reinforcement is to be calculated in the same manner as slabs having supports on two sides only. The spacing of rods so determined may safely be increased somewhat for the portions of the slabs between the edges and the quarter points.

Beams and Girders. The arrangement of columns, girders and beams is determined according to the same principles as in steel construction. Where the spacing of girders is not large (12 to 15 ft.) and where cross beams are not needed to secure lateral stiffness, the latter may be entirely omitted or used only at columns so as to form a panel which is square or nearly so. Reinforcement in two directions is not economical for oblong panels. Generally where cross beams are used they should be spaced from 5 to 8 ft. apart.

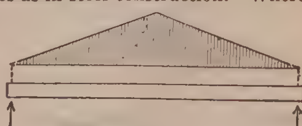


Fig. 49

If the panels are square or nearly so, the distribution of load on the beam may be assumed in accordance with Fig. 49. Under this assumption the bending moment at the center is equal to $wl^3/12$, where w = load per square foot and l = length of beam. Strictly speaking, the distribution is somewhat more uniform than here indicated, so that the bending moment given by this formula is somewhat on the safe side.

36. Effect of Continuity in Floors

Continuity of beams and slabs over their supports affects the moment factors that may be used in design. The following are the usual approximate rules for equal spans. For unequal spans special analysis is required. Beams and slabs built to act integrally with beams, girders or other **slightly** restraining supports and carrying uniform loads:

Single span: Maximum possible moment near center $= \dfrac{Wl^2}{8}$

Two spans: (Fig. 50)

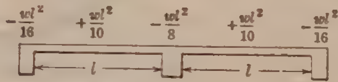


Fig. 50

More than two spans: (Fig. 51)

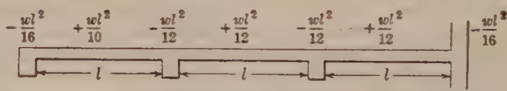


Fig. 51

For beams and slabs built to act integrally with columns, walls or other restraining supports and assumed to carry uniform loads, the customary approximate moment factors depend upon the relative stiffness factors of beam and supports: that is relative $\dfrac{I}{l} = \dfrac{\text{moment of inertia}}{\text{length of member}}$

Refer to Fig. 52

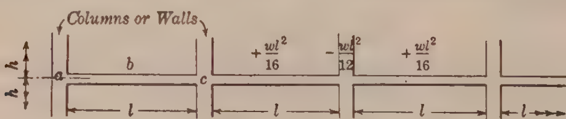


Fig. 52

Case 1. $\dfrac{I}{l}$ for beam $< \dfrac{1}{2} \times \dfrac{I}{l}$ for col. $\begin{cases} M_a = -\dfrac{wl^2}{12} \\ M_b = +\dfrac{wl^2}{12} \\ M_c = -\dfrac{wl^2}{12} \end{cases}$

Case 2. $\dfrac{I}{l}$ for beam $\gtreqless$ $\dfrac{I}{l}$ for col. $\begin{cases} M_a = -\dfrac{wl^2}{16} \\ M_b = +\dfrac{wl^2}{10} \\ M_c = -\dfrac{wl^2}{10} \end{cases}$

For moment and shear factors in continuous beams for particular conditions of loading see Sect. 7, Art. 14.

37. Distribution of Concentrated Loads in Floor Slabs

Various tests by Ohio State Highway Commission and the University of Illinois show that a slab having a width at least twice the span length and simply supported will distribute a point load at the center in such manner that the bending moment produced will be the same as for an independent slice of the slab having a width equal to 2/3 the span length and supporting the load as a line load across its width. From this it may be concluded that a load concentrated on a line parallel to the supports (Fig. 53) will be distributed over an "effective width" equal to the length of line plus 2/3 span length. Thus for a 5000-lb. load on a line 2 ft. long parallel to supports of a slab of 9-ft. span, "effective width" $= 2/3 \times 9 + 2 = 8$ ft. and bending moment $= \frac{5000}{8} \times \frac{9}{4} = 14\,000$ ft-lb.

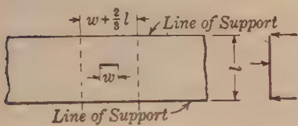


Fig. 53

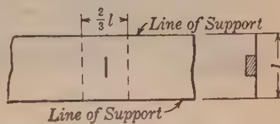


Fig. 54

A center load concentrated on a line perpendicular to supports (Fig. 54) may be considered as distributed along the length of line in direction of span and across the width of a slab having a width equal to 2/3 span length.

Theoretical considerations indicate that for a point-load off center the "effective" lateral distribution at each support will be $4/3 \times$ distance to support. In calculating moment due to such a load the effective width at load may be assumed to be the average of the lateral distribution at both supports or $2/3\,l$ as for center load. (See Fig. 55.)

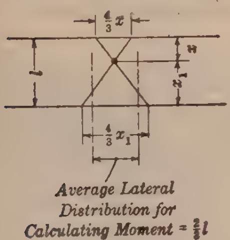


Fig. 55

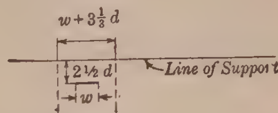
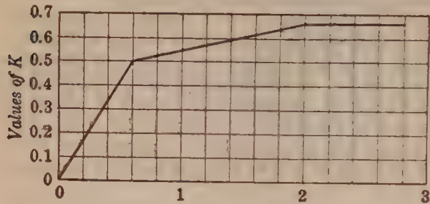


Fig. 56

Critical shears for a concentrated load approaching a support have not been determined experimentally but existing data indicate that a slab properly designed for moment in accordance with the above will fail by punching shear rather than by diagonal tension. In the absence of more definite experimental data it may be safer to follow one of the rules adopted as good practice. The specification of the American Railway Engineering Association is as follows: Calculate shear at support by placing the load a distance from support equal to 2-1/2 times effective depth of slab and assume lateral distribution equal to 4/3 this distance as indicated in Fig. 56.

It is usually the case that rules for shear control the proportioning of the slab and require thickening the slab as designed for moment, or haunching at the supports which increase the cost of formwork. Experiments are now under way at Iowa State College, Ames, Iowa, to determine more definitely the laws of load distribution in floor slabs in relation to shear as well as moment and the results may lead to considerable economy in the design of floor slabs for moving concentrated loads.



Ratio = $\dfrac{\textit{Width of Slab}}{\textit{Span Length}}$

Lateral Distribution of Point Load at Mid Span = Kl

Fig. 57

For narrow slabs (width less than twice span length) the following rules are supported by experimental data, tests being made for point center load at middle of width. If width is 0.6 span length, "effective width" = half span length. Straight line variation of "effective widths" may be assumed for intermediate widths between this case and $\frac{w}{l} = 2$, and between this case and $\frac{w}{l} = 0$. (See Fig. 57.)

38. Flat-Slab Floors

The flat-slab floor is a type of floor in which a slab is supported directly upon the columns without the use of beams and girders, the slab acting as a continuous plate built into the column supports. Three general arrangements of slab reinforcement are in common use as illustrated in Figs. 58, 59

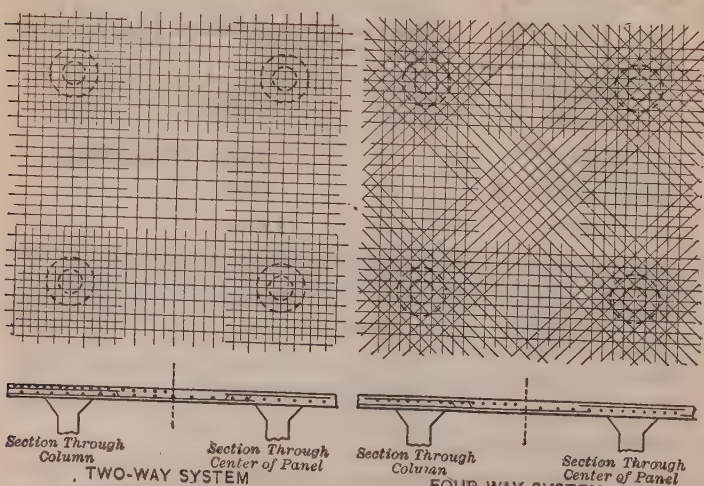


Fig. 58 Flat Slab Reinforcement Fig. 59

and 60. Columns are usually enlarged at the top into capitals. "Dropped panels" are often formed by thickening the slab in the vicinity of the columns as shown in Fig. 62.

Bending moments in the slab are approximated. When not governed by municipal building codes, the Joint Code of the American Concrete Institute and Concrete Reinforcing Steel Institute may be followed as quoted in part in the text following.

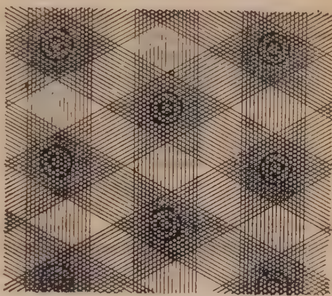
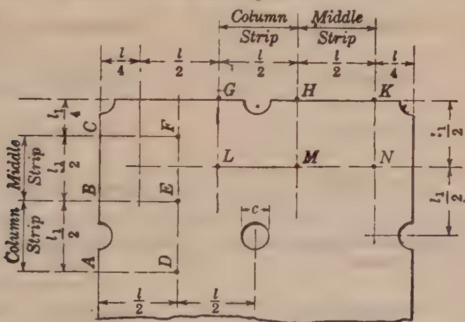


Fig. 60

Fig. 61 shows the sections for the critical moments given in the table for the interior panels of a slab having three or more rows of panels in each direction, and in which the panels are approximately uniform in size.

Dropped panels should have a length or diameter b not less than 0.35 of the panel length l in that direction and a thickness t_2 not greater than 1-1/2 times nor less than 1-1/4 times the thickness t_1 of the slab proper.

In calculating reinforcement all bars crossing the section are considered effective. For diagonal bars in 4-way reinforcement the effective area is taken as the cross-sectional area multiplied by the sine of the angle between the bar and the section on which moments are figured.



Column Head Sections AB and GH Follow the Circumference of the Column Capital

Fig. 61

Due attention must be paid to bending moments at sections intermediate to the critical sections. The lines of inflection may be assumed as follows, distances being measured from center lines of panel:

	Without drop	With drop
Column strip................................	$0.33\ (l - c)$	$0.3\ (l - c)$
Middle strip................................	$0.33\ l$	$0.3\ l$

Allowance for shifting of points of inflection due to varying loads is made by carrying **all** bars in rectangular or diagonal bands, each side of sections of critical positive or negative moment, to a line at least 20 diameters beyond the lines of inflection. At least 0.4 of all bars in each band provided for positive moment should extend continuously in either direction to act as reinforcement for the opposite critical sections for negative moment. At least 1/3 of bars for positive moment in the column strip should extend into the dropped panel at least 20 diameters of the bar, or, if there are no dropped panels, to a point not more than 1/8 l from the center line of the column or support.

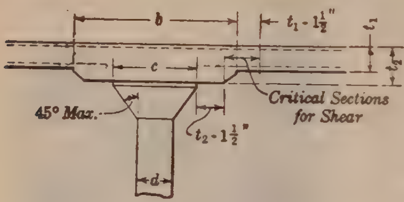


Fig. 62

Critical Moments for Interior Panels

W = Total uniform load on a single panel area.

Section	2-way reinforcement, no drop panel	2-way reinforcement, dropped panel
AB	$-0.0414\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$-0.0450\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$
BC	$-0.0144\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$-0.0135\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$
DE	$+0.0198\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$+0.0180\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$
EF	$+0.0144\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$+0.0135\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$
Section	**4-way reinforcement, no drop panel**	**4-way reinforcement, dropped panel**
AB	$-0.045\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$-0.0486\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$
BC	$-0.009\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$-0.0072\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$
DE	$+0.018\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$+0.0171\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$
EF	$+0.018\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$	$+0.0171\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$

For moments on GH, HK, LM and MN substitute l_1 for l in the above moment factors for AB, BC, DE and EF respectively.

For square panels $l_1 = l$.

The numerical sum of positive and negative moments for the column strip and middle strip in each direction is the same for all cases, namely,

$$0.09\ Wl\left(1 - \frac{2}{3}\frac{c}{l}\right)^2 \quad \text{and} \quad 0.09\ Wl_1\left(1 - \frac{2}{3}\frac{c}{l_1}\right)^2.$$

The ratio of $\frac{l}{l_1}$ should not exceed 4/3.

In wall panels and other panels in which the slab is discontinuous at edge of panel, the maximum positive moments on the strips perpendicular to the discontinuous edge should be calculated as 25% greater than those given in the table for interior panels, and all positive moment reinforcement should extend to the discontinuous edge and be embedded into the supports. The negative moments at the discontinuous edge are assumed as follows: in the column strip 90% of that for interior panels and in the middle strip, 5/8 of that for interior panels.

In panels having a marginal beam on one edge or two adjacent edges, the beam should be designed to carry at least the directly superimposed loads, exclusive of panel load. A marginal beam having a depth greater than 1-1/2 times minimum slab thickness should be designed to carry, in addition, a uniformly distributed load equal to at least 1/4 the total load for which the adjacent panel or panels are designed. Slabs supported by marginal beams on opposite edges should be designed as freely supported slabs for the entire load.

Minimum thickness t_1 of slab without drop

$$= 0.038\left(1 - 1.44\frac{c}{l}\right)l\sqrt{w} + 1\text{-}1/2,$$

in which w equals total load in pounds **per square foot.** Minimum thickness of dropped panel $t_2 = 0.02\ l\sqrt{w} + 1$. In addition, the slab thickness t_1 or t_2 should not be less than $\frac{l}{32}$ for floor slabs or $\frac{l}{40}$ for roof slabs (based on 2000-lb. concrete).

Shearing Stresses. Let r = proportional amount of total negative reinforcement in the column strip which directly crosses the dropped panel; or if there is no drop, the proportional amount directly crossing the column capital. Unless the slab is reinforced for shear, the unit shearing stresses calculated on the critical sections indicated in Fig. 61 should not exceed 40 $(l + r)$ or 60 lb. per sq. in., whichever governs (2000-lb. concrete being assumed).

Three-Way System. In 3-way systems, negative amount on the column head sections may be assumed as $0.032\ Wl\left(1 - \frac{2}{3}\frac{c}{l}\right)^2$ for each sextant strip around the column, and positive moment midway between the columns as $0.016\ Wl\left(1 - \frac{2}{3}\frac{c}{l}\right)^2$ for each sextant. l = distance center to center of columns and W = the total load on the **parallelogram panel.**

39. Reinforced-Concrete Columns

The rules of design below follow the Joint Code of the American Concrete Institute and Concrete Reinforcing and Steel Institute and may be used when building codes do not govern.

Concrete Columns may be: (a) reinforced by longitudinal bars (at least 4 having a minimum diameter of 5/8 in.) and separate hoops or ties. Longitudinal bars should have an effective cross-sectional area not less than 0.5% nor more than 2% of the total area of the column, and be embedded at least 2 in. clear from the face. The hoops or lateral ties should be not less than 1/4 in. in diameter spaced not more than 12 in.

(b) reinforced by longitudinal bars (at least 6 having a minimum diameter of 1/2 in.) and closely spaced continuous spirals enclosing a circular core the diameter of which may be measured c. to c. of the wire. The longitudinal bars should have an effective cross-sectional area not less than 1% nor more than 6% of that of the core. The spiral reinforcement should be not less than 1/4 the volume of the longitudinal reinforcement and the spirals spaced not more than 1/6 the diameter of the core nor more than 3 inches. Spirals should be evenly spaced and held by at least 3 vertical spacer bars.

Notation

n = ratio modulus of elasticity, steel to concrete.
p = steel ratio (area of steel to area of concrete core) for spirally reinforced column.
A_s = effective cross-sectional area of longitudinal reinforcement.
A'_c = net area of concrete in columns without spiral reinforcement (total column area less area of steel).
A = area of concrete core of spirally reinforced column.
$A_c = A(1 - p)$ = net area of core of spirally reinforced column.
f_c = allowable compression in concrete not to exceed $0.225 f'_c$.
f'_c = the ultimate strength at 28 days.

(a) Safe axial load for reinforced concrete columns without spiral reinforcement $= (A'_c + A_s n) f_c$.

(b) Safe axial load for spirally reinforced columns

$$= (A_c + npA) \times [300 + (0.1 + 4p) f'_c].$$

The above safe loads are for columns having a ratio of length, h, to radius of gyration, R, less than 40.

If the ratio exceeds 40 the safe load should be reduced, as follows: P being the safe load for the shorter column:

Safe load for long columns $= P\left(1.33 - \frac{h}{120\,R}\right)$, R for spirally reinforced columns should be calculated for the radius of gyration of the core.

Columns subjected to bending should be designed for the combined stresses, and additional reinforcement may be added to provide for the bending. For columns without spiral reinforcement the allowable compressive stress on the concrete may be increased 50%. The total amount of reinforcement considered in the calculations should not exceed 4% of the total column area.

For spirally reinforced columns the allowable compressive stress on the concrete core may be increased 20% over that shown in the bracket of the formula for safe axial load.

Principal columns in buildings should have a minimum diameter or thickness of 12 inches. Posts that are not continuous from story to story should have a minimum diameter or thickness of 6 inches.

Composite Columns. Safe load on composite columns in which a structural-steel or cast-iron column is thoroughly encased in a circumferentially reinforced concrete core may be based on the allowable unit stress of the metal column plus a unit stress of $0.25 f'_c$ on the area within the spiral core. The diameter of a cast-iron section should not exceed half of the diameter of the spiral core. Spiral reinforcement should be not less than 0.5% of the volume of the core.

40. Column and Wall Footings

Reinforced Concrete is especially well suited for the construction of footings for walls and columns and the foundations of any structure where the bearing area must be considerably larger than the superstructure resting thereon, such as towers, chimneys, lighthouses, arches, and, frequently, bridge piers and abutments. To secure the necessary bearing area by the use of plain concrete or other masonry requires great depth of foundation, increasing the expense and, often in the case of buildings, occupying much valuable space. Grillage foundations, consisting of steel I-beams protected by concrete, are more expensive than slabs of reinforced concrete, as they do not utilize the compressive resistance of the concrete; generally, therefore, the reinforced slab is to be preferred. Reinforced concrete is also used extensively for bearing piles and also for sheet piling, caissons, and other forms of foundation work.

Footings. The problem of the design of reinforced-concrete footings is in general the same as that of floors. For single footings of ordinary size a single square slab is most convenient. For larger footings and for footings carrying more than one column, a combination of beam and slab, similar to floor construction, is often most economical. For very soft soils this may need to spread as a floor over the entire foundation area, forming a monolithic structure.

For a **square slab** (Fig. 63) the pressure should be carried as directly as possible from the extremities to the center. Two sets of main reinforcing rods aa' and bb' should be used, as shown in the figure. The reinforcing of the remaining corners can best be done by sets of diagonal rods dd'. If these do not cover the area, then a few short cross-rods may be used. Reinforced in this way the total pressure on the area $ABCD$ may be assumed to be carried to the line BC, where the bending moment and shear will be a maximum. Figured as a free cantilever the resulting stresses will be higher than actually exist. If the entire square is reinforced by rods in two directions only, as ee', then a considerable part of such rods in the corners of the square are ineffective.

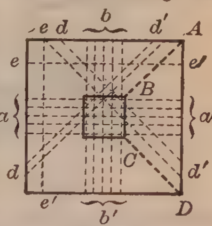


Fig. 63

In **cantilever beams,** used as footings, the maximum shearing stress is near the base of the wall where the moment is large. Shear cracks tend to form on the dotted lines a, a, Fig. 64. Bent rods, if used, should be bent just outside the column base, and not at the end of the beam, and stirrups must be spaced closely at this point. The beam being short the bond stress may require special attention. The shearing stresses in the concrete are likely to be high and exceed the limit generally allowed for beams. In this case, since the depth of the beam is relatively great, the dangerous section will be some distance from the face of the well. This distance may be taken equal to the effective depth of the beam and the rules for maximum shear then applied.

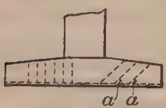
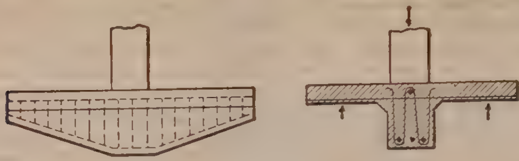


Fig. 64

For large **individual footings** a beam and slab may be economical. To secure the benefit of a T-section and to give a flat upper surface the beam may be placed under the slab as shown in Fig. 65. This arrangement requires some attention to the connection of slab to beam, as the upward pressure against the slab tends to pull it away from the beam. The use of a horizontal rod in the top of the main beam, bonded by stirrups, will give a thoroughly good anchorage for the transverse rods of the slab. For still larger areas a system of girders and beams may be adopted, constituting a floor reversed as



(a) (b)

Fig. 65. Large Individual Footings

to loads. For **wall footings** a simple slab construction with transverse reinforcement is suitable.

Stresses in Footings Eccentrically Loaded. If for any reason (eccentric load, earth pressure, wind pressure, etc.) the resultant of the forces acting above the foundation level intersects that level eccentrically, the pressures upon the earth below are not uniform but vary in intensity from a minimum at one edge to a maximum at the other. It is customary, and sufficiently accurate for the purpose, to assume that in such a case the pressure varies uniformly from one side to the other. In Fig. 66 let e = eccentricity of resultant pressure, l = length of base, W = total vertical load per lineal foot on founda-

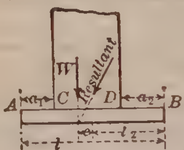


Fig. 66

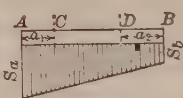


Fig. 67

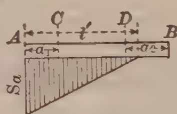


Fig. 68

tion, and S = pressure per unit area at any point. Then for rectangular areas, if e is less than $1/6\ l$, the variation in pressure is as represented in Fig. 67. The values of the intensity of pressure at A and B are as follows:

For edge A, $$S_a = \frac{W}{l}\left(1 + \frac{6e}{l}\right)$$

For edge B, $$S_b = \frac{W}{l}\left(1 - \frac{6e}{l}\right)$$

If e is greater than $1/6\ l$ the pressure is distributed as in Fig. 68 over a length l'_1 equal to $3\ (1/2\ l - e)$, and the pressure at A is $S_a = 2\ W/l' = 4\ W/3\ (l - 2e)$. The bending moments at C and D are then given by the formulas:

For Fig. 67, $$M_C = \frac{W}{2\,l^3}(l^2 + 6\,el - 4\ ea_1)\,a_1^2$$

and $$M_D = \frac{W}{2\,l^3}(l^2 - 6\,el + 4\,ea_1)\,a_1^2$$

For Fig. 68, $$M_C = \frac{2\,Wa_1^2}{3\,(l - 2\,e)^2}(9\,l - 18\,e - 2\,a_1)$$

and $$M_D = \frac{W}{54\,(l - 2\,e)^2}(2\,a_2 + 1 - 6\,e)^3$$

41. Arch Bridges

In ordinary masonry or concrete arches tensile stresses are not permissible. The ring must therefore be designed so that the line of pressure will not pass outside the middle-third. In reinforced arches this limitation does not exist. The arch rib is a beam, and if properly reinforced it may carry heavy bending moments involving tensile stresses in the steel. Besides this advantage, reinforcement renders an arch a much more secure and reliable structure, it greatly aids in preventing cracks due to any slight settlement, and by furnishing a form of construction of greater reliability makes possible the use of working stresses in the concrete considerably higher than those used in plain masonry. Furthermore, in long-span arches where the dead load constitutes by far the larger part of the load, any increase in average working stress counts greatly towards economy. It affects not only the arch itself but also the abutments and foundations.

Since the arch is a beam subject to either positive or negative bending moment it is essential that it should be reinforced on both sides, but the shearing stresses due to beam action are relatively small, so that little is needed in the way of web reinforcement. It is desirable, however, that the inner and outer reinforcements be tied together, somewhat as in a column, although in this case the necessity therefor is much less.

Technical terms are illustrated in Fig. 69. Abutment, pier, arch ring are the main parts of the structure. Soffit, back, and skewback are surfaces,

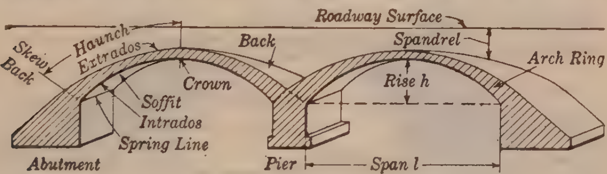


Fig. 69

the skewback being the surface of the abutment or pier from which the arch ring proper springs. Intrados and extrados are the lines on a longitudinal section, being the intersection of the cutting plane with the soffit and back respectively. The spandrel is the space between the back of the arch and the roadway over. The application of other terms is self-evident. "Span" and "rise" may also be applied to center-to-center distances on the neutral axis of the longitudinal section. The rise-ratio is ratio of rise to span $= \frac{h}{l}$.

The several types and sub-types of arch structure are as follows:

(1) **Solid barrel arch** in which the arch ring is continuous for full width of bridge.

(a) Spandrel filled (Fig. 69). The roadway is carried over the bridge on earth fill placed over the arch. Spandrel retaining walls at the two sides of arch retain the roadway fill.

(b) **Open Spandrel** (Fig. 70). The arch ring supports small spandrel arches or transverse walls or columns which in turn carry the floor system.

(2) **Ribbed Arches** consist of a number of relatively narrow arch ribs springing from the abutments and piers, supporting small spandrel arches or columns which in turn support the floor system.

Classification by Static Condition. A second classification of arches may be made with reference to their static condition, namely, three-hinged arch (statically determinate), two-hinged arch (indeterminate to first degree), and fixed or hingeless arch (indeterminate to third degree). The single hinge arch (hinge at crown) is rarely used.

The three-hinged arch is unaffected by temperature change, as regards stresses — excepting in a minute degree due to slight change in rise-ratio caused by rise or fall at the crown. Temperature stresses in a two-hinged arch of large rise-ratio are relatively small and may sometimes be neglected altogether. As the rise-ratio decreases temperature stresses become more important. Temperature stresses in the fixed arch are relatively large and increase in proportion as the rise-ratio decreases; the minimum rise-ratio may be determined by the temperature stresses.

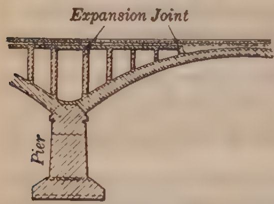


Fig. 70

The above considerations are important in selecting the type of structure, particularly for grade crossing elimination when it is necessary to keep distance from roadway under to roadway over at a minimum both on account of limitations of grades and cost of approaches. In such cases the **rigid-frame** bridge, to be discussed later, is preferable to either the arch or beam type structure.

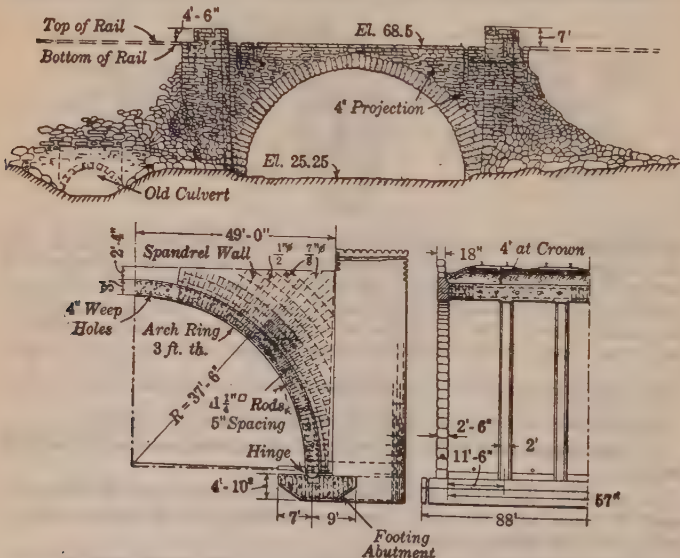


Fig. 71

Fig. 71 shows a two-hinged solid barrel reinforced-concrete arch designed to carry railroad traffic on an existing high embankment over a highway cut through the embankment.

Hinges. One type of hinge for concrete arch, designed for the above bridge, is shown in Fig. 72. Several other types of hinge are illustrated in Fig. 73.

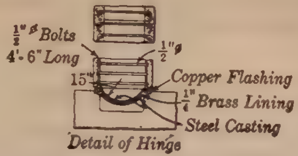


Fig. 72

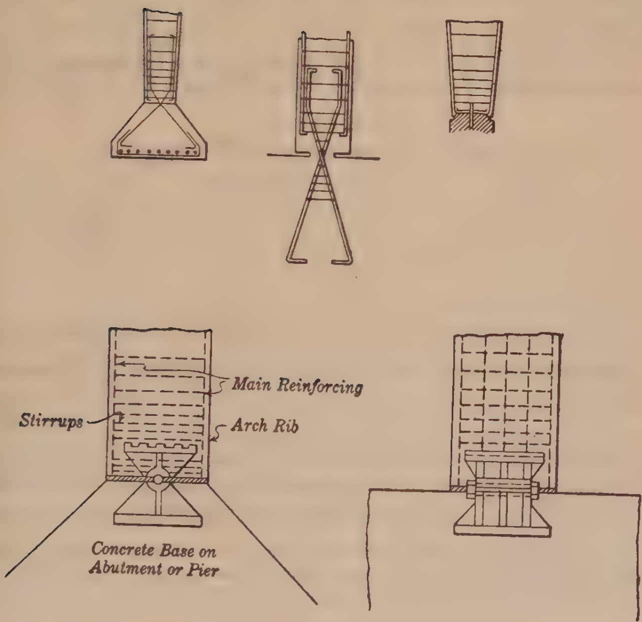


Fig. 73

42. Details of Design

The various parts of the arch structure should be well bonded together to resist temperature changes. All walls should be reinforced both vertically and horizontally with 0.1% to 0.2% of steel in order to prevent contraction cracks. Spandrel retaining walls should be provided with expansion joints above or on each side of the pier or at the edge of abutment to prevent cracking due to rise and fall of the crown caused by temperature changes.

Arches may safely be constructed on a good earth foundation that is not too compressible and that offers good resistance to sliding; also on pile foundations if the piles are driven to proper resistance and batter piles are driven to take the horizontal thrust. If the driving of the piles indicates uneven bearing capacity, the arch and abutments should be well reinforced transversely and any construction joints should be well keyed. Transverse reinforcement is a safeguard to any arch not built on ledge rock.

43. Intrados Curves

The intrados curves of arches should be true curves; preferably circular segments, elliptical segments, or (for "full-centered" arches) semicircles, or semi-ellipses. Parabolic and transformed catenary curves and their intermediates are not so pleasing and their use is justified only for structures of large size when strict economy is urgent. The use of "multiple-centered" curves invariablyde tracts from the appearance of the structure.

Clearance requirements as in the case of an arch over a highway sometimes require a modification of the elliptical curve as indicated in Fig. 74. The lower

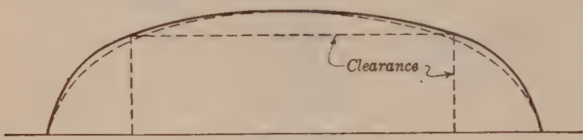


Fig. 74

curve is a true ellipse of equation form $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$. In which a and b are respectively semi-major and semi-minor axes and coincident with the axes of reference. The upper curve has the equation $\left(\frac{x}{a}\right)^{2.2} + \left(\frac{y}{b}\right)^{2.2} = 1$ and provides more clearance at the sides of the roadway under, and, as it is a truly mathematical curve, irregularities are avoided. Different results may be obtained by applying different fractional exponents to the equation $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^n = 1$. Such curves may be rapidly calculated by use of the log. slide rule, and have been used for bridges of the Westchester County (New York) Parkways with pleasing effect in contrast with nearby bridges having multiple-centered curves or "spline" curves.

44. Preliminary Design

The design of the two-hinged and fixed arch is ordinarily a process of approximation being the mathematical analysis of a predetermined arch section. Various methods of direct design have been developed which are useful for large structures, notably that of Whitney described in Trans. Am. Soc. C. E. 1925. For small and moderate span bridges the methods of analysis illustrated by examples following are simpler. Various formulas for determining crown thickness for preliminary design have been derived but are not particularly useful as different formulas give different results. Judgment based on experience will enable the designer to lay out a section for analysis that will not require a re-analysis.

45. Three-Hinged Arch

The three-hinged arch is merely a truss frame *abc* (Fig. 75) in which the stress in the omitted tie member *ac* is replaced by thrust *H* at the abutments. Loads may be applied along the curved members *ab* and *bc* which act as beams transferring their reactions to the joints of the truss frame. Thus in Fig. 75 the vertical component of reaction of *bc* at $b = \frac{Pa}{1/2l} = \frac{2\,Pa}{l}$. Taking moments about b, $\frac{V_L\,l}{2} = \frac{Pa}{l} \times \frac{l}{2} = Hh$ from which $H = \frac{Pa}{2h}$. Further discussion of analysis is unnecessary.

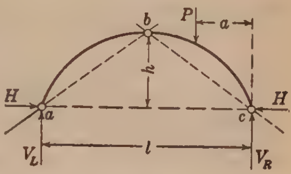


Fig. 75

46. Theory of Indeterminate Reinforced-Concrete Structures

The reader is referred to textbooks for a general discussion of the criteria for statical determinateness or indeterminateness of structure. In statically indeterminate structures the direction and magnitude, or direction, magnitude and points of application of the reactions cannot be found by the laws of statics alone, but depend also upon the relative elastic properties of different parts of the structure itself. There are no evident lever arms of the reaction from which the bending moments in the structure can be determined.

The methods explained in this section for attacking the problem depend upon a few simple fundamental laws of flexture which are illustrated, in the pages immediately following, by a few simple examples in beam deflection. The treatment is not rigorous. Shear and direct stress deformation is neglected in the development although correction for direct stress deformation may be made as " rib-shortening " as indicated for the arch. Other approximations are made to which attention will be called, but for all practical purposes the methods shown are sufficiently accurate for the design of the structures illustrated. Involved in these calculations are quantities which are a measure of the elastic properties of the structure, all containing values for moment of inertia. In such cases gross moments of inertia should be used, although the unit stresses are finally calculated on the occasional " cracked section," neglecting tension in the concrete.

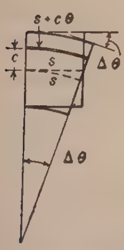


Fig. 76

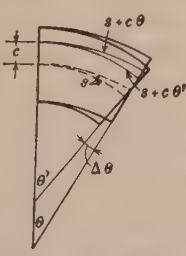


Fig. 77

Assume a very small length *s* of a beam originally straight as indicated in Fig. 76. After strain due to flexure alone, the change in length of a fiber distant *c* from the neutral axis will then be (in circular measure): $s + c\Delta\theta - s = c\Delta\theta$, $\Delta\theta$ being the measure of the change in direction of the tangents at the two ends of *s*. The strain (change in length) of this fiber may also be expressed as $\frac{Mc}{EI}$ in which $\frac{Mc}{I}$ is the stress on the fiber, due to the moment M of all external forces acting on the beam, as determined by the well-known beam formula. Comparing the two equations above, $\Delta\theta = \frac{Ms}{EI}$. The change in direction of the tangent between any two points will be the sum of the small changes for all the small lengths s between the points considered, or $\Sigma \frac{Ms}{EI}$.

Assume next a small length s of the neutral axis of a beam originally curved and subtending an angle θ before flexure and θ' after flexure (Fig. 77). The change in length of a fiber distant c from the neutral axis (due to flexure alone) will then be $(s + c\,\theta') - (s + c\,\theta) = c\,\Delta\theta$. The strain (change in length) of this fiber may also be expressed as $\frac{f(s + c\,\theta)}{E}$ in which f is the **unit** stress on this fiber due to the external moment M on the section. We then have $c\,\Delta\theta = \frac{f\,(s + c\,\theta)}{E}$ from which $\mathbf{f} = \frac{\mathbf{c\,\Delta\theta\,E}}{\mathbf{s + c\theta}}$. If a = the cross sectional area of this fiber, the moment of the stress about the neutral axis $= fac = \frac{ac^2\,\Delta\theta E}{s + c\theta}$.

Summing over the entire cross-section, $\mathbf{M} = \Sigma\,\mathbf{fac} = \frac{\Sigma \mathbf{ac^2\,E\Delta\theta}}{\mathbf{s + c\,\theta}} = \frac{\mathbf{IE\Delta\theta}}{\mathbf{s + c\theta}}$ from which $\Delta\theta = \frac{\mathbf{M\,(s + c\,\theta)}}{\mathbf{EI}}$. For the curvature occurring in ordinary arches, $s + c\,\theta$ may be assumed equal to s. Hence $\Delta\theta = \frac{\mathbf{Ms}}{\mathbf{EI}}$ approximate. Summing over any given length of the axis the change in direction of the tangents at the end of such length $= \Sigma\Delta\theta = \theta = \Sigma \frac{Ms}{EI}$.

Fig. 78 represents a straight cantilever beam supporting a load. Assume origin of coordinates (**at the point where displacements are to be measured**) in this case at a. The angle between the tangents at the ends of any small division s of the axis after flexure is $\Delta\theta = \frac{Ms}{EI}$ in which M is the bending moment (due to the external loads) at such point. The deflection at a contributed by the flexure in s is in circular measure, $x\Delta\theta = \frac{Mxs}{EI}$. The total angular change θ is the sum of effects of all the small divisions between the points considered; that is, $\Sigma \frac{Ms}{EI} = \frac{I}{E} \Sigma \frac{Ms}{I}$; E being placed outside the summation sign if the beam is homogeneous; that is of constant E. Likewise the total displacement δ between the points considered, is $\Sigma\,x\,\Delta\theta = \frac{1}{E} \Sigma \frac{Mxs}{I}$. The length considered may be from the support to any other point or between any two points away from the support. In the numerical example following, the total deflection of point a from a line perpendicular to the support is measured.

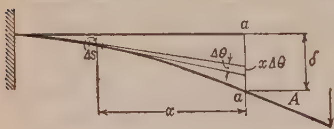
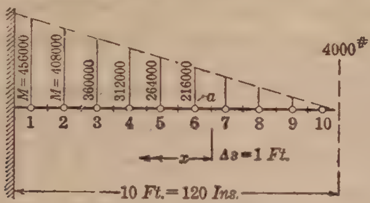


Fig. 78

In this example also s is constant and could have been placed outside the sign of summation, but was retained to illustrate the method when it is convenient to vary the lengths of divisions. Moments of inertia I and bending moments at centers of divisions are as indicated. (Fig. 79.)



x is Measured from the Point at which Displacements are to be Figured, to the Centers of Divisions

Fig. 79

Point	M	s	I	x	$\frac{Ms}{I}$	$\frac{Mxs}{I}$	$\frac{M}{EI}$
1	456 000	12 in.	200	66 in.	27 400	1 810 000	.0000760
2	408 000	12 in.	180	54 in.	27 200	1 470 000	.0000756
3	360 000	12 in.	160	42 in.	27 000	1 135 000	.0000750
4	312 000	12 in.	140	30 in.	26 700	800 000	.0000743
5	264 000	12 in.	120	18 in.	26 400	475 000	.0000733
6	216 000	12 in.	100	6 in.	26 000	156 000	.0000720
Σ	....	....	....	....	160 700	5 846 000	

$E = 30\,000\,000$ (steel beam assumed). Approximate angular change from support to $a = \Sigma \frac{Ms}{EI} = \frac{160\,700}{30\,000\,000} = 0.00535$ radian. Approximate deflection at $a = \Sigma \frac{Mxs}{EI} = \frac{I}{E} \Sigma \frac{Mxs}{I} = \frac{5\,846\,000}{30\,000\,000} = 0.2$ in. The smaller the divisions, the closer will be the approximation.

From what precedes it is seen that the angular change will be the area of the $\frac{M}{EI}$ diagram plotted to scale as **indicated** in Fig. 80 although not here reproduced to scale. Also the deflection will be the moment of the $\frac{M}{EI}$ diagram about the point of displacement A.

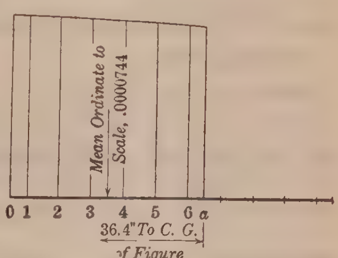


Fig. 80

Angular change 0 to $A = .0000744 \times 6 \times 12 = 0.00535$ radian.

Deflection at $A = 0.00535 \times 36.4 = 0.2$ inch.

The Curved Cantilever Beam. (Fig. 81.) The angular change contributed by the flexure of any division s of the axis of a member may be measured by the change in angle between the tangents at the ends of the division or between the radii or between any lines straight or broken, attached to the ends of the divisions. Making use of this geometric principle the deflection of a point in any desired direction may be readily calculated as shown. Assume as before, origin of coordinates at the point a whose calculated displacements are desired. From the figure it is seen that the vertical displacement of point a contributed by the flexure in s is $x\,\Delta\theta = \frac{Mxs}{EI}$ and the horizontal displacement is $y\,\Delta\theta = \frac{Mys}{EI}$ in which M is the bending moment on division s due to the loads acting on the beam. Summing the effects of all points on the axis (from support to a) total vertical displacement $= \Sigma\,\frac{Mxs}{EI}$ and total horizontal displacement is $\Sigma\,\frac{Mys}{EI}$. Likewise the total angular change (from support to a) is $\Sigma\,\frac{Ms}{EI}$. Note that in the expressions $\frac{Mxs}{EI}$ above (for straight or curved cantilever beams) x is the moment on the several divisions s due to **vertical unit load placed at the point where vertical displacements are to be measured.** Whence the expressions for vertical deflections may be written $\Sigma\,\frac{MM_as}{EI}$, in which M_a is the moment on the several divisions s due to unit vertical load at a,

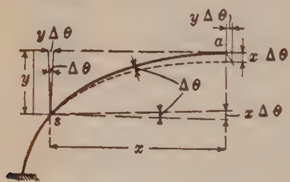


Fig. 81

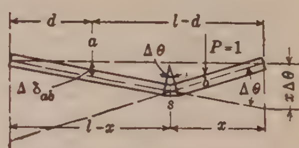


Fig. 82

likewise for horizontal deflection at a in the curved beam, $\Sigma\,\frac{Mys}{EI}$ may be written $\Sigma\,\frac{MM'_as}{EI}$, in which M'_a is the moment on the several divisions due to unit **horizontal** load at a.

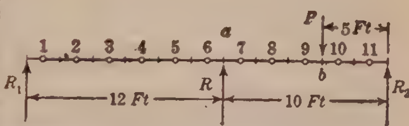
Simple Span Beam. (See Fig. 82.) For simplicity of demonstration the flexure contributed by a small length s only is shown to exaggerated scale in the figure. It will be convenient in what follows to find the constants for a load unity (say 1 lb.). Let δ_{ab} indicate deflection at a due to unit load at b and indicate the increment of deflection contributed by s as $\Delta\,\delta_{ab}$. By geometry $\Delta\,\delta_{ab} = \frac{x\Delta\theta d}{l}$. From what has preceded $\Delta\theta = M_b\frac{s}{EI}$ in which M_b = moment on s due to unit load. Then $\Delta\,\delta_{ab} = \frac{xd}{l}M_b\frac{s}{EI}$. Summing the effects of flexure on all elements s we have $\delta_{ab} = \Sigma M_b\frac{s}{EI}\cdot\frac{xd}{l}$. For load P greater than unity we have of course $P\,\delta_{ab} = P\Sigma M_b\frac{s}{EI}\cdot\frac{xd}{l}$. Note now that the expression $\frac{xd}{l}$ is the same as for moment on s due to unit load at $a = M_a$. The equation for deflection may then be expressed as $P\delta_{ab} = P\Sigma M_a M_b \frac{s}{EI}$. It is obvious from the form of this equation that $P\delta_{ab} = P\Sigma M_a M_b \frac{s}{EI} = P\delta_{ba}$. (Maxwell-Mohr's Theorem of Reciprocal Displacements.)

Application. Find R for the continuous beam shown in Fig. 83. If there were no reaction R the downward deflection at a for the 22-ft. span beam would be $P\delta ac = P\Sigma M_a M_b \frac{S}{EI}$. The upward deflection at a due to R acting on the 22-ft. span beam (that is $R\delta aa = R\Sigma M_a M_a \frac{S}{EI}$) must be equal and opposite; whence

$$R = \frac{P\,\delta ab}{\delta aa} = \frac{P\Sigma M_a M_b \frac{S}{EI}}{\Sigma M_a M_a \frac{S}{EI}}$$

In the numerical calculation I is, for simplicity of demonstration, assumed constant and equal divisions of 2 ft. are taken. $\frac{s}{EI}$, being constant, may then be placed outside the sign of summation and appearing in both numerator and denominator, it will cancel out. The method is just as simple for variable I and unequal divisions excepting that these quantities would be retained in the summations as illustrated in the example for the cantilever beam. The results are accurate to within 1%.

Point	M_a	M_b	M_aM_b	M_aM_a
1	0.45	0.23	0.10	0.20
2	1.36	0.68	0.93	1.85
3	2.27	1.14	2.59	5.16
4	3.17	1.59	5.04	10.05
5	4.09	2.05	8.39	16.70
6	5.09	2.50	12.72	25.85
7	4.90	2.95	14.45	24.00
8	3.81	3.41	13.00	14.51
9	2.72	3.86	10.50	7.40
10	1.64	2.32	3.81	2.69
11	0.54	0.77	0.42	0.29
Σ	....	....	71.95	108.70



$$R = \frac{71.95}{108.7} = 0.66\,P \qquad R_1 = \frac{5\,P - 6.6\,P}{22} = -\,0.07\,P$$

$$R_2 = 0.41\,P$$

Fig. 83

Referring to Fig. 83 and the equation above for R, note that $M_b = \frac{x'x}{l}$. Then $P\delta_{ac} = P\Sigma\left(\frac{M_a s}{EI}\cdot\frac{x}{l}\right)x' = P$ multiplied by the moment at the load point b of all the quantities $\frac{M_a s}{EI}$ considered as loadson the beam in their proper position as shown in Fig. 84.

The quantities $\frac{M_a s}{EI}$ when used in this way are termed "elastic" loads and the moments due to them will be designated by m. M_a in all cases is the moment on any division s due to a unit load acting at the point where the displacement is to be measured and in the direction in which the displacement is required.

Applying this principle to the continuous beam, influence line for the reaction $R = \dfrac{P\Sigma \frac{M_a s}{EI} \cdot \frac{x'x}{l}}{\Sigma M_a M_a \frac{s}{EI}}$ may be calculated as the load P moves over the span as illustrated in the numerical example. In this example $\frac{s}{EI}$ being constant, may be placed outside the summation sign, and appearing in both numerator and denominator it cancels out.

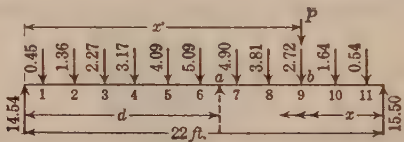


Fig. 84

M_{11} = 15.5	P at 11	$Ra = P\,(15.5 \div 108.7) = 0.14\,P$
M_{10} = 45.4	P at 10	$Ra = P\,(45.4 \div 108.7) = 0.42\,P$
M_9 = 72.0	P at 9	$Ra = P\,(72.0 \div 108.7) = 0.66\,P$
M_8 = 93.3	P at 8	$Ra = P\,(93.3 \div 108.7) = 0.86\,P$
M_7 = 107.8	P at 7	$Ra = P\,(106.8 \div 108.7) = 0.98\,P$
Ma = 108.7	P at a	$Ra = P\,(108.7 \div 108.7) = P$
M_6 = 110.6	P at 6	$Ra = P\,(110.6 \div 108.7) = 1.01\,P$
etc.	etc.	etc.

47. Two-hinged Arch

Refer to Fig. 85. Assume a curved beam resting on rollers at one end and simply supported at the other. Flexure in any small division s of the beam will cause a horizontal displacement at the free end $= y\Delta\theta = \frac{Mys}{EI}$ in which M is the bending moment on the division s due to the external loads neglecting the horizontal thrust H. The total displacement due to flexure in all the divisions $s = \Sigma \frac{Mys}{EI}$.

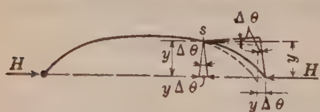


Fig. 85

Now assume a horizontal thrust applied to the end of the beam sufficient to counteract the horizontal displacement. New moments Hy are introduced acting on the individual divisions and we have: Total horizontal displacement due to thrust $= \Sigma y\Delta\theta = \Sigma \frac{(Hy)ys}{EI} = \Sigma \frac{Hy^2s}{EI}$. The horizontal displacement due to external loads and that due to the thrust being equal and opposite we have

$$\Sigma \frac{Mys}{EI} = \Sigma \frac{Hy^2s}{EI}, \quad \text{or} \quad \frac{I}{E}\Sigma \frac{Mys}{I} = \frac{H}{E}\Sigma \frac{y^2s}{I}.$$

From which $H = \dfrac{\Sigma \dfrac{Mys}{I}}{\Sigma \dfrac{y^2 s}{I}}$ for a curved beam hinged at the ends but ends fixed in location. (Two-hinged arch.)

In the above equations y is also an expression for moment in the several divisions due to unit horizontal load acting like H. Whence the expression for H may be written $\dfrac{\Sigma M M_a \frac{s}{I}}{\Sigma M_a M_a \frac{s}{I}}$ in which M_a is the moment in the several divisions due to such unit load.

Temperature Stresses. c = coefficient of expansion (0.0000065). t = temperature change in degrees Fahrenheit. l = span length. E = modulus of elasticity of concrete.

$$\Sigma y \Delta\theta = ctl = \frac{H}{E}\Sigma\frac{y^2 s}{I}; \text{ from which } H = \frac{Ectl}{\Sigma\frac{y^2 s}{I}}$$

Rib Shortening. fc = approximate average stress on the concrete throughout the arch due to thrust from the dead load.

$$\Sigma y \Delta\theta = \frac{fcl}{E} = \frac{H}{E}\Sigma\frac{y^2 s}{I}; \text{ from which } H = \frac{fcl}{\Sigma\frac{y^2 s}{I}}$$

48. Rigid-Frame Bridges

The rigid-frame bridge developed by the Westchester County (New York) Park Commission, A. G. Hayden, Designing Engineer, has proven its advan-

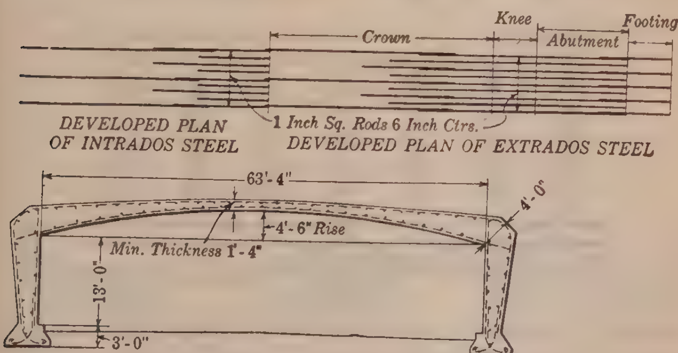


Fig. 86

tages both from an economic and architectural point of view and is peculiarly adapted to structures separating intersecting highways.

Fig. 86 shows typical details of a single-span solid-section frame; Fig. 87, two-span solid-section frame. Fig. 88 shows a ribbed type frame consisting of a series of 5 rigid frames supporting a floor slab and having a reinforced-concrete cutoff wall at each end to retain the approach fill. Although certain effects of skew exist in the skewed ribbed type frame and were accounted for in design, it was used for heavily skewed bridges before other effects had been

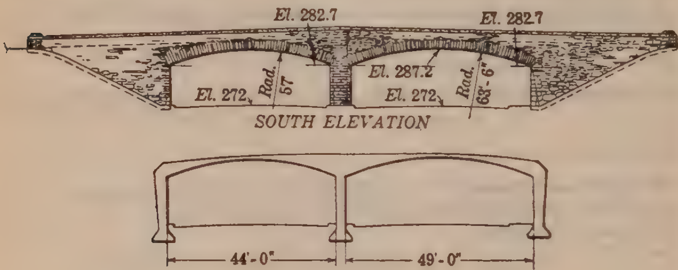


Fig. 87

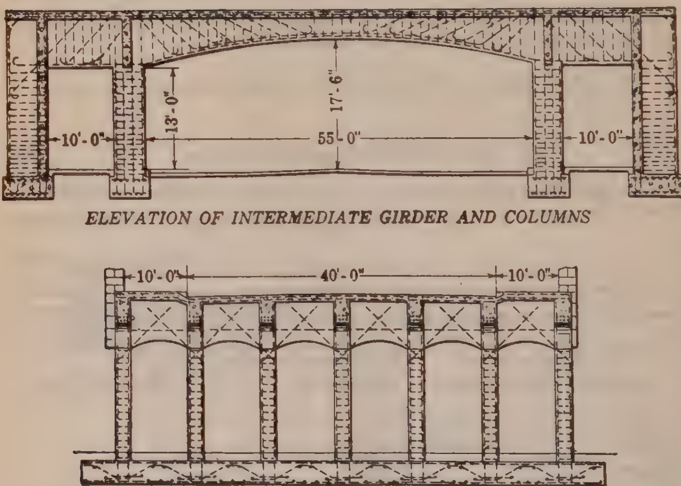


Fig. 88

evaluated. The complete solution of problems of design relating to skewed arches and frames of solid section now makes it possible to design such structures with certainty and supersede the ribbed type. Fig. 89 shows the adaptation of rigid-frame design to structural steel, which will not be treated in this section.

The intrinsic economy of the rigid-frame bridge lies in the fact that the external work and therefore equivalent internal work done under load is less for the rigidly connected structure than for the beam separately supported on its abutments. Moreover, the entire structure is effective for resisting this smaller amount of external work. There are no idle members. Still further, the horizontal loads due to earth pressure counteract to a degree the effect of the vertical loads.

In design, the structure may be made safe as a self-supporting unit discounting earth pressure. Passive earth pressure developed by flexure in the structure may also be taken into account though this must be assumed. Stresses are easily calculated for an assumed uneven settlement of the footings.

In addition to the intrinsic economy of the frame bridge further economy is realized in the approaches since the distance from roadway under to roadway over may be made less than for other types on account of the slender proportions of the structure. The fixed arch of like span and equal area-way requires greater rise in order to reduce temperature stresses. Temperature stresses in frame bridges of the proportions illustrated are negligible, although in special cases they should be examined.

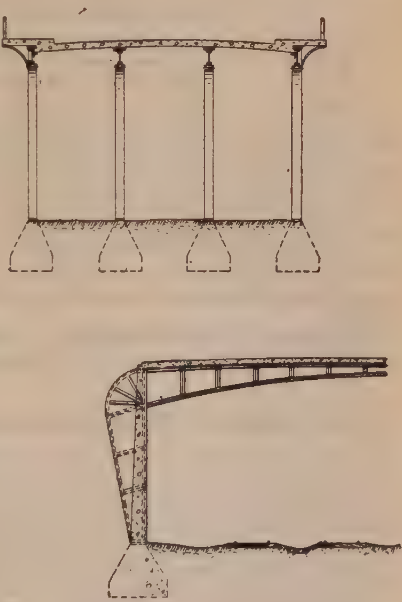


Fig. 89

Methods of Design. Frame bridges of the Westchester County Parkways have been designed by a variety of methods, some entirely new, but all giving the same results. Later methods have been greatly simplified. It is beyond the scope of this work to do more than indicate the line of attack which will be illustrated by a partial design of

Fig. 90

a double-span solid-section reinforced-concrete rigid-frame bridge by the influence-line method.

Before proceeding with the illustrative example some general observations will be made. The calculation given is based on the assumption of hinged condition at the bottom of the footings. Fixed condition might as well have been assumed although for a structure resting free on the soil such condition could not be realized. The absence of a definite hinge at the bottom of the footing does, however, introduce a small amount of restraint here which may be taken into account as follows, though this is generally unnecessary. Fig. 91 shows one type of base, resting free on the soil. It is obvious that the resultant reaction cannot come at either edge, since this would presume infinite soil pressure at these points. If it could suddenly come very close to the edge causing extremely high local pressure, slight yielding of the soil would automatically throw it inward again. There is, therefore, a reasonable range on the base within which the resultant reaction must come, and theoretical critical positions controlling the stresses in any region of the frame may be easily determined for any particular case. Detailed discussion, however, is beyond the scope of this work. It is usually sufficient for all practical purposes to make analysis as for hinge at a, since the footing reinforcement extending upward into the leg and the uncalculated soffit reinforcement will adequately provide for uncalculated restraint.

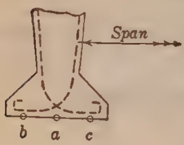


Fig. 91

If the structure is seriously restrained at the footings, as by embedded piles, calculation for both hinged and fixed conditions will cover all contingencies.

49. Design of Double-Span Continuous Frame

If the statically indeterminate structure shown in Fig. 92 is designed for hinged conditions at the footings (resting free on the soil) there will be, for any system of loads, six unknown horizontal and vertical components of reactions as shown. These cannot be solved by the laws of statics. Suppose the supports are arbitrarily altered by the removal of redundant reactions C, D and F as shown in Fig. 93. This "transformed" structure is statically determinate and for any given condition of loading all the stresses and therefore all the deflections can be found by statics. The problem resolves itself into finding in Fig. 93 the vertical and horizontal deflections at (2) and horizontal deflections at (3), due to the load P by the methods indicated in the preceding text, and then finding what unknown forces C, D and F will bring these deflections back to zero; that is, what reactions will restore the conditions to those shown in Fig. 92 when the problem will be solved. When C, D and F have been determined, the application of them to the structure in Fig. 93 causes reaction in the direction of H_1, V_1, and V_3 which, added to the latter, gives the final reactions A, B and G as will be shown.

Let δ_{cp} = deflection in direction and at point of application of C due to unit load applied like P.

δ_{cc} = deflection in direction and at point of application of C due to unit load applied like C.

δ_{cd} = deflection in direction and at point of application of C due to unit load applied like D.

δ_{cf} = deflection in direction and at point of application of C due to unit load applied like F.

Then the deflections in the direction and at point of application of C due to the actual loads and unknown reactions are

$$P\delta_{cp}, \quad C\delta_{cc}, \quad D\delta_{cd}, \quad F\delta_{cf}$$

Since the reactions C, D and F must be such that the deflection $P\delta_{cp}$ caused by the load P in the transformed structure (Fig. 93).

$$P\delta_{cp} + C\delta_{cc} + D\delta_{cd} + F\delta_{cf} = 0$$

By like reasoning we have also the following equations:

$$P\delta_{dp} + C\delta_{dc} + D\delta_{dd} + F\delta_{df} = 0$$
$$P\delta_{fp} + C\delta_{fc} + D\delta_{fd} + F\delta_{ff} = 0$$

As demonstrated for the simple beam, the quantities δ_{cp}, δ_{cc}, δ_{cd}, δ_{cf} and so on, are equal to $\Sigma M_c M_p \frac{s}{EI}$, $\Sigma M_c M_c \frac{s}{EI}$, $\Sigma M_c M_d \frac{s}{EI}$, $\Sigma M_c M_f \frac{s}{EI}$ and so on in which M_p is the moment on any division s due to the load P and M_c, M_d, M_f (and so on) are moments on any division s due to load unity acting like the reaction components. Whence we have the final equations

$$-P\Sigma M_c M_p \frac{s}{EI} = C\Sigma M_c M_c \frac{s}{EI} + D\Sigma M_c M_d \frac{s}{EI} + F\Sigma M_c M_f \frac{s}{EI}$$
$$-P\Sigma M_d M_p \frac{s}{EI} = C\Sigma M_d M_c \frac{s}{EI} + D\Sigma M_d M_d \frac{s}{EI} + F\Sigma M_d M_f \frac{s}{EI}$$
$$-P\Sigma M_f M_p \frac{s}{EI} = C\Sigma M_f M_c \frac{s}{EI} + D\Sigma M_f M_d \frac{s}{EI} + F\Sigma M_f M_f \frac{s}{EI}$$

When the coefficients of C, D and F have been determined numerically, the above three equations may be solved for the three unknown components of reactions C, D and F. If there is more than one load on the structure M_p is the moment on any division s due to the system of loads.

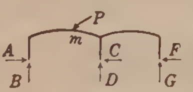


Fig. 92

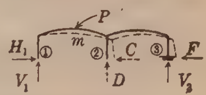


Fig. 93

This method of determining the redundant reaction components C, D and F is well adapted for cases where a fixed loading is being considered, as was illustrated in the example of the 2-span beam. If, however, moving load is to be considered and influence lines for the redundant reaction components are desired the method of placing the "elastic loads" on the structure may be used as was illustrated for the 2-span continuous beam in the **text preceding.** The labor of calculation is thereby reduced. For example, the expression $P\Sigma M_c M_p \frac{s}{EI}$ above may be put $P\Sigma\left(M_c \frac{s}{EI}\right) M_p$ in which M_p is the moment on any particular division s of the transformed structure, due to load $P = 1$, that is $\frac{xd}{l}$ (l being total span of transformed structure) and $M_c \frac{s}{EI}$ is a quantity pertaining to such division. The arithmetical process in calculating $M_c \frac{s}{EI} M_p$ a is the same as calculating the moment at P due to the quantity $M_c \frac{s}{EI}$ acting as a load on its particular division; and $\Sigma M_c \frac{s}{EI} M_p$ is the moment at P due to the several quantities $M_c \frac{s}{EI}$ in their appropriate positions. It follows that the calculation of moments at all points of the transformed structure under such loading will give a complete influence line for the influence load unity moving over the span.

If we designate moments due to the quantities $M_c\frac{s}{EI}$ treated as loads by the symbol m_c we have as the final equation

$$Pm_c = C\Sigma M_c M_c\frac{s}{EI} + D\Sigma M_c M_d\frac{s}{EI} + F\Sigma M_c M_f\frac{s}{EI}$$

Likewise

$$Pm_d = C\Sigma M_d M_c\frac{s}{EI} + D\Sigma M_d M_d\frac{s}{EI} + F\Sigma M_d M_f\frac{s}{EI}$$

and

$$Pm_f = C\Sigma M_f M_c\frac{s}{EI} + D\Sigma M_f M_d\frac{s}{EI} + F\Sigma M_f M_f\frac{s}{EI}$$

The use of these equations will be illustrated in the example following:

Nomenclature: Terms not defined in the text or indicated on the figures are:

M_E and N_E: Moment and thrust at any point in the structure due to earth pressure.

N_p, N_c, N_d, N_f: Thrusts at any point in the structure due to the loads and reactions P, C, D and F.

M_t and N_t: Movement and thrust at any point in the structure due to temperature change.

t = total depth of section of arch ring or frame.

In the following calculations the divisions s are all 5 ft.; s and E (modulus of elasticity) being constant cancel out of all equations, excepting temperature equations.

Fig. 94 shows the structure to be analyzed and Figs. 95*a*, 95*b*, 95*c* and 95*d* show the method of calculating M_c, M_d, M_f and M_p used in the equations.

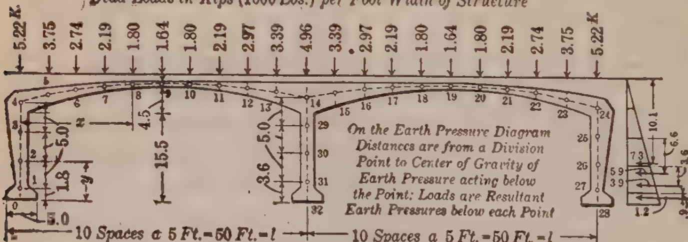
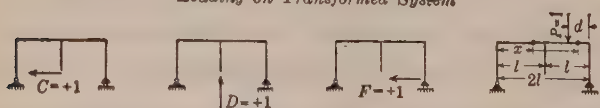


Fig. 94

Loading on Transformed System



$M_c = -y$ from 0 to 14: 14 to 32 $= 0$ from 14 to 28

Fig. 95*a*

$M_d = -\frac{1}{2}x$ from 4 to 14 $= -\frac{1}{2}(2l-x)$ from 14 to 24

Fig. 95*b*

$M_f = -y$ from 0 to 28 $=0$ in Center Post

Fig. 95*c*

$M_p = \frac{xd}{2l}$ for Points to Left of P

$M_p = \frac{(2l-d)(2l-x)}{2l}$ for Points Right of P

Fig. 95*d*

Table of Frame Constants and Elastic Loads

$S = 5.0$ ft.

Point	x	y	t	$\frac{1}{I}$		$\frac{M_c}{I}$	$\frac{M_d}{I}$	$\frac{M_f}{I}$
1	0	1.8	2.00	1.50		− 2.7	− 0	− 2.7
2	0	6.8	2.6	.68		− 4.6	− 0	− 4.6
3	0	11.8	3.3	.33		− 3.9	− 0	− 3.9
4	0	16.8	3.75	.23		− 3.9	− 0	− 3.9
5	5	18.5	2.8	.55		− 10.2	− 1.4	− 10.2
6	10	19.2	2.0	1.50		− 28.8	− 7.5	− 28.8
7	15	19.9	1.5	3.57		− 71.0	− 26.8	− 71.0
8	20	20.4	1.1	9.09		−185.4	− 90.9	−185.4
9	25	20.5	1.0	12.50		−256.2	−156.2	−256.2
10	30	20.4	1.2	7.14		−145.7	−107.1	−145.7
11	35	20.2	1.8	2.04		− 41.2	− 35.7	− 41.2
12	40	19.8	2.6	.68		− 13.5	− 13.6	− 13.5
13	45	19.1	4.0	.19		− 3.6	− 4.3	− 3.6
14	50	18.6	5.7	.06	elastic loads	− 0.6	− 1.5	− 1.1
15	55	19.1	4.0	.19		− 0	− 4.3	− 3.6
16	60	19.8	2.6	.68		− 0	− 13.6	− 13.5
17	65	20.2	1.8	2.04		− 0	− 35.7	− 41.2
18	70	20.4	1.2	7.14		− 0	−107.1	−145.7
19	75	20.5	1.0	12.50		− 0	−156.2	−256.2
20	80	20.4	1.1	9.09		− 0	− 90.9	−185.4
21	85	19.9	1.5	3.57		− 0	− 26.8	− 71.0
22	90	19.2	2.0	1.50		− 0	− 7.5	− 28.5
23	95	18.5	2.8	.55		− 0	− 1.4	− 10.2
24	100	16.8	3.75	.23		− 0	− 0	− 3.9
25	100	11.8	3.3	.33		− 0	− 0	− 3.9
26	100	6.8	2.6	.68		− 0	− 0	− 4.6
27	100	1.8	2.0	1.50		− 0	− 0	− 2.7
29	50	13.6	2.0	1.50		− 20.4	− 0	− 0
30	50	8.6	2.0	1.50		− 12.9	− 0	− 0
31	50	3.6	2.0	1.50		− 5.4	− 0	− 0

Point	$\frac{(M_c)^2}{I}$	$\frac{(M_d)^2}{I}$	$\frac{(M_f)^2}{I}$	$\frac{M_f M_c}{I}$	$\frac{M_f M_d}{I}$	$\frac{M_c M_d}{I}$	$\frac{x}{2}$
1	5	0	5	5	0	0	0
2	31	0	31	31	0	0	0
3	46	0	46	46	0	0	0
4	65	0	65	65	0	0	0
5	189	3	189	189	26	26	2.5
6	553	38	553	553	144	144	5.0
7	1 413	201	1 413	1 413	533	533	7.5
8	3 782	909	3 782	3 782	1 854	1854	10.0
9	5 252	1 953	5 252	5 252	3 203	3203	12.5
10	2 971	1 607	2 971	2 971	2 185	2185	15.0
11	832	625	832	832	721	721	17.5
12	267	272	267	267	270	270	20.0
13	69	96	69	69	81	81	22.5
14	10	38	21	10	28	14	25.0
15	0	96	69	0	81	0	....
16	0	272	267	0	270	0	....
17	0	625	832	0	721	0	....
18	0	1 607	2 971	0	2 185	0	....
19	0	1 953	5 252	0	3 203	0	....
20	0	909	3 782	0	1 854	0	....
21	0	201	1 413	0	533	0	....
22	0	38	553	0	144	0	....
23	0	3	189	0	26	0	....
24	0	0	65	0	0	0	....
25	0	0	46	0	0	0	....
26	0	0	31	0	0	0	....
27	0	0	5	0	0	0	....
29	277	0	0	0	0	0	....
30	111	0	0	0	0	0	....
31	19	0	0	0	0	0	....
	15 903	11 446	30 971	15 485	18 062	9031	....

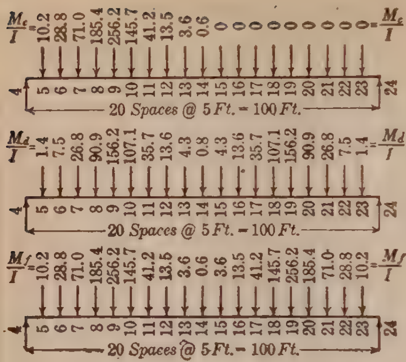


Fig. 96

$\frac{M_c}{I}$, $\frac{M_d}{I}$ and $\frac{M_f}{I}$ are the "elastic loads" derived in the preceding table, whose use was illustrated in the example for the continuous beam, page 1080.

Table of "Elastic Load" Moments

(Calculations not shown)

Point	mc	md	mf	Point	mc	md	mf
4	0	0	0	15	8127	11 443	18 059
5	2 878	2 222	3 781	16	7224	11 418	18 038
6	5 705	4 436	7 511	17	6321	11 324	17 950
7	8 388	6 613	11 097	18	5418	11 052	17 655
8	10 716	8 656	14 328	19	4515	10 245	16 632
9	12 117	10 245	16 632	20	3612	8 562	14 328
10	12 237	11 052	17 655	21	2709	6 613	11 097
11	11 629	11 324	17 950	22	1806	4 436	7 511
12	10 814	11 418	18 038	23	903	2 222	3 781
13	9 932	11 443	18 059	24	0	0	0
14	9 032	11 447	18 062				

Calculation for Reactions C, D and F

From equations heretofore derived (p. 1085) but canceling out s and E which are constant:

$$Pm_c = C\Sigma\frac{M_cM_c}{I} + D\Sigma\frac{M_cM_d}{I} + F\Sigma\frac{M_cM_f}{I}$$

$$Pm_d = C\Sigma\frac{M_dM_c}{I} + D\Sigma\frac{M_dM_d}{I} + F\Sigma\frac{M_dM_f}{I}$$

$$Pm_f = C\Sigma\frac{M_fM_c}{I} + D\Sigma\frac{M_fM_d}{I} + F\Sigma\frac{M_fM_f}{I}$$

Substituting the numerical valves for terms that have been calculated in the table we have for $P = 1$:

$$m_c = 15903\ C + 9031\ D = 15485\ F$$

$$m_d = 9031\ C + 11446\ D + 18062\ F$$

$$m_f = 15485\ C + 18062\ D + 30971\ F$$

Solving these equations,

$$C = -\ 0.00006115\ m_f + 0.00012235\ m_c$$

$$D = -\ 0.00063834\ m_f \qquad\qquad + 0.00109462\ m_d$$

$$F = -\ 0.00043514\ m_f - 0.00006115\ m_c + 0.00063834\ m_d$$

In the table that follows the values of m_c, m_d amd m_f are substituted and reactions C, D and F obtainéd.

Point	$-\frac{6.115}{100000}$ mf	$-\frac{12.235}{100000}$ mc	C	$-\frac{63.834}{100000}$ mf	$-\frac{109.642}{100000}$ md	D
5	− .231	.352	+.121	− 2.413	2.430	.017
6	− .459	.698	+.239	− 4.794	4.855	.061
7	− .678	1.027	+.349	− 7.082	7.241	.159
8	− .878	1.311	+.435	− 9.147	9.474	.327
9	−1.017	1.482	+.465	−10.616	11.214	.598
10	−1.080	1.497	+.417	−11.270	12.096	.826
11	−1.098	1.423	+.325	−11.458	12.397	.939
12	−1.103	1.323	+.220	−11.516	12.501	.985
13	−1.104	1.215	+.111	−11.528	12.527	.999
14	−1.104	1.105	+.001	−11.528	12.527	.999
15	−1.104	.994	−.110	−11.528	12.527	.999
16	−1.103	.884	−.219	−11.516	12.501	.985
17	−1.098	.773	−.325	−11.458	12.397	.939
18	−1.080	.663	−.417	−11.270	12.096	.826
19	−1.017	.552	−.465	−10.616	11.214	.598
20	− .876	.442	−.434	− 9.147	9.474	.327
21	.678	.331	−.347	− 7.082	7.241	.159
22	− .459	.221	−.238	− 4.794	4.855	.061
23	− .231	.110	−.121	− 2.413	2.430	.017

Point	$-\frac{45.514}{100\ 000}$ mf	$-\frac{6.115}{100\ 000}$ mc	$-\frac{63.834}{100\ 000}$ md	F	C + F
5	1.645	−.176	−1.417	+.052	+.173
6	3.268	−.349	−2.831	+.088	+.327
7	4.828	−.513	−4.223	+.092	+.437
8	6.236	−.655	−5.525	+.056	+.491
9	7.236	−.741	−6.540	−.045	+.420
10	7.682	−.748	−7.053	−.119	+.298
11	7.811	−.711	−7.229	−.129	+.196
12	7.850	−.661	−7.290	−.101	+.119
13	7.859	−.607	−7.306	−.054	+.057
14	7.859	−.552	−7.306	+.001	+.002
15	7.859	−.497	−7.306	+.056	−.054
16	7.850	−.442	−7.290	+.118	−.101
17	7.811	−.386	−7.229	+.196	−.129
18	7.682	−.331	−7.053	+.298	−.119
19	7.236	−.276	−6.540	+.420	−.045
20	6.236	−.221	−5.525	+.490	+.055
21	4.828	−.166	−4.223	+.439	+.092
22	3.268	−.110	−2.831	+.327	+.089
23	1.645	−.055	−1.417	+.173	+.052

Influence Table for Moments

$M = M_p + M_cC + M_dD + M_fF = M_p - (C + F)\,y - 0.5\,Dx$ for left span)

Load at	Point 2	Point 4	Point 7				Point 9			
	−6.8 (C+F)	−16.8 (C+F)	Mp	−19.9 (C+F)	−7.5 D	M	Mp	−20.5 (C+F)	−12.5 D	M
5	−1.18	−2.91	4.25	−3.44	− .13	+ .68	3.75	− 3.54	− .21	.00
6	−2.22	−5.49	8.50	−6.51	− .46	+1.53	7.50	− 6.70	− .76	+ .04
7	−2.97	−7.34	12.75	−8.70	−1.19	+2.86	11.25	− 8.96	− 1.99	+ .30
8	−3.34	−8.25	12.00	−9.77	−2.45	− .22	15.00	−10.06	− 4.09	+ .85
9	−2.86	−7.06	11.25	−8.36	−4.48	−1.59	18.75	− 8.61	− 7.47	+2.67
10	−2.02	−5.01	10.50	−5.93	−6.20	−1.63	17.50	− 6.11	−10.32	+1.07
11	−1.33	−3.29	9.75	−3.90	−7.04	−1.19	16.25	− 4.02	−11.74	+ .49
12	− .81	−2.00	9.00	−2.36	−7.39	− .75	15.00	− 2.44	−12.31	+ .25
13	− .39	− .96	8.25	−1.13	−7.49	− .37	13.75	− 1.17	−12.49	+ .09
14	0	0	7.50	0	−7.50	0	12.50	0	−12.50	0
15	+ .37	+ .91	6.75	+1.07	−7.49	+ .33	11.25	+ 1.11	−12.49	− .13
16	+ .69	+1.70	6.00	+2.01	−7.39	+ .62	10.00	+ 2.07	−12.31	− .24
17	+ .88	+2.17	5.25	+2.57	−7.04	+ .78	8.75	+ 2.64	−11.74	− .35
18	+ .81	+2.00	4.50	+2.37	−6.20	+ .67	7.50	+ 2.44	−10.32	− .38
19	+ .31	+ .76	3.75	+ .89	−4.48	+ .16	6.25	+ .92	− 7.47	− .30
20	− .37	− .92	3.00	−1.09	−2.45	− .54	5.00	− 1.13	− 4.09	− .22
21	− .63	−1.55	2.25	−1.83	−1.19	− .77	3.75	− 1.89	− 1.99	− .13
22	− .61	−1.50	1.50	−1.77	− .46	− .73	2.50	− 1.82	− .76	− .08
23	− .35	− .87	.75	−1.03	− .13	− .41	1.25	− 1.07	− .21	− .03

Load at	Point 11				Point 14			
	Mp	−20.2 (C+F)	−17.5 D	M	Mp	−18.6 (C+F)	−25.0 D	M
5	3.25	−3.49	− .30	− .54	2.50	−3.22	− .42	− 1.14
6	6.50	−6.60	− 1.07	−1.17	5.00	−6.08	− 1.53	− 2.61
7	9.75	−8.83	− 2.79	−1.87	7.50	−8.13	− 3.98	− 4.61
8	13.00	−9.92	− 5.72	−2.64	10.00	−9.13	− 8.17	− 7.30
9	16.25	−8.48	−10.46	−2.69	12.50	−7.81	−14.95	−10.26
10	19.50	−6.02	−14.46	− .98	15.00	−5.54	−20.65	−11.19
11	22.75	−3.96	−16.43	+2.36	17.50	−3.65	−23.48	− 9.63
12	21.00	−2.40	−17.24	+1.36	20.00	−2.21	−24.62	− 6.83
13	19.25	−1.15	−17.48	+ .62	22.50	−1.06	−24.97	− 3.53
14	17.50	0	−17.50	0	25.00	0	−25.00	0
15	15.75	+1.09	−17.48	− .69	22.50	+1.00	−24.97	− 1.47
16	14.00	+2.04	−17.24	−1.20	20.00	+1.88	−24.62	− 2.74
17	12.25	+2.61	−16.43	−1.57	17.50	+2.40	−23.48	− 3.58
18	10.50	+2.40	−14.46	−1.56	15.00	+2.21	−20.65	− 3.44
19	8.75	+ .91	−10.46	− .80	12.50	+ .84	−14.95	− 1.61
20	7.00	−1.11	− 5.72	+ .17	10.00	−1.02	− 8.17	+ .81
21	5.25	−1.86	− 2.79	+ .60	7.50	−1.71	− 3.98	+ 1.81
22	3.50	−1.80	− 1.07	+ .63	5.00	−1.66	− 1.53	+ 1.81
23	1.75	−1.05	− .30	+ .40	2.50	− .97	− .42	+ 1.11

Influence Table for Thrust in End Post

Load at Point	4	5	6	7	8	9	10
N_p $0.5\ D$	1.000 0.000	0.950 0.008	0.900 0.030	0.850 0.079	0.800 0.163	0.750 0.299	0.700 0.413
$N_p - 0.5\ D$	1.000	0.942	0.870	0.771	0.637	0.451	0.287
Load at Point	**11**	**12**	**13**	**14**	**15**	**16**	**17**
N_p $0.5\ D$	0.650 0.469	0.600 0.492	0.550 0.499	0.500 0.500	0.450 0.499	0.400 0.492	0.350 0.469
$N_p - 0.5\ D$	0.181	0.108	0.051	0	−0.049	−0.092	−0.119
Load at Point	**18**	**19**	**20**	**21**	**22**	**23**	**24**
N_p $0.5\ D$	0.300 0.413	0.250 0.299	0.200 0.163	0.150 0.079	0.100 0.030	0.050 0.008	0 0
$N_p - 0.5\ D$	−0.113	−0.049	+0.037	+0.071	+0.070	+0.042	0

For horizontal member $\mathbf{N} = C + F$. For end post $\mathbf{N} = \mathbf{N_p} - 0.5\ D$.

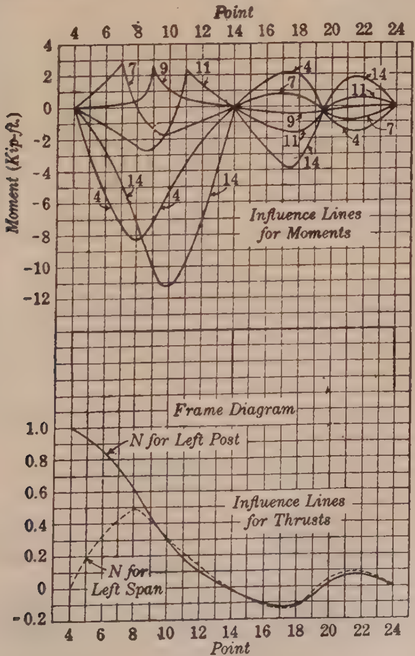
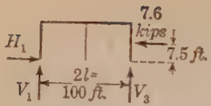


Fig. 97



$H_1 = 7.6$

$V_1 = - V_3 = 0.57$

Calculations for Earth Pressure, Right Side Only

Point	$-H_1y$	V_1x	$M_p = -H_1y + V_1x$	$\frac{M_fM_p}{I}$	$\frac{M_dM_p}{I}$	$\frac{M_cM_p}{I}$
1	13.6	0	− 13.6	37	0	37
2	51.7	0	− 51.7	239	0	239
3	89.7	0	− 89.7	350	0	350
4	127.7	0	−127.7	498	0	498
5	140.6	2.8	−137.8	1 406	193	1 406
6	145.9	5.7	−140.2	4 038	1 052	4 038
7	151.2	8.6	−142.7	10 132	3 824	10 132
8	155.0	11.4	−143.6	26 623	13 053	26 623
9	155.8	14.2	−141.6	36 278	22 118	36 278
10	155.0	17.1	−137.9	20 092	14 769	20 092
11	153.5	20.0	−133.5	5 500	4 766	5 100
12	150.5	22.2	−128.3	1 732	1 745	1 732
13	145.2	25.7	−119.5	430	514	430
14	141.4	28.7	−112.7	124	169	68
15	145.2	31.4	−113.8	410	489	0
16	150.5	34.2	−116.3	1 570	1 582	0
17	153.5	37.1	−116.4	4 796	4 155	0
18	155.0	40.0	−115.0	16 756	12 316	0
19	155.8	42.8	−113.0	28 951	17 651	0
20	155.0	45.6	−109.4	20 283	9 944	0
21	151.2	48.5	−102.7	7 292	2 752	0
22	145.9	51.4	− 94.5	2 722	709	0
23	140.6	54.2	− 86.4	881	121	0
24	127.7	57.0	− 70.7	276	0	0
25	....	....	− 39.0	152	0	0
26	....	....	− 14.0	64	0	0
27	....	....	− 1.1	3	0	0
29	....	....	0	0	0	0
30	....	....	0	0	0	0
31	....	....	0	0	0	0
Σ	....	....	....	191 635	111 922	107 023

$$-C = -\frac{6.115}{100\,000}\Sigma\frac{M_fM_p}{I} + \frac{12.235}{100\,000}\Sigma\frac{M_cM_p}{I} - 0$$

$$-D = -\frac{63.834}{100\,000}\Sigma\frac{M_fM_p}{I} \qquad + \frac{109.462}{100\,000}\Sigma\frac{M_dM_p}{I}$$

$$-F = +\frac{43.514}{100\,000}\Sigma\frac{M_fM_p}{I} - \frac{6.115}{100\,000}\Sigma\frac{M_cM_p}{I} - \frac{63.834}{100\,000}\Sigma\frac{M_dM_p}{I}$$

$C = -1.38 \qquad D = -0.18 \qquad F = -5.40$

Point	$M_E = M_p + M_fF + M_dD + M_cC$				$N_E = N_p + N_fF + N_dD + N_cC$				
	M_cC	M_dD	M_fF	M_E	N_cC	N_dD	N_fF	N_p	N_E
1	2.48	0	9.72	− 1.40	0	+.09	0	+ .57	+ .66
2	9.38	0	36.72	− 5.60	0	+.09	0	+ .57	+ .66
3	16.28	0	63.72	− 9.70	0	+.09	0	+ .57	+ .66
4	23.18	0	90.72	−13.80	....	....	....	....	....
5	25.53	.45	99.90	−12.15	−1.38	0	−5.40	+7.60	+ .82
6	26.50	.90	103.68	− 9.12	−1.38	0	−5.40	+7.60	+ .82
7	27.46	1.35	107.46	− 6.43	−1.38	0	−5.40	+7.60	+ .82
8	28.15	1.80	110.16	− 3.49	−1.38	0	−5.40	+7.60	+ .82
9	28.29	2.25	110.70	− .36	−1.38	0	−5.40	+7.60	+ .82
10	28.15	2.70	110.16	+ 3.11	−1.38	0	−5.40	+7.60	+ .82
11	27.88	3.15	109.08	+ 7.33	−1.38	0	−5.40	+7.60	+ .82
12	37.32	3.60	106.92	+ 9.54	−1.38	0	−5.40	+7.60	+ .82
13	26.36	4.05	103.14	+13.92	−1.38	0	−5.40	+7.60	+ .82
				+17.91	−1.38	0	−5.40	+7.60	+ .82
14	25.67	4.50	100.44	− 7.80	0	0	−5.40	+7.60	+2.20
15	0	4.05	103.14	− 6.61	0	0	−5.40	+7.60	+2.20
16	0	3.60	106.92	− 5.78	0	0	−5.40	+7.60	+2.20
17	0	3.15	109.08	− 3.45	0	0	−5.40	+7.60	+2.20
18	0	2.70	110.16	− 2.14	0	0	−5.40	+7.60	+2.20
19	0	2.25	110.70	− .05	0	0	−5.40	+7.60	+2.20
20	0	1.80	110.16	+ 2.56	0	0	−5.40	+7.60	+2.20
21	0	1.35	107.46	+ 6.11	0	0	−5.40	+7.60	+2.20
22	0	.90	103.68	+10.53	0	0	−5.40	+7.60	+2.20
23	0	.45	99.90	+13.95	0	0	−5.40	+7.60	+2.20
24	0	0	90.72	+20.02	0	....	....	....	....
25	0	0	63.72	+24.72	0	+.09	0	− .57	− .48
26	0	0	36.72	+22.72	0	+.09	0	− .57	− .48
27	0	0	9.72	+ 8.62	0	+.09	0	− .57	− .48
29	18.75	0	0	+18.75	0	−.18	0	0	− .18
30	11.90	0	0	+11.90	0	−.18	0	0	− .18
31	4.97	0	0	+ 4.97	0	−.18	0	0	− .18

Temperature Stresses

c = coefficient of linear expansion = elongation per unit length per degree Fahrenheit rise in temperature.

t = temperature change in degrees Fahrenheit.

l = span length.

Table of moments and thrusts gives values for 50° rise.
For 50° drop in temperature, the signs will change.

Fundamental equations already derived are:

$$P\delta_{cp} + C\delta_{cc} + D\delta_{cd} + F\delta_{cf} = 0$$

$$P\delta_{dp} + C\delta_{dc} + D\delta_{dd} + F\delta_{df} = 0$$

$$P\delta_{fp} + C\delta_{fc} + D\delta_{fd} + F\delta_{ff} = 0$$

in which $P\delta_{cp}$, $P\delta_{dp}$ and $P\delta_{fp}$ are deflections of points in the transformed system. An increase of temperature causes deflection of these same points in like directions and these temperature deflections may be substituted for the deflections $P\delta_{cp}$, $P\delta_{dp}$ and $P\delta_{fp}$ in the above equations. Substituting the proper flexual equivalents in the remaining terms and multiplying through E/s, we have

$$-\frac{E}{s}lct = C\Sigma\frac{M_c M_c}{I} + D\Sigma\frac{M_c M_d}{I} + F\Sigma\frac{M_c M_f}{I}$$

$$0 = C\Sigma\frac{M_d M_c}{I} + D\Sigma\frac{M_d M_d}{I} + F\Sigma\frac{M_d M_f}{I}$$

$$-\frac{E}{s} \times 2\,lct = C\Sigma\frac{M_f M_c}{I} + D\Sigma\frac{M_f M_d}{I} + F\Sigma\frac{M_f M_f}{I}$$

After substituting numerical values in these equations and solving

$$C = 0.$$

$$D = -\ 0.00063834\frac{E}{s} \times 2\,l \times ct.$$

$$F = \ 0.00040457\frac{E}{s} \times 2\,l \times ct.$$

$$\frac{E}{s} \times 2\,lct = \frac{144 \times 2\,500\,000}{5} \times 100 \times 0.0000065 \times 50 = 2\,340\,000.$$

Then $C = 0$ $\quad D = -\ 1.494$ kips. $\quad F = 0.947$ kip.

Table of Moments, Kip. ft., 50° F. Temperature Rise

Point	1	2	3	4	5	6	7
$\frac{Dx}{2}$	0	0	0	0	3.7	7.5	11.2
yF	1.7	6.4	11.2	15.9	17.5	18.2	18.8
$M_p = \frac{Dx}{2} - yF$	−1.7	−6.4	−11.2	−15.9	−13.8	−10.7	−7.6
Point	**8**	**9**	**10**	**11**	**12**	**13**	**14**
$\frac{Dx}{2}$	14.9	18.7	22.4	26.2	29.9	33.6	37.4
yF	19.3	19.4	19.3	19.1	18.7	18.1	17.6
$M_p = \frac{Dx}{2} - yF$	−4.4	−0.7	+3.1	+7.1	+11.2	+15.5	+19.8

In center post $M_t = 0$. $N_t = -\ 1.49$ kips.

In end posts $N_t = -\frac{D}{2} = +\ 0.75$ kip.

In top (horizontal member) $N_t = F = +\ 0.95$ kip.

Live load: Bridge is designed for uniform live load of 70 lb. per linear foot width of structure and in addition a single concentration of 3300 lb. per foot width of structure.

Uniform live load on a 5-ft. division = 5 × 70 = 350 lb. = 0.35 kip.
Concentration as above 3.30 kips.

The following calculations are for the design of section at Point 11. Moment and thrust due to earth pressure are brought forward from the calculation sheet for "earth pressure." Dead load and live load moments and thrusts for actual loads are derived in the following table. Column 1 shows the load point. Columns 2 and 3 tabulate respectively the influence line ordinates for moment and thrust as scaled from the influence diagram for point 11. Column 4 tabulates the actual dead loads at the various points as shown on the structure diagram. Column 5 tabulates the live loads with concentration placed at Point 9 for negative moment and at point 11 for positive moment. Actual dead and live load moments and thrusts are obtained by multiplying across columns 2 × 4, 2 × 5, 3 × 4 and 3 × 5 to obtain results in columns 6 to 11. The summations at the bottom give total effect of all the loads on the structure.

Sample Calculations of Actual Dead and Live Load Moments and Thrusts for Point 11

1	2	3	4	5	6	7	8	9	10	11
Load point	Ordinates		Dead loads	Live loads	Dead load		Live load		Live load	
	M	N			M	N	M	N	M	N
4	0	0	5.22	0.35	0	0	......	......	0	0
5	−0.54	+0.173	3.75	0.35	−2.0	+0.7	......	......	−0.2	+0.06
6	−1.17	+0.327	2.74	0.35	−3.2	+0.9	......	......	−0.4	+0.11
7	−1.87	+0.437	2.19	0.35	−4.1	+1.0	......	......	−0.7	+0.15
8	−2.64	+0.491	1.80	0.35	−4.8	+0.9	......	......	−0.9	+0.17
9	−2.69	+0.420	1.64	3.65	−4.4	+0.7	......	......	−9.8	+1.53
10	−0.98	+0.298	1.80	0.35	−1.8	+0.5	......	......	−0.3	+0.10
11	+2.36	+0.196	2.19	3.65	+5.2	+0.4	+8.6	+0.72	......	......
12	+1.36	+0.119	2.97	0.35	+4.0	+0.4	+0.5	+0.04	......	......
13	+0.62	+0.057	3.99	0.35	+2.5	+0.2	+0.2	+0.02	......	......
14	0	+0.002	4.96	0.35	0	0	......	......	......	......
15	−0.69	−0.054	3.99	0.35	−2.8	−0.2	......	......	−0.2	−0.02
16	−1.20	−0.101	2.97	0.35	−3.6	−0.3	......	......	−0.4	−0.04
17	−1.57	−0.129	2.19	0.35	−3.4	−0.3	......	......	−0.5	−0.05
18	−1.56	−0.119	1.80	0.35	−2.8	−0.2	......	......	−0.5	−0.04
19	−0.80	−0.045	1.64	0.35	−1.3	−0.1	......	......	−0.3	−0.02
20	+0.17	+0.055	1.80	0.35	+0.3	+0.1	+0.1	+0.02	......	......
21	+0.60	+0.092	2.19	0.35	+1.3	+0.2	+0.2	+0.03	......	......
22	+0.63	+0.089	2.74	0.35	+1.7	+0.2	+0.2	+0.03	......	......
23	+0.40	+0.052	3.75	0.35	+1.5	+0.2	+0.1	+0.02	......	......
24	0	0	5.22	0.35	0	0	0	0	......	......
Σ	−15.71	+1.698	.....	.....	−17.7	+5.3	+9.9	+8.8	−14.2	+1.95
	+ 6.14	+0.660	.....	.....	......	......	......	......	......	......

Moments in kip-ft. (1000 ft-lb.)
Thrusts in kips (1000 lb.)

Summary of Maximum Forces Acting on Section at Point 11

Note. Forces have been calculated for the various sections due to earth pressure acting at the right end of the structure. Forces at any section due to earth pressure left will be the same as forces on a symmetrically placed section in the other span due to earth pressure right. For example, forces at point 11 for earth pressure left will be the same as forces at point 17 calculated for earth pressure right. The sum of the calculated forces for points 11 and 17 will therefore give the forces due to balanced earth pressure for either point.

	Moment kip-ft.	Corresponding thrust, kips
Dead load	−17.7	+5.3
Earth pressure, right	+ 7.3	+0.3
Earth pressure, left	− 3.4	+1.6
Sub total	−13.8	+7.2
Live load	−14.2	+2.0
Sub total	−28.0	+9.2
Temperature	− 7.1	−1.0
Total	−35.1	+8.2

Having the maximum moment and thrust acting on the section, the required area of reinforcing steel and the stresses in the concrete may be calculated by the methods described elsewhere in this Section or by any other convenient method.

Similar calculations needed for all other parts.

50. Analysis of a Symmetrical Fixed Arch

The symmetrical fixed arch is statically indeterminate to the third degree. The analysis is therefore similar to that of the two-span rigid frame given above.

One-half of the transformed structure used in this analysis is shown in Fig. 98. The problem is to find the redundants B, C and D, such that under applied loads or deformations the ends of the two cantilevers formed by cutting the arch at the crown will coincide. To simplify this problem, the redundants are applied at the end of a rigid bracket directly below and at z distance from the crown. The equations taken directly from the previous analysis express the conditions here:

$$\delta_b = B\delta_{bb} + C\delta_{bc} + D\delta_{bd} + \Sigma P\delta_{bk} = 0$$
$$\delta_c = B\delta_{cb} + C\delta_{cc} + D\delta_{cd} + \Sigma P\delta_{ck} = 0$$
$$\delta_d = B\delta_{db} + C\delta_{dc} + D\delta_{dd} + \Sigma P\delta_{dk} = 0$$

From the condition of symmetry $\delta_{bd} = \delta_{db} = \Sigma\dfrac{xs}{EI} = 0$ and $\delta_{cd} = \delta_{dc} = \Sigma\dfrac{xys}{EI} = 0.$

In addition, $\delta_{bc} = \delta_{cb} = \Sigma\dfrac{ys}{EI} = 0$ when $Z = \dfrac{\Sigma y''s/I}{\Sigma s/I}$. (See Fig. 98.)

The equation then becomes:

$$B\delta_{bb} + \Sigma P\delta_{bk} = 0$$
$$C\delta_{cc} + \Sigma P\delta_{ck} = 0$$
$$D\delta_{dd} + \Sigma P\delta_{dk} = 0$$

Temperature and rib shortening effects are determined by substituting, for the loading deflections, the corresponding deflections due to temperature change and shortening of the arch caused by axial thrust. The equations become:

Temperature	Rib Shortening
$B\delta_{bb} = 0$	$B\delta_{bb} = 0$
$C\delta_{cc} - ctL = 0$	$C\delta_{cc} + \dfrac{f_c}{E}L = 0$
$D\delta_{dd} = 0$	$D\delta_{dd} = 0$

The minus sign in the temperature equation expresses the fact that for a rise (positive) in temperature, the movement of point O is opposite to the direction of C.

In the rib shortening formula, f_c = thrust at the crown divided by area of crown cross-section. This method for determining the effect of rib shortening is approximate, but the results are sufficiently near those determined by more accurate analysis to make the design safe for the usual arch under live load.

The solution of a problem is shown. A load of unity is applied (see Fig. 98) and the redundants determined. Any system of loads in the plane of the arch axis can be applied and B, C and D determined in the same manner. The remaining reactions can be determined by laws of statics.

This method applies equally to concrete or to steel structures, the only difference being in the calculation of I.

Influence lines are particularly useful in presenting a problem clearly and a systematizing of computations will save time and labor.

General Equations

Loads: $B\delta_{bb} + \Sigma P\delta_{bk} = 0$

$C\delta_{cc} + \Sigma P\delta_{ck} = 0$

$D\delta_{dd} + \Sigma P\delta_{dk} = 0 \qquad M_b = 1$

Temperature: $B = 0 \qquad M_c = y$

$C\delta_{cc} - ctl = 0 \qquad M_d = x$

$D = 0$

$$z = \frac{\Sigma y''\frac{s}{I}}{\Sigma\frac{s}{I}} = \frac{104.5}{20} = 5.23$$

M = moments due to load P

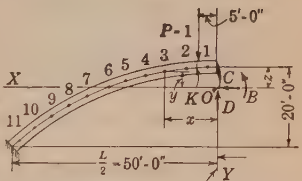


Fig. 98

Point	These quantities apply for load P in any position									
	x	y''	t	Ic	15 Is	I = Ic + 15 Is	s	s/I	y'' s/I	y
1	2.5	0.04	2.55	1.38	0.44	1.82	5.0	2.75	0.11	−5.19
2	7.5	0.40	2.63	1.52	0.47	1.99	5.0	2.52	1.00	−4.83
3	12.4	1.07	2.73	1.70	0.52	2.22	5.0	2.25	2.41	−4.16
4	17.3	2.10	2.82	1.87	0.56	2.43	5.0	2.06	4.33	−3.13
5	22.1	3.46	2.91	2.05	0.60	2.65	5.0	1.89	6.54	−1.77
6	26.9	5.15	3.00	2.25	0.65	2.90	5.0	1.73	8.90	−0.08
7	31.4	7.16	3.09	2.46	0.70	3.16	5.0	1.58	11.30	1.93
8	35.8	9.49	3.18	2.68	0.75	3.43	5.0	1.46	13.85	4.26
9	40.0	12.10	3.27	2.91	0.80	3.71	5.0	1.35	16.35	6.87
10	44.1	15.00	3.36	3.16	0.85	4.01	5.0	1.25	18.70	9.77
11	48.0	18.20	3.45	3.42	0.91	4.33	5.0	1.16	21.00	12.97
Σ	......	......	......	......	......	......	.....	20.00	104.5	....

Point	These quantities apply for load P in any position				These quantities only vary with load position			
	$y\frac{s}{I}$	$x\frac{s}{I}$	$y^2\frac{s}{I}$	$x^2\frac{s}{I}$	M	$M\frac{s}{I}$	$My\frac{s}{I}$	$Mx\frac{s}{I}$
1	−14.28	6.88	74.1	17	0	0	0	0
2	−12.17	18.90	58.8	142	− 2.5	− 6.3	30.4	− 47
3	− 9.35	27.90	38.9	346	− 7.4	− 16.7	69.1	− 207
4	− 6.45	35.60	20.2	616	−12.3	− 25.4	79.3	− 438
5	− 3.34	41.70	5.9	924	−17.1	− 32.3	57.1	− 714
6	− 0.14	46.50	0	1 250	−21.9	− 37.8	3.1	− 1 018
7	3.05	49.60	5.9	1 557	−26.4	− 41.7	− 80.5	− 1 310
8	6.22	52.30	26.5	1 874	−30.8	− 45.0	− 191.8	− 1 610
9	9.27	54.00	63.7	2 160	−35.0	− 47.2	− 324.0	− 1 890
10	12.21	55.10	119.3	2 430	−39.1	− 48.9	− 478.0	− 2 150
11	15.02	55.70	194.6	2 670	−43.0	− 49.9	− 646.0	− 2 400
Σ	0	444.2	607.9	13 986	....	−351.2	−1481.3	− 11 784

$$E\delta_{bb} = \Sigma M_b{}^2\frac{s}{I} = 2\,\Sigma\frac{s}{I} = 2 \times 20 = 40$$

$$E\delta_{cc} = \Sigma M_c{}^2\frac{s}{I} = 2\,\Sigma y^2\frac{s}{I} = 2 \times 607.9 = 1\,215.8$$

$$E\delta_{dd} = \Sigma M_d{}^2\frac{s}{I} = 2\,\Sigma x^2\frac{s}{I} = 2 \times 13\,986 = 27\,972$$

$$E\delta_{bk} = \Sigma MM_b\frac{s}{I} = \Sigma M\frac{s}{I} = -\ 351.2$$

$$E\delta_{ck} = \Sigma MM_c\frac{s}{I} = \Sigma My\frac{s}{I} = -\ 1\,481.3$$

$$E\delta_{dk} = \Sigma MM_d\frac{s}{I} = \Sigma Mx\frac{s}{I} = -\ 11\,784$$

$$B \times 40 - 351.2 = 0;\ B = 8.77$$

$$C \times 1215.8 - 1481.3 = 0;\ C = 1.22$$

$$D \times 27\,972 - 11\,784 = 0;\ D = .421$$

Temperature effect 40° rise or fall.

$$CE\delta_{cc} - EctL = 0$$

$$C = \frac{288\,000\,000 \times .000006 \times 40 \times 100}{1215.8} = \pm\ 5700$$

51. Integral Arch Action

In the design of open spandrel reinforced-concrete arches it has been customary to design the arch rib as carrying the dead load of the spandrel columns or walls and the floor system without being otherwise affected by the superstructure, and to design the superstructure as such without being affected by the action of the arch rib. Recent investigations both analytical and experimental in connection with structures in service show that large discrepancies, not always on the safe side, are introduced by neglecting in design the continuity of the structure. Reinforced-concrete does not lend itself to a complete segregation of the parts, and the usual assumptions made in

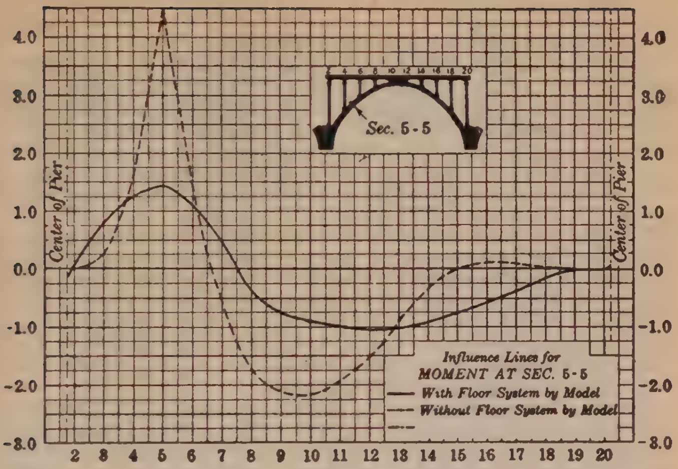


Fig. 99

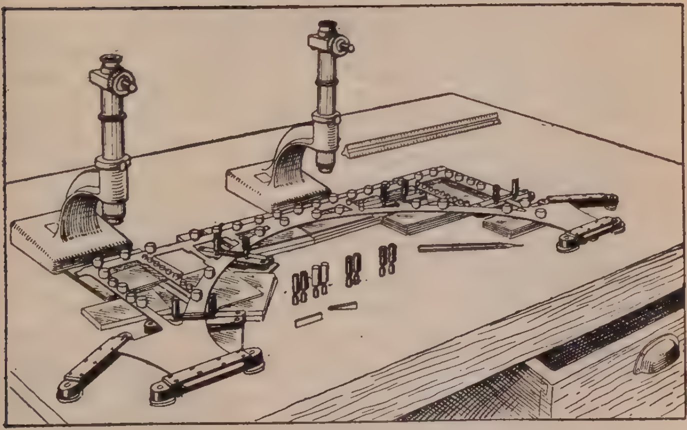


Fig. 100

design are often at serious variance with the conditions realized in construction. This caution is introduced here, without detailed discussion of the correct methods of design, in order that certain important points shall be recognized and proper means taken for their solution.

Fig. 99 shows a comparison of the calculated influence line for moment near the quarter point of an arch rib for the two cases. The solid line is the influence line obtained by assuming the entire structure to act integrally as usually constructed; the dotted line is that obtained by assuming the arch rib to carry the superstructure as dead load without inter-action between the two. Comparisons for thrust and shear at this point and for moment, thrust and shear at other points show like discrepancies, not always on the safe side.

The influence lines were obtained by the deformeter method of Professor George E. Beggs of Princeton University. This is a process of obtaining a ratio between an indeterminate reaction or between the stress in a member of an indeterminate structure and the load producing it, by measuring, in a celluloid or cardboard model of the structure, the relation between an arbitrarily imposed displacement at the section under consideration and the resulting displacement at the point of application of the load and in the direction of its application as described in Sect.

The structure for which the above comparisons are given is the Ashtabula Viaduct in Ohio as analyzed by D. H. Overman, Bridge Designer, Ohio Department of Highways, of which J. R. Burkey is Chief Engineer of Bridges. Fig. 100 shows Professor Beggs's deformeter apparatus mounted over the celluloid model of this structure. The model is shown weighted down upon small ball-bearings resting upon plate glass to reduce friction acting against free motion in the model.

52. Skewed Arches

Demonstration of methods of analysis for the skew arches beyond the scope of this book but a brief general discussion is pertinent.

The failure of many skewed reinforced-concrete arch bridges and concern for the safety of others have led many engineers to condemn such construction. The rapid growth of automobile traffic, however, and the economic pressure for elimination of grade crossings have brought this problem to the fore. It is doubtful if the public with its growing appreciation of esthetics in bridge construction will tolerate the construction of old-time girder and truss types for the many over-passes and under-passes that will have to be built in the near future, and the nature of modern traffic precludes kinking one highway, crossing another, in order to avoid the construction of a skewed arch or frame structure. The Arch Committee of the American Society of Civil Engineers has gone far toward a solution of this problem, taking advantage of the mathematical researches of Professor J. Charles Rathbun of the South Dakota School of Mines * and the investigations of Professor George E. Beggs of Princeton University by deformeter methods of mechanical analysis.

Fig. 101 shows graphically the principal forces acting on a skewed single span arch like structure (rigid-frame bridge) that do not exist in the "right" structure. A comparison of Figs. 102 and 103 shows the effect, on the principal system of reinforcing, of the forces introduced by the skew. The transverse rods in Fig. 102 are merely tie-rods whereas the transverse rods in Fig. 103 are principal reinforcement. This is sufficient evidence of the importance of accounting for structural effects of skew in the safe design of important structures.

* Trans. Am. Soc. C. E., 1924.

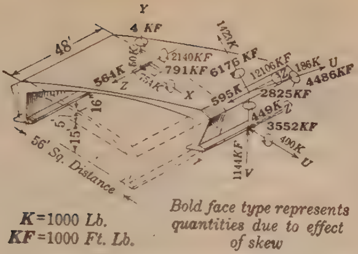


Fig. 101

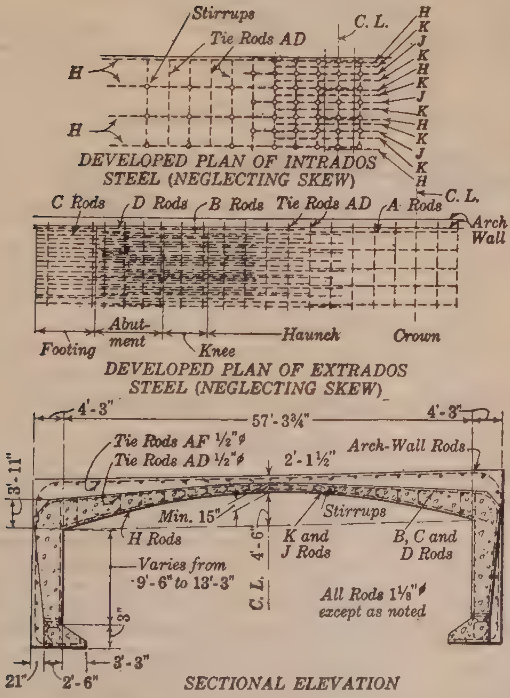
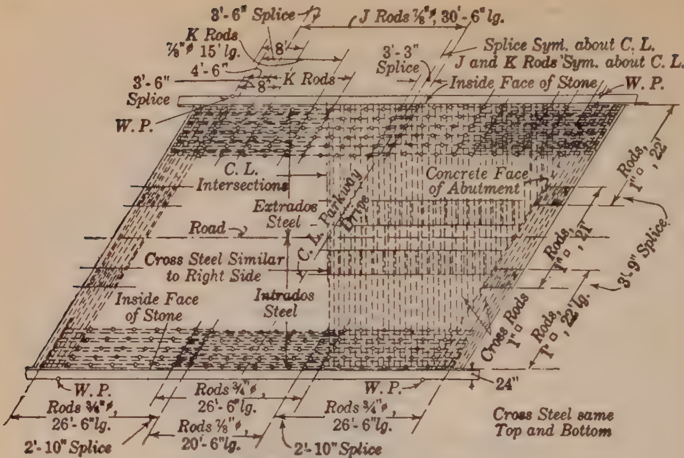
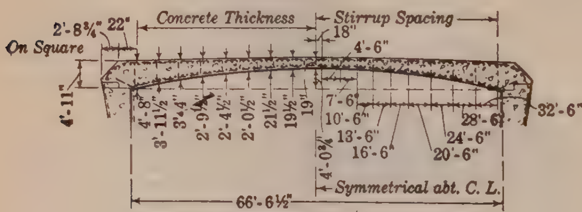


Fig. 102



PLAN OF STEEL



TYPICAL SECTION PARALLEL TO ROAD

Fig. 103

MISCELLANEOUS STRUCTURES

53. Retaining Walls

Forms of Walls built of reinforced concrete are made such that advantage is taken of the weight of the material supported to increase the stability of the wall against overturning. Fig. 104 represents in outline the usual type of reinforced wall. It consists of a vertical wall AE attached to a floor DC. For low walls the upright part of AE may act simply as a cantilever; and likewise the parts EC and ED. For higher walls the part AE is tied to EC at intervals by back walls, AEC, in the form of thin transverse walls with tension reinforcement. The projecting portion ED may still act as a cantilever, or it, also, may be connected to the vertical wall AE by means of buttresses. In either case the earth pressures act in essentially the same manner, and the necessary width of base is found

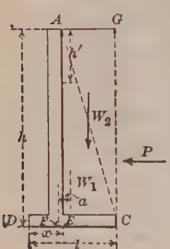


Fig. 104

in the same way. The first type is adapted to heights of 12 to 18 ft. For higher walls the second type will be more economical.

Stability of Wall. The stability of a retaining wall is usually determined with reference to its overturning. For safety in this regard the common requirement is that the resultant line of pressure shall intersect the base within the middle-third. This gives a distribution of pressure such that there is some compression over the entire section. Besides stability against overturning, the pressure on the foundation must be investigated and if found excessive the footings must be extended until the pressures are reduced to safe values.

In Fig. 104 let h = height of wall, l = width of base, x = distance from toe to back of wall AE, $k = x/l$, P = total horizontal pressure of earth against the vertical section GC, w = weight of earth filling per cubic foot, W_1 = weight of masonry per lineal foot, W_2 = weight per lineal foot of earth above the floor EC, a = lever arm of W_1 about point F, the edge of the middle-third. Then in order that the resultant pressure shall cut the outer edge of the middle-third the value of l is given by

$$l = \sqrt{\frac{2\,Ph - 6\,W_1a}{wh\,(1 + 2\,k - 3\,k^2)}}$$

If the moment of the masonry, W_1a, be neglected the value of k which will give a minimum value of l is 1/3. Using this value of k and taking w = 100 lb. per cu. ft. the value of l becomes $l = 1/8\,\sqrt{P}$ in which l is to be expressed in feet and P in pounds.

For other values of x/l, or k, the width of base must be increased over that required for k = 1/3 by the following percentages:

For $x/l = k$ =	0.5	0.25	0.15	0.10	0.0
Per cent increase =	2	2	4	6	15

Compared to masonry walls the reinforced wall with l/h = 1/4 to 1/3 has about the same stability as a solid masonry wall of the same width of base, and of the common type in which the outer face is battered 1 to 2 in. per foot. Under ordinary conditions a width of 0.4 to 0.5 the height will be required.

Foundation Pressures. If the pressures on the foundations need to be considered the value of these pressures is obtained as described for footings, after first determining where the resultant of W_1, W_2 and P cuts the base. This feature of the design is too often neglected and most of the failures of retaining walls have been due to excessive pressures at the outer toe. The pressure can be reduced to safe limits by widening the foundation or the foundation may be strengthened by means of piles.

Stresses in the Wall. Form (1), Fig. 105 or 106. The bending moment at the base of the upright portion AE is $Ph/3$. At any other section, F, it is $P'h'/3$, where P' is the pressure for depth h'.

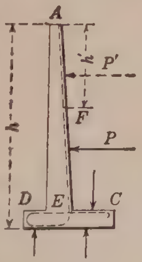


Fig. 105

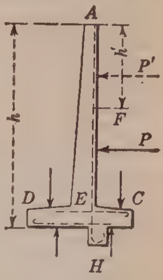


Fig. 106

Only a portion of the reinforcing rods need be carried up the full height. At the base the vertical rods must have sufficient anchorage to develop their full strength.

The cantilever DE must be treated in the same manner as the upright cantilever. The pressures will be much heavier and the shear and bond stress may need attention. The cantilever EC is acted upon by an upward and a downward force as shown in the figure. The maximum moment will be at E and will be negative. It is provided for by reinforcement as shown, which may be bent around into the toe DE or down into the projection EH if such is necessary to prevent sliding. The bending moments in DE and EC, due to upward earth pressure, can be readily found by means of the moment formulas given under the subject of footings for foundations.

Form (b), Fig. 107. The external pressures are practically the same as in the case previously considered. The pressure against the longitudinal wall AE is carried laterally for the most part and given over to the inclined back

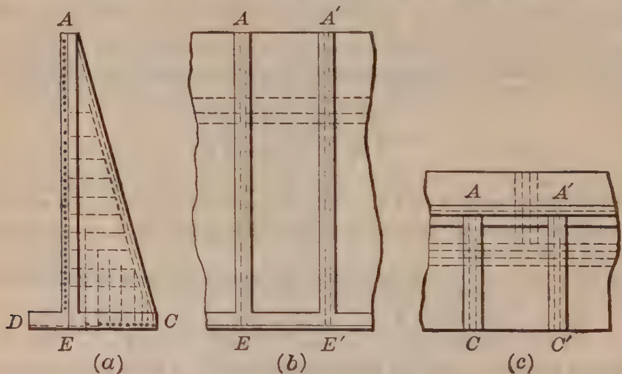


Fig. 107. Retaining Walls

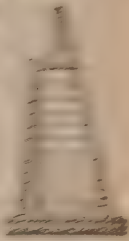
walls. The wall AE must therefore be designed as a slab supported along the lines AE, $A'E'$, and EE', Fig. (b).

The floor EC is supported by the back wall AEC and is therefore reinforced longitudinally as a floor slab in accordance with the resultant pressure at any point. The back wall ACE acts as a cantilever beam anchored to the floor. It is also a T-beam, the flange being the longitudinal wall AE. The tension along the edge AC is carried by rods near the edge, whose stress at any point is found with sufficient accuracy by an equation of moments taken about the center of the front wall. The maximum stress will be at the bottom. The main tension rods in AC should be well anchored in the reinforcing rods of the floor EC. A few additional vertical rods should also be put in to insure thorough bonding of floor to wall.

A horizontal beam may be made of the coping at A, thus giving some support to the wall AB along its upper edge. A downward projection may be necessary at the toe D, or at some other point in the base, in order to increase the resistance against forward sliding.

To prevent unsightly cracks, long walls must be provided with expansion joints at intervals of 50 to 75 ft., or must be reinforced longitudinally by 0.1 to 0.2% of steel. Long walls have thus been successfully built without expansion joints.

The form of retaining wall shown in Fig. 107 was extensively used on the Great Northern R. R. at Seattle, Wash. (Eng. News, Vol. 53, 1905). An estimate by C. F. Graff of the amounts of material per lineal foot required in reinforced and plain con-



is less likely to be injured in driving than a wooden pile. Where the bearing capacity depends largely upon frictional resistance the strength of the concrete pile may be taken the same as that of the wooden pile. Tapered piles appear to have a bearing capacity in soft material considerably greater than a straight pile of the same average diameter.

In the construction of piers in sandy beaches molded piles having an enlarged base have been very successfully used. These have been jetted into place. The same form is well adapted generally to foundations in sand where the jet process can be used.

In driving molded piles a driving head is used in which a cushion of sand, rope, or some other convenient material is interposed between a driving block of wood and the concrete. Experience indicates that any injury caused in driving rarely extends more than a few inches from the base of the pile. The jet process may often be used as in the case of wooden piles. In this case the end of the pile is preferably made square in order to provide a maximum of bearing surface. Where piles of considerable length are required the lower portion may often be advantageously made of wood and the upper portion of concrete doweled to the wooden pile. A considerable saving in cost is thus possible. In open water a hollow concrete pile driven over a wooden pile and to a firm bearing into the ground has given very satisfactory results.

The cost of concrete piles is generally from two to four times as much as that of wooden piles, so that the former are not likely to be economical where the conditions are favorable to the life of the wooden pile.

Sheet piling may also be made of concrete to advantage wherever it is desirable to leave it in place as a part of the permanent structure.

Repairs to Wooden Pile Structures have been successfully made by enclosing the upper portion of the wooden pile within a concrete box constructed of slabs of reinforced concrete.

55. Pipe and Box Culverts

Pipe and Box Culverts. For small openings the monolithic pipe or box form is very advantageous. Considerable settlement, as a whole, may be permissible, and hence solid foundations may not be needed. The cross-section may be circular, eliptical or rectangular. Theoretically, the elliptical form is the best, as it corresponds more nearly to the requirements for resisting the earth pressure. The circular is practically as good for small openings, whereas for large openings the rectangular form will often be the best on account of its simplicity and the smaller head-room required. Where the culvert is manufactured at a shop and transported to the site, the circular or the elliptical form will usually be the most advantageous.

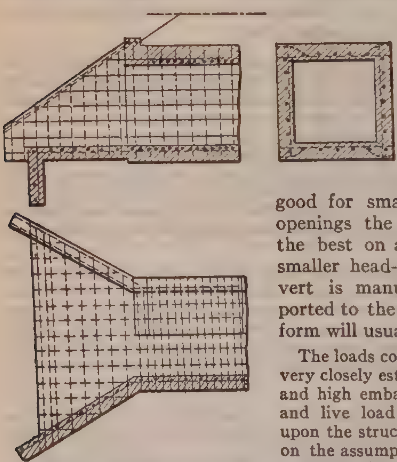


Fig. 109. Standard Box Culvert

The loads coming upon such structures cannot be very closely estimated, but except for small spans and high embankments the entire weight of filling and live load is usually assumed to act directly upon the structure. The stresses may be calculated on the assumption that this load acts vertically and is uniformly distributed over the width or diameter of the culvert. The lateral pressure should not be considered as an aid in supporting the vertical load. On this basis the maximum bending moment in a circular culvert is given by $M = 1/16pd^2$, where M =

bending moment at crown and at center of base for a unit length of culvert, p = load per unit area and d = diameter of culvert. For rectangular culverts the bending moments may be taken as for a simply supported beam, and in the vertical walls the moments may be calculated for a lateral pressure equal to 1/8 the vertical load.

Reinforcement of Culverts. In the circular form a wire mesh is convenient, especially for small diameters. A single mesh is sufficient, placed near the extrados at the sides and crossing the central axis at about the quarter point. In the rectanglar form, if reinforcement for negative moments at the corners is omitted, then the four sides will act as simple beams.

Longitudinal reinforcement should be provided to some extent. Where foundations are good a very small amount will be sufficient, but if settlement is likely to occur the longitudinal reinforcement becomes of importance. The entire culvert will act as a beam subjected mainly to positive bending moments. Most of the reinforcement should therefore be placed along the bottom of the culvert. Fig. 109 is a standard design for a rectangular box railroad culvert.

Tests of reinforced culvert pipe by A. N. Talbot showed that under concentrated loads the actual strength was fully equal to the theoretical strength; and that under a load distributed by means of a sand box the actual effective strength was relatively somewhat greater than for concentrated loads. Complete failure in the latter tests was generally prevented by lateral support from the sand box.

56. Dams

Two Types of Reinforced-Concrete Dams are shown in Figs. 110 and 111. The first is suitable for locations where little or no water passes over the crest, the second is designed to act as a spillway. In both cases the dam consists

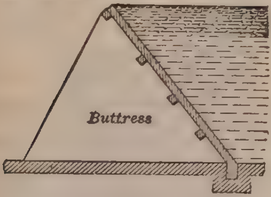


Fig. 110

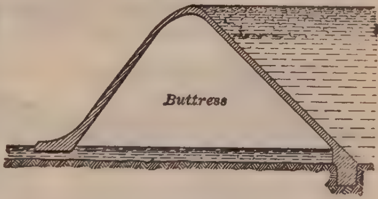


Fig. 111

of a water-tight reinforced floor on the upstream side, supported by solid buttresses placed 10 to 15 ft. apart and 12 to 24 in. thick. The upper wall may be supported directly on the cross walls and reinforced with longitudinal rods, or longitudinal beams may be used as shown and the slab supported on these. The pressure on the foundation is determined by considering the resultant of water pressure and weight of dam. The buttresses or cross walls are subjected to compressive stresses only. Ample longitudinal reinforcement should be provided to bind the structure together thoroughly.

Where the foundation is solid rock the buttresses may rest directly upon it, but where a pile foundation is required a continuous floor should be provided under the entire structure and covering the heads of the piles. If well bonded together a reinforced-concrete dam is exceedingly strong and stable and must fail as a unit if at all. Great care must be exercised in securing a thorough connection between the foundation and dam by means of a deep concrete connection at the upper toe. When founded on piles all seepage of water underneath the dam must be cut off by a line of sheet piling at the upper toe. This piling should be well covered by the concrete. To prevent upward pressure on the floor, or upon the lower face, drain openings should be provided connecting the interior with the tail-water level.

57. Dock and Harbor Walls; Breakwaters

Forces to be Considered. Dock and harbor walls not subjected to wave action are designed as retaining walls subjected to the earth pressure on the land side with surcharge due to loads on docks or quays. The action of heavy ice may require consideration, both with respect to its pressure and its wearing effect. In the case of outer protecting works, such as breakwaters, piers or jetties, the action of the waves is the principal force to be considered.

Force of Waves. The pressure or force exerted by waves depends upon the velocity of the impinging water and its depth. The former depends upon the exposure, wind velocity, depth of water and shape of bottom. The velocity in shallow water is approximately proportional to the square root of the height of wave and the depth of water. The pressure due to impact is proportional to the square of the velocity and therefore is also approximately proportional to the height of wave and depth of water.

Pressures due to impact of high waves have been measured in several cases. For waves 10 ft. high, pressures of 1800 to 3000 lb. per sq. ft. have been observed, and for extremely high waves of 25 to 30 ft. pressures of 6000 to 7000 lb. per sq. ft. have been noted. In exposed locations large blocks of concrete weighing 3000 tons have been moved, requiring an estimated average pressure of over 2 tons per sq. ft. The maximum intensity occurs at about the mean water level, is zero at the crest and gradually diminishes towards the bottom. The impact of waves on either vertical or sloping walls causes large masses of water to be thrown upward which, on falling, strikes severe blows upon the upper surface of the wall or breakwater, tending to rupture it.

Breakwaters are constructed in various ways. Timber cribs filled with stone are suitable for moderate wave action. The same construction, capped

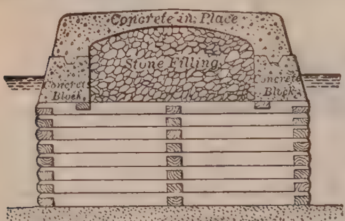


Fig. 112

Fig. 113

with concrete, as shown in Fig. 112, forms a strong combination. The structure is built to above the water-level by the use of large concrete blocks. The whole is then capped with mass concrete. Fig. 113 illustrates the same general type in which the mass is built of mass concrete deposited under water within temporary frames. Separate concrete blocks are often dovetailed or clamped together to give additional stability.

Foundations for Breakwaters, not located on solid rock, are commonly prepared by first depositing a broad layer of rubble stone at a flat slope and leveling up the top by the aid of careful soundings or by means of divers. Piles are also used, which are cut off below water level and protected by heavy riprap. Bags of cement have also been frequently employed for the purpose.

The Reinforced-Concrete Caisson furnishes perhaps the most satisfactory method of construction yet devised. Fig. 114 illustrates such a form used by W. V. Judson on Lake Michigan. The caisson is built at shore and towed to place. It is then sunk by filling one or more compartments with stone, and the whole is capped with concrete, sufficiently reinforced to preserve the monolithic character of the structure. Similar caissons have been employed for quay walls and breakwaters in European works. At Rotterdam, caissons were employed of dimensions 131 ft. × 32 ft. and 43 ft. high. These were built in dry docks, towed to place and brought into alinement by means of a tongue-and-groove joint. Some cells were filled with concrete, others with sand or rock. Reinforced concrete possesses great advantages for

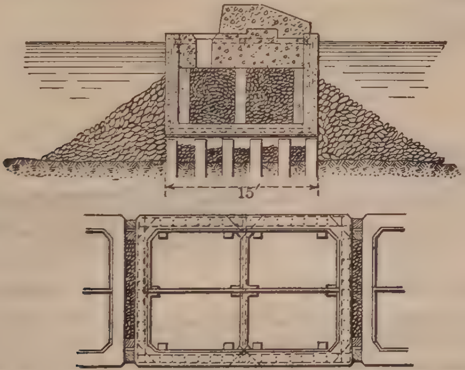


Fig. 114. Reinforced-Concrete Caisson

construction of this sort over ordinary masonry or concrete construction by reason of its monolithic character.

Concrete sheet piling, fitted with tongue and groove, has been used successfully for breakwaters. Two rows of such piling are driven, after which the intervening space is filled with sand and the whole capped with a superstructure of reinforced concrete.

Sea Wall. Fig. 115 illustrates the Galveston sea wall. It is founded on piles and constructed of mass concrete, but this is reinforced near the front face, as shown.

Dock and Harbor Walls are built, in the main, like ordinary retaining walls. Frequently a pile foundation is necessary. If the piles are driven in soft mate-

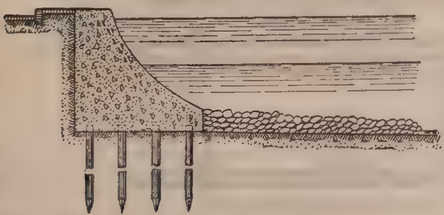


Fig. 115. Galveston Sea Wall

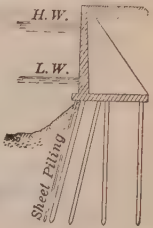


Fig. 116

rial, one or more rows should be driven at a considerable inclination in order to resist the horizontal component of the pressure. The walls are commonly constructed of concrete, either in loose blocks of mass concrete, plain or reinforced. Where the wear from ice is heavy a concrete wall may well be faced with granite or with other hard stone. Reinforced concrete possesses the same advantages in this type of wall as for ordinary retaining walls and is built of the same general form. Fig. 116 shows the essential features.

Anchored walls of reinforced concrete are often advantageous, especially in the construction of new walls along a river front where anchorage is readily secured. Fig. 117 illustrates such a construction as carried out at Berne, Switzerland. The wall consists

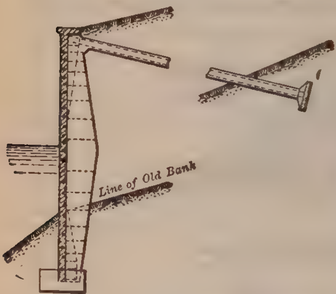


Fig. 117. An Anchored Wall

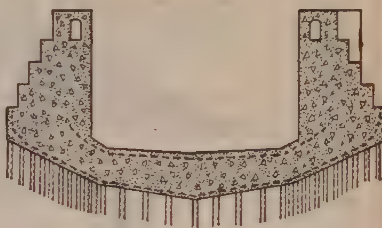


Fig. 118. Canal Lock

of a series of anchored beams connected by slab construction. The stability of the wall is measured by the stability of the entire mass between the wall and the anchorage.

Dry Docks and Canal Locks. In this construction reinforced concrete is of advantage, not only in the construction of the walls, but also in the floors, enabling them to be readily designed against upward pressure and in one piece with the walls. Fig. 118 is a section through the lock on the Charles River, Boston, showing this feature.

58. Reservoirs, Tanks, Bins

For Covered Reservoirs reinforced concrete is very well adapted. The rectangular form with flat cover is usually the most convenient; its design involves the same features as building design with the additional one of imperviousness. Fig. 119 illustrates a typical design of this kind for Indianapolis (Eng. News, Oct. 15, 1908). Especial care is required in such structures to prevent contraction cracks by the use of a liberal amount of reinforcement.

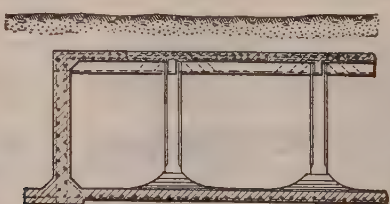


Fig. 119. Covered Reservoir

Fig. 120 illustrates the general form of construction used in a large elevator at Baltimore. The walls of the bins are 8 in. thick and are reinforced with vertical rods of 1/2-in. round steel, spaced 3 ft. apart, and with horizontal bars 3/8 in. by 1 to 1-3/4 in. spaced 12 in. apart. The hopper bottoms, the tunnels for the conveying machinery and the foundation floor resting upon the piles are all of reinforced concrete. (Eng. Record, Feb. 20, 1909.)

Elevated towers and tanks may also be made of concrete, but high pressures are difficult to deal with. Bins and coal pockets are structures for which concrete is well adapted. For the storage of coal bins of unprotected steel are not durable, but reinforced concrete furnishes an almost ideal material, lending itself readily to the necessary form for strength and furnishing the desired durability.

Reinforced concrete is advantageously used in other minor forms of structures and structural elements. Noteworthy among such uses is its employment for fence posts and poles for various purposes. In building construction it is found that sills, lintels, steps and staircases are usually cheaper in reinforced concrete than in stone or metal.

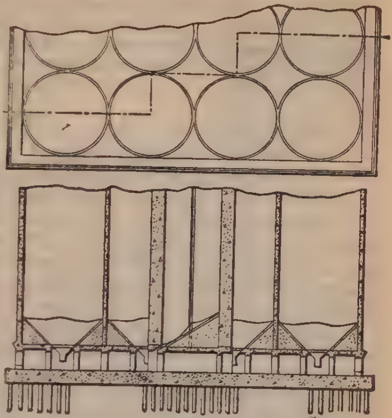


Fig. 120. Grain Elevator

59. Conduits, Sewers, Pipes

For Conduits not under pressure, large sewers and the like, reinforced concrete lends itself to convenient and economical construction. As to the analysis and design, these structures are only special cases of the monolithic pipe or box which was discussed in Art. 55. The character of the foundation and

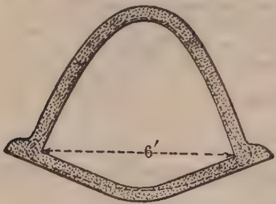


Fig. 121

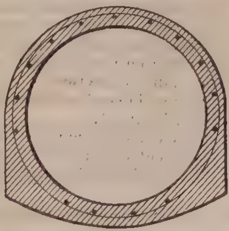


Fig. 122

convenience in construction will lead to various modifications of design. Fig. 121 is a typical cross-section of a large sewer for Harrisburg, Pa. A mesh of expanded metal is used for reinforcement, arranged to resist positive moments excepting at bottom and corners. Fig. 122 illustrates a design of a circular pipe or conduit molded in place.

Reinforced concrete has also been used to some extent for pipes under pressure, but it is very difficult to secure imperviousness under heads of considerable magnitude.

In pressure pipes the tensile stress is entirely taken by the steel, the concrete furnishing merely for the impervious layer and resisting bending due to earth loading.

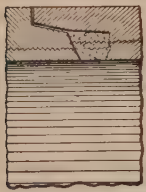


Fig. 123

Pipes of reinforced concrete separately molded in sections have been successfully used for moderate pressures with a considerable economy over metal. The joining of such pipes is accomplished by overlapping of the reinforcement. One method of doing this is illustrated in Fig. 123.

60. Chimneys

General Design. A reinforced-concrete chimney consists of the outer main shell, the inner shell or lining, and the base. The outer shell is made 4 to 6 in. thick at the top and 8 to 12 in. at the bottom, depending upon the height and diameter. The inner shell is generally made 4 in. in thickness, and of concrete or fire brick, the latter being preferable for high temperatures. The amount of reinforcement required is small, as the load is purely vertical and of small amount. Half-inch rods spaced 1 to 2 ft. apart are ample. The lining is generally carried up about one-third the height, but the large amount of cracking which has occurred in some chimneys just above the lining indicates the desirability of more extensive linings. The outer shell is designed to resist wind pressure and also with some reference to the temperature stresses. The base must be of such an area that the maximum earth pressure due to vertical load and wind pressure will not exceed safe limits. The thickness is then determined with reference to the bending and shearing stresses which are caused by these pressures.

Wind Stresses. The stresses at any given section, distance h below the top, will be determined. The notation is as follows:

A = area of chimney section under consideration.
A_s = total area of steel sections.
W = weight of superincumbent portion of chimney.
P = wind pressure on that portion.
M = bending moment at the section.
e = distance from the center of the section to where the resultant of the weight and wind pressure cuts the section (" eccentric distance ").
f_c = unit stress in concrete adjacent to the steel at lee side.
f_c' = unit stress in concrete adjacent to steel at windward side.
h = distance from top to section considered.
r = mean radius of section.
f_s = unit stress in steel at the windward side.
p = steel ratio, A_s/A.
m = a coefficient such that $f_c = mW/A$.
m' = a coefficient such that $f_c' = m'W/A$.
n = ratio of modulus of elasticity of steel to that of concrete, taken at 15.

The value of e is first to be determined by the formulas:

$$M = 1/2\ Ph \qquad e = M/W = Ph/2\ W$$

If e is greater than $1/2\ r$ there will be tension on the windward side. In this case it is assumed that the steel will carry all the tension. If the tensile stress is small, however, the concrete will not be overstressed and it may be desirable to calculate its stress on this assumption. Two cases will therefore be considered: (1) Where the stresses are all compressive, or the tension on the windward side is small (less than 50 lb. per sq. in., say), and (2) where there are large stresses on the windward side and the concrete is neglected.

Case 1. Having found e as above described the unit stresses are found by

$$f_c = \frac{1 + 2\,e/r}{1 + np}\,\frac{W}{A}, \quad \text{and} \quad f_c' = \frac{1 - 2\,e/r}{1 - np}\,\frac{W}{A}$$

Case 2. Practical formulas for this case cannot be stated, but the curved lines in Fig. 124 give values of m for different values of e/r and of p. Then

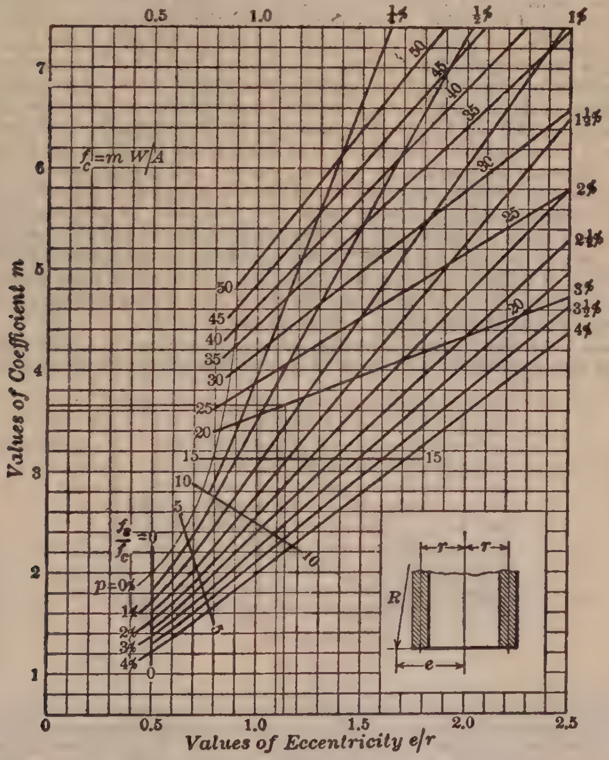


Fig. 124. Diagram for Reinforced-Concrete Chimneys

$f_c = m\,W/A$. The straight lines give the ratio of $\frac{f_s}{f_c}$ for the given value e/r and p; from this ratio the value of S_s is determined.

The maximum concrete stress is slightly greater than the values calculated by the above formulas, as these give the stress adjacent to the steel. The results are, however, sufficiently accurate.

The Wind Pressure commonly assumed is 25 to 50 lb. per sq. ft. on the projected area, but this is probably higher than necessary. Experiments made at the Eiffel Tower and at the National Physical Laboratory of England show that the pressure per square foot on square, flat surfaces, from 10 to 100 sq. ft. in extent, is 0.0032 times the square of the wind velocity in miles per hour. Taking the velocity at 100 miles per hour and the pressure on a cylindrical surface at 2/3 that on the projected plane, a value of 20 lb. per sq. ft. is deduced, which should be ample.

Temperature Stresses. Both theoretical considerations and evidence from practice indicate that temperature stresses are likely to be high, especially near the top of the lining. They occur in direction both vertically and circumferentially and are due to the high temperature of the interior as compared to the exterior. Both vertical and horizontal cracks of the exterior are of frequent occurrence and are doubtless due to this cause. Assuming the coefficient of expansion to be the same at all temperatures, a theoretical analysis indicates that for 1% of circumferential reinforcement the compressive stress in the concrete on the interior is as much as 350 lb. per sq. in. for each 100° difference in temperature between exterior and interior surfaces; the corresponding stress in the steel is about 5000 lb. per sq. in. Approximately the same stresses are caused in the vertical reinforcement. The stresses in the concrete increase somewhat with increase in reinforcement, but decrease in the steel; they are proportional to the differences in temperature of outer and inner surfaces. A difference in temperature of 200° will therefore bring the compressive stress well up to safe limit. So far as temperature stresses are concerned it is advantageous to use a small amount of steel.

Bases. The bases of chimneys are designed in the same manner as column footings. In this case, however, the earth pressures must be determined with reference to the

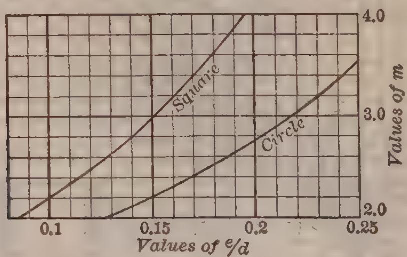


Fig. 125

effect of wind pressures and vertical load combined. The resultant of the wind pressure, the weight of chimney, the earth filling above the base and the base itself is to be first determined. Suppose the resultant cuts the base at a distance e from the center. Let W = total vertical load, M = wind moment at bottom of base, A = area of base, d = maximum diameter of rectangular base, or diameter of circular base, and p = maximum pressure on foundation per unit area. Then $e = M/W$. The values of p are as follows:

For rectangular bases, wind acting in the direction of the diagonal, and for values of e less than $d/12$, $p = W/A + 12\,M/Ad$. For circular bases, and values of e less than $d/8$, $p = W/A + 8\,M/Ad$. For either rectangular or circular bases, and values of e greater than the limits above given, $p = mW/A$, in which m is to be taken from Fig. 125.

APPENDICES

The American Society for Testing Materials (1926)

Appendix 1. Cement Manufacture

Cement. In engineering literature the term cement is understood to mean the finely pulverized product obtained by the burning of a suitable mixture of argillaceous and calcareous materials, or by an artificial mixture of such materials after burning, which will possess the property of hardening into a solid mass when mixed with water. The essential ingredients in the manufacture of cement are calcium carbonate ($CaCO_3$), silica (SiO_2) and alumina (Al_2O_3); the last two, combined in various proportions, constitute the argillaceous material. The characteristic property of cement is that of hardening by the addition of water, and therefore its ability to harden when excluded from the air.

Portland Cement is the finely pulverized product resulting from the calcination to incipient fusion of an intimate, artificial mixture of properly proportioned argillaceous and calcareous materials. It has a definite chemical composition varying within comparatively narrow limits. There are three distinct stages in the process of its manufacture: (1) the preparation of the correct mixture by the selection, mixing and grinding of the ingredients, (2) the burning of the mixture to a clinker, and (3) the pulverizing of the burned clinker to a fine powder. Portland cement is distinguished by uniformity of quality and high strength which it rapidly acquires.

Portland cement should be used in structures where strength is of special importance, or where the structure is exposed to the action of the elements. It should be used invariably in reinforced concrete construction.

Manufacture of Portland Cement. A great variety of materials is available for the manufacture of portland cement. The principal combinations in use are as follows:

	Argillaceous.	Calcareous.
(1)	Argillaceous limestone (cement rock).	Pure limestone.
(2)	Clay or shale.	Pure limestone.
(3)	Clay or shale.	Chalky limestone or chalk.
(4)	Clay.	Marl.
(5)	Clay	Alkali waste.
(6)	Blas urnace slag.	Pure limestone.

Two general processes are used in the grinding and mixing of the raw material, the dry and the wet. In the dry process, the material is thoroughly dried and then finely ground, the latter stages of grinding being done after the mixing. Where slag is used it is first granulated by being run into water, which causes it to form into small, easily crushed particles. In the wet process the material is mixed and ground together into a " slurry " containing 60 to 70% of water. This slurry is then pumped directly to the burning kiln. The wet process is generally used with clay and marl.

After this preparation the material is burned to a clinker in a kiln of which there are two general types, the fixed vertical kiln and the continuous rotary kiln. The latter is used in most American plants, and to its development is due, largely, the growth of the cement industry in this country. From 100 to 200 lb. of coal are required per barrel of product, and the production per kiln is generally from 100 to 300 barrels.

The final grinding of the burned clinker is of the greatest importance. As it is only the very finest particles (the " flour ") that possess active properties, the advantage of fine grinding is obvious, this is done in two or three stages. The necessary calcium sulfate is generally added before the final grinding.

Natural Cement is the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature below fusion. The proportions of lime and clay in the raw material may vary between much wider limits than in the case of portland cement. Natural cement does not develop its strength as quickly nor is it as strong or uniform in composition as portland cement. It is now little used in the United States.

Natural cement is suitable for use in structures where mass and weight rather than strength are the essential features. It is thus adapted for use in many underground structures, such as sewers, conduits, massive foundations and in the unexposed portions of large and massive structures above ground, such as dams, retaining walls, piers and abutments. Used alone, or mixed with lime mortar, it also makes a suitable mortar for ordinary brick and stone masonry.

Puzzolan or Slag Cement is the finely pulverized product resulting from grinding a mechanical mixture of fused argillaceous material and hydrated lime. The argillaceous substance may consist of natural puzzolanic material, such as volcanic ash, trass or allied igneous material, or of artificial materials, such as blast-furnace slag or burnt clay. Puzzolan cement is not as strong or reliable as portland or natural cement and is not extensively used in the United States. Its use should be confined to unexposed work where great strength is not required.

Appendix 2. Testing of Cement

Standard Cement Tests. In all important work the cement should be fully specified and regularly tested. For unimportant work, however, there is little danger to be feared in using well-known brands of cement without testing. Results of test depend largely upon the method of making them; it is hence very important that definite and uniform methods be used. The kinds of tests usually made are as follows: (1) Fineness. (2) Time of setting. (3) Tensile strength of neat cement and of sand mortar. (4) Soundness, or constancy of volume. A chemical analysis is also desirable on large work. Of much importance in securing uniform and reliable results are the methods of sampling, the consistency of the mortar, temperature of the material, character of sand used and of apparatus employed. The standard methods of testing given below have come into general use in the United States.

Standard Methods of Cement Testing devised and recommended by the American Society for Testing Materials (1926) are given here (chemical analysis omitted).

Fineness. Wire cloth for standard sieves for cement shall be woven (not twilled) from brass, bronze, or other suitable wire, and mounted without distortion on frames about 2 in. below the top of the frame. The sieve frames shall be circular, approximately 8 in. in diameter, and may be provided with a pan and cover.

2. A standard No. 200 sieve is one having nominally an 0.0029-in. opening and 200 wires per inch standardized by the U. S. Bureau of Standards, and conforming to the following requirements:

The No. 200 sieve should have 200 wires per inch, and the number of wires in any whole inch shall not be outside the limits of 192 to 208. No opening between adjacent parallel wires shall be more than 0.0050 in. in width. The diameter of the wire should be 0.0021 in. and the average diameter shall not be outside the limits 0.0019 to 0.0023 in. The value of the sieve as determined by sieving tests made in conformity with the standard specification for these tests on a standardized cement which gives a residue of 25 to 20% on the No. 200 sieve, or on other similarly graded material, shall not show a variation of more than 1.5% above or below the standards maintained at the Bureau of Standards.

3. The test shall be made with 50 grams of cement. The sieve shall be thoroughly clean and dry. The cement shall be placed on the No. 200 sieve, with pan and cover attached, if desired, and shall be held in one hand in a slightly inclined position so that the sample will be well distributed over the sieve, at the same time gently striking the side about 150 times per minute against the palm of the other hand on the up stroke. The sieve shall be turned every 25 strokes about one-sixth of a revolution in the same direction. The operation shall continue until not more than 0.05 gram passes through in 1 minute of continuous sieving. The fineness shall be determined from the weight of the residue on the sieve expressed as a percentage of the weight of the original sample.

4. Mechanical sieving devices may be used, but the cement shall not be rejected if it meets the fineness requirement when tested by the hand method described in Par. 3.

5. A permissible variation of 1 will be allowed, and all results in excess of the specified limit but within this permissible variation shall be reported as 22%.

Mixing Cement Pastes and Mortars. 6. The quantity of dry material to be mixed at one time shall be 500 grams for neat cement mixtures and 1000 grams for mortar mixtures. The proportions of cement or cement and sand shall be stated by weight in grams of the dry materials; the quantity of water shall be expressed in cubic centimeters (1 cc. of water = 1 gram). The dry materials shall be weighed, placed upon a non-absorbent surface, thoroughly mixed dry if sand is used, and a crater formed in the center, into which the proper percentage of clean water shall be poured; the material on the outer edge shall be turned into the crater by the aid of a trowel. After an interval of 1/2 minute for the absorption of the water the operation shall be completed by continuous, vigorous mixing, squeezing and kneading with the hands for at least a minute. During the operation of mixing, the hands should be protected by rubber gloves.

7. The temperature of the room, apparatus, materials and mixing water shall be maintained as nearly as practicable at 21° C. (70° F.).

Normal Consistency. 8. The Vicat apparatus consists of a frame bearing a movable rod weighing 300 grams, one end being 1 cm. in diameter for a distance of 6 cm., the other having a removable needle 1 mm. in diameter, 6 cm. long. The rod is reversible, and can be held in any desired position by a screw, and has midway between the ends a mark which moves under a scale (graduated to millimeters) attached to the frame. The paste is held in a conical, hard-rubber ring 7 cm. in diameter at the base, 6 cm. at the top, and 4 cm. high, resting on a glass plate about 10 cm square.

9. In making the determination, 500 grams of cement, with a measured quantity of water, shall be kneaded into a paste, as described in paragraph 6, and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained about 6 in. apart; the ball resting in the palm of one hand shall be pressed into the larger end of the rubber ring held in the other hand, completely filling the ring with paste; the excess at the larger end shall then be removed by a single movement of the palm of the hand, the ring shall then be placed on the larger end on a glass plate and the excess paste at the smaller end sliced off at the top of the ring by a single oblique stroke of a trowel held at a slight angle with the top of the ring. During these operations care shall be taken not to compress the paste. The paste confined in the ring, resting on the plate, shall be placed under the rod, the larger end of which shall be brought in contact with the surface of the paste; the scale shall then be read, and the rod quickly released. The paste shall be of normal consistency when the rod settles to a point 10 mm. below the original surface in 1/2 minute after being released. The apparatus shall be free from all vibrations during the test. Trial pastes shall be made with varying percentages of water until the normal consistency is obtained. The amount of water required shall be expressed in percentage by weight of the dry cement.

10. The consistency of standard mortar shall depend on the amount of water re-

quired to produce a paste of normal consistency from the same sample of cement. Having determined the normal consistency of the sample, the consistency of standard mortar made from the same sample shall be as indicated in the table, the values being in percentage of the combined dry weights of the cement and standard sand:

Percentage of Water for Standard Mortars

Percentage of water for neat cement paste of normal consistency	Percentage of water for one cement, three standard Ottawa sand	Percentage of water for neat cement paste of normal consistency	Percentage of water for one cement three standard Ottawa sand
15	9.0	23	10.3
16	9.2	24	10.5
17	9.3	25	10.7
18	9.5	26	10.8
19	9.7	27	11.0
20	9.8	28	11.2
21	10.0	29	11.3
22	10.2	30	11.5

Soundness. 11. A steam apparatus, which can be maintained at a temperature between 98 and 100° C. is recommended.

12. A pat from cement paste of normal consistency about 3 in. in diameter, 1/2 in. thick at the center, and tapering to a thin edge, shall be made on clean glass plates about 4 in. square, and stored in moist air for 24 hours. In molding the pat, the cement paste shall be flattened on the glass and the pat then formed by drawing the trowel from the outer edge toward the center.

13. The pat shall then be placed in an atmosphere of steam at a temperature between 98 and 100° C. upon a suitable support 1 in. above boiling water for 5 hours.

14. Should the pat leave the plate, distortion may be detected best with a straight-edge applied to the surface which was in contact with the plate.

Time of Setting. 15. The following are alternate methods, either of which may be used as ordered:

16. The time of setting shall be determined with the Vicat apparatus.

17. A paste of normal consistence shall be molded in the hard-rubber ring as described in paragraph 8 and placed under the movable rod, the smaller end of which shall then be carefully brought in contact with the surface of the paste, and the rod quickly released. The initial set shall be said to have occurred when the needle ceases to pass a point 5 mm. above the glass plate in 1/2 minute after being released; and the final set, when the needle does not sink visibly into the paste. The test pieces shall be kept in moist air during the test. This may be accomplished by placing them on a rack over water contained in a pan and covered by a damp cloth, kept from contact with them by means of a wire screen, or they may be stored in a moist closet. Care shall be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point may increase the penetration. The time of setting is affected not only by the percentage and temperature of the water used and the amount of kneading the paste receives, but by the temperature and humidity of the air, and its determination is therefore only approximate.

18. The time of setting shall be determined by the Gillmore needles.

19. The time of setting shall be determined as follows: A pat of neat cement paste about 3 in. in diameter and 1/2 in. in thickness with a flat top mixed to a normal consistency shall be kept in moist air at a temperature maintained as nearly as practicable at 21° C. (70° F.). The cement shall be considered to have acquired its initial set when the pat will bear, without appreciable indentation, the Gillmore needle 1/12 in. in diameter, loaded to weight 1/4 lb. The final set has been acquired when the pat will bear, without appreciable indentation, the Gillmore needle 1/24 in. in diameter,

loaded to weigh 1 lb. In making the test, the needles shall be held in a vertical position, and applied lightly to the surface of the pat.

Tension Tests. 20. The form of test piece shown in Fig. 126 shall be used. The molds shall be made of non-corroding metal and have sufficient material in the sides to

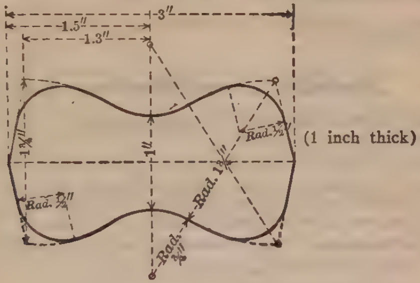


Fig. 126

prevent spreading during molding. Molds shall be wiped with an oily cloth before using.

21. The sand to be used shall be natural sand from Ottawa, Ill., screened to pass a No. 20 sieve and retained on a No. 30 sieve. This sand may be obtained from the Ottawa Silica Co., Ottawa, Ill.

22. This sand, having passed the No. 20 sieve, shall be considered standard when not more than 5 grams pass the No. 30 sieve after one minute continuous sieving of a 500-gram sample.

23. The sieves shall conform to the following specifications:

The No. 20 sieve shall have between 19.5 and 20.5 wires per whole inch of the warp wires and between 19 and 21 wires per whole inch of the shoot wires. The diameter of the wire should be 0.0165 in. and the average diameter shall not be outside the limits of 0.0160 and 0.0170 in.

The No. 30 sieve shall have between 29.5 and 30.5 wires per whole inch of the warp wires and between 28.5 and 31.5 wires per whole inch of the shoot wires. The diameter of the wire should be 0.0110 in and the average diameter shall not be outside the limits 0.0105 to 0.0115 in.

24. Immediately after mixing, the standard mortar shall be placed in the molds, pressed in firmly with the thumbs and smoothed off with a trowel without ramming. Additional mortar shall be heaped above the mold and smoothed off with a trowel; the trowel shall be drawn over the mold in such a manner as to exert a moderate pressure on the material. The mold shall then be turned over and the operation of heaping thumbing and smoothing off repeated.

25. Tests shall be made with any standard machine. The briquets shall be tested as soon as they are removed from the water. The bearing surfaces of the clips and briquets shall be free from grains of sand or dirt. The briquets shall be carefully centered and the load applied continuously at the rate of 600 lb. per minute.

26. Testing machines should be frequently calibrated in order to determine their accuracy.

27. Briquets that are manifestly faulty, or that give strengths differing more than 15% from the average value of all test pieces made from the same sample and broken at the same period, shall not be considered in determining the tensile strength.

Storage of Test Pieces. 28. The moist closet may consist of a soapstone, slate or concrete box, or a wooden box lined with metal. If a wooden box is used, the interior should be covered with felt or broad wicking kept wet. The bottom of the moist closet should be covered with water. The interior of the closet should be provided with nonabsorbent shelves on which to place the test pieces, the shelves being so arranged that they may be withdrawn readily.

29. Unless otherwise specified all test pieces, immediately after molding, shall be placed in the moist closet for from 20 to 24 hours.

30. The briquets shall be kept in molds on glass plates in the moist closet for at least 20 hours. After 24 hours in moist air the briquets shall be immersed in clean water in storage tanks of non-corroding material.

31. The air and water shall be maintained as nearly as practicable at a temperature of 21° C. (70° F.).

Appendix 3. Specifications for Cement

Standard Specifications for Portland Cement. American Society for Testing Materials, Standards, 1926.

Definition. Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion, an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum.

Chemical Properties. The following limits shall not be exceeded:

Loss on ignition, per cent	4.00
Insoluble residue, per cent	0.85
Sulfuric anhydride (SO^3), per cent	2.00
Magnesia (MgO), per cent	5.00

Physical Properties

(1) **Fineness.** The residue on a standard No. 200 sieve shall not exceed 22% by weight.

(2) **Soundness.** A pat of neat cement shall remain firm and hard, and show no signs of distortion, cracking, checking, or disintegration in the steam test for soundness.

(3) **Time of Setting.** The cement shall not develop initial set in less than 45 minutes when the Vicat needle is used or 60 minutes when the Gillmore needle is used. Final set shall be attained within 10 hours.

(4) **Tensile Strength.** The average tensile strength in pounds per square inch of not less than three standard mortar briquets composed of one part cement and three parts standard sand by weight, shall be equal to or higher than the following:

Age at test, days	Storage of briquets	Tensile strength, lb. per sq. in.
7	1 day in moist air, 6 days in water	225
28	1 day in moist air, 27 days in water	325

7. The average tensile strength of standard mortar at 28 days shall be higher than the strength at 7 days.

Packages, Marking and Storage. 8. The cement shall be delivered in suitable bags or barrels with the brand and name of the manufacturer plainly marked thereon, unless shipped in bulk. A bag shall contain 94 lb. net. A barrel shall contain 376 lb. net.

9. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

Inspection. 10. Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 12 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 33 days shall be allowed for the completion of the 28-day test. The cement shall be tested in accordance with the methods hereinbefore prescribed. The 28-day test shall be waived only when specifically so ordered.

Rejection. 11. The cement may be rejected if it fails to meet any of the requirements of these specifications.

12. Cement remaining in storage prior to shipment for a period greater than 6 months after test shall be retested and shall be rejected if it fails to meet any of the requirements of these specifications.

13. Cement shall not be rejected on account of failure to meet the fineness requirement if upon retest after drying at 100° C. for 1 hour it meets this requirement.

14. Cement failing to meet the test for soundness in steam may be accepted if it passes a retest using a new sample at any time within 28 days thereafter.

15. Packages varying more than 5% from the specified weight may be rejected; and if the average weight of packages in any shipment, as shown by weighing 50 packages taken at random, is less than that specified, the entire shipment may be rejected.

Standard Specifications for Natural Cement. Adopted by the American Society for Testing Materials, 1904; revised 1908, 1909:

Definition. 1. Natural cement is the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas.

Physical Properties. 2. The residue on a standard No. 100 sieve shall not exceed 10%, and on a standard No. 200 sieve shall not exceed 30%, by weight.

3. Pats of neat cement about 3 in. in diameter, 1/2 in. thick at center, tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(a) A pat shall then be kept in air at normal temperature.

(b) Another pat shall be kept in water maintained as near 70° F. as practicable.

These pats shall be observed at intervals for at least 28 days, and, to satisfactorily pass the tests, shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

4. The cement shall not develop initial set in less than 10 minutes, using the Vicat needle. Final set shall be attained in not less than 30 minutes nor more than 3 hours, using the Vicat needle.

5. The minimum requirements for tensile strength for briquets 1 sq. in. in cross-section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

Age	Neat cement	Strength
24 hours in moist air		75 lb.
7 days (1 day in moist air, 6 days in water)		150 lb.
28 days (1 day in moist air, 27 days in water)		250 lb.
	One part cement, three parts standard Ottawa Sand	
7 days (1 day in moist air, 6 days in water)		50 lb.
28 days (1 day in moist air, 27 days in water)		125 lb.

Packages, Marking and Storage. 6. The cement shall be delivered in suitable bags or barrels with the brand and name of the manufacturer plainly marked thereon. A bag shall contain 94 lb. net. A barrel shall contain 282 lb. net.

7. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

Inspection. 8. (a) Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 10 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 31 days shall be allowed for the completion of the 28-day test.

(b) The cement shall be tested in accordance with the methods contained in the Standard Specifications and Tests for Portland Cement of the American Society for Testing Materials.

Rejection. 9. The cement may be rejected if it fails to meet any of the requirements of these specifications.

10. Cement failing to meet the 7-day requirements may be held awaiting the results of the 28-day tests before rejection.

Note. For specification for High-Early-Strength Cement see Appendix 7.

American Society for Testing Materials.

Appendix 4. Tentative Specifications for Concrete Aggregates

Serial Designation: C 33 — 28 T

This is a Tentative Standard, published for the purpose of eliciting criticism and suggestions, and as such is subject to annual revision.

Issued, 1921; Revised, 1923, 1926, 1928

Quality of Aggregates

It is recognized that for certain purposes satisfactory results may be obtained with materials not conforming to these specifications. In such cases the use of fine and coarse aggregate not conforming to these specifications should be authorized only under special provisions based upon laboratory studies of the possibility of designing a mixture of materials to be used on the job that will yield concrete equivalent to the specified mixture made with material complying with these specifications in all respects.

Fine Aggregate

General Characteristics. 1. Fine aggregate shall consist of sand or other approved inert materials with similar characteristics, or a combination thereof, having hard, strong, durable particles and shall conform to the requirements of these specifications.

Deleterious Substances. 2. (a) The maximum percentages of deleterious substances shall not exceed the following values:

	Per cent by weight
Removed by decantation	3
Shale	1
Coal	1
Clay lumps	1
Other local deleterious substances (such as alkali, mica, coated grains, sift and flaky particles)	—

Note. It is recognized that under certain conditions maximum percentages of deleterious substances less than those shown in the table should be specified.

(b) The sum of the percentages of shale, coal, clay lumps, soft fragments and other deleterious substances shall not exceed 5 per cent by weight.

(c) All fine aggregate shall be free from injurious amounts of organic impurities. Aggregates subjected to the colorimetric test for organic impurities and producing a color darker than the standard shall be rejected unless they pass the mortar strength test as specified in Sect. 4.

Grading. 3. (a) Fine aggregate shall be well graded from coarse to fine and when tested by means of laboratory sieves * shall conform to the following requirements:

* For detail requirements for these sieves, see the Standard Specifications for Sieves for Testing Purposes (Serial Designation: E 11) of the American Society for Testing Materials, 1927 Book of A.S.T.M. Standards, Part II, p. 917.

	Per cent
Passing a 3/8-in. sieve	100
Passing a No. 4 sieve	(85) to (100)
Passing a No. 16 sieve	(45) to (80)
Passing a No. 50 sieve	(2) to (30)
Passing a No. 100 sieve	(0) to (5)

Note. Figures in parentheses are suggested as limiting percentages but they may be altered within these limits to suit local conditions.

(b) In case the concrete resulting from a mixture of aggregates approaching the extreme limits for gradation is not of a workable character, or when finished does not exhibit a proper surface, due to an excess of particles approximately 1/8 to 1/2 in. in size, either a fine aggregate having a sufficiently greater percentage of fine material, or a coarse aggregate having a sufficiently smaller percentage of fine material must be used.

Mortar Strength. 4. Fine aggregates, when subjected to the mortar strength test, shall have a tensile or compressive strength at the age of 7 and 28 days of not less than (100)%* of that developed by mortar of the same proportions and consistency made of the same cement and standard Ottawa sand.

Coarse Aggregate

General Characteristics. 5. Coarse aggregate shall consist of crushed stone, gravel, blast-furnace slag, or other approved inert materials of similar characteristics, or combinations thereof, having hard, strong, durable pieces, free from adherent coatings and conforming to the requirements of these specifications.

Deleterious Substances. 6. (a) The maximum percentages of deleterious substances shall not exceed the following values:

	Per cent by weight
Removed by decantation	1
Shale	1
Coal	1
Clay lumps	1/2
Soft fragments	5
Other local deleterious substances (such as alkali, friable, thin, elongated, or laminated pieces)	..

Note. It is recognized that under certain conditions maximum percentages of deleterious substances less than those shown in the table should be specified.

(b) The sum of the percentages of shale, coal, clay lumps and soft fragments shall not exceed 5% by weight.

Grading. 7. (a) Coarse aggregate shall be well graded, between the limits specified, and shall conform to the following requirements:

		Per cent by weight
Passing ...... in. sieve † or screen	(maximum size)	not less than (95)
Passing ...... in. sieve † or screen	(intermediate size)	not less than (40)
	(one-half maximum size)	not more than (75)
Passing ...... in. sieve † or screen	(intermediate sizes)	not less than .....
	(as needed)	not more than ....
Passing No. 4 sieve or a 1/4-in. screen		not more than (10)

Note. Where a range is shown the engineer should use an appropriate figure within the limits recommended. The percentages in parentheses are recommended but may need to be altered to suit local conditions.

* Percentages in parentheses are recommended but they may be altered to suit local conditions.

† The question whether round or square openings shall be used for testing aggregates is now being studied by the Section on Coarse Screens of the Technical Committee on Size and Shape of Committee E-1 on Methods of Testing.

(b) In case the concrete resulting from a mixture of aggregates approaching the extreme limits for gradation is not of a workable character, or when finished does not exhibit a proper surface, due to an excess of particles approximately 1/8 to 1/2 in. in size, either a fine aggregate having a sufficiently greater percentage of fine material, or a coarse aggregate having a sufficiently smaller percentage of fine material shall be used.

Weight of Slag. 8. Blast-furnace slag that meets the grading requirements of these specifications shall conform to the following minimum weight requirements:

Base concrete.......................... 65 lb. per cu. ft.
Surface concrete (subject to abrasion)........ 70 lb. per cu. ft.

Durability. 9. Coarse aggregate shall pass a sodium sulfate accelerated soundness test, except that aggregates failing in the accelerated soundness test may be used if they pass a satisfactory freezing and thawing test.

Appendix 5. Reinforcing Steel

Excerpts from Specifications for Billet Steel Concrete Reinforcement Bars and Rail Steel Concrete Reinforcement Bars, American Society for Testing Materials Standards.

Billet Steel Bars

The material for billet steel bars may be made by the Bessemer or open-hearth process. The bars shall be rolled from new billets; no re-rolled material will be accepted.

Chemical composition: Phosphorus..... { Bessemer; not over 0.10%
Open-hearth; not over 0.05%

An analysis of each melt of steel shall be made by the manufacturer to determine the percentages of carbon, manganese, phosphorus and sulphur. The analysis shall be made from a test ingot taken during the pouring of the melt. The chemical composition thus determined shall be reported to the purchaser or his representative.

Analysis may be made by the purchaser from finished bars representing each melt of open-hearth steel and each melt or lot of tons of Bessemer steel.

One tension and one bend test shall be made from each melt of open-hearth steel and from each melt or each lot of 10 tons of Bessemer steel except that if material from one melt differs 3/8 inch or more in thickness or diameter, one tension and one bend test shall be made from both the thinnest and thickest material rolled.

Rail Steel Bars

Bars shall be rolled from standard section Tee-rails.

One tension and one bend test shall be made from each lot of 10 tons or less of each size of bar, rolled from rails varying not more than 10 lb. per yd. in nominal weight.

Billet Steel and Rail Steel Bars

Tension and bend test specimens for plain and deformed bars shall be taken from the finished bars, and shall be of the full thickness or diameter of bars as rolled; except that the specimens for deformed bars may be machined for a length of at least 9 in. if deemed necessary by the manufacturer to obtain uniform cross-section. Tension and bend test specimens for twisted bars shall be taken from the finished bars without further treatment.

Twisted bars shall have one complete twist in a length not over 12 times the thickness of the bar.

Properties considered	Tensile properties billet steel bars							Rail steel bars	
	Plain bars			Deformed bars			Cold twisted bars	Plain bars	Deformed and not twisted bars
	Structural steel grade	Intermediate grade	Hard grade	Structural steel grade	Intermediate grade	Hard grade			
Tensile strength, lb. per sq. in.	55 000 to 70 000	70 000 to 85 000	80 000 min.	55 000 to 70 000	70 000 to 85 000	80 000 min.	Recorded only	80 000	80 000
Yield point, min., lb. per sq. in.	33 000	40 000	50 000	33 000	40 000	50 000	55 000	50 000	50 000
Elongation in 8 in., min., per cent *	1 400 000 / Ten. str.	1 300 000 / Ten. str.	1 200 000 / Ten. str.	1 250 000 / Ten. str.	1 125 000 / Ten. str.	1 000 000 / Ten. str.	5	1 200 000 / Ten. str.	1 000 000 / Ten. str.

* See modification for thickness below.

The yield point shall be determined by the drop of the beam of the testing machine.

For plain and deformed bars over 3/4 in. in thickness or diameter, a deduction of 1/4% from the percentages of elongation specified shall be made for each increase of 1/32 in. in thickness or diameter above 3/4 in. For plain and deformed bars under 7/16 in. in thickness or diameter, a deduction of 1/2% from the percentages of elongation specified shall be made for each decrease of 1/32 in. in thickness or diameter below 7/16 in.

Bend-test Requirements. The test specimen shall bend cold around a pin without cracking on the outside of the bent portion, as follows:

Thickness or diameter of bar	Billet steel bars							Rail steel bars	
	Plain bars			Deformed bars			Cold twisted bars	Plain bars	Deformed and not twisted bars
	Structural steel grade	Intermediate grade	Hard grade	Structural steel grade	Intermediate grade	Hard grade			
Under 3/4 in.	180° $d = t$	180° $d = 2t$	180° $d = 3t$	180° $d = t$	180° $d = 3t$	180° $d = 4t$	180° $d = 2t$	180° $d = 3t$	180° $d = 4t$
3/4 in or over	180° $d = t$	90° $d = 2t$	90° $d = 3t$	180° $d = 2t$	90° $d = 3t$	90° $d = 4t$	180° $d = 3t$	90° $d = 3t$	90° $d = 4t$

Explanatory Note.— d = the diameter of pin about which the specimen is bent; t = the thickness or diameter of the specimen.

Appendix 6. Compression Tests for Concrete

Making and Storing Test Cylinders. The quality of the concrete in any portion of a structure should be based on the average of tests of not less than three cylinders, and better five. Test cylinders should be made and stored in accordance with the " Standard Methods of Making and Storing Specimens of Concrete in the Field " of the American Society Testing Materials, Serial Designation: C 31-21.

The concrete for testing must be taken from the forms immediately after placing, whenever possible. Shovel the concrete from the forms into a bucket and mix with a trowel before placing in the molds. If the concrete is being placed with buggies, the sampling should preferably be done by means of a " buggy sampler," designed especially for the purpose. This is a trough-shaped receptacle made of metal and provided with hooks at the ends which fit over the front and rear edges of the buggy. The top of the " trough " is partially closed by means of two inclined baffles converging upward and inward from the sides. Thus a long, narrow opening is formed which allows only a portion of the concrete entering the buggy to pass into the sampler. When the buggy is in position under the hopper or spout, the opening into the sampler takes a representative portion of the batch as it enters the buggy. This eliminates the inaccuracy of sampling the concrete with a scoop after the buggy has been filled, partial segregation of aggregates and cement making such procedure unreliable.

Breaking Test Cylinders. When the cylinders arrive at the laboratory they should be unpacked at once and stored in damp sand or in a moist room until broken. If the cylinders have not been capped on the job they should be capped upon arrival so that both ends shall be smooth, parallel and at right angles to the axis of the cylinder.

Compression tests should be made soon after removal of the specimens from the curing room, that is, while still in a damp condition.

The cylinders should be carefully placed in the testing machine so as to give an even bearing surface. The metal bearing plates should be placed in contact with the ends of the specimen.

The load should be applied through a spherical bearing block placed on top of the test piece. The diameter of the bearing block should be approximately the same as that of the test cylinder. The upper section of the bearing block should be kept in motion as the head of the testing machine is brought to a bearing on the test piece. The load should be applied smoothly and evenly, without shock. The moving head of the machine should travel at the rate of about 0.05 in. per minute.

Results of Compression Tests. In general, only the ultimate compressive strengths need be observed. The load indicated by the testing machine, at failure of the test piece, should be recorded and the unit compressive strength calculated in pounds per square inch, the cross-sectional area having been computed from the average diameters of the specimen. The type of failure and appearance of the concrete should also be noted.

Report Forms. A record of the investigation and design of the concrete mixtures together with the history and results of the concrete tests should be kept. Such report forms may be made to suit the particular requirements at hand.

From " Design and Control of Concrete Mixtures " Portland Cement Association.

Standard Method of Making and Storing Specimens of Concrete in the Field

Serial Designation: C 31-27. A. S. T. M.

1. The methods herein specified apply to molding and storing of test specimens of concrete sampled from concrete being used in construction.

2. The test specimens shall be cylindrical in form with the length twice the diameter. The standard shall be 6 by 12-in. cylinders where the coarse aggregate does not exceed 2 in. in size; for aggregates larger than 2 in. 8 × 16 in. cylinders shall be used; 2 by 4 in. cylinders may be used for mixtures without coarse aggregates.

3. (a) The molds shall be cylindrical in form, made of non-absorbent material, and shall be substantial enough to hold their form during the molding of the test specimens. They shall not vary in diameter more than 1/16 in. in any direction, nor shall they vary in height more than 1/16 in. from the height required. They shall be substantially water tight so that there will be no leakage of water from the test specimen during molding.

(b) Each mold shall be provided with a base plate having a plane surface and made of non-absorbent material. This plate shall be large enough in diameter to properly support the form without leakage. Plate glass or planed metal are satisfactory for this purpose. A similar plate should be provided for covering the top surface of the test specimen after being molded.

(c) Suggestions for suitable forms are shown in the accompanying figures.

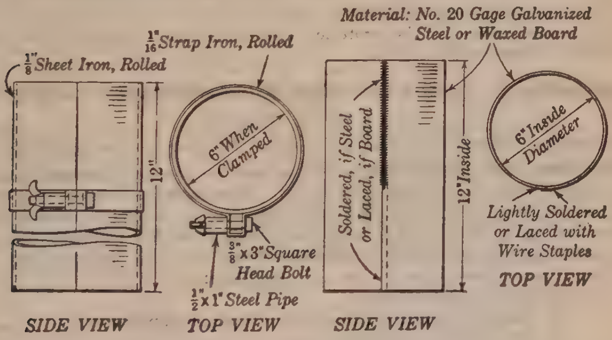


Fig. 127

4. (a) Concrete for the test specimens shall be taken immediately after it has been placed in the work. All the concrete for each sample shall be taken from one place. A sufficient number of samples — each large enough to make one test specimen — shall be taken at different points so that the test specimens made from them will give a fair average of the concrete placed in that portion of the structure selected for tests. The location from which each sample is taken shall be noted clearly for future reference.

(b) In securing sample, the concrete shall be taken from the mass by a shovel or similar implement and placed in a large pail or other receptacle, for transporting to the point of molding. Care shall be taken to see that each test specimen represents the total mixture of the concrete in that place. Different samples shall not be mixed together but each sample shall make one specimen.

5. (a) The pails or other receptacles containing the samples of concrete shall be taken as quickly as possible to the place selected for molding test specimens. To offset segregation of the concrete occurring during transportation, each sample shall be dumped into a non-absorbent water-tight receptacle and, after slight stirring, immediately placed in the mold.

(b) The test specimens shall be molded by placing the concrete in the form in layers approximately 4 in. in thickness. Each layer shall be puddled with 25 strokes with a 5/8 bar about 2 ft. long, tapered slightly at the lower end. After puddling the top layer, the surface concrete shall be struck off with a trowel and covered with the top cover plate which will later be used in capping the test specimens.

6. Two to four hours after molding, the test specimens shall be capped with a thin layer of stiff neat cement paste in order that the cylinder may present a smooth end for testing. The cap shall be formed by means of a piece of plate glass 1/4 in. thick or a machined metal plate 1/2 in. thick and of a diameter 2 or 3 in. larger than that of the mold. This plate is worked on the fresh cement paste until it rests on top of the form. The cement for capping should be mixed to a stiff paste some time before it is to be used in order to avoid the tendency of the cap to shrink. Adhesion of the concrete to the top and bottom plates can be avoided by oiling the plates or by inserting a sheet of paraffined tissue paper.

7. At the end of 48 hours the test specimens shall be removed from the molds and buried in damp sand except in case the molds shown in Fig. 3 are used; in this case test specimens may be buried in damp sand without removal of the mold, thus permitting shipping of the test specimens in the molds.

8. (a) The test specimens shall remain buried in damp sand until 10 days prior to the date of test. They shall then be well packed in damp sand or wet shavings and shipped to the testing laboratory, where they shall be stored either in a moist room or in damp sand until the date of test.

(b) Should a 7-day test be required, the test specimens shall remain at the work as long as possible to harden and then shall be shipped so as to arrive at the laboratory in time to make the test on the required date.

(c) Test specimens shall be protected from drying after removal from damp storage and before testing.

Appendix 7. Notes on High-Early-Strength Cements

By Duff A. Abrams

Director of Research, International Cement Corporation, New York City

High-Early-Strength Cements are being used for many varieties of concrete work where time is an important factor. The principal features of these cements are that concrete made from them may be placed in service after curing for 1 to 3 days. They have gained widespread recognition for construction of pavements, bridges, buildings, floors, concrete products, etc.

Three types of these cements are now available:

(1) True portland cement.
(2) Portland cement with certain chemical admixtures.
(3) High-alumina cements.

The specification for these cements has not been fully standardized. The Committee on Cement recently recommended to the American Society for Testing Materials a specification for High-Early-Strength Portland Cement which differs from the present " Standard Specification and Tests for Portland Cement " only in that the following requirements are substituted for tensile strength and SO_3 content:

Tensile Strength 1–3 Standard Briquets
24 hr. — 275 lb. per sq. in.
3 days — 375 1–3 Standard Briquets
SO_3 not more than 2.50%

One American manufacturer produces a high-early-strength portland cement by a double-burning process. Cements of this type are ground to such a fineness that 90 to 99% will pass the standard 200-mesh sieve.

For the use of high-early-strength cement in connection with concrete pavements see Sect. 20, Art. 17, p. 1158.

SECTION 12

STEEL STRUCTURES

ASSOCIATE EDITOR

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ROOFS AND BUILDINGS †

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† Written by Robins Fleming, American Bridge Company.

ROOFS AND BUILDINGS

1. Data for Roof Trusses

The constituent parts of a roof are the **covering** (including covering proper and any sheathing or slab that is used), the **purlins** and the **trusses**.

Roof Coverings are of many kinds, the most familiar being wood, slate, tin, sheet metal, tile, concrete, gypsum and the various composition roofings. It is highly important that the choice of covering be adapted to the slope of the roof or vice versa. The lack of this adaptation is a frequent cause of leaky roofs.

The **Slope** of a roof is the natural tangent of the inclination with the horizontal and is best expressed in inches of rise to one horizontal foot. The preferable slope for tar and gravel on sheathing is from 3/4 to 1-1/4 in. per ft.; reinforced cement tile of the "Bonanza" and similar types, 5 in. per ft.; corrugated sheet metal, 6 in. per ft.; slate, 7 in. per ft. The kind of waterproofing used on concrete and gypsum roofs will determine the slope.

Purlins are either wooden or steel beams extending horizontally from truss to truss and carrying the roof loads to the trusses. For long spans it is sometimes economical to space trusses far apart and to use deep purlins supporting intermediate beams called **jack rafters**.

Trusses are used to support roofs covering openings of from 20 to 200 ft., while for spans exceeding 200 ft. arches are generally used. A **truss** is an assemblage of bars, or members, which forms a structure to carry transverse loads and which under vertical loads has vertical reactions, the bars being so joined together at their ends that they bear only direct tension or compression when the external loads are applied at the joints. A **riveted truss** (Fig. 6) is one in which the members are connected together at the joints by being riveted to connecting steel plates, and a pin truss or **pin-connected truss** has the members joined together at their ends by means of one steel cylinder or pin at each joint. Purlins usually rest on the top surface of the upper chord, which is the upper bar of a truss, and should be placed directly over the joints whenever possible. In some roofs without jack rafters the purlins must be spaced so close together that some of them rest on the upper chord between the joints, thus causing the upper chord to act as a beam in addition to its direct stress.

A Panel of a roof truss is the space between two consecutive joints of the top chord, and a panel length is the distance from one of these joints to the next. A **bay** is the space between two adjacent trusses, and the bay length is the distance from center to center of trusses.

Forms of Trusses vary to suit the span length, outline of roof and clear head-room required underneath. The Fink and Warren trusses shown in Figs. 1 and 2 are suitable for spans up to about 120 ft. When the short web member of a Fink truss is replaced by two members, the Fink truss is converted into a fan truss as shown at (*a*) and (*b*) Fig. 2.

Loads on Roof Trusses are: dead, snow, wind; also ceilings, a crowd of people on a gallery or floor, or machinery suspended from the truss. Dead loads consist of weight of roof covering, sheathing rafters, purlins, trusses and suspended ceilings.

Weights of roofing per square foot of roof surface.

1. Coverings: wood shingles, 2 to 3 lb.; tin sheets or shingles, 1 lb.; corrugated steel, 2 to 3 lb ; slates, 7 to 10 lb.; tiles, 8 to 25 lb.; felt and gravel, 8 to 10 lb.; skylight glass 3/16 to 1/2 in., including iron frames, 4 to 10 lb.
2. Sheathing: spruce, white pine or hemlock boards, 1 in. thick, 3 lb.; Southern pine, 1 in. thick, 4 lb.
3. Rafters: wood, 2 by 4 to 2 by 8 in. spaced 16 to 24 in. on centers, 1.5 to 5 lb.
4. Purlins: wood, 1 to 3 lb.; steel, 1-1/2 to 5 lb.

5. Ceilings: plastered, 10 lb. per square foot of ceiling area.

6. Cinder concrete, 9 lb. per in. of thickness; stone concrete 12 lb. per in. of thickness.

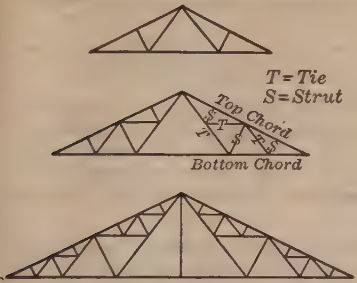


Fig. 1. Fink Roof Trusses

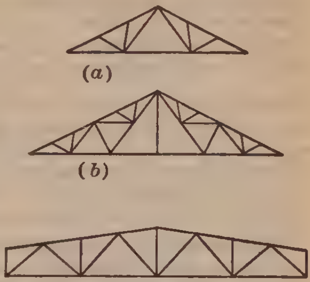


Fig. 2. Fan and Warren Trusses

Weights of Fink Trusses In Pounds

Slope of Top Chord, 6 in. on 1 ft.

Span c. to c. of Bearings, ft.	Load in lb. per linear ft. of Top Chord, Uniformly Distributed					
	500	600	700	800	900	10000
25	610	620	650	730	740	820
30	760	860	880	960	1030	1140
35	1050	1100	1250	1440	1500	1650
40	1250	1400	1620	1670	1830	1920
45	1500	1740	1920	1970	2220	2320
50	1840	2100	2170	2400	2650	3000
55	2100	2340	2420	2850	3050	3340
60	2360	2620	2920	3200	3600	3800
65	2700	3000	3500	3650	4140	4200
70	3100	3450	3930	4260	4540	4900
75	3460	4100	4560	4800	5200	5560
80	3920	4430	4800	5500	5960	6350

Weights of Warren Trusses in Pounds

Slope of Top Chord, 3/4 in. on 1 ft. for 25 to 35-ft. spans; 1 in. on 1 ft. for 40 to 55-ft. spans; 1-1/4 in. on 1 ft. for 60 to 80-ft. spans

Span c. to c. of Bearings, ft.	Load in lb. per linear ft. of Top Chord, Uniformly Distributed					
	500	600	700	800	900	1000
25	680	700	730	840	900	980
30	840	1000	1020	1150	1200	1350
35	1100	1270	1360	1530	1640	1870
40	1230	1420	1620	1760	1940	2100
45	1600	1800	2080	2180	2420	2640
50	1950	2230	2400	2660	2920	3280
55	2230	2680	2730	3180	3600	3700
60	2380	2720	3270	3360	3840	3900
65	2920	3300	3620	4170	4220	4770
70	3230	3920	4030	4620	4980	5350
75	3770	4360	5000	5280	5730	5800
80	4200	4940	5320	6060	6120	6850

Weights of Steel Trusses are often determined from empirical formulas, but such formulas are not trustworthy as they neglect many of the variable factors entering into the weights. The preceding tables are for the two most common forms of trusses:

The weights of ordinary Fink Trusses with slopes 5 in. and 4 in. on 1 ft. are from 5 to 15% more than those with 6 in. on 1 ft. given in the table.

Snow Loads on roofs vary with the latitude and slope of roof. The following values for weight of snow per horizontal square foot are commonly specified: New England and Michigan, 30 lb.; New York City and Chicago, 20 lb.; Baltimore, Cincinnati and St. Louis, 10 lb. Snow loads need not be used on roof surfaces having an inclination of 45 degrees or more from the horizontal, if there are no snow guards.

Wind Loads for roofs vary with the locality and slope of the roof and should be taken as acting horizontally at 30 lb. per sq. ft. on vertical surfaces of the most exposed structures and 15 to 25 lb. per sq. ft. on vertical surfaces of less exposed structures. On inclined surfaces only the normal component of the wind pressure need be considered, and this varies with the inclination of the roof.

Duchemin formula for wind pressure on inclined surfaces:

$$N = P\frac{2 \sin \theta}{1 + \sin^2 \theta}$$

in which N = normal pressure in pounds per square foot;
P = horizontal pressure on a vertical plane (taken at 20 lb. per sq. ft.);
θ = degrees of inclination to the horizontal.

θ	N	Slope	θ	N
5°	3.46	1 in. on 1 ft.	4° 45′ 49″	3.30
10°	6.76	2 in. on 1 ft.	9° 27′ 45″	6.39
15°	9.63	3 in. on 1 ft.	14° 2′ 10″	9.14
20°	12.25	4 in. on 1 ft.	18° 26′ 6″	11.50
25°	14.35	5 in. on 1 ft.	22° 37′ 12″	13.42
30°	16.00	6 in. on 1 ft.	26° 33′ 54″	14.88
35°	17.28	7 in. on 1 ft.	30° 15′ 24″	16.06
40°	18.20	8 in. on 1 ft.	33° 41′ 25″	16.95
45°	18.88			
to 90°	20.00			

For a pressure other than 20 lb. per sq. ft. the above values are to be changed proportionally.

The Total Load for an ordinary roof truss may be either

Dead + Wind + 1/2 Snow

or

Dead + 1/2 Wind + Snow

It is improbable that both the maximum snow and wind loads will be upon the roof at the same time. Sufficient accuracy is obtained in designing roofs up to 100-ft. span by assuming a total load per square foot of exposed surface large enough to include the dead, wind and snow loads. This combined load may be considered applied vertically. Unless governed by a local building code, for spans up to and including 90 ft. and in climates corresponding to that of New York, the total minimum uniform load in lb. per sq. ft. of roof surface for different kinds of covering may be taken as follows:

Corrugated metal on steel purlins	40
Gravel or composition on wood sheathing	50
Slate on boards	50
Tile on steel purlins	55
Gravel or composition on cinder concrete	65
on stone concrete	75
Slate or tile on cinder concrete	70
Fireproof buildings of the mill-building class when slope is less than 2 in. per ft.	90

For roof spans over 90 ft. the above cited loads should be increased 1% for every 2 ft. increase of span. For roofs in climates where snow is excessive 5 to 10 lb. should be added, and in climates where there is not liable to be snow 10 lb. may be deducted. Additions should be made for any special loads such as shafting, ceilings or trolleys carried by the bottom chord, and tanks, elevator supports or similar loads above the roof.

Stresses may be found by analytic or by graphic methods, the latter being usually employed. The analytic method can be profitably used as a check when advisable. For roof trusses the simplest analytic solution is by the method of joints or method of moments. In applying the **method of joints** a section is passed around a joint in such a manner as to cut all the bars meeting at that joint and then applying the relation from statics that the algebraic sum of all the vertical components meeting at the joint must = 0, or that the algebraic sum of the horizontal components meeting at the joint must = 0. Thus, by passing a section around joint A, Fig. 3, it is seen at once that the vertical component of P_1 = 3-1/2 W and therefore the stress in P_1 = 3-1/2 $W \sec\theta$, where θ is the angle the bar P_1 makes with the vertical. Similarly, the horizontal component of P_8, that is, stress in P_8 = horizontal component of P_1 = 3-1/2 $W \tan\theta$. The **method of moments** is that method of computing stresses in which a section is first passed through the truss, cutting in general not more than three members, one of which shall contain the stress desired; and then the equation of moments about the intersection of the other two bars cut is written and solved for the desired stress. Thus, in Fig. 3, to compute stress P_{10} by method of moments, pass a vertical section through P_4, P_{14}, P_{10} and consider the left side of this section. The equation of moments about B, the origin of moments for P_{10}, is $(3\text{-}1/2\ \mathbf{W})\boldsymbol{l}/2 - \mathbf{W}6\boldsymbol{l}/8 - P_{10}\mathbf{h} = \mathbf{0}$. In this equation P_{10} is assumed to be in tension, and since in the solution of the equation both sides are positive the bar P_{10} is in tension and its value $= Wl/h$. The section passed may be straight or curved, vertical or inclined. In Fig. 3 the dotted section for P_{12} is passed in such a way as to cut 6 bars, but by taking the origin of moments at B and writing the equation of moments of all forces above the section there results $-\mathbf{W}\boldsymbol{l}/8 + \mathbf{P}_{12}\,\mathbf{h}/2 = \mathbf{0}$; whence $\mathbf{P}_{12} = \mathbf{W}\boldsymbol{l}/4\,\mathbf{h}$.

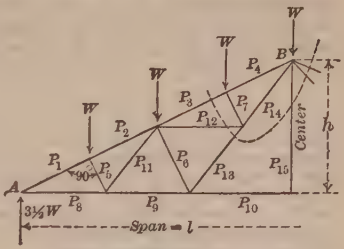


Fig. 3

By using the method of moments or method of joints the stresses in the Fink truss of Fig. 3 have been computed and the accompanying table prepared for a span of l, a height of h and a vertical load W at each upper joint. The load W is called a panel or apex load.

Coefficients for Stresses in Fink Truss. Fig. 3

Bar	Kind of Stress	$\frac{l}{h} = 3$	$\frac{l}{h} = 3.46$ 30°	$\frac{l}{h} = 4$	$\frac{l}{h} = 5$	$\frac{l}{h} = 6$
P_1	Comp.	6.31	7.00	7.83	9.42	11.07
P_2	"	5.75	6.50	7.38	9.05	10.75
P_3	"	5.20	6.00	6.93	8.68	10.43
P_4	"	4.65	5.50	6.48	8.31	10.12
P_5	"	0.83	0.87	0.89	0.93	0.95
P_6	"	1.66	1.73	1.79	1.86	1.90
P_7	"	0.83	0.87	0.89	0.93	0.95
P_8	Tension	5.25	6.06	7.00	8.75	10.50
P_9	"	4.50	5.19	6.00	7.50	9.00
P_{10}	"	3.00	3.46	4.00	5.00	6.00
P_{11}	"	0.75	0.87	1.00	1.25	1.50
P_{12}	"	0.75	0.87	1.00	1.25	1.50
P_{13}	"	1.50	1.73	2.00	2.50	3.00
P_{14}	"	2.25	2.60	3.00	3.75	4.50
P_{15}	"	0.00	0.00	0.00	0.00	0.00

The upper chord panel loads are equal. To obtain stress in pounds in any bar of the truss, multiply the coefficient by the panel load in pounds.

2. Design of Roof Trusses

Working Stresses. For designing the structural steel of roof trusses and buildings (when not governed by a local building code) the following unit or working stresses in lb. per sq. in. are recommended:

	(a) Acceptable Steel	(b) Standard Steel
Tension, net section, rolled steel	16 000	18 000
Axial compression, rolled steel and steel castings, short lengths	16 000	18 000
Bending: on extreme fibers of rolled shapes, built sections, girders, and steel castings	16 000	18 000
Bending, on extreme fibers of pins	24 000	27 000
Shear on power-driven rivets and pins	12 000	13 500
Shear on bolts and hand-driven rivets	10 000	11 200
Shear, average, on webs of plate girders and rolled beams, gross section	10 000	11 200
Bearing pressure on power-driven rivets and pins	24 000	27 000
Bearing on bolts and hand-driven rivets	20 000	22 500
Axial compression on gross section of columns and struts	16 000–70 l/r	18 000–80 l/r
With a maximum of	13 000	14 500

Where l = effective length of member in inches;
r = least radius of gyration of section in inches.

By " acceptable steel " is meant steel which is acceptable to the building official, but of which the origin and physical characteristics are not definitely determined. By " standard steel " it is meant that the steel to be used is guaranteed to conform to the American Society of Testing Materials Standard Specification for Structural Steel for Buildings (serial designation A9–24).

For combined stresses due to wind and other loads the above-mentioned unit stresses in column (a) may be increased 50%, provided the section thus obtained is not less than that required if wind forces are neglected. In column (b) they may be increased 33-1/3%.

The effective length of main compression members should not exceed 125 times their least radius of gyration, and those for wind and lateral bracing 200 times their least radius of gyration.

Any portion of the cross-section of a compression member may be neglected in computing the radius of gyration, provided this portion is neglected in the design of the member. Eccentricity, shock, rust, the bending of lateral struts due to their own weight, should all be considered.

Spacing of Trusses. Theoretically, the economic spacing of roof trusses is about one-quarter the span, but in practice the spacing may be governed by conditions peculiar to the structure under consideration. If the covering or sheathing rests on purlins a spacing of about 16 ft. is commonly used for spans up to 65 ft.; beyond that the spacing may be one-quarter of the span. Where plank sheathing rests directly on the trusses the spacing may be 6 to 8 ft. for 2-in. and 8 to 10 ft. for 3-in. tongued and grooved plank. If spans are of unusual length or roof loads are very heavy it may be found economical to use jack rafters resting on latticed or beam girders framed into the roof trusses. Sub-purlins can rest on top of the jack rafters or wood sheathing can extend from rafter to rafter.

Proportioning Purlins. Steel purlins are usually made of ∟'s, [’s or I beams, although for long spans they are made of truss form. Where the purlin is placed so as to deflect in the plane of the resultant load acting upon it, as when an I beam is set with its web vertical to carry a vertical load, the size of the purlin is computed by the formula, $\mathbf{M/S = I/c}$, where M is the maximum bending moment on the purlin in inch-pounds; S, the allowable unit stress in pounds per square inch; I, the moment of inertia about axis perpendicular to web; c, half depth of beam in inches. Having computed M, S is assumed, the value of I/c (called the section modulus) is found by dividing M by S, and the size of a beam having a section modulus equal to or slightly greater than this is chosen from a table giving properties of structural shapes.

If the purlin is placed so that the deflection is not in the plane of the resultant load carried by it, as when an I beam supports a vertical load and rests on an inclined chord of a truss, hence having its web inclined, a trial size of the beam is taken and the maxmum fiber stress in the outermost fiber is computed by the formula

$$\mathbf{S} = \frac{\mathbf{M_1c_1}}{\mathbf{I_1}} + \frac{\mathbf{M_2c_2}}{\mathbf{I_2}},$$

where S = maximum fiber unit stress, M_1 = bending moment due to that component of load which is normal to plane of roof, I_1 = moment of inertia of section of purlin about a gravity axis parallel to roof. c_1 = the distance from this axis to fiber on which stress is maximum, M_2 = bending moment due to component of load which is parallel to roof, I_2 = moment of inertia of section about a gravity axis normal to roof, and c_2 = the distance from this axis to fiber on which stress is a maximum. This method does not apply to purlins of ∟ section, but the moments must be resolved into the planes of their principal axes. By connecting the purlins together with sag rods running up both sides of a sloping roof to the same connection at the peak of the roof the second term of the preceding equation is negligible in the design of all purlins

Section Moduli in Plane of Vertical Loading of Angles and Channels Placed at Right Angles to Roofs as Indicated in Fig. 4

Purlin	Slope of Roof in Inches on 1 Foot								
	0	1	2	3	4	5	6	7	8
2 × 2 × 1/4 L	0.18	0.19	0.20	0.21	0.22	0.24	0.26	0.29	0.31
2 × 2 × 3/8 L	0.27	0.28	0.29	0.31	0.33	0.35	0.37	0.40	0.42
2-1/2 × 2 × 1/4 L	0.30	0.31	0.33	0.35	0.38	0.41	0.44	0.46	0.49
2-1/2 × 2 × 3/8 L	0.40	0.42	0.45	0.48	0.52	0.57	0.61	0.65	0.69
2-1/2 × 2-1/2 × 1/4 L	0.31	0.32	0.33	0.35	0.37	0.39	0.42	0.45	0.48
2-1/2 × 2-1/2 × 3/8 L	0.42	0.44	0.46	0.49	0.52	0.55	0.60	0.64	0.68
3 × 2-1/2 × 1/4 L	0.44	0.46	0.49	0.52	0.56	0.60	0.65	0.69	0.74
3 × 2-1/2 × 3/8 L	0.64	0.67	0.71	0.76	0.82	0.89	0.95	1.00	1.06
3-1/2 × 2-1/2 × 1/4 L	0.56	0.59	0.64	0.70	0.76	0.83	0.89	0.96	0.84
3-1/2 × 2-1/2 × 3/8 L	0.84	0.89	0.96	1.04	1.14	1.22	1.29	1.37	1.19
4 × 3 × 1/4 L	0.75	0.80	0.86	0.93	1.02	1.11	1.18	1.26	1.27
4 × 3 × 3/8 L	1.11	1.18	1.26	1.35	1.47	1.58	1.70	1.81	1.76
5 × 3-1/2 × 5/16 L	1.48	1.55	1.66	1.80	1.96	2.12	2.30	2.39	2.06
5 × 3-1/2 × 3/8 L	1.76	1.86	1.99	2.15	2.34	2.52	2.71	2.80	2.42
6 × 4 × 3/8 L	2.50	2.66	2.87	3.10	3.41	3.70	4.00	3.64	3.17
6 × 4 × 1/2 L	3.30	3.52	3.79	4.10	4.52	4.84	5.18	4.65	4.09
4 × 5.4 ⊏	1.9	1.2	0.9	0.72	0.62	0.55	0.48	0.45	0.42
5 × 6.7 ⊏	3.0	1.85	1.3	1.05	0.86	0.76	0.67	0.61	0.57
6 × 8.2 ⊏	4.3	2.5	1.8	1.42	1.15	1.01	0.9	0.83	0.78
7 × 9.8 ⊏	6.0	3.3	2.3	1.8	1.5	1.38	1.17	1.07	1.0
8 × 11.5 ⊏	8.1	4.3	3.0	2.3	1.85	1.6	1.45	1.25	1.2
9 × 13.4 ⊏	10.5	5.6	3.8	2.9	2.4	2.05	1.95	1.7	1.6
10 × 15.3 ⊏	13.4	6.7	4.7	3.5	2.8	2.9	2.2	2.0	1.9
12 × 20.7 ⊏	21.4	10.5	7.0	5.4	4.3	3.8	3.4	3.0	2.8
15 × 33.9 ⊏	41.5	19.8	18.0	9.8	8.0	6.5	6.0	5.4	5.0

except the one or two at the peak carrying the pull from the sag rods. Sag rods are usually made 5/8 or 3/4 in. in diameter and are spaced 6 to 7 ft. apart. To prevent undue deflection the depth of a rolled beam purlin should not be less than **1/32** of its span length. For fiber stress not exceeding 10 000 it may be 1/40.

To determine the strength of the ordinary sizes of L's and ⊏'s for purlins without sag rods and free to bend in any direction, the section moduli can be taken from the preceding table and used in the fundamental formula for flexure.

It should be noted that L purlins set as in Fig. 4 are stronger than those of Fig. 5. This table is based on purlins being set as shown in Fig. 4.

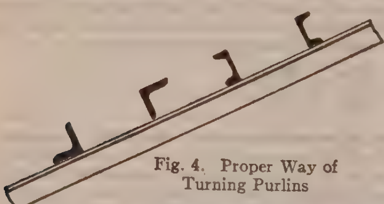


Fig. 4. Proper Way of Turning Purlins

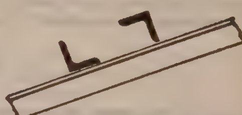


Fig. 5. Improper Way of Turning Purlins

Proportioning Tension Members. A tension member or bar of a truss, also called a tie, is one that carries a pull or tensile stress. For riveted trusses 2 L's, and for pin trusses 2 eyebars are commonly used for each tension member. For design of eyebars see Art. 27. In riveted trusses carrying light loads the 2 ∟'s should be placed side by side with a space between them equal to the thickness of the gusset plates, Fig. 6, and should have their vertical legs up. These ∟'s should be connected together at points from 2 to 3 ft. apart throughout their lengths by placing a circular steel washer between the ∟'s and driving a rivet through the 2 ∟'s and the washer. These rivets are called tack or **stay rivets,** and simply hold the ∟'s together during transportation and erection and make the ∟'s act together when under stress.

To proportion the cross-section of a tension member of 2 ∟'s divide the total stress in the member by the allowable unit tensile stress, thus obtaining the required net area of the 2 ∟'s, and then choose from a table of properties of ∟'s, 2 ∟'s of the same size, each having a net area equal to or slightly larger than 1/2 the required net area. The net area of an ∟ is found by deducting from the gross area of the cross-section the greatest sectional area cut out by rivet holes in any one section. In **computing net areas** the diameter of a rivet hole is assumed 1-8 in. larger than the diameter of the cold rivet before driving. An angle is sometimes connected to a gusset plate by rivets in one leg only, in which case there would usually be only one row of rivets in the connecting leg, and hence only one rivet hole area, equal to diameter of hole times thickness of piece, would be deducted from the gross section. Where there are two rows of rivets in the ∟ at one end the rivets should be staggered so that there would be only one rivet hole in any one section.

Proportioning Compression Members. A compression member or column in a truss is one which carries a compressive stress, and for roof trusses is made of one or two ∟'s, Fig. 6, two ∟'s with a plate between, or of two [’s. In light work one or two ∟'s are nearly always used for each member, while for heavy trusses the top chord is composed of two [’s or two or more ∟'s. To proportion the cross-section of a compression member subjected to compression only, the simplest way is to assume a trial cross-section and compare the actual average stress per square inch on this section with the allowable average stress per square inch as computed from a column formula. If the actual and allowable stresses are equal or if the actual is only slightly less than the allowable the section is used, but if the actual is greater than the allowable a new trial area is chosen and the process repeated.

The Actual Average Stress per square inch is found by dividing the total stress in the member by the gross area of the cross-section. The allowable average stress is found from the column formula of the specifications (see p. 1184).

The following example will illustrate the method of proportioning the inclined top chord of a Fink truss having a panel length along this chord of 7 ft., a direct compression of 46 500 lb., and made of 2 ∟'s spaced 3/8 in. apart for the gusset plates, the allowable unit stress being given by the formula $S = 16\,000 - 70\,l/r$. Assume two 4 by 3 by 5/16 in. ∟'s and place the 4 in. legs vertical and 3/8 in. apart. From a steel company's handbook (or p. 676), the area of the 2 ∟'s is found to be 4.18 sq. in. and the least radius of gyration that about the gravity axis parallel to the short legs. Dividing 46 500 lb. by the area 4.18, the actual average unit stress is found to be 11 120 lb. per sq. in., and by substituting $l = 84$ in. and $r = 1.27$ in. in the column formula the value of the allowable unit-stress, S, is 11 370 lb. per sq. in. Hence the 2 ∟'s assumed are of correct size. These ∟'s must be riveted together at intervals not exceeding $0.65 \times 84/1.27 = 43$ in.; 0.65 being the least radius of gyration of one ∟ and 1.27 the least for 2 ∟'s. It is customary to place tack rivets every 2 or 3 ft. apart throughout the length of these members.

Purlins resting between joints on the top chord of a truss cause **cross-bending** therein, which, in addition to the compression acting, must be considered in the design of the chord. In such cases the shortest method for designing the chord is to assume a section, usually two ∟'s or two ∟'s and a vertical plate for light trusses, and compare the maximum unit stress with the allowable. If the section thus assumed is too large or too small a new trial section

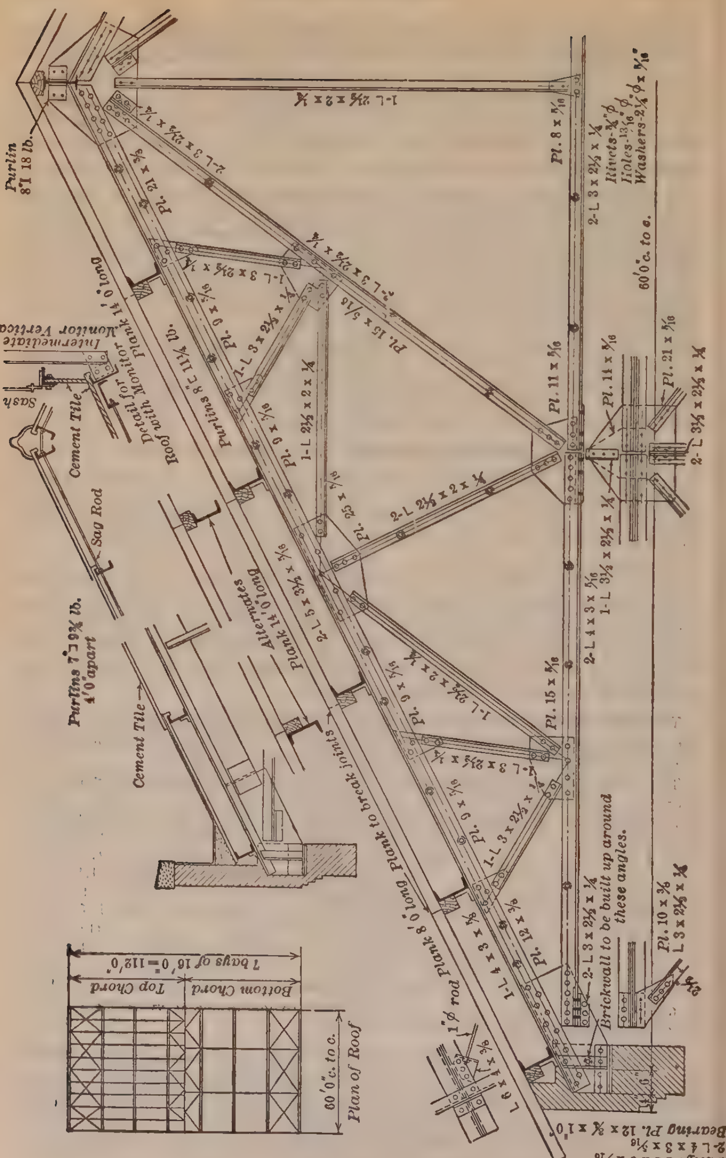


Fig. 6. Fan Roof Truss with Details

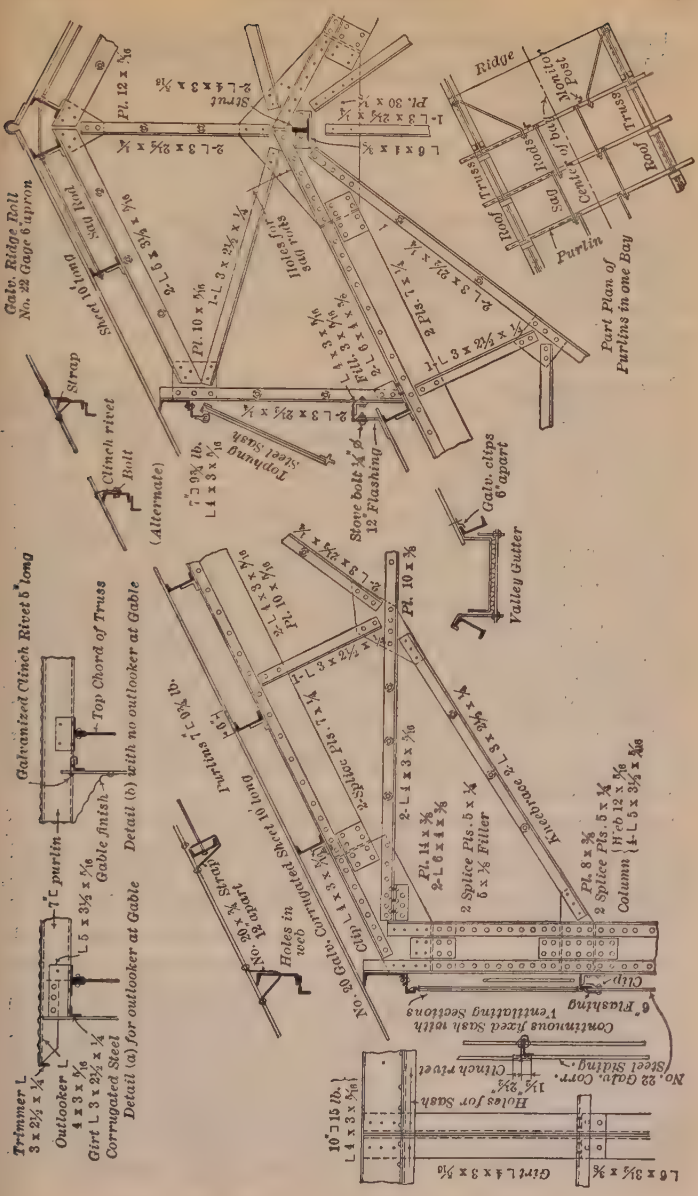


Fig. 7. Details for Corrugated Steel Covered Building

is chosen and the process repeated until the section is satisfactory. The maximum unit-stress in a continuous chord due to combined direct compression and bending is with sufficient accuracy given by

$$S = \frac{P}{A} + \frac{\frac{Mc}{I}}{1 - \frac{Pl^2}{10\,EI}},$$

where S is the maximum unit stress on the most strained fiber, in pounds per square inch; P the direct compressive load in pounds; A the area of cross-section in square inches; M the maximum bending moment on the chord acting as a beam of panel length l and continuous over the joints; c the distance from gravity axis to most strained fiber under consideration; I the moment of inertia of cross-section; E the modulus of elasticity, for steel say 30 000 000 lb. per sq. in.

Designing Joints. Since most roof trusses are of the riveted type, only riveted joints will be considered. For the design of pin joints see Art. 29. In laying out the members of the truss their center lines should meet at a point at each joint and should coincide with the truss diagram. It is the usual practice in roof work to use the gage lines of the angles for the center lines, instead of center of gravity lines. The error is negligible. Rivets should be placed in each member to transmit the stress from that member into the gusset plate. As an illustration of the method of computing the number of rivets, consider a member carrying a direct stress of 46 500 lb., and made of 2 L's 4 by 3 by 5/16 in., one on either side of a 3/8 in. gusset. The rivets connecting the L's to the gusset are in double shear, that is, they must be sheared on two sections before failure can occur; they are in bearing on the 3/8-in. gusset plate and in bearing in the oppostie direction on 2 thicknesses of 5/16 in. L. The **double shearing value** of one rivet is equal to twice the area of its cross-section multiplied by the permissible shearing stress per square inch, or, assuming rivets 3/4 in. in diameter, the double shearing strength is 2 × 0.44 × 12 000 = 10 560; the 12 000 being the permissible shear in pounds per square inch. The **bearing value** of a rivet on a piece is the product of the diameter of the rivet in inches, the thickness of the piece in inches and the permissible bearing pressure in pounds per square inch. In the case here assumed the bearing value of the 3/4-in. rivet on the 3/8-in. gusset plate is 3/4 × 3/8 × 24 000 = 6750 lb., and since this is smaller than either the double shearing value or the bearing on the two 5/16-in. L's, 6750 lb. is the allowable stress on one rivet; 24 000 being the permissible bearing strength per square inch. The number of rivets in the member at this joint is 46 500/6750, or 7.

The Permissible Unit-Stresses used in the problem just solved, namely 12 000 lb. per sq. in. for shearing and 24 000 lb. per sq. in. for bearing, are safe values for shop, that is, machine-driven rivets. For field rivets driven by hand, or for bolts, the safe shearing and bearing values are 10 000 and 20 000 lb. per sq. in., respectively. Allowable shearing and bearing values for rivets of different diameters are given in Art. 21.

Fig. 6 shows a fan roof truss with details.

Fig. 7 shows details of an ordinary steel building with covering of corrugated steel.

Details. Even though to make what is usually considered the design of a structure may be easy, to detail it may be difficult. The ideal draftsman is a designer, a designer of details. Seldom has a structure given way for lack of strength in the main members but faulty details have invited and brought disaster. Theoretically, in laying out the members of a roof truss the center of gravity lines should meet at a point at each joint and should coincide with the truss diagram. However, in practice the rivet lines are generally used. Details are thus simplified. Eccentric rivet connections should be reduced to a minimum. Economy will be gained by having as many duplicate parts as possible. Connections that induce secondary stresses should be avoided. Details should be designed in accordance with shipping facilities, cost of transportation and ease of erection. While it may lessen the cost of field work to fabricate trusses in few sections, the transportation charges may be enough larger to consume all saving.

Again, while it may lessen freight charges to ship a truss in many pieces, the cost of field riveting will be increased.

3. Steel Mill-Building Frames

Steel Mill-buildings in general are divided into three types. Those of the first type have steel frames carrying the roof loads as well as the weight of the walls which protect the frames; the walls being constructed of corrugated steel sheets or of concrete supported by the steel frames. The essential parts of the steel work are the roof trusses, columns and bracing; a truss together with the columns which support it constitutes a bent, of which Figs. 8, 9, and 10 represent typical cases. In the second type of steel mill-building the columns are braced principally by thin masonry walls, thus eliminating most

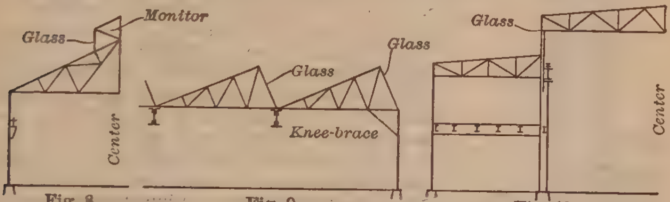


Fig. 8 Fig. 9 Fig. 10

Steel Mill-building Frames

of the steel bracing between columns, while in the third type the steel roof trusses are supported on masonry walls instead of on steel. Light is admitted to the interior of mill-buildings through windows in the side and end walls and in addition through skylights in the roof proper or in the monitor roof, or through windows in clerestory of the monitor. **A monitor,** (Fig. 8), is that portion of building extending above the main roof for the purpose of ventilating or lighting the interior of a building. Fig. 9 shows **saw-tooth** roof construction. Roofs of this form of construction are extensively used for wide areas. The glazed portions should of course face the north; a diffused light thus illuminates the floor below without casting shadows.

Columns should be proportioned to resist (1) direct stress due to roof, crane or other loads, (2) stress due to wind and (3) stress due to eccentric loads. A great number of column formulas have been suggested for the design of steel members in compression. The "straight-line" formula $\mathbf{S = 16\,000 - 70\, \mathit{l}/r}$, except where noted otherwise, is used throughout this section.

Rivets in tension should be avoided when practicable. Carrying the crane load on a column flange directly under the web of the runway girder is preferable to carrying it on a bracket. By connecting roof trusses to columns with gussets greater stiffness is obtained than by resting them on column caps. Various column details are shown in Figs. 11 and 12.

Girts, sometimes called side purlins, are beams, usually of angles or channels fastened to the columns to support the side covering and to resist wind pressure. For corrugated steel siding the wind pressure is the greater load and the longer leg of the angle or the web of the channel should be laid

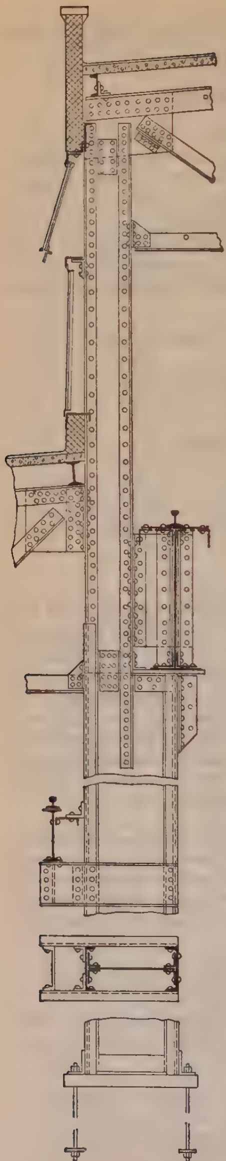


Fig. 11. Typical Column Details

horizontal. To prevent deflection sag rods 5 to 8 ft. apart fastened to the girts and running to the eave struts are used.

The Bracing of a mill-building should be so designed that wind and vibratory stresses are carried to the foundations. Diagonal bracing should be introduced into the plane of both top and bottom chords for stiffness as well as for calculated stresses. This applies also to roof trusses resting on brick walls. Adjustable rods may be used for top chord bracing, the purlins acting as struts, but the bracing of the bottom chord should be angles or other rigid shapes. Diagonals between the steel columns in each end of the building and in an occasional side bay are usually sufficient to carry the induced stresses to the foundations. When it can be done an excellent way is to carry the transverse thrust from the wind as well as that from the cranes to the ends of the building and thence by diagonals to the ground. The usual way is to design each bent as a unit for both the vertical and horizontal loads of one-half the adjoining bays. In such cases the kneebrace is an important member. A **kneebrace** is a short diagonal, usually a pair of angles, used to connect a truss or beam to a column. Traveling cranes running through a building often bar the use of transverse kneebraces. The gusset plates connecting the trusses to the columns should then be as large as possible and calculations made accordingly. Angles may be added to stiffen the edges of the plates. Kneebraces should be used between crane girders and their supporting columns.

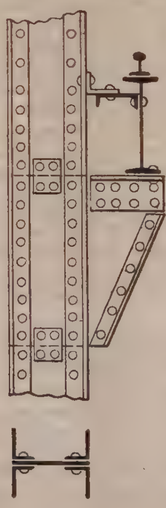


Fig. 12. Connection of Crane Bracket to Column

Electric Traveling Cranes are an important part of the equipment of most mill-buildings and are often the dominant factor in the design. Ten-ton cranes are the most common. Cranes of 50 short tons capacity and less are usually carried on a two-wheel carriage at each end. Those of 60 to 125 short tons capacity should have four-wheel carriages in order to avoid excessively concentrated loads on the runway girders. Cranes of 150 short tons capacity and more are built with end carriages of both four and eight wheels. The eight-wheel carriage reduces the concentrated load on the runway girders. The wheel bases, maximum wheel loads and clearance of cranes vary with the design of the maker. The figures given in the table on next page represent average values.

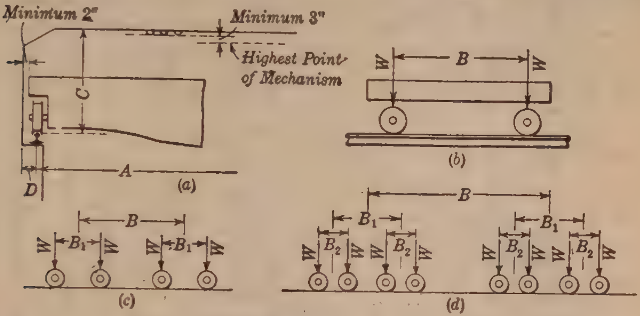


Fig. 13. Wheel Diagrams for Traveling Cranes

Stresses due to the wheel loads given in the table should be increased 25% for vibration and impact except for hand-power cranes, where 10% is sufficient. For two cranes in action on the same girder, no impact need to added provided the stress obtained is larger than the stress due to a single crane with impact. In addition to the vertical loads the top flanges of crane girders should be designed to resist a transverse horizontal thrust on each carriage, applied to the wheels, of 10% of the lifting capacity of the crane. The traction stress due to starting or stopping the crane should be assumed at 10% of each wheel load and may be considered as distributing itself along the entire length of the runway.

Floor Loads. Live Loads in pounds per square foot on floors of the mill-building class should not be assumed at less than the following:

Mold lofts, pattern and template shops	60
Machine shops, light machinery	120
Machine shops, heavy machinery	150 to 200
Factories	100 to 250
Foundries, charging floor	300 to 800
Power houses	200

It is important that industrial processes be understood sufficiently to make proper provision for their loads and stresses. Provision should also be made for the support of all engines, boilers, tanks or other concentrated loads carried by the steel construction.

The Working Stresses given in Art. 2 may be used in proportioning members.

Wind Stresses. For buildings not more than 25 ft. to the eave line a horizontal wind pressure should be assumed at not less than 15 lb. per sq. ft. on the sides and the corresponding normal component on the roof according to the Duchemin formula for wind pressure on inclined surfaces. For buildings more than 25 ft. to the eave line the horizontal pressure should be taken at not less than 15 lb. for the lower 25 ft., and 20 lb. for the side surface above 25 ft. and the corresponding normal component on the roof. The steel framework should be designed to carry wind pressure to the ground.

The stresses in the framework due to wind acting on the vertical and inclined sides of a building depend to a great extent on the manner of fixing the ends of the columns. If the ends are hinged, that is, free to turn, the

Loads and Clearances for Electric Cranes. See Fig. 13

Capacity in tons	Span in ft. c. to c. Rails (A)	Wheel Base B	Wheel Base B_1	Wheel Base B_2	Maximum Load in lb. for each Wheel (W)	Vertical Clearance (C)	Side Clearance (D)	Am. Soc. C. E. Rail in lb. per yd.
		ft. in.	ft.	ft. in.		ft. in.	in.	
5	40	9 0	....	....	13 000	6 0	10	50
	60	10 0	....	....	15 000	6 0	10	50
	80	11 0	....	....	18 000	6 0	10	50
10	40	9 0	....	....	20 000	6 6	12	60
	60	10 0	....	....	23 000	6 6	12	60
	80	11 0	....	....	26 000	6 6	12	60
15	40	10 0	....	....	30 000	7 0	12	60
	60	11 0	....	....	33 000	7 0	12	60
	80	12 0	....	....	36 000	7 0	12	60
20	40	10 6	....	....	35 000	7 6	12	80
	60	11 6	....	....	39 000	7 6	12	80
	80	12 0	....	....	43 000	7 6	12	80
30	40	11 0	....	....	50 000	8 0	13	80
	60	12 0	....	....	54 000	8 6	13	80
	80	13 0	....	....	59 000	8 6	13	80
40	40	12 0	....	....	66 000	8 6	14	100
	60	13 0	....	....	71 000	9 0	14	100
	80	14 0	....	....	77 000	9 0	14	100
50	40	13 0	....	....	80 000	9 6	15	100
	60	13 6	....	....	85 000	10 0	15	100
	80	14 0	....	....	90 000	10 0	15	100
75	60	12 0	5	....	65 000	12 0	16	100
	80	14 0	5	....	70 000	12 6	16	100
100	60	12 0	5	....	90 000	14 0	18	150
	80	14 0	5	....	95 000	15 0	18	150
125	60	13 0	5	....	110 000	15 0	20	175
	80	15 0	5	....	120 000	16 0	20	175
150	60	18 0	5	....	130 000	16 0	22	175
	80	20 0	5	....	140 000	17 0	22	175
175	60	14 0	7	3 6	65 000	17 0	24	150
	80	19 0	7	3 6	75 000	18 0	24	150
200	60	14 0	7	3 6	75 000	17 0	24	150
	80	19 0	7	3 6	85 000	18 0	24	150
Loads and Clearances for Hand Cranes								
2	30	4 0	....	....	3 500	4 0	7	30
	50	5 0	....	....	4 000	4 0	7	30
4	30	4 0	....	....	5 500	4 6	8	30
	50	5 0	....	....	6 500	4 6	8	30
6	30	6 0	....	....	8 000	5 0	9	40
	50	7 0	....	....	9 500	5 0	9	40
8	30	6 0	....	....	10 500	5 0	10	50
	50	7 0	....	....	12 000	5 0	10	50
10	30	7 0	....	....	13 000	5 0	10	50
	50	8 0	....	....	14 500	5 0	10	50

stresses throughout the frame are different from those existing when the ends of the columns are fixed. A **column is fixed** at the end when it is so rigidly held there that the axis of the column cannot change its direction at that point. In the case of a fixed-end column the point of **contra-flexure**, that is, the point where the direction of curvature changes, is taken halfway between the base of the column and the foot of the kneebrace. In light buildings the columns should not usually be assumed fixed at the bottom.

The case of an intermediate transverse bent of a kneebraced mill-building will be considered. The example taken will be a bent of 60-ft. span, height 18 ft. to foot of kneebrace, 24 ft. to bottom chord and slope of roof 6 in. to 1 ft. Trusses are 16 ft.

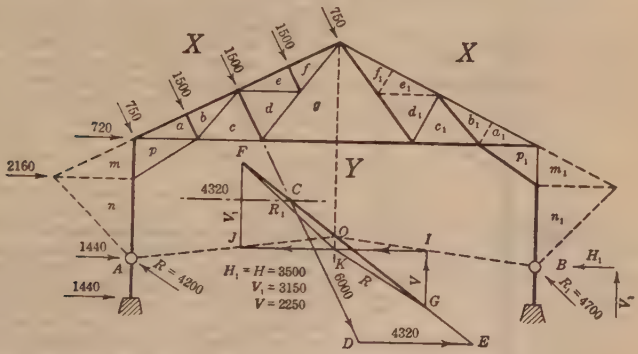


Fig. 14. Load and Reaction Diagram.

apart c. to c. The wind pressure will be taken at 15 lb. per sq. ft. perpendicular to the sides of the building and at 11.2 lb. (Duchemin formula, Table, Art. 1) normal to the roof. The columns are assumed partially fixed at the lower end, with the point of contra-flexure at one-third the distance between the lower end and the foot of the kneebrace; the upper ends are considered supported.

Fig. 14, the load and reaction diagram, shows the arrangement of the forces and the method of finding the reactions at the points of contra-flexure in the columns. The

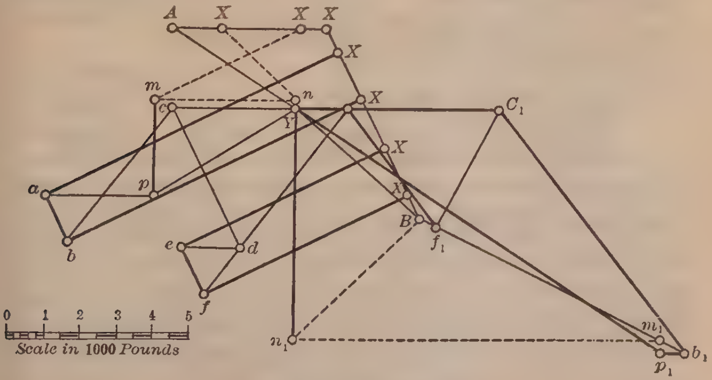


Fig. 15. Stress Diagram

wind shear is assumed to be divided equally between the two columns, making the horizontal reactions at A and B equal. CE is the resultant of CD and DE, the wind on the roof and the sides. The wind resultant is laid off, FG, so that it is bisected by the mid-vertical line at O. The perpendiculars dropped from F and G upon the dotted lines OA and OB gives the vertical components of the reactions at B and A respectively. Draw the horizontal line JI intersecting the mid-vertical at K. Then $KF = R_1$ and $GK = R$, the reactions at B and A respectively.

Fig. 15 is the stress diagram. The heavy lines indicate compressive and the light lines tensile stresses. Dotted lines indicate stresses in imaginary truss members which are inserted to make the stresses statically determinate. Removing these members will produce bending stresses in the columns but will not affect the stresses in truss members and kneebraces. The stress in any member can also be determined by the method of moments or sections. For instance, let S = required stress in the kneebrace. Take M (Fig. 15a) as center of moments and since the sum of moments of all forces about M = zero; $(4200 \times 15.0) - (4320 \times 9.0) - (S \times 5.0) = 0$ from which S = 4800 lb. tension. This checks with the stress obtained by the graphical method of Fig. 15.

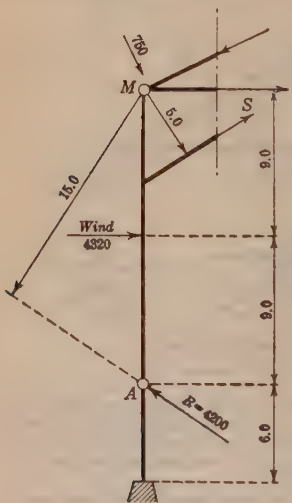


Fig. 15a. Stress in Kneebrace

The stresses in the columns due to wind are as follows: The direct stresses in the portion below the kneebrace are 2250 lb for the windward and 3150 lb. for the leeward column, being the vertical reactions V and V_1, scaled from the reaction diagram. The horizontal shear at the point of contra-flexure of each column scaled from the reaction diagram is 3500 lb. The direct stress in the portion of the windward column above kneebrace equals the vertical reaction V plus the vertical component of the kneebrace and in the leeward column the vertical reaction V_1, minus the vertical component of the kneebrace. The horizontal shear in the portion of the windward column above the kneebrace equals the horizontal component of the stress in the kneebrace (3960) + wind (4320) − H (3500) = 4780. The horizontal stress in the portion of the leeward column above the kneebrace equals the horizontal component of the stress in the kneebrace (10 200) − H_1 (3500) = 6700.

The bending moments in the columns are:

	Ft.-lb.
At foot of leeward column, 3500 × 6	= 21 000
At foot of leeward kneebrace, 3500 × 12	= 42 000
At foot of windward column (3500 × 6) + (1440 × 3)	= 25 320
At foot of windward kneebrace (3500 × 12) − (1440 × 12)	= 24 720

It is seen that the maximum bending moment is at the foot of the leeward kneebrace.

To Design a Mill-Building Bent proceed as follows: Determine stresses in truss due to a total uniform load over the entire roof surface, as in Art. 1. The total load includes the wind load which may be assumed at 10 lb. per sq. ft. for roofs of 3 in. slope and more and at 5 lb. for slopes of less than 3 in. Proportion the members for these stresses using the working stresses recommended in Art. 2. Then find the wind stresses due to the normal wind forces by the methods of Figs. 14 and 15. If the wind stress in any member from Fig. 15 is greater than the wind stress (obtained by interpolation) due to a vertical load of 10 or 15 lb. per sq. ft. over the entire roof surface, that member is proportioned for the maximum wind plus the stress from the uniform loads other than wind, using working stresses 50% more than in the first calculation, but in no case is a less section to be used than that first obtained. The members bc and b_1c_1 will generally need to be increased; often cd and c_1d_1; occasionally gd, gf, gf_1 and gd_1. Compressive stresses are noted in certain tension members, particularly b_1c_1 and the lower chord. The diagonals bc and b_1c_1 can be made of two angles instead of one as when designed for tension alone. The compressive stresses in the lower chord are overcome by the tensile stresses due to the dead load. The kneebraces have wind stresses only, and are proportioned for the larger working stresses.

The Columns may be designed as follows:

(1) Proportion for the direct stress from the specified total uniform load on the roof surface using the column formula specified. (See p. 1184.)

(2) Proportion for combined compressive and bending stresses using the formula of Art. 2,

$$S = \frac{P}{A} + \frac{\frac{Mc}{I}}{1 - \frac{Pl^2}{10\,EI}}.$$

S in this formula is the actual maximum fiber stress which should not exceed by more than 50% the S obtained from the column formula in (1). The direct stress, P, is that due to the roof loads (other than wind) and the vertical reaction from wind. For M the maximum bending moment due to wind is to be used.

The greater section obtained by (1) or (2) is to be used.

For example, if the roof of Fig. 14 be of corrugated steel, the total direct load on a column is 31 ft. × 16 ft. × sec 26° 34′ × 40 lb. = 22 200 lb. The column is proportioned for this load according to (1) above. As with truss members it may be assumed that 10 lb. of the 40 lb. uniform load is from wind leaving 16 650 lb. from loads other than wind. The reaction diagram shows a direct stress in the leeward column of 3150 lb. In proportioning the column according to (2) the value of P in the formula is therefore 16 650 + 3150 = 19 800 lb. The value of M is the maximum bending moment, 42 000 ft.-lb. = 504 000 in.-lb. The greater section in this particular case is that obtained by (2) and it should be used.

4. Tall Building Frames

The Steel Frames used in the construction of modern tall buildings such as office buildings carry the entire weight of walls, floors and, in fact, all dead and live loads. To these buildings have been given the name **skeleton construction.**

Columns should always be made of steel, although in cheap construction cast iron is sometimes used. Fig. 16 shows various common sections employed for columns in buildings. Where two [’s or four L’s are used to form a column they should be connected together by **lattice bars**, which are flat bars from 1-3/4 to 2-1/2 in. in width riveted first to one [or pair of L’s and then to the other. Columns made of L’s and plates must be riveted together with rivets in the body of the column spaced not over 16 times the thickness of the thinnest outside piece connected, the maximum distance used being 6 in. on centers. At ends of such columns the rivet pitch, which is the distance of rivets on centers, should not exceed four diameters of the rivet for a length equal to 1-1/2 times the greatest width of the member. Columns should be continuous over two stories, and splices should be placed just above the floor levels. Cast-iron bases or pedestals are often used under columns to distribute the load over the foundations, although rolled steel slabs are preferable in most cases and have come into quite general use. For heavy loads foundations are made of one or more tiers of grillage beams on concrete footings. All columns must be encased in concrete, terra cotta or other fireproof materials, Fig. 16, and no pipes should be placed within such casings. Two inches of concrete or 2 to 4 in. of terra cotta is the minimum thickness for casings, and the space between the terra cotta and the columns should be filled with concrete or other similar filling.

In tall buildings **wind bracing** is a prime requisite. No other feature of their design has called forth a greater variance of opinion. Dependence for bracing is often placed upon the walls, floors and partitions in buildings less than 100 ft. high where the height is not more than twice the minimum horizontal dimension. The columns are strongly spliced and secured to the floor framing. For greater limits the wind stresses are carried to the ground by the steel frame. This often presents a difficult problem. Kneebraces are excellent when they do not project into rooms and corridors. A system of diagonal bracing, ∟'s or ⊏'s, as shown in Fig. 17 (*a*), is ideal but it can seldom be used. In a system without diagonals the connections of floor girders and beams to columns must be designed to take bending stresses and at the same time come within prescribed architectural limits. Connections that have often been used are shown in Fig. 17 (*b*), (*c*), (*d*). For the lower floor connections details (*b*) or (*d*) are used and for the upper floors where wind stresses are light (*c*) is used.

See Art. 7. Wind Stresses in Tall Buildings.

Live Loads. Tall buildings are usually within the limits of a city and must be designed in accordance with the provisions of a municipal building code. When not governed by a building code the following loads and working stresses are recommended:

Minimum Live Load in Pounds per Square Foot of Floor Area

Dwellings (private residences) first floor	40
Dwellings (private residences) upper floors	30
Apartment houses, first floor	50
Apartment houses, upper floors	40
Hotels, first floor	80
Hotels, upper floors	40
Office buildings, first floor	100
Office buildings, upper floors	50
School buildings, class rooms	50
School buildings, assembly rooms	75
Churches and theatres	75
Places of public assembly where floors are used for drilling or dancing	120
Where not so used	100
Retail stores, ordinary	100
Private garages, pleasure vehicles only	60
Public garages, pleasure vehicles only	90
Garages, motor trucks 1 to 3 tons capacity	150
Garages, motor trucks 3-1/2 to 5 tons capacity	200
Warehouses	200 to 500

Every steel beam in any floor should be capable of sustaining a live load concentrated at its center of not less than 3000 lb. if used for business purposes; of 2000 lb. if used as a private garage storing pleasure vehicles only; of 3000 lb. if used as a public garage storing pleasure vehicles only; of 8000 lb.

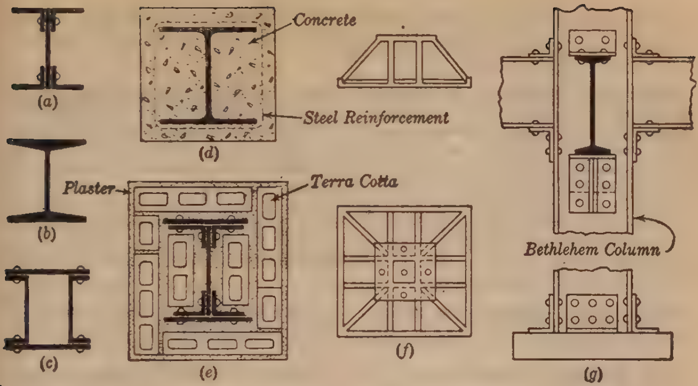


Fig. 16. Columns for Fireproof Buildings

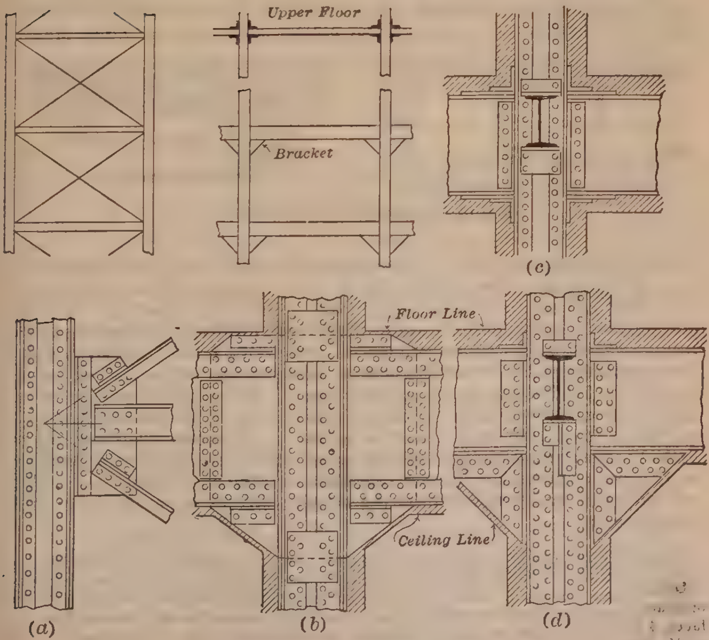


Fig. 17. Types and Details of Wind Bracing

rf motor trucks of 1 to 3 tons capacity are stored and of 12 000 lb. if motor trucks of 3-1/2 to 5 tons capacity are stored. The kind of material stored in a warehouse will determine the floor load.

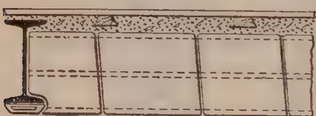
Reduction of Live Load. In computing stresses for columns in such buildings as warehouses and factories which are likely to be fully loaded on all floors simultaneously the full specified live load should be used. In types of buildings such as apartment houses and office buildings the specified floor loads may be reduced in computing column stresses as follows: for the roof the specified live load should be used, for the top floor 90% of the live load, for each succeeding lower floor the live load may be reduced by 5% until 50% of the live load floor loads is reached, when such reduced loads should be used for all remaining floors, except the first or ground floor for which the full specified live load should be used. Girders of office building type carrying more than 200 sq. ft. of floor may have the specified live load reduced 10%. Dead loads must be computed in all cases.

Wind Pressure should be assumed at not less than 20 lb. per sq. ft. on the sides of building and the corresponding normal component on the roof.

The Working Stresses given in Art. 2 should be used in proportioning, members.

5. Fireproof Floors

Floors for Steel Buildings consist of a wearing surface of wood, concrete marble or tile laid on hollow-tile blocks, brick or concrete arches which are carried by I beams, called joists or floor beams, spaced from 4 to 8 ft. apart,


Terra Cotta Arch (End Construction)

Reinforced Concrete

Fig. 18. Fireproof Floor Construction

though usually from 5 or 6 ft., depending on the kind of arches used and the span of the beams. Floor girders, that is, deep I beams or plate girders running from column to column, support the joists, and if between exterior columns, they also support the walls. **Hollow-tile blocks** made of porous terra cotta, and used as flat arches, are placed between the I-beam joists and set in cement mortar so as to cover the bottom flanges of the beams, thus affording protection of the beams against fire underneath. A layer of concrete is spread over the blocks, covering the top flanges of beams to a depth of about 2 in., and on this concrete the wearing surface is placed. **Concrete floors** are either arched or flat. In the latter case they consist of a slab in which steel netting or rods are embedded. The concrete surrounds the steel beams, and a flat ceiling is usually suspended from the beams. Fig. 18 shows details of fireproof flooring.

Dead or Fixed Loads on floors should be computed for each case. The following are ordinary values in pounds per square foot for fireproof flooring: wooden wearing surfaces, 4 to 6; screeds or nailing strips, 2; concrete filling, 15 to 30; arch or slab, 20 to 50; steel floor joists and girders, 8 to 12; ceiling construction and plaster, 3 to 10. The total dead weight of fireproof floors varies from 50 to 100 lb. per sq. ft. The accompanying table gives weights of hollow-tile floor arches.

Weight of Hollow-tile Flat Floor Arches

Exclusive of Weight of Steel, Filling or Wearing Surface

End Construction			Side Construction		
Depth of Tile, in.	Beam Spacing, ft.	Weight, lb. per sq. ft.	Depth of Tile, in.	Beam Spacing, ft.	Weight, lb. per sq. ft.
6	3 to 4	25	6	3.5 to 4.0	27
7	4 to 5	26	7	4.0 to 4.5	29
8	5 to 6	27	8	4.5 to 5.0	32
9	6 to 7	29	9	5.5 to 6.0	36
10	7 to 8	33	10	6.0 to 6.5	39
12	8 to 9	38	12	6.5 to 7.0	44

Reinforced Concrete Floor Slabs weigh about 9 lb. per sq. ft. per inch of thickness if of cinder concrete and 12 lb. if of stone concrete. Cinder concrete fireproofing around beams weighs from 40 lb. per lin. ft. for 8-in. to 170 lb. for 24-in. standard beams of light weight. These weights will be increased for beams of wider flanges.

The Spacing, or distance between centers of floor beams, varies with the type of fireproofing, the length of span of the beams and character of loading. After determining the live load and the type and weight of fireproofing the total live and dead load can be easily computed and the correct size and spacing of I beams for this loading and span can be taken from tables in steel companies' handbooks, or the spacing may be assumed and the proper beams to be used for the spacing is found by taking from pages 666 to 669 a size and weight having a section modulus which equals the value of I/c in the following formula:

$$\frac{I}{c} = \frac{3\,dwl^2}{2\,S},$$

where I/c is the section modulus in inch units, I being the moment of inertia and c the half depth of beam; d, the distance center to center of beams in feet; w, total live and dead load in pounds per square foot; l, span of beam in feet; S, allowable unit fiber stress in pounds per square inch. If a beam cannot be found with a section modulus close to that required, a slight change in the spacing may be advisable. In this case a beam may be chosen and the distance $d = \dfrac{2\,SI}{3\,cwl^2}$ computed. The heaviest section of a given depth of I beam should be avoided, as it is not as economical as a lighter beam of a greater depth. The size of a beam required to carry a uniform load may also be chosen by multiplying the load per running foot of beam by the span and selecting from a handbook a beam corresponding, and in case the load on the beam is concentrated at the center it can be reduced to an equivalent uniform load by multiplying it by two. For loadings not uniform or not concentrated at the center of the span the section modulus **I/c** must be found from the formula **I/c = M/S,** where M is the bending moment in inch-pounds and S the unit stress which is usually taken at 16 000 lb. per sq. in.

The Deflection of Floor Beams carrying plastered ceilings underneath should not exceed 1/360 of the span. For I-beams carrying uniform loads, simply supported at the ends and stressed to 16 000 lb. per sq. in., this limit of deflection will be exceeded if the ratio of span length to depth of beam exceeds 24.

Compression Flanges of I Beams or girders should be stayed laterally at distances not exceeding 12 times the flange width. If this ratio is exceeded the allowable unit stress should be found by the formula, $S = 19\,000 - 250\, l/b$, where S = allowable unit stress in pounds per square inch, l = unsupported length and b = width of flange. A corresponding reduction should be made in the tables of safe loads given in steel companies' handbooks. For example, the tabular allowable uniform load for an 18 in. 55-lb. beam 20 ft. long based on a maximum bending stress of 16 000 lb. per sq. in. is 47 000 lb. The beam length l, is 240 in. and the flange width, b, is 6 in. If the top flange is unsupported laterally the allowable unit stress is $19\,000 - 250\, l/b = 9000$ and the safe load is $9/16 \times 47\,000$ lb. = 26 500 lb.

Tie Rods from 5/8 to 7/8 in. in diameter are used to connect the webs of I beams in buildings to assist in construction, in bracing the beams laterally and to take the thrust from the floor arches. For uniformly loaded floor arches the horizontal thrust is given approximately by $T = 3\, wl^2/2r$, and the spacing of the tie rods by $l_2 = 10\,000\, ra/wl^2$, where T = thrust in pounds per linear foot of floor beam; w = total live and dead load per square foot; l = span of arch in feet; r = rise of arch in inches; l_2 = distance between rods in feet; a = net area of tie rods in square inches. For flat arches r is approximately 6/10 to 8/10 the depth of the arch.

The Safe Unit Shearing Strength of I-beam webs or of plate-girder webs for buildings is given by

$$S_s = \frac{12\,000}{1 + \dfrac{h^2}{3000\, t^2}}$$

where S_s is in pounds per square inch; d = depth of beam or web of plate girder, t = thickness of web, h = vertical distance between flanges of I beams, and for plate girders horizontal distance between stiffeners or vertical distance between flanges, whichever is the smaller. All dimensions are in inches. To obtain the total safe shearing strength V of a web multiply the above unit stress by the gross cross-sectional area of web, or $V = S_s dt$.

Safe Unit Shearing Strength of I-Beam Webs for Buildings

Pounds per Square Inch

$\frac{h}{t}$	S_s	$\frac{h}{t}$	S_s	$\frac{h}{t}$	S_s	$\frac{h}{t}$	S_s	$\frac{h}{t}$	S_s
10	11 610	20	10 590	30	9230	40	7830	50	6550
12	11 450	22	10 330	32	8950	42	7560	52	6310
14	11 260	24	10 070	34	8660	44	7290	54	6090
16	11 060	26	9 790	36	8380	46	7040	56	5870
18	10 830	28	9 510	38	8100	48	6790	58	5660

6. Fireproof Walls and Partitions

Walls for Steel Mill Buildings are made of brick, concrete, tile, corrugated steel, hy-rib and concrete, metal lath and plaster, wood or metal sash. Two or more kinds of siding are often used in the same building. If corrugated steel is used for the roof the sides are often made of it also but of lighter weight. Less than No. 22 should not be used for roofs nor less than No. 24 for siding. For the roof the usual method of fastening is by straps to [purlins and for sides by straps or clinch rivets to [or L girts. Though more expensive, galvanized sheets are preferable to painted sheets. Walls between columns are usually made 12 in. thick if of brick and 8 in. thick if of tile or concrete. A greater thickness may be required when the walls carry all loads or when the columns are set free from the walls.

Where a **rectangular opening** is to be made in a solid masonry wall, and there are no windows or other openings in the wall above, one or more I beams placed side by side, and connected together by cast-iron or rolled steel ⊏ separators fitting between the beams, are used to support the masonry. Assuming that the beams carry a triangular portion of the wall having for its base the span and for its altitude 1/2 the span, the **section modulus of the combined beams** $= wtl^3/32\,000$, where w is the weight of the masonry in pounds per cubic foot; t = thickness of wall in feet; l = span of opening in feet. If there are windows above the opening, the entire weight vertically above the opening should be taken on the beam. In either case the beams should have a depth of not less than about 1/24 the span.

The question of lighting has been much discussed in recent years. It is important that natural light should be utilized to the best advantage. The location of windows as well as their area should be considered. The types of sash most used for mill buildings are the ordinary wooden sash and the rolled steel sash. The latter is more satisfactory and is used almost entirely for the best modern buildings.

Walls in Office Buildings having steel frames are made of brick, stone or terra cotta and are carried by girders or I beams at each floor level, thus separating the wall into parts each one story in height. The thickness of these walls is usually fixed by building laws, 12 in. being the minimum. Terra cotta facing with brick backing for exterior walls is good construction. Cement mortar should be used. Hollow-tile furring blocks, usually 2 to 6 in. in thickness, attached to the inside surfaces of exterior walls, are used as a protection against dampness and noise, and serve to hold the plaster. **A spandrel,** which is that part of an outside wall in an office building over a window and under the window vertically above, is carried on I beams, or on ⊏'s and ∟'s. The terra cotta blocks are anchored to the steel. Fig. 19 shows spandrel construction.

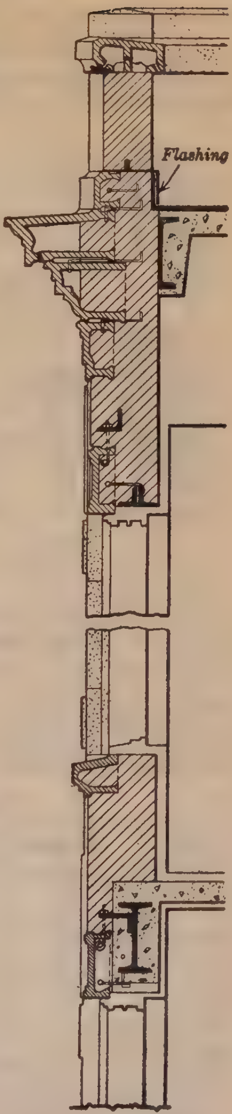


Fig. 19. Spandrel and Cornice Sections

Partitions in Fireproof Buildings, Fig. 20, are made of hollow terra cotta blocks, plaster, plaster block, brick and of various patented materials. Of these materials terra cotta hollow blocks are commonly employed. Partitions of this material are made from 4 to 12 in. thick, 4 in. being the usual thickness for the blocks, which for the body of the partition are 12 by 12 in. in size but for the top and sides are 6 to 8 by 12 in. to fit in with the square blocks. Terra cotta should be wet before being laid and also before being plastered, and should be laid in a mortar composed of one part lime-putty, two parts cement and two to three parts sand. **Plaster** on both sides of a terra cotta partition increases the thickness by about 1-1/2 in., that is, 3/4 in. per side Twelve feet for 3-in., 16 ft. for 4-in., 20 ft. for 6-in., are the usual heights for the thicknesses of partitions given. Solid plaster partitions are made from 2 to 4 in. thick and are supported on

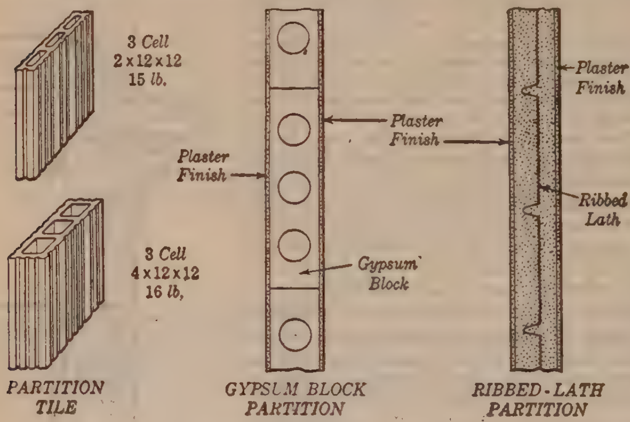


Fig. 20. Fire-resisting Partitions

wire lath on steel framework. For the 4-in. partitions a central body of cinder concrete supported between two such wire nettings is built, then plastered on both sides. **Hollow plaster partitions** consist of a steel framework of [’s to which metal lath is attached on each side, and to this the plaster is applied, leaving a hollow space between the two layers of plaster.

The accompanying table gives weights of various forms of partitions, the weight in each case of terra cotta and plaster blocks not including the plastered surfaces. For each plastered surface used add 5 lb. per square foot of surface. The weights given are per square foot of partition.

Weights of Partitions

Kind of Partition	Thickness, in.	Weight, lb. per sq. ft.	Kind of Partition	Thickness, in.	Weight, lb. per sq. ft.
Porous or hard-burned terra cotta	2	10 to 14	Solid plaster	2	20
	3	11 to 16		4	32
	4	12 to 19	Hollow plaster	4	22
	5	17 to 22	Plaster blocks	2	7
	6	22 to 24		4	12
	8	28 to 33		8	22

Wood stud and plaster partitions weigh from 80 to 150 lb. per lin. ft., depending upon height and thickness of plaster; gypsum block partitions may be assumed at 10 lb. per sq. ft. for 3-in. thickness with an increase of 2-1/2 lb. for each additional inch of thickness. To this should be added the weight of the plaster.

The question of whether partitions, especially those subject to relocation, should be considered as dead or live load is perplexing. An approach to a solution is to consider them equivalent to an addition of 15 lb. per sq. ft. to the dead load of the floor.

7. Wind Bracing in Tall Buildings

Wind Pressure for many-storied buildings when not specified by a building code should be assumed at not less than 20 lb. per sq. ft. on the sides of build-

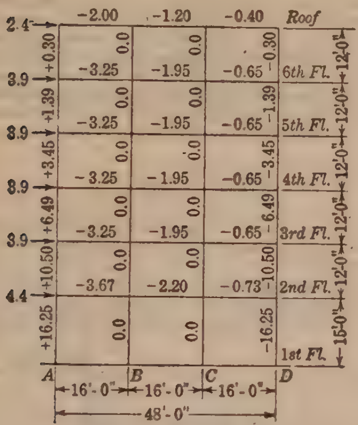
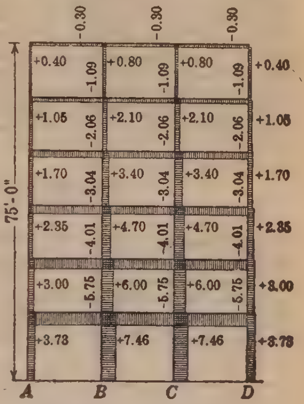


DIAGRAM OF LOADS AND AXIAL STRESSES
Loads and Stresses Given in Thousands of Pounds

Fig. 21*a*



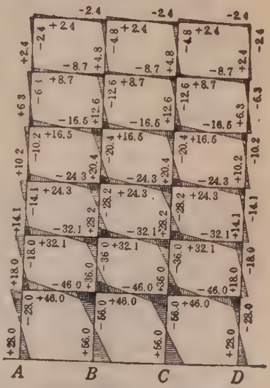
SHEAR DIAGRAM
Shears Given in Thousands of Pounds

Fig. 21*b*

ings and the corresponding normal pressure on the roof. The steel frame should be designed to resist the stresses caused by these pressures. It is inadvisable to allocate a certain amount of wind force to the walls, partitions, and floors, and the remainder to the steel framework.

The gusset-plate type of wind bracing is now used almost exclusively in the bracing of high tier buildings. The large bending moments in girders and columns do not make it economical with the use of steel. Its great advantage is that it interferes much less with the development of architectural features than other types. Different methods of calculating wind stresses have been proposed. A method in quite common use is the one here presented.

The three-aisle bent of a six-story building shown in Fig. 21*a* will be taken for illustration. Loads and direct stresses are given throughout. The shear diagram (Fig. 21*b*), gives shears and Fig. 21*c* gives bending moments for both girders and



MOMENT DIAGRAM
Moments Given in Thousands of Foot-Pounds

Fig. 21*c*

columns. It is evident that for horizontal wind pressure the bent is a cantilever beam or truss with fixed end at the foundation. Wind loads are assumed as given in the figure. It is also assumed that the point of contra-flexure of each column is at mid-height of the story, that the point of contra-flexure of each girder is at its center of span and that the joints are perfectly rigid.

The structure is regarded as equivalent to a series of independent portals. The total horizontal shear is divided equally between the number of aisles. An outer column thus takes but one-half the shear of an interior column. For equal transverse spacing of columns the direct or axial stress due to the overturning moment of the wind is taken by the outside columns. The interior columns belong to adjacent portal systems. The compressive stress from one portal balances the tensile stress from the other, making the resultant axial stress zero. The direct stress in the outer columns of any story is then equal to the wind moment at the point of contra-flexure divided by the width of the building. Numerically: in the fourth story outer columns the direct stress is

$$\frac{(2.4 \times 30) + (3.9 \times 18) + (3.9 \times 6)}{48} = 3450.$$

The difference in axial stress between any two abutting sections of an outside column acts as a shear at the point of contra-flexure of each of the row of girders of the abutting sections.

Considering in detail the fourth floor, Fig. 22, we have as the shear in each girder, 6487 − 3450 = 3037. The maximum bending moment in each girder is 3037 × 8 = 24 296 ft-lb. In the same way the maximum bending moment in a third floor girder

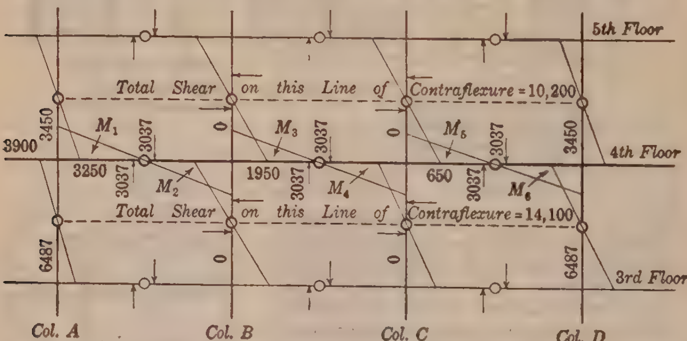


Fig. 22

is (10 500 − 6487) × 8 = 32 104. The bending moments in columns are the shears at points of contra-flexure multiplied by one-half the story heights.

Since panels are equal and each portal assumed to take one-third the shear, each leg of the portal takes one-sixth of the shear. The windward columns therefore reduce the panel loads by one-sixth, leaving five-sixths as the direct stress in the windward girder. The direct stress in the middle girder is further reduced by two-sixths, leaving the direct stress three-sixths. The leeward girder is further reduced by two-sixths, leaving a direct stress of one-sixth. Numerically: the direct compressing stesses in the fourth floor girders are, from A to B, $3900 - \frac{3900}{6} = 3250$; from B to C, $3250 - \frac{3900}{3} = 1950$; from C to D, $1950 - \frac{3900}{3} = 650$.

It will be observed that the bending moments in girders of the same floors are alike and equal to the sum of those in the outer columns above and below the girder. Their values are not affected by the width of the aisle. The bending in an outer column in any story equals the total shear in that story multiplied by the story height and the product divided by four times the number of aisles. The bending moment in an interior column equals twice that in an outer column. Of course the algebraic sum of the moments about any point is zero.

If the transverse spaces between columns are unequal, the wind shear taken by each aisle may be assumed proportional to its span. Though not theoretically correct, the direct stresses in columns will thus be brought to the outer rows. The bending moments at each end of a floor girder will be the same though they will not be alike for all the girders of a floor.

Column splices should be made in lengths of two or more stories and care taken that they are spliced sufficiently to make them continuous as retards transverse bending. Column splices and the connections of girders and beams to columns should be riveted. As a general rule unless the height of a building exceeds twice the least width the wind stresses require but little special consideration.

8. Cost Data and Building Laws

The following Costs are average values for buildings. The **cost of shop work** in cents per pound for light building work is as follows: roof trusses 1.25 to 2.50, hip and valley framing 2.50 to 5.00, beam work 0.50 to 1.00, columns from 0.60 for rolled sections to 1.75 for riveted plate and ∟ sections, plate girders 0.90 to 1.50. Steel mill-building frames, shop work about $35.00 per short ton including drafting. **Erection of Steel:** bolted work $16.00 to $24.00 per short ton, riveted work $18.00 to $30.00 for office buildings with riveted connections $20.00 per short ton is a common figure. **Corrugated steel** $12.00 to $18.00 per square, furnished and laid. **Roofs** per square: slate $15.00 to $20.00, three- or four-ply gravel, not including sheathing, $4.00 to $6.00. **Structural Steel** for building costs approximately from 5 to 8 cents per lb. in place complete, including painting. Two-inch cement mortar walls on metal fabric $2.00 to $3.00 per sq. yd.

Material. The " Standard Specifications for Structural Steel for Buildings " of the American Society for Testing Materials are in general use. Among required properties of structural steel mentioned in these specifications are: ultimate tensile strength, 55 000 to 65 000 lb. per sq. in. of cross-section; maximum phosphorus, 0.06%; minimum per cent of elongation in 8 in., 1 400 000 divided by the ultimate tensile strength, for material over 3/4 in. in thickness a deduction of 1 from the above percentage of elongation shall be made for each increase of 1/8 in. in thickness above 3/4 in. to a minimum of 18%, for material under 5/16 in. in thickness a deduction of 2.5 from the above percentage of elongation shall be made for each decrease of 1/16 in. in thickness below 5/16 in.; cold bends without fracture 180 degrees flat for plates shapes, and bars 3/4 in. or under in thickness, for thickness over 3/4 in. and including 1-1/4 in. around a pin the diameter of which is equal to the thickness of the specimen, and for material over 1-1/4 in. in thickness around a pin the diameter of which is equal to twice the thickness of the specimen.

Rivet steel shall have an ultimate tensile strength of 45 000 to 56 000 lb. per sq. in. of cross-section; maximum phosphorus, 0.06%; maximum sulfur, 0.045%; minimum per cent of elongation in 8 in., 1 400 000 divided by the ultimate tensile strength; cold bends without fracture 180 degrees flat.

Standard Specification for Steel Castings and for Gray Iron Castings have also been issued by the A. S. T. M.

Workmanship. All workmanship should be first-class in every respect and in accordance with practice followed by the best bridge shops. Before being worked, material should be thoroughly straightened by methods that will not injure it. Punching should be done accurately, but occasional inaccuracies in matching of holes may be corrected with reamer. The diameter of the punch should not be more than 1/16 in. larger, nor that of the die 1/8 in. larger than the diameter of the rivet. Rivets should be driven by pressure tools wherever possible. Abutting surfaces of compression members except where joints are fully spliced, should be planed to even bearing so as to giv close contact throughout.

Painting. Red Lead or Graphite Paints are commonly used for painting steel in buildings, one coat being applied at the shop and one after erection. Linseed oil may be used instead of paint for the shop coat. Red-lead paint should be mixed just before using and should be kept thoroughly stirred and should be applied with round, not flat, brushes. Painting should not be allowed in wet or freezing weather nor on any but clean surfaces.

Cast iron need not be painted until after erection. Steelwork for foundations to be entirely embedded in concrete need not be painted but should be free of dirt, grease or other matter which would impair the bond of the concrete.

Inspection. All inspection and tests are usually made at the option and expense of the purchaser, the contractor furnishing necessary test pieces and the free use of a testing machine.

Erection. Care should be taken that all steelwork be level and plumb before bolting or riveting and that proper provision be made for resisting stresses due to erection operations. Field connections are better when riveted than when bolted but specifications are often needlessly severe in calling for all connections to be riveted. Field connections may be bolted throughout for one-story buildings carrying no concentrated loads, shafting or cranes. Other buildings should have connections riveted for column splices, trusses, girders and beams to columns, chord splices of trusses and for bracing. In structures subject to heavy loads and vibrations rivets should be used throughout, except that purlins and girts that do not form part of the bracing may be bolted. Drift pins should be used only to bring parts together. Unfair holes should be made to match by reaming.

Building Laws. In the United States there are 80 or 100 cities having a population of over 100 000 each and about 200 cities with a population of from 30 000 to 100 000 each in which are located the great majority of buildings other than mill buildings requiring steel in their construction, the design of which must usually be in accordance with a municipal building code. The engineer should acquaint himself with these regulations before beginning

Allowable Live Loads for Floors According to Building Laws

Class of Building	Maximum Live Load, pounds per square foot			
	New York	Chicago	Philadelphia	Boston
Dwellings, Apartment Houses, Hotels. Tenement Houses or Lodging Houses	40, 60	40, 50	40	50
Office Buildings, First Floor	60	50	60	125
Office Buildings, above First Floor	60	50	60	60
Schools or Places of Instruction	75	75	75	50–100
Stables or Carriage Houses	....	40* or 100†	....	50–150
Buildings for Public Assembly	100	100	120	100
Buildings for Ordinary Stores, Light Manufacturing and Light Storage	120	100	120	125
Stores for Heavy Materials, Warehouses and Factories	....	....	150	250
Roofs, pitch 20° or less	40	25	30	....
Roofs, pitch more than 20°	30	25	30	....
Sidewalks	300	....	....	250
Public Buildings	....	....	....	75–100

* Stables less than 500 sq. ft. in area. † Stables over 500 sq. ft. in area.

his work. As the data given in handbooks may have been superseded, all information should be obtained from the building code itself. When codes are incomplete, as many are, the engineer must supply lacking data from such authorities as he deems advisable.

Cast-iron Columns to be used in construction of buildings in New York City must not have diameter less than 5 in. or thickness of metal less than 3/4-in.; nor shall they have an unsupported length of more than 70 times their least radius of gyration or 20 times their least lateral dimension or diameter except by special permission or except as modified by column formula; top and bottom flanges, seats and lugs must

Allowable Unit Loads for Columns According to Building Laws

Material of Column	Allowable Unit-load, pounds per square inch			
	New York	Chicago	Philadelphia	Boston
Medium Steel......	$16\,000 - 70\frac{l}{r}$	$16\,000 - 70\frac{l}{r}$	$\dfrac{16\,250}{1 + \dfrac{l^2}{11\,000\,r^2}}$	$\dfrac{18\,000}{1 + \dfrac{l^2}{20\,000\,r^2}}$
Cast Iron..........	$9\,000 - 40\frac{l}{r}$	$10\,000 - 60\frac{l}{r}$	$\dfrac{11\,600}{1 + \dfrac{l^2}{400\,d^2}}$	$9000 - 40\frac{l}{r}$

l = length, r = radius of gyration, d = diameter or least side, all in inches.

be of ample strength reinforced by fillets and brackets, and shall be at least 1 in. thick when finished. Column joints secured by not less than 4 bolts of at least 3/4-in. diameter. The core of a column below a joint shall be not larger than the core of the column above and the metal shall be tapered down for a distance of at least 6 in., or a joint plate of sufficient strength may be inserted to distribute the load. If the core of a column shifts more than one-fourth the thickness of the shell, the strength shall be computed by assuming the thickness of metal all around equal to the thinnest part, and the column will be condemned if this computation shows the strength to be less than that required by the New York building code. Whenever blowholes or imperfections are found which reduce the cross-sectional area of the column more than 10 per cent, such column will be condemned. Posts and columns not having one open side or back to be drilled with 3/8-in. hole in shaft to show thickness.

Wind Bracing. Building codes vary greatly in their provisions for wind bracing. The **New York** code requires that all buildings over 150 ft. in height and all buildings or parts of buildings in which the height is more than four times the minimum horizontal dimension, be designed to resist a horizontal wind pressure of 30 lb. for every square foot of exposed surface measured from the ground to the top of the structure. When the stress in any member due to wind does not exceed 50% of the stress due to live and dead loads, it may be neglected. When such stress exceeds 50% of the stress due to live and dead loads, the specified working stresses may be increased by 50% in designing such members to resist the combined stresses. The **Chicago** code specifies that all buildings shall be designed to resist a horizontal wind pressure of 20 lb. per sq. ft. and allows the same increase of working stresses as the New York code. In **Philadelphia** a wind pressure of not less than 30 lb. per sq. ft. is called for on all buildings erected in open spaces or on wharves. On tall buildings erected in built-up districts the wind pressure is to be figured for not less than 25 lb. at tenth story, 2-1/2 lb. less on each succeeding lower story, and 2-1/2 lb. additional on each succeeding upper story to a maximum of 35 lb. at the fourteenth story and above. In proportioning members subject to stresses due to wind loads the working stresses may be increased 30%. The code of **Boston** requires on a structure a wind pressure of not less than 10 lb. per sq. ft. for the first 40 ft. in height above ground, a pressure of 15 lb. per sq. ft. for from 40 to 80 ft. and 20 lb. per sq. ft. for portions more than 80 ft. above ground. An increase of 20% in working stresses is allowed for wind loads combined with other loads.

Allowable Unit Stresses According to Building Laws

Kind of Material	Allowable Unit-stress, lb. per sq. in.			
	New York	Chicago	Philadelphia	Boston
Compression:				
Rolled Steel	16 000	14 000	14 500, 16 250	18 000
Cast Steel	16 000	14 000	....	....
Wrought Iron	....	10 000	12 500	16 000
Cast Iron (in Short Blocks)	16 000	10 000	11 600	....
Steel Pins and Rivets (Bearing)	24 000	25 000	....	24 000
Tension:				
Rolled Steel	16 000	16 000	14 500, 16 250	18 000
Cast Steel	16 000	16 000	....	....
Wrought Iron	....	12 000	12 500	12 000
Cast Iron	3 000	....	2 500	....
Extreme-fiber Stress, Bending:				
Rolled-steel Beams	16 000	16 000	16 250	18 000
Rolled-steel Pins, Rivets and Bolts	20 000	25 000	22 000	27 000
Riveted Steel Beams (Net Flange Section)	16 000	16 000	....	....
Cast Iron, Compression Side	16 000	10 000	....	10 000
Cast Iron, Tension Side	3 000	3 000	....	4 000
Shear:				
Steel Web Plates	10 000	10 000	8 800, 10 000	12 000
Steel Shop Rivets and Pins	12 000	12 000	11 000	13 500
Steel Field Rivets and Pins	8 000	10 000	8 800	....
Steel Field Bolts	7 000	....	8 800	10 000
Cast Iron	3 000	2 000	....	2 000

Under Philadelphia, the first value is for mild and the second for medium steel.

A number of provisions besides those for floor and wind loads affect the design of the steelwork. **Fireproofing around exterior columns** in New York must be 8 in. thick on the outer face while in Chicago 4 in. will answer. **Thickness of enclosure walls** when carried by the steelwork is specified as 12 in. for New York, Chicago and many other cities. A large number have a thickness of 12 in. for a top section of 75 ft. and a greater thickness for sections below. Still other variations are found.

Limitations upon the **heights of buildings** ranging from 120 to 250 ft. are placed in many cities. For fireproof commercial buildings 2-1/2 times the width of the adjoining street is quite common. A number of cities have limits of 150 to 175 ft. **Special requirements** must not be overlooked. One such, found in a few codes is that prohibiting the spacing of floorbeams of fireproof buildings more than 5 ft. on centers for stores, warehouses and factory buildings and 8 ft. on centers for other buildings. This eliminates "long-span" construction. Other requirements are confined to the individual code in which they are found. As stated previously, the structural engineer should be acquainted with the municipal code by which the construction of his building is governed.

GENERAL DATA FOR BRIDGES

9. Dead Loads

Simple Bridges include all beam, girder or truss bridges supported at both ends only. **Continuous** bridges include those which continue unbroken over two or more spans and which under vertical loads have vertical reactions. A cantilever beam is one that is fixed at one end and unsupported at the other and a **cantilever** bridge is therefore one having one or more cantilevers. An **arch** bridge may be defined as one which under vertical loads produces inclined pressures on its supports, thus including **suspension** bridges which have the end reactions outward as well as the more common forms of arched bridges which have reactions upward and inward.

The first important bridge of metal was a cast-iron arch of 100-ft. span built at Coalbrookdale in England in 1779, and the first iron truss bridge in the United States is believed to be the one built at Frankfort, N. Y., in 1840. The largest cast-iron bridge in America was built in 1866, in Chestnut St., Philadelphia, its longest span being an arch of 185 ft. Steel was first used in bridge construction in the arches of the Eads bridge at St. Louis in 1874. The modern steel truss span is the outgrowth of the wooden and the combination truss, the latter being a truss in which some members are of wood and some of iron.

A Deck Bridge is one in which the floor system is supported on or near the top chord of the trusses or girders, and a **through bridge** is one in which the floor is on or near the lower chord, but the depth of which is sufficient to permit the use of overhead bracing. A **half through** bridge is a **through** bridge which is so shallow, or whose floor is so near the top chord that no overhead bracing can be used. Bridges are **square** or **skew**, the former including those having the ends of the span at right angles to the length, hence having the trusses parallel and of the same length; while skew spans have one or both ends at oblique angles with the length and the trusses of a span may be of the same or different lengths.

Economical Types of simple bridges include rolled I-beam bridges for short spans up to 40 or 50 ft.; plate girders for longer spans up to 65 or 70 ft. for highway and up to 100 or 125 ft. for railway bridges: riveted half through trusses for highway bridges between 50 and 100 ft; riveted deck or through trusses up to about 150 ft. for highway, and to about 200 ft. for railway bridges; pin-connected trusses for longer spans.

Loads on Bridges include the dead or fixed, the live or movable, impact due to live loads, wind, and lateral dynamic forces due to irregularities of track, centrifugal force due to electric cars or trains moving on curved tracks, snow, longitudinal forces due to friction, and forces due to changes of temperature.

The Dead Load of railroad bridges includes the weight of the girders and trusses, of the top, bottom and transverse lateral bracing, and of the floor system which may be of either the open or solid type. The following weights are approximate only but are sufficient for use in preliminary designing and they should in every case be checked from the finished design. For ordinary open floors the weight of track is from 400 to 600 lb. per ft. for one track including rails, guard timbers, ties and all fastenings. The weight of steel per foot of span in single-track railroad bridges of the several types named below, having open floors and designed for Cooper's Class E-40, E-50 and E-60 loadings (Art. 10) in accordance with the American Railway Engineering Association Specifications, 1925, is approximately expressed by:

Deck plate girders. . . . . $w = k\,(13\,l + 100)$
Through plate girders. . . . . $w = k\,(15\,l + 500)$
Through pin-connected or riveted trusses . . . . $w = k\,(9\,l + 700)$

where l = length of span; k = 0.88 for E–40; k = 1.00 for E–50 and k = 1.12 for E–60 loadings.

The weight of steel in a double-track girder bridge is about 100% greater, and in a double-track truss bridge from 80 to 90% greater than for a similar single-track structure.

The Total Weight of Material in one single-track span is found by adding the weight of the track, 400 to 600 lb. per ft. to the weight of steel as given by the above formulas and multiplying by the span in feet. A **dead panel load** on one truss is the weight on one panel point from one panel length of truss and floor system. For a single-track bridge the panel load on one truss is found by multiplying the total dead load per foot of truss by the panel length. For example, if a single-track through bridge designed for a Cooper E–50 live load, has a span of 200 ft. divided into 8 equal panels of 25 ft. the above formula, $w = 9\,l + 700$ gives 2500 lb. per ft. as the weight of steel in the bridge. If the track weighs 450 lb. per foot of bridge the total load per foot of one truss = 1/2(2500 + 450) = 1475 lb., and the panel load per truss is 1475 × 25 = 36 875 lb. It is customary to assume two-thirds of the panel load on the loaded and one-third on the unloaded chord panel points, but a better method is to divide the weight of the truss equally between the upper and lower chords and apply the weight of track and floor after the latter has been obtained from the floor design at the loaded chord.

Highway Bridge Dead Loads include in addition to the weight of the sidewalks, if any, the weight of the roadway and the steel in the stringers, floor beams, trusses and bracing. Highway bridges vary greatly in size and design and no formula will give accurate results for the dead weight. The floor is designed first and its weight computed. The weight of the steel in the trusses or girders and the lateral system may be assumed. After the sections of the main members have been found for the live, impact and trial dead load, the assumed dead load may be checked, a percentage being added for details. More than one or two trials will seldom be necessary. After the bridge has been completely designed an accurate final estimate of the dead weight should be made and the sections corrected if necessary. For computing the loads on highway bridges the following **weights of materials** in pounds per cubic foot will be found useful: timber, creosoted, 60; oak, untreated, 54; pine, untreated, 48; concrete, 150; paving brick, 150; granite blocks, 160; asphalt, 135; macadam, 130; sand or earth, 100; stone ballast, 100; steel, 490; cast iron, 450. The following weights are in pounds per linear foot: ordinary latticed sidewalk railings, 30; rails and fastenings, street railway tracks, 100 per track; rails and fastenings, railroad tracks, 150 per track.

Snow Loads for railroad bridges are usually negligible, but for highway bridges in cold climates it is necessary to consider the snow load in some cases, especially for drawbridges when open. The maximum allowance is 20 lb. per sq. ft.

10. Live Loads

Live or Movable Loads ordinarily used for the design of railroad bridges consist of one or two locomotives followed by a uniform load representing the weight of the heaviest cars. To allow for future increase the weights specified by this loading should be greater than the heaviest locomotives and cars in service on the road for which the bridge is designed. Wheel spacings and weights of actual rolling stock are so variable that typical loads are largely used, those specified by Cooper being most common. Fig. 23 shows Cooper's

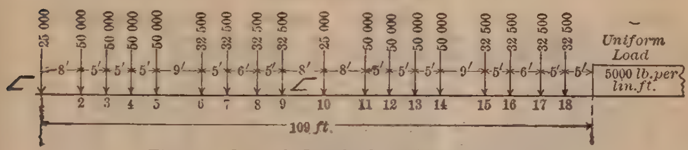


Fig. 23. Cooper's Standard Loading, Class E-50

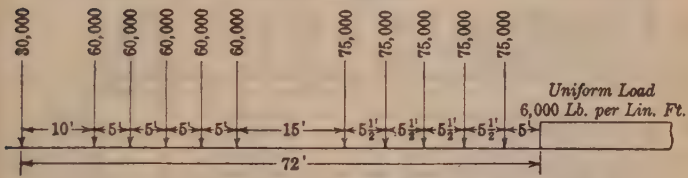
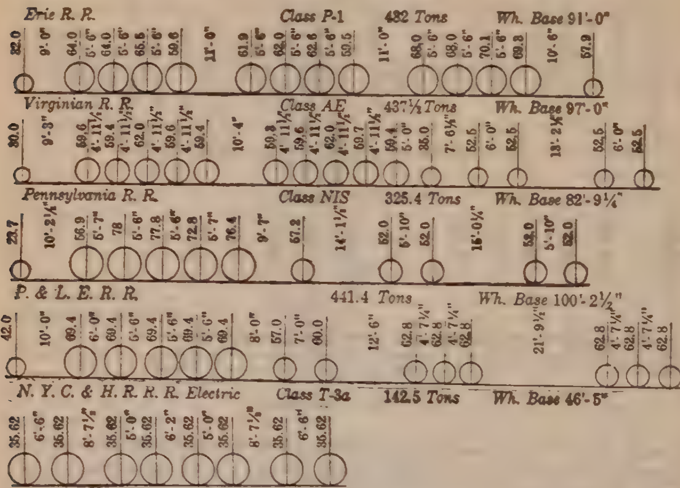


Fig. 23a. Class M-60

Class E-50 standard loading, consisting of two consolidation locomotives followed by a uniform load of 5000 lb. per lin. ft. The loads given are axle loads. Cooper's standards provide for heavier traffic by using Classes E-60 and E-55 and for lighter traffic by using Classes E-45, E-40, and E-30, each of which has the same wheel spacing as Class E-50, but the weights, and hence all functions of these weights such as reactions and stresses, are less, being proportional to the class number; thus the weights of E-60 are 12/10 those of E-50 and of E-45 are 9/10 those of E-50; E-40, 8/10 of E-50; E-30, 6/10 of E-50. At the present time, 1929, most of the important bridges on main lines are designed for E-60, and, in some cases, especially for floor systems, for as high as E-75 to E-90. In the latter case higher unit stresses than those ordinarily allowed are used.

The Cooper System of Loading was introduced about thirty years ago when the heaviest loads were represented by E-40. Since then there has been a continuous change in type, as well as in weight, of locomotives. Fig. 24 shows four of the heaviest existing locomotives arranged in the general order of their stress-producing effect. The first two are of the Mallet, or articulated, type, with 12 and 10 driver axles, respectively; the last two are the Santa Fé type, with 5 driver axles. No. 1 produces maximum stresses in spans above, roughtly, 150 ft.; No. 2 in spans between about 70 and 150 ft.; and No. 3 in shorter spans. The stresses produced by these locomotives differ from those obtained by the use of any single Cooper loading. Thus No. 1 will produce a center moment in a 100-ft. span equivalent to that from E-72; in a 200-ft. span, to E-66; in a 300-ft. span, to E-63; 400-ft. span, to E-62; 2000-ft. span, to E-60. Again, a 140-ft. span designed for No. 3 has ratings varying between E-54.8, for the center chord members, to E-69.5, for the center panel diagonals, and E-75 for the floor beams.

To overcome these inconsistencies, and more nearly to fit modern loading conditions, Dr. D. B. Steinman has proposed a new conventional wheel load diagram, M–60, (Fig. 23a), which very closely produces the same maximum stresses in all spans as those produced by the heaviest existing locomotives. This proposed loading is a composite of the first three locomotives shown in



All loads are in thousand pounds per axle.

Fig. 24. Wheel Diagrams of Heavy Locomotives in Service.

Fig. 24. While it is of the Mallet type, it conforms very closely, in stress-producing effects, with the Santa Fé, Decapod, and other heavy locomotives. As in the case of the Cooper E series, lighter or heavier loadings can be provided by using a different class number, such as M-50, or M–70, and changing weights and stresses proportionately.

Equivalent Uniform Live Loads which will give stresses as close as possible to those computed for actual wheel loads are sometimes used, the most common for computing chord stresses being the uniform load per foot which produces a moment at the center of the span equal to the maximum moment produced on the span by two locomotives followed by a uniform train load. The equivalent uniform load for computing web stresses is the uniform load per foot which produces an end shear equal to that produced by the locomotive loading. The table on p. 1165 gives for Cooper's E–50, maximum moments; end, quarter point and center shears; pier reactions for equal spans; and the equivalent uniform loads based upon center moments, end shears and pier reactions. The table on pages 1168 and 1169 gives the same data for M-10 loading. For M–60 multiply by 6.

The latest type of passenger **electric locomotives**, Class T–3a, used by the N. Y. C. & H. R. R. R., weighs about 285 000 lb. distributed nearly equally on 8 axles, as shown in the last wheel diagram in Fig. 24.

Maximum Moments, Shears, Floor Beam or Pier Reactions, and Equivalent Uniform Loads for Cooper's Class E-50

For One Rail

Span, ft.	Max. Moment, M	Max. Shears, V			Max. Pier React. R	Equivalent Uniform Load		
		End	1/4 Pt.	Cent.		For Max. Moment	For Max. End Shear	For Max. Pier React.
10	70.4	37.5	25.0	12.5	50.0	5625	7500	5000
11	82.1	40.9	26.1	13.6	54.5	5430	7450	4960
12	100.0	43.8	27.1	14.6	58.4	5560	7290	4860
13	118.8	46.2	27.9	15.4	61.6	5625	7100	4740
14	137.5	48.2	29.5	16.2	65.2	5610	6900	4660
15	156.3	50.0	31.3	16.6	68.3	5560	6670	4555
16	175.0	53.1	32.9	17.1	71.1	5470	6640	4445
17	193.8	55.9	34.3	17.3	73.5	5360	6580	5325
18	212.5	58.3	35.4	17.4	75.9	5250	6490	4210
19	233.3	60.5	36.5	17.5	78.6	5170	6370	4140
20	257.9	62.5	37.5	17.5	81.9	5160	6250	4100
21	282.5	64.3	39.2	18.1	84.9	5100	6125	4040
22	307.1	65.9	40.9	18.8	87.6	5050	5990	3980
23	331.8	67.4	42.4	19.3	90.2	5000	5860	3920
24	356.5	69.3	43.8	19.8	92.4	4940	5775	3850
25	381.3	71.0	45.0	20.2	94.6	4880	5680	3780
26	406.0	72.6	46.1	20.6	97.1	4820	5585	3735
27	430.8	74.0	47.2	21.1	100.1	4750	5480	3710
28	456.9	75.5	48.2	21.4	102.8	4690	5390	3670
29	485.0	76.9	49.1	21.8	105.4	4625	5300	3635
30	513.0	78.8	50.0	22.1	107.9	4560	5255	3595
31	541.1	80.5	50.9	22.7	110.6	4500	5195	3570
32	569.3	82.1	51.8	23.4	113.7	4450	5130	3555
33	597.4	83.7	52.5	24.0	116.7	4390	5070	3535
34	625.8	85.1	53.5	24.6	119.4	4330	5005	3510
35	653.8	86.5	54.4	25.1	122.0	4275	4945	3485
36	685.8	88.2	55.1	25.8	124.4	4240	4900	3455
37	717.9	89.8	56.0	26.2	126.9	4200	4855	3430
38	750.0	91.4	56.7	26.6	129.7	4170	4810	3410
39	783.3	92.9	57.5	27.1	132.3	4135	4765	3390
40	819.5	94.3	58.5	27.5	135.0	4100	4715	3375
41	855.8	96.0	59.4	27.9	137.6	4070	4680	3355
42	892.0	97.6	60.2	28.3	140.2	4040	4650	3340
43	928.3	99.2	61.1	28.6	142.9	4010	4615	3325
44	964.5	100.7	61.9	29.0	145.6	3980	4580	3310
45	1000.8	102.1	62.6	29.3	148.3	3960	4540	3295
46	1037.3	103.5	63.4	29.6	150.9	3930	4500	3280
47	1073.3	104.9	64.2	29.9	153.4	3900	4465	3265
48	1109.5	106.3	65.1	30.2	156.0	3870	4430	3250
49	1148.5	107.7	66.0	30.6	158.5	3840	4395	3235
50	1188.6	109.0	66.8	31.1	161.0	3810	4360	3220

For units see note at end of table.

Maximum Moments, Shears and Uniform Loads—Continued

Span, ft.	Max. Moment, M	Max. Shears, V			Max. Pier React. R	Equivalent Uniform Load		
		End	1/4 Pt.	Cent.		For Max. Moment	For Max. End Shear	For Max. Pier React.
51	1228.9	110.4	67.6	31.5	163.6	3790	4330	3210
52	1269.0	111.8	68.5	31.9	166.6	3770	4300	3205
53	1309.2	113.1	69.2	32.3	169.6	3750	4270	3200
54	1351.8	114.5	70.1	32.6	172.5	3730	4240	3195
55	1396.1	115.8	71.0	33.0	175.4	3710	4215	3195
56	1440.5	117.2	71.8	33.3	178.5	3690	4185	3195
57	1484.9	118.5	72.7	33.6	181.8	3670	4160	3195
58	1529.2	119.8	73.5	34.0	185.1	3650	4130	3195
59	1576.2	121.2	74.4	34.4	188.4	3630	4110	3195
60	1624.5	122.5	75.2	34.9	191.5	3610	4080	3195
61	1672.9	123.9	76.0	35.2	194.7	3600	4060	3190
62	1721.2	125.2	76.6	35.6	197.7	3585	4040	3190
63	1769.5	126.6	77.4	36.0	200.7	3570	4020	3185
64	1819.4	128.2	78.5	36.4	203.6	3560	4005	3180
65	1871.9	129.7	78.8	36.8	206.7	3550	3990	3180
66	1924.4	131.2	79.5	37.1	209.7	3535	3975	3175
67	1976.9	133.0	80.3	37.5	212.7	3520	3970	3175
68	2029.4	134.8	81.0	37.8	215.6	3510	3965	3170
69	2081.9	136.5	81.7	38.1	218.5	3500	3955	3165
70	2134.4	138.1	82.4	38.4	221.3	3485	3945	3160
71	2186.6	139.8	83.1	38.8	224.1	3475	3940	3155
72	2241.2	141.7	83.8	39.2	226.9	3460	3935	3150
73	2292.4	143.5	84.4	39.6	229.6	3450	3930	3145
74	2349.0	145.3	85.0	40.0	232.4	3440	3925	3140
75	2407.3	147.1	85.7	40.4	235.2	3430	3920	3135
76	2465.0	148.8	86.5	40.8	238.0	3420	3915	3130
77	2523.9	150.5	87.4	41.1	240.7	3410	3910	3125
78	2581.2	152.1	88.2	41.5	243.3	3400	3900	3120
79	2640.4	153.8	88.9	41.7	245.9	3385	3895	3115
80	2700.6	155.3	89.6	42.1	248.6	3375	3885	3110
81	2759.6	157.0	90.4	42.5	251.1	3370	3875	3100
82	2820.9	158.6	91.2	43.0	253.6	3360	3870	3090
83	2883.1	160.3	92.1	43.4	256.1	3350	3860	3085
84	2945.4	161.8	93.0	43.7	258.7	3345	3855	3080
85	3008.6	163.4	93.9	44.1	260.8	3335	3850	3070
86	3074.5	165.1	94.3	44.5	263.0	3325	3840	3060
87	3138.3	166.8	95.7	44.9	265.6	3320	3830	3055
88	3205.3	168.4	96.5	45.2	268.3	3310	3825	3050
89	3269.9	170.0	97.4	45.6	270.8	3300	3820	3040
90	3338.1	171.5	98.4	45.9	273.2	3295	3810	3035

For units see note at end of table.

Maximum Moments, Shears and Uniform Loads—Continued

Span, ft.	Max. Moment, M	Max. Shears, V			Max. Pier React. R	Equivalent Uniform Load		
		End	1/4 Pt.	Cent.		For Max. Moment	For Max. End Shear	For Max. Pier React.
91	3403.7	173.1	99.4	46.2	275.6	3290	3805	3030
92	3470.9	174.7	100.4	46.6	278.0	3280	3800	3020
93	3539.3	176.4	101.2	46.9	280.3	3270	3795	3015
94	3606.6	178.0	102.1	47.3	282.7	3265	3790	3005
95	3674.3	179.5	103.1	47.5	285.1	3260	3780	3000
96	3743.1	181.0	104.1	47.9	287.5	3250	3770	2995
97	3811.2	182.7	105.1	48.1	289.7	3240	3765	2985
98	3883.1	184.3	106.2	48.5	292.0	3235	3760	2980
99	3952.9	186.0	107.2	48.9	294.2	3225	3755	2970
100	4024.9	187.5	108.2	49.2	296.5	3220	3750	2965
101	4097.0	189.0	109.1	49.5	298.6	3220	3745	2955
102	4169.9	190.6	110.1	49.9	300.8	3220	3740	2950
103	4263.3	192.1	111.0	50.1	303.0	3215	3730	2940
104	4344.0	193.6	111.9	50.5	305.3	3215	3725	2935
105	4422.0	195.1	112.7	50.7	307.5	3215	3715	2930
106	4500.4	196.6	113.6	51.1	309.8	3215	3710	2920
107	4583.3	198.1	114.5	51.5	312.0	3215	3700	2915
108	4681.6	199.5	115.5	51.7	314.2	3210	3695	2910
109	4773.0	201.0	116.4	52.0	316.3	3210	3690	2900
110	4858.5	202.5	117.4	52.3	318.5	3210	3680	2895
111	4947.7	204.0	118.2	52.5	320.7	3210	3675	2890
112	5033.6	205.5	119.1	52.7	322.8	3210	3670	2880
113	5123.8	207.0	120.0	53.1	324.9	3210	3665	2875
114	5215.0	208.4	121.0	53.5	327.0	3210	3655	2870
115	5306.2	209.9	121.9	53.9	329.0	3210	3650	2860
116	5398.5	211.3	122.9	54.2	331.1	3210	3645	2855
117	5486.9	212.8	123.7	54.6	333.3	3205	3640	2850
118	5579.7	214.2	124.6	54.9	335.6	3205	3630	2845
119	5673.5	215.7	125.5	55.3	337.8	3205	3625	2840
120	5767.6	217.1	126.4	55.6	340.0	3205	3620	2835
121	5858.1	218.6	127.2	55.9	342.2	3205	3610	2830
122	5953.4	220.0	128.1	56.2	344.5	3205	3605	2825
123	6045.2	221.4	129.0	56.5	346.7	3200	3600	2820
124	6146.7	222.8	130.0	57.0	349.0	3200	3595	2815
125	6245.5	224.2	130.9	57.5	351.2	3200	3590	2810
150	8827.9	259.2	152.2	68.0	406.7	3140	3455	2710
175	11690.6	293.1	172.9	78.2	464.6	3055	3350	2655
200	14841.2	326.3	191.8	88.0	523.8	2965	3265	2620
250	21990.6	391.5	229.6	106.3	644.0	2815	3130	2575

For span lengths L greater than 284 ft., max. moment = **5/16 L^2 + 2375.**
For span lengths L greater than 100 ft., max. end shear = **1.25 L + 90 − 2750/L.**
For span lengths L each greater than 142 ft., max. pier reaction = **2.5 L + 4750/L.**

Moments are in thousand pounds per foot; shears are given in thousand pounds; pier reactions are given in thousand pounds and are for piers between two spans each equal to the tabulated span; equivalent loads are in pounds per linear foot.

Maximum Moments, Shears, Reactions, and Equivalent Uniform Loads. M-10 Loading

Per Track

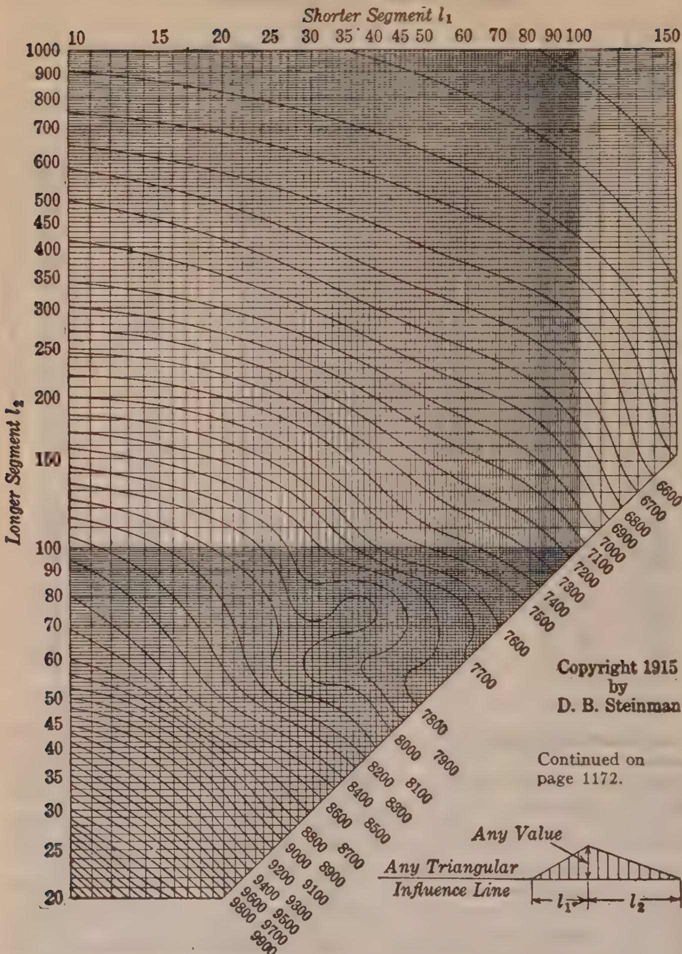
Span, ft.	Maximum Moment, ft.-lb.	Maximum Shear, lb.	Maximum Floor Beam Reaction, lb.	Equivalent Uniform Load		
				M	S	R
10	35 156	18 750	25 000	2813	3750	2500
11	41 051	20 455	28 409	2714	3719	2583
12	50 000	21 875	31 250	2778	3646	2604
13	59 375	23 077	33 654	2811	3550	2589
14	68 750	24 107	35 714	2806	3444	2551
15	78 125	25 000	37 500	2778	3333	2500
16	87 500	26 562	39 094	2734	3320	2443
17	96 850	27 941	40 559	2681	3287	2386
18	106 250	29 167	41 917	2624	3241	2329
19	116 610	30 263	43 184	2584	3186	2273
20	128 910	31 250	44 375	2578	3125	2219
21	141 220	32 738	45 500	2562	3118	2167
22	156 250	34 091	46 568	2583	3099	2117
23	171 880	35 326	47 587	2599	3072	2069
24	187 500	36 458	48 562	2604	3038	2023
25	203 120	37 500	49 500	2600	3000	1980
26	218 750	38 481	50 788	2589	2960	1953
27	234 380	39 426	52 019	2572	2920	1927
28	250 000	40 339	53 196	2551	2881	1900
29	265 620	41 224	54 328	2527	2843	1873
30	281 250	42 083	55 417	2500	2806	1847
31	296 940	42 919	56 790	2472	2769	1832
32	312 780	43 734	58 109	2444	2733	1816
33	328 770	44 530	59 379	2415	2699	1799
34	344 880	45 309	60 603	2387	2665	1782
35	361 150	46 071	61 786	2359	2633	1765
36	377 550	46 819	63 347	2331	2601	1760
37	394 110	47 716	64 851	2303	2579	1753
38	410 790	48 276	66 303	2276	2541	1745
39	427 620	48 987	67 705	2249	2512	1736
40	444 590	49 688	69 062	2223	2484	1727
42	478 620	51 059	72 131	2171	2431	1717
44	513 880	52 398	75 011	2124	2382	1705
46	549 340	53 707	77 728	2077	2335	1690
48	585 390	54 990	80 302	2033	2291	1673
50	622 060	56 250	82 750	1991	2250	1655
52	660 270	57 788	85 279	1954	2223	1640
54	702 280	59 444	87 694	1927	2202	1624
56	744 770	61 161	90 009	1900	2184	1607
58	787 950	62 931	92 233	1874	2170	1590
60	833 550	64 583	94 375	1852	2153	1573

Maximum Moments, Shears, Reactions, and Equivalent Uniform Loads. M-10 Loading—Continued

Span, ft.	Maximum Moment, ft.-lb.	Maximum Shear, lb.	Maximum Floor Beam Reaction, lb.	Equivalent Uniform Load		
				M	S	R
62	881 910	66 129	96 605	1835	2133	1558
64	930 860	67 578	98 758	1818	2112	1543
66	980 420	69 015	100 840	1801	2091	1528
68	1 032 690	70 441	102 860	1787	2072	1513
70	1 086 320	71 786	104 820	1774	2051	1497
72	1 144 230	73 056	106 730	1766	2029	1473
74	1 202 850	74 257	108 590	1757	2007	1467
76	1 263 040	75 395	110 400	1749	1984	1453
78	1 326 600	76 474	112 170	1744	1961	1438
80	1 390 950	77 500	113 910	1739	1938	1424
82	1 455 720	78 476	115 730	1732	1914	1411
84	1 521 520	79 405	117 510	1725	1891	1399
86	1 587 760	80 291	119 250	1717	1867	1387
88	1 654 710	81 136	120 960	1709	1844	1375
90	1 722 230	81 944	122 640	1701	1821	1363
92	1 790 320	82 722	124 290	1692	1798	1351
94	1 859 160	83 457	125 910	1683	1776	1340
96	1 928 500	84 745	127 510	1674	1766	1328
98	1 998 520	86 066	129 080	1665	1756	1317
100	2 071 830	87 375	130 620	1658	1747	1306
125	3 036 310	102 900	149 920	1555	1646	1199
150	4 109 130	117 417	170 770	1461	1566	1139
175	5 286 100	131 360	192 800	1381	1501	1102
200	6 555 170	144 940	215 580	1311	1449	1078
250	9 370 310	171 450	262 460	1199	1372	1050

An Exact Equivalent Uniform Load is one which gives exactly the same stress in a member as the actual wheel loading. The following charts and tables, based upon Cooper and Steinman loadings, were constructed by Dr. D. B. Steinman. Fig. 25 is a chart from which may be obtained the exact equivalent load, based upon Cooper's E–60 loading, for any stress function having a triangular influence line the lengths of whose segments are l_1 and l_2. Since, for end reactions or intermediate shears in a girder without floor beams the influence diagram is a triangle with $l_1 = 0$, instead of extending the chart the values of the corresponding equivalent uniform loads are given in a separate table. Fig. 25*M* is a similar chart for the *M*–60 loading. Fig. 26 gives the governing wheel of a Cooper loading for maximum of any stress having a triangular influence line. Fig. 26*M* gives the same information for an *M* loading. The use of these charts will be explained in Arts. 17, 24, 25 and 26 in connection with stresses in girders and trusses.

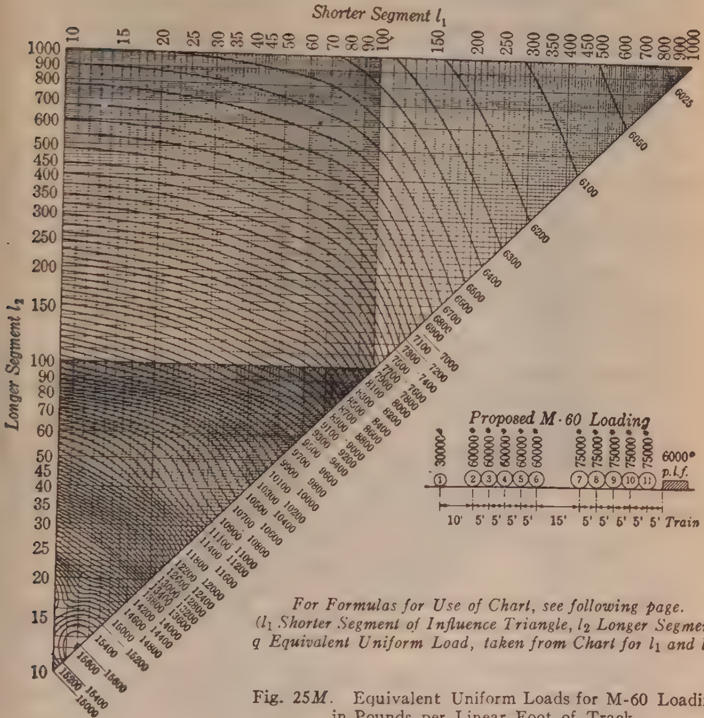
A **Conversion Table,** (Fig. 27*a*), is given on page 1175 by which stresses computed for an E loading may be compared with those for an *M* loading. Thus, for a 100-ft. span designed for E–60, the center moment is equivalent to an *M*



Continued on page 1172.

Fig. 25. Equivalent Uniform Loads for Cooper's E-60 in Pounds per Linear Foot of Track

(.776 × 60) = M–46.56; the end shear to an M (.858 × 60) = M–51.48. Conversely, if the span is designed for M–60, the center moment is equivalent to that from an E $\left(\frac{60}{.776}\right)$ = E–77.40; the end shear from E–70; the pier reaction from E–66. A still more useful conversion chart, applicable to any



For Formulas for Use of Chart, see following page.
(l_1 Shorter Segment of Influence Triangle, l_2 Longer Segment, q Equivalent Uniform Load, taken from Chart for l_1 and l_2)

Fig. 25*M*. Equivalent Uniform Loads for M-60 Loading in Pounds per Linear Foot of Track.
Continued on page 1174

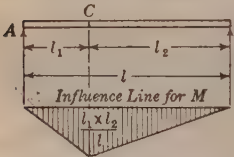
stress whose influence line is a triangle, is given in Fig. 27*b*. It is recommended that all stresses, whether from an E or an M loading, be computed from the equivalent uniform load corresponding to the shorter and longer segments, l_1 and l_2 respectively, of the influence line.

The Live Loads for a Highway Bridge should be chosen to correspond with the location of the structure, suitable provision being made for future traffic. Most of the large cities have their own specifications. In many states the design of highway bridges must conform with the specifications issued by the Highway Commissions of those states. The Final Report, 1924, of the Special Committee of the Am. Soc. C. E. on Specifications for the Design of

Formulas for Use of Fig. 25 and 25M

l_1 = Shorter segment of influence triangle.
l_2 = Longer segment.
q = Equivalent uniform load, taken from chart for l_1 and l_2.

BENDING MOMENTS
(and Chord Stresses)

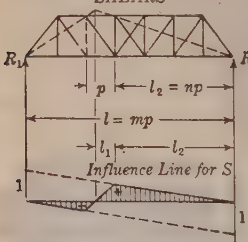


Max. Moment at C in Span AB.

$$M = 1/2\, q l_1 l_2$$

(l_1 and l_2 = The Two Segments of the Span)

SHEARS

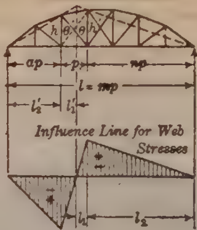


Max. Shear in Truss Panel:

$$V = \frac{1/2\, q l_1 l_2}{p}$$

$$\left(l_1 = \frac{l_2}{m-1}\right) \quad (l_2 = np).$$

WEB STRESSES



$$l_1 = \left(\frac{m}{ar + b} - 1\right) l_2 \qquad r = \frac{h'}{h}$$

For Member shown full:

Max. + Web Stress $\Big\}$ $= 1/2q\, l_1 l_2 \frac{\sec\theta}{rp}$;

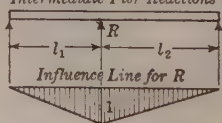
Max. − Web Stress $\Big\}$ $= 1/2\, q' l'_1 l'_2 \frac{\sec\theta}{p}$.

For Member shown dotted:

Max. − Web Stress $\Big\}$ $= 1/2\ q_1 l_2 \frac{\sec\theta'}{p}$;

Max. + Web Stress $\Big\}$ $= 1/2\, q' l'_1 l'_2 \frac{r.\sec\theta'}{p}$.

FLOORBEAM REACTIONS
Intermediate Pier Reactions

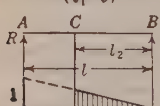


Max. Intermediate Reaction:

$$R = 1/2\, q\,(l_1 + l_2)$$

(l_1 and l_2 = The Two Adjoining Spans.)

$(l_1 = 0)$



Max. Shear at C: $\ {}^*V = 1/2\, q l_2 \frac{l_2}{l}$

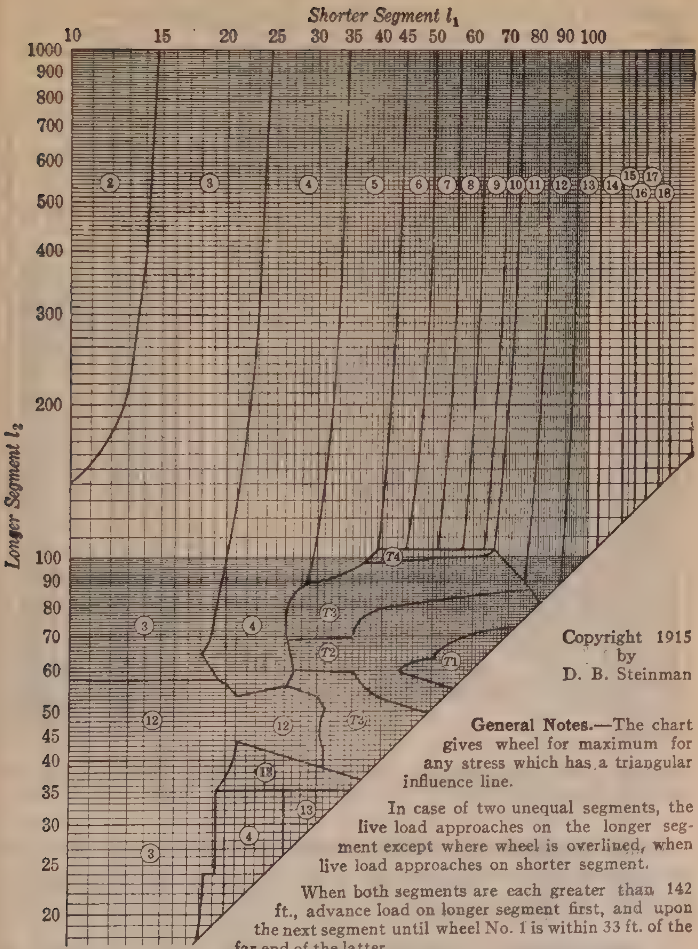
Max. Reaction at A: $(l_2 = l)$.

$$R = 1/2\, q l$$

l_2	q	l_2	q	l_2	q	l_2	q	l_2	q	l_2	q	l_2	q	l_2	q	l_2	q
1000	6418	825	6504	650	6634	475	6852	300	7295	150	8300	90	9140	55	10100	20	15000
975	6429	800	6519	625	6658	450	6896	275	7400	140	8420	85	9230	50	10450	15	16000
950	6440	775	6535	600	6683	425	6945	250	7520	130	8540	80	9315	45	10900	10	18000
925	6451	750	6552	575	6711	400	7000	225	7660	120	8680	75	9410	40	11310	5	24000
900	6463	725	6571	550	6741	375	7060	200	7830	110	8840	70	9470	35	11830	..	.....
875	6476	700	6591	525	6775	350	7130	175	8035	100	9000	65	9600	30	12650	..	.....
850	6490	675	6612	500	6812	325	7205	160	8185	95	9060	60	9800	25	13625	..	.....

* Note: In the formula for "V" the effect of wheel ○ is neglected. To correct for this, subtract $30\,000\left(1 - \frac{l_2 + 8}{l}\right)$ from the above value of V.

Fig. 25*a*. Equivalent Uniform Loads for Cooper's E-60, for Shears and Reactions ($l_1 = 0$), per Linear Foot of Track.



General Notes.—The chart gives wheel for maximum for any stress which has a triangular influence line.

In case of two unequal segments, the live load approaches on the longer segment except where wheel is overlined, when live load approaches on shorter segment.

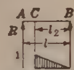
When both segments are each greater than 142 ft., advance load on longer segment first, and upon the next segment until wheel No. 1 is within 33 ft. of the far end of the latter.

Fig. 26. Wheel Determining Maximum Stress. Cooper's Loadings

Steel Highway Bridges, and the Standard Specifications, 1926, for Highway Bridges of the A. A. S. H. O. (American Association State Highway Officials) represent the best modern practice which is being followed in many states. Railway and highway bridge specifications differ little in unit stresses and details of design, workmanship and materials. The principal difference lies in loadings and impact.

	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	100	110
1000 to 300	2	2	3	3	3	4	4	5	5	6	6	7	7	8	8	9	9	9	10	10	11
290 to 250	2	2	3	3	3	4	4	5	5	6	7	7	7	8	8	9	9	9	10	10	11
240	2	2	3	3	3	4	4	5	6	6	7	7	7	8	8	9	9	9	10	10	11
230	2	2	3	3	3	4	4	5	6	6	7	7	7	8	8	9	9	9	10	10	11
220	2	2	3	3	3	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	11
210	2	2	3	3	3	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	11
200 to 160	2	2	3	3	3	4	5	5	6	6	7	7	8	8	8	9	9	9	10	10	11
150	2	2	3	3	3	4	5	5	6	6	7	7	8	8	8	9	9	9	10	10	11
140	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	11
130	2	2	3	3	4	4	5	5	6	7	7	8	8	8	9	9	9	10	10	10	11
120	2	2	3	3	4	4	5	5	6	7	7	8	8	8	9	9	9	10	10	10	11
110	2	2	3	4	4	4	5	5	6	7	7	8	8	8	9	9	9	10	10	10	11
100	2	2	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	10	11
95	2	2	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	10	
90	$\overline{11}$	$\overline{11}$	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10		
85	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	7	7	7	8	8	9	9	9	9	10			
80	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	7	8	8	9	9	9	10				
75	11	11	11	10	10	10	10	10	10	10	9	8	8	9	9	9					
70	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	8	8	9	9						
65	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	9							
60	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	9	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	8								
55	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	9	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$									
50	7	7	$\overline{10}$	$\overline{10}$	9	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$	$\overline{8}$										
45	7	7	8	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$	$\overline{8}$											
40	7	7	8	8	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$												
35	7	7	8	8	9	$\overline{9}$	$\overline{9}$	$\overline{8}$													
30	7	7	8	9	9	9	$\overline{9}$														
25	7	7	8	9	9	9															
20	7	7	8	9	9																
15	7	7	8	9																	
10	7	7	8																		
5	7	7																			

The Shorter Segment is Ahead followed by the Longer One except where Wheel is Overlined. For l_1>117.5 place Head of Uniform Load 117.5 *from Head of Span*



Max. Shear at C. $S = \frac{1}{2} q l_2 \frac{l_2}{l}$
Max. Reaction of A
$(l_2 = l)$ $R = \frac{1}{2} q l$

l_2	q	l_2	q
1000	6612	250	8230
975	6627	225	8441
950	6643	200	8696
925	6660	175	9007
900	6678	160	9229
875	6696	150	9398
850	6716	140	9574
825	6737	130	9772
800	6759	120	9990
775	6783	110	10227
750	6808	100	10485
725	6834	95	10620
700	6863	90	10926
675	6894	85	11273
650	6926	80	11625
625	6962	75	11973
600	7000	70	12306
575	7041	65	12604
550	7085	60	12917
525	7134	55	13140
500	7187	50	13500
475	7246	45	14148
450	7310	40	14906
425	7382	35	15796
400	7462	30	16833
375	7551	25	18000
350	7652	20	18750
325	7767	15	20000
300	7898	10	22500
275	8051	5	30000

Fig. 26*M*. Wheel Determining Maximum Stress. *M* Loadings. Equivalent Uniform Loads for Shears and Reactions ($l_1 = 0$), *M*-60 Loading per Linear Foot of Track.

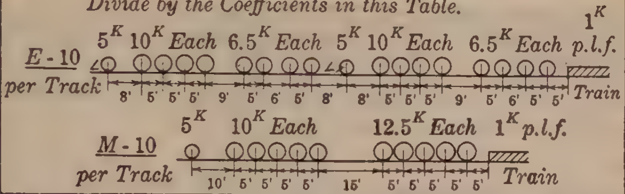
Modern Requirements in Highway Bridge Loadings are clearly shown in the following excerpts from the Am. Soc. C. E. 1924 Specifications. The Standard Specifications of the A. A. S. H. O. for 1926 differ only slightly from these loading requirements.

9. **Traffic Classification of Bridges.** Bridges shall be classified, on the basis of traffic, as follows:

- Class A. City bridges or other bridges carrying a highway traffic of exceptionally heavy load units.
- Class B. Bridges on primary roads.
- Class C. Bridges on secondary roads.
- Class D. Bridges carrying electric railway traffic in addition to highway traffic.

CONVERSION TABLE

To Convert E-Rating to Equivalent M-Rating Multiply by the Coefficients in this Table.
To Convert M-Rating to Equivalent E-Rating Divide by the Coefficients in this Table.



Span Length in Feet	Maximum Moment	Maximum Shear	Maximum Floor Beam Reaction	Span Length in Feet	Maximum Moment	Maximum Shear	Maximum Floor Beam Reaction
10	.800	.800	.800	46	.755	.771	.777
11	.801	.799	.769	48	.759	.771	.777
12	.800	.800	.747	50	.765	.775	.778
13	.800	.800	.732	52	.769	.772	.782
14	.801	.799	.730	54	.769	.767	.786
15	.800	.800	.729	56	.774	.760	.793
16	.800	.800	.727	58	.777	.759	.803
17	.801	.800	.725	60	.779	.756	.812
18	.800	.800	.724	62	.781	.756	.818
19	.800	.799	.729	64	.782	.759	.825
20	.800	.800	.739	66	.785	.760	.831
21	.800	.786	.746	68	.785	.766	.837
22	.786	.773	.754	70	.786	.770	.845
23	.772	.763	.757	72	.784	.775	.850
24	.760	.760	.761	74	.781	.784	.857
25	.751	.758	.764	76	.782	.790	.862
26	.743	.756	.765	78	.778	.795	.867
27	.735	.752	.770	80	.776	.801	.873
28	.731	.749	.774	82	.775	.808	.877
29	.729	.746	.778	84	.775	.815	.881
30	.730	.748	.778	86	.775	.825	.885
31	.729	.750	.779	88	.774	.828	.887
32	.728	.752	.782	90	.774	.839	.893
33	.726	.751	.786	92	.774	.850	.895
34	.726	.752	.788	94	.777	.854	.898
35	.724	.751	.789	96	.777	.855	.901
36	.726	.754	.786	98	.777	.859	.906
37	.728	.753	.782	100	.776	.858	.909
38	.731	.757	.781	125	.823	.874	.937
39	.732	.758	.782	150	.859	.885	.953
40	.738	.759	.782	175	.886	.893	.964
42	.745	.768	.779	200	.905	.902	.971
44	.752	.768	.776	250	.939	.913	.981

Fig. 27a

10. **Width of Roadway.** The minimum clear width of roadway shall be 9 ft. for each lane of traffic, with a minimum of 12 ft. for one lane of traffic. Bridges designed for two lines of highway traffic preferably shall have a clear width of roadway of not less than 20 ft.

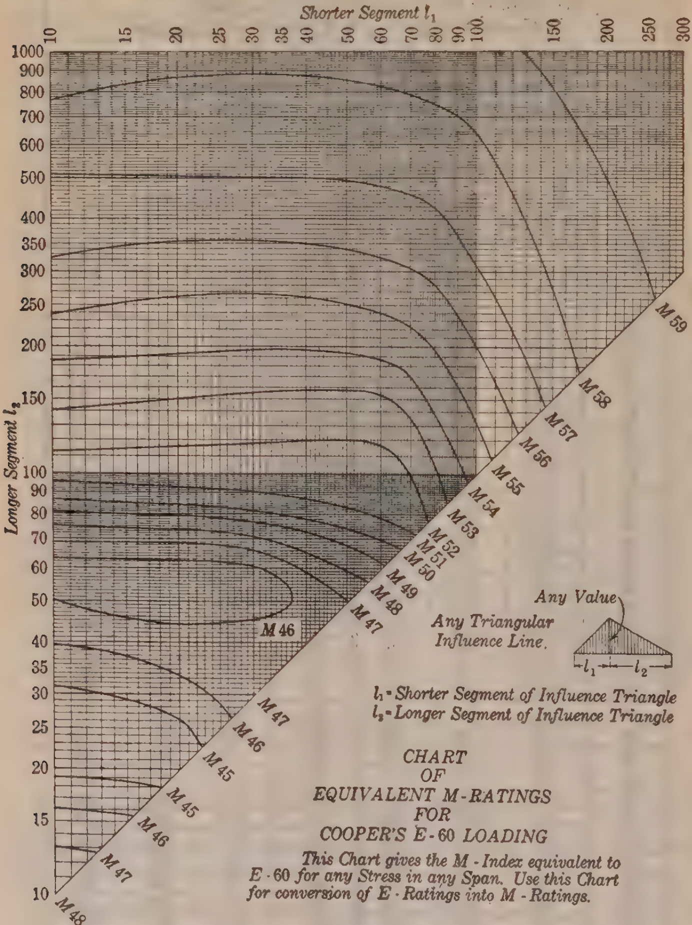


Fig. 27b

103. **Roadway Live Load.** The live loads for roadway shall be represented by typical truck loadings. Each typical truck loading shall be considered as occupying one lane of traffic 9 ft. wide. Typical truck loading shall be designated by the letter H, followed by a numeral indicating the weight in tons of the typical truck loads.

104. **Typical Truck Loadings.**

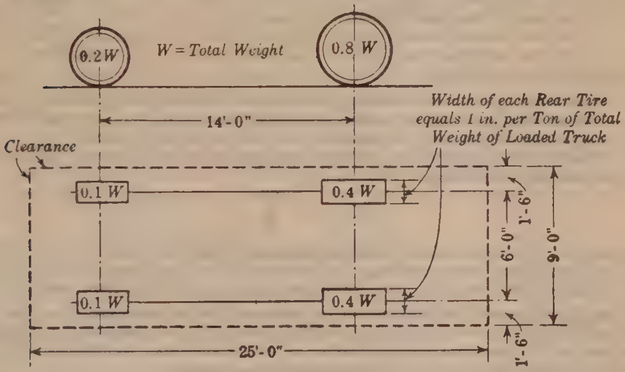
For Floor System:

H–20 .... 20-ton trucks
H–15 .... 15-ton trucks, or one 20-ton truck
H–13 .... 13-ton trucks, or one 15-ton truck

For Girders and Trusses:

H–20 .... 600 lb. per lin. ft. and 28 000 lb. concentrated
H–15 .... 450 lb. per lin. ft. and 21 000 lb. concentrated
H–13 .... 390 lb. per lin. ft. and 18 200 lb. concentrated

The concentrated loads are to be placed so as to cause maximum effect. Typical trucks shall have total loaded weights distributed as in Fig. 2.

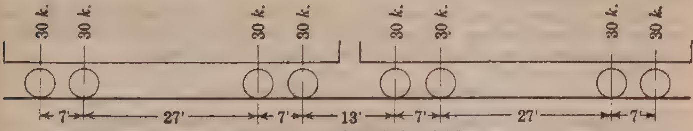


TYPICAL TRUCK

Fig. 2

(Of Report)

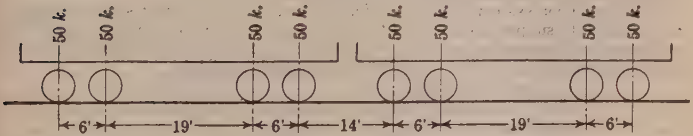
105. **Typical Electric Cars.** A train on each track shall be composed of two double-truck cars coupled together, with wheel concentrations and spacing of axles as shown in Fig. 3 or Fig. 4.



TYPICAL PASSENGER CAR

Fig. 3

(Of Report)



TYPICAL FREIGHT CAR

Fig. 4

(Of Report)

106. Selection of Live Loads.

Class A Bridges	H-20 loads
Class B Bridges	H-15 loads
Class C Bridges	H-13 loads

107. Application of Loads.

To Floor System:

Floor systems shall be designed for as many trucks, side by side, and headed in the same direction, as the width of roadway will accommodate, with a maximum of four. For floors of Class B and Class C bridges the alternate single-truck load shall also be considered.

In the design of floor beams and floor beam hangers, the following percentages of the resultant live load stresses shall be used:

For one, or two, loaded lanes	100
For three loaded lanes	90
For four loaded lanes	80

To Girders and Trusses:

For the purpose of determining the amount of live load delivered to each girder or truss, each foot of roadway width for roadways 18 ft. or less in width, shall be assumed to carry one-ninth of the load on one traffic lane.

For roadways greater than 18 ft. in width, the number of lanes of traffic shall be determined by the equation:

$$K = 1 + \frac{W}{18}$$

in which K = the number of lanes of traffic, expressed in a whole or mixed number, distributed uniformly over the entire width of roadway; W = the width of roadway that must be loaded to cause maximum stress in the girder or truss.

Class D bridges shall be designed for the highway live loads as herein specified, on any portion of the roadway; or the electric live loads on the car track and the highway live loads on the remaining lanes of traffic. For this condition, the clearance line for electric cars shall be 5 ft. 0 in. from the center line of track.

108. **Sidewalk Loading.** Sidewalk live loads for Class A bridges shall be determined by the following formula:

$$P = \left(25 + \frac{2250}{L}\right)\left(\frac{W + 5}{W}\right)$$

in which P = live load in pounds per square foot; L = loaded length in feet; W = width of sidewalk in feet.

For Class B and C bridges use 80% of this loading. For foot-bridges use the loading for Closs A bridges, making W equal to one-half the width of the roadway.

The maximum value of P shall in no case exceed 100 lb. per sq. ft.

Distribution of Truck Wheel Loads

109. **Shear.** In calculating end shears and end reactions of stringers and floor beams, no lateral or longitudinal distribution of wheel loads shall be assumed.

110. **Bending Moment in Stringers.** In determining bending moments in stringers, each wheel load shall be assumed to be concentrated at a point.

When the floor system is designed for one truck, each interior stringer shall be proportioned to support that part of one rear-wheel load, or those parts of one front-wheel load and one rear-wheel load, represented by a fraction the numerator of which is the stringer spacing, in feet, and the denominator of which is:

- 4 ft. 0 in. for plank floors;
- 5 ft. 0 in. for 4-in. and 6-in. strip floors and wood blocks on a 4-in. plank sub-floor;
- 6 ft. 0 in. for reinforced concrete floors.

When the floor system is designed for two trucks, each interior stringer shall be proportioned to support that part of one rear-wheel load, or those parts of one front-wheel load and one rear-wheel load, represented by a fraction the numerator of which is the stringer spacing, in feet, and the denominator of which is:

3 ft. 6 in. for plank floors;
4 ft. 0 in. for 4-in. and 6-in. strip floors and wood blocks on a 4-in. plank sub-floor;
4 ft 6 in. for reinforced concrete floors.

The live load supported by the outside stringers shall be the reaction of a wheel in the most unfavorable position, assuming the flooring to act as a simple beam, but this live load shall in no case be less than would be required for interior stringers under these requirements.

These distribution rules govern only when the stringer spacing is not greater than the denominator which applies to the particular case under consideration. When the stringer spacing is greater than this distance, the stringer loads shall be determined by the reactions of the truck wheels placed in the most unfavorable position, assuming the flooring between stringers to act as simple beams.

The combined load capacity of the stringers in a panel shall be not less than the total live and dead load in the panel.

111. **Bending Moment in Floor Beams.** In determining bending moments in floor beams, each wheel load shall be assumed as concentrated at a point.

When stringers are omitted and the floor is supported directly on the floor beams, the latter shall be proportioned to carry that fraction of one axle load, when the floor system is designed for one truck, or of two axle loads, when the floor system is designed for two trucks, the numerator of which is the floor beam spacing, in feet, and the denominator of which is:

4 ft. 0 in. for plank floors;
5 ft. 0 in. for 4-in. and 6-in. strip floors and wood blocks on a 4-in. plank sub-floor;
6 ft. 0 in. for reinforced concrete floors.

When the spacing of floor beams exceeds the denominator given, but is less than the axle spacing (14 ft. 0 in.), each beam shall be proportioned to carry the full axle load or loads.

When the floor-beam spacing exceeds the spacing of axles, the load supported on each floor-beam shall be the maximum reaction due to the axle loads, assuming the flooring between floor beams to act as a simple beam.

112. **Distribution of Electric Railway Wheel Loads.** Electric railway wheel loads may be assumed to be uniformly distributed longitudinally over a length of 3 ft. 6 in. In the case of ballasted floors, a lateral distribution of 10 ft. for an axle load may be assumed.

Example. Maximum bending moment at the center of a Class A bridge, span 100 ft., width of roadway 20 ft., is computed as follows: For one lane of traffic, load the entire span with 600 lb. per lin. ft. and place the 28 000 lb. concentration at the center. $M = 600 \times 100^2/8 + 28\,000/2 \times 50 = 1\,450\,000$. $K = 1 + 20/18 = 2.11$. Moment in one girder = $1.05 \times 1\,450\,000 = 1\,525\,000$. For the maximum shear at the 1/4 point, place the 28 000-lb. load at that point, and the uniform load over the part of the span lying to the right of that point. Then maximum shear at 1/4 point in one girder = $1.05\ (600 \times 75^2/200 + 21\,000) = 37\,875$. A 16-ft. roadway will carry $16 \times 1/9 = 1.77$ lanes of traffic, divided equally between the two girders, and the stresses from one lane of traffic will be multiplied by 0.885, instead of 1.05. If there is a 6-ft. sidewalk the adjacent girder will carry the following additional sidewalk loading, placed so as to produce the maximum effect: for center moment, $P = (25 + 2250/100)\ 11/6 = 87.1$ lb. per sq. ft. = 523 lb. per lin. ft. of girder; for shear at the 1/4 point, $P = (25 + 2250/75)\ 11/6 = 100$ lb. per sq. ft. = 600 lb. per lin. ft. For shear in any section to the right of the 1/4 point, use 600 lb. per lin. ft. over the portion of the bridge between the section and the right end.

If the bridge has a concrete floor supported on longitudinal stringers 4 ft. apart, the wheels on one side of a truck, assumed to be moving longitudinally, are placed over the stringer in the position to produce maximum stress, and the stress thus found is multiplied by 4/6, if there is but one lane of traffic, and by 4/4.5 if there are two or more lanes.

Impact is the dynamic effect on a bridge due to the moving load and allowance should be made for it. The usual method is to increase the live load stress by a certain per cent of itself. The Am. Soc. C. E. specifications for highway bridges require, for floor beams and stringers, that this increment be taken as 30%, and for floor beam hangers as 60% of the live load stresses, provided these percentages are not less than would be given by the following formula for girder and truss members of highway bridges:

$$I = \frac{S}{3}\left(\frac{2000 - l}{1600 + 10\,l}\right)$$

where I = impact or dynamic increment to be added to live load stress; S = computed maximum live load stress; l = loaded length of lane of traffic, or of track, in feet, producing the maximum stress in the member. For bridges carrying more than one lane or track, the aggregate loaded length of all lanes or tracks producing the stress shall be used.

The A. A. S. H. O. specifications for highway bridges require an impact allowance of 60% for floor beam hangers. For all other portions of the structure the percentage is determined by the following formulas: for electric railway loads, $I = \dfrac{L + 900}{12\,L + 1200}$; for highway loads, $I = \dfrac{L + 250}{10\,L + 500}$ (when $W \ngtr 18$ ft.), or, $I = \dfrac{36}{W + 18} \cdot \dfrac{L + 250}{10\,L + 500}$ (when $W > 18$ ft.). For highway loads the maximum value to be used is not to exceed 30%. L = loaded length of span producing the maximum stress; W = width of roadway between girders or trusses; I = impact coefficient. Impact is not added to stresses produced by longitudinal, lateral or centrifugal forces, nor generally to those due to sidewalk loads.

Other Forces acting upon a highway bridge are covered by the Am. Soc. C. E. Specifications as follows:

115. **Centrifugal Force.** Bridges carrying electric railway traffic on a curve shall be designed to resist a lateral force equal to 10% of the moving railway load without impact allowance. This load shall be assumed to act 4 ft. above the rail.

116. **Lateral Forces.** Spans of 150 ft. and less shall be designed to resist a lateral force of 300 lb. per lin. ft. on the loaded chord and 150 lb. per lin. ft. on the unloaded chord. For spans of more than 150 ft., for each additional 30 ft. of span there shall be added 10 lb. per lin. ft. for the loaded chord and 5 lb. per lin. ft. for the unloaded chord. (Compare with corresponding A. A. S. H. O. Specifications, below.)

All lateral forces are to be considered as moving loads.

117. Viaduct towers shall be designed for a horizontal wind force of 50 lb. per sq. ft. on one and one-half times the vertical projection of the structure unloaded.

118. **Longitudinal Force.** Provision shall be made for the starting and stopping of electric railway trains, with a coefficient of friction of 15%. (Compare with corresponding A. A. S. H. O. Specifications, below.)

119. **Temperature.** Provision shall be made for movement due to temperature. For thermal stresses in fixed arch bridges provisions shall be made for stresses induced by a temperature range of 120° F.

120. **Alternate Stresses.** Members subject to alternate stresses of tension and compression shall be proportioned for the kind of stress requiring the larger section. If the alternate stresses occur in succession during the passage of live load, as in stiff counters, each stress shall be increased by 50% of the smaller. Connections shall be designed for the sum of the original stresses.

121. **Combined Stresses.** Members subject to both axial and bending stresses shall be proportioned so that the combined fiber stresses will not exceed the unit stresses given in Art. 201 (page 1184). In members continuous over panel points, three-fourths of the bending stress computed as for simple beams shall be added to the axial stress.

122. For stresses produced by combination of longitudinal or lateral forces with live load, dead load, impact, and centrifugal force, the unit stresses may be increased 25% over those specified in Art. 201. When secondary stresses are included the unit stress may be increased 33-1/3%. In no case shall the section be less than that required for deal load, live load, impact, and centrifugal force at the unit stresses specified in Art. 201, or less than that required if secondary stresses are not considered.

The following A. A. S. H. O. Specifications differ slightly from Sections 116 to 118, above:

Traction, or longitudinal force, due to electric trains, 20% of the load on one track only. Provision for wind is as follows:

Lateral Force. The force due to wind and lateral vibrations shall consist of a horizontal moving load equal to 30 lb. per sq. ft. on the side area of any floor construction, the side area of all railings, and 1.5 times the side area of each truss, girder or arch. In addition to the foregoing, a moving load of 150 lb. per lin. ft. shall be considered as acting in the plane of the loaded chord, on highway bridges, and 300 lb. per lin. ft. upon bridges for combined highway and electric railway service. However, in the case of structures having a reinforced concrete floor slab, effectively anchored to the supporting structure, this additional loaded chord load need not be considered.

The transverse bracing and anchorage of trestle towers and bents shall be proportioned to resist the above specified lateral forces acting upon the superimposed spans, plus a pressure of 30 lb. per sq ft. on 1.5 times the side areas of all columns, struts and bracing.

The longitudinal bracing of trestle towers shall be proportioned to resist not less than 0.7 of the transverse lateral forces above described, in addition to the tractive or longitudinal force due to electric railway traffic, when the latter exists.

11. Specifications for Loads and Stresses

The General Specifications for Steel Railway Bridges, 1925, of the American Railway Engineering Association have been adopted, either in their exact or in some modified form, by the majority of American railways. The essential clauses relating to unit stresses, proportioning, detailing, materials, workmanship and inspection apply and are much used in the design of other classes of steel structures, such as highway bridges and buildings (compare unit stresses for buildings, Art. 2). On account of their importance the most important clauses relating to design are here given nearly in full. The Final Report, 1923, of the Am. Soc. C. E. Committee on Specifications for Bridge Design does not differ essentially from the A. R. E. A. Specifications. Where important differences do occur, both clauses will be given, that of the Am. Soc. C. E. being indicated by an *.

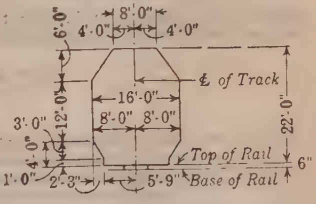


Fig. 1
(Of Report)

(II) General Features of Design

Materials Used. 8. Structures shall be made wholly of structural steel except where otherwise specified. Rivet steel shall be used for rivets only. Cast steel preferably shall be used for shoes and bearings. Cast iron may be used only where specifically authorized by the engineer.

Spacing of Trusses, Girders, and Floor Beams. 12. The width center to center of girders or trusses shall be not less than one-fifteenth of the effective span, and not less than is necessary to prevent overturning under the assumed lateral loading. Panel lengths shall not exceed 1-1/2 times the width center to center of trusses or girders

Clearances. 13. If the alignment is straight, clearances shall be not less than as shown on the diagram, Fig. 1. If the alignment is curved, the width of the diagram shall be so increased as to provide the same minimum clearances for a car 80 ft. long, 14 ft. high and 60 ft. center to center of trucks, allowance being made for curvature and superelevation of rails. The height of rail shall be assumed as 6 in.

(III) Loads

18. Members shall be proportioned for that combination of stresses which gives the maximum total stress, except as otherwise provided.

Dead Load. 19. The dead load shall consist of the estimated weight of the entire suspended structure. Timber shall be assumed to weight 4-1/2 lb. per ft. B. M.; ballast, assumed level with the base of rail and including track ties embedded therein, 120 lb. per cu. ft.; reinforced concrete, 150 lb. per cu. ft.; waterproofing, 150 lb. per cu. ft.; and rails and fastenings, 150 lb. per lin. ft. of track.

***Live Loads.** 103. The live load for each track shall consist of typical engines followed by a uniform train load, according to either Class E series, or Class M series, as may be specified by the engineer.

Loading E–60 or Loading M–50 is recommended for main-line bridges of American railways.

22. A train load of 1200 lb. per lin. ft. of one track shall be used in determining the stability of spans and towers against overturning.

Multiple Tracks. 23. In calculating the maximum stresses due to live load and centrifugal force when two, three or four tracks are loaded simultaneously, the following percentages of the specified live load shall be used:

For two tracks	90%
For three tracks	80%
For four tracks	75%

Floors. 24. Timber cross-ties shall be designed for the maximum wheel load distributed over three ties and with 100% impact added. The fiber stress shall not exceed 2000 lb. per sq. in. The ties shall be not less than 10 ft. in length. They shall be spaced with openings not exceeding 4 in. and shall be secured against bunching. The maximum dap of ties shall be 1-1/4 in.

25. Floors consisting of beams transverse to the axis of the structure shall be designed for a uniform live load of 15 000 lb. per lin. ft. for each track, with 100% impact added, when the minimum live loads specified in Sect. 20 (Cooper's E-60, or two axle loads of 75 000 lb. each spaced 7 ft. apart), is used. When heavier loadings are used, this uniform load shall be increased proportionately.

26. Floors consisting of longitudinal beams shall be designed for the wheel loads specified.

27. In ballasted floor bridges, the live load shall be considered as uniformly distributed laterally over a width of 10 ft.

Impact. 28. The dynamic increment of the live load shall be added to the maximum computed live load stresses and shall be determined by the formula,

$$I = S\frac{300}{300 + \frac{L^2}{100}},$$

in which I = impact or dynamic increment to be added to the live-load stress;
S = computed maximum live-load stress;
L = the length in feet of the portion of the span which is loaded to produce the maximum stress in the member.

***Impact.** 104. The dynamic increment of the live load shall be added to the maximum computed live load stresses and shall be determined by the formula:

$$I = S\frac{2000 - L}{1600 + 10\,L},$$

* No reduction given for multiple tracks as is done in A. R. E. A.
* A. S. C. E. Specifications.

in which I = impact or dynamic increment to be added to live load stresses;
S = computed maximum live-load stress;
L = loaded length of track, in feet, producing the maximum stress in the member. For bridges carrying more than one track, the aggregate length of all tracks producing the stresses shall be used.

29. For bridges designed exclusively for electric traction, the impact stresses shall be taken as one-half of those given by the formula in Sect. 28.

* 105. For bridges designed exclusively for electric traction, impact shall be taken as one-third of that given by the impact formula.

30. Impact shall not be added to stresses produced by longitudinal or lateral forces, or by the train load specified in Sect. 22.

* **Allowance for Increase of Live Load.** 106. Wherever an additional counter would be required, or reversal of stress would be caused, or heavier dimensioning would result, a member and its details shall be designed for a live load 50% greater than that given in Art. 103, with an allowance of 50% increase in unit stresses.

Eccentricity of Load on Curves. 31. For bridges on curves, provision shall be made for the increased load carried by any truss, girder or stringer due to the eccentricity of the load.

Lateral Forces. 32. The wind force on the structure shall be a moving load of 30 lb. per sq. ft. on 1-1/2 times its vertical projection on a plane parallel with its axis, but not less than 200 lb. per lin. ft. at the loaded chord or flange, and 150 lb. per lin. ft. at the unloaded chord or flange.

The wind force on the train shall be a moving load of 300 lb. per lin. ft. on one track, applied 8 ft. above the base of rail.

33. The lateral force to provide for the effect of the sway of the engines and train, in addition to the wind loads specified in Sect. 32, shall be a moving load equal to 5% of the specified live load on one track, but not more than 400 lb. per lin. ft., applied at the base of rail.

34. The lateral bracing between compression chords or flanges and between the posts of viaduct towers shall be capable of resisting a transverse shear in any panel equal to 2-1/2% of the total axial stress in the chords or posts in that panel.

35. In proportioning the bracing, Sects. 32 and 33 shall be combined or Sect. 34 used alone, whichever gives the greater section.

* **Lateral Forces.** 108. All spans shall be designed for a lateral force on the loaded chord of 200 lb. per lin. ft. plus 10% of the specified train load on one track, and 200 lb. per lin. ft. on the unloaded chord, these forces being considered as moving.

* **Wind Force.** 109. Viaduct towers shall be designed for the one of the following loads that causes the greater stress:

Viaduct towers shall be designed for a force of 50 lb. per sq. ft. on 1-1/2 times the vertical projection of the structure unloaded; or 30 lb. per sq. ft. on the same surface plus 400 lb. per lin. ft. of structure applied 7 ft. above the rail for the assumed wind force on the train when the structure is either fully loaded or loaded on either track with empty cars assumed to weigh 1200 lb. per lin. ft., whichever gives the larger stress.

Centrifugal Force. 36. On curves, the centrifugal force (assumed to act 6 ft. above the rail) shall be taken equal to a percentage of the live load, including impact, according to the following table:

Degree of Curve....	0° 20′	0° 40′	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°
Percentage.........	2-1/2	5	7-1/2	10	10	10	10	10	10	10	10	10	10	10
Speed in miles per hour	80	80	80	65	53	46	41	38	35	33	31	29	28	27

* **Centrifugal Force.** 107. Structures on curves shall be designed to resist the centrifugal force of the live load applied 7 ft. above base of rail, as computed by the following formula:

$$C = \frac{0.067\,W V^2}{R},$$

* A. S. C. E. Specifications.

in which C = horizontal centrifugal force;
W = live load, including impact;
R = radius of curve (5730 ÷ degree of curve);
V = speed, in miles per hour = (60 − 2-1/2 times the degree of curve).

Longitudinal Force. 37. Provision shall be made in the design for the effect of a longitudinal force of 20% of the live load on one track only, applied 6 ft. above the top of the rail. In structures (such as ballasted deck bridges of only three or four spans) where, by reason of continuity of members or frictional resistance, the longitudinal force will be largely directed to the abutments, its effect on the superstructure shall be taken as one-half that specified above.

* **Longitudinal Force.** 110. Provision shall be made for the starting and stopping of trains, with a coefficient of friction on the engine drivers at 20% and on the remainder of the train at 10%.

(IV) Unit Stresses and Proportioning of Parts

38. The several parts of structures shall be so proportioned that the unit stresses will not exceed the following, except as modified in Sects. 46 and 47:

	Lb. per sq. in.
Axial tension, net section	16 000 †
Axial compression, gross section	$15\,000 - 50\frac{l}{r}$
but not to exceed	12 500

l = the length of the member in inches.
r = the least radius of gyration of the member in inches.

Tension in extreme fibers of rolled shapes, built sections and girders, net section	16,000
Tension in extreme fibers of pins	24 000
Shear in plate girder webs, gross section	10 000
Shear in power-driven rivets and pins	12 000
Bearing on power-driven rivets, pins, outstanding legs of stiffener angles, and other steel parts in contact	24 000

Rivets driven and bucked by pneumatically or electrically operated hammers are considered power-driven.

The above mentioned values for shear and bearing shall be reduced 25% for countersunk rivets, hand-driven rivets and turned bolts.

Bearing on expansion rollers, per linear inch	$600d$

d = the diameter of the rollers in inches.

	Lb. per sq. in
Bearing on granite masonry	800
Bearing on sandstone and limestone masonry	400
Bearing on concrete masonry	600
* 201. Compression (one diameter)	16 000

Compression on columns:

$$p = \frac{16\,000}{1 + \dfrac{l^2}{13\,500\,r^2}},$$

but not to exceed the value for $\frac{l}{r} = 40$.

* A. S. C. E. Specifications.

† Many engineers favor an increase in unit stresses to 18 000 or 20 000 lb., and in exceptional cases even higher. Accompanying higher unit stresses there should be a liberal allowance for all external forces, impact and all possible combinations of stresses, in consequence of which a more consistent, economical and safe design will result. High unit stresses have been used in some notable bridges. Thus, 20 000 for Blackwell's Island bridge, ordinary loading, and 24 000 congested loading; 20 000 for the Quebec bridge, exclusive of secondary stresses, and 22 000 inclusive of secondary stresses.

Proportioning Web Members. 43. Web members shall be so proportioned that an increase of live load which will increase the total unit stresses in the chords 50% will not produce total unit stresses in the web members more than 50% greater than the designing stresses.

Reversal of Stress. 44. Members subject to reversal of stress under the passage of the live load shall be proportioned as follows:

Determine the resultant tensile stress and the resultant compressive stress and increase each by 50% of the smaller; then proportion the member so that it will be capable of resisting either increased resultant stress. The connections shall be proportioned for the sum of the resultant stresses.

Combined Stresses. 45. Members subject to both axial and bending stresses (including bending due to floor beam deflection) shall be so proportioned that the combined fiber stresses will not exceed the allowed axial stress. In members continuous over panel points, only three-fourths of the bending stress computed as for simple beams shall be added to the axial stress.

46. Members subject to stresses produced by a combination of dead load, live load, impact, and centrifugal force, with either lateral or longitudinal forces, or bending due to lateral action, may be proportioned for unit stresses 25% greater than those specified in Sect. 38; but the section shall not be less than that required for dead load, live load impact, and centrifugal force.

Secondary Stress. 47. Designing and detailing shall be done so as to avoid secondary stresses as far as possible. In ordinary trusses without sub-paneling, no account usually need be taken of the secondary stresses in any member of which width measured in the plane of the truss is less than one-tenth of its length. Where this ratio is exceeded, or where sub-paneling is used, secondary stresses due to deflection of the truss shall be computed. The unit stresses specified in Sect. 38 may be increased one-third for a combination of the secondary stresses with the other stressses, but the section shall not be less than that required when secondary stresses are not considered.

Compression Flanges. 48. The gross area of the compression flanges of plate girders and rolled beams shall not be less than the gross area of the tension flanges, but the stress per square inch of gross area shall not exceed

$$16\,000 - 150\frac{l}{b},$$

in which l = the length in inches of the unsupported flange between lateral connections or knee braces;

b = the flange width in inches.

12. Specifications for Details

(V) Details of Design

Slenderness Ratios. 49. The ratio of length to least radius of gyration shall not exceed:

100 for main compression members;
120 for wind and sway bracing;
140 for single lacing, and for double lacing not riveted at intersections;
170 for double lacing riveted at intersections;
200 for riveted tension members.

Depth Ratios. 50. The depth of trusses preferably shall be not less than one-tenth of the span. The depth of plate girders preferably shall be not less than one-twelfth of the span. The depth of rolled beams used as girders and the depth of solid floors preferably shall be not less than one-fifteenth of the span. If smaller depths that these are used, the section shall be so increased that the maximum deflection will not be greater than if these limiting ratios had not been exceeded.

Effective Area of Angles. 54. The effective area of single angles in tension shall be assumed as the net area of the connected leg plus 50% of the area of the unconnected leg. Single angles connected by lug angles shall be considered as connected by one leg.

Counters. 55. If web members are subject to reversal of stress, their end connections preferably shall be riveted. Adjustable counters shall have open turnbuckles.

Strength of Connections. 56. Connections shall have a strength at least equal to that of the members connected, regardless of the computed stress. Connections shall be made, as nearly as practicable, symmetrical about the axes of the members.

Limiting Thickness of Metal. 57. Metal shall be not less than 3/8 in. thick, except for fillers. Metal subject to marked corrosive influences shall be increased in thickness or protected against such influences.

Sizes of Rivets. 58. Rivets shall be 3/4 in., 7/8 in. or 1 in. in diameter as specified.

Pitch of Rivets. 59. The minimum distance between centers of rivet holes shall be three diameters of the rivet, but the distance preferably shall be not less than 3-1/2 in. for 1-in. rivets, 3 in. for 7/8-in. rivets and 2-1/2 in. for 3 4-in. rivets. The maximum pitch in the line of stress for members composed of plates and shapes shall be 7 in. for 1-in. rivets, 6 in for 7/8-in. rivets, and 5 in. for 3 4-in. rivets. For angles with two gage lines and rivets staggered, the maximum pitch in each line shall be twice the amounts given above. If two or more web plates are used in contact, stitch rivets shall be provided to make them act in unison. In compression members, the stitch rivets shall be spaced not more than 24 times the thickness of the thinnest plate in the direction perpendicular to the line of stress, and not more than 12 times the thickness of the thinnest plate in the line of stress. In tension members, the stitch rivets shall be spaced not more than 24 times the thickness of the thinnest outer plate in either direction. In tension members composed of two angles in contact, a pitch of 12 in. may be used for riveting the angles together.

Edge Distance. 60. The minimum distance from the center of any rivet hole to a sheared edge shall be: 1-3/4 in. for 1-in. rivets, 1-1/2 in. for 7/8-in. rivets, and 1-1/4 in. for 3/4-in. rivets; to a rolled edge 1-1/2 in., 1-1/4 in. and 1-1/8 in. respectively. The maximum distance from any edge shall be 8 times the thickness of the plate, but shall not exceed 6 in.

Sizes of Rivets in Angles. 61. The diameter of the rivets in any angle whose size is determined by calculated stress shall not exceed one-fourth of the width of the leg in which they are driven. In angles the size of which is not so determined 1-in. rivets may be used in 3-1/2-in. legs, 7/8-in. rivets in 3-in. legs, and 3/4-in. rivets in 2-1/2-in. legs.

Long Rivets. 62. Rivets which carry calculated stress and the grip of which exceeds 4-1/2 diameters shall be increased in number at least one per cent for each additional 1/16 in. of grip. If the grip exceeds 6 times the diameter of the rivet, specially designed rivets shall be used.

Pitch of Rivets at Ends. 63. The pitch of rivets at the ends of built compression members shall not exceed 4 diameters of the rivet for a distance equal to 1-1/2 times the maximum width of the member.

Compression Members. 64. In built compression members, the metal shall be concentrated in the webs and flanges. The thickness of each web shall be not less than 1/30 of the distance between the lines of rivets connecting it to the flanges. The thickness of cover plates shall be not less than 1/40 of the distance between the nearest rivet lines.

Outstanding Legs of Angles. 65. The width of the outstanding legs of angles in compression (except when reinforced by plates) shall not exceed the following:

(a) For stringer flange angles, 10 times the thickness.
(b) For main members carrying axial stress, 12 times the thickness.
(c) For bracing and other secondary members, 14 times the thickness.

Stay Plates. 66. The open sides of compression members shall be provided with lacing bars and shall have stay plates as near each end as practicable. Stay plates shall be provided at intermediate points where the lacing is interrupted. In main members, the length of the end stay plates shall be not less than 1-1/4 times the distance between the lines of rivets connecting them to the outer flanges, and the length of intermediate stay plates shall be not less than 3/4 of that distance. Their thickness shall be not less than 1/50 of the same distance.

67. Tension members composed of shapes shall have their separate segments stayed together. The stay plates shall have a length not less than two-thirds of the lengths specified for stay plates on compression members.

Lacing. 68. The lacing of compression members shall be proportioned to resist a shearing stress of 2-1/2% of the direct stress. The section shall be made as required by Sects. 38 and 49, in which l shall be taken as the distance between connections of the lacing to the main sections.

The minimum width of lacing bars shall be 3 in. for 1-in. rivets, 2-3/4 in. for 7/8-in. rivets, 2-1/2 in. for 3/4-in. rivets, and 2 in. for 5/8-in. rivets.

69. In members composed of side segments and a cover plate, with the open side laced, one half the shear shall be considered as taken by the lacing. Where double lacing is used, the shear in the plane of the lacing shall be distributed equally between the two systems.

70. Lacing bars of compression members shall be so spaced that the $\frac{l}{r}$ of the portion of the flange included between their connections will be not greater than 40, and not greater than two-thirds of the $\frac{l}{r}$ of the members.

71. In connecting lacing bars to flanges, 5/8-in. rivets shall be used for flanges less than 2-1/2 in. wide, 3/4-in. rivets for flanges from 2-1/2 to 3-1/2 in. wide, and 7/8-in. rivets for flanges 3-1/2 or more inches wide. Lacing bars with at least two rivets in each end shall be used for flanges over 5 in. wide.

72. The angle of lacing bars with the axis of the member shall be not less than 45 deg. for double lacing, and 60 deg. for single lacing. If the distance between rivet lines in the flanges is more than 15 in. and a single-rivet bar is used, the lacing shall be double and riveted at the intersections.

*315. The latticing of compression members shall be proportioned to resist shearing stress normal to the member not less than that calculated by the formula:

$$R = \frac{Pl}{4000\,y},$$

in which R = normal shearing stress, in pounds;
P = strength of column as a compression member, expressed in pounds;
l = length of column, in inches;
y = distance from neutral axis to extreme fiber, in inches.

In a compression member with a cover-plate, the cover-plate will be assumed to take one-half the shear.

*316. The diameter of the rivets shall not exceed one-third the width of the bar.

Splices. 73. Abutting joints in compression members faced for bearing shall have their component parts spliced. The gross area of the splice material shall be not less than 50% of the gross area of the smaller member. In determining the number of rivets in compression splices, the stress in the splice material shall be taken as 15 000 lb. per sq. in. of gross area.

74. Joints in riveted work not faced for bearing shall be fully spliced.

***Built Chord Splices.** 310. Built chords subjected to compression only, when faced for bearings, shall be spliced on four sides sufficient to hold the abutting members accurately in place and to transmit at least 25% of the stress through splice-plates. Generally, they shall be spliced as near to panel points as practicable. All other joints shall be fully spliced.

Net Section at Pin Holes. 75. In pin-connected riveted tension members, the net section across the pin hole shall be not less than 140% and the net section back of the pin hole not less than 100% of the net section of the body of the member, and there shall be sufficient rivets to make the material effective.

* A. S. C. E. Specifications.

Net Section Defined. 76. The net section of riveted members shall be the least area which can be obtained by deducting from the gross sectional area the areas of holes cut by any plane perpendicular to the axis of the member and parts of the areas of other holes on one side of the plane within a distance of 4 in., which are on gage lines 1 in. or more from those of the holes cut by the plane, the parts being determined by the formula:

$$A\left[1-\frac{P}{4}\right],$$

in which A = the area of the hole;

P = the distance in inches of the center of the hole from the plane.

77. In determining the net section, the diameter of the rivet hole shall be taken 1/8 in. larger than the nominal diameter of the rivet.

Pin Plates. 78. Where necessary to give the required section or bearing area, pin holes shall be reinforced on each segment by plates, one of which on each side must be as wide as the outstanding flanges will permit. These plates shall contain enough rivets and be so connected as to transmit and distribute the bearing pressure uniformly over the full cross-section and to reduce the eccentricity of the segment to a minimum. At least one full-width plate on each segment shall extend to the far edge of the stay plate and the others not less than 6 in. beyond the near edge.

Indirect Splices. 79. If splice plates are not in direct contact with the parts which they connect, rivets shall be used on each side of the joint in excess of the number required in the case of direct contact to the extent of two extra lines for each intervening plate.

Fillers. 80. Where rivets carrying stress pass through fillers, the fillers shall be extended beyond the connected member and the extension secured by additional rivets sufficient to develop the value of the filler.

Forked Ends. 81. Forked ends on compression members will be permitted only where unavoidable. Where forked ends are used, a sufficient number of pin plates shall be provided to make the jaws of twice the sectional area of the member and they shall be extended as far as necessary in order to carry the stress of the main member into the jaws, but shall not be shorter than required by Sect. 78.

Upset Ends. 85. Bars with screw ends shall be upset so that the area at the root of the thread will be at least 15% greater than in the body of the bar.

Expansion Bearings. 88. Spans more than 70 ft. in length shall have rollers at one end. Shorter spans shall be arranged to slide on smooth surfaces.

Fixed Bearings. 89. Bearings and ends of spans shall be secured against lateral motion.

Rollers. 90. Expansion rollers shall be not less than 6 in. in diameter. They shall be coupled together with substantial side bars, which shall be so arranged that the rollers can be cleaned readily. Rollers shall be geared to the upper and lower plates.

Pedestals and Shoes. 91. Pedestals and shoes preferably shall be made of cast steel. The difference between the top and bottom bearing widths shall not exceed twice the depth. For hinged bearings, the depth shall be measured from the center of the pin. Where built pedestals and shoes are used, the web plates and the angles connecting them to the base plate shall be not less than 3/4 in. thick. If the size of the pedestal permits, the webs shall be rigidly connected transversely. The minimum thickness of the metal in cast-steel pedestals shall be 1 in. Pedestals and shoes shall be so constructed that the load will be distributed uniformly over the entire bearing. Spans more than 70 ft. in length shall have hinged bearings at each end.

(VI) Bracing

Spacing of Stringers. 98. Stringers usually shall be spaced 6 ft. 6 in. center to center. If four stringers are used under one track, each pair shall be placed symmetrically about the rail.

I-Beam Girders. 99. Rolled beams supporting timber decks shall be arranged with not more than four, and preferably not less than two beams under each rail.

The beams in each group shall be placed symmetrically about the rail, and shall be spaced far enough apart to permit cleaning and painting. They shall be connected by solid web diaphragms near the ends and at intermediate points, spaced not over 12 times the flange width. Bearing plates shall be continuous under each group of beams. End stiffeners shall be used if required by Sect. 38.

Floor Beam Connections. 100. Floor beams preferably shall be square to the girders or trusses. They shall be riveted directly to the girders or between the posts of through and deck truss spans.

End Connection Angles. 101. The legs of stringer connection angles shall be not less than 4 in. in width, and not less than 5/8 in. in thickness before facing. Shelf angles shall be provided to support the stringers during erection, but the connection angles shall be sufficient to carry the whole load. Stringers in through spans shall be riveted between the floor beams.

Stringer Frames. 102. Where two lines of stringers are used under each track in panels more than 20 ft. in length, they shall be connected by cross frames.

(VII) Bracing

Design of Bracing. 105. Lateral, longitudinal, and transverse bracing shall be composed of shapes with riveted connections. Lateral bracing shall have concentric connections to chords at end joints, and preferably throughout. The connections between the lateral bracing and the chords shall be designed to avoid, as far as practicable, any bending stress in the truss members.

106. When a double system of bracing is used, both systems may be considered simultaneously effective if the members meet the requirements as both tension and compression members.

Lateral Bracing. 107. Bottom lateral bracing shall be provided in all bridges except deck plate girder spans less than 50 ft. long. Continuous steel or concrete floors shall be considered lateral bracing.

108. Top lateral bracing shall be provided in deck spans, and in through spans having sufficient head room.

Portal and Sway Bracing. 109. Deck truss spans shall have sway bracing at every panel point. The top lateral loads preferably shall be carried to the supports by means of a complete top lateral system, or the loads may be considered as transferred to the bottom lateral system at each sway frame.

110. Through truss spans shall have portal bracing, with knee braces, as deep as the specified clearance will allow.

111. Through truss spans shall have sway bracing at every intermediate panel point if the height of the trusses is enough to allow a depth of 6 ft. or more for the bracing. When the height of the trusses will not allow that depth, the top lateral struts shall be of the same depth as the chord and shall have knee braces.

Cross-Frames. 112. Deck plate girder spans shall be provided with cross-frames at each end proportioned to resist centrifugal and lateral forces, and shall have intermediate cross-frames at intervals not exceeding 18 ft.

Laterals. 113. The smallest angle to be used in lateral bracing shall be 3-1/2 by 3 by 3/8 in. There shall be not less than three rivets at each end connection of the angles. Angles shall be connected at their intersections by plates.

(VIII) Plate Girders

Spacing of Girders. 115. The girders of deck bridges usually shall be spaced 6 ft. 6 in. between centers, except that:

(a) In single-track deck spans 75 ft. or more in length, girders shall be spaced in accordance with Sect. 12, but not less than 7 ft. 6 in. between centers.

(b) In bridges on curves, the girders shall be spaced as shown on the plans.

Design of Plate Girders. 116. Plate girders shall be proportioned either by the moment of inertia of their net section including compression side; or by assuming that the flanges are concentrated at their centers of gravity. In the latter case, one-eighth of the gross section of the web, if properly spliced, may be used as flange section. For girders having unusual sections, the moment of inertia method shall be used.

Flange Section. 117. The flange angles shall form as large a part of the area of the flange as practicable. Side plates shall not be used except when flange angles exceeding 1 in. in thickness otherwise would be required.

118. Flange plates shall be equal in thickness, or shall diminish in thickness from the flange angles outward. No plate shall have a thickness greater than that of the flange angles.

119. Where flange cover plates are used, one cover plate of the top flange shall extend the full length of the girder. Other flange plates shall extend at least 18 in. beyond the theoretical end.

Thickness of Web Plates. 120. The thickness of web plates shall be not less than $\frac{1}{20}\sqrt{D}$, where D represents the distance between flanges in inches.

Flange Rivets. 121. The flanges of plate girders shall be connected to the web with a sufficient number of rivets to transfer to the flange section the horizontal shear at any point combined with any load that is applied directly on the flange. Where ties rest on the flange, one wheel load shall be assumed to be distributed over 3 ft.

Flange Splices. 122. Splices in flange members shall not be used except by special permission of the engineer. No two members shall be spliced at the same cross-section. If practicable, splices shall be located at points where there is an excess of section. The net section of the splice shall exceed by 10% the net section of the member spliced. Flange angle splices shall consist of two angles—one on each side.

Web Splices. 123. Web plates shall be spliced symmetrically by plates on each side. The splice plates for shear shall be the full depth of the girders between flanges. The splice shall be equal to the web in strength in both shear and moment. There shall be not less than two rows of rivets on each side of the joint.

End Stiffeners. 124. Plate girders shall have stiffener angles over end bearings, the outstanding legs of which will extend as nearly as practicable to the outer edge of the flange angles. These end stiffeners shall be proportioned for bearing of the outstanding legs on the flange angles, and shall be arranged to transmit the end reaction to the pedestals or distribute it over the masonry bearings. They shall be connected to the web by enough rivets to transmit the reaction. End stiffeners shall not be crimped.

Intermediate Stiffeners. 125. The webs of plate girders shall be stiffened by angles at intervals not greater than:

(a) Six feet.
(b) The depth of the web.
(c) The distance given by the formula

$$d = \frac{t}{40}(12\,000 - S),$$

in which d = the distance between rivet lines of stiffeners in inches;
t = the thickness of the web in inches;
S = web shear in pounds per square inch at the point considered.

126. If the depth of the web between the flange angles or side plates is less than 50 times the thickness of the web, intermediate stiffeners may be omitted.

127. Stiffener angles shall be placed at points of concentrated loads, and shall not be crimped.

128. Intermediate stiffeners shall be riveted in pairs to the web of the girder. The outstanding leg of each angle shall be not less than 2 in. plus 1/30 of the depth of the girder, nor more than 16 times its thickness.

Gusset Plates in Through Girders. 129. In through plate girder spans, the top flanges shall be braced by means of gusset plates or knee-braces with solid webs connected to the floor beams and extending usually to the clearance line. If the unsupported length of the inclined edge of the gusset plate exceeds 18 in., the gusset plate shall have one or two stiffening angles riveted along its edge. The gusset plate shall be riveted to a stiffener angle on the girder. Preferably it shall form no part of the floor beam web.

(IX) Trusses

Type of Truss and Sections of Members. 136. Trusses shall have single intersection web systems and, preferably, inclined end posts. The top chords and end posts shall be made usually of two side segments with one cover plate and with stay plates and lacing on the open side. The bottom chords of riveted trusses shall be made symmetrical, usually of vertical side plates with flange angles. Web members shall be symmetrical in section.

Camber. 137. The length of members of truss spans shall be such that the camber will be equal to the deflection produced by the combined dead and live loads without impact.

Riveted Members in Pin-connected Trusses. 138. In pin-connected trusses, hip verticals (and members with similar functions) and, in single-track spans, the two panels at each end of the bottom chords shall be riveted members.

Eye Bars. 139. The thickness of an eye bar shall be not less than one-eighth of the width nor less than 1 in., and not greater than 2 in. The cross-sectional area of the head through the center of the pin hole shall exceed that of the body of the bar by at least 37-1/2%. The form of the head shall be submitted to the engineer for approval before the bars are made. The diameter of the pin shall be not less than seven-eighths of the width of the widest bar attached.

Packing. 140. The eye bars of a set shall be packed symmetrically about the plane of the truss and as nearly parallel as practicable. In no case shall the inclination of any bar to the plane of the truss exceed 1/16 in. to the foot. They shall be packed close, held against lateral movement, and so arranged that bars in the same panel will not be in contact.

Gusset Plates. 141. The thickness of gusset plates connecting the chords and web members of the truss shall be proportionate to the stress to be transferred, but shall not be less than 1/2 in.

(X) Viaducts

Type of Viaduct. 144. Viaducts shall consist usually of alternate tower spans and free spans of plate girders or riveted trusses supported on bents. The tower spans usually shall be not less than 30 ft. long.

Bents and Towers. 145. Viaduct bents shall be composed preferably of two supporting columns, and the bents usually shall be united in pairs to form towers. In towers having more than two vertical panels, horizontal bracing shall be placed at alternate intermediate panel points. In double track towers, provision shall be made for the transmission of the longitudinal force to both sides.

Single Bents. 146. Where long spans are supported on short single bents, such bents shall have hinged ends, or else have their columns and anchorages proportioned to resist the bending stresses produced by changes in temperature.

Bottom Struts. 147. The bottom struts of viaduct towers shall be proportioned for the calculated stresses, but in no case for less than one-fourth of the dead load reaction on one pedestal, considered as compressive stress. Provision shall be made in the column bearings for expansion of the tower bracing.

Batter. 148. The columns usually shall have a batter transversely of one horizontal to six vertical for single track viaducts, or one horizontal to eight vertical for double track viaducts.

Depths of Girders. 149. The depths of the girders in a viaduct preferably shall be uniform.

Spacing of Girders. 150. In single-track viaducts, the girder spacing usually shall be uniform throughout, and shall be determined by the spacing for the longest span in the viaduct, according to the rules specified for deck plate girder spans.

151. In double-track viaducts, the girders under each track usually shall be spaced 6 ft. 6 in. between centers, and the inner lines of girders shall be supported by cross-girders framed between and riveted to the posts.

Girder Connections and Bracing. 152. The girders of tower spans shall be fastened at both ends to the tops of the posts or cross-girders. Girders between towers shall have one end riveted, and shall be provided with an effective expansion joint at the other end. No bracing or sway frame shall be common to abutting spans.

153. If neither of the girders under a track rests directly over a tower post, bracing shall be provided to carry the longitudinal force into the tower bracing without producing lateral bending stress in the cross-girders or posts.

Sole and Masonry Plates. 154. Sole and masonry plates shall be not less than 3/4 in. thick.

Anchorage for Towers. 155. Anchor bolts for viaduct towers and similar structures shall be designed to engage a mass of masonry the weight of which is at least 1-1/2 times the uplift.

13. Highway Bridge Floors

A Highway Bridge Floor consists of the floor proper, stringers and floor beam (Fig. 28). The upper surface of the floor is the wearing surface. Stringers are beams placed lengthwise of the bridge which carry the loads from floor to floor beams, the latter being transverse to the length of the bridge and connected to the trusses or girders at opposite panel points. Sidewalks are usually of plank supported on stringers which are in turn carried either by brackets at the panel points of the trusses or by extensions of the floor beams. The hand railing at the outer edge of the sidewalk is supported by the brackets, and when the panels are long, an intermediate railing support is also provided at the middle of each panel. The economic panel length for light bridges having plank flooring is from 13 to 15 ft., whereas for heavy traffic it is from 15 to 20 ft., and even as much as 25 ft. for heavy structures of long spans.

Floors on Secondary Country Road Bridges generally consist of one or two layers of planking; one layer is usually 3 in. thick, laid transversely to the length of the roadway; and for two layers the lower is 2 to 4 in. thick and laid transversely, while the 2-in. upper planking is either transverse or diagonal. The lower planking should be pressure treated with creosote oil.

Weights of Floors. For 3-in. yellow pine planking on 3 by 12-in. yellow pine stringers, spaced 2 ft. on centers, the total weight of planking and stringers is 21 lb. per sq. ft.; 2-in. spruce on 3-in. pine with 3 by 12-in. stringers, 28 lb. per sq. ft.; 3-in. yellow pine on 4 by 14-in. stringers spaced 2-1/2 ft., 23 lb.; 3-in. pine on 4 by 14-in. stringers spaced 3 ft., 22 lb., 2-in. spruce on 3-in. pine with 6 by 14-in. stringers spaced 3-ft., 32 lb. per sq. ft. Asphalt laid 2 in. thick on 1 in. of sand and 4 in. of concrete on 5/16-in. buckled plates gives a weight of 85 to 90 lb. per sq. ft. not including stringers.

Steel Stringers are usually made of I beams which are set on the top flanges of the floor beams or connected to the floor-beam webs by hitch or connecting ∟'s. To find the size of I beam compute the required section modulus M/S, and look up in a table of properties of I beams the size and weight of beam corresponding. M = maximum bending moment in inch-pounds, S = allowable unit stress, generally 16 000 lb. per sq. in.

Each stringer should have two hitch ∟'s at each end to connect stringer to floor-beam web. The number of rivets connecting the two ∟'s to the stringer web equals the maximum end shear on one stringer divided by the value of one shop rivet. The rivets through the floor-beam web, connecting hitch ∟'s from a stringer on each side of the floor beam, are field-driven, and their number is obtained by (1) dividing the maximum end shear on one stringer by either the single shearing value of the rivet or the bearing on one thickness of hitch ∟ whichever is the smaller, or (2) by dividing the maximum load going the through connection from both sides of the floor beam by the bearing value of a field rivet on the floor-beam web, or on two thicknesses of

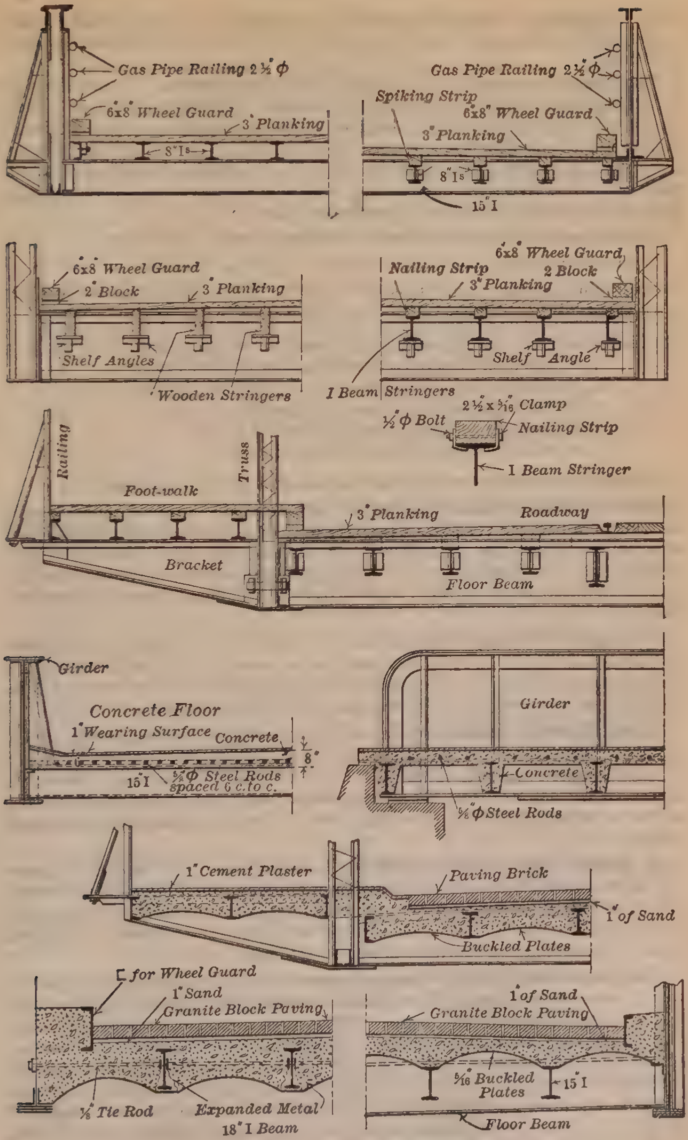


Fig. 28. Floors for Highway Bridges

hitch L if smaller. The greater number of rivets from (1) and (2) should be used. Some engineers claim that double the number just found should be used to allow for bending.

Floor Beams are made of I beams or of built-up beams. I beams require little shop work and should be used if possible. A built-up beam is a plate girder (Arts. 20 and 21) and is designed as such. In any case the maximum moments and maximum shear must be computed. The following formulas are applicable to floor beams where there is a sidewalk on each side of the roadway. The live load per running foot of floor beam and of sidewalk is properly placed to produce the maximum effects.

$$\text{Maximum moment on floor beam} = 1/8\,(p + w)\,d^2 - 1/2\,w_1 e^2$$

$$\text{Maximum moment on sidewalk bracket} = 1/2\,(p_1 + w_1)\,e^2$$

$$\text{Maximum reaction at one truss} = 1/2\,(w + p)\,d + (w_1 + p_1)\,e + p_1 e^2/2d$$

$$\text{Maximum shear on floor beam} = 1/2\,(w + p)\,d + p_1 e^2/2\,d$$

in which w = dead load in pounds per linear foot of floor beam;
p = live load in pounds per linear foot of floor beam;
w_1 = dead load per linear foot of sidewalk brackets;
p_1 = live load per linear foot of sidewalk brackets;
d = distance center to center of trusses in feet;
e = width of each of two sidewalks in feet.

Moments are in foot-pounds and reaction and shear in pounds.

Floors on Primary Road Bridges are now generally built of concrete with some form of wearing surface. The slabs may be supported upon longitudinal stringers, as in the case of wooden floors, or, when the panel length is short, directly upon the floor beams, the stringers being omitted. Wearing surface may be concrete, brick, asphalt, asphalt block, bituminous carpet, or creosoted block.

A Concrete Wearing Surface consists either of an additional thickness (usually about 3/4 in.) of rich concrete placed monolithically with the slab, or of a separately placed wearing surface of rich concrete, varying from 2-1/2 in. in thickness at the edges of the roadway to 4 in. at the center.

A Brick Wearing Surface consists of paving brick laid on edge in successive courses. They are placed, after the concrete is cured and dampened, upon a dry sand-cement bedding course of 1/2 to 1-in. finished thickness. Joints are filled with hot asphalt filler, the surface dressed with a thin surface of dry sand or fine screenings, and a final rolling given to bind the dressing in the asphalt coating.

An Asphalt Block Wearing Surface is constructed in a similar manner. The mortar bed is spread out to a uniform thickness of 1/2 in. and the blocks are immediately laid with close joints and uniform top surface. Clean dry sand is swept into the joints, the residue being allowed to remain 30 days or longer. No rolling is permitted during construction.

The Bituminous Carpet Wearing Surface is placed either upon a wood or upon a concrete sub-floor. The former is thoroughly sealed by calking with oakum, or by other suitable means. Either type must be thoroughly cleaned. The prime coat of thinned tar is applied at the rate of 1/4 gal. per sq. yd. of surface. After 12 hours a second tar coat of slightly different quality, and heated to about 200° F. is applied at the rate of 1/2 gal. per sq. yd. Immediately after application of the tar the surface is covered to a depth of about 1/2 in. with gravel, stone chips or slag aggregate of about 1/2 in. or less in size, and hand tamped or rolled with a heavy hand roller. Successive applications of tar and aggregate are made until a total thickness of about 3/4 in. is reached.

The Creosoted-wood Block Wearing Surface likewise may have either a wood or a concrete sub-floor. The former is calked, cleaned, and mopped with a coat of hot filler upon which is placed a layer of 3-ply roofing felt, laid with 6 in. laps and thoroughly rolled into contact with the filler. The concrete sub-floor is prepared by cleaning and drying, and spreading a thin smooth coating of hot filler of about 1/2 in. in thickness. The creosoted-wood blocks are laid upon this surface in straight parallel courses. After rolling, the joints are filled with hot bituminous filler. The surface is covered with about 1/2 in. of clean, dry sand which is allowed to remain under traffic for several weeks.

Floors on City Bridges differ from those on primary roads in general only in having wider roadways and sidewalks, harder and more durable surfaces, and heavier loading conditions, usually including one or two street-car tracks. The live load will be Class A loading, together with 50 or 60-ton electric railway cars, when tracks are present. The sub-floor may consist of concrete arches enclosing I beam stringers and arching the spaces in between; of steel buckled plates riveted to the stringer flanges and supporting a covering of concrete; or of a reinforced-concrete slab similar to the one used on primary country roads, Fig. 27.

Buckled Plates (Fig. 28) made by pressing dome-shaped buckles in flat steel plates are used for supporting brick, asphalt, or granite blocks. These pavements are laid on 1-in. sand spread on concrete which is laid on the buckled plates. Plates are 1/4, 5/16, 3/8, 7/16 in. thick, 5/16 being most common for roadways and 1/4 for sidewalks; and should be riveted, not bolted, to the stringer flanges with 5/8 or 3/4-in. rivets spaced not over 6 in. Buckles are from 2 to 3-1/2 in. deep, from 2.0 to 5-1/2 ft. on a side, and from 1 to 15 buckles are used in one long plate. Plates are stronger with the buckles turned down.

The Wearing Surface of City Bridges may be any of the types already described, but generally consists of brick, asphalt, or granite blocks. Creosoted-wood blocks have also been much used in some cities, but are not as durable as the more metallic surfaces. Sidewalks of city bridges frequently have a concrete sub-floor and a concrete or granulithic wearing surface.

The Weights of Floors in pounds per cubic foot are: Yellow pine, 48; spruce or white pine, 30; creosoted yellow pine paving blocks, 65; paving bricks, 150; concrete, 150; bituminous concrete, 125; Trinidad asphalt, 140. **Waterproofing** (felt, roofing pitch, sand and road pitch) weighs 12 lb. per sq. ft. Street-car tracks about 100 lb. per lin. ft. Snow, freshly fallen, 8 lb. per cu. ft.; moistened and compacted, 15 lb. Weights of finished floors, exclusive of stringers and floor beams, vary considerably, from 30 to 100 (or more) lb. per sq. ft., and should be computed in each case.

14. Railroad Bridge Floors

In Open Floors of through bridges (Fig. 29) the ties rest directly on steel stringers, one or two of these being under each rail. The stringers are supported by the floor beams. In deck bridges the ties rest on the top flanges of the girders. In solid or ballasted floors the ties are embedded in stone ballast carried either by trough floors or by solid floors of buckled or flat plates, or reinforced-concrete slabs which are supported on I beams. The

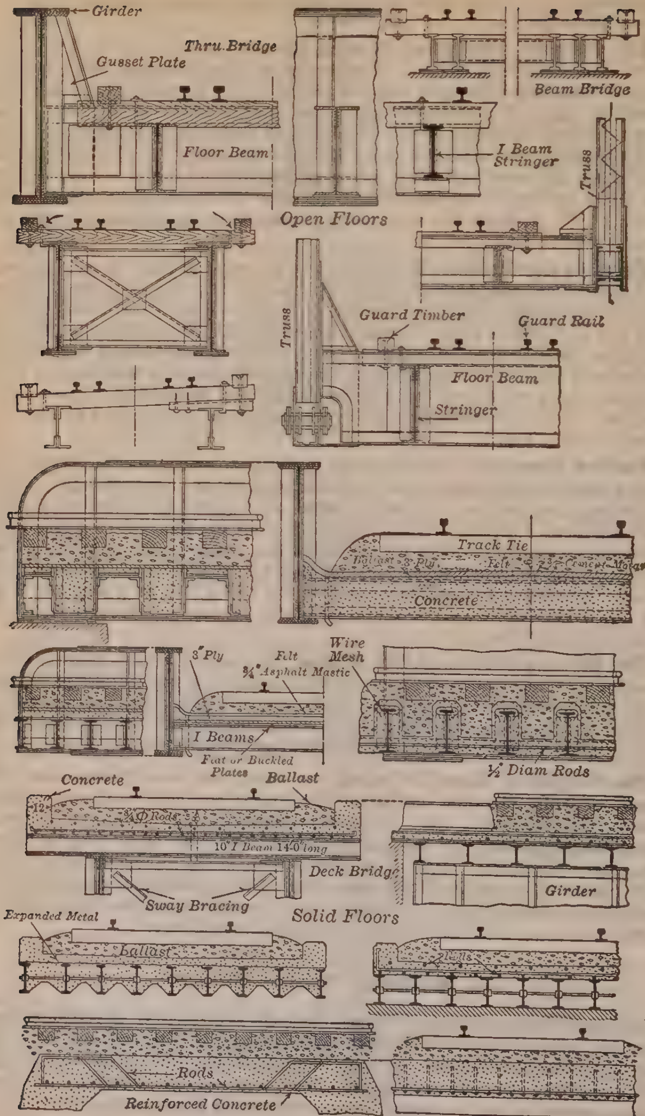


Fig. 29. Floors for Railroad Bridges

I beams for short spans run lengthwise of the track and rest on the abutments, while for long spans they are placed transversely and rest on girders or trusses. On tracks the guard timbers should be notched 1 in. over each tie, should be bolted to every fourth tie and at splices, and spaced at least 20 in. outside of the main rail. **Ties** should be not less than 8 by 8 in., by 10 ft. in length, should be spaced with not more than 6-in. openings, notched over the stringers 1 in. and secured to the same by hook bolts every third tie. Ties should be designed to carry the maximum axle load, with 100% impact, assumed as distributed over three ties with allowable unit stress not over 2000 lb. per sq. in. **Steel guard rails** are spaced 10 in. clear inside of each main rail and extend across the bridge 50 ft. beyond the abutments and around curves if any exist at the bridge. They should be spiked to every tie on the approaches and at least to every other tie on the bridge, and should be provided with a metal cap at the outer ends where they meet.

For Solid Floors troughs were first used to hold the ballast, but more recent bridges have the ballast carried by flat plates 3/8 or 7/16 in. thick riveted to I beams laid transversely to the track. The I beams are spaced 14 to 24 in. on centers, and in through bridges are connected to the girder webs and in deck spans rest on the top flanges of the girders with fascia girders or deep ∟'s along their ends to retain the ballast. The ties are spaced from 18 to 24 in. on centers and the guard rails and timbers are usually omitted. Buckled plates have been used instead of the flat plates; they are stronger and the I beams can be spaced 3 ft. apart, thus diminishing their number. Floors have been built by supporting the flat or buckled plates on the lower flanges of the beams and embedding the ties in ballast between the beams, but in this construction the I beams are almost entirely covered and cannot be inspected or painted without removing the track and ballast, so that best practice places the plates on the top of the I beams. Ties are usually 6 in. deep and have 6 to 8 in. of ballast underneath. Reinforced concrete is also used in place of the floor plates.

A Preservative should be placed between the ballast and steel of ballasted floors. A layer of asphalt-concrete not less than 2 in. thick is sufficient, and the upper surface of the coating should be sloped to allow for drainage. The asphalt should cover all steel parts which otherwise would be in contact with the ballast, and it should be applied only to clean, dry steel surfaces, and if the steel is cold or damp a layer of hot sand should be placed on it and swept off just before the asphalt is applied. The best results are obtained by first coating the steel with hot asphalt, then adding a layer of at least 1 in. of a mixture of one part hot asphalt and four parts hot sand and finally applying a 1/4-in. coating of hot asphalt sprinkled with sand. The layers are each tamped or ironed. Examination of steel surfaces thus treated has been made after 11 years' service and the steel found in good condition.

Stringers for through bridges are I beams or built-up beams; if the latter, the depth should be approximately 1/8 the panel length. When only one line is used per rail the stringers should be not less than 6 ft. 6 in. nor more than 8 ft. on centers, and if two lines are used per rail they should be spaced symmetrically about the rail and from 9 to 15 in. from its center. Sometimes the main stringer is placed under or near the center of the rail and an additional stringer of smaller size is placed outside near the ends of the ties. This outer is called a safety stringer. Deck plate girder bridges generally have no stringers, the ties resting on the top flanges.

Built-up Stringers are designed in the same manner as deck plate girders except that flange plates are not generally needed. Where flange plates are used the ties must be notched to fit over the rivet heads or else the rivets in the top flange plate must be countersunk. Unless the stringer is directly under the rail, flange ∟'s with unequal legs should be used and the long leg should be against the web, or else two rows of rivets must be used in the vertical leg. Placing the long leg horizontal increases the effective depth but also increases the bending due to the deflection of the ties. If the ratio of the length of top flange to its width exceeds 12, cross frames should be added to reduce the unsupported length.

In Proportioning the Cross-section of I-beam stringers or flanges of built-up stringers the **absolute maximum moment** (Art. 17) for the live load should be used and to it should be added the dead load moment at the middle. This is not exact but is on the side of safety and the error is very small. The following formulas are useful in computing the absolute live moments on stringers. They apply for equal loads spaced 5 ft. on centers, and since consolidation locomotives are generally used in design and since more than four wheels would rarely be on a stringer at a time, the formulas cover most panel lengths. There are four cases:

With one wheel on the stringer, $\mathbf{M} = 1/4\, \mathbf{W}l$;

With two wheels on the stringer, $\mathbf{M} = \mathbf{W}\,(2l - 5)^2/8l$;

With three wheels on the stringer, $\mathbf{M} = \mathbf{W}\,(3/4l - 5)$;

With four wheels on the stringer, $\mathbf{M} = \mathbf{W}\,(2l - 5)^2/4l - 5\mathbf{W}$;

where W = the weight on one wheel in pounds;
l = length of stringer in feet;
M = absolute maximum live bending moment in foot-pounds per rail.

With one or with three wheels on the stringer the absolute maximum moment occurs at the middle; with two or with four wheels the absolute maximum is at a wheel 1-1/4 ft. from the middle when the wheel spacings are 5 ft. as assumed above.

Floor Beams are ordinarily built-up beams riveted between the webs of girders or the posts of trusses. In connecting them to pin-connected trusses it is advantageous to place the beams opposite the pins even though this necessitates cutting away the lower corners of the beams (Fig. 29). In addition to its own weight, which is practically uniformly distributed, a floor beam is subjected to a concentrated load at each line of stringers. The distance between centers of girders varies, for different roads and for different girder depths, from 12 to 16 ft. on straight track. For double tracks the stringers are frequently placed 6 ft. on centers and the two girders 28 to 30 ft. on centers. The economic depth of floor beams is much larger than for stringers, but the economic depth usually cannot be used since in through bridges the floor must be kept as shallow as possible and the floor beams are frequently made only slightly deeper than the stringers. In through plate girder bridges a portion of each end of each floor-beam web plate is replaced by a gusset plate of the same thickness but having greater depth at the girder. These gussets extend to the top flanges of the girders to support the flanges laterally (Fig. 29). The gusset and web should be spliced with plates on each side of the web and with sufficient rivets to transmit the shear at the splice and to carry that portion of the moment that the web is allowed to carry. For deep girders the inclined edge of each gusset should be stiffened with two ∟'s. The lower flange of the floor beam should rest on the lateral connecting plate, which in turn should rest on and be riveted to the outstanding leg of the lower flange ∟ of the girder, but there must be sufficient rivets in the hitch ∟'s to carry all the floor-beam reaction into the girder web. The stringers should either rest on the lower flange of the floor beam or else on a small shelf ∟ to aid in erection, but neither the flange nor the shelf should be counted in computing the rivets required to connect stringers to floor beam.

If there is a flange plate on a symmetrical floor beam of a single-track bridge with two lines of stringers the length of the plate can be determined by applying the expression $\frac{2\,ba}{A} + c + 2$, which gives the total length of the plate in feet; b being the distance from girder to stringer and c the distance between stringers, both in feet, a, net area of plate in square inches, A, net area of bottom flange at point of maximum section plus 1/8 gross section of web in square inches. If there are two plates on each flange the above expression also holds for the length of the plate next to the ∟'s if the a is made to include the net area of the second plate as well as of the first.

For a uniform live and dead load the maximum load on a floor beam at a stringer connection is twice the maximum live and dead end shear on one stringer, the panels being all equal, but with a series of wheel loads this is not true. For wheel loads the position of the wheels to give maximum load on floor beam is the same as will give the maximum bending moment at the corresponding point on a span whose length equals the sum of the two adjacent panels. For an end floor beam the maximum load on the floor beam at a stringer connection is equal to the maximum live and dead end shear on one stringer.

15. Transportation and Erection

In Loading on Cars care must be taken not to get the steel so high that it will strike overhead obstructions. In some sections of this country the **clear head room** above top of rail is as low as 14 or 15 ft., but generally more room is available. Posts, chords and other similar members when piled on cars should be braced and bolted in position to prevent displacement. Wherever possible the length of a piece should be such as to overhang one car length by just a few feet.

Long Plate Girders require special attention in loading so that they will not be too high and will be free to ride around curves. They are loaded on flat cars, generally with their webs vertical, and supported at two points only. If a girder is longer than two cars one or more idler or spacer cars carrying no load are used between the end cars on which the whole weight of the girder is carried. Each support consists of a transverse bolster or sill made of one or two timbers laid on the floor of each end car and bolted to the same with one large bolt 2 or 3 in. in diameter, passing through the center of the sill and countersunk in its top. Under the center and each end of the sill is a steel track plate upon which the sill at one support can rotate and by means of a slotted pin hole can rotate or move longitudinally at the other support. The top flange of the girder is braced to the ends of the sills by inclined wooden braces, 6 or 8 in. square, connected together at the top by yoke plates passing over the top flange and connected to the ends of the sills by bent plates and bolts. The ends of the sills are tied to the bottom flange of the girder by diagonal rods, two or four rods being used at each support. For the heaviest girders steel gun-truck cars of 100 000 lb. capacity are used and the distance from top of a vertical girder to top of rail is about 5-1/2 ft. more than the depth of the girder. Girders as long as 125 to 130 ft. and from 9 to 12 ft. deep have been shipped as described above or with slight modifications.

In Designing the following items should be observed to facilitate erection. Avoid details which necessitate telescoping one member into another, as in top chords, connections of floor beams to girders and stringers to floor beams; swinging a member vertically or rotating it horizontally into place is much easier. Place shelf ∟'s or the equivalent on floor beams under all stringer connections where two stringers connect at the same point on opposite sides of the beam. All through spans should be designed so that either trusses and girders or the floor system may be erected first in final position. Avoid notching ties to clear rivet heads and laterals. Ship lateral plates loose or bolt them to members so they will not be bent or broken. Deep gusset plates on floor beams should be stiffened with ∟'s. Tack "loose" fillers with countersunk rivets. Allow clearances for drilling anchor-bolt holes in masonry and for setting the bolts after the steel is all in place. Top chord sections nearest the center should contain at least two panel points. Provide a separate bed plate for each shoe. In riveted bridges the gusset plate at the top of the end post should be shop-riveted to the end post. At pin-connected joints, tie plates should be kept far enough away from the joints and enough rivets should be countersunk to allow the members to swing in place.

The Erection of a Bridge consists in assembling its various parts to complete the whole. A **derrick car** is a car upon which a derrick or similar device is placed and operated, usually by power carried on the car. **Falsework** is the temporary steel or wooden supports upon which the final steel work is erected. A **traveler** is a steel or wooden frame with which the parts of the structure are handled during erection, and which is supported by and moves back and forth upon either the falsework as in ordinary truss bridges, or upon a portion of the finished structure as in cantilever erection. The traveler was formerly much used but is now seldom employed, derrick cars or locomotive cranes being used instead. Very high towers are often erected by means of a creeper derrick, which moves up the columns as the structure increases in height.

Girders and Small Trusses are Erected with gin poles, derrick cars, derricks or gallows frames. Where derrick cars are available they should be used, for with them a girder of ordinary size and weight can be taken from the cars, moved to the proper position and lowered upon its supports. A **gallows frame** consists of a transverse wooden bent guyed at the top and of such shape that a train can pass through it. A frame of this kind is placed at each end of a span and the cars carrying the girder to be erected is run out on falsework, or on an old bridge as the case may be, and the girder is raised from the cars with two tackles at each frame. The cars are then removed and the girder is lowered into position or is placed alongside of its final position and later slid laterally into place. A **gin pole** is a solid or framed pole guyed at the top with at least four guys and with its lower end resting on a support. They are set slightly inclined so that the upper end overhangs the load, which is lifted by hoist lines passing over a sheave at the top. Gin poles are useful for erecting simple girders, beams or small riveted bridge or roof trusses. A framed gin pole 146 ft. long was used in erection of a tower at Manitowoc, Wis., but ordinarily they are single masts 30 to 60 ft. long. Girders are sometimes put in place by blocking and jacks.

Falsework of wood consists of pile or trestle bents spaced from 10 to 50 ft. apart, although the distance apart should generally be equal to the panel length of the truss to be erected. Where piles can be used they make the best falsework and four or more are placed in a bent, the bents being braced transversely and longitudinally. If the height above water is only 10 to 20 ft. the piles are sawed off just below that height and capped, but for greater heights framed bents are built upon the piles, which are sawed off just above water level. Posts in framed bents should be 10 by 10 or 12 by 12 in. and bracings 3 by 10 or 3 by 12. In swift water the bents should be braced in pairs to form towers and the long openings between towers spanned with trusses. If piles cannot be driven the bents must be framed and rest on sills or small timber cribs to distribute the pressure over the bottom. Wooden stringers are bolted to the bents to carry the steel and the tracks for the traveler.

Stringers should be placed about one foot below level of steel to allow for blocking.

Arches, Cantilever Arms, and suspended spans of cantilever bridges are usually erected without falsework by building out, panel by panel, from fixed shore anchorages, anchor arms or adjacent fixed spans. Suspended spans of cantilever bridges are made temporarily continuous with the anchor and cantilever arms. Closure at the center is effected by means of a mechanical device, usually a toggle placed in the line of the chord in the panel at the end of the cantilever arm, or at the ends of the arch, which affords control of the position of the ends of the meeting trusses, both vertically and horizontally. After closure the suspended span is converted into a simple span by relieving one of the end horizontal members. In the case of the Quebec bridge, the 640-ft. suspended span was erected on barges in a sheltered cove several miles from the bridge, floated to the site and hoisted bodily into position. Similarly the 433-ft. suspended span of the Carquinez Straits, Cal., cantilever was lifted into place by means of counterweights.

Where Falsework cannot be used, simple trusses are sometimes erected as cantilevers by building out from either end, but in such cases the members must be specially designed to allow this. Bridges are sometimes erected along the shore and then floated out into position on barges. Long span simple trusses are sometimes erected upon falsework for the first few panels and continued as a cantilever for an equal or greater number of panels, the free end then being supported upon a temporary steel bent. The process of cantilevering and supporting, at intervals, the free end upon bents is continued until the opposite pier is reached. The falsework and temporary bents are then removed. This method was used in erecting the 400 and 600-ft. spans of the Hudson River bridge of the N. Y. C. & H. R. R. R. at Castleton. The 400-ft. span was erected first, the three easterly panels upon timber falsework, and the sixth and eighth panel points upon steel bents. The first four panels of the 600-ft. span were then cantilevered out from the 400-ft. span, used as a temporary anchorage, and later supported under the fourth, eighth, and tenth panel points upon steel bents. The two spans have 14 and 18 panels, respectively.

Paint for Steel Bridges. Parts of bridge members in contact should be given one coat of red-lead paint before being riveted together in the shop. All other surfaces should have one coat of raw linseed oil or one coat of red-lead paint before leaving the shop. After erection two coats of different colors should be applied, the last after the first is dry. Red-lead paint should be mixed 20 to 25 lb. red lead to one gallon raw linseed oil, although some engineers specify as much as 35 lb. per gallon of oil. Graphite paint, made by grinding 12.5 lb. graphite in one gallon linseed oil, is good for either the first or priming coat or for subsequent coats. For one short ton of structural material allow one-half gallon of paint for the first and three-eighths for the second coat.

BEAM AND GIRDER BRIDGES

16. General Arrangement

Beam Bridges are used for short spans up to 35 or 40 ft. in length both for railroad or highway and electric railway structures. They consist of I beams running parallel with the railroad or highway, resting at their ends on abutments and carrying open or solid floors in railroad bridges and plank or paved roadways for highways. The beams must be computed from their section moduli, but their depth for railroad bridges should not be less than 1/15 the span, for electric railway bridges not less than 1/20, and for ordinary highway traffic not less than 1/20. The following spacing of beams is desirable for **railroad spans** with open floors, that is, ties resting directly on the beams:

two I beams under each rail, 12 in. between webs and symmetrically under the rail with a safety stringer 2 ft. from each outermost track stringer. At each end and at center of the span channel braces should be riveted between the beams, and at intervals of 5 ft. channel separators should connect the two beams under each rail. For spans over 10 ft. diagonal braces should also be used. For double-track bridges there should be one safety stringer between tracks. For beam bridges with solid floors the I beams should be spaced apart 18 to 30 in., covered with flat steel plates and ballast, and should have braces at ends and center of span. For **highway bridges** the beams are spaced apart 2 to 3 ft. for wooden floors, and up to 5 or 6 ft. for concrete slab floors, and should be connected together at their ends with ∟'s or ⊏'s.

A Plate Girder is a beam made by riveting a flange, consisting of ∟'s alone or ∟'s and plates, along the upper and lower edges of a solid vertical plate called the **web**. Upright pairs of ∟'s called **stiffeners** are usually riveted to the web at the supports and at certain intermediate points. Plate girders are most suitable for spans between 25 and 100 ft., and even for spans as high as 125 or 130 ft. they make excellent bridges, expensive in first cost but low in maintenance charges.

Tubular Bridges were built by Robert Stevenson, who in 1850 completed the Britannia bridge in Wales and in 1859 the tubular bridge at Montreal, the latter being replaced in 1898 by a truss bridge. The Britannia bridge consists of two parallel independent wrought-iron tubes through which the trains run, each tube having two vertical solid webs connecting to top and bottom chords formed by a series of cells made of plates and ∟'s. Each tube is 15 ft. wide and from 23 ft. deep at the ends to 30 ft. at the center. There are two 460-ft. spans and two 230-ft. spans.

Deck Plate Girder Bridges usually consist of two girders under each track connected by two systems of lateral bracing, one in the plane of the top and the other in the plane of the bottom flanges, and by transverse sway frames at each end and at intermediate points from 15 to 20 ft. apart. For double-track structures the two inner girders should be connected together at the panel points of the top lateral systems by angle struts only without any transverse or lateral bracing between these inner girders. The two girders in single-track deck bridges are spaced 6-1/2 ft. for spans up to 60 ft., about 8 ft. for 120-ft. spans, and proportionate distances for spans between 60 and 100 ft. The deeper the girders the greater should be the spacing. For double-track spans having four girders, the tracks being from 12 to 13 ft. on centers, the inner girders should be 5-1/2 to 6-1/2 ft. apart respectively if the two girders under a track are 6-1/2 ft. apart. Deck spans having floor beams and stringers can be well arranged for single track by placing the girders 9 to 10 ft. apart and connecting the floor beams to the girders at such an elevation that the tops of stringers and girders are in one horizontal plane, the stringers resting on top of the floor beams and spaced 5 or 6 ft. apart. The ties are supported by stringers and girders, the latter serving as safety stringers in case of a derailment, and the stringers are held laterally to the girders by diaphragm plates, that is, vertical plates placed over the floor beams and between the stringers and girders. One lateral bracing system is used at the level of the lower flanges of floor beams.

Through Plate Girder Bridges generally cost more under the same conditions than deck, but they are used where head room under the bridge is essential as in the case of grade-crossing elimination. While for short spans the distance between centers of girders on single track may be as small as 12 ft., this distance is ordinarily 13 to 15 ft. For clearances see Art. 11. Double-track bridges may have two or three girders, the center girder carrying weight from both tracks. Panel lengths vary from 9 to 15 ft., the smaller value being used to secure shallow floor systems. Plate girders for deck spans have the upper corners square, but for long through railroad, and especially for through highway bridges, it is desirable to have them rounded to prevent accidents in the one case and for appearance in the other. The radius of the curve is usually made equal to the length of the bed plate.

Economical Depth for both deck and through girders varies between 1/6.5 of the span length, for short spans, and 1/12 for 125-ft. spans. The latter is not the most economical, but it is the greatest practical depth ratio. For spans up to 80 ft., expansion may be provided by sliding bearing plates. For longer spans, pedestals with pin bearing should be used at each end, and rollers at the expansion end.

Highway Plate Girder Bridges should have two girders for ordinary through spans, and for heavy construction with two sidewalks and one roadway three girders, and in extreme widths even more, are used. Where two girders are used in through spans they should be placed at the curbs to avoid interference with vehicular traffic, and for this same reason deck spans should be used if possible where the width of roadway necessitates three or more girders. The central girder of through bridges projecting up into a roadway interferes with traffic, and where two street railway tracks cross over the bridge the distance between track centers must be increased from the usual 9 or 10 ft. to 13 to 15 ft., thus making undesirable curves in the tracks. For deck spans the tops of floor beams are placed near the top of the girders and the stringers are framed between the floor beams. The **sidewalks** are carried on brackets riveted to the outside girder webs, and since the upper flanges of the brackets are in tension some provision other than tension on rivets must be made to transfer the stress into the floor beam upper flange. This is best done by connecting top of bracket to top of floor beam with a tie plate passing either through or over the main girder.

The Top Lateral System for deck plate girder railroad bridges should be a Warren truss with panel lengths as nearly equal to the distance between the girders as possible and with an angle strut at each panel point, thus making the slope of the diagonals about 45 deg. At alternate panel points the ∟ strut is the top bar of the cross frame, and this spaces the cross frames at the ends and at intermediate points at distances apart equal to twice the distance between girders. No ∟'s less than 3-1/2 by 3 by 3/8 should be used for bracing in railroad, nor less than 2-1/2 by 2-1/2 by 5/16 for highway bridges. For intermediate cross frames of deck bridges two horizontal and two diagonal ∟'s 3-1/2 by 3-1/2 by 3/8 are commonly used, while the end cross frames should be computed to carry the end reaction of the top lateral system. These end frames should have two horizontal ∟'s at top and two at bottom, with two diagonal ∟'s or [’s in each frame. All frames must be riveted to the horizontal lateral bracing plates and to vertical connecting plates which are riveted to stiffeners on the girders. The lower bracing for deck spans should consist of a Warren truss, the ∟ struts being omitted except where they constitute the lower members of the cross frames. For through single-track spans the lateral system should consist of two diagonal ∟'s in each panel riveted to plates which are connected to lower flanges of girders and floor beams; and if there are no end floor beams a strut should connect the stringers and girders together at each end of the span. The laterals are generally connected to the bottom flanges of the stringers. For through double-track railroad or through highway bridges the laterals sometimes run over two panels instead of one.

17. Girders without Floor Beams

Notation. V = shear at section of girder, V_0 = maximum positive live shear in end panel, M = moment at any section, M_C = moment at center of span, l = length of span, a = distance from section to right end of span, g = distance from center of span to any section, w = uniform dead load per unit of length, w_1 = uniform live load per unit of length, W = total load on span, D = increase in shear due to loads coming on span, p = panel length, n = number of panels to right of a given panel, m = number of panels in span, x = distance from resultant load on span to right support.

Long Spans for American Plate Girders

Location	Railway or Highway	Length, feet	Height, feet	Weight of One Girder, short tons
St. Catherine's, Can..	G. T. Ry.	105.7	9.0	64.5
Newark, N. J.	C. R. R. of N. J.	109.5	10.0	77.4
Lyons, N. Y.	N. Y. C. & H. R.	110.7	11.2	96.0
Janesville, Wis.	C. M. & St. P.	114.5	9.5	42.0
Near Jordan, N. Y.	N. Y. C & H. R.	116.0	10.0	77.5
Albany, N. Y.	N. Y. C. & H. R.	116.5	10.0	85.0
Bradford Division	Erie R. R.	128.0	9.0	49.0
Towanda, Pa.	L. V. R. R.	129.5	10.0	56.0
Hubbard, O.	Erie R. R.	131.3	9.6	66.9
Terre Haute, Ind.	Highway	121.5	9.8	30.3
Sixth St., Phila.	Highway	123.0	9.5	50.0
Worcester, Mass.	B. & A. R. R.	122.5	11.0	170.0
Chicago, Ill.	N. Y. C. & St. L. R. R.	131.75	9.7	98.0
Chicago, Ill.	N. Y. C. & St. L. R. R.	125.7	10.8	130.5
New Durham, N. J..	West Shore, N. Y. C. R. R..	131.50	10.0	105.0
Newark Bay, N. J.	Central R. R. of N. J.	125.0	10.0	54.0
Joliet, Ill.	Elgin, Joliet & Eastern R. R	126.83	11.4	72.0

The Shear on a section is the algebraic sum of all outer forces or components of same acting on one side of and parallel with the given section; it is positive when the resultant force on the left is upward and on the right downward. **Bending moment**, or moment, at a section of a beam is the algebraic sum of the moments of all forces acting on either side of the section. It is positive when the resultant moment on left of section is clockwise and on the right is anti-clockwise, causing the beam to bend convex downward and hence causing compression in upper and tension in lower portion of beam.

Dead Load Shears. With uniform load over entire span the shear at any section distant a from right end is $\mathbf{V} = \mathbf{w}\,(\mathbf{a} - 1/2\,l)$. This is the equation of a straight line having for its ordinate at the left end of span the maximum positive shear of $wl/2$, and for the right end an ordinate of equal amount which is the maximum negative shear. Shear at center is zero. The above equation shows that the shear at any section is also equal to the amount of load between the section and the center of span. If an addition to the uniform load there are one or more concentrated fixed loads the shear at any section is found by subtracting from either reaction the loads, uniform and concentrated, between the reaction and the section, and the moment is a maximum where the shear passes through zero.

Dead Load Moments. With uniform load over entire span the moment at any section distant a from the right support is $\mathbf{M} = 1/2\,\mathbf{wa}\,(l - \mathbf{a})$. This is the equation of a parabola with axis vertical and vertex over the center of the span. The moment is positive at all points and is maximum when $a \doteq l/2$, in which case the maximum moment is $\mathbf{M_c} = \mathbf{w}l^2/8$; if w is in pounds per foot and the span l in feet M is in foot-pounds; where it is necessary to reduce this to inch-pounds multiply by 12, not by 144. If M_C is the moment at center and g is the distance from center to any point, the moment at that point equals $\mathbf{M} = \mathbf{M_c} - 4\,\mathbf{M_c g^2}/l^2$.

Live Load Shears and Moments. With a uniform load the maximum shear at either end occurs for full loading and may be found by use of above formulas for dead load shear. The maximum positive live shear at any point is $\mathbf{V} = \mathbf{w_1 a^2}/2\,l$, and occurs when the live load covers the portion of the girder to right of section. This is the equation of a parabola with axis vertical and vertex at right support. The maximum negative live shear at any section occurs when the left portion of girder is loaded, and may be found by calling a the distance from left instead of from right support. The maximum moment at any point occurs for full loading, and may be found by use of formulas for dead load moments.

Live Load Shears. With a single concentrated load the maximum positive shear at a given section occurs when the load is just to right of section and is equal to the left reaction; the maximum negative occurs when the load is just to the left of section and is equal to the right reaction. With a system of concentrated loads the maximum positive shear at any section occurs when one of the heavy loads is just to the right and most of the remaining loads are also to the right of the section. The problem of finding the greatest shear at a section A (Fig. 30) involves therefore first the location of the loads on the span to produce the maximum. Starting with first wheel at A, the shear will then be the left reaction; and if all the loads are moved to the left a distance d_1 until the second wheel is at A the shear is increased if $\frac{\mathbf{W}\mathbf{d}_1}{l} + \mathbf{D} > P_1$, (see Fig. 30). W is the total load on span at beginning of movement, d_1 the distance loads are moved, D is left reaction due to loads that may come on the right of span during the movement. This same relation holds for any other similar movement and for any section provided no loads leave the span during the operation. To apply to the end of span W must include all loads on except the one which is at the end and which therefore leaves the span during the movement. If $d_1 > l - a$, P_1 will leave the span when P_2 is moved to A, (Fig. 30), in which case the shear at A is greater when P_2 is there if $(W - P_1)d_1/l + D < P_1 a/l$. There is one more case to be considered. Suppose P_2 at A and P_1 on the span and that when P_3 is moved to A, P_1 leaves the span. Then the shear at A will be increased if

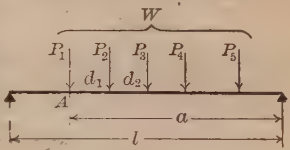


Fig. 30

$$\frac{(\mathbf{W} - \mathbf{P}_1)\,\mathbf{d}_2}{l} + \mathbf{D} + \mathbf{P}_1\frac{(l - \mathbf{a} - \mathbf{d}_1)}{l} > \mathbf{P}_2.$$

Minimum Shear at End of Span. Let it be required to find the maximum shear at the left end of a 40-ft. span with Cooper's E-50 loading (Fig. 23). Placing wheel 1 at the left end the total load on the span exclusive of 1 is 132.5, and moving the wheels 8 ft. to the left until wheel 2 lies at the end the shear at the end is increased, since $132.5 \times 8/40 + D > 12.5$. Moving the loads 5 ft. farther to the left the third wheel lies at the end, and the shear is decreased during this second movement because $140 \times 5/40 + D < 25$, the 140 being total load on span exclusive of wheel 2 when wheel 2 is at the end. Hence with wheel 2 at the end the maximum shear exists there and equals the left reaction or $65 \times 8/40 + 100 \times 32.5/40 = 94.25$ per rail. This shear may be obtained also with the aid of the **moment diagram** (Fig. 34) as follows. The moment of all wheels to the left of the right support about that support is 4370, and, deducting $12.5 \times 48 = 600$, there remains the moment about right support of all wheels that are on the span or 3770 thousand foot-pounds. Dividing this by the span length, 40, the left reaction or shear at left end is 94 250 lb. In the case of a Cooper or similar loading (where a light wheel precedes a series of heavy ones) it is unnecessary to apply the criterion for end shear, since P_1 never gives a maximum and P_2 usually does, the exceptions being unimportant.

Live Load Moments. For a single concentrated load P the maximum moment at a given section will occur when the load is at that section. With a system of concentrated wheel loads the **maximum moment** will occur at a given fixed section when there are as many loads on the span as is possible, consistent with the heavy loads being near, and one of the loads at the section; and this load must be such that when it is just at the right of the section the average load on the left must be less, and when just at the left the average on the left must be greater than the average load on the entire span. Usually several positions of the loads satisfy this criterion, in which case the greatest moment must be determined by inspection or by computing all cases.

The Absolute Maximum Moment is the greatest moment that can occur on a given span under a system of concentrated wheel loads. It will always occur at a wheel which lies at or near the center of the span and the criterion for its determination is as follows. When the center of the span is halfway between the center of gravity of loads on the span and one of the wheels near the center of gravity, then the moment under the wheel where the shear changes sign is a maximum and is the greatest which this combination of loads can produce on the span. When the center of gravity of loads on the span coincides with one of the loads the absolute maximum will occur at the center of span. The method of procedure is then: Assume some loads on the span so as to bring the heavy loads near the center and as many loads on the span as possible. Find the center of gravity of these loads and place the center of span midway between the center of gravity and the nearest load. If all the loads assumed on are now really on the span and the shear changes sign at this load, compute the moment at the nearest load just mentioned. If when the above assumed loads are placed in the position just cited some of the loads assumed on are really off the span, it is then necessary to try again, using a new resultant. Unless one can tell by inspection it is necessary to compute in this manner the moment at several of the wheels until the greatest possible moment is found. In Fig. 31 the maximum occurs at P_2 when the center of span bisects b and therefore $x = 1/2\ (l - b)$ and the expression for the moment at P_2 is

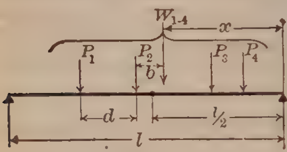


Fig. 31

$$M_2 = \frac{W_{1-4}(l-b)^2}{4l} - \Sigma Pd$$

where W_{1-4} is the resultant of loads on span and ΣPd is the sum of the moments of loads to left of P_2 about P_2. If now the dead load is to be taken into account with the wheel loading, the value of

$$x = \frac{1/2\ l - b(wl + W_{1-4})}{wl + 2\,W_{1-4}}$$

and the maximum moment accurs at P_2. It is however sufficiently accurate to compute the absolute maximum due to live load only as above stated and add the dead moment at the center.

The Exact Equivalent Uniform Live Load, obtained from Art. 10, may be employed conveniently to obtain maximum moments and shears due to a Cooper loading. For maximum moment at a section of a girder, the equivalent uniform load q may be obtained from the chart, Fig. 25, by taking $l_2 = a$ and $l_1 = l - a$, the two segments into which the section divides the span length. The bending moment $\mathbf{M} = 1/2\ \mathbf{q}l_1l_2$.

For maximum positive shear $l_1 = o$ and $l_2 = a$. Instead of extending the chart to include values of q for $l_1 = o$, the latter are given in a separate table, Fig. 25a for the corresponding values of l_2. The q thus obtained is correct for end reactions, and $R = 1/2\ ql$; but for intermediate sections wheel 2 is presupposed to be placed at the section considered and the effect of wheel 1 is disregarded. The maximum shear at a section, upon the above assumption, is $\mathbf{V} = \dfrac{1/2\ \mathbf{q}l_2^2}{l}$. To correct for the effect of wheel 1 subtract $30\,000\left(1 - \dfrac{(l_2 + 8)}{l}\right)$ from the above value of V. This correction and the values of q in Art. 10 are based upon loads per track. For loads per rail divide by two.

For example, to find the maximum shears in an 80-ft. plate girder due to E-60 loading per rail; at the end, $l_2 = 80$, $q = 4657$, $R = 1/2\ ql_2 = 166\,300$; at the 1/4 point, $l_2 = 60$, $q = 4900$, $V = 1/2ql_2 3/4 = 110\,250$; at the 1/2 point, $l_2 = 40$, $q = 5655$, $V = 1/2\ ql_2 1/2 = 56\,550$. To correct for the effect of wheel 1 subtract $1/2 \times 30\,000$ $(1 - (l_2 + 8)/80)$ from the last two values of V or 2250 and 6000 lb. respectively.

18. Girders with Floor Beams

For Notation see beginning of Art. 17

Dead Load Shears. The uniform load here assumed is that due to the floor system only or, in the case of a truss bridge, due to floor system and weight of trusses; in other words, the dead load is assumed to act at the panel points only. For the shears and moments due to the dead weight of a plate girder itself the formulas in Art. 17 may be used. The panels are of equal length unless otherwise stated. The shear in any panel is

$$\mathbf{V} = \mathbf{wp}\,(\mathbf{n} - 1/2\,\mathbf{m} + 1/2)$$

where n is the number of panels to the right of the one in question. The maximum dead positive shear is in the left end panel; that is, when $n = m - 1$, and is $V_0 = wp\,(m - 1)/2$. This is of course equal to the gross left reaction minus the 1/2 panel load at the support. From this maximum positive value the shear decreases by **wp** for every panel towards the right, reaching a maximum negative shear in the right end panel.

Dead Load Moments. The moments at panel points are equal to the moments existing at the corresponding points on a girder without floor beams, and the moments between floor beams are slightly less than without floor beams. Without floor beams all ordinates are those of a parabola, while with floor beams only the ordinates at floor beams fall on a parabola.

Live Load Shears. The neutral point in a panel is that point at which if a single concentrated load is placed, and no other loads act on the span, the shear in the panel is zero. Every panel has a neutral point and its distance from the right end of the panel $= np/(m - 1)$. For maximum positive live shear the load must extend from the neutral point to right end of span, and for maximum negative shear from the neutral point to left end of span. The exact maximum positive live shear in any panel is $\mathbf{V} = \dfrac{V_0 n^2}{(m-1)^2} = \dfrac{w_1 p n^2}{2\,(m-1)}$. The exact equivalent uniform live load for Cooper's loadings for maximum positive live load shear in any panel may be obtained from the chart, Fig. 25 (Art. 10), by letting l_1 = the distance from the neutral point to the right end of the panel and $l_2 = a$ = the distance from the end of the panel to the right end of the span. The maximum negative shear for a given panel is the same as the maximum positive shear in the corresponding panel on the opposite side of center of span. The above is the exact method for finding live shears, but the approximate method is mostly used for uniform loads; this assumes that the full live panel load is applied to each panel point to the right of a panel to produce maximum positive live shear and that no load acts to the left of the panel, thus neglecting the one-half panel load which acts at the left end of the panel. By this method the maximum positive shear is $\mathbf{V} = \dfrac{w_1 p n\,(n+1)}{2m}$.

Live Load Shears. For maximum positive shear in any panel under a system of wheel loads most if not all of the loads should lie to the right of the panel and one of the wheels should be on the floor beam at the right end of the panel and this must be such a wheel that when it is just to the right of the panel the total load on the span must be more than m times the load on the panel, and when this wheel is placed just within the right end of the panel the total load on the bridge must be less than m times the load on the panel. If two or more positions satisfy this criterion the one giving the greatest shear may be determined by inspection, by computing the shear for all the positions that satisfy the criterion, or by computing the change in shear due to a movement of the load system. Thus in Fig. 32 to find the change in shear due to a slight movement suppose that the above criterion for panel A is satisfied by either wheels 1 or 2. Place P_1 at right end of panel A and move the loads till P_2 comes to that point; then if $d_1 < p$ the shear in A is greater for position 2 than 1 if $\dfrac{Wd_1}{l} + D > \dfrac{P_1 d_1}{p}$. If $d_1 > p$ but P_1 does not leave the span while the movement is taking place then the shear is increased if $\dfrac{Wd_1}{l} + D > P_1$. When P_2 is at right end of A and P_3 is then moved to that point the shear is increased if $\frac{Wd_2}{l} + D > \frac{(P_1 + P_2)d_2}{p}$, where $d_1 + d_2 < p$; and is increased if $\frac{Wd_2}{l} + D > \frac{P_2 d_2}{p} + \frac{P_1(p - d_1)}{p}$, where $d_1 + d_2 > p$ but $d_1 < p$ and $d_2 < p$. These expressions do not hold if a load leaves the span during the movement, but similar ones may be written for this case.

Live Load Moments. The criterion for determining the position of loads to give a maximum moment at a panel point is the same as if there were no floor beams, see Art. 17. Clearly the maximum live moment at a panel point next the end as B, (Fig. 32),

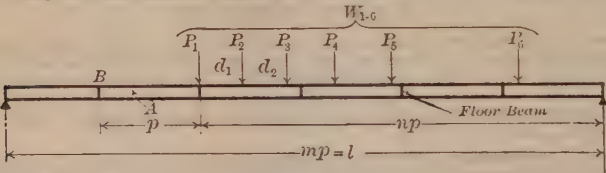


Fig. 32

equals $V_0 p$, and for any point in the end panel the maximum live moment is the maximum live shear in that panel times the distance from the end of span to the point in question. The criterion for maximum moment for a point within a panel such as C (Fig. 33) is $W_1 + \frac{W_2 q}{p} = \frac{W a_1}{l}$, where W_1 is the total load on panel to left of, and W_2 the total load on the panel in question.

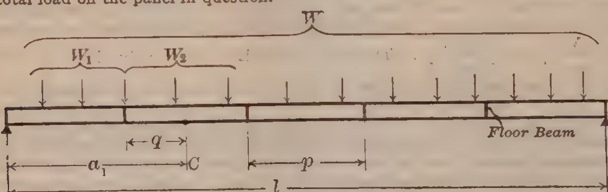


Fig. 33

The Moment Diagram (Fig. 34) aids in computing shears and moments. Its essential parts are weights and wheel spacings for a concentrated load system, distance of each wheel from the first, sum of all loads from the first

Wheel	Wheel Loads	Sum of Loads	Sum of Distances	Moments	Distances
1	12.5	12.50			8.0
2	25.0	37.50	8	100.00	5.0
3	25.0	62.50	13	287.50	6.0
4	25.0	87.50	18	600.00	5.0
5	25.0	112.50	23	1037.50	9.0
6	16.25	128.75	32	2050.00	5.0
7	16.25	145.00	37	2693.75	6.0
8	16.25	161.25	43	3563.75	5.0
9	16.25	177.50	48	4370.00	8.0
10	12.5	190.00	56	5790.00	8.0
11	25.0	215.00	64	7310.00	5.0
12	25.0	240.00	69	8385.00	5.0
13	25.0	265.00	74	9585.00	5.0
14	25.0	290.00	79	10910.00	9.0
15	16.25	306.25	88	13520.00	5.0
16	16.25	322.50	93	15051.25	6.0
17	16.25	338.75	99	16986.25	5.0
18	16.25	355.00	104	18680.00	5.0
	2.5 per linear foot	355.00	109	20455.00	

Loads are in thousands of pounds per rail

Fig. 34. Moment Diagram for Wheel Loads for Cooper's E–50 Loading

to each succeeding wheel inclusive, sum of moments about each wheel of all loads to the left of that wheel. The wheel spacings should be plotted to scale and the span lengths to the same scale.

To illustrate let it be required to compute maximum moment at center of a 40-ft. span, using the diagram in Fig. 34. The fourth wheel satisfies the criterion in Art. 17 and by inspection gives the greatest moment. To compute this moment find the moment of left reaction about center and deduct the sum of moments of wheels 1, 2 and 3. To do this find the moment of all wheels on the span about right support, 2693.75 + 145(1) = 2838.75 in thousands of foot-pounds; then divide this by the span, 40 ft., and multiply the quotient by the half span, 20 ft.; whence 1419.375 is the moment of left reaction about center. Subtract 600, the moment of wheels 1, 2 and 3 about wheel 4, and 819 375 ft-lb. is the moment at center of span.

19. Plate Girder Webs

Notation. M = bending moment, t = thickness of web, h = depth of web, V = vertical shear on web, S = allowable tensile unit stress, S_s = allowable shearing unit stress, S_b = bearing unit stress on rivets, S_c = allowable compressive unit stress, A = gross area of cross-section of web plate, d = horizontal distance between stiffeners, H = horizontal force on outside rivet in a vertical row, t_1 and h_1 = thickness and depth of web splice plate, M_r = bending moment carried by one row of rivets in web splice, M_w = net moment carried by web, p = pitch in inches, W = number of rivets in a row. Forces in pounds, dimensions in inches, moments in inch-pounds, stresses in pounds per square inch.

The Function of the Web of a girder is to resist the shearing forces acting on the various sections of the girder and also to carry some of the bending moment. The depth and thickness of a web plate must be such that its cross-sectional area will withstand the shear and buckling and at the same time give the girder an economic depth, that is, a depth requiring the least material in the girder. Girders are sometimes made with curved flanges, giving greater depths at the center than at the ends.

The Economic Depth of a Girder varies with the kind and amount of loads, the length of span and the allowable unit stresses. A common rule for girders up to about 40-ft. span is that the depth should be such as to make the weight of flanges equal to the total weight of web with its splice plates and stiffening angles and, in proportion as the span exceeds 40 ft., to make the weight of flanges greater than that of the web and its details. Unless limited by such considerations as headroom, girders having constant cross-section should be given a depth equal to 1/8 or 1/9 their length, and girders of variable sections 1/10 to 1/12 their length. For short built-up girders used as stringers the depth must generally be greater, more nearly 1/6 their length.

The Thickness of Webs must be such as to resist the shear at various sections and also to prevent buckling. The gross cross-sectional area of the web at any section is found by dividing the total shear by the allowable unit shearing stress. The required thickness for shear is $t = V/S_s h$.

Buckling of the Web. The horizontal and vertical shears acting in a web plate produce a tension and a compression of equal intensities on planes making an angle of 45 deg. with the neutral axis. This compression tends to buckle the web plate, and the web must be thick enough to withstand the buckling or else stiffeners must be riveted to the web to increase its compressive strength or, what amounts to the same thing, its shearing strength. Stiffeners are used over bearings, at points of concentrated loading and at other points where the width of the unsupported web between flange angles is greater than 50 times its thickness. These stiffeners are usually made of ∟'s riveted vertically to the web. Having found the required thickness for shearing from the formula just given, the proper **distance between stiffeners** must then be determined; and if the distance in the clear between stiffeners, d, comes equal to or greater than the clear vertical distance between horizontal angles the stiffeners are spaced equal to this vertical distance, with 6 ft. as a maximum. If d comes out very small so that stiffeners are required too close together, a thicker web should be chosen to reduce the number of stiffeners. The horizontal distance in the clear between stiffeners is given by $\mathbf{d} = \dfrac{(12\,000 - \mathbf{s})\mathbf{t}}{40}$, in which s is the average shear per square inch in the web at the section where d is desired.

The Minimum Thickness for Webs of plate girders in buildings is 1/4 in., for highway bridges 5/16 and preferably 3/8 in., and for railroad bridges not less than 3/8 for lightest traffic and 7/16 for heavy traffic. The thickness near the ends of heavy girders is sometimes greater than near the center, and the increase may be made either by riveting reinforcing plates on the sides of the main web plate or by using a thicker plate near each end. In the latter case thin fillers must be used between the flanges and the thin web plates. Webs as thick as 1 in. have been used, but ordinarily they are from 7/16 to 3/4 for railroad bridges, 5/16 to 7/16 for highway, and from 1/4 to 3/8 for buildings. Sheared plates may be obtained as wide as 10 ft., with lengths of 10 or 11 ft., and universal mill plates up to 4 ft. wide by 70 to 80 ft. long. Plates of great width or length are subject to special prices.

Intermediate Stiffeners (Fig. 42) used to stiffen the web of a railway plate girder vary from 3-1/2 × 3-1/2 × 3/8 in. for spans under 50 ft. to 6 × 4 × 3/8 in. for 100-ft. spans. By the A. R. E. A. Specifications the outstanding legs should not be less than 1/30 of the depth of the girder plus 2 in. For girders in buildings and highway bridges they are frequently made lighter. They should be used in pairs, one on each side of the web. In deck bridges intermediate stiffeners should be proportioned to carry the greatest wheel load in compression, should be connected to the web with sufficient rivets to transmit this load, should not be crimped, that is, not bent around the vertical legs of the flange ∟'s and should have outstanding legs as wide as possible and not extend over the edge of the horizontal leg of flange ∟'s. For through spans the connecting ∟'s at floor beams should have outstanding legs of at least 4 in., with rivets arranged so that floor beams may be placed without spreading girders.

End Stiffeners used to connect one girder web to another, as at junction of stringer to floor beam, must be capable of carrying the full end shear on a vertical section through the line of rivets. Where a girder rests upon a support as at abutments the end stiffeners (Fig. 42) usually consist of two pairs of ∟'s, one pair at the extreme end of girder and the other over the bed plate and about 1 in. from its inner edge. For girders with bearings arranged on rockers each pair of ∟'s should be proportioned for one-half the maximum end reactions, but for those resting on flat bearings each pair should be assumed to carry at least three-quarters of the reaction. Stiffeners at the ends and at points of concentrated loading should be proportioned as long columns, in which l is taken equal to one-half the depth of the girder in inches and r is the radius of gyration of the pair of stiffeners about their gravity axis lying in the central plane of the web plate. The outstanding legs should be as wide as the flange angles will allow and should fit tightly against them.

Thus, given an end shear of 319 500 lb., depth of girder 80-1/2 in., thickness of web plate 7/16 in., and thickness of fillers between stiffeners and web 3/4 in. Assume two pairs of 5 × 3-1/2 × 11/16-in. ∟'s with the 5-in. legs outstanding. The 3-1/2-in. legs are separated by the 7/16-in. web and the two 3/4-in. fillers, or 1-15/16 in. in all. For their combined section about the axis in the web r is 3.12 in. $\mathbf{S} = \mathbf{16\,000 - 70}\ l/\mathbf{r} = \mathbf{16\,000 - 70 \times 40.25/3.12 = 15\,000}$ (nearly). Required area = 319 500 ÷ 15 000 = 21.4 sq. in. The actual area of the 4 ∟'s is 21.5 sq. in. The number of rivets in double shear required to transfer the end shear from the angles to the tight fillers is 319 500 ÷ 14 430 = 23. The number used is 30, not counting those through the flange angles. The number of rivets in bearing on the web required to transfer the end shear from fillers to web is 319 500 ÷ 9190 = 35. The number used is 48, making the filler plates a rigid part of the web plate.

For intermediate stiffeners, 1/30 of the depth of girder plus 2 in. requires the outstanding leg to be 5 in. Accordingly each pair of intermediate stiffeners will consist of 5 × 3-1/2 × 3/8-in. L's. The spacing should not exceed 6 ft. nor be greater than $\frac{t\,(12\,000 - S)}{40}$, where t = 7/16 in. and S = total shear at section ÷ gross area of web. The exact position of the several pairs of stiffeners is governed by the location of the cross frames.

End Bearings. Expansion and contraction longitudinally due to changes in temperature must be allowed at one end of all spans, a range of 150° F. being the amount generally used. Taking the coefficient of linear expansion of steel as 0.000007 per degree Fahrenheit and a range of 150°, the expansion amounts to 1/8 in. for 10 ft. of length. Spans less than 80 ft. may have planed sliding bearings at one end (Fig. 42). For spans over 80 ft., long turned

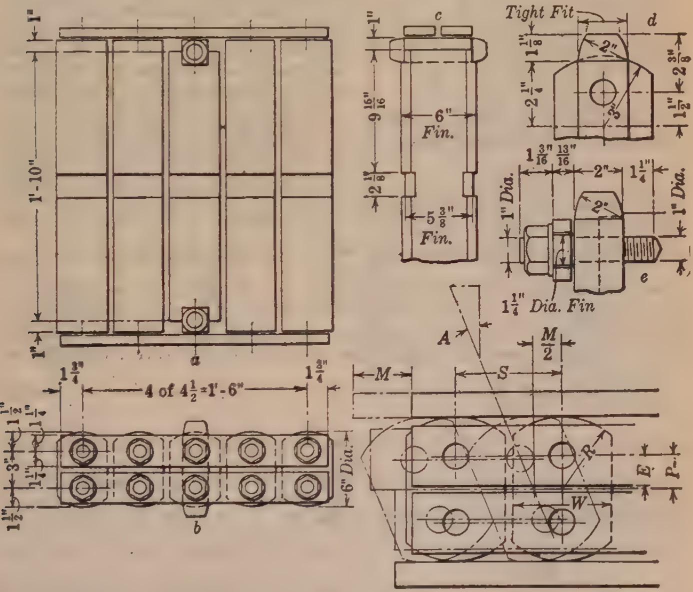


Fig. 35

rollers not less than 6 in. in diameter should be used and each end should rest on a rocker or bolster arranged with a pin to fix the point of application of the reaction. Allowable **pressure per linear inch of rollers** is variously taken at from 600 D to 300 D depending on whether impact is or is not allowed; D is diameter of roller in inches. **Bearing on Masonry** may be safely taken at from 400 to 800 lb. per sq. in. **Segmental rollers** as used for long spans are made by removing portions of cylindrical rollers as shown in Fig. 35.

Fig. 35 is typical of the 6-in. rocker nest of the American Bridge Co. To prevent sliding of the structure on the rollers or base plate, the ends of the middle roller are provided with vertical teeth which mesh into corresponding holes in the top and bottom plates. Lateral slipping is prevented by the groove at the middle of each roller, which exactly fits over a corresponding longitudinal projection left in the planed surfaces of the bearing plates. Fig. 35 shows the locking of rollers and side bars under an extreme longitudinal movement M. Let M = movement of bridge in either direction from the initial vertical position of rollers. Then $A = 28.7\ M/R$ in degrees. Sidebars will lock when $E = P \cos A$. Rollers will lock when $W = S \cos A$. Both will lock at the same time when $E/P = W/S$. W must not be less than M.

Web Splices. Girders over 40 to 50 ft. in length usually have their webs in two or more pieces, the web plates being spliced generally with one plate on each side of the web at each joint, and sometimes in addition thereto a plate riveted at each joint to the vertical leg of each of the four flange angles.

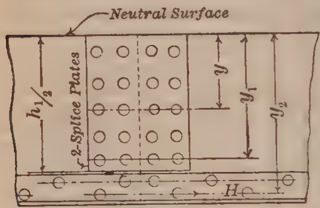


Fig. 36

Web plate splices should always be designed for moment, and when shear acts simultaneously it should likewise be considered, although generally when a splice is sufficient to carry the full bending moment of the net section of the web shear may be neglected. To design a web splice consisting of two plates (Fig. 36), one on each side of the web: (1) Make the two splice plates as deep as the vertical distance in the clear between flange ∟'s, and then compute their thickness so that the combined net moment of resistance of the two plates equals that of the web; but no plate less than 3/8 in. should be used. Thickness of each splice plate is $t_1 = \frac{th^3}{2\,h_1^3}$. (2) Next space the rivets in one vertical row uniformly if possible; if any uneven spaces are required place the larger spaces near the neutral axis of the girder. The pitch in a row should not be less than 3 and not over 4 in. (3) Compute the bending moment that one vertical row of rivets in the splice plates can carry, using $\mathbf{M_r} = \left(\frac{2\,\mathbf{H}}{\mathbf{y_2}}\right)\sum_0^{y_1}\mathbf{y}^2$, where H is the value of the outermost flange rivet in bearing on the web (Fig. 36) and $\sum_0^{y_1}\mathbf{y}^2$ is the sum of the squares of the distances from neutral axis to the rivets on one side of this axis only. (4) Compute the net moment of resistance of the web $= \mathbf{M_w} = \frac{\mathbf{Sth^2}}{8}$, S being the allowed unit stress in the tension flange. (5) The number of rows of rivets on each side of the joint is $\frac{\mathbf{M_w}}{\mathbf{M_r}}$. If the number of rows thus found involves a fraction of a row, as 2-1/2 rows, the rivet spacing in each row may be changed accordingly, or better, use only the full rows for each side of joint, two in this case, and place the splice at a point where the flanges have sufficient excess area to carry a moment equal to the difference between the net moment of the web and the moment of the two full rows. In case the splice cannot be so placed, then additional plates having a net area equal to that just mentioned should be placed on each flange.

With a uniform pitch, the bending moment a row of rivets will carry is given by $\mathbf{M_r} = \frac{\mathbf{Hp(N+1)(N+2)}}{6}$, where H is the horizontal force on the rivet farthest from the neutral surface and which is included as carrying moment. If H is in pounds, p inches, and N the number of rivets in one full row, then M_r is in inch-pounds.

20. Plate Girder Flanges

Notation. M = bending moment in inch-pounds, d_1 = distance in inches between center of gravity of flanges, S = allowable unit tensile or compressive stress in pounds per square inch, A = net area of cross-section of tension flange in square inches, t = thickness and h = depth of web, l = unsupported length and w = width of compression flange, p = pitch of rivets all in inches, V = vertical shearing force on girder, r = value of rivet in pounds, v = resultant shear per running inch of flange in pounds, W = maximum wheel load in pounds, v_1 = exact longitudinal shear per running inch of flange in pounds, m = statical moment in inch units, I = moment of inertia of entire cross-section of girder in inch units.

The Flanges of a beam or plate girder carry the greater part of the bending moment acting on the girder, and as they receive stress from the web they serve to prevent its deformation. One flange, usually the lower, is in tension and the other is in compression. The latter is a horizontal column receiving its stress from the web at points throughout its length, and held in a vertical direction by the web, and in a horizontal direction, usually, though not always, by bracing. The function of the flanges of a girder, then, is to carry moment and they must be designed for this purpose.

The Composition of Flanges varies in different girders and also usually at different sections of the same girder. For light loads or for short spans each flange may be made of two ∟'s alone (Fig. 37) or for heavier work two ∟'s with one or more horizontal flange or cover plates riveted to the outstanding legs of the ∟'s (Fig. 38). To secure larger cross-sections vertical flange

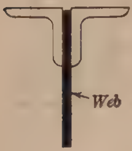


Fig. 37

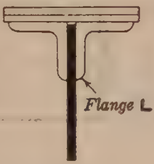


Fig. 38

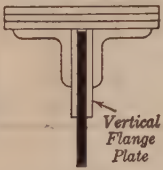


Fig. 39

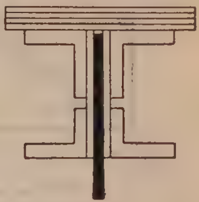


Fig. 40

plates between the ∟'s and web are used, and these plates are run the entire length of the span, while the horizontal flange plates are made of different lengths (Fig. 39). For heavy sections an arrangement such as is shown in Fig. 40 may be used for the top flange. In this case the lower flange is generally given the more usual section consisting of two ∟'s with cover and vertical plates (Fig. 39). Railroad bridge girders with over 150 sq. in. of gross cross-section in each flange have been built.

Proportioning Tension Flanges. The design of a flange (Fig. 42) consists in finding the required cross-section at the point of maximum bending moment, selecting proper sizes of ∟'s and plates and then finding the lengths of the plates. If the web could carry no moment the compression in one flange and the tension in the other would resist the full moment and would form a couple the arm d_1 of which is the distance between centers of gravity of the two flanges, and each flange stress would be $\mathbf{M/d_1}$. The flange stress divided by the allowable tensile unit stress S would give the required net area for the tension flange, that is, $\dfrac{M}{d_1S}$. But since the web does carry a moment $= \dfrac{Sth^2}{6}$ if there is no vertical row of rivets at or near the section considered, and approximately $Sth^2/8$ if there is, then the equivalent area, which if placed in each flange will carry the same moment as the web can carry, is $th/6$ or $th/8$ respectively. The correct required net area of the tension flange is usually based on the latter value and is $\mathbf{A} = \frac{\mathbf{M}}{\mathrm{d_1 S}} - \frac{\mathbf{th}}{8}$. In applying this formula care must be taken to use the proper numerical values. Thus if d_1, t and h are inches and S is pounds per square inch, then M is inch-pounds, and the result A will be square inches.

With following data, to proportion the cross-section of a tension flange: maximum M in foot-pounds = 1 500 000 live, 360 000 dead, 1 210 000 impact, total 3 070 000; web 84 by 1/2 in., distance back to back of flange ∟'s 84-1/2 in., maximum tensile stress, 16 000 lb. per sq. in. Assume the distance center to center of gravity of flanges, called the effective depth, as 83.0 in., then the required net flange area = $\frac{3\,070\,000\ (12)}{83.0\ (16\,000)} - \frac{84}{2\ (8)} = 22.5$. Two 6 by 6 by 5/8 ∟'s with gross area of 14.2 sq. in. have a net area of 14.2 − 4 (1) 5/8 = 11.7 sq. in., which deducted from 22.5 leaves 10.8 as the required net area of plates. Assuming plates 14 in. wide with two 7/8

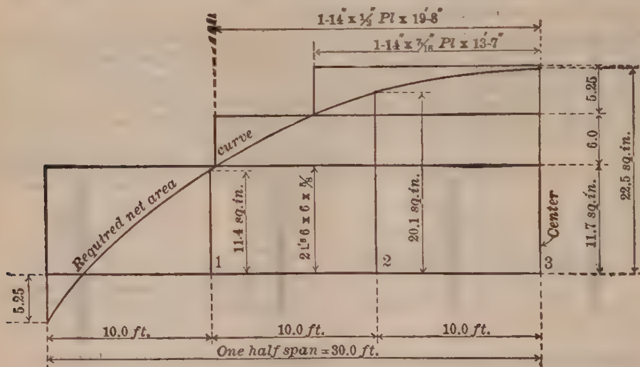


Fig. 41

rivets in one section the net width is 12 in. Required thickness for plates = 10.8/12 = 0.9 in. Hence one plate 14 by 1/2 and one 14 by 7/16 and two 6 by 6 by 5/8 ∟'s are to be used. In computing the net area of the two ∟'s the two holes for 7/8 rivets in each ∟ are taken as 1/8 larger in diameter than the rivet, namely 1 in.; similarly in the plates. The position of the center of gravity of this flange must be computed to check the effective depth, and if the assumed effective depth is in appreciable error the design must be revised.

The Compression Flange should not be of less and is generally of the same gross section as the tension flange. It is usually supported laterally at points with distance apart not exceeding 12 times its width. When the compression flange consists of angles, or of angles and flat cover plates, it is proportioned by the column formula, $S = 16\,000 - 150\ l/b$, where S = allowable unit stress on the gross section of the flange, l = unsupported length, and b = flange width.

Lengths of Flange Plates. Following is a graphic solution for a single-track through railroad girder bridge of 60-ft. span divided into 6 equal panels. By means of above formula for A compute the required net flange area at each of the panel points, 1, 2, 3 (Fig. 41), these being the points at which the live and dead moments are previously computed; suppose their net areas to be 11.4, 20.1 and 22.5 sq. in. respectively. Plot these values at points 1, 2, 3 to any convenient scale, such as 1/8 in. = 1 sq. in. For the horizontal scale 1/4 in. = 1 ft. is convenient. At the end lay off on opposite side of the horizontal axis the web equivalent, that is, $th/8$, here assumed 5.25 sq. in., and then draw a smooth curve through the four points located. Proportion bottom flange at point of maximum moment, finding the sizes of ∟'s and plates as described above, and, beginning at the horizontal axis, lay off first the net area of the two ∟'s, 11.7 sq. in., and then of each succeeding plate, placing the thickest plate next to the ∟'s. The half lengths of plates may now be scaled from the diagram, and to each half length should be added 1 ft. or enough to get in at least two rows of rivets. The plates on the top flange may be made the same length as on the bottom except that the one next to the ∟'s should run the entire length of the girder.

With a uniform load over span L and a uniform depth of girder the length x of any plate is $\mathbf{x = L\sqrt{a/A_1}}$, where A_1 is the total area of the flange at the maximum section plus the web equivalent, and a is the area of the plate being cut plus the areas of all plates outside of it. If there are several plates, a is first the area of outside plate; second, this plus the area of the next plate; third, these two plus the area of third plate.

The Pitch of Flange Rivets (Fig. 42) is the distance from the center of one rivet to the center of the next whether in the same row or not. In bottom flanges of deck girders and in both flanges of through girders the rivets connecting the flanges to web plates transmit longitudinal shear only, whereas these rivets in the top flanges of deck girders or stringers carry, in addition to the longitudinal, a vertical shear due to the pressure from the wheel loads and track. The required pitch for horizontal flange rivets which connect the flange ∟'s to the web in girders without loads on the top flange is, according to the usual approximate method, $\mathbf{p = rd_1/V}$, where r is the value of the rivet, namely, the bearing value on web plate or bearing on two ∟'s or double shear whichever is the least. If the rivets carry vertical loads in addition to the longitudinal shear, as in top flanges of stringers or deck girders, the resultant shear per running inch is $\mathbf{v} = \sqrt{\left(\frac{\mathbf{V}}{\mathbf{d_1}}\right)^2 + \left(\frac{\mathbf{W}}{\mathbf{36}}\right)^2}$, in which W is the maximum wheel load, here assumed distributed over 36 in. of flange length. And the pitch is $\mathbf{p = r/v}$. This approximate method for finding the pitch of horizontal rivets in flanges gives spacing slightly smaller than the true value and is on the safe side. The exact **longitudinal shear** existing between the flange and web or between the flange plates and ∟'s is $\mathbf{v_1 = Vm/I}$, in which m is the statical moment about the neutral axis of that part of the flange outside of the section on which the horizontal shear is desired, in other words, that part of the flange section which is connected to the remainder of the flange or to the web by means of the rivets the pitch of which is required.

In Spacing Rivets in Flanges the pitch is computed at intervals of from 2 to 5 ft. for stringers and deck girders, and the spacing is varied in accordance with those computed values. For through girders the pitch throughout any one panel is made constant since the shear is constant except for dead weight of girder. Vertical rivets which connect flange plates to flange ∟'s are generally given the same pitch as the horizontal rivets. The pitch can be computed by the above exact formula.

A Good Method for Finding Pitch of flange rivets is illustrated by the following modification of the above approximate method. Required the pitch of horizontal flange rivets at the end of a through girder having a web 60 by 1/2 in.; at the end each flange consists of two 6 by 6 by 1/2 ∟'s and one cover plate 14 by 1/2. Rivets 7/8 in. in diameter; bearing 24 000 lb. per sq. in.; shearing 12 000 lb. per sq. in. End shear including live, impact and dead, 200 000 lb. Effective depth 58.6 in. The gross area of two ∟'s and one cover plate is 11.5 + 7.0 = 18.5 sq. in. and including the web equivalent is 18.5 + 3.75 = 22.25 sq. in.; hence the longitudinal shear per running inch between flange ∟'s and web is **(18.5/22.25) 200 000/58.6 = 2850 lb.** The least value of the rivet is 7/8 × 1/2 × 24 000 = 10 500 lb., and the required pitch at end of girder is **10 500/28.0** = 3-3/4 in.

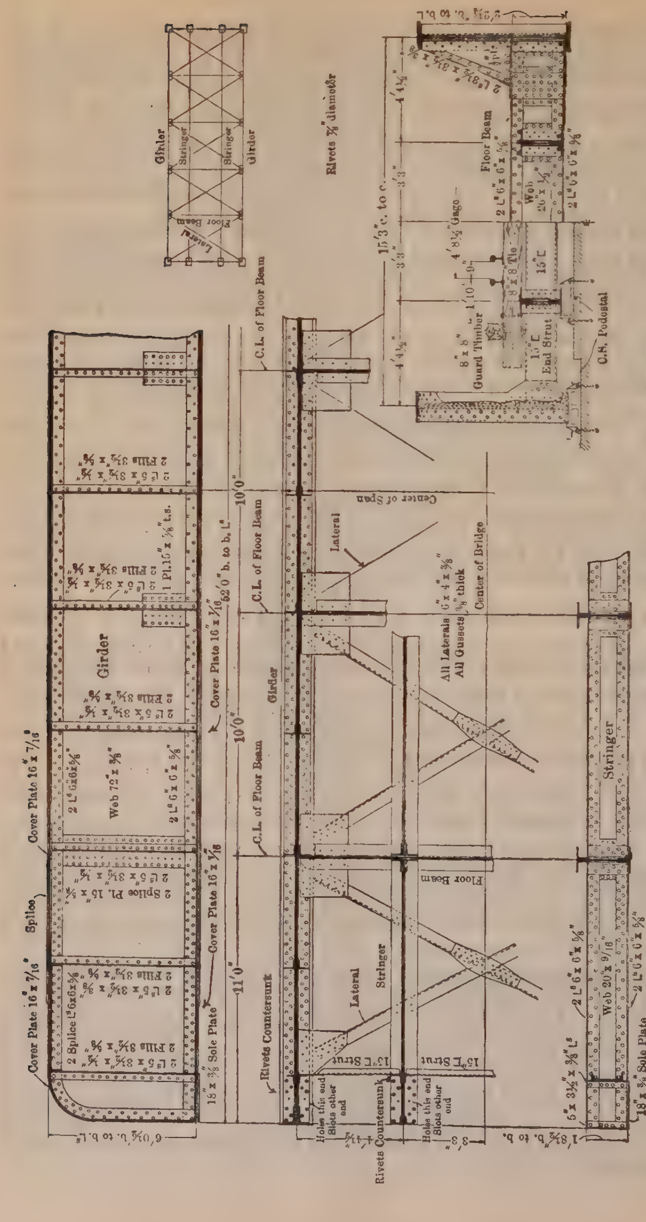


Fig. 42. Design of a Single Track Through Plate Girder Bridge

21. Riveted Joints

Notation. S_c, S_s and S_t are the allowable unit bearing, shearing and tensile stresses in pounds per square inch; d, diameter of rivet; l, end distance; t, thickness, and w width of main plate; all dimensions in inches; n, number of rivets on each side of joint.

Before being driven, a rivet consists of a shank, or cylindrical portion, and a head which may be either conical or nearly hemispherical, the latter form being called buttonheads. The process of riveting consists of heating the rivet and while it is hot placing it in the hole and upsetting the shank end to form a second head. A countersunk rivet is used where it is desired that the head of the rivet must be flush with the surface of the part into which it is driven, but to secure a surface free from obstruction the countersunk head must be slightly chipped.

Lap Riveted Joints are shown in Figs. 43 and 44, the first being single and the second double riveted. Butt joints are shown in Figs. 45–48, those with two cover plates being better because of less bending in the rivets. Butt

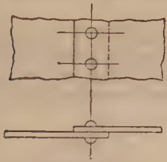


Fig. 43

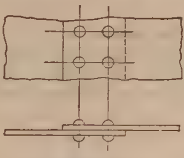


Fig. 44

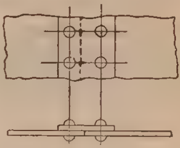


Fig. 45

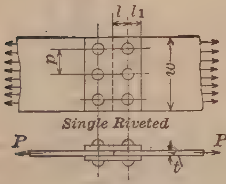


Fig. 46

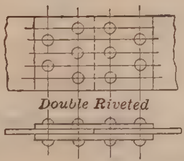


Fig. 47

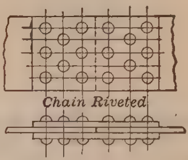


Fig. 48

joints with a single strap require twice as many rivets as lap joints to transfer a stress from the one main plate into the other. In lap joints, and in butt joints with a single strap or cover, the rivets are in single shear, while in butt joints with double covers all rivets are in double shear.

Design of Riveted Joints. In Fig. 46 the main plate has a tension of P and is spliced with two covers each of which must have a thickness of at least $t/2$, although usually each would be made from $2/3\,t$ to $3/4\,t$. There must be sufficient net area in the main plate on its cross-section through the row of rivets to carry the force P in tension, that is, $(\mathbf{w} - \mathbf{nd})\,\mathbf{tS_t} = \mathbf{P}$. The number of rivets on each side of the joint must transmit the force P in double shear, that is, $\mathbf{n} = \dfrac{2\,\mathbf{P}}{\pi \mathbf{d}^2 \mathbf{S_s}}$; or in bearing on the main plate, $\mathbf{n} = \dfrac{\mathbf{P}}{\mathbf{tdS_c}}$. The distances l and l_1, Fig. 46, must be sufficient to prevent shearing or tearing of the main plate and two covers respectively. For bridge work, rivets of 3/4 and 7/8-in. diameter are commonly used; for buildings, generally 3/4-in. The minimum pitched used for structural work is $3\,d$, and the minimum $\mathbf{l} = \mathbf{1.5\,d}$. Distance between rows in Fig. 47 should not be less than $\mathbf{2\,d}$. Bearing value of one rivet on plate having thickness $\mathbf{t} = \mathbf{tdS_c}$. Single shearing value of one rivet is $\dfrac{\mathbf{S_s} \pi \mathbf{d}^2}{\mathbf{4}}$.

The Efficiency of a Joint is the ratio of the strength of a given width of joint to that of a solid plate of equal width, and is usually expressed as per cent; for example, single-riveted lap joints show efficiencies of from 50 to 70% and double-riveted lap joints 60 to 80%. Joints with drilled or reamed holes show higher efficiencies than those with punched holes. Values of from 60 to 70% are what may usually be expected. By careful design the rivets may be arranged so that higher efficiencies may be obtained. Tests on full-size angles show that tensile strength on the net area varies from 75 to 90% depending on the arrangement of rivets and on whether or not both legs of the angles are connected to the end plates by which stress is transmitted into the angles. Both legs of angles should be riveted at connections. **Friction** between the main and splice plates in butt joints and between the two main plates of lap joints should not be relied upon, for although under quiescent loads it may continue to exist, it is uncertain and for movable loads it may be destroyed altogether.

The American Bridge Co.'s Standards show for button-head rivets: diameter of head 1-1/2 times diameter of shank plus 1/8 in.; height of head = 0.425 times diameter of head; for countersunk heads the slope is 30 deg. from axis of shank and the depth of head is 1/2 the diameter of shank. **Cambria Steel Company's standards** give for

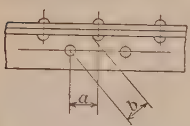


Fig. 49

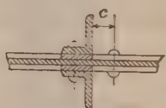


Fig. 50

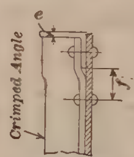


Fig. 51

button-heads: height of head = 6/10 times diameter of shank, radius of head = 3/4 times diameter of shank + 1/16 in.; for countersunk heads the diameter is the same as for button-heads and the slope is 30 deg. For required clearances see Figs.

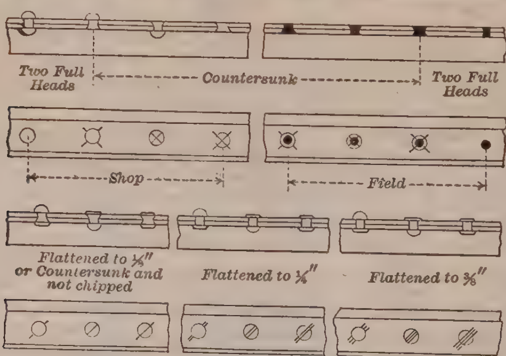


Fig. 52. Conventional Signs for Rivets

49, 50 and 51. In Fig. 49, a must be sufficient to make b at least 1-1/4 and 1-3/8 in. for 3/4 or 7/8 rivets respectively. In Fig. 50, minimum c is 1-1/4 and 1-3/8 for 3/4 or 7/8 rivets respectively. In Fig. 51 f is 1-1/2 in. plus 2 e but never less than 2 in.

Shearing and Bearing Values of Rivets in Pounds

Stress, lb. per sq. in.	Diam. Rivet in.	Value for Single Shear	Bearing Values for plate thickness in inches								
			1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
10 000 for Shearing 20 000 for Bearing	3/8	1100	1880	2340	2810						
	1/2	1960	2500	3130	3750	4 380	5 000				
	5/8	3070	3130	3910	4690	5 470	6 250	7 030	7 810		
	3/4	4420	3750	4690	5630	6 560	7 500	8 440	9 380	10 310	11 250
	7/8	6010	4380	5470	6570	7 660	8 750	9 840	10 940	12 030	13 130
	1	7850	5000	6250	7500	8 750	10 000	11 250	12 500	13 750	15 000
12 000 for Shearing 24 000 for Bearing	3/8	1320	2250	2810	3380						
	1/2	2360	3000	3750	4500	5 250	6 000				
	5/8	3680	3750	4690	5630	6 560	7 500	8 440	9 380		
	3/4	5300	4500	5630	6750	7 880	9 000	10 130	11 250	12 375	13 500
	7/8	7220	5250	6560	7880	9 190	10 500	11 810	13 130	14 430	15 750
	1	9430	6000	7500	9000	10 500	12 000	13 500	15 000	16 500	18 000

22. Welded Joints

Welded Joints. The art of welding joints has, within the past few years, developed with great rapidity and may now be economically applied in repair work, in reinforcing old structures in the field, and in replacing difficult riveted connections both in shop and field. Growth in use for light industrial trusses and for joints in steel pipes is very rapid at present. It cannot yet compete, economically, with riveting for heavy building and bridge work. Elimination of the noise of riveting in cities is an important advantage which is causing an urge for the use of welding.

Welded joints are made by fusion, using either oxy-acetylene gas or an electric arc. The former, on account of the comparatively low first cost, is well adapted to repair work where the amount of welding is not large, but the high operating cost makes it less economical for large work. In electric welding the heat required to fuse the edges of the parts to be joined is produced by a sustained electric spark or arc formed between a metallic wire clamped to a negative electrode, held in the hand and guided by the operator, and the work itself, which forms the positive electrode. In the process the wire is melted and fuses with the adjoining edges of the parts, forming the joint.

Figs. 53, 53*a*, 54, 54*a* show four different kinds of weld. For the butt weld, Figs. 53 and 53*a*, the edge of two plates are butted together and the weld is made by depositing welding metal between the abutting edges. The edges of the plates may be scarfed either to a single or to a double weld. In the single and double welds, Figs 54 and 54*a*, a fillet weld is deposited along the ends of the plates, as shown. If one of the plates is narrower than the other, fillet welds may also be made along the side edges. Fillets are designated by the height of their sides, as 5/16, 3/8 in., etc. The strength of the weld is based upon the strength of the fillet per lineal inch. A working value of 2500 lb. per lin. in. of a 5/16-in. weld is equivalent to 8000 lb. per sq. in. of contact weld surface. Tests have been made by the Westinghouse Co. and other electric companies interested in the development of electric welding and it has beeen shown conclusively that it is possible to develop the full strength of the members joined. The Lincoln Electric Co., in "Arc Welding," 1926, publishes a chart which shows that for fillets varying in thickness from 1/8 in. to 9/16 in. the rupturing force ranges from 5000 to 20 000 lb. per inch length of weld, respectively, in direct tension, Fig. 53; 3000 to 14 600 lb. in single shear, Fig. 54; and 2500 to 11 300 lb. in longitudinal shear when the fillets are placed along the side edges of the plate. These correspond to about 40 000, 25 000, and 20 000 lb. per sq. in. of surface of contact, respectively. With a factor of safety of 4, a 5/16-in. fillet in cross shear should have a safe strength of about 2000 lb. per lin. in. The application of the art of welding to industrial construction is too new for working stresses to have become standardized, but the American Welding Society and other national organizations, through their research committees, are seeking tangible data for use in structural design.

Fig. 53

Fig. 53a

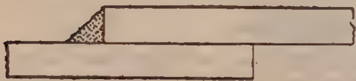


Fig. 54

Fig. 54a

The obvious economical **advantages of electric welding** are: Use of the entire gross section of tension members, since there are no rivet holes to be subtracted; less material for end connections, since webs of beams and girders can be welded direct to other girders or to columns without the use of connecting angles; use of outstanding plates instead of angles for stiffeners; and reduced cost of making shop drawings, since required welds can be indicated in a much simpler manner than can the location and spacing of rivets. In some cases, because of the fixed end condition of beams and girders due to the ability of end welds to take direct tension, it is possible to design such members as continuous girders in flexure, thus effecting some economy in material. On the other hand, the weight of welding wire electric consumed is considerable, amounting in the case of the Sharon, Pa., building to about one per cent of the weight of the structural steel. There is more difficulty in plumbing the framework and holding it in proper position during erection, since there are fewer connection angles to bolt to. Overhead welding is difficult and should be avoided.

A number of important structures have been erected by the all-welded process. As examples may be mentioned the five-story 70 × 220-ft. building for the Westinghouse Co. at Sharon Pa.; numerous shops and manufacturing buildings; the reinforcement of a railroad bridge across the Missouri River at Leavenworth; the addition of a second deck to a highway bridge across the Susquehanna River at Havre de Grace; and a skew railway truss bridge at Chicopee Falls.

SIMPLE TRUSS BRIDGES

23. Types of Trusses

Historical. The following are important steps in the development of American metallic truss bridges. The first iron truss bridge was built in 1840 over the Erie Canal by Earl Trumbull; some members were cast and some were wrought iron. In 1840 Squire Whipple built the first bowstring truss and in 1847 published the first work on stresses in bridge trusses; in 1852–53 he built a railroad span of 146 ft. on the Rensselaer and Saratoga R. R. which had the first double intersection trusses. In 1845 the first iron truss railroad bridge was built on the Reading R. R. by R. B. Osborne. In 1844 Thomas and Caleb Pratt patented the Pratt combination truss of wood and iron, and by 1850 Pratt trusses were made entirely of metal The first Fink truss in 1852 at Fairmont, W. Va., had three spans each 205 ft., with cast chord and wrought-iron diagonals. Longest Fink truss at St. Charles, Mo., built 1871, span

Fig. 55. Fink

Fig. 56. Bollman

Fig. 57. Whipple

Fig. 58. Through Howe

Fig. 59. Through Pratt

Fig. 60. Through Warren

Fig. 61. Deck Howe

Fig. 62. Deck Pratt

Fig. 63. Deck Warren

Fig. 64

Fig. 65

Fig. 66

Fig. 67

Fig. 68. Baltimore

Fig. 69. Post.

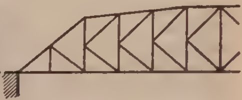


Fig. 70. K Truss

Fig. 71. Bowstring

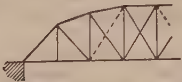


Fig. 72. Parker

Fig. 73. Pennsylvania

306-1/2 ft. First long Bollman truss built in 1852 at Harper's Ferry, Va.; span 124 ft. In 1858–59 John W. Murphy built for Lehigh Valley R. R. a Whipple truss with pin connections throughout, and in 1863 he first used all wrought iron compression and tension members in a pin-connected bridge with cast-iron chord blocks and pedestals. Howard Carroll began building riveted lattice trusses entirely of wrought iron on New York Central R. R. in 1859. In 1861 J. H. Linville first used wide forged eye-bars and wrought-iron web posts, the upper chords being cast iron. First Baltimore or Pettit truss 1871 at Mt. Union, Pa., designed by bridge department of Pennsylvania R. R. The first Post truss 1865; built by S. S. Post for Erie R. R. About 1870 C. H.

Long Span American Simple Bridges

Span center to center of end pins (Ft. In.)	Kind of truss	Over what river	Location	Railroad tracks: Single	Railroad tracks: Double	Date of completion
720 0	Pennsylvania	Ohio	Metropolis, Ill.	......	*	1917
668 0	"	Mississippi	St. Louis (Municipal)		*	†1912
643 10-1/2	"	Ohio	Louisville, Ky.	......	*	1918
640 0	"K" truss	St. Lawrence	Quebec	......	*	1917
598 6	Pennsylvania	Hudson	Castleton, N. Y.	......	*	1924
586 0	"	Great Miami	Elizabethtown, Ohio	......	......	†1906
552 0	"	Ohio	Metropolis, Ill.	......	*	1917
546 6	"	Ohio	Louisville and Jeffer-ville	*	......	1894
542 6	"	Ohio	Cincinnati and Covington	......	*	†1889
533 0	"	Delaware	Philadelphia	......	*	1896
531 0	"	Allegheny	Pittsburgh	......	......	†1914
523 0	"	Ohio	Pittsburgh (Brunot's I.)	......	*	1915
522 0	"	Ohio	Wheeling	*	......	1885
521 11-3/4	Warren	Ohio	Henderson, Ky.	*	......	.....
519 2-1/2	Pennsylvania	Ohio	Wheeling	......	......	†1892
518 1-3/4	Whipple	Ohio	Cairo	*	......	1889
518 0	Pennsylvania	Ohio	Kenova, W. Va.	*	......	1913
518 0	"	Susquehanna	Havre de Grace, Md.	......	*	1909
517 8-1/2	"	Monongahela	Glenwood, Pa.	......	......	†1895
517 6	"	Mississippi	St. Louis (Merchant's)	......	*	1890
517 6	"	Mississippi	St. Louis (McKinley)	......	......	†1910
515 0	"	Monongahela	West Braddock, Pa.	......	......	†1897
515 0	"	Monongahela	Webster to Donora, Pa.	......	......	†1909
506 8	"	Ohio	Newport and Cincinnati	*	......	†1896
504 0	"	Susitria	Alaska	......	*	1921
500 0	"	Missouri	Sioux City	*	......	†1895
498 0	"	Monongahela	Clairton, Pa.	......	*	1903
495 8-1/8	"	Monongahela	Rankin, Pa.	......	*	1900
489 3	"	Monongahela	West Braddock, Pa.	......	......	†1897
484 6	"	Ohio	Cincinnati and Covington	......	*	†1889
465 0-1/4	Parker	Miami	New Baltimore, Ohio	......	......	†1901
453 10	Pennsylvania	Monongahela	Pittsburgh (So. 10th St.)	......	......	†1904
450 0	"	Brazos	Waco, Tex.	*	......	†1902
447 0	"	Allegheny	Mossgrove, Pa.	*	......	1899
440 0	Baltimore	Missouri	Bellefontaine, Mo.	......	*	1893
439 3	Pennsylvania	Ohio	Pittsburgh (Neville's Island)	......	......	†1927
435 10	Baltimore	Miami	Hamilton, Ohio	......	......	†1895
434 0	Warren	Allegheny	B. & O. R. R., Pittsburgh	......	*	1920
433 2-3/8	Pennsylvania	Carquinez Straits	San Francisco, Calif.	......	......	†1927
430 0	"	Mississippi	Red Wing, Minn.	......	......	†1896

† Highway traffic

Long Span American Simple Bridges—(*Continued*)

Span center to center of end pins		Kind of truss	Over what river	Location	Railroad tracks		Date of completion
					Single	Double	
Ft.	In.						
430	0	Warren.....	Monongahela	Coal Valley, Pa......	......	*	1919
425	6-1/2	Pennsylvania	Missouri....	Kansas City........	......	*	†1911
425	0	"	Missouri....	Mobridge, S. Dakota.	*	......	1907
416	6	"	Columbia....	Near Rock Island, Wash............	*	......	§1893
416	0-1/4	"	Missouri....	St. Charles, Mo.....	......	......	†1904
416	0	"	Ohio........	Pittsburgh (Brunot's Is.)............	......	*	1915
415	6	Whipple.....	Ohio........	Point Pleasant, W. Va.	*	......	1885
413	0	Pennsylvania	Allegheny...	New Kensington, Pa.	......	......	†1901

† Highway traffic. § Reconstructed in 1926

Parker modified the Pratt truss, using inclined upper chords, thus originating the Parker truss. Steel was first used 1874 in Eads' arch bridge at St. Louis. In 1879 first bridge was built entirely of steel at Glasgow over Missouri River. Pegram truss introduced by G. H. Pegram, 1887. The longest simple truss bridge is one of 720-ft span built over the Ohio River at Metropolis, Ill., in 1917; it has Pennsylvania trusses.

Types of Bridge Trusses are the Pratt (Fig. 59), Parker (Fig. 72), Warren (Figs. 60 and 63), Pennsylvania (frequently called Pettit) (Fig. 73). The Post, Bollman and Whipple are no longer built, and the Fink, in modified form, is used only for roof trusses. When the span is long enough to require subdivided panels, it is economical to curve the top chord, using the Pettit rather than the Baltimore type. In the Howe truss the verticals are in tension and the diagonals in compression. It was formerly much employed for wood, or combination wood-iron, bridges, but, like the Fink, is now principally used in roof construction. In the Pratt the verticals are in compression except the end suspender or **Hip Vertical**, *s* in Fig. 59, which is a tension member; and the diagonals are in tension except the two end ones called end posts. Pratt trusses are sometimes made with vertical end posts even in through bridges, in which case the diagonal in each end panel slopes downward toward the center and is in tension. The dotted bars in Fig. 59 are counters, or counter diagonals, and they come into action when the live load shear in the panel is greater than and of opposite kind from the dead shear in that panel. For greater rigidity it is better practice to omit the counter diagonals and to make the main diagonals in the panels requiring counter bracing, compression members. In Fig. 60 the dotted lines represent **sub-verticals** which are frequently used to reduce the panel length, and the short verticals and diagonals of the Baltimore truss (Fig. 68) are for the same purpose. The **collision strut**, *c* in Fig. 58, is used to brace the end post, but is now seldom employed. **Pratt** and **Warren** trusses, of riveted construction, are economical types for spans up to 150 to 200 ft. For spans larger than about 200 ft., the inclined or curved chord trusses with pin connections are most economical, and the Parker (Fig. 72) and Pennsylvania (Fig. 73) types are the best forms. For very long spans, necessitating great depth of truss and long panel length, the **Pettit truss** with subdivided panels is used. When a limited underclearance requires a shallow floor the economical lower span limit for this type may be as low as 300 ft., but under ordinary conditions it is generally much higher. The most recent type, suitable for long spans, is the K truss. It has been used for the 230-ft. span built by the Atchison, Topeka & Santa Fe Ry. across the Arkansas River at Pueblo, Colo. The webbing is arranged as shown in Fig. 70. Trusses of this type are economical only for spans of 300 ft. or over. The most notable use of this type of webbing is in the cantilever and anchor arms of the Quebec bridge. The chief superiority of the K type over the Pennsylvania is in the lower secondary stresses and in the avoidance of the use of the short horizontal struts required, in the Pennsylvania truss, to brace the vertical posts.

The Economic Depth of a Truss is that which makes the material in the bridge a minimum. It varies from one-fifth to one-eighth of the span depending on the form of truss, the length and number of panels and the allowable unit stresess. For short-span through bridges the minmum depth is fixed by the required clear head room. As the depth of a truss increases the weights of the chords become less and the weights of the web members greater, hence there must be a depth for each bridge for which the weight of material is the least. Ordinary spans of 250 ft. or less usually have depths of about one-sixth the span. Longer spans are generally relatively shallower in order to avoid excessively long web members and the increased cost of handling and erecting the deeper truss. The greater the live load and the wider the bridge the greater may the relative depth be made. In the 523-ft. spans of the Ohio Connecting bridge the center depth is 91 ft. or 1/5.75 of the span; in the 552-ft. spans of the Metropolis bridge it is 83 ft. or 1/6.65 of the span; in the 720-ft. span of the same bridge it is 110 ft. or 1/6.54 of the span; and in the 640-ft. suspended span of the Quebec bridge it is 110 ft. or 1/5.82 of the span. End depth of trusses is fixed by the clearance plus depth of portal, which latter increases with the length of span. The parabola gives a satisfactory curve for the chord.

The Economic Length of Panel is more or less independent of the truss depth since in the longer spans, subdivided panels may be used, whereas for the shorter trusses without subdivided panels there is a practical advantage in the use of long panels, inasmuch as the heavier floor system is stiffer and freer from vibration. The inferior limit of span length for trusses with subdivided panels varies from 240 ft. for highway to about 300 ft. for railway bridges. Panel lengths for ordinary highway bridges vary between 15 and 25 ft.; for railway bridges between 25 and 40 ft. Panels are generally kept of uniform length. In the Municipal bridge at St. Louis the panel lengths vary from 30 ft. near the end to 48 ft. at the center of the span. By varying the panel length greater freedom in truss design is gained at the expense of greater shop cost of the floor system.

24. Stresses in Pratt Trusses

Notation. V = shear on a given panel, θ = angle between diagonal and vertical, W = live plus dead panel load, W_1 = live panel load, p = panel length, m = number of panels in span, n = number of panels to right of a given panel, M_n and M_{n+1} = moments at right and left end of a panel respectively, q = equivalent uniform live load, l = length of a bar, A = area of cross-section, E = modulus of elasticity.

Stresses in Bridge Trusses are determined graphically (Sect. 2) or algebraically; for the latter there are two general methods: that of moments and that of resolution of forces. The second method may be subdivided into method of joints and method of sections. The **method of moments** consists in passing a section through the truss so as to cut the bar in which the stress is to be found, then writing an equation of moments for all forces on one side of the section and solving this equation for the unknown stress. The **method of joints** consists in passing a section around each joint and finding the forces acting at each joint successively. In the **method of sections** a section is passed through the bar in question, and by applying to one side of the section either or both of the conditions that the algebraic sum of horizontal forces must equal zero or that the algebraic sum of the vertical forces must equal zero, the unknown stresses in the bars cut by the section may be found. To illustrate: consider the single-track through Pratt truss bridge in Fig. 74 having a dead load of 400 lb. per ft. for the track and 1000 + 10 L = 1000 + 10 (120) = 2200 lb. per ft. for the steel in the bridge; total 2600 lb. per ft. of bridge or 1300 × 20 = 26 000 lb. per panel per truss. Assume all on loaded chord. Live load 3000 lb. per ft. per truss or 60 000 lb. per panel per truss. Find maximum stress in bar 8 by method of moments; bar 2 by joints; and bar 5 by sections. The loads in Fig. 74 and the following computations are expressed in units of 1000 lb.

Method of Moments. For bar 8 pass a section vertically through 8 and consider forces on left of section. The origin of moments is at intersection of other two main bars cut, namely, at lower panel point 3. Maximum moment at 3 and therefore maximum stress in 8 occur for full live and dead load. $R_1 = 215$. Assume 8 in tension and place algebraic sum of moments of all forces on left of section a–a equal to zero; 215 (60) − 86 (20 + 40) + 24 $P_8 = 0$. $P_8 = -322.5$. The minus sign denotes that the stress is opposite to that assumed, and is compression.

Method of Joints. For bar 2 pass a section around lower end of 2 and consider the vertical forces acting upon the part below the section. Since the panel load is the only vertical force other than the stress P_2 acting on this part of truss the stress P_2 must be equal to this force; hence P_2 is maximum for full panel load and equals 86. It is tension since it acts away from the lower joint.

Method of Sections. For bar 5 pass a section vertically through 5. Imagine P_5 resolved into horizontal and vertical components, and since the algebraic sum of the outer forces on left of a–a, that is, the shear on the panel, together with the vertical component of P_5 equals zero, it follows that $P_5 = V \sec\theta$. For a truss having horizontal chords the vertical component of a diagonal is equal to the shear in the panel. When shear is positive P_5 is tension, and its maximum value occurs for maximum positive shear, which by the approximate method, Art. 18, paragraph 3, occurs when all panel points to right of the panel in question are loaded with live load and dead load covers whole span. Under this loading the positive shear due to 86 on points, 1, 2, 3 is 86 (1 + 2 + 3)/6 = 86, and the negative shear due to 26 on 4 and 5 is 26 (1 + 2)/6 = 13, hence maximum positive shear = 86 − 13 = 73 and maximum $P_5 = 73 \times 31.24/24 = 95.0$ tension. If negative shear occurs in panel 3–4, P_5 will be in compression, or if it cannot stand compression a counter as shown by the dotted line in panel 3–4 will be needed, and for symmetry one will also be needed in panel 2–3. For maximum negative shear place live load on points 4 and 5 only, then negative shear on panel 3–4 = 17, and P_5 will be 17 $\sec\theta$ = compression, or else a counter must be used, in which case maximum stress in counter = 17 $\sec\theta$ = tension.

For Concentrated Wheel Loadings the computation of stresses is the same in principle as with uniform or panel loads and the determination of the various moments and shears is simplified with the aid of the moment diagram (Art. 18). For trusses with parallel chords the maximum stress in a diagonal is the maximum shear on the panel containing the diagonal, multiplied by $\sec\theta$, and the maximum chord stress in a bar is the maximum moment about its origin of moments divided by the lever arm of the bar. For the verticals of trusses with parallel chords the maximum stress is the greatest shear on an inclined plane cutting the vertical and not cutting any diagonal in action for the loading producing the shear.

Let it be required to find maximum live load tension in diagonal 5 (Fig. 74) using Cooper's loading E–50 (Fig. 34). To produce the greatest shear the loads must lie partly on panel 3–4 and principally to right of 3, and the total load on the bridge must equal m (here 6) times the load on panel 3–4. Wheel 1 being small, place wheel 2 just to right of joint 3, then load on span is 215.0, which is greater than 6 × 12.5 = 75.0; now with wheel 2 just to left of joint 3, 215.0 is less than 6 × 37.5 = 225.0. Hence wheel 2 and joint 3 satisfies criterion for maximum shear in panel 3–4. With wheel 3 at joint 3 the criterion is also satisfied. Starting with wheel 2 at joint 3 and moving up the loads till wheel 3 is at joint 3 the shear is increased because 215 × 5/120 + 25 × 4/120 > 37.5 × 5/20, hence position 3 gives more shear than position 2.

Fig. 76 gives the influence lines for shear in the several panels of the truss of Fig. 74. An **influence line** shows the variation in a stress function, such as a moment, shear, floorbeam load or stress as a single load rolls across the structure. In Fig. 76 the triangles $0–3_1–g$ and $g–4_2–6$ are the influence diagrams for the positive and negative shears, respectively, in the panel 3–4. g is the neutral point in the panel; l_1, l_2 and l'_1, l'_2 are the segments of the positive and negative triangles, respectively. The position of the live load for

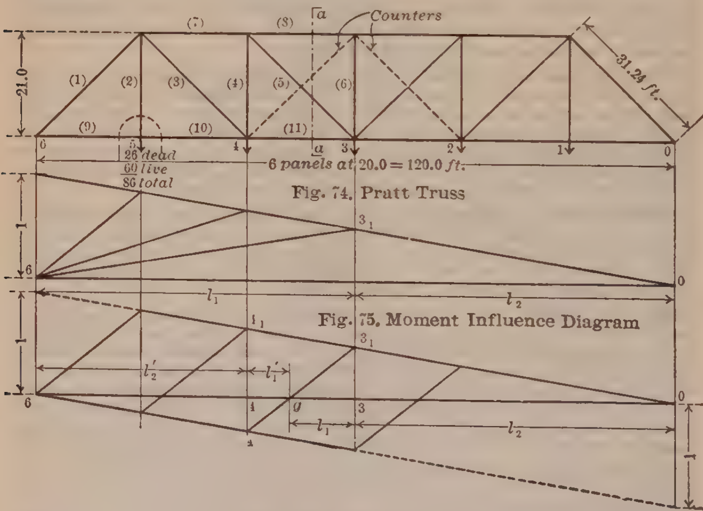


Fig. 74. Pratt Truss

Fig. 75. Moment Influence Diagram

Fig. 76. Shear Influence Diagram

maximum tension in the diagonal 5 may be obtained from the chart, Fig. 26 (Art. 10). The distance of the neutral point from joint 3 = $np/(m - 1)$ = 3 × 20/5 = 12 (Art. 18). Entering the chart for $l_1 = 12$ and $l_2 = np = 60$, it is found that wheel 3 should stand at 3.

The Shear in Panel 3–4 (Fig. 74) with wheel 3 at joint 3 = left reaction minus floorbeam reaction at 4. Using the moment diagram the shear = (8385 + 240 × 4)/120 − 287.5/20 = 63.5. Maximum tension in diagonal 5 = 63 500 sec θ = 63 500 × 31.24/24 = 82 700 lb. In this manner live load stresses in all diagonals may be found, and after adding impact and dead load stresses the diagonals can then be designed. The maximum stress in vertical 4 equals the maximum shear on the inclined section cutting bars 7–4–11. This occurs for the same loading, and is of the same magnitude as for maximum shear just found, hence compression in bar 4 = vertical component of diagonal 5 or 63 500 lb.

The shear in panel 3–4 can also be found by multiplying the corresponding triangular influence area, Fig. 76, by the exact equivalent uniform load obtained from Fig. 25 (Art. 10). This shear may be expressed by $V = 1/2 q l_1 l_2 / p$. Thus for maximum positive shear in panel 3–4, $l_1 = 12$, $l_2 = 60$. From Fig. 25 (Art. 10) the exact equivalent uniform load $q = 3600$ for E–50, and $V = 1/2 \times 3600 \times 12 \times 60/20 = 64\,800$. The small error is due to wheel 1, being 1 ft. in the negative area. Similarly the maximum negative shear $= 1/2 \times q'l'_1l'_2/p$, where q' is obtained from Fig. 25 for $l'_1 = 8$ and $l'_2 = 40$. Actual stresses are obtained by multiplying the shear by sec θ.

The hip-vertical bar 2 has a maximum tension equal to the floorbeam reaction and hence receives its stress only for loads on the two adjacent panels 4–5 and 5–6. The position of the loads must be the same as will cause maximum bending moment at center of a span of length 4–6, and after the loads are so placed the stress in bar 2 = $(M_4 - 2\,M_5)/p$, where M_4 is the moment about 4 of all wheels on distance 4–6 and M_5 the moment about 5 of loads on 5–6. This position can be obtained from the chart, Fig. 26 (Art. 10), by taking l_1 and l_2 each equal to 20 ft., the adjacent panel lengths. It is found that wheel 3 should stand at joint 5. The live load tension in the hip-vertical can be taken directly from the table in Art. 10. It is the maximum pier (or floor-beam) reaction for two adjacent 20-ft. spans and equals 81 900 lb.

The Moment Influence Lines are shown in Fig. 75, that for member 8 being 0–3_1–6, with the apex at 3_1 under the center of moments at joint 3. $l_1 = 60$, $l_2 = 60$. Fig. 26 shows that wheel 11 at joint 3 is the governing position for maximum moment. From Fig. 25 is obtained the equivalent uniform load $q = 3210$ for E–50. The bending moment for a triangular influence diagram is $\mathbf{M = 1/2\, q l_1 l_2}$. Whence $P_8 = 1/2 \times 3210 \times 60 \times 60/24 = 240\,750$.

The Moment at a Joint due to any load P standing anywhere on the span is equal to the product (called the load product) of the load and the influence ordinate at P multiplied by the distance of the center of moments from the left end of the span. If several wheels stand on the span the moment is equal to the algebraic sum of all of these products. For any series of wheel loads the position of the loading producing a maximum moment can be found by trial and is the one which makes the algebraic sum of the load products a maximum. The stress is equal to the sum of the load products multiplied by e/h, where e = the distance of the center of moments from the left support and h = the lever arm of the stress with reference to the center of moments.

25. Warren and Baltimore Trusses

Notation the same as that at beginning of Art 24.

To Compute Maximum Stresses for all bars of the Warren truss in Fig. 77, let dead panel load on each bottom chord joint be 20 (thousands of pounds) and live panel load 40. Place dead load of 20 at each bottom joint, and beginning at center and using method of joints write on each web member and then on each chord member a coefficient from which the true stresses for uniform full loading may be found. Since the truss is symmetrical each diagonal meeting at panel point 2 has a vertical component of 10.0 and this component or coefficient is written on the bar as shown. Then the joint at the upper end of this diagonal is considered and the vertical component or coefficient for the next diagonal (3) obtained, 10.0 in this case, and so on to the end where the vertical component of the end post should equal the reaction. By applying the method of joints to point 4 the horizontal component of P_7 is found to be 30.0 tan θ, so the coefficient 30.0 is written on the lower chord member; and similarly at joint 3 the actual tension $P = (30.0 + 30.0 + 10.0)$ tan θ = 70.0 tan θ and the coefficient 70.0 is written on the bar. All diagonal coefficients must be multiplied by sec θ = 22.36/20; all chord coefficients by tan θ = 10/20. Thus dead load stress in end post = 30 × 22.36/20 = 33.5 compression; and $P_7 = 30.0 \times 10/20 = 15.0$ tension. For live load chord stresses multiply dead chord stresses by ratio of live panel load to dead panel load. For live web stresses find maximum live shears by method of sections, Art. 24, and multiply these shears by sec θ. Thus for panel 3–4 maximum shear = 40 (1 + 2 + 3)/4 = 60; for panel 2–3, maximum positive shear = 40 (1 + 2)/4 = 30 and maximum negative shear = 40 (1)/4 = 10. The following table shows the final stresses.

Stresses for Truss in Fig. 77 in Thousands of Pounds

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Dead......	− 33.5	+ 33.5	−11.2	+11.2	−30.0	− 40.0	+15.0	+ 35.0
Live.......	− 67.1	+ 67.1	+11.2 −33.5	−11.2 +33.5	−60.0	− 80.0	+30.0	+ 70.0
Max.......	−100.6	+100.6	−44.7	+44.7	−90.0	−120.0	+45.0	+105.0
Min.......	− 33.5	+ 33.5	0.0	0.0	−30.0	− 40.0	+15.0	+ 35.0

− denotes compression. + denotes tension.

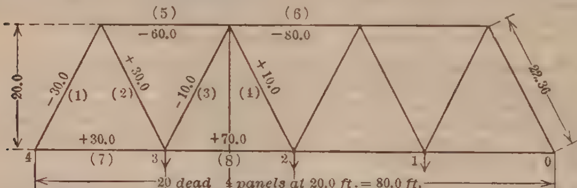


Fig. 77. Warren Truss

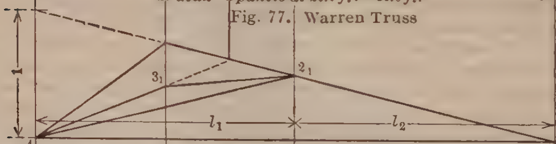


Fig. 78. Moment Influence Diagram

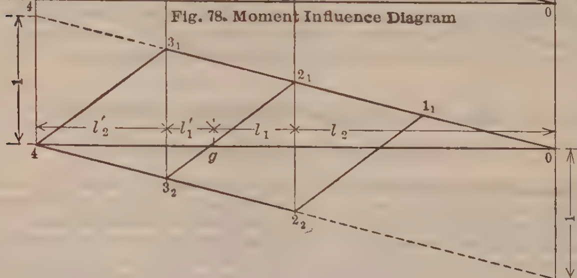


Fig. 79. Shear Influence Diagram

The influence lines for shears, (Fig. 79), and for moments at the loaded joints, (Fig.78), for the Warren truss, (Fig.77), are the same as for a Pratt truss of equal length and number of panels. For example, that for the moment for P_6 is $0–2_1–4_1$ (Fig. 78); l_1 and l_2 are the segments of the influence triangles; maximum moment $\mathbf{M} = 1/2\ q l_1 l_2$; maximum shear in a panel, $\mathbf{V} = \dfrac{1/2\ q l_1 l_2}{p}$. The influence line for moments about the unloaded joint opposite P_8 is $0–2_1–3_1–4$ (Fig. 78). P_6 and P_8 may be found by multiplying the respective influence areas by e/h, where h = depth of truss; e = horizontal distance from the center of moments to the left support (= 2 p for P_6 and 1-1/2 p for P_8). The charts, Figs. 25 and 26, apply exactly only when the influence diagram is a triangle. When the latter is not a triangle, as in the case of $0–2_1–3_1–4$ for P_8,

(Fig. 78), the charts may still be applied with sufficient accuracy by assuming a triangular influence diagram coinciding as nearly as possible with and having the same area as the actual influence diagram. l_1 and l_2 to be used in the charts are then the two segments of this triangle, and the governing wheel obtained from Fig. 26 stands at the loaded joint nearest the apex of the triangle.

The Baltimore Truss (Fig. 68) may have its sub-verticals and sub-diagonals arranged as in Fig. 80 or in Fig. 81 which represent panels without counters on left of center of truss. In Fig. 80 maximum $\mathbf{P_1} = \dfrac{\mathbf{Wp \sec \theta_1}}{\mathbf{(p + p_1)}}$ and is compression; if $p_1 = p$ (as is usual) then $P_1 = \dfrac{(\mathbf{W \sec \theta})}{2}$; in other words the vertical component of $P_1 = 1/2\ W$, as may be easily seen by studying the forces below the dotted section and taking moments at a. In Fig. 81, for members in action as shown, $P_4 = (W \sec \theta)/2$ and is tension. Maximum tension in P_2 (Fig. 80) occurs when live load extends from right end of span up to and including b, and if $p_1 = p$, $P_2 = (V - W/2) \sec \theta$. Maximum tension, P_3 (Fig. 81), occurs when the live load extends from right end of span up to and including c and $\mathbf{P_3} = \left(\dfrac{\mathbf{V + W}}{2}\right) \mathbf{\sec \theta}$. If n = number of panels to right of b (Fig. 80) or to right of c (Fig. 81), and m = number of panels in

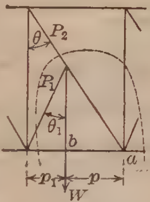


Fig. 80

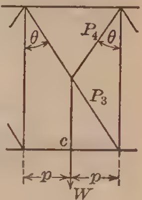


Fig. 81

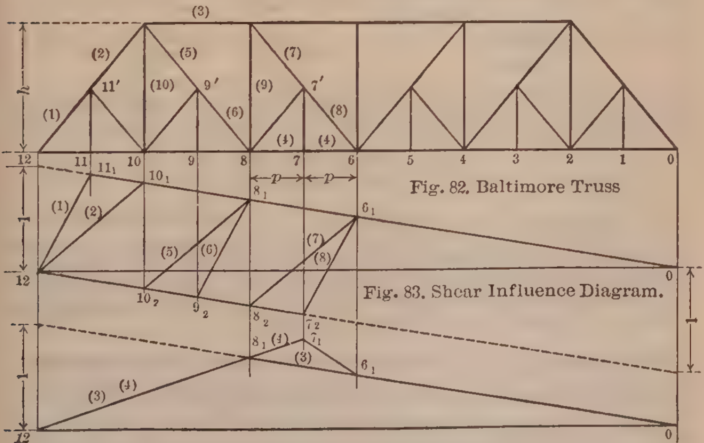


Fig. 82. Baltimore Truss

Fig. 83. Shear Influence Diagram.

Fig. 84. Moment Influence Diagram

the span, all equal, then maximum live load tension in P_2 and also in P_3 is given by the formula $\mathbf{1/2\ W} \left[\dfrac{\mathbf{n(n+1)}}{\mathbf{m}} - \mathbf{1}\right] \sec \theta$.

Influence lines for shears in the main web members of the Baltimore truss, (Fig. 82), are shown in Fig. 83, and for moments, in Fig. 84. Maximum live load shear in the upper diagonals (of type P_2, P_5), and in the vertical posts (of type $\mathbf{P_9}$) $= \mathbf{V} = \frac{1/2\, q l_1 l}{2p}$; that in the lower half of the main diagonals (of type P_1, P_6) $= \mathbf{V} = \frac{1/2\, q l_1 l_2}{p}$. For actual stress multiply by sec θ.

The influence line for moments about joint 8, for P_3, is $0 - 8_1 - 12$, Fig. 84. That for P_4 is $0 - 6_1 - 7_1 - 12$. Maximum live load stresses may be found by multiplying these areas by the equivalent uniform load q and by e/h; where e = distance from center of moments to the left support; h = depth of truss. $e = 4\,p$ for either P_3 or P_4. e/h = sec θ for all web members of a truss with horizontal chords. l_1 and l_2, for use in the chart, Fig. 25 (Art. 10), are the lengths of the two segments of the triangular influence diagram, or of the triangle approximating in outline and area the actual influence diagram (see also Warren truss, above). For all triangular moment influence diagrams $\mathbf{M} = 1/2\, q l_1 l_2$.

26. Other Types of Trusses

Notation the same as that given at beginning of Art. 24

Stresses in Trusses having Inclined Chords are found by methods of moments. Maximum chord stresses occur for full live and dead loads over entire span. Referring to Fig. 85, showing a through Parker truss, the maximum live and dead load stresses in upper and lower chord bars P_1 and P_2 are given by the following formulas:

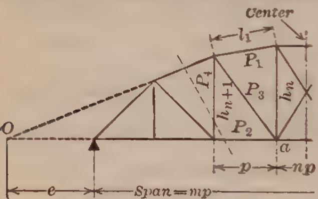


Fig. 85

$$P_1 = \frac{M_n}{h_n}\frac{l_1}{p} = \frac{W l_1 n(m-n)}{2\,h_n}$$

$$P_2 = \frac{M_{n+1}}{h_{n+1}} = \frac{Wp(m-n-1)(n+1)}{2\,h_{n+1}}$$

Maximum tension in P_3 occurs for live panel loads on all panels from right end of span up to and including a, with dead load over whole span, and

$$P_3 = \left(\frac{M_n}{h_n} - \frac{M_{n+1}}{h_{n+1}}\right)\frac{l_3}{p}$$

where l_3 is length of P_3, and M_n and M_{n+1} are simultaneous moments at right and left ends of panel respectively. Maximum compression in P_4 occurs for same loading as for maximum tension in P_3 and is found by methods of moments using all forces to left of dotted section with origin of moments at O. Some verticals in inclined chord bridges are in tension for certain positions of live load and these stresses must be computed.

Chord Stresses due to Wheel Loads are found by method of moments. For example, in Fig. 85 the maximum chord stress P_1 would be found by dividing the maximum moment at the origin of moments a by the lever arm of the bar P_1, that is, by the perpendicular distance from a to the bar. Instead of using the lever arm of the bar, however, it is often more convenient to use the arm of one of the components of the stress and then find the stress from the component. Thus in the case of P_1 its horizontal component $= \mathbf{M_a/h_n}$ and its stress $= \mathbf{M_a l_1/p h_n}$. The criterion for determining the position of loads to secure maximum moments is given in Art. 18.

Influence for Moments in the Parker truss, (Fig. 86), are similar to those for a Pratt truss and are not here shown. The influence line for a web member, (Fig. 87), is constructed by laying off h'_0/h_0 downward from 0; where h'_0 and h_0 are the vertical distances at the right and left supports, respectively, to the continuation of the top chord member cut by the section passing through the web member. For example, to find the stress in the diagonal 9 due to Cooper's E–60 a vertical section a–a through the diagonal also cuts the upper chord member 8, the continuation of which gives the vertical intercepts $h'_0 = 44$ ft. and $h_0 = 30$ ft.; $h'_0/h_0 = 44/30 = 1.467$. The influence line for P_9 is $0-4_1-5_2-7$. g is the neutral point in the panel. The distance $g-4 = l_1 = np/(n + (m - n - 1)\, h'_0/h_0) = 4 \times 28.57/(4 + 1.467 \times 2) = 16.5$. $l_2 = 4 \times 28.57 = 114.28$. From Fig. 25 (Art. 10), $q = 4020$. The area of the triangle $0-4_1-g-0 = 1/2 \times 4/7 \times 130.78 = 37.37$. The center of moments for P_9 lies 428.55 ft. to the left of the left support; the lever arm for P_9 about the center of moments = 393.7 ft. $e/h = 428.55/393.7 = 1.088$. $P_9 = 37.37 \times 4020 \times 1.088 = 164\,000$.

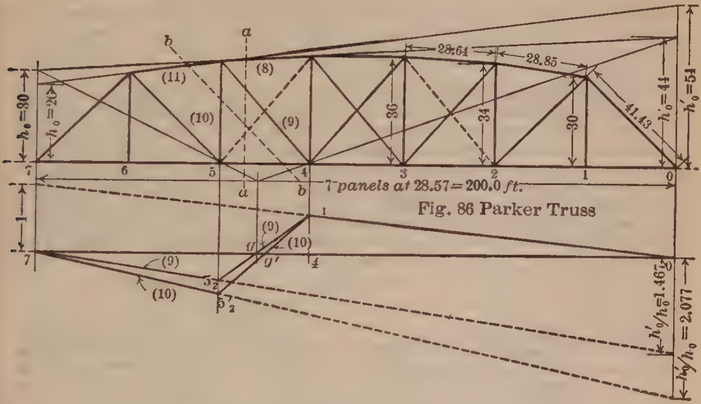


Fig. 86 Parker Truss

Fig. 87. Influence Diagram for Web Members

For the vertical 10, the section $b-b$ cuts the top chord member 11, and $h'_0/h_0 = 54/26 = 2.077$ which is laid off downward from 0, (Fig. 87), to furnish the influence line $0-4_1-5'_2-7$. The distance from the neutral point g' to the right end of the panel $= np/(n + (m - n - 1)\, h'_0/h_0) = 4 \times 28.57/(4 + 2.077 \times 2) = 15.6 = l_1$. $l_2 = 114.28 : q = 4030$. The area of the triangle $0-4_1-g'-0 = 1/2 \times 4/7 \times 129.88 = 37.11$. $e/h = 185.71/242.84 = 0.765$. $P_{10} = 37.11 \times 4030 \times 0.765 = -114\,000$.

The neutral point for a web member can also be found graphically. Thus, to find the neutral point g, for the diagonal 9, connect the tops of the end verticals, $h'_0 = 44$, $h_0 = 30$, with the right and left panel points, 4 and 5, respectively, at the ends of the cut panel. The vertical through the intersection of these lines gives g. To find g', the neutral point for the vertical 10, join the tops of $h'_0 = 54$, $h_0 = 26$, with 4 and 5 respectively, the ends of the panel cut by $b-b$.

Statically Indeterminate Trusses are those in which the stresses cannot be found by the principles of statics. Some trusses are statically determinate as regards the outer forces but are statically indeterminate as regards the inner forces or stresses. A **redundant member** in a truss is one that is not necessary for the stability of the truss. The condition that a truss is statically determinate is $\mathbf{b = 2\,j - 3}$, where b is total number of necessary bars, j the number of joints in the truss. The equation applies to the whole or any part of the truss. If $b > 2j - 3$ the system is statically indeterminate; if $b < 2j - 3$ it is unstable. Stresses in statically indeterminate trusses are found either by making the assumption that the truss is divided into its separate systems and that each system acts independently, or by the principle of least work.

The Double-system Warren Truss in Fig. 88, also called a lattice truss, is statically indeterminate because 26 bars > 2 (14) − 3 = 25. Assume the web to be divided into two systems as shown by heavy and light lines and let it be required to compute

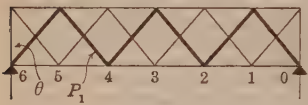


Fig. 88

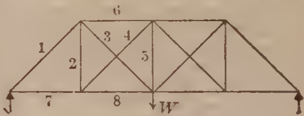


Fig. 89

the maximum tension in P_1 under a dead panel load of 6 and a live panel load of 12 thousands of pounds on the lower chord. Maximum positive shear on panel 4–6 of web with heavy lines occurs for live panel load on points 2 and 4 and stress P_1 = [(2 + 4) 18/6] sec θ.

The Principle of Least Work requires that the total internal work performed by the stresses in the members of a truss must be a minimum; hence an outer load acting on a statically indeterminate truss will be distributed on the different systems of the truss in such a way that the total internal work done by the stresses will be a minimum. **Work** is the product of force times its displacement. The work performed by a stress P on a bar having length l, area A, and modulus of elasticity E is **1/2 P^2l/AE** and total internal work in truss is **1/2 $\Sigma P^2l/AE$.** The truss in Fig. 89 carrying the load W at the center can be separated into two trusses as in Figs. 90 and 91, one with the force kW and the other with a force $(1 - k) W$ at the center respectively. The value of k

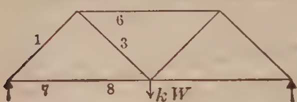


Fig. 90

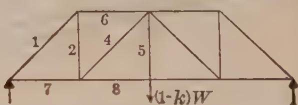


Fig. 91

must be such as to make the total internal work a minimum. The stresses in various bars of Fig. 90 and Fig. 91 in terms of the unknown loads **kW** and **(1 − k) W** respectively can be found and substituted in the formula. The differential coefficient of the work with respect to k is then placed equal to zero and solved for k. The final expression for k involves the dimensions of the truss and the areas of the bars. This process can be applied for a load of unity at each of the panel points in succession and a table of stresses prepared from which the stresses for any panel load may be determined. Since the areas of the bars must be known before the principle of least work can be applied, the process of design is a tentative one.

27. Design of Tension Members

Notation. S = allowable tensile unit stress, P = direct tension in a member in pounds, S_1 = maximum stress in pounds per square inch in eyebar due to combined tension and bending, t = thickness, l = length, and h = width of eyebar; all in inches.

Pin-connected Trusses were generally used in America until about 1890 for spans of all lengths. Since that date the **riveted truss** has come to be the standard for railroad bridges having spans less than 150 to 250 ft. The railroad bridge over the Missouri River at Kansas City has riveted trusses of 425 ft. 6-1/2-in. span. The longest riveted simple truss is the 720-ft. channel span of the Metropolis bridge. The 552-ft. spans of this same bridge are likewise riveted. For highway bridges the pin-connected type is often used for shorter lengths. Short pony trusses for country highway bridges are now invariably riveted, for greater lateral stiffness. The riveted structure is the stiffer form and the individual members thereof are more nearly free from objectionable vibration and rattling. The pin-connected truss usually is somewhat lighter than the riveted and can be more easily and more quickly erected. The ideal truss has hinged joints without friction. Neither of the types used fulfills this requirement, for as the truss deflects the rigidity of the riveted joints and the friction on the pins of a pin joint cause bending or secondary stresses in the truss members. The most complete analysis of secondary stresses that has as yet been made for any riveted truss is given in the paper " Stress Measurements on the Hell Gate Arch Bridge," by Dr. D. B. Steinmann, Proc. Am. Soc. C. E., Oct., 1917.

Tension Members may be eyebars or built-up sections. An **eyebar** has a uniform rectangular cross-section throughout the greater part of its length and at each end has a head or enlarged portion through which the pin passes. It is the most efficient means of transmitting tension and is economical of material but deficient in rigidity. Eyebars should not be used where the minimum stresses are light or where compression is possible, as in the first two lower chord panels, for hip verticals, or for diagonals subject to counter stress. Counters are sometimes used in light highway bridges and are composed of one or two adjustable eyebars or square rods with looped ends, but whenever possible adjustable bars should be avoided.

Built-up or Riveted Tension Members may be made in any of the forms suitable for compression members but less emphasis need be placed upon lateral stiffness. For light construction one ∟, or two ∟'s placed with their backs together, or separated to form a built channel (Fig. 92), or four ∟'s (Fig. 93), are used. In the last two cases the ∟'s may be separated by tie plates, but lacing or a solid web plate should be used for good work. For heavier construction some form of box section composed of rolled or built channels connected by lacing and tie plates (Fig. 94, 95) is used. The sections may be further enlarged by the addition of one or more side plates riveted to the web of each channel. In riveted trusses the flanges are often turned in to permit the member to go inside of the connecting joint gusset plates. In built-up tension members the rivets should be so placed as to reduce the section as little as possible. The net section will generally occur at the first row of rivets in a splice, end gusset or pin plate. In computing net sections the diameter of the rivet holes should be taken 1/8 in. larger than the nominal diameter of rivet.

Eyebars vary in size from 2 in. in width having a minimum thickness of 5/8 in. to 10 in. in width for usual construction, and even more for the largest structures; 16-in. bars have been used in several of the largest bridges, the maximum size used in the 668-ft. span of the Municipal bridge at St. Louis being 16 × 2-1/16 in. For wide bars the thickness rarely exceeds 2-1/2 in. Generally the ratio of width of bar to thickness lines between the limits of 4 and 7. If bars are made too thin their heads may buckle, due to the compression at the pins, and if made too thick the bending moments on the pins are increased unnecessarily. The dimensions of eyebar heads as given in steel companies' handbooks are such that the bars break in the body when tested to destruction. Eyebar heads are forged or upset in a die, and the net cross-sectional area through the pin hole is from 33 to 40% larger than that of the body. The heads are finished the same thickness as the body.

To Proportion tension members subjected to direct stress the required net area of the cross-section is P/S. For an eyebar under the tension P the maximum unit stress S_1 on the bottom fibers due to combined tension and bending caused by the weight of the bar is

$$\mathbf{S_1 = \frac{P}{th} + \frac{4\,700\,000\,h}{P/th + 22\,400\,000\,(h/\mathit{l})^2}}$$

28. Design of Compression Members

Notation. S = maximum or allowable unit stress, A = cross-section of a member, P = direct stress in a member, M = bending moment, c = distance from gravity axis to most stressed fiber, I = moment of inertia, l = length of column, r = radius of gyration of column, e = distance from center of gravity of cross-section to point of application of eccentric load, E = modulus of elasticity, w = weight of compression member per unit of length, I_D and I_C = moments of inertia of one ⊏ about gravity axis perpendicular and parallel to web respectively, A_1 = area of one ⊏, x = distance from back of ⊏ to center of gravity.

Compression Members should be proportioned, whenever possible, to be of equal strength about the two principal axes A–A and B–B (Fig. 94), if there is only direct compression in the member; and if there is transverse bending in addition, the strength about the axis around which bending takes place should be the greater. The cross-sectional area should be as far removed

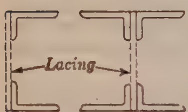


Fig. 92 Fig. 93

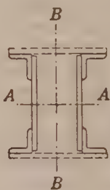


Fig. 94

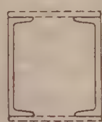


Fig. 95

Fig. 96

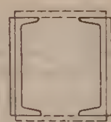


Fig. 97

from the center of gravity of the section as possible, consistent with proper thickness of materials and proper bracing of the different segments. Figs. 92 to 97 show typical sections of web compression members and Figs. 98 to 101 of compression chords and end-posts for bridge trusses, the smaller ones being suitable for light highway or railroad construction and the larger ones

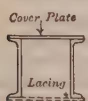


Fig. 98

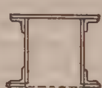


Fig. 99

Fig. 100

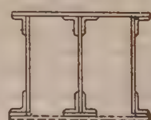


Fig. 101

for heavy work. All of the sections shown except the H-section in Fig. 96 require lacing or lattice bars to connect the segments together. In Fig. 99 and Fig. 101, flat bars are riveted to the lower ∟'s to keep the center of gravity of the section at or near the mid-height axis. Chord sections as in Fig. 99 are called **box sections,** the vertical plates are called **webs** and the upper horizontal plate is the **cover plate.** See also Art. 36. Compression members should not have l/r exceed 100 for main members nor 120 for laterals.

Short Columns are those having l/r less than 40; if the load is concentric, that is, applied at the centers of gravity of the end cross-sections or uniformly over these sections, the stress is one of simple compression distributed uniformly over the various cross-sections of the column, and the unit compression is $\mathbf{S = P/A}$. If the load is eccentric, that is, not applied at the axis of the column, the maximum unit stress is $\mathbf{S = P/A + Mc/I}$ and the minimum unit stress on the same section may be found from the same formula by changing the + to a − sign, using for c in each case the distance from the gravity axis to the maximum or minimum stressed fiber. $M = Pe$, where e is the eccentricity of the load, and I is the moment of inertia of the section about the axis through the center of gravity and $\perp$ to the plane of M.

The strength of a short column depends primarily upon the elastic limit of the steel and also in large measure upon the effectiveness with which its segments are tied together. The ultimate strength of a column never exceeds and is often considerably less than the yield point of the steel. The results of extensive tests made at the Bureau of Standards under the direction of the Special Committee on Steel Columns and Struts of the Am. Soc. C. E. (Final Report, Procs. Am. Soc. C. E., Dec., 1917) indicate that the elastic limit of columns made of carbon steel having a tensile strength of 60 000 lb. per sq. in. may be as low as 19 500 lb. per sq. in. This committee recommends 12 000 lb. per sq. in. as the maximum unit stress to be used when $l/r < 60$. The A. R. E. A. specifications permit the use of 12 500 up to $l/r = 50$, and the Am. Soc. C. E. specifications, 14,300 up to $l/r = 40$.

Long Columns are additionally weakened by their tendency to buckle in the plane of the maximum l/r. Their strength, for ordinary values of l/r, is affected by too many accidental and uncertain conditions to permit its expression by a rational formula. Two types of empirical formulas have been used, the straight line and the Rankine formulas. The former gives good results between $l/r = 50$ and $l/r = 240$, and it has been adopted by the A. R. E. A. (1925) in the form, $S = 15\,000 - 50\,l/r$, with a maximum value of $S = 12\,500$, for $l/r = 50$ or less. The Rankine formula also gives good results for $l/r > 40$, and approaches the Euler curve for very large values of l/r. It has been adopted by the Am. Soc. C. E. Committee in the form $S = 16\,000/(1 + l^2/13\,500\,r^2)$, with a maximum value of 14 300, for $l/r = 40$ or less. With the A. R. E. A. formula, $S = 9000$ for $l/r = 120$, $S = 10\,000$ for $l/r = 100$, $S = 11\,500$ for $l/r = 70$, $S = 12\,500$ for $l/r = 50$. With the Am. Soc. C. E. formula, $S = 7740$ for $l/r = 120$, $S = 9190$ for $l/r = 100$, $S = 11\,740$ for $l/r = 70$, $S = 13\,500$ for $l/r = 50$, $S = 14\,300$ for $l/r = 40$. The A. R. E. A. formula gives the larger unit stresses for the higher values of l/r, and the smaller unit stresses for the lower values of l/r. The two curves cross at $l/r = 76$ and $S = 11\,200$. On account of the deflection of structures and the distortions at the joints all columns are regarded as pin-ended.

For columns subjected to combined compression and bending, and for tension members under combined tension and bending, the maximum unit compression or tension is

$$\mathbf{S} = \frac{\mathbf{P}}{\mathbf{A}} + \frac{\mathbf{Mc}}{\mathbf{I} \pm \mathbf{P}l^2/\mathbf{a}_2\mathbf{E}},$$

in which a_2 is a constant depending on the condition of the ends and the kind of loading causing bending moment M. For members having hinged ends $a_2 = 9.6$ for uniformly distributed loading and 12 for a concentrated load at the middle; for members having fixed ends $a_2 = 32$ for uniform loading and 24 for a concentrated load at middle. The minus sign in the denominator must be used when P is compression and the plus sign when P is tension. This formula may also be used for members having eccentric loads, in which case M is the bending moment due to the eccentric loads and a_2 should be taken for a uniformly distributed loading.

Proportioning Top Chord. Let it be required to proportion the top chord of the 7 panel, 200-ft. typical pin-connected truss of the American Bridge Co., shown in Fig. 109.

The maximum compressions from dead, live and impact stresses are: $U_1U_2 = 870\,000$; $U_2U_3 = 980\,000$; $U_3U_4 = 978\,000$. The lengths are 28.85, 28.64 and 28.57 ft. respectively. The section for U_2U_3 will also answer for U_3U_4. Allowable unit stress $= 16\,000 - 70\,l/r$.*

* The A. R. E. A. Specifications, 1910 and 1920.

Design of U_1U_2. Let the chord be of box section as in Fig. 102. The least radius of gyration is approximately 4/10 the depth of the section which is here assumed to be 25 in., or $r = 10$. With $l = 346$ and a trial r of 10 the allowable average unit stress is 13 580 lb. per sq. in. A trial section must have 870 000/13 580 = 64 sq. in. The section shown in Fig. 102 with 66.44 sq. in. as determined below has its gravity axis $A–A$ above the mid-height axis $B–B$ a distance $d = 169/66.44 = 2.53$ in., 169 being the statical moment about axis $B–B$. $I_{A-A} = 6602 - 66.44 \times (2.53)^2 = 6177$ and radius of gyration about gravity axis $A–A = (6177/66.44)^{1/2} = 9.65$ and allowable

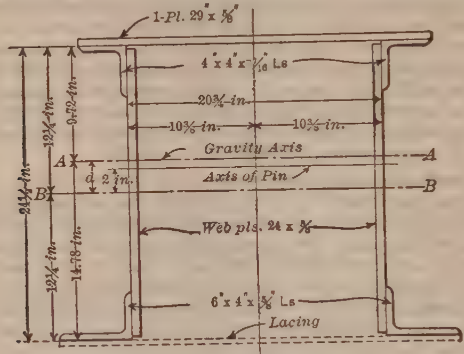


Fig. 102. Section Top Chord U_1U_2

average unit stress is 13 480 lb. per sq. in. The column has an actual average stress of 870 000/66.44 = 13 100 lb. per sq. in. and is therefore of sufficient size for the compression. The statical moment and moment of inertia about mid-height axis $B–B$ are found as follows:

	A	Stat. Mom.	I_{B-B}
1 Pl. 29 × 5/8	= 18.10 × 12.56 =	227.5 × 12.56 =	2862
2 ∟'s 4 × 4 × 7/16	= 6.62 × 11.09 =	73.5 × 11.09 =	815
2 ∟'s 6 × 4 × 5/8	= 11.72 × 11.22 =	− 132.0 × 11.22 =	1485
2 Pls. 24 × 5/8	= 30.00	00.0	1440
	66.44	169.0	6602

In this computation the moments of inertia of the ∟'s and horizontal plates about their gravity axes have been omitted. The radius of gyration about axis $A–A$ is less than that about the vertical gravity axis, and since $A–A$ is a principal axis, the radius used, 9.65, is the least radius of the section.

The center of the pin = 2.53 − 2.0 = 0.53 in. below axis $A–A$. The upward moment due to eccentricity = 870 000 × 0.53 = − 461 000 in-lb.; the downward moment due to the weight of the member = $wl^2/8$ = about 302 000 in-lb. The resultant moment = − 159 000 in-lb. and the additional compressive stress in the lowest fiber of the section is $S = Mc/I = 159\,000 \times 14.78/6177 = 380$ lb. per sq. in. The required sectional area = 870 000/(13 480 − 380) = 66.3 sq. in. Actual area = 66.4 sq. in.

Design of U_2U_3 and U_3U_4. The same section as for U_1U_2 will be used except that the web plates will be 13/16 in. thick instead of 5/8 in. By the method of computation employed above the following data are obtained: $I_{A-A} = 6534$; $r = 9.3$; $l/r = 37$; $S = 16\,000 - 70 \times 37 = 13\,410$; required area = 980 000/13 410 = 73 sq. in.; actual area = 75.48 sq. in.; $d = 2.24$ in. and the center of the pin = 2.24 − 2.0 = 0.24 in. below axis $A–A$. The moment from the weight of the member is 348 000 in-lb. and the resultant moment = 348 000 − 980 000 × 0.24 = 103 000 in-lb. The additional compressive stress on the upper fiber of the section is $S = Mc/I = 103\,000 \times$

10.63/6534 = 170 lb. per sq. in. The required sectional area = 980 000/(13 410 − 170) = 74.0 sq. in. Actual area = 75.48 sq. in. By the specifications the thickness of the cover plate must be at least 1/50 the distance between the rivets connecting it to the flanges = 1/50 × 25.25 = 0.50 in.; and the thickness of the web plate must be at least 1/40 the distance between the rows of rivets = 1/40 × 20 = 0.50 in. The sections chosen satisfy these requirements.

Proportioning Compression Vertical. Let it be required to proportion the vertical U_2L_2 composed of two ⊏'s (Fig. 103), carrying a maximum concentric dead, live and impact stress of 221 000 lb. The center of the pins is in the axis D–D. Assume two ⊏'s 15 in. 40 lb. having a combined gross area of 23.52 sq. in. and a radius of gyration for the two ⊏'s about a gravity axis perpendicular to the web of 5.4 in. l/r = 408/5.4 = 76; S = 16 000 − 70 × 76 = 10 680 lb. per sq. in. Required area = 221 000/10 680 = 20.7 sq. in. The ⊏'s are spaced 12.5 in. back to back. To make r about axis B–B equal that about D–D (Fig. 103), the distance z must be $x + \sqrt{(I_D - I_C)/A_1}$ which in this case is less than 12 in. If the flanges of the ⊏'s are turned outward z is the same as above except that x is negative. z may also be obtained directly from the Steel Co. handbooks. Since the floorbeam and the top strut reduce the unsupported length of the column for bending about the axis B–B, z may be less than this value for equal values of l/r. Axis D–D therefore decidedly governs in this present case. With flanges turned in as in Fig. 103 the distance in the clear between flanges should not be less than 2-1/2 in. if lattice bars are to be hand riveted nor less than 4 in., if machine riveted.

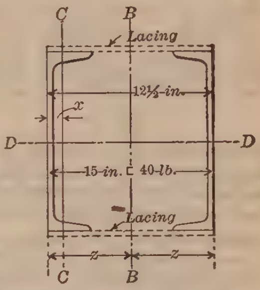


Fig. 103. Section Vertical Post U_2L_2

Inclined End-posts of Through Bridges are subjected to a direct compression from live, dead and impact stresses and also, at times, to a slight increased direct compression due to wind and to a considerable transverse bending due to wind in a plane perpendicular to the plane of the truss, these wind stresses being due to the action of the wind on the upper portion of the truss. The end-post is a part of the portal system and as such carries to the supports the wind pressure brought to the hip joint by the top lateral system. The cross-sectional area of the post must be such that the actual average unit stress P/A, due to live, dead and impact stresses shall not exceed that given by the column formula $\mathbf{P/A = 16\,000 - 70\ l/r}$,* where l is the length in inches between pin centers and r the radius of gyration of cross-section about the gravity axis parallel to the cover plate, as axis A–A, Fig. 102. The maximum unit stress on the outermost fiber due to live, dead, impact and wind stresses may exceed by not more than 25% the allowed axial unit stress; that is, for railroad bridges the maximum combined unit stress, S in formula below, should not exceed 16 000 + 25% of 16 000 = 20 000 lb. This maximum fiber stress due to axial compression, the tendency to buckle in the plane of the truss and the bending moment in the plane of the portal due to transverse wind forces is given with sufficient accuracy by

$$\mathbf{S} = \frac{\mathbf{P}}{\mathbf{A}} + 70\frac{l}{\mathbf{r}} + \frac{\mathbf{Mc}}{\mathbf{I}}\dagger$$

In this formula P must include the direct compression due to all loads,

* A. R. E. A. Specifications, 1910 and 1920. Changed to P/A = 15 000 − 50 l/r in 1925.

† A. R. E. A. Specifications, 1910 and 1920. With the 1925 specifications the middle term becomes 50 l/r.

including wind; c is 1/2 the extreme width of end-post, I is moment of inertia of section about gravity axis perpendicular to cover plate, l_1 is length of end-post from bottom of knee-brace to lower bearing of end-post; $\mathbf{M} = \frac{\mathbf{W}l_1}{4}$ or $\frac{\mathbf{W}l_1}{2}$ depending on whether the bottom of the post is fixed or is not fixed, l/r has the value used in the column formula for direction compression. The post may be considered as fixed at lower end if the end floorbeam is rigidly connected to it, or if the moment due to one-half the direct compression in post acting with a lever arm equal to the distance between bearings on the pin is greater than $Wl_1/4$. W is the total wind load in pounds acting at the hip joint including that brought to that joint by the top lateral system.

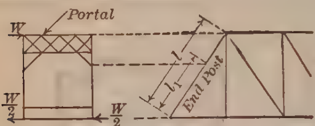


Fig. 104

If the weight of the member is also considered S will containan other Mc/I term in which M is the bending moment due to the weight of the post, c is the distance from the gravity axis parallel to the cover plate to the uppermost fiber, and I is the moment of inertia about the same axis.

Design of L_0U_1. The section of U_1U_2 will be used except that the web plates will be increased to 3/4 in. P = 873 000; l = 41.4 ft.; l_1 = 31.4 ft. By the preceding method of computation it is found that I_{A-A} = 6501; r = 9.5; l/r = 52.5; S = 16 000 − 70 × 52.1 = 12 325; required area = 873 000/12 325 = 70.7 sq. in.; actual area = 72.44 sq. in. d = 2.31 in. and the center of the pin = 2.31 − 2.0 = 0.31 in. below the gravity axis $A-A$. The negative moment from eccentricity = 873 000 × 0.31 = −270 630; the positive moment from its own weight = about 370 000, and the resultant moment = about 100 000 in-lb. The additional compression on the upper fiber = Mc/I = 100 000 × 10.88/6501 = 170 lb. per sq. in. The required sectional area = 873 000/(12 325 − 170) = 72.0 sq. in., which is less than the actual area.

It is necessary to test the section when the wind stresses are included. The wind load along the top chord is 200 lb. per lin. ft. W = 200 × 3 × 28.57 = 17 200; $Wl_1/4$ = 17 200 × 31.4 × 12/4 = 1 630 000 in-lb. The direct compression from dead and live load is 610 000 lb. and from wind 26 400 lb.; the distance between bearings on the shoe = 21 in. and the available moment of restraint at the shoe = 636 400 × 21/2 = 6 700 000. Since this exceeds the moment from the wind the end may be considered fixed and the point of contraflexure taken at the mid-point between pin and foot of knee-brace. The moment at the knee-brace likewise = 1 630 000 in-lb. Further, it is found that I_{C-C} (about the axis perpendicular to the cover plate) = 7600; c (to the extreme fiber of the bottom flange ∟'s) = 16.37 in.; l/r (in the plane of the truss) = 52.5; 70 l/r = 3675. The maximum unit stress upon the outer fiber of the lower flange ∟ (neglecting effect of weight of member and eccentricity, which is slight) is

$$S = \frac{873\,000 + 26\,400}{72.44} + 3675 + \frac{1\,630\,000 \times 16.37}{7600} = 19\,485 \text{ lb. per sq. in.}$$

Since this is less than the allowable 16 000 + 25% = 20 000 the section does not need to be increased on account of the wind stresses.

29. Design of Details

Notation. D = diameter of pin, S_b = allowable unit fiber stress for bending, S_c for bearing on pins, S_t for tension, S_s = actual average unit shearing stress on pins, t = required thickness for bearing of a built-up member, w = width of the widest eyebar on a pin, P = direct tension or compression in a member.

In Pin-connected Bridges (Fig. 109) there is at each joint a cylindrical steel pin which has at each end screw-threads upon which hexagonal nuts are placed, the Lomas nut, Fig. 106, being most common form. Cotter pins, Fig. 105, are seldom used. Pilot nuts, Fig. 107, and driving nuts, Fig. 108, are placed on pins to protect threads during erection. For ordinary trusses, pins with Lomas nuts vary from 2 to 9 in. in diameter, but pins of greater diameters are used for large trusses. In **packing,** that is, arranging bars on a pin, the bars should be packed as closely as possible, allowing clearances of 1/16 in. between two eyebars and from 1/8 to 1/4 in for built-up members; bars having horizontal components should be close together, similarly those having vertical components; bars should run parallel to plane of truss if possible and maximum deviation should not exceed 1/8 in. per foot, better 1/16 in. per foot; generally a bar with small stress should be on the outside; bars running in same direction should have a space of at least 1 in. between them.

Fig. 105

Fig. 106

Fig. 107

Fig. 108

To Determine Diameter of Pin after all joints of the truss are packed, assume the diameter, then compute thicknesses of reinforcing or pin plates for built-up members, and then, taking the forces as concentrated at the centers of the eyebars and at the centers of the bearings of built-up members, compute the diameter of pin to withstand the maximum bending moment. If this diameter agrees with the assumed diameter the maximum shear on the pin should be investigated; if not, the computation should be revised. Not more than one or two diameters should be used in a truss. The trial and final diameter of pin must be equal to or larger than $(S_t/S_c)\, w$ or otherwise the bearing on the eyebar is excessive. The value of S_t/S_c is often taken at 3/4. In designing pins to carry moments it is best to assume forces concentrated at centers of bearings, but in investigating the strength of an existing structure this assumption may in some cases lead to a resulting extreme fiber stress larger than the ultimate strength of the material in bending and yet the structure stands. In such a case if an assumption can be made which results in safe unit stresses throughout, the joint is safe. The total required thickness of a built-up member in bearing on a pin is $\mathbf{t = P/S_cD}$.

The **maximum bending moment,** M, at a section of a pin is the resultant of the moment at that section due to horizontal forces, M_h, and that due to vertical forces, M_v, or $M = \sqrt{M_h^2 + M_v^2}$. And the diameter of pin required to withstand this moment is $D = \sqrt[3]{10.2\, M/S_b}$. Tables showing bending moments of pins of various diameters are given in structural handbooks, so that the value of D can be easily taken from the tables as soon as M is known. The **maximum shear,** V, at a section of a pin is the resultant of the shear at that section due to horizontal forces, V_h, and that due to vertical forces, V_v, or $\mathbf{V = \sqrt{V_h^2 + V_v^2}}$. The shear is usually assumed distributed, in which case $\mathbf{S_s = 1.27\, V/D^2}$, but the intensity of maximum shearing stress on the neutral plane is $\mathbf{1.70\, V/D^2}$.

In Riveted Trusses (Fig. 114) the members are connected together at the joints by gusset plates which must be of sufficient size and thickness to transmit the stresses, and there must be a sufficient number of rivets in each member to develop its full strength. The stress is assumed to be equally distributed over the various rivets, hence they should be spaced with this in view. **Gusset plates** are frequently made too thin. They must be computed to carry the compression and tensile stresses brought on them by the main truss members, and as these stresses are frequently applied eccentrically to the gussets the eccentricity must be allowed for. In such cases as the lower chord joints of Pratt trusses the gussets must have sufficient net area along the horizontal row of

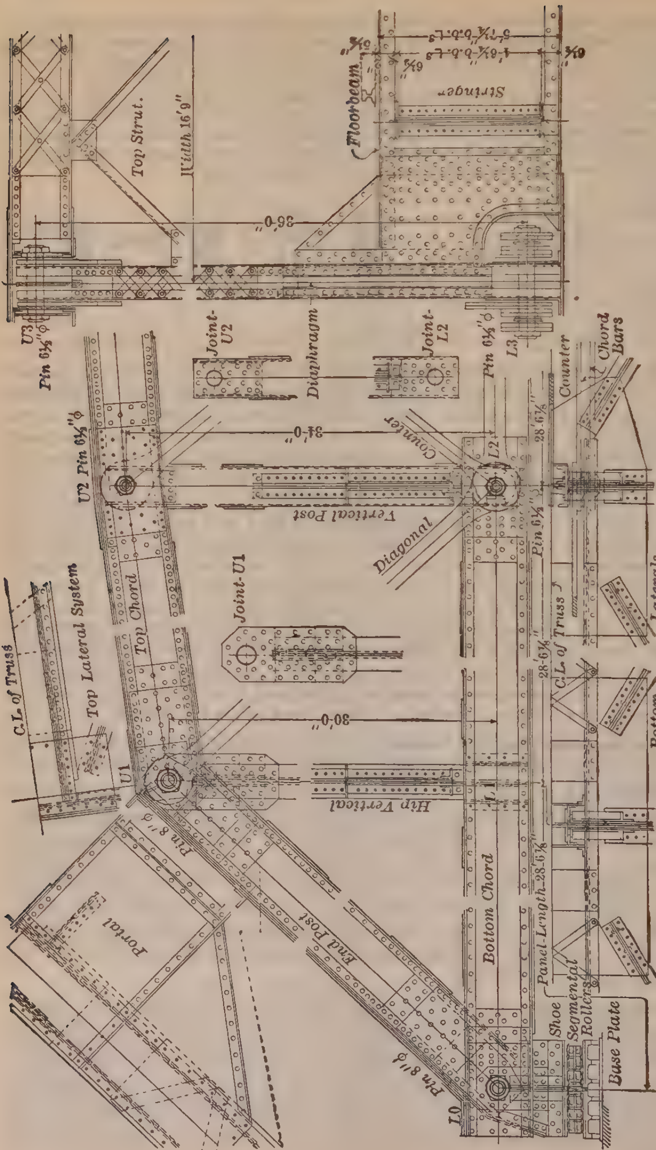


Fig. 109. 200-ft. Through Pin Connected Span

rivets connecting the plates to the chord to carry the shearing force on that section and similarly at the vertical row of rivets connecting gusset plates to the vertical. Ordinarily two gusset plates are used at each joint, although in light trusses only one is used. Center of gravity lines of all members should meet at a point, and the center of gravity of rivets connecting a member to a gusset plate should coincide with the center of gravity of the member.

Lattice Bars must be of sufficient strength to brace the segment of a compression member so that the segments will act as a unit and in doing this they are subjected to tension or compression due to shear resulting from the bending of the column. Formerly the design of the lacing of a column was largely a matter of judgment, but since the failure of the first Quebec bridge, by column shear, many attempts have been made to devise a rational formula. The best known method is based upon the analogy between the bent column, under axial load, and a beam of equal length and section and bearing a uniformly distributed transverse load of such an amount as to deflect the beam the same amount at its center, as the column is deflected under buckling action. The maximum end shear of the beam is assumed to be the maximum shear in the column. The Am. Soc. C. E. formula, $R = Pl/4000\,y$ (Art. 11), was obtained in this manner. About the only thing that is certain about maximum shear in a column is that it is equal to $P \sin\theta$, where P = axial load on column and θ = maximum slope of bent column to the originally straight axis. The A. R. E. A. specifications (Art. 11) assume a maximum value for $\sin\theta = .025$, corresponding to $\theta = 1.5$ deg. nearly. Maximum θ may occur either at the ends, in case of single curvature, or at either the ends or the center, in case of double curvature. The shear in the column is assumed to be carried equally by all of the lattice bars cut by a normal section, when the column is open on both sides; and half the shear is assumed to be so carried by the latticing when the column is open only on one side and a cover plate is used on the other. The stress in any bar is **V** cosec θ, where V is the shear carried by the bar, and θ is the angle which the bar makes with the axis of the column.

30. Deflection, Camber, and Bracing

Notation. P = direct tension or compression in a member, l = length of member, A = area of cross-section of member, E = modulus of elasticity, u = stress in member due to load of unity, L = span of truss, p = panel length, p_1 = increase in panel length, H = depth and R = radius of truss, c = camber of truss.

Deflection of a Truss may be due to elastic deformation of the members caused by stresses within them, to imperfect workmanship such as the play in pin holes of a pin-connected truss, and to deformation of the members caused by changes of temperature. The deflection in any direction of any joint of a truss due to any cause may be computed by the following method provided the change in length of each bar can be determined. Place a load of unity at the joint and acting in the direction in which the deflection is wanted and compute the stress in all bars due to this load; call this stress u; then the deflection is given by $\Delta = \Sigma u l_1$, where Δ denotes the deflection of the joint in the direction desired, and Σ denotes the summation of the various products of the stress u due to load unity by the change in length l_1 for each bar successively. The change of length l_1, in any bar due to a series of loads upon a truss is $\mathbf{P}l/\mathbf{AE}$, hence the deflection for this case is given by $\Delta = \Sigma \mathbf{Pu}l/\mathbf{AE}$. In applying this formula, tensile stresses should be given the positive sign and compressive stresses negative, both for stresses due to the loads on the truss and for the imaginary load of unity. If all the bars are of the same material the E may be placed on the outside of the summation sign. A is usually the gross area of the cross-section, even for tension bars. To find the **absolute deflection** of a joint with respect to its original position it is necessary first to find the vertical movement by placing the load of unity vertically at the joint, then the horizontal movement by placing it horizontally at the same joint, and the resultant of these two component movements will give the magnitude of the true deflection.

The **Camber of a Truss** is the distance the center of a chord is raised above the ends to allow for the deflection of the truss. It is usually expressed in inches and varies from 1/1000 to 1/2000 of the span length. Camber is used so that when the maximum load is on the bridge the truss will have the form assumed in the design, and is usually obtained by making the panel lengths of the top chords, or their horizontal projections, longer than the corresponding panels of lower chord in the proportion of 1/8 in. in 10 ft. Camber is sometimes made equal to the deflection due to full live and dead loading by increasing or decreasing the lengths of the truss members by amounts equal to their corresponding deformations. When the camber is made equal to this maximum deflection the radius, R, to which the lower chord of the truss is curved and the increase, p_1, in panel length of the upper chord are given with sufficient accuracy by the following formulas respectively:

$$\mathbf{R = L^2/8\,c} \quad \text{and} \quad \mathbf{p_1 = 8\,cpH/L^2}$$

Bracing for Deck Bridges usually consists of a horizontal truss in the plane of the top and also of the bottom chords, together with a vertical transverse system at each panel point. The transverse bracing is also called sway bracing, and the horizontal trusses are called the top and bottom lateral systems. The chords of the main trusses constitute the chords of the lateral system, and the floorbeams usually act as the struts of the top system. With a sway bracing at each panel point the bottom laterals are not really necessary, although desirable, and sometimes are omitted in all or in alternate panels, in which case the sway bracing must be designed to carry the wind loads from bottom to top panel points. When both lateral systems are used, the function of the sway bracing is to equalize the deflection of the trusses when the latter are subjected to unequal loading as when only one track of a double-track bridge is loaded. Since the wind loads on top and bottom panel points are unequal the sway bracing receives stress due to unequal lateral deflection of these points. The sway or portal bracing at each end of the bridge must be heavier than at intermediate points.

Through Bridges have top and bottom lateral systems and also have sway bracing at each top panel point extending down as low as the required head-

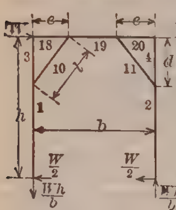


Fig. 110

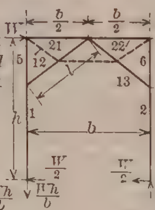


Fig. 111

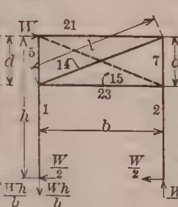


Fig. 112

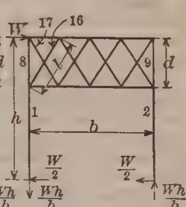


Fig. 113

room allows. When the end-posts are inclined the end sway bracing or **portal** is placed in the inclined plane of the end posts. In the best construction all members of the bracing systems are rigid, that is, made of angles or other similar shapes. Simple portals are shown in Figs. 110, 111, 112 and 113, and the following formulas give the stresses due to wind load W when the posts are hinged at bottom:

For P_1 tension $= Wh/b$, shear $= W/2$, moment $= W(h - d)/2$. For P_2 compression $= Wh/b$, shear $= W/2$, moment $= W(h - d)/2$. For P_3 compression $= Wh(1/2\,e - 1/b)$, shear $= W(h - d)/2\,d$, moment $= W(h - d)/2$. For P_4 tension

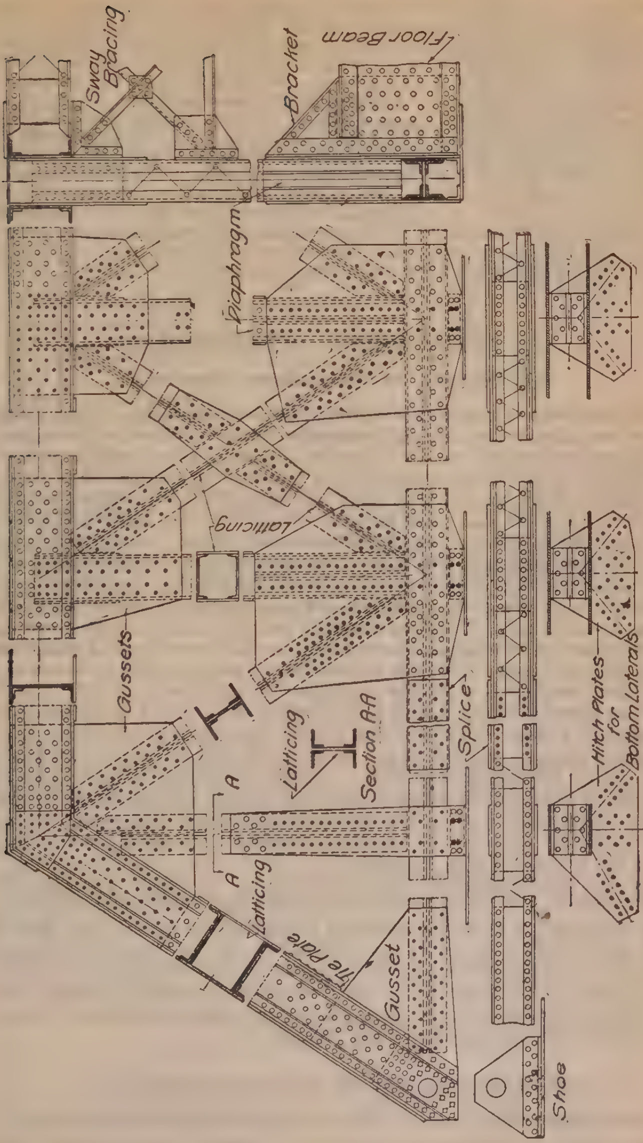


Fig. 114. Partial Detail Drawing of a Through Riveted Railroad Bridge.

$= Wh\,(1/2\,e - 1/b)$, shear $= W(h-d)/2\,d$, moment $= W(h-d)/2$. For P_5 and P_6 direct stress = zero, shear $= W(h-d)/2\,d$, moment $= W(h-d)/2$. For P_7 compression $= Wh/b$, shear $= W(h-d)/2\,d$, moment $= W(h-d)/2$. For P_8 tension $= Wh/2\,b$, shear $= W(h-d)/2\,d$, moment $= W(h-d)/2$. For P_9 compression $= Wh/2\,b$, shear $= W(h-d)/2\,d$, moment $= W(h-d)/2$. For P_{10} tension $= Whl/2\,de$. For P_{11} compression $= Whl/2\,de$. For P_{12} tension $= Whl/bd$. For P_{13} compression $= Whl/bd$. For P_{14} tension $= Whl/bd$. For P_{15} assumed zero. For P_{16} tension $= Whl/2\,bd$. For P_{17} compression $= Whl/2\,bd$. For P_{18} compression $= W(h+d)/2\,d$, shear $= Wh\,(1/2\,e - 1/b)$, moment $= Wh\,(1/2 - e/b)$. For P_{19} compression $= W/2$, shear $= Wh/b$, moment $= Wh\,(1/2 - e/b)$. For P_{20} tension $= W(h-d)/2\,d$, shear $= Wh\,(1/2\,e - 1/b)$, moment $= Wh\,(1/2 - e/b)$. For P_{21} compression $= W(h+d)/2\,d$. For P_{22} tension $= W(h-d)/2\,d$. For P_{23} compression $= Wh/2\,d$. Chord stresses in Fig. 113 may be obtained by use of moments. Dotted bars in Fig. 111 have no stress. If posts are fixed at the lower ends, h in above formulas should be replaced by distance from top of post to point of contraflexure, which is usually $h/2 + d/2$.

SECONDARY STRESSES

31. Nature of Secondary Stresses

The members of a truss form the sides of a series of triangles. Under stress one or more sides in each triangle are shortened by compression and the others lengthened by tension. As the angles of a triangle depend on the relative length of its sides, the angles at joints of a truss tend to change. This change in angle is wholly prevented by rivet and gusset-plate connections, mostly prevented by rivets alone, and partially—often to a very material extent—prevented by friction on pins of pin-connected joints. The members are therefore bent instead of continuing to be straight sides of true triangles and the resulting bending stresses are called secondary stresses. In an ideal truss with frictionless pins every member would remain straight and no secondary stresses would result. The **total stress** in any fiber is the algebraic sum of the secondary, or bending, and the primary, or axial, stress and is a maximum in certain outer fibers. The ratio of secondary to primary stress in chords and main web members is ordinarily not large, generally less than 20%, but in short members, and those having small primary stresses, such as the auxiliary members of trusses with subdivided panels, the ratio may be several times the above amount. For this reason trusses of the Baltimore or Pennsylvania types should be avoided, if possible. Likewise short bracing struts, such as are used in Pennsylvania trusses to brace the vertical columns at mid-height, or the collision struts sometimes used in end panels of Pratt trusses, create high secondary stresses in all adjoining members. The latter are now used much less commonly than formerly.

Mention has already been made of the growing movement to increase the present unit stresses. If higher unit stresses are adopted it will become increasingly important to know the maximum stress that can occur in any fiber of any member—that is, the worst combination of secondary and primary stresses. Maximum secondary and maximum primary stresses do not necessarily occur at the same time—with the same condition of loading—but generally maximum primary stress, with its accompanying secondary stress, gives the maximum or governing combination.

32. Computation of Secondary Stresses

Exact Computation of the secondary stresses in an ordinary structure is a tedious affair, necessitating the solution of as many simultaneous equations as there are joints, for each governing position of the live load. As there are many such positions the work becomes staggering. It is not surprising, therefore, that designers avoid such laborious computations and seek safety in the use of low unit stresses.

Approximate Method. The method of successive approximations is so simple, and gives results so closely approximating those by the exact method that it will be illustrated here by an example occurring in "Statically Indeterminate Stresses," by Parcel and Maney. The bending moments in the members meeting at the hip joint, C, of the 518-ft. span of the Kenova bridge, N. & W. R. R., will be determined (Fig. 114*a*).

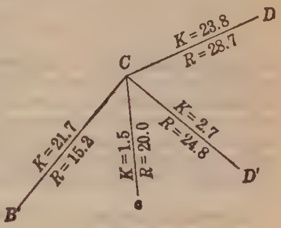


Fig. 114*a*

$K = 4\,EI/L$; $R = D/L$; where $E =$ 29 000 000, I = moment of inertia about the axis perpendicular to plane of truss, in inch-pounds, L = length of member in inches, D = deflection in inches of far end of member, relative to C, measured normal to the axis of the member. The values of K given in Fig. 114*a* should be multiplied by 1 000 000 000, and those of R divided by 10 000. Thus for CD, $I = 83\,000$, $D = 1.16$, $L = 405$, $K = 4 \times 29\,000\,000 \times 83\,000/405 = 23.8 \times 1\,000\,000\,000$, $R = 1.16/405 = 28.7/10\,000$. D can be most readily obtained from a Williot diagram, as will be explained later. K and R are thus known in advance for every member of the truss.

The assumption is first made that the angles of rotation ϕ of the joints at C, D, D', c, B' are all equal. Let ϕ_1 be this angle, in radians, the subscript meaning first approximation. Then $\phi_1 = \Sigma KR/\Sigma K$, the summations to include all of the members meeting at C, thus,

$$\begin{aligned}
CD &= 23.8 \times 28.7 = 683\\
CD' &= 2.7 \times 24.8 = 67\\
Cc &= 1.5 \times 20.0 = 30\\
CB' &= 21.7 \times 15.2 = 330\\
\Sigma K &= 49.7 \qquad \Sigma KR = 1110\\
\phi_1 &= 1110/49.7 = 22.3
\end{aligned}$$

Taking account of the units assumed for R, the true value of ϕ_1 at $C = 22.3/10\,000$. A similar calculation is made for each joint of the truss. Designating the angle of rotation at each of the joints adjacent to C by ϕ'_1 to distinguish it from ϕ_1 at C, the following values of ϕ'_1 were obtained: ϕ'_1 at $D = 23.2$; at $D' = 18.4$; at $c = 23.8$; at $B' = 28.4$.

For the second approximation at C use the formula, $\phi_2 = \phi_1 - 1/2\, \Sigma K(\phi'_1 - R)/\Sigma K$, where ϕ_1 = first approximate value at C; ϕ_2 = second approximate value at C; ϕ'_1 = first approximate value at each of the adjacent joints. Thus, $1/2\, K(\phi'_1 - R)$ at

$$\begin{aligned}
D &= 1/2 \times 23.8 \times (23.2 - 28.7) = -\ 65\\
D' &= 1/2 \times 2.7 \times (18.4 - 24.8) = -\ 9\\
c &= 1/2 \times 1.5 \times (23.8 - 20.0) = +\ 3\\
B' &= 1/2 \times 21.7 \times (28.4 - 15.2) = +\ 143\\
\Sigma K &= 49.7 \quad 1/2\, \Sigma K(\phi'_1 - R) = +\ 72\\
\phi_2 &= 22.3 - 72/49.7 = 22.3 - 1.4 = 20.9
\end{aligned}$$

A similar operation is performed for each joint of the truss. The results at the joints adjacent to C may be designated by ϕ'_2 to distinguish them from ϕ_2 at C. These values were found to be as follows: ϕ'_2 at $D = 25.3$; at $D' = 17.2$; at $c = 22.2$; at $B' = 25.0$.

For the third approximation use the formula $\phi_3 = \phi_2 - 1/2\ \Sigma K(\phi'_2 - \phi'_1)/\Sigma K$, where ϕ_2 and ϕ_3 are the values of ϕ at C resulting from the second and third approximations, and ϕ'_1 and ϕ'_2 the values of ϕ at each of the joints adjacent to C resulting from the first and second approximations, respectively. Thus, $1/2\ K\ (\phi'_2 - \phi'_1)$ at

$$
\begin{aligned}
D\ &= 1/2 \times 23.8 \times (25.3 - 23.2) = +25 \\
D'\ &= 1/2 \times 2.7 \times (17.2 - 18.4) = -2 \\
c\ &= 1/2 \times 1.5 \times (22.2 - 23.8) = -1 \\
B'\ &= 1/2 \times 21.7 \times (25.0 - 28.4) = -37
\end{aligned}
$$

$$\Sigma K = 49.7 \quad 1/2\Sigma K(\phi'_2 - \phi'_1) = -15$$

$$\phi_3 = 20.9 + 15/49.7 = 21.2$$

In a similar manner ϕ_3 is found for every joint of the truss. It is thus seen that the values of ϕ at C have converged practically to a constant and that nothing would be gained by further successive substitutions. This method of solution is not applicable to every set of simultaneous linear equations, but it is generally available for secondary stresses when there is a degree of uniformity in the values of ϕ at adjacent joints. The test of its practicality is the rapidity with which the successive values of ϕ converge to a constant quantity.

The **bending moment** at the C end of CD is expressed in general terms by

$$\mathbf{M_{CD} = 1/2K_{CD}\,[2\,\phi_C + \phi_D - 3\,R_{CD}]}.$$

Using the results of the third approximation, $\phi_C = 21.2$, $\phi_D = 25.4$, $M_{CD} = 1/2 \times 23.8\ [42.4 + 25.4 - 3 \times 28.7] = -220$. Since the scale is 1 000 000 000/10 000 = 100 000, $M_{CD} = -$ 22 000 000 in-lb. $f = Mc/I$ = 22 000 000 $\times$ 24/83 000 = $-$ 6400 lb. f = extreme fiber stress per square inch from secondary effects. The primary unit stress for this member under the same condition of loading is 13 600 lb. Percentage of secondary to primary stress = 6400/13 600 = 47. This truss is of the Pennsylvania type with many short, relatively low stressed members. The percentages run high throughout the truss, especially in the vicinity of the auxiliary members, showing clearly the damaging effect of the latter.

The **Deflection D** of one end of a member relative to the other and its original direction, due to the distortion of the truss under the load, can readily be scaled from a Williot displacement diagram (Art. 40 and Figs. 132, 133, 134, page 1267). For example in the cantilever truss shown in Fig. 132, let it be desired to find the deflection of G, relative to F, normal to the horizontal line passing through the new position of F. The displacement diagram, (Fig. 133, p. 1267), was drawn on the assumption that the joint A and the bar AB are fixed in space, the pier at e being removed to allow free movement of the rest of the truss. $A'F'$ then represents the motion of the point F, and $A'G'$ the motion of the point G. The distance $F'G'$ is the actual motion of G relative to F, and the projection of $F'G'$ upon a line normal to FG (of the truss), a vertical in this case, is the required value of D. D is positive when the deflection of one end, relative to the other, is clockwise. Thus, standing at joint F and imagining the other end of FG to move in a direction parallel to $F'G'$, if FG turns in a clockwise direction, as in the present case, D is positive. For the member cd, D is the projection of $c'd'$ (in the displacement diagram) upon a normal to cd. The lines representing these projections are all present in the diagram and may at once be scaled. Standing at c and putting in the displacement $c'd'$ at d normal to cd, the member cd will swing about c in an anti-clockwise direction, and D is minus. The sign of ϕ is obtained from the numerical solution of the equations and is to be interpreted by the same convention as for D. Thus, when ϕ is positive the joint is twisted in a clockwise direction. Negative M means tension in the outer fiber first met with, in a member, as one passes around the joint in a clockwise direction. Thus, $M_{CD} = -$ 22 000 000 in-lb. means a tension of 6400 lb. per sq. in. in the upper fiber of CD at the C end.

MOVABLE AND CANTILEVER BRIDGES

33. Types of Movable Bridges

A Draw or Movable Bridge is one which can be moved from its normal position to allow the unobstructed passage of vessels. Movable bridges are classified into the five following types: swing, bascule, retractile, lift, and ferry bridges. Those of the **swing type** revolve horizontally about a vertical axis, which may be at the center, at one end or at an intermediate point of the span. **Bascule bridges** include all movable bridges which move in a vertical plane, rolling back from the opening or rotating about a horizontal axis. **Retractile drawbridges** are those which are supported on wheels resting on tracks on shore, and which are rolled horizontally backward from the opening either along the approach of the bridge or at an angle thereto. **Lift bridges** can be raised vertically to a sufficient height to allow vessels to pass underneath. **Ferry bridges** consist of a movable car which is suspended from, and which moves back and forth under an overhead fixed bridge supported on towers, the overhead portion being so high above water level as to afford proper clearance for navigation.

Swing Bridges generally consist of two or more plate girders or trusses supported on a center or pivot pier in such a manner that they can be revolved on a circular track or pivot resting upon and attached to the pier. When the bridge is open there is a passageway for vessels on either side of the pier and the draw, and the trusses act as cantilevers overhanging the middle support. To prevent the open bridge from being struck by a vessel approaching or passing through the channel, a **fender pier** of piles and timber is built around the supporting pier, thus protecting the open draw span and guiding the vessel into the channel. The fender pier extends lengthwise of the stream and must be longer and wider than the open draw. Sometimes the pivot is placed on one side of the center between the center and the end of the draw, and the bridge then has two arms of unequal length, the shorter arm being counterweighted to balance the longer. Unequal arms may be used where the stream is too narrow for a central pier; the short counterweighted arm hanging over the shore and the long arm over the water when the bridge is closed. **Double swing bridges** have been used in a few instances. They consist of two swing spans each supported by a pier on each side of the stream and having their outer ends connected at the center of the channel when the bridges are closed. Swing bridges are the simplest and most economical of the various types and where there is ample room for operation they should generally be used. In locations where the pivot pier takes up too much space in the stream, or where room is not available for turning, the swing bridge cannot be used and some form of bascule bridge is preferable.

The **Jack-knife or Folding Drawbridge** is a swing bridge used only for railroad traffic, and as usually built has one Howe truss or steel truss under each rail, the rail being fastened directly to the upper chord of the truss. The trusses are connected together by small pivoted bars every few feet apart along the top chords, and each truss revolves horizontally about a pivot at one end, while the movable end is supported during operation by a needle beam which is guyed to the top of a tower or bent located on the shore at the pivot end of the span. When the bridge is open the trusses are parallel with the bank and are folded closely together. Since jack-knife draws cannot have floors, such bridges should not be used.

Bascule Bridges of various designs are in use, some being patented. Their merits lie in the speed with which they are operated and in the small amount of room required for their erection and operation. One to two minutes is the time often required for opening or closing a bridge although many structures cannot be made ready and opened in this time. Bascule bridges are used when conditions are unfavorable for a swing bridge, such as when a waterway is too narrow for a center pier in the river, and the abutting property is too valuable to afford space for swinging the span when the pier is placed on one shore (as is the case in Chicago). As such conditions arise only in cities, most bascule bridges are of the highway type. They are well adapted for railway service when the number of tracks is likely to increase in the future, for several parallel bridges may be built side by side at different times. A bascule bridge may have one or two leaves. When the required opening is not too great the former is to be preferred on account of economy in first cost and because the free end has a firm support when the leaf is closed. When two leaves are used they must be locked together at the center when the bridge is closed.

Bascule bridges are generally either of the trunnion or the rolling lift type. **Trunnion bridges** rotate about a horizontal and, usually, fixed axis placed near one end. There are several types differing in the manner of support of the counterweight and the mode of operation. The main trusses or girders may be continued back of the trunnion a short distance, and the counterweight be placed in this extension below the floor as far back of the trunnion as possible (as in the City of Chicago type); or the counterweight may be

Fig. 115

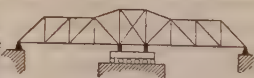


Fig. 116

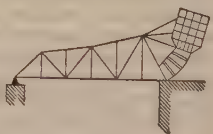


Fig. 117

carried overhead at one end of a moving framework pivoted to and supported by an independent tower placed back of the trunnion, and operating somewhat as a walking beam (as in the Strauss and Brown types). The river end of the walking beam is attached to the moving leaf at some point beyond the trunnion, by linkages or cables. In some cases, as in the earlier Brown type, the counterweight moves vertically and is supported by cables which pass over sheaves at the top of a fixed vertical tower and thence down to the point of attachment on the moving leaf. In all cases the counterweights are of such size and move in such paths that the power required for opening or closing the draw is only that necessary to overcome inertia, frictional resistance, wind and snow. The Scherzer **rolling lift** bridges, (Fig. 117), are of the bascule type, but instead of rotating on trunnions they roll back on a horizontal track, the ends of the main girders or trusses being curved for this purpose. As the curved portion rolls backward on the track the outer portion or bascule leaf rises, leaving the channel clear. **Bascule bridges** are operated by racks and pinions placed in the most favorable position for speed of operation and minimum expenditure of power.

Retractile, Direct Lift and Ferry Bridges are not common types. Retractile bridges may have one or two leaves overhanging the stream when closed and extending back over the abutments far enough to serve as a counterweight. This type is applicable to short spans only. The direct lift bridge consists of a simple truss structure which when closed rests on the abutments similar to any fixed span, but it is opened by being raised vertically by means of cables or chains passing over sheaves at the top of a tower at either end of the span. Counterweights are used to reduce the work of moving the bridge. The only ferry bridge in the United States is that across the ship canal at Duluth. It has two riveted steel trusses of about 390-ft. clear span supported on a steel tower on either side of the canal. The clear height is about 135 ft. above the water level. These two fixed trusses carry the track from which are suspended a steel frame and a ferry car, both of which travel back and forth across the canal. The retractile, direct lift and ferry bridges are used for highway traffic only.

Swing Bridges, which are generally supported at the center of the span, are either center-bearing, (Fig. 115), or rim-bearing (Fig. 116). While in motion a **center-bearing structure** is supported at the pivot pier on a turntable consisting of some transverse girders which carry the dead weight of the bridge to a center pivot or casting about which the structure revolves. When the bridge is closed wedges are inserted under the trusses at the center so as to carry no dead load but to carry all the live load reaction on the center pier. Trailing wheels placed under the turntable steady the bridge while swinging but receive no load otherwise. From 4 to 8 wheels varying in diameter from 12 to 20 in. are used. The center pivot consists of an upper and a lower casting, the upper one supporting the weight of the bridge by means of cross or diaphragm girders, the lower one being fixed to the masonry center pier. The pivot turns on 2 or 3 disks, 15 to 50 in. in diameter. When two disks are used the lower one is of hard tempered steel, with an upper spherical concave surface of slightly smaller radius than the lower disk. When three disks are used, the top and bottom ones are of hardened steel, with concave surfaces, and the middle one is of hard bronze, with both surfaces convex. An oil ring on the lower pedestal feeds grooves cut in the turning surfaces, and permits the disks to run in a bath of boil. **Rim-bearing** swing bridges have their trusses supported at the center pier directly on the drum or on distributing girders from which the load is carried to the drum. The **drum** is a circular girder resting on conical wheels which in turn are supported on a circular track of the pier. The center-bearing swing bridge, except for very wide city bridges, is distinctly superior to the rim-bearing type, and has nearly superseded the latter. **Combined rim- and center-bearing** swing bridges are those in which part of the weight is carried by the center pivot. This arrangement is preferable to that where all the weight is taken by the drum, as the center load helps to anchor the center pivot and prevent lateral motion under horizontal dynamic forces.

The heaviest swing bridge in the United States, up to 1926, and at present, 1928, the heaviest center-bearing bridge, is the double-track, double-deck structure of the Southern Pacific Co. across the Sacramento River at Sacramento, Calif. The span is 390 ft. 3 in., center to center of end supports; the maximum load on the center pivot is 6 748 000 lb.; that on each end wedge is 868 000 lb.; that on each center wedge is 1 528 000 lb. The center disks are 52 in. in diameter. It is believed that heavier spans of this type are practicable. The present longest and heaviest swing bridge in the United States is the double-track, double-deck railway and highway bridge of the At. T. & S. F. Railway System across the Mississippi River at Fort Madison, Iowa. The span is 525 ft. and the weight of the swinging parts is 10 000 000 lb. It is a combination rim- and center-bearing structure.

Center-bearing swing bridges, when closed and ends raised ready for the passage of the live load, are continuous girders over three points of support.

Rim-bearing swing bridges are nearly always built as partially continuous girders over four points of support. The diagonals in the panel between the

Long American Movable Bridges

Type	Location	Nature of crossing	Span Ft.	Span In.	Railroad or highway	Built
Swing	St. John's, Ore.	Willamette R.	521	0	R.R.	1908
	East Omaha, Neb.	Missouri R.	520	0	R.R.	1893
	East Omaha, Neb.	Missouri R.	519	4-1/8	R.R.	1903
	New London, Conn.	Thames R.	497	7	R.R.	1889
	Staten Island	Arthur Kill	496	6	R.R.	1888
	Duluth, Minn.	St. Louis Bay	485	7-1/2	R.R. Hy.	1897
	C. M. & N. R.R., Chicago	Drainage Can.	474	3-1/2	R.R.	1899
	Sioux City, Iowa	Missouri R.	469	11-3/8	R.R. Hy.	1895
	Willamette Pacific R.R.	Coos Bay	458	0	R.R.	
	Middletown, Conn.	Connecticut R.	447	0	Hy.	1896
	Sacramento, Calif.	Sacramento R.	390	3	R.R.	1911
	New York, N. Y.	Harlem R.	389	0	R.R.	1895
Scherzer lift	Terminal Ry., Chicago	Chicago R.	275	0	R.R.	1901
	B. & O. R.R., Cleveland	Cuyahoga R.	230	0	R.R.	1907
	22d St., Chicago	Chicago R.	216	0	Hy.	1906
Vertical lift	Fratt Br., Kansas City	Missouri R.	428	0	R.R. Hy.	1911
	Cincinnati, Ohio	Ohio	365	0	R.R.	1922
	Chattanooga, Tenn.	Tennessee R.	310	0	R.R.	1920
	G. N. R.R., Mont.	Missouri R.	296	0	R.R.	1914
	Portland, Ore.	Columbia R.	275	0	Hy.	1916
	Pa. R.R., Chicago	Chicago R.	272	10	R.R.	
	Fairview, Mont.	Yellowstone R.	271	0	R.R.	1814
	Yankton, S. Dak.	Missouri R.	250	0	R.R. Hy.	1924
	Hawthorne Ave., Portland	Willamette R.	244	3	Hy.	
	Pine Bluff, Ark.	Arkansas R.	239	4	R.R. Hy.	1916
	Portland, Ore.	Willamette R.	220	0	R.R.	1916
Trunnion bascule	Saulte Ste. Marie	Ship Canal	336	0	R.R.	1914
	Chattanooga, Tenn.	Tennessee R.	310	0	Hy.	1917
	*16th St., Chicago	Chicago R.	260	0	R.R.	1918
	Wells St., El. R.R., Chicago	Chicago R.	268	0	Hy. St. Ry.	1921
	Mich. Blvd., Chicago	Chicago R.	256	0	Hy.	1920
	Portland, Ore.	Willamette R.	252	0	Hy.	1926
	Lake St., Chicago	So. Chicago R.	245	0	Hy. El.	1916
	Fremont Ave., Seattle	Lake Washington Canal	242	0	Hy.	1917
	*South Chicago	Calumet R.	235	0	R.R.	1914
	Rio Vista, Calif.	Sacramento R.	226	0	Hy.	1919
	Isleton, Calif.	Sacramento R.	226	0	Hy.	1922
	Troy–Cohoes, N. Y.	Hudson R.	224	0	Hy.	1924
	Badger Ave., Los Angeles	East Basin Channel	220	0	Hy.	1924
	15th Ave., Seattle	Lake Washington Canal	218	0	Hy.	1917
	East Lake Ave., Seattle	Lake Washington Canal	218	0	Hy.	1919
	*Seattle, Wash.	Salmon Bay	206	7	R.R.	1914

* Single leaf; other bascule bridges not marked, double leaf.

two central supports are omitted, or are very light, thus allowing moment but no shear to be transmitted across this panel.

Drawbridge Trusses whether center or rim-bearing may have their ends simply supported, latched, or raised. The **ends** are **simply supported** when the end supports are so placed that the ends of the trusses will just come to a bearing without producing any dead load reactions and the entire dead weight is carried by the center pier either when the bridge is open or closed. The live load can thus produce upward end reactions only, and when it is in such a position as to cause a downward end reaction the truss will rise from the bearing, and, in such cases, when the live load passes over the bridge the ends hammer on the bearings. This is not a good arrangement. To prevent this the **ends** should either be **latched** or **raised** by suitable machinery until upward end reactions are created equal to the greatest possible downward live load reaction. Efficient latches are complicated and it is the general practice to raise the ends until the upward dead load reaction is equal to 1.5 times the maximum negative live load reaction when one arm is loaded.

34. Design of Movable Bridges

The determination of stresses in continuous swing bridges is based on the usual method for continuous beams but the members have also to be designed for the loads when the ends of the bridge are swung off the support. The extra height of truss over center supports is owing to this latter condition. Trunnions and pivots offer special problems in bearing and friction.

For a full explanation see Treatise on Movable Bridges, by Otis E. Hovey, Wiley, 1927.

Notation. W = equals total uplift of swing bridge at both ends in pounds, HP = total horsepower required for raising ends of swing bridge, h = total lift in feet, t = time for lifting in seconds, R = radius of center line of track in feet, t_1 = time for opening in seconds, W_1 = weight on rollers under drum of rim-bearing or total revolving weight on pivot of center-bearing bridge in pounds, r = radius of pivot in feet, b = width and l = length of bridge in feet.

Machinery for Operating a Swing Bridge includes that necessary for turning the bridge and for lifting and lowering the ends of the truss or girders. All swing bridges should be so arranged that they can be operated by hand power, and all but the smallest that are opened infrequently should be provided with motive power. The motor may be a steam engine, an electric motor or a gasoline engine. Where electric current is available the electric motor is the simplest and most economical, for it takes up little room, is easily installed and the cost of maintenance is low. For the larger bridges where electric current is not available the steam engine gives the most satisfactory power, especially if the bridge is to be opened frequently, but with light spans and infrequent openings the gasoline engine is best.

Turning Machinery. Whatever form of power is used for turning the bridge it is usually applied through a vertical shaft which is attached to the drum of a rim-bearing turntable and to some girder or floorbeam of the center-bearing type. On the lower end of the shaft is a pinion engaging in the circular rack which is attached to the pivot pier or to the lower track on which the turntable revolves. The rotation of the shaft causes the bridge to revolve. Electric motors are usually set on top of the drum or on a platform projecting from the side of the drum, and they are usually of the railway type, series wound and waterproof. The armature speed varies in different bridges from about 450 to 650 r.p m. and generally should not exceed 600 r.p.m. The controllers for the motors should be placed in the operator's house, together with the switchboard and necessary switches, meters, circuit-breakers and fuses. Friction brakes are used for most movable bridges, although where the motor is close to the rack of swing bridges they are not absolutely necessary but even here are desirable. When two motors are used, as in most heavy bridges, they are placed on opposite sides of the drum. Fig. 118 shows the turning mechanism for the swing span of the Charlestown

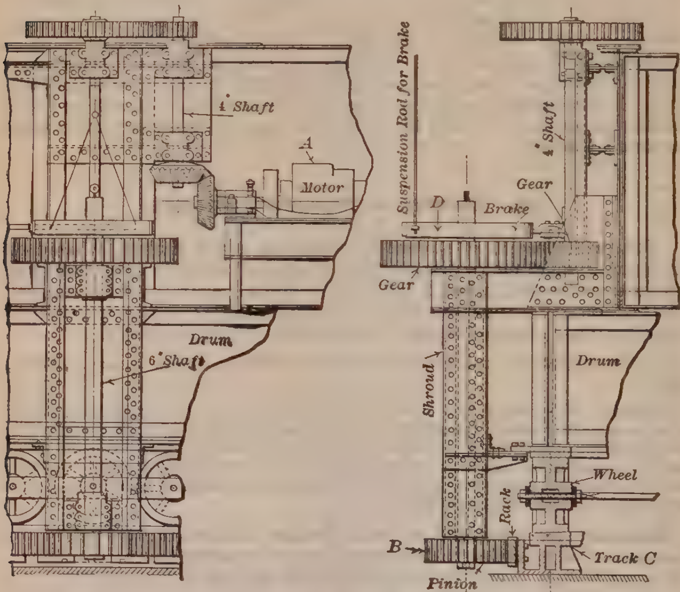


Fig. 118. **Turning Mechanism, Charlestown Bridge, Boston. Mass.**

bridge in Boston, Mass., which has two trains of gears driven by electric motors, one of which is shown. The motor *A* is attached to a steel supporting frame and drives the reduction gearing which transmits the power through the pinion shaft to pinion *B* engaging the circular rack bolted to the base casting or track *C*. A brake *D* is attached to each pinion shaft and is operated by compressed air.

End-Lifting Mechanisms. Swing bridges require some form of end-lifting device which will lift the ends of the trusses or girders when the bridge is closed and which lowers them when the bridge is to be opened. Lifting may be done in small bridges by allowing the ends to run up on wheels which are mounted on the fixed piers, but for ordinary cases some better arrangement, such as **toggle joints** or **hydraulic jacks**, is necessary for raising the ends sufficiently to allow **wedges** or other similar supports to be inserted.

The present practice is to make the end lifting device as simple as possible and to depend upon wedges, operated by a toggle mechanism, both to lift the bridge and to support the loading after the ends are lifted.

Figs. 119 and 119*a* are reproduced, with the author's permission, from Hovey's Movable Bridges. Fig. 119 shows a diagrammatic sketch of the principle of the end

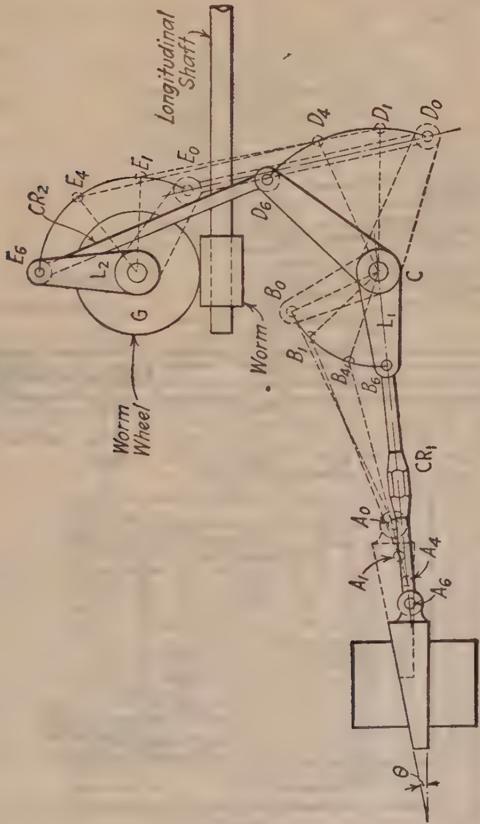


Fig. 119

wedge operation. The longitudinal shaft may run to the center of the bridge, but with electrically operated bridges a motor is placed very near the end lifting mechanism. Fig. 119*a* shows the end wedging mechanism of the Kalan bridge, Columbia River Crossing of the Union Pacific System. The upper operating lever in section *A–A* is a crank which will pass its dead center and withdraw the wedges if the motor is not stopped when the wedges are fully driven. The bridge is a single-track center-bearing swing span, 280 ft. between end supports. The center has two disks, 24 in. in diameter, the lower of hard steel and the upper of hard bronze. The turning and wedging motors have a normal capacity of 35 hp., and there is a gasoline auxiliary with a nominal rating of 53 hp.

Rail-lifts for railroad bridges are generally provided to raise the ends of

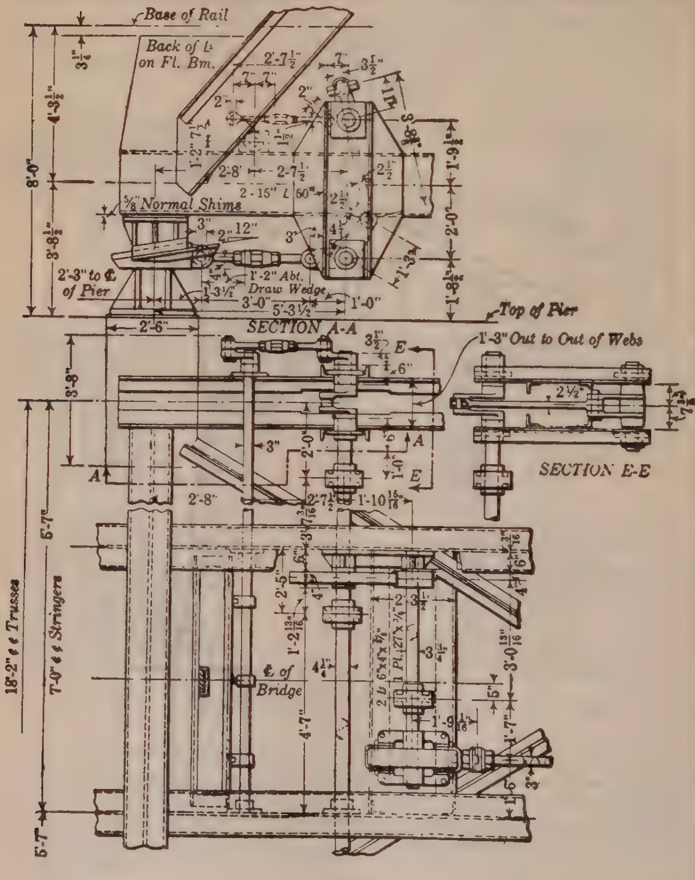


Fig. 119a

the rails on the swing span so that as the bridge revolves they will be above the rails on the adjoining fixed spans. This is usually done by means of shafting connected with the end-lifting device, and there should be automatic home and distant **signals** on each side of the bridge interlocked with the end-lifting mechanism and rail lifts so that the signals can be set at safety only when the ends of the bridge and the rails are in their proper positions. Rail-lifts are objectionable because the rails are not properly held during the passage of trains, and those forms of **rail-locks** which insure proper alignment of rails and allow the rails to be securely fastened to the track are better. **End latches** to bring the bridge into proper alignment when closed should be arranged to work automatically but only when the ends of the bridge are slowly approaching the supports.

The Power Required for operating a swing bridge is that for lifting the ends and for turning the bridge. These two operations are not performed at the same time, so that the maximum power necessary is the greater of the two, not their sum. To **lift the ends** the resistance varies from zero at the beginning to a maximum at the end of the operation, and the total horsepower required, excluding the frictional resistances of shafting and all other movable parts of the end-lifting mechanism, is $\mathbf{HP = Wh/550\ t}$. In this formula W must be taken as the total end uplift; for example, if the uplift at each end of a truss is 30 000 lb. and there are two trusses, W would be 120 000 lb. The frictional resistances to be overcome in lifting the ends vary greatly and are estimated at from 100 to 250% of the above value of P. If friction at 100% is allowed, then the horsepower required for lifting and overcoming frictional resistances in the lifting mechanism is $\mathbf{Wh/275\ t}$.

In Turning the Bridge the resistances to be overcome are those due to friction, unequal wind pressure and inertia of the mass. The frictional resistance in a rim-bearing bridge is caused by friction between the wheels and the upper and lower tracks, also at the axles of the wheels and at the center pivot. The **coefficient of the total friction** for rim-bearing bridges was found by C. Shaler Smith to be from 0.004 to to 0.008 of the load on the wheels; by Boller and Schumacher to be 0.0035 for the Thames River bridge in Connecticut, and by Theodore Cooper to be 0.0038 for the Second Avenue bridge in New York. In center-bearing structures the friction is principally at the center pivot, and the coefficient of frictional resistance at the circumference of the pivot was found by C. Shaler Smith to be 0.09 of the weight turned; and by C. C. Schneider to be 0.067 at the start and 0.045 to maintain motion at uniform speed for a bridge with a center-bearing of hardened steel and phosphor-bronze disks carrying a pressure of 3000 lb. per sq. in. The last-named engineer also found the highest coefficient of total friction on new bridges, including that of shafts and gearing for hand operation, to be 0.115 for starting and 0.08 for maintaining motion. The resistance due to an **unbalanced wind pressure** of from 4 to 5 lb. per square foot of surface of one arm of the draw is sometimes allowed for in determining the power required, and sometimes this is provided for by using larger coefficients of friction than those given above. To overcome the inertia of the bridge and to accelerate the motion the power required depends on the time allowed for opening the bridge. The motion may be accelerated during the first half and retarded during the last half of the movement, as is usual; or it may be accelerated during the first third, then maintained at a uniform velocity for the second third, and retarded for the last third.

The Horsepower required for overcoming all frictional and wind resistances of a rim-bearing bridge while turning is given approximately by $\mathbf{WR/11671\ t_1}$, in which the coefficient of resistance is assumed to be 0.015. And for a center-bearing bridge while turning with an assumed coefficient of resistance of 0.15 the horsepower is $\mathbf{W_1 r/1167\ t_1}$. The horsepower required for overcoming inertia is approximately $\mathbf{W_1(b^2 + \mathit{l}^2)/10\,767\ t_1{}^3}$. In the three formulas just given it is assumed that the motion is accelerated during the first half and retarded during the last half of the time of swinging; and in the first two formulas the force is applied at the center of the track and in the last it is applied at the center of gyration.

35. Cantilever Trusses

Historical. Bridges which have cantilever, that is, projecting, arms are called cantilever bridges and the principle involved in their construction is very old. The first European cantilever structure of note was a highway bridge of 124-ft. span designed and built by Gerber in 1867 over the river Main at Hassfurt, and the first cantilever railroad bridge was a span of 148 ft. completed in 1876 over the Warthe at Posen. **In America the first iron cantilever** bridge was the Kentucky River bridge of the Cincinnati Southern R.R., built in 1876–7 by C. Shaler Smith of the Baltimore Bridge Co. The Michigan Central R.R. bridge over Niagara River was the second important

Longest American Cantilever Bridges

Span		Crossing	Location	Railroad tracks	Highway	Deck or Through	Date of completion
Ft.	In.						
1800	0	St. Lawrence River	Quebec, Ont.	2	.....	Through	1917
1182	0	East River	*Blackwell's Id., N. Y.	.....	*	"	1909
1100	0	Carquinez Straits	San Francisco, Calif.	.....	*	"	1927
1097	0	Montreal Harbor	Montreal, Quebec	.....	*	"	1927
812	0	Monongahela River	Pittsburgh, Pa.	2	.....	"	1904
790	5	Mississippi River	Memphis, Tenn.	1	*	"	1892
790	5	Mississippi River	Memphis, Tenn.	2	*	"	1915
775	0	Ohio River	†Sciotoville, Ohio	2	.....	"	1918
769	0	Ohio River	Beaver, Pa.	2	.....	"	1910
750	0	Outer Bridge Crossing	Staten Island, N. Y.	.....	*	"	1928
750	0	Ohio River	Sewickley, Pa.	.....	*	"	1910
725	0	Ohio River	Ironton, Ohio	.....	*	"	1922
700	0	Ohio River	Mingo Junction, Ohio	2	.....	"	1904
672	0	Elizabeth, Howland Hook	Staten Island, N. Y.	.....	*	"	1928
671	0	Mississippi River	Thebes, Ill.	2	.....	"	1905
670	0	Allegheny River	Pittsburgh, Pa.	.....	*	"	1927
660	0	Colorado River	Red Rock, Calif.	1	.....	"	1890
650	0	Ohio River	Marietta, Ohio	.....	*	"	1903
555	9	Ottawa River	Ottawa, Ont.	1	*	"	1900
525	0	Long Lake	Hamilton Co., N. Y.	.....	*	"	1901
523	0	Hudson River	Poughkeepsie, N. Y.	2	.....	Deck	1888
520	3	Allegheny River	†Bessemer, L. E. R.R.	2	.....	"	1918
520	0	Kentucky River	Tyrone, Ky.	1	.....	"	1889
520	0	Ohio River	Cincinnati and Newport	.....	*	Through	1891
516	3	Ohio River	†Cincinnati, Ohio	2	.....	"	1921
483	0	Ohio River	Louisville, Ky.	1	*	"	1886
480	0	Mississippi River	Burlington, Iowa	.....	*	"	1917
480	0	Kanawha River	Point Pleasant, W. Va.	1	.....	"	1888
477	0	St. John River	St. John, N. B.	1	.....	"	1885
450	0	Allegheny River	Highland Park, Pittsburgh	.....	*	"	1900
442	0	Mississippi River	Muscatine, Iowa	.....	*	"	1889
420	0	Mississippi River	Clinton, Iowa	.....	*	"	1892
420	0	St. Lawrence River	Cornwall, Ont.	1	.....	"	1899
413	0	Allegheny River	Bet. Reno and Oil City, Pa.	.....	*	"	1902

* Known as Queensboro bridge. † Continuous bridge.

American cantilever and was completed in 1883. The bridge of this type having the longest span in the world at the time of its construction in 1918 is the Quebec bridge, which has a span of 1800 ft. center to center of river piers. The Quebec bridge which collapsed Aug. 29, 1907, during construction also was being built with a span of 1800 ft. Until the successful construction of the present Quebec bridge, the largest bridge of the cantilever type and the one having the longest span was the Forth bridge in Scotland, which has among others two spans each of 1700 ft. center to center of bearings.

The Modern Cantilever Truss is the outgrowth of the continuous truss, the change being brought about by the use of hinges which fix the points of contraflexure in the cantilever truss, thus removing some of the uncertainties in the calculations. The ordinary cantilever bridge, Fig. 120, of three spans on four supports consists of two **anchor arms,** one at each shore, two **cantilever arms,** and one **suspended span** and the suspended span is supported at the outer ends of the cantilever arms, whereas for bridges of great lengths, one or more intermediate spans, Fig. 122, are used in combination with some of the parts just mentioned. An **intermediate span** is one which extends unbroken between two piers and beyond which it projects to form a cantilever arm in

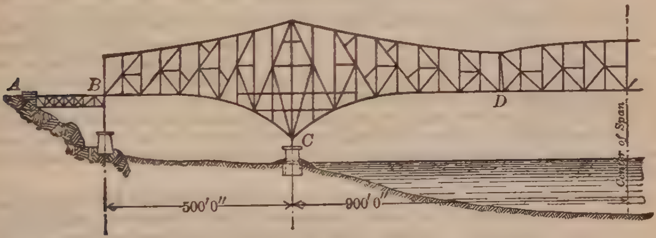


Fig. 120. Quebec Bridge, 1907

each of the adjoining spans. A **hinge** is a junction between a cantilever arm and a suspended span where the bending moment is zero after the bridge is complete, and for trusses is usually made by connecting the two portions by a pin at one chord and either omitting the opposite chord bar of the truss or arranging it so that it cannot carry stress after the bridge is self-supporting. Hinges can transmit shear but no moment after the bridge is complete, but during erection they are arranged to carry both. In **plate girder cantilever** bridges links connecting the suspended span to the cantilever arm have been used as hinges. Cantilever bridges have the advantage over simple trusses that the span containing the cantilever arm and suspended span can be erected without falseworks by being built out from the adjacent spans piece by piece, and the advantage over continuous trusses that the stresses are more accurately determined and are not altered by any reasonable settlement of the foundations.

Typical Trusses of five large American cantilever structures are shown in Figs. 120–124. Fig. 120 shows one half side elevation of the Quebec bridge which collapsed on account of the failure of the lower chord in the anchor arm, while being built in 1907 over the St. Lawrence River about 6 miles above Quebec. It consisted of two trusses 67 ft. between centers, each having two anchor arms *BC*, two cantilever arms *CD* and one suspended span hung from the cantilever arms at *D*. At each end was a short approach span *AB*. The structure was designed to carry two railroad tracks, two electric railway tracks, two highways and two sidewalks. Clearance above highest tide 150 ft.; maximum depth of water about 180 ft. Main trusses were pin-connected with pins as large as 12 in. in diameter and eyebars as wide as 18 in. Depth of truss: suspended span, at center 130 ft., at hinge D about 97 ft.; anchor arm, at end B about 97 ft., at pier C 315 ft. Panel lengths: anchor arm, 10 at 50 ft.; cantilever arm, 10 at 56 ft. 3 in.; suspended span, 12 at 56 ft. 3 in.

The New Quebec Bridge, Fig. 121, is built at the same site and has the same channel span, 1800 ft. center to center of river piers, as the bridge which collapsed. The length of the main structure is 2830 ft. and the total length, face to face of abutments is

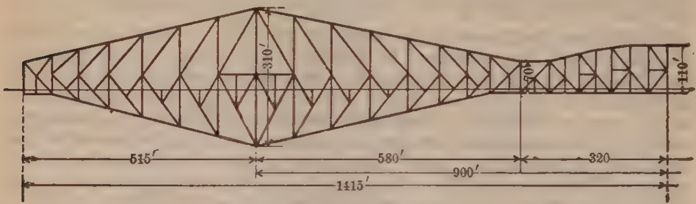


Fig. 121. Quebec Bridge, 1917

3239 ft. The depth of truss at the river pier is 310 ft.; at the ends of the anchor and cantilever arms, 70 ft.; at the center of the suspended span, 110 ft. The main structure consists of two trusses of the K type, 88 ft. between centers. It carries two railroad tracks and two 3-ft. walks, one on the side of each track. It is designed for a live load consisting of two E–60 engines and 5000 lb. per lin. ft. on each track. Main trusses are pin-connected with the pins varying from 8 in. to 30 in. in diameter and maximum width of eyebars 16 in. There are thirty-two 16 in. by 2-3/16-in. eyebars, with a net cross-sectional area of 1120 sq. in., in the top chord panel adjoining the vertical post over the river pier.

The Thebes Cantilever Bridge, Fig. 122, crosses the Mississippi River at Thebes, Ill., and consists of a steel structure of five spans with several approach spans of concrete arches at each shore. One-half the steel structure in shown in Fig. 122. The

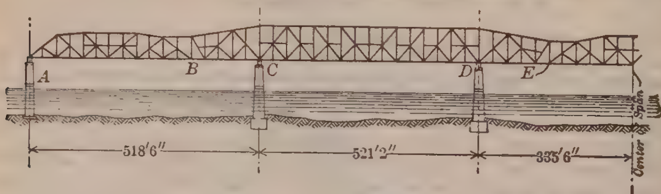


Fig. 122. Thebes Bridge

suspended span AB is supported on pier A and on the cantilever arm BC. Span CD is an intermediate span and DE is another cantilever arm which supports the central suspended span. Clearance above high water 65 ft., above low water 103 ft. There are two lines of trusses 32 ft. on centers carrying two railroad tracks. Each of the three suspended spans is 366 ft. long, each of the four cantilever arms 152 ft. 6 in. long and each of the two intermediate spans is 521 ft. 2 in. center to center of bearings. Depth of truss: suspended span, at center 55 ft.; cantilever arms at B and E 50 ft.; intermediate spans, 27 ft. throughout.

The New Memphis Bridge is located about 200 ft. upstream from the structure built in 1892. It has the same length of channel span as the older bridge, 790 ft. 5 in. In arrangement of spans, outline and type of truss it is very similar to the portion of the Thebes bridge shown in Fig. 122. But whereas the latter is symmetrical about the center of the channel span, the Memphis bridge terminates with a 186-ft. anchor arm on the right of the channel span. The span lengths are, beginning at the left, Fig. 122;

suspended span, 417 ft. 9-3/4 in.; cantilever arm, 186 ft. 3-3/4 in.; fixed span, 621 ft.; cantilever arm, 186 ft. 3-3/4 in.; suspended span, 417 ft. 9-3/4 in.; cantilever arm, arm, 186 ft. 3-3/4 in.; anchor arm, 186 ft. 3-3/4 in. Total length of main structure, not including a 345-ft. simple approach deck span, is 2201 ft. 10-1/2 in. It consists of two lines of trusses, 32 ft. between centers, carrying two railroad tracks and two 14-ft. cantilever highways. Superstructure is 76 ft. in the clear above high water level. Depths of truss are: at center of suspended spans, 67 ft.; at ends of cantilever arms, 55 ft.; throughout fixed span, 88 ft. It was designed for Cooper's E–50 loading on each of the railroad tracks, and a 17-1/2-ton road roller and 100 lb. per sq. ft. on the highways. Maximum diameter of pin is 20 in. and width of eyebars is 16 in.

The Wabash R. R. Bridge over Monongahela River at Pittsburgh, Pa., is a cantilever bridge 1504 ft. long exclusive of the steel viaduct approach on each shore. Fig. 123 shows one-half of the main structure which consists of two trusses 32 ft. on centers carrying two railroad tracks spaced 13 ft. on centers. The anchor arm AB is 346 ft. long, the cantilever arm BC 226 ft., and the suspended span, which is supported at

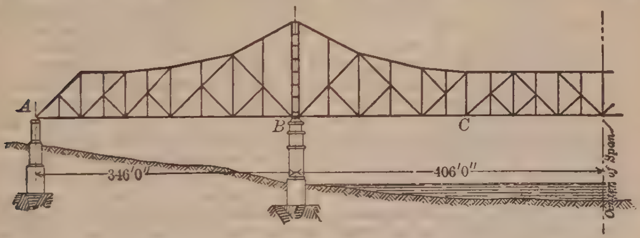


Fig. 123. Wabash R.R. Bridge, Pittsburgh, Pa.

C, is 360 ft. in length. Depth of truss: at portals, 60 ft.; suspended span, 60 ft. throughout; at main piers, 126 ft. 6 in. The tracks are on a 1% grade, so that the clearance under the central span near one pier is 70 ft., and near the other is 77.86 ft. above "full pool" level, which is below extreme high water. Panel lengths vary from 30 to 40 ft. Eyebars are from 12 to 14 in. wide, the latter having heads 33 in. in diameter. Pins 12 and 14 in. in diameter.

Queensboro or Welfare Island Bridge spans the East River and Welfare Island at New York and is a cantilever bridge on six masonry piers, two on each shore and two on the island. Fig. 124 shows the outline of that part of truss from the center of the island span at A to the end E in Borough of Queens. The total length of the cantilever

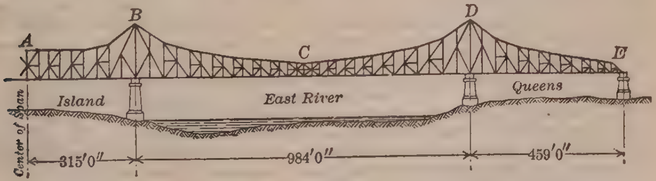


Fig. 124. Queensboro Bridge

portion of the bridge is 3724 ft. 6 in., divided into an intermediate span of 630 ft. on the island, two cantilever arms each of 492 ft. on the east, two cantilever arms each of 591 ft. on the west, one each anchor arm of 459 ft. and one west anchor arm of 469 ft. 6 in. Longest span 1182 ft. Depth of truss over each of the four intermediate piers such as B and D, 185 ft. There are two trusses 60 ft. on centers, made partially continuous by having the two cantilever arms joined together by rockers at C, thus making the structure statically indeterminate.

36. General Data for Cantilevers

Live Loads on Large Bridges. Firth of Forth: 2240 lb. per lin. ft. for each of two tracks. **Memphis**: 4000 lb. per lin. ft. per track; single track. **New Memphis**: Cooper's E–50 loading per track, 17-1/2-ton road roller and 100 lb. per sq. ft. on highways. **Wabash R.R.** over Monongahela River at Pittsburgh: two consolidation locomotives followed by 4500 lb. per lin. ft. for each of two tracks. **Thebes**: for floor system, one concentrated load of 50 000 lb. followed by 5000 lb per lin. ft. for each of two tracks; for trusses 20% less. **First Quebec**: trusses in general, (a) on each steam railway track 3000 lb. per lin. ft. for unlimited length, or (b) on each steam railway track two Cooper's Class E–33 locomotives followed by a train of 3300 lb. per lin. ft., total length of train 900 ft., or (c) on each steam railway track one Cooper's Class E–40 locomotive followed by a train of 4000 lb. per lin. ft., total length of train 550 ft.; no loading on electric railway tracks, roadways or sidewalks; hangers, subdiagonals and floor system, (a) on each steam railway track two Cooper's Class E–40 locomotives followed by a train of 4000 lb. per lin. ft., and (b) on each electric railway track 50 000 lb. on two axles 10 ft. apart with 20 ft. to leading axle of car following, and (c) on each roadway 24 000 lb. on two axles 10 ft. apart. **New Quebec**: two Cooper's E–60 engines and 5000 lb. per lin. ft. on each track. **Queensboro**: main trusses, original specifications for "congested" load of 12 600 lb. per lin. ft. of bridge as follows, two elevated railway tracks at 1700 lb. per lin. ft. each, four trolley tracks at 1000 lb. per lin. ft. each, 35.5-ft. roadway at 100 lb. per sq. ft. or 3550 lb. per lin. ft. of bridge, two 11-ft. sidewalks at 75 lb. per sq. ft. or 1650 lb. per lin. ft. of bridge, and a "regular" live load of 6300 lb. per lin. ft.; two elevated railway tracks added later increased original truss live loading from 12 600 to 16 000 and from 6300 to 8000 lb. per lin. ft. of bridge for "congested" and "regular" loadings respectively; floor systems and secondary trusses, on each elevated railway track cars of four axles spaced 6–10–6 ft. with 26 000 lb. per axle, on each street car track cars of two axles spaced 10 ft. with axle load of 26 000 lb. or 1800 lb. per lin. ft. of track, on any part of roadway 48 000 lb on two axles 10 ft. apart and 5-ft. gage covering a space 12 ft. by 30 ft. and on the remaining roadway surface 100 lb. per sq. ft., on the sidewalks 100 lb. per sq. ft. When nearly completed the Queensboro bridge was found to be too weak to carry the above loads, so that it is not being used as originally planned.

Unit Stresses in Pounds per Square Inch specified for some large cantilever bridges. **Firth of Forth**: maximum compression 17 000; maximum tension 16 350; ultimate tensile strength of steel 67 000 to 74 000 and ultimate compressive strength of 76 000 to 83 000. **Monongahela River** at Pittsburgh: dead load compression 21 000 where $l/r < 40$; dead load tension 22 000; live load stresses one-half these values; ultimate tensilve strength for eyebar steel 63 000 to 73 000 and for plates and angles 60 000 to 70 000; shearing on rivets 10 000 for rivet steel of ultimate tensile strength of 52 000 to 63 000. **Queensboro**: compression for structural steel under ordinary loads $20\,000 - 90\,l/r$, and for congested loads $24\,000 - 100\,l/r$; tension for nickel steel eyebars under ordinary loads 30 000 and under congested loads 39 000; tension for structural steel in main members of trusses, towers and bracing 20 000 for ordinary and 24 000 for congested loads respectively; shear on shop rivets 13 000 and 16 000 for ordinary and congested loads; full sized annealed nickel steel eyebars up to a maximum size of 16 in. × 2.5 in. were required to show a minimum elastic limit of 48 000, a minimum ultimate tensile strength of 85 000 and an elongation of 9% in 18 ft.; open-hearth structural steel eyebars, minimum elastic limit 28 000 and minimum ultimate strength of 56 000 and elongation of 10% in body of bar. **First Quebec**: compression, ordinary, $12\,000\,(1 + \text{min./max.})$, extreme 24 000, both for $l/r < 50$; tension, ordinary $12\,000\,(1 + \text{min./max.})$, extreme 24 000; structural steel with ultimate tensile strength of 62 000 to 70 000.

Compression Members of large cantilever bridges in America are latticed box sections as shown in Figs. 125, 126 and 127. The Forth bridge members are hollow cylinders some of which are 12 ft. in diameter and made of plates, 1-1/4 in. in thickness, riveted together and stiffened by inside stiffening frames. Fig. 125 shows a section of a lower chord member of the ill-fated Quebec bridge, Fig. 126 of the Queensboro, and Fig. 127 of the Monongahela River bridge. These are typical sections for the three bridges respectively. The segments of the Quebec chord were connected by latticing only, and except at splices there were no diaphragms, while the segments of the Queensboro and the Monongahela chords are braced by both latticing and diaphragms. The latticing and diaphragms are not completely shown here. The largest

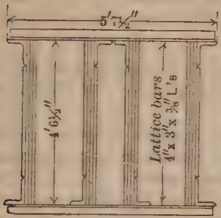


Fig. 125

Fig. 126

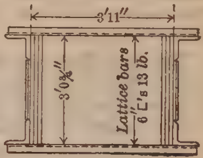


Fig. 127

cross-sectional area of any compression chord in the Queensboro bridge is 1072.1 sq. in.

Typical Compression Chords of Large Cantilever Bridges

Bridge	Area of cross-section	Sectional area of lattice *	Sectional area of lattice rivets †	Length of chord l	Radius of gyration r ‡	l/r
	Sq. In.	Sq. In.	Sq. In.	Ft. In.	In.	
Quebec (first)...	781	10	4.8	57 0-9/32	19.7	35
Memphis (old)..	213	10	4.8	28 2-3/4	14.8	23
Queensboro.....	852	25	4.8	31 6-1/8	22	17
Thebes........	189	11-1/4	4.8	30 6-5/32	20.1	18
Monongahela...	262	14-1/2	7.2	30 6-1/32	25.4	14

* Total area (measured at right angles to axis of lattice bar) of lattice bars cut by cross-section of chord. † Area of rivets connecting all lattice bars cut by a cross-section chord to outside web, one end only. ‡ Axis normal to latticing; not necessarily the least radius of gyration.

The Lower Chord, L13-L14 of the New Quebec bridge, Fig. 128, adjacent to the main pier, has a cross-sectional area of 1902 sq. in., being about 42 ft. 11 in. long and 10 ft. 3-1/4 in. wide. The depth of the lower chord diminishes in accordance with a regular taper of 1/32 in. in 12 in., from about 7 ft. 2 in. at the main shoe to 4 ft. 1-3/4 in. at the end of the anchor arm. The chord section has four vertical ribs each consisting (L 13–L 14) of 4 web plates with an aggregate thickness of 3-3/4 in.; 4 flange L's, 8 in. × 8 in. × 1 in.; 2 cover plates, 20 in. × 1-1/8 in.; 2 cover plates, 20 in. × 1-3/16 in. Each outer pair of ribs is connected at top and bottom flanges by 8-1/2 in. × 1 in. lattice bars, and at mid-height by a horizontal diaphragm consisting of 1 web plate, 33 in. × 11/16 in., and 4 flange L's, 8 in. × 8 in. × 5/8 in. The areas of these longitudinal diaphragms are included in the cross-section of the member. The webs are also strengthened transversely by cross (vertical) diaphragms about 15 ft. apart, having 10 in. × 16-in. manholes. The 8-1/2 in. × 1-in. lattice bars have 2 rows of 3 rivets each in each end and 4 rivets at the intersection. The inner pairs of ribs are tied together by vertical and mid-horizontal diaphragms and by tie plates in the planes of the flanges. The weight of L 13-L 14 is about 400 short tons. In order to facilitate shipment and erection this member was spliced at the center and each of the two sections divided longitudinally. The **Main Vertical Post** is 310 ft. between centers of end pins. Its unsupported length is 145 ft. It is composed of four separate columns latticed together, its outside dimensions being about 9 ft. by 10 ft. Its cross-sectional area is 1903 sq. in.; its weight 1500 short tons and it was shipped in 26 pieces.

The New Memphis Bridge compression chords also have four webs 42 in. deep reinforced with side plates, and 8 flange L's 8 in. × 8 in. The maximum cross-sectional area is 445 sq. in.

The Length of the Cantilever Arm for a given location may be varied considerably, but the location of the abutments and piers is generally limited by local conditions. In such a bridge as the Monongahela, Fig. 123, the central span between piers is determined by the width of the waterway, and the abutments are frequently located by other similar requirements. If the abutments are fixed, the position of the piers should be such as to make the total cost of the bridge a minimum, which will practically be when the material in the trusses is a minimum. On account of the varying live and dead loads, different shapes of trusses and different unit stresses, the relation between the length of anchor arm and the entire length of bridge between abutments can be expressed only approximately. With the general arrangement shown in Fig. 123, the anchor arm is usually about 0.20 L, the cantilever arm about 0.17 L and the suspended span about 0.26 L, where L is the total length between abutments. These values are average values only, and many cantilever structures have been built in which the ratios of lengths do not agree with the fractions given.

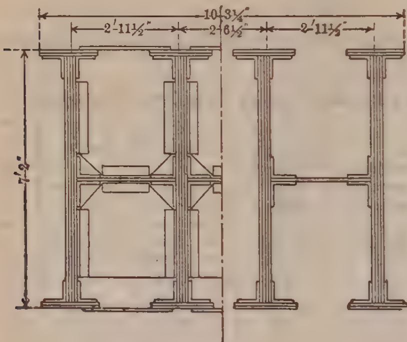


Fig. 128

Anchorages. Most cantilevers with a central suspended span require anchoring at each shore end of the bridge to produce a downward reaction on the anchor-arm truss. These anchorages usually consist of a series of eyebars attached to the end pin of the truss and extending down into the masonry where they are connected to girders. The anchorage for each end of each anchor-arm truss of the first Quebec bridge consisted of sixteen 10 in. by 2-1/16-in. eyebars connected by pins to a series of eight plate girders each 6 ft. deep and 17 ft. 6 in. long. Above these girders was another layer of girders 8 ft. 6 in. deep and 36 ft. long and above this a layer consisting of twelve 15-in. I-beams 22 ft. long. The eyebars and the grillage of beams and girders were embedded in the masonry and grouted. In several of the large cantilevers the anchorages are placed in inspection galleries.

37. Reactions of Cantilevers

Reactions may be determined by ordinary methods when the structure is statically determinate as regards the outer forces, and by deflections when statically indeterminate. A cantilever bridge having the points of application of all reactions known and having all bearing points but one arranged so that they can exert only vertical reactions on the truss is **statically determinate** as regards outer forces when the number of equations given by the manner of construction is equal to $n - 2$ where n is number of supports. Thus the reactions on truss in Fig. 129 may be found by statics because the

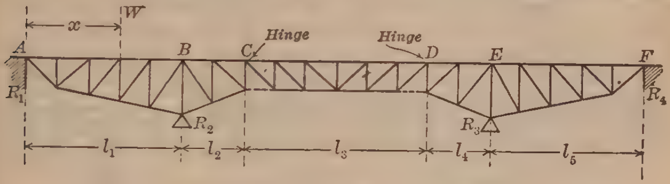


Fig. 129

number of equations given by the method of construction, that is, by the insertion of a hinge at C and D, is equal to $n - 2 = 2$. Similarly the truss in Fig. 130 is statically determinate because the two hinges, P and T, and the omission of diagonals in panel LM and the corresponding panel on right of center give the $n - 2 = 4$ equations required. Each hinge makes the bending moment at the section where it is inserted equal to zero. The dotted members in Fig. 129 and Fig. 130 receive stress only during erection.

Reactions for Fig. 129, having a central suspended span and four supports. For a load on the anchor arm AB, R_1 and R_2 are upward; for a load on the cantilever arm BC, R_1 is downward and R_2 upward; in either case R_3 and R_4 are each zero and R_1 and R_2 are as follows:

$$\mathbf{x} \leq l_1 + l_2 \quad \mathbf{R}_1 = \mathbf{W}(l_1 - \mathbf{x})/l_1 \quad \mathbf{R}_2 = \mathbf{W}\mathbf{x}/l_1$$

The suspended span is a simple truss supported at C and D, and for a load on that span the pressure on cantilever arm BC at $C = W(l_1 + l_3 + l_3 - x)/l_3$ and on cantilever arm DE at $D = W(x - l_1 - l_2)/l_3$; in this case R_1 and R_4 are downward, R_2 and R_3 are upward, and their values are as follows:

for $\mathbf{x} > l_1 + l_2$ for $\mathbf{x} < l_1 + l_2 + l_3$

$$\mathbf{R}_1 = -\frac{\mathbf{W}(l_1 + l_2 + {}_3 - \mathbf{x})l_2}{l_3 l_1} \qquad \mathbf{R}_2 = \frac{\mathbf{W}(l_1 + l_2 + l_3 - \mathbf{x})(l_1 + l_2)}{l_3 l_1}$$

$$\mathbf{R}_3 = \frac{\mathbf{W}(\mathbf{x} - l_1 - l_2)(l_4 + l_5)}{l_3 l_5} \qquad \mathbf{R}_4 = -\frac{\mathbf{W}(\mathbf{x} - l_1 - l_2)l_4}{l_3 l_5}$$

With a load on the portion DF, Fig. 129, there are reactions at R_3 and R_4 only, and if x be taken as the distance from load to F, their values are the same as for R_2 and R_1 respectively when the load is on the portion AC.

With a **uniformly distributed live load** the maximum downward R_1 (Fig. 129) and maximum downward R_4 occur for live load covering the central span B to E; maximum upward R_1 and R_4, load spans l_1 and l_5 respectively; maximum R_2, load spans l_1, l_2 and l_3; maximum R_3, load spans l_3, l_4 and l_5.

Reactions for Fig. 130, having a central suspended span and six supports, only three of which are shown. Hinges at P and T make the bending moments at those points zero; and the omission of diagonals in panel LM makes the shear in that panel zero; when the top and bottom chords in panel LM are horizontal, and therefore $M_2 = M_3$. If one of the chords in panel LM were inclined, the shear in that panel would be equal to vertical component of the inclined chord, and $\mathbf{M}_2/\mathbf{h}_2 = \mathbf{M}_3/\mathbf{h}_3$, where M_2 and M_3, h_2 and h_3 are the bending moments and heights at supports 2 and 3 respectively. For a concentrated load on the anchor arm GL reactions exist at R_1 and R_2 only, and this arm is a simple truss supported at R_1 and R_2. With the load on the cantilever arm MP distant x_1 from M there are reactions at R_1, R_2 and R_3; R_1 being downward, R_2 and R_3 upward, and their values are

$$x_1 \leq l_3 \qquad R_1 = -\frac{Wx_1}{l_1} \qquad R_2 = \frac{Wx_1}{l_1} \qquad R_3 = W$$

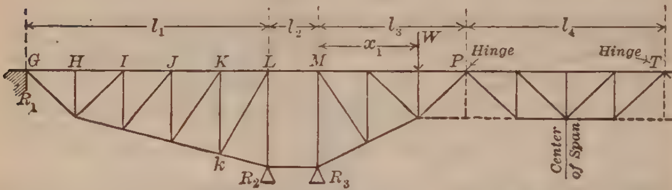


Fig. 130

For a load on the suspended span PT, reactions exist at the six supports, the end ones (at the two abutments) being downward and the four central ones (at the piers) being upward.

38. Stresses in Cantilevers

Stresses in Truss Members may be computed directly or found with aid of influence lines. Let it be required to compute maximum tension, P_{L-M}, in top chord member LM in Fig. 130, for a dead panel load of 10, all on top chord, and a live panel load of 20 thousands of pounds. Let the panel length be 20 ft.; depth of truss at L and M 40 ft., and at H 20 ft. Pass a vertical section through LM, consider forces on right of the section and write moment equation about the origin of moments, namely, R_3. The live load should be placed on all top chord panel points between M and T. The moment equation for dead loads is:

$$10(20 + 40) + 2.5 \times 10 \times 60 - 40P_{L-M} = 0,$$

and P_{L-M} for dead loads is 52.5 tension. Live $P_{L-M} = 2 \times 52.5 = 105.0$ thousands of pounds tension.

Maximum Stresses in Top Chord Member IJ (Fig. 130). Pass vertical section through IJ, consider left side of section and take origin of moments at lower chord joint 25 ft. under I. Effective dead load $R_1 = -10\ (20 + 40)/100 - 2.5\ (10)\ 60/100 + 2\ (10) = -1.0$, the minus sign denoting a downward reaction. Moment equation for dead loads about origin of moments is

$$25\ P_{I-J} - 10 \times 20 - 1 \times 40 = 0$$

from which P_{I-J} for dead loads is 9.6 tension. For live tension, load all panel points between M and T, and for live compression load joints H, I, J, K.

$$\text{Live tension, } P_{I-J} = \frac{[20\ (20 + 40) + 20 \times 2.5 \times 60]40}{100 \times 25} = 67.2$$

$$\text{Live compression, } P_{I-J} = \frac{40 \times 40 - 20 \times 20}{25} = 48.0$$

Combining the above live and dead stresses: maximum tension in $IJ = 76.8$ and maximum compression = 38.4 thousands of pounds.

Maximum Stresses in Vertical Kk (Fig. 130). Origin of moments 60 ft. to left of R_1. Effective dead load R_1 is 1.0 downward, as previously found, hence dead load causes compression in Kk. For live load in any position on the bridge there is compression in Kk. Dividing the moment above the origin by the lever arm of the bar, 140 ft.,

$$\text{Dead compression, } P_{K-k} = \frac{1 \times 60 + 10\,(80 + 100 + 120 + 140)}{140} = 31.86$$

Live compression = 2 × 31.86 = 63.72, and the maximum compression is 95.58. There can be no tension in Kk.

The Maximum Uplift at Abutment G (Fig. 130) occurs when the live load covers the central span from M to T. Under dead loads alone the effective reaction R_1 is 1.0 downward, but the total dead R_1 is really upward and is 5.0 − 1.0 = 4.0 if the dead load on the joint G be 5.0. The maximum live uplift, that is, the maximum downward live R_1, is 20(20 + 40) + 20 × 2.5 × 60/100 = 42.0, hence the true maximum uplift is 42.0 − 4.0 = 38.0.

39. Influence Lines for Cantilevers

Definition of Influence Line. If a load of unity is allowed to pass over a structure, and at each position of the load there is plotted the shear, moment, reaction or stress which exists at some fixed section of the structure, the

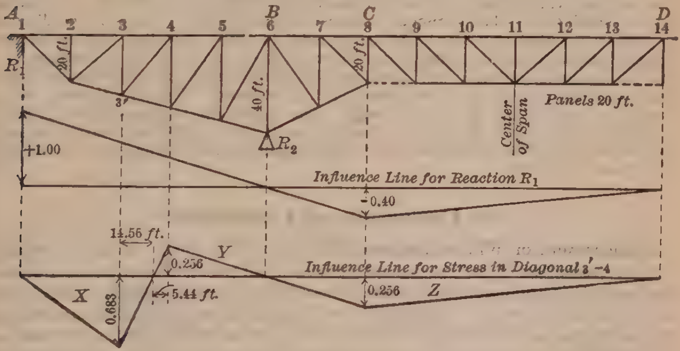


Fig. 131

resulting curve is called an influence line. These lines are useful in determining what portion of a structure should be loaded with a live load to produce the maximum or the minimum effect, and in determining the amount of the effect. For example, Fig. 131 shows the influence line for reaction R_1 and the influence line for stress in diagonal 3′–4.

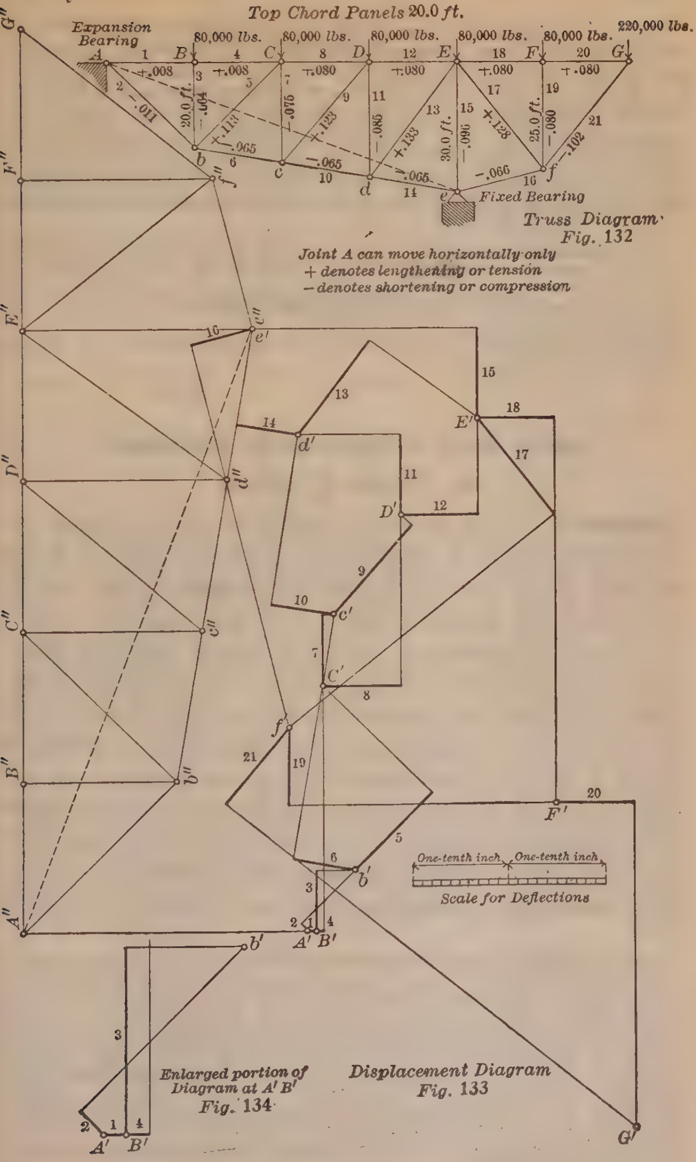
Influence Line for Reaction R_1 (Fig. 131). Let the load of unity approach from the right. There is no reaction at R_1 until the load is to the left of hinge D, and as the load moves from D to C, R_1 gradually increases until the load gets to C, when its value is − 1 × 40/100 = − 0.40. As the load moves from C to A, R_1 gradually changes from a downward force of 0.40 to an upward one of 1.00, passing through zero at B. Upward reactions are called positive and are plotted above the horizontal axis in the diagram. With a uniform dead load the value of R_1 is found from the influence line by multiplying the load per foot by the algebraic sum of the areas of the positive and negative triangles, and with a uniform live load the maximum upward or downward live R_1 is found by multiplying the live load per foot by the area of the positive and negative triangles respectively, the load in the first case being placed only over the positive triangles and in the second case over the negative triangles.

Influence Line for Stress in Diagonal 3′–4 (Fig. 131). As the load of unity approaches from the right the stress in the diagonal is zero until the load passes D, when the stress gradually increases until the load is at C. With load at C the stress is most easily found by considering forces to the left of a vertical section through the diagonal 3′–4 and taking moments about the origin, which is 60 ft. to the left of R_1. This stress is $0.40 \times 60 \times 32.02/120 \times 25 = 0.256$ tension, and is plotted at the load, namely at C. As the load moves from C to joint 4 the stress changes uniformly from tension to compression, and when it is at 4 the compression in the diagonal is 0.256 which is plotted at the load at 4. When the load crosses the panel 4–3 the stress changes from compression to tension and during the remainder of the movement to A there is tension in the bar 3′–4 as shown. Given a dead and a live load of 1000 and 2000 lb. per ft. per truss respectively, let it be required to find the maximum tension, $P_{3'-4}$, in bar 3′–4. The area of triangle X, Fig. 131, is $54.55 \times 0.683/2 = 18.63$, of Y 5.82 and of Z 20.48. The units used are the foot and the pound. For tension in 3–4 the live load must cover the distances represented by the bases of the triangles X and Z, hence live $P_{3'-4} = 2000\ (18.63 + 20.48) = 78\ 220$ lb. tension; dead $P_{3'-4} = 1000\ (18.63 + 20.48 - 5.82) = 33\ 290$ lb. tension; total tension, $P_{3'-4} = 111\ 510$ lb.

Stresses from Concentrated Loadings are most readily found from the influence lines. The latter are nearly always triangular. (Exceptions: the Queensboro Bridge and continuous bridges.) See X, Y, Z, Fig. 131. The position of the loading lying between the joints 14 and 6, producing maximum tension in the diagonal 3′–4, is the same as that which would produce the maximum moment at a point 8 in a simple span of length 6–14. The triangular influence lines enclosing the areas Y and X may be used in a similar way for the groups of loads in these respective portions of the bridge, those in X causing additional tension in 3′–4, and those in Y compression. The L.L. stress is obtained still more quickly when a table of equivalent uniform loads, such as is given for M–60, page 1171, is at hand. Thus, for the part 14–6, $l_1 = 40$, $l_2 = 120$. For M–60, $w = 8617$. The maximum tension $= 20.48 \times 8617 = 176\ 650$. For loads in X, $l_1 = 14.56$, $l_2 = 40$, $w = 12\ 000$, and maximum tension $= 18.63 \times 12\ 000 = 223\ 560$. The total tension in the diagonal 3–4 $= 176\ 650 + 223\ 560 = 400\ 210$.

40. Deflection of Cantilevers

Displacement or Williot Diagram (Figs. 132, 133, 134). This is a graphical determination of the exact displacement of all points of a truss for a given loading. It is applicable to all cases where the changes in lengths of the truss members can be found. There are three steps in the solution as illustrated by the determination of deflections of all joints of truss in Fig. 132, which shows the diagram of truss drawn to scale, the joints lettered, the bars numbered in order in which their deformations are laid off in Fig. 133 and the changes in lengths of the various bars due to the loading shown, these changes due to stress being found by dividing the actual unit stress for each bar by the modulus of elasticity. The second step is to draw the displacement diagram by beginning at any joint and assuming this joint fixed in position and one of the bars at the joint fixed in direction. Usually start with a bar which appears to have a small movement. On lines parallel to the bars in the truss diagram lay off the changes in length for the bars and erect perpendiculars to these deformations. Thus, in Fig. 133, starting with joint A as fixed in position and bar AB fixed in direction lay off to scale and to the right from A' the change $+ .008$ in. for the bar AB, thus locating $A'B'$; then from A' lay off $- .011$ in. for bar Ab, marked 2 in Fig. 133, and erect a perpendicular to this line, and similarly with bar Bb, or number 3, thus locating b', $A'b'$ showing the actual movement to scale of point b with respect to A and the fixed direction AB. Continuing in this way the displacement diagram is finally ended by locating G'. Care must be taken to lay off the deformations in proper directions; thus, bar Ab is in compression and its deformation is laid upward and to the left



Figs. 132, 133, 134. Displacements for a Cantilever from A'. This diagram has been drawn on the assumption that A is fixed, whereas e is really fixed and A can and does move horizontally. The third step then is to rotate the truss about e, the real fixed joint, till all bars in the truss in Fig. 133 are at right angles to the corresponding bars in Fig. 132. This is here done by moving A' horizontally until $A''e'$ is at right angles to Ae, then completing truss $A''G''\epsilon''$. The true displacement of any joint is shown in Fig. 133; for example, joint A has moved to the right a distance $A''A'$; E has moved to the right and downward from E'' to E'. Fig. 134 shows a part of Fig. 133 on an enlarged scale, and is here made necessary on account of the reduction in printing.

The displacement diagram not only shows the final resultant motion of each joint but it also shows the horizontal and vertical motion as well. Thus, while joint A has moved only horizontally to the right a distance equal to $A''A'$, joint B has moved vertically a distance equal to the vertical projection of $B''B'$ and horizontally a distance equal to the horizontal projection of $B''B'$.

Two scales are necessary, one of distances for the truss diagram in Fig. 132 and the other for the deflections in Fig. 133.

ARCH AND SUSPENSION BRIDGES

41. Types of Arches

Classification. An arch is a structure which under any and all loads produces inclined reactions at the supports. Arches are classified as to the number of hinges used in one rib, there being three-hinged, two-hinged, one-hinged and hingeless arches. When **three hinges** are used there is one at each support and one at the crown; when only **two hinges** are used, one is placed at each support; and if an arch has only **one hinge** it is located at the crown. **Hingeless arches** are also called fixed or continuous. Structures with only one hinge are seldom built, but the other types are common. As to the arrangement of the ribs, arches are also classified into two types, namely, those having solid webs extending between and connected to the flanges and those in which the webs are open and consisting of web members connecting the upper and lower chords in the same manner as in ordinary trusses. In ribs having **solid webs** both flanges are usually (though not always) curved, thus making the rib a curved plate girder. Ribs with **open webs** may be either spandrel-braced, in which case the upper chord is horizontal and the lower is curved, or they may have the two chords curved, either parallel or lune-shaped, and be connected by the web members, thus forming what are sometimes called trussed-arch ribs. There are, then, **spandrel-braced, trussed-arch,** and **solid ribs,** and any one of these may be hinged or fixed.

Historical and Descriptive. The **earliest metallic arch** of note in the United States is the cast-iron hingeless arch bridge carrying Chestnut St. over the Schuylkill River in Philadelphia. This bridge was completed in 1866, and has spans of 185 ft., and the ribs, bracing and floor plates are all of cast iron. The Eads bridge in St. Louis having a span of 519 ft. 9-3/8 in. and the Kaiser Wilhelm bridge over the Wupper River in Germany with a span of approximately 558 ft. are the longest hingeless arches in the United States and Europe respectively. **Longest three-hinged arch-bridge** in the United States is the 618-ft. span at the Grand Canyon, Ariz. **Longest arch span** is the two-hinged highway bridge across the Kill van Kull, Staten Island, N. Y., with a span of 1675 ft. **Longest three-hinged arches** for supporting roofs were used in the Manufactures and Liberal Arts Building, Columbian Exposition, Chicago, 1893, the span being 368 ft.

42. Two-hinged Arches

Notation. Δx and Δy, horizontal and vertical movements respectively of origin of coordinates of a curved beam: $\Delta\phi$, angular movement of axis of curved beam at origin of coordinates. M, bending moment of vertical forces at point on arch axis (gravity axis), x and y, coordinates of a point on arch axis. ds, elementary length of arch axis. E, modulus of elasticity of material. I, moment of inertia of cross-section of rib. I_0, moment of inertia of cross-section of rib at crown. c, coefficient of linear expansion, t, change in temperature in degrees Fahrenheit from normal. S, average value of compressive unity stress. L, length. A, area of cross-section of bar. U, stress in bar due to load of unity acting horizontally at one hinge of braced arch. P, stress in bar of braced arch due to vertical loads and vertical reactions.

Important American Arch Bridges

Span	Nature of crossing	Location	R.R. or Hy.	Number of hinges	Date of completion
Ft. In.					
1675 0	Kill Van Kull.....	Staten Island, N. Y........	Hy.	2	1932
977 6	East River........	Hell Gate, N. Y...........	R.R.	2	1917
840 0	Niagara River....	Niagara Falls, N. Y........	Hy.	2	1898
640 0	Niagara River....	Niagara Falls, N. Y........	R.R.	2	1924
618 0	Colorado River....	Grand Canyon, Ariz........	Hy.	3	1928
592 0	Colorado River....	Topock, Ariz..............	Hy.	3	1916
591 0	Cuyahoga River...	Cleveland, Ohio...........	Hy.	3	1917
550 0	Niagara River....	Niagara Falls, N. Y........	R.R.	2	1897
540 0	Connecticut River.	North Walpole, N. H.......	Hy.	3	1905
535 0	Willamette River..	Portland, Ore.............	Hy.	3	1926
519 9-3/8	Mississippi River..	St. Louis, Mo.............	R.R. Hy.	0	1874
508 9-5/8	Harlem River.....	Washington Bridge, N. Y...	Hy.	2	1889
501 8-1/8	Mississippi River..	St. Louis, Mo.............	R.R. Hy.	0	1874
456 0	Mississippi River..	Minneapolis, Minn........	Hy.	3	1889
448 8-1/4	Rio Grande River.	Costa Rica...............	R.R.	2	1902
440 0	Pittsburgh Jn. R.R.	Oakland, Pittsburgh, Pa....	Hy.	0	1907
428 0	Genesee River....	Driving Park Ave., Rochester	Hy.	3	1890
423 6	Niagara River....	Buffalo–Ft. Erie..........	Hy.	3	1927
400 0	Whitewater River.	Richmond, Indiana........	Hy.	3	1886
360 0	Magdalena River..	Honda, U. S. of Colombia...	R.R.	3	1884
360 0	Gulley and 3 streets	Hawk St., Albany, N. Y....	Hy.	3	1890
360 0	Panther Hollow...	Schenly Park, Pittsburgh...	Hy.	3	1898
355 0	Oak Orchard Creek	Main St., Waterport, N. Y..	Hy.	2	1900
340 0	Fraser River......	Lillooet, B. C............	Hy.	3	1888
336 0	Stony Creek......	Near Bear Creek Sta., B. C..	R.R.	3	1893
330 8	Crooked River.....	Oregon...................	Hy.	2	1926
327 0	Papolopen Creek..	Palisade Interstate Park, N.Y.	Hy.	2	1916
290 4-1/2	Surprise Creek....	Near Bear Creek Sta., B. C..	R.R.	3	1897
270 0	Salmon River.....	Near Keefers Sta., B. C....	R.R.	3	1893
258 0	Mississippi River..	Hennepin Ave., Minneapolis	Hy.	3	1888
258 0	Mississippi River..	Hennepin Ave., Minneapolis	Hy.	2	1891
240 5	Spokane River....	Post St., Spokane, Wash...	Hy.	3	1893
240 0	Spokane River....	Post St., Spokane, Wash...	R.R.	3	1903
240 0	Canyon..........	Near Skagway, Alaska.....	R.R.	3	1901
234 0	Six Mile Creek....	Stewart Ave., Ithaca, N. Y..	Hy.	3	1896
216 0	Salmon River.....	Pulaski, N. Y.............	Hy.	3	1888
210 6-7/8	Mahoning River...	Youngstown, Ohio.........	Hy.	2	1899

Arches with Two Hinges have less deflection at the crown due to changes in temperature, and are more rigid than those having three hinges, but the temperature stresses are large and any horizontal movement of one support produces stress throughout the arch. The two hinges are placed at the supports, and the arch may be spandrel-braced or may have two or more ribs with solid or open webs. The two arches over Niagara River are typical structures, the Grand Trunk Railway bridge being a deck spandrel-braced structure with span of 550 ft., carrying two railroad tracks on the upper deck and a highway immediately under this deck. The Niagara and Clifton bridge is near the Grand Trunk bridge and carries a highway on the upper deck, the highway being supported on vertical columns which are carried by the curved trussed ribs.

Deflection of a Curved Beam. If beam *OA* (Fig. 135) is fixed at *A* and is free at *O*, the horizontal, vertical and angular movements of the axis of the beam at the origin of coordinates *O* are

$$\Delta x = \int_0^A \frac{Myd_s}{EI} \qquad \Delta y = \int_0^A \frac{Mxd_s}{EI} \qquad \Delta\phi = \int_0^A \frac{Md_s}{EI}$$

Reactions on Arch with a Solid Web (Fig. 136). An arch with two hinges is statically indeterminate as regards the outer forces, one condition or equa-

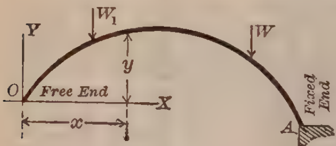


Fig. 135

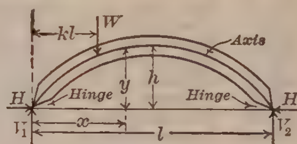


Fig. 136

tion being required in addition to those of statics. This additional equation may be obtained by the principle of least work or by deflections.

$$V_1 = W(1-k) \qquad V_2 = Wk \qquad H = \frac{\int \frac{Myd_s}{EI}}{\int \frac{y^2 d_s}{EI}}$$

This equation for *H* can be integrated only when the equation of arch axis is known, in which case *M*, d_s and *I* must be expressed in terms of the coordinates *x* and *y*. The integration extends over entire length of rib. If the material and the cross-section of rib are uniform for whole length the value of *H* becomes $H = \frac{\int Myd_s}{\int y^2 d_s}$. In case the arch axis is not a regular curve so that the integration cannot be performed, the value of *H* for an arch of uniform material and cross-section may be determined approximately by dividing the axis into a number of equal parts and letting *M* be the bending moment due to vertical loads and reactions and *y* the ordinate at the center of each division, $H = \frac{\Sigma My}{\Sigma y^2}$, where ΣMy denotes the summation of the various products of *M* and *y* for the center of each division and Σy^2 the summation of the squares of the several ordinates to these centers. In all of the above equations for H the effect of shear and axial force is neglected and only the effect of the moment is considered.

Parabolic Arch Rib with Solid Web (Fig. 137). A parabolic arch rib is one which has a parabola for an axis, the vertex being at the crown. The formula for H gives its value for bending only and assumes a rib of uniform material but with a moment of inertia varying from a minimum at the crown to a maximum at the skew-back hinges; the moment of inertia at any section equals that at the crown times the secant of the angle the arch axis makes with the horizontal at that section.

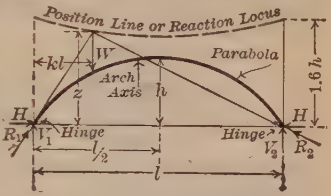


Fig. 137

$$V_1 = W(1 - k) \qquad V_2 = Wk$$

$$H = \frac{5Wl}{8h}(k - 2k^3 + k^4) \qquad z = \frac{1.6\,h}{1 + k - k^2}$$

Effect of Temperature on Arch Ribs with Solid Webs. A rise in the temperature tends to increase the span length and a fall in temperature to decrease it. If the span cannot change, a horizontal reaction is exerted on the arch at each support, and temperature stresses are produced within the rib. For the general case, Fig. 148, the horizontal reaction is

$$H = \frac{EI_0clt}{\int^1 y^2dx}$$

For a **parabolic rib,** (Fig. 137), with solid web this equation for horizontal reaction becomes $H = 15EI_0ct/8h^2$. The temperature range t is frequently taken as $\pm$ 75° Fahr. from the mean temperature of 50° Fahr. For a rise in temperature above normal, t is positive and the reactions H act inwardly and for a fall in temperature below normal, t is negative and the reactions H act outwardly. The bending moment at any section due to temperature is Hy, being positive for a fall and negative for a rise in temperature. I is here assumed to increase from the crown towards the abutments.

Rib Shortening in Arch Ribs with Solid Webs. The compression of the rib due to the thrust acting along the arch axis shortens the arch axis and produces outward horizontal reactions at the supports. The horizontal reaction is

For general case (Fig. 136) $\quad H = -\dfrac{SI_0l}{\int_0^1 y^2dx}$

For the parabolic rib (Fig. 137) $H = -\dfrac{15\,SI_0}{8\,h^2}$

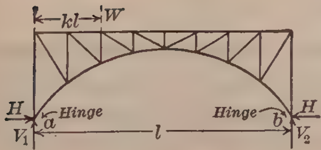


Fig. 138

Rib shortening is similar in effect to a fall in temperature. The moments are all positive. For a rib having a small rise the effect of rib shortening should not be neglected.

Reactions on a Braced Arch due to Vertical Loads. If in Fig. 138 the arch

[illegible]

$$[illegible]$$

[illegible]

Three-hinged Arches

[illegible] **increase in temperature** [illegible] **decrease in temperature** [illegible]

Reactions [illegible] algebraically [illegible] graphical methods are [illegible]

$$V_1 = \frac{Wx}{l} \qquad V_2 = \frac{W(l-x)}{l}$$

[illegible] with a single load on [illegible]

$$\text{when } x < \frac{l}{2} \quad H = \frac{Wx}{2h} \qquad \text{when } x \geq \frac{l}{2} \quad H = \frac{W(l-x)}{2h}$$

[illegible]

$$[illegible] \qquad H = \frac{wl^2}{16h}$$

[illegible]

$$V_1 = V_2 = 1/2\, wl \qquad H = \frac{wl^2}{8h}$$

[illegible] Reactions and Stresses in Roof Arches. [illegible] than the equi- [illegible] Draw [illegible] in the force polygon parallel to the closing line ad till it intersects $8k$ which is parallel to R_2. $8k$ and ko give the magnitude and direction of R_2 and R_1 respectively. With the reactions known, a stress polygon showing all stresses in both portions of the rib is

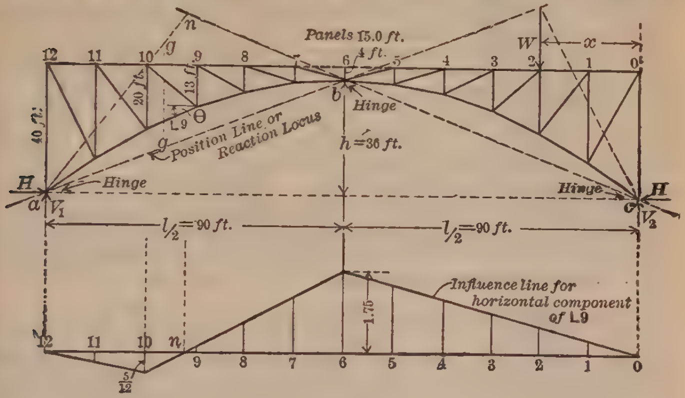


Fig. 139

drawn in the usual way by beginning at joint a. These stresses may be checked algebraically by method of moments, using the true equilibrium polygon; this polygon being started through a, using the first force polygon

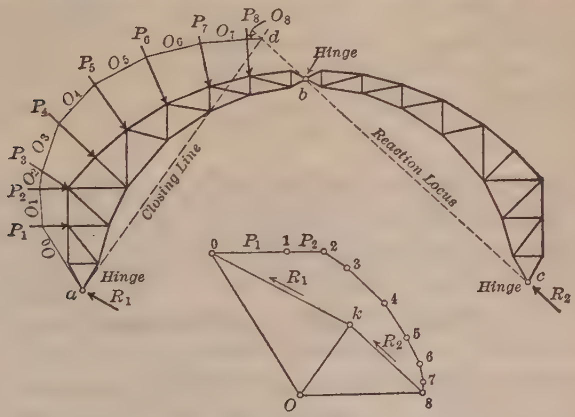


Fig. 140

with k as the pole, the reaction $R_1 = ko$ being drawn through a till it intersects P_1, then the string $k1$ till it meets P_2, and so on. The true polygon finally passes through the three hinges. By method of moments the stress in a bar is equal to moment at the origin of moments divided by the lever arm.

Computation of Reactions and Stresses for Spandrel-braced Arch. With a dead panel load of 20 and a live panel load of 40 thousands of pounds per truss, required the maximum stresses in lower chord L_9 (Fig. 139). Passing a vertical section g–g through L_9, taking origin of moments at 10 and drawing the line an through 10, it is seen that for a load to right of n the bar L_9 is in compression, for a load at n L_9 is zero, and when the load is to left of n L_9 is in tension. For maximum compression, load joints 1 to 9 inclusive, and for maximum tension joints 10 and 11. Loads at 0 and 12 produce stresses in end-posts only. Consider first the dead load. For this loading the thrust at crown hinge is horizontal, and by taking moments about a for left half of rib is found to be $6 \times 20 \times 3 \times 15/36 = 150$. With origin at 10 and considering the part of rib between b and section g–g the dead load stress in $L_9 = \sec \theta\,[150 \times 4 + 20\,(15 + 30 + 45) + 10 \times 60]\,.20 = 165.53$ = compression; where θ is angle L_9 makes with horizontal. For live load tension, loads on joints 10 and 11 cause $V_2 = 40\ (1 + 2)/12 = 10$ and $H = V_2 \times 90/36 = 25$, and considering the part of structure between c and g–g with origin at 10, the moment equation is $20\ L_9/\sec \theta - 10 \times 150 + 25 \times 40 = 0$; $L_9 = 27.59$ = tension. The live load compression in L_9 is most easily found as follows. Since ratio of live to dead panel load is 2, the compression in L_9 for full live load only $= 2 \times 165.53 = 331.06$, and compression for live loads on joints 1 to 9 inclusive is full live load compression plus live load tension due to live loads on 10 and 11, or maximum live compression $L_9 = 331.06 + 27.59 = 358.65$. Results: maximum compression = 524.18 and minimum compression = 137.94.

Influence Line for horizontal component of stress in L_9 is shown in Fig. 139. This influence line consists of three straight lines the ordinates to which represent the horizontal components of stress in L_9 due to a load of unity on the portions 0–6, 6–10, and 10–12, respectively. It is only necessary to compute the ordinate at 6 and 10. With load at 6 the moment at 10, Fig. 151, divided by 20 ft. gives the horizontal component of $L_9 = 1.75$ compression and this is plotted at joint 6, similarly with load of unity at 10 the ordinate at 10 is 5/12 tension. The horizontal component of L_9 may now be found from the diagram either by multiplying the areas of the triangles by the load per foot, which gives exact results, or by multiplying the ordinates to the influence line at the panel points by the panel load giving approximate results that are slightly larger than the exact. For example, using dead and live panel loads of 20 and 40 respectively, the following results verify those obtained in the preceding paragraph, which are computed by the usual approximate methods. Live tension in $L_9 = [40(1 + 1/2)5/12] \sec \theta = 27.59$. Live compression in $L_9 = [40(1/8 + 2/3 + 29/24) + 40(1 + 5/6 + 4/6 + 3/6 + 2/6 + 1/6)1.75] \sec \theta = 358.65$.

The Position of Live Load for Maximum Stress in a given bar must be determined for each case. For example, the point n, Fig. 139, is the neutral point for bar L_9, and if a load is placed on the right of n as at joint 8, the left reaction acts through a and through the intersection of position line bc with the vertical at 8, and since the left reaction is the only force acting on left at section g–g, the moment at origin of moments 10 is negative, hence L_9 must cause a positive moment about 10 and is compression. Similarly for a load to left of n, P_9 is tension. With a system of **concentrated loads** the influence line is useful. Referring to Fig. 139, the part of the influence line from O to n is the same shape as an influence line for bending moments at joint 6 on a simple span of length O–n, hence the position of loads for maximum compression in L_9 is the same as the position of loads that will cause maximum moment at 6 on span O–n, and the maximum tension in L_9 is caused by the same position of loads as will cause maximum moment at 10 on span 12–n.

44. Fixed Arches

Notation. y_1 and y_2 = ordinates of points of application of reactions at left and right ends respectively, ds = elementary length of arch axis, E = modulus of elasticity of material, I = moment of inertia of cross-section of rib, I_0 = moment of inertia of cross-section of rib at crown, M_L = moment at any point on left half of arch axis of all outer loads between that point and the crown, M_R = moment at any point on right half of arch axis of all loads between that point and the crown, m = number of divisions into which each half of arch axis is divided; H_C, V_C, M_C, respectively, are the thrust, shear and moment at crown, z = ordinate of the reaction locus, M_1 = moment at each support, c = coefficient of linear expansion, t = change in temperature in degrees Fahrenheit from normal, S = average compressive unit stress.

A Fixed or Continuous Arch has no hinges, and it is so rigidly held at its ends that the arch axis cannot change its direction at those points. Arches without hinges are more rigid than those having hinges, have greater temperature stresses and are more affected by settlements of the supports.

Reactions for Fixed Arches. With vertical load W as shown in Fig. 141 there are five unknown quantities V_1, V_2, H, y_1, y_2 to be determined to completely find the reactions. The following five equations give relations between these unknowns:

$$V_1 + V_2 - W = 0 \qquad Wkl - V_2 l + H(y_1 + y_2) = 0$$

$$\int_0^1 \frac{Myd_s}{EI} = 0 \qquad \int_0^1 \frac{Mxd_s}{EI} = 0 \qquad \int_0^1 \frac{Md_s}{EI} = 0$$

The first two of the above equations are from statics and the last three from the conditions that the horizontal, vertical and angular movement of the arch axis at the origin of coordinates A is each equal to zero. Moment at A is Hy_1, at B is Hy_2, and at any point D is Hu.

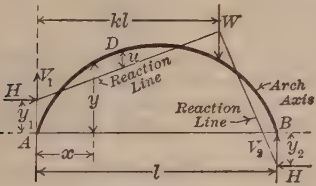


Fig. 141

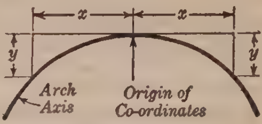


Fig. 142

The Thrust, Shear and Moment at Crown of a fixed arch (Fig. 142) having a shape such that the integration of the preceding paragraph cannot be performed, are given approximately by the following formulas, which assume the arch axis to be divided into short lengths such that the ratio of the length of each division, d_s, to the moment of inertia, I, at center of division is a constant.

$$H_c = \frac{m\Sigma M_R y + m\Sigma M_L y - \Sigma M_R \Sigma y - \Sigma M_L \Sigma y}{2\,[m\Sigma y^2 - (\Sigma y)^2]}$$

$$V_c = \frac{\Sigma M_L x - \Sigma M_R x}{2\,\Sigma x^2} \qquad M_c = \frac{\Sigma M_R + \Sigma M_L - 2\,H_c \Sigma y}{2\,m}$$

In these equations numerical values of M_R, M_L, x, and y are positive; the summations cover one-half the length of the span only; a positive value of M_C denotes a positive moment at the crown and a negative value denotes a negative moment there; for a positive value of V_C the line of pressure at the crown slopes upward towards the left and for a negative value upwards towards the right.

Reactions for a Parabolic Arch Rib having fixed ends and a variable moment of inertia of cross-section (Fig. 143). The modulus of elasticity is assumed constant and the moment of inertia assumed to vary from the crown to the abutments; at any section being equal to the moment of inertia at the crown times the secant of angle of inclination of arch axis at that section,

$$V_1 = W(1 + 2k)(1 - k)^2 \qquad V_2 = W(3k^2 - 2k^3)$$

$$H = \frac{15\,Plk^2}{4\,h}(1 - k)^2 \qquad y_1 = \frac{10\,k - 4}{15\,k}h$$

$$z = \frac{6\,h}{5} \qquad y_2 = \frac{6 - 10\,k}{15(1 - k)}h$$

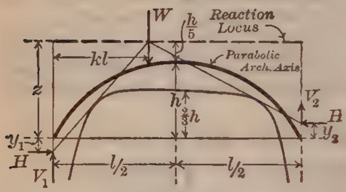


Fig. 143

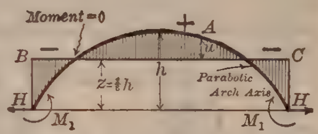


Fig. 144

The lines of reactions are tangent to a curve which consists of parts of two hyperbolas.

Temperature Stresses in a Parabolic Rib (Fig. 144).

$$H = \pm\frac{45\,EI_0ct}{4\,h^2} \qquad z = \frac{2\,h}{3} \qquad M_1 = \pm\frac{2\,Hh}{3}$$

For a fall in temperature the moment at each support is negative and the reaction H acts outward as shown in Fig. 144; moment at any point such as A is Hu, and is negative at all points below line BC and positive at all points above BC. For a rise in temperature the reaction H and the moments are opposite to those in Fig. 144.

Rib Shortening for a Parabolic Rib causes outward horizontal reactions H and bending moments like those for a fall in temperature.

$$H = -\frac{45\,SI_0}{4\,h^2} \qquad z = \frac{2\,h}{3} \qquad M_1 = -\frac{2\,Hh}{3}$$

45. Suspension Systems

Historical and Descriptive. A suspension bridge consists of a floor suspended from cables or chains which are supported by passing over towers and by being anchored to the earth at their ends. In the early suspension bridges chains were used, but they were soon superseded by wire cables. To prevent undue oscillation or deflection the cables are provided with stays or stiffening trusses or else are trussed. When the cables are trussed they are usually made of eyebars, and the system then becomes an inverted arch suspended between the tops of the towers and having the inward pull from the main arch span resisted by an outward pull from the back stays extending from the top of each tower to the anchorage in the earth. The **first suspension bridge** of note in the United States was built in 1801 by James Finley at Greensburg, Pa., and had a span of 70 ft. carried by chains. The Chain Bridge near Newburyport, Mass., was built in 1810 by John Templeman on Finley's design, was strengthened in 1900 and again in 1910. The **first wire suspension bridge** in America was a foot bridge of 408-ft. span built in 1816 over the Schuylkill River; in Europe the first one was built in 1834 at Freiburg. In 1855 John A. Roebling completed the Niagara suspension bridge, which had a span of 821 ft. 4 in. and carried a railroad on the upper and a highway on the lower deck; wooden stiffening trusses 16 ft. deep were

American Suspension Bridges

Span	Crossing	Location	Number of cables	Date of completion
Ft. In.				
3500 0	Hudson River	New York (Ft. Lee)	4	1932
1850 0	Detroit River	Detroit, Mich	2	1930
1750 0	Delaware River	Philadelphia–Camden	2	1926
1632 0	Hudson River	Bear Mt., N. Y.	2	1924
1600 0	East River	New York (Williamsburgh Bridge)	4	1904
1595 6	East River	New York (Brooklyn Bridge)	4	1883
1500 0	Hudson River	Poughkeepsie, N. Y.	2	1929
1470 0	East River	New York (Manhattan Bridge)	4	1909
1200 0	Mt. Hope Bay	Rhode Island	2	1930
1113 9	Strait of Atlantic O.	Florianopolis, Brazil	2	1926
1057 0	Ohio River	Cincinnati, Ohio	4	1867*
1030 0	Ojuela River	Mampimi, Mexico	2	1900
1010 0	Ohio River	Wheeling, W. Va.	4	1849†
940 0	Cauca River	Occidente, Antioquia, Colombia	4	1894
800 0	Ohio River	Rochester, Pa.	2	1897
800 0	Niagara River	Lewiston, N. Y.; Queenston, Ont.	2	1899
775 0	Ohio River	Parkersburg, W. Va.	2	1916
705 0	Ohio River	East Liverpool, Ohio	2	1896
705 0	Rondout Creek	Kingston, N. Y.	2	1922
700 0	Ohio River	Galliopolis, Ohio	2	1928
700 0	Ohio River	Steubenville, Ohio	2	1905
510 0	New River	Capertown, W. Va.	2	1903
470 0	Brazos River	Waco, Texas	2	1870
470 0	Allegheny River	Warren, Pa.	2	1871
450 0	Guyan River	Guyandotte, W. Va.	4	1848
442 1	Allegheny River	Pittsburgh, Pa.	2	1926
400 0	Railroad tracks	Grand Avenue, St. Louis, Mo.	2	1891
400 0	Kennebec River	Waterville, Maine	2	1903

* Rebuilt 1895–98, two new cables being added above the two old ones. † Rebuilt 1854 and 1862. Before the reconstruction in 1862 the bridge had twelve cables arranged in two groups of six each, placed side by side.

used. In 1880 iron trusses replaced the wooden ones in this structure, in 1886 wrought-iron towers replaced the stone towers, and in 1897 the structure was replaced by a two-hinged spandrel-braced arch. The largest suspension bridge in the world is now under construction across the Hudson River at New York. It will have a central span of 3500 ft. The present (1928) longest suspension bridge is the one across the Delaware River at Philadelphia. It has a central span of 1750 ft.

46. Cables

Notation. w_1 = intensity of load at point having coordinates x and y. H = horizontal component of tension in cable. T = tension in cable at any point. w = uniform load per horizontal unit of length. l = horizontal length of span of cable. h = sag or vertical distance from lowest point of cable to its support at top of tower. l_1 = length of cable between towers.

Cables for small bridges are made of wire rope, and for large structures they are made by compacting a number of steel wires into cylindrical form and clamping them in this shape by means of steel clamps placed above the panel points of the stiffening trusses thus serving as saddles for the suspenders. Each of the four main cables of the Manhattan bridge in New York is composed of 9472 parallel wires arranged in 37 strands, each wire being 0.192 in. in diameter before and not over 0.197 in. in diameter after galvanizing. These cables are approximately 20-3/4 in. in diameter, and are wrapped with galvanized-iron wire and painted. The Williamsburgh bridge in New York has four main cables, each 18-3/4 in. in diameter and each composed of 7696 wires arranged in 37 strands clamped together and covered between clamps with cotton duck soaked in oxidized linseed oil and varnish gum composition and with a sheet-iron shell. The Delaware River bridge has two main cables, each 30 in. in diameter composed of 18 666 wires arranged in 61 strands of 306 wires each. Each wire galvanized has a diameter of 0.196 in. Each of the two Bear Mountain bridge cables is made up of 37 strands of 196 wires each, making a total of 7252 wires, and has a finished diameter, after compacting and wrapping, of 18 in. The new Hudson River bridge at New York has four main cables, each 30 in. in diameter, of 26 474 wires arranged in 61 strands of 434 No. 6 galvanized wire, 0.196 in. diameter over galvanizing. Each cable will have a total steel area of 800 sq. in. and will carry a maximum tension of 65 400 000 lb. Cables are cradled when the horizontal distance between the two cables at a pier is greater than that at the center of the span.

Linear Arch. A linear arch for a given set of forces is the true equilibrium polygon for those forces, and hence it may be defined as an arch in which there is only direct tension or compression, and is incapable of carrying bending moment. The general equation of the linear arch is $d^2y/dx^2 = w_1/H$, where the left side of the equation denotes the second differential. A cable of uniform cross-section and material suspended at its ends and subjected only to its own weight assumes the form of a catenary.

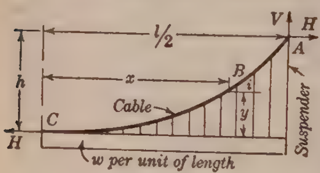


Fig. 145

Cable Carrying a Uniform Load. Fig. 145 shows one-half of such a cable, and for this loading the curve of the cable becomes a parabola with axis vertical and vertex at lowest point of curve, C, and its equation is

$$y = \frac{4\,hx^2}{l^2} \quad \text{or} \quad y = \frac{wx^2}{2\,H}$$

The **tension in the cable** at the lowest point C is the horizontal component H, this component being constant throughout the length of the span, and is $H = wl^2/8\,h$. Tension at any point such as B is given by the following:

$$T_B = \sqrt{H^2 + w^2x^2} = \frac{w}{8\,h}\sqrt{l^4 + 64\,h^2x^2}$$

Maximum tension exists at the tower A and its value is

$$T_A = \frac{w}{8\,h}\sqrt{l^4 + 16\,h^2l^2}$$

Tangent of angle of slope ($\tan i$) at any point B and at the upper A is $8\,hx/l^2$

and $4\,h/l$ respectively. **Length of cable** between supports, that is, twice the length of curve AC, Fig. 145, is

$$l_1 = 2\int_0^{\frac{l}{2}}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2}dx = l\left(1+\frac{8}{3}\frac{h^2}{l^2}-\frac{32}{5}\frac{h^4}{l^4}+\ldots\right)$$

The **Sag Ratio** or **Dip Ratio** is the ratio of the sag of the cable, h in Fig. 145, to the span l, and this ratio varies from about 1/8 to 1/15. The term versine is also used to denote the sag h, but this is not a proper use since the curve is not a circle. The greater the sag the smaller the tension in the cables but the larger the towers. Sag ratio for Williamsburgh bridge is 1/9; for Brooklyn bridge, about 1/12.5; for the Bear Mountain bridge, about 1/8; for the Delaware River bridge, about 1/8.75, and for the Hudson River bridge at New York, about 1/10.75.

Steel Wires for Cables, Suspenders and Hand Ropes of Manhattan bridge were required to be made in an open-hearth furnace lined with silica. Maximum percentages for chemical requirements: C, 0.85%; Mn, 0.55%; Si, 0.20%; P, 0.04%; S, 0.035%; Cu, 0.02%; Strength: 215 000 and 200 000 lb. per sq. in. before and after galvanizing respectively; minimum elongation in 12 in., 2% and 4% before and after galvanizing. For the Williamsburgh bridge the following were the maximum chemical requirements: P, 0.04%; *S*, 0.03%; Mn, 0.05%; Si, 0.1%; Cu, 0.02%. Minimum ultimate strength, 200 000 lb. per sq. in.; elongation of 2-1/2% in 5 ft. and of 5% in 8 in. The wire reached an ultimate strength of 225 000 lb. per sq. in. The minimum ultimate strength for the Bear Mountain and Delaware River bridges was about 215 000 lb. per sq. in.

47. Stiffening Trusses

Notation. w = uniform live load per unit of length. w_1 and w_2 = live loads per unit of length borne by the cable and stiffening truss respectively. h = sag of cable. d = deflection at center of span. A = cross-sectional area of cable. E = modulus of elasticity of material in cable and truss. I = moment of inertia of truss. V = shear in truss. M = bending moment on truss.

Stiffening Trusses are used to distribute a concentrated load or a partial uniform load over the whole or a part of the length of the cable and they are usually steel riveted trusses, although in small bridges are sometimes made of wood. Where riveted trusses are used they are of the Warren type with one, two or three web systems. For highway bridges, where permissible deflections are large and the loads light, the stiffening trusses are light, whereas for railroad bridges, which are subjected to heavy and unequal loads and where the allowable deflection is small, the stiffening system must be heavy. The trusses are usually suspended from the cables by hangers or suspenders, which are wire rope or rods attached to the trusses at their panel points. These trusses may be continuous from anchorage to anchorage, may extend unbroken from tower to tower only or may extend from tower to tower with a hinge at the center and at each tower.

Stiffening Trusses without a Center Hinge. Under a uniformly distributed load covering the entire span there are no moments and shears in the truss. With a uniformly distributed live load extending from the left tower, A (Fig. 146) a distance x, the following approximate functions of this load exist. Upward pull on truss per unit of length is wx/l.

$$R_1 = \frac{1/2\,wx(l-x)}{l}, \qquad R_2 = -\frac{1/2\,wx(l-x)}{l}$$

The shear at any section distant z from A is

$$z < x, \qquad V = \frac{w}{2\,l}(xl - x^2 - 2\,zl + 2\,zx)$$

$$z > x, \qquad V = \frac{w}{2\,l}(-\,xl - x^2 + 2\,zx)$$

When $z = 0$, x, or l, $V = wx(l - x)/2\,l$, being positive at the towers A and B and negative at D. The **greatest shear** possible occurs when the half span is loaded, that is, when $x = l/2$ and maximum $V = 1/8\,wl$. This maximum shear occurs at the towers and at the center of span. The moment at any section distant z from A, (Fig. 146), is

$$z < x, \qquad M = \frac{wz}{2l}(xl - x^2 - zl + xz)$$

this moment being positive with a maximum $wx^2(l - x)/8\,l$ when $z = x/2$;

$$z > x, \qquad M = \frac{wx}{2l}(xl - zl - xz + z^2)$$

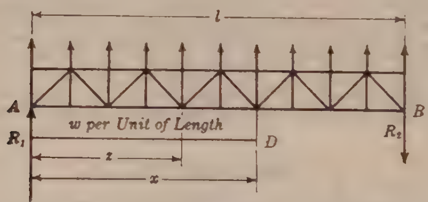


Fig. 146

and these are negative moments with a maximum value of $M = -wx(x - l)^2/8l$ when $z = (l + x)/2$. The **greatest positive moment** occurs at the center of the loaded part when the live load extends 2/3 across the span from A and is $M = wl^2/54$; the **greatest negative moment** has this same value at the center of the unloaded part when the load covers the left third of the span.

More Accurate Methods for Determining Stresses in cables and stiffening trusses make use of deflections; for example, with a stiffening truss without a center hinge the deflections of the cable and of the truss at any vertical section such as the center are equal. The **deflection of the cable** at the center of a span is approximately

$$d = \frac{w_1 l^2(3\,l^2 + 16\,h^2)}{128\,h^2AE}$$

And the deflection of the stiffening truss which is supported at the towers is

$$d = \frac{5}{384}\,\frac{w_2 l^4}{EI}$$

The ratio of the load borne by the cable to that borne by the truss is

$$\frac{w_1}{w_2} = \frac{5\,l^2h^2A}{3\,I(3\,l^2 + 16\,h^2)}$$

The total load $w_1 + w_2$ is here assumed to cover the entire span.

Stiffening Trusses Having a Center Hinge (Fig. 147). Shear but no moment can be transmitted across the hinge. For a uniform live load of w per unit of length maximum positive shear occurs at R_1 when the load extends from R_1 to $X = 1/3\, l$. Then $R_1 = wl/6$ and acts upward. The maximum negative shear has the same value and occurs at the same point when the load extends from $X = 1/3\, l$ to R_2. The maximum shear at the center hinge occurs when the load covers half the span, and is equal to $wl/8$. Maximum positive moment occurs at $x = 0.234\, l$ from R_1 when the load extends from R_1 to $X = 0.395\, l$, and is equal to $0.019\, wl^2$. Maximum negative moment occurs at the same point and has the same value, $0.019\, wl^2$ when the load covers the remaining portion of the span. When the load covers the entire span the cable takes the entire load and the stiffening truss is without stress.

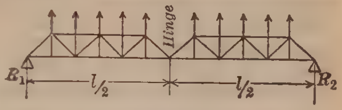


Fig. 147

Trussed or Stiffened Cables are also used to prevent excessive deformation of the cables, the stiffening trusses being omitted. The cables are made

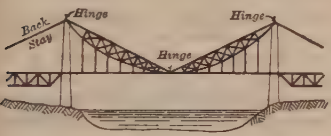


Fig. 148

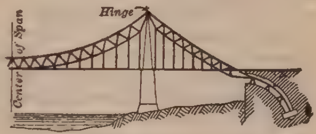


Fig. 149

of eyebars or links, and serve as members of the trusses from which the roadway is suspended as in Figs. 148 and 149. These structures are inverted arches, the first being three-hinged and the second two-hinged.

48. Stays and Anchorages

A Back Stay is that portion of the main cable which extends from the anchorage to the top of the tower. It may be a continuation of the main cable, as is usual, or it may be a separate member, in which case the main cable extends from tower to tower only. That part of the roadway under the back stay is sometimes suspended from the back stay as in the Manhattan and Brooklyn bridges, or it may be carried on separate trusses as in the old (now replaced) Point bridge in Pittsburgh, Pa., or on the stiffening trusses as in the Williamsburgh bridge. When the main cable passes over a movable saddle on the tower the maximum horizontal components of the stresses in the main cable and back stay are equal, and the maximum stresses are equal if the two parts are inclined at equal angles. The friction between the saddle and the tower produces a horizontal force on the tower equal to the vertical weight on the saddle times the coefficient of friction of the roller bearing. It has been found by experience that this friction is often great enough to prevent horizontal motion of the saddle. Stresses and deflections are then set up in the towers for which they were not designed. In several of the most recent bridges the cables are fixed to the top of the steel towers, which deflect horizontally under the unbalanced pull of the cables (from temperature and loading). With the Bear Mountain and Delaware River bridges the base of the tower is fixed and the top deflects elastically. With the Florianopolis bridge the base of the tower is hinged and the tower rocks about this hinge.

Anchorages for the back stays are provided by attaching them to eyebar chains and girders or castings which are set in masonry at each end of the bridge. The chain of eyebars may be inclined in a straight line or in a series of broken lines, and they may be placed in an inspection tunnel as in the Williamsburgh bridge or may be completely embedded in concrete. The eyebars of the anchor chains in the Point bridge in Pittsburgh were embedded in a poor quality of masonry, and when examined after 27 years of service were found to be in good condition except that they were pitted considerably for 5 or 6 ft. near the surface of the ground.

Hangers or Suspenders are vertical wire cables or members built of structural shapes which connect the stiffening trusses to the main cables. When the main cables are made of wire the suspenders are also of wire rope, as in the Manhattan bridge, where the suspenders consist of four wire ropes 1-3/4 in. in diameter at each panel point of each stiffening truss, that is, two loops over the main cable. These ropes rest in two grooves in the cast-steel cable bands which clasp the main cables, and the stiffening trusses are suspended from their lower ends. In the Budapest suspension bridge the cables are made of links of eyebars connected by pins, and the stiffening trusses are suspended from these chain cables by suspenders consisting of two channels. **Stay-cables** are inclined cables extending from the tops of the towers to the stiffening trusses, where they are attached at the suspender connections. They were used in the main and side spans of the Brooklyn bridge and in the main span of Roebling's bridge at Niagara Falls, but in recent suspension bridges they have not been used.

Saddles are castings resting on the top of the towers to support the main cables at these points. They usually consist of a steel casting on each tower, and each cable is laid in a groove in the casting, the groove and cable being covered with a cast or rolled steel cover. In the Manhattan bridge the tower saddles are rigidly connected to the towers, whereas in the Williamsburgh bridge each saddle rests on a nest of forty cylindrical steel rollers each 2-1/4 in. in diameter and 7 ft. 6 in. long. In the latter bridge each saddle is in one piece, weighs about 72 000 lb. and is 7 ft. 8 in. wide and 19 ft. long. The chain cables of the Budapest bridge are connected to the tops of the steel towers by links and pins and the towers are hinged at their bases.

Piers. There are usually two piers in a suspension bridge, one at either shore. They may be either of masonry or of steel. Masonry gives a better appearance, but is more costly and takes up greater space. A steel towers or pier may be a single bent of two supports hinged at the bottom or a framed tower with four or more supports. The forces on the pier depend on the arrangement of the cables. It was proposed by Morison, Trans. Am. Soc. C. E., Dec., 1896, to connect the cable of the main span and the back stay of the end span in two separate sockets at the top of the tower. In this case there would at times be great inequalities in the horizontal components of the stresses in the main cable and the back stay, and provision would have to be made for these differences in the design of the pier. When saddles on rollers are used to support the cables the maximum longitudinal force which can act on the top of a tower may be taken approximately at 1/100 the total load on the rollers. This assumes the rollers to be in good condition and free from excessive rust. The horizontal force in this case acts just before and until the rollers move. The movement equalizes the horizontal components of the stresses on the two sides of the saddle and the pressure is then vertical on the pier, except in so far as wind pressure and cradling of the cables produce horizontal forces.

MISCELLANEOUS STRUCTURES

49. Viaducts

A Viaduct is a bridge consisting of a series of spans supported on towers, the spans being usually plate girders, though sometimes trusses are used. Viaducts are used to carry railroads or highways over valleys or rivers. They are also called trestles. Each tower usually has four legs or columns, although six or two may be used. When two are used they are connected by transverse bracing only, thus forming a **rocker tower**; and when four or more legs are

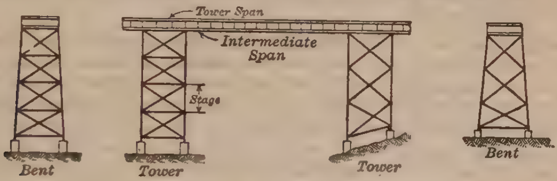


Fig. 150. Viaduct Towers

used in one tower they are connected by transverse and longitudinal bracing. A **stage** or story in a tower is that portion of the tower between the level of one horizontal brace and the level of the one above it. Fig. 150 shows the two usual types of viaduct towers.

Length of Span. For a given viaduct the tower span is usually made constant, 30 ft. for low and 45 ft. for high structures being common. The intermediate spans should increase in length as the heights of the adjacent towers increase, but to simplify construction not more than two or three different lengths should be used in any one structure. The intermediate span should be approximately twice as long as the tower span, 60 ft. for intermediate spans being common practice for ordinary viaducts. The Boone viaduct has alternate spans of 75 and 45 ft. throughout its length except

High Railroad Viaducts

Viaduct	Railroad	Length, Ft.	Maximum height, Ft.	Number of tracks	Weight of metal, Short Tons	Tower spans, Ft.	Ordinary intermediate spans, Ft.
Boone	Chicago & N.W.	2685	185	2	5680	45	75
Gokteik *	Burmah State	2260	335	1	4310	40	60–120
Pecos	So. Pacific	2180	321	1	1820	35	65
Kinzua	Erie	2053	301	1	3352	38.5	61
Panther Creek	Wil. B. & E.	1650	161	1	830	30	40–65
Loa †	Antofagasta	800	336.5	1	1115	32	80
Spokane River	Ore. Wash. & Nav.	3003	187	1	3822	40	80
Snake River	Ore. Wash. & Nav.	3920	300	1	8100	60	80

* Burmah. † Chile.

for the 300-ft. truss span near the center. In the Gokteik structure all tower spans are 40 ft., with intermediate truss spans of 120 ft., except those near the ends, which are plate girder spans of 60 ft. The girders may all be of the same depth, even though the spans are of various lengths as in the Boone viaduct or of different depths as in the second Kinzua viaduct built in 1900. The length of the tower span should be sufficient to prevent an upward pull on the anchorages due to traction.

Lateral Stability. A tower consisting of two transverse vertical bents has in each bent two inclined columns which are connected by bracing. The width of the bent at the top varies from 8 to 11 ft. for single-track railroad structures, and the two lines of girders are usually spaced the same distance apart as the columns. The double-track Boone viaduct has four lines of girders spaced with three equal spaces of 6.5 ft., and the outer girders are vertically over the tops of the columns. The batter of columns is frequently made such that the width of the bent at the bottom is sufficient to prevent an upward pull on the foundation when the structure is loaded with the lightest vertical load consistent with the greatest wind pressure. The batter varies from 1.5 to 3 in. per foot.

Bracing. The main girders should be connected together with transverse sway frames and with top and bottom horizontal lateral bracing. The diagonals and horizontal members of the transverse and of the longitudinal bracing of the towers may be made of latticed angles or channels connected to the tower legs with gusset plates which are usually shop-riveted to the columns. Rods and adjustable members should not be used for laterals. The second Kinzua viaduct has ordinary longitudinal bracing, but the transverse bracing in the bents consists of horizontal portal girders at intervals of about 30 ft. without any diagonals.

Connections of Longitudinal Girders to the bents are usually made in single-tract viaducts by resting them directly upon the horizontal caps of the inclined columns, although in some cases, the second Kinzua viaduct for example, the tower and intermediate span girders are riveted to the vertical faces of the columns, except at expansion joints, where one end of the intermediate girder rests in an expansion bearing or pocket which is attached to the vertical side of the column. The expansion joints in this structure are from 199 to 260 ft. apart. The longitudinal girders may be connected to a transverse girder which is riveted between the columns of each bent. Fig. 151

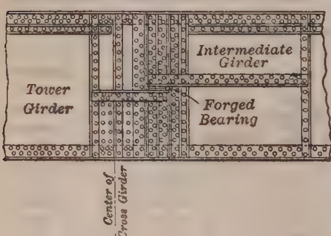


Fig. 151

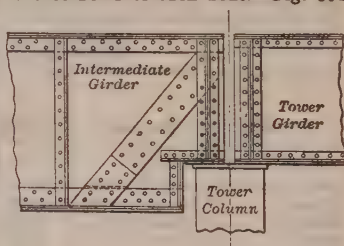


Fig. 152

shows the design used in the Boone viaduct where the tower girders rest upon the upper flanges of transverse girders and are notched to receive the ends of the intermediate girders which have drop ends. The tower and intermediate girders here have a uniform depth of 7 ft. A forged-steel bearing between the lower and upper girders allows for deflection and temperature displacements. In Fig. 152 is shown another form of drop end where the girders have different depths.

Column Bases. In high towers provision should be made for expansion at the bottom of the towers in a direction at right angles to the length of the viaduct. In the Boone viaduct this is accomplished by bolting the base of one column in a bent to the cast pedestal which is anchored to the masonry, while the other column in the same bent has slotted holes for the anchor bolts in its base so that it may move on a phosphor-bronze sliding plate which is inserted between column base and pedestal. The Kinzua viaduct has a nest of rollers under one column in each bent.

Live Loads. **First Kinzua** viaduct was designed and built in 1882 for locomotives weighing 161 340 lb. on a wheel base of 54.25 ft.; maximum wind pressure 30 lb. per sq. ft. **Second Kinzua**: built in 1900; two locomotives each of 274 000 lb. followed by 4000 lb. per linear foot; longitudinal traction of 0.2 of the maximum live load; temperature stresses due to 150° variation; wind pressure for the loaded structure of 30 lb. per square foot of vertical surface of train, floor and girders and 100 lb. per vertical foot of bent, and for the unloaded structure 50 lb. per square foot of vertical surface of floor and girders and 160 lb. per vertical foot of bent. **Boone** viaduct: built 1900–01; 75-ft. spans, 6100 lb. live and 1400 lb. dead per linear foot; 45-ft. spans, 7600 lb. live and 1250 lb. dead per linear foot. **Fort Dodge** viaduct: two locomotives each of 308 000 lb. followed by 4000 lb. per linear foot.

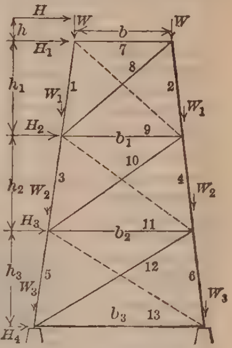


Fig. 153

Stresses in Towers due to vertical loads. The vertical loads consist of the weights of the locomotives and train, and the dead loads. In Fig. 153 let W be the total live and dead load acting at the top of each post and W_1, W_2, and W_3 the dead loads at the joints. Stress in bar 1 is designated by P_1 and so on. θ is angle between any bar and the vertical.

$$P_1 = P_2 = W \sec\theta \qquad P_3 = P_4 = (W + W_1)\sec\theta$$
$$P_5 = P_6 = (W + W_1 + W_2)\sec\theta$$
$$P_7 = W\tan\theta \qquad P_9 = W_1\tan\theta$$
$$P_{11} = W_2\tan\theta \qquad P_{13} = (W + W_1 + W_2)\tan\theta$$

The diagonals have no stress under vertical loading. All of the above stresses are compression except P_{13}.

Stresses in Towers Due to Wind. In Fig. 153 the horizontal forces shown are those due to wind, H being the wind pressure on the train and H_1, H_2, H_3 the pressures on the structure. + denotes tension, − compression. The diagonals are tension members and the dotted bars are not in action for the forces shown. The stresses are

$$P_1 = Hh\sec\theta/b$$
$$P_2 = -[H(h + h_1) + H_1h_1]\sec\theta/b_1$$
$$P_3 = [H(h + h_1) + H_1h_1]\sec\theta/b_1$$
$$P_4 = -[H(h + h_1 + h_2) + H_1(h_1 + h_2) + H_2h_2]\sec\theta/b_2$$
$$P_5 = -P_4$$
$$P_6 = -[H(h + h_1 + h_2 + h_3) + H_1(h_1 + h_2 + h_3) + H_2(h_2 + h_3) + H_3h_3]\sec\theta/b_3$$

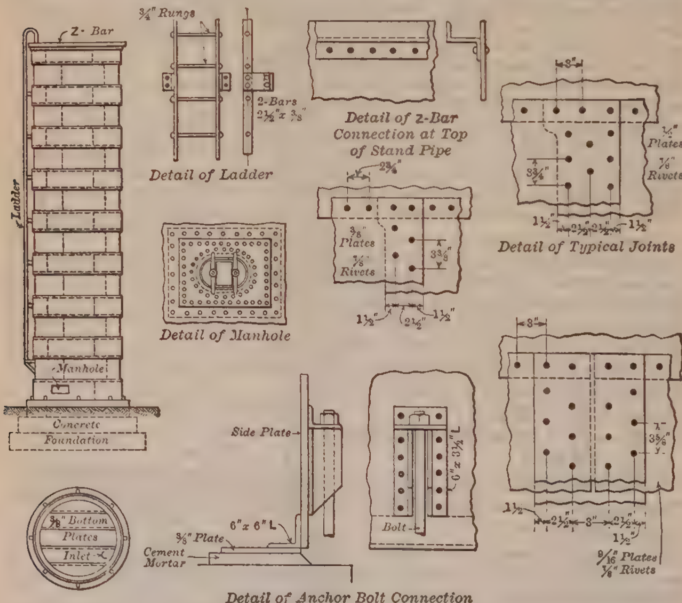
For the horizontal and diagonal members the stresses are most easily found by taking moments about the origin, which is at the intersection of the columns produced. The stresses due to traction and temperature should be found.

50. Standpipes

Notation. H = height in feet from a point to surface of water in stand-pipe. D = internal diameter of stand-pipe in feet. t = thickness of side plate in inches. S =

allowable tensile unit stress in pounds per square inch. e = efficiency of vertical riveted joint. b = circumferential distance in inches between centers of anchor bolts. W = weight of metal in stand-pipe above a given horizontal joint in pounds. p = pitch of rivets in horizontal joint in inches. M = moment in foot-pounds of wind pressure at a horizontal joint. V = allowable stress on a rivet in pounds.

A Standpipe is a metallic tank, usually of cylindrical form, with a flat bottom resting directly upon a masonry or sand foundation and used for the storage of liquids, usually water or oil. For storing water standpipes vary in diameter from 12 to 30 ft. and in height from 35 to 100 ft., although there has



Detail of Anchor Bolt Connection

Fig. 154. Stand-pipe, with Details

been constructed a standpipe with a diameter of 150 ft. and a height of 20 ft. and also one with a diameter of 4 ft. and a height of 210 ft.; the latter being surrounded by a masonry tower. Standpipes for the storage of water should generally have no roofs in cold climates but in warm climates should be covered. One pipe may be used to serve as an inlet as well as an outlet and this may connect with the standpipe in the horizontal base plate or in the side near the base. Failures of standpipes have been caused by defective foundations, by wind or ice pressure, or by hydrostatic pressure. Fig. 154 shows details of stand-pipe construction.

The Thickness of Side Plates must be computed to withstand the tension due to water pressure, proper allowance being made for weakening of plates at the vertical seams or joints. Side plates less than 1/4 in. in thickness should not be used. The maximum thickness required seldom exceeds 1-1/8 in. Tension in side plates at a point H distance down from water surface = 2.6 HD per vertical linear inch, and the thickness of plate required at that depth for the sides of a cylindrical standpipe containing water is $t = 2.6HD/Se$, in which S should be taken at from 10 000 to 12 000 lb. per sq. in. for structural steel. The efficiency e depends on the design of the vertical joints, which should be so made as to develop an efficiency of from 60% to 75%. With a depth of water of 60 ft., an internal diameter of 18 ft., allowable tension of 12 000 lb. per sq. in., and efficiency of 66-2/3%, the thickness of plate required at that depth = 2.6 × 60 × 18/12 000 × .66-2/3 = .35 in., or practically 3/8 in.

Side Plates in cylindrical standpipes are arranged in horizontal rings or courses about 5 ft. deep, with adjacent courses of different diameters so that the courses lap over each other. Plates should be bent by cold-rolling, then punched and finally planed to a bevel along all edges for calking. Rivet holes in plates having thickness of 9/16 or less should be punched 1/16 in. larger than the diameter of the cold rivet; those in 5/8 to 3/4 in. inclusive should be sub-punched 3/16 in. less in diameter than the cold rivet and then reamed to 1/16 in. larger than the rivet; those in plates over 3/4 in. in thickness should be drilled 1/16 in. larger than the rivet. Plates should be open-hearth "structural" steel and rivets of "rivet" steel or wrought iron.

Bottom Plates for flat-bottom standpipes should be not less than 5/16 in. in thickness when laid on concrete foundations; when laid on sand as in the case of oil-tanks of large diameters bottom plates are frequently made 1/4 in. If on a concrete foundation they should be completely riveted and then laid on a 2-in. layer of portland cement mortar before the mortar hardens, or else they should be bedded by pouring cement grout through 1-1/2-in. temporary iron pipes screwed into holes in the plates, the pipes being replaced with screw plugs. Side and bottom plates are connected by one or two ∟'s having beveled edges for calking. When only one ∟ is used its thickness should be approximately equal to that of the lowest side plate.

A Steel Ladder, extending from the top to within 8 or 9 ft. of the ground should be attached to the outside of the standpipe. The rungs may be made 15 in. long, 3/4 in. in diameter, from 12 to 14 in. apart and attached to two vertical bars 2-1/2 in. by 3/8 in. A manhole is necessary near the bottom, the side plates being reinforced around this opening by plates or steel forgings. Openings for supply pipes, whether in the bottom or side, must be similarly reinforced.

Wind Pressure may generally be taken at 30 lb. per sq. ft. on vertical surfaces, except in the most exposed locations, where 40 lb. per sq. ft. should be used. On a cylindrical surface it should be taken as 2/3 that on a plane surface equal to the diameter of cylinder multiplied by its height, and this pressure is assumed to act horizontally in any direction. Unless anchored to the foundation an empty standpipe will overturn when the overturning moment of wind pressure taken about the base, that is, 10 H^2D ft.-lb. for above pressure of 30 lb. per sq. ft., is greater than the moment of the weight of the empty standpipe taken about the outer edge of the base. **Anchor bolts** should always be used and should be computed for a tensile unit stress of not more than 15 000 lb. per sq. in.; minimum diameter 1-1/4 in., and they should be attached to anchor plates embedded in the foundation at sufficient depth to take at least 1-1/2 times the computed tension in the bolts. Maximum tension in the anchor bolts may be found by regarding the standpipe as a cantilever subjected to the overturning moment of the wind and the righting moment of the weight of the empty tank, about the neutral axis. This neutral axis can best be found by trial. On its windward side all of the tension is taken by the bolts; on its leeward side all of the compression is taken by the portion of the base plate immediately under the edge of the tank. It is assumed that the anchor bolt nuts are just brought to a bearing.

In Fig. 155, N–A is the neutral axis taken close to the leeward edge. A_B = area of bolts on the right of N–A; A_T = area of base angle on the left of N–A,

$A = A_B + A_T$, G–G = axis through center of gravity of the two areas A_B and A_T; c_1 and c_2 = distances from G–G to extreme points in tension and compression respectively; g = distance from G–G to the center of the tank; I = moment of inertia of A, the section under stress, about G–G; S_B = maximum unit tension at extreme point on the windward side; S_c = maximum unit compression at extreme point on the leeward side. $\mathbf{S_B = \frac{(M - Wg)c_1}{I} - \frac{W}{A}}$. $\mathbf{S_c = \frac{(M - Wg)c_2}{I} + \frac{W}{A}}$. If all dimensions are taken in feet, S_B and S_c will be pounds per square feet. By proportion, from the extreme fiber stresses S_B and S_c, the correct position of the neutral axis can be found. If it does not agree with the trial one a new position of the line N–A should be taken and the process repeated until the assumed and the computed positions of N–A agree closely. S_B may be found with sufficient accuracy by assuming the gravity axis G–G to coincide with the tangent G'–G' at the inner edge of the rim circle, and I to be the moment of inertia of the bolt areas about G'–G' = sum of the products of the areas of the bolts into the square of their distances to G'–G'. The W/A term, being small, may be neglected. If included, $A = A_B +$ the bearing area of about 1/8 of the rim base angle. Then $\mathbf{S_B = (M - Wr)D/I}$, where $r = D/2$.

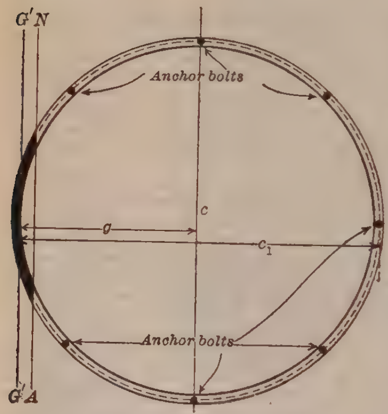


Fig. 155. Section at Base of Tank

For example, let H = 80 ft.; D = 20 ft.; W = 75 000 lb.; wind = 30 lb. per sq. ft. $M = 2/3 \times 20 \times 80 \times 30 \times 40 = 1\,280\,000$ ft-lb. $Wr = 75\,000 \times 10 = 750\,000$. $M - Wr = 530\,000$ ft-lb. Assume eight 1-1/4-in. anchor bolts upset to 1-5/8 in. Area of each bolt = 1.227 sq. in. = .00854 sq. ft. $I = 2 \times 0.00854\,(1/2 \times 2^2 + 1.7^2 + 1^2 + 0.3^2)\,10^2 = 10.2$. Neglecting W/A, $S_B = 530\,000 \times 20/10.2 = 1\,040\,000$ lb. per sq. ft. = 7230 lb. per sq. in. The area of 1/8 of the rim bearing, assuming the effective bearing width of base angle to be 3 in. = 282.6 sq. in. $A = 1.227 \times 7 + 282.6 = 291.2$ sq. in. $W/A = 75\,000/291.2 = 260$ lb. per sq. in. Including W/A $S_B = 7230 - 260 = 6970$ lb. per sq. in. Total stress S in extreme windward bolt = $6970 \times 1.227 = 8550$ lb. The total tension in the bolts from $M - Wr = 2 \times 7230 \times 1.227\,(1/2 \times 20 + 17 + 10 + 3)/20 = 35\,500$ lb. The average compression over 1/8 the area of the rim from $M - Wr = 35\,500/282.6 = 125$ lb. Approximate average compression on the masonry $= 260 + 125 = 385$ lb. per sq. in. The actual maximum on outer edge of angle is slightly greater than this but well within the safe bearing strength of the concrete. Assume it to be 500 lb. By proportion the neutral axis lies $500 \times 20/(6970 + 500) = 1.4$ ft. from the tangent $G'-G'$. The assumed position $N-A$ was 0.8 ft. from $G'-G'$, indicating that a small portion only of the rim circle takes all of the compression and justifying the approximate assumptions made.

The Pitch of Rivets connecting the base angle to the rim of the tank should also be computed. Assuming the maximum compression to be 500 lb. per sq. in., the compression per lineal inch of angle $= 500 \times 3 = 1500$ lb. Safe single shearing strength of a 3/4-in. rivet = 4400. $p = \frac{4400}{1500} = 3$ in.

Anchor bolts must be connected to the sides, not to the bottom plate of the structure. To prevent buckling of the side plates under wind pressure a ⊏ or a heavy ∟ must be riveted around the upper edge. In addition ∟'s are sometimes riveted circumferentially on the outside at different elevations.

The Stress on a Rivet in a horizontal joint in pounds due to the weight of the standpipe above that joint = $0.027\ pW/D$; and due to wind pressure acting on the cylindrical surface above the joint = $0.106\ pM/D^2$. With a pressure of 30 lb. per sq. ft. this stress on a rivet due to wind becomes $1.06\ pH^2/D$. Or the pitch of these rivets in inches required to withstand the combined stresses due to weight and wind = $DV/(.027\ W + 1.06\ H^2)$. The stresses due to ice are indeterminate. All joints must be calked after riveting.

Allowable stresses per square inch on steel field rivets: shearing 9000, bearing 18 000. For 1/4-in. plates 5/8-in. rivets should be used; for 5/16-in. plates, 3/4-in. rivets; for 3/8 to 7/8-in. plates, 7/8-in. rivets; for 15/16 to 1-1/8-in. plates, 1-in. rivets. For horizontal joints use lap-joints, usually single riveted. For vertical joints use double-riveted lap-joints for 1/4, 5/16 and 3/8-in. plates; triple-riveted lap-joints for 7/16 and 1/2-in. plates; double-riveted butt-joints for 9/16 to 3/4-in. plates inclusive; triple-riveted butt-joints for 13/16 to 1-in. plates inclusive.

51. Water Tanks on Towers

Notation. W_1 = weight of water in pounds vertically above and below a horizontal section together with the weight of the conical or hemispherical bottom below the section. W = weight of water in pounds in the tank together with the weight of bottom. θ = angle between element of conical bottom, or between tangent to spherical bottom, and horizontal. D = diameter of cylindrical portion of tank in feet. d = diameter of a horizontal joint in a conical bottom in feet. h = distance in feet from surface of water to any point in a radial joint of a conical bottom. l = unsupported length and r = least radius of gyration of cross-section of column, both in inches. M = moment at a horizontal plane due to wind pressure on structure above that plane. r_1 = radius of circle passing through column centers.

A Water Tank on a Tower consists of a steel tank, usually cylindrical, supported on a steel tower which may have four or more columns connected to the tank at or near the bottom and connected to each other by horizontal and diagonal braces. The bottom of the tank may be flat, conical or hemispherical; the latter is the best and is riveted directly to the lowest cylindrical side plates at intersection of center line of column with side plate. Elevated tanks are more economical and of more pleasing appearance that standpipes. Ice pressure causes fewer failures of elevated tanks than of standpipes, and the tanks are usually covered with conical or curved roofs. Fig. 156 shows typical details of construction of a water tank and tower. The cylindrical part is similar in action to the corresponding portion of a standpipe.

The Riser or Inlet Pipe is the supply pipe, which usually also serves as an outlet; enters the tank at the center of the bottom, at which place the tank must be reinforced. An **expansion joint** is placed in the riser at or near the junction with the tank. Risers should be braced laterally at each panel point of the tower and in cold climates should be covered with a **frost-proof casing** consisting of three thicknesses of planking and two of tarred paper with air spaces between the thicknesses of planking.

A Conical Roof made of 1/8-in. sheet steel is self-supporting for diameters of 20 ft. or less if the roof plates are riveted to the upper edge of the top side plate; for diameters greater than 20 ft. ∟ rafters must be used. Iron ladders extend from a trap-door in the lower part of the roof down the outside and inside of the tank, ending on the balcony on the outside. Another ladder, attached to one of the tower columns, should run from the balcony to within 8 or 9 ft. of the ground. The balcony should be

3 ft. wide and its floor should be a horizontal plate girder placed at intersection of bottom and side plates.

Conical Bottoms. Tension in pounds per linear inch of any horizontal joint or section $= \dfrac{0.0265\ W_1}{d \sin \theta}$; at intersection of bottom and side plates this tension becomes $0.0265\ W/D \sin \theta$. Tension at any point of a radial joint

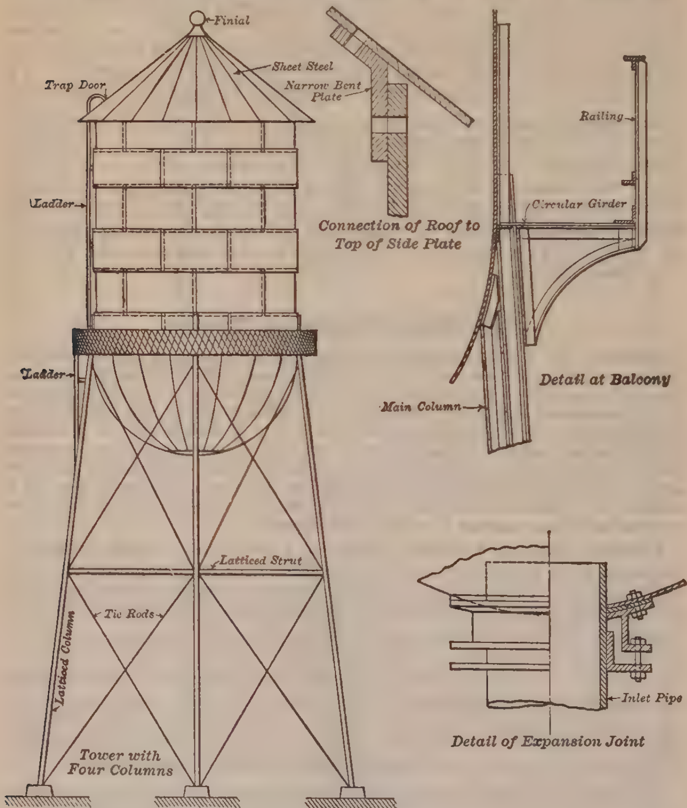


Fig. 156. Water Tank on a Tower

in pounds per linear inch, that is, of a joint coinciding with an element of cone $= \dfrac{2.6\ dh}{\sin \theta}$. A horizontal outside girder must be used at the intersection of side plates with the bottom to resist the annular compression caused by the inward pull from the bottom. Horizontal component of inward pull from bottom $= 0.0265\ W/D \tan \theta$ pounds per circumferential inch; annular compression $= 0.0132\ W/\tan \theta$; and vertical component $= 0.0265\ W/D$ pounds per circumferential inch. This vertical component is resisted by a vertical circular girder.

Spherical Bottoms. Tension in pounds per linear inch of any horizontal joint or section $= \dfrac{0.0265\ W_1}{D \sin^2 \theta}$. The spherical bottom exerts only a vertical pull on the cylindrical side, amounting to 0.0265 W/D pounds per circumferential inch. Best design requires the thickness of spherical bottoms to be equal to that of the lowest side plate in the cylindrical part. The cylindrical side plates must be reinforced at column connections.

Allowable Unit Stresses in pounds per square inch. Tension: in tank plates, 12 000; other parts, 16 000. Compression, 16 000 − 70 l/r; the ratio l/r should in no case exceed 120. Shear: on field rivets in tank, and bolts, 9000; on shop rivets and pins, 12 000. Bearing: field rivets in tank, and bolts, 18 000; shop rivets and pins, 24 000. Bending in pins, 24 000. For combined wind and other loads the above unit stresses may be increased 25%. All steel except rivets to be of "structural" grade; rivets of "rivet" steel.

Stresses in Columns are due to weight of structure, weight of water in tank, and overturning effect of the wind. The vertical component of stress in any column due to the first two forces is equal to the total weight divided by the number of columns.

52. Coal, Ore, and Grain Bins

A Steel Bin consists of a tank supported on columns and is used for handling or storing materials such as coal, ashes, ore, broken stone, sand and grain. Bins for coal, ashes or other similar materials are usually shallow, and admit of a variety of designs, while grain bins are deep and of cylindrical form. There are three types of bins, the hopper bin, the suspension bin or bunker and the cylindrical bin. For handling coal and ore the construction is such that these materials are dumped into the top of the bins from cars, or conveyors above and when needed are taken from the bottom or side by means of gates and chutes.

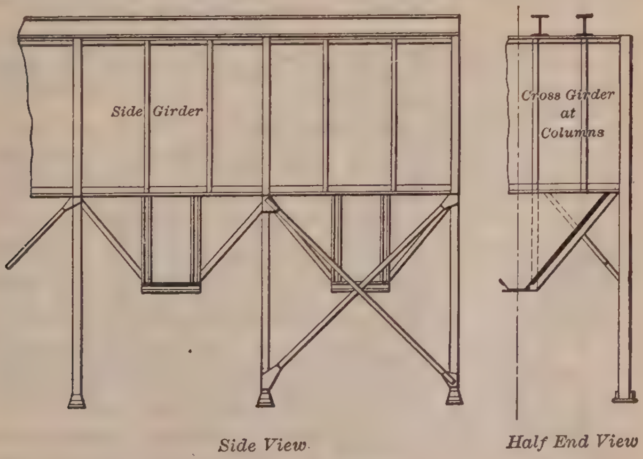


Fig. 157. Hopper Bin

Hopper Bins (Fig. 157) are generally arranged in units, each unit consisting of a square or rectangular box with four vertical sides and having an

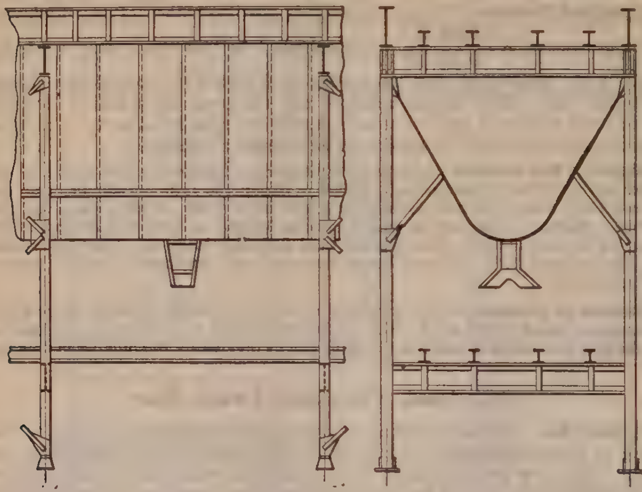


Fig. 158. Suspension Bin.

inverted frustum of a pyramid for a bottom. This bin is mounted on columns, which may be braced longitudinally with diagonal bracing, and transversely with knee-braces or diagonals. The gate through which the bin is emptied is placed at the bottom of each hopper, and the inclination of the sides of the hopper should be such that the bin can be emptied by gravity alone.

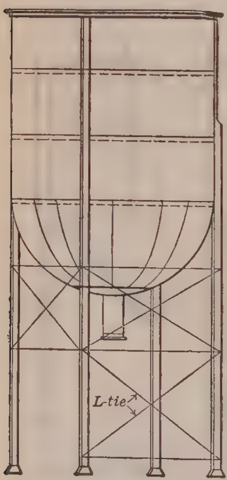


Fig. 159

Suspension Bins (Fig. 158) are steel bunkers supported on steel columns, the bunker being made of plates suspended from two parallel girders which are supported by the columns. The plates are subjected to tension only, and the side girders should be placed with their webs in the plane of this tension, or if placed vertically some provision must be made for resisting the horizontal component of pull from the plates. The ends of the bunker are made of plates stiffened by I beams or ∟'s. To protect the steel against action of materials stored in the bunker, the inside is covered with a layer of concrete reinforced with expanded metal or other similar fabric.

Circular Bins (Fig. 159) are cylindrical, with hemispherical bottoms, and with supporting columns riveted to the cylinder at the junction of the hemisphere. They are used principally for the storage or handling of ore, the outlet gate being placed at the center of the bottom. The construction is similar to that of water tanks on towers, except that the columns are vertical, and no balcony, with its accompanying horizontal circular girder, is necessary.

Steel Grain Bins are usually cylindrical, with either flat or hemispherical bottoms, and for large grain elevators are arranged in groups. The Great Northern elevator at Buffalo, N. Y., has thirty bins 38 ft. in diameter and 85 ft. deep set in three rows of ten bins each, and between these are eighteen smaller ones each 15.5 ft. in diameter arranged in two rows. All of these have hemispherical bottoms tapering into cones and are supported on circular girders resting on columns.

Weight and Angle of Repose of Dry Materials

Material	Weight, lb. per cu. ft.	Angle of repose, degrees	Material	Weight, lb. per cu. ft.	Angle of repose, degrees
Anthracite coal. .	52	27	Wheat.	50	28
Bituminous coal.	47 to 55	35	Barley.	39	27
Ashes.	40	40	Oats.	28	28
Sand.	90 to 115	34	Corn.	44	28

53. Catenary Bridges

A Catenary Bridge, (Fig. 160), is a light truss bridge which crosses the track of an electrified railroad, and the main function of which is to support an overhead wire over the center of each track. It is supported at the sides of the outside tracks, and occasionally at an intermediate point when many tracks have to be crossed, by latticed posts or towers to which it is securely riveted. The end towers extend above the bridge to support cross arms which carry the feed, signal and grounding wires.

The trolley wire is supported by a system of suspension cables, called the **catenary**, which carries the weight of the trolley and maintains the latter in a plane parallel to the tracks. The types of catenaries and their supporting bridges described in this article are those in use on the N. Y., N. H. & H. R. R. The two systems of catenaries are known as the single and the compound. The single consists of a main **messenger cable** which supports, by hangers and clips, the trolley or contact wires. The latter consists of two wires, the top one of copper (to secure proper conductivity), the bottom one of steel for contact purposes, it having been found that copper wire is too rapidly abraded. The copper feeds the steel wire at intervals of ten feet. The entire system is alive and is suspended below the bridges from which it is insulated. The advantages of this system are its lightness, ease of erection and permanency of position on curves. It weighs about 1200 lb. per track for spans of 280 to 300 ft. between bridges. The disadvantage is the non-protection of the track wire, in electrical storms, by an overhead wire. The compound system consists of a main messenger cable, supported by saddles on top of the bridges, Fig. 160, from which, at the quarter points of the cable span, is supported and insulated from it a single catenary system. In this type the main cable is dead. The advantage of this system is the protection afforded by the main cable by being grounded where it passes over each bridge. The disadvantages are its weight, which is about 2400 lb. per track for spans of 300 ft., the additional cost of erection,

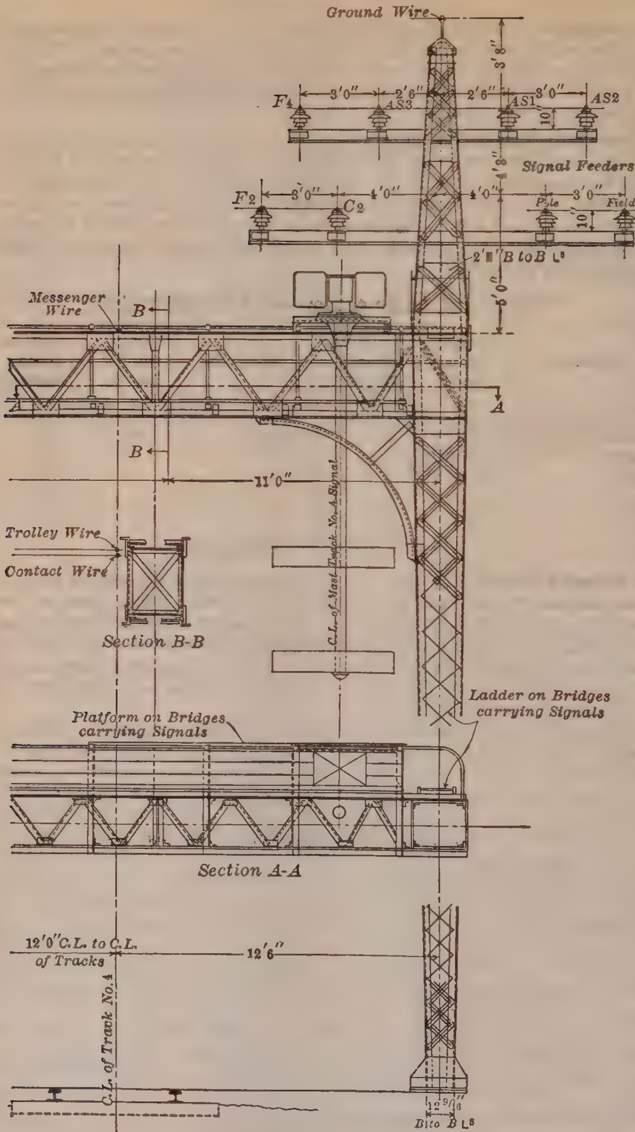


Fig. 160. One End and Tower, Four-track Catenary Bridge

the effect of temperature in changing alignment on curves, and the greater maintenance cost.

The Economical Spacing of the bridges on the New Haven road is from 150 to 315 ft. In addition to carrying the catenary system a secondary function of the catenary bridges is to support signals where they are required by the necessities of the block system sidings, bridges, etc., and the platforms needed for the maintenance of the signals. Visibility requires that the signals should project downward and frequent changes in tracks, location of cross-overs, etc., necessitating the relocation of signals make it advisable to design each bridge with sufficient strength to carry the signal loadings. The approximate weight of one automatic signal and its platform is 2400 lb.

The Loads for which the bridges are designed are (1) a signal over each track, (2) the vertical and horizontal loads from the catenary over each track, (3) the weight of the bridge, (4) the vertical and horizontal loads from the feed, signal and grounding wires on each cross arm, (5) the wind on the bridge surface, acting either along the track or laterally, and (6) the wind acting laterally upon the wires regarded either as bare or covered with ice.

The Catenaries are strung at definite tensions which depend upon the sag and accordingly vary with the temperature (Art. 46). When the catenaries are taken around curves the messenger cable is pulled away from the track in order to maintain the trolley wire over the center line. The catenary then lies in an inclined surface and the hangers resist the tendency of the trolley wire to straighten out. The lateral force required to maintain this condition is the **curve pull** which has to be supported by the catenary bridge as a horizontal load. The curve pull is very approximately equal to the cable tension multiplied by the span length and divided by the radius of the curve. Wind is assumed at 30 lb. per sq. ft. on the full windward and half the leeward surfaces of bridges, and upon two-thirds the projected areas of all bare wires. It is taken at 8 lb. per sq. ft. upon two-thirds the projected areas of wires coated with 1/2 in. of ice all around. A higher wind does not permit the ice to form, or will break it off if already formed. The wind blowing across the tracks on the catenaries and cross-arm wires constitutes a lateral load upon the catenary bridges.

Stresses in trusses and towers of the usual two-post type may be found approximately by the method employed for mill-building bents (Art. 3). The towers may be regarded as hinged or as partially fixed at the base according to the degree of efficiency of the anchorage. Equal horizontal reactions may be assumed at the feet of the towers (or at the points of contraflexure when the feet are regarded as partially fixed).

The Allowable Unit Stresses adapted by the New Haven road are as follows: tension from vertical loads, 20 000 lb. per sq. in.; tension from combined wind and vertical loads, 25 000 lb. per sq. in.; compression from all conditions of loading, $21\,000 - 70\,l/r$. Open-hearth steel with an ultimate strength of 60 000 lb. per sq. in. is used.

54. Transmission Towers

(Much of the following Article has, with the permission of the American Bridge Company, been taken from its Handbook, Transmission Towers.)

Steel towers, bents and poles are used to support high-tension transmission lines. The spacing depends upon the strength of the cables and the topography of the country, and varies from 200 or 300 ft. to several thousand. The average span is 400 or 500 ft., but river crossings may be much longer. The longest span at present is that of the Cushman Project over the Narrows, near Seattle, Wash., which is 6240 ft. The highest towers are those of the Pacific Gas & Electric Co. at the Vaca-Contra-Costa Crossing near San Francisco, with a maximum height of 459 ft. from the ground. When topographical conditions permit, the span and sag are made such that the tension in the cables, under the worst conditions of dead load, minimum temperature, wind and ice coating, will be slightly below the elastic limit of the cables. The choice of cables, and consequent spacing and height of towers, is an economic problem depending for its solution upon many attendant conditions. Properties of solid and stranded cables made of copper, aluminum or aluminum reinforced with steel, and copper-clad steel wire are given in the handbooks of structural and wire companies.

Cables. The greater the ratio of sag to span the greater is the allowable span for a given cable. On the other hand, the greater will be the height of the towers and the absolute length of cable needed. The economic sag ratio is about 3 to 4%. For this small sag ratio the curve of the cable very closely approximates a parabola, and that assumption is generally made.

For supports on the same level, Fig. 161:

$$H = \frac{WL^2}{8S}, \quad S = \frac{WL^2}{8H} = \sqrt{\frac{(L_c - L)\,3L}{8}}, \quad L_c = L + \frac{8S^2}{3L}.$$

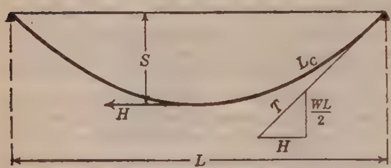


Fig. 161

L = span length in feet; L_c = cable length in feet; S = sag in feet; H = tension in cable at center of span; W = weight of cable in pounds per linear foot. With ice on the cable W = weight of cable plus weight of ice per linear foot. With horizontal wind on bare or ice-covered cable, W = square root of the sum of the squares of the vertical and horizontal loads. The Joint Committee of the National Electric Light Association recommends the following classes of loading on cables:

Class A. A horizontal wind load of 15 lb. per sq. ft. on projected area of bare cable, combined with its dead load.

Class B. A horizontal wind load of 8 lb. per sq. ft. on projected area of cable when covered with a coating of 1/2 in. of ice (total 1 in., plus diameter of wire), combined with the vertical load of cable and ice.

When severe sleet conditions may occur:

Class C. A horizontal wind load of 11 lb. per sq. ft. on projected area of cable when covered with a coating of 3/4 in. of ice (total 1-1/2 in. plus diameter of wire), combined with the vertical load of cable and ice. These wind loads are equivalent to about 60% of the actual normal pressure upon a flat diametral surface.

When the supports are at different levels, (Fig. 162), the actual weight of the cable WLc does not differ appreciably from WL_0, but may differ considerably from WL. To bring the parabolic formulas to the same degree of approximation as those for supports on the same level, the total load is taken as WL_0, making the average load per horizontal foot $= WL_0/L = W/\cos a$. The following formulas are based upon this assumption.

$$S_0 = WLL_0/8\,T = WL^2/8\,T \cos a$$
$$S_1 = WL_1^2/8\,H \qquad L_1/2 = L/2 - hH \cos a/WL$$
$$S_2 = WL_2^2/8\,H \qquad L_2/2 = L/2 + hH \cos a/WL$$
$$Lc = L + 4/3\,(S_1^2/L_1 + S_2^2/L_2).$$

Further, $S_0 = S$. That is, the normal sag of the inclined cable, loaded with $W/\cos a$ per horizontal foot, equals the vertical sag of a cable with the same horizontal span, with supports at the same level, and loaded with W pounds per horizontal foot.

The maximum tension occurs at the lowest temperature at which the maximum loading of ice and wind is possible, and this will vary with the locality. The maximum tension actually occurs, at the supports, but with such small sag ratios the error in using the more convenient center tension is negligible. Given the span, the maximum allowable tension in the cable, and the loading, the sag and length of cable can be found by the above formulas. For any other temperature or condition of loading the corresponding sag and cable length are most readily obtained from charts or it may quickly be computed by the method of successive approximations.

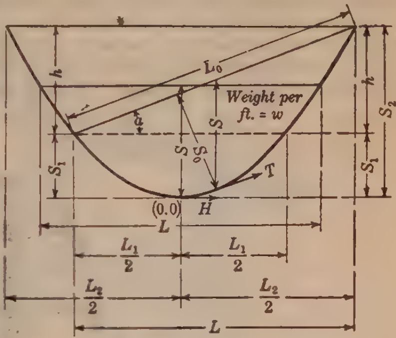


Fig. 162. Supports at Different Elevations.

Example. Span = 1000 ft.; diameter of cable = 0.63 in.; area of cable = 0.236 sq. in.; ultimate strength of cable = 14 154 lb.; maximum allowable tension = 7000 lb.; lowest temperature at which Class B loading may occur = 20° F. Find the sag, length and tension in the bare cable, without wind, at 120° F. Weight of bare cable = .93 lb. per lin. ft. Weight of 1/2 in. of ice = .64 lb. per lin. ft. Total vertical load = 1.57 lb. per lin. ft. Resultant load from dead weight, ice and horizontal wind = $\sqrt{(1.10^2) + (1.57)^2}$ = 1.90 lb. per lin. ft.

Sag at 20° = $1000^2 \times 1.90/8 \times 7000$ = 34 ft.

$Lc = 1000 + 8 \times 34^2/3000 = 1003.09$. Increase in length due to 100 degrees rise of temperature = $1000 \times .0000095 \times 100 = 0.95$ ft. Assume a decrease in tension at 120° of 3580 lb. due to increased sag and no ice or wind. Corresponding decrease in length = stress per square inch $\times$ length/E = $3580 \times 1000/.236 \times 16\,000\,000$ = 0.95 ft., where E = modulus of elasticity of the cable. Total length = 1003.09 + .95 − .95 = 1003.09 ft. Corresponding sag = $\sqrt{3.09 \times 3000/8}$ = 34 ft. Tension = $WL^2/8\,S = 1000^2 \times .93/8 \times 34 = 3420$ lb., which agrees with the assumed tension. It is merely a coincidence that, in this example, the lengths and sags, respectively, at the two temperatures are the same. The length and sag at any stringing temperature can be found in a similar manner.

Towers. Self-supporting towers with three or four legs, capable of resisting an unbalanced pull in the direction of the line from one or more conductors, as well as a transverse wind load from the wires, are generally used, although flexible bents and even steel latticed poles are sometimes employed on secondary lines. In the latter case self-supporting towers should be introduced frequently along the line.

Ordinary towers, (Fig.163), are called **suspension or line towers** when placed on tangents, or when the angle in the line does not exceed 5°, and **angle towers** when the angle is greater than 5°. The latter are designed for the same loads as the former with the addition of the transverse pull due to the angle in the line. **Dead-end** or **Anchor towers,** (Fig. 164), are designed to take the dead-end pulls from all of the cables on one side.

Loads. In addition to the weight of the tower itself, the insulators and the bare or ice-covered cables, the tower must be designed for the lateral pressure of the wind on tower and cables, and for an unbalanced pull in the direction of the line from the cables due to dead-ending, or to the breaking of one or two wires in one of the adjacent spans. The wind on the towers should be taken at 25 lb., or 13 lb. per sq. ft. on 1.5 times the projected area of one face, depending upon whether Class A or Class B loading is used for the cables. When the tower is at an angle in the line the transverse pull from all or part of the conductors must be included. The necessary combinations of possible simultaneous loadings should be considered.

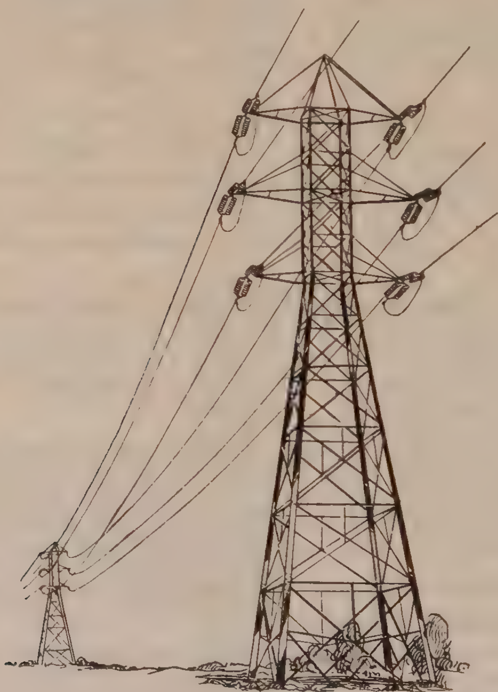


Fig. 163. Line Tower

Stresses. The towers should be made as simple in design as possible. The stresses in line or angle towers due to vertical and transverse loads will occasion no difficulty. The external forces are resolved into components lying in the planes of the faces and the stresses in each plane may be found either by graphical or analytic methods as in planar bents. At every change of slope of the faces the stresses in the preceding zone which act at the top joints of the new faces should likewise, together with the external forces at the same joints, be resolved into components lying in the planes of the new slope. The lower slope is then treated as a planar bent for the forces lying in its plane.

Provision should always be made for a possible unbalanced pull in the direction of the line, due to the dead-ending or breaking of one or more wires on one side of the tower. Such a pull will tend to twist the tower about its vertical axis and will create stresses in all of the faces. Fig. 165 is a standard

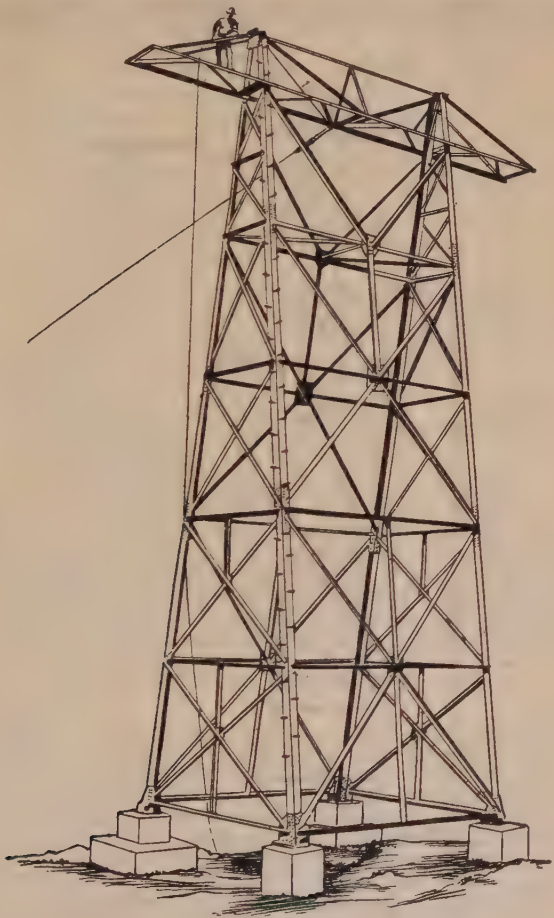


Fig. 164. Anchor Tower, River Crossing

four-legged, square, line or angle tower of the American Bridge Company. Assume an unbalanced horizontal pull P at one end of a cross arm (Fig. 165). There is created a torsion Pc about the vertical axis, and a longitudinal shear P at the center of the tower. The shear tends to overturn the tower longitudinally and may be replaced by two forces, each, $S = P/2$, acting in faces 1 and 3. If two diagonals, X, each capable of taking either compresssion or tension, be placed in the cross frame, $S/2$ will act at each corner of faces 1 and 3.

If the tower is square and all faces alike and equally rigid, the torsion Pc may be replaced by two equal couples, Tb, acting in the faces 1, 3, and 2, 4, respectively, or

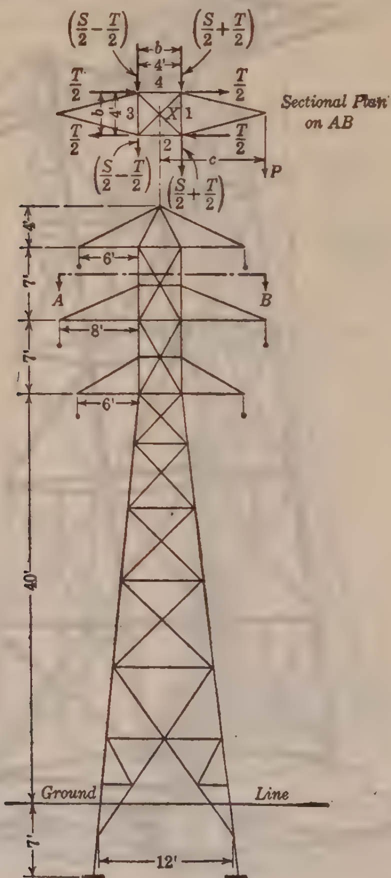


Fig. 165

$Pc = 2\ Tb$, $T = Pc/2\ b$. With two stiff diagonals X, $T/2$ will act at each corner of each face, all tending to rotate the tower about its vertical axis in the same direction as P. The stresses in the transverse faces 2 and 4 are then due to the two equal horizontal forces, $T/2$, applied in the plane of the cross frame at each corner. Similarly, the stresses in the longitudinal faces 1, 3, are due to a pair of horizontal forces, $S/2 + T/2$, and $S/2 - T/2$, respectively, likewise applied at the corners. The stresses in the columns due to these forces will be small, as they tend to neutralize each other in adjacent faces. The diagonal in the cross frame must transmit from face 1 to face 3 the total shear $S-T$. With two stiff diagonals in a square panel the stress in each is $0.7\ (S-T)$. A similar analysis should be made for every cross-frame bearing an unbalanced pull, P. If the tower is rectangular, or the faces are not equally rigid, the torsion is not resisted equally by each pair of faces, and the problem becomes indeterminate. An approximate assumption, however, may be made.

Unit Stresses. It is customary to use higher unit stresses for transmission towers than for bridges or buildings. The specifications of the American Bridge Company recommend:

For ordinary transmission towers:

Axial tension on net section....................20 000
(a) Axial compression on gross section (Max. 15 000)..20 000 − 85 l/r
(b) Axial compression on gross section...............15 500 − 55 l/r

Use (a) when l/r does not exceed 150.
Use (b) when l/r exceeds 150.

Shear on bolts or rivets........................13 500
Bearing on bolts or rivets......................27 000

l/r shall not exceed, for legs, 140; for all other members having calculated stress, 200; for members having nominal stress only, 250.

For river crossing, and other important structures:

(a) Axial compression on gross section (Max. 14 000)..18 000 − 80 l/r
(b) Axial compression on gross section...............13 500 − 50 l/r

Use (a) when l/r does not exceed 150.
Use (b) when l/r exceeds 150.

Shear on bolts or rivets........................12 000
Bearing on bolts or rivets......................24 000

Transmission towers are made relatively light. The bracing generally consists of single angles which are riveted or bolted directly to the tower legs. The latter are also either single angles, or very simple channel or angle sections, arranged so as to avoid latticing or expensive shop work. The same grade of steel and of workmanship should be used as for bridges. Towers are painted or galvanized, but the latter is preferable except for large towers and special structures where galvanizing is impracticable. When bolts are used for connecting galvanized members they should be "hot-dip" galvanized.

Anchors. The design of the foundations under the tower legs is an economic problem of great importance, requiring the best engineering judgment. In addition to the vertical loads, these foundations or anchors under the legs of the towers have to resist vertical uplift, and the horizontal shear from the unbalanced horizontal forces applied at the top of the tower. Anchors may be either of concrete, or of steel, or a combination of both. Concrete anchors may be either of the solid pedestal type, or a concrete slab buried several feet below the surface of the ground and connected therewith by a narrow concrete shaft. In the former case the tower rests on the top of the pedestal and is anchored thereto by long bolts that extend down nearly to the bottom. In the latter case the angle leg of the tower extends into the concrete slab, terminating at the bottom with angle lugs to give a good bond (Fig. 166). The angle leg is cut and spliced a little above the top of the concrete shaft, in order that the stub part may be set in the concrete when the latter is poured. If the projection of the slab beyond the shaft is more than its thickness steel reinforcing bars must be used.

Fig. 167 shows the all-steel anchor of the American Bridge Company. The vertical force and part of the horizontal is carried, by means of a stub angle, to a steel grillage. The bracing of the tower is attached to the stub angle at a point below the surface where sufficient earth resistance is met to take the major part of the horizontal force. If necessary to increase the resistance short channel lugs are bolted to the stub angle legs at this point. All-steel anchors should be galvanized for protection against corrosion. Sometimes, to insure further protection, the grillage and stub angles are embedded in concrete. Resistance of an anchor to uplift may be assumed equivalent to all of the weight contained in the inverted frustum of a pyramid whose base is the grillage and whose sides make an angle of 30° with the vertical. The weight of earth may be assumed at 90 lb. and that of the concrete at 140 lb. per cu. ft.

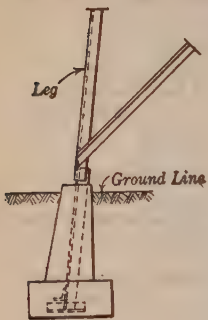


Fig. 166

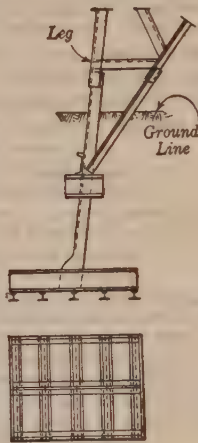


Fig. 167

SECTION 13

HYDRAULICS, PUMPING, WATER POWER

BY

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PRINCIPLES OF HYDROMECHANICS

1. Definitions

Hydromechanics deals with the equilibrium and motion of fluids and of bodies in or surrounding them. **Fluids** are those bodies which offer very small resistance to deformation. **Liquids** are those fluids whose particles tend to cling together. **Gases** are those fluids whose particles tend to separate from each other.

Hydromechanics is divided into **Hydrostatics,** which treats of fluids at rest, and **Hydrodynamics,** which treats of fluids in motion. **Hydraulics** is that part of Hydrodynamics which treats of water in motion.

Pressure is the force with which one body presses or acts upon another. The pressure acting on any surface is understood to be the sum of all the normal pressures and normal components of pressures acting on that surface. **Intensity of pressure** at any point is the pressure exerted upon a unit area at that point.

Pressure Head, at any point in a fluid at rest, is the vertical distance of the point from the free surface of the fluid. $\mathbf{h = p/w}$, where h is the head, p is the pressure in pounds per square foot, and w is the weight per cubic foot of the fluid. A pressure of one pound per square inch is approximately equal to that of a column of water 2.31 ft. in height.

Velocity Head. If, through a small area, the velocity of a fluid is v feet per second, the velocity head is $\mathbf{v^2/2\,g}$, g being the acceleration due to gravity in feet per second per second.

Head of Elevation is the height of a point above any assumed datum. If any point is z ft. above any convenient datum level, the head of elevation at that point above the given datum is said to be z ft.

Friction Head is the head absorbed in a fluid in motion, among its own particles and between itself and its bounding surfaces.

Density is the number of units of mass in unit volume of a substance. **Relative density** is the ratio of a given density to some density taken as a standard. When pure water at its maximum density is taken as a standard, the relative density becomes the specific gravity. Resistance to flow in liquids increases with density.

One Atmosphere gives a pressure equal to 14.7 lb. per sq. in. This is equal to the pressure at the base of a column of water 34.0 ft. high or of a column of mercury 30 in. high.

2. Properties of Water

Compressibility of water is very slight and varies greatly with the temperature and amount of air in solution. M. Grassi obtained the following results with distilled water:

Temperature Fahr.	Maximum pressure, atmospheres	Mean compressibility per atmosphere	Temperature Fahr.	Maximum pressure, atmospheres	Mean compressibility per atmosphere
32°	7.4	0.0000502	56°	8.4	0.0000476
35°	10.0	0.0000515	79°	7.2	0.0000455
51°	5.1	0.0000480	128°	6.3	0.0000444

At a pressure of 65 000 lb. per sq. in., water is said to be compressed over 10%. (Eng. News, Oct. 4, 1900.)

Modulus of Compressibility or of Elasticity in Compression

Lord Kelvin (Enc. Brit., IX Ed., Vol. VII, p. 818) on the authority of Grassi gives the following values:

Temperature		Modulus of compressibility or elasticity		
Cent.	Fahr.	Grams per sq. cm.	Lb. per sq. in.	Lb. per sq. ft.
0°	32°	20 600 000	293 000	42 192 000
1.5°	34.7°	20 200 000	287 000	41 373 000
4.1°	39.4°	20 700 000	294 000	42 397 000
10.8°	51.4°	21 500 000	306 000	44 036 000
13.4°	56.1°	21 600 000	307 000	44 240 000
18°	64.4°	22 400 000	319 000	45 879 000
25°	77.0°	22 600 000	321 000	46 289 000
34°	93.2°	22 800 000	324 000	46 698 000
43°	109.4°	23 300 000	331 000	47 722 000
53°	127.4°	23 500 000	334 000	48 132 000

Impurities in water slightly affect the specific gravity, as shown by the following values from various authorities:

Rivers: Garonne, France	1.000149
Thames, England	1.0003
Mississippi, U. S. (filtered)	1.00025
Springs	1.0003 to 1.006
Pacific Ocean	1.0265
Dead Sea	1.172

Tests at the Cornell Hydraulic Laboratory showed turbid water to be lighter than the water from the same source when clear, probably because of air adhering to the material causing turbidity.

Air ordinarily dissolved in water varies with the temperature. The amounts of oxygen and of air dissolved at different temperatures under a pressure of one atmosphere, as determined at the Lawrence Experimental Station, are as follows:

Oxygen and Air Dissolved in Water under a Pressure of One Atmosphere

In parts per 100 000 by weight

Temperature Fahr.	Oxygen	Air	Temperature Fahr.	Oxygen	Air	Temperature Fahr.	Oxygen	Air
32°	1.470	6.38	51°	1.117	4.84	70°	0.899	3.91
33°	1.445	6.28	52	1.103	4.78	71	0.889	3.87
34	1.422	6.19	53	1.089	4.72	72	0.880	3.83
35	1.400	6.08	54	1.076	4.67	73	0.871	3.79
36	1.379	5.99	55	1.063	4.62	74	0.862	3.75
37°	1.358	5.89	56°	1.050	4.56	75°	0.853	3.71
38	1.338	5.80	57	1.038	4.51	76	0.844	3.67
39	1.318	5.72	58	1.026	4.46	77	0.835	3.63
40	1.299	5.64	59	1.014	4.41	78	0.826	3.59
41	1.280	5.56	60	1.003	4.36	79	0.817	3.55
42°	1.262	5.49	61°	0.992	4.31	80°	0.808	3.51
43	1.244	5.41	62	0.981	4.26	81	0.800	3.48
44	1.227	5.33	63	0.970	4.21	82	0.792	3.44
45	1.210	5.26	64	0.959	4.16	83	0.784	3.41
46	1.193	5.19	65	0.949	4.12	84	0.776	3.38
47°	1.177	5.12	66°	0.939	4.08	85°	0.768	3.34
48	1.161	5.05	67	0.929	4.04	86	0.760	3.30
49	1.146	4.98	68	0.919	3.99	....	....	......
50	1.131	4.91	69	0.909	3.95	....	....	......

Effect of Temperature. The following table gives relative density or specific gravity and the weight of a cubic foot of water at various temperatures.

Specific Gravity and Weight of Water

Temperature Fahr.	Specific gravity	Pounds per cu. ft.	Log of weight per cu. ft.	Temperature Fahr.	Specific gravity	Pounds per cu. ft.	Log of weight per cu. ft.
15°	0.99831	....	....	80°	.99669	62.217	1.79391
20	.99898	....	....	85	.99592	62.169	1.79357
25	.99947	....	....	90	.99510	62.118	1.79322
30	.99979	....	....	95	.99418	62.061	1.79282
32	.99987	62.416	1.79529	100	.99318	61.998	1.79238
35°	.99996	62.421	1.79533	110°	.99105	61.865	1.79144
39.3	1.	62.424	1.79535	120	.98870	61.719	1.79042
40	0.99999	62.423	1.79534	130	.98608	61.555	1.78926
45	.99992	62.419	1.79532	140	.98338	61.386	1.78807
50	.99975	62.408	1.79524	150	.98043	61.203	1.78673
55°	.99946	62.390	1.79511	160°	.97729	61.006	1.78537
60	.99907	61.366	1.79495	170	.97397	60.799	1.78390
65	.99859	62.336	1.79474	180	.97056	60.586	1.78237
70	.99802	62.300	1.79449	200	.96333	60.135	1.77913
75	.99739	62.261	1.79422	212	.95865	59.843	1.77701

3. Hydrostatics

Laws of Perfect Liquids. (1) At any point in a liquid at rest the hydrostatic pressure is the same in all directions. (2) At all points in a horizontal plane in a liquid at rest the pressure is the same. (3) The pressure of water at rest, upon the surface of the vessel containing it, is normal to that surface at every point of it. (4) If a body is immersed in a liquid, the pressure upon it will be normal at every point of the surface of the body. (5) Every external pressure upon a liquid in a vessel will be transmitted with equal intensity to all parts of the liquid and of the containing surface.

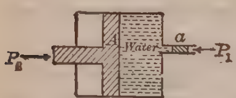


Fig. 1

The Hydrostatic Press, often called the hydraulic press, is used to exert heavy pressures generated in accordance with law (5). Let P_1 be a force applied through the small area a; its intensity per unit of area is P_1/a, and this acts with equal intensity upon all parts of the area A of a large piston in a strong cylinder. Thus the total pressure P_2 on the large piston is $\mathbf{P_2 = P_1 A/a}$. If $A = 1000a$, then $P_2 = 1000\ P_1$. The work of the resisting force P_2 cannot, however, be greater than the work of the applied force P_1.

Head and Pressure. In still water the unit pressure at any point is directly proportional to its distance below the surface. A head of 1 ft. gives a pressure of 0.4335 lb. per sq. in.; 1 lb. per sq. in. pressure is produced by a head of 2.3068 ft. These values are for fresh water at its maximum density (39.3° F.)

The Center of Pressure for any plane surface acted upon by a fluid is the point of action of the resultant pressure acting upon the surface. The resultant presssure $\mathbf{P}$ on any submerged plane area $= \mathbf{Az_gw}$, where A is the area, z_g is the vertical distance from the free surface of the fluid to the center of gravity of the area, and w is the weight of unit volume of the fluid.

To find the location of the center of pressure of a plane area, extend the plane until it cuts the free surface of the liquid and call the trace OY then

$$x_c = \frac{I_y}{Ax_g} = \frac{r^2{}_{gy}}{x_g}$$

where x_c is the perpendicular distance from the center of pressure to OY, I_y is the moment of inertia of the area about OY, A is the area, x_g is the perpendicular distance from the center of gravity of the area to OY, and r_{gy} is the radius of gyration of the surface about the axis Y, and $r^2{}_{gy} = x_g^2 + r_g^2$. Values of x_g and r_g^2 are found from Sect. 4, for common plane areas, x_g being equal to the disrance from the center of gravity to the upper edge plus the distance from that edge to the axis OY, and r_g is radius of gyration corresponding to axis through G. For example, let a triangle of base b and altitude d be immersed vertically in water with its vertex at a distance $2\ d$ below the surface and its base parallel to the surface; then $x_g = 2\ d + 2/3\ d$, and $r_g^2 = 1/18\ d^2$, whence $x_c = 129/48\ d$.

Flotation. When a body floats in a fluid, the surface of the body in contact with the fluid is subject to hydrostatic pressures, the intensity of pressure on any element of the surface depending upon its depth below the surface. The resultant of the vertical components of these hydrostatic pressures is called **buoyancy.**

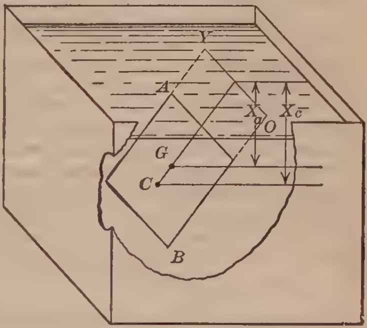


Fig. 2

Archimedes' Principle. The resultant pressure of a fluid on a body immersed in it acts vertically upward through the center of gravity of the displaced fluid and is equal to the weight of the fluid displaced.

Center of Buoyancy is the center of gravity of the displaced fluid and is the point of application of the resultant of all the upward forces acting on the body.

Stability of Floating Bodies. If the weight of the solid is not equal to the weight of the displaced fluid, that is, to the buoyancy effort, or if its center of gravity does not lie in the same vertical line with the center of buoyancy, the two forces form an unbalanced system and motion begins. If, when the forces are equal, the center of gravity of the solid lies in the same vertical line as the center of buoyancy and underneath the latter, the equilibrium is stable, whereas, if above, the equilibrium is unstable, and if the two centers coincide, the equilibrium is indifferent.

The equilibrium may be stable throughout a limited displacement with the center of gravity above the center of buoyancy depending on the shape of the body.

Viscosity is the evidence of cohesion between the particles of a fluid and is that physical property of a fluid which causes it to offer a resistance, analogous to friction, to the relative sliding motion of two adjacent particles. This property, chiefly noticeable when the fluid is in motion, is the cause of all so-called fluid friction.

The viscosity of a liquid is assumed to be derived from Poiseuille's equation:

$$h_f = \frac{32\ \mu L v_m}{gD^2}$$

wherein h_f = loss of head, in feet of water, in length of pipe;
L = length of pipe in feet;
v_m = mean velocity in pipe in feet per second;
D = diameter of pipe in feet;
g = acceleration due to gravity = 32.16 ft. per sec., per second;
μ = absolute viscosity in pounds per second per square foot.
= at 32° Fahr. 0.001203
= at 50° Fahr. 0.000880
= at 68.4° Fahr. 0.000672
= at 100° Fahr. 0.000464
= at 160° Fahr. 0.000269
for water.

Viscosity decreases as temperature increases. To convert **absolute viscosity** in the C. G. S. system to the English system, multiply its value by 0.0672. **Kinematic viscosity** is the quotient obtained by dividing the absolute viscosity by the specific gravity of the liquid.

Surface Tension is caused by the cohesion of the molecules of a fluid and gives it the appearance of having an elastic skin at its surface of separation from a gas or any other fluid.

The Specific Gravity of a solid or liquid is the ratio of its weight to the weight of an equal volume of water. Thus, the specific gravity of a stone which weighs 150 lb. per cu. ft. is 150/62.5 = 2.4. For a body heavier than water the specific gravity is also the ratio of its weight to the loss of weight in water when entirely submerged. Thus, if W is the weight in air, and W' the weight in water, then **specific gravity** = $W/(W - W')$; for example, if a piece of lead weighing 6.45 lb. in air weighs 5.88 lb. when submerged, then the specific gravity of lead is 6.45/0.57 = 11.3. For a body lighter than water, sink it by means of a heavier body and then deduct the weight of the latter. For porous substances and for liquids special methods are used; see Sect. 11, Art. 3, for cement tests.

4. Hydrodynamic Laws

Torricelli's Theorem. The velocity of a jet of liquid discharging under a head H is the same as that acquired by a body falling through the same height.

Bernouilli's Theorem. In steady flow the sum of the velocity head, pressure head, and head of elevation at any point is equal to the sum of the corresponding heads at any other point ± the losses of head due to friction, + if the last point is downstream from the first point, − if upstream. This may be expressed by the following equation, known as Bernouilli's Theorem. Let two points be 1 and 2, 2 being downstream from 1; p_1 and p_2 be the fluid pressures at 1 and 2; v_1 and v_2 be the velocities at 1 and 2; h_{e1} and h_{e2} be the elevations of 1 and 2 above any convenient datum level; w be the weight of a cubic unit of the fluid; g be the acceleration due to gravity; h_f be the loss of head due to friction between 1 and 2. Then

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + h_{e1} = \frac{v_2^2}{2g} + \frac{p_2}{w} + h_{e2} + h_f$$

Conservation of Energy. In steady flow the sum of kinetic and potential energies at any point in a stream is equal to the sum of the kinetic and potential energies at any other point in the stream ± the loss of energy due to friction between the two points, + if the second point is downstream from the first point, − if upstream. If both sides of the equation of Bernouilli are multiplied by the mass passing either of the two points in unit time, the two sides can be reduced to the form of the summation of kinetic and potential energies.

Charles' or Gay-Lussac's Law. The pressure remaining constant, the volume of a given mass of gas varies directly with the absolute temperature. If the pressure is constant, the heaviness (and therefore the specific gravity) varies inversely with the absolute temperature. The volume and heaviness remaining constant, the pressure of a given mass of gas varies directly with the absolute temperature.

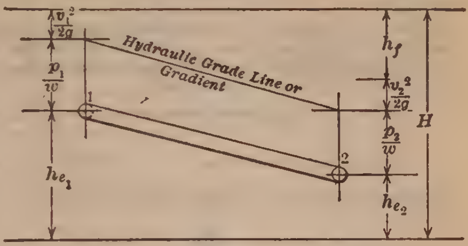


Fig. 3. Bernouilli's Theorem

Boyle's or Mariotte's Law. The temperature remaining constant, the pressure of a given mass of gas varies inversely with the volume and directly with the heaviness.

Dalton's Laws. In a mixture of different gases when there is equilibrium, each gas behaves as a vacuum to all the rest. If one or more liquids are introduced into a space of given volume, the amount of vapor given off by each depends only upon the temperature and pressure and is independent of any other gas or vapor present, and therefore the total pressures of the gases and vapors contained in the space is the sum of the separate pressures which each would exert if it were the only one present. The law does not hold good when the respective liquids and gases act chemically with one another.

Avogadro's Law. In equal volumes of gases having the same temperature and pressure the numbers of molecules are the same.

5. Jets and Vortices

Impact and Reaction. When a stream of a fluid impinges on a solid surface, it presses on the surface with a force equal and opposite to that by which the velocity and direction of motion of the fluid are changed. Generally in problems on the impact of fluids it is necessary to neglect the effect of friction between the fluid and the surface on which it moves and in the jet itself.

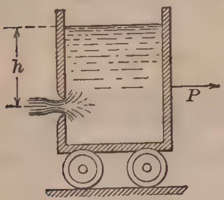


Fig. 4. Reaction of Jet

Reaction of Jet upon a vessel from which it issues. Let A be the area of the orifice, h be the head on the orifice, w be the weight of unit volume of the fluid. Then the reaction of jet is $P = 2\,Ahw$.

If the orifice is in a thin plate with a coefficient of velocity = 0.96 and a coefficient of contraction 0.64, then $P = 0.96 \times 0.64 \times 2\,Ahw$, or $P = 1.23\,Ahw$.

Impulse of a Jet on a Fixed Curved Vane with borders. Let θ be the angle through which the jet is turned, Q the volume of discharge of the jet per unit of time, U the absolute mean velocity of the jet. Taking the X axis as the axis of the jet before deflection and the Y axis at right angles to that in the plane of the jet, then,

$$P_x = \frac{Qw}{g} U (1 - \cos\theta) \qquad P_y = \frac{Qw}{g} U \sin\theta \qquad P = \frac{Qw}{g} U \sqrt{2\,(1 - \cos\theta)}$$

If ψ is the angle the force P makes with the X axis, $\tan\psi = \sin\theta/(1 - \cos\theta)$. For a flat vane perpendicular to axis of jet, $P = P_x = QwU/g$.

Impulse of a Jet on a Fixed Solid of Revolution whose axis coincides with the axis of the jet is the same as in the preceding case except that $Py = 0$. Then $P = P_x = Q_w U/g\ (1 - \cos\theta)$. The direction of the force is along the axis of the jet.

Impulse of a jet upon a curved vane with flanges at the sides to prevent water leaving it except in the direction of the tangent to its tip, and the vane moving with a uniform velocity in the direction of the jet, is found as follows: Let V be the absolute velocity of the vane, A the area of the jet; then

Force with which vane is moved is $P_x = \frac{Aw}{g}(U - V)^2(1 - \cos\theta)$

Work done on the vane, $W = \frac{Aw}{g}(U - V)^2\ V(1 - \cos\theta)$

Maximum efficiency is obtained when $\theta = 180°$ and $V = U/3$, and is 16/27, or 59.3%.

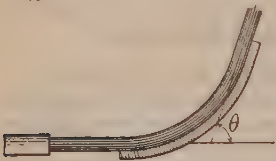


Fig. 5. Impulse upon Curved Vane

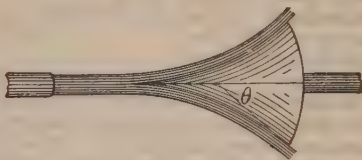


Fig. 6. Solid of Revolution

Impulse of a jet upon a solid of revolution, whose axis coincides with the axis of the jet, moving with a uniform velocity in the direction of the jet, is the same as in the above case of a moving curved vane.

Impulse of a jet upon a series of curved vanes with borders, or upon a series of surfaces of revolution, whose axes coincide with the axis of the jet; moving in the direction of the jet, is found as follows:

The force and work are

$$P = \frac{Aw}{g} U\ (U - V)\ (1 - \cos\theta) \qquad W = \frac{Aw}{g} U\ (U - V)\ V\ (1 - \cos\theta)$$

Maximum efficiency is obtained when $\theta = 180°$ and $V = U/2$, and is unity.

Impulse of a jet upon a flat vane without borders, moving in the direction of the jet is as follows:

Force and work are

$$P = \frac{Aw}{g}\ (U - V)^2 \sin^2\theta \qquad W = \frac{Aw}{g}\ (U - V)^2\ V \sin^2\theta$$

In this case θ is the minimum angle between the vane and the jet, and is 90° when vane is normal to axis of jet. The maximum efficiency is obtained when $\theta = 90°$ and $V = U/3$, and is 29.6%.

Impulse of a jet upon a series of flat vanes without borders, moving in the direction of the jet, is as follows:

Force and work are

$$P = \frac{Aw}{g}\ (U - V) \sin^2\theta \qquad W = \frac{Aw}{g}\ (U - V)\ V \sin^2\theta$$

The maximum efficiency is obtained when $\theta - 90°$ and $V - U/2$, and is 50%.

When the distance between nozzle and vane is less than the diameter of the nozzle, the discharge is reduced and the reaction on the vane is **decreased** whereas the pressure within the nozzle is increased.

Radiating Current. If water flows from a center outward between two parallel plates, and friction be neglected, the pressure head at any point distant R_2 from the center is

$$\frac{P_2}{w} = H - \frac{R_1{}^2 V_1{}^2}{2\ gR_2{}^2} \quad \text{and} \quad R_1V_1 = R_2V_2$$

where V_1 is the velocity at radius R_1 and V_2 that at R_2. If the discharge from between the plates is into air, the pressure at the outlet is atmospheric, and as the pressures decrease toward the center the pressure on the top of the upper plate is greater than that on the bottom of it. This explains the phenomenon of the so-called "ball nozzle."

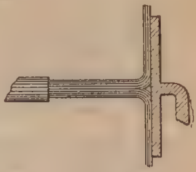


Fig. 7. Flat Plate

Vortices. A Free Circular Vortex is a revolving mass of water in which the stream lines are concentric circles and in which the total head for each stream line is the same, or $R_1V_1 = R_2V_2$ as in a radiating current.

A Free Spiral Vortex is a revolving mass of water having a radiating flow combined with a circular flow, in both of which

$$R_1V_1 = R_2V_2$$

and the same equation holds for the spiral motion in which the direction of the current will make a constant angle with the radius to its axis, or when the path of the current is a logarithmic spiral. The centrifugal pump delivers water into its shell in a free vortex of this kind, the velocity diminishing and the pressure increasing as it flows outward.

A Forced Vortex is a revolving mass of water in which, by the application of some force or forces, the law of velocity variation is caused to be different from the above. The simple case is that in which all the particles have equal angular velocity. Then if ω = angular velocity and $v = R\omega$, the pressure head at any point is

$$\frac{P}{w} = \frac{R^2\omega^2}{2g} + \text{a constant}$$

and the relations at any two points, 1 and 2, in the same horizontal plane are

$$\frac{P_2 - P_1}{w} = \frac{\omega^2}{2g}(R_2^2 - R_1^2) = \frac{V_2^2 - V_1^2}{2g}$$

This is the equation of the surface assumed by the water in a revolving vessel, and indicates that radial planes will cut the surface in parabolas with vertices downward and at the center. The height z_x of the surface at any point x above a tangent to the parabola at its vertex o is

$$z_x = \frac{P_x - P_o}{w} = \frac{\omega^2 R_x^2}{2g} = \frac{V_x^2}{2g}$$

Rectilinear Translation and Acceleration. When a mass of water is contained in a vessel and moves in a straight line with uniform velocity, the pressure upon each surface along horizontal planes will be uniform, but if the motion is uniformly accelerated, then the pressure decreases from the front to the rear of the vessel for positive acceleration and vice versa for negative acceleration or retardation.

If points 1 and 2 are distant x apart horizontally, and j is the acceleration,

$$(P_1 - P_2)/w = z_1 - z_2 = xj/g$$

and the surface has an angle of inclination ϕ whose tangent is j/g.

6. Hydraulic Computations and Tables

Units of Measure used in this Section are the second, the foot, and the pound-weight. For some minor quantities such as pressures and diameters the inch is employed instead of the foot, but the latter is to be used in all standard hydraulic formulas. (Art. 9 employs some metric units.)

Powers of Numbers, Velocities, Velocity Heads

n	$n^{3/2}$	$n^{2/3}$	$n^{1.25}$	$n^{0.8}$	$n^{1.87}$	$n^{0.585}$	$\sqrt{2gn}$	$n^2/2g$
0.01	0.0010	0.0464	0.0032	0.0251	0.0002	0.0851	0.802	....
0.02	0.0028	0.0737	0.0075	0.0437	0.0007	0.1233	1.134	....
0.03	0.0052	0.0965	0.0125	0.0605	0.0014	0.1532	1.389	....
0.04	0.0080	0.1170	0.0179	0.0761	0.0024	0.1787	1.604	....
0.05	0.0112	0.1357	0.0236	0.0910	0.0037	0.2013	1.793	....
0.06	0.0147	0.1533	0.0297	0.1053	0.0052	0.2219	1.965	....
0.07	0.0185	0.1699	0.0360	0.1191	0.0069	0.2410	2.122	....
0.08	0.0226	0.1857	0.0425	0.1326	0.0089	0.2589	2.269	....
0.09	0.0270	0.2008	0.0493	0.1457	0.0111	0.2757	2.406	0.0001
0.10	0.0316	0.2155	0.0562	0.1585	0.0135	0.2917	2.537	0.0002
0.2	0.0894	0.3420	0.1338	0.2759	0.0493	0.4227	3.587	0.0006
0.3	0.1643	0.4481	0.2220	0.3817	0.1052	0.5251	4.393	0.0014
0.4	0.2530	0.5429	0.3181	0.4804	0.1802	0.6125	5.073	0.0025
0.5	0.3536	0.6300	0.4205	0.5743	0.2736	0.6902	5.671	0.0039
0.6	0.4648	0.7114	0.5281	0.6645	0.3847	0.7609	6.213	0.0056
0.7	0.5857	0.7884	0.6403	0.7518	0.5133	0.8263	6.710	0.0076
0.8	0.7155	0.8618	0.7566	0.8365	0.6588	0.8875	7.174	0.0099
0.9	0.8538	0.9322	0.8766	0.9192	0.8212	0.9452	7.609	0.0126
1.0	1.000	1.000	1.000	1.000	1.000	1.000	8.021	0.0155
1.1	1.154	1.066	1.127	1.079	1.195	1.052	8.412	0.0188
1.2	1.315	1.129	1.256	1.157	1.406	1.102	8.786	0.0224
1.3	1.482	1.191	1.388	1.233	1.633	1.151	9.145	0.0263
1.4	1.657	1.251	1.523	1.309	1.876	1.197	9.490	0.0305
1.5	1.837	1.310	1.660	1.383	2.135	1.242	9.823	0.0350
1.6	2.024	1.368	1.799	1.456	2.408	1.286	10.15	0.0398
1.7	2.217	1.424	1.941	1.529	2.697	1.328	10.46	0.0449
1.8	2.415	1.480	2.085	1.600	3.002	1.370	10.76	0.0504
1.9	2.619	1.534	2.231	1.671	3.321	1.410	11.06	0.0561
2.0	2.828	1.587	2.378	1.741	3.655	1.449	11.34	0.0622
2.1	3.043	1.640	2.528	1.810	4.005	1.487	11.62	0.0686
2.2	3.263	1.692	2.679	1.879	4.369	1.524	11.90	0.0752
2.3	3.488	1.742	2.832	1.947	4.747	1.561	12.16	0.0822
2.4	3.718	1.793	2.987	2.015	5.140	1.597	12.43	0.0895
2.5	3.953	1.842	3.144	2.081	5.548	1.633	12.68	0.0972
2.6	4.192	1.891	3.302	2.148	5.970	1.667	12.93	0.1051
2.7	4.437	1.939	3.461	2.214	6.407	1.701	13.18	0.1133
2.8	4.685	1.987	3.622	2.279	6.858	1.734	13.42	0.1219
2.9	4.939	2.034	3.784	2.344	7.323	1.768	13.66	0.1307
3.0	5.196	2.080	3.948	2.408	7.802	1.800	13.89	0.1399
3.1	5.458	2.126	4.113	2.472	8.296	1.832	14.12	0.1494
3.2	5.724	2.172	4.280	2.536	8.803	1.863	14.35	0.1592
3.3	5.995	2.217	4.448	2.599	9.324	1.894	14.57	0.1693
3.4	6.269	2.261	4.617	2.662	9.860	1.924	14.79	0.1797

The last column contains values of the velocity head $n^2/2g$ for the values of the velocity n and the last column but one gives values of the theoretic velocity due to a head n. The value of g used in these columns is 32.162 ft. per sec. per sec.

Powers of Numbers, Velocities, Velocity Heads—Continued

n	$n^{3/2}$	$n^{2/3}$	$n^{1.25}$	$n^{0.8}$	$n^{1.87}$	$n^{0.535}$	$\sqrt{2gn}$	$n^2/2g$
3.5	6.548	2.305	4.787	2.724	10.41	1.955	15.01	0 1904
3.6	6.831	2.349	4.959	2.786	10.97	1.984	15.22	0.2015
3.7	7.117	2.392	5.132	2.848	11.55	2.014	15.43	0.2128
3.8	7.408	2.435	5.306	2.910	12.14	2.043	15.64	0.2245
3.9	7.702	2.478	5.481	2.971	12.74	2.071	15.84	0.2364
4.0	8.000	2.520	5.657	3.031	13.36	2.099	16.04	0.2487
4.1	8.302	2.562	5.834	3.092	13.99	2.127	16.24	0.2613
4.2	8.607	2.603	6.013	3.152	14.64	2.155	16.44	0.2742
4.3	8.916	2.644	6.192	3.212	15.30	2.182	16.63	0.2874
4.4	9.229	2.685	6.373	3.272	15.97	2.209	16.82	0.3009
4.5	9.546	2.726	6.554	3.331	16.65	2.236	17.01	0.3148
4.6	9.866	2.766	6.737	3.390	17.35	2.262	17.20	0.3289
4.7	10.19	2.806	6.920	3.449	18.07	2.289	17.39	0.3434
4.8	10.51	2.846	7.105	3.507	18.79	2.315	17.57	0.3582
4.9	10.85	2.885	7.290	3.566	19.53	2.340	17.75	0.3732
5.0	11.18	2.924	7.477	3.624	20.28	2.366	17.93	0.3886
5.1	11.52	2.963	7.664	3.682	21.05	2.391	18.11	0.4043
5.2	11.86	3.001	7.852	3.739	21.82	2.416	18.29	0.4203
5.3	12.20	3.040	8.042	3.797	22.62	2.441	18.46	0.4367
5.4	12.55	3.078	8.232	3.854	23.42	2.465	18.64	0.4533
5.5	12.90	3.116	8.423	3.911	24.24	2.489	18.81	0.4702
5.6	13.25	3.153	8.615	3.968	25.07	2.514	18.98	0.4875
5.7	13.61	3.191	8.807	4.024	25.91	2.537	19.15	0.5051
5.8	13.97	3.228	9.001	4.081	26.77	2.561	19.32	0.5231
5.9	14.33	3.265	9.195	4.137	27.64	2.585	19.48	0.5411
6.0	14.70	3.302	9.391	4.193	28.52	2.608	19.65	0.5596
6.1	15.07	3.338	9.587	4.249	29.41	2.631	19.81	0.5784
6.2	15.44	3.375	9.783	4.304	30.32	2.654	19.97	0.5975
6.3	15.81	3.411	9.981	4.360	31.24	2.677	20.13	0.6170
6.4	16.19	3.447	10.18	4.415	32.18	2.700	20.29	0.6367
6.5	16.57	3.483	10.38	4.470	33.12	2.722	20.45	0.6568
6.6	16.96	3.519	10.58	4.525	34.08	2.744	20.61	0.6771
6.7	17.34	3.554	10.78	4.580	35.06	2.767	20.76	0.6978
6.8	17.73	3.589	10.98	4.635	36.04	2.789	20.92	0.7188
6.9	18.13	3.624	11.18	4.689	37.04	2.811	21.07	0.7401
7.0	18.52	3.659	11.39	4.743	38.05	2.832	21.22	0.7617
7.1	18.92	3.694	11.59	4.797	39.07	2.854	21.37	0.7836
7.2	19.32	3.729	11.79	4.851	40.10	2.875	21.52	0.8058
7.3	19.72	3.763	12.00	4.905	41.15	2.896	21.67	0.8284
7.4	20.13	3.797	12.21	4.959	42.21	2.918	21.82	0.8512
7.5	20.54	3.832	12.41	5.012	43.29	2.939	21.97	0.8744
7.6	20.95	3.866	12.62	5.066	44.37	2.960	22.11	0.8979
7.7	21.36	3.899	12.83	5.119	45.47	2.980	22.26	0.9217
7.8	21.79	3.933	13.04	5.172	46.58	3.001	22.40	0.9458
7.9	22.20	3.967	13.24	5.225	47.70	3.022	22.54	0.9702

These tables give values of three-halves powers and two-thirds powers of numbers which are useful in weir computations, and also other powers which occur in exponential formulas for flow in pipes and channels. The last

Powers of Numbers, Velocities, Velocity Heads—Continued

n	$n^{3/2}$	$n^{2/3}$	$n^{1.25}$	$n^{0.8}$	$n^{1.87}$	$n^{0.535}$	$\sqrt{2gn}$	$n^2/2g$
8.0	22.63	4.000	13.45	5.278	48.84	3.042	22.69	0.9949
8.1	23.05	4.033	13.67	5.331	49.99	3.062	22.83	1.020
8.2	23.48	4.066	13.88	5.383	51.15	3.082	22.97	1.045
8.3	23.91	4.099	14.09	5.436	52.32	3.102	23.11	1.071
8.4	24.34	4.132	14.30	5.488	53.51	3.122	23.25	1.097
8.5	24.78	4.165	14.51	5.540	54.70	3.142	23.38	1.123
8.6	25.22	4.198	14.73	5.592	55.91	3.162	23.52	1.150
8.7	25.66	4.230	14.94	5.644	57.14	3.182	23.66	1.177
8.8	26.10	4.262	15.16	5.696	58.37	3.201	23.79	1.204
8.9	26.55	4.295	15.37	5.748	59.62	3.221	23.93	1.231
9.0	27.00	4.327	15.59	5.800	60.87	3.240	24.06	1.259
9.1	27.45	4.359	15.81	5.851	62.14	3.259	24.20	1.287
9.2	27.91	4.391	16.02	5.902	63.43	3.278	24.33	1.316
9.3	28.36	4.422	16.24	5.954	64.72	3.297	24.46	1.344
9.4	28.82	4.454	16.46	6.005	66.03	3.316	24.59	1.374
9.5	29.28	4.486	16.68	6.056	67.35	3.335	24.72	1.403
9.6	29.74	4.517	16.90	6.107	68.68	3.354	24.85	1.433
9.7	30.21	4.548	17.12	6.158	70.03	3.372	24.98	1.463
9.8	30.68	4.580	17.34	6.209	71.38	3.391	25.11	1.493
9.9	31.15	4.611	17.56	6.259	72.75	3.409	25.24	1.524
10.0	31.62	4.642	17.78	6.310	74.13	3.428	25.36	1.554
10.5	34.02	4.795	18.90	6.561	81.21	3.518	25.99	1.714
11.0	36.48	4.946	20.03	6.810	88.59	3.607	26.60	1.881
11.5	38.99	5.095	21.18	7.056	96.27	3.694	27.20	2.056
12.0	41.57	5.241	22.34	7.300	104.25	3.779	27.78	2.238
12.5	44.19	5.386	23.50	7.543	112.52	3.862	28.36	2.429
13.0	46.87	5.529	24.68	7.783	121.08	3.944	28.92	2.627
13.5	49.60	5.670	25.88	8.022	129.93	4.024	29.47	2.833
14.0	52.38	5.809	27.08	8.259	139.08	4.103	30.01	3.047
14.5	55.21	5.946	28.29	8.494	148.51	4.181	30.54	3.268
15.0	58.09	6.082	29.52	8.727	158.23	4.258	31.06	3.498
15.5	61.02	6.217	30.75	8.959	168.24	4.333	31.58	3.735
16.0	64.00	6.350	32.00	9.190	178.53	4.407	32.08	3.979
16.5	67.02	6.481	33.25	9.419	189.10	4.481	32.58	4.232
17.0	70.09	6.611	34.52	9.647	199.96	4.553	33.07	4.492
17.5	73.21	6.741	35.79	9.873	211.10	4.624	33.55	4.761
18.0	76.37	6.868	37.08	10.098	222.52	4.694	34.03	5.037
18.5	79.57	6.995	38.37	10.321	234.21	4.764	34.50	5.320
19.0	82.82	7.120	39.67	10.544	246.19	4.832	34.96	5.612
19.5	86.11	7.245	40.97	10.765	258.44	4.900	35.42	5.911
20.0	89.44	7.368	42.29	10.986	270.97	4.967	35.87	6.218

column contains values of the velocity-head $n^2/2g$ for various values of n, and the last column but one gives values of the theoretic velocity $\sqrt{2gn}$ for a head n. The value of g used for these columns is 32.162 ft. per sec. per sec.

See Sect. 1, Art. 9. for a more extended table of three-halves powers.

FLOW IN PIPES AND CHANNELS

7. Orifices and Short Pipes

Small Orifice in a Thin Plate. When the efflux takes place through an orifice in a thin plate whose diameter is not greater than 1/8 the head, a contracted vein, or "vena contracta," is formed, the filaments of water not becoming parallel until they reach a plane at right angles to the axis of the jet and about 1/2 its diameter from the orifice; and not until reaching this plane does the internal fluid pressure become the same as the pressure of the surrounding medium. If the vessel were large enough so that the velocity at the surface were zero, the theoretical velocity at the place of greatest contraction would be $\mathbf{v} = \sqrt{2\,gh}$, where h is the head at its center of pressure and g is the acceleration due to gravity. For orifices where the diameter is not greater than **1/8 h,** the center of pressure and center of gravity are assumed to coincide, and, in the case of orifices in a vertical wall, the centers of gravity of the orifice and of the contracted vein are assumed at the same elevation. Practically, however, the velocity $\mathbf{v} = C_v \sqrt{2\,gh}$, where C_v, the coefficient of velocity, is determined experimentally. The discharge then would be $\mathbf{Q} = C_cA\mathbf{v}$, where A is the area of the orifice, C_c the coefficient of contraction, and v the actual velocity at the place of greatest contraction. Then:

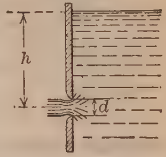


Fig. 8. Small Orifice in Thin Plate

$$Q = C_cC_vA\sqrt{2\,gh} = CA\sqrt{2\,gh}$$

where C is the coefficient of discharge.

For ordinary work very satisfactory results are obtained by using $C_v = 0.97$ and $C_c = 0.62$. C then $= C_cC_v = 0.61$. The values of C generally range from 0.63 to 0.60, the lower values occurring with the higher heads. Square and rectangular orifices have slightly higher discharges than circular ones, the increases being between 1% and 1-1/2%.

Submerged Orifices. The theoretical discharge is $Q = A\sqrt{2\,gh}$, where h is the difference in level of the free surfaces on the two sides of the orifice. Coefficients in this case are, according to Weisbach, 1/75 less than for free discharge into the air.

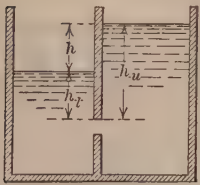


Fig. 9. Submerged Orifice

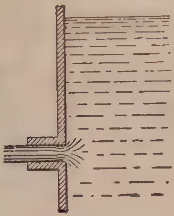


Fig. 10. Short Tube

Short Tube. If an external tube of the same diameter as the orifice and at least 2-1/2 diameters long is attached to it, the effluent stream, after forming a **"vena contracta,"** re-expands and fills the tube, and the discharge is increased. To attain this result, however, the tube must be full of water before the outer end is unstopped, and must not be oily; nor must the head h be greater than about 40 ft. for efflux into the air. The coefficient of velocity is then the coefficient of discharge, since the coefficient of contraction is unity. $C = 0.815$ for tubes from **2-1/2** to **3** diameters long. For other lengths the coefficients for tubes and short pipes are:

Length in diameters....	1	3	5	10	25	50	75	100
Coefficient...........	0.62	0.815	0.79	0.77	0.71	0.64	0.59	0.55

Borda's Tube is a short tube, of the same diameter as the orifice, attached to the orifice on the inside of the wall. If this tube is short enough so that the water is discharged without filling the tube, the theoretical coefficient C is 0.5, which is very nearly that obtained by experiment.

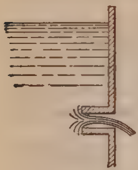


Fig. 11.
Borda's Tube

Converging Mouthpieces. The discharge through an orifice is increased as the filaments of water are made to approach it in a more nearly normal direction. Hence, if walls of an orifice converge downstream, the flow is increased until it reaches a maximum when the angle between two opposite sides is about 13-1/2°, and C then is about 0.94. For other angles the value of C is

For angle........	0°	10°	13.5°	20°	30°	40°	60°
C..............	**0.84**	**0.93**	**0.94**	**0.93**	**0.90**	**0.87**	**0.82**

Rounded Mouthpiece. Rounding the entrance further reduces the contraction and when the curve approximates to the "vena contracta," **C = unity** and $Q = C_v A \sqrt{2gh}$. Experimentally $C = C_v$ varies from 0.96 to 0.99.

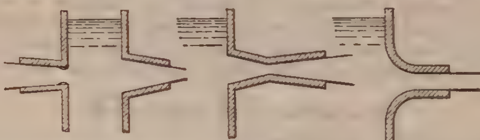


Fig. 12. Divergent, Convergent, Compound and Rounded Mouthpieces

When the upstream edge of an orifice is rounded, the discharge is increased. Schoder and Dawson ("Hydraulics," McGraw-Hill, N. Y.) give the following from the Cornell Hydraulic Laboratory, on discharge of orifices into air.

Values for 6 Ft. Heads

C = circular S = square

Orifice, in. shape	Radius of upstream edge in per cent of diameter	Coefficient of discharge	Per cent increase over sharp edge	Orifice, in. shape	Radius of upstream edge in per cent of diameter	Coefficient of discharge	Per cent increase over sharp edge
3/8 C	0	.609	0	1-3/8 C	1.56	.633	5.5
11/16 C	0	.601	0	8 S	1.56	.6365	4.7
1 C	0	.601	0	1 C	1.80	.638	6.2
1-3/8 C	0	.600	0	6 S	2.08	.650	6.4
6 S	0	.611	0	8 S	3.12	.662	8.9
8 S	0	.608	0	6 S	4.16	.693	13.4
1-3/8 C	0.65	.615	2.5	3/8 C	4.70	.680	11.6
1 C	.70	.620	3.2	8 S	6.25	.725	19.3
8 S	.78	.627	3.13	6 S	8.33	.772	26.4
6 S	1.04	.632	3.4	6 S	16.67	.850	39.1
11/16 C	1.16	.627	4.3	6 S	33.33	.867	41.9
3/8 C	1.20	.631	3.6				

Divergent Mouthpieces, if they flow full and are more than 5 diameters in length, will increase the value of C above that for the orifice at the inlet and in some cases to as much as 1.4 times.

Compound Mouthpieces. If a divergent tube, whose angle of divergence between opposite sides is about 10°, is attached to a rounded mouthpiece and caused to flow full, the head at the throat becomess less than the atmospheric pressure and may practically fall as low as − 24 ft. of water. The total head producing flow through the throat then is $\mathbf{h} +$ 24 ft. and $\mathbf{v} = \mathbf{C}\sqrt{2\,\mathbf{g}(\mathbf{h} + 24)}$, where C is for the rounded orifice less a small allowance for friction in the diverging tube. Theoretically it would be possible to have C as high as 8.0 or 9.0 for heads less than 1 ft. but experimentally 2.43 is the highest value obtained.

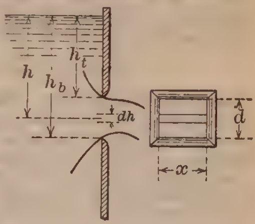


Fig. 13. Large Orifices

Large Orifices. This case differs from the discharge of " small orifices " in that the difference of head between the top and bottom of the orifice is so great that it must be taken into account. Let $\mathbf{h_t}$ be the distance from the free surface of the water to the top of the orifice; $\mathbf{h_b}$ be the distance from the free surface of the water to the bottom of the orifice; $\mathbf{x}$ be the width and $\mathbf{d} = \mathbf{h_b} - \mathbf{h_t}$ the depth of the orifice; $\mathbf{h}$ be the distance from the free surface of the water to any horizontal layer through the orifice of $\mathbf{dh}$ depth and width $\mathbf{x}$. Then the velocity of water through this thin layer is $\sqrt{2\,\mathbf{gh}}$, the small discharge $\mathbf{dQ}$ for this area is $\sqrt{2\,\mathbf{gh}} \cdot \mathbf{xdh}$, and the total theoretical discharge is

$$\mathbf{Q} = \int_{\mathbf{h_b}}^{\mathbf{h_t}} \sqrt{2\,\mathbf{gh}} \cdot \mathbf{xdh}$$

The width x may be either constant or variable, and if the latter, must be expressed in terms of h before integrating.

For a rectangle with $\mathbf{x} =$ **constant,** $\mathbf{Q} = 2/3\,\mathbf{x}\sqrt{2\,\mathbf{g}}(\mathbf{h_b}^{3/2} - \mathbf{h_t}^{3/2})$

For a vertical triangle, base x at the top and horizontal.

$$\mathbf{Q} = \frac{2}{15}\frac{\mathbf{x}\sqrt{2\,\mathbf{g}}}{\mathbf{h_b} - \mathbf{h_t}}[2\,\mathbf{h_b}^{5/2} - 5\,\mathbf{h_b h_t}^{3/2} + 3\,\mathbf{h_t}^{5/2}]$$

These equations are not exact theoretically, because the head is completely changed into velocity head at the " vena contracta " and not at the orifice as was assumed; because the assumption is that the velocity is constant for any area $x \cdot dh$, when it is known that at the edges the velocity is materially decreased; and because it is assumed that the water flows in plain layers at the orifice, which is not the case. A correct result could be more nearly approached were a point taken at the " vena contracta " instead of at the orifice, but accurate measurement is there impossible. However, by the introduction of an experimental coefficient, which is less variable than the C in the preceding articles, these equations answer every purpose when correction for the velocity v_a with which the water approaches the orifice is made by adding to the observed head the **head** to which the **velocity of approach** is due, $= \mathbf{v_t}^2/2\,\mathbf{g} = \mathbf{h_v}$. For heads above 2 ft., the value of C for square orifices varies from 0.60 to 0.62 and for circular orifices from 0.59 to 0.61.

Submerged Rectangular Orifices. Let h_u be the depth of the top of the orifice on the incoming or upper side; h_l be the depth of the top of the orifice on the discharge or lower side; A be the area of the orifice; $h_v = v_a^2/2\,g$ the equivalent head of the velocity of approach. Then (Fig. 9):

$$\mathbf{Q} = \mathbf{CA}\sqrt{2\,\mathbf{g}(\mathbf{h_u} + \mathbf{h_v} - \mathbf{h_l})}$$

The value of C for a submerged orifice may be taken as about 1% less than its value for the same orifice under the same effective head when discharging freely into the air.

Partially Submerged Orifices. Let h_t and h_b be as before, b the width of the orifice, h the difference in elevation of water on the two sides of the orifice. Then values of C are closely the same as for submerged orifices, and

$$Q = C\sqrt{2g}\,b\,[2/3\,(h^{3/2} - h_t^{3/2}) + (h_b - h)h^{1/2}]$$

Incomplete Contraction. The foregoing matter on orifices is based upon the assumption that the contraction is complete. Suppression of contraction is accomplished by adding an internal projection at the edge of the orifice extending over the entire perimeter. If the projection does not extend over the entire perimeter, or if it is a short distance from the orifice, partial contraction results. Suppressing the contraction increases the discharge, but decreases the energy of the jet. If k is the ratio of the periphery of the orifice with a border to the whole periphery, the coefficient of discharge for partial contraction is, according to Bidone,

$$\text{For rectangular orifices, } C_s = C\,(1 + 0.152\,k)$$
$$\text{For circular orifices, } C_s = C\,(1 + 0.128\,k)$$

If the sides of a vessel are less than 2.7 times the corresponding width of the orifice distant from any side of the orifice, their influence is felt in partially reducing the contraction.

The effect of suppression of contraction is less when it is produced by extending the plane of the sides upstream from the aperture than when it is caused by a curved entrance.

Increase of Discharge of Square Orifices by Suppression of Contraction

Suppression	Authority: Lesbros * — Suppression by planes. Free discharge. Orifice 0.656 ft. sq., per cent increase	Authority: Stewart, Wisconsin experiments — Suppression by curved surfaces. Submerged discharge. Orifice 4.0 ft. sq., per cent increase
Bottom	3.4	4.0
Bottom and one side	....	12.0
Two opposite sides	6.1	....
Bottom and two sides	12.7	27.0
Four sides	....	54.0

* See Hamilton Smith: "Hydraulics," and Wisconsin Engineer, Dec. 1907.

The suppression in the Wisconsin Experiments was produced by a quarter of an elliptic cylinder so set that its major semi-axis, which was 3.0 ft., was parallel to the flow and its minor semi-axis, 2 ft. 10-1/2 in., was perpendicular thereto.

Two Orifices Adjacent, separated by a narrow bar, discharge more than the two considered separately, because of mutual velocity influences. When the width of bar is less than the least dimension of the orifices the discharge will nearly equal that through an orifice of area and form like orifices and bar combined less that of an orifice of area and form equal to the bar with contractions suppressed next the orifices.

8. Long Pipes

Resistances. When water flows through a pipe of such length that the particles come in contact with the wall, resistance is produced by what is commonly termed fluid friction. For short pipes this is inconsiderable and is provided for in the coefficients of discharge, but when a pipe is more than 50 diameters in length this element becomes important, and as the length increases it makes up the major part of the loss encountered.

Hydraulic Grade Line. When water flows from rest into a channel of any sort, an amount of head $\mathbf{h_v}$ equal to $\mathbf{v^2/2\,g}$ is absorbed in creating the velocity with which it discharges, and a small amount more is used up in frictional resistances. As the water passes through the channel, an amount of head $\mathbf{h_f}$ is used up in overcoming the resistance to flow. If the pipe expands the velocity is reduced and a part of the h_v is returned and appears as pressure head $\mathbf{h_p}$. If the pipe contracts, some of the h_p is converted into h_v. The pressure head measures the height of a column of water which would exert the same pressure as does the water against the walls of the channel. The **Hydraulic Grade Line, or Hydraulic Gradient,** is the locus of the tops of all such columns of water that can be placed along the channel, and the distance from the grade line to the center of the pipe at any point measures the pressure head at the latter. In an open channel the surface of the stream is the Hydraulic Grade Line.

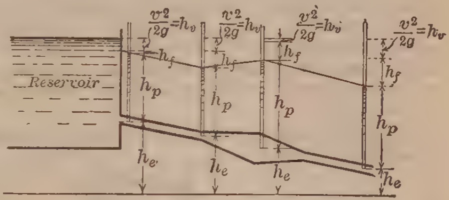


Fig. 14. Hydraulic Grade Line

The total head producing flow is equal to the sum of the velocity head $v^2/2\,g$ at discharge and all losses of head along the line.

Conditions of Flow. Normal flow is a flow such that the components of the velocities in the direction of flow at any section vary in approximate accordance with the ordinates of a cylinder with an ellipsoidal end, such that the maximum ordinate is approximately 1.19 times the mean, the lengths of the cylinder being the component of the velocity in the direction of flow at the surface of contact and the lengths of the ellipsoidal end being the difference between the components of the velocities in the direction of flow at the axis of the pipe and at the surface of contact. This condition of flow is disturbed by contraction, expansion or curvature of the stream, and the water requires a length of about 40 diameters of the pipe to readjust itself after passing a contraction, about 50 to 60 diameters after passing an expansion, and from 100 to 300 diameters after passing a curve. In any region of abnormal flow the pressure indicated by an orifice in the pipe wall will be different for the same mean velocity from that indicated in case of normal flow. In the cases of contractions and expansions, the indication of pressure will usually be greater than that for normal flow, and after a curve, if the pressure be read on the concave side of the stream, the indication will be high, whereas if read on the opposite side it will be low, as also to a less degree if read at right angles to the direction of curvature. It is therefore essential, when making accurate measurements of loss of head, to take observations only at points where the flow is known to be normal. In very rough pipes, on account of the retardation of velocity at the wall, the ratio between the mean and the maximum is reduced and may possibly be as low as 0.60. After a contraction, on the other hand, the ratio may be increased to 0.95, and in the case of a jet from a tapering nozzle to 0.995.

Under normal flow the absorption of head in overcoming the resistances to the motion of the water, called the friction head, h_f, varies as a power of the mean velocity between the 1.7 and the square.

Low Velocity or Viscous Flow. When water flows in capillary tubes or between plates close together, the friction head has been found to vary as the first power of the velocity, and at very low velocities in larger pipes the same law holds. The velocity below which this form of flow occurs is called the **critical velocity.** The critical velocity occupies a considerable range depending upon whether the flow is changing from sub-critical velocity to super-critical velocity flow, or vice versa. In either case the existing condition of flow tends to continue itself, so that for an increasing velocity the critical velocity will be higher than for a decreasing one. Experiments by means of coloring matter in the water have shown that as long as the friction head varies as the first power of the velocity the flow is taking place in straight-line filaments, but as soon as the critical velocity is past the flow has become involved in eddies or vortices, and the exponent of v is at once increased to above 1.70.

The critical velocity may be determined by equating the straight line or viscous flow equation for h_f,

$$h_f = \frac{32\ \mu L v_m}{g D^2} \qquad \text{(Art. 3)}$$

with a similar equation for normal or turbulent flow as:

$$h_f = \frac{0.38\ v_m^{1.87}\ L}{1000\ D^{1.25}} \qquad \text{(Art. 11)}$$

and solving for v_m which will be the value of the critical velocity.

To determine whether the flow in any case is above or below the critical velocity solve for h_f in the above equations and the one giving the higher value is that of the kind of flow existing.

9. Capillary Tubes and Soils

For Capillary Tubes let T_c = temperature centigrade, s = slope, D = diameter of tube, v = velocity. The formula deduced by Poiseuille is

$$v = 52.500\ (1 + 0.03368\ T_c + 0.000221\ T_c^2)\ D^2 s$$

which has been approximately confirmed by other experimenters, and may be assumed to apply to sub-critical velocity flow. The critical velocity varies inversely as some power of the diameter. Experiments (Trans. Am. Soc. C. E., Dec., 1903, Vol. 51, p. 298) have given

$$\text{Critical velocity} = 0.088/D^{0.784}$$

as an appropriate formula, for a temperature of 70° Fahr.

From the above it is evident that sub-critical velocity flow will be encountered in practice only in small pipes, but it appears to extend considerably farther at low than at high temperatures. In a 2-in. pipe with a temperature of 40° F. or 4.4° C. the critical velocity appeared to be about 0.35 ft. per sec.

Flow in Soils. For the flow of water through soils, let T_F = temperature Fahrenheit, s = slope, d_s = effective size of the sand in millimeters, V =

velocity through the soil in meters per day, considering the entire area of the bed as effective in carrying water, and C = a coefficient depending upon the uniformity coefficient of the sand, the shape of the grains, the chemical composition and the closeness and packing of the material, and ranging in value for sands from 370 to 1200, varying inversely as the uniformity coefficients. The Lawrence Experiment Station has derived the formula

$$V = Cd_s^2 \left(\frac{T_F + 10°}{60°}\right) s$$

This is seen to be similar to the formula for small pipes in that temperature plays an important part and that h or s varies as the first power of v.

The Effective Size of a sand or gravel is the size of grain such that 10% of the particles by weight are smaller and 90% greater. The size of a sand grain is determined by the diameter of a sphere of equal volume. The **Uniformity Coefficient** of a sand or gravel is the ratio of that size of grain than which 60% of the sample is finer, to the effective size.

Evidently a high uniformity coefficient indicates a large variation in size of grains, while a coefficient of unity would indicate that substantially all the grains were of one size.

Temperature. In small pipes and in the flow through soils the temperature of the water is a very important factor. For smooth pipes of 2-in. diameter and under, the loss of head has been found to increase about 4% for each 10° fall of temperature from 70° to 40° F. (Trans. Am. Soc. C. E., 1903, vol. 51, p. 290.) Other experiments have shown that near the boiling point the loss of head increases with the temperature. In large pipes the effect of temperature may be safely overlooked, except in cases where great refinement is necessary.

In some weir experiments where the discharge was about 200 cu. ft. per sec. it has appeared that the effect of a change from 76° to 33° F. was to decrease the discharge about 3/4 of 1%. With small weirs, laboratory experiments have shown that the influence of temperature between 32° and 45° F. is greater per degree than higher up the scale. Accurate data covering these points are, however, wanting.

At low velocities in small pipes the loss of head may be less with cold than with warm water, due to the fact that the former does not pass from the condition of viscous to turbulent flow as readily as does the latter and hence the h_f may continue to vary as v beyond the point where its variation with v^2 would give the greater loss.

10. Formulas for Long Pipes

The Chezy Formula. If v is the velocity in the pipe, C a coefficient dependent upon roughness, density, velocity, and diameter, r the **Hydraulic Radius,** namely the cross-sectional area divided by the wetted perimeter, h_f the frictional loss of head in a length L, and if h_f/L is designated by s, the inclination or slope, then

$$v = C\sqrt{\frac{rh_f}{L}} \quad \text{or} \quad v = C\sqrt{rs}$$

in which for new pipes C ranges from 95 to 152 and for old pipes from 60 to 120, the value increasing both with the diameter and the velocity, as shown in the tables herewith.

Values of C in Chezy Formula for Cast-iron Pipes

Diameter of pipe, inches	Velocities in feet per second							
	For new pipes				For old pipes			
	1	3	6	10	1	3	6	10
3	95	98	100	102	63	68	71	73
6	96	101	104	106	69	74	77	79
9	98	105	109	112	73	78	80	84
12	100	108	112	117	77	82	85	88
15	102	110	117	122	81	86	89	91
18	105	112	119	125	86	91	94	97
24	111	120	126	131	92	98	101	104
30	118	126	131	136	98	103	106	109
36	124	131	136	140	103	108	111	114
42	130	136	140	144	105	111	114	117
48	135	141	145	148	106	112	115	118
60	142	147	150	152	....	....	....	....

For steel riveted pipes, see next table. Chezy's formula is also used for conduits and streams but the coefficient C for such cases is generally expressed in terms of r and s (see Art. 15 for formulas of Bazin and Kutter).

Darcy's Formula. The original form of Darcy's equation was $rs = av + bv^2$, where a and b were coefficients. This Darcy later reduced to $\mathbf{rs} = \mathbf{Cv^2}$, where $C = c_1 + c_2/r$, where c_1 and c_2 are constants. For new cast-iron and for wrought-iron pipes of the same roughness, Darcy's vales of these constants are $c_1 = 0.0000773$ and $c_2 = 0.00000162$. The formula then reduces to

Values of C in Chezy Formula for Steel Riveted Pipes

Diameter of pipes, inches	Velocity in feet per second			
	1	3	5	10
3	81	86	89	92
11	92	102	107	115
11	93	99	102	105
15	109	112	114	117
38	113	113	113	113
42	102	106	108	111
48	105	105	105	105
72	110	110	111	111
72	93	101	105	110
103	114	109	106	104

Values of C in Darcy's Formula $CV^2 = Ds$

Diameter, inches	Rough pipes	Smooth pipes
3	0.00080	0.00040
4	0.00076	0.00038
6	0.00072	0.00036
8	0.00068	0.00034
10	0.00066	0.00033
12	0.00066	0.00033
14	0.00065	0.000325
16	0.00064	0.00032
24	0.00064	0.00032
30	0.00063	0.000315
36	0.00062	0.00031
48	0.00062	0.00031

$$\mathbf{h_f} = 0.00000642\,\frac{(12\,D + 1)}{D}\,\frac{v^2L}{r} \quad \text{or} \quad v = 394\sqrt{\frac{D}{12\,D + 1}}\sqrt{rs}$$

where D is the diameter of the pipe in feet. For rough pipe Darcy reduced the velocity one-half. Darcy's formula may be transposed to $CV^2 = Ds$, in which case C has an average value of 0.00032 for clean pipes of diameters from 8 to 48 in. inclusive, the variation being only 3% from the mean for all except the 8-in. The preceding table gives more accurate values of C for Darcy's formula in the last form.

Fanning's formula for flow in pipes is

$$h_f = \frac{4\,fL}{D}\,\frac{v^2}{2\,g} \qquad \text{or} \qquad v = \sqrt{\frac{2\,gDh_f}{4\,fL}}$$

where f is a coefficient which ranges from 0.0071 to 0.0028 for new pipes and from 0.0152 to 0.0046 for old ones, the value decreasing as diameter and velocity increase. The other notation is the same as that at the beginning of this article.

Values of f in Fanning's Formula for Cast-iron Pipes

Diameter of pipe, inches	Velocity in feet per second							
	For new pipes				For old pipes			
	1	3	6	10	1	3	6	10
3	.0071	.0067	.0064	.0062	.0152	.0139	.0128	.0122
6	.007	.0063	.006	.0057	.0135	.0117	.0108	.0103
9	.0067	.0058	.0055	.0051	.0122	.0105	.010	.0092
12	.0064	.0056	.0051	.0048	.0108	.0096	.0089	.0084
15	.0062	.0053	.0048	.0053	.0099	.0087	.0081	.0078
18	.0058	.0051	.0045	.0041	.0087	.0078	.0073	.0069
24	.0053	.0045	.0040	.0037	.0076	.0067	.0063	.0060
30	.0046	.0040	.0037	.0035	.0067	.0061	.0057	.0055
36	.0042	.0037	.0035	.0033	.0061	.0056	.0052	.0050
42	.0038	.0035	.0033	.0031	.0058	.0052	.005	.0048
48	.0036	.0032	.0031	.0029	.0057	.0051	.0049	.0046
60	.0032	.0030	.0029	.0028	....	....	....	....

Kutter. The Kutter formula was designed for open channels and will be treated under that head. It is sometimes used for pipes, but the results from it, since the coefficients, like those of the Chezy and Fanning formulas, change with the velocity in the same pipe, are usually erroneous, except for a very small range of velocity. For this reason it is not to be recommended for general use in computations for pipes.

11. Exponential Formulas for Pipes

One form of an exponential formula for flow of water in pipes is

$$h_f = \frac{Kv^N L}{D^{1.25}}$$

where the notation is the same as that at beginning of Art. 10, but where the coefficient K and exponent N may vary with the kind and condition of the pipe. To avoid zeros in the coefficient a unit length of 1000 ft. may be taken, when the formula becomes

$$h_f = \frac{Kv^N}{D^{1.25}} \cdot \frac{L}{1000}$$

in which N has a mean value of 1.87 and K ranges from 0.28 to 0.48 with an average value of 0.38 for ordinarily clean pipes. For rough or tuberculated pipes K may become as high as 0.70. The advantage of the exponential formula is that the coefficient for the same pipe is nearly constant and, if the exponent, its range being from 1.70 to 2.00, is properly selected, absolutely so; and the variation in all cases is much less than with other formulas, so that with a few average coefficients for different classes of channels, all hydraulic flow problems may be solved with reasonable accuracy without reference to any tables of coefficients. The foregoing formula with the coefficient 0.38 may be expected to give results within 20% of accuracy for any pipes likely to be encountered which have diameters from 1 in. to 15 ft., except those extremely tuberculated, and with velocities from 1 ft. to 20 ft. per sec. For ordinary cast-iron or riveted pipe with any diameter and velocity, the results may be expected to be within 6% of accuracy.

The exponential formula is derived from experiment in the following manner: Any plane curve passing through the origin of coordinates can be represented by an equation of the form $y = mx^N$, in which m and N may be either constant or variable. If the curve is one of single curvature such that the change of inclination of its tangents either continuously increases or continuously decreases, both m and N become constants. All curves which are loci of equations expressing the relation between velocity and loss of head in flowing water are of this latter class, and consequently $h_f = mv^N$ is a general expression for the loss of head in either a pipe or an open channel. If for any pipe line the values of m and N be determined, the equation of flow in that line is established. Expressing the above equation for logarithmic computations, it becomes

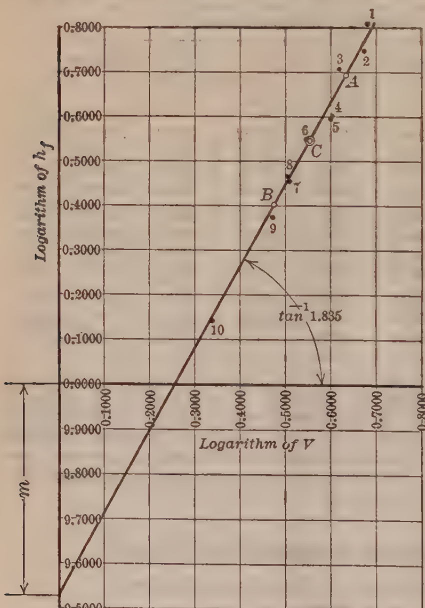


Fig. 15. Logarithmic Plotting

$$\log h_f = \log m + N \log v$$

and considering the logarithms as mere quantities, this is at once seen to be the equation of a straight line in which $\log m$ is the intercept on the h_f axis and N is the tangent of the angle which the line makes with the v axis. Both m and N may be found by determining two points in the line representing the plottings of the logarithms. If it is desired to draw the straight line which most nearly coincides with a number of points, it must pass through their center of gravity and also through the centers of gravity of the two groups into which the center of gravity of the whole divides them. Having the last two points, the equation of the line is readily determined.

The following example will serve to illustrate the process (Fig. 15). The data are from observations on a 12-in. cast-iron water main very carefully laid in a tangent some 3500 ft. long, the loss in 1000 ft. of which was measured.

C = center of gravity or mean point of the whole group
A = center of gravity of part of group above C
B = center of gravity of part of group below C

$C_h = h_f$ coordinate of C, $\quad C_v = v$ coordinate of C
$A_h = h_f$ coordinate of A, $\quad A_v = v$ coordinate of A
$B_h = h_f$ coordinate of B, $\quad B_v = v$ coordinate of B

Observed Data

No.	v Ft. per sec.	hf Ft. of water	log v (Logarithms)		log hf (Logarithms)	
1	4.794	6.515	0.6807		0.8139	
2	4.667	5.577	.6690	Sum = 3.1667	.7464	Sum = 3.4641
3	4.155	5.100	.6186	mean = 0.63334	.7076	mean = 0.69282
4	3.998	4.002	.6018	$= A_v$	.6023	$= A_h$
5	3.950	3.926	.5966		.5939	
6	3.519	3.566	.5464		.5522	
7	3.252	2.888	.5122	Sum = 2.3715	.4606	Sum = 2.0046
8	3.208	2.942	.5062	mean = 0.47430	.4686	mean = 0.40092
9	2.943	2.374	.4688	$= B_v$	.3755	B_h
10	2.177	1.405	.3379		.1477	
			Sum = 5.5382		Sum = 5.4687	
			mean = 0.53382 = C_v		mean = 0.54687 = C_h	

$A_v - C_v = 0.63334 - 0.55382 = 0.07952 \qquad A_h - C_h = 0.69282 - 0.54687 = 0.14595$
$C_v - B_v = 0.55382 - 0.47430 = 0.07952 \qquad C_h - B_h = 0.54687 - 0.40092 = 0.14595$

Since $A_v - C_v = C_v - B_v$ and $A_h - C_h = C_h - B_h$, the three points A, C, and B are in a straight line, which fact checks the accuracy of the work.

N = tangent of inclination of line ACB

$$= \frac{A_h - C_h}{A_v - C_v} = \frac{C_h - B_h}{C_v - B_v} = \frac{A_h - B_h}{A_v - B_v} = \frac{0.14595}{0.07952} = 1.835$$

Since $\log m = \log h_f - N \log v$, using the coordinates of C

$$\log m = 0.54687 - 1.835 \times 0.55382$$
$$= 0.54687 - 1.01626 = 9.53061 = \log 0.3393, \text{ and } m = 0.3393$$

The equation for this 12-in. pipe is therefore $h_f = 0.3393\, v^{1.835}\, L/1000$.

Remark: Evidently the v coordinates must be divided between the same pair of observations as the h_f coordinates. The mathematical determination of what group should include a point whose v coordinate is on one side of C and whose h_f coordinate is on the other, depending on whether the point itself is above or below a normal to the line ACB through C. This can usually be established by plotting the logarithms on ordinary cross-section paper, or the observations on logarithmic paper.

To introduce the diameter into the equation, a series of values of m and N for pipes of different diameters must be obtained. The range of N is relatively small, the limits for all reliable pipe experiments on record being from 1.70 to 2.08, and if the pipes are of the same character of surface and alignment the value of n will be constant. It is, therefore, only necessary to consider the variation of m, which depends upon the area of the cross-section or upon D and upon the roughness. Evidently m varies inversely as some power of the diameter, and for $1/D = 0$, $m = 0$ for any velocity, so the curve representing the relation between m and D will be $m = KD^{-x}$. Proceeding in the same manner as before, an average value of $x = 1.25$ will be obtained, and the formula for pipe of the same character as that in the above experiment is

$$h_f = \frac{Kv^{1.835}}{D^{1.25}} \cdot \frac{L}{1000} \quad \text{or} \quad h_f = \frac{0.38\, v^{1.87}}{D^{1.25}} \cdot \frac{L}{1000}$$

for average conditions, and this may be transposed to

$$v = 73.54\, D^{0.668}\, s^{0.535} \quad \text{or} \quad v = 185\, r^{0.668}\, s^{0.535}$$

Scobey's Formula for **Wood-stave Pipe** is of the above form and is

$$v = 1.62\, D^{0.65}\, h_f^{0.555}$$

wherein h_F is loss of head per 1000 ft. The U. S. Dept. of Agriculture Formula (Bulletin 854) for tile drains of clay or concrete, 4 in. to 12 in. diameter, is:

$$v = 138\, r^{2/3}\, s^{1/2}$$

Hazen and Williams Formula. A commonly accepted exponential expression is that known as the Hazen and Williams formula (see Hydraulic Tables, Williams and Hazen, John Wiley & Sons, New York City), which is:

$$V = cr^{0.63} s^{0.54} 0.001^{-0.04}$$

The last term of this formula $0.001^{-0.04}$ was introduced to make the co efficient c correspond in value with that of the Chezy formula for a slope s of 1 ft. in 1000 ft. This formula was originally devised to furnish the basis for the Hazen and Williams Hydraulic Slide Rule by which ordinary problems of discharge and loss of head in either closed or open channels may be readily solved, and from which the above named "Hydraulic Tables" have been derived.

The Hiram F. Mills Formulas. The most scientific and rational formulas yet presented for the flow of water in pipes are those devised by the late Hiram F. Mills, C. E., of Lawrence, Massachusetts, and posthumously published in 1923 through the interest of John R. Freeman, C. E., and Karl R. Kennison, C. E., by the American Academy of Arts and Sciences, of Boston. The Mills formulas are of the form:

$$s = \frac{C_1v + C_2v^2}{r^{1.25}}$$

wherein C_2 is zero for flow below the so-called critical velocity. The expression is quite similar in appearance to the old Darcy formula $rs = av + bv^2$, a form originating with Dr. Young and Prony about the beginning of the 19th century.

The theory of these formulas is that the flow of water takes place throughout a part of the pipe section in approximately parallel filaments, under which condition the loss of head h_f varies as the first power of v, and through the rest under conditions of disturbances inducing impacts which cause the loss to vary as v^2, the latter being considered the probable limiting condition under extreme disturbance. As such maximum disturbances cannot occur in small pipes it follows that the average exponent of v, which is that derived for the exponential formulas, will be smaller for small pipes than for larger ones. Also by reason of the disturbances caused by the roughness of the pipe wall the exponent of v for rough pipes will be more largely influenced by the v^2 condition and the mean exponent will be greater. The exponent of r in the Mills formula corresponds to that of D or r in the exponential formula previously considered.

The flow of water in a pipe or other channel, starting from rest as set forth by Mr. Mills, passes through four different stages or conditions.

First. It starts as viscous flow with the velocity increasing from zero at the pipe wall to a maximum at the center, and h_f varies as C_1v, with C_1 a constant.

Second. A velocity is reached at which the water begins to move at the pipe wall and eddies starting there move out toward the center. The first effect is to reduce the value of C_1, and at the same time a resistance varying as C_2v^2 appears with C_2 increasing from zero, approximately as the ordinates to a quarter ellipse and becoming constant, while C_1 begins to increase from its decreased value.

Third. With C_2 constant, C_1 continues to increase approximately as the ordinates to a quarter ellipse starting from the point of zero velocity until the maximum disturbance of the stream due to eddies has been reached, after which C_1 becomes constant and this constitutes the

Fourth condition, when both C_1 and C_2 are constant with C_1 considerably smaller than C_2.

To provide for all these conditions and the additional effect of temperature, which in small pipes and at low velocities is important, is evidently impossible in a single formula, since for each class of pipe at least four equations would be required, two of which must involve a variable coefficient. Omitting all of the first three conditions, Mr. Mills presents the following equations for the fourth:

For	v above	$r^{1.25}$ s
Seamless drawn brass tubes....	6 ft. per sec.	$= 0.0000593\,v + 0.0000220\,v^2$
Riveted sheet-iron pipe coated with bitumen..............	4 ft. per sec.	$= 0.0000542\,v + 0.0000317\,v^2$
Wrought-iron pipe coated with asphalt....................	4 ft. per sec.	$= 0.0000527\,v + 0.0000252\,v^2$
Tin pipes....................	2.5 ft. per sec.	$= 0.0000398\,v + 0.0000257\,v^2$
Glass pipes..................	2.5 ft. per sec.	$= 0.0000386\,v + 0.0000294\,v^2$
Wrought-iron pipes...........	4 ft. per sec.	$= 0.0000370\,v + 0.0000337\,v^2$
Lead pipes...................	2.5 ft. per sec.	$= 0.0000340\,v + 0.0000264\,v^2$
Galvanized-iron pipes.........	4 ft. per sec.	$= 0.0000318\,v + 0.0000483\,v^2$
Tarred cast-iron pipes.........	2 ft. per sec.	$= 0.0000201\,v + 0.0000414\,v^2$
Bored wooden pipe...........	1 ft. per sec.	$= 0.0000149\,v + 0.0000836\,v^2$
New uncoated cast-iron pipes..	1 ft. per sec.	$= 0.0000144\,v + 0.0000497\,v^2$
Old cast-iron pipes — cleaned..	1 ft. per sec.	$= 0.0000080\,v + 0.0000572\,v^2$
Old cast-iron pipes with thin calcareous deposit..........	1 ft. per sec.	$= 0.0000072\,v + 0.0001121\,v^2$
Old rusty and tuberculated wrought-iron pipes..........	1 ft. per sec.	$= 0.0000032\,v + 0.0001185\,v^2$

For velocities of 6 ft. per sec., where the effect of C_1v is negligible, the pipes are arranged according to resistance, as follows:

Material	$r^{1.25}$ s	Material	$r^{1.25}$ s
Seamless drawn brass....	$0.0000319\,v^2$	Tar coated cast iron......	$0.0000447\,v^2$
Lead..............	$.0000320\,v^2$	New cast iron (uncoated).	$.0000521\,v^2$
Tin................	$.0000323\,v^2$	Galvanized iron.........	$.0000549\,v^2$
Asphalt-coated wrought iron.................	$.0000340\,v^2$	Old cast iron — cleaned...	$.0000585\,v^2$
Glass.............	$.0000358\,v^2$	Bored wooden pipe......	$.0000863\,v^2$
Wrought iron.......	$.0000399\,v^2$	Old cast iron with calcareous deposit..........	$.0001133\,v^2$
Bitumen-coated sheet iron................	$.0000408\,v^2$	Old rusty tuberculated wrought iron..........	$.0001163\,v^2$

The group of tables now presented give:

(a) The loss of head in smooth brass pipes by the Mills formulas, which may be used for all very smooth pipes up to 4 in. in diameter.

(b) The loss of head in wrought-iron pipes by the Mills formulas, which may be used for ordinary pipes up to 5 in. in diameter.

(c) The loss of head in ordinary pipes from 6 in. to 72 in. in diameter by the Hazen and Williams formula, using a coefficient of 100, corresponding to poorly laid cast iron several years old but not badly tuberculated.

For pipes as smooth as those included in (*a*) the coefficient varies from 130 to 150, increasing with the diameter. For ordinary cast-iron pipe when new a coefficient of 120 is reasonable and for riveted steel pipe under similar conditions 110 is recommended. Fire hose ranges from 140 for the smoothest to 89 for unlined linen hose.

Loss of Head in Smooth Brass Pipe

By Hiram F. Mills Formulas

Based on Experiments of Saph and Schoder, Mills and Freeman. In Feet of Water per 1000 Ft. of Pipe. From Plottings. Temperature of Water about 70° F.

Pipe diameters	I 0.107 in. 0.0892 ft.	II 0.180 in. 0.0150 ft.	III 0.221 in. 0.0184 ft.	IV 0.282 in. 0.0235 ft.	V 0.321 in. 0.0268 ft.	VI 0.376 in. 0.0313 ft.
Velocity, ft. per sec.	h_f Feet	h_f Feet	h_f Feet	h_f Feet	h_f Feet	h_f Feet
0.5	71	11	8.4	6	5.3	5.5
1.0	145	30	23	22	21.5	19.9
1.5	207	85	73	59	51.5	43
2.0	269	162	136	104	86.5	71
2.5	368	260	203	149	128	104
3.0	665	365	280	206	176	144
3.5	915	477	365	272	232	188
4.0	1165	603	465	344	291	240
4.5	1420	742	575	424	359	296
5.0	1720	894	690	510	433	356
5.5	2040	1060	820	608	513	422
6.0	2370	1235	955	708	600	491

Pipe diameters	VII 0.540 in. 0.0450 ft.	VIII 0.630 in. 0.0525 ft.	IX 0.820 in. 0.0683 ft.	X 1.054 in. 0.0878 ft.	XI 1.237 in. 0.1031 ft.
Velocity, ft. per sec.	h_f Feet	h_f Feet	h_f Feet	h_f Feet	h_f Feet
0.5	2.7	2.5	2.3	1.7	1.5
1.0	13.9	11.8	8.1	5.8	4.9
1.5	27.7	22.6	16.3	12.0	10
2.0	45.4	37.4	26.9	19.6	16
2.5	67	55	39.6	29.3	24
3.0	92	76	54.5	40	33
3.5	119	99	73.5	54	43
4.0	149	127	90.5	68	54
4.5	183	155	112	83	67
5.0	220	187	135	99	80
5.5	259	241	160	116	95
6.0	305	257	186	136	112

Pipe diameters	XII 1.498 in. 0.1248 ft.	XIII 2.090 in. 0.1742 ft.	XIV 2.108 in. 0.1757 ft.	XV 3.067 in. 0.2556 ft.	XVI 4.00 in. 0.3333 ft.
Velocity, ft. per sec.	h_f Feet	h_f Feet	h_f Feet	h_f Feet	h_f Feet
0.5	1.2	0.75	0.75	0.5	0.34
1.0	3.8	2.5	2.5	1.6	1.4
1.5	7.7	5.0	5.0	3.3	2.3
2.0	12.6	8.4	8.4	5.4	3.8
2.5	18.6	12.5	12.5	8.0	5.7
3.0	26.0	17.0	17.0	11.0	7.8
3.5	33.6	22.0	22.0	14.4	10.4
4.0	42.6	28.0	28.0	18.2	13.0
4.5	52.5	34.6	34.6	22.5	16.1
5.0	63.2	42.0	42.0	27.0	19.3
5.5	75.0	49.5	49.5	32.0	22.9
6.0	87.5	58.0	58.0	37.4	26.8

Note. Pipes XIV, XV and XVI were slightly rougher than the others.

The 10-ft.-diameter brick intake tunnel at Detroit gave a value ranging from 80 to 90.

Experiments on a 42-in. cast-iron sewer at Baltimore reported by Keefer and Register (Eng. News-Record, Mar. 1, 1928) wherein the pipe was coated with a layer of slime about 1 in. thick, gave a coefficient of 103 computed for the full diameter and 116 when the diameter is reduced to 40 in. on account of the slime layer.

Experiments by the U. S. Dept. of Agriculture, reported by C. E. Ramser on Flow through the St. Frances Floodway, Arkansas (Eng. News-Record, April 5, 1928), wherein the flow took place through a wooded area generally free from undergrowth, at depths ranging from 4 to 9.5 ft. and with average velocities from 0.4 to 0.62 ft. per sec., give values of the coefficient ranging from 15 to 20.

Experiments by the same personnel on Flows in the Little River Drainage in Missouri (Eng. News-Record, Oct. 12, 1922) give for depths of 4.5 to 5.0 ft. in a floodway obstructed by scattered brush and small tree growths at velocities from 0.5 to 0.75 ft. a coefficient of about 24, and for a floodway cleared of all brush and trees, but not of stumps, for similar depths and velocities the coefficient was about 40.

The rock section of the Chicago Drainage Canal 160 ft. wide and 22 ft. to 26 ft. deep gave $C = 77$ to 97.

Loss of Head in Commercial Wrought-iron Pipe

By Hiram F. Mills Formulas

Based on Experiments of John R. Freeman, 1892. In Feet of Water per 1000 Ft. of Pipe. From "Flow of Water in Pipes," Mills. At Summer Temperatures.

Pipe diameters	1/4 in. 0.030 ft.	1/2 in. 0.052 ft.	3/4 in. 0.068 ft.	1 in. 0.0884 ft.	1-1/4 in. 0.1156 ft.	1-1/2 in. 0.1356 ft.
Velocity, ft. per sec.	h_f Feet	h_f Feet	h_f Feet	h_f Feet	h_f Feet	h_f Feet
1	24.8	12.5	8.93	6.43	4.61	3.78
1.5	51.5	25.9	18.5	13.36	9.56	7.85
2	86.9	43.8	31.3	22.5	16.1	13.2
2.5	130.8	65.8	47.1	33.9	24.3	19.9
3	182.9	92.1	65.8	47.4	34.0	27.9
3.5	242.9	122.3	87.4	63	45.1	37
4	311	156.6	111.9	80.6	57.7	47.4
5	465	234	167.3	120.6	86.3	70.9
6	649	327	233.7	168.5	120.6	99

Pipe diameters	2 in. 0.1744 ft.	2-1/2 in. 0.2086 ft.	3 in. 0.2596 ft.	4 in. 0.3436 ft.	5 in. 0.4268 ft.
Velocity, ft. per sec.	h_f Feet	h_f Feet	h_f Feet	h_f Feet	h_f Feet
1	2.75	2.20	1.67	1.18	0.882
1.5	5.72	4.57	3.47	2.45	1.86
2	9.65	7.72	5.86	4.13	3.19
2.5	14.5	11.6	8.81	6.22	4.84
3	20.3	16.2	12.3	8.68	6.82
3.5	27	21.6	16.4	11.5	9.12
4	34.5	27.6	21	14.8	11.74
5	51.6	41.3	31.3	22.1	17.70
6	72.1	57.6	43.8	30.9	25.03

Discharge and Loss of Head in Pipes

By Hazen and Williams Formula with $C = 100$ h_f in Feet of Water per 1000 Ft. of Pipe.

For new straight cast-iron pipe $C = 130$ to 140, whence the velocity and discharge will be increased 30 to 40% from the above values, for the given h_f.

For badly tuberculated pipe C may be as low as 70, whence the velocity and discharge may be decreased 30% from the above values, for the given h_f.

Size...	6 in.		8 in.		10 in.		12 in.	
Velocity, ft. per sec.	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet
0.5	.098	.37	.175	.265	.273	.203	.393	.165
1.0	.196	1.33	.349	.96	.545	.74	.785	.595
1.5	.299	2.83	.524	2.02	.818	1.56	1.178	1.27
2.0	.393	4.80	.698	3.45	1.091	2.65	1.571	2.15
2.5	.491	7.25	.873	5.20	1.364	4.03	1.963	3.25
3.0	.589	10.20	1.047	7.30	1.636	5.61	2.356	4.55
3.5	.688	13.60	1.222	9.75	1.909	7.50	2.749	6.05
4.0	.785	17.40	1.396	12.45	2.182	9.60	3.142	7.75
4.5	.883	21.65	1.571	15.45	2.454	11.90	3.534	9.62
5.0	.982	26.20	1.745	18.80	2.727	14.50	3.927	11.70
5.5	1.080	31.30	1.920	22.40	3.000	17.30	4.320	14.00
6.0	1.178	36.80	2.094	26.30	3.272	20.20	4.712	16.40
6.5	1.276	42.70	2.269	30.50	3.545	23.50	5.105	19.00
7.0	1.374	48.90	2.443	35.00	3.818	27.00	5.498	21.80
7.5	1.472	55.70	2.618	39.80	4.091	30.70	5.890	24.80
8.0	1.571	62.80	2.793	44.80	4.363	34.60	6.283	27.95
8.5	1.668	70.50	2.967	50.10	4.636	38.80	6.676	31.25
9.0	1.767	78.30	3.142	55.00	4.909	43.05	7.069	34.80
9.5	1.865	86.00	3.316	62.00	5.181	47.60	7.461	38.40
10.0	1.963	95.00	3.491	68.50	5.454	52.10	7.854	42.25

Size...	16 in.		20 in.		24 in.		30 in.	
Velocity, ft. per sec.	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet
0.5	.698	.118	1.091	.092	1.571	.059	2.454	.054
1.0	1.396	.425	2.182	.329	3.142	.266	4.909	.205
1.5	2.094	.90	3.272	.695	4.712	.562	7.363	.433
2.0	2.793	1.54	4.363	1.180	6.283	.958	9.817	.739
2.5	3.491	2.32	5.454	1.780	7.854	1.45	12.272	1.120
3.0	4.189	3.25	6.545	2.515	9.425	2.02	14.726	1.565
3.5	4.887	4.33	7.636	3.33	10.996	2.70	17.181	2.08
4.0	5.585	5.27	8.727	4.27	12.566	3.46	19.635	2.67
4.5	6.283	6.88	9.817	5.32	14.137	4.30	22.089	3.32
5.0	6.981	8.39	10.908	6.46	15.708	5.22	24.544	4.02
5.5	7.679	10.00	11.999	7.70	17.279	6.22	26.998	4.81
6.0	8.378	11.65	13.090	9.05	18.850	7.32	29.452	5.32
6.5	9.076	13.60	14.181	10.50	20.420	8.50	31.907	6.55
7.0	9.774	15.60	15.272	12.05	21.991	9.75	34.361	7.50
7.5	10.472	17.75	16.362	13.70	23.562	11.10	36.816	8.55
8.0	11.170	19.95	17.453	15.40	25.133	12.50	39.270	9.61
8.5	11.868	22.35	18.544	17.35	26.704	13.95	41.724	10.75
9.0	12.566	24.90	19.635	19.20	28.274	15.50	44.179	11.95
9.5	13.264	27.50	20.726	21.20	29.845	17.20	46.633	13.40
10.0	13.963	30.20	21.817	23.30	31.416	19.80	49.087	14.60

Discharge and Loss of Head in Pipes—Continued

By Hazen and Williams Formula with $C=100$ h_f in Feet of Water per 1000 Ft. of Pipe.

Size...	36 in.		42 in.		48 in.		54 in.	
Velocity, ft. per sec.	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet
0.5	3.534	.046	4.811	.0384	6.283	.0328	7.952	.0286
1.0	7.069	.166	9.621	.134	12.566	.118	15.904	.103
1.5	10.603	.350	14.432	.301	18.850	.250	23.856	.219
2.0	14.137	.597	19.242	.498	25.133	.425	31.809	.372
2.5	17.671	.902	24.053	.753	31.416	.645	39.761	.532
3.0	21.206	1.27	28.863	1.06	37.699	.901	47.713	.785
3.5	24.740	1.68	33.674	1.41	43.982	1.205	55.665	1.045
4.0	28.274	2.16	38.485	1.80	50.265	1.54	63.617	1.34
4.5	31.809	2.68	43.295	2.24	56.549	1.92	71.569	1.67
5.0	35.343	3.25	48.106	2.73	62.831	2.32	79.522	2.02
5.5	38.877	3.89	52.916	3.25	69.115	2.78	87.474	2.42
6.0	42.411	4.57	57.727	3.81	75.398	3.26	95.426	2.83
6.5	45.946	5.14	62.537	4.42	81.681	3.79	103.378	3.30
7.0	49.480	6.08	67.348	5.08	87.965	4.33	111.330	3.78
7.5	53.014	6.90	72.158	5.78	94.248	4.92	119.282	4.30
8.0	56.549	7.78	76.969	6.50	100.531	5.55	127.234	4.83
8.5	60.083	8.70	81 780	7.25	106.814	6.21	135.187	5.41
9.0	63.617	9.67	86.590	8.07	113.097	6.90	143.139	6.05
9.5	67.152	10.70	91.401	8.95	119.381	7.64	151.091	6.65
10.0	70.686	11.80	96.211	9.80	120.564	8.40	159.043	7.33

Size...	60 in.		66 in.		72 in.	
Velocity, ft. per sec.	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet	Discharge, cu. ft. per sec.	h_f Feet
0.5	9.817	.0252	11.879	.0227	14.137	.0204
1.0	19.635	.0915	23.758	.0819	28.274	.0719
1.5	29.452	.193	35.637	.173	42.411	.156
2.0	39.270	.329	47.517	.293	56.549	.266
2.5	49.087	.498	59.396	.444	70.686	.402
3.0	58.905	.695	71.275	.623	84.823	.563
3.5	68.722	.926	83.154	.828	98.960	.748
4.0	78.540	1.18	95.033	1.06	113.097	.96
4.5	88.357	1.47	106.912	1.32	127.234	1.19
5.0	98.175	1.79	118.791	1.61	141.372	1.45
5.5	107.992	2.13	130.671	1.92	155.509	1.73
6.0	117.810	2.51	142.550	2.25	169.646	2.03
6.5	127.627	2.92	154.429	2.62	183.783	2.37
7.0	137.445	3.34	166.308	2.99	197.920	2.70
7.5	147.262	3.80	178.187	3.40	212.057	3.08
8.0	157.080	4.28	190.066	3.82	226.195	3.47
8.5	166.897	4.79	201.945	4.28	240.332	3.87
9.0	176.715	5.35	213.825	4.77	254.469	4.32
9.5	186.532	5.88	225.704	5.25	268.606	4.77
10.0	196.350	6.45	237.583	5.80	282.743	5.22

Cubic Feet per Second Equivalents

Cu. ft. per sec.	Million gals. daily	Gals. per min.	Cu. ft. per sec.	Million gals. daily	Gals. per min.	Cu. ft. per sec.	Million gals. daily	Gals. per min.
1	0.6463	448.2	16	10.3403	7 171.1	35	22.6195	15 686.8
2	1.2925	896.4	16.5	10.6635	7 395.2	37.5	24.2351	16 807.3
3	1.9388	1 344.6	17	10.9866	7 619.3	42.5	27.4665	19 048.3
4	2.5851	1 792.8	17.5	11.3097	7 843.4	45	29.0826	20 168.7
5	3.2314	2 241.0	18	11.6328	8 067.5	47.5	30.6978	21 289.2
6	3.8776	2 689.2	18.5	11.9560	8 291.6	52.5	33.9292	23 530.2
7	4.5240	3 137.4	19	12.2791	8 515.7	55	35.5449	24 650.7
8	5.1702	3 585.6	19.5	12.6023	8 739.8	57.5	37.1605	25 771.2
9	5.8164	4 033.7	20.5	13.2486	9 188.0	62.5	40.3919	28 012.2
10.5	6.7858	4 706.0	21	13.5717	9 412.1	65	42.0076	29 132.6
11	7.1090	4 930.1	21.5	13.8948	9 636.2	67.5	43.6232	30 253.1
11.5	7.4321	5 154.2	22	14.2179	9 860.3	72.5	46.8546	32 494.1
12	7.7552	5 378.3	22.5	14.5411	10 084.4	75	48.4703	33 614.6
12.5	8.0784	5 602.4	23	14.8642	10 308.5	77.5	50.0859	34 735.1
13	8.4015	5 826.5	23.5	15.1873	10 532.6	82.5	53.3173	36 976.0
13.5	8.7246	6 050.6	24	15.5105	10 756.7	85	54.9330	38 096.5
14	9.0478	6 274.7	24.5	15.8336	10 980.8	87.5	56.5486	39 217.0
14.5	9.3709	6 498.8	25	16.1568	11 204.9	92.5	59.7800	41 458.0
15	9.6941	6 722.9	27.5	17.7724	12 325.3	95	61.3957	42 578.4
15.5	10.0172	6 947.0	32.5	21.0038	14 566.3	97.5	63.0113	43 698.9

Million Gallons Daily Equivalents

Million gals. daily	Gals. per min.	Cu. ft. per sec.	Million gals. daily	Gals. per min.	Cu. ft. per sec.	Million gals. daily	Gals. per min.	Cu. ft. per sec.
1	694	1.547	16	11 111	24.757	35	24 305	54.157
2	1 389	3.095	16.5	11 458	25.531	37.5	26 042	58.025
3	2 083	4.642	17	11 806	26.305	42.5	29 514	65.762
4	2 778	6.189	17.5	12 153	27.078	45	31 250	69.630
5	3 472	7.737	18	12 500	27.852	47.5	32 986	73.499
6	4 167	9.284	18.5	12 847	28.626	52.5	36 458	81.235
7	4 861	10.831	19	13 194	29.399	55	38 194	85.104
8	5 556	12.379	19.5	13 542	30.173	57.5	39 931	88.972
9	6 250	13.926	20.5	14 236	31.720	62.5	43 403	96.709
10.5	7 292	16.247	21	14 583	32.494	65	45 139	100.577
11	7 639	17.021	21.5	14 931	33.268	67.5	46 875	104.445
11.5	7 986	17.794	22	15 278	34.041	72.5	50 347	112.182
12	8 333	18.568	22.5	15 625	34.815	75	52 083	116.051
12.5	8 681	19.342	23	15 972	35.589	77.5	53 819	119.919
13	9 028	20.115	23.5	16 319	36.362	82.5	57 292	127.656
13.5	9 375	20.889	24	16 667	37.136	85	59 028	131.524
14	9 722	21.663	24.5	17 014	37.910	87.5	60 764	135.092
14.5	10 069	22.436	25	17 361	38.684	92.5	64 236	143.129
15	10 417	23.210	27.5	19 097	42.352	95	65 972	146.997
15.5	10 764	23.984	32.5	22 569	50.289	97.5	67 708	150.866

Gallons per Minute Equivalents

Gals. per min.	Gals. per day	Cu. ft. per sec.	Gals. per min.	Gals. per day	Cu. ft. per sec.	Gals. per min.	Gals. per day	Cu. ft. per sec.
1	1 440	0.13370	13.5	19 440	1.80495	21.5	30 960	2.87455
2	2 880	.26740	14	20 160	1.87180	22	31 680	2.94140
3	4 320	.40110	14.5	20 880	1.93865	22.5	32 400	3.00825
4	5 760	.53480	15	21 600	2.00550	23	33 120	3.07510
5	7 200	.66850	15.5	22 320	2.07235	23.5	33 840	3.14195
6	8 640	.80220	16	23 040	2.13920	24	34 560	3.20880
7	10 080	.93590	16.5	23 760	2.20605	24.5	35 280	3.27565
8	11 520	1.06960	17	24 480	2.27290	25	36 000	3.34250
9	12 960	1.20330	17.5	25 200	2.33975	27.5	39 600	3.67675
10.5	15 120	1.40385	18	25 920	2.40660	32.5	46 800	4.34525
11	15 840	1.47070	18.5	26 640	2.47345	35	50 400	4.67950
11.5	16 560	1.53755	19	27 360	2.25403	37.5	54 000	5.01375
12	17 280	1.60440	19.5	28 080	2.60715	42.5	61 200	5.68225
12.5	18 000	1.67125	20.5	29 520	2.74085	45	64 800	6.01650
13	18 720	1.73810	21	30 240	2.80770	47.5	68 400	6.35075

12. Variations in Diameter, Material and Conditions

Relation of Diameter of Pipe to Quantity Discharged. In terms of C for Chezy's formula

$$Q = \pi/8\ C\ \sqrt{sD^5}$$

in terms of Fanning's coefficient f, $Q = \pi/4\ \sqrt{\frac{2g}{4f} \cdot \frac{h_f}{L} \cdot D^5}$

Approximately for rough pipe $\quad Q = 1000\ D^{5/2}\ s^{1/2}$

and for smooth pipe $\quad Q = 2 \times 1000\ D^{5/2}\ s^{1/2}$

Material. Most pipes experimented upon previous to 1900 were of cast iron. Experiment since that date and others previous thereto indicate that the exponential formulas (Art. 11) may be safely used for wooden stave pipe, and that for riveted steel pipe the effect of rivets and joints is to reduce the discharge from 10 to 12% below that of cast iron when pipes are new.

Roughness. The effect of roughness in a water pipe is in general to retard the flow or increase the loss in head. This is accomplished by reducing the velocity of the water at the surface of contact, thus producing a general reduction in velocity and also causing cross currents or eddies which use up the energy in the stream. Roughness decreases C and increases f and K in the foregoing formulas. It also increases N somewhat in the exponential formula.

Curvature. The effect of curvature is to increase the loss of head. This increased loss is partly due to the cross currents and eddies set up in the bend, but also to the changes of velocity along the stream lines and increased friction along the walls of the channels due to increased velocities over part of the circumference. The loss of head due to a curve may be stated in terms of the velocity head h_v or, better, in terms of the equivalent length of straight pipe which would give the same loss as the curve.

The loss in a curve should be measured as the excess over that in an equal length of straight pipe. An examination of the experiments upon curve resistance by W. E. Fuller, Mem. Am. Soc. C. E. (Jour. N. E. W. W. A., Dec., 1913) has led to the following conclusions:

1. The excess loss of head due to bends is greater for large than for small pipes.
2. For large pipes a 6-ft. radius has given the minimum excess resistance except where very long radii are used.
3. If the radius can be made very long, the least excess resistance will be from the curve of longest radius.
4. For small pipes, with long radii, the total loss of head may be less than that in a straight pipe of length equal to the sum of the two tangents to the curve.
5. Approximate formulas:

For 90-deg. bends of a radius between 1.5 ft. and 10 ft. the excess loss of head due to the curve $h_b = 1/4 \left(\frac{V_2}{2g}\right)$

For tees = bends of zero radii, $h_b = 1\text{-}1/4 \left(\frac{V^2}{2g}\right)$

For 90-deg. bends of 6-in. radius $h_b = 1/2 \left(\frac{V^2}{2g}\right)$

For loss in 45-deg. bends use 3/4 of loss in similar 90-deg. bends.
For loss in 22-1/2-deg. bends use 1/2 of loss in similar 90-deg. bends.
For loss in a Y-branch use 3/4 of loss in similar tee.

The above formulas though convenient to apply are evidently inaccurate because they take no account of differences in radius of curvature.

The following values of curve and special resistance were obtained experimentally from tests of 30-in., 16-in., and 12-in. cast-iron pipes, and the excess

Resistance in Pipe, and Equivalent Loss of Head, Due to Curve or Special Connection

Obstruction: 90-deg. curves: Pipe diameter	Radius of curve	No. of diameters equal to Radius	Authority	Equivalent length of straight pipe, measured in pipe diameters, producing same loss of head as excess due to curve or special: 30-in. pipe, No. diameters	16-in. pipe, No. diameters	12-in. pipe, No. diameters
30 in.	15 ft.	6	Williams, Hubbell and Fenkell	30.0	....	....
30 in.	10 ft.	5	Williams, Hubbell and Fenkell	20.9	....	....
12 in.	4 ft.	4	Williams, Hubbell and Fenkell	......	....	15.3
16 in.	5 ft.	3.76	Williams, Hubbell and Fenkell	......	19.8	....
12 in.	3 ft.	3	Williams, Hubbell and Fenkell	......	....	13.92
30 in.	6 ft.	2.4	Williams, Hubbell and Fenkell	14.4	....	....
12 in.	2 ft.	2	H. F. Mills................	......	....	12.38
12 in.	13 in.	1.08	Williams, Hubbell and Fenkell	......	....	19.7
30-in. gate............			Williams, Hubbell and Fenkell	62.2	....	....
30-in. by 10-in. Y or 30-in. by 6-in. gross or two 30-in. by 6-in. T's			Williams, Hubbell and Fenkell	1.25	....	....
30-in. by 30-in. Y......			Williams, Hubbell and Fenkell	20. ±	....	....

resistance is expressed in equivalent lengths of straight pipe, measured in pipe diameters. (Trans. Am. Soc. C. E. Vol. XLVII, pp. 360 and 205.)

Unfortunately most laboratory experiments on curve resistance appear to have been made with measuring apparatus insufficiently delicate to detect the small differences of loss of head due to the introduction of curves in the experimental lines.

A length of about 100 diameters of pipe seems to be required to eliminate the effect of a curve on the distribution of velocities in a smooth pipe, and a somewhat shorter length as rougher pipe is involved.

Expansions when sudden always produce eddies which increase the loss of head. Consider two sections of a pipe, 1 and 2; 1 to be taken at a point where normal condition of flow exists before expansion and 2 after expansion. If v_1 and v_2 are the velocities and A_1 and A_2 the areas at the two sections then the loss of head due to this sudden enlargement

$$h_{fe} = \frac{(v_1 - v_2)^2}{2\,g} \quad \text{or} \quad h_{fe} = \left[\frac{A_2}{A_1} - 1\right]^2 \frac{v_2^2}{2\,g}$$

According to St. Venant, this quantity should be increased by $v^2/18\,g$, but this correction is so small as a rule that it can be neglected, and more recent experiments indicate that the formula is as likely to give results in excess as otherwise.

Experiments by Prof. T. J. Rodhouse at the Cornell Hydraulic Laboratory in 1905 gave:

$$h_{fe} = k\left[\frac{A_2}{A_1} - 1\right]^2 \frac{v_2^2}{2\,g}$$

wherein $k = 0.967$ for an expansion from 1.069 in. to 2.096 in. and 0.987 for one from 1.5 in. to 2.096 in. in circular brass pipe. A length of 35 diameters of the larger pipe was required to eliminate the effect of the expansion.

Contraction, when sudden, produces an effect upon a stream very similar to that of a sharp orifice; that is, just beyond the contraction occurs the point of minimum cross-section of the stream or the "vena contrata." There result not only the loss of head due to the contraction of the stream, but also that due to the reenlargement of it after passing the "vena contracta." If v is the velocity under conditions of normal flow in the pipe after passing the contraction and C is the coefficient of contraction, the same in this case as for a sharp orifice, then the loss of head due to the contraction is

$$h_{fc} = \frac{v^2}{2\,g}\left[\frac{1}{C} - 1\right]^2$$

According to St. Venant this quantity should be increased by $v^2/18\,g$. Also it may be written $h_{fc} = C_c v^2/2\,g$, where C_c varies from 0.42 to 0.53. A fair assumption to make is $C_c = 0.5$. This may also be taken as the loss of head due to sharp-edged entrance into a pipe. The value of C is probably too high for small pipes and too low for large pipes.

A length of about 30 diameters appears necessary to eliminate the effect of a contraction.

Obstructions. If the sectional area of a pipe be gradually decreased and then gradually increased as in the case of a Venturi meter, the loss of head for moderate velocities is not much increased over that due to normal flow. When the obstruction causes a sudden contraction or expansion of the stream or there is discontinuity of the pipe wall, the loss of head is increased.

The disturbances caused by the various obstructions extend for considerable distances downstream. Those producing symmetrical distortions of the velocity curve disappear sooner than the others.

Valves. The losses due to valves in pipe lines have been investigated with accuracy in only a few instances. From these experiments it appears that a fully open gate valve in a pipe causes a loss of head corresponding to about 6 diameters of length of the pipe.

Siphons. A siphon is a pipe whose center line rises above the hydraulic grade line at some portion of its course and hence operates through a partial vacuum.

The laws governing flow through siphons are the same as those for other pipes, but if the siphon be short, allowance must be made for losses at entry and due to curvature. Unless the velocity is maintained at about or above 2 ft. per sec. air will accumulate at the summit and may materially retard or absolutely check the flow. On this account the siphon should be carefully graded up to a high point at which air can be conveniently removed by an ejector or through a filling chamber from which point the drop should be quite abrupt to the outlet. On account of the tendency of air to expand and separate from the water at low pressure the lift of the siphon should be as low as circumstances will permit, and not above 20 ft. unless an ejector or a high velocity is used.

A siphon installed by George S. Pierson, M. Am. Soc. C. E., of 24-in.-diameter cast-iron pipe about 2900 ft. long with a 20-ft. lift has been in use for many years connecting a distant well with one near the pumping station at Kalamazoo, Michigan.

A 24-in.-diameter spiral riveted pipe siphon installed by the writer at Taughannock Falls, N. Y., in 1904, has an initial lift of 9 ft., a run of 411 ft., with a rise of 6 in. and a drop of 88 ft. to a waterwheel which is supplied through it, and drives a lighting plant furnishing current to the village of Trumansburg. The wheel has a draft tube giving a further drop of 14 ft., and no difficulty has been encountered from accumulation of air when running, the operation head being from 90 to 93 ft.

Flow Through Culverts. Experiments at the Hydraulic Laboratory of the University of Iowa by the Bureau of Public Roads have resulted in the following formulas for the flow through such structures when flowing full (Studies in Engineering Bulletin Univ. of Iowa, Feb. 1926):

Q = discharge in cubic feet per second;
A = cross-sectional area of pipe in square feet;
D = diameter of pipe in feet;
L = length of culvert in feet;
H = loss or fall of head through culvert including velocity head absorbed at entrance;
r = hydraulic radius in feet.

Concrete pipe with square cornered entrance

$$Q = \frac{A\sqrt{2gH}}{\sqrt{1 + 0.31\,D^{0.5} + \dfrac{0.026\,L}{D^{1.2}}}};$$

or for $L = 30.6$ ft., $Q = 4.40\,D^{2.09}\,H^{0.50}$.

Concrete pipe with beveled lip entrance

$$Q = \frac{A\sqrt{2gH}}{\sqrt{1.1 + \dfrac{0.026\,L}{D^{1.2}}}};$$

or for $L = 30.6$ ft., $Q = 4.61\,D^{2.18}\,H^{0.50}$.

Vitrified clay pipe, regular bell upstream

$$Q = \frac{A \sqrt{2\,g\,H}}{\sqrt{1 + 0.023\,D^{1.9} + \dfrac{0.022\,L}{D^{1.2}}}};$$

or for $L = 30.6$ ft., $Q = 5.07\,D^{2.05}\,H^{0.50}$.

Corrugated metal pipe — corrugations 2-3/4 in. apart and 1/2 in. deep

$$Q = \frac{A \sqrt{2\,g\,H}}{\sqrt{1 + 0.16\,D^{0.6} + \dfrac{0.106\,L}{D^{1.2}}}};$$

or for $L = 30.6$ ft., $Q = 3.10\,D^{1.31}\,H^{0.50}$.

Box culverts with square cornered entrance

$$Q = \frac{A \sqrt{2\,g\,H}}{\sqrt{1 + 0.4\,r^{0.3} + \dfrac{0.0045\,L}{r^{1.25}}}}.$$

Box culverts with rounded entrances of 6 = in. radius

$$Q = \frac{A \sqrt{2\,g\,H}}{\sqrt{1.05 + \dfrac{0.0045\,L}{r^{1.25}}}}.$$

Flaring the outlet end of a 36-ft.-long culvert having a rounded entrance by diverging the sides at an angle of 6° 30′ with axis to double area at outlet increased its discharge 60% and gave a discharge 86% above that of a similar culvert without flare and having square cornered entrance.

13. Oil, Sand and Air

Oil Pipe Lines. With heavy viscous oils the loss of head becomes so large that transportation by pipe lines is difficult. Raising the temperature of the oil increases its fluidity, but this is satisfactory only for short lines, because to maintain a sufficient fluidity throughout a long line the initial temperature must be raised to such a point that disintegration is likely to result. Experiments have been tried with oil mixed with water, but cross currents and eddies in the pipe line produce at the discharge an emulsion, from which it is very difficult to separate the water. It is, however, possible by mixing about 10% of water with the oil and forcing it through a pipe line having rifling grooves or guides along its walls, to facilitate the transportation. The rotation of the liquid by the rifling causes the water to form a thin film or sheet between the oil and the pipe. Emulsification does not take place, and practically all the water can be separated by allowing the mixture to stand for a short time in a tank.

The rifling of an 8-in. line used by the Southern Pacific Railway Company consists of six helical grooves in the circumference, making a complete revolution every 10 ft. of axial length. This line was laid with a valley every 400 ft. of length, the depth being equal to the diameter of the pipe, in order to facilitate starting the flow after the line had been out of service and the expedient seems to have served its purpose.

The results of tests made by the Southern Pacific Railroad Company give the following values of K for the formula $h_f = Kv^2L/1000\,D$. (Engr. News, June 7, 1906, and Eng. Record, May 23, 1908.)

	K
8-in. plain pipe carrying oil only	134
8-in. plain pipe carrying 90% oil and 10% water	79
8-in. rifled pipe carrying 90% oil and 10% water	0.95
3-in. plain pipe carrying oil only	284
3-in. rifled pipe carrying 90% oil and 10% water	0.64
Mean of above pipes for water only	0.35

Filter Plant Sludge or Clay Slurry. The losses of head due to pumping sludge from the New Orleans Filtration Plant were investigated by Prof. W. B. Gregory of Tulane University in 1926, and the results were published in Mechanical Engineering, June, 1927.

The loss of head appeared to be substantially constant and independent of velocity at values below the critical velocity and it therefore follows that when such material is to be pumped the most economical rate of flow is at the critical velocity.

Flow of Clay Slurry in 4-in. Pipe

Experiment No.	Per cent solids by weight	Specific gravity of slurry	Critical velocity V_c in ft. per sec.	H_f in ft. of water	H_f in ft. of slurry	Apparent * viscosity μ	Average temperature degrees Fahr.	Corresponding H_f for water, H. and W. formula C = 140
1	18.6	1.13	2.25	5.25	4.65	0.0162	88+	5.18
2	23.4	1.175	3.60	13.00	11.05	0.0251	88+	12.40
3	29.05	1.225	6.00	28.50	23.25	0.0331	88+	31.80
4	32.5	1.255	7.00	48.00	38.25	0.0477	88+	42.20
5	35.3	1.285	8.50	70.00	54.50	0.0572	88+	57.50
6	23.6	1.17	1.5	3.85	3.29	0.0178	65	2.45
7	23.6	1.17	2.0	5.00	4.28	0.0174	65	4.15

* This fluid is not truly viscous.

The approximate composition of the sludge was:

	Per cent
Clay and sand	70.29
Carbonate of lime	23.96
Hydrate of magnesia	2.90
Hydrated oxide of iron	2.32
Undetermined	0.53
Total	100.00

At velocities above the critical the loss of head corresponded closely with that of clear water as given by the Hazen and Williams formula with a coefficient of 140.

Transportation of Solids. The percentage of solid matter that can be transported in flowing water in a pipe after a velocity has been attained sufficient to cause the suspension of the solids, is independent of the velocity. At the lower velocities the solids are rolled or dragged along the bottom with a relatively large loss of head; as the velocity increases they are picked up by the water and retained in suspension. The velocity at which the material becomes suspended is dependent upon the size of the grains and their weight. The velocity at which the material becomes approximately all suspended is the one at which the minimum loss of head will occur; and when the cost of pressure is great compared with the cost of the channel, this will be the most economical velocity to use. If the material varies in the size of its grains the condition of minimum resistance may occur before the heavier particles are in suspension. Ordinarily, however, pressure is of less importance than the line cost, and hence higher velocities will be used in order that the delivery per unit of time may be as great as possible.

The accurate experiments bearing on the subject have been made with sands having an effective size varying from 0.16 mm. to 0.75 mm. For a 1-in. pipe the velocity of suspension for such material is about 3.5 ft. per sec.; for a 3-in. pipe about 4 ft., and for a 32-in. pipe about 9 ft. The loss of head due to the transportation of these sands may be taken as about 25% of the loss due to water alone at the velocity of suspension, for each 1% of sand added to the water. For the higher velocities with fine sands, the resistance appears to increase somewhat more rapidly than with water alone, whereas with the coarser material the increase is slightly less rapid. For average results the loss of head for the mixture may be taken to vary from the velocity of suspension as v^2, until more complete investigation shall discover the true law. As much as 52% of sand has been transported through a 4-in. pipe. (Trans. Am. Soc. C. E., 1906, vol. 57, p. 400.)

Flow of Compressible Fluids in Pipes. When ai · flows along a pipe there is necessarily a fall of pressure due to the resistance of the pipe, and consequently the density of the air decreases and its veclocity increases along the pipe in the direction of motion. The effect of the resistance is to create eddying motions, which, as they subside, give back to the air the heat equivalent of the work expended in producing them. The result is that, apart from conduction through the walls of the pipe, the flow is isothermal.

Flow of Gas in Pipes under Small Differences of Pressure. Let p_1 and p_2 be the absolute unit pressures at the inlet and outlet of the pipe, w_g the weight of the gas in pounds per cubic foot, h_{fg} = head lost in feet of the gas, h_f = head lost in feet of water, and f = coefficient. If $p_1 - p_2$ be small so that the change in density may be neglected, then

$$p_1 - p_2 = w_g h_{fg} = w h_f$$

The law of flow may be expressed by the Fanning formula

$$h_f = \frac{4\,fL}{D}\,\frac{v^2}{2\,g}$$

and f has an average value of $0.0044\,\frac{w_g}{w}\left(1 + \frac{1}{7\,D}\right)$. Or when h is measured in feet of gas the coefficient becomes $f' = 0.0044\left(1 + \frac{1}{7\,D}\right)$. Experiments on compressed air give

$$f = 0.0027\,\frac{w_a}{w}\left(1 + \frac{3}{10\,D}\right) \quad \text{and} \quad f' = 0.0027\left(1 + \frac{3}{10\,D}\right)$$

Head Lost in an Inclined Gas Main. Let h_{p1} and h_{p2} be the heights of water columns at points 1 and 2, z_1 and z_2 be the elevations of the points, w_a the unit weight of air and let the flow be from 1 to 2, then

$$h_{fg} = \frac{1}{w_g}\left\{w(h_{p1} - h_{p2}) - w_a\,(z_1 - z_2)\right\} + z_1 - z_2$$

or

$$h_f = (h_{p1} - h_{p2}) - \frac{w_a - w_g}{w}\,(z_1 - z_2)$$

When Variation in Density is taken into account, the flow of air in a long uniform pipe is modified. Let v_1 be the initial velocity in the pipe whose diameter remains uniform; then the true equation as given by Unwin (Hydraulics, London) is

$$v_1 = \sqrt{\left\{222\,900\,\frac{D}{fL}\,\frac{w_a}{w}\,\frac{p_1^2 - p_2^2}{p_1^2}\right\}},$$

and an approximate formula for ordinary temperature is

$$v_1 = \left(1.132 - 0.726\,\frac{p_2}{p_1}\right)\sqrt{222\,900\,\frac{Dw_a}{fLw}}$$

The **terminal pressure** p_2 in terms of the **initial pressure** p_1 is given by

$$p_2 = p_1 \sqrt{1 - \frac{f \frac{w}{w_a} v_1^2 L}{229\,000 D}}$$

The distribution of velocity in an air main is very similar to that in a water main; that is, it approximates to a cylinder surmounted by an ellipsoid. The ratio of the mean to the velocity at the center is given as 0.873.

14. Flow over Weirs

Sharp-edged Weirs. When an obstruction is placed in an open channel, so that water is caused to flow over it, it is called a dam or weir. If the top of the weir is a thin straight edge, the conditions of flow are similar to those that would exist in an orifice in a thin wall if the side contractions were suppressed and the head fell so low that the water did not fill the orifice to its top. If the portions of the dam near the walls of the channel are raised above the level of the rest so that water does not flow over them, the overflowing jet is contracted at the sides as in the case of an orifice. The general expression for the discharge of water over a weir is

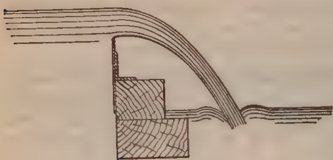


Fig. 16. Sharp-edged Weir

$$Q = 2/3\, CLH \sqrt{2\, gH}$$

wherein H is the height above the crest of the weir to the level of still water and L is the length of the crest over which the water flows. Practically, it is not possible to measure H, but a head h may be observed to the surface of the stream above the curve of depression caused by the weir, and to this the velocity head h_v due to the velocity v_a with which the water approaches the weir, may be added when the result is approximately equal to H. If the velocity of approach is small, h as observed may be treated as equal to H. C is a coefficient which depends upon the height and form of the weir, whether or not there be end contractions, the character of the weir surface and the condition of the water on the downstream side. In weir formulas it is customary to combine one or more of the factors 2/3, C, and 2 g into a single coefficient.

Four Recognized Formulas for the discharge of weirs are as follows, but the first and the fourth are the most important.

The Francis formula

$$Q = 3.33\, LH^{3/2} \quad \text{or} \quad Q = 3.33\, L\, [(h + h_v)^{3/2} - h_v^{3/2}]$$

The Fteley and Stearns formula

$$Q = 3.31\, LH^{3/2} + 0.007\, L \quad \text{or} \quad Q = 3.31\, L\, (h + 1.5\, h_v)^{3/2} + 0.007\, L$$

The Hamilton Smith formula

$$Q = 3.29\, (L + H/7)\, H^{3/2} \quad \text{or}$$

$$Q = 3.29 \left(L + \frac{h + 1\text{-}1/3\, h_v}{7}\right) h + 1\text{-}1/3\, h_v)^{3/2}$$

The Bazin formula

$$Q = mLh\sqrt{2gh},$$

where $m = \left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{a+h}\right)^2\right]$

in which a is the height of the crest of the weir above the bottom of the channel of approach. For weirs with end contractions, Francis concluded that L in the above formulas should be replaced by $L - 0.1\,nH$, where n is the number of full end contractions. This correction has been generally accepted, but it is by no means accurate, and for exact work in measuring water a weir without end contractions is to be preferred. These formulas all apply to a weir with a vertical upstream face, a sharp edge and with free access of air the under side of the overfalling sheet of water.

Rehbock's formula, published in 1912, is based on measurements over weirs ranging in height from 0.41 ft. to 1.64 ft. in a channel 1.64 ft. wide, with heads from 0.03 ft. to 0.75 ft., and is:

$$Q = 2/3\, mLh\sqrt{2gh} \quad \text{where} \quad m = 0.605 + \frac{1}{320\,h - 3} + 0.08\,\frac{h}{a}$$

Hazen's formula for sharp-edged weirs without end contraction (Trans. Am. Soc. C. E., Vol. LXXVII, p. 1289 *et seq.*) is:

$Q = \left(3.27 + 0.5\,\frac{h}{a}\right) h^{3/2}$, and for heads above 0.3 ft. gives results within 1% of those obtained by Francis and Fteley and Stearns.

The Schoder and Turner formula

$$Q = 3.33\,L\left[\left(h + \frac{v_a^2}{2g}\right)^{3/2} + h\,\frac{v_b^2}{2g}\right]$$

wherein v_a is the mean velocity in the channel above the level of the crest of the weir, and v_b is that in the portion below it, is the most accurate yet devised. (Trans. Am. Soc. C. E., Vol. XCII.)

Triangular or V-shaped Weir. This form of weir, suggested by Prof. Thomson of Dublin, possesses the peculiarity that, whatever the heads, the sections of the stream are similar, and hence it may be expected to have a coefficient more nearly constant than the ordinary weir and be particularly well adapted to the measurement of water where the flow varies through a considerable range. The coefficient will vary for different inclinations of the sides of the notch. For a sharp-edged weir in which the sides make an angle of 90 deg. with each other, since $L = 2\,h$, the discharge is $Q = 2.6\,h^{5/2}$.

Prof. H. W. King (Michigan Technic, October, 1916), gives $Q = 2.52\,h^{2.47}$ deduced from a series of experiments covering a range of h from 0.15 ft. to 1.8 ft.

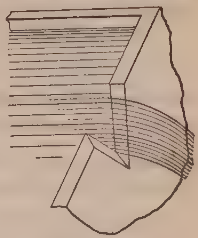


Fig. 17. Triangular Weir

Trapezoidal or Cippoletti Weir. By combining the rectangular notch or weir with end contractions, with the two halves of a triangular weir, it is possible to so proportion the triangular portions that they shall compensate for the effect of contraction and the result is approximately obtained when the inclination of the ends is one horizontal to four vertical. The formula

$Q = 3.367\ Lh^{3/2}$ is commonly used for such a weir without **velocity of approach.** Professor H. W. King recommends

$$Q = 3.34\ Lh^{1.47}\left[1 + 0.56\left(\frac{h}{a+h}\right)^2\right]$$

which takes account of **velocity approach.**

No experiments have been made upon weirs of this type when other than sharp-edged with vertical faces, but the effects of inclination and rounding may be expected to affect them similarly to rectangular weirs.

Rounding the Upstream Corner of the crest of a weir increases the discharge. With flat-crested weirs Bazin found this effect to amount to as much as 13% where the radius of the rounding was 4 in. and the breadth of crest 6.56 ft. Fteley and Stearns, with weirs up to 1 in. in breadth, found the rounding to be equivalent to increasing the head by $h_R = 0.7\ R$, where R is the radius of the rounding.

Inclining the Upstream Face away from the current decreases the contraction and increases the discharge as much as 10% when the slope is one of 45 deg. If the inclination is in the opposite direction, the contraction is increased and the discharge decreased. With a 45-deg. slope, the decrease may be as much as 7%. Inclining the **downstream face** does not materially alter the discharge until the slope becomes at least 3 horizontal to 1 vertical, when the discharge is reduced.

Rounding the Entire Crest reduces the discharge for low heads, but increases it for those wherein the curve of the crest approaches the curve of the natural under side of the sheet. By a combination of a rounded crest and an inclined upstream slope, the discharge may be increased 20% above that of the sharp-edged weir.

Flat Crests decrease the discharge until the head becomes so high that the sheet jumps clear of the downstream corner, when they have no effect. A broad flat crest may reduce the discharge 25% below that of the sharp edge.

The Sheet of Water adhering to the downstream face of a vertical sharp-edged weir has increased the discharge about 28%. The sheet being wetted, that is depressed and the space between it and the weir filled with water, due to the formation there of a partial vacuum, has increased the discharge about 15%. The sheet being depressed, but the space only partially filled with water, has increased the discharge about 6%.

Submerged Weirs. When water on the downstream side of the weir rises above the level of the crest, the weir is said to be submerged. If h_u is the head observed on the upstream side and h is the difference of head on the two sides, the usual formula for the discharge of a submerged weir is

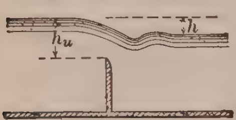


Fig. 18. Submerged Weir

$$Q = CL\sqrt{2\,gh}\,(h_u - h/3)$$

where C for a sharp edge varies from 0.58 to 0.63. On account of the difficulty of measuring h, the head in the lower pool, because of the turbulence there, accurate results with this formula are impossible.

15. Open Channels

Hydraulic Radius or Hydraulic Mean Depth. The Hydraulic Radius, also called the Hydraulic Mean Depth because it is the depth of a rectangle whose area is the same as that of the section under consideration and whose width is equal to the wetted perimeter of the latter, is equal to the area divided by the wetted perimeter and is represented by r. In open channels it takes the place of D in the pipe formulas, and for a circular pipe flowing full or half full is equal to $D/4$. The cross-sections of natural channels, particularly in soft materials, approximate to one or to two parabolic segments. It may therefore be useful to utilize the relation of the area to the arc of the parabola in approximating the hydraulic radius. Let y_1 = semi-width of channel if symmetrical about a vertical, or the distance from the bank to the vertical of greatest depth, d = the greatest depth, S_1 = length of the arc from the surface to the foot of the deepest vertical. The area of the semi-parabola $A_1 = 2/3\ y_1 d$, and the length of the arc when $d < 0.25 y_1$ is approximately $S_1 = y_1 + 2/3\ d^2/y_1$. If the channel is symmetrical, $r = A_1/S_1$, but if the other part be made up of a different parabola, then a similar expression may be written for A_2 and S_2 and $r = (A_1 + A_2)/(S_1 + S_2)$.

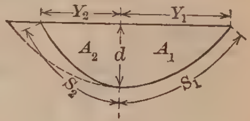


Fig. 19

Long Channels. In the term "open channel" are included all rivers, artificial canals, aqueducts and conduits and, in addition, sewers and pipes of whatever section which run partially full. The force producing flow cannot now be provided for by any external head, but is solely due to the slope or gradient of the channel.

Normal Flow. When the magnitude and distribution of velocities at various sections over a considerable length of the stream are the same, the flow is said to be normal. The curve of the distribution of velocities in a vertical section parallel to the direction of motion through any point of an open channel approximates an ellipse, being tangent to the bottom and with its axis in or near the surface of the stream. Wave action, eddies, etc., cause the velocities near the surface to be decreased, and this effect may extend to nearly half the depth. The actual curve then follows an ellipse halfway to the surface, where it leaves the ellipse and becomes somewhat flattened, giving a maximum velocity in the upper two-tenths of the depth. The mean velocity on any vertical is normally about 0.9 the maximum and occurs about 0.577 d from the surface, where d is the depth of the stream. The loss of head in open channels, probably on account of their greater irregularity, varies as a somewhat higher power of v than in pipes. It, therefore, is possible to use formulas involving v^2 with a less variation in the coefficient than in pipes, though an exponent of 1.9 gives more constant coefficients.

The Chezy Formula is $\mathbf{v = C\sqrt{rs}}$, where v is the velocity, C a coefficient, s the slope and r the hydraulic radius = area ÷ wetted perimeter.

The Bazin Formula is probably one of the best yet devised for the flow of water in open channels, although the same careful scrutiny has not been given to its roughness factors as to those of Kutter. It is

$$\mathbf{v} = \frac{87}{0.552 + \frac{m}{\sqrt{r}}}\sqrt{rs} = C\sqrt{rs}$$

where r is the hydraulic radius in feet and v is the velocity in feet per second. Values of the roughness factor m are as follows:

Very smooth surfaces of cement and planed boards	0.06
Smooth surfaces of boards, bricks, concrete	0.16

For brick sewers and dirty concrete	0.28
Ashlar or rubble masonry	0.46
Earthen channels, very regular or pitched with stones, tunnels and canals in rock	0.85
Earthen channels in ordinary condition	1.30
Earthen channels presenting an exceptional resistance, the wetted surface being covered with detritus, stones or weeds	1.75

Bazin's Formula. Values of C in $v = C\sqrt{rs}$

Hydraulic radius r, feet	(a) m = 0.06	(b) m = 0.16	(c) m = 0.28	(d) m = 0.46	(e) m = 0.85	(f) m = 1.30	(g) m = 1.75
0.2	127	96	74	55	35	25	19
0.3	131	103	82	63	41	30	23
0.4	135	108	88	68	46	32	26
0.5	137	112	92	72	50	37	29
0.6	139	116	96	76	53	39	31
0.8	141	119	101	82	58	43	35
1.0	142	122	105	86	62	47	38
1.3	144	126	109	91	67	51	42
1.5	145	128	112	94	70	54	44
1.75	146	130	114	97	73	57	46
2.0	147	132	116	99	76	59	49
2.5	148	134	119	103	80	64	53
3.0	149	136	122	107	84	67	56
4.0	150	138	126	111	89	72	61
5.0	151	140	129	115	94	77	65
6.0	151	142	131	118	98	80	69
8.0	152	144	134	122	102	86	74
10.0	153	145	136	125	106	90	79
12.0	....	....	....	....	109	94	82
15.0	....	....	....	....	113	98	87
20.0	....	....	....	....	117	103	92
30.0	....	....	....	....	123	110	100
50.0	....	....	....	....	129	119	108

(a) For very smooth cement and planed boards.
(b) For smooth boards, brick, concrete, glazed earthenware pipes.
(c) For smooth but dirty brick or concrete.
(d) For ashlar masonry.
(e) For earth canals in very good condition and canals pitched with stones.
(f) For earth canals in ordinary condition.
(g) For earth canals exceptionally rough.

The Kutter Formula, though largely used, is probably not entirely satisfactory for large streams with slight slopes. It depends for its accuracy almost entirely upon the experimental determination of its coefficient of roughness n, which changes for different velocities in the same channel, and great care must therefore be exercised in extending computations beyond the limits of actual experiment. Let r = hydraulic radius, s = slope, and v = mean velocity; then the formula is, for the English system of measures,

$$v = \left[\frac{41.6 + \frac{1.811}{n} + \frac{0.00281}{s}}{1 + \left(41.6 + \frac{0.00281}{s}\right)\frac{n}{\sqrt{r}}}\right]\sqrt{rs} = C\sqrt{rs}$$

Kutter's Formula. Values of Coefficient C

Hydraulic radius r, ft.	Coefficient n .010	.015	.020	.025	.030	.040	Coefficient n .010	.015	.020	.025	.030	.040
	Slope = 0.000025						Slope = 0.00005					
0.1	57	33	23	17	14	10	67	39	26	20	16	11
0.2	75	45	31	24	19	14	87	51	35	26	21	15
0.4	97	59	42	32	26	19	109	66	46	35	28	20
0.6	112	69	49	38	31	22	122	76	53	41	33	24
1.0	131	83	60	47	38	28	140	89	64	49	40	29
1.5	148	95	69	55	45	33	154	99	72	57	47	34
2.0	160	104	77	61	50	37	164	107	79	62	51	38
3.28	181	121	90	72	60	45	181	121	90	72	60	45
6.0	206	142	108	88	74	57	199	137	105	85	72	56
10.0	225	159	124	102	87	68	212	149	116	96	82	64
16.0	242	174	138	115	100	79	223	160	126	106	91	73
30.0	261	193	157	133	117	95	236	172	139	118	103	84
50.0	274	207	170	147	130	107	245	181	148	127	112	93
	Slope = 0.0001						Slope = 0.0002					
0.1	78	44	30	22	17	12	85	48	32	24	18	12
0.2	98	57	39	29	23	16	105	61	42	31	25	17
0.4	119	72	50	38	31	22	125	76	53	40	32	23
0.6	131	81	57	44	35	25	138	85	60	46	37	26
1.0	147	98	67	52	42	31	151	96	69	54	44	32
1.5	159	103	75	59	48	35	162	105	77	60	49	36
2.0	168	109	81	64	53	39	170	111	82	64	54	40
3.28	181	121	90	72	60	45	181	121	90	72	60	45
6.0	195	134	102	84	71	54	193	132	100	82	69	53
10.0	205	143	111	92	78	62	201	140	108	89	76	60
15.0	212	150	118	98	85	68	207	145	113	95	82	65
30.0	222	160	128	108	95	77	215	154	122	103	89	73
50.0	227	166	134	114	100	83	220	158	126	108	94	78
	Slope = 0.0004						Slope = 0.001					
0.1	89	50	34	25	19	13	94	54	36	27	21	14
0.2	110	65	44	32	25	18	113	66	45	34	27	18
0.4	129	79	55	42	33	23	131	80	56	43	34	24
0.6	140	87	62	47	38	27	142	88	63	48	39	27
1.0	154	98	70	55	45	32	155	99	71	56	45	33
1.5	164	106	78	61	50	37	165	108	78	62	50	37
2.0	170	112	83	65	54	40	171	112	83	66	54	40
4.0	184	124	94	76	63	48	184	124	93	75	63	48
6.0	191	130	99	81	69	53	190	130	99	81	68	52
10.0	199	138	107	88	75	59	197	136	105	87	74	58
20.0	207	146	115	96	83	66	205	144	113	94	81	65
50.0	215	154	123	104	91	75	212	151	120	101	89	72
	Slope = 0.01											
0.1	95	54	36	27	21	14						
0.2	114	67	46	34	27	19						
0.4	133	82	57	44	35	24						
0.6	143	90	64	49	39	28						
1.0	156	99	72	56	45	33						
1.5	165	107	79	62	51	37						
2.0	171	112	83	66	55	40						
3.28	181	121	90	72	60	45						
6.0	190	129	99	81	68	52						
10.0	196	136	105	86	74	58						
20.0	204	143	112	93	80	64						
50.0	210	150	119	100	87	71						

For slopes steeper than 1 ft. per 100 ft., the coefficient C remains practically constant for a given n and r and has the values given in the adjacent columns for $S = 0.01$.

When $r = 3.28$ ft. the value of C is independent of the slope and its value is $1.811/n$.

where the coefficient of roughness n has the following values:

Rectangular wooden flume (very smooth)	0.0098
Neat cement, glazed pipes and very smooth iron pipes	.010
Plaster, 1 : 3 mixture, iron pipes in best order	.011
Unplaned timber, ordinary iron pipe	.012
Brick washed with cement, basket-shaped sewer:	
6 by 6.7 ft., nearly new	.0130
6 by 6.7 ft., one year old	.0148
6 by 6.7 ft., four years old	.0152
Brick washed with cement, 9 ft. in diameter:	
Nearly new	.0116
Four years old	.0133
Old Croton aqueduct, brick lined	.015
Sudbury aqueduct	.01
Glasgow aqueduct, cement lined	.0124
Steel pipe, riveted, clean, 1897 (mean)	.0144
Steel pipe, riveted, clean, 1899 (mean)	.0155
Rough brickwork, incrested or tuberculated iron	.015
Brickwork or ashlar in bad condition, rubble in cement in good order	.017
Rough rubble in cement, stone pitching, very firm gravel	.020
Earth of tolerably uniform cross-section, slope and direction, in moderately good order and regimen, and free from stones and weeds; or stone pitching in bad condition	.025
Earth, having stones and weeds occasionally	.030
Gravel in bad condition, earth in bad order and regimen, overgrown with vegetation, and strewn with stones and detritus	.035

Manning's Formula (Trans. Inst. C. E. of Ireland, Vol. 20, 1890) is

$$v = \frac{1.486}{n} r^{2/3} s^{1/2}$$

in which the values of n are very near those for Kutter's formula given above.

The Hazen and Williams formula

$$v = cr^{0.67} s^{0.54} 0.001^{0.04}$$

may be used for open channels with the following coefficients:

For very smooth channels	C = 135 to 140
For ordinary unplaned plank	C = 104 to 118
For ordinay sewer crock	C = 100 to 127
For ordinary brick sewers	C = 100 to 125
For large canals in rock	C = 75 to 95
For ordinary earthen channels	C = 35 to 84
For rough natural channels	C = 30 to 70
For floodways through brush	C = 25 to 40
For floodways through trees	C = 15 to 20

16. Variations in Sections

Roughness. In general, the effects of roughness in a stream bed have been taken care of by coefficients. The action in streams is similar to that in pipes, causing eddies and cross currents which absorb head and so reduce velocity.

Curvature disarranges the distribution of velocities and prevents the continuance of normal conditions of flow. In general, the tendency is for the velocity on the concave side of the stream to decrease, thus causing some of the material in suspension to be deposited. This decrease in velocity on the concave side is accompanied by an increase in velocity on the convex side, giving rise to erosion. In general terms, the conditions of scour will prevail where the hydraulic axis is convergent to the bank, produce fill where this axis diverges from the bank, and cause no action where the axis and bank are parallel; the hydraulic axis is taken as the locus of the center of gravity of the section of the stream.

Principles which govern a stream freely forming its own channel in an alluvial plane are: (1) A plastic mass moving in a resisting medium will assume the form in which it encounters least resistance. (2) The transverse section of a body adjusted to the form of least resistance will have the maximum ratio of area to perimeter. (3) The form of a fluid mass will vary, whenever the direction of movement is changed, by virtue of the unequal inertia of particles in different parts of the mass. (4) The form being unsymmetrical and the direction of movement not a straight line, if the mass is variable the path described by the center of gravity, or the hydraulic axis, will be variable also in position and length. (5) With given limits of mass variation, the vagation of the paths will lie within a zone of certain width.

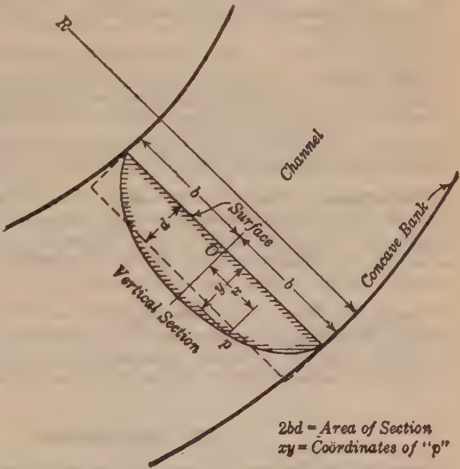


Fig. 20

Thus a river traversing a homogeneous soil will form a bed whose width and depth will be largely determined by the variations in volume, being wider and shoaler as the vagation of the hydraulic axis is greater; narrower and deeper as the volume becomes constant and the vagation of the hydraulic axis becomes less.

Cross-section and Curvature. The relation between the cross-section of the stream and the curvature in soft material when the channel has reached a condition of stability is given by H. C. Ripley, Trans. Am. Soc. C. E., Vol. XC, by the following equations from which the section may be plotted:

For streams where the channel occupies the full width of the waterway:

$$\text{(I)}\quad y = 1.445\,d\left(1 - \frac{x^2}{b^2}\right) + 1.445\,d\left[\frac{17.52}{R}\left(1 + \frac{x^2}{b^2}\right)x\right]$$

For channels not occupying the entire width of the waterway and to channels at harbor entrances created by the action of a single curved jetty:

$$\text{(II)}\quad y = 1.65\,d\left(1 + \frac{x^2}{b^2}\right) + 1.65\,d\left[\frac{26.28}{R}\left(1 - \frac{x^2}{b^2}\right)x\right]$$

Wherein d = mean depth of channel in feet;
b = half-width of channel in feet;
R = radius of curvature of the concave side of the channel, in feet;
x is the abscissa and y the ordinate of a point on the bottom, the origin of coordinates being taken at the center of the channel at the water surface.

When R is less than 40 $\sqrt{\text{area of section}}$ no further deepening of the channel results from increased sharpness of curvature.

When R is greater than about 50 $\sqrt{\text{area of section}}$ the shape of the channel does not conform strictly to that given by the equations.

Formula II is not applicable to straight channels nor to curved ones when R exceeds about 110 $\sqrt{\text{area of section}}$.

On a cross-over bar when the channel is neither on a curve nor in a straight reach the maximum depth is about 14-1/2% less than the computed value.

Formula II gives a width of channel at mean depth about 20% greater than the actual width.

Hence it follows: (1) That every artificial channel, whether it is a flume, an open aqueduct, a ditch or a drainage canal, to discharge the greatest volume with a given slope should have the form determined by formula I both in straight and curved reaches; and (2) That bends in streams when R is greater than 40 $\sqrt{\text{area of section}}$ are constructive bends and tend to stability whereas sharper bends are destructive, tending to shift the channel.

Obstructions. The loss in velocity, due to obstructions, arises mainly in the loss in expansion from the contracted section to the normal section of the stream. Wherever an obstruction occurs, the velocity is increased and the surface level generally lowered. After the obstruction is passed, the velocity decreases and the surface rises to normal.

Change of Section. (See also General Laws under Curvature.) In general a change of section is accompanied by a change in the distribution of velocity. In a stream through a homogeneous alluvial soil, change in the distribution of velocity is accompanied by either erosion or deposition of material.

17. Backwater

Formulas are frequently given for computing the extent and rise of backwater caused by an obstruction in a stream, but in order that a formula may be applicable to such cases as occur in practice it must be extremely complicated. Those usually given consider only a change in depth, without the necessarily corresponding change in width. For practical application, except in the extremely rare case of backwater in an artificial channel with vertical sides, such formulas are valueless. The practical method of solving the problem is to divide the channel in which the backwater occurs into reaches such that both the velocity and the hydraulic radius will be substantially constant throughout each reach, and to compute for each reach, beginning with that next the obstruction, for a mean v and r, the h_f. This h_f when added to elevation of the surface at the downstream end gives the elevation at the beginning of the next reach. By making the several sections sufficiently short, any desired degree of accuracy may be obtained within the limits of the formula used for loss of head.

Bridge Piers. An experimental investigation on models of bridge piers having widths of 6 in. and lengths from 18 to 33 in. in a flume 26 in. wide, the depth of water being about 2 ft., by Floyd A. Nagler, Jun. Am. Soc. C. E. (Trans. Am. Soc. C. E., Vol. LXXXII), has developed the following formula for backwater due to such piers:

Let C = a coefficient;

d_1 = depth of water at point of measurement of velocity of approach above piers in feet;

d_2 = depth of water at point of measurement of velocity of retreat below piers in feet;
h = observed difference of head between water surfaces above and below piers = backwater effect of piers in feet;
K = a coefficient;
Q = flow in channel in cubic feet per second;
V_1 = velocity of approach above piers in feet per second;
V_2 = velocity of retreat below piers in feet per second;
W = unobstructed width of channel at piers in feet.

Then assuming the slope of the channel throughout the area considered to be zero:

$$h = \frac{Q^2}{2g\left[CW\left(d_2 - \frac{0.3\,V_2^2}{2g}\right)\right]^2} - K\frac{V_1^2}{2g}$$

in which C varies from 0.86 for a rectangular pier to 0.94 for a pier with boat-shaped nose and fish-shaped tail.

If there is a perceptible fall in the channel from the point of measurement of d_1 to that of d_2 the value of d_2 must be decreased by this fall, and if there is a rise the value of d_2 must be increased by the rise.

The coefficient K varies with the percentage of the original channel that is obstructed, approximately as follows:

Per cent obstructed..	0	5	10	15	20	30
Coefficient K.........	1.00	1.04	1.27	1.50	1.70	1.93
Per cent obstructed..	40	50	60	70	80	90
Coefficient K.........	2.00	2.03	2.04	2.05	2.06	2.07

Example. Let Q = 19 500 c.f.s.
Area of unobstructed stream = 3680 sq. ft.
Area of obstruction = 351 sq. ft. = 9.5%
Area of clear waterway = 3329 sq. ft.
Depth of channel d_2 = 16.4 ft.

$$V_1 = \frac{19\,500}{3680} = 5.3 \text{ ft. per sec.}$$

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} = 0.4362 \text{ ft. and } 0.3\frac{V_2^2}{2g} = 0.131 \text{ ft.}$$

Assume C = .894 from shape of obstruction
K = 1.25 from area of obstruction

Substituting in formula there results:

$$h = \frac{19\,500^2}{64.4 \times 0.894^2 \times 203^2\,(16.4 - 0.13)^2} - 1.25 \times 0.4362$$

$$= 0.677 - 0.545 \text{ ft.} = 0.132 \text{ ft.}$$

Jump. When an obstruction is present in a stream, and the slope is so steep that the velocity v_1 before the obstruction is greater than $\sqrt{gd_1}$, where d_1 is the depth before the obstruction, a jump is likely to occur, producing a depth d_2, and if the velocity head $v_1^2/2g$ be called h, the value of d_2 is slightly less than that given by

$$d_2 = 1/2\,h + \sqrt{h\,(d_1 + 1/4\,h)}$$

and the height of the jump is then $d_2 - d_1$.

Investigations by Prof. S. M. Woodward of the State University of Iowa

(Miami Conservancy District Technical Reports, Part III, Dayton, Ohio, 1917) have developed the following formula for the **hydraulic jump** which is in close accord with the experimental results:

Let d_1 = depth of water above jump;
d_2 = depth of water below jump;
V_1 = velocity of stream before jump occurs.

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2\,V_1^2\,d_1}{g}}$$

Waves. When the water in a stream passes under or over some obstruction and is then thrown upward, a standing wave is produced. When a stream of high velocity discharges into a large body of water moving with a slower velocity, the curve of heading up does not always extend back till it reaches the free surface of the water, but ends abruptly at the point of change of velocity and a standing wave may be there produced. Under these conditions very little of the velocity head is converted into head of elevation, but is absorbed in impact and wave formation.

Whirlpools, eddies, and waves all indicate a useless expenditure of force and are like to cause damage to banks and bed. Where a minimum of loss of head is desired they should be as far as possible eliminated, which may usually be accomplished by smoothing the channel and removing abrupt changes of section.

18. Transportation of Sediment

Sediment can be transported by a stream in two ways: By being dragged or rolled along the bottom, or by being carried along in suspension. The weight of particles which will be dragged or rolled along by a stream is supposed to vary as the sixth power of the velocity. The experimental data usually quoted to determine the velocity at which different materials are moved were made before the beginning of the last century. These with some more added since by various observers gave a limiting bottom velocity for stability of material as follows:

Material	Bottom velocity, ft. per sec.
Soft earth	0.25
Soft clay	0.50
Sand	1.00
Gravel	2.00
Sea pebbles (1.06 in. in diameter)	2.20
Brickbats (4.76 cu. in.)	2.25 to 2.50
Slate (9.06 cu. in.)	2.75 to 3.00
Broken stone	4.00

Observations made in India show that for any section of channel and character of silt there is a certain **critical velocity** $\mathbf{v_s}$ at which silt is neither picked up nor deposited. If the velocity is increased, scour results; and if it is decreased, silting occurs. This critical velocity is found to depend upon the depth and to be given by the equation $\mathbf{v_s = md^{.64}}$, where d is the depth of the channel and m is a coefficient whose values are given as follows:

For fine light sandy silt of northern India,	$m \doteq 0.82$
For somewhat coarser light sandy silt,	$m = 0.90$
For a sandy loam,	$m = 0.99$
For a rather coarse silt, as débris of hard soils,	$m = 1.07$

The silt-transporting power of a stream is said to vary as $v^{2.56}$. (Kennedy, Proc. Inst. C. E., 1894–5, vol. 119.)

Velocities of Descent of certain fine materials through still water are:

Materials	Velocity, ft. per sec.
Brick clay (mixed with water and allowed to settle half an hour)	0.009
Fresh-water sand	0.166
Sea sand	0.196
Rounded pebbles (size of peas)	1.000

The heads corresponding to these velocities, h_v, may be taken to measure the amount of pressure head h_p acting upward that is necessary to keep them in suspension. If the change of velocity in any vertical is such as to cause the excess of h_p at the lower level over that next above to be equal to or greater than the h_v obtained above for the particular material, there will be no deposition. As the rate of change of velocity is greatest near the bottom, the greatest amount of suspended matter will be found in that region, because material which may settle through the upper layers will have its descent stopped by the more rapidly increasing pressures.

The flow of ground water into a river through its bed aids in suspension by giving an upward velocity to the water near the bottom; and obstructions in the bottom which direct the current upward aid in suspension temporarily; but the main factor in the transmission of suspended matter is the increase of pressure from point to point on a vertical as the velocity decreases. Whenever due to a disturbance of any sort the velocities in a vertical are equalized, the pressures become equalized also and the material deposits.

The Phenomena of Scour, in the case of beds along which the water passes tangentially, are also due to differences of pressure caused by variations of velocity. The velocity in the semi-fluid layers of sediment next the bottom is relatively very low, whereas that in the water just above may be considerable. The pressure in the water is then less than that in the saturated material, and the excess in the latter lifts the particles of it up into the current until the velocity of the latter is reduced to a point where equilibrium is established, or else all the loose materials are carried away, and in either case the scour ceases.

The proportion of silt in various waters, in parts per 100 000 by weight, is as follows, the amounts for the Rhine, Vistula, and Rhone being very exceptional.

Mississippi, near mouth, in flood	175
Mississippi, near mouth, ordinary	67
Mississippi, above Ohio, ordinary	20 to 30
Ohio, at Louisville, ordinary	35
Ohio, at Cincinnati, ordinary	23
Allegheny, at Pittsburgh, ordinary	5
Interior rivers in Illinois, ordinary	1 to 8
Colorado, at Yuma, average annual	560 to 1420
Colorado at Yuma, flood, Jan., 1916	2990
Rhine, Germany, in flood	1000
Vistula, Germany, in flood	2000
Maas, Holland, ordinary to flood	1 to 30
Danube, Austria, ordinary	33
Rhone, France, in flood	2000
Po, Italy, in flood	330
Nile, Egypt, in Flood	150
Ganges, India, in flood	130 to 800
Indus, India, in flood	420
Sutlej, India, in flood	300 to 900
Roorkee, canal, India, in flood	3000
Indian rivers, average for year	61
Yellow, China, normal	400
Yellow, China, average flood	6500
Yellow, China, high flood	9000 to 10 000

Bank Slopes. The angle ϕ at which materials stand when submerged varies considerably. Very light material, as might be expected, assumes the steepest slope when the current impinges against the bank in flowing past. The following are given as results of observations in India.

Alluvial soil, soft rock, and very firm gravel; slope 1/2 horizontal to 1 vertical, or $\phi = 63° 20'$.

Stiff earth or clay, or alluvial banks with stone pitching; slope, 1 to 1, or $\phi = 45°$.

Ordinary earth; slope 1-1/2 horizontal to 1 vertical, or $\phi = 33° 40'$.

Loose earth and soft slippery clay; slopes, 2 horizontal to 1 vertical to 3 horizontal to 1 vertical, or $\phi = 26° 30'$ to $18° 20'$.

MEASUREMENT OF WATER

19. Weight and Volume

The methods of measuring water may be classed as absolute and inferential. The absolute methods are two: by weight and by volume. The inferential methods embrace the nozzle, the Venturi meter, the orifice, the weir, the float, the current meter, the waterwheel, the hydrometric pendulum, coloring matter and chemicals, as well as variations of these methods. In some cases the method may appear to occupy an intermediate position between the two classes, but generally will fall in one or the other.

By Weight. The most accurate method of measuring water is by weight, and when the weight of a quantity of water and its temperature are known, its volume can be readily computed. This method is necessarily limited, by the capacity of weighing apparatus, to moderate or small quantities. The following table gives the volume of 100 lb. of water at various temperatures.

Volume in Cubic Feet of 100 Lb. of Water

Temperature, degree Fahr.	Volume	Temperature, degree Fahr.	Volume	Temperature, degree Fahr.	Volume	Temperature, degree Fahr.	Volume
32	1.6021	50	1.6024	90	1.6098	160	1.6392
35	1.6020	60	1.6035	100	1.6129	180	1.6505
39.3	1.6019	70	1.6051	120	1.6202	200	1.6629
45	1.6020	80	1.6073	140	1.6290	212	1.6709

By Volume. The method ranking second in accuracy is by volume. The volume that can be so handled is limited by the capacity of the measuring vessel or tank, which may vary from a pipette to a reservoir or a natural lake. The accuracy of the observation depends upon the accuracy with which the areas and change of level of the surface are determined. If V = the volume, L = its length, A_1, A_2, and A_c = the areas respectively of the two ends and the mid-section of the body, and A_m = the mean area, then the volume is given by the prismoidal formula $\mathbf{V = 1/6\,L\,(A_1 + 4\,A_c + A_2)}$, and the mean area perpendicular to L is $A_m = 1/6\,(A_1 + 4\,A_c + A_m)$. These formulas apply to the sphere, the hemisphere, any segment of a sphere, a ring or torus, a cone, a pyramid, a wedge, a frustum, a paraboloid, an ellipsoid or any segment of one, and to practically all solids having parallel ends and bounded by plane surfaces.

When measurements are made in tanks with irregular bottoms, the bottom should be covered with water and the difference of elevation of the water surfaces, before and after filling, be measured by means of a hook gage, a point gage, a tube gage, or a scale, depending on the accuracy desired, the devices being here enumerated in the order of their accuracy.

Approximate Volumetric Measurements. The methods under this head include the various water meters in which a liquid fills a space and is then expelled through ports or valves by a piston, as in the water cylinder of a pumping engine — which falls in this class. On account of leakage through the valves and past the piston and the possibility of the space not being completely filled when operating at high speeds, the volume of the liquid does not exactly coincide with that of the chamber, and where exactness is required the relation between the two must be experimentally determined. The amount of liquid discharged may vary from the volume of the chamber by from 1 to 20% of the latter. A device used for measuring water at Lowell early in the last century belongs to this class; it consisted of a large paddle-wheel arranged with its paddles closely fitting a specially prepared channel through which the water to be measured was caused to flow. The paddle, completely obstructing the channel, was driven ahead by the water. The wheel therefore assumed the velocity of the water, and so long as the paddle was in close contact with the sides and bottom of the channel the volume swept through by it below the plane of the water surface measured the quantity of water passing in that time, subject to a correction for leakage past the paddle, which relatively to the total volume could be made very small. A modification of this device has been utilized in Europe at some of the turbine-testing plants, where a very easy-running car, traveling on a track over a channel, carries a screen which can be lowered into the water and which fills the cross-section of the channel. The car is then driven forward at practically the velocity of the current, and the volume swept through by the screen below the water surface measures the water discharged, with a small correction for leakage around the edges. With a sufficiently long channel, the possibilities of accuracy in the apparatus considerably exceed those of any of the inferential or of the other semi-inferential devices.

To this class belong the various automatic flush tanks which are designed to discharge their contents when the water reaches a certain elevation, either by causing the tank to turn on its axis, or by bringing a siphon into action, or by operating valves with the aid of a float. These are sometimes designated as low-pressure meters because they use up or dissipate the major part of the pressure energy of the stream. This class also includes those of the pressure water meters in which there is a definite space filled by the water, whether it is that formed by the displacement of a reciprocating piston, a vibrating disk, or a revolving chamber, in all of which the moving part is essentially a piston. All such apparatus must be calibrated before its indications can be relied upon in cases where exact determinations are essential.

20. Nozzles and Venturi Meters

By the Nozzle. The measurement of water by means of a nozzle requires first an accurate determination of the area of the outlet and of the section at which the pressure is read, and thereafter the observation of the pressure at some point as close as possible to the inlet of the nozzle. This pressure should be observed as communicated through at least four orifices situated 90 deg. apart in a plane perpendicular to the axis of the nozzle. The orifices should be cut with their walls normal to the axis of the pipe where the elements of the interior surface are parallel, or at least the tangents to the elements at the plane of the orifice should be parallel to each other and to the axis of the channel. The edges of the orifices should be smooth without projecting slivers or burrs, and the several orifices should be united to a common equalizing chamber to which the pressure gage is connected. Frequently this chamber is an annular space surrounding the pipe, being made in a special casting designed for the purpose. Fairly good results may be obtained by connecting the gage to a single orifice at the base of the nozzle, but in this case it is important to remove to a considerable distance upstream, ten diameters of the pipe at least, any curvature or obstruction causing an unsymmetrical disturbance of the velocities. In any event a curve should not be within five diameters of the place where the pressure is read, unless a screen or baffle is used to equalize the velocity distribution before the gage is reached. Let A_g = area at the gage section, A_n = area at the nozzle outlet, h_p = pressure head at gage section, h_{vg} = velocity head at gage section, h_{vn} = velocity head at nozzle outlet, Q = quantity of water discharged, v_g = velocity at gage section, v_n = velocity at nozzle outlet, C = coefficient of discharge. When the nozzle is horizontal,

$$h_{vn} = \frac{h_p}{1 - (A_n/A_g)^2}$$

When the center at the gate section is a vertical distance z below the center of the nozzle outlet, h_p in the above equation must be decreased by the distance z, and if the gage section be above the outlet by the distance z, h_p must be increased by z. Then

$$Q = CA_nv_n = CA_n\sqrt{2\,gh_{vn}} = CA_n\sqrt{2\,g\frac{h_p \pm z}{1 - (A_n/A_g)^2}}$$

With a smooth tapering nozzle, 2 in. or more in diameter at its outlet, the angle of convergence between the two sides being from 10 to 15 deg., and a waterway leading to it of such form as to prevent swirls and eddies, and a carefully made piezometer connection having at least four orifices as above described, connected to an accurate gage for measuring the head, the coefficient C may be taken at 0.995 and the resulting measurement may be within 1/4 of 1% of the actual discharge. In ordinary work, however, a coefficient of 0.98 is as high as it is safe to rely upon, and the discharges so computed, with careful observation, should be within 1% of correctness. In smaller nozzles the coefficient will be somewhat lower than in large ones, and may be expected to increase for very large sizes.

If the nozzle outlet is submerged, h_p in the foregoing expressions must be decreased by the depth of water over the center of the nozzle outlet, and in the event of discharge into a vacuum, h_p must be increased by the height of water column equivalent to the measure of the vacuum. (Trans. Am. Soc. C. E., 1891, vol. 23, p. 492.)

By the Venturi Meter. This device consists of a short truncated converging cone united by a short cylindrical or forge section, called the throat, to a diverging truncated cone, the latter being much longer than the former and having therefore a more gradual taper. The pressure head is read through piezometer connections at the inlet of the upstream cone and at the throat, the difference of pressure head at the two points representing the increase in velocity head at the smaller section over that at the larger and the frictional loss between the two. As the upper cone is short, usually not

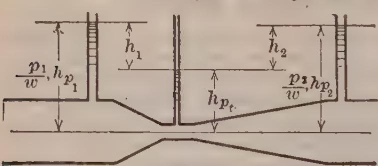


Fig. 21. Venturi Meter

Fire Streams

Size of nozzle, in.	Pressure at base of play pipe	Best fire stream		Gallons per minute	Required hydrant pressure for 2-1/2-in. best rubber-lined fire hose of lengths as indicated						
					Feet						
		Height of jet, ft.	Reach of jet ft.		50 lb.	100 lb.	200 lb.	300 lb.	500 lb.	800 lb.	1000 lb.
	10	17	19	52	10	11	11	12	13	15	16
	20	33	29	73	21	22	23	24	26	30	32
	30	48	37	90	31	32	34	36	40	45	49
	40	60	44	104	42	43	46	48	53	60	65
3/4	50	67	50	116	52	54	57	60	66	75	81
	60	72	54	127	63	65	68	72	79	90	97
	70	76	58	137	73	75	80	84	92	105	114
	80	79	62	147	84	86	91	96	106	120	130
	100	83	68	164	105	108	114	120	132	150	163
	10	18	21	71	11	11	13	14	16	19	22
	20	34	33	100	22	23	25	27	32	39	43
	30	49	42	123	33	34	38	41	48	58	65
	40	62	49	142	43	46	50	55	64	78	87
7/8	50	71	55	159	54	57	63	69	80	97	108
	60	77	61	174	65	69	75	82	96	116	130
	70	81	66	188	76	80	88	96	112	136	152
	80	85	70	201	87	91	101	110	128	155	173
	100	90	76	224	109	114	126	137	160	194	216
	10	18	21	93	12	12	14	16	20	26	30
	20	35	37	132	23	25	29	33	41	52	60
	30	51	47	161	34	37	43	49	61	79	90
	40	64	55	186	46	50	58	66	81	105	120
1	50	73	61	208	57	62	72	82	102	131	151
	60	79	67	228	69	75	87	98	122	157	181
	70	85	72	246	80	87	101	115	142	183	211
	80	89	76	263	92	100	115	131	162	209	241
	100	96	83	295	115	125	144	164	203	.....	......
	10	18	22	119	12	14	17	20	27	36	43
	20	36	38	168	25	28	34	41	54	73	85
	30	52	50	206	37	42	52	61	80	109	128
	40	65	59	238	50	56	69	81	107	145	171
1-1/8	50	75	66	266	62	70	86	102	131	181	213
	60	83	72	291	74	84	103	122	160	218	256
	70	88	77	314	87	98	120	143	187	254	......
	80	92	81	336	99	112	138	163	214	.....	......
	100	99	89	376	124	140	172	204	.....	.....	......
	10	19	22	148	14	16	21	26	36	51	61
	20	37	40	209	27	32	42	52	72	101	121
	30	53	54	256	41	49	63	78	108	152	182
	40	67	63	296	55	65	84	104	144	203	243
1-1/4	50	77	70	331	68	81	106	130	180	254	......
	60	85	76	363	82	97	127	156	216	.....	......
	70	91	81	392	96	113	148	182	252	.....	......
	80	95	85	419	110	129	169	208	.....	.....	......
	100	101	93	468	137	162	211	261	.....	.....	......
	10	20	23	182	16	19	27	34	49	71	86
	20	38	42	257	31	39	53	68	98	143	173
	30	55	56	315	47	58	80	103	147	214	259
	40	69	66	363	62	77	107	137	196	.....	......
1-3/8	50	79	73	406	78	96	134	171	245	.....	......
	60	87	79	445	93	116	160	205	.....	.....	......
	70	92	84	480	109	135	187	239	.....	.....	......
	80	97	88	514	124	154	214	.....	.....	.....	......
	100	103	96	574	156	193	.....	.....	.....	.....	......

over two diameters in length, and may be made quite smooth, the latter element is ordinarily negligible. The discharge is computed by the same formula as that of the nozzle when h_p is replaced by the difference of the pressures read at the inlet and the throat. Let A_i = area at inlet to meter, A_t = area at throat, h_{pi} = pressure head at inlet and h_{pt} = pressure head at throat. Then

$$Q = CA_t\sqrt{2g\frac{h_{pi} - h_{pt}}{1 - (A_t/A_i)^2}}$$

The coefficient C has an average value of 0.96 to 0.98 and is increased when the inlet is near a curve over the condition when following a straight pipe. Ordinarily the accuracy of measurement should be within 3%. The loss of head in passing through the meter is usually from 10 to 15% of the difference of pressure head between inlet and throat. A special recording device is furnished with the meter castings by the manufacturers. The commercial size of the meter is that of the inlet, or the same as that of the pipe in which it is to be set. (Trans. Am. Soc. C. E., 1887, vol. 17, p. 228.)

Let d and D be diameters of the throat and inlet pipe in inches, and h the so-called "head on the meter," or $h_{pi} - h_{pt}$, in feet. Then the discharge, in U. S. gallons per 24 hours, is

$$Q = 28276\,C\frac{d^2\sqrt{h}}{\sqrt{1 - (d/D)^4}}$$

In Eng. News, July 31, 1913, Allen Hazen concludes that 0.99 is the usual value of C, and J. W. Ledoux gives 0.97 as derived from tests on 2- and 8-in. throats for high velocities. In Eng. News, Oct. 2, 1913, A. T. Safford gives 0.97 for small discharges and nearly 1.00 for large ones. Tests at Cornell University by E. W. Schoder in 1915 on a 12 by 5-in. meter gave about 0.99 for C when the discharge was about 9000 gal. per minute.

Fire Streams. The preceding table gives the characteristics of fire streams from smooth conical nozzles, as established by the experiments of John R. Freeman, Mem. Am. Soc. C. E. (Trans. A. S. C. E., Vol. XXI). The pressures indicated are those existing while the stream is flowing. For ring nozzles the characteristics correspond to those of the next size smaller smooth nozzle.

21. Orifices

For Small Orifices 1 in. in height or less, the coefficient 0.61 for the discharge from circular orifices and 0.62 for square and rectangular ones, if the head be more than eight times the vertical dimension, will give results sufficiently accurate for ordinary cases. If more accurate gagings are required, the orifices should be calibrated in place by weighing or volumetrically measuring the water discharged under different heads. When the vertical dimension of the orifice is more than one-sixth the head over the top, the variation of pressure between the top and the bottom causes the center of pressure on the orifice to be appreciably below the center of gravity, and the center of pressure at the vena contracta is below that at the plane of the orifice. Let the notation be

A = area of orifice; A_c = area of jet at vena contracta;
C_v = coefficient of velocity; C_c = coefficient of contraction;
C or C_vC_c = coefficient of discharge;
h_b = head above bottom of orifice; h_{bc} = head above bottom of vena contracta;
h_g = head above center of gravity of orifice; h_t = head above top of orifice;
h_{tc} = head above top of vena contracta; h_v = head due to velocity of approach;

L = length of sill of rectangular orifice; v = velocity through orifice;
v_c = velocity through vena contracta; Q = quantity discharged;
R = radius of a circular orifice;
x = width of orifice at any point; x_c = width of vena contracta at any point;
z = height of water surface above any point of the orifice;
z_c = height of water surface above any point in the vena contracta.

The General Formula for discharge per second is

$$Q = Av = A_c v_c = C_v \sqrt{2g} \int_{h_{tc} + h_v}^{h_{bc} + h_v} x_c z^{1/2} \, dz \tag{1}$$

in which, if x_c is not constant, it must be expressed as a function of z. The application of this formula requires the determination of the dimensions of the vena contracta, which is a difficult operation, and therefore the dimensions of the orifices may be substituted for those of the vena contracta and the coefficient of contraction C_c may be introduced, when, since $C_v C_c = C$, the above equation becomes,

$$Q = Av = C \sqrt{2g} \int_{h_t + h_v}^{h_b + h_v} x z^{1/2} \, dz \tag{2}$$

If the orifice is horizontal $h_b = h_t$ and z is constant. For a rectangular orifice in which $x = L$, after integrating,

$$Q = Av = 2/3 \, CL \sqrt{2g} \, [(h_b + h_v)^{3/2} - (h_t + h_v)^{3/2}] \tag{3}$$

The best experiments show the value of the coefficient with full contraction on all sides to be practically constant and for square orifices $C = 0.604$, for circular orifices $C = 0.597$.

If contraction is suppressed on one side by bringing the wall of the channel of approach into coincidence with the plane of the side of the orifice, the contraction on the opposite side is increased somewhat, but not by an amount equal to the original contraction on the side suppressed.

For a rectangular orifice 0.656 ft. high and 2.624 ft. wide with end contractions suppressed, Bazin's experiments give the coeficient C in equation (3) as 0.624, showing about 3% increase of discharge above that for full contraction on all sides.

For square orifices the experiments of Lebros give: Contraction suppressed on one side, C is increased 2.9%. Contraction suppressed on two sides, C is increased 5.25%. Contraction suppressed on bottom, C is increased 3.25%. Contraction suppressed on bottom and one side, C is increased 7.25%. Contraction suppressed on bottom and two sides, C is increased 11.5%.

For circular orifices Bidone deduced the effect of full suppression as increasing C or the discharge 12.8% and partial suppression correspondingly. It is to be noted the suppression at the top would have less effect than at the bottom, and may be taken as increasing the discharge of a square 2.6%. From this it may be expected that the suppression of contraction over the lower half of a circular orifice would increase the discharge about 8%, over the upper half about 5%, and over one side half about 6%.

Stewart's experiments at the University of Wisconsin on 4 ft. sq. submerged orifices and pipes give considerably higher results where contraction is suppressed by a curved entry. See Art. 7.

For the discharge of sluice gates where the bottom and sides coincide with those of the channel, the coefficient for the corresponding orifice may be used provided there is no apron beyond the gate and the discharge is allowed to fall away freely. If there is an apron or chute or continuation of the channel, the friction along it will reduce the coefficient, particularly at low heads. When h_g was about 2-1/2 times the height of the opening, the discharge has been found to be as much as 12% less than for the orifice with full contraction.

22. Weirs

The Weir affords the most commonly used method of measuring the volume of water in moderately large quantities. The standard weir, or sharp-edged weir, consists of a vertical partition across a channel with its top edge horizontal, sharp cornered and narrow enough so that at the heads used the overflowing sheet jumps from the upstream edge clear of the downstream corner. Such weirs may be either with or without end contractions. A weir with end contractions is one whose crest extends only part way across the channel and is terminated by partitions in its plane, with their vertical edges rising above the level of the water on the upstream side. Such a weir may be compared to a rectangular orifice upon which the head has fallen below the top. A weir without end contractions is one which extends entirely across the channel. If a = height of crest of weir above bottom of channel of approach, A_w = area of stream in the plane of the weir, H = height above the crest of the surface of still water upstream from the weir, h = head above crest as observed, v_w = velocity in and perpendicular to the plane of the weir, then the formula for the discharge is similar to that for the orifice and is

$$Q = A_w v_w = 2/3\ CL\sqrt{2g}\cdot H^{3/2} = 2/3\ CL\sqrt{2g}\,(h + h_v)^{3/2}$$

Since $LH = L(h + h_v)$ = area of the stream above the crest level at the plane of still water and CLH is the area in the plane of the crest, the total **head** producing flow is $2/3\sqrt{2g}\,(h + h_v)$. (h_v = head due to velocity of approach.)

The Francis Formula. The coefficient C in this formula was determined experimentally by James B. Francis as about 0.62, and by combining this with $2/3\sqrt{2g}$, the well-known coefficient of the Francis formula 3.33 is obtained. This formula was considered by its inventor to be reliable between heads of 0.5 ft. and 2.00 ft. Later investigators have modified it into the form:

$$Q = 3.33\ L\,(h + 1.4\ h_v)^{3/2}$$

In applying this formula the process is as follows: Having measured the head h at a point above the surface curve to the weir, compute an approximate value of the discharge by the equation $Q_1 = 3.33\ Lh^{3/2}$. Find the approximate velocity at the plane where the head is observed by the equation $v = Q_1 \div L(h + a)$ and the velocity head by $h_v = v^2/2\ g$. Then Q is obtained by substitution in the above formula and should be within 3 to 4% of correctness if the head is not more than 30% of a and has been properly measured, and the sheet is fully aerated underneath.

For weirs with end contractions Francis recommended reducing the length L in the above formula by 0.1 H for each full end contraction. This correction is only an approximation and, for accurate gagings, weirs with end contractions should not be used.

The Machinery Builders Society in October, 1917, in connection with the Testing Code for Hydraulic Turbines, adopted the following coefficients for use with the Francis formula in the form $Q = CLh^{3/2}$ which include the effect of velocity of approach.

Table of Values of C for Various Heads and Heights of Crest a

Head h	Height of crest a, in feet									
	4	5	6	7	8	9	10	12	14	16	20
1.0	3.376	3.356	3.344	3.335	3.329	3.325	3.322	3.317	3.314	3.311	3.308
1.2	3.391	3.366	3.350	3.339	3.332	3.326	3.322	3.316	3.311	3.308	3.305
1.4	3.409	3.378	3.359	3.346	3.336	3.330	3.324	3.316	3.311	3.307	3.303
1.6	3.429	3.392	3.370	3.354	3.343	3.334	3.328	3.319	3.312	3.308	3.302
1.8	3.450	3.408	3.382	3.363	3.350	3.340	3.333	3.322	3.315	3.309	3.303
2.0	.....	3.425	3.394	3.373	3.358	3.347	3.338	3.325	3.317	3.311	3.304

The above coefficients are the averages of values computed by the following three formulas:

(1) Bazin,

$$Q = \left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\frac{h^2}{(a+h)^2}\right]\sqrt{2g}\,Lh^{3/2}$$

(2) Rehbock,

$$Q = \left[0.605 + \frac{1}{320h-3} + 0.08\frac{h}{a}\right]\frac{2}{3}\sqrt{2g}\,Lh^{3/2}$$

(3) Fteley-Stearns,

$Q = 3.31\ L\ (h + 1.5\ h_v)^{3/2} + 0.007\ L$, in which

h_v = head due to velocity of approach.

The formula of Rehbock is based upon rather small scale laboratory experiments, and in the writer's opinion is given undue weight in this connection.

The Bazin Formula has been considered the most accurate one for wide ranges of head, and it has been commonly applied between heads of 0.2 ft. and 6 ft.; it does not require a correction for velocity of approach, as it is based upon the observed head h and not on the **theoretical** or **total** head $H = h + h_v$ as are the others. It applies only to weirs without end contractions and is

$$Q = \left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{a+h}\right)^2\right]Lh\sqrt{2gh}$$

The tables, on pp. 1360 and 1361, give values of Q for a weir 1 ft. long and for various values of h and a. The value of g used in computing these tables is 32.17 ft. per sec. per sec.

Recent investigations indicate that it gives too high values for heads below one foot when applied to truly sharp edged weirs.

When the weir is so high that the velocity of the approaching water is practically zero, Bazin's formula reduces to

$$Q = \left(0.405 + \frac{0.00984}{h}\right)Lh\sqrt{2gh}$$

At low heads, less than 0.2 ft., Bazin's formula gives discharges somewhat too high and the formula proposed by Fteley and Stearns is recommended, which is:

$$Q = 3.31\ LH^{3/2} + 0.007\ L$$

The results by this formula are within 4 to 6% of the experimental values for heads ranging from 0.2 to 0.007 ft., and the actual discharges were generally in excess of those given by the formula. It holds only so long as the sheet jumps free of the crest and the space behind it is fully aerated.

Schoder and Turner Formula. From several series of experiments involving 2438 individual determinations, carried on at the Cornell Hydraulic Laboratory, in which the flow was very accurately measured volumetrically, the range of head covered being from 0.012 to 2.75 ft., and weirs ranging in height from 0.5 to 7.5 ft., and comparisons with the work of previous experimenters, Francis, Fteley and Stearns, Bazin and Rehbock, Professors E. W. Schoder and K. B. Turner derived a more accurate expression for the discharge of a vertical smooth sharp-edge weir than any of the foregoing. Their formula involves both the velocity of approach above the level of the weir crest v_a and that below it, v_b (Trans. Am. Soc. C. E., Vol. XCII, Proceedings, Sept., 1927) and requires the distribution of velocity in the channel of approach to be investigated by current meters.

Discharge in Cubic Feet per Second per Foot of Length over Sharp-edged Vertical Weirs without end Contractions

Computed by Bazin's Formula

Head h, ft.	Height in feet of crest of weir above bottom of channel of approach						
	a = 2	a = 3	a = 4	a = 5	a = 6	a = 7	a = 8
0.2	0.33	0.33	0.33	0.33	0.33	0.33	0.33
0.3	0.58	0.58	0.58	0.58	0.58	0.58	0.58
0.4	0.88	0.88	0.88	0.87	0.87	0.87	0.87
0.5	1.23	1.21	1.21	1.21	1.21	1.21	1.21
0.6	1.62	1.59	1.59	1.58	1.58	1.58	1.58
0.7	2.04	2.01	1.99	1.98	1.98	1.98	1.98
0.8	2.50	2.45	2.43	2.42	2.41	2.41	2.41
0.9	3.00	2.93	2.90	2.88	2.88	2.87	2.86
1.0	3.53	3.44	3.40	3.38	3.36	3.36	3.35
1.2	4.68	4.55	4.48	4.47	4.42	4.41	4.40
1.4	5.99	5.78	5.68	5.62	5.58	5.56	5.54
1.5	6.68	6.44	6.30	6.23	6.20	6.18	6.16
1.6	7.40	7.12	6.97	6.89	6.84	6.80	6.78
1.8	8.93	8.56	8.37	8.25	8.18	8.13	8.09
2.0	10.58	10.12	9.87	9.72	9.62	9.55	9.51
2,2	12.34	11.77	11.46	11.27	11.14	11.06	10.99
2.4	14.20	13.53	13.15	12.91	12.75	12.64	12.56
2.5	15.17	14.45	14.03	13.76	13.59	13.47	13.38
2.6	16.16	15.38	14.92	14.63	14.44	14.30	14.20
2.8	18.23	17.32	16.79	16.44	16.21	16.04	15.92
3.0	20.39	19.36	18.74	18.33	18.06	17.86	17.71
3.2	22.64	21.48	20.77	20.31	19.98	19.75	19.58
3.4	24.98	23.70	22.89	22.36	21.99	21.72	21.52
3.5	26.20	24.83	24.00	23.43	23.01	22.73	22.48
3.6	27.41	25.99	25.09	24.49	24.06	23.75	23.52
3.8	29.94	28.38	27.38	26.70	26.22	25.87	25.60
4.0	32.54	30.84	29.74	28.99	28.45	28.05	27.74
4.2	35.22	33.39	32.18	31.35	30.75	30.30	29.96
4.4	37.99	36.01	34.70	33.78	33.12	32.62	32.24
4.6	40.83	38.71	37.29	36.29	35.56	35.01	34.58
4.8	43.75	41.49	39.96	38.87	38.07	37.46	37.00
5.0	46.71	44.31	42.67	41.49	40.62	39.96	39.44
5.2	49.81	47.27	45.50	44.23	43.29	42.57	42.01
5.4	52.94	50.23	48.38	47.02	46.00	45.22	44.60
5.6	56.15	53.33	51.34	49.88	48.79	47.94	47.28
5.8	59.42	56.45	54.34	52.79	51.62	50.71	49.99
6.0	62.77	59.65	57.43	55.78	54.53	53.55	52.78

The formula is for weirs without end contractions:

$$Q = 3.33\,L\left[\left(h + \frac{va^2}{2\,g}\right)^{3/2} + h\,\frac{vb^2}{2\,g}\right]$$

They also found the effect of a roughening of the upstream face of a weir from the condition of a smooth brass plate to that of a coarse file for a distance of 1 ft. from the crest to be to increase the discharge nearly 2% at a 0.5-ft. head, and to over 1% for a 1.35-ft. head.

Discharge in Cubic Feet per Second per Foot of Length over Sharp-edged Vertical Weirs without End Contractions—Continued

Computed by Bazin's Formula

Head h, ft.	Height in feet of crest of weir above bottom of channel of approach						
	a = 9	a = 10	a = 12	a = 16	a = 20	a = 25	a = 30
0.2	0.33	0.33	0.33	0.33	0.33	0.33	0.33
0.3	0.58	0.58	0.58	0.58	0.58	0.58	0.58
0.4	0.87	0.87	0.87	0.87	0.87	0.87	0.87
0.5	1.21	1.21	1.21	1.21	1.20	1.20	1.20
0.6	1.57	1.57	1.57	1.57	1.57	1.57	1.57
0.7	1.97	1.97	1.97	1.97	1.97	1.97	1.97
0.8	2.40	2.40	2.40	2.40	2.40	2.40	2.40
0.9	2.86	2.86	2.86	2.86	2.85	2.85	2.85
1.0	3.35	3.34	3.34	3.33	3.33	3.33	3.33
1.2	4.39	4.38	4.38	4.37	4.36	4.36	4.36
1.4	5.53	5.52	5.51	5.49	5.49	5.48	5.48
1.5	6.14	6.13	6.12	6.11	6.10	6.09	6.09
1.6	6.76	6.74	6.73	6.71	6.69	6.69	6.69
1.8	8.07	8.05	8.02	7.99	7.98	7.97	7.96
2.0	9.47	9.44	9.40	9.36	9.34	9.33	9.32
2.2	10.95	10.91	10.86	10.81	10.78	10.76	10.75
2.4	12.50	12.45	12.39	12.32	12.28	12.25	12.24
2.5	13.31	13.26	13.18	13.10	13.06	13.03	13.01
2.6	14.13	14.07	13.99	13.90	13.85	13.82	13.80
2.8	15.83	15.76	15.66	15.54	15.48	15.44	15.42
3.0	17.60	17.52	17.39	17.25	17.18	17.13	17.10
3.2	19.45	19.34	19.19	19.02	18.93	18.87	18.83
3.4	21.36	21.24	21.06	20.86	20.75	20.68	20.63
3.5	22.38	22.22	22.00	21.83	21.69	21.62	21.60
3.6	23.34	23.20	22.99	22.75	22.62	22.53	22.48
3.8	25.39	25.23	24.99	24.71	24.56	24.45	24.39
4.0	27.51	27.32	27.05	26.72	26.55	26.42	26.35
4.2	29.69	29.48	29.17	28.79	28.59	28.45	28.36
4.4	31.94	31.70	31.34	30.92	30.66	30.52	30.42
4.6	34.25	33.98	33.58	33.10	32.84	32.65	32.53
4.8	36.62	36.33	35.88	35.35	35.05	34.83	34.70
5.0	39.03	38.70	38.21	37.61	37.28	37.03	36.88
5.2	41.56	41.20	40.65	39.99	39.61	39.33	39.17
5.4	44.11	43.71	43.12	42.38	41.96	41.66	41.47
5.6	46.74	46.31	45.65	44.84	44.38	44.04	43.83
5.8	49.41	48.94	48.22	47.33	46.83	46.45	46.22
6.0	52.15	51.64	50.86	49.90	49.34	48.92	48.67

The effect of slightly rounding the upstream corner of the weir crest (by about 1/24 in. or 1 mm.) was to increase the discharge 2% for a head of 0.5 ft. and 0.5% for a head of 1.35 ft. A rounding to a radius of 1/8 in. increased the discharge about 3% and to 1/4 in. about 5-1/2% at 0.5 ft. head. In one case the effect of altering the distribution of velocities in the channel of approach changed the discharge as much as 25% for the same head on the weir.

The Flow over Irregular Crests may be computed by multiplying the discharge of a standard weir of the same height and length and at the same head by a factor depending on the form of the crests. The following tables give the multipliers for various forms of weirs (Fig. 22) as determined from experiments upon full-size models at the Hydraulic Laboratory of Cornell University:

Multipliers for Flat-topped Weirs (Fig. 22*A*)

Head h, ft.	Width of flat crest in feet							
	b = 0.48	b = 0.93	b = 1.65	b = 3.17	b = 5.84	b = 8.98	b = 12.24	b = 16.30
0.5	0.902	0.830	0.795	0.790	0.785	0.783	0.783	0.783
1.0	0.972	0.904	0.810	0.797	0.800	0.798	0.795	0.792
1.5	1.000	0.957	0.875	0.797	0.807	0.803	0.802	0.793
2.0	1.000	0.989	0.930	0.815	0.805	0.800	0.798	0.791
2.5	1.000	1.000	0.970	0.842	0.800	0.795	0.792	0.790
3.0	1.000	1.000	1.000	0.870	0.796	0.791	0.787	0.789
3.5	1.000	1.000	1.000	0.896	0.793	0.787	0.783	0.787
4.0	1.000	1.000	1.000	0.925	0.790	0.783	0.780	0.785

Multipliers (m) for Triangular Weirs (Fig. 22*B*)

Head h in feet,	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
For b = 6.65 ft., m =	1.060	1.079	1.091	1.086	1.076	1.067	1.060	1.054
For b = 11.25 ft., m =	1.060	1.079	1.092	1.097	1.096	1.095	1.094	1.093

Multipliers for Compound Weirs (Fig. 22)

Head h, ft.	Type F	Type G	Type H	Type I	Type J	Type K	Type L
0.5	0.964	0.932	0.934	0.968	0.971	0.971	0.971
1.0	1.026	0.982	1.000	1.008	1.040	1.040	0.983
1.5	1.064	1.015	1.040	1.030	1.083	1.092	1.022
2.0	1.066	1.031	1.061	1.034	1.113	1.126	1.040
2.5	1.025	1.038	1.073	1.038	1.118	1.146	1.057
3.0	0.992	1.044	1.082	1.042	1.120	1.163	1.072
3.5	0.966	1.049	1.090	1.046	1.122	1.177	1.085
4.0	0.944	1.053	1.097	1.050	1.125	1.190	1.097

Imperfectly Aerated and Submerged or Drowned Weirs. Bazin recognizes four types of overfalling sheet or nappe.

(a) Free nappe, with the space under the sheet thoroughly aerated, being the condition assumed in the ordinary weir formulas.

(b) Depressed nappe, or the condition when the space under the sheet is partially aerated and the sheet is pressed toward the downstream face of the weir by the atmospheric pressure outside.

(c) Drowned nappe, or that when the space underneath the sheet is entirely filled with water though the surface downstream may be below the crest level.

(d) Adhering nappe, or when at low heads the water flows over the crest and down its lower face in contact therewith, allowing no air to accumulate underneath the sheet.

Experimentally, with a weir 2.46 ft. high and 6.56 ft. long and a head 0.656 ft., Bazin found:

Nappe	Increased discharge of free nappe, Per cent
Depressed	6
Drowned	15
Adhering	28

The effects increase with increasing head, and vice versa.

For Submerged or Drowned Weirs when h_u is the head upstream of the weir and h_l that downstream, Bazin gives the following multiplier to be applied to the discharge over a sharp-edged weir with a free nappe:

$$\left[1.05\left(1+\frac{1}{5}\cdot\frac{h_l}{a}\right)\sqrt[3]{\frac{h_u-h_l}{h_u}}\right]$$

Experiments by the author on a submerged weir with a rounded crest (Type *L*, Fig. 22) showed that until the submergence h_l amounted to 30% of h_u, the reduction of discharge was less than 10%. So long as the overfalling sheet plunges below the water downstream, the following formula may be applied,

$$h^{3/2} = h_u{}^{3/2} - h_l{}^{5/2}/h_u$$

and the resulting value of $h^{3/2}$ used in the ordinary weir formulas.

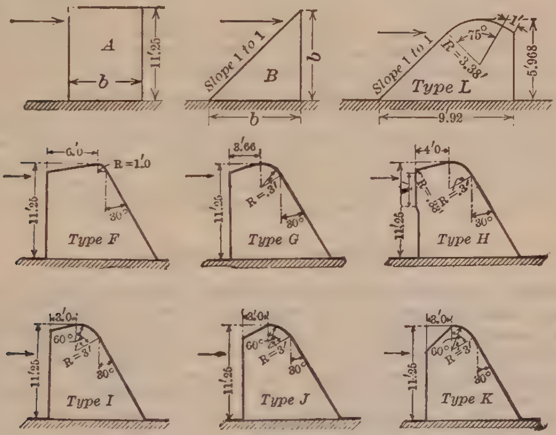


Fig. 22. Types of Weirs and Dams

Measurement of Head. When the head on a weir exceeds 1 ft. the means used for observing the head may cause an appreciable variation in the computed discharge.

The following table gives the percentage variation of heads observed in various ways, from those indicated by a device *A* similar to that used by Bazin:

Variations of Head Observations on Weirs

(Experimental)

A. Head observed by hook gage in pit connected at bottom of channel of approach by 4-in.-diameter pipe flush with wall, 29.88 ft. upstream from weir 6.55 ft. high.

B. Head observed by tape and bob over center of 16-ft.-wide channel 14 ft. upstream from weir 6.55 ft. high.

Head observed by 1-in. glass tube gage connected to 1/4-in.-diameter orifice 6 ft. upstream from weir 6.55 ft. high:

C. In brass plate flush with channel wall 2-1/4 in. below level of crest.

D. In iron cap flush with timber channel wall 1 ft. above bottom of channel.

E. In brass plug flush with face of 2 in. by 12 in. by 18 ft. plank on center line of channel, 3 in. above bottom.

F. Head observed by 1-in. glass tube gage connected to 1-in. pipe with 1/8-in. orifices, 2 in. apart on under side, set transversely across channel 1 ft. above bottom and 10.2 ft. upstream from weir 5.85 ft. high.

G. Head observed by hook gage in barrel connected to five 5/8-in. orifices in weir bulkhead 9-3/8 in. below crest of weir 6 ft. high and 65 ft. long.

H. Head observed by 1-in. glass tube connected to 1-in. pipe with 1/4-in. orifices, 6 in. apart on underside set transversely across channel 8 in. above bottom and 37 ft. upstream from weir 5.2 ft. high.

Variation of Observed Head in Per Cent of A

Head A	B	C	D	E	F	G	H
Feet	%	%	%	%	%	%	%
0.0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	−0.68	−0.4
1.0	0	0	0	0	−0.28	−1.97	−0.8
1.5	0	+0.40	+1.10	+1.30	−0.50	−2.41	−1.2
2.0	0	+0.33	+1.25	+1.28	−0.82		−1.75
2.5	0	−0.08	+1.22	+0.80			−2.48
3.0	−0.7	−0.40	+1.10	+0.08			−3.17
3.5	−2.0	−0.57	+0.92	−0.35			−3.85
4.0							−4.62
4.5							−5.23
5.0							−6.10
5.5							−6.74

23. Current Observations

By Floats. In shallow streams approximate discharges may be obtained by surface floats, the mean of whose velocity when multiplied by 0.80 will give the average velocity probably within 10%. For more accurate gagings the **rod float** may be used, and when the conditions of the channel are ideal the accuracy of this method places it next to the weir. Rod floats may consist of tin tubes or bamboo poles loaded with shot, or of wooden rods wrapped at their bottom with sheet lead or otherwise weighted. They should be so loaded as to stand vertically in the water, and the point to which they sink should be marked by a ring of paint. Wooden floats, if to be used in accurate work for more than a very few minutes at a time, should be thoroughly oiled, or coated with a waterproof varnish, to prevent absorption of water and the consequent change of buoyancy. In starting the floats they should be immersed to the submergence mark with the bottom inclined slightly against the current and allowed to swing to a vertical position under the influence of the current while held with the thumb and finger as trunnions whose axis is perpendicular to the current and parallel to the surface. As soon as the float comes to a vertical position it should be released without disturbing its submergence or its motion, and allowed to float far enough to take up the velocity of the stream before passing the point at which the upstream observation is taken. The course selected should be in a straight reach of as nearly uniform depth and section as possible, free from weeds and having a straight channel leading to it.

The cross-section should be accurately determined and a length for the course laid off on the bank parallel to the axis of the current. If the stream is narrow, two cords or wires situated in a vertical plane a foot or more apart should be stretched across at the upper and lower ends of the course, and either one of the wires or a separate line should be tagged at intervals to assist in locating the float. The observer should take the time as the float passes the plane of the wires and the recorder or the starter its location along the tagged line. If the stream is wide, the floats should be observed with a transit whose telescope is at right angles to the course and the position determined by means of the vertical angle to the water surface, the elevation of the telescope axis above the water being measured. Where less accurate results are required a single line or a pair of ranges may be used to locate the upper and lower ends of the course and the position may be estimated. If but a single observer and one or two floats are employed, the time may be best taken by means of a stop watch, but for rapid work a large number of floats may be used and with an observer using an ordinary watch and a recorder, at each end of the course, fully as accurate work may be done. The two watches must be set together and frequently compared during the observations. The floats should be so started as to cover as far as possible the entire cross-section of the channel throughout the course, and where the depth of the cross-section varies, floats of different lengths should be used. It is customary to consider the course of the float as that indicated by the mean of its positions at the upper and lower wires and that its velocity is uniform throughout the run and that the area of the stream is the mean area throughout the course.

The floats should be made to pass as close to the bottom as possible without dragging or striking on obstructions, but it is never possible in ordinary streams to have them run closer than several inches above stream bed. The velocity of the float will therefore be greater than the mean velocity of the water in the whole vertical in which the float travels. The float gives practically the mean velocity in that part of the vertical through which its submergence extends, and may be taken to represent the velocities horizontally for half the distance each way to the next float. When d = depth of water, i = immersion of float, u = mean velocity of water, v_f = velocity of float, then

$$u - v_f\left\{1.0 - 0.116\left[\frac{d-i}{d} - 0.1\right]\right\} \text{and } Q = Au$$

In a rectangular channel 10 ft. deep and 16 ft. wide it was found that float discharges agreed with weir discharges within 2% for ranges of immersion from 60 to 98% of the depth. Double floats and subsurface floats are of little value for measuring water.

By Screen or Diaphragm. This is a special form of float and is mounted on a truck running on rails along a straight and uniform section of the channel and is so devised that it can be quickly lowered into the water and made to occupy nearly the whole area of the channel during the run and be lifted out at the end. A proper distance being allowed for starting and stopping and the screen occupying practically the whole cross-section of the channel, its velocity corresponds very nearly to that of the water. If, however, the screen does not approximately fill the cross-section of the channel the Francis float formula of the preceding paragraph may be applied. As the screen is usually run in a rectangular channel of considerable depth it may be desirable to make a second further application of the formula to cover the spaces between the screen and the side walls by considering the distance from the center of the channel to the edge of the screen as i, half the width of the channel as d, and the first computed u as v_f for each half of the channel. The depth of water then becomes the width. The sum of the discharge as computed for each half then will be the true discharge.

By Current Meter. The current meter consists usually of a revolving device, driven by the current, the revolutions of which bear some relation to the velocity of the water, and are transmitted to some form of a recording or sounding apparatus whereby the number occurring in a given time may be observed.

Two types of meter may be distinguished: the screw or direct acting-meter, and the cup or differential meter. The Haskell, Fteley-Stearns and Ott meters are examples of screw or direct acting meters. The Price meter is the best example of a cup meter, and has been adopted as a standard by the U. S. Geological Survey.

The meter should be rated to determine the ratio between its revolutions and the velocity of the passing water, which rating is usually accomplished by drawing the meter at known speeds through still water, either attached to a truck running on rails over a channel or to a boat propelled at a uniform speed, usually by a person walking along the shore. In gaging, the meter is attached to a line and sunk by a weight, or in shallow water attached to a rod, and held at various points in a vertical line until sufficient observations are obtained to give the mean velocity in the vertical line with the required degree of precision. The meter is then moved to another vertical line and the mean velocity determined there. Since it is not possible to operate the meter close to the bottom, the mean of the velocities indicated by it in any vertical line will be greater than the mean velocity of the water in that vertical line, as in the case of the float, and the same formula may be used for reducing the observations, where v_f becomes the mean velocity determined by the meter; or the vertical curve may be drawn from the observations and extended to the bottom by the eye, when the mean velocity of the water can be found directly with the aid of a planimeter or any integrating or area-measuring process.

As the meterwheel may easily become clogged by weeds or by floating matter getting into the bearings, it should be examined from time to time during a gaging. As soon as a meter is rated, what is known as the hand test should be applied. That is, the instrument is held firmly in the position in which it stands when in operation and the wheel is spun by a single effort of the hand, the length of time that elapses before it ceases its revolutions being observed and recorded. This should be repeated several times, and thereafter before starting a gaging and whenever the meter is taken from the water the hand test should be applied and the result recorded. In this way it will be possible to detect both the existence and time of a change in the rating of the instrument.

Comparisons between meter and weir gagings show them to agree under most favorable conditions for the former within from from 3 to 4%. It is to be noted that during

Resistance of Meters, Cables and Rods to Dragging through Still Water

(Experiments made for Author at University of Michigan Naval Tank)

Velocity, ft.	Meters only. No weights or cables included		Cables per foot		Rods per foot	
	Ritchie-Haskell direction current meter 8-in. wheel, weight 24.7 lb.	Haskell current meter 8-in. wheel, weight 10.3 lb.	3/8-in. steel cable d = 0.400 to 0.392 in.	1/4-in. steel cable d = 0.264 to 0.259 in.	3/8-in. steel rod d = 0.382 in.	1/4-in. steel rod d = 0.267 in.
	Resistance, lb.	Resistance, lb.	Resistance, lb.	Resistance, lb.	Resistance, lb.	Resistance, lb.
3.0	1.70	0.45	0.40	0.26	0.36	0.29
4.0	3.75	2.40	0.72	0.47	0.64	0.52
5.0	5.75	4.15	1.12	0.75	0.99	0.78
6.0	8.40	6.00	1.52	1.04	1.40	1.07
7.0	11.60	8.05	1.91	1.37	1.88	1.38

an observation the meter should be stationary in the current or only moving as impelled thereby. The method of gaging sometimes used in which the meter is moved continuously about the cross-section is entirely untrustworthy.

Cup meters may over-register in considerably perturbed water 25%.

Screw meters may under-register in considerably perturbed water 10%.

The 3/8-in. cable used, 10 ft. 1/2 in. long, weighed 1 lb. 7-1/2 oz. in air and 1 lb. 2 oz. in water and had a 3-strand rope center 3/16 in. in diameter, covered by 6 strands 0.121 in. in diameter, each composed of 6 strands of 0.037 in. in diameter wires on a 1/16-in. rope center.

The 1/4-in. cable used, 11 ft. long, weighed 1 lb. 2 oz. in air, and 15 oz. in water, and had an insulated electric wire 0.038 in. in diameter for a center covered by 6 strands 0.086 in. in diameter composed of 7 wires 0.030 in. in diameter.

By a Waterwheel. Since practically all submerged waterwheels are composed of a series of orifices or nozzles, they may be used for measuring water whenever the discharge through them for any head and number of revolutions is known, as the velocity, and hence the discharge for constant gate, increases as $\sqrt{2\,gh}$, and the speed of the wheel for similar relative discharges must vary as that of the water.

By the Hydrometric Pendulum. This apparatus consists of a metal ball suspended upon a cord. When the ball is lowered into a current the cord is inclined away from the vertical in some relation to the force of the current. With a suitable scale for measuring the inclination, this becomes a cheap but crude current meter.

A method somewhat akin to the hydrometric pendulum is that of the torsion disk. A disk or plate is attached by its vertical edge rigidly to a small rod which is supported in a vertical position by guides and carries a pointer at the top which travels over a graduated arc. A zero reading is taken with the disk out of the water and at right angles to the direction of the current. The disk is then immersed parallel to the current and revolved by means of an arm on the rod above the pointer until the disk is again perpendicular to the current. The variation of the pointer from its first position measures the torsion of the rod, which is itself a measure of the moment of the force of the current on the disk.

By Coloring Matter, Brand, Sawdust, and Chemicals. These materials are deposited in the water and partake of the nature of floats which disperse themselves throughout the whole body of the stream. The various anilines or even ordinary bluing may be used as colors, and are suitable for use not only in open channels and pipes but also in subterranean streams. The observations are ocular. In the case of chemicals, as lithium or common salt, the observations depend usually upon chemical analyses. The latter method is often valuable in determining the amount of water from a particular source that occurs in a mixed stream, as knowing the dilution at inlet and at outlet the added untreated water can be estimated. The use of chemicals for the measurement of water passing through turbine wheels has been very fully developed by B. F. Groat, M. Am. Soc. C. E. (Trans. Am. Soc. C. E., Vol. LXXX). The essentials for accuracy in this process are the thorough mixing of a known solution with the influent water and a sufficiently distributed sampling of the effluent, care being taken that eddy and back currents do not dissipate the chemical into portions of the water that is not utilized in the measurement.

By a Cord. The measurement of surface velocities by the curve formed by a loose cord attached to the two banks and floating in the stream is possible.

By Ripple Formation. Surface velocities may be measured by the ripples formed by two pins allowed to scratch the surface at known distances apart in a line at right angles to its axis. The distance from this line to the intersection of the ripples varies with the velocity.

24. The Pitot Tube

The Pitot Tube is an instrument having two orifices or two sets of orifices one of which may be so directed as to receive the impact of a stream, and the other opening either at right angles thereto, or at some other angle so as to receive only the pressure energy of the water. These orifices are connected to a gage composed of two parallel glass tubes with a scale between or behind them, so that the difference of height of the two columns of liquid supported by the pressures on the orifices may be observed. This difference is then a function of the velocity head of the flowing water. The impact opening may be a hole bored in a piece of small-diametered pipe, the bottom of which is closed, and which is inserted in the stream of water flowing through a conduit or other channel, and the pressure orifice may be an opening in the channel wall normal to its interior surface. Experiment shows that for ranges of

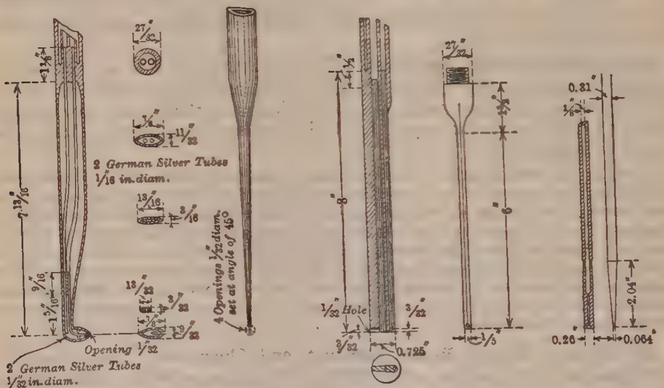


Fig. 23. Details of Pitot Tube

velocity up to 10 ft. per sec. and for pipes of diameters up to 8 in. the indication of a water column gage, when the openings are as last described, appears to give the velocity head at the impact opening within 1 or 2%, assuming the indication of the gage to be $v_2/2\ g$. In large open channels this statement does not appear to hold true and for work in the latter it is necessary that the instrument be rated. This rating may be accomplished with fair accuracy in a manner similar to that of rating current meters. A better rating can be obtained by measuring with the instrument the flow through a pipe, the discharge of which is simultaneously determined volumetrically or gravimetrically. The method of gaging is to locate the impact opening of the instrument successively at different points in the cross-section of the channel and read the indication of the gage, from which the velocity at the several points may be computed and thereby the discharge of the pipe or channel be determined. Since in the case of normal flow in a circular pipe flowing full, the central velocity is about 1.19 times the mean velocity, if the existence of normal flow at the point of gaging is established, the impact orifice of the instrument may be placed at the center of the pipe and the observed velocity divided by 1.19, or multiplied by its reciprocal 0.84 to get the mean velocity of the water. Such a determination when the area of the cross-section is accurately known and the condition of normal flow established should be correct within 3%. If normal flow does not exist but the distortion of the velocity curve is symmetrical about the axis of the pipe, the ratio of the mean to the maximum velocity, sometimes called the coefficient of the section, may be obtained by observations across a diameter and the mean velocity thereafter obtained by multiplying the observed central velocity by the thus obtained coefficient. The accuracy of the results will depend upon the accuracy with which the coefficient has been established, but with careful work should be within 5%. Observations with the Pitot tube in closed channels where the velocity distribution is distorted unsymmetrically as by a curve or gate cannot be relied upon for establishing the mean velocity unless they cover several diameters of the pipe.

The formula for reducing Pitot tube observations is usually misunderstood. It may be deduced as follows for the case of the pressure orifice in the wall of a circular pipe:

h_d = observed difference of head by the gage;
p = pressure head at any point in the cross-section of the pipe;
p_c = pressure head at the center of the cross-section of the pipe;
p_w = pressure head at the wall of the cross-section of the pipe;
v = velocity at any point in the cross-section of the pipe;
v_c = velocity at the center of the cross-section of the pipe;
v_w = velocity at the wall of the cross-section of the pipe.

By Bernouilli's theorem, for all points in a horizontal diameter of the section $p + v^2/2g$ is constant and

$$p_c + v_c^2/2g = p_w + v_w^2/2g = p + v^2/2g$$

The impact opening of the tube receives and transmits the pressure of the stream at that point, plus twice the head to which the velocity is due by the theory of impact, or its gage column represents $p + v^2/g$ for the point at which the orifice is situated. The opening in the wall transmits to its gage column the pressure p_w. Then the difference of the columns of the gage is

$$h_d = p + v^2/g - p_w$$

and for the point at the center of the pipe

$$h_d = p_c + v_c^2/g - p_w$$

But by Bernouilli's theorem

$$p_w = v_c + v_c^2/2g - v_w^2/2g$$

and the gage difference at the center then is

$$h_d = p_c + v_c^2/g - (p_c + v_c^2/2g - v_w^2/2g) = \frac{v_c^2 + v_w^2}{2g}$$

When $v_w = v_c/5$, then $h_d = 1.04\, v_c^2/2g$; and when $v_w = v_c/10$, then $h_d = 1.01\, v_c^2/2g$.

From the foregoing it appears that when the observed differences of head of the instrument with the pressure orifice in the wall of the pipe give the correct mean velocity on the theory that $h_d = v^2/2g$, it indicates the actual wall velocity to be about or less than one-tenth of that at the center. The closest measurement to the wall yet observed in the author's experience was at a point about 3/5 of 1% of the diameter from the wall in a 30-in. pipe, at which point the velocity was about 1/2 that at the center on the $h_d = v^2/2g$ theory. Although it is not impossible to conceive the rate of reduction of velocity between this point and the wall to bring the wall velocity down to 10% of that at the center it is difficult to understand. It therefore seems that the formula must be $h_d = v^2/g$ as theory has indicated, and it then follows that the velocity of the center v_c instead of being twice that at the wall v_w, will be about the $\sqrt{2}$ or 1.414, times v_w.

When the instrument carries its own pressure opening it does not seem possible to deduce its coefficient theoretically for the reason that the pressure orifice is always so situated that the velocity of the water next the instrument is considerably retarded, and hence the pressure increased, in passing from the impact to the pressure opening. Consequently the pressure transmitted is not that corresponding to the velocity to which the impact opening is subjected. The head indicated on the gage will therefore always be less than v^2/g, and as the velocity past the pressure opening is never less than that at the pipe wall, the head on the gage will always be greater than $v^2/2g$. Therefore when, as is commonly the practice, hd is assumed to represent $v^2/2g$, the indication must be multiplied by a coefficient less than unity, usually between 0.7 and 0.9; while using v^2/g, the coefficient of the instrument will be greater than unity.

Practically it is of little importance which formula be used so far as measuring the flow is concerned, as the tube must be rated in any case; but when a study of the laws of flow is involved the use of the correct formula is essential. (Trans. Am. Soc. C. E., vol. 47.)

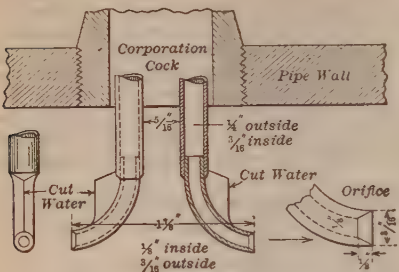


Fig. 24. Cole-Flad Pitometer

A Pitot tube of special form combined with a photographic recording apparatus is known commercially as the Cole-Flad Pitometer. In this instrument the pressure opening points downstream or opposite to the impact opening.

25. Fountain Jets

When water discharges over the horizontal circumference of a vertical pipe, the conditions partake in part of the nature of the flow over a weir and in part of the discharge through a nozzle, at low heads the former and at higher heads the latter predominating, with a range of transition from one condition to the other between. The head on such a device may be most satisfactorily read by an orifice at the center of the pipe directed against the current and located a short distance below the outlet. The head to be used is that measured by a water column attached to the orifice, less the height of the column of water from the orifice to the discharging lip of the pipe.

From a series of experiments made by Lawrence and Braunworth at the Hydraulic Laboratory of Cornell University (see Trans. Am. Soc. C. E., Dec., 1906, Vol. 57, p. 265) in which the pipes had sharp edges tapered on the outside and the head was read as above, the diameters used being 2, 4, 6, 9, and 12 in., the following formulas were deduced, which may be expected to give average results within 5% of the truth:

For Weir Flow, $Q = 8.8\ D^{1.29}h^{1.29}$, which is applicable for heads below $h = 0.028\ D^{1.04}$ with the jet falling free from the pipe; that is, aerated underneath.

For Jet Flow, $Q = 5.84\ D^{2.025}h^{0.53}$, which is applicable for heads above $h = 0.107\ D^{1.04}$. For the range between these two heads the result lies between those given by the above formulas whose lines intersect at $h = 0.045\ D^{1.04}$.

For ordinary computations these formulas, with the same limitations, may be used as:

For the weir, $\mathbf{Q = 8.8\ D^{1.25}h^{1.25}}$

For the jet, $\mathbf{Q = 5.84\ D^{2}h^{1/2}}$

Where D is diameter of jet in feet.

26. Observations of Heads

In Open Channels the most common method of reading head is by a graduated scale upon the face of which the water rises. If this is situated in running water the sides should be tapered to their edges to prevent a depression of the surface due to an eddy at the upstream corner. On account of capillarity the reading will be higher than the true surface of the water and the observer may be expected to read to about one-half the smallest division that is clearly visible. The United States Geological Survey uses for its stream-gaging stations the **ball and chain gage,** which consists of a chain running over a pulley and carrying a ball at its lower end which may be read by allowing the ball to scratch the surface in running water or to be just immersed in still water. The scale is usually horizontal, and a pointer on the chain indicates thereon the position of the ball.

A more accurate apparatus and one which may be readily provided in the field, consists of a plumb bob suspended from a steel tape which is passed over a block cut at the edge to an arc of about 2- or 3-in. radius, having on the top a mark against which the reading of the tape is taken. By fastening an ordinary leveling rod target to the block and causing the graduated edge of the tape to lie next the vernier, very accurate readings of the position of the bob may be obtained. In running water the bob should be allowed to scratch the surface and cause a fine ripple. When the surface undulates, readings of the high and low of the vibration should be taken. In still water the bob may be swung until it barely cuts the surface, or it may be slowly lowered until contact with the water is indicated by the rising of the water to the point due to capillarity. The **point gage** consists of a point attached to a graduated rod sliding past a vernier. It is read similarly to the plumb bob, and is only second in accuracy to the hook gage. The ordinary leveling rod with its upper section fastened to a post and the lower carrying a metal point makes an excellent and easily obtained point gage.

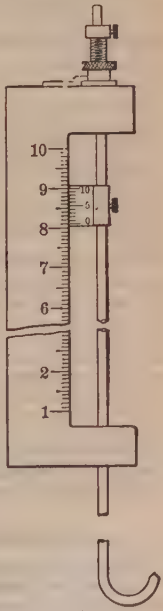


Fig. 25. Hook Gage

The Hook Gage, an apparatus invented by Uriah Boyden, consists of a hook attached to a graduated rod sliding past a vernier, or to a vernier sliding past a fixed scale. The hook is lowered below the surface of the water and slowly raised until its point comes in contact with the surface film of the water when the latter is distorted, and if the point in contact is in the reflection of a light a black spot appears; if in diffused light, a bright spot. In still water the delicacy of the instrument is such that variations of the surface of 1/10000 of a foot may be readily observed. For best results the point of the hook should be a cone with a vertex angle of about 90 degrees.

The Water Column which rises in a glass tube connected to the channel may be utilized for reading head, but in this case the attraction of the walls of the tube for the water causes the liquid to rise higher than the height due to pressure alone. If the tube is less than 0.5 in. in internal diameter this effect must be allowed for in accurate work. Under no circumstances should the height of the column be measured to the intersection of the water and the glass, but a line tangent to the extreme edge of the curve of the meniscus at the center of the tube should be taken. Except on a vibrating column during the rise of the liquid the water meniscus is always concave upward.

The Mercury Column, owing to the lack of affinity between the glass and the mercury, has its meniscus convex upward and should be read on a tangent to the top of the curve. As the specific gravity of mercury is 13.58 at 60° Fahr., one foot of mercury is the equivalent of 13.58 ft. of water.

The Bourdon Pressure Gage, though sometimes used in hydraulic work, is not sufficiently delicate to give satisfactory results except where the pressure to be measured exceeds 5 lb., and is unsuited for determining small differences of pressure.

The Differential Gage is used for measuring differences of head at two points in a stream. This gage consists of two vertical glass tubes connected at one end, with a scale between or behind them, the free ends being connected respectively by hose or pipes to the two points between in which the difference is to be observed. For measuring large differences of head the tubes are connected at the bottom and are then filled about half full of **mercury** or for smaller differences of head, with **carbon-tetra-chloride.** The difference of height of the mercury or tetra-chloride columns, when multiplied by the specific gravity of the liquid used minus one, gives the height of the water column that would measure the difference in pressure head at the two connections to the channel.

For small differences of head the tubes may be connected at the top and the water allowed to rise in the tubes entrapping air above it. Unless the pressure is so great as to compress the air sufficiently to cause its weight to be appreciable, the difference in height of the water columns measures the difference of pressure heads at the connections.

For very small differences the air in the upper portion of the gage may be replaced with oil. If G_o is the specific gravity of the oil and h_u and h_l the height of the upstream and downstream water columns, the difference of pressure head indicated will be $c\ (1 - G_o)\ (h_u - h_l)$ where c is a coefficient less than unity depending on the kind of oil. When a gage of this kind is used it should be calibrated by comparison with a water-column gage to determine the value of c, or better the value of one unit of the scale. For kerosene oil with $G_o = 0.7879$ at a temperature of 60° Fahr., the reading of the gage has been found to be 4.895 times the head; with gasolene having $G_o = 0.7159$ at the same temperature, the multiplication was 3.52; and with ordinary sperm oil it was 9.05. (Trans. Am. Soc. C. E., vol. 47, pp. 78-90, April, 1902.) Carbon-tetra-chloride being only a little heavier than water has also the effect of magnifying the observed reading. When suitably colored by dye to cause it to register on photographic paper it is used in the Cole-Flad Pitometer recording apparatus.

27. Early Methods of Measuring of Water

Before the methods and apparatus above described were introduced the methods of measurement were oftentimes somewhat indefinite. For milling and manufacturing purposes the quantity of water conveyed or to be taken was frequently described as that which will flow through an aperture of a certain area, usually measured in square inches, under a certain specified head, or those existing in the particular structure from which it was to be taken.

An old grant made in the late eighteenth century specified:

> "Three sluices of water to issue out of the canal, each sluice to contain thirty-five square inches, and the bottom of the water in each sluice to be four feet and one-half foot below the top of the water in the canal at its common level."

Later grants from the same source were less specific, usually reading a certain number of "square inches of water" under a certain head, but meaning thereby the amount of water that would flow through an aperture of the named area. Sometimes the word "square" is omitted.

In other localities grants of a certain number of square inches or inches of water to be drawn from a specified or understood dam or flume are common, by which was meant the amount of water that could be drawn with the usual appurtenances through an aperture of the size named from the pond or flume as it existed When the existent conditions at the time of the grant are known the quantity of water so granted is usually not difficult to determine.

Experiments by the writer have shown that the coefficient of discharge for the ordinary apertures with square-edged gates and their accompanying chutes to overshot wheels varied from 0.50 to 0.65 for the type of apparatus commonly in use, depending on the extent of the suppression of contraction by the side walls. Beveling the edge of the gates to an angle of 30 deg. with the horizontal increased the coefficients about 30% where the height of opening was from 1/8 to 1/10 of its width.

Other grants specified sufficient water to propel a **run of stone** under the conditions existent at the site of the grant. The quantity required depended on the head utilized, and the value of a **run of stone** was influenced by the size of stones in use in the particular locality. According to Whitham ("Water Rights Determination," John Wiley & Sons, Inc.) a **run of stone** might vary from 10 to 25 hp. Where the specification includes "the usual machinery connected therewith" as much as 40 hp. per run of stones has been estimated. Whitham finally concludes:

A run of stone in a first-class merchant mill yielding 85 to 100 barrels per day requires 30 to 40 hp.

A run of stone in a second-class merchant mill yielding 50 to 60 barrels per day requires 20 to 25 hp.

A run of stone in a common country grist mill requires 12 to 15 hp.

In the early days of milling when the stones were smaller, the amount of auxiliary machinery used was not so extensive and the flour ground coarser, the power required per run of stone was from 2-1/2 to 6 hp.

Miner's Inch. The measurement of water through apertures was applied by the early Californians in apportioning water for mining uses. Hence came into being the miner's inch as a unit of measurement; it was the quantity of water that would flow through an aperture one inch square in a plank under conditions which varied in different localities, but the head was usually about 6 in.

In California and Montana 40, in Colorado 38.4, and in Arizona, Idaho, Nevada and Utah 50 miner's inches = 1 cubic foot per second.

TURBINES AND WATERWHEELS

28. Classifications

Turbines are of two types, reaction or pressure turbines and tangential or impulse turbines. In the former the wheel is completely filled with water, which acts by its pressure, and in the latter the head is converted into velocity and the wheel should never be filled.

A **reaction** or **pressure turbine** is an assembling of curved channels or nozzles revolving uniformly about a fixed axis, usually either horizontal or vertical, with suitable stationary or movable guides to direct the flow into the revolving channels. The revolving part of such an assemblage is called the **runner** or **wheel.**

Runners are designated as Right-Hand or Left-Hand, according to the direction of their rotation. When the observer stands in the line of the shaft of the wheel with the inlet end toward him, or is looking in the direction along the shaft in which the water flows to enter or leave the wheel, a wheel which appears to revolve in a clockwise direction is Right-hand, and one revolving in a counter clockwise direction is Left-hand.

Turbines are classed according to the direction in which the water passes through them, as: (1) **Outward flow**, or **Fourneyron** wheels. (2) **Inward flow**, or **Francis** wheels. (3) **Parallel** or **axial flow**, or **Jonval** wheels. (4) **Mixed flow**, or **American** wheels. In the last named class the flow is first inward, then axial. When standing still the laws of flow through nozzles apply to this apparatus without modification, but when revolving the centrifugal force of the water inclosed in the wheel complicates the equations.

Nomenclature

b = barometric column of water;
d_i = depth of wheel at inlet;
d_o = depth of wheel at outlet;
e = efficiency of turbine;
e_h = hydraulic efficiency;
h_c = centrifugal head;
h_{ci} = height of center of inlet above tailwater;
h_{eo} = height of center of outlet above tailwater;
h_{pi} = pressure head at inlet;
h_{po} = pressure head at outlet;
H = total head acting;
H_n = head corresponding to a runaway speed of n revolutions;
k_1 = power constant;
k_2 = capacity constant;
k_3 = speed constant;
K = wheel characteristic;
n = number of revolutions per minute;
Q = quantity of discharge per unit of time;
R_i = radius of wheel at inlet;
R_o = radius of wheel at outlet;
u_i = radial component of velocity at inlet of wheel;
u_o = radial component of velocity at outlet of wheel;
U_i = absolute velocity of water at inlet of wheel;
U_o = absolute velocity of water at outlet of wheel;
U_t = absolute velocity of water at outlet of draft tube;
v_i = velocity of wheel at inlet;
v_o = velocity of wheel at outlet;
V_i = velocity of whirl at inlet;
V_o = velocity of whirl at outlet;
V_{ri} = relative velocity of water at inlet;
V_{ro} = relative velocity of water at outlet;
w = weight of one cubic unit of water;
W = power;
ϕ = speed coefficient.

Fundamental Reactions. Bernouilli's equations for the flow through a turbine:

At inlet, $$H - h_{ei} + b = h_{pi} + \frac{U_i^2}{2g}$$

Between inlet and outlet the wheel impresses upon the water a head due to centrifugal force $= (v_i^2 - v_o^2)/2g$, so that within the wheel:

$$h_{ei} + h_{pi} + \frac{V_{ri}^2}{2g} = h_{po} + \frac{V_{ro}^2}{2g} + \frac{v_i^2 - v_o^2}{2g} + h_{eo}$$

and between outlet and tail-water:

$$h_{po} = \frac{U_o^2}{2g} + h_{eo} = b + \frac{U_t^2}{2g}$$

To relate V_{ri} and V_{ro} to U_i, U_o, v_i and v_o, referring to Fig. 26, where

$$AB = U_i \quad ab = U_o \quad AC = v_i \quad ac = v_o \quad AD = V_i \quad ad = u_o$$
$$BC = V_{ri} \quad bc = V_{ro}$$
$$BD = u_i \quad bd = V_o \quad R_i = \text{radius at inlet} \quad R_o = \text{radius at outlet}$$

α = angle between absolute velocity of water and tangent to periphery of wheel at inlet;
β_i = angle between tangent to inlet of bucket and tangent to periphery of wheel;
β_o = angle between tangent to outlet of bucket and tangent to interior of bucket circle;
θ = angle through which the jet is diverted or angle between U_i and U_o.

$$V_{ri}^2 = (v_i - U_i \cos\alpha)^2 + (U_i \sin\alpha)^2$$
$$V_{ro}^2 = [v_o - U_o \cos(\alpha + \theta)]^2 + [U_o \sin(\alpha + \theta)]^2$$

Or otherwise, since the flow is radial, the product of the radial velocity and the area of the circumferential sections of the channels must be constant, whence: $\pi R_i d_i U_i \sin\alpha = \pi R_i d_i u_i = \pi R_o d_o u_o$

or $u_o = R_i d_i u_i / R_o d_o$

The total work done by the water in passing through the turbine is measured by the change in kinetic energy and equals $W = Qw\,(U_i^2 - U_t^2)/2\,g$, and the energy carried away in the water is $QwU_t^2/2\,g$. The work is also equal to $W = Qw\,(V_i v_i - V_o v_o)/g$, and by the aid of these equations all the factors may be interrelated.

When U_0 is radial or axial, as it should be for maximum efficiency, the relations are materially simplified, as U_0 is then equal to u_0.

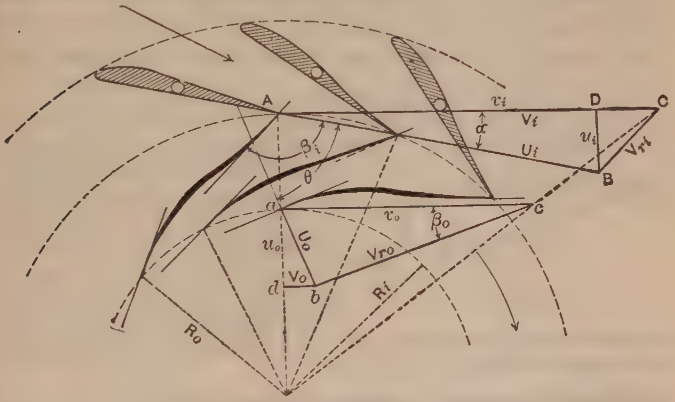


Fig. 26. Turbine Diagram

29. Definitions

The theoretical power of a fall of water is the force, or weight of the water, multiplied by the space passed through, or the height of the fall; when H = the height through which the water falls, Q = the discharge in cubic feet per second, w = the weight of a cubic foot of water, the theoretical horsepower is $= QwH/550$. A turbine wheel converts a part of this theoretical power into effective energy, and the **mechanical horsepower** of the wheel $= Wh_p = cQwH/550$, where c is the efficiency of the wheel.

The Efficiency of the wheel is the ratio of its output in power to the total power of the water and head used. As c is usually about 80%, the horsepower of the wheel $= QH/11$ approximately, or 11 cu. ft. of water per second falling through 1 ft. will yield 1 hp. at the turbine shaft.

The Hydraulic Efficiency of a turbine is the quotient obtained by dividing the change in kinetic energy by the total potential energy of the fall utilized and is

$$e_h = \frac{Qw}{2\,g}\,\frac{(U_i^2 - U_t^2)}{QwH} = \frac{U_i^2 - U_t^2}{2\,gH}$$

This differs from the actual efficiency, and is what would be its limiting value if there were no frictional or impact losses either in the water itself or in the mechanism. The actual efficiency, or the efficiency of turbines as shown at the Holyoke Testing Flume, ranges from 70% to 93%.

The Discharge Q of a wheel varies as the velocity and is therefore proportional to $\sqrt{H}$.

The Mechanical Power of a given wheel varies as the head H and also as the discharge Q, which in turn varies as $\sqrt{H}$. Therefore, the power varies as $H\sqrt{H} = H^{3/2}$.

Unit Power is the power at 1-ft. head = horsepower$_1$ = $\dfrac{\text{horsepower}}{H\sqrt{H}}$, designated HP_1.

The Speed at which a wheel will give its maximum efficiency under a given head depends upon the radius of the inlet circle and the form of the buckets or vanes. For best effect the receiving edge of the bucket must be parallel to the direction of the velocity of the inflowing water relative to the moving vane, and at discharge such that the water leaves either axially or radially.

In Fig. 26, if AB represents in direction and magnitude the absolute velocity of the water entering the wheel and AC that of the bucket, then CB is the relative velocity of the water on the bucket, and the inlet edge should be tangent to it. If the velocity of the water is increased to AD, then for the same bucket the velocity of the wheel must be increased to AE, to keep ED parallel to CB, so that the water may still glide upon the bucket without shock. It follows, then, that for any given wheel a certain fixed ratio must exist between its speed and that of the water, for maximum efficiency. The speed of the water is evidently a function of the head, and hence for every head there is some speed at which a given wheel will yield the maximum efficiency. The best speeds are therefore proportional to $\sqrt{H}$.

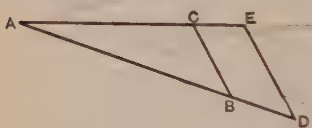


Fig. 27

Unit Speed is the speed of a wheel at 1-ft. head = $n_1 = \dfrac{n}{\sqrt{H}}$.

The Speed Coefficient, usually designated ϕ, is the ratio of the linear velocity of the runner periphery to the spouting velocity of the water under the given head;

$$\phi = \frac{\pi D_1 n}{60 \times \sqrt{2gH} \times 12}$$

where D_1 = outside diameter of runner in inches, measured at center line of inlet.

The Discharging Capacity, and hence the **power** of a wheel, is dependent upon the area of the cross-section of the water passages through it, which are themselves a function of the diameter of the wheel. Therefore in similar wheels the power at a given head will vary as the square of the diameters.

When the performance of a given wheel has been determined for a particular head as to horsepower, discharge and efficiency, the performance at any other head may be computed, and the performance of any similar wheel at any head may be determined with reasonable accuracy by the foregoing relations. By similar wheel is meant one in which all dimensions are in the same ratio to those of the first wheel, all angles being the same.

By changing the form of the vanes or buckets the speed at which the wheel gives its best efficiency may be varied. Starting with a normal speed equal to the tangential component of the velocity of the inflowing water in which case the inlet edge of the bucket would be radial, the speed may be either increased or decreased 25% without appreciably affecting the efficiency. If the speed is increased the bucket tends to become convex toward the incoming water, and is inclined backward toward the outlet; if the speed is decreased the bucket becomes more concave toward the jet and has less backward inclination toward the outlet. The power of a given sized wheel is decreased by varying its speed from the normal, by reason of the resulting contraction of the waterways.

30. Selection of Runners

Type Characteristic. Turbine runners of the same general class may differ widely in their characteristics of speed and capacity, or power, depending upon the size and shape of the blades. In order that runners of different types, or of different manufacture, may be compared on a common basis, a type characteristic has been adopted. *The type characteristic is defined as the speed in r.p.m. at which a wheel would run if it were reduced proportionally in all dimensions so as to develop* 1 *hp. under* 1-*ft. head.* This characteristic is also called the **characteristic speed** or the **specified speed.** It is designated K and is made up of:

(a) A power constant k_1 = horsepower/QH, which is the power of a similar runner using 1 cu. ft. of water under a 1-ft. head.

(b) A capacity constant, $k_2 = Q/D_1^2\sqrt{H}$, which is the discharge of a similar runner of 1-ft.-diameter under a head of 1 ft., where D_1 = diameter of 1 ft.

(c) A speed constant, $k_3 = \sqrt{H} = \pi D_1 n/60\sqrt{H}$, which is the speed of the runner itself under a head of 1 ft. Then

$$\mathbf{K} = \frac{60\,\mathbf{k}_3\sqrt{\mathbf{k}_1\mathbf{k}_2}}{\pi} \quad \text{or} \quad \mathbf{K} = \frac{\mathbf{n}\sqrt{\mathbf{horsepower}}}{\mathbf{H}^{5/4}}$$

For 1-ft. head $\mathbf{K} = \mathbf{n}_1\sqrt{\mathbf{Hp}_1}$ where n_1 = unit speed $= \dfrac{n}{\sqrt{H}}$, Hp_1 = unit horsepower $= \dfrac{Hp}{H^{3/2}}$.

Obviously this characteristic combines the considerations of power and speed and enables a complete comparison to be made between wheels of known efficiency.

The values of K for American, Francis or pressure type turbines vary from 12 to 160, as shown by the table on p. 1379.* For impulse wheels the maximum value is about 6, where a single jet is used. The interval between 6 and 12 is not covered by either the Francis type or a true impulse wheel, though the old Girard turbine would apply between these limits.

The turbines included in the following table are all of recent design and high efficiencies. There are other modern high-efficiency wheels made and the manufacturers listed make other types than the ones shown, but these are all on which data have been furnished to the author.

In general, high values of K indicate relatively " high-speed " wheels, or those adapted for operation under low heads of water. Experience has shown that for any given head there is a limiting value of K beyond which it is impracticable to go, as the resulting speeds would cause pitting of the runner

* The **speed** value used in computing K is that at which the best efficiency is attained by the runner, usually referred to as " best speed " or " normal speed." The **horsepower,** for the sake of definiteness and uniformity in practice should preferable be taken at the actual best efficiency point or gate-opening. However, as this can by arrived at only by a plotting of the test, it has become the practice of most manufacturers to figure K from the full gate horsepower at the best efficient speed. The former method is of course the more conservative, but the latter the more readily applied.

blades and develop excessive centrifugal forces. The limiting values of K as determined by best American practice are approximately:

For heads above 600 ft., $K = 12$ to 25
For heads from 100 to 600 ft., $K = 25$ to 55
For heads below 100 ft., $K = 55$ to 150

Fig. 28 shows graphically the relation between K and H, the curve being an average of the practice of several leading turbine manufacturers.

In using this characteristic K, its value is computed from the requirements of the plant under consideration. If this value falls within the allowable limit for the given head as shown by Fig. 28, the turbine best suited to the place

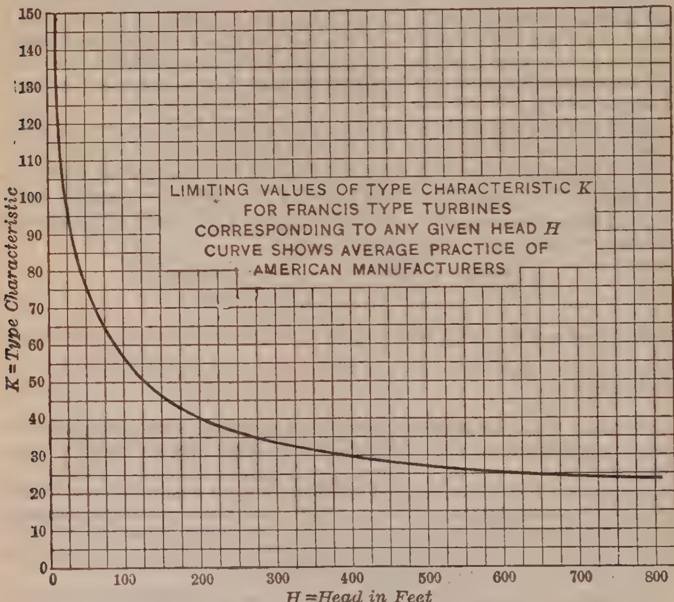


Fig. 28. Limiting Values of Type Characteristic K

will be the one having a value of K nearest that computed. If the value of K computed from the plant requirements considerably exceeds the limiting values, it will be necessary to reduce the power per runner, or reduce the speed, in case this be permissible and K may then be recomputed, using a value of the horsepower equal to the total power desired, divided by the number of wheels assumed, and the runner selected as before. This will be illustrated by several typical examples.

Example 1. "It is desired to install a 60-cycle, 2300-volt, hydroelectric unit, with 450 hp. available under 25-ft. head."

For this low head installation a high-speed turbine (one having the highest permissible value of K) should be used. From the curve (Fig. 28) K_{max} for 25-ft. head is about 95.

Table of Type Characteristics

Manufacturer	Name of runner	K *
Allis-Chalmers Mfg. Co.	Type A	13.55
	Type B	20.3
	Type C	29.4
	Type D	40.7
	Type E	51.7 to 60.5
	Type F	72
	Type G	82
	Type H	92.5
	Type I	105
	Type Nx_1	130
	Type Nx_2	160
Hydraulic Turbine Corp.	Camden, Type 4	53.2
	Camden, Type 8	71.3
	Camden, Type 11	83.5
J. and W. Jolly	McCormack Special	68.2
James Leffel & Co.	Samson-Standard	73.0
	Leffel-Sparks, Type F	81.0
	Leffel, Type Z	105.0
Platt Iron Works Co.	Victor, Type 1	84
	Victor, Type 2	75
	Victor, Type 3	60
	Victor, Type 4	45
	Victor, Type 5	32
	Victor, Type 6	20
	Victor, Type 7	12
S. Morgan Smith Co.	Type E	25.1
	Type F	33.8
	Type G	46.1
	Type H	52.0
	Type K	60.0
	Type N	71.3
	Type O	76.6
	Type R	86.0
	Type S	95.5
Wellman-Seaver-Morgan Co.	Runner No. 42	84.5
	Runner No. 48	71.5
	Runner No. 40	65.5
	Runner No. 11	50
	Runner No. 18	27.2

* K computed using horsepower values at full gate instead of at best efficiency point.

$Hp_1 = \dfrac{Hp}{H^{3/2}} = \dfrac{450}{125} = 3.6$, $n_1 = K/\sqrt{Hp_1} = 95/\sqrt{3.6} = 50$, $n = 50 \times \sqrt{25} = 250$ r.p.m. As this is not a synchronous speed, 240 will be assumed.

Then, $n_1 = \dfrac{240}{\sqrt{25}} = 48$ and $K = 48\sqrt{3.6} = 91.1$.

At least four of the wheels shown in the foregoing table have a K above 91.

The performance of a wheel of given size being known, it is possible to determine the approximate size required to fulfil the assumed conditions, remembering that speed varies inversely with D and power varies as D^2. Or, from such tables as are given on pp. 1382 to 1387 the size under any type that comes nearest to the required conditions may readily be determined.

Thus, from Table (p. 1383) a type of ($K = 92.5$) Allis-Chalmers Co. turbine shows the following performance:

Diameter	$HP._1$	n_1	Q_1	HP.	R.p.m.	Q
30 in.	3.40	50.0	37.4	425	250	187
34 in.	4.36	44.2	48.0	545	221	240

Obviously the required diameter is between 30 and 34 in.

Example 2. Required to develop 6000 hp. at 360 r.p.m. under a head of 125 ft.

$$H.P._1 = \frac{6000}{125\sqrt{125}} = 4.3, \quad n_1 = \frac{360}{\sqrt{125}} = 32.2, \quad K = 32.2\sqrt{4.3} = 66.8.$$

The allowable limit for K at 125-ft. head is only about 50, from Fig. (27*a*).* Therefore, two units will be assumed developing 3000 hp. each.

Then $H.P._1 = \dfrac{3000}{125\sqrt{125}} = 2.145$ and $K = 32.2 \times \sqrt{2.145} = 47.2$, which falls below the limiting value.

Referring to the table on pp. 1385 and 1386 we find that a 42-in. Victor type 4($K = 45$) turbine develops 3026 hp. at 342 r.p.m. under 125-ft. head, and a 42-in. Camden type 4 ($K = 51$) develops 3400 hp. at 365 r.p.m. Both of these wheels very closely approximate the assumed conditions and by a slight modification of the design would be able to meet them exactly.

Example 3. Required to develop 3000 hp. with one hydroelectric unit installed under 900-ft. head.

For economical generator construction, the speed for a unit of this size should not exceed about 450 r.p.m. On this assumption, $n_1 = \dfrac{450}{\sqrt{900}} = 15$ Hp. $= \dfrac{3000}{900\sqrt{900}} = 0.111$, $K = 15\sqrt{0.111} = 5$. As this value of K falls below the minimum of 12 recommended for Francis turbines, the pressureless type, or impulse wheel, should be used.

Assuming 80% efficiency, $Q = \dfrac{3000 \times 11}{900} = 36.6$ c.f.s.

Velocity of free jet $= .97\sqrt{2\,gh} = .97\sqrt{64.4 \times 900} = 233.5$ ft. per sec.

Area of jet $= \dfrac{Q}{V} = \dfrac{36.6}{233.5} = .157$ sq. ft. $= 22.55$ sq. in. Diam. jet = 5-3/8 in.

For maximum efficiency, the peripheral velocity of the runner buckets should theoretically be one-half the spouting velocity of the jet. In actual practice, however, this ratio is taken at about $.47\sqrt{2\,gh}$. $.47\sqrt{64.4 \times 900} = 113$ ft. per sec.

At 450 r.p.m., $D = \dfrac{113 \times 60}{450 \times \pi} = 4.8$ ft. diam. or 57.6 in.

* It will be necessary, then, to use more than one runner. Where it is desired to install only one unit, a multiple-runner turbine may be used. This expedient is, however, being resorted to less and less frequently in best practice and should be avoided whenever possible.

$$\frac{57.6}{5.375} = \frac{10.7}{1} = \text{ratio of wheel diameter to diameter of jet.}$$

This is about the minimum ratio allowable in present practice and it is considered better to increase it to 16 or 18. Assuming 16:

$D = 5.375 \times 16 = 86$ in. A 7-ft. wheel would probably be used. Then r.p.m. $= \dfrac{113 \times 60}{7 \times \pi} = 308$, and the nearest synchronous speed would be 300 r.p.m.

31. Turbine Performances

Variation of Discharge. The quantity of water discharged for an inward-flow turbine is greatest when the wheel is still, and on account of centrifugal force it decreases as the speed increases.

For the other types the discharge is less when still than at a velocity near that giving maximum efficiency, on account of the loss due to shock at entrance, and increases until the wheel gets up to speed; from the maximum it decreases, on account of the interference of the revolving buckets with the stream flowing through to a theoretical zero at a speed of infinity.

Gates. The quantity of water passing through a turbine, and hence its power, may be regulated by opening or closing the gates. In the older turbines, the gate was a cylinder sliding parallel to the axis and reducing the height of the inlet openings. This form is objectionable, as it does not permit the bucket to be uniformly filled at part gate-openings, and the more generally adopted type with recent wheels is the so-called wicket gate, which swings

Performances of McCormack Wheels at Over and Under Speeds

Per cent of original head	Per cent of original power at best speed	Per cent of original power at original speed	Per cent of best power at original speed	Per cent of best speed at original speed	Approximate per cent of best efficiency at original speed
150	183.7	180	97.5	81.6	96.5
145	174.6	172	98.4	83.0	97.1
140	165.7	164	99.0	84.5	97.6
135	156.9	156	99.4	86.1	98.0
130	148.2	148	99.7	87.7	98.4
125	139.8	140	100.1	89.4	98.7
120	131.5	132	100.2	91.3	99.0
115	123.4	124	100.3	93.3	99.2
110	115.4	116	100.5	95.3	99.4
105	107.6	108	100.4	97.6	99.7
100	100	100	100	100	100
95	92.6	92	99.3	102.6	99.3
90	85.4	84	98.3	105.4	98.4
85	78.4	76	96.9	108.5	97.0
80	71.5	68	95.2	111.8	94.5
75	64.9	60	92.4	115.5	92.5
70	58.6	52	88.8	119.5	89.0
65	52.4	44	84.0	124.1	85.0
60	46.5	36	77.5	129.0	78.5
55	40.8	28	68.5	134.8	69.5
50	35.4	20	56.5	141.4	57.5
45	30.2	12	39.8	149.0	41.0
40	25.3	4	15.8	158.2	17.0
37.5	22.9	0	0.0	163.4	0.0

on a pivot set parallel to the axis of the wheel, and forms not only the gate but the guide for directing the water upon the wheel.

Constant Speed. In the operation of electrical machinery a constant speed is necessary, and hence if the head falls below the normal the turbine must be run at a higher speed than that for which it was designed, in which case the power and efficiency are rapidly reduced and the outlet edges of the buckets are liable to corrode.

If, on the other hand, the head increases, the wheel must run at a lower speed than that for which it was designed, which, though not reducing power and efficiency nearly so rapidly as overspeeding, nevertheless renders the buckets liable to be cut out by the water, particularly if it carry sand or grit.

By **Runaway Speed** is meant the speed at which the wheel would rotate under any given head, with no load; that is, its highest speed, under that head. From this it follows that a wheel with a runaway speed, that is relatively high as compared with the best speed, will fall off in power rapidly as it is overspeeded, and vice versa. For low-speed turbines the runaway speed varies between 1.6 to 1.7 times the best speed and for high-speed turbines is about 1.5 times the best speed. Hence high-speed turbines decrease less rapidly in power when overspeeded than do low-speed ones.

The table given above shows the performance of a pair of McCormack wheels with $K = 68.2$ computed from tests. Other American wheels will give results varying slightly from these figures, which may however be safely used in approximate calculations.

32. Manufacturer's Tables

Manufacturer's Tables. The following tables give the performances at 1-ft. head of various turbines commonly met with. The values have been taken from the makers' catalogs, both old and new, and do not necessarily show the best performances or latest types developed. They are partly intended for reference in connection with old installations and, wherever possible, the date of the catalog from which the tables were taken is shown, to indicate the age of the design.

All values have been computed for 1-ft. head. At any head H:
To obtain horsepower of wheel multiply its tabular horsepower by $H^{3/2}$
To obtain discharge of wheel multiply its tabular discharge by $H^{1/2}$
To obtain speed of wheel multiply its tabular speed by $H^{1/2}$

Turbine Performances at One-foot Head

Allis-Chalmers Manufacturing Company, Milwaukee, Wisconsin
From Tables Furnished by Makers in 1916

Diameter	Type A $K = 13.55$ $\phi = 0.585$			Type B $K = 20.3$ $\phi = 0.625$			Type C $K = 29.4$ $\phi = 0.665$			Type D $K = 40.7$ $\phi = 0.70$		
	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
15	0.0358	0.394	71.7	0.0705	0.776	76.6	0.150	1.43	81.4	0.226	2.49	85.7
21	0.0705	0.776	51.2	0.138	1.523	54.7	0.225	2.48	58.2	0.442	4.86	61.3
27	0.116	1.276	39.8	0.229	2.52	42.5	0.423	4.65	45.2	0.731	8.04	47.6
34	0.184	2.024	31.6	0.363	3.99	33.8	0.668	7.35	35.9	1.158	12.74	37.8
42	0.280	3.08	25.6	0.551	6.06	27.4	1.016	11.18	29.1	1.765	19.4	30.6
50	0.398	4.38	21.5	0.79	8.69	23.0	1.450	15.95	24.4	2.50	27.5	25.7
60	0.573	6.30	17.9	1.13	12.43	19.1	2.08	22.88	20.4	3.61	39.7	21.4
70	0.785	8.64	15.4	1.53	16.83	16.4	2.82	31.00	17.5	4.90	53.9	18.4

Turbine Performances at One-foot Head—Continued

Allis-Chalmers Mfg. Co. From Maker's Catalogs.

Diameter	Type E, K = 51.7–60.5, ϕ = 0.75			Type F, K = 72, ϕ = 0.80			Type G, K = 82, ϕ = 0.77			Type H, K = 92.5, ϕ = 0.815		
	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
14	0.277	3.05	98.4	0.47	5.17	105.0	0.66	7.26	101.0	0.74	8.15	107.0
18	0.471	5.18	76.5	0.775	8.52	82.0	1.09	12.0	78.5	1.22	13.4	83.5
22	0.731	8.04	62.6	1.15	12.65	67.0	1.63	17.9	64.0	1.82	20.0	68.5
26	1.055	11.60	53.0	1.62	17.8	56.5	2.27	25.0	54.5	2.55	28.1	58.0
30	1.436	15.80	46.0	2.15	23.6	49.0	3.02	33.2	47.0	3.4	37.4	50.0
34	1.89	20.82	40.5	2.76	30.4	43.5	3.88	42.7	41.5	4.36	48.0	44.2
38	2.42	26.60	36.3	3.44	37.8	39.0	4.85	53.3	37.0	5.45	60.0	39.5
42½	3.09	34.0	32.4	4.32	47.5	34.5	6.06	66.6	33.3	6.8	74.8	36.3
47½	4.01	44.1	29.0	5.4	59.4	31.0	7.6	83.6	29.7	8.5	93.5	31.6
52½	4.95	54.5	26.3	6.6	72.5	28.0	9.28	102.0	27.0	10.6	116.5	28.3
57½	6.10	67.1	24.0	7.9	87.0	25.6	11.1	122.0	24.5	11.8	130.0	26.8
64½	7.63	83.9	21.6	9.75	107.0	23.0	13.75	151.5	22.0	15.4	169.5	23.5
72	9.58	105.4	19.2	12.3	135.0	20.4	17.4	191.5	19.6	19.5	214.5	20.8
80	12.30	135.3	17.3	15.3	168.0	18.4	21.5	237.0	17.6	24.1	265.0	18.8
90	......	......	......	......	......	......	......	27.2	299.0	15.7	30.5	336.0	16.7
100	......	......	......	......	......	......	......	33.6	370.0	14.1	37.7	415.0	15.0

All values of the discharge Q_1 are calculated from Unit Horsepower Hp_1, using an efficiency of 80%.

Jas. Leffel & Co., Springfield, Ohio
From 1916 Catalog

Type	Diameter	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
Standard Samson (Double buckets) $K = 73$	17 in.	0.616	6.71	92.8
	20	.808	8.8	81.4
	23	1.064	11.63	70.8
	26	1.368	14.87	62.6
	30	1.816	19.79	54.2
	35	2.464	26.83	46.4
	40	3.232	35.19	40.6
	45	4.088	44.54	36.2
	50	5.048	54.99	32.4
	56	6.328	68.97	29.
	62	7.76	84.55	26.2
	68	9.336	101.7	24.
Z (Single buckets) $K = 105$	12 in.	0.66	7.21	125
	15	1.042	11.29	100
	18	1.512	16.35	83.3
	24	2.763	29.45	62.5
	30	4.415	46.45	50
	36	6.36	66.85	41.6
	42	8.67	91.1	35.75
	48	11.31	118.7	31.25
	54	14.31	150.3	27.75
	60	17.67	186.0	25.0

Trump Mfg. Co., Springfield, Ohio
From 1914 Catalog

Type	Diameter	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
Trump Standard $K = 64$	17 in.	0.57	6.28	79.2
	23	1.21	13.31	58.6
	30	2.135	22.61	44.4
	40	3.646	40.1	33.6
	48	5.247	57.7	28.
	56	7.136	78.6	24.
	66	10.38	114.3	20.
Small Capacity For High Heads $K = 33$	10 in.	.0607	.669	137.8
	13	.1008	1.113	104.0
	15	.1314	1.446	90.4
	17	.1771	1.950	78.8
	20	.2276	2.505	68.9
	23	.3024	3.340	59.8

Turbine Performances at One-foot Head.—Continued

Dayton Globe Iron Works, Dayton, O. From 1909 Catalog

Type	Diameter	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
New	[illegible] in.	[illegible]	2.9[illegible]	[illegible]
American	[illegible]	[illegible]	7.[illegible]	[illegible]
K = [illegible]	[illegible]	[illegible]	11.[illegible]	[illegible]
	30	[illegible]	18.[illegible]	42.[illegible]
	36	2.488	27.[illegible]	35.[illegible]
	42	3.[illegible]24	33.[illegible]	30.[illegible]
	48	4.[illegible]8	46.[illegible]	[illegible]
	54	[illegible]	[illegible]	[illegible]
	60	[illegible]	[illegible]	22.[illegible]
Improved	[illegible] in.	0.[illegible]	[illegible]	102.
New	[illegible]	[illegible]	[illegible]	87.
American	22	1.[illegible]	[illegible]	73.
K = 79	25	1.[illegible]	14.[illegible]	66.8
	29	1.[illegible]	[illegible]	54.
	34	2.[illegible]	[illegible]	[illegible]
	39	3.[illegible]	[illegible]	43.6
	44	4.[illegible]	[illegible]	38.8
	49	5.[illegible]	[illegible]	34.8
	54	6.[illegible]	[illegible]	31.6
	60	7.[illegible]	[illegible]	29.
	70	9.[illegible]	[illegible]	[illegible]

Stillwell-Bierce Co., Platt Iron Works, Dayton, Ohio. From 1904 Catalog

Type	Diameter	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
Victor A	12 in	0.296	3.26	117.4
Obsolete	18	.666	7.34	78.6
K = 63.5	24	1.183	13.04	58.6
	30	1.849	20.39	47.
	36	2.662	29.36	39.
	42	3.624	39.97	33.6
	48	4.741	52.2	29.
	54	5.99	66.1	25.6
	60	7.39	81.6	23.
Victor	12 in	0.325	3.59	117.4
Increased	18	.732	8.07	78.6
Capacity	24	1.292	14.35	58.6
K = 66.6	30	2.036	22.43	47.
	36	2.928	32.30	39.
	42	3.986	43.96	33.6
	48	5.206	57.42	29.
	54	6.59	72.68	25.6
	60	8.13	89.72	23.

Risdon-Alcott Turbine Co., Mt. Holly, N.J.

Type	Diameter	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
Alcott	[illegible] in.	0.2[illegible]	[illegible]	94.2
High Duty	18	.460	[illegible]	[illegible]
Special	24	.8[illegible]	[illegible]	52.
K = 46.7	30	1.2[illegible]	[illegible]	42.
	42	2.4[illegible]	[illegible]	30.
Risdon	[illegible] in.	0.2[illegible]	[illegible]	78.4
Double	20	.[illegible]	[illegible]	66
Capacity	25	.[illegible]	[illegible]	54.4
K = [illegible]	30	.[illegible]	[illegible]	47.[illegible]
	40	1.[illegible]	18.[illegible]	35.[illegible]
	50	2.[illegible]	[illegible]	[illegible]
	60	[illegible]	[illegible]	22.4
	72	[illegible]	[illegible]	[illegible]
Leviathan	[illegible] in	0.608	[illegible]	95.
K = 74	[illegible]	.82[illegible]	[illegible]	[illegible]
	24	[illegible]	[illegible]	71.[illegible]
	27	[illegible]	[illegible]	63.4
	30	[illegible]	[illegible]	57.
	36	[illegible]	[illegible]	47.6
	42	[illegible]	[illegible]	40.8
	48	4.[illegible]	47.43	35.[illegible]
	54	5.[illegible]	[illegible]	31.6
	60	[illegible]	[illegible]	28.[illegible]
	[illegible]	[illegible]	[illegible]	26.
	72	[illegible]	[illegible]	23.8

S. Morgan Smith Co., York, Pa. From 1894 and 1910 Catalogs

Type	Diameter	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
McCormack	18 in	0.56	6.17	64.4
(1894)	24	1.045	11.53	50.6
K = 51.4	30	1.595	17.59	41.6
	36	2.24	24.71	35.4
	42	3.233	35.67	30.
	48	3.885	42.85	24.
	57	5.843	64.45	22.
New	18 in.	0.543	5.99	70.8
Success	24	1.014	11.19	55.6
(1894)	30	1.547	17.06	45.6
K = 55	36	2.173	23.97	38.8
	42	3.136	34.6	33.
	48	3.768	41.57	28.6
	54	5.051	55.12	25.
	60	7.34	81.03	22.6
	72	11.18	123.37	18.6
Smith	18 in.	0.76	8.28	92.5
(1910)	24	1.353	14.73	69.3
K = 80.6	30	2.113	23.01	55.5
	36	3.047	33.19	46.2
	42	4.143	45.1	39.5
	48	5.362	58.43	34.7
	54	6.85	74.9	30.8
	60	8.45	92.1	27.8
	72	12.18	132.6	23.

Turbine Performances at One-foot Head

Victor Turbines, Platt Iron Works, Dayton, Ohio

Tables Submitted by Maker in 1918

Diameter	Type 1 K = 84 ϕ = 0.88 Max. H = 40			Type 2 K = 75 ϕ = 0.84 Max. H = 60			Type 3 K = 60 ϕ = 0.76 Max. H = 125			Type 4 K = 45 ϕ = 0.70 Max. H = 200		
	Hp_1	Q_1 c.f.s.	R.p.m.	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
15	....	.....	....	.531	5.84	103.0	.414	4.56	93.3	.276	3.04	85.7
18	....	.....	....	.762	8.38	85.9	.593	6.53	77.8	.396	4.36	71.5
24	....	.....	....	1.36	14.95	64.3	1.060	11.65	58.3	.705	7.75	53.6
30	2.42	26.70	54.0	2.12	23.30	51.5	1.640	18.05	46.7	1.100	12.10	42.9
36	3.48	38.40	45.0	3.06	33.70	42.8	2.375	26.15	38.9	1.585	17.45	35.7
42	4.74	52.20	38.6	4.16	45.70	36.8	3.24	35.65	33.3	2.160	23.80	30.6
48	6.20	68.30	33.7	5.43	59.70	32.2	4.22	46.40	29.2	2.810	30.90	26.8
54	....	.....	....	6.87	75.60	28.6	5.36	59.00	25.9	3.570	39.30	23.8
60	....	.....	....	8.50	93.50	25.7	6.58	72.40	23.4	4.42	48.60	21.4
68	....	.....	....	10.89	120.00	22.7	8.48	93.30	20.6	5.67	62.40	18.9
76	....	.....	....	13.60	149.50	20.3	10.65	117.00	18.4	7.09	78.00	16.9

Diameter	Type 5 K = 32 ϕ = 0.66 Max. H = 400			Type 6 K = 20 ϕ = 0.62 Max. H = 600			Type 7 K = 12 ϕ = 0.59 Max. H = 800		
	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
15	.157	1.73	80.9	.0693	.836	76.0	.0274	.302	72.4
18	.226	2.49	67.4	.0996	1.10	63.3	.0396	.436	60.3
24	.402	4.43	50.5	.1770	1.95	47.5	.0704	.775	45.2
30	.625	6.87	40.5	.277	3.05	38.0	.1110	1.22	36.2
36	.901	9.91	33.7	.398	4.38	31.7	.1575	1.74	30.2
42	1.220	13.40	28.9	.544	5.99	27.1	.2160	2.38	25.8
48	1.700	18.70	25.3	.706	7.77	23.8	.2820	3.10	22.6
54	2.020	22.20	22.5	.897	9.87	21.1	.3560	3.92	20.1
60	2.505	27.60	20.2	1.105	12.20	19.0	.4400	4.84	18.1
68	3.230	35.50	17.8	1.415	15.60	16.8	.5630	6.20	16.0
76	4.050	44.50	15.9	1.770	19.50	15.0	.7150	7.86	14.2

Turbine Performances at One-foot Head

Camden Turbines, Hydraulic Turbine Corporation, Camden, N. J.

Table Submitted by Maker in 1918

Diameter	Type 4, K = 53.2, ϕ = .755, Maximum power 9% above tabulated Hp.			Type 8, K = 71.3, ϕ = .75, Maximum power 17% above tabulated Hp.			Type 11, K = 83.5, ϕ = .72, Maximum power 12% above tabulated Hp.		
	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
15	.311	3.05	91.4	.51	5.10	92.0	.82	8.20	87.4
18	.448	4.39	76.2	.73	7.35	76.7	1.180	11.80	72.8
21	.610	5.97	65.2	1.00	9.73	65.7	1.607	16.07	62.4
24	.796	7.80	57.1	1.31	13.00	57.5	2.099	20.99	54.6
30	1.25	12.20	45.7	2.05	20.40	46.0	3.280	32.80	43.7
36	1.80	17.56	38.0	2.45	29.29	38.3	4.723	47.23	36.4
42	2.44	23.91	32.6	4.01	39.81	32.8	6.428	64.28	31.2
48	3.19	31.63	28.5	5.24	52.03	28.7	8.396	83.96	27.3
54	4.03	39.52	25.3	6.64	65.63	25.5	10.627	106.27	24.2
60	4.98	48.80	22.8	8.20	81.60	23.0	13.120	131.20	21.8
66	6.03	58.04	20.7	9.92	98.50	20.9	15.875	158.75	19.8
72	7.17	70.27	19.0	11.80	117.17	19.1	18.892	188.92	18.2
75	7.78	76.25	18.2	12.81	127.20	18.4	20.400	204.00	17.4

Turbine Performances at One-foot Head

S. Morgan Smith Company, York, Pa.

From Data Submitted by Maker in 1918

The following table shows the performance of the various types of S. Morgan Smith Company turbines of 30-in.-diameter at 1-ft. head, and the maximum heads under which they are designed to operate. Except for special conditions, sizes are made at intervals of 3 in., as 27 in., 30 in., 33 in., etc. Also for special conditions turbines are designed with characteristics differing from those tabled, but the characteristics must be within certain limits of power and speed, as indicated in the table, according to the head under which the turbines are to be placed.

Type	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Maximum head, ft.	K	Type	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Maximum head, ft.	K
E	0.375	3.89	41	750′	25.1	N	2.25	23.35	47.5	65	71.3
F	0.600	6.22	43.6	300	33.8	O	2.45	25.40	49.0	55	76.6
G	1.08	11.2	44.3	150	46.1	R	3.08	32.00	49.0	40	86.0
H	1.2	12.45	47.5	120	52.0	S	3.65	37.90	50.0	30	95.5
K	1.6	16.60	47.5	90	60.0						

Turbine Performances at One-foot Head

J. and W. Jolly, Holyoke, Mass.

From Catalog issued about 1910

Type	Diameter, In.	Hp_1	Q_1 c.f.s.	R.p.m.$_1$	Type	Diameter, In.	Hp_1	Q_1 c.f.s.	R.p.m.$_1$
McCormack	12	0.240	2.65	99.6	McCormack	42	3.234	35.67	30.0
Holyoke	18	0.539	6.17	64.3	Holyoke	48	3.885	42.85	24.5
$K = 54$	24	1.045	11.53	50.6	$K = 54$	54	4.959	54.69	22.8
	30	1.595	17.59	41.7		60	6.122	67.52	20.6
	36	2.241	24.71	35.5		66	7.407	81.70	18.7

Performances of Samson Wheels. From 1908 Catalogs

Size, In.		Heads in feet												
		4	6	8	10	12	14	16	20	25	30	35	40	50
17	Power	4.9	9.1	13.9	19.5	25.5	32.3	39.5	55.0	77.0	101	128	156	218
	Water	13.4	16.4	19.0	21.3	23.2	25.1	26.8	30.0	33.6	36.8	39.7	42.4	47.4
	Speed	186	228	264	294	322	348	372	416	464	510	550	588	657
20	Power	6.4	11.9	18.3	25.5	33.6	42.3	51.5	72.2	101	133	167	204	285
	Water	17.6	21.6	24.9	27.8	30.5	32.9	35.2	39.3	44.0	48.2	52.1	55.6	62.2
	Speed	162	199	230	257	282	304	325	364	407	445	481	514	575
23	Power	8.5	15.7	24.2	33.8	44.4	55.9	68.3	95.5	133	175	221	270	377
	Water	23.2	28.5	32.9	36.8	40.3	43.5	46.5	52.0	58.1	63.7	68.8	73.6	82.3
	Speed	141	173	200	224	245	265	283	316	354	387	418	447	500
26	Power	10.9	20.1	30.9	43.2	56.7	71.5	87.3	121	171	224	283	345	482
	Water	29.7	36.4	42.1	47.0	51.5	55.6	59.5	65.3	74.3	81.4	88.0	94.0	105
	Speed	125	153	177	198	217	234	250	280	313	343	370	396	442
30	Power	14.5	26.7	41.1	57.5	75.5	95.2	116	162	227	299	376	460	642
	Water	39.6	48.5	56.0	62.6	68.6	74.1	79.2	88.5	99.0	108	117	125	140
	Speed	108	132	153	171	188	203	217	242	271	297	321	343	381
35	Power	19.7	36.2	55.7	77.9	102	129	158	220	308	405	510	623	....
	Water	53.7	65.1	75.9	84.9	93.0	100	107	120	134	147	159	170	....
	Speed	93	114	132	147	161	174	186	208	232	255	275	294	....
40	Power	25.8	47.5	73.1	102	134	169	207	289	404	531	668	817	....
	Water	70.4	86.2	99.5	111	122	131	141	157	176	193	208	223	....
	Speed	81	100	115	129	141	152	163	182	203	223	240	257	....
45	Power	32.7	60.1	92.5	129	170	214	262	366	511	672	847	1034	....
	Water	89.0	109.1	126	141	154	167	178	199	223	244	263	282	....
	Speed	72	88	102	114	125	135	145	162	181	198	214	229	....
50	Power	40.5	74.2	114	160	210	264	324	451	631	829	1045	....	....
	Water	110	134.7	156	174	190	206	220	245	275	301	325	....	....
	Speed	65	80	92	103	113	122	130	145	162	178	192	....	....
56	Power	50.6	93	143	200	263	332	405	566	791	1040	1314	....	....
	Water	138	169	195	218	239	258	276	308	345	378	408	....	....
	Speed	58	71	82	92	101	109	116	130	145	159	172	....	....
62	Power	62.1	114	176	245	323	407	497	694	970	1275	1634	....	....
	Water	169	207	239	267	293	316	338	378	423	463	502	....	....
	Speed	52	64	74	83	91	98	105	117	131	144	155	....	....
68	Power	74.7	137	211	295	388	489	597	835	1167	1494	....	....	....
	Water	203	249	288	322	352	381	407	455	509	557	....	....	....
	Speed	48	59	68	76	83	89	96	107	120	130	....	....	....
74	Power	88.5	162	250	350	460	579	708	992	1382	1805	....	....	....
	Water	242.1	795	341	381	417	451	482	539	602	659	....	....	....
	Speed	44	54	62	70	76	82	88	99	110	120	....	....	....

For each size the first line gives the horsepower, the second the discharge in cubic feet per second, and the third the speed in revolutions per minute.

Turbine Windage and Friction. The approximate power absorbed by windage and friction in modern turbines in vertical settings may be computed by the following formula:

$$\text{Windage and friction in horsepowers} = 0.000154\ B\ D^4\ n^3;$$

where B = height of gate or guide vane opening, in feet;

D = diameter of wheel at middle of guide vane opening, in feet;

n = revolutions per minute.

33. Tangential or Impulse Turbines

The Girard Turbine may be either radial inward or outward flow, or axial flow. The buckets are so shaped that on leaving the wheel the water has a relative velocity in the direction opposite to that of motion, being received at inlet from a series of nozzles pointed as nearly as may be in the direction of motion. Air is admitted to the wheel to prevent spraying of the jets, and the turbine cannot run submerged. Regulation of speed and power is accomplished by closing part of the inlet nozzles. This motor is very satisfactory for high heads, but its speed cannot be varied appreciably without loss of efficiency unless the head also changes. The efficiencies range 70 to 80%.

The Pelton Turbine is usually mounted on a horizontal shaft and consists of a series of cups which successively receive a jet from one or more nozzles. The jet on striking the cup is divided and turned through nearly 180° and is then discharged with a low absolute velocity. Regulation is accomplished by closing or deflecting the nozzle. The maximum efficiency in this type of turbine occurs when the velocity of the cup V is about one-half that of the jet and when θ = 180 deg., in accordance with the law of curved vanes, and varies from 70 to 85%.

Wheels of the Pelton type are frequently equipped with the Doble nozzle which is closed by a pin or needle moving longitudinally in the axis of the jet from the inside and which may be adjusted to give a symmetrical jet of any effective area from that due to the full nozzle opening to zero. This apparatus affords opportunity for excellent control of the discharge, and hence greatly facilitates regulation without causing a waste of either water or energy.

Waterwheels. This term is applied to the overshot, the breast and the undershot wheels, all of which have become practically obsolete in America. The efficiencies of overshot and breast wheels ranged from 60 to 88%. In these wheels the work was done principally by the weight of the water. The efficiencies of straight-bladed undershots were from 25 to 35%, and for the curved-bladed or Poncelet wheels from 50 to 65%. In these wheels the work was done by the impulse of the stream.

Spiral Wheels are similar to axial-flow turbines except that the runner blades are helical surfaces and they are usually mounted on a horizontal or slightly inclined shaft. For low heads and large quantities of water this form is very satisfactory, efficiencies as high as from 80 to 85% being reported.

PUMPS AND PUMPING

34. Centrifugal Pumps and Screw Pumps

The Work in foot-pounds done by a pump is the product of the weight in pounds of the liquid pumped and the height in feet through or against which it is lifted. The **power** of a pump is the work done in unit time, and the **horse-power** is the work per second in foot-pounds divided by 550, or the work per minute divided by 33 000.

The Centrifugal Pump is essentially an inward-flow turbine reversed, the power being applied to rotate the runner and the water being admitted at the center or eye and discharged at the outer periphery. The earlier centrifugal pumps consisted of a spider made up of three or more arms, either straight or curved, which rotated in a concentric chamber to which water was admitted at the center and discharged at some point in the circumference. The efficiency of this apparatus rarely exceeded 30%, and it was only used for low lifts. By shifting the enclosing chamber so that the area outside the runner increases toward the outlet in proportion to quantity of water that must pass the several sections, by encasing the blades in a revolving shell, Fig. 29, thus reducing the friction of the revolving water on the fixed shell, and by more rational proportioning of the blades and casing, the pump has been greatly improved until its efficiency may slightly exceed 80%, and by passing the water successively through a series of runners any desired lift may be accomplished. As the apparatus contains no valves or parts, it is particularly adapted to the handling of water containing sand or gravel, and of viscous liquids and sewage; and as its discharge is continuous it has an advantage over reciprocating pumps in freedom from water hammer in the suction and discharge pipes.

Fig. 29

In operation it is necessary that the pump, when set above the level of its supply, be primed before it will start, and in such cases a foot valve is advantageous. It does not begin to deliver water until a certain speed of revolution is attained depending upon the height of the lift. When delivery has commenced the speed may be lowered somewhat below this point before it will cease on account of the inertia of the flow when once established, but the conditions of flow are unstable in this region.

As the water passes through the pump, the impeller impresses upon it a head h_c due to centrifugal force; this is equal to the static head that the pump would maintain when running at its normal speed, but at which it would deliver no water were it not that the rotation of the water in the eye of the wheel and in the outer casing has the effect of increasing h_c from 1 to 12%. This increase is greater with vanes having radial

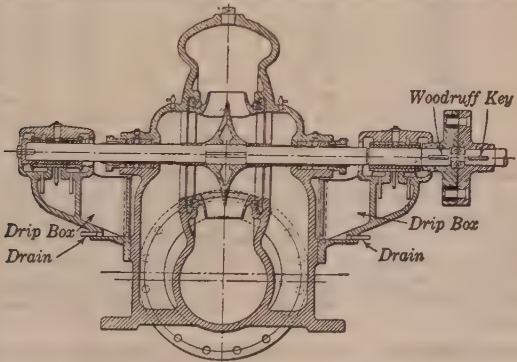


Fig. 30

tips than those which are turned back, and is raised when the wheel is surrounded by a large vortex chamber.

If the head is dropped slightly below the true value of h_c, the discharge will commence, and will continue so long as it remains below this amount. The discharge cannot, however, be reduced below that corresponding to the head h_c after flow has started without danger of its stopping altogether, so the statement is generally made that a centrifugal pump will begin to deliver against a certain head at a certain speed and cannot deliver less than a certain quantity.

Lift for Single-stage Centrifugals. A single-stage centrifugal pump has been made to lift against a head of 936 ft. and such pumps with a lift as high as 350 ft. are in service, but it is usually considered that 100 ft. is the practical limit without a second stage. For higher lifts a series of pumps is used on the same shaft, delivering water from one pump to the next, by which any desired head may be overcome.

The following table gives the approximate average performance of a centrifugal pump so designed that the discharge ceases when the head becomes 50% greater than that for which the pump is speeded.

Average Performance of a Centrifugal Pump

At constant speed					At constant head
Per cent of designed head	Per cent of designed discharge	Per cent of designed brake horsepower or input	Per cent of designed water horsepower or output	Per cent of best efficiency	Per cent of best speed
150	....	....	0	0	81.6
140	31.5	91	44	48.2	84.5
130	55	94.5	71.5	75.6	87.6
120	74	97	89	91.6	91.3
110	88	99	97	98	95.4
100	100	100	100	100	100
90	109	101	98.5	97.4	105.2
80	116	101.5	92.6	91.4	112
70	123	102	86.1	85	119.5
60	129	102.5	77.5	75.6	129
50	134.5	103	67.5	65.5	141
40	140	103.3	56	54.1	158
30	144	103.4	43.2	41.6	182
20	148	103.5	29.6	28.6	222
10	151	103.5	15.1	14.1	316
0	....	....	0	0	....

The Suction Lift of a pump is dependent upon the pressure at which the entrained air in the water separates. This separation occurs at lower pressures in rapidly moving than in slowly moving water. The lift also depends upon the velocity in the suction pipe, being less for high and greater for low velocities. The head at which the air separates is usually between 24 and 28 ft. below atmospheric pressure, and 26 ft. may be taken as an average safe value. The velocity effect may be taken as $h = v^2/g$ including inlet losses from the suction basin.

Power and Efficiency. Let Q = volume of water lifted per second, w = weight of a cubic unit of water, g = acceleration of gravity, v = velocity of outer circumference of pump wheel, β_0 = angle which vane makes with outer circumference, u_0 = radial component of velocity at outer circumference, H = total head = height water is raised plus all frictional heads. Then

$$\text{Power required to drive pump} = \frac{Qw}{g} v_0 (v_0 - u_0 \cot \beta_0)$$

$$\text{Hydraulic efficiency of pump} = gH/v_0 (v_0 - u_0 \cot \beta_0)$$

The hydraulic efficiency is the quotient of the gross work done by the pump, divided by the work done on the pump wheel. The speed v_0 required to pump against a given head must increase as β_0 decreases, and it becomes a minimum when $\beta_0 = 90°$. In this case the vanes are radial at the outer circumference, and for perfect efficiency $v_0^2 = 2\,gH$. As the efficiency decreases v_0 must increase so that $v_0^2 = k \cdot 2\,gH$, where k is a coefficient whose value ranges from 1.2 to 1.8.

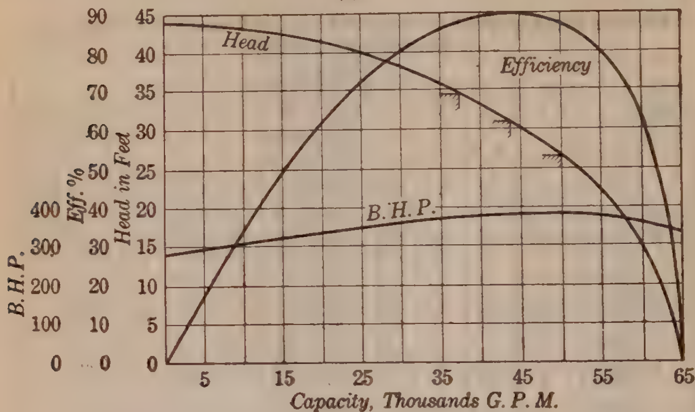
Performances at Best Speeds of Allis-Chalmers Co.'s Standard Centrifugal Pumps

Size, In.		Lifts in feet										
		5	10	15	20	25	30	40	50	60	80	100
	Power	0.25	0.75	1.00	1.5	2.25	3	4.5	6	8	12	17
3	Water	76	107	131	152	170	185	214	240	262	303	339
	Speed	447	632	774	893	998	1095	1265	1415	1550	1790	2000
	Power	.60	1.75	2.75	4.5	6	8	12	17	24	33	46
5	Water	205	290	356	411	460	504	582	650	712	823	920
	Speed	268	380	465	536	600	658	760	850	930	1075	1200
	Power	1.00	3	5	8	11	14	21.5	30	40	61	....
6	Water	415	586	718	830	926	1015	1170	1310	1435	1660	....
	Speed	190	270	330	381	426	467	539	602	660	762	....
	Power	1.75	4.5	8	12	17	22	34	47	62	96	....
8	Water	650	918	1125	1300	1450	1590	1835	2050	2250	2600	....
	Speed	161	227	278	322	359	394	455	508	556	643	....
	Power	2	5.5	9.5	15	21	27	42	59	77	.....	....
10	Water	850	1200	1400	1700	1900	2080	2400	2680	2940	.....	....
	Speed	143	202	247	285	318	350	403	450	494	.....	....
	Power	3	7.5	13.5	21	28.5	38	58	80	106	.....	....
12	Water	1165	1645	2030	2330	2600	2850	3300	3680	4030	.....	....
	Speed	128	178	222	256	286	314	363	405	444	.....	....
	Power	4	9.5	17.5	27	37.5	50	76	106	140	.....	....
14	Water	1595	2250	2760	3190	3560	3910	4510	5040	5520	.....	....
	Speed	122	173	212	245	274	300	346	387	424	.....	....
	Power	4.5	12	22	34	47	62	95	132	.....	.....	....
16	Water	2085	2950	3610	4170	4660	5110	5900	6600	.....	.....	....
	Speed	117	165	202	233	261	286	330	368	.....	.....	....
	Power	6	16	29	44	62	81	125	.....	.....	.....	....
18	Water	2830	4070	4900	5660	6330	6925	8000	.....	.....	.....	....
	Speed	112	158	193	223	250	274	316	.....	.....	.....	....
	Power	7.5	20.5	37	57	80	105	162	.....	.....	.....	....
18	Water	3675	5200	6360	7350	8220	8980	10380	.....	.....	.....	....
Special	Speed	112	158	193	223	250	274	316	.....	.....	.....	....

Power = brake horsepower to drive pump shaft. Water = gallons per minute delivered. Speed = revolutions per minute.

Screw Pumps. The screw pump is closely related to the centrifugal but has fewer vanes and hence has larger passages for a given capacity and is in consequence particularly fitted for drainage and sewage pumping where the lifts are low and the liquid pumped liable to contain a considerable amount of floating matter. The range of head is limited to not over 45 ft. Screw pumps now on the market are of two distinct types, one known as the Wood pump, consisting of a series of scoop-shaped buckets arranged around the periphery of a wheel which pick up the water as the wheel revolves very much as the tank of a locomotive is filled when in motion. These pumps are frequently provided with a trash cutter near the inlet of the buckets to prevent large pieces of floating material from getting in and clogging them. The second type has its propeller built on the lines of a screw and thus may be made with much larger passageways and is more closely related to the centrifugal.

(a)



(b)

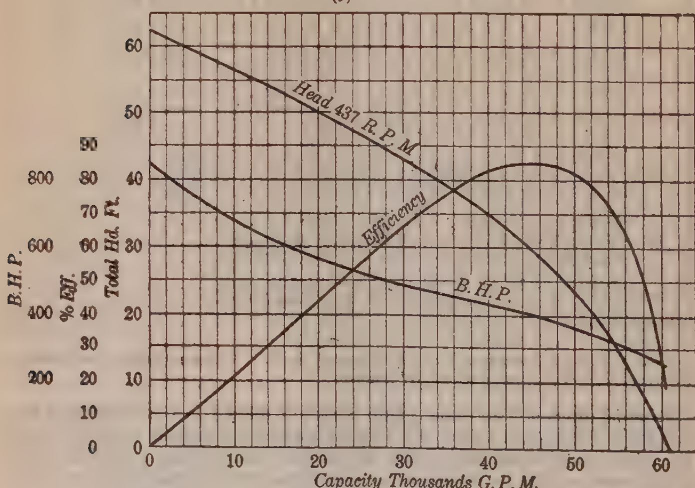


Fig. 31. Curves of Centrifugal Pump and Screw Pump
(a) 42-in. Centrifugal (Allis-Chalmers)
(b) 42-in. Screw Pump (Worthington)

Pumps of both types are built of very large capacities. The efficiencies now obtainable with screw pumps are somewhat lower than those of centrifugals of the same capacity.

The rules of **speed variation effects**, within working limits for centrifugal and screw pumps are:

Head varies as the square of the speed.
Capacity varies as the first power of the speed.
Brake Horsepower varies as the cube of the speed.

The **specific speed**, K_p, of a centrifugal or screw pump is

$$K_p = nQ^{1/2}/H^{3/4} = n/\sqrt{H} \cdot \sqrt{Q/\sqrt{H}}$$

where n = revolutions per minute;
Q = discharge capacity in c.f.s.;
H = head of water pumped against.

Although this expression is somewhat similar to that for the specific speed of turbines, the two equations are not interchangeable.

The **size** of centrifugal and screw pumps is designated by the diameter of the discharge opening.

Figs. 31*a* and 31*b* represent respectively the characteristic performance curves of a centrifugal and a screw pump of the second type designed to meet similar requirements.

Considering the working range of each pump to be from 40 000 to 50 000 gal. per min., the range of **head** is seen to be 11 ft. in both cases, with the **efficiency** of the centrifugal ranging from 89 to 88%, and that of the screw from 83 to 82%. For the centrifugal the **power** required (B. hp.), is nearly constant throughout a wide range and decreases slowly in both directions from a maximum at a discharge of 50 000 gal. at a head of 19.4 ft., while for the screw it decreases rapidly with increase of discharge from the starting point. The **speed** of the centrifugal is given as 225 r.p.m., whereas that of the screw is 437 r.p.m., or nearly twice as great. Both pumps, being motor-driven, are designed for constant speed. The **capacity** of the motor required to start the screw pump with its casing full of water is nearly twice that of the centrifugal.

The **specific speed** of screw pumps is much higher than that of centrifugals.

35. Reciprocating Piston Pumps

Types. Reciprocating piston pumps may be classed as to operation as either single- or double-acting. The former takes water on one stroke and discharges it on the other; the double-acting pump takes and discharges during both strokes. As to construction they are divided into inside-packed and outside-packed pumps, an example of the latter being shown in Fig. 32.

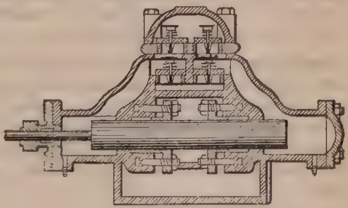


Fig. 32

A continuous discharge from a single-acting pump may be obtained by either of the devices shown in Fig. 33, in which when suction takes place on the upstroke the chamber below the piston is filled with water and at the same time the volume of water in the discharge chamber above the piston is delivered. The volume of the discharge chamber being less than that of the suction chamber, as the piston descends part of the water in the suction chamber is delivered through the discharge chamber and part of it flows into the space above the piston to be delivered on the suction stroke.

Displacement Curves. If the piston velocities as ordinates are plotted to time, or to space passed through by a crank moving at uniform velocity, as abscissas, the area enclosed between the curve, the axis of the abscissas, and a vertical ordinate will represent the displacement of the piston at any time. For uniform effort and uniform flow this displacement should be uniform, but in a reciprocating pump this can only be approximated.

Long Suctions. If the suction of a reciprocating pump is long, even though the lift is low, on account of the variable velocity, a considerable portion of the suction head is absorbed in accelerating the water at the beginning of the stroke. A part of this force is recovered at the end of the stroke as pressure forcing the piston ahead, but unless the head h_f is available at the beginning of the stroke the pump will not lift. If the speed of the piston is such that it moves more rapidly than the inertia of the water can be overcome, the water

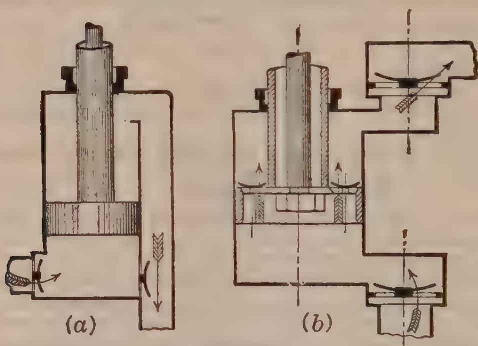


Fig. 33

separates from the piston and may ultimately overtake it, as the latter is retarded in the second half of its stroke, causing severe shock or hammer.

The head at inlet of the suction for a maximum is equal to the depth of the suction opening below the water surface in the suction well plus the height of the water barometer, and the head at outlet is that corresponding to a perfect vacuum less the effective lift of the pump. When, as is usually the case, the pump is located above the level of water in the suction well the head available for producing flow is reduced.

Separation of Air. The suction lift of a pump is dependent upon the pressure at which the entrained air in the water separates. The head at which the air separates is usually taken at about 26 ft. below atmospheric pressure. Separation of air also may occur on the discharge side of the piston during the second half of the stroke when the piston is being retarded due to the velocity previously imparted to the water being so great as to carry it away from the piston.

Air Chambers. To prevent separation an air chamber may be installed on either the suction or the discharge pipe and should be located as near as possible to the pump. The value of the air chamber increases with its size, cross-section being more effective than height. It should be so located that the water is drawn as low as possible without actually emptying the chamber at each stroke.

Flywheel pumps, kind	Ratio of change of air volume in air chamber to volume of	
	1 stroke of piston	1 revolution of pump
Single-acting	0.557	0.557
Double-acting	0.211	0.106
Two double-acting 90 deg. apart	0.048	0.012
Three-throw single-acting 120 deg. apart	0.0032	0.0011
Three-throw double-acting 120 deg. apart	0.0032	0.0011

The duplex pump without a flywheel, if the piston were accelerated uniformly, would give a uniform delivery; in practice, however, the variation is about the same as that of a three-throw flywheel pump.

36. Pumping Engines

Types. Pumping engines are a combination of a pump with a steam engine in a single machine, and the term is usually restricted to machines of at least one million gallons daily capacity. There are four general types:

(a) The Common Duplex Pumping Engine, consisting of two parallel double-acting pumps directly connected to two steam engines, and taking strokes alternately or 90 deg. apart, each engine operating the valves of the other. In this type the length of stroke is variable, and the steam must follow throughout the full stroke, and can be used expansively only by adding a low-pressure cylinder into which it expands continuously during the discharge stroke of the high-pressure cylinder.

(b) The Duplex Pumping Engine with the Worthington High-Duty Attachment, which is a duplex pump having attached to each piston rod a pair of auxiliary cylinders filled with oil and mounted on trunnions through which they are connected to the delivery main. These cylinders rotate in such a manner that at the beginning of the stroke they oppose the movement of the piston and thus prevent racing while the steam follows at full pressure. At midstroke just after cutoff they neutralize each other and toward the end of the stroke assist the movement of the piston when the steam is working expansively. This device enables the steam to be expanded in one, two, or three cyclinders, and thereby increases the economy of operation.

(c) The D'Auria Pumping Engine, in which the use of steam expansively is accomplished by circulating through the frame of the pump a body of water which is acted upon by an auxiliary piston and accelerated during the first half of the stroke, and then acts upon this piston during its retardation in the second half of the stroke.

(d) The Crank and Flywheel Pumping Engine, in which the expansive use of steam is provided for by the inertia of the flywheel.

Performance. The performance of pumping engines is usually stated as **Duty,** which was originally defined as the number of foot-pounds of work delivered for each 100 lb. of coal burned under the boilers. Owing to the variation in evaporative power of coal, and also of boiler efficiency, the coal basis for computing duty is more or less unsatisfactory for accurate work, and a new basis was adopted using an assumed evaporation of 10 lb. of water per lb. of coal, so that duty is also defined as the number of foot-pounds of work delivered for each 1000 lb. of dry steam, this being determined by the amount of water evaporated from and at 212° F. with corrections for moisture in the steam delivered to the engine. On account of the varying dryness of the steam and the condensation between boiler and engine it has been suggested that duty be based upon each 1 000 000 B.t.u. in the steam delivered to the engine and this value is now usually reported, sometimes alone and sometimes with the others. This would correspond to the preceding if the steam were supplied to the engine at an absolute pressure of about 5 lb., corresponding to a temperature of about 162° F. It follows that the duty on the last-named basis will in practice be less than that on the steam basis and usually less than that on the coal basis.

The duty of pumping engines and steam pumps ranges from about 3 to 8 million foot-pounds in the single-cylinder boiler-feed pumps to 160 million foot-pounds in the best high-duty, direct-acting and flywheel pumping engines, and in turbo-driven centrifugals, all on the 1000 lb. of dry steam basis. The coal consumed per horsepower per hour on a basis of 10 lb. of water evaporated per pound of coal may be obtained by dividing 198 by the duty in million foot-pounds.

Duty and Coal Consumption of Steam Pumping Machinery

Kind of pumping machinery	Duty in 1 000 000 ft-lb. per 1000 lb. of dry steam	Pounds of coal per horsepower per hour
Single-cylinder boiler-feed pumps and air pumps	3 to 5	66 to 40
Steam fire engines:		
Silsby rotary	6.5 to 8	30 to 25
Amoskeag	6 to 9	33 to 22
Clapp and Jones	7.5 to 14	27 to 14
High-pressure duplex pumps	15 to 20	13 to 10
Duplex compound non-condensing pumps	25 to 40	8 to 5
Duplex compound condensing engines	30 to 45	6.6 to 4.4
Duplex triple condensing engines	40 to 60	5 to 3.3
Cornish flywheel engines	60 to 70	3.3 to 2.8
Walking-beam compound condensing engines	60 to 85	3.3 to 2.3
Steam turbines and centrifugal pumps	80 to 90	2.5 to 2.2
Worthington high-duty duplex engines	100 to 160	2.0 to 1.25
Compound condensing flywheel engines	100 to 130	2.0 to 1.5
Triple-expansion flywheel engines	110 to 160	1.8 to 1.25
Steam-turbine-driven centrifugals (large)	100 to 160	2.0 to 1.25

Direct-acting Water Motor. When the steam end of a duplex pump is replaced by a similar end with large valve openings, which may be connected to a supply of water under pressure, the device is called a direct-acting water motor or pump. On account of the high frictional losses the device is not so efficient as a turbine connected to a centrifugal or a reciprocating pump. As the apparatus requires little attention, it may be advantageously used as a continuous relay at the end of a long line of pressure pipe to increase the pressure in a following line where a smaller amount of water is needed.

37. Jet Pump and Hydraulic Ram

The Jet Pump. In this apparatus (Fig. 34) a small jet of water at high velocity is discharged through a throat the upstream end of which connects to a suction pipe and the downstream end to the discharge. The velocity of the jet reduces its pressure below that of the atmosphere and thus creates a partial vacuum at the throat into which water from the pump well rises and is then mingled with the jet and carried along at reducing velocity by it. The efficiency of this apparatus is greatest

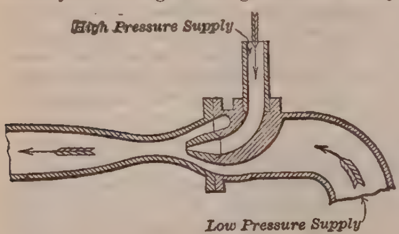


Fig. 34

with a high suction lift and low pressure on the discharge, when it may reach 25 or 30%. When a steam jet replaces the water jet, the ordinary steam injector is obtained in which the vacuum is formed not only by the velocity of the jet but by the condensation of the steam.

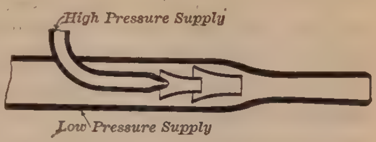


Fig. 35

The water-jet pump may be advantageously used to produce a large discharge through a nozzle at a low pressure for a fountain, or for fire fighting, by means of the apparatus shown in Fig. 35, which is sometimes called an injector nozzle. (Proc. Inst. Mech. Eng. of Great Britain 1879.)

The following table gives the quantity of water at a head of 700 lb. per sq. in. required to deliver through a one-inch nozzle a jet of 150 g.p.m., the head at the nozzle being 100 ft.

Low-pressure supply, Pressure, lb. per sq. in.	60	50	40	30	20	10
High-pressure supply at 700 lb. per sq. in. G.p.m.	3.7	10.9	18.1	25.2	32.4	39.6

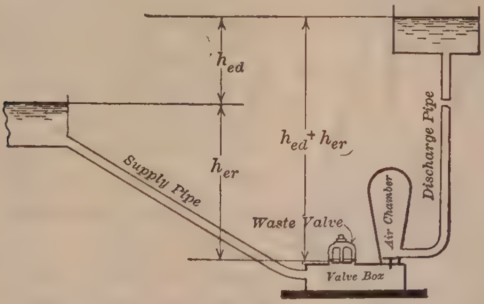


Fig. 36

The Hydraulic Ram (Fig. 36) consists of a drive pipe delivering into a chamber whence the water escapes to waste until the waste valve is closed by the discharging water, and then into the air chamber, in which it compresses the air until the flow stops, when the air expands, closing the valve at the chamber inlet and forcing water up the delivery pipe. When the flow is stopped there is a rebound of pressure toward the inlet. This creates a reduction of pressure and causes the waste valve to drop open and flow is

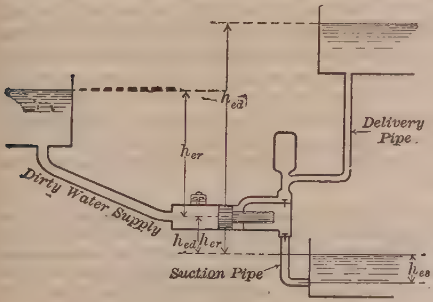


Fig. 37

started again. The apparatus thus intermittently utilizes the momentum in the drive pipe, and the longer this pipe the more work the machine will do.

If h_{er} is the height of the supply reservoir above the ram, the efficiency is $e = Q_d(h_{ed}+h_{er})/Q_s h_{er}$ when a portion of the water used to drive is pumped, the inlet of the drive pipe being taken as the suction inlet. When a separate supply is pumped, as may be done with the machine shown in Fig. 37, $e = Q_d h_{ed}/Q_s h_{er}$. The efficiency decreases as the ratio of h_{ed}/h_{er} increases, and becomes very small for high lifts. The value of e in the above expressions may be as high as 70%. Rankine gives $e = 1.12 - 0.2\sqrt{(h_{de} + h_{er})/h_{er}}$.

38. Pulsometer and Air Lift

The Pulsometer (Fig. 38) has a chamber with two parts, each connected to a suction pipe at the bottom and to a steam pipe at the top with a discharge at the side near the bottom. Steam enters and fills one chamber and then condenses when the steam inlet valve shifts, allowing steam to enter the other chamber. As the steam condenses, a vacuum is formed which lifts water into the first chamber. When the second chamber is filled with steam, the valve shifts again and steam enters the first chamber, forcing out the water through the discharge valves and then condensing again, making the process continuous. Tests of a pulsometer reported by DeVolson Wood, Trans. Am. Soc. Mech. Engs., 1892, vol. 13, p. 211, gave a duty ranging from 9 to 13 million foot-pounds. The apparatus is practically adapted to pumping out cofferdams, on account of the ease with which it can be set up, as it may be simply suspended from a ring at the top and connected with steam, suction and discharge hose lines.

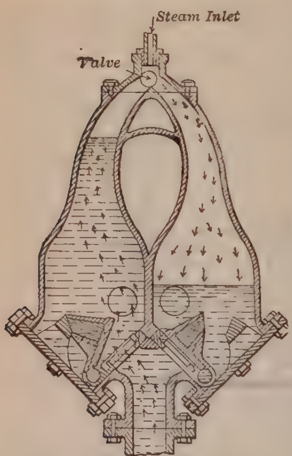


Fig. 38

The Air Lift. This device involves the discharge at the bottom of the well or at least a considerable distance below the water surface, of air into the mouth of the delivery tube. The air as small bubbles mixes with the water, and the specific gravity of the mixture is so reduced that the pressure of the water outside the delivery tube causes the mixture to overflow at the top. Evidently the greater the length of the pipe below the surface the greater the difference between the weight of the columns within and without the tube and the higher the water can be lifted. Generally the depth of submersion is made 1.5 to 2 times the lift.

The air lift is especially adapted to raising liquids from great depths, and is more efficient in handling hot liquids on account of the greater expansion of the air. It is particularly easy to install when the well has sufficient depth below the water surface. The air at entrance should be distributed over the area of the pipe rather than concentrated at the center, and small bubbles give better efficiencies than those which fill the entire delivery tube.

Experimentally the following relations are found to give the best results:

Head in feet	10	20	30	50	100
Qa/Qw	1.0	1.5	2.0	2.5	3.0

Of the indicated horsepower of the air chamber of the compressor the lift under favorable conditions may return as much as 45%, but ordinarily not more than 25 and frequently only 10 or 12%. The duty of air compressors ranges usually from 6 to 40 million foot-pounds, and the duty of the whole plant may therefore range from less than one million to perhaps 20 million foot-pounds as an upper limit. (See Proc. Inst. Civ. Engs. of Gt. Britain, 1900, vol. 140, p. 323, and 1906, vol. 163, p. 353.)

During the operation of the air lift the following equations apply:

a_p = area of eduction pipe, above air inlet, in square feet;
c_e = coefficient of entrance = 0.97. (See Art. 7 of this Section.)
c_p = coefficient due to pipe friction and average slip varying from 0.70 to 0.99;
d = diameter of eduction pipe in inches;
h_l = lift in feet;
h_s = submergence of air discharge in feet;
p_b = barometric pressure on water in well and at upper end of discharge pipe in feet of water;
p_t = absolute pressure at inlet in foot piece in feet of water;
q_b = discharge of air at pressure p_b in cubic feet per second;
q_w = discharge of liquid in cubic feet per second;
u_w = density of liquid pumped;
v_l = velocity of liquid in eduction pipe below air inlet;
W = work output in foot-gallons per second.

$$h_s - \frac{p_t - p_b}{u_w} = \frac{v_l^2}{2g}(1 + c_e) \tag{1}$$

$$\frac{q_b}{q_w}\frac{p_b}{u_w}\log_e\frac{p_t}{p_b} = h_l + \left[\frac{\left(1 + c_p\frac{W}{d}\right)(q_b + q_w)^2 + c_e q_w^2}{2g\,a_p^2}\right] \tag{2}$$

$$\frac{p_b}{q_w\,u_w}\log_e\frac{p_t}{p_b} = \frac{1 + c_p\frac{W}{d}}{2g\,a_p^2}(q_b + q_w) \tag{3}$$

$$\left(1 + c_p\frac{W}{d}\right)(q_b^2 - q_w^2) = 2gh_l\,a_p^2 + c_e q_w^2 \tag{4}$$

Example. To determine the air required to pump water at a rate of 2 cu. ft. per sec. from a well 10 in. in diameter and 330 ft. deep, in which the air pipe inlet is submerged 64.33 ft. and the lift is 25.67 ft., the air pipe being 2-1/2 in. in diameter and passing through the 10-in. pipe.

a_p = area of 10-in. pipe − area of outside of 2-1/2-in. pipe = 0.5 sq. ft.;
c_e = 0.97;
c_p = 0.80;
d = 10 in. − 3 in. = 7 in. = net diameter of 10-in. pipe enclosing 2-1/2-in. pipe;
h_l = 25.67 ft.;
h_s = 64.33 ft.;
p_b = 14.7 lb. = 33.96 ft.;
p_t = 64.33 + 33.96 = 98.29 ft. = pressure of water over air outlet + atmospheric pressure;
q_w = 2.0 cu. ft. per sec.;
q_b = air required in c.f.s. atmospheric pressure.

u_w = density of mixture of air and water $= \dfrac{62.5\,q_w + 0.08\,q_b}{62.5\,(q_w + q_b)}$;

$v_t = \dfrac{2}{0.545} = 3.67$ = discharge in cubic feet ÷ area of 10-in. discharge pipe;

$W = 2 \times 7.48 \times 25.67 = 384.8$ ft. gal. = product of gallons pumped and lift.

Applying equation (4):

$$\left(1 + c_p \frac{W}{d}\right)(q_b^2 - q_w^2) = 2\,gh_l\,a_p^2 + C_e q_w^2$$

and substituting therein:

$$\left(1 + 0.80\,\frac{384.8}{7}\right)(q_b^2 - 2^2) = 64.4 \times 25.67 \times 0.5^2 + 0.97 \times 2^2$$

or
$$44.97\,q_b^2 = 179.88 + 413.2 + 3.88$$

and
$$q_b^2 = \frac{596.6}{44.97} = 13.25 \quad \therefore \quad q_b = \sqrt{13.25} = 3.64 \text{ c.f.s.}$$

If the submergence were not given its proper amount could be computed from equation (3) by determining the value of p_s, the only unknown therein, which can then be introduced in equation (1) to obtain h_s, as follows:

Equation (3)
$$\frac{p_b}{q_w\,u_w}\log_e\frac{p_t}{p_b} = \frac{\left(1 + c_p \dfrac{W}{d}\right)(q_b + q_w)}{2\,g\,a_p^2}$$

Substituting values:

$$\frac{33.96}{2 \times \left(\dfrac{62.4 \times 2 + 0.08 \times 3.64}{62.4 \times 5.64}\right)}\log_e\frac{p_t}{33.96} = \frac{44.97\,(3.64 + 2)}{64.4 \times 0.5^2}$$

or
$$\frac{33.96}{0.706}\log_e\frac{p_t}{33.96} = \frac{254.5}{16.1}$$

$$\therefore \log_e\frac{p_t}{33.96} = \frac{0.706}{33.96} \times \frac{254.5}{16.1} = \frac{180.5}{546.76} = 0.33$$

0.33 being the hyperbolic or natural logarithm of 1.38

$$\frac{p_t}{33.96} = 1.38, \quad \text{and} \quad p_t = 1.38 \times 33.96 = 46.86 \text{ ft.}$$

By equation (1)
$$h_s - \frac{p_t - p_b}{u_w} = \frac{v_t^2}{2\,g}(1 + c_e),$$

and substituting:
$$h_s - \frac{46.86 - 33.96}{0.353} = \frac{(3.67)^2}{64.4}(1 + 0.97)$$

or
$$h_s = \frac{12.90}{0.353} + \frac{13.47 \times 1.97}{64.4} = 36.54 + 0.49 = 37.03 \text{ ft.}$$

This submergence being that given the highest efficiency is about 60% of the total length of eduction, or sum of submergence and lift, and indicates the submergence used in the apparatus as specified was too great for best results.

Davis and Weidner (see Bull. 450 Univ. of Wisconsin) conclude

1. The central air tube pump has the greatest theoretical capacity for a given size of well.

2. The coefficient of pipe friction and slip decreases as the discharge increases, and decreases as the ratio of volume of air to volume of water increases.

3. The coefficient of pipe friction and slip varies with the length of pump, but seems to be independent of the percentage of submergence and of the lift.

4. The length of pump, the percentage of submergence, and therefore the lift remaining constant, there is a definite quantity of air causing the maximum discharge. This quantity of air for maximum discharge, as also the ratio of volume of air to volume of water, differs for different percentages of submergence and lift, the length of the pump remaining constant.

5. The length of pump remaining constant, the maximum output (e.g., foot-gallons) occurs at about the same percentage of submergence for all rates of air consumption, being at from 61 to 65% for the pump used in the Wisconsin experiments. At other submergences the output varies as the ordinates of a parabola having a vertical axis. Under these conditions the lift does not remain constant as the percentage of submergence varies.

6. The length of pump and percentage of submergence remaining constant, and therefore constant lift, the efficiency increases as the input decreases, that is, the highest efficiencies are obtained at the lowest rates of pumping.

7. By varying the percentage of submergence, and therefore the lift, the length of pump remaining constant, the maximum efficiency is obtained at approximately 63% submergence for all rates of input or discharge.

8. The lift remaining constant, the efficiency increases as the percentage of submergence increases, for all rates of input and all practical percentages of submergence.

9. With the same size and type of pump, the percentage of submergence remaining constant, the efficiency increased as the lift increased for the small lifts experimented on, that is, up to about 24 ft. From a theoretical study, however, the indications are that a point will be reached from which the efficiency will decrease as the lift increases.

10. Other conditions remaining constant, there is no advantage to be gained by introducing compressed air above the surface of the water in the well.

11. The type of the foot-piece has very little effect on the efficiency of the pump, so long as the air is introduced in an efficient manner and the full cross-sectional area of the eduction pipe is realized for the passage of the liquid. Anything in the shape of a nozzle to increase the kinetic energy of the air is detrimental.

12. A diverging outlet which will conserve the kinetic energy of the velocity head increases the efficiency.

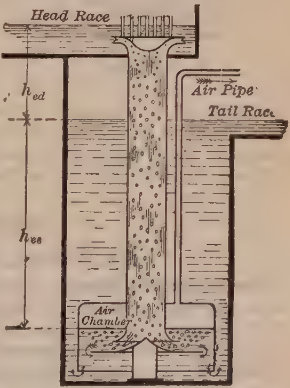


Fig. 39

The Hydraulic Air Compressor. This apparatus (Fig. 39) is practically the reverse of the air lift. Water is caused to pass vertically downward through a tube or shaft at a moderately high velocity, and by means of small pipes or an open surface at the top opportunity is given for the water to absorb as much air as possible. At the bottom the water is discharged against a considerable head and is made to pass underneath a collecting hood which is connected with a chamber for storing air. The reduction of velocity at outlet causes the water to release a portion of the entrained air, which rises through the collecting hood and passes to the air chamber under a pressure equal to h_{es} the head of water above the free surface underneath the hood.

The velocity at which bubbles of air separate themselves from running water is about 0.75 ft. per sec., and the flow down the tube or shaft must exceed this. Experimentally a velocity between 12 and 16 ft. per sec. gives the best results. (Eng. and Min. Jour., Jan. 19, 1907.)

39. Miscellaneous Pumps

The Archimedean Screw has been largely used in Holland against heads not exceeding 10 ft. It consists of an inclined shaft carrying one or more helices of considerable diameter, which are rotated with little clearance in a circular or semicircular channel connecting head and tail water. The angle of inclination of the channel should be less than the angle of the helix so that the water always tends to run down the helix and up the channel. The best angle for the helix is found to be between 30 and 40 deg., and an efficiency of 75% has been reported.

The Scoop Wheel is used for drainage pumping, and consists of a series of flat vanes revolving in a curved channel, and is practically a reversed undershot waterwheel. It may be used for lifts as high as 6 ft. and has been constructed to deliver 160 cu. ft. per sec. An efficiency of 70% is claimed under favorable circumstances.

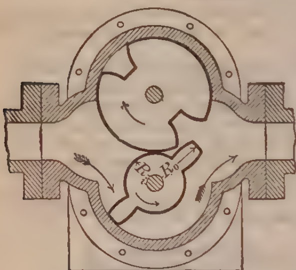


Fig. 40

The Positive Rotary Pump (Fig. 40) is intermediate between the centrifugal and the reciprocating piston type. It has long been familiar in the Silsby fire engine, where both the steam and water ends are of this class. The same construction is used in the ordinary rotary blowers for delivering free air. The chief objection to this pump is the difficulty of keeping the revolving pistons tight. Like the centrifugal it requires no valves, and like the reciprocating pump is positive in action. The discharge of such a pump per revolution is $\mathbf{Q} = \pi \mathbf{d}(\mathbf{R_i}^2 + \mathbf{R_0}^2)$ where d = axial length of the piston, or distance between crowns, R_i = radius of piston hub, and R_0 = radius of piston tip. The discharge as computed from this formula must be reduced for leakage past the pistons.

WATER POWER

40. Power, Slope, and Discharge

The Power of a Stream is the product of the fall H and the weight Qw of water flowing per second, Q being the discharge and w the weight of a cubic unit of water. When H is feet, Q cubic feet per second, and w pounds per cubic foot, then **work per second in foot-pounds = QwH** and **horsepower = QwH/550.** Assuming a plant to deliver 80% of the power of the stream the horsepower developed at the wheel shaft is simply $QH/11$. Assuming an efficiency of generating apparatus of 90%, since 1 kilowatt = 1.34 horsepower, the electrical output at the station switchboard is $QH/16$ in kilowatts. Since a stream of water flows continuously, its power should be stated on the basis of 24 hours per day and 365 days in a year, and a horsepower year is $24 \times 365 = 8760$ horsepower-hours, or 525 600 horsepower-minutes, or 31 536 000 horsepower-seconds.

The International Advisory Committee on Rating of Rivers (1928) recommended that:

1. The **Kilowatt** be adopted as the unit of power to be used in rating rivers.
2. The **full power of the river** and **gross head** be used as a basis for rating rivers.
3. The **flow duration curve** and also the **month of occurrence** of minimum flow are desirable as a basis for rating rivers, and where this information can not be given there should be supplied: (a) the minimum flow and its occurrence; (b) the flow which is available for six months of the year; and (c) the **arithmetic mean flow.**

In ordinary industrial operations it is usually estimated that a plant will operate during 310 days in a year and from 10 to 16 hours per day, depending on the character of the industry. The number of horsepower-hours in a year required for each horsepower capacity is therefore from 3100 to 4960. If the water which would flow during the remaining 14 to 8 hours and the remaining 55 days can be stored for use during the time the plant runs, the capacity of the plant per hour for the tunning time will be from 8760/3100 to 8760/4960 or from 2.83 to 1.76 hp. per horsepower of average stream capacity. In using the term horsepower in connection with water-power development the unit of time upon which it is based should be stated, and when no unit is designated 24-hour 365-day power should be understood. Evidently a stream will yield twice as much power on a 12-hour basis as it will on a 24-hour basis if the flow of the other 12 hours can be stored.

The Slope. The distance through which the water falls vertically per unit of horizontal distance traveled is sometimes called the slope per unit of distance, and is for uniform fall the tangent of the inclination of the water surface from the horizontal. Since the slope of natural streams is ordinarily small, the inclined length and the horizontal length practically coincide.

In ordinary natural streams, except in the vicinity of an obstruction, the low-water slope is greater than the flood slope. For streams with nearly vertical side-walls, the reverse will ordinarily be true. Upstream from and in the vicinity of a dam over which the water flows, the flood slope will be greater than the low-water slope, except in channels which enlarge their cross-section very rapidly by overflowing extensive flats as the water rises. Generally for the estimation of power the natural slope may be taken as practically the same at all times unless the location is upstream from an obstruction which may cause the water to back up from below against the base of the dam without causing a similar rise above its crest.

Backwater. The distance upstream to which the backwater effect of a dam or other obstruction extends in an ordinary stream will be greater at low water than in time of flood.

The Discharge or Runoff of all streams varies from day to day, month to month, and year to year, and bears some relation to the rainfall on the drainage area. This relation is by no means a simple one, and stream-flow measurements are essential to the reliable determination of the flow, unless a comparison can be made with the known discharge from a similar watershed similarly located; and even then great care must be taken in estimating to allow for the influence of a range of hills if one exist between the two basins considered, as greater precipitation and hence greater runoff may be expected on the windward side of a summit of land than on the leeward side.

The water that falls on the surface of the earth is partly evaporated by the direct rays of the sun and partly by the aid of vegetation, partly absorbed by the soil to reach the streams through their banks and beds, and partly flows away over the surface. The quantity of discharge of a stream is therefore dependent upon both the rainfall and the evaporation, and ordinarily amounts to the surface runoff plus the subsurface flow, and of this discharge the continuous as distinguished from the intermittent flood flows, when corrected for the effect of visible storage in ponds and lakes, represents the subsurface supply, or the percolation, which generally ranges from 25 to 60% of the total supply, being greater on flat, sandy, and porous drainages than on steep, impervious ones.

Occasionally it happens that percolation from an area whose surface is tributary to

Conversion Table

Head-discharge to Horsepower and Kilowatts

Head × discharge Q.H.	Theoretical Horsepower T.Hp.	Mechanical Horsepower at Wheelshaft. Eff. 80% M.Hp.	Electrical Kilowatts at Switchboard. Eff. of Gen. 92% Kw. at S.B.
100	11.3456	9.0765	6.2269
125	14.1820	11.3456	7.7836
150	17.0184	13.6147	9.3403
175	19.8548	15.8838	10.8970
200	22.6912	18.1530	12.4538
225	25.5276	20.4221	14.0105
250	28.3640	22.6912	15.5672
375	31.2004	24.9603	17.1239
300	34.0369	27.2295	18.6806
325	36.8733	29.4986	20.2374
350	39.7097	31.7678	21.7941
375	42.5461	34.0369	23.3508
400	45.3825	36.3060	24.9075
425	48.2189	38.5751	26.4642
450	51.0553	40.8442	28.0210
475	53.8917	43.1134	29.5777
500	56.7281	45.3825	31.1344
525	59.5645	47.6516	32.6911
550	62.4009	49.9207	34.2478
575	65.2373	52.1898	35.8046
600	68.0738	54.4590	37.3613
625	70.9012	56.7282	38.9180
650	73.7466	58.9973	40.4747
675	76.5830	61.2664	42.0314
700	79.4194	63.5355	43.5882
725	82.2558	65.8046	45.1449
750	85.0922	68.0738	46.7016
775	87.9286	70.3429	48.2583
800	90.7650	72.6120	49.8150
825	93.6014	74.8811	51.3718
850	96.4378	77.1502	52.9285
875	99.2742	79.4194	54.4852
900	102.1107	81.6886	56.0419
925	104.9471	83.9577	57.5986
950	107.7835	86.2268	59.1554
975	110.6199	88.4959	60.7121
1000	113.4563	90.7650	62.2688
	= $QH \times .113456$	= $QH \times .090765$ = T.Hp. × .80	= $QH \times .062269$ = T.Hp. × .54884 = M.Hp. × .68604

one stream is delivered by underground strata into another, but such conditions are relatively rare. It will be found that whereas the percentage of rainfall appearing as runoff is greater for precipitous and rocky drainages, the amount of the runoff that can be utilized for power will be greater from flat, sandy areas.

Runoff Data. The most useful as well as most accurate data available for American conditions are those of Desmond FitzGerald, obtained at Boston on the Sudbury and Cochituate watersheds. (Trans. Am. Soc. C. E., 1892, Vol. 27, p. 253.)

Runoffs in Percentages of the Rainfall

Period	Sudbury, 1875 to 1890			Cochituate, 1863 to 1891
	Mean	Max.	Min.	
January.....	49.1	88.8	7.6	53.1
February....	78.2	206.9	42.9	71.9
March......	109.6	261.7	74.0	84.6
April.......	109.1	188.3	48.5	83.8
May........	62.3	260.2	39.9	47.9
June........	29.1	54.7	14.0	27.9
July........	8.9	20.9	3.6	13.1
August......	13.0	61.2	4.1	17.7
September...	14.2	30.9	6.1	23.5
October.....	23.1	71.4	4.8	23.6
November...	39.5	174.7	11.3	34.0
December...	52.5	127.3	9.6	49.9
Year......	49.5	62.2	31.9	43.8

The Sudbury watershed has an area of 75.199 square miles, is hilly, with steep slopes having some large swamps within its borders, and the material is costly unmodified drift. The Cochituate watershed has an area of 18.87 square miles, with flat, sandy slopes and the surface mostly modified drift. The two are adjacent to each other.

The records of the Sudbury watershed are given in more detail than those of Cochituate, and the more important data are in the following table:

Sudbury River Watershed Record of 16 Years

Period	Rainfall in inches			Runoff in cubic feet per second per square mile			Evaporation from water, inches			Mean temperature, Fahr.
	Mean	Max.	Min.	Mean	Max.	Min.	Mean	Max.	Min.	
Jan.....	4.179	6.36	1.83	1.779	4.305	0.159	0.96	......	......	27°
Feb.....	4.062	6.55	0.74	3.023	7.428	1.469	1.05	......	......	28
Mar....	4.578	8.36	1.07	4.354	7.448	2.071	1.70	......	......	34
Apr.....	3.320	5.79	1.82	3.247	5.094	1.342	2.97	3.12	2.78	44
May....	3.203	5.21	0.96	1.732	2.526	0.796	4.46	5.89	3.35	57
June....	2.985	6.24	1.47	0.779	1.346	0.271	5.54	7.01	3.94	67
July....	3.784	9.13	1.41	0.292	0.980	0.096	5.98	7.50	4.82	71
Aug....	4.227	7.18	0.74	0.478	2.216	0.086	5.50	7.41	4.25	69
Sept....	3.232	8.74	0.32	0.412	1.786	0.068	4.12	5.13	3.08	62
Oct.....	4.413	10.51	0.81	0.882	3.518	0.109	3.16	4.13	2.51	52
Nov....	4.107	7.23	1.15	1.453	4.267	0.271	2.25	3.00	0.66	40
Dec.....	3.710	6.37	0.87	1.689	4.708	0.271	1.51	......	......	30
Year..	45.800	53.00	32.78	1.669	2.626	0.824	39.20	41.51	34.05	48.4

When the rainfall is high, a greater portion appears as runoff and when low a less percentage than the mean, since the vegetation takes its share before the water gets away, except in severe storms, and when there is little rain during the growing season vegetation takes nearly all. With rainfall records as a basis and the data of FitzGerald's experiments, it is possible to estimate with a fair degree of accuracy the annual runoff of streams in the humid portions of the United Stations, and also to approximate the low-water flow, particularly if cognizance is taken of the character of the surface and the influence of the vegetation on the evaporation.

In working from rainfall to runoff it is to be noted that a given amount of precipitation concentrated in a few sharp showers will appear to a larger extent as runoff than the same amount falling during the same period in a more continuous but mild storm. Also when the soil, of a clayey nature, is baked by a drouth and vegetation is drooping, a larger proportion of the precipitation will run off than when the soil is slightly moist and vegetation thriving. Rain falling at night contributes more to the runoff than that falling in the daytime by reason of the direct evaporation in the latter case by the sun's rays, and a mild shower falling on a hot surface may be almost wholly evaporated without the aid of vegetation. Rain falling on frozen ground appears almost wholly as runoff. The water which runs off over the surface reaches the streams within a few hours, but the subsurface flow is likely to continue for at least two weeks under ordinary conditions before it all appears in the open channels.

41. Rainfall in the United States

The table on the following pages, compiled from the records of the U. S. Weather Bureau, gives the normal monthly and annual precipitation in inches at 190 places in the United States, as established from records prior to 1922. In the arid regions, the minimum annual precipitation may be as low as 25% of the normal, but in the humid portions, while records as low as 40% occur, an annual minimum less than 50% of the normal is rare. The terms Rainfall and Precipitation are usually synonymous, meaning the vertical depth of rain and melted snow.

42. Percolation and Runoff

Percolation, or subsurface runoff, varies greatly with the character of the soils and their covering. The accompanying data, from both European and American sources, gives the percolation in percentages of the rainfall:

	Per cent
Sand, bare	65 to 85
Ordinary ground, bare	29 and 29.2
Ordinary ground, cultivated, no crops	36
Loam, bare	33
Peat, bare	44
Sand, with grass growing 6 months	14
Loam with grass growing 6 months	1.3
Peat with grass growing 6 months	14.6 and 8.7
Ordinary soil with sod	33
Bare ground	45
Bare ground covered with 1 cm. sand	82
Bare ground covered with 1 cm. straw	94
Bare ground covered with 5 cm. dry leaves	93
Bare ground with grass growing	−34*
Meadow grass, short	0 to 1.15
Meadow grass, long	0 to −100*
Beans and peas	5
Clover	−12*
Indian corn	40 to −50*
Wheat	25
Rye	40 to 18
Oats	38
Potatoes	40
Ash forest	37
Alder forest	43
Beech forest	44
Elm forest	50
Maple forest	57
Oak forest	58 to 70
Vineyards	66
Mixed forest	74
Norway spruce forest	91
Pine forest	93
Fir forest	94

These data so far as they deal with crops and forests must be accepted as giving only relative relations. It is, however, safe to say that the percolating runoff from evergreen forests will be about twice that from deciduous forests and three times that from cropped areas during the growing season.

* Water in excess of rainfall taken from subsurface runoff of adjacent areas.

Normal Precipitation in United States

From Beginning of Records to 1920 or 1922. (U. S. Weather Bureau)

State	Station	Elevation, ft.	Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Alabama	Anniston	728	29	5.01	5.02	5.59	4.66	4.12	4.08	5.65	4.35	3.34	2.32	2.81	4.83	51.78
	Decatur	573	42	5.16	4.83	6.06	4.60	3.89	3.63	4.67	3.57	2.77	2.59	3.20	4.34	49.31
	Greensboro	220	49	5.09	5.13	5.43	4.28	4.00	3.70	4.55	4.65	3.13	2.28	3.42	5.77	50.83
	Mobile	84	50	4.74	5.29	6.33	5.01	4.31	5.44	7.03	7.06	5.32	3.54	3.72	4.77	62.56
	Montgomery	240	48	5.07	5.55	6.25	4.78	3.84	4.02	4.70	4.13	3.09	2.38	3.21	4.85	51.86
Arizona	Phoenix	1108	56	0.85	0.84	0.63	0.33	0.11	0.07	1.11	0.98	0.64	0.43	0.66	0.88	7.53
	Yuma	141	52	0.46	0.46	0.34	0.08	0.04	0.01	0.21	0.57	0.28	0.21	0.27	0.42	3.35
Arkansas	Fort Smith	457	41	2.46	2.54	3.17	3.82	4.63	3.71	3.35	3.49	2.84	2.83	2.93	2.58	38.35
	Little Rock	357	43	4.70	3.91	4.71	4.98	4.71	3.86	3.43	3.76	3.16	2.75	4.14	4.12	48.23
California	Eureka	64	36	7.59	6.80	5.78	3.29	2.07	0.88	0.14	0.18	1.04	2.33	5.38	6.37	41.85
	Fresno	293	41	1.83	1.50	1.75	0.82	0.50	0.10	0.01	0.01	0.24	0.54	1.01	1.51	9.82
	Independence	3957	54	1.42	0.77	0.58	0.17	0.24	0.06	0.16	0.15	0.15	0.28	0.18	1.16	5.32
	Los Angeles	361	42	3.34	3.19	2.92	0.89	0.40	0.08	0.01	0.02	0.17	0.68	1.21	2.67	15.62
	Mt. Tamalpais	2375	25	5.81	4.98	3.93	1.39	1.22	0.27	0.02	0.02	0.65	1.52	3.32	4.42	27.55
	Red Bluff	307	46	4.86	3.83	3.41	1.66	1.14	0.47	0.03	0.04	0.84	1.31	2.85	4.42	24.86
	Sacramento	71	74	3.82	2.83	2.77	1.46	0.75	0.12	0.02	0.01	0.27	0.78	1.93	3.80	18.56
	San Diego	87	69	1.93	1.93	1.49	0.61	0.31	0.06	0.06	0.10	0.08	0.37	0.94	1.75	9.63
	San Francisco	206	73	4.90	3.63	3.20	1.49	0.71	0.15	0.02	0.02	0.33	0.96	2.53	4.58	22.52
	San Jose	95	48	3.05	2.45	2.66	1.08	0.55	0.11	T	0.02	0.35	0.72	1.50	2.64	15.13
	San Luis Obispo	201	52	4.11	3.99	3.57	1.35	0.54	0.09	T	0.03	0.26	0.92	1.68	3.84	20.38
Colorado	Denver	5292	51	0.44	0.52	0.96	2.13	2.41	1.35	1.78	1.40	1.01	0.98	0.59	0.71	14.28
	Pueblo	4685	54	0.29	0.50	0.62	1.40	1.57	1.37	1.99	1.55	0.76	0.68	0.36	0.47	11.56
Connecticut	Hartford	159	59	3.44	3.48	3.91	4.09	4.38	4.10	4.15	4.47	3.58	3.69	3.55	3.38	46.25
	New Haven	127	73	3.80	3.96	4.04	3.54	3.88	3.15	4.30	4.45	3.71	3.70	3.57	3.67	45.77
Delaware	Milford	20	37	3.44	3.59	4.05	3.35	3.83	3.77	4.22	4.62	3.51	3.41	3.07	3.81	44.67
Dist. of Col.	Washington	75	82	3.17	2.99	3.46	3.32	3.59	3.88	4.40	4.04	3.07	3.05	2.51	3.05	40.53

Normal Precipitation in United States—Continued

State	Station	Elevation, ft.	Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Florida.......	Jacksonville.....	222	67	2.64	3.14	3.17	2.46	3.95	5.63	6.33	6.52	7.49	4.42	2.12	2.95	50.82
	Merritts Island..	20	42	2.99	2.71	2.44	2.66	3.67	6.27	5.68	5.51	7.72	5.83	2.41	2.40	50.29
	Key West......	16	75	1.90	1.36	1.48	1.51	3.72	4.04	3.36	4.06	6.98	5.10	2.02	1.76	37.29
	Miami.........	83	36	2.92	2.23	2.67	3.44	6.30	7.42	6.24	6.48	8.99	8.38	2.78	2.09	59.94
	Pensacola......	151	44	3.93	4.38	4.87	4.00	3.45	4.58	6.72	7.99	5.46	4.23	3.58	4.64	57.83
	Tampa..........	103	59	2.53	2.60	2.57	1.91	2.72	7.37	9.02	8.77	6.22	2.42	1.82	2.24	50.19
Georgia......	Atlanta........	1218	55	4.74	4.92	5.53	4.01	3.48	3.91	4.39	4.49	3.35	2.57	3.21	4.70	49.30
	Augusta........	180	55	3.78	5.18	4.53	3.23	2.96	4.56	5.00	5.25	3.48	2.45	2.67	3.39	46.48
	Macon.........	370	45	3.66	4.44	5.02	3.73	2.89	3.90	4.85	4.34	3.02	2.27	2.77	3.99	44.88
	Savannah......	65	62	2.92	3.08	3.58	2.78	3.33	5.37	6.44	7.78	5.21	3.04	2.22	3.01	48.76
	Thomasville....	273	39	3.80	4.79	4.05	3.47	3.71	5.37	6.73	6.00	4.77	3.08	2.58	4.32	52.80
Idaho........	Boise..........	2739	55	1.98	1.43	1.64	1.25	1.50	0.86	0.26	0.18	0.46	1.09	1.34	1.74	13.73
	Lewiston.......	757	41	1.39	1.20	1.10	0.98	1.52	1.36	0.51	0.52	0.81	1.13	1.44	1.41	13.37
	Pocatello.......	4477	21	1.39	1.29	1.57	1.40	1.67	1.13	0.66	0.73	0.88	1.22	0.92	1.02	13.88
Illinois.......	Cairo..........	356	49	3.92	3.28	3.83	3.78	3.67	4.00	3.30	2.88	2.60	2.70	3.59	3.40	40.95
	Chicago........	824	50	2.09	2.16	2.51	2.80	3.64	3.42	3.39	3.02	3.02	2.54	2.36	2.04	32.99
	Minonk........	751	26	1.67	1.71	2.38	3.01	3.91	3.37	2.94	3.17	3.60	1.91	1.88	1.69	31.24
	Peoria.........	609	65	1.80	1.96	2.69	3.25	3.91	3.70	3.77	3.12	3.70	2.29	2.27	2.04	34.50
	Springfield......	644	41	2.25	2.51	2.91	3.40	4.50	4.13	3.02	3.05	3.32	2.44	2.44	2.11	36.08
	Whitehall......	578	50	2.61	2.32	2.94	3.55	4.63	3.69	3.58	3.06	3.57	2.45	2.37	2.04	36.81
Indiana......	Evansville......	431	46	3.70	3.29	4.28	3.93	3.83	4.07	3.58	3.46	3.19	2.74	3.80	3.49	43.36
	Indianapolis....	822	59	2.99	2.69	4.10	3.68	3.98	3.84	3.93	3.17	3.05	2.78	3.27	2.94	40.42
	Lafayette.......	617	52	2.65	2.52	3.22	3.53	4.36	4.18	3.80	3.53	2.94	2.72	2.96	2.60	39.01
	Logansport.....	620	45	2.41	2.54	2.85	3.24	4.35	3.73	3.20	3.13	3.25	2.61	3.05	2.52	36.88
Iowa.........	Charles City....	1015	46	1.04	1.12	1.86	2.52	4.42	4.85	3.82	3.34	3.36	2.48	1.61	1.28	31.70
	Davenport......	606	49	1.60	1.54	2.19	2.79	4.15	4.01	3.49	3.48	3.30	2.38	1.84	1.49	32.26
	Des Moines.....	861	44	1.15	1.18	1.64	3.02	4.72	4.66	3.62	3.35	3.42	2.59	1.49	1.27	32.11

Normal Precipitation in United States—Continued

State	Station	Elevation, ft.	Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Iowa—*cont.*	Dubuque	698	67	1.59	1.53	2.16	2.66	4.06	4.41	3.90	3.31	3.79	2.62	1.82	1.52	33.37
	Keokuk	614	49	1.69	1.54	2.29	3.10	4.19	4.29	3.85	2.93	3.86	2.39	1.92	1.59	33.64
	Sioux City	1135	42	0.66	0.77	1.18	2.77	3.94	3.96	3.71	3.17	2.97	1.84	1.07	0.89	26.93
Kansas	Concordia	1392	37	0.57	0.91	1.23	2.28	4.35	4.35	3.53	2.88	2.60	1.99	0.96	0.63	26.38
	Dodge City	2509	48	0.42	0.75	0.84	1.83	2.90	3.32	3.00	2.61	1.76	1.30	0.71	0.65	20.09
	Topeka	997	35	0.93	1.54	2.20	2.64	4.83	4.51	4.46	4.18	4.03	2.19	1.51	0.98	34.00
	Wichita	1377	34	0.74	1.40	1.81	2.80	4.80	4.18	3.22	3.32	3.08	2.33	1.45	0.99	30.14
Kentucky	Lexington	989	64	4.17	3.86	4.33	3.77	3.89	4.16	3.85	3.56	3.21	2.46	3.37	3.88	44.51
	Louisville	525	54	4.00	3.67	4.44	4.11	3.70	4.02	3.91	3.33	2.66	2.67	3.68	3.81	44.00
Louisiana	New Orleans	51	75	4.48	4.19	4.42	4.76	4.25	5.43	6.58	5.75	4.67	3.67	3.72	4.68	56.60
	Shreveport	249	49	3.82	3.34	4.03	4.70	4.09	3.39	3.38	2.67	2.94	2.86	3.60	4.23	43.05
Maine	Eastport	76	50	3.84	3.41	4.06	3.01	3.27	3.18	3.23	3.06	2.94	3.76	3.56	3.70	41.02
	Portland	99	53	3.76	3.64	3.90	3.32	3.54	3.24	3.46	3.49	3.27	3.41	3.53	3.65	42.21
Maryland	Baltimore	115	50	3.33	3.33	3.74	3.36	3.45	3.87	4.66	4.53	3.63	2.82	2.71	3.25	42.68
Massachusetts	Amherst	222	85	3.38	3.27	3.60	3.18	3.80	3.61	4.40	4.36	3.66	3.71	3.62	3.58	44.17
	Boston	124	103	3.67	3.50	4.03	3.78	3.55	3.11	3.50	4.01	3.38	3.54	3.98	3.70	43.75
	Nantucket	47	47	3.82	3.17	3.68	3.28	3.14	2.80	2.63	3.16	2.83	3.52	3.30	4.05	39.38
	New Bedford	88	107	4.03	3.89	4.27	3.94	3.94	3.12	3.30	4.15	3.45	3.86	4.14	4.12	46.21
Michigan	Alpena	609	48	2.01	1.68	1.91	2.21	3.18	3.39	2.72	3.07	3.12	3.20	2.56	2.13	31.18
	Calumet	1246	33	2.64	1.61	1.68	2.17	3.24	3.24	2.80	3.02	3.56	3.07	2.70	2.99	32.72
	Detroit	730	50	2.13	2.18	2.41	2.44	3.30	3.68	3.36	2.77	2.65	2.41	2.40	2.32	32.05
	Escanaba	612	48	1.48	1.43	1.92	2.12	3.18	3.49	3.32	3.44	3.35	2.96	2.18	1.67	30.54
	Grand Haven	628	49	2.54	2.09	2.27	2.66	3.34	3.30	2.61	2.81	3.64	3.03	2.77	2.45	33.51
	Kalamazoo	955	44	2.24	2.13	2.30	2.66	4.07	4.02	3.19	2.89	3.22	2.83	2.78	2.71	35.04
	Lansing	863	57	1.85	1.82	2.35	2.51	3.35	3.71	3.16	2.84	2.77	2.44	2.23	1.94	30.97
	Marquette	734	49	2.16	1.76	2.08	2.31	3.10	3.51	3.11	2.76	3.21	3.01	3.00	2.47	32.48
	Port Huron	639	46	1.91	1.98	2.27	2.16	3.05	2.91	2.75	2.81	2.66	2.62	2.40	2.05	29.57
	Sault Ste. Marie	614	33	1.90	1.45	1.82	2.25	2.89	2.74	2.62	2.97	3.39	3.14	2.89	2.21	30.27

Normal Precipitation in United States—Continued

State	Station	Elevation, ft.	Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Minnesota...	Duluth........	1133	50	1.04	0.97	1.56	2.03	3.41	4.16	3.78	3.27	3.48	2.53	1.52	1.18	28.93
	Minneapolis....	918	59	1.05	0.95	1.58	2.41	3.61	4.22	3.51	3.51	3.37	2.17	1.41	1.25	29.04
	Moorhead......	935	40	0.72	0.71	0.96	2.16	2.86	4.01	3.64	2.99	2.19	1.76	0.90	0.72	23.62
	New Ulm......	791	42	1.06	1.02	1.58	2.09	3.64	4.75	3.81	3.48	3.05	2.19	1.18	0.79	28.64
	St. Paul........	940	84	0.92	0.82	1.39	2.33	3.41	4.07	3.46	3.38	3.23	2.04	1.37	0.98	27.40
Mississippi....	Columbus......	191	62	5.07	5.53	6.17	5.34	3.94	4.12	4.71	4.34	3.09	2.44	4.19	5.12	54.06
	Meridian.......	375	31	5.08	5.30	4.96	4.52	4.44	4.62	5.35	4.87	3.38	2.16	2.96	5.05	52.69
	Vicksburg......	247	68	5.15	4.76	5.38	5.28	4.22	3.91	4.57	3.50	3.24	2.76	4.12	5.16	52.05
Missouri.....	Columbia......	784	36	2.06	1.89	3.03	3.97	4.52	4.44	3.31	3.65	4.56	2.39	2.08	1.70	37.60
	Hannibal.......	534	44	1.73	1.72	2.54	3.16	4.48	3.81	3.29	3.19	3.77	2.37	1.95	1.42	33.43
	Kansas City....	963	52	1.24	1.72	2.50	3.11	4.73	4.85	4.18	4.06	4.24	2.95	1.91	1.35	36.84
	St. Louis.......	567	84	2.32	2.60	3.46	3.68	4.54	4.64	3.67	3.50	3.18	2.86	2.85	2.47	39.78
	Springfield......	1302	44	2.39	2.48	3.31	3.92	5.39	4.80	4.29	4.06	3.33	3.09	2.76	2.35	42.17
Montana.....	Havre.........	2505	41	0.80	0.52	0.50	0.90	1.89	2.76	1.79	1.22	1.27	0.70	0.62	0.58	13.55
	Helena.........	4110	41	0.94	0.67	0.77	1.06	2.15	2.32	1.14	0.70	1.25	0.90	0.72	0.77	13.39
	Miles City......	2371	43	0.66	0.51	0.89	1.02	2.18	2.74	1.58	1.08	0.94	0.87	0.51	0.48	13.46
	Missoula.......	3225	51	1.35	0.86	1.02	1.03	2.12	2.15	1.02	0.86	1.34	1.20	1.18	1.43	15.56
Nebraska.....	Lincoln........	1189	40	0.64	1.02	1.22	2.57	4.32	4.34	3.89	3.67	2.93	2.03	1.03	0.84	28.50
	North Platte....	2841	46	0.42	0.55	0.85	2.15	2.87	3.22	2.79	2.37	1.41	1.11	0.45	0.59	18.78
	Omaha.........	1103	62	0.71	0.90	1.32	2.83	4.03	4.74	4.22	3.46	3.09	2.44	1.15	0.99	29.88
	Valentine.......	2598	35	0.51	0.59	1.15	2.28	3.00	3.04	2.88	2.51	1.41	1.07	0.53	0.53	19.50
Nevada......	Reno..........	4532	33	1.79	1.21	0.87	0.46	0.70	0.31	0.31	0.29	0.33	0.34	0.36	1.08	8.35
	Winnemucca....	4344	38	1.14	0.88	0.96	0.80	0.88	0.61	0.18	0.17	0.37	0.65	0.76	0.97	8.37
New Hampshire	Concord........	350	67	3.03	2.77	3.11	2.93	3.13	3.20	3.67	3.74	3.45	3.47	3.23	3.07	38.80
New Jersey...	Atlantic City...	16	47	3.45	3.28	3.69	3.10	3.06	3.08	3.63	4.33	2.89	3.14	2.97	3.90	40.52
	New Brunswick.	110	67	3.79	3.62	3.81	3.74	3.89	3.76	5.04	5.15	3.60	3.58	3.48	3.67	47.13
New Mexico..	Santa Fé.......	7013	72	0.67	0.86	0.77	0.92	1.07	1.17	2.65	2.40	1.53	1.11	0.75	0.73	14.63

Normal Precipitation in United States—Continued

State	Station	Elevation, ft.	Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
New York....	Albany.........	97	95	2.55	2.45	2.76	2.69	3.43	3.86	3.98	3.77	3.32	3.33	2.91	2.65	37.70
	Buffalo........	600	64	3.20	2.91	2.83	2.53	3.11	2.87	3.13	3.04	3.16	3.48	3.10	3.29	36.65
	Cooperstown....	1200	67	2.77	2.62	2.86	2.79	3.66	4.33	4.61	4.37	3.61	3.44	3.02	2.86	40.94
	Ithaca.........	928	62	2.08	1.86	2.34	2.36	3.42	3.62	3.44	3.17	3.05	2.91	2.36	2.27	32.88
	New York......	314	95	3.28	3.29	3.44	3.34	3.46	3.43	4.06	4.28	3.39	3.44	3.42	3.33	42.16
	Ogdensburg.....	258	37	2.22	2.21	2.40	2.13	2.90	3.29	3.07	2.95	2.91	2.75	2.48	2.39	31.70
	Oswego........	335	72	3.01	2.62	2.78	2.44	3.04	3.25	3.21	2.76	2.94	3.57	3.34	3.50	36.46
	Rochester......	500	67	2.60	2.39	2.58	2.46	3.08	3.09	3.16	2.85	2.77	2.92	2.65	2.64	33.19
North Carolina	Charlotte.......	779	44	3.98	4.28	4.27	3.28	3.76	4.31	5.29	5.29	2.97	3.07	2.59	3.77	46.86
	Hatteras.......	11	45	4.33	4.11	4.54	3.90	3.66	4.43	5.43	5.73	5.15	5.20	3.65	4.55	54.68
	Murphy........	1614	48	5.60	5.97	6.47	4.76	3.89	5.34	5.97	5.35	3.44	3.41	3.81	5.20	59.21
	Raleigh........	390	44	3.25	3.93	3.95	3.45	4.21	4.52	5.54	5.72	3.55	2.95	2.34	3.23	46.64
	Wilmington.....	52	50	3.27	3.41	3.38	2.78	3.61	5.36	6.73	6.74	4.94	3.48	2.11	2.98	48.79
North Dakota.	Bismarck.......	1674	46	0.52	0.48	1.03	1.63	2.43	3.41	2.21	1.99	1.25	0.95	0.62	0.58	17.10
	Buford.........	1950	52	0.57	0.49	0.55	1.01	2.27	2.89	1.66	1.37	0.98	0.87	0.47	0.54	13.67
	Pembina.......	789	49	0.64	0.71	0.90	1.41	2.22	3.28	2.38	2.13	1.69	1.34	0.68	0.76	18.14
Ohio.........	Cincinnati......	628	86	3.37	3.08	3.74	3.25	3.92	4.09	3.73	3.55	2.85	2.73	3.03	3.29	40.63
	Cleveland......	762	49	2.58	2.50	2.74	2.46	3.19	3.33	3.51	3.01	3.19	2.81	2.51	2.37	34.20
	Columbus......	918	44	3.10	2.79	3.54	2.93	3.60	3.30	3.56	3.30	2.47	2.50	2.75	2.68	36.52
	Marietta.......	627	102	2.37	3.13	3.48	3.41	3.88	4.21	4.61	3.84	3.07	2.89	2.92	3.41	41.22
	Sandusky......	629	53	2.24	2.20	2.61	2.68	3.20	3.57	3.47	3.46	3.01	2.62	2.44	2.23	33.73
	Toledo.........	769	60	2.23	2.01	2.42	2.52	3.46	3.41	3.10	2.87	2.53	2.43	2.45	2.30	31.73
Oklahoma....	Fort Gibson....	556	45	2.27	2.02	2.76	4.41	4.73	3.63	2.63	3.56	3.03	3.34	2.76	2.10	37.24
	Oklahoma......	1247	30	1.17	1.10	2.10	3.14	5.46	3.49	2.87	2.88	2.62	2.57	2.07	1.56	31.03
Oregon.......	Astoria.........	24	72	12.16	9.15	8.75	5.42	3.76	3.06	1.21	1.26	3.46	5.65	11.08	12.03	76.99
	Baker City.....	3471	31	1.27	1.19	1.19	0.99	1.47	1.20	0.58	0.48	0.76	0.82	1.00	1.31	12.26
	Portland.......	57	52	6.55	5.48	4.82	3.05	2.30	1.63	0.54	0.64	1.86	3.27	6.46	6.91	43.56
	Roseburg.......	510	45	5.43	4.49	3.50	2.29	1.92	1.13	0.36	0.30	1.22	2.48	4.69	5.46	33.27

Normal Precipitation in United States—Continued

State	Station	Elevation, ft.	Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Pennsylvania	Erie	714	47	2.96	2.69	2.67	2.65	3.55	3.57	3.09	3.37	3.43	3.81	3.27	2.84	37.90
	Harrisburg	361	51	3.07	2.82	3.04	3.10	3.78	3.91	3.79	4.18	3.35	2.97	2.25	3.14	39.40
	Philadelphia	156	49	3.27	3.31	3.48	3.16	3.33	3.31	4.24	4.74	3.33	2.94	2.99	3.33	41.43
	Pittsburgh	842	78	2.70	2.42	2.94	3.05	3.33	3.72	3.86	3.32	2.77	2.72	2.43	2.89	36.15
	Wellsboro	1419	56	2.57	2.35	3.25	3.58	4.05	4.05	4.00	3.71	3.08	3.13	2.58	2.55	38.90
Rhode Island	Block Island	26	40	3.77	3.88	3.99	3.63	3.56	2.64	3.17	3.35	2.78	3.65	3.66	3.90	41.98
	Providence	160	89	4.01	3.79	4.06	3.72	3.58	3.19	3.30	3.96	3.31	3.54	3.81	3.89	44.16
South Carolina	Aiken	527	53	3.58	4.05	3.96	3.09	3.46	4.74	4.84	5.43	3.38	2.63	2.57	3.61	45.34
	Camden	222	71	3.38	3.97	3.70	3.08	3.21	4.48	5.23	5.46	3.66	2.62	2.35	3.40	44.54
	Charleston	48	129	2.99	3.09	3.28	2.41	3.31	5.16	6.20	6.54	5.18	3.80	2.60	3.24	47.80
South Dakota	Huron	1306	39	0.56	0.56	0.91	2.38	3.06	3.85	3.06	2.47	1.58	1.34	0.59	0.58	20.94
	Pierre	1572	29	0.47	0.54	1.04	1.90	2.51	2.62	2.80	2.12	1.18	0.88	0.43	0.51	17.00
	Rapid City	3251	33	0.40	0.50	1.05	1.97	3.45	3.34	2.41	1.63	1.26	0.90	0.38	0.42	17.71
	Yankton	1234	48	0.56	0.85	1.28	2.89	4.00	3.96	3.67	3.02	2.67	1.51	0.77	0.84	26.02
Tennessee	Chattanooga	762	42	5.28	4.93	5.83	4.78	3.79	4.17	4.25	4.05	3.25	2.92	3.16	4.86	51.27
	Knoxville	996	53	4.79	4.57	5.18	4.48	3.77	4.40	4.33	4.07	2.84	2.61	3.22	4.21	48.47
	Memphis	409	58	4.86	4.40	5.21	5.10	4.25	4.01	3.27	3.41	2.98	2.83	4.14	4.35	48.81
	Nashville	654	58	4.84	4.18	5.00	4.43	3.92	4.26	4.06	3.64	3.62	2.58	3.58	3.82	47.93
Texas	Abilene	1738	35	0.86	0.91	1.15	2.50	3.87	2.59	2.03	2.54	2.67	2.56	1.40	1.23	24.31
	Amarillo	3676	29	0.49	0.81	0.59	1.72	3.00	2.43	2.95	3.12	2.28	1.52	1.03	0.89	20.83
	Austin	593	60	1.97	2.33	2.31	3.51	4.28	2.48	2.38	2.40	3.45	3.00	2.50	2.57	33.18
	Corpus Christi	20	34	1.55	1.57	1.56	1.98	3.00	2.34	1.56	1.90	3.83	2.13	2.08	1.34	24.84
	El Paso	3762	70	0.42	0.45	0.31	0.21	0.26	0.63	1.72	1.83	1.49	0.79	0.55	0.48	9.14
	Fort Worth	670	34	1.77	1.86	2.49	3.74	4.62	3.28	2.95	2.67	2.54	2.83	2.67	1.96	33.38
	Galveston	69	49	3.24	2.84	2.69	3.24	3.64	4.24	3.70	4.80	5.55	4.60	3.77	3.85	46.20
	Palestine	510	40	3.29	2.98	3.36	4.27	4.52	3.43	2.75	2.44	2.93	3.35	3.52	3.91	40.75
	San Antonio	701	50	1.36	1.60	1.63	2.90	3.07	2.53	2.34	2.58	3.30	2.44	2.18	1.77	27.70

Normal Precipitation in United States—Concluded

State	Station	Elevation, ft.	Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Utah........	Modena........	5479	20	0.91	1.15	1.18	0.83	0.85	0.30	1.45	1.37	1.10	0.94	0.53	0.58	11.19
	Salt Lake City..	4408	57	1.35	1.47	2.07	2.01	2.00	0.81	0.52	0.77	0.97	1.54	1.37	1.35	16.23
Vermont.....	Burlington.....	404	79	1.83	1.58	2.01	2.05	3.02	3.29	3.97	3.47	3.58	3.25	2.50	1.94	32.49
	Woodstock.....	700	48	2.78	2.73	3.07	2.74	3.31	3.25	3.87	3.25	3.34	3.25	2.93	2.93	37.45
Virginia......	Dale Enterprise.	1350	41	2.91	2.70	3.17	3.03	4.20	5.25	4.40	4.03	3.28	2.77	2.13	2.68	40.55
	Fortress Monroe.	8	53	3.06	2.99	3.57	3.12	3.67	3.68	4.63	4.76	3.67	3.02	2.96	3.47	42.60
	Lynchburg.....	681	49	3.47	3.23	3.62	3.11	3.71	3.97	4.14	4.07	3.54	3.15	2.63	3.11	41.75
	Norfolk........	91	50	3.18	3.51	4.03	3.43	3.89	4.24	5.83	5.46	3.66	3.35	2.47	3.38	46.43
	Richmond......	144	49	3.05	3.14	3.84	3.50	3.84	3.83	4.30	4.56	3.19	3.11	2.41	3.09	41.86
	Wytheville.....	2293	43	3.04	3.14	3.59	3.10	3.61	4.41	4.20	4.34	3.33	2.70	2.19	2.75	40.40
Washington...	North Head....	211	55	8.98	7.13	6.26	4.28	2.98	2.38	1.17	0.99	2.92	5.10	9.18	9.59	60.96
	Port Townsend..	80	60	2.56	1.84	1.68	1.51	1.77	1.51	0.77	0.74	1.11	1.60	2.65	2.64	20.38
	Seattle.........	248	31	4.76	3.66	2.83	2.48	1.93	1.49	0.66	0.53	1.71	2.72	5.69	5.45	33.91
	Spokane.......	1943	40	2.14	1.75	1.26	1.13	1.46	1.34	0.66	0.56	0.85	1.21	2.16	2.15	16.63
	Tacoma........	213	36	6.11	4.31	3.58	2.93	2.23	1.79	0.65	0.66	2.14	3.22	6.85	6.92	41.39
	Tatoosh Island..	86	50	12.22	8.71	8.00	5.69	3.87	3.52	1.46	2.07	5.20	7.64	12.62	13.08	84.08
	Walla Walla....	1000	49	1.95	1.74	1.69	1.60	1.75	1.12	0.39	0.43	0.88	1.53	2.12	1.98	17.18
West Virginia.	Morgantown....	1250	47	3.87	3.11	3.71	3.35	3.62	4.54	5.07	4.20	3.06	3.16	2.90	3.14	43.73
	Parkersburgh...	638	36	3.63	3.08	3.43	3.22	3.43	4.14	4.38	3.48	2.57	2.45	2.49	2.84	39.14
Wisconsin....	Eau Claire.....	800	29	1.13	1.20	1.96	2.41	4.39	4.82	3.62	3.52	3.64	3.18	1.68	1.34	32.89
	Green Bay.....	617	34	1.53	1.45	2.12	2.54	3.77	3.48	3.36	2.81	3.34	2.43	2.05	1.59	30.50
	La Crosse......	714	48	1.15	1.06	1.64	2.41	3.79	4.39	3.90	3.54	3.87	2.42	1.61	1.31	31.09
	Madison.......	974	58	1.57	1.49	2.01	2.54	3.78	3.88	3.71	3.12	3.36	2.36	1.78	1.58	31.18
	Milwaukee.....	681	50	2.02	1.79	2.50	2.72	3.57	3.62	2.90	2.70	3.14	2.37	1.85	1.79	30.97
Wyoming.....	Cheyenne......	6088	50	0.38	0.57	0.94	1.81	2.39	1.47	2.03	1.48	1.19	0.84	0.44	0.45	13.99
	Lander.........	5372	29	0.50	0.81	1.37	2.21	2.25	1.12	0.73	0.54	1.06	1.45	0.67	0.82	13.53
	Yellowstone Park	6200	32	1.99	1.51	1.97	1.31	2.04	1.72	1.34	1.03	1.21	1.37	1.30	1.50	18.29

Runoff Data of Various American Rivers

River and Station	Area sq. mi.	Years	Mean yearly: Rainfall, in.	Mean yearly: Runoff: Per cent rainfall	Mean yearly: Runoff: Cu.ft. per sec. per sq. mi.	Year of min. flow: Rainfall, in.	Year of min. flow: Runoff: Per cent rainfall	Year of min. flow: Runoff: Cu.ft. per sec. per sq. mi.	Min. monthly flow, cu. ft. per sec. per sq. mi.
Sudbury: Boston, Mass.	75.2	1875–97	45.77	48.6	1.637	32.78	34.1	0.825	....
Cochituate: Boston, Mass.	18.9	1863–96	47.08	43.2	1.498	31.20	31.3	0.719	....
Mystic: Boston, Mass.	26.9	1878–96	43.79	45.6	1.471	31.22	29.8	0.687	....
Connecticut: Hartford, Conn.	10 234	1871–85	44.69	56.5	1.860	40.02	45.6	1.345	....
Housatonic: Gaylordsville, Conn.	1 020	1900–05	47.86	61.6	2.170	39.77	59.8	1.750	.354
Croton: Old Croton Dam	338	1870–94	48.38	50.8	1.810	38.52	37.8	1.071	....
Upper Hudson: Mechanicville, N.Y.	4 500	1888–96	39.7	59.0	1.720	33.49	52.2	1.286	....
Genesee: Mount Morris, N. Y.	1 060	1894–96	39.82	32.5	0.955	31.00	21.5	0.492	....
Passaic: Dundee Dam, N. J.	822	1877–93	47.08	54.0	1.875	35.64	42.7	1.122	....
Perkiomen: Philadelphia, Pa.	152	1844–97	47.98	49.2	1.741	38.67	40.4	1.154	....
Tohickon: Philadelphia, Pa.	102	1884–97	50.17	56.7	2.095	38.34	49.0	1.381	....
Neshaminy: Philadelphia, Pa.	139	1884–97	47.88	48.5	1.712	36.30	44.3	1.192	....
Susquehanna: Harrisburg, Pa.	28 030	1891–05	39.38	53.6	1.553	31.62	51.7	1.203	.141
Wilkes-Barre, Pa.	9 810	1899–05	39.85	58.2	1.708	31.77	47.7	1.116	.106
Williamsport, Pa.	5 640	1895–05	40.02	55.6	1.640	37.46	44.3	1.221	.133
Ohio: Wheeling, W. Va.	23 820	1884–05	41.71	54.4	1.670	33.47	48.7	1.200	.106
Potomac: Pt. of Rocks, Md.	9 650	1895–05	36.86	38.6	1.047	37.25	21.9	0.602	.124
Shenandoah: Millville, W. Va.	3 000	1895–05	38.33	35.6	1.005	30.47	25.8	0.579	.177
James: Cartersville, Va.	6 230	1898–05	42.98	42.4	1.342	30.58	35.0	0.788	.186
Buchanan, Va.	2 060	1895–05	41.17	41.1	1.246	30.45	37.6	0.844	.159
North: Glasgow, Va.	830	1895–05	40.76	39.2	1.177	36.49	33.3	0.896	.150
Appomattox: Mattoak, Va.	745	1900–05	42.98	38.4	1.214	30.80	35.5	0.805	.239
Roanoke: Roanoke, Va.	390	1896–05	42.68	41.5	1.303	35.21	25.2	0.654	.194
Randolph, Va.	3 080	1900–05	43.80	42.6	1.375	34.00	32.4	0.810	.265

Runoff Data of Various American Rivers—Concluded

River and Station	Area, sq. mi.	Years	Mean yearly: Rainfall, in.	Mean yearly: Runoff, Per cent rainfall	Mean yearly: Runoff, Cu. ft. per sec. per sq. mi.	Year of min. flow: Rainfall, in.	Year of min. flow: Runoff, Per cent rainfall	Year of min. flow: Runoff, Cu. ft. per sec. per sq. mi.	Min. monthly flow, cu. ft. per sec. per sq. mi.
Savannah: Augustus, Ga.	7 294	1884–91	45.41	48.9	1.635	43.10	37.7	1.197	....
Des Plaines: Riverside, Ill.	630	1889*	30.56	21.6	0.487	32.38	9.9	0.235	....
Huron: Geddes, Mich.	757	1905–08	34.02	32.5	0.813	32.15	21.8	0.517	.107
Grand: Grand Rapids, Mich.	4 900	1899†	28.85	40.8	0.867	31.35	26.8	0.618	.255
Upper Mississippi: Pokegama Falls	3 265	1885–99	26.57	18.4	0.361	22.86	7.1	0.119	....
Tennessee: Chattanooga, Tenn.	21 400	1874–06	.....	.....	1.860	.....	.....	1.272	....
Colorado: Yuma, Ariz.	225 000	1879–06	.....	.....	0.045	.....	.....	0.012	....
Arkansas: Canyon, Colo.	3 060	1888–92	.....	.....	0.266	.....	.....	0.141	.059
Rio Grande: Del Norte, Colo.	1 400	1890‡	.....	.....	0.644	.....	.....	0.311	.049
Embudo, N. Mex.	10 900	1889§	.....	.....	0.107	.....	.....	0.036	.012
El Paso, Tex.	30 000	1890‖	.....	.....	0.035	.....	.....	0.002	.000
Bear River: Battle Creek, Idaho	4 500	1900¶	.....	.....	0.283	.....	.....	0.163	.060

* Also 1893–95. † Also 1902–06 and 1908. ‡ Also 1891–93 and 1901–02. § Also 1890–92 and 1901–03. ‖ Also 1891–92 and 1901–03. ¶ Also 1891–93 and 1901–02.

Evaporation and Reservoirs. The final runoff of a stream is also affected considerably by the extent of water surface from which evaporation may take place. The evaporation from water surfaces in the United States varies from 18 in. to over 100 in. annually, being greatest in the arid regions and least at high altitudes in the north. For water-power calculations in the humid portions, the evaporation during the summer months is usually all that needs to be considered, as at other times it is insignificant, or else a portion of flow is necessarily wasted. During summer in the northern states this evaporation may amount to about 60% of the annual rainfall in open country, but where the surrounding shores are forested and the water surfaces narrow so as to be little exposed to wind the evaporation is much less.

The evaporation, particularly from water surfaces, is considerably influenced by the wind, as shown by the following observations:

Wind velocity in miles per hour	0	5	10	15	20	25	30
Relative evaporation	1.0	2.2	3.8	4.9	5.7	6.1	6.3

The aquatic plants exercise a considerable influence upon the evaporation from water surfaces. From experiments made under the direction of the author, it appears that the bulrushes, like long grass, exhale much more moisture than would be evaporated from open water, whereas the pond lily exhales considerably less, the common arrow-leaf occupying an intermediate position.

Runoff from Typical Streams in various parts of the United States has been shown in the preceding pages with other appurtenant data.

In applying the data of this table to cases not therein included, cognizance must be taken of both the rainfall and the topographical conditions of the watershed to which it is applied.

Runoff from Great Lakes Drainage Basins

From Local Drainage (by Author)

By Local Stream Gagings within Period from 1892 to 1913

Month	Huron-Michigan, c.f.s. per sq. mi.	Erie-St. Clair, c.f.s. per sq. mi.	Ontario, c.f.s. per sq. mi.
January	0.806	0.987	1.512
February	0.849	0.748	1.402
March	1.380	2.073	2.682
April	1.763	1.438	3.328
May	1.195	0.867	2.019
June	0.910	0.639	1.223
July	0.642	0.306	0.804
August	0.539	0.219	0.681
September	0.534	0.258	0.638
October	0.619	0.241	0.902
November	0.689	0.292	1.058
December	0.666	0.598	1.259
Full year	0.883	0.722	1.459
April to November, inclusive	0.861	0.533	1.332
June to November, inclusive	0.655	0.326	0.884
Percent of drainage area covered by gagings	32	33	37

By U. S. Lake Survey Gagings of Outlets

Average of 1900 to 1907

River	From total drainage		From local drainage	
	c.f.s.	c.f.s. per sq. mi.	c.f.s.	c.f.s. per sq. mi.
St. Lawrence	247 762	0.84	47 690	1.45
Niagara	200 072	0.785	6 544	0.16
St. Clair	193 528	0.905	115 328	0.84
St. Marys	78 200	1.028	78 200	1.028

From Local Drainage (by J. R. Freeman)

River	U. S. L. S. reports		Local stream gagings	
	c.f.s.	c.f.s. per sq. mi.	c.f.s. per sq. mi.	Percentage of drainage covered
Niagara 1915–24	21 850	0.587	0.64	32
St. Clair 1915–24	113 000	0.820	0.71	85
St. Marys 1921–24	49 343	0.648	0.47	25

43. Flood Discharge of Rivers

The flood discharge of a stream will be greater per square mile for small drainage areas than for large ones on account of the greater intensity of precipitation on the former in time of storm. Although many flood formulas for computing the runoff per square mile in time of storm have been suggested, the data in the following table of observed flood flows in rivers of the United States as established by the Hydrographic Branch of the U. S. Geological Survey will be found, for those rivers, much more valuable and useful than any formula can ever be.

In using this table to estimate flood flows the character of the drainage area must be carefully considered together with the rainfall conditions. The ordinary storm rate of precipitation in the humid regions is less than 0.4 in. per hour. Excessive rates are from 0.4 to 0.7 in. per hour, but on rare occasions extreme rates of 2 in. are recorded for periods of about an hour over areas of 20 to 100 square miles. In the arid regions, as much as 17 in. in 24 hours has been recorded over a considerable area. The average slope of the watershed and that of the stream itself also influence the maximum discharge, as also whether the ground be frozen or not.

Maximum Rate of Discharge of Streams in the United States

Stream and place	Drainage area, sq. mi.	Date	Cu. ft. per sec. per sq. mi.
Beacon Brook, near Fishkill, N. Y.	0.25	1897	3.200
Budlong Creek, Utica, N. Y.	1.13	1904	120.40
Sylvan Glen Creek, New Hartford, N. Y.	1.18	1904	56.58
Pequest River, Hunts Pond, N. J.	1.70	1904	25.30
Starch Factory Creek, New Hartford, N. Y.	3.40	1904	109.62
Starch Factory Creek, near New Hartford, N. Y.	3.40	1905	209.00
Wilson Creek, near Carbondale, Pa.	3.40	1922	300.00
Reels Creek, Deerfield, N. Y.	4.40	1904	48.36
Mad Brook, Sherburne, N. Y.	5.00	1905	262.00
Skinner Creek, Mannsville, N. Y.	6.40	1891	124.20
Coldspring Brook, Mass.	6.43	1886	48.40
Croton River, South Branch, N. Y.	7.80	1869	73.90
Woodhull Reservoir, Herkimer, N. Y.	9.40	1869	77.80
Mill Brook, Edmeston, N. Y.	9.40	1905	241.00
Fall Brook, near Carbondale, Pa.	11.4	1922	146.00
Stony Brook, Boston, Mass.	12.7	....	121.00
Great River, Westfield, Mass.	14.0	....	71.40*
Smartswood Lake, N. J.	16.0	....	68.00
Williamstown River, Williamstown, N. Y.	16.5	....	34.00
Ridley Creek, Delaware Co., Pa.	20.0	1843	750.00
Croton River, West Branch, N. Y.	20.5	1874	54.40
Beaverdam Creek, Altmar, N. Y.	20.7	....	111.00
Crum Creek, Delaware Co., Pa.	22.0	1843	410.00
Trout Brook, Centerville, N. Y.	23.0	....	50.60
Pequonnock River, Bridgeport, Conn.	25.0	1905	157.00
Wantuppa Lake, Fall River, Mass.	28.5	1875	72.00
Pequest River, Huntsville, N. J.	31.4	....	19.30
Sawkill, near mouth, N. J.	35.0	....	228.60
Jail Branch, Winooski River, E. Barre, Vt.	38.0	1927	303.00
Whippany River, Whippany, N. J.	38.0	1896	84.20
Cuyadutta Creek, Johnstown, N. Y.	40.0	1896	72.40

* Average flow for day of maximum discharge.

Maximum Rate of Discharge of Streams in the United States—Cont.

Stream and place	Drainage area, sq. mi.	Date	Cu. ft. per sec. per sq. mi.
Sugar River, above Watertown, N. Y.	42.0	1927	52.70
Six Mile Creek, Ithaca, N. Y.	46.0	1905	185.00
Dog River, Northfield, Vt.	47.0	1913	72.2
West Canada Creek, Motts Dam, N. Y.	47.5	....	34.10
Darby Creek, Delaware Co., Pa.	48.0	1843	580.00
Sauquoit Creek, New York Mills, N. Y.	51.5	....	53.40
Dog River, Northfield, Vt.	52.0	1927	154.00
Rockaway River, Dover, N. J.	52.5	....	43.00
Oneida Creek, Kenwood, N. Y.	59.0	1890	41.20
Flat River, R. I.	61.0	1843	120.90
Camden Creek, Camden, N. Y.	61.4	1889	24.10
Chester Creek, Delaware Co., Pa.	62.0	1843	1000.00
Ottor Creek, above Lowville, N. Y.	62.0	1928	34.8
Nine Mile Creek, Stittville, N. Y.	62.6	1898	124.90
Wissahickon Creek, Philadelphia, Pa.	64.6	1898	43.50
Winooski River, N. Branch, Writesville, Vt.	67.0	1927	257.00
Sandy Creek, Allendale, N. Y.	68.4	1891	87.70
Rock Creek, Washington, D. C.	77.5	....	126.30
Sudbury River, Farmington, Mass.	78.0	1897	41.38
Pequanock River, Pompton, N. J.	78.0	1902	55.78
Hockanum River, Conn.	79.0	....	78.10
Nashua River, Mass.	84.5	1850	71.04
Independence Creek, Crandall, N. Y.	93.2	1869	66.50
Moose River, N. Branch, above Lyons Falls, N. Y.	94	1928	21.20
Passaic River, Chatham, N. J.	100	1903	17.20
Deer River, Deer River, N. Y.	101	1869	78.10
Wanaque River, N. J.	101	1882	66.00
Tohicken Creek, Mount Pleasant, Pa.	102	1894	138.30
Fisk Creek, East Branch, Point Rocks, N. Y.	104	1897	80.50
Onondaga Creek, Syracuse, N. Y.	108.0	1913	30.00
Nashua River, Mass.	109	1848	104.53
Sandy Creek, North Branch, Adams, N. Y.	110	1897	67.30
Scantic River, North Branch, Conn.	118	....	51.80
Ramapo River, Mahwah, N. J.	118	1903	105.09
Rockaway River, Boonton, N. J.	125	1902	22.24
Patuxent River, Laurel, Md.	127	1915	40.02
Neshaminy Creek, below forks, Pa.	139	1894	97.60
Oriskany Creek, Colemans, N. Y.	141	1888	55.80
Mad River, Moreton, Vt.	143	1927	161.00
Oriskany Creek, Oriskany, N. Y.	144	1904	29.00
Perkiomen Creek, Frederick, Pa.	152	1894	115.80
Mohawk River, Ridge Mills, N. Y.	153	....	46.40
Mohawk River, State dam, Rome, N. Y.	158	1904	27.34
Ramapo River, Pompton, N. J.	160	1882	65.88
Westfield River, Knightville, Mass.	162	1924	65.00
Fish Creek, E. Branch, Taberg, N. Y.	169	1913	65.09
Stillwater Reservoir Water Shed, N. Y.	178	1928	26.70
Fish Creek, W. Branch, McConnellsville, N. Y.	187	1885	32.70
Pawtucket River, Providence, R. I.	190	1867	56.85
Catskill Creek, S. Catro, N. Y.	210	1901	100.00
Unadilla River, New Berlin, N. Y.	204	1905	40.00
Moose River, S. Branch, above Lyons Falls, N.Y.	216	1928	47.80
Salmon River, Altmar, N. Y.	221	....	27.60
Black River, Forestport, N. Y.	268	....	39.00
Piscataquis River, Foxcroft, Me.	286	1909	77.62

Maximum Rate of Discharge of Streams in the United States—Cont.

Stream and place	Drainage area, sq. mi.	Date	Cu. ft. per sec. per sq. mi.
Antietam Creek, Sharpsburg, Md.	295	1902	23.17
Croton River, Croton Dam, N. Y.	338.8	....	74.87
Great River, Westfield, Mass.	350	1878	151.40
East Canada Creek, Dolgeville, N. Y.	356	1898	24.70
West Canada Creek, Hinckley, N. Y.	372	1869	104.57
Pompton River, Two Bridges, N. J.	380	1903	61.60
Winooski, Montpelier, Vt.	396	1927	14.40
Moose River, Ayers Mill, N. Y.	407	....	31.00
Winooski, Montpelier, Vt.	420	1912	48.00
Stony Creek, Johnstown, Pa.	428	....	70.00
Ausable River, Ausable Forks, N. Y.	444	1913	56.30
Mississippi River, below Richford, Vt.	445	1913	23.00
Deerfield River, Shelburne Falls, Mass.	501	1909	42.51
West Canada Creek, Middleville, N. Y.	518	1898	24.90
Farmington River, Conn.	584	....	41.70
Hoosic River, Johnsonville, N. Y.	605	1913	38.01
Pemigewasset River, Plymouth, N. H.	615	1923	46.00
Monocacy River, near Frederick, Md.	660	1902	31.00
White River, West Hartford, Vt.	687	1913	44.00
Passaic River, Little Falls, N. J.	773	1882	24.20
North River, Port Republic, Va.	804	1896	29.80
Passaic River, Dundee, N. J.	823	1903	43.56
North River, Glasgow, Va.	831	1896	44.80
Raritan River, Boundbrook, N. J.	806	1882	64.52
Potomac, North Branch, Cumberland, Md.	891	1897	22.80
Black River, Lyons Falls, N. Y.	897	1869	46.00
Schoharie Creek, Fort Hunter, N. Y.	909.3	1913	44.56
Winooski River, Richmond, Vt.	985	1904	29.80
Winooski River, Essex Junction, Vt.	1 016	1927	114.00
Genesee River, Mount Morris, N. Y.	1 070	1896	39.20
Penobscot River, E. Branch, Grindstone, Me.	1 100	1909	23.36
Mohawk River, Little Falls, N. Y.	1 306	1913	26.25
Youghiogheny River, Connellsville, Pa.	1 320	1907	41.20
Greenbrier River, Alderson, W. Va.	1 344	1913	44.76
Black River, Carthage, N. Y.	1 812	1869	21.20
Black River, Watertown, N. Y.	1 878	1928	18.00
Schuylkill River, Fairmount, Pa.	1 915	1898	12.20
Chemung River, Elmira, N. Y.	2 055	1889	67.10
James River, Buchanan, Va.	2 058	1896	15.60
Androscoggin River, Rumford, Me.	2 220	1869	25.00
Susquehanna River, Binghamton, N. Y.	2 350	1902	26.60
Genesee River, Rochester, N. Y.	2 365	1865	17.00
Kennebec River, betw. Forks and Waterville, Me.	2 700	1901	48.56
Hudson River, Fort Edward, N. Y.	2 825	1900	15.60
Shenandoah River, Millville, W. Va.	2 995	1896	46.65
Connecticut River, Fairlee, Vt.	3 100	1913	18.50
Mohawk River, Rexford, N. Y.	3 384	1892	23.10
Mohawk River, Cohoes, N. Y.	3 472	1913	28.50
Merrimac River, Lowell, Mass.	4 085	....	19.80
Kennebec River, Waterville, Me.	4 270	1901	35.36
Susquehanna, W. Branch, Williamsport, Pa.	4 500	....	11.60
Hudson River, Mechanicville, N. Y.	4 500	1913	26.67
Merrimac River, Lawrence, Mass.	4 553	....	23.40
Potomac River, Dam No. 5, Md.	4 640	....	22.20

Maximum Rate of Discharge of Streams in the United States

Stream and place	Drainage area, sq. mi.	Date	Cu. ft. per sec. per sq. mi.
Delaware River, Lambertville, N. J.	6 500	....	53.80
Delaware River, N. J.	6 750	....	50.00
Delaware River, Stockton, N. J.	6 790	1841	37.59
Susquehanna River, Northumberland, Pa.	6 800	1889	17.50
Juniata River, Newport, Pa.	3 380	1889	53.80
Connecticut River, Sunderland, Mass.	8 000	1913	17.00
Connecticut River, Holyoke, Mass.	8 660	1854	21.10
Potomac River, Point of Rocks, Md.	9 654	1889	48.90
Connecticut River, Hartford, Conn.	10 234	1854	20.00
Potomac River, Md.	11 043	....	42.60
Allegheny River, Freeport, Pa.	11 400	1891	26.60
Potomac River, Great Falls, Md.	11 427	1889	41.15
Potomac River, Chain Bridge, D. C.	11 545	1893	17.20
Susquehanna River, Harrisburg, Pa.	24 030	1889	30.60
Camp Brach River, Ensley, Ala.	7.43	1909	68.77
Cane Creek, Bakersville, N. C.	22	1901	1341.00
Bear Grass Creek, Louisville, Ky.	27.5	1908	100.00
Elkhorn Creek, Keystone, W. Va.	44	1901	1363.00
Tocca River, Blueridge, Ga.	231	1901	53.20
Middle Oconee River, Athens, Ga.	395	1902	49.52
Pacolet River, Spartanburg, S. C.	400	1903	88.90
Tygart Valley River, Belington, W. Va.	403	1907	40.88
Hiwassee River, Murphy, N. C.	410	1899	54.54
Coosawattee River, Carters, Ga.	532	1901	31.86
Occoquan Creek, Occoquan, Va.	546	1915	38.30
Tugaloo River, Madison, S. C.	593	1905	36.86
Etowah River, Canton, Ga.	604	1895	31.50
Tuckasegee River, Bryson, N. C.	662	1899	58.23
Little Tennessee River, Judson, N. C.	675	1901	85.30
Broad River, Carlton, Ga.	762	1902	38.22
Holston River, S. Fork, Bluff City, Tenn.	828	1902	39.80
Shenandoah River, N. Fork, Riverton, Va.	1 037	1901	20.86
Saluda River, Waterloo, S. C.	1 056	1903	18.00
Flint River, near Woodbury, Ga.	1 090	1913	32.35
Greenbriar River, Alderson, W. Va.	1 344	1913	44.76
Catawba River, Catawba, N. C.	1 535	1901	61.89
Chattahoochee River, Oakdale, Ga.	1 560	1899	27.92
Shenandoah River, S. Fork, Front Royal, Va.	1 570	1902	48.92
Ocmulgee River, Macon, Ga.	2 425	1902	20.97
New River, Radford, Va.	2 725	1900	63.78
Catawba River, near Rock Hill, S. C.	2 987	1901	50.50
Shenandoah River, Millville, W. Va.	2 995	1896	46.65
Chattahoochee River, West Point, Ga.	3 300	1901	26.86
Yadkin River, Salisbury, N. C.	3 399	1899	38.30
Tallapoosa River, Milstead, Ala.	3 840	1901	18.23
Coosa River, Rome, Ga.	4 001	1901	16.04
Broad River, Alston, S. C.	4 609	1901	28.44
Black Warrior River, Tuscaloosa, Ala.	4 900	1895	38.80
New River, Fayette, W. Va.	6 200	1899	17.83
Coosa River, Riverside, Ala.	6 850	1898	10.53
Savannah River, Augusta, Ga.	7 294	1884	42.50*
Roanoke River, Old Gaston, N. C.	8 350	1877	32.90
Kanawha River, Charleston, W. Va.	8 900	1875	13.50

*Average flow for day of maximum discharge.

Maximum Rate of Discharge of Streams in the United States—Cont.

Stream and place	Drainage area, sq. mi.	Date	Cu. ft. per sec. per sq. mi.
Yazoo River, Miss	13 850	....	10.04
Tennessee River, Chattanooga, Tenn	21 382	1867	34.37
Ohio River, Wheeling, W. Va	23 800	1884	20.80
Tennessee River, Florence, Ala	30 800	1897	16.20
Ohio River, Louisville, Ky	90 600	1913	8.49*
Mississippi River, Columbus, Ky	930 540	1858	1.59
Mississippi River, Miss	1 244 000	....	1.19
Cherryvale Creek, Cherryvale, Kan	2	....	930.00
Portage Creek, Kalamazoo, Mich	48	1916	31.00
Loramie Res., Outlet, O	72	1913	97.22
Devils Creek, Viele, Ia	143	1905	1300.00
Whiteface River, below Meadowlands, Minn	446	1916	13.15
Little Wolf River, Royalton, Wis	485	1914	11.00
Olentangy River, Columbus, O	520	1913	115.30
Silver Creek, near Lebanon, Ill	335	1908	15.64
Des Plaines River, Riverside, Ill	630	1889	20.80†
Milwaukee River, Milwaukee, Wis	661	1915	7.98
Kettle River, near Sandstone, Minn	825	1912	7.15
Scioto River, Columbus, O	1 047	1913	80.82
Elkhorn River, near Norfolk, Neb	2 470	1903	3.24
Sangamon River, Riverton, Ill	2 560	1911	7.50
Saline River, Beverly, Kan	2 730	1896	5.86
Verdigris River, Liberty, Kans	3 067	1904	16,45
Wabash River, Logansport, Ind	3 163	1904	17.99
Neosho River, Iola, Kans	3 670	1904	20.30
Grand River, Grand Rapids, Mich	4 900	1905	10.00
St. Croix River, Minn	5 950	1883	6.00
Fox River, Rapide Croche Dam, Wis	6 200	1895	2.49
Cedar River, Cedar Rapids, Ia	6 320	1917	9.00
Chippewa River, Eau Claire, Wis	6 740	1905	9.00
Smoky Hill River, Ellsworth, Kans	7 980	1903	1.43†
Blue River, Manhattan, Kans	9 490	1903	9.13
Illinois River, Peoria, Ill	13 480	1904	5.94
Loup River, Columbus, Neb	13 540	1896	5.17
Republican River, Junction, Kans	25 837	1903	1.80†
Mississippi River, St. Paul, Minn	35 700	1867	3.27
Mississippi River, Prescott, Wis	44 070	1881	2.50
Platte River, Columbus, Neb	56 900	1905	0.83
Kansas River, Lecompton, Kan	58 550	1903	3.98
Kansas River, Lawrence, Kans	59 841	1903	3.80
Mississippi River, Grafton, Ill	171 570	1883	2.10
Missouri River, Sioux City, Ia	323 462	1881	1.64
Baker Creek, Baker, Nev	10	1914	17.00
Willow Creek, Heppner, Ore	20	1903	1800.00
Pinal Creek, Globe, Ariz	25	1904	560.00
Chalk River, Fillmore, Utah	38	1914	12.85
Gallinas River, Las Vegas, N. Mex	90	1904	129.10
Asay Creek, Hatch, Utah	96	1913	16.70
Ohanapecosh River, near Lewis, Wash	116	1909	64.70
Rio Mora, below Mora, N. Mex	159	1904	139.70
Sapello River, Los Alamos, N. Mex	221	1904	36.70
Miller Creek, Lovelia, Ore	270	1907	24.93

* Maximum daily. † Average flow for day of maximum discharge.

Maximum Rate of Discharge of Streams in the United States—Cont.

Stream and place	Drainage area, sq. mi.	Date	Cu. ft. per sec. per sq. mi.
St. Regis River, St. Regis, Mont	278	1913	22.40
Rapid Creek, Rapid, S. Dak	320	1904	2.85
Carson River, E. Fork, Gardnerville, Nev	381	1904	8.69
Rio Mora, Weber, N. Mex	422	1904	65.70
Grand River, N. Branch, Haley, N. D	500	1913	11.60
Yakima River, Cle Elum, Wash	500	1915	51.20
Price River, Helper, Utah	530	1913	8.50
Moyie River, Snyder, Idaho	717	1913	11.15
Purgatory River, Trinidad, Colo	742	1904	61.20
Clearwater River, S. Fork, Grangeville, Id	940	1912	10.46
Cut Bank Creek, Cut Bank, Mont	971	1908	9.07
Redwater River, Belle Fourche, S. D	1 006	1904	8.00
Virgin River, Virgin, Utah	1 020	1912	11.90
Truckee River, Reno, Nev	1 070	1913	7.02
Cowlitz River, Mossy Rock, Wash	1 170	1906	43.50
Wenetchee River, Dryden, Wash	1 200	1913	20.10
Heart River, Richardton, N. D	1 250	1906	6.40
Hondo River, reservoir, N. Mex	1 387	1904	4.56
San Juan River, Arboles, Colo	1 390	1911	28.80
Canadian River, French, N. Mex	1 478	1904	105.56*
Rogue River, Tolo, Ore	2 020	1909	23.90
Truckee River, Clark, Nev	1 740	1911	3.00
Yellowstone River, Corwin Springs, Mont	2 630	1911	8.67
Pecos River, Santa Rosa, N. Mex	2 649	1904	17.56
Canadian River, Taylor, N. Mex	2 832	1904	32.11†
Salt Creek, at mouth, N. Mex	3 052	1904	4.10
Spokane River, Spokane, Wash	4 000	1894	8.80
White River, near Interior, S. D	4 090	1905	4.03
Clearwater River, Kamiah, Id	4 850	1913	15.80
Willamette River, Albany, Ore	4 860	1861	62.20
Guadalupe River, near Cuero, Tex	5 020	1903	14.20
Yakima River, Kiona, Wash	5 520	1906	11.50
Salt River, Roosevelt, Ariz	5 756	1893	36.0
Verde River, McDowell, Ariz	6 000	1893	24.05‡
Pecos River, Fort Sumner, N. Mex	6 191	1904	7.29
Gunnison River, Whitewater, Colo	7 863	1905	3.67
Rio Grande, Rio Grande, N. Mex	11 250	1904	2.75
Canadian River, Logan, N. Mex	11 440	1904	12.29§
Salt River, Ariz	12 000	1891	24.69
Yellowstone River, Huntley, Mont	12 000	1907	4.03
Salmon River, Whitebird, Idaho	13 600	1913	5.97
Humboldt River, Oreana, Nev	13 800	1897	0.22
Pecos River, Roswell, N. Mex	14 840	1904	3.75
Grand River, Fruita, Colo	16 800	1909	3.81
Gila River, Florence, Ariz	17 750	1891	7.50
Missouri River, Cascade, Mont	18 300	1908	2.70
Big Horn River, Hardin, Wyo	20 700	1908	1.97
Red River, Grand Forks, N. D	25 000	1897	1.70
Clark Fork River, Metaline Falls, Wash	25 600	1913	4.33
Colorado River, Austin, Tex	34 200	1900	3.57
Red River, Ark	97 000	....	2.32
Arkansas and White Rivers, Ark	189 000	....	0.84

* Rate for 0.5 hour. † Rate for 7 hours. ‡ Rate for 24 hours. § Rate for 12 hours.

Maximum Rate of Discharge of Streams in the United States—Cont.

Stream and place	Drainage area, sq. mi.	Date	Cu. ft. per sec. per sq. mi.
Colorado River, Yuma, Ariz	225 000	1909	0.67
Columbia River, The Dalles, Ore	237 000	1894	1.89
Missouri River, St. Charles, Mo	530 810	1883	1.13
Mississippi River, St. Louis, Mo	702 380	1883	1.28
Switzer Canyon, San Diego, Calif	3.55	1916	188.00
Grand Central River, below Forks, Alaska	14.6	1906	100.00
Arroyo Seco River, Pasadena, Calif	16.4	1916	192.00
Yuba River, Bowman Dam, Calif	19	....	31.60
Sweetwater River, Descanso, Calif	43.7	1916	226.00
San Vicente Creek, Foster, Calif	74.9	1916	248.00
Kruzgamepa River, Outlet Salmon, Alaska	84	1902	51.20
Otay River, Lower Otay Reservoir, Calif	98.6	1916	379.00
San Jacinto River San Jacinto, Calif	108	1916	278.00
Sweetwater River, Jamacho, Calif	172	1916	250.00
Santa Ana River, Mentone, Calif	182	1914	46.70
Sweetwater River, Sweetwater Dam, Calif	186	1916	177.00
Santa Ynez River, Santa Barbara, Calif	207	1907	45.65
San Gabriel River, Azusa, Calif	222	1916	180.00
Calaveras River, Jenny Lind, Calif	395	1911	176.20
Chatanika River, below Poker Creek, Alaska	456	1911	7.62
San Luis Rey River, Oceanside, Calif	565	1916	169.00
Stony Creek, Fruto, Calif	760	1904	29.21*
San Joaquin River, Friont, Calif	1 170	1927	31.50
Kings River, Sanger, Calif	1 191	1927	31.40
Yuba River, near Smartsville, Calif	1 220	1909	90.91
Tuolumne River, Lagrange, Calif	1 501	....	30.60
San Joaquin River, Hamptonville, Calif	1 637	1881	36.51*
King River, State Point, Calif	1 742	1901	25.22
American River, Fair Oaks, Calif	1 910	1907	62.20
Birch Creek, Fourteen Mile House, Alaska	2 150	1911	6.90
Kern River, Rio Bravo, Calif	2 345	1897	2.3 *
Feather River, Oroville, Calif	3 640	1907	51.37*
Sacramento River, Iron Canyon, Calif	9 295	1904	23.47*
Sacramento River, Red Bluff, Calif	10 400	1909	24.42
Yukon River, Eagle Alaska	122 000	1911	2.08
Chagres River, Alhajuela, Panama	427	1909	398.10
Chagres River, Bohio, Panama	779	1909	115.50
Chagres River, Gatum, Panama	1 320	1909	93.90

* Mean for day when discharge was a maximum.

44. Storage of Water

The Mass Diagram. To determine either the amount of storage necessary to utilize at an average or variable rate of consumption the entire flow of a stream or any part of it, or the amount of water that can be continuously utilized from a stream, the flow for a considerable period being known, the most simple process is by the aid of the so-called mass diagram. The reliability of this process depends upon the completeness and accuracy of the information as to the runoff of the stream. A comparison of the runoff data of the Ohio, Tennessee, Sudbury, and Colorado rivers by John C. Hoyt in Eng. News, April 23, 1908, Vol. 59, p. 459, shows that a single year's record is entirely insufficient for reliable prognostication and that five-year periods in the humid sections usually give mean results within 10% of the normal average, but may be in error as much as 30%, and in the arid region in error from 4 to 88%. Ten-year averages in the humid region are usually accurate within less than 10%, but on the Colorado the error ranged from 1 to 48%. It may therefore be concluded that at least a five-year record should be secured or a much longer one constructed from rainfall data, to obtain results of reasonable accuracy in the humid regions, and that a prediction of runoff in the arid region should be adopted with extreme caution.

Having the desired record, the beginning for purposes of computation may advantageously be taken at the last of the latest low-water month of the earliest year, but such a starting point is not essential. The diagram is usually constructed with a time unit of one month, although when only a part of the flow is to be stored a weekly unit may be preferred. The record should be arranged chronologically in a table, and after each item the sum of it and all the preceding items should be entered. These sums then represent the total quantity of water that has been discharged by the stream at the end of any period since the beginning of the tabulated observations, and may be expressed in cubic feet per second, or any other convenient unit. With unit of time as the abscissa and total discharge as the ordinate, the mass curve is plotted, and if the lowest possible line is drawn tangent to the curve at two points an ordinate to

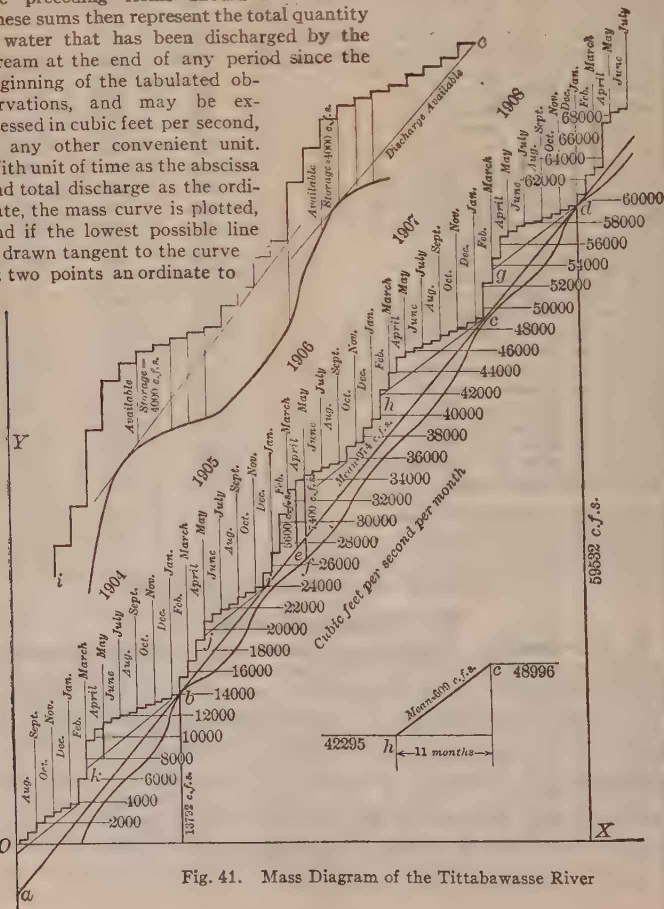


Fig. 41. Mass Diagram of the Tittabawasse River

this line will represent the maximum quantity of water that could have been used from the stream for the period between the ordinate considered and the intersection of the tangent with the axis of abscissa, assuming the quantity to be used uniformly during the period and a similar use to continue indefinitely. This ordinate divided by the time above included will then give the quantity to be so used per unit of time. The amount of storage required to equalize this flow will then be represented by the maximum partial ordinate between the tangent and the curve.

Fig. 37 shows a mass curve for a river in Michigan. In this case, for the sake of distinctness, the mass data have been plotted as a stepped curve rather than as a smooth one, the discharge being in cubic feet per second per month. When this is done it must be remembered that the ordinates considered are always those to the right-hand end of the step. The tangent line *bd* shows the average daily available yield to be, by the ordinate at *d*, 59 532 cu. ft. per sec. divided by the time from the last of December, 1903, to January, 1909, inclusive, about 61.2 months, or 974 cu. ft. per sec. The partial ordinate between the point *e* on the tangent and the right end of the step of the mass curve for April, 1906, gives 5605 cu. ft. per sec. for one month, or $5605 \times 60 \times 60 \times 24 \times 30 = 14\,528\,160\,000$ cu. ft. as the storage required to provide a constant flow of 974 cu. ft. per sec.

If instead of a constant use a variable one is desired, a mass curve of the variable use should be plotted in the same manner as the original diagram, but this variable flow curve must be of such dimensions that it falls either below or is tangent to the original mass curve. Such curves will usually have an opposite phase to the mass curve and hence must fall below or tangent to the line *bd*, as shown by the lower sinuous curve of the figure. The storage required to provide a supply of the nature represented by this curve is shown by the partial ordinate at *f* between the consumption and the mass curves to be 7400 cu. ft. per sec. for one month, or 19 180 800 000 cu. ft.

If it is desired to determine the amount of storage necessary to develop continuously a less quantity than the total flow of the stream, draw from the low points of the mass curve a line having an inclination parallel to the mass consumption line for the quantity desired. In the figure the lines *dg*, *ch*, *bk*, etc., represent such consumption lines, and the maximum storage required for the corresponding uniform consumption is represented by the partial ordinate for May, 1904, between the consumption line *bk* and the mass curve. The intersection of the consumption line, with the vertical of the mass curve, indicates the time when storage should have begun, in the particular year, to yield the consumption represented by the line, and the ordinates between consumption and mass curves represent the amount of water in storage at the time shown by each ordinate. The triangle in the lower right of the figure shows the rate represented by the lines *dg*, etc.

To determine the equalizing value upon the flow of the stream of a given amount of storage, plot downwards from the mass curve for the six high months of each year the magnitude of the storage in the unit of the diagram. Join the points so located in each year by a curve as in the upper diagram of the figure, in which *ic* is an enlarged representation of the mass curve from *i* to *c*, and from each of the succeeding low points of the mass curve draw a tangent to the curve just located. The minimum inclination of such tangents which do not cut the mass curve will give the rate of consumption that may be uniformly utilized with the storage in question. This method fails when the storage considered is so great as to exceed the maximum partial ordinate between the tangent *bd* and the mass curve, as in that case the storage could not be filled by the stream between dry periods.

Corrections for Evaporation. When from the amount of storage needed as shown by the mass diagram and a study of the topography of the watershed the area of water surface of the required reservoir is determined, the evaporation may be compiled and the net yield of the stream determined. A corrected mass diagram may then be constructed for more accurate computations.

Bank Storage. The visible capacity of reservoirs is very materially augmented in sandy or gravelly districts by the water held in the adjacent ground between the high and low water planes. The volume of such storage is dependent on the slope of the natural water table and on the proportion of voids in the soil. The former can frequently be determined by observations on wells in the territory affected, and the latter for the most porous soils may amount to 40% of the total volume, but for ordinary sands and gravels may be safely taken at about 20%. For clay it is inconsiderable, probably about 3 or 4%, and for loam about 10%. Sandstone rocks take up about 15% of their volume of water, limestone 5%, shales 4%, and granites 1%. The difficulty of estimating the value of bank storage lies in the uncertainty as to the actual character of the strata below the surface. The water stored in the ground is given out much less rapidly than that from an open reservoir, and hence is of little value for sudden fluctuations of draft, but in a long gradual lowering of the surface of the reservoir will add materially to the quantity available.

Emptying of Reservoirs. The time required to empty a pond or reservoir between known levels through a certain opening, or over a weir of known dimensions, is obtained from the general formula

$$t = \int_{H_2}^{H_1} \frac{A_r}{CA_0 \sqrt{2\,gh}}\,dh \tag{1}$$

in which A_r is the area of the reservoir at any elevation and A_0 is the area of the orifice through which discharge takes place, while H_1 is the elevation of the water surface, H_2 a lower elevation, h the height of the water surface above the center of the orifice, and C the coefficient of discharge. If A_r is variable it must be expressed in terms of h. When A_r is constant and $H_2 = 0$ this becomes $t = 2\,A_r \sqrt{H_1}/CA_0 \sqrt{2\,g}$, which is twice the time required for the same quantity to be discharged under a constant head H_1. For a weir, the expression for weir discharge is substituted for that of the orifice in the denominator of (1).

(a) In practice the area of reservoirs will be variable, and where definite information as to the relation between Ar and h is not available the reservoir volume may be assumed as semi-parabolic in longitudinal section and parabolic in cross-section and plan. With X as the length, Y as the half width, and Z as the depth, the area of the surface is 4/3 XY and the area at any elevation is $A = 4/3\ X_1Y_1Z/Z_1$ where X_1, Y_1 and Z_1 apply to the original surface considered. If a is the height of the center of the orifice or of the crest of the weir above the reservoir bottom $Z = h + a$ and $Ar = 4/3\ X_1Y_1\ (h + a)/Z_1$, which may be substituted in (1) before integration. (b) Another assumption which may be made is that X is directly proportional to Z, which is equivalent to saying that the reservoir has a uniform slope lengthwise. In this case $Ar = 4/3\ X_1Y_1\ (h + a)^{3/2}/Z_1^{3/2}$. (c) If it is assumed that Y also varies directly with Z, or that the sides of the reservoir slope uniformly to the center, which is, however, a condition never met with in natural ponds, then $Ar = 4/3\ X_1Y_1\ (h + a)^2/Z_1^2$. Of the above assumptions that under (b) is the most likely to conform to natural conditions, except where the reservoir has silted considerably, when assumption (a) will be more accurate. It may be added that since 4/3 X_1Y_1 in cases (a), (b), and (c) gives the area of the pond surface, if this is known it may be substituted at once for Ar, when the only assumption involved is that of the form of the vertical sections.

The time required to empty a vessel of rectangular cross and longitudinal section is twice that required to discharge its volume at the initial head. The time required to discharge one-half the contents of such a vessel into one of the same size and shape with their bottoms at the same level is equal to the time required to empty the vessel under the initial head.

The time required to empty a wedge-shaped vessel with parallel ends, vertex down, or a right paraboloid of revolution, vertex down, or any vessel in which the areas at different heights vary as the first power of their distance above the bottom is 4/3 of that required to discharge its volume under the initial head.

For a pyramid or cone with vertex down the factor is 6/5.
For a bottom hemisphere 7/5.
For a sphere 8/5.

When the reservoir is of irregular shape so that the foregoing assumptions are not sufficiently accurate, recourse must be had to Simpson's Rule and the distance $H_1 - H_n$ through which the water falls may be divided into an even number n of equal spaces and the area of the surface A_r at each of the so located positions determined by survey or computation from the known shape of the reservoir, calling them $A_{r1}, A_{r2}, A_{r3} \ldots . A_{rn}$. The approximate time required then is:

$$t = \frac{H_n - H_1}{3\,CA_0\sqrt{2g} \times n}\left[\frac{A_{r1}}{H_1^{1/2}} + 4\left(\frac{A_{r2}}{H_2^{1/2}} + \frac{A_{r4}}{H_4^{1/2}} + \frac{A_{r6}}{H_6^{1/2}} + \ldots \frac{A_{rn-1}}{H_{n-1}^{1/2}}\right) + 2\left(\frac{A_{r3}}{H_3^{1/2}} + \frac{A_{r5}}{H_5^{1/2}} + \ldots \frac{A_{rn-2}}{H_{n-2}^{1/2}}\right) + \frac{A_{rn}}{H_n^{1/2}}\right]$$

where H_1, H_2, etc., are the heights of the surfaces.

The volume, V, discharged may be similarly obtained from:

$$dV = -A_r\,dH, \text{ whence } V = \int_n^1 A_r\,dH, \text{ or approximately:}$$

$$V = \frac{H_1 - H_n}{3n}[A_{r1} + 4(A_{r2} + A_{r4} + \ldots A_{rn-1}) + 2(A_{r3} + A_{r5} + \ldots A_{rn-2}) + A_{rn}]$$

For a prismatic tank of rectangular section, the volume varies as the head and the time required to empty portions of it varies as the square root of the head, whence the last quarter will empty in half the time and the last one-ninth in one-third of the time.

For a wedge-shaped vessel the volume varies as the square of the remaining head and the time varies as $h^{3/2}$.

For the time required to empty a pond over a weir from a head on the weir of h_1 to a head h_2 the equation is

$$t = 2\frac{A_r}{CL}\left(\frac{1}{\sqrt{h_2}} - \frac{1}{\sqrt{h_1}}\right)$$

the weir discharge being represented by $Q = CL\,h^{3/2}$, C being considered constant.

45. Auxiliary Steam Plants

Desirability. When sufficient storage cannot be economically secured to equalize the flow sufficiently to yield the required quantity of power, the deficiency may be made up by a steam plant. It is to be remembered that a steam plant depreciates about as fast when idle as when running, and that the interest charges are to be carried during the whole year. When operation, interest, depreciation, and maintenance of a steam auxiliary are capitalized it will often be discovered that storage would have been a better investment.

Capacity. The capacity of a steam plant necessary to make up the deficiency of flow is best determined by the use of a diagram of graded discharge, as in Fig. 42, in which, with the time unit as the abscissa, the discharges, either natural or equalized by storage as the plan may be, as ordinates, are plotted, not chronologically but in the order of their magnitudes from least to greatest. A horizontal line drawn through the diagram, whose ordinates represent the desired total capacity, limits between itself and the curve the total amount of water that must have been replaced by auxiliary power, and for each month the amount is given by its individual partial ordinate between the capacity line and the curve.

Operation of Combined Steam and Water Plants. When a steam auxiliary is used, it will usually be advantageous to run it continuously during the periods when it must run at all and use the water power to provide for the fluctuations, if the size of the pond at the plant permits this without wasting water. For the reason that a steam plant when idle is radiating heat, and hence not only wasting interest and depreciation charges but also a part of the fuel cost, it is better to let the water power lie idle while the pond fills up, as by such procedure nothing is lost, the full value of the water being utilized. On this account the importance of a large pond at the station is apparent.

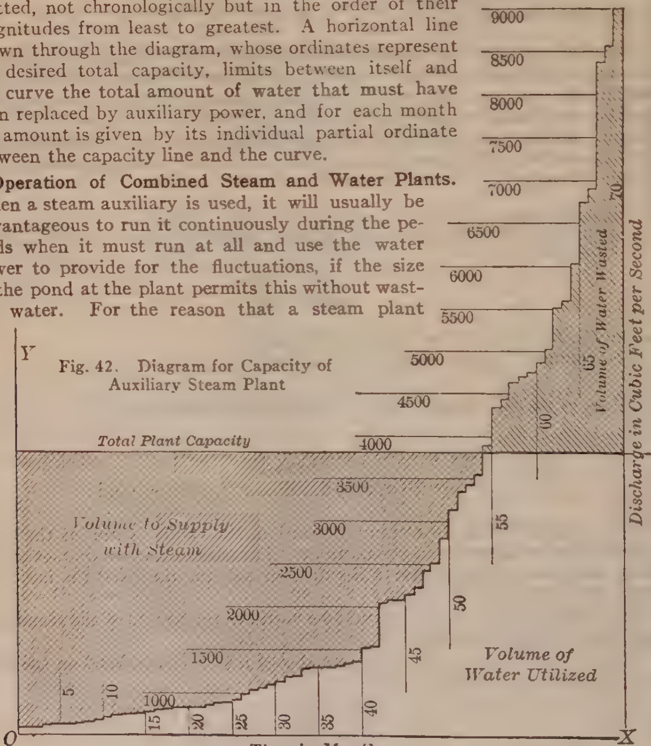


Fig. 42. Diagram for Capacity of Auxiliary Steam Plant

46. Design of a Power Plant

Power House. The Power House should be so located as to be protected from floating ice and other debris. When situated in a gorge, the position should be such as to permit a maximum of sunlight to reach the building. The intakes, whether direct to the wheelpits or to a closed flume or pipe line, should be so located that debris will readily pass by them. If water is flowing over the spillway or through the waste gate, these structures may be advantageously put near by, in order that they may be controlled to keep the intakes clear.

To avoid ice troubles the intake to the wheelpits or closed flumes may best communicate directly with the pond, rather than with a canal or head race, and a curtain wall may wisely be carried a few feet below the water surface at the entrance. To facilitate inspection the entrance to the wheelpits should be provided with easily operated gates.

See paper by Benj. F. Groat in Trans. Am. Soc. C. E., Vol. LXXXII, for special construction to divert ice.

Wheelpit. For low and moderate heads, up to 40 or 50 ft., the open wheelpit is to be preferred to the closed flume or pipe line. Of this head 20 ft. may be covered by the draft tube. Vertical wheel and umbrella type generators give higher efficiencies than horizontal shaft installations, due mainly to the reduction of bearings. The passages leading to the wheels should be so designed that the water is never retarded in its velocity as it passes toward the wheels but if possible is gradually accelerated. The losses between head and tail water in the best-designed plants, excluding the turbine loss, is about 4% of the total output for plants with heads below 20 ft., and may frequently amount to 12%. For higher heads the percentage is less, as there is little change in the rack losses and friction in flume, wheelpit, and draft tubes, while the power increases with the head. For approximate calculations a uniform loss of 1 ft. may be taken for all heads.

The **Draft Tube** should expand gradually from the outlet of the wheel until an area of outlet is attained such as to require a velocity through it a little greater than that of the water into which it discharges.

Draft Tubes may be classed as **Straight**, having the sides parallel, i.e. cylindrical, **expanding** when they form the frustum of a cone; **spreading** when they terminate in a trumpet mouth discharging radially around the full circumference; and **elbow** when the stream is turned within them, through an angle usually 90 deg. The straight draft tube is to be found only in old installations.

The proper design of the draft tube is one of the most important features of a hydraulic power plant as it may affect the efficiency and output of the turbines from 5 to 10% on low heads.

W. M. White, M. Am. Soc. C. E., has determined by very careful experiment the form assumed by a circular jet striking a flat plate perpendicularly and flowing off along it in all directions. (See Journal Assoc. Engr. Socs., Aug., 1901, and Trans. Am. Soc. Mech. Engrs., Vol. 43, May, 1921.)

It may therefore be expected that the form of draft tube that would offer the least resistance to the discharge of water through it would be of such form as that shown in Fig. 43. Such a tube, however, would not produce a regain of the velocity head,

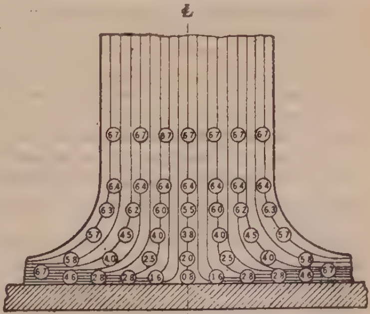


Fig. 43. Section of jet impinging on plate. Figures in circles indicate velocities. After White.

which is normally high at the outlet of the wheel. Therefore an advantage may be obtained by expanding the jet as it falls and the limiting angle of inclination of the sides of the tube has been found experimentally to be about 8 deg. from the axis when the jet is under less than atmospheric pressure and about 11 deg. when it is under atmospheric or greater pressures.

The hydrucone, Fig. 44, designed by Mr. White, embodies these conditions, its outlet being shaped similarly to the jet profile and water being discharged horizontally in all directions from the plate below its outlet.

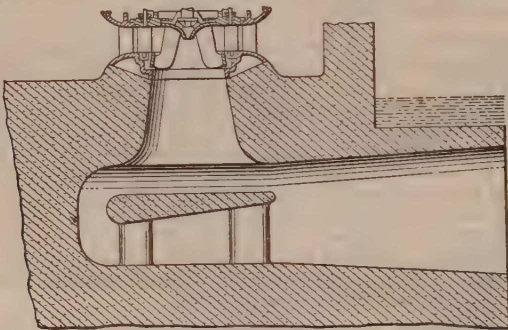


Fig. 44. White's Hydrucone

This device utilizes to some extent the centrifugal force of the discharging water and on tests has increased the efficiency about 6% over the best obtainable with the ordinary form of tube.

The same principles may be utilized in the design of an **elbow** draft tube by considering the flow through it to represent that through a circle whose diameter is the radius of the jet, as in Fig. 45.

The water passing through the circle whose diameter is *od* will be one-fourth of the total of the jet whose radius is *od* and, when the filaments have become parallel along the plate as at g' c', occupies the circle whose diameter is *oc* and would pass through the portion of the circle *vcw* covered by the arc *acb* if properly directed. But as some of the water in the circle *od* falls outside of the quadrant *oacb* it cannot follow the lines *oa* and *ob* but must be brought to the plane *ab* by paths bounded by some such lines as *oea* and *ofb*, and *ab* becomes the chord whose arc is the required width of the draft tube. If there is to be no further lateral expansion of the jet after passing the arc *acb*, then the direction of the outside filaments must be changed to tangency with the sides *am* and *bn* and this may be well accomplished by allowing a greater spread to the jet before it reaches *acb*; for this design curves such as *oka* and *ojb* give good results.

As the water is assumed to be traveling in the direction *oc* when it reaches *ab* it follows that to pass this plane it must be accelerated or else the chord *ab* must be extended to a length equal to the arc *acb*, giving the plan of the tube a form represented by the semicircle *pkojq* in which the filaments of water will have directions in planes indicated by the arrows. The line *pq* then becomes the horizontal projection of the throat of the draft tube and may fall anywhere between *t* and *c*. Experiment indicates that the area of the throat should not be greater than that of the jet at the plane where its curvature begins. Beyond the throat the tube may be caused to expand vertically along a line $g'r'$, making an angle with the bottom of the tube of not to exceed 22 deg., but better, not over 16 deg., until an area of tube is reached giving the desired outlet velocity.

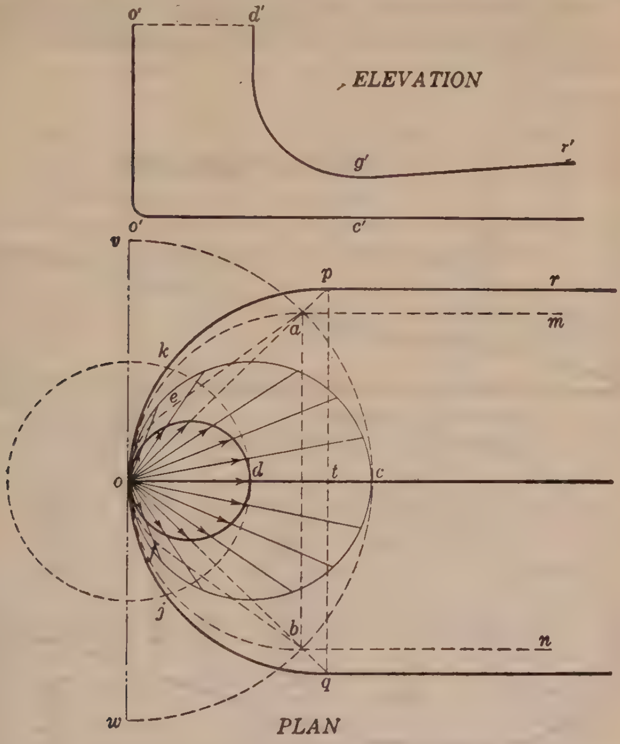


Fig. 45. Draft tube diagram

Having a tube of the general design just outlined, the jet being expanded from the outlet of the wheel downward (Fig. 46), it may be found that the width is too great for the space available; in this case the floor of the tube may be rolled up at the sides giving a U-shaped section having the same area as the original full-width one as shown in the lower half inverted plan. The length of the horizontal part of the tube must be increased to give the area at bb_2 that existed at rs in the full-width tube. The top of the upturned portion is represented by the broken line $a'm'j'b'$ in elevation and its boundaries by $ambb_2g_2a$ in the lower half inverted plan. The tube may be given a partial **spreading** characteristic by moving the center o downstream a short distance and raising the curve $a'm'$ to allow discharge into the upper projections of the tube directly from the whole of the upstream half of the descending jet. A tube of the latter type gave the highest efficiency of those tested for the Alabama Power Company at the Alden Hydraulic Laboratory, Worcester, Mass.; the report was published in Trans. Am. Soc. C. E., Vol. LXXXVII, pp. 892-926. For **spreading** draft tubes it is advantageous to insert a steep cone extending up the axis nearly to the outlet of the wheel.

The height of the draft tube, that is, the distance from the outlet of the wheel to the level of tail water, may theoretically be 34 ft., less the head corresponding to the change of velocity from the top of the tube to the outlet.

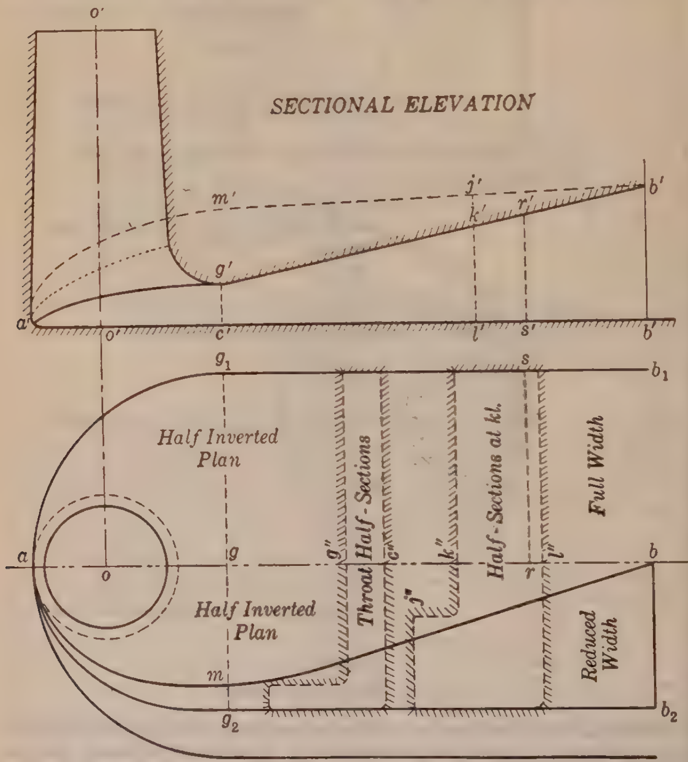


Fig. 46. Elbow Draft Tube

On account of the tendency of water to vaporize as it approaches the condition of flow in a vacuum it is found inadvisable to utilize this full height in the draft tube. In general practice a height of 20 ft. is found satisfactory with an upper limit of 24 ft., or possibly 26 ft. if the turbine will always run at or near full discharge.

Head or Fall Increasers are devices connected with the draft tube to increase the discharge by reducing the tail-water pressure. The most effective device of this kind appears to be that designed by Clemens Herschel, which is an application of the principle of the Venturi tube through the throat of which water is passed from the pond drawing into itself through apertures in the throat, water discharged by the wheel. (See Eng. News, Vol. 59, June, 1908, p. 635.)

Another device has involved the surrounding of the discharge from the wheel in the vertical part of its course with a stream from headwater.

By the introduction of a jet from headwater at the bottom of a curved draft tube the author has increased the output of a plant in flood time between 5 and 6% and by a utilization of the flow over the dam, if adjacent to the power-house, a considerable further advantage can be secured. The author's tests were made with the flow into the elbow at the center of the bottom of the draft tube. It has since been found to be more effective when introduced at the sides. Tests by the Power Corporation of Canada wherein the flow was introduced at the sides and somewhat above the bottom, showed increases of power as high as 30% due to the ejector flow.

The Generating Unit. In the earlier installations horizontal-shaft generators were largely used, either belted or geared to the turbines, or driven directly from a shaft on which several turbines were mounted. Tests of such installations have shown them to be considerably less efficient than the vertical-direct-connected units, and the multiple-wheel unit, whether vertical or horizontal, is not now looked upon with favor except under very special conditions.

The direct-connected unit requires that both turbine and generator shall be adapted to the same speed of rotation.

In selecting machinery the best speed of rotation is one slightly less than one-half the so-called spouting velocity, or velocity due to the head, but by specially designed wheels this can be increased as much as 50%, giving a fairly wide range of generator speeds from which to select.

The table on the following page gives data as to the vertical waterwheel generators, developed by the Allis-Chalmers Manufacturing Company, and may be taken as fairly representative of the machines of other manufacturers.

Current. Except where current is to be used in the immediate vicinity of the plant or for arc lighting or electric railway service, direct-current generators are not commonly used in the United States. For incandescent lighting, 3-phase 60-cycle alternating-current generators have a preference, but for power work 25- or 30-cycle machinery is considered more satisfactory. The current is frequently generated at about 2300 volts pressure, and is transformed upward to the line voltage for transmission. The line voltage may be anything between 440 and 150 000 volts; the higher the pressure the more power can be delivered over a given size of conductor, and consequently the low voltages are only used in local distribution or inside buildings.

Bearings. The bearings should be accessible at all times, and in the case of a vertical-shaft installation the entire load may be wisely concentrated on a single thrust bearing of special design. Such bearings may be placed either above or below the generator, and according to the loads to be carried may be of the simple plate type, roller bearings, the Kingsbury or the Spring bearing, or for very heavy loads the oil pressure bearing.

Of these the **plate bearing** consists of two plates, the lower of which is supported in a fixed position while the upper is fastened to the shaft and revolves with it, resting upon a film of oil between it and the lower plate, both plates being immersed in a bath of oil. Such bearings are satisfactory for speeds encountered in low-head plants.

The **roller bearing** consists of two plates as above with the film of oil replaced by a cage of rollers, either conical and of considerable length, or cylin-

Vertical Water Wheel Generators

Allis-Chalmers Manufacturing Company

60 Cycles, 2300 Volts or Less, 2 or 3 Phase

K.V.A.	R.P.M.	Outside diameter in inches	Approximate total weight	K.V.A.	R.P.M.	Outside diameter in inches	Approximate total weight
65	277	73.5	9 000	500	100	148	45 000
75	277	73.5	9 100	500	90	185	65 000
85	277	73.5	9 850	500	60	225	90 000
100	277	73.5	10 000	525	300	95.5	22 000
110	277	73.5	12 500	530	150	137	36 500
125	277	73.5	13 000	600	300	95.5	24 000
150	720	48	5 500	625	150	137	42 000
150	150	107	20 000	625	120	148	50 000
150	120	128	24 000	625	100	148	55 000
187-1/2	720	48	5 600	675	180	138	38 000
187-1/2	150	107	21 000	750	150	137	44 000
187-1/2	120	128	24 500	750	120	148	53 000
200	150	107	22 500	750	100	148	57 000
200	120	128	28 500	800	100	195	63 000
250	150	107	24 500	800	180	138	40 000
250	120	128	29 500	850	128	194	67 000
250	90	151	34 000	850	82	195	83 000
250	64	195	48 000	940	180	171	58 000
300	90	151	35 000	940	100	195	70 000
300	64	195	50 000	1 000	82	195	85 000
312	300	95.5	19 000	1 080	150	192	72 000
312	150	107	28 000	1 080	128	194	77 000
312	120	128	31 000	1 250	180	171	68 000
312	100	148	35 000	1 250	150	192	75 000
325	180	138	28 000	1 250	128	194	80 000
375	300	95.5	19 500	1 250	100	195	90 000
375	180	138	30 000	1 750	150	192	102 000
375	150	137	33 000	1 875	164	194	90 000
375	120	128	33 000	2 000	150	192	105 000
375	100	148	35 000	2 200	164	194	92 000
425	120	128	38 000	2 400	150	192	116 000
425	100	148	45 000	2 500	164	194	100 000
425	90	185	60 000	2 800	164	194	105 000
425	60	225	87 000	2 800	150	192	120 000
437	180	138	32 000	3 120	150	214	132 000
450	150	137	35 000	3 750	150	214	135 000
450	300	95.5	22 000	5 000	150	214	220 000
500	180	138	33 500	6 250	150	214	230 000
500	120	148	40 000	8 500	144	242	280 000
				10 000	144	242	280 000

drical and of length not greater than their diameter. The plates and rollers of this bearing are all immersed in an oil bath.

The **Kingsbury bearing** replaces the cage of rollers with a series of segments or shoes supported by a pivoting surface near one end and slightly beyond the center of gravity in the direction of rotation. This causes the space between the segment and the upper plate to be more open at one end than the other so that a wedge-shaped film of oil is drawn in by the revolving disk. The parts of this bearing are immersed in an oil bath and the bearing operates satisfactorily under surface pressures of 400 to 500 lb. per sq. in. in high-speed turbine installations.

The **spring bearing** introduces a series of spiral springs fixed on the lower plate and carrying above them a rubbing plate which takes the load of the upper plate on a film of oil. On account of the ready adjustment of the springs to irregularity of load the bearing can be made of smaller diameter than the others and hence occupies less space. Like the preceding, the parts are immersed in a bath of oil.

The **oil thrust bearing** provides for the introduction of oil under a high pressure to a chamber in the plates near the shaft from which it flows out between the plates, insuring complete lubrication. The loads carried depend upon the pressure of the oil which is circulated by a positive piston pump. These bearings are used for the heaviest loads and the highest speeds, and are particularly adapted to steam turbine work.

The Exciter may be most economically driven from the main turbine shaft, but in many plants an independent exciter driven by a separate wheel is installed. After the plant is once started, and as long as it continues in operation, the excitation may be provided by a motor generator or rotary converter transforming the alternating current of the main generators into direct current for their excitation. The economy of this is slightly less than that of the separate exciter so far as operation is concerned, but when interest is considered may prove more economical, so that a single separator exciter may be provided to excite the first machine of a series and the rest be operated from the main current as above indicated. The exciting power usually ranges from 0.5 to 3% of the power at full load, being nearly constant in amount whether the main generator is loaded to full capacity or not.

The Switchboard should be so located as to enable the operator to command a view of the machines which he controls, and should have the following instruments: (a) On generator panel, 1 voltmeter, 1 ammeter for each phase, 1 frequency meter and 1 field ammeter. (b) On feeder panel, 1 wattmeter. (c) On exciter panel, 1 voltmeter and 1 ammeter. (d) On a bracket 1 synchroscope when more than one generator is installed. To which are added the several switches and connections, and for studying the station output an integrating wattmeter should be placed on the main circuits.

The Governor for turbines operates upon the gates and must be capable of exerting a large amount of power at times. This is usually accomplished by a piston driven by oil which is maintained at a high pressure in a reservoir with compressed air by the aid of a force pump operated from the turbine shaft. The action of the piston is controlled by valves moved by the governing mechanism proper. For less rapid operation governors depending on simple mechanical friction are utilized, but these governors are not sufficiently delicate for combined lighting and power loads. In governing impulse wheels, less power is needed, and the nozzle is frequently deflected away from the bucket without altering the flow. In the Doble Nozzle a tapered pin is pushed forward through the center to close and pulled back to open, either operation requiring a relatively small amount of power. The governor should be so located as to be under the eye of the attendant at the switchboard, unless a second attendant is available to look after it.

The usually accepted equation for determining the torque T of the regulating shaft resulting from the operation of the gates, is:

$$T = \frac{C \times \text{horsepower of turbines}}{\sqrt{\text{head}}}$$

Where C has a maximum value of 50 and a minimum of 25.

For low-head open-flume plants the percentage temporary change in speed for load thrown off is given by:

$$d = 81\,000\,000 \frac{(\text{Hp.}) \times t}{WR^2 \times (\text{r.p.m.})^2};$$

d = percentage change of speed;
Hp. = maximum horsepower of turbine;
t = time in seconds occupied by governor in moving gates through their range;
W = weight of rotating parts;
R = radius of gyration of rotating parts;
r.p.m. = normal revolutions per minute.

Remote Control. By the use of special apparatus designed by either the Allis-Chalmers or the General Electric Company the operation of water power plants may be controlled from a distance so that a single operating crew may handle several plants and the apparatus may be in part operated by a float so as to be thrown on and off as the pond rises and falls.

Crane. As this piece of apparatus is only used occasionally, it may wisely be of the hand-operated type where first cost is an item of importance. Cranes operating with chains are preferable to those using cable for power-house work.

The Racks. The loss of head through the racks is of considerable importance, particularly in low-head plants, and may be considerably reduced by using lenticular or fish-shaped rack bars. A platform from which to rake debris from the racks and a chute or other device for removing it is important.

47. Operation of Plant

The Load Factor of a plant is the ratio of the average to the maximum load carried. For economical operation this should be brought as near unity as possible, and it becomes good policy to sell the output at a low rate during the low-demand hours to stimulate consumption at such times.

In ordinary plants supplying municipal, domestic, manufacturing and commercial service the load factor is usually between 40 and 50%, which means that for a large part of the time the plant runs at part capacity, but late in the afternoon it has for 1 or 2 hours a peak load of nearly the maximum amount, the actual maximum covering only a few minutes. The result of this is that such a plant must be designed to deliver very much more than the average capacity of the water utilized. Usually the machinery installed is made capable of delivering the entire daily output in from 6 to 8 hours.

The Plant or Capacity Factor is the ratio of the average load to the rated capacity of the plant.

The Diversity Factor is a term used to indicate the diversity of uses in which the output is absorbed and may be defined as the ratio of the sum of maximum demands of the subdivisions of the load to the maximum demand of the whole system, measured at the point of supply.

The Connected Load is the combined continuous rating of all the receiving apparatus on the customer's premises connected to the system.

The Demand Factor is the ratio of the maximum demand to the total connected load.

The Power Factor of a plant is the ratio of the effective power in watts to the volt-amperes, all measured at the switchboard. The existence of a power factor, which is peculiar to alternating-current installations, is due to the fact that there usually exists in a line an induced current which neutralizes in part the power current. The effect of the induced current may be reduced by special expedients, but with ordinary motor loads is likely to cause the power factor to drop to 0.85, and in some distributions it may fall as low as 0.30. The power factor is determined by the load and not by the generating apparatus, and so far as the latter is concerned should be taken as unity unless otherwise specified.

Except as the efficiency of the generator may be affected by a change in power factor, there is no effect upon the input to the generator, hence the turbine requirement is always based on the kilowatts delivered and not on the volt-amperes.

Regulation. If the speed of the generator changes, the voltage or pressure changes and the brilliancy of illumination and power of the output varies. Reducing speed reduces voltage, and vice versa. To maintain constant speed with a varying load requires a reduction or increase of gate opening, and where the wheels are supplied through a pipe line, a consequent change in velocity and pressure through the conduit.

Water-Hammer and Surge Tanks (see Warren, Trans. Am. Soc. C. E., Vol. LXXIX, p. 238). When power plants are supplied through long pipe lines it becomes necessary to provide against the water-hammer and surges occurring when the wheels are stopped and started if by manipulating gates. In plants using impulse wheels this is sometimes met by deflecting the nozzle so that the jet passes off the wheel when the latter is to be stopped and thus the flow of water is not interfered with, but water is wasted.

The magnitude of the water-hammer and surge increases as the rapidity of the stoppage or start of the flow. The **maximum hammer** occurs when the time t of closing the gate is instantaneous or less than the critical time $\frac{2L}{a}$, where L is the length of the pipe in feet and a is the velocity of vibration along the pipe in feet per second.

Then
$$h_h = \frac{av}{g} = \frac{145\,v}{\sqrt{1 + \frac{Kd}{Eb}}} \tag{1}$$

where h_h = head due to hammer, in excess of static head, g = the acceleration due to gravity, v = mean velocity in pipe in feet per second, d = inside diameter of pipe in feet, b = thickness of pipe wall in feet, K = modulus of elasticity of water in compression, taken as 42 400 000 lb. per sq. ft., and E = modulus of elasticity for material of pipe in tension, taken as 4 000 000 000 lb. per sq. ft. or 28 000 000 lb. per sq. in. for steel plate. Then

$$a = \frac{4660}{\sqrt{1 + \frac{Kd}{Eb}}}$$

For **ordinary water-hammer** that is, when the time of closing is greater than $\frac{2L}{a}$ the equation is:

$$h_h = \frac{Lv}{g\left(t - \frac{L}{a}\right)} \tag{2}$$

To counteract or reduce the hammer, standpipes or surge tanks are connected to the pipe line at or near the lower end.

For the **maximum surge** S_{max}, in a standpipe or surge tank either in starting or stopping the plant, the equation is:

$$S_{max} = H_f \left[1 + \frac{1}{2\,me} m' \left(\pi - \cos^{-1} m \right) \right] \tag{3}$$

where $m^1 = \frac{m}{\sqrt{1 - m^2}}$ and H_f = total loss of head in feet between feeding reservoir and standpipe, $m = \frac{H_f}{2\,Q} \sqrt{\frac{A_t\, g\, A_p}{L}}$ wherein Q = steady discharge in cubic feet per second, taken before gate starts to close or after gate is fully open and flow established. A_t is the area of the standpipe or surge tank and A_p the area of the pipe line, both in square feet. e = Napierian logarithmic base = 2.718.

In starting, S is measured from the water surface under static conditions and in stopping from a distance below this equal to H_f.

The foregoing equations do not take into account the effect of the operation of the waterwheel governor.

The following table gives a comparison of Initial Velocity, Time of Closure and Pressure rise by various authorities. See Kerr, Trans. A. S. M. E., Vol. 51, No. 6.

Initial velocity, ft. per sec.	Closure intervals 2 L/a	Rise in pressure in feet		
		By de Sparre-Gariel formulas	By R. S. Quick chart	By Gibson's arithmatical integration
10	10	51.8	51.8	51.5
9	9	51.8	51.8	....
8	8	51.8	51.0	....
7	7	51.8	51.0	....
6	6	51.8	51.2	....
5	5	51.8	53.8	54.1
4	4	58.2	59.6	....
3	3	66.5	67.4	68.0
2	2	77.7	78.5	....
1	1	93.3	93.2	93.26
0	0	0.0	0.0	0.0

To determine the **economical size** of penstock or pipe line the following formula may be used:

$$d = \sqrt[6]{\frac{3250\ QI}{C^2\ PB}} \tag{4}$$

wherein I = income in dollars per year per foot of head, C = the coefficient in the Chezy formula, $V = C\sqrt{rs}$, P = percentage return on investment including profit, depreciation, maintenance and taxes, and B cost of pipe in place per foot of diameter per foot of length, other symbols as before.

Differential Surge Tanks. From formula (3) it is evident that an increase in the area of the standpipe or surge tank reduces the surge, but for economic reasons it is not usually possible to make this area very large and often not so large as is desirable. To overcome this situation the differential surge tank has been introduced, which consists of a standpipe of about the same diameter as the pipe line, freely connected to it, and a storage tank of larger dimensions surrounding the standpipe and connected to the pipe line by a restricted passage (see Johnson, Trans. A. S. M. E., Vol. 30, p. 443, and Trans. Am. Soc. C. E., Vol. LXXVIII, p. 760), the whole being in such a condition that with a heavy surge water may overflow the standpipe into the surge tank whence it flows back to the pipe line through the restricted passage unsynchronously to the surges in the line itself.

48. Testing Code for Hydraulic Turbines

(Abstract)

(Approved by Machinery Builders' Society, Oct. 11, 1917)

Introduction. This code is intended to apply to acceptance tests of hydraulic machinery.

Efficiency of the plant takes account of all losses of energy between head water and tail water outside the plant as far as, or through the switchboard.

Efficiency of the machinery takes account of all losses of energy between the water in the wheelpit and tail water and as far as the generator terminals or switchboard.

Efficiency of the turbine takes account of all losses of energy between the water in the wheelpit and tail water and as far as the coupling on the turbine shaft. Losses through racks, in intake to penstock and in penstock shall not be charged against the turbine, nor shall the head necessary to discharge the water from the end of the draft tube. The net or effective head acting on the turbine shall be measured from a point near the intake to the turbine casing in turbines equipped with casings, or from a point immediately over the turbine in turbines having an open flume setting, to a point in the tail-race, and a correction for the velocity head required to discharge the water into the tail-race shall be added to the tail-water elevation; and a similar correction applied at the intake to encased turbines. The power developed by the turbine shall be taken as the mechanical power delivered on the turbine shaft and transmitted by the turbine shaft to the generator or other driven machine or system.

General. When the contract calls for the performance of the guide vanes, runner, draft chest and draft tube only, the velocity head at entrance to the casing shall be excluded, and the pressure shall be measured by piezometers so connected to the casing as to avoid velocity effects, instead of at entrance to the casing.

Apparatus shall be inspected before, during and after tests.

Testing apparatus must not interfere with operation of unit under test, which shall have been operated under load for at least 3 days prior to test.

Leakage of air into wheel or draft tube or of water out of wheelpit or penstock must be prevented or measured and allowed for.

Tests should not be made when variations of load exceed 3%; of head 2%, and of speed 1%, above or below the average of each.

Important instruments shall be installed in duplicate and calibrated before and after test.

Power Output. Power output may be measured by the electrical generator, or by a Prony brake or Dynamometer. If by a generator, the latter must be tested either in the shop or in the field and its efficiency curve established for the range covered by the tests, in accordance with the Standardization Rules of the Am. Inst. E. E. of September, 1916. The output of the wheel then is the kilowatt output of the generator divided by its efficiency and reduced to horsepower, the exciting current not being charged against the turbine. If the exciter is driven from the turbine, then its output similarly computed shall be added to that of the main generator.

When a dynamometer, either of the Prony brake, friction disk or other type is used, the dynamometer is to be so arranged as to avoid imposing either end thrust or side thrust on the turbine shaft and bearings, or to avoid adding any friction load which is not measured, and the brake must be capable of operating with the weighing beam floating free of the stops during the entire duration of a run.

Power Input or Water Horsepower. The turbine shall be tested, if possible, under the effective head stated in the contract, and at the speed specified in the contract.

Variation of 10% in head may be allowed if the speed is adjusted to correspond according to the law that

Speed varies as $h^{1/2}$
Power varies as $h^{3/2}$

Errors in adjustments of speed to head may be corrected for up to 2%, on the basis of test curves of the same or a similar turbine. The hydraulic equivalent of the speed is equal to the specified speed multiplied by the square root of the ratio of the effective head existing during the test to the specified effective head. The hydraulic equivalent of the horsepower is equal to the specified horsepower, multiplied by the three-halves power of the ratio of the effective head existing during the test to the specified effective head. Tests shall not be run when the head differs from the specification by more than 10%, or when the total draft head approaches within 5 ft. of the height of the barometric water column.

For turbines having closed casings the head is to be measured by at least two, and when possible not less than four piezometers connected to a straight portion of the penstock near the turbine casing intake, and by two or more board, rod or float gages in the tail-race, placed at points reasonably free from local disturbances.

The conditions of measurement, including velocity distribution, length of straight run of penstock, and conditions of piezometer orifices shall be such that no piezometer shall vary in its readings by more than 20% of the velocity head from the average of all the piezometers in the section of measurement. The piezometer orifices shall be flush with the surface of the penstock wall, the passages shall be normal to the wall, and the wall shall be smooth and parallel with the flow in the vicinity of the orifices. The piezometer orifices shall be approximately 1/4 in. in diameter.

The effective head on the turbine is taken as the difference between the elevation corresponding to the pressure in the penstock near the entrance to the turbine casing, and the elevation of the tail water at the highest point attained by the discharge from the unit test, the above difference being corrected by adding the velocity head in the penstock at the point of measurement and subtracting the residual velocity head at the end of the draft tube. The velocity head in the penstock shall be taken as the square of the mean velocity at the point of measurement, divided by 2 g; the mean velocity being equal to the quantity of water flowing in cubic feet per second, divided by the cross-sectional area of the penstock at the point of measurement in square feet. The residual velocity head at the end of the draft tube shall be taken as the square of the mean velocity at the end of the draft tube, divided by 2 g;

the mean velocity being equal to the quantity of water flowing in cubic feet per second, divided by the cross-sectional area of the penstock at the point of measurement in square feet. The residual velocity head at the end of the draft tube shall be taken as the square of the mean velocity at the end of the draft tube, divided by 2 g; the mean velocity being equal to the quantity flowing in cubic feet per second, divided by the final cross-sectional discharge area of the closed or submerged portion of the draft tube in square feet.

For turbines set in open flume, the head is to be measured by board, rod or float gages, located above the center of the turbine, and by board, rod or float gages in the tail-race, all gages being placed at points reasonably free from local disturbances, and not less than two gages being installed in the flume and not less than two in the tail-race.

Such gages are to be free of velocity effects, and if this is not obtainable when the gages are set in the open channel, they shall be placed in properly arranged stilling boxes.

The effective head on the turbine is to be taken as the difference between the elevation of the free water surface immediately above the center of the turbine, and the elevation of the tail water at the highest point attained by the discharge from the unit under test, the above difference being corrected by subtracting the residual velocity head at the end of the draft tube, computed as in the previous case.

Quantity of Water. The quantity of water discharged from the turbine is to be measured by weir, current meter, Pitot tube, screen or diaphragm, or by the chemical method, in accordance with the methods described in Arts. 22, 23 and 24 of this section.

When the quantity of water is measured by weir, weirs with suppressed end contractions shall be used.

The weir or weirs shall if possible be located on the tail-race side of the turbine, and care shall be taken that smooth flow, free from eddies, surface disturbances or the presence of considerable quantities of air in suspension, exists in the channel of approach. To insure this condition the weir should not be located too close to the end of the draft tube, and stilling racks and booms should be used when required. The channel of approach should be straight, of uniform cross-section and unobstructed by racks and booms, for a length of at least 25 ft. from the crest. The racks should be arranged to give approximately uniform velocity across the channel of approach. The uniformity of velocity should be verified by current meter or otherwise.

When the discharge is measured by current meter, observations shall be taken by two different types of meter, one type having preferably such characteristics that it will slightly over-register under conditions of turbulent or oblique flow, and the other type having characteristics such that it will under-register under similar conditions. The true velocity obtained by reducing the meter readings on the basis of their still-water ratings may then be taken as a weighted mean between the two series of observations.

The point method of observation shall be used and sufficient points shall be obtained to enable both vertical and horizontal velocity curves to be plotted for all portions of the section of measurement. The average velocity shall be determined from these curves by planimeter.

When the Pitot tube method is used, the Pitot tube shall be located in a straight run of penstock or conduit, at a distance equal to at least ten pipe diameters from any upstream bend and at least five diameters from a downstream bend. When the observation is made in a circular pipe or penstock, at least two Pitot tubes shall be arranged to traverse two relatively perpendicular diameters, but in the case of very large penstocks or those having unsymmetrical flow. Pitot tubes shall be arranged to traverse completely or partially the intermediate diameters, giving traverses at 45-deg. intervals.

When the screen method is used, the length of run of the screen shall be sufficiently in excess of the portion used for measurement to provide ample space for starting and stopping the screen so as to insure uniform conditions over the measured portion of the run. In determining the discharge the velocity of the screen shall be multiplied by an area intermediate between the net immersed area of the moving screen and the average area of stream cross-section of the portion of the channel traversed. The variation of the level in the flume shall be observed during the course of the run and the average elevation shall be used in determining the area.

When the chemical method is used, samples shall be taken from points distributed over the entire sampling section.

SECTION 14

WATER SUPPLY

BY

ALLEN HAZEN

MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

COLLECTION OF WATER

Much of this material was originally prepared by the author's partner, the late George C. Whipple, M. Am. Soc. C. E.

COLLECTION OF WATER

1. Intakes

Intakes are structures built out into a body of water for the purpose of drawing water for use. The position of intakes is often affected by considerations of local pollution when sewage is allowed to flow into the same body of water from which the supply is taken. This is commonly the case in cities located upon rivers and great lakes. The depth of intake is frequently a matter of importance where water of different qualities is to be obtained at different levels. There are three type of intakes: (1) Unprotected intakes, (2) Submerged intakes, (3) Exposed or tower cribs.

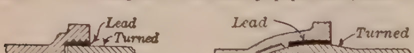
Unprotected intakes are used for small supplies. The pipe is allowed to terminate at the desired point, sometimes being protected by a coarse screen. A fine screen is not permissible because it will be clogged by matters carried by the water.

A Submerged Crib is a structure built on the bottom or built at the surface and sunk to the bottom of the lake or river from the interior of which the water is taken. It serves the purpose of roughly screening the water and also of protecting the end of the intake pipe from damage. The opening is frequently covered by a rack of wooden bars spaced an inch or so apart. Sometimes a steel or brass screen is used in addition to the rack. Exposed or **tower cribs** are structures built on the bottom of the river or lake and extending above high water. They are frequently provided at different levels with ports controlled by gates, and screens may be located in their interiors. Tower cribs have many advantages for large supplies. The ports may be closed and the water pumped out of the intake pipe and everything inspected for tightness and condition. Screens in them may be reached for cleaning and repairs. Tower cribs require excellent foundations and they must be built strong enough to withstand ice pressures. In cold climates they are used only for large supplies. In warmer climates where ice pressure is not effective they are also used for small supplies.

Intake Pipes or Conduits are the connecting channels between the intakes and the shore. Intake pipes up to 36 in. in diameter are generally of cast iron. From 48 to 72 in. in diameter the most common material is steel pipe. For larger intakes tunnels driven under the bed of the lake reaching from the shore to the bottom of an exposed crib are most common.

☞ Long intakes in lakes should be of ample size to avoid friction and excessive suction on the pumps. Short intakes in sediment-carrying rivers should be smaller with velocities to prevent them from filling up with silt. Where pipe is laid on the bottom of a lake or river a channel should be dredged for it so that the top of the pipe is well below the natural bottom, but such dredging may be omitted in lakes deeper than about 30 ft.

Cast-iron Pipe for intakes is commonly of a special type. From four to eight 12-ft. lengths of ordinary pipe are joined in the ordinary way and then a special joint is used (Figs. 1 and 2) which is made flexible with a turned spherical casting moving in a bed of lead which has been run inside of an enlarged bell. Such joints, well made, give a considerable amount of flexibility while remaining nearly water-tight. The pipe is put together above water and gradually lowered to position, the flexible joints making this possible.



(a) (b)

Fig. 1. Special Joints, Cast-iron Pipe

Where the water is not too deep to allow a diver to work comfortably, the flexible joints are sometimes omitted and flange joints bolted together by a diver are used. Numerous other kinds of joints are also used.

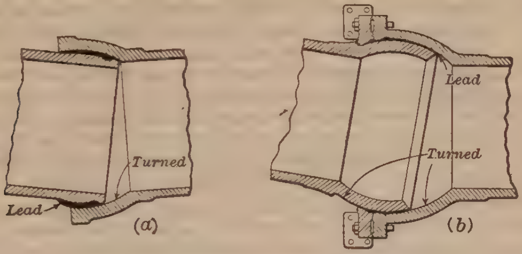


Fig. 2. Special Joints, Cast-iron Pipe

Trouble from leaks in intake pipes has frequently been experienced where the water through which they passed was subject to pollution. Where the intake passes under highly polluted water it is safer to have it below the bed of the river or lake, well covered with sand or other protecting material.

Steel Pipes for intakes are laid in much the same way as cast-iron pipes. Flexible joints are riveted to the ends of the steel pipes where required (Figs. 3 and 4), but the length of steel pipe between such joints may be greater, as steel pipe is stronger than cast-iron pipe. Steel pipe is frequently designed to fit closely the contour of the bottom and it can then be put together with flange joints bolted up by a diver.

The difficulty of making such joints with divers increases rapidly with the depth. Up to 30 or 40 ft. there is but little difficulty. Beyond this depth the difficulty increases and the joints practically become impossible before 100 ft. of water is reached.

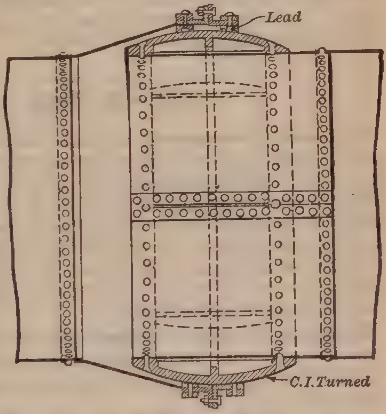


Fig. 3. Flexible Joint of Rochester Steel Pipe

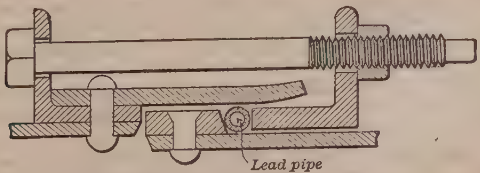


Fig. 4. Flexible Joint of Erie Steel Pipe

Tunnels under lake or river bottoms are most safely and cheaply driven in rock. Intakes under the lakes at Cleveland and Chicago have been driven in clay. The greatest difficulty has been from marsh gas contained in the clay, which comes into the tunnels during construction, forming an explosive mixture. There has been great loss of life and property from accidental explosions. Where clay is free from marsh gas it is a good material to tunnel through. Tunnels may be driven through sand and gravel, but with greater difficulty and expense. Air pressure must always be used corresponding to the distance that the tunnel is below the surface of the water. The length of time that men can work under pressure decreases rapidly as the pressure increases, so that such tunnels cannot be driven at too great a depth. The ordinary limit is 60 or 80 ft. and about 100 ft. is the extreme limit.

Anchor Ice is troublesome in northern climates in the case of intakes taking their supply near the surface or in shallow water. Much trouble has been experienced from this cause. A steam boiler furnishing steam to be taken to the seat of trouble through a hose is the most effective means of maintaining the supply.

Pumping Stations are commonly at the shore end of an intake pipe or tunnel. The pumps must be so placed as to be able to take water at the lowest water level and they must also be protected from water during the highest floods. In the case of rivers having a wide range of water level the protective structures are frequently very expensive.

2. Data of Yield and Storage

Impounding Reservoirs are artificial lakes built on upland streams for the purpose of storing freshet flows for use during those times when the natural flow of the stream is insufficient to maintain the supply. Impounding reservoirs are formed by the construction of a dam, preferably across a narrow valley at a point where there is an enlargement of the valley above to give large storage capacity. A reservoir on a tributary is substantially as useful in maintaining the supply as a reservoir on the main stream, up to the point where the reservoir on the tributary can be filled from bottom to top by the runoff of an ordinarily dry year.

Storage and Yield. To estimate the yield of an actual reservoir, or of a reservoir to be built, or to estimate the size of reservoir required to develop a given output, the best starting point is a record of the flow of the stream feeding it.

The **storage ratio** is the net available storage divided by the mean flow for one year. The **percentage yield** is the percentage of the mean flow that can be made available for use when needed.

From the flow record make a mass diagram by the method described in Sect. 13, Art. 44. Assume a rate of draft and find the maximum depletion that would have occurred at that rate during each year. These maximum annual depletions form a series. The series may be examined by probability methods, and the amount of storage that will be exceeded on 5% of the years may be accepted as practically sufficient. The calculation is repeated for other rates of draft until enough points are found to make a storage diagram which will serve as a basis for the calculation.

95% Dry Year Used. The 95% dry year is used as a basis for municipal water supplies. It is defined as the year of such dryness that 5% of the years are drier and 95% of the years are wetter than it. The drier years are the years that require more storage. For the 5% years that are drier than the normal less than full supply will be available. On Eastern data about 94% of the full supply on an average will be available in these drier years.

For extra conservative estimates, a 98% dry year may be used. This in a general way will call for 14% more storage. If the 99% dry year were used, 25% more storage would be required.

American cities have experienced moderate water shortages at intervals, and building storage that will be used less than once in twenty years is only warranted where the cost of storage capacity is unusually low.

Probable Error. The probable error (due to variation in annual quantities) in the mean flow of a stream is found by the formula

$$\text{Per cent of probable error} = \frac{67.5 \text{ Coefficient of Variation}}{\sqrt{T}}$$

T being the number of years in the record. Thus, for a 30-year record on an Eastern stream having a coefficient of variation of 0.25, the probable error is 3%; and for a Western stream a 20-year record with a c. v. of 0.70, the probable error is 10.6%. The errors in gaging must also be considered. No estimate can be more accurate than the data on which it rests, and available data do not often permit precision.

Data for Other Streams. If the record of the stream for which an estimate is to be made were sufficiently long and accurate, no other data would be needed for computing storage; but in most cases records are short or absent. A good short-term record is useful but the probable error is large. It is best to give weight to available records of flow and storage of other streams as similar as may be to the one for which an estimate is required.

Rainfall. Runoff comes from rainfall and follows it. Rainfall data are more abundant than runoff data. With meager runoff data there is a temptation to try to find relations between rainfall and runoff and to build up theoretical runoff tables and use them as a basis of estimate. Something can be done but great skill is required, for the relations are exceedingly complex and depend upon unknown and only partly known factors. Years ago when runoff data were less common there was more need of such expedients. Now, the records of the U. S. Geological Survey are numerous and widely distributed. These and other gagings will nearly always serve as a safer starting point.

Size of Catchment. In general runoff is proportional to catchment area and the required storage for a similar development will be in the same ratio. No doubt there will be small differences in special cases but in the absence of clear proof of difference the assumption is at least approximate that variations are in direct proportion to area. Areas high in elevation usually produce more water proportionately than low ones.

Cycles. A close study of rainfall data discloses cycles. Corresponding cycles in runoff are to be expected. As a practical matter not much attention need be given to them. But if a short term record is used, get the record of the nearest stream for which the data have been kept for a long time, and from it find whether the short term period has been unusually dry or wet. Numerical corrections for such differences are difficult, but a general idea of relative conditions is obtained.

Daily and Annual Storage. The required storage is made up of two parts: (1) daily storage required to balance fluctuations of flow within a single year; and (2) annual storage required to carry over surplus of wet years and make it available in dry years.

Methods of collecting, combining and stating the results are given in Transactions, Am. Soc. C. E., 77, p. 1539, 1914; but since that publication more data and some improvements in methods have become available and are used in preparing the tables that follow.

Practical Procedure. The mean flow and other data to be described are obtained for each stream flow from its record. With these data at hand, an estimate of the mean flow is made for the given stream. The required storage or the available output may then be computed with the aid of the tables that follow.

The mean annual flow is conveniently expressed in inches of runoff. To find the mean flow in gallons, multiply the area in square miles by inches of runoff and the product by 47 581 to obtain the mean flow in gallons per day, or by 17 380 000 to obtain the mean flow in gallons per annum.

Owing to the variations in flow and the irregularity of the variations, the whole of the mean flow of a stream can never be utilized. Some of it will be lost in flood flows. The more storage provided, the higher the percentage of the mean flow that can be utilized.

Three tables are given showing the storages required to yield various percentages of the mean flow for different parts of the country and with different degrees of development. These are followed by Tables 4 and 5, giving statistics of actual water supplies and stream flows for comparison and to aid in selecting the proper starting points for the estimate.

Table 1 is for streams north of the Potomac and east of the Alleghanies, and for low developments with not more than 50% of the mean flow utilized. In this table the second line of classification is according to ground storage.

Ground Storage. Ground storage is an important element in stream flow. Streams having extensive deposits of sand and gravel on their catchment areas have better maintained flows than where the materials are impervious. Ground storage may amount to several inches of runoff. Where ground storage exists, artificial storage may be less, and vice versa.

Ground storage is best expressed as day's supply at the rate of draft and represents the reduction in needed artificial storage below the amount otherwise required. This is called ground storage, but in reality it represents all the variations that there are from whatever cause; and seasonal distribution of rainfall, snow conditions and other matters contribute to a total of which ground storage is usually the largest element. Table 4 shows the computed ground storage in days for a number of actual supplies. In Table 1 four grades of ground storage are shown. In judging of the amount of ground storage in the absence of definite data, a stream that goes dry in droughts has no ground storage; one that has very well maintained flows in droughts has abundant ground storage.

Table 1 may also be used for streams on the western slope of the Alleghanies and on the South Atlantic Coast, but with less confidence, and greater variations from it must be expected. It should not be used for any other part of the country. For corresponding low developments in the West, data are not available to permit general statement, and local data must be studied by the mass diagram method.

Table 2 is for high developments, where more than 50% of the mean flow is made available, and when the reservoir will not refill in dry years and stored water must be carried over from year to year. In this table the second line of classification is the relative variation in annual flow. Some streams vary from year to year much more than others, and the amount of storage is dependent more upon this characteristic than any other. The coefficient of variation is the best index of this variation and is used as a basis for classification.

Coefficient of Variation. To compute the coefficient of variation, make a table of annual flows in any convenient units and find the mean. The difference between each term and the mean, without regard to sign, is the variation. Each variation is squared and the sum of the squares is found. Divide this by the number of terms less 1. The square root of the quotient is the **standard variation.** The **coefficient of variation** (c. v.) of annual flows is the standard variation divided by the mean.

The coefficient of variation of annual flows should be calculated for each record of ten years and over that is to be used. It is well to take into account records of neighboring streams, and it will be safer to use the indications of good long-term records of neighboring streams than a value derived from a short-term record of the stream under consideration.

For streams having coefficients of variation under 0.50, use Table 2. This applies to all that part of the United States east of the Mississippi River and also to Oregon and Washington. It also applies to eastern Canada. It is not recommended that a value below 0.20 should be used for any estimate; and the table shows no lower values. North of the Potomac and east of the Alleghanies values from 0.20 to 0.25 prevail. South of the Potomac the values are somwehat higher. West of the Alleghanies they increase more rapidly.

If ground-water storage is indicated, deductions from the storage shown in the table may be made in the same amounts that would be used in classifying the stream under Table 1. The last column shows the deduction for thirty days' ground storage. Other amounts are in proportion.

Table 3 is for streams having coefficients of variation above 0.50. These are most common in that part of the United States west of the Mississippi River excepting Oregon and Washington. Storage equal to ninety days has been added in this table to cover the seasonal distribution of rainfall and runoff which is usual in the western part of the country.

Table 4 shows data of mean flow, coefficients of variation in annual flows and ground storage for selected streams.

In making estimates, all local data should be examined. Unless local data are very good and records of 20 years and upward are available, it will not pay to make mass diagrams. More reliable estimates will be reached by the shorter process of mean flows and coefficients of variation and by the aid of Tables 1 to 3.

Table 5 shows statistics of runoff and storage for some of the more important municipal water supplies in the United States.

Example. Compute the storage required to develop 35 million gallons of water per day from an area of 48 square miles of mountain country with little ground storage, the mean runoff being assumed from data for neighboring areas to be 25 in., and the coefficient of variation in mean annual flow to be 0.20.

The mean annual flow is $48 \times 25 \times 17.379 = 20\,855$ m.g., or 57.1 m.g. per day. The required delivery, 35 m.g.d., is 61% of the mean flow. As this is more than 50%, Table 2 is used. Under c.v. = 0.20, 60% and 65% of mean flow available call for storages of 0.31 and 0.35. By interpolation 61% calls for 0.318 of the mean annual flow. $0.318 \times 20\,855 = 6632$ m.g. This is the required storage. To this must be added an allowance to cover loss by evaporation, by a method to be explained below.

From the same area for a first installment of 20 m.g.d. equal to 35% of the mean flow, Table 1 is used. No ground storage assumed. Storage required, 0.128 of the mean flow, or 2669 m.g. An allowance for evaporation must be made.

Evaporation. The loss of water by evaporation from the surface of the water in the reservoir must be allowed for, except that when the records of flow upon which the estimate is based represent conditions after reservoir construction, no correction is necessary.

(1) Where the evaporation from a water surface is greater than the rainfall, compute the yield for the full capacity of the reservoir and deduct the net loss by evaporation, computed as a draft, which is the amount by which evaporation from water area is greater than evaporation from land area. The net loss in runoff is equal to the evaporation from water surface plus runoff from land surface less rainfall. The net loss in inches is applied to the average water area, and this for practical purposes may be taken as 0.9 of the area at the flow line of the reservoir. The amount so found is to be deducted from the computed daily capacity of the source.

Example. What will be the average daily loss by evaporation from a reservoir of 850 acres, where the rainfall is 28 in., the runoff is 8 in., and the evaporation from the water surface is 40 in.?

The net loss in inches is $40 + 8 - 28 = 20$. 20 in. in depth on 0.9 of 850 acres is $20/12 \times 0.9 \times 850 \times 325\,851 = 415$ m.g. per annum or 1.14 m.g. per day.

(2) Where the average rainfall is greater than the evaporation from a water surface, proceed as follows:

(a) From the natural mean flow deduct the estimated loss by evaporation computed as under (1) to obtain the mean flow after development.

(b) From the reservoir capacity deduct an amount sufficient to equalize the evaporation loss throughout the year calculated for a dry year. An allowance ranging from 6 in. in depth over the area of the reservoir for northern New England and New York to 8 in. in the latitude of Philadelphia and to 12 in. or more on the south Atlantic slope and increased 50% or more for shallow reservoirs with high water temperatures in summer. The adjusted values for mean flow and storage are then used in the calculation.

Economical Development. The economical development of a catchment area is reached when the dam has been built so high that the cost of making it higher is more than the value of the additional water secured. The height of dam corresponding to economical development may increase with the demand

(1) Northeastern States North of the Potomac and East of the Alleghanies

Partial Development. Reservoir Filling Each Winter: Ground Storage Used for Second Line of Classification

Per cent of mean flow used	Storage ratio			
	Impervious soils. No ground storage	Average soils. 30 days' ground storage	Deep gravel and sand. 60 days' ground storage	Greatest natural storage. 90 days' ground storage
50	0.229	0.188	0.147	0.106
45	0.192	0.155	0.118	0.081
40	0.159	0.126	0.093	0.060
35	0.128	0.099	0.070	0.042
30	0.098	0.073	0.049	0.024
25	0.072	0.052	0.031	0.010
20	0.048	0.032	0.015	0
15	0.029	0.017	0.004	0
10	0.014	0.006	0	0

(2) East of the Mississippi River: Also Oregon and Washington

High and Complete Developments: Reservoir not Refilling Each Winter. Relative Variation in Annual Flows Measured by the Coefficient of Variation Used as the Second Line of Classification.

Per cent of mean flow available	Storage ratio									Deduction for 30 days' ground storage *
	c.v. = 0.20	c.v. = 0.22	c.v. = 0.24	c.v. = 0.26	c.v. = 0.28	c.v. = 0.30	c.v. = 0.35	c.v. = 0.40	c.v. = 0.45	
95	1.21	1.33	1.46	1.60	1.74	1.90	2.30	2.70	3.10	0.078
90	0.85	0.92	1.00	1.09	1.20	1.31	1.60	1.88	2.20	0.074
85	0.66	0.71	0.77	0.83	0.91	1.00	1.23	1.47	1.70	0.070
80	0.54	0.57	0.61	0.66	0.71	0.78	0.97	1.19	1.39	0.066
75	0.45	0.47	0.50	0.53	0.57	0.62	0.77	0.95	1.13	0.062
70	0.39	0.40	0.41	0.44	0.47	0.50	0.62	0.76	0.92	0.058
65	0.35	0.35	0.35	0.37	0.39	0.41	0.50	0.61	0.74	0.053
60	0.31	0.31	0.31	0.32	0.33	0.34	0.40	0.49	0.60	0.049
55	0.27	0.27	0.27	0.27	0.28	0.28	0.33	0.39	0.49	0.045
50	0.23	0.23	0.23	0.23	0.23	0.24	0.26	0.32	0.39	0.041

* See tables 1 and 4 for classification and data regarding ground storage. For larger or smaller amounts the deductions are in proportion to the number of days' storage.

(3) West of the Mississippi River, Except Washington and Oregon

Per cent of mean flow available	Storage ratio								
	c.v. = 0.50	c.v. = 0.60	c.v. = 0.70	c.v. = 0.80	c.v. = 0.90	c.v. = 1.00	c.v. = 1.10	c.v. = 1.20	c.v. = 1.50
90	3.00	3.80	4.70	5.60	6.40	......	......	......	......
85	2.30	3.00	3.70	4.50	5.30	6.10	7.00	......	......
80	1.85	2.40	3.10	3.70	4.40	5.10	5.90	6.70	9.30
75	1.55	2.00	2.60	3.15	3.70	4.40	5.00	5.70	8.10
70	1.28	1.70	2.20	2.70	3.20	3.80	4.40	5.00	7.20
65	1.05	1.44	1.85	2.30	2.85	3.40	3.90	4.50	6.50
60	0.89	1.21	1.60	2.00	2.50	3.00	3.50	4.00	6.00
55	0.74	1.02	1.35	1.75	2.20	2.65	3.10	3.60	5.50
50	0.61	0.86	1.15	1.50	1.90	2.35	2.80	3.25	5.00
45	0.51	0.72	0.98	1.30	1.70	2.10	2.50	2.90	4.40
40	0.42	0.61	0.84	1.12	1.45	1.80	2.15	2.50	3.80
35	0.34	0.51	0.72	0.96	1.22	1.50	1.80	2.15	3.30
30	0.27	0.42	0.61	0.80	1.00	1.25	1.50	1.80	2.75

for and value of water, and this often leads to raising dams. On the other hand, full probable economic development of any supply that is utilized should be taken into account to the end that partial developments first made may be arranged to be increased without too much difficulty.

In a general way, in the northeastern states the economical limit is reached by the storage of an amount equal to at least half the mean annual flow, and not greater than the mean annual flow, and when between 75% and 90% of the mean flow is made available for use. If water is low in value and conditions of storage difficult, the limit will be lower. Where water is especially valuable and the site is favorable for cheap storage, additional amounts may be advantageous.

In the western states more storage is required and the proportion of the mean flow that can be used is less. Storages, of three times the mean annual flow are not uncommon.

(4) Statistics of Flow for a Few Selected Streams

Arranged in order of size of coefficient of variation

(In any particular case look for local data and recent data, bringing old records up to date before using them.)

River	Place of measurement or use	Number of years in record	Last year of record	Area square miles	Mean annual flow, inches	Coefficient of variation in annual flow	Days to be deducted for ground water storage
Hudson	Mechanicville	29	1916	4 500	23.5	0.16	73
Susquehanna	Harrisburg	26	1916	24 100	21.1	0.18	....
Ohio	Wheeling	21	1905	23 800	22.7	0.19	....
Manhan	Holyoke, W. W.	23	1919	13	25.7	0.20	25
Arkansas	Canon City, Colo.	34	1921	3 060	3.41	0.20	....
Columbia	Dallas, Ore.	37	1915	237 000	13.0	0.21	....
Pequannock	Newark, W. W.	27	1918	62	27.4	0.22	....
Wachusett	Boston, W. W.	27	1923	109	23.1	0.22	61
Perkiomen	Philadelphia	28	1912	152	22.6	0.22	33
Neshaminy	Philadelphia	28	1912	139	22.6	0.22	10
Merrimack	Lawrence	37	1916	4 634	20.1	0.23	66
Croton	New York, W. W.	53	1920	375	22.6	0.24	23
Potomac	Pt. of Rocks	23	1919	11 460	14.1	0.25	....
Willamette	Albany, Ore.	24	1915	4 860	40.3	0.26	....
Tohicken	Philadelphia	29	1912	102	27.7	0.27	2
Sudbury	Boston, W. W.	49	1923	75	20.6	0.27	25
Sacramento	Jelleys Ferry	20	1915	10 400	18.1	0.29	....
Gunpowder	Baltimore, W. W.	29	1911	308	19.2	0.30	91
Colorado	Yuma	16	1917	225 000	1.42	0.32	....
Tuolumne	La Grange, Calif.	20	1915	1 500	26.2	0.41	....
Mississippi	Pekegama Falls	30	1914	3 265	5.5	0.45	....
Cheesman	Denver, W. W.	19	1921	1 680	1.62	0.47	....
Red River	Grand Forks, N. D.	38	1919	25 755	1.52	0.49	....
Bear Creek	Denver, W. W.	21	1921	170	5.60	0.53	....
Colorado	Austin, Texas	20	1917	37 000	0.75	0.57	....
Rio Grande	Elephant Butte	20	1914	32 000	0.66	0.58	....
San Leandro	S. L. Reservoir, Calif.	36	1914	42	8.10	0.66	....
Crystal Springs	San Francisco	30	1919	36	11.1	0.69	....
Alameda Creek	San Francisco	30	1919	620	4.78	0.73	....
Sweetwater	San Diego	31	1918	186	1.64	1.90	....

3. Effects of Storage

Storage in an impounding reservoir may have important effects on the quality of the water, both for good and for harm. By reason of the opportunity given for sedimentation, storage tends to remove suspended mineral matter such as silt and clay and thus clarifies the water. Bacteria are also reduced in number by being carried to the bottom with the sediment, by natural death, by the destructive action of sunlight and by the effect of other organisms, so that the sanitary quality of the water is improved. Colored waters stored for long periods in impounding reservoirs tend to become bleached. On the other hand, storage offers exceptional opportunities for the growth of algae and protozoa which give rise to objectionable odors and tend to increase the turbidity and sediment present in the water. At the bottom of large reservoirs also decomposition of organic matter may take place, with the production of foul gases and of carbonic acid. One result of this is that the water is more liable to attack lead pipes.

Period of Storage. The normal period of storage of large reservoirs is expressed in days and is taken to be the capacity of the reservoir divided by the average daily flow through it. This expression is useful for comparing different reservoirs, but it means little as far as indicating the length of time that the water is actually stored on account of the variations in volume of the water entering the reservoirs. For example, in a reservoir that has a normal period of storage of thirty days there may be one or more times each year when the stream flow is sufficient to displace all the water in the reservoirs in two or three days. Furthermore, the water entering a reservoir at one end reaches the outlet at the other end not always after the displacement of all the waters, for, by reason of difference in the density of water due to difference in temperature, the action of the wind, etc., part of it may cross the reservoir in a period much shorter than that required for displacement; so that even in a reservoir that has a normal storage period of thirty days the actual time of transit of water from the inlet to the outlet may be reduced to a period of a few hours. For this reason storage reservoirs cannot be depended upon alone to protect the water supply from a sanitary standpoint.

The period of storage is based on the entire capacity of the reservoir, while the storage ratio is based on net available storage that can be drawn for use. Many reservoirs have dead water below the intakes, and the two terms, while similar, must be clearly distinguished.

Horizontal Currents Induced by the Wind. Wind blowing over the surface of a body of water causes a movement of the surface water in the same direction. The velocity of such induced currents of water varies according to the intensity and duration of the wind. Experiments made at Lake Erie have shown that for winds blowing in one direction for at least ten hours the velocity of the surface water is from 3 to 7% of that of the wind. For winds blowing long from one direction this per cent increases. The depths of such induced currents or the velocity of currents below the surface are not well known, but the former are doubtless 10 to 20 ft.

Stagnation. The lower portions of deep reservoirs are not affected by wind action, and during certain periods remain stagnant. Where the volume of water is large and the reservoir is clean, the dissolved oxygen in the water during the stagnant period is sufficient to take care of organic changes and keep the water sweet. This is true of such reservoirs as Ashokan and Kensico Reservoirs of New York City and of Wachusett Reservoir of the Boston

(5) Statistics of Some of the Larger Storage Systems for Municipal Supply in the United States, January 1, 1927

Data in **heavy type** are estimated from the best available data.

System	Storage in billions of gallons, including reservoirs now building	Number of reservoirs	Area in square miles tributary to or used in connection with reservoirs	Mean flow of tributary area		Coefficient of variation in annual flows	Storage ratio
				M.G.D.	Inches runoff		
New York City	285	21	967	1138	24.7	0.22	0.69
Boston Met. W. W.	70	10	202	207	21.5	0.24	0.93
San Diego	64	7	654	**67**	**2.15**	**1.50**	**2.62**
San Francisco S. V. W. Co.	62	5	665	145	4.6	0.70	1.17
Denver	40	3	2860	335	2.45	**0.50**	0.33
Oakland E. B. W. Co.	33	4	80	322	8.0	0.70	2.80
Providence	27	5	93	117	26.5	0.24	0.63
North Jersey District	26	1	94	**112**	**25**	**0.24**	**0.64**
Los Angeles	23	4	2740	340	2.6	**0.65**	0.19
Baltimore, Md.	20	1	308	282	19.2	0.30	0.19
New Haven, Conn.	16	14	145	129	18.6	**0.24**	0.34
Portland, Ore.	13	1	102	540	112	0.20	0.07
Bridgeport, Conn.	12	15	95	**86**	**19.0**	**0.24**	**0.35**
Troy, N. Y.	12	1	67	**64**	**20**	**0.20**	**0.51**
Newark, N. J.	12	4	64	81	27.0	0.24	0.40
Hartford, Conn.	11.6	7	44	48	23.0	**0.22**	0.66
Seattle, Wash.	11.5	2	142	483	71	**0.20**	0.07
Wilkes-Barre S. B. W. Co.	11.4	14	149	**133**	**20**	**0.24**	**0.23**
Rochester, N. Y.*	10.8	2	66	37	12.0	0.27	0.80
Jersey City, N. J.	8.6	1	121	**144**	**25.0**	**0.22**	**0.16**
Columbus, Ohio	6.9	2	1052	532	10.6	**0.40**	0.03
Worcester, Mass.	6.0	8	22	24	23	**0.20**	**0.68**
St. Paul, Minn.*	5.7	7	138	29	4.4	0.45	0.53
Norfolk, Va.	5.5	2	56	**53**	**20**	0.30	**0.28**
New Bedford, Mass.*	5.0	2	28	**27**	**20**	**0.24**	**0.51**
Reading, Pa.	4.0	3	218	**200**	**20**	**0.25**	**0.05**
Cambridge, Mass.	3.8	4	24	**23**	**20**	**0.24**	**0.45**
Springfield, Mass.	2.5	1	48	62	27.0	0.20	0.11
Akron, Ohio	2.4	1	207	140	14.2	**0.40**	0.05
Holyoke, Mass.	2.3	4	18	226	26.2	0.21	0.28

* Natural lakes raised and controlled.

Note. There are impounding reservoirs of considerable size not in the above table at Forth Worth, Texas, Decatur, Ill., Scranton, Johnstown and Altoona, Pa., Waterbury, Conn., Lynn and Pittsfield, Mass., and no doubt at many other places.

Metropolitan district and a few others. In smaller and shallower reservoirs the stagnant waters usually become foul in taste and odor, due to putrefaction after the oxygen originally contained in the water is exhausted.

Water of better quality may be drawn from such reservoirs by intakes located near the surface, but even such intakes draw some water from a depth, and more or less bottom water is usually mixed with the surface water and tends to impair its quality.

Turnovers. Once in the spring and once in the fall there takes place a thorough mixing of the top and bottom waters. This is due to the change in temperatures. During warm weather the top water is warmer and lighter than the bottom water. During cold weather it is colder and lighter. In both cases when the maximum density of water at a temperature of 39° F. is reached, overturning takes place. A strong wind will bring about overturning before the complete temperature adjustment is reached. Following the spring and fall turnovers, the water has increased color, taste and odor, and becomes objectionable in quality for a time, followed by gradual improvement.

If there are deposits of organic matter at the bottom of a reservoir bacterial putrefaction will take place at the bottom during the period of stagnation. The oxygen will become exhausted and the water impregnated with carbonic acid, sulfureted hydrogen, carbureted hydrogen, and compounds of ferrous iron and organic matter. Crenothrix and fungi may develop. During the following periods of circulation this stagnant water becomes mixed with the rest of the water in the reservoir and may temporarily increase its color. The ferrous iron becoming oxidized tends to act as a coagulant and thus to exert a purifying effect and facilitate subsequent filtration and decolorization.

During the period of circulation the spores of various algae and other organisms are distributed through the water together with food for their growth, and under the influence of sunlight in the upper layers they grow and remain near the surface by reason of gases evolved during the process of growth. Diatoms, in particular, are likely to be abundant in large impounding reservoirs after the spring and fall "turnovers."

Organisms. Some of the most troublesome forms of algae, such as Anabaena, occur only during hot weather. They seldom develop when the temperature is less than 65° or 70°, although in the fall they may linger in the water even though the temperature is lower than that. It is because of the lower summer temperature that algae growths are less prevalent in England than in America.

The organisms growing in reservoirs are both animal and vegetable. The latter, by the aid of sunlight, break up carbonic acid forming carbon and organic matter, which is the basis of their structure, exactly as do plants growing in air. Other very small organisms of the animal type feed upon the vegetable organisms, and larger ones upon the smaller, and as a result there is a series of organisms up to and including the fishes and other forms of aquatic life.

Organisms grow in all waters exposed to sunshine but the strength of the growths depends upon many conditions, among which may be mentioned temperature, the amount of food supply, represented by mineral matters in the water, and wind action. The latter is quite important because wind keeps the water in agitation and renders it unfavorable for the development of many kinds of organisms.

Organisms by their growth, and especially by their death and decay, have a profound influence upon the quality of water obtainable from reservoirs.

In some cases a moderate control of organisms may be secured by the use of copper sulfate, but such control usually serves to change the kind of growth rather than to reduce its total amount. Filtration of water containing growths of such organisms is the only adequate treatment. Strong growths make the filtration more difficult and expensive and they are therefore undesirable even where the water is filtered.

In the tropics and warm climates growths are most troublesome, and in cool northern latitudes they are less important.

Soil Stripping. In order to prevent the growths of algae reservoir sites are sometimes cleaned by removing the vegetation and top soil. This practice has been followed extensively in Massachusetts reservoirs. It results in a temporary benefit, but ultimately deposits of material occur on the bottom that contain as much organic matter as that found in the soil, and the advantage of stripping is lost. The organic matter that has the greatest effect on the quality of impounding waters is derived from grass, weeds and other vegetation on the reservoir site. This should be removed by cutting and burning just before the reservoir is filled. For the sake of appearance and to prevent growths of water weeds and filamentous algae the shores of the reservoir from 2 to 5 ft. vertically above the high-water mark and for 10 to 20 ft. or more below, according to circumstances, should be cleared of stumps and roots. Elsewhere the stumps should be cut to 12 in. or less above the mean surface of the ground.

Drainage of Swamps. The color of a stored water may be sometimes reduced by draining swamps on the catchment area. This also tends to reduce the danger of the reservoir becoming seeded with algae. The advantage is less where the water is to be purified.

Penetration of Sunlight into Water. On account of the rapid absorption of the sun's rays by water, the disinfecting effect of sunlight and its bleaching action on the coloring matter are limited to a shallow layer near the surface. In clear water there is little bleaching effect below a depth of five feet, and in turbid waters the effect of the sunlight may be felt only a few inches. For this reason growths of organisms are much less likely to occur in turbid, silt-bearing waters than in clear waters.

4. Ground-water Supplies

Ground Water is that part of the rainfall that has accumulated in the ground, either in soil or in rock. Its upper surface is called the water table, or the ground-water level. The water is actually present in the pores of the soil or granular rock or in fissures, crevices and seams. These may be considered as underground storage reservoirs.

Classification. The water in the ground, above an impermeable stratum, and relatively near the surface, is termed the upper ground water, and wells for obtaining it are called shallow wells. Water taken from beneath an impervious stratum is termed artesian, or deep-seated water, and wells for taking it are artesian wells, or deep wells. Originally the term " artesian " was applied only to deep-seated water that was under a sufficient head to cause it to flow naturally to the surface without pumping. All ground water is derived primarily from the rainfall. Geologically there are three principal classes of ground water, namely, those occurring in underlying stratified porous rock covering large areas, those occurring in old lake or river beds, and those occurring in deposits of sand and gravel, that is, in the drift.

Ground-water supplies are obtained from (1) sand and gravel deposits; (2) sandstone rock; (3) limestone rock. In the first two cases the water is present in the pores of a more or less homogeneous material. In the last it is present in caverns and fissures, usually extending indefinitely into the limestone rock, and replenished by surface waters that flow into them or by ground waters from sand and gravel deposits above the limestone. Water from either sand or sandstone is well filtered and of good hygienic quality. Water from limestone is often surface water that has flowed simply through a limestone cave and may be of inferior hygienic quality.

Percolation is the downward flow of rain water through the ground under the influence of gravitation and capillarity. It varies in amount according to the rainfall, porosity of the soil, temperature, etc. Under specially favorable conditions, illustrated by the sands of Long Island and the sand dunes of Holland, percolation may amount to from 30 to 60% of the rainfall. With less pervious material, such as is found in the South and Middle West, percolation may be as low as 10 to 20%.

Velocity and Direction of Subterranean Flow. The slope of the water table can be determined by measuring the elevation of the water in a series of bore holes, enough measurements being made to determine not only the amount of slope but also the direction of greatest declivity. The direction of flow is the direction of greatest declivity. The velocity may be estimated approximately from the slope for clean sands and gravels where samples can be secured, by making mechanical analyses and determining the average effective size of the material; but such calculations, based upon conditions largely unknown, are to be used with caution. The formula for the flow of water through sand is given in Art. 11.

Measurement of the Velocity of Flow of Ground Water. The actual velocity of ground water may be measured by introducing a solution of salt into one of two wells and noting the time required for its flow to the second well, the time being determined by making frequent analyses of the water. A more accurate method is that of Professor Slichter, who used an electrical device by which the conductivity of the water in the lower well and in the ground between the two wells could be determined. Ammonium chloride or some other electrolyte was placed in the upper well and the time required for it to flow to the lower well was determined by noting the increase in conductivity of water. Ground water in sandy soils often has a velocity of 5 to 10 ft. per day, although velocities of 50 ft. or more per day are not unknown. The deep-seated water found in underlying rock often moves with extreme slowness. In some cases calculations have shown the velocity to be as small as 10 ft. per year.

Quantity of Underground Flow. The quantity of underground flow is obtained by multiplying the velocity by the area of the cross-section under consideration and by the percentage of voids in the material. These usually are from 30 to 40% in natural gravels and somewhat less in sandstones.

Flow of Water into Wells. If a well is sunk into the ground water and pumped, the surface of the ground water adjacent to the well will be depressed and assume a form similar to that shown in Fig. 5. The shaded area is commonly called the cone of the depression, and the area within which the water table is appreciably affected is termed the **circle of influence.** If pumping were continued and the ground water received no acquisitions, the circle of influence would widen without limit. Usually a ground water has a natural slope and a flow in a definite direction, so that, when the circle of influence has broadened until the ground-water flow tributary to the area equals the amount of water pumped, a condition of equilibrium is obtained. The curve assumed by the water table near the well is parabolic in form. Formulas have been worked out to show the relations existing between the various elements of the problem, but they are valuable chiefly as

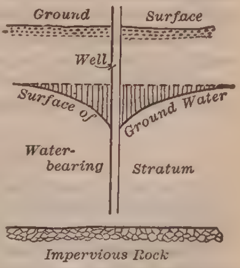


Fig. 5. Ground Water

indicating relative conditions caused by drawing down the water table to different depths. They cannot be depended upon to determine with any degree of accuracy the capacity of a well or the radius of the circle of influence, except in special cases where unusual attempts have been made to determine the values of the constants involved.

Interference with Wells. Wells placed in the same stratum near together mutually interfere according to the size and spacing of the wells, the radius of the circle of influence and the lowering of the ground-water table. As an illustration of this interference Slichter has given the following example. If in place of a 6-in. well that has a radius of the circle of influence of 600 ft., and a lowering of 10 ft. in the water table by pumping, there were two 6-in. wells 200 ft. apart, the yield of each would be 85% as much as a single well. while if there were a number of wells 100 ft. apart the yield of each would be only about one-third as much as that of a single well of same size.

Tests of Wells are frequently carried out by putting down test wells and pumping them for a limited period. Such tests have little value beyond showing the perviousness of the material about the test well. If there is water in the underground reservoir, water can be pumped at any rate at which the well is capable of taking it from the reservoir as long as the supply lasts. This may be a day, a week, a year, or permanently, according to the supply. Actual use through dry periods is usually the best test. Wells extending below sea level, as on Long Island and in New Jersey, may draw fresh water in excess of their permanent capacity for several years, while sea water is gradually filling the voids previously occupied by the fresh water.

Yield of Ground Water. The yield of ground-water collecting works depends upon the size of the catchment area, the rainfall, percolation, etc., and also upon the efficiency of the devices employed for collecting the flow. The conditions governing the amount are the same as those governing surface flow with storage. It is always difficult and frequently impossible to estimate the amount of underground storage utilized by the works and for this reason close estimates of yield can seldom be made. Where the underground storage is very large, as on Long Island, a development as high as could be obtained from corresponding surface sources is secured and in that case a yield of 900 000 gallons per square mile per day is obtained.

Conditions on Long Island are exceptionally favorable for the collection of ground water. In other localities where the rainfall is the same the safe yield might not amount to 500 000 gallons per day per square mile. In some parts of the country the safe yield is as low as 100 000 gallons per day per square mile. It is seldom that more than about 3 million gallons per day can be economically obtained from sand or sandstone at any one place, and often the limit is as low as one million gallons or less. When larger quantities are required it is customary to have several stations with pumps operated separately or by power transmitted from a central station.

Temperature of Ground Water. One of the advantages of a ground water supply is its cool and equable temperature. In the case of ground water found in the drift the ordinary temperature of the water is usually not far from the average temperature of the air in the same place.

5. Wells and Pumping

Ground water is made available for public water supplies by large dug wells, driven wells, infiltration galleries, and springs. There are three principal types of driven wells, that is, shallow, tubular wells; deep, bored wells in soft material; and rock wells.

Wells of Large Diameter. The chief advantages of the well of large diameter are, the storage that it affords and the possibility of placing the pumps at a low level and short suction pipes. The effect of the size of the well on the yield is comparatively small. Large wells are useful where the pumping is variable and especially in cases where the ground water flows through fine material with low velocity. Dug wells avoid the clogging that occurs in driven wells located in iron-bearing sands. As the cost of large wells increases rapidly with the depth, they are seldom made more than 50 ft. Large wells vary in diameter up to 50 or 100 ft. They are commonly lined with brick, concrete, or masonry, openings being left for the entrance of water. They are covered to exclude light and dirt. The capacity of large wells is sometimes increased by driving small wells or galleries horizontally into the ground, or by sinking vertical wells in the bottom of the large well.

Shallow, Tubular Wells. The ordinary driven well consists of a wrought-iron or steel tube, 2 to 8 in. in diameter, with a strainer near the bottom. It is forced into the ground by a hammer or by the use of a falling weight or with the aid of a jet of water carried through a small pipe to loosen the material in advance of the point. The strainers may be merely holes or slots in pieces of brass pipe, or larger holes in the pipe, covered with brass gauze. Porcelain strainers and tile wells also are used. The use of different metals in a strainer is objectionable, as it gives opportunity for galvanic action, causing corrosion and clogging. The size of the openings must be adjusted to the texture of the soil. They must be small enough to prevent the entrance of any large quantity of sand but large enough to reduce the entrance velocity to a point where the friction will not be excessive, that is, less than about 0.2 ft. per second.

Tube wells usually afford only a limited space for the entrance of water, and as the rate of flow, depending upon the fineness of the material, is usually slow, a great number of wells must be used to obtain a considerable supply of water. It is common to connect such wells up in batteries extending in lines across the valley or strata from which water is to be obtained.

Three methods of connecting up such batteries of wells may be mentioned. First, **direct connections;** where the tops of the wells are connected with a suction pipe laid on the surface of the ground or a short distance below it, extending to the suction end of the pump. This method of connection is common, but with it any air that leaks into the wells and to the suction pipe is carried to the pump and must be handled with the water and frequently interferes with the operation of the pump, and any fine sand that enters the well with the water is also carried forward to the pump, and if a reciprocating pump is used it scores and wears the moving parts and its efficiency is rapidly destroyed. In the second or **siphon method,** independent suction pipes are run down inside each well. For instance, 4-inch suctions are used in 8-in. wells. The suction pipes must be carried below the lowest level from which water is to be drawn. The suction pipes are connected to the main suctions outside the well, and the best arrangement is to carry that pipe to the bottom of a large receiving well near the pumping station as Fig. 6. The pipe then acts as a siphon, and is limited in its draft to about 25 ft. The suction pipe must be graded to a summit just outside this well and the air that would otherwise accumulate at this point must be removed by an air pump. The sand is carried into the large suction well and deposits on its bottom. The pump suction in the large well draws water free from both air and sand. The third or **pumping method** may be used where the material is deep and coarse enough to permit fewer and larger wells to be used. A centrifugal pump with vertical shaft is placed in each well. Such pumps have the motors above the surface of the ground with long shafts reaching to the pumps below the lowest water in the well and throw water to the surface where it may be carried off under pressure to storage tanks or to a central pumping station. This method in general is not applicable to wells in fine-grained materials, but may be extended somewhat to finer materials by the use of larger wells surrounded with specially prepared material brought in or put around the well as it is placed, or prepared from the natural material by a construction pump introducing a strong back and forth current which churns out the finer material and leaves the well surrounded with only the coarser parts of the natural material.

Suction lines for wells on either the first or second system must be made and maintained air-tight and the joints must be made more carefully than are required for ordinary water pipes. Flange pipes with rubber or other compressible gaskets have been widely used. In other cases bell and spigot joints have been made tight by the use of lead wool, which has the advantage over poured lead in that the whole mass of the joint may be calked and made tight, as distinguished from a poured joint where the influence of calking only extends back a quarter of an inch or so, leaving a small annular space which will be certain to leak after the 1/4 in. tightened by calking has been loosened by a number of small movements due to temperature changes or settlements.

Conditions of obtaining underground water vary greatly in different parts of the country and it is not possible to lay down general rules that will apply in all cases. Many cases are found which seem to differ widely from what may be regarded as usual over average practice.

Deep, Bored Wells. Deep wells in soft material are bored. The loosened material is brought up by the use of a water jet or sand bucket; such wells are usually cased. Wells are bored in rock by the use of a cutting drill which is raised, revolved, and let fall with a blow. Where strict artesian conditions prevail a casing is put down to the impervious stratum.

Infiltration Galleries. Infiltration galleries are closed conduits of masonry, wood, iron, brick or vitrified pipe, laid with numerous small openings to allow the inflow of water. They serve the double purpose of collecting the ground water and conveying it to the pump well. To prevent the entrance of fine material through the openings the latter are surrounded on the outside with graded gravel and sand. Infiltration galleries are placed at right angles to the general line of flow of the ground water, although sometimes they are placed near the shores of streams. In the latter case, as the bed of the stream is generally covered with silt, the greater proportion of water is derived from the land side. The attempt to use the bank of the stream as a filter and collect the stream water in a filter gallery beside it has been frequently attended with failure.

Pumping is required to utilize most ground waters. It is possible to lift water by a suction pump not more than 25 ft., and special effort should be made to locate the works so that water can be obtained without more than this amount of suction.

An air pump on the suction pipe frequently facilitates pumping with high lifts and long suction lines. An air chamber is placed on the suction just before connection is made with the pump, and the air that leaks into the system is separated and removed by an air pump, so that the main pump always draws water without air.

A Deep-well Pump is a small pump of peculiar construction lowered through the pipe to the bottom of the well and operated by a connecting rod reaching to an engine above the surface of the ground.

A Screw Pump is a screw propeller, or a series of them, placed in the pipe and lifting the water in the well by being revolved rapidly, motive power being applied to the shaft above the surface of the ground.

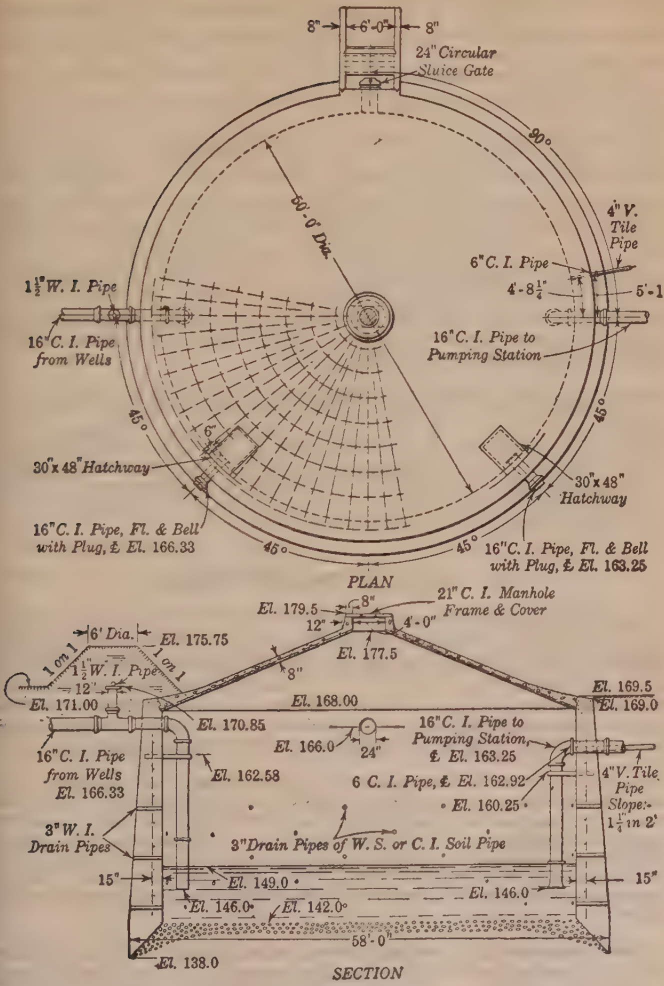

8"
6'-0"
8"
24" Circular Sluice Gate
90°
60'-0" Dia.
4" V. Tile Pipe
6" C. I. Pipe
4'-8¼"
5'-1¼"
1½" W. I. Pipe
16" C. I. Pipe from Wells
16" C. I. Pipe to Pumping Station
45°
6"
30"x 48" Hatchway
30"x 48" Hatchway
16" C. I. Pipe, Fl. & Bell with Plug, ℄ El. 166.33
46°
45°
16" C. I. Pipe, Fl. & Bell with Plug, ℄ El. 163.25
PLAN
El. 179.5
8"
21" C. I. Manhole Frame & Cover
12"
4'-0"
6' Dia.
El. 175.75
1 on 1
1 on 1
1½" W. I. Pipe
El. 177.5
8"
12"
El. 171.00
El. 168.00
El. 169.5
El. 169.0
El. 170.85
16" C. I. Pipe to Pumping Station, ℄ El. 163.25
El. 166.0
24"
16" C. I. Pipe from Wells El. 166.33
El. 162.58
6 C. I. Pipe, ℄ El. 162.92
4" V. Tile Pipe Slope:- 1¼" in 2'
El. 160.25
3" W. I. Drain Pipes
3" Drain Pipes of W. S. or C. I. Soil Pipe
15"
El. 149.0
15"
El. 146.0
El. 146.0
El. 142.0
58'-0"
El. 138.0
SECTION

Fig. 6

The Air Lift is a process of taking water out of wells by the pressure of compressed air. The well is extended in depth to a considerable distance below the water level. Compressed air is carried to the bottom of the well through a small pipe and allowed to escape. The air mixes with the water in the discharge pipe, which must not be too large in diameter. The column of mixed material is lighter than solid water, and it becomes higher in proportion until water mixed with air overflows at the surface into a separating device.

All three of these means of getting water out of deep wells are of very low efficiency from a mechanical standpoint. The expense of lifting water from wells 30 to 50 ft. deep frequently exceeds the cost of pumping it afterwards against a much higher lift.

Hardness is common in ground waters. Waters obtained from sands and gravels free from lime, as in New England, Long Island, and parts of New Jersey, are soft. The glacial drift from central New York westward contains lime, and waters obtained from it are hard. The hardness depends more upon the fertility of the overlying soil than upon the amount of lime in the sand, because rich soil produces carbonic acid, which is taken up by the water as it passes through the soil and this promotes the solution of lime. Waters from limestone rock are always hard.

Among the largest ground-water supplies in the United States may be mentioned the Brooklyn Works, and other supplies upon Long Island for suburbs of New York, and the works at San Antonio, Houston and Galveston, Texas; Memphis, Tenn.; Lowell, Mass.; Camden, N. J.; Jackson, Michigan; and Jacksonville and Miami, Florida. Ground-water supplies have advantages for small works and a large majority of all public supplies are of this character, although the aggregate amount of water produced is much less than from surface supplies. Ground-water supplies are also largely used by industries and for domestic purposes.

6. Tests for Potable Water

Sanitary Inspection. No surface water is entirely without danger of infection. The closer the proximity of the source of pollution to the intake of the waterworks, the greater is the danger, although it is a question of time of flow rather than distance of flow. The population on catchment areas may be classed as, urban (population above 4000 per sq. mile), village (population between 4000 and 1000), and rural (population below 1000). The figures are best expressed in population per square mile of drainage area as above.

Sanitary regulations for the prevention of contamination of water supplies are in force in many states. The guiding principle should be: to make the sewage pass through the ground and not over the ground, and if this cannot be done, to remove it from the catchment area or purify and disinfect it. In considering the danger of privies located in proximity to watercourses, the character of the soil and the slope of the surface should be taken into account, and also the method of cleaning. Cases of typhoid fever or dysentery existing on a catchment area should receive special consideration.

Water Analysis. A complete water analysis consists of four sections, physical, chemical, bacteriological, and microscopical. The standard methods of analysis are given in " Standard Methods for the Examination of Water and Sewage," American Public Health Association. (Fourth Edition may be obtained from the Secretary of the A. P. H. A., Boston, Mass.) The standard method of expressing the results of chemical analysis is in parts per million by weight, which is practically equivalent to milligrams per liter. To change such results to " parts per 100 000," divide by 10; to " grains per

U. S. gallon," divide by 17.1; to "grains per Imperial gallon," divide by 14.3. Bacteria and microscopic organisms are expressed in "number per cubic centimeter." The expression "degrees of hardness" usually refers to "grains per gallon." Degrees of hardness on Clark's scale are "grains per Imperial gallon." On account of the variety of conditions there are no accepted standards of purity to either chemical or bacteriological analysis.

Samples of Water for physical, chemical, and microscopical analysis should be collected in clean, glass-stoppered bottles holding two quarts or a gallon. Bacteriological examinations demand special sterilized, glass-stoppered bottles holding at least two ounces. Bottles may be sterilized by heating in an oven for one hour at 160° C. Bacteriological examinations must be ordinarily made within six hours after collection, and the sample must be packed in ice for transportation. Tests for gases, such as dissolved oxygen and carbonic acid, should be made at the time of collection of the sample. Chemists usually prefer to furnish their own bottles.

Turbidity is caused by clay, silt, microscopic organisms and other fine particles. A perfectly clear water has a turbidity of zero. Turbidities higher than 2 to 5 parts per million are noticeable in drinking water and are objectionable. Turbidities as high as 50 or 100 make a water "muddy." In the Mississippi River and elsewhere in the South and Middle West, below the region of the glacial drift, turbidities of 2000 or 3000 are not uncommon. The turbidity of a river varies more or less directly with the stream flow.

The Standard of Turbidity is a water which contains 100 parts per million of silica in such a state of fineness that a bright platinum wire 1 mm. in diameter can just be seen when the center of the wire is 100 mm. below the surface of the water and the eye of the observer is 1.2 meters above the wire, the observation being made in the middle of the day in the open air, but not in sunlight, and in a vessel so large that the sides do not shut out the light so as to influence the results. The turbidity of such a water is 100. For actual use a standard suspension is prepared from a diatomaceous earth, to contain 1 gram of silica per liter and have a turbidity of 1000. From it silica standards are prepared by dilution with distilled water. For turbidity readings below 20, gallon bottles of clear white glass are used and smaller bottles for higher turbidities. The turbidity of a sample of water is determined by comparing it with these standards, looking through the bottles sidewise and noting the distinctness of some object, such as a series of ruled parallel lines, seen through them, or looking at a black surface while standing with back to the light.

For field use the U. S. Geological Survey turbidity rod consists of a rod with a platinum wire, 1 mm. in diameter near the end, projecting at right angles about one inch. At the other end of the rod, at a distance of 1.2 meters (about 4 ft.) is placed a wire ring through which the observer looks when making the examination. The rod is graduated, so that the distance from the wire to any mark indicates the turbidity when that mark is at the water surface and the wire just disappears from view. This graduation is determined by experiment.

The Color of water is caused chiefly by organic matter derived from decayed vegetation. Swamp waters are usually high colored. Ground waters are usually colorless. The color of the water in large lakes is usually below 10. River waters are colored in proportion to the area of swamps on the catchment area. If the color is as high as 20, water will look unsightly in a porcelain bathtub or in a glass on a white tablecloth. Higher colors are objectionable and warrant purification of the water.

Color is Measured by comparing the sample of water with artificial standards by dissolving platinum and cobalt chlorides in distilled water. The unit of color is that produced by one part per million of metallic platinum. In the field the glass disk method of the U. S. Geological Survey is used. Disks of colored glass, standardized against the platinum solution, are placed at the end of a metallic tube, comparison being made with the sample placed in a similar tube.

Odor. Fishy, grassy, and aromatic odors are caused by microscopic organisms such as Asterionella, Anabæna, Synura, and are due to oily secretions. Moldy and musty odors are due to decomposing organic matter. Peaty odors are due to the same substances that give water its color. Ground waters sometimes have sulfurous odors, due to dissolved gases.

Hardness is caused by carbonates, sulfates, and chlorides of calcium and magnesium. Hard water destroys soap. It has been estimated that the loss due to waste of soap for domestic purposes amounts to ten cents per million gallons for each part per million of hardness. This is based on average conditions and a use of 100 gallons per capita daily. A hardness of ten parts per million is practically unnoticeable, and it requires a hardness of 20 or 30 parts per million to produce curdling with soap. Water may be called hard if the hardness is above 100; very hard, if above 200; and excessively hard, if above 300. Hardness is due to the solvent action that water containing carbonic acid has on soil containing lime. Sewage pollution increases hardness. For temporary hardness see Art. 7.

Chlorine in water represents salt and may be due to sewage pollution, or to proximity to the sea, or, in the case of deep wells, to deposits of salt. The normal chlorine in waters decreases from the seacoast inland, and normal chlorine maps have been prepared by the U. S. Geological Survey for New England, New York, and New Jersey. Sewage pollution is indicated by excess above the normal. If the chlorine exceeds 15 or 20 parts per million it is apt to cause corrosion in boilers and plumbing fixtures. Chlorine is not removed by filtration or by any artificial process.

Iron in water is apt to cause trouble if present in quantities of more than 0.3 to 0.5 part per million. It is usually present as carbonate or hydrate, occasionally as sulfate, and often in organic combination. Manganese causes similar trouble but is less common.

Dissolved Gases. Carbonic acid is exceedingly soluble in water, but is easily reduced in amount by exposure to the atmosphere. Oxygen also dissolves readily in water, but the amount that can be present depends upon temperature and pressure. In winter waters normally contain nearly twice as much dissolved oxygen as in summer. (See Art. 3.)

Microscopical Examinations are required in studying the algae and protozoa in connection with the subject of odors. The method is in general that of concentration by filtration through a small sand filter in a glass funnel and examination of the concentrate with a microscope that magnifies about 100 diameters. Special cells are required, but the operations are not difficult.

Bacteriological Examinations require special methods and the use of sterilizers, incubators, and delicate pieces of apparatus. Several days are required for making the tests. The reports usually give the number of bacteria per cubic centimeter and state the presence or absence of the colon bacillus (B. coli) in 0.1, 1.0, and 10.0 cu. cm. of the sample.

Ground waters contain very few bacteria, seldom more than 100 per cu. cm. Surface waters often contain bacteria in large numbers, several hundred and sometimes many thousands per cubic centimeter. In silt-bearing streams the numbers vary approximately with the turbidity. Pollution generally causes an increase in the number of bacteria. Lake waters generally contain fewer bacteria than river waters. Most of the bacteria found in water are probably harmless. High numbers, however, are objectionable, because among them some objectionable species are more likely to be present. Filtered waters contain few bacteria, and the reduction in the number of bacteria by filtration is commonly taken as an indication of the efficiency of the process. The germs

Hardness of Water Supplied in American Cities

(Parts per million)

	Alkalinity	Incrustants	Total
Albany, N. Y.	42	25	67
Akron, Ohio	58	30	88
Baltimore, Md.	28	9	37
Birmingham, Ala.	.....	.....	37
Boston, Mass.	.....	.....	15
Bridgeport, Conn.	.....	.....	20
Buffalo, N. Y.	97	20	117
Cambridge, Mass.	16	15	31
Chicago, Ill.	110	20	130
Cincinnati, Ohio	36	60	99
Cleveland, Ohio	84	24	108
Columbus, Ohio (After softening)	50	40	90
Dayton, Ohio	275	64	339
Denver, Colorado	.....	.....	159
Detroit, Mich.	.....	.....	101
Grand Rapids, Mich. (After softening)	60	47	107
Hartford, Conn.	.....	.....	14
Indianapolis, Ind.	227	62	289
Jersey City, N. J.	29	37	66
Kansas City, Mo.	146	67	213
Los Angeles, Calif. (Old supply)	180	76	256
(Owens River)	125	0	125
Louisville, Ky.	50	30	80
Lowell, Mass.	.....	.....	23
Memphis, Tenn.	63	0	63
Milwaukee, Wis.	113	5	118
Minneapolis, Minn.	145	30	175
New Haven, Conn.	28	4	32
Newark, N. J.	.....	.....	19

	Alkalinity	Incrustants	Total
New Orleans, La. (After softening)	41	27	68
New York City:			
Croton	30	10	40
Catskill	7	9	16
Norfolk, Va.	50	7	57
Oakland, Calif.: E. B. Water Co.	140	45	185
Omaha, Neb.	149	76	225
Paterson, N. J., E. Jersey Water Co.	21	19	40
Philadelphia, Pa.:			
Schuylkill	41	59	100
Delaware	24	24	48
Pittsburgh, Pa.	25	25	50
Portland, Ore.	.....	.....	10
Providence, R. I.	5	10	15
Reading, Pa.	70	23	93
Richmond, Va.	41	9	50
Rochester, N. Y.	58	4	62
St. Louis, Mo. (After softening)	47	49	96
St. Paul, Minn.	162	12	174
San Francisco, Calif., S. V. Water Co.	.....	.....	154
Seattle, Wash.	.....	.....	14
Spokane, Wash.	117	30	147
Syracuse, N. Y.	91	3	94
Toledo, Ohio	146	68	214
Wilkes-Barre, Pa., S. B. Water Co.	13	7	20
Washington, D. C.	62	10	72
Worcester, Mass.	.....	.....	12
Wilmington, Del.	.....	.....	60

of typhoid fever, Asiatic cholera, dysentery, etc., may exist in water, but there are no reliable methods for detecting their presence, although certain recent methods are promising. The *bacillus coli communis*, commonly referred to as B. coli, is a constant inhabitant of the intestines of man and warm-blooded animals. It can be detected in water with comparative ease, and its presence is often taken as an indication of fecal contamination.

The Typhoid Fever Death Rate of a community is the number of deaths per year from typhoid fever per hundred thousand. In cities possessing satisfactory water supplies the rate is not often above 10, but typhoid fever is carried in other ways, and higher rates with good water sometimes occur. The substitution of a filtered water, or other pure supply, for a polluted water usually reduces the typhoid fever death rate.

7. Water for Boilers

A Steam Boiler requires good water as much as it does good coal. Bad boiler waters cause corrosion, scale, foaming, overheating, and leaks, resulting in loss of heat, increased labor of attendance, increased cost of operation and repairs, a shortened life of the boiler, and increased danger of explosion. All natural waters are more or less corrosive. Magnesium chloride and other salts cause corrosion, especially when concentrated in a boiler. Galvanic action sometimes causes corrosion. Local corrosion is termed "pitting" or "grooving."

Boiler Scale is formed by the precipitation from the water of the carbonates and sulfates of calcium and magnesium, together with smaller amounts of other salts and suspended matter. Calcium carbonate is quite insoluble after its extra molecule of carbonic acid has been driven off by heat; calcium sulfate becomes almost insoluble above 250° F.; magnesium carbonate is changed to magnesium hydrate and precipitated. Besides these, boiler scale often contains iron, silica, alumina, organic matter, etc. Carbonates often separate as soft mud which is comparatively unobjectionable; they may also form a hard scale. Sulfates always form a hard scale. Calcium sulfate precipitates in a compact, crystalline form, removed by hammering and chipping. It may happen that different kinds of scale occur in the same boiler, due to the different temperatures of the sheet in different parts and to the circulation of the water. The scale in the tubes is often different from that on the sheets.

Hard Waters invariably form scale, and comparatively soft waters may also do so if the boiler is used too long without being emptied. Concentrated soft waters are almost as bad in their effects as water naturally hard. The greater the hardness, however, the more troublesome the waters. **Foaming** is caused chiefly by an excess of alkaline salts, which cause the water to form suds, as if soap had been added. This makes a boiler unmanageable and affects the quality of the steam. If grease is present in the water the sludge or scale may become very sticky. In this condition it adheres tenaciously to the plates and causes overheating, which usually occurs in spots.

The **care of a boiler** has very much to do with the effects of hard waters. The frequent blowing off of a boiler tends to reduce the amount of sludge, and to that extent is advantageous, but in the process of blowing off only a part of the water is removed. Better results are obtained by allowing the boiler to cool, emptying it and cleaning it if necessary. Corrosion due to gases can be eliminated to some extent by allowing a thin scale to form in the boiler.

Boiler Compounds are often employed as a remedy and have their legitimate use. In serious cases of acid corrosion, lime or caustic soda may be used with advantage. Nothing will effectually prevent the precipitation of calcium carbonate, but the use of soda will cause some of the calcium sulfate to settle as carbonate instead of sulfate, thus making the character of the scale less objectionable. To prevent adherence of the scale to the boiler shell many substances have been used, such as potatoes, kerosene, and all sorts of nostrums, organic and mineral. Most of these are practically worthless. Of the boiler compounds more commonly sold none have given more general satisfaction than those which have soda and some form of tannic acid as primary constituents. Tannic acid has a slight action on the iron of the boiler, and is reasonably efficient in preventing scale from sticking, while if properly used its action on the iron is not serious.

One gallon of hemlock extract with two gallons of water and three pounds of soda ash forms a good compound. Hemlock extract costs from 3 to 5 cents a pound in barrel lots and soda ash costs less than 2 cents a pound. Tri-sodium phosphate is also used with excellent results with many waters.

Chemicals for Water Softening. The following table gives the number of pounds of commercial lime (85% available) and soda ash (58% Na_2O) which are required to remove each part per million of the substances mentioned in the first column from one million gallons of water.

	Lime	Soda ash
Free carbonic acid	12.5	
Free sulfuric acid (as H_2SO_4)		9.03
Alkalinity (in terms of $CaCO_3$)	5.5	
Incrustants (in terms of $CaCO_3$)		8.85
Magnesium	22.9	

Temporary Hardness is that part of hardness (Art. 6) due to carbonates. It produces scale in boilers, but is partly removed by boiling, so that treating the water before it goes to the boiler reduces the amount. In most waters the temporary hardness is equivalent to the "alkalinity." Sulfates and chloride of lime and magnesia produce hard scale in boilers and are not removed by boiling. They comprise the permanent hardness, or incrustants and ordinarily are equal to the difference between the alkalinity and the total hardness. For purposes of water softening it is necessary to know the total hardness, the alkalinity or acidity, the amount of magnesia, and the free carbonic acid.

Red Water Troubles. This term is used to designate the troubles that grow out of distributing water that corrodes iron. The red water is caused by the iron which is first taken into solution by the water and afterwards deposited by oxidation. Waters that do this are called active. Active waters also attack lead pipes and may poison those who use water drawn through them.

Active waters may be made quiet by a number of methods, among them removal of free carbonic acid by aeration and treatment with caustic soda, soda ash or lime to neutralize acids and increase alkalinity.

Hard waters in which the hardness is largely alkalinity or temporary hardness are usually quiet even if they contain considerable free carbonic acid, while soft waters of low alkalinity are active even with minimum quantities of free carbonic acid.

Soft waters that are yellow from swamp drainage are especially active by reason of organic matter and acids. Such waters require coagulation, filtration, aeration and frequently a lime treatment, which make them harder but less active.

The "pH" value of a water represents the composite effect of its acid and alkaline constituents, both mineral and organic. It is easily determined by the use of special equipment and in conjunction with knowledge of alkalinity and free carbonic acid serves to indicate corrosive properties.

To be completely quiet waters with alkalinities below 50 p. p. m. must have a pH above 8 which requires practically complete elimination of carbonic acid. With higher alkalinities waters may be quiet with lower pH values and some free carbonic acid. The pH, however, must remain above 7 to be quiet with alkalinities as high as 300 p. p. m.

Soft waters relatively free from carbonic acid are actually distributed by many water works plants with pH values of about 7, and while not entirely quiet some activity is tolerated in preference to a further increase in hardness. Values below 7 are undesirable in all cases. This is a relatively new subject. Conditions are improving and there is room for further improvement.

PURIFICATION OF WATER

8. Auxiliary Processes

Water Purification is frequently spoken of as " filtration," but the processes employed are much broader than are properly covered by the word " filtration," and include such auxiliary processes as aeration, straining, coagulation, disinfection, and sedimentation.

The Capacity of purification works must be sufficient to meet the requirements at the maximum rate of use, and it is usually necessary to provide reserve parts so that the full supply may be maintained while cleanings, repairs, etc., are carried out. As a general rule the capacities of purification plants should be 50% greater than the greatest expected annual rate of use, but the ratio varies according to circumstances.

Aeration consists in bringing water into intimate or violent contact with air for the double purpose of introducing oxygen and of removing objectionable gases. Aeration is used as a preliminary treatment in all cases where oxygen is deficient in the raw water. Oxygen is very easily introduced. Water falling in drops through a height of two or three feet will take up more than half the quantity of oxygen that it will take up by the fullest exposure. The fall of water over a dam, or the play through a jet of a fountain, will serve to fully aerate it as far as introducing oxygen is concerned.

More vigorous aeration is required to remove the gases or substances that produce tastes and odors resulting from the growth and decay of organisms in quiet water in the light. Sufficient aeration greatly reduces or removes these tastes and odors.

There are many kinds of aerators, but those that throw the water upward through numerous small jets are most efficient. For raw water the jets must not be so small as to clog rapidly and must be arranged to be easily cleaned. The best dispersion is obtained when the water is revolved by guides as it approaches the jet. The spacing of the jets must be arranged to properly disperse the water so that it will have ample contact with new air. Two types of aerator that have been successful are shown in Fig. 7.

Filtered water is sometimes aerated to remove carbonic acid and tastes and odors. Effluent aerators are not subject to clogging and may have more numerous and smaller jets. In some recent designs effluent aerators are combined with filter regulators arranged to use for aeration that part of the necessary maximum head not required at the moment.

Screens are used for removing dead leaves, sticks, etc. Stationary inclined screens raked off at intervals are most commonly used. Revolving screens of various types are used for closer screening. Screening through brass wire cloth with as many as 60 meshes per inch is used in paper mills. Screening as a preliminary to filtration is advantageous within certain limits, but close screening is unnecessary.

Coagulation consists in the addition of some substance to the water that reacts with substances in the water, producing a flocculent precipitate which surrounds minute suspended particles in the water and draws them together into aggregates that can be removed by subsequent processes which would not serve to remove the individual particles. Coagulation is an essential part of the treatment of (1) all waters containing large amounts of very finely divided mineral matter, or turbidity, and (2) all waters highly colored by vegetable stain. It is also frequently used for other waters, but is not necessarily essential.

Dry Feed. Coagulants were formerly dissolved, making standard solutions which were fed through adjustable orifices at the desired rates. These devices have been mainly replaced with dry feed machines in which the pulverized coagulant is fed by controlled screw feeds or other devices at the

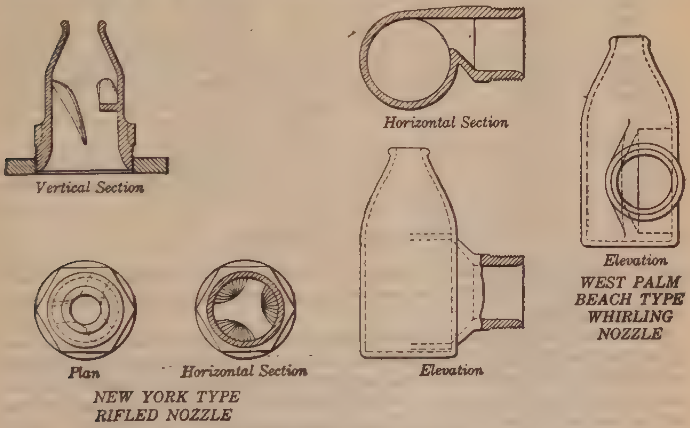


Fig. 7. Aerator Nozzles

desired rate into a dissolving chamber through which an unmeasured quantity of water flows on its way to mix with the main body of water.

Sulfate of Alumina, commonly called alum, is the most widely used coagulant. It is clean to handle, easily dissolved, and efficient in its action. It requires lime or other alkalinity to decompose it. In most cases sufficient lime is present in the water treated. Otherwise lime or soda ash must be added. The approximate quantities of sulfate of alumina (17% Al_2O_3) required to coagulate waters of various degrees of turbidity and color, and the alkalinity required to react with them are given in the table on p. 1470.

Ferric Salts have sometimes been used as coagulants. They are nearly equivalent in action to sulfate of alumina but less convenient to apply and have been used much less widely.

Ferrous Sulfate or Copperas is extensively used as a coagulant for hard waters. It is cheaper than sulfate of alumina and equally efficient in removing turbidity, but requires a more alkaline water. For this reason it is always necessary to use lime in connection with it, and the amount of lime must be closely adjusted to the chemical condition of the water.

Copperas cannot be successfully used except with adequate chemical supervision. The process is therefore preferred principally in large plants where continued close expert supervision can be given. By a judicious regulation of the amount of lime it is possible to partially soften hard waters by this treatment.

Softening. Water is softened for municipal use by two processes: first, conversion of incrustants to temporary hardness by treatment with nine

Turbidity	Alum		Alkalinity necessary for reaction, p. p. m.
	Grains per gallon	Pounds per million gallons	
0	0.50	72	5
50	1.00	143	9
100	1.35	193	12
200	1.78	254	16
300	2.05	293	18
500	2.50	357	22
700	2.87	413	25
1000	3.32	474	30
1500	3.90	557	34
2000	4.40	628	38
Color			
50	1.00	143	9
100	2.00	286	17
200	4.00	572	35

Different kinds of turbidity and color vary considerably in the amounts of coagulant that they require, so that figures varying considerably from the above average will be found. Deep reservoir waters containing iron in solution are more easily coagulated. It is well to keep the alkalinity of the treated water as high as 10 or 12 parts per million to guard against corrosion of metals.

At some large filter plants alum is manufactured from bauxite.

pounds of soda-ash per million gallons for each part of incrustants to be converted. This is a relatively expensive process often omitted and seldom carried to completion.

Second and more important, temporary hardness or carbonates are removed by adding lime to the water, followed by mixing and agitation and by settling and filtering. The added lime takes up the carbonic acid, which holds the lime in the water in solution, and the added lime, together with that in the original water, is precipitated. This is known as the Clark process. Some of the magnesium is also precipitated under favorable conditions. The dose of lime is in proportion to the carbonic acid and does not bear a quantitative relation to the softening effected. A small amount of coagulant aids and is usually added.

Water that has been softened by lime must be **recarbonated** to restore the natural balance of lime and carbonic acid before filtration and use. Products of combustion are cleaned by washing with water in a coke tower and then passed through the water before filtering. Otherwise the filter sand " grows," that is to say, the sizes of the grains are increased by deposits and there are objectionable deposits of crystalline calcium carbonate in valves, water meters, and other parts of the distribution system.

Among important municipal softening plants may be mentioned those at Grand Rapids, Mich., Columbus, Ohio, and Tampa and Miami, Florida. Partial softening is also carried out with lime treatments of river waters at St. Louis, New Orleans and elsewhere. Many railroads have softening plants for their locomotives.

Softening water by lime under certain conditions results in **sterilization** of the water. The results are fully equal to those obtained with chlorine and no other treatment is necessary where lime is used.

The **cost of softening** with soda-ash and lime is more expensive than ordinary filtration. At Columbus, Ohio, the operating costs are about $20 per million gallons for a reduction of 200 parts of hardness, or about 10 cents per million gallons for each part of hardness removed.

The **zeolite** process consists in passing hard water through a material that has the capacity of temporarily absorbing the hardness from the water. The material is regenerated by passing a salt solution through it. Almost complete softening can be obtained. The process is too expensive for municipal plants, but is frequently advantageous for boiler feedwater, water for laundries, and in hotels or houses for water for domestic purposes.

Disinfection consists in the addition to the water of some substance to kill objectionable organisms in it, which substance must be so adjusted as not to be injurious in the other uses to which the water is put.

Liquid Chlorine. Chlorine gas furnished liquefied in steel cylinders has come into general use as a sterilizing agent. It has completely displaced the hyperchloride of lime previously used for the same purpose and is so cheap and satisfactory that the rival processes of ozone treatment and ultra-violet rays have not found extended use.

The chlorine is fed out through an automatic control apparatus, is dissolved in a relatively small quantity of water and this water is afterward mixed with the main flow. Gravity apparatus is best but it can be applied under pressure with special equipment. Chlorine may also be applied directly as a gas to the water by the use of so-called diffusers. Chlorine is very poisonous and destructive and the equipment is best placed in small rooms having an outside door only, entirely separated from other parts of the waterworks plant.

Water after being mixed with the chlorine should be kept by itself as in a pipe or compartment or a reservoir for 15 minutes or more during which the chlorine has a chance to act upon the water without dilution. If the treated water is at once discharged into a large reservoir the effect is largely lost.

The required dose depends upon various matters and especially upon the amount and character of the organic matter in the water treated. Organic matter absorbs chlorine and the dose must be sufficient to satisfy, first, the organic matter and afterward to leave an amount sufficient for sterilization. The size of the dose is determined by noting the excess of chlorine after the period of treatment and this is regulated to an amount shown by experience to be satisfactory under local conditions.

Application of Chlorine to Raw Water. This is commonly practiced where no filtration is used. Excellent results have followed the use of chlorine where filtration was not practicable, but over-confidence in the method has sometimes led to unfortunate results. The sterilization is incomplete when the water is turbid or contains floating organic matter. A water ordinarily treated with success may occasionally change its character and fail to respond, as, for instance, at a time of the sudden rise of a river in a flood or the agitation of a lake by a sudden high wind.

The **application of chlorine to water before filtering** is occasionally used. It requires a relatively large dose because of the impurities in the water. When used in advance of filters, tastes and odors resulting from by-products of the action of chlorine on certain impurities may be eliminated through the biological action in the sand beds, but an excess of chlorine will sterilize the filter sand and reduce its biological action with unfortunate results. In general, such application should be made only where there are large settling or coagulation basins between the point of application and the filters.

Application to Filter Effluents where the raw water is polluted is almost universal. The filtered water receiving the chlorine has less organic matter than raw water and the amount is less variable, and the required dose is smaller and the action more efficient and reliable.

Tastes and Odors from Chlorine Treatment. Chlorine in itself has a taste and odor. If the water passes through reservoirs or long pipes after being treated and before being used it will tend to the elimination of chlorine. It is usually possible to operate satisfactorily. Some kinds of organic matter when present in the water form compounds with the chlorine having stronger and more disagreeable tastes and odors than were the original substances or the chlorine that is added. Phenol compounds in waste products of some industrial establishments are incredibly potent and are not removed by ordinary filtration processes.

Dechlorinizing Processes. A considerable excess of chlorine may be used to permit a stronger reaction and the excess afterwards removed by aeration to drive it off, or by the addition of sulfur dioxide or thiosulphate of soda to absorb it with the formation of harmless products. These methods have been occasionally recommended for special cases but are not at present in common use.

Sulfate of Copper. Sulfate of copper is extensively and successfully used to kill algae and other organisms growing in the light in impounding reservoirs. The common method of application is to drag bags of the material through the water by boats until dissolved. It is also used to treat water in swimming pools. It is added continuously by dry feed to aqueducts or by immersing containers with a hole in the bottom to varying depths in flowing streams or inlets to reservoirs, the rate of solution varying with the depth of immersion. In reservoir treatment the chemical should be distributed as uniformly as possible by triangulation of the surface or by parallel paths first in one direction and then at right angles. Large reservoirs (30 billion gallon reservoirs have been treated) should be subdivided and the volume of each section determined and the chemical apportioned accordingly. Fast launches are used for such service. In the winter period it has been applied through holes in the ice, but distribution is difficult and considerable skill is required. Dosage depends upon the organism present which should be determined by microscopical examination since all forms are not amenable to treatment. Happily the taste- and odor-producing organisms are usually readily destroyed. Usual dosage varies from 1 to 3 lb. per million gallons. In absence of exact information 2 lb. is a safe quantity to use. Dosage in winter is usually double that required in the summer. Fish are rarely killed if the chemical is properly distributed and the organisms are not allowed to accumulate to heavy amounts before treatment. The treated water should not reach the consumer under two days, in order to allow the disagreeable odors produced by treatment to escape from the water. A distribution reservoir should be by-passed or turned out of service. If it has several days' storage capacity, the upper half only may be treated and the treated water allowed to gradually displace the untreated.

Attention is called to the increasing use of chlorination to control algae. Many of those forms not readily destroyed by copper sulfate succumb to chlorine. Particularly is prechlorination used to lighten the load on filter plants and lengthen runs, to prevent growths in sedimentation basins and swimming pools. Dosage varies from 0.5 to 1 or 2 parts per million.

9. Sedimentation

Sedimentation consists in taking water through basins in which the velocity of flow is reduced to a point that permits the heavier suspended matters to settle to the bottom by gravity. Sedimentation is widely used as a preliminary process and is the cheapest way of removing those relatively large particles that will settle out in a moderate length of time. **Baffles** consisting of light dividing walls separating one basin into parts through which the water passes successively, increase the efficiency by preventing the partially cleared water from mixing with uncleared water. Too many baffles increase the length of the horizontal courses and the velocity to a point where deposition of the finest particles is prevented. **Cleaning** is usually accomplished hydraulically, by opening a gate and flushing out the sediment. To facilitate this, drains are built, and the whole bottom slopes to them. Jets of water from hose are used to facilitate the movement.

Sedimentation was formerly used as the principal process in clarifying water from many turbid rivers. Under the most favorable conditions the results reached fell short of present standards for public water supplies and sedimentation as an independent process has almost entirely disappeared. Sedimentation basins today are usually also **coagulating basins** and the determination of size and other conditions depends upon the time required for the chemical reactions to be completed. Chemical rather than physical conditions control, but a coagulating basin must also be a sedimentation basin, and if it is well designed for this it will make for better results in the process as a whole.

Basins may be large open basins with masonry floors and walls, cut up into units to facilitate cleaning. Vaulted covers like those used for sand filters and reservoirs have the advantage of keeping out wind action which is occasionally disturbing. Natural basins of larger size are sometimes availed of.

Time Required for Reactions. Some chemical reactions take place promptly when the chemicals are applied. In such cases moderate sized basins suffice. In other cases and especially in the coagulation of soft waters by aluminum sulfate a very considerable period of time may be required for complete coagulation. There has been a tendency to increase basin sizes for such waters. Basins holding several days' supply work where those holding a few hours' supply are unsatisfactory. If the period is too short the water passes to and through the filters before the chemical reactions are complete. The reactions may be finished in part in the filter sand and in part in the pipes of the distribution system; in the latter case with the formation of objectionable deposits. This is called secondary coagulation. Such deposits are usually colored red by iron taken up from the pipes even though the bulk of the material is white aluminum hydrate. These deposits are stirred up in the pipes at times of extra draft of water and give rise to conditions similar to those caused by active waters and noted under the heading "Red Water Troubles."

In the Intermittent System of operation a basin is filled and allowed to stand and then drawn off. In the **continuous system** of operation water flows constantly in at one end and out at the other.

The Efficiency of a Basin depends upon its area and upon the system of baffling employed. A deep basin is not more efficient than a shallow one of the same area, but a certain depth is necessary in order to hold an accumulation of sediment and to prevent the velocity of flow through the basin from becoming too great to permit full deposition. As a rough rule the depth may be one-sixtieth of the average course that the water will follow when baffled.

Settlement of Particles in Still Water at 50° F.

Kind of material	Diameter of particles in mm.	Rate of settlement, mm. per second
Coarse sand....	1	100
	0.20	21
Fine sand.....	0.10	8
	0.06	3.8
	0.04	2.1
	0.02	0.6
Silt...........	0.01	0.15
Coarse clay....	0.001	0.0015
Fine clay......	0.0001	0.000015

Bacteria and other organisms settle more slowly if at all because their specific gravity is so near to that of water. The rate of settling is greater as the temperature is higher. Twice as much water can pass through the basin with the corresponding results in summer as in winter. The limit of size of particles removed by settling basins can be computed approximately (Trans. Am. Soc. C. E., vol. 53, p. 45) as follows:

The Diameter of Particles in millimeters, such that 75% will be removed with continuance of operation, may be computed by

$$d = 0.0027\,f\,\sqrt{\frac{\text{million gallons daily}}{\text{area of basin in acres}}}\,\sqrt{\frac{60}{t+10}}$$

in which f is a factor depending upon the arrangement of basins and baffling. Use 1.73 for a basin with one inlet and one outlet well separated; 1.41 for two basins through which the water passes successively; 1.22 for a well-baffled basin or other specially good arrangement. $f = 1.00$ is a theoretical limit not reached in practice. In the last term t is temperature in degrees Fahrenheit. For comparisons use $t = 50$ in all cases. The rule does not apply for separations above 0.05 millimeter. It is not precise, but it affords a convenient basis for comparing sedimentation and coagulating basins.

In a general way basins holding six hours' supply, well baffled, in connection with mechanical filters, remove particles more than 0.02 mm. in diameter. Sedimentation basins for sand filters, 24 hours' supply, remove particles more than 0.007 mm. At Washington, D. C., in a succession of three reservoirs holding a week's supply, particles larger than 0.003 mm. are removed.

The Amount of Sediment removed in settling basins at St. Louis amounts to 12 cu. yd. per million gallons; with less turbid waters it is much less. For the Hudson at Albany only about 0.15 cu. yd. is removed.

Scrubbers, or preliminary filters, are rapid coarse-grained filters or their equivalent. Substantially they take the place of sedimentation basins, doing the same work but doing it more quickly and in less space, though usually at greater cost.

It is easy to design a scrubber that is efficient in operation. It is difficult to design one that can also be economically cleaned and kept continuously in efficient working order. From the standpoint of design and construction the cleaning devices are the most important parts of a scrubber. The object of scrubbers is to lighten the work of the filters to which the water is subsequently applied and especially to increase the length of run of the final filter. They are most useful for waters that are highly polluted or difficult to treat, and add to the certainty and reliability of operation.

The size of particles in millimeters, such that 75% will be removed by a scrubber or preliminary filter of sand or gravel 40 in. deep, may be computed by

$$d = 0.0003\,\sqrt{\left\{\begin{matrix}\text{effective size of}\\ \text{sand or gravel}\end{matrix}\right\} \times \left\{\begin{matrix}\text{rate of filtration, mill}\\ \text{gallons per acre daily}\end{matrix}\right.}\,\sqrt{\frac{60}{t+10}}$$

The scrubbers and preliminary filters that have given most satisfactory results are cleaned and operated practically as mechanical filters, but with simpler arrangements.

10. Sand for Filters

A Filter consists of a horizontal layer of sand through which water is passed to underdrains beneath, together with the containing structure and all auxiliaries. Filters act primarily as strainers, the interstices between the sand grains being small and serving to stop all particles too large to pass through them. They also serve to purify the water in other ways. Filters are classified as mechanical filters, sand filters, intermittent filters, and special kinds of filters according to the construction, rates at which they are operated, and the methods used for cleaning.

The Sand for filtering purposes is best clean quartz sand, free from gravel and large particles, and also free from excessive quantities of fine particles and dirt of every description. The presence of a small amount of fine material often aids the action of filter sand. For filtering river waters and any waters carrying carbonic acid, filter sand should be free from lime, as otherwise the water will be hardened. Waters containing less carbonic acid, such as lake waters, will not dissolve lime and sand containing lime may be used.

The Size of Sand Grains is determined by sifting through a set of rated sieves. About 110 grams of moist sand are put in a small iron dish and dried over a lamp. After cooling, 100 grams are put in the coarsest of a set of sieves, and the sieves are put in a mechanical shaker. A definite number of turns found by experience to be sufficient, is given. The shaking is not continued until no more passes, but only until the amount passing is small, so that doubling the number of shakes would not greatly change the result. The sieves are then taken apart, the material that has passed all the sieves is first put upon the pan of the scale and weighed, then the material remaining on the finest sieve is added to it and again weighed. The process is repeated until all the material is on the scale, when it should equal the original weight. The percentages finer than the sizes corresponding to the several sieves are then plotted on a diagram, from which the required data are taken.

The Effective Size of sand is that size such that 10% of the sand grains by weight are finer than it. The size of a sand grain is always taken as the diameter of a sphere of equal volume.

The Uniformity Coefficient is the ratio between the effective size and that size such that 60% of the sand is finer than it.

In rating sieves an ordinary sand is put upon them and the shaking is performed with the usual number of revolutions. The sieves are then taken apart, each sieve is taken separately, and is given a further slight shaking. A small additional amount of sand passes. The grains so passing are substantially larger than all the grains that have previously passed and smaller than those that remain. This small quantity of sand represents the size of separation of the sieve. A certain number of sand grains are counted out and weighed on an assay balance and the average weight is obtained. The diameter is obtained by the formula

$$\text{D in mm.} = 0.9\sqrt[3]{w} = \sqrt[3]{\frac{6}{\text{Sp. Gr.} \times \pi}}\sqrt[3]{w}. \qquad w = \text{weight in milligrams.}$$

There is but little difference in the results of using round-grained and sharp-grained sands, and between grains of different shapes, but the rating is best carried out with various representative sands. Rating of sieves once made does not change appreciably with use until some openings become enlarged or some wires become broken. When this happens the sieves should be at once replaced. When a set of sieves is rated a lithographed sheet is made for plotting the results, in which one line represents each sieve in the set. The best results are with a mixed plotting, partly logarithmic and partly natural, laid out so that normal sands plot as nearly straight lines. On this system accuracy is obtained with a smaller number of sieves. Enough sieves so that each has a size of separation not more than twice as great as the next below will suffice, except where the uniformity coefficient is under 2. In that event one intermediate sieve should be used between each two.

The following represents approximately the relation between commercial brass wire cloth and the sizes of separation:

Meshes per inch	200	140	100	50	40	30	20
Separation in mm	0.10	0.13	0.17	0.33	0.48	0.63	0.95

Owing to variations in weaving, individual sieves will vary 15% either way, and no sieve should be used without rating. In selecting wire cloth make sure that the spacing of the wires is even and that the number of wires per inch in one direction is not more than 10% greater than the number in the other direction Coarser sieves are best of brass plates perforated with round holes and rated in the same way as wire cloth sieves.

Sieves are now manufactured of standard wire cloth and may be purchased with the size of each sieve stated in millimeters. These sizes usually relate to the average size of opening and this differs from the size of separation which is based on the actual average diameters of particles which do and do not pass. For the coarser sieves it is about 10% less and for the finer sieves the variation is greater and not in any constant proportion. All sieves should be rated before being used for analysis of filter sand, and the size of separation must always be used and not the average size of opening.

Turbidity of Sand is the measure of clay in it. Put 10 grams of moist sand into a glass vessel holding one liter and fill with clear water. Agitate vigorously until all fine matter is in suspension. Allow to settle one minute, take the turbidity with a rod. Multiply the turbidity so found by 100 to obtain the parts per million of turbidity in the sand. The weight of clay is from one-half to two-thirds the turbidity, depending upon the size of the clay particles. Turbidity of sand prepared from stock containing clay should always be taken, but when there is no clay in the stock it is unnecessary to do it. At the Washington filtration plant the turbidity of the filter sand was not allowed to exceed 4000 in parts per million, or 0.4%, corresponding to about 0.2% actual clay.

11. Use of Filter Sand

Loss of Head is the frictional resistance of the sand to the passage of water. It is measured by the vertical distance between the level of the raw water over the sand and the level of the water in a small standpipe connected with the drains below the sand. Filters are provided with loss-of-head indicators. For any given condition of the filter bed the loss of head is directly proportional to the rate of filtration. The loss of head is made up of two parts, the frictional resistance of the clean sand and the resistance due the accumulation of dirt on the surface of the filter. The initial loss of head is that at the beginning of a run, but even this is more than for clean sand because there is some dirt on the surface from the very start.

The Frictional Resistance of sand to water when closely packed, with the pores completely filled with water, and in the entire absence of clogging, is indicated by

$$v = cd^2 \frac{h}{l} \left(\frac{t^\circ + 10^\circ}{60}\right)$$

where v is the velocity of the water in meters daily in a solid column of the same area as that of the sand, or approximately in million gallons per acre daily; c is a factor depending upon the uniformity coefficient, the shape of the sand grain, the chemical composition, the cleanness, and closeness of packing, commonly varying from 600 to 1200 for new sand and from 400 to 800 for old sand; d is the effective size in millimeters; h is the loss of head; l is the thickness of sand through which the water passes; t° is the temperature (F.).

Accuracy depends more upon good work in determining d, the effective size in millimeters, than upon any other matter. The formula applies to filter sands with uniformity coefficients less than 3, but also less closely to those with uniformity coefficients to 6, and even 10. It does not apply to coarse gravels in which the viscosity of water is no longer controlling.

Required Head in Feet for Clean Sand One Foot Thick
$c = 800$, $t = 50°$ F.

The adjacent table covers the ranges most commonly used in filtration. It applies to clean sand with no surface clogging. For thicker sand layers the head is greater in direct proportion.

Rate of filtration, million gallons per acre daily	Effective size of sand, millimeters			
	0.20	0.30	0.40	0.50
3	0.09	0.04	0.02	0.01
5	0.15	0.06	0.04	0.02
10	0.29	0.13	0.07	0.05
20	0.59	0.26	0.15	0.09
60	1.76	0.78	0.44	0.28
125	....	1.62	0.91	0.59

Preparation of Filter Sand. Occasionally natural sand is found suitable for use in filters, especially sea sand. Most bank and river sands require treatment. Coarse particles are removed by screening. Fine particles are removed by washing. If the stock contains clay it must first go through a pug mill or otherwise be agitated to break up all the lumps of clay and loosen the particles from the sand grains. It is then passed through a washing box in which the sand moves horizontally and the water vertically at a rate corresponding to the rate of settlement of sand grains of the size of the required separation. The washed sand is drawn off with but little water. The box should have one square foot of area for each cubic yard per hour to be handled.

Filter sand can be prepared from a great variety of raw stocks. The essential requirement is that the stock contain a sufficient proportion of sand grains of the right size. To find the amount of filter sand that can be prepared from a given raw stock make a mechanical analysis of it and find the percentages finer than the desired effective size and 60% line. The difference between these percentages multiplied by 2 is the theoretical amount of filter sand that can be obtained. Actually from 75 to 95% of this amount should be obtained by a suitable plant.

Stock can be **screened wet** under a spray of water to exclude particles larger than about 3 mm. in diameter. If finer screens are used wet they become clogged and the process stalls.

In preparing sand for mechanical filters where the uniformity coefficient must not exceed about 1.6, it is frequently necessary to make finer separations. Any desired separation such as 1 mm. or 1.2 mm. may be made by drying the stock in a furnace and sifting it completely dry and still warm through screens of appropriate size that are mechanically agitated. Such sand must usually be subsequently washed to remove the excess of fine particles.

It will not often pay to work stock from which more than 10% of fine material must be removed because of the difficulty of washing, nor stock that has more than 50% of gravel to be excluded.

The size of separation in sand washing depends principally upon the area of the boxes containing the mixed sand and water to which the mixture is coming, and from which the sand is drawn to the bottom, while the dirty water overflows, and upon the volume of water, taken always as the volume of the waste that overflows at the top. The approximate size of separation such that 75% of the particles of that size will be retained may be computed by

$$D \text{ in mm.} = 0.0065\, f\, \frac{\text{gallons per minute of water overflowing}}{\text{square feet of box area}}$$

in which f ranges from 3.0 for ordinary single boxes to 1.5 for specially designed boxes with water entering steadily at the bottom and overflowing at well-distributed points at the top. The rule does not apply for sand grains less than 0.10 mm. in diameter.

Voids in filter sand range from 35 to 45%, according to the uniformity coefficient and the method of packing. Close packing is obtained with sand either perfectly dry or saturated with water. Sand packed moist always has more voids and settles from 4 to 8% when it is filled with water. Voids in sand are determined by driving a cylinder of sheet iron into the sand in the filter, carefully cutting away the adjoining material with a mason's trowel, and taking out sand equal to the exact contents of the cylinder. The sand is dried, weighed, and the volume of the solid particles computed, taking into account the specific gravity, which for nearly all filter sands is 2.65. This is compared with the volume of the cylinder.

Results obtained by filling voids of dry sand with water are invariably too low because all the air is not driven out, even when filled in the most careful manner, from below.

Gravel is required in filter construction, usually in several sizes, prepared by screening. Crushed rock is used where gravel is not available. Gravel, especially in finer grades, is often obtained as a by-product from the preparation of the filter sand. Gravel should always be washed free from sand, clay, and fine particles. Where the water contains carbonic acid it is preferable free from limestone.

Washing and Handling Sand. Filter sand is commonly handled and washed hydraulically. Water through a jet under a hundred pounds pressure passes through an open space and enters a throat a little greater in diameter than the jet and carries with it sand loosened by water which surrounds the space. The mixture of sand and water will flow as a liquid through pipes to the washers and from the washers to the points where it is stored.

One volume of water is required to take up one volume of sand to make slush, which slush has about 60% of the solid sand and a specific gravity of 1.59.

Ejectors form the basis of the washing and transporting system of sand filters. They are also used with mechanical filters for the occasional sand handling and washing that is required. With a pressure of 100 lb. on a 0.7 in. jet discharging through 50 ft. of 3-in. hose to 4-in. main pipes, 10 cu. yd. of sand per hour can be lifted vertically about 50 ft. or carried horizontally 700 ft. or any proportionate combination of the two. The ejector tables given herewith permit calculations to be made for other pressures and quantities and pipe sizes.

12. Sand Ejectors

The Resistance to be overcome in the discharge piping is made up of actual lift and friction. For the lift, multiply the actual lift in feet by the specific gravity of the mixture. For friction of sand and water in 3-in. and 4-in. pipes, compute the friction for water alone, and add 3.5 ft. per thousand for each per cent of sand in the mixture. For 6-in. or larger pipe add 2.5 ft. and for 2.5-in. hose, add 4.5 ft. per thousand. These figures will be close enough for velocities 5 ft. per second or over. Sand and water mixtures will flow well at all velocities above 5 ft. per second and fairly well from 4 to 5 ft. per second. Between 3 and 4 ft. per second there will be more friction than calculated and some stoppages; and below 3 ft. per second sand and water mixtures will not flow.

Sand Ejectors and Flow of the Water and Sand in Discharge Piping

Pounds pressure, feed-water	Diameter jet, in.	Best diameter for throat, in.	Per cent sand in discharge by volume	Cubic yards sand per hour	Pressure of discharge, ft.	Friction in feet per 1000 in discharge piping			
						2-1/2 in.	3 in.	4 in.	5 in.
60	0.5	0.94	20	5.0	28	150	140	.....	....
	0.5	1.06	25	7.2	20	176	150	.....	....
	0.5	1.19	30	10.0	14	206	168	.....	....
	0.6	1.13	20	7.2	28	178	124	90	....
	0.6	1.27	25	10.3	20	222	144	110	....
	0.6	1.42	30	14.3	14	275	170	120	....
	0.7	1.15	15	6.5	39	200	114	70	....
	0.7	1.32	20	9.7	28	250	140	88	....
	0.7	1.48	25	14.0	20	325	175	102	....
	0.8	1.32	15	8.5	39	298	145	71	....
	0.8	1.50	20	12.7	28	370	180	89	73
	0.8	1.70	25	18.4	20	480	240	109	82
	0.9	1.30	10	6.5	52	350	160	61	42
	0.9	1.48	15	10.8	39	435	202	80	56
	0.9	1.70	20	16.1	28	540	250	102	71
80	0.5	0.94	20	5.7	38	154	130	.....	....
	0.5	1.06	25	8.3	27	186	145	.....	....
	0.5	1.19	30	11.5	18	225	160	.....	....
	0.6	1.13	20	8.3	38	208	128	90	....
	0.6	1.27	25	11.9	27	260	153	107	....
	0.6	1.43	30	16.5	18	320	187	117	....
	0.7	1.15	15	7.5	52	245	127	70	....
	0.7	1.32	20	11.2	38	305	158	86	....
	0.7	1.48	25	16.2	27	400	204	104	71
	0.8	1.32	15	9.8	52	370	176	76	56
	0.8	1.50	20	14.7	38	465	222	95	71
	0.8	1.70	25	21.2	27	600	290	115	84
	0.9	1.30	10	7.5	70	450	200	70	43
	0.9	1.48	15	12.4	52	550	248	91	58
	0.9	1.70	20	18.6	38	680	315	114	73

Velocity in feet per second

$$= \frac{137.5 \text{ cu. yd. per hour}}{\text{Per cent of sand in discharge} \times (\text{diameter of pipe in inches})^2}$$

Cubic yards per hour

$$= \frac{\text{Per cent sand} \times \text{velocity} \times \text{diameter}^2}{137.5}$$

Hydraulics of Ejector. The best form of throat is one approaching the shape of a Venturi meter. The calculations in this section are for a throat of this shape but somewhat worn by use and not quite in the best condition. With a carefully turned new throat the results may be higher and with worn throats lower. With the best size and shape of throat for any given condition, the tables in this section show the approximate relations. Fig. 8 shows the working parts of a sand ejector, and Fig. 9 shows the method of use in connection with the sand washing equipment and storage of clean sand.

Sand Ejectors and Flow of the Water and Sand in Discharge Piping

Pounds pressure, feed-water	Diameter jet, in.	Best diameter for throat, in.	Per cent sand in discharge by volume	Cubic yards sand per hour	Pressure of discharge, ft.	Friction in feet per 1000 in discharge piping: 2-1/2 in.	3 in.	4 in.	5 in.
100	0.5	0.94	20	6.4	47	162	126	.....	....
	0.5	1.06	25	9.2	34	200	143	.....	....
	0.5	1.19	30	12.9	23	250	164	120	....
	0.6	1.13	20	9.2	47	230	133	87	....
	0.6	1.27	25	13.3	34	300	165	103	...
	0.6	1.43	30	18.5	23	380	210	118	105
	0.7	1.15	15	8.4	65	292	144	71	....
	0.7	1.32	20	12.6	47	360	180	88	71
	0.7	1.48	25	18.1	34	470	235	108	83
	0.8	1.32	15	11.0	65	450	208	82	56
	0.8	1.50	20	16.4	47	550	260	104	71
	0.8	1.70	25	23.7	34	.....	340	130	86
	0.9	1.30	10	8.3	87	.....	240	78	45
	0.9	1.48	15	13.9	65	.....	300	100	60
	0.9	1.70	20	20.8	47	.....	370	128	76
150	0.5	0.94	20	7.8	71	195	124	90	....
	0.5	1.06	25	11.4	51	240	150	110	....
	0.5	1.19	30	15.8	35	305	180	118	....
	0.6	1.13	20	11.3	71	310	160	86	75
	0.6	1.27	25	16.3	51	400	205	104	83
	0.6	1.43	30	22.7	35	520	260	125	100
	0.7	1.15	15	10.3	98	410	190	78	56
	0.7	1.32	20	15.4	71	500	236	98	71
	0.7	1.48	25	22.2	51	640	310	120	85
	0.8	1.32	15	13.5	98	.....	285	99	59
	0.8	1.50	20	20.0	71	.....	350	123	75
	0.8	1.70	25	29.0	51	.....	450	160	93
	0.9	1.30	10	10.2	131	.....	335	100	50
	0.9	1.48	15	17.0	98	.....	410	129	67
	0.9	1.70	20	25.5	71	.....	510	163	85

Per cent of sand in water thrown by volume	5	10	15	20	25	30
Specific gravity of mixture	1.05	1.10	1.15	1.20	1.25	1.30
Per cent slush by volume	8.3	16.7	25.0	33.3	41.7	50.0
Per cent nozzle water by volume	91.7	83.3	75.0	66.7	58.3	50.0
Weight of slush per part water from nozzle	0.15	0.32	0.53	0.80	1.14	1.59
Q = ratio total weight of discharge to weight of jet water	1.15	1.32	1.53	1.80	2.14	2.59
P = proportion of jet pressure developed in discharge	0.50	0.37	0.28	0.20	0.14	0.10
T = ratio of diameter of throat to diameter of jet	1.22	1.44	1.65	1.88	2.12	2.38
V = ratio of velocity in throat to velocity in jet	0.73	0.58	0.49	0.43	0.38	0.35

The formula $(P + V)T^{1.5} = 1.65$ applies to well-shaped Venturi throat ejectors throwing sand. The best results are obtained when $QV = 0.9$ with 5% or 10% variation either way, and within this approximate range $PQ^2 = 0.65$. This is the most convenient equation for comparing efficiencies.

The slush will not enter at a velocity of more than about 40 ft. per second, and at such a velocity wears out the parts rapidly, and with high pressures this compels the use of throats slightly larger than computed as most efficient. The above proportions are suitable for pressures up to 150 lb., and may be used for all lower ones without material loss in efficiency.

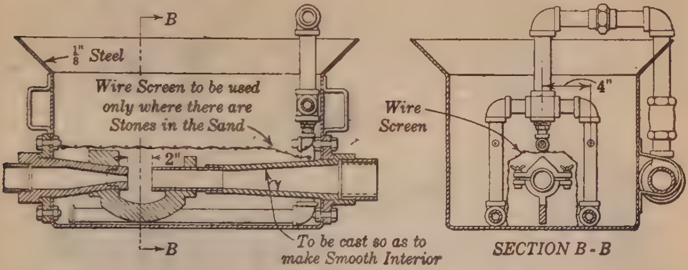


Fig. 8

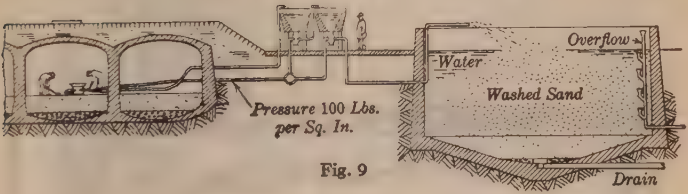


Fig. 9

13. Mechanical Filters

Mechanical filters are filters operating at a high rate with mechanical appliances for cleaning the sand without removing it from the filter. The filter tank containing the sand was formerly of wood or sometimes of steel, but reinforced concrete is now used almost exclusively. It is essential that the filter tank should be air-tight as well as water-tight, to prevent the entrance of air from the outside in the last portion of the run. Fig. 11 shows the arrangement of one mechanical filter in a group of such filters, Fig. 10 shows the arrangement of filters in the group and the main connections but does not indicate the extent of the coagulating basins.

The Sand is commonly from 30 to 36 in. deep, and with effective sizes ranging from 0.35 to 0.50, commonly about 0.40 mm. The uniformity coefficient should be as low as possible and must not exceed 1.6. Sand with high uniformity coefficients is separated into coarse and fine parts in the act of washing, and full filtering efficiency cannot then be obtained. The sand is supported at the bottom by layers of gravel, which must be heavy enough and open enough to allow the wash water to pass without lifting.

The Strainer System serves the double purpose of introducing wash water to wash the sand with a reverse current and of carrying off the effluent. In order to serve its purpose of carrying off the effluent the frictional resistance of the system must not exceed one-fourth the frictional resistance of the sand at the beginning of the run. Otherwise the near parts of the filter would operate at higher rates than the remote parts.

The fundamental requirement of the strainer system when used for washing is that it shall give an equal dispersion of wash water over the whole area of the filter. There is a tendency for the sand to lift at one place. When this happens the pressure on the strainers at that place is reduced and there is a tendency for all the wash water to go

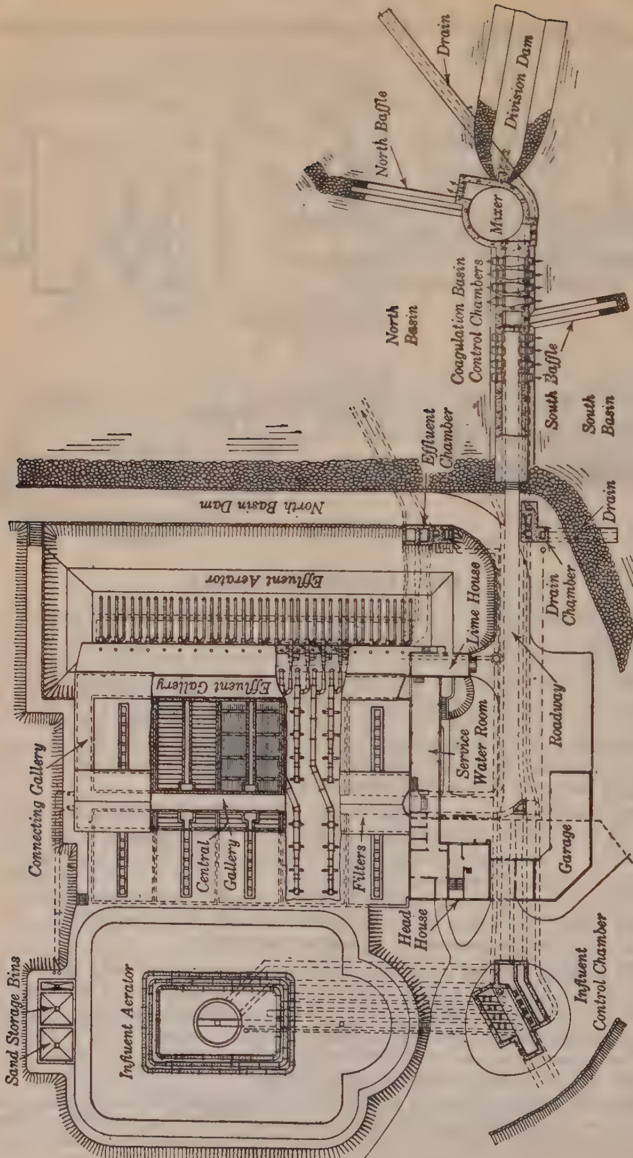


Fig. 10. Layout of a Mechanical Filter

out at one place. This is controlled by putting restrictions on the outlets having resistance at the rate of use greater than the resistance offered by the sand, so that these throats control the position of overflow of the water. The throats must be small enough to give the desired rate of wash with a frictional resistance that will insure the even distribution of the wash water, and large enough so that it does not exceed the economical head applicable to this purpose. The area of main drains should equal twice the area of all the throats supplied by them, and lateral piping should have 2-1/2 times the area of all the throats supplied. In a large filter the ratios should be a little higher.

In a typical design for a mechanical filter, the wash water is supplied at a minimum

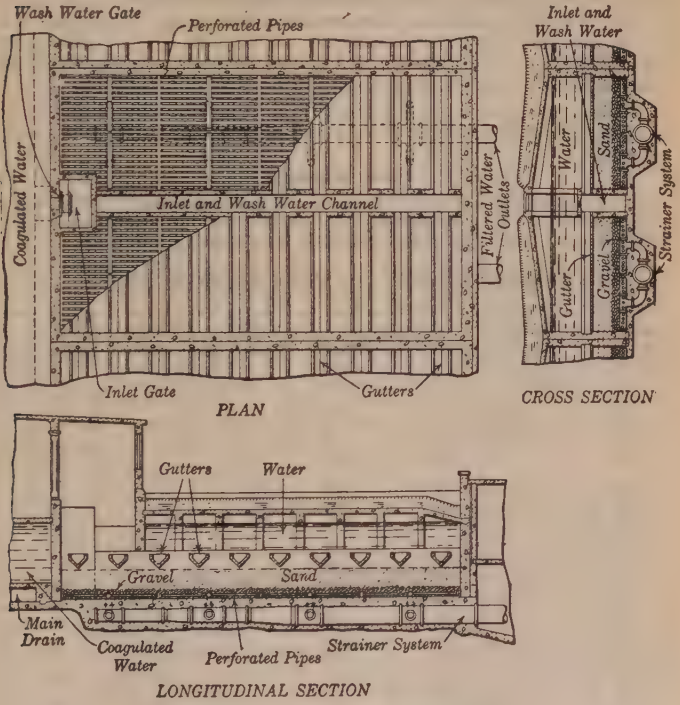


Fig. 11. One Unit of a Mechanical Filter

head of 10 lb. or 23 ft. measured from the level of the gutters, of which 3 ft. are required to lift the sand, 8 are consumed in pipe and gate losses and 12 are effective on the openings in the perforated pipes. For a rate of wash of 26 in. per minute and a coefficient of discharge through the openings of 0.65, the required area of holes in the aggregate is equal to 0.2% of the filter area. To correspond with this and to keep the friction of the pipe system to a proportionate amount, the main drain must equal 0.4% in a small filter or 0.5% in a large one, and the laterals and intermediate piping must equal about 0.6% of the filter area. If these pipes are too small, the distribution of wash water will be bad. In filtering at the ordinary rate which is about one-eighth of the above stated rate of wash, the loss of head, varying approximately with the square of the quantity, will be about 4 in. for the entire pipe system from the gravel to the regulating apparatus.

Mud Balls. Mud balls are accumulations of dirty sand on the bottom of the filter just above the gravel, resulting from settling of caked sand from the top of the filter at the beginning of the wash. Mud balls were prevented in early designs of circular filters by revolving rakes. In other cases air washing, that is to say, blowing air into the bottom of the filter through a separate system of pipes, has been used to break up mud balls. They may be sometimes broken up and pulled out by wire nets on long poles drawn through the sand while the filter is being washed. A relatively high rate of wash is favorable to keeping a filter clear of mud balls, but going too far results in loss of fine sand grains and changing the character of the filter sand. Under any conditions some mud balls accumulate in the bottom of the filter and tend to reduce its efficiency. It is recommended that all the sand be removed from the filter and washed with close screening annually to keep it in good order.

Washing. As a general rule that rate of wash that will expand the filter sand by about 30% is sufficient and, on the other hand, will not wash out too many

Filter Washing

Table showing the rate of wash in inches per minute required to lift the sand in a mechanical filter at various temperatures and with several sizes of sand and to give various degrees of expansion of the sand in the filter during the wash.

$$\textbf{Basis: Rate} = 30d^{1.5}\,(1 + 0.06\%\ \text{expansion})\,\frac{t+30}{80}$$

Temperature	Size of sand	10% expansion	20% expansion	30% expansion	40% expansion	50% expansion
32	0.35	7.7	10.6	13.4	16.3	19.2
35		8.1	11.1	14.1	17.1	20.2
40		8.7	11.9	15.2	18.5	21.7
45		9.3	12.8	16.3	19.8	23.3
50		9.9	13.7	17.4	21.1	24.8
55		10.5	14.5	18.5	22.4	26.4
60		11.2	15.3	19.5	23.7	27.9
65		11.8	16.2	20.6	25.0	29.5
70		12.4	17.1	21.7	26.4	31.0
75		13.1	17.9	22.7	27.7	32.6
32	0.40	9.4	12.9	16.5	20.8	23.7
35		9.9	13.6	17.3	21.8	24.6
40		10.6	14.6	18.6	23.4	26.5
45		11.4	15.6	19.9	25.1	28.4
50		12.1	16.7	21.2	26.8	30.4
55		12.9	17.7	22.6	28.5	32.2
60		13.7	18.8	23.9	30.2	34.1
65		14.4	19.8	25.2	31.8	36.0
70		15.2	20.9	26.6	33.5	37.9
75		15.9	21.9	27.9	35.2	39.8
32	0.45	11.2	15.4	19.6	23.9	28.1
35		11.8	16.2	20.6	25.0	29.4
40		12.7	17.4	22.2	26.9	31.7
45		13.6	18.7	23.7	28.9	34.0
50		14.5	19.9	25.3	30.8	36.2
55		15.4	21.2	26.9	32.7	38.5
60		16.3	22.4	28.5	34.6	40.8
65		17.2	23.7	30.1	36.6	43.0
70		18.1	24.9	31.7	38.5	45.3
75		19.0	26.1	33.2	40.4	47.5

of the fine particles and change the character of the filter sand. The rate required to produce this result depends, among other things, upon the grain size of the sand and upon the temperature of the water, the required rate being higher in summer and lower in winter and higher as the filter sand is coarser.

Viscosity is the dominant element in the conditions that determine the rate of filter washing; but the phenomena come in a middle ground of change, corresponding to that observed above the critical velocity of flow of water in small pipes, and the conditions deviate materially from those of viscous flow.

Under these conditions no simple formula can be found, but within certain limits the empirical formula used in preparing the table given herewith is a close approximation. It must not be extended to conditions far different from those indicated.

As a general rule the rate of washing that gives 30% expansion is believed to be sufficient and best adapted to average conditions.

Gutters to carry off the wash water should not be less than 12 nor more than 18 in. at their edges above the surface of the sand. The eddying action of the water entering them reduces velocity, and the size of the gutters must be larger than is computed by the usual hydraulic formulas.

Regulating Apparatus. The regulating apparatus for rate control is now almost always of the Venturi type arranged so that change in rate as the filter becomes clogged is automatically met by opening the control gate to maintain the desired rate. In a few recent designs, aerator nozzles are attached in such a way that an automatic apparatus sends as much of the filtered water as possible through them, thus utilizing the surplus head of the filter for aeration.

Loss of head ranges from 2 ft. or less at the beginning of a run to the limit that is allowed, which may be anything from 4 to 10 ft. With the loss of head limited to 4 ft., no suction exists in the filter, and the whole operation is said to be performed by a positive head. With loss of head going to 6 ft. and over the outlet pipe acts as a draft tube and produces suction in the filter in the latter part of the run. This is known as negative head.

Negative head is efficient in making a filter operate, but practically its action is limited because with it air is extracted from the passing water which fills the voids in the sand and soon cuts off the suction. The first foot of negative head is only a little less useful than the last foot of positive head, but each additional foot utilized yields a smaller return than the last. The full effect of negative head may be utilized by putting an air chamber on the outlet pipe and pumping the air from it in the latter part of the run, thereby making the full negative head available.

The Frequency of Washing depends upon the character of the water and especially on the size and manner of operation of the coagulating basin, and commonly ranges from 8 to 48 hours. Washing requires about five minutes and the auxiliary operations more than as much more. From 2 to 5% of the water filtered is required for wash water.

The Rate of Filtration is generally 2 gal. per square foot per minute, equal to 125 million gal. per acre, or 117 cm. per square meter, per day, 347 sq. ft. of filter pass water at the rate of one million gallons per 24 hours.

Leaks Along the Sides. One of the commonest causes of inferior work in mechanical filters is the passage of unfiltered water through a space between the filter sand and the filter walls and this water mixes with the water which has passed through the sand and impairs the quality of the effluent.

As the filter run progresses, and head increases, the sand is compressed. Most of the compression is vertical, but some of it is horizontal. The horizontal compression opens a crack between the caked sand and the filter wall and the crack may pass unfiltered water while it is still too small to be observed. Dirty filter sand compresses more than clean sand and with old filter sand there is more deterioration in quality than with new sand. This used to happen with sand filters but for many years a design for the sides at the bottom has been used which prevents it. The same method unfortunately cannot be applied to mechanical filters because it would leave an unwashed strip around the edge; but roughening of the filter walls and the use of narrow offsets or ledges is advisable.

The erroneous idea has often been expressed that the filter sand or the sediment layer on its top "breaks" under pressure as the loss of head increases. The effect has been widely observed, but the true explanation less often recognized.

14. Sand Filter Beds

Sand Filters differ from mechanical filters in that the rate of filtration is much lower and the filter is cleaned when dirty by scraping off the surface layer of dirty sand, which is washed and ultimately replaced. The sand should have an effective size between 0.25 and 0.35 and a uniformity coefficient not exceeding 3.0. Many old filters had sands with uniformity coefficients up to 5, but such sand is much more difficult to keep in good order. Sand suitable for sand filters is more cheaply obtained than sand with a low uniformity coefficient required for mechanical filters.

The Gravel is commonly placed in three layers. The lowest layer is 7 in. thick and has 3/4-in. to 2-in. screens with an effective size of about 20 mm.; the second layer 3 in. deep, 3/8 to 3/4 in. screens with an effective size of about 8 mm.; and the top layer 2 in. deep, with an effective size of from 2 to 3 mm.

Underdrains must be designed so that, at the proposed rate of filtration, the frictional resistance of the whole system will not be more than about one-fourth of the frictional resistance of the sand when clean. Otherwise when the filter is started that part of the filter near the outlet would do most of the work.

Underdrains for Sand Filters

Rate of filtration, million gallons per acre daily	5	6	8	10	15
Average resistance of clean sand in feet	0.150	0.180	0.240	0.300	0.450
Total allowable friction and velocity head in underdrainage system	0.037	0.045	0.060	0.075	0.112
Approximate ratio of filter area to area of main drain	5100	4700	4200	3800	3200
Approximate maximum velocity in main drain (varying somewhat with size)	0.90	1.00	1.18	1.34	1.68
Approximate maximum velocity in laterals (varying somewhat with size)	0.55	0.61	0.72	0.82	1.04

Masonry covers are usually provided for sand filters. In the north they are necessary as a protection from cold winter weather which would interfere with sand cleaning operations. Open filters were formerly used in warmer climates but covered filters have proved to be more advantageous in most places in the United States where sand filters are used. The net height inside the masonry structure is 12 ft. to accommodate the filtering materials and give convenient headroom for cleaning operations.

Filters are built in units ranging from an acre in the largest plants to half an acre and less in small ones. There must be enough units so that one unit can be out of service at any time for cleaning. As resanding operations frequently run through several weeks there must also be enough units so that the supply can be maintained with two units out for such periods.

Maximum Areas of Filter Beds Drained in Square Feet

Diameter of drain, in.	Shape and kind of drain	Rate of filtration, million gallons per day				
		5	6	8	10	15
4	Round lateral...	264	245	218	200	168
5	Round lateral...	420	390	345	316	266
6	Round lateral...	610	570	500	460	390
8	Split lateral.....	520	490	430	400	320
10	Split lateral.....	830	770	680	630	530
12	Split lateral.....	1 200	1 120	1 000	910	770
10	Round main....	2 700	2 500	2 200	2 000	1 700
12	Round main....	3 900	3 600	3 200	2 900	2 400
15	Round main.....	6 200	5 800	5 100	4 600	3 900
18	Round main.....	9 000	8 300	7 400	6 700	5 600
21	Round main.....	12 300	11 400	10 000	9 100	7 600
24	Round main.....	16 100	14 900	13 200	12 000	10 000
36	Round main.....	37 000	34 000	30 000	27 000	22 000

Automatic controllers are sometimes used on the outlets of sand filters, but as the change in loss of head is slow, hand control in connection with adequate devices for indicating at all times the rate of filtration and loss of head is sufficient.

The Period of a sand filter is the time between cleanings expressed in millions of gallons per acre. That is to say, if a filter operates 20 days at a 5-million rate the period is 100. The average period for a plant is found by dividing the number of million gallons filtered in one year by the total number of acres of filter surface cleaned.

Periods are increased by drawing the water off when the loss of head has reached the allowed limit and raking the surface of the sand and then proceeding. It does not usually pay to rake more than once in one period. Thorough sedimentation lengthens the period. The application of coagulant lengthens it if applied sufficiently long before filtration, but with a short period of settling after application it shortens it. Preliminary filters lengthen the period. Periods commonly range from 50 to 200. If they average less than 50 there is something wrong with the arrangements.

The Loss of Head in sand filters is commonly limited to 4 ft., but this is arbitrary and losses up to 5 or 6 ft. are sometimes permitted. The **rates** employed with sand filters filtering river waters without preliminary chemical treatment are about 3 million gallons per acre daily. For lake and reservoir waters rates twice as high are used. For filtering river waters with preliminary chemical treatment and settling when the water requires it, corresponding rates can be used.

Cleaning. At the end of a period the dirty sand to a depth of from 1/2 to 1-1/2 in. is removed. It is best shoveled to movable ejectors throwing the water through the piping system to sand washers outside. Scrapings are repeated for a considerable period, averaging 2 years, when washed sand is restored bringing it back to the original level. Cleanings in very cold weather may be done with the dirty sand piled up on the surface of the filter, where it remains until spring. The quantity of water under pressure required for handling and washing sand is commonly from 0.4 to 1.0% of the quantity filtered.

An old filter is commonly more efficient within limits than a new one, and the water filtered during the last part of the period is frequently better than in the first part. It has sometimes been recommended that the first water from sand filters be wasted. It is better practice, however, to start the filter at a lower rate, at which the first effluent will be good, and gradually increase the rate after processes of purification become established.

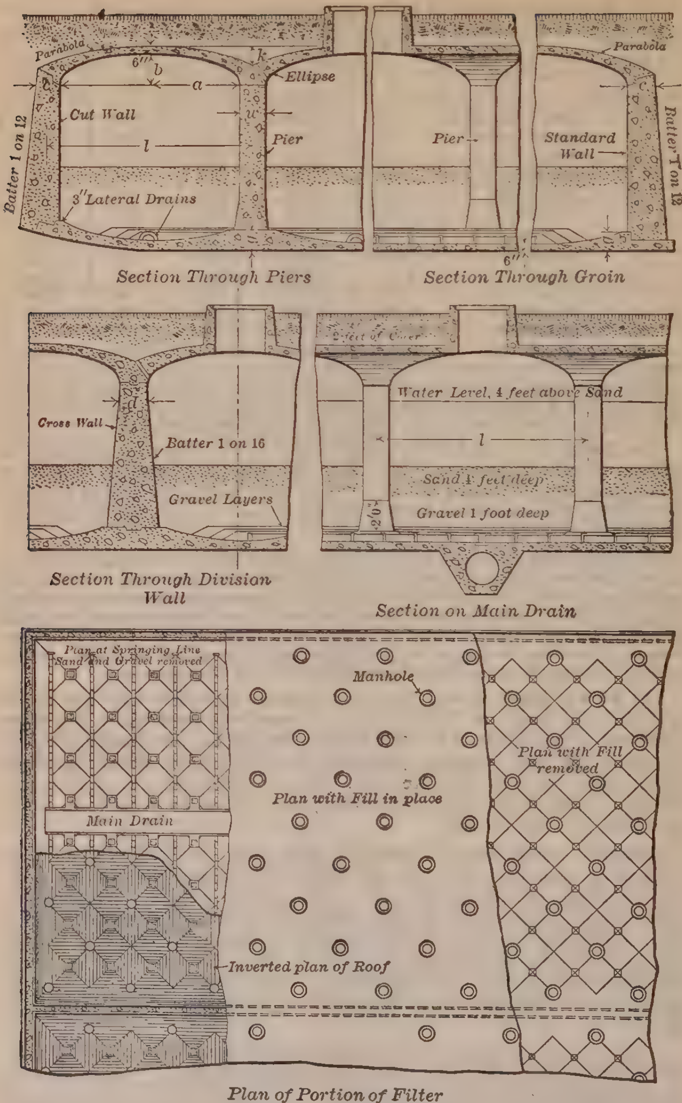


Fig. 12. Slow Sand Filter with Groined Roof

15. Results of Purification

The Cost of installing filters at post war prices may range from $20 000 to $60 000 per million gallons daily capacity, figures between $30 000 and $40 000 being most common. As the filters must usually be 50% greater in capacity than the average output, the cost based on the average output will be correspondingly greater. Operating costs range from $2 to $4 per million gallons and less for sand filters treating clear waters to $6 or $8 per million gallons for mechanical filters with turbid and colored waters. The whole cost of filtration, including both operating expenses and interest and depreciation charges on the capital, commonly ranges from $12 to $20 per million gallons.

Sand filters are often more expensive to build but cheaper to operate than mechanical filters. Lake waters are cheaper to purify than river waters, and reservoir waters are intermediate. Large amounts of color and turbidity add to the cost of treatment. Softening is more expensive than ordinary filtration.

In filtering river waters 99% removal of bacteria is commonly obtained. The efficiency is usually somewhat higher in summer than in winter. Growths of bacteria in the underdrains occasionally occur. These have no hygienic significance but reduce the percentage of apparent removal. These are most likely to occur in filters operating at low rates. Nearly all of the remaining bacteria are now removed by disinfection with chlorine. Purification as practiced does not absolutely remove infection, but it reduces the risk to one so extremely small that it may be considered negligible. Experience with filtration extending over half a century has shown that well-designed, well-constructed, and well-operated filter plants can be relied upon to furnish a safe and acceptable supply of water.

It is good practice to select a raw water for filtration as free from pollution as possible and to take all reasonable precautions to prevent the increase in pollution of existing supplies.

Intermittent Filters are sand filters in which the oxidizing action of the air is more important than the straining action of the sand. They are built of coarse sand with large underdrains freely open to the outlet and with no controlling or throttling devices. Water is applied generally once a day and generally for a period of not more than 12 hours. It is essential that the water on the filter should be drawn down, leaving the surface entirely exposed for some time before the next application of water. This fills the pores of the filtering material with air and the water next filtered is brought in contact with this air. The oxidizing action of an intermittent filter is similar to the oxidizing action of an intermittent filter or sprinkling filter which is used for treating sewage.

Intermittent filters are used for treating badly smelling waters, especially from shallow reservoirs and reservoirs that have at times high temperatures, as in the tropics. Such waters may contain large quantities of organic matter and be incapable of being filtered satisfactorily by either mechanical filters or sand filters of the ordinary type. The rate of filtration depends upon the amount of organic matter in the water, and may be from 3 to 6 million gallons per acre per day. Cleaning the sand is done by the same methods as those used for sand filters.

DISTRIBUTION OF WATER

16. Systems of Distribution

A Waterworks System must secure its supply when and where it can be gotten, and must deliver it when and where required by its customers, or "takers." Waterworks structures are required to collect the water; to hold it from times when it is available until it is required; to pump it to a higher elevation; to convey it from the point where it is available to the points where it is required, and to allow the water to be measured and controlled at all points. Water is required by the takers at very unequal rates, following the requirements and emergencies that arise in their business or domestic uses, and the fundamental requirement controlling the design of works is to secure the ability to supply water wherever and whenever required and in whatever reasonable amounts may be needed.

The Distribution System includes all the main pipes and lateral pipes, the standpipes and distributing reservoirs, gates, meters, all services and connections as far as owned by the water department within and near the area that is actually served with water. The piping in a distribution system must be designed so that water can be supplied to any point at any time at the greatest rate at which water may be fairly demanded at that place.

Gridiron System. This is a system in which all pipes are connected with all other pipes at street intersections, so that in case of a fire at any point water comes to that point through pipes from all directions. This arrangement is more advantageous in supplying water for fire protection than the branching system, which would be sufficient and often best for supplying water for all purposes except fire service. The gridiron system is practically universal in American cities.

An economical system for the distribution of water for routine uses only would consist of a system of branching pipes, each branch being made sufficiently large to supply the water to the territory served by it at the time of day when use is greatest.

High-service Systems. Where there are considerable parts of a city where the ground is high, so that the service and pressure used in most of the area do not suffice to supply them properly, it is common to cut them off and make special high-service systems for them. Each high-service system has its own distribution and distributing reservoir (or else direct pumping), and it is to be considered as if it were an entirely separate system of waterworks. Water is commonly taken from the low-service mains, and those mains must be large enough to supply the high service water in addition to their other functions. In all other respects they may be treated as if the high service did not exist. In nearly all cases water is pumped specially into the high-service pipes.

The Direct Pumping System is used in the middle states and in general wherever the ground is flat. With it there is no distributing reservoir, but water is pumped into the pipes at all times as needed. Pumps of special design are required for this service. When but little water is used the pump must go slowly and automatic regulators must close the throttle so that it will not go too fast and increase the pressure. When water is to be used rapidly the pump must increase its speed and deliver the increased volume of water at full pressure.

Pumps operating on this system are not as efficient as pumps operating at constant speed with a distributing reservoir connected with the system. The pumps must have much greater capacity than where there is a distributing reservoir. The capacity of the pumps must always be equal to three times the average annual rate of consumption, and in small works the capacity of the pumps must be four or five or more times as great. In general the capacity of pumps must be sufficient to maintain the fire service and the domestic service at the same time. It is further good business when possible to have one pumping unit in reserve beyond this capacity.

Reservoirs with Direct Pumping. Wherever the water is filtered or ground water obtained there is a limit to the rate at which pure water can be provided in emergency, and it is commonly necessary to have water during large fires at a greater rate than can be furnished by the filters or wells. To meet this condition pure-water reservoirs at the pumping stations are provided. The size of reservoir is to be determined by the same considerations that determine the size of an elevated distributing reservoir. With a reservoir of this kind kept ordinarily full or nearly full the supply works may operate at a continuous rate throughout the twenty-four hours, and there will always be a reserve of pure water that can be pumped as fast as the pumps can take it in case of fire or other emergency. Such reservoirs are always covered.

Auxiliary Fire Service Systems consist of high-pressure pumping station and pipes laid in central portions of cities where the greatest values of property subject to fire hazard are concentrated. These pipes are built to withstand pressures up to 300 lb., and are connected with hydrants of special design and painted some other color than the hydrants connected with the ordinary pipes. The pumps may be operated by gas engines, electricity, or by other power. Such systems are used only in case of fire. In case of need the pumps are started and the pressure is raised on the system to such a point that efficient fire streams are obtained by attaching lines of hose to the hydrants. No steam fire engines are used.

Salt Water is sometimes used and is believed to be more efficient in extinguishing fires than fresh water. It also results in more damage to goods and property not destroyed by fire with which it comes in contact. Otherwise water is taken from the nearest lake, river, or reservoir or from the ordinary water mains. Where auxiliary fire systems are used there is obviously less demand upon the regular supply system for fire service, and its capacity for rendering fire service may therefore be reduced somewhat. Frequently more money is spent on auxiliary fire service systems than would suffice to reinforce the regular waterworks system to the extent that good service could be obtained from it.

Increased Pressure for Fire Service is obtained in some direct pumping systems by having additional boilers kept under steam and turning on more steam when a fire alarm is sounded. By this means the pressure throughout the city is increased and hose streams are obtained from hydrants without the use of fire engines. This system is much used with direct pumping in the middle states. It is best adapted to small places. The expense of the system and the disturbance that comes to the distribution from increasing the pressure in the whole city are too great in large cities where fire alarms are sounded frequently.

17. Water Consumption

Per Capita Consumption is the amount of water used per day for each person living in the city or area supplied on the basis of the annual average figures. In other words, it is the whole quantity of water supplied in gallons in one year, divided by 365 and divided by the total population of the district supplied with water. Domestic and public purposes will usually take from 50 to 70 gal. per capita daily, a little more where water is very cheap, and a little less where it is comparatively dear. To this must be added industrial uses which include railways, shipping, etc. Industrial uses are very variable and are sometimes larger than domestic uses. In hot, arid climates much water is used for irrigation and lawn sprinkling.

Maximum Monthly Rate of Consumption. During that month in the year when the consumption is highest, from 10 to 25% more water is used than the average for the year. In some cases 40% more water is used.

High monthly rates of consumption are usually associated with either a very dry period, with more than the usual sprinkling of streets and lawns, or an exceptionally cold month, with a continued draft of water through many services to keep exposed and imperfectly protected pipes from freezing. Where services are metered the excess consumption in cold weather largely disappears. It is cheaper to cover the pipes or otherwise to protect them from freezing than to pay for the water that it is necessary to allow to run in order to protect them.

Maximum Weekly and Daily Rates. There will be some weeks and some days when the quantities will considerably exceed the average for the maximum month. Generally a maximum daily consumption of 10 or 15 gal. per capita in excess of the average for the maximum month must be expected.

Hourly Fluctuations in Flow. Water is required primarily for domestic and manufacturing purposes, and for these purposes is required in quantities that are fairly well determined and at times that do not vary very much from day to day. The greatest normal use of water is in the morning hours. The afternoon use is a little less. The night use of water is comparatively small.

Leakage goes on all the time. The ordinary range from 25% above to 25% below the average covers the fluctuations for 12 hours per day. On an

Part I—Fully Metered Systems

Place	Average output, millions of gallons daily	Population supplied in 1926	Number of services	Gallons per capita daily
Detroit, Mich.	212	1 680 000	265 640	126
Cleveland, Ohio	169	1 135 000	139 412	149
Boston Metropolitan District	130	1 326 000	192 337	98
Milwaukee, Wis.	75	587 000	83 135	128
Newark, N. J.	54	540 000	56 232	100
Passaic Water Co., N. J.	53	424 000	35 253	125
Cincinnati, Ohio	50	420 000	77 479	119
San Francisco, Calif., S. V. W. Co.	49	650 000	100 000	75
New Orleans, La.	48	433 000	76 898	110
Minneapolis, Minn.	46	430 000	94 000	106
Seattle, Wash.	41	433 000	67 000	95
Toledo, Ohio	35	320 000	61 000	110
Portland, Ore.	34	351 000	83 146	96
East Bay Water Co., Calif.	32	531 000	117 000	60
Rochester, N. Y.	28	305 000	58 500	92
Atlanta, Ga.	28	285 000	51 000	98
Columbus, Ohio	27	300 000	52 979	91
Hackensack Water Co., N. J.	25	400 000	59 205	62
Providence, R. I.	24	315 000	40 106	76
St. Paul, Minn.	22	297 000	55 000	74
Akron, Ohio	19	220 000	39 000	85
Reading, Pa.	18	115 000	25 500	160
Hartford, Conn.	17	198 000	20 736	84
Worcester, Mass.	16	196 000	27 084	80
Memphis, Tenn.	15	176 000	38 000	87
Springfield, Mass.	14	145 000	20 890	96
San Diego, Calif.	14	140 000	31 045	100

Part II—Partially Metered Systems

Place	Average output, millions of gallons daily	Population supplied in 1926	Number of services	Percentage of services metered	Gallons per capita daily
Chicago, Ill.	900	3 325 000	378 729	15	270
New York, N. Y.	823	5 646 000	553 681	25	147
Philadelphia, Pa.	342	2 025 000	438 000	32	169
Pittsburgh, Pa.	125	610 000	105 037	42	205
St. Louis, Mo.	119	819 000	133 697	8	145
Buffalo, N. Y.	118	575 000	....	48	205
Baltimore, Md.	109	820 000	180 000	21	132
Washington, D. C.	69	498 000	91 588	86	139
Wilkes-Barre, Pa., S. B. W. Co.	60	371 000	80 000	1	162
Denver, Colo.	58	335 000	59 423	2	173
Kansas City, Mo.	51	409 000	84 643	82	125
Louisville, Ky.	44	340 000	57 934	16	139
Indianapolis Water Co.	34	380 000	73 259	34	90
New Haven Water Co.	27	260 000	38 000	45	105
Bridgeport Water Co.	27	205 000	27 900	24	130
Wilmington, Del.	15	124 000	20 778	87	123
Norfolk, Va.	13	201 000	28 000	85	65
Cambridge, Mass.	12	120 000	16 049	50	103

average the output will exceed 40% above the average 3 hours per day, 50% above the average 1 hour per day and 60% above the average 15 minutes per day, and 80% above the average very rarely. On Sundays and holidays the fluctuations are less than on business days. For unmetered systems the percentage fluctuations are smaller and the above figures for per cent may be used as per capita variations, i.e., 40 gal. per capita daily above the average will be exceeded 3 hours per day. Pipes for distribution with no fire service should have a capacity of 70% above the mean to meet peak loads.

Waste of Water. The amount of water that must be supplied at the source is always greater than that actually needed for use. There is leakage from the main pipes, and from the service pipes, and from a considerable percentage of the plumbing fixtures in the houses and other buildings, and water is allowed to run uselessly from many openings. It is mainly for this reason that there are such great differences in the per capita consumption in American cities.

Waste from Service Pipes. Bad plumbing and the unnecessary draft of water from fixtures are best checked by putting meters on the services and adopting a schedule of rates by which the payments of takers are dependent upon the amount of water passing through the meters. If the water that is wasted is paid for at a fair price, then the water department has no occasion to object to the waste. The waste beyond the meter becomes entirely a matter for the taker to consider.

Loss of Water by Leakage from Mains is best detected by dividing the system by closing the valves into comparatively small sections and by supplying water to each small section in rotation through a meter. Meters may be put on the mains temporarily or permanently for this purpose.

It is often possible to shut all the gates on a section and maintain the supply to it through a line of fire hose, connecting one hydrant inside the area with another hydrant outside, a meter being placed in the line of hose.

By opening and closing gates, a number of adjoining sections may be tested with the same equipment and in the course of a short time. Tests of this kind are best made between midnight and five o'clock in the morning, when the normal use of water is at a minimum and there are practically no fluctuations in the rate of draft. Such tests show the relative tightness of different parts of the system, and the lengths of pipe where the greatest leakage occurs are then to be dug up and repaired or replaced with pipes that do not leak.

Amount of Growth to be Anticipated. In designing pipe lines, it is necessary to anticipate growth to a certain extent in order to avoid the necessity of duplicating the lines at an early date. On the other hand, anticipating future growth to an unreasonable extent results in burdening present takers with the cost of facilities provided for the future to an unreasonable extent. In general, all new pipe lines should be designed to serve a population 50% greater than the present population, and in cases of special difficulty, where an additional line would be specially difficult or expensive, a greater growth than this should be anticipated.

Increasing the diameter of the pipe 1% increases the carrying capacity 2.63%, and increases the cost of the pipe from 1 to 1.5%, according to the size and class of pipe and the conditions under which it is laid. On this basis adding 1% to the investment adds from 1.75 to 2.63% to the carrying capacity, $100 invested now in increasing the size of the pipe adds as much to the capacity as from $175 to $263 invested in a new pipe line at some time in the future if the new line is of the same size as the present one. $100 invested now at 5% will amount to $175 in 11 years and $263 in 20 years; at 4% the increase will be reached in 14 years and 25 years, respectively. In general these represent economical limits of time to be anticipated.

As a general rule design should be made for ten or fifteen years only where the growth is over 3% per annum or where money is hard to get, and design for twenty or twenty-five years where growth is under 2% per annum or where money is obtainable at a low rate, and also in all cases where pipe is less than 12 in. in diameter or where pressure is light. There are many exceptions to this rule under peculiar conditions and it must be applied with caution.

18. Fire Protection

The Requirements of Fire Service vary greatly. In European cities, with fireproof buildings, but little water is required for the extinguishment of fire. In tropical countries, where buildings are widely separated and represent but small value, and often in wet climates, it does not pay to furnish fire service. It is better to let buildings burn now and then than to provide long and larger pipes and other equipment that would be required for fire service. In American cities wooden construction is common and wooden floors are used in many buildings having brick walls. A large pipe capacity is required to provide the water which is required for extinguishing fires in such buildings.

The Amount of Water required for extinguishing fires is not very large in the aggregate, but when fires occur it is wanted at a high rate, and pipes must therefore be provided of large capacity to meet this demand. Pipe sizes required for fire protection in American cities are always larger than those required for other uses, and the size of pipe to be selected within the area of the distribution system, and between it and the distributing reservoir or pumping station where direct pumping is used, is mainly controlled by questions of fire protection.

Water Required for Fire Service. The amount of water to be provided for fire service depends upon many matters; among others, the size of buildings, the materials and methods of construction; upon how near the buildings are together; the pressure at which the water is available; upon whether auxiliary fire systems are available; upon how great a loss of life and property might result from a bad fire, and upon the cost of making a given quantity of water available and the financial ability of the system or community to pay for doing it.

For Average American Conditions, take the square root of the population in thousands and this indicates the rate in millions of gallons of water per day at which water should be provided for fire service.

For example: If the population is 9 thousand allow water at a rate of 3 million gallons per day for fire service. If the population is 25 thousand allow 5 million gallons per day, and if 100 thousand allow 10 million gallons of water per day.

The pipes must be designed large enough so that the quantity of water for fire service will be available even though the fire occurs at a time when water is being used at a high rate for other purposes. It is not necessary to assume the extreme maximum rate of draft for other purposes; some chances can be taken. To find the required capacity add, first, the average annual rate of consumption; second, 40 gal. per capita to cover ordinary fluctuations; third, the amount of water allowed for fire protection.

Concentration of Water for Fire Service. In the case of cities up to 100 000 inhabitants it is generally necessary to provide pipe capacity so that the whole amount of water provided for fire protection can be delivered with some loss of pressure in the neighborhood of the closest, largest, highest and most valuable buildings, and at each of such points if there are several; elsewhere piping capable of delivering smaller quantities varying with the kind and value of construction and the proximity of the various buildings.

Population That Can Be Supplied by Pipes of Various Sizes

Based on an average use of one hundred gallons per capita daily

Diameter of one pipe line, in.	For two or more pipes, sum of areas in sq. in. Sectional area of pipe, sq. in.	With an average amount of fire service *			With no fire service. Maximum draft 170 gallons per capita daily		
		Flat slopes and long lines V = 2 †	Average conditions V = 3	Steep slopes and short lines V = 4	Flat slopes and long lines V = 2	Average conditions V = 3	Steep slopes and short lines V = 4
4	13	13	28	50	660	990	1 330
6	28	64	130	220	1 490	2 240	2 950
8	50	180	380	660	2 650	3 980	5 320
10	79	410	860	1 450	4 150	6 190	8 280
12	113	820	1 660	2 750	5 950	8 950	12 000
16	201	2 250	4 400	7 000	10 600	15 900	21 300
20	314	4 700	9 000	13 700	16 500	24 800	33 200
24	452	8 200	15 300	24 000	23 900	35 800	47 800
30	707	16 300	29 000	44 000	37 400	56 100	74 800
36	1018	27 500	48 000	70 000	53 800	80 500	108 000
42	1385	42 000	71 000	103 000	73 200	110 000	146 000
48	1810	60 000	102 000	145 000	95 300	142 000	190 000
54	2290	81 000	135 000	192 000	121 000	181 000	242 000
60	2827	107 000	176 000	250 000	148 000	224 000	299 000

* Gallons daily $= 140$ pop. $+ 1\,000\,000\sqrt{\text{Pop. in thousands}}$.

† V = velocity, feet per second.

This table may be used as a very general guide. With high per capita consumption and bad fire conditions the sizes should be increased. Under opposite conditions they may be reduced. It will often pay to make pipe sizes a little smaller in the distribution and larger in the supply mains without changing the total capacity of the system.

A Standard Fire Stream is one flowing 250 gal. per minute through a smooth nozzle 1-1/8 in. in diameter, with a pressure at the base of the tip of 45 lb. Such a stream is effective to a height of 70 ft. above the ground or with a horizontal carry not exceeding 63 ft. When fed through the best quality 2-1/2-in. rubber-lined hose the hydrant pressure required to throw such a stream taken while the stream is running is as follows:

Feet of hose	= 50	100	200	400	600
Pounds per square inch	= 56	63	77	106	135

The hydrant pressure is less during the fire than at other times, because more head is lost in friction in the pipes, and the ordinary pressure must be greater to insure standard conditions during the fire. The best hydrant pressure for general use is considered to be from 80 to 100 lb., but as other conditions are frequently controlling, fire service must be largely adapted to what is available.

The best statement of the hydraulics of fire streams and nozzles is in a paper by John R. Freeman, Trans. Am. Soc. C. E., vol. 21, p. 303.

Pressure for Domestic Service Only. At the street line 20 lb. per sq. in. will raise water to the upper floor of three-story residences and allow a fair service, but generally 40 lb. per sq. in. is the least allowance for fair domestic service. For business blocks and higher buildings higher pressures are needed; 60 or 70 lb. per sq. in. is not too much to give fair service in mills and business blocks that are not especially high. High steel buildings generally pump their own water and no effort is made to supply them without such pumping.

Pressure for Fire Service. If steam fire engines are used and depended upon as in many American cities, the only requirement for pressure is that during fires and with the heaviest draft the pipes shall have sufficient capacity to supply water to the steam fire engines and at the same time retain as much pressure as is needed for domestic service. If the pressure is higher, hose streams can be obtained from the hydrants without the use of the fire engines. The additional pressure to permit this to be done is very desirable. A pressure of 70 lb. during fires is the lowest pressure that permits effective hose streams to be obtained for use on buildings of moderate size. If only residences are involved, 50 or 60 lb. will give fair streams. In business districts with large buildings better hose streams are obtained with higher pressures, and in general the higher the pressure the better the fire service. 100 lb. gives a good working service without steam fire engines. Higher pressures up to 150 lb. and more are available in some cities.

19. Reservoirs and Standpipes

Distributing Reservoirs are connected immediately with the distribution system and as near as possible to the center of population supplied. Their function is to take water when it comes and to make it available when it is needed. They are especially to maintain the service at times of fire and on other occasions when water is drawn rapidly. Frequently they also serve the purpose of allowing the pumps supplying the service to be shut down during certain hours of the day or at night, thereby economizing labor. This is especially the case in small plants. **Open reservoirs** with earth embankments or masonry walls have been frequently used. Ground waters and filtered waters always deteriorate in quality in such reservoirs, owing to the growth of certain organisms in the sunlight. **Covered reservoirs** are always to be preferred for distributing reservoirs. Roofs are sometimes used to exclude the light and keep the water from deteriorating. A light roof not necessarily water-tight serves this purpose.

Masonry Covers for distributing reservoirs are often used. At Washington, D. C., Springfield, Mass., Cleveland, Ohio, and elsewhere, groined arch construction has been used. Floors to carry the weight of the roof and distribute it over the whole base are built as inverted groined arches. The piers are of concrete, as thick as 12% of the span on centers, and not more than 12 times as high as thick. If the reservoir is deep, large piers will be required to meet this condition, and the span of the arches is increased to correspond. The roof is of groined arch vaulting without reinforcing. The outside walls, with a minimum thickness of about 12% of their height at top and 16% at bottom, are braced at the bottom by the floor blocks and at the top by the roof blocks, and are calculated as reinforced beams, with breaking moment at about 43% of the distance from the bottom to the top, equal to $4\,h^3$, h being the height of the wall in feet. In deep reservoirs economy is secured by carrying the floor on a slope of about 1 in 6 to the raised base of the walls, thereby reducing the height of the walls. The masonry is backed up by solid earth embankment, and 2 ft. of soil is placed over the top to keep frost from the masonry. Ventilators are provided to allow the passage of air as water rises and falls in the reservoir. The top is covered with grass and shrubbery, but trees or any plants with strong heavy roots should not be planted.

Data for Steel Standpipes

Diameter in ft.	Height in ft.	Capacity in thousand gallons	Required thickness of lowest plate with stress not exceeding 10 000 lb. per sq. in.	Thickness of bottom, in.	Approximate weight in net tons	Approximate relative costs			
						Tank at 8c. per lb.	Foundation 5 ft. deep at $15 per cu. yd.	Total with 10% added for appurtenances and connections	Per thousand gallons
30	20	106	1/4	1/4	15	$2 400	$2 520	$5 410	51
	40	211	5/16	1/4	26	4 160	2 520	7 350	35
	60	317	1/2	3/8	47	7 520	2 520	11 040	35
	80	423	5/8	3/8	72	11 520	2 520	15 440	36
	100	528	13/16	1/2	105	16 800	2 520	21 250	40
	120	634	15/16	1/2	144	23 040	2 520	28 120	44
40	20	188	1/4	1/4	21	3 360	4 230	8 350	44
	40	376	7/16	3/8	45	7 200	4 230	12 570	33
	60	564	5/8	3/8	77	12 320	4 230	18 200	32
	80	752	7/8	1/2	125	20 000	4 230	26 650	35
	100	940	1-1/16	5/8	184	29 440	4 230	37 040	39
	120	1128	1-1/4	5/8	251	40 160	4 230	48 830	43
50	20	293	5/16	1/4	30	4 800	6 380	12 300	42
	40	587	9/16	3/8	68	10 880	6 380	18 990	32
	60	881	13/16	1/2	124	19 840	6 380	28 840	35
	80	1175	1-1/16	5/8	198	31 680	6 380	41 870	36
60	20	423	5/16	1/4	38	6 080	8 950	16 530	39
	40	846	5/8	3/8	91	14 560	8 950	25 860	31
	60	1269	15/16	1/2	170	27 200	8 950	39 760	31
	80	1692	1-1/4	5/8	276	44 160	8 950	58 420	35
70	20	575	3/8	3/8	61	9 760	12 000	23 940	42
	40	1151	3/4	1/2	133	21 280	12 000	33 280	29
	60	1727	1-1/8	5/8	241	38 560	12 000	55 620	32

Overflows should invariably be provided for distributing reservoirs and should have sufficient capacity to discharge all the water that the pipes or pumps are capable of bringing to them. Many reservoirs have been lost and great damage done by failure to provide sufficient overflow capacity.

The cost of covered masonry reservoirs under conditions of 1927 in a general way would range from $15 000 to $25 000 per million gallons capacity according to size and conditions of construction, with small reservoirs costing considerably more.

Capacity of Distributing Reservoirs. With the water coming to the city through a gravity line at a steady rate, a storage of about 0.15 of one day's supply serves to balance all the fluctuations in hourly consumption, not including fire drafts. With a pumping system where all the pumping is done during the day a reservoir should hold at least one day's supply. Considerations of fire service frequently determine the size of reservoirs, so that in addition to meeting the ordinary service, drafts amounting to not more than one-quarter or one-half of a day's supply can be made to maintain full fire service for a period of two, four, eight, or some other number of hours, depending upon the size of the city and the amount of property that is to be protected.

The Elevation of distributing reservoirs is a matter of great importance, as it controls the pressure of water in the entire distribution system. Reservoirs have been generally built at levels to give necessary pressures at the time that they were built and in the district to be served. As time has gone on larger buildings and higher buildings and buildings upon higher ground have been erected, and there is demand for, and need of, more pressure than is available. This sometimes involves the abandonment of old reservoirs and the construction of newer ones at higher levels. Abandonment of distributing reservoirs because of insufficient elevation has been common.

Standpipes are elevated reservoirs built of sheet steel entirely above the surface of the ground, and are commonly used where the desired water level is a considerable distance above the surface of the ground. The limitations of steel construction do not in general allow standpipes to be used in large works. Roofs should be provided on all standpipes holding waters deteriorating in the sunshine, that is, in general, for ground waters and filtered waters.

Reinforced concrete standpipes have been used, but it has been found hard to keep them in good order under frost action. Towers of masonry are frequently built about standpipes for ornamental purposes, and to protect them from wind pressure, and to make very tall standpipes small in diameter safe.

Elevated steel tanks with spherical bottoms, supported on steel trestles, have largely displaced standpipes in recent years. They have the advantage that the whole of the water is available and that, for the same effective storage, the weight on the foundation is less. The East Providence, R. I., tank holding one million gallons with a flow line 205 ft. above the ground built in 1904 was the first of a great number of such tanks. The largest tank of this kind at Charleston, S. C., holds two million gallons, with a flow line 115 ft. above the ground.

The accompanying table shows dimensions and relative costs of a number of sizes.

The ratio of height to diameter may be varied within limits. A shallower tank of larger diameter maintains pressure better as water is drawn, but will usually cost a little more. The figures for cost are intended to give relative costs for tanks erected, including foundations on average hard soil at current prices in 1927, the steel work alone costing about 8 cents a pound.

Wooden tanks are frequently used in railway supplies and in industrial operations, but are seldom to be recommended for public water supply.

Dimensions and Costs of Elevated Steel Tanks
(Based on information from Chicago Bridge & Iron Works)

	Capacity in million gallons				
	0.1	0.3	0.5	1.0	2.0
Approximate diameter, feet..	29	41	45	60	80
Height of cylinder..........	16	24	32	34	35
Total height of tank........	23	34	47	54	62
Weight of water, tons.......	416	1250	2100	4160	8330
Height 100 ft. to flow line:					
Weight of tank, tons......	50	108	165	315	620
Cost....................	$10 000	$22 000	$34 000	$66 000	$130 000
Height 150 ft.:					
Weight of tank, tons......	65	145	215	425	850
Cost....................	$12 000	$28 000	$42 000	$83 000	$165 000
Height 200 ft.:					
Weight of tank, tons......	82	175	265	530	1040
Cost....................	$15 000	$34 000	$52 000	$102 000	$200 000

20. Cast-Iron Pipes

Cast-iron pipe is one of the oldest and most generally used materials in waterworks construction. It is made in lengths of 12 ft., but the smallest sizes are also made in lengths of 16 ft. and 16.4 ft., the latter being 5 meters on the metric system.

At present it is mainly used in sizes from 6 in. to 36 in. Four-in. pipe is now little used, and sizes above 36-in. under pressure in city streets are too hazardous because of the great damage that results from sudden breakage. Designs and specifications have been adopted by the American Water Works Association for four classes of pipe, called A, B, C and D, intended to serve working pressures up to 100, 200, 300 and 400 ft., respectively. Actually the smaller sizes of each class are tougher and stronger than the larger ones. In practice, B pipe is commonly used from 6 in. to 20 in. or 24 in., and C pipe for the larger sizes to 36 in. Each length of pipe is tested in the foundry under a pressure far above working pressure, but this does not prevent a small percentage of pipes from breaking in service.

Among the common causes of breakage are the weight of fill over the pipe, settlement, unequal support, unbalanced pressures at bends and temperature changes. The jar from a passing heavily loaded truck may be the last straw. Breaks are nost common where there are rocks, large boulders or other unyielding substances in the material under and near the pipe, and are less frequent in soils of clay and even grained sand which yield slightly under pressure.

The weight of the earth above the pipe results in pressures and stresses, that can be avoided only in part by careful backfilling. In deep cuts and other difficult places large pipe of any kind should be surrounded with concrete.

Coating. Cast-iron pipe is usually coated by dipping in coal tar at a temperature of about 300° F. Redistilled coal tar of good quality is to be used. With continued heating it loses its more volatile parts and becomes brittle. To prevent this a heavy oil obtained in the distillation of the tar is added to the dipping tank from time to time.

Cement-lined Pipe is cast-iron pipe lined with a layer of cement mortar which is poured soft into the rapidly revolving pipe in which it distributes it-

Cast-Iron Pipe—Table Showing Heads to be Allowed with Various Soils, Depths of Cover and Classes of Pipe (Original)

Diameter, in.	Class, American Water Works Association	Thickness, in.	Feet of fill over top of pipe						
			Clay or sand	2.0	4.0	6.0	8.0	10.0	12.0
			Average soil	1.5	3.0	4.5	6.0	7.5	9.0
			Rocks and boulders	1.0	2.0	3.0	4.0	5.0	6.0
				Maximum working head in feet					
12	B	0.62	....	345	302	259	217	174	131
12	C	0.68	....	406	369	331	294	256	219
16	B	0.70	....	298	250	202	154	106	58
16	C	0.80	....	376	336	296	256	216	176
20	B	0.80	....	283	233	183	133	83	33
20	C	0.92	....	358	317	275	234	192	151
24	B	0.89	....	267	215	163	111	59	7
24	C	1.04	....	345	302	260	217	174	131
24	D	1.16	....	405	368	330	293	255	218
30	B	1.03	....	252	198	144	90	36	.....
30	C	1.20	....	325	280	235	190	145	100
30	D	1.37	....	395	356	318	279	241	203
36	B	1.15	....	236	179	122	65	8	.....
36	C	1.36	....	310	264	217	171	124	78
36	D	1.58	....	387	348	309	270	231	192

self before it hardens. The cement layer should not be too thin, usually not less than 1/4 in. Natural cement alone, or mixed with portland, seems to have an advantage over portland cement for this purpose. Because of the greater smoothness of the cement lining there is no loss in capacity even as compared with new unlined pipe. Such pipes are not subject to corrosion and retain their carrying capacity better than tar-dipped pipe.

Pipe is now being made and sold with thinner linings, and the manufacturers report difficulty in getting lining as thick as 1/4 in. that will not crack. Very thin lining is undesirable from the standpoint of permanency. Linings from 1/4 in. to 1/2 in. thick have been successfully used in some jobs.

Delavaud or **centrifugal** pipe is made of cast iron distributed by special machine on the interior of a rapidly revolving mold so that the material is spun as it solidifies into a pipe. The unit strength of the material is greater than of ordinary cast iron and the pipe walls are thinner. The outside diameter is made of standard dimensions. Such pipe is sold by the foot and the cost at present is about the same as ordinary cast-iron pipe.

Depth of Cover. In the latitude of New York and Philadelphia pipe is laid with a cover of about 4 ft. of earth, but large pipe, that is, pipe 2 ft. in diameter and over, may be safely laid with a little less cover than smaller pipe. In the latitude of Boston, Albany and Chicago the depth of cover is increased to 4-1/2 or 5 ft. Further north still greater depths of cover are required. In the latitude of North Carolina and south it is only necessary to give the pipes sufficient cover so that they will not become exposed in the street, or, in other words, to protect them from physical injury aside from frost. In city streets a minimum depth of 30 or 36 in. is recommended in all warm climates. Shallower pipes are broken by excavation and traffic in city streets. In countries where there is no frost main pipes outside the city are sometimes laid on top of the surface of the ground. In other cases they are just covered with soil.

The former arrangement leaves them open to inspection for leakage. The latter arrangement protects them against excessive heat. The cost of pipe laying is greater as the cover is greater.

Cost of Cast-iron Pipe. Cast-iron pipe is sold by the net ton. The maker guarantees the weight. Pipes underrunning the standard weight by more than a certain percentage (the percentage depending upon the diameter and class of pipe) are rejected, and pipes overrunning the standard weight by more than a certain percentage are accepted, but the weight beyond the allowed excess is not paid for. From 1875 to 1909 the price of cast-iron pipe in the northeastern states ranged from $17 to $35 per net ton. The average price was not far from $25, and this was approximately the selling price in 1909. Prices of over $30 per ton ruled in 1875, 1880, 1883, 1884, and 1907. Prices under $20 ruled from 1895 to 1898 inclusive. In 1917 to 1926, the ordinary range was between $50 and $70 per ton. In 1927 it fell somewhat and $40 is a representative figure. Current quotations should be consulted in the engineering papers.

Maximum Bends in Cast-Iron Pipe Joints

Size of pipe in inches	Bend in one joint	Deflection of 12-ft. length, in inches	Approximate radius in feet of curve produced by succession of joints
4	4°	10.0	170
12	3°	7.5	230
16	2° 41′	6.8	260
20	2° 9′	5.4	320
24	1° 47′	4.5	390
30	1° 26′	3.6	480
36	1° 12′	3.0	570
42	1° 2′	2.6	660
48	0° 55′	2.3	750

Special bends are used for shorter curves than those in the table. Specials are all castings other than straight pipe in 12-ft. lengths. Bends, reducers, increasers, crosses, tees, branches, are some of the common forms of specials. Standard designs and weights for all the more frequently used specials have been adopted by the New England Water Works Association and the American Water Works Association.

The weight of specials in long lines of supply pipe in the country is less than 1% of the weight of the pipe. In the distribution system in villages and small cities the weight of the specials will average 2-1/2% of the weight of the pipe. In city work, with short blocks and many obstacles to be passed, specials will average 4% of the weight of the pipe.

Specials cost twice as much per pound as straight pipe, or a little more.

Joints in cast-iron pipe were formerly made with lead and this still remains an important joint material. The following table shows proper dimensions and weight of lead joints. The lead is melted and poured into the joint between temporary gaskets and is then calked with a tool to take up the shrinkage of the material as it solidifies. The effect of the calking only extends a short distance; and that part of the joint depended on for tightness is not more than 1/2 in. deep. Back and forth temperature movements will loosen this in time and result in small leakages.

Joints are made deeper where the pressure is high, and not so deep where it is light. Workmen often make the joints heavier or lighter than specified. Some lead is oxidized in melting and pouring, and otherwise wasted and lost. Variations in the diameter of bell and spigot make a difference in the amount of lead. A small fraction of an inch variation makes a large percentage difference in the amount of lead required. For good management find out how much lead is required, and require inspectors to hold the dimensions of bells and spigots to the design, and the workmen in the trench to the depth of joint required, and investigate closely any large discrepancy between the computed and required amounts of lead.

Amount of Lead Required for Joints

Dimensions in inches				Lead required, pounds per joint		
Diameter of pipe	Approximate diameter lead ring	Depth of lead	Thickness of lead	Theoretical weight including bead	Range in weights reported for various works	To be used for estimate
4	5.2	2.00	0.4	6.5	6.5	7
6	7.3	2.00	0.4	9.0	6–10	10
8	9.5	2.00	0.4	11.7	8–12	12
10	11.8	2.00	0.4	14.6	10–16	16
12	13.9	2.25	0.5	19.1	13–20	20
16	18.3	2.25	0.5	30.3	24–32	32
20	22.4	2.25	0.5	37.0	30–40	40
24	26.6	2.50	0.5	48.5	38–50	50
30	32.9	2.50	0.5	59.6	56–75	62
36	39.2	2.75	0.5	77.8	68–115	80

Data for Cast-Iron Pipe

Diameter, in.	Class, American Water Works Association	Thickness, in.	Weight per lineal foot	Weight per 12-ft. length	Relative cost per lineal foot for average conditions: 4-ft. cover, 3 % rock, pipe at $40 per ton, labor at 50 cents per hour			
					Country trenching machine	Country hand work	Village or suburban, oiled macadam pavements	City conditions, concrete or asphalt pavements
6	B	0.48	33	400	1.33	1.65	2.34	3.42
8	B	0.51	48	570	1.77	2.12	2.83	3.98
10	B	0.57	64	765	2.26	2.63	3.38	4.58
12	B	0.62	82	985	2.79	3.19	3.98	5.24
16	B	0.70	125	1500	4.13	4.61	5.51	6.99
16	C	0.80	144	1725	4.63	5.11	6.01	7.49
20	B	0.80	175	2100	5.61	6.17	7.19	8.89
20	C	0.92	208	2500	6.49	7.05	8.07	9.79
24	B	0.89	233	2800	7.33	7.98	9.13	11.05
24	C	1.04	279	3350	8.54	9.19	10.34	12.26
30	C	1.20	400	4800	12.00	12.80	14.20	16.63
30	D	1.37	450	5400	13.32	14.12	15.52	17.95
36	C	1.36	546	6550	16.19	17.15	18.79	21.69
36	D	1.58	625	7500	18.28	19.24	20.88	23.78

A lead joint tight for its whole depth and therefore more reliable and free from leakage may be made by calking lead wool into the joint with a pneumatic tool in thin layers. The cost is greater and such joints are used only in places of special difficulty.

Leadite and Lead-Hydrotite are proprietary mixtures, of which sulfur is the most important element, and are often used in place of lead. They are cheaper and do not require calking. The joints leak slightly when first made but later become tight.

Cement Joints are widely used, especially on the Pacific Coast and their use is extending in Texas, Florida and elsewhere. They are cheap and satisfactory and when well made remain permanently tight. Ordinary portland cement is mixed with very little water and calked in layers with a tool, as hard as lead is calked. Pipes laid with cement or leadite joints may be stiffer than pipe laid with lead joints, but extended experience shows sufficient flexibility to meet practical conditions.

Leakage in pipe should always be tested under full working pressure. If it is practicable to hold the trench open so long it is best to make the test before the trench is backfilled, and to calk all joints showing leakage until they are tight. It is impossible to keep lines of cast-iron pipes permanently tight. The expansion and contraction from temperature changes are accompanied by a slight slipping of the lead on the iron at each joint; settlements cause movements in the joints.

With well-tested work under average conditions in a waterworks system a leakage of three gallons per 24 hours per lineal foot of lead joint under a pressure of 100 lb. per square inch may be anticipated. On this basis the leakage per thousand feet of pipe in gallons per 24 hours will be as follows:

Inches diam..	6	8	10	12	16	20	24	30	36	48
Gallons......	500	650	775	900	1200	1500	1800	2200	2600	3400

Lines of pipe when first laid should have less leakage, and those that have been down for many years may be expected to leak more. It will pay to dig up pipe and calk joints whenever and as far as the value of water saved during an assumed period of five years exceeds the cost of locating and stopping the leakage.

Cost of Cast-iron Pipe Laid. The table in this section shows the approximate cost of pipe of all the sizes and thicknesses in most common use, laid under average conditions. These prices include the cost of the pipe, specials, lead, and all costs of laying, assuming 3% of the trench rock excavation; pipe $40 per ton and labor 50 cents per hour. The column for country work may be used for outside pipe lines where there is no special difficulty. The column for village and suburban work includes taking up and replacing oiled macadam pavements. The column for city conditions includes taking up and replacing concrete or asphalt pavements.

In Congested Districts of cities where there are many existing pipes, conduits, sewers, and other structures to be met and passed, the cost per foot may be increased very greatly above the figures given, which are intended to cover only ordinary city conditions. Many local conditions affect the cost of the work, and prices fluctuate, and the costs will often be found differing considerably from those given, which are intended to represent average conditions, and to show approximately the relative cost of pipe of different sizes and the thicknesses under ordinary conditions of use.

21. Steel Pipes

Steel pipe is used principally in long supply mains, leading from the source of supply to the city. It is not generally adapted to use in city streets or in the distribution system.

Steel pipe is generally cheaper than cast-iron pipe in sizes 36 in. in diameter and upward. It has occasionally been used for 30-in. pipe and is seldom to be recommended for smaller sizes.

Riveted Pipe. The older lines of steel pipe were all riveted. Generally the sheets of steel are 7 ft. or 8 ft. wide, and are bent so that one sheet goes entirely around, forming one section of pipe. Four of these sheets are riveted together in the shop, making a length of pipe 28 to 30 ft. long. This is tested for tightness and dipped in protective coating, and then shipped to the place where it is to be used.

The circular seams may be single riveted. The longitudinal seams which alone are required to carry the stress due to the pressure of the water are double riveted, in all cases, except where the pressure is very low.

In-and-out courses are used, alternate rings being larger and smaller. **Taper lengths** are also used in which one end of each pipe is smaller than the other end and will slip into the large end of the next length. Pipes have also been made with all the lengths the same size fastened together with butt straps on the outside, but as this is a more expensive method it has not been often used.

Lead Joints have been used in Australia on long lines at Coolgardie and Sydney. Cast-iron or steel sleeves are used in making them. The lead acts as an expansion joint and each length of pipe is free to expand and contract with the temperature changes, the steel slipping over the lead to permit this.

Place	Year installed	Diameter, inches	Thickness, inches	Length, feet	Kind
Rochester, N. Y.	1873	36	3/16	51 000	Riveted wrought iron
Rochester, N. Y.	1873	24	3/16–1/4	15 400	
E. Jersey Water Co.	1891	48	1/4–3/8	116 000	Riveted steel
E. Jersey Water Co.	1891	36	1/4	26 000	
Rochester, N. Y.	1893	38	1/4–3/8	140 000	
Cambridge, Mass.	1895	40	1/4–5/16	24 000	
New Bedford, Mass.	1896	48	5/16	42 500	
Allegheny City, Pa.	1896	60	....	50 000	
Brooklyn, N. Y.	1896	66	....	79 200	
E. Jersey Water Co.	1896	48	....	26 400	
E. Jersey Water Co.	1896	42	....	89 700	
Adelaide, Australia	1897	15–24–26	1/4	63 472	Lock-bar steel
Minneapolis, Minn.	1897	50	5/16–3/8	34 400	Riveted steel
Passaic Water Co., N. J.	1897	42	....	21 000	
Albany, N. Y.	1898	48	5/16	7 900	
E. Jersey Water Co.	1899	51	....	47 500	
Atlantic City, N. J.	1901	30	1/4	(?)27 000	
Pittsburgh, Pa.	1901	51–42	....	26 000	
Jersey City, N. J.	1902	72	5/16–7/16	91 000	
Coolgardie, Australia	1903	30	1/4–5/16	1 848 000	Lock-bar steel
Newark, N. J.	1903	60	....	39 300	Riveted steel
Troy, N. Y.	1903	33	....	35 300	
Schenectady, N. Y.	1903	36	....	24 000	
Lynchburgh, Va.	1905	30	....	15 000	Lock-bar steel
Wilmington, Del.	1905	43–48	....	19 350	
Passaic Water Co., N. J.	1905	42–48	....	11 000	
Pittsburgh, Pa.	1905	50	....	26 500	
Springfield, Mass.	1909	42	1/4–7/16	61 000	

Continuous Riveting has been used in nearly all American steel pipe lines; that is, each length of pipe in the field has been tightly riveted to its neighbors. Practical experience with the system of construction has been satisfactory.

Temperature Stresses. Under conditions in the northern states the temperature of water and consequently of the pipe will range from 32 to 75 or 80° F., or for average conditions the range is about 45°. The pipe must be held so that it will not move with expansion and contraction. Under these con-

Data for Steel Pipe

Diameter in inches	Thickness of plate in inches	Approximate weight in pounds per linear foot	Relative cost laid complete under average conditions per lin. ft.	Lock-bar pipe			Double-riveted pipe			Greatest allowable depth of cover in feet	Temperature stress in net tons
				Gross pressure without allowance for water ram and deterioration	Safe working pressure after allowing for average water ram and corrosion: Lb. per sq.in.	Safe working pressure after allowing for average water ram and corrosion: Ft. of head	Gross pressure without allowance for water ram and deterioration	Safe working pressure after allowing for average water ram and corrosion: Lb. per sq.in.	Safe working pressure after allowing for average water ram and corrosion: Ft. of head		
30	3/16	80	$ 9.60	180	100	231	144	86	199	5	92
	1/4	104	10.90	240	140	323	192	118	273	8	122
	5/16	127	12.20	300	180	416	240	150	347	12	152
	3/8	151	14.50	360	220	508	288	182	420	18	183
	7/16	174	14.80	420	260	600	331	209	483	25	214
36	1/4	123	13.00	200	113	261	160	97	224	5	147
	5/16	151	14.50	250	147	339	200	123	284	9	183
	3/8	179	16.00	300	180	416	240	150	347	12	220
	7/16	207	17.50	350	213	491	280	177	408	17	257
	1/2	235	19.00	400	247	570	320	203	468	22	294
42	1/4	141	14.90	171	94	217	137	81	187	4	172
	5/16	174	16.70	214	123	284	171	104	240	6	214
	3/8	207	18.50	257	151	349	206	127	293	9	257
	7/16	240	20.30	300	180	416	240	150	347	12	300
	1/2	273	22.10	343	209	482	274	173	400	16	343
48	1/4	160	16.80	150	80	185	120	70	161	3	196
	5/16	198	18.90	187	105	242	150	90	208	5	244
	3/8	235	21.00	225	130	300	180	110	254	7	293
	7/16	273	23.10	262	155	358	210	130	300	9	342
	1/2	310	25.20	300	180	416	240	150	347	12	391
54	5/16	221	21.10	167	91	210	133	79	182	4	275
	3/8	263	23.50	200	113	260	160	97	224	6	330
	7/16	305	25.90	233	135	312	187	115	265	8	386
	1/2	348	28.30	267	158	365	213	132	305	10	441
	5/8	432	34.70	333	202	466	267	168	388	15	551
60	5/16	244	23.40	150	80	185	120	70	161	3	305
	3/8	291	26.10	180	100	231	144	86	201	4	366
	7/16	338	28.80	210	120	277	168	102	235	6	430
	1/2	385	31.50	240	140	323	192	118	273	8	488
	5/8	479	38.70	300	180	416	240	150	347	12	612
72	5/16	291	28.10	125	63	146	100	57	132	2	367
	3/8	347	31.30	150	80	185	120	70	161	3	440
	7/16	404	34.50	175	97	224	140	83	191	4	515
	1/2	460	37.70	200	113	261	160	97	224	6	588
	5/8	573	46.00	250	147	339	200	123	284	9	735

ditions the change in temperature, which tends to expand or contract the pipe, uses all its force in putting it under stress. The stresses thus produced may range from nothing at the low temperature to the maximum amount in compression at the higher temperature, or from nothing at the higher temperature to the maximum amount in tension of the lower temperature, or they may be divided, coming partly in tension at the lower temperature and partly in compression at the higher temperature. This depends on the temperature at which the pipe is laid and finally connected up. Under the most unfavorable conditions the stresses produced by temperature in this climate are about 9000 lb. per sq. in., and as this comes well within the strength of the steel no difficulty is occasioned by them. Ordinarily they are less than this.

Anchorages are built to keep the pipe from moving at all free ends and at all sharp bends. These anchorages are formed by riveting angle irons to the steel pipe, usually four angles being attached to one 30-ft. length, with a sufficient number of rivets to hold the temperature stresses, and these lengths of pipe are surrounded with concrete reinforced with steel rails of such a shape as to be capable of withstanding the computed amount of stress. In some lines anchorages have been omitted and expansion joints provided at all gates, ends, and other places when continuity of riveted connection cannot be maintained. The temperature push or pull on the anchorage in tons of 2000 lb. for steel pipes of various diameters and thicknesses is shown in the last column of the preceding table.

Gates and Steel Pipes. Flanged gates are usually considerably smaller in diameter than the pipe (Sect. 7, p. 693) and are bolted on one side directly to a flange on the steel pipe and on the other to a nipple connected with the steel pipe through a sleeve with lead joints. In this way the gate is protected from temperature stresses of the steel pipe, and on the other hand is held from being moved by unbalanced water pressure as is the case where lead joints are used on both sides. It is usual to build anchorages on the steel pipe on either side of gates or otherwise the two ends of the steel pipe may be held from movement by an enclosing steel and concrete structure.

Lock-bar Steel Pipe is made by upsetting the edges of the plates and connecting them by a lock bar in the shape of an H going over the opposite edges and being forced down over them by hydraulic pressure. This takes the place of the riveting in the longitudinal joints. The circular joints may be made by riveting or otherwise as for riveted pipe. While double riveting develops only about 72% of the strength of the steel plate, the lock bar is capable of developing 100%. Owing to occasional defects in material or workmanship on the lock bars, in making calculations it is recommended that only 90% of the strength of the plate should be used for lock-bar pipe. Two sheets of steel each 30 ft. long are joined with two bars at their edges to make a 30-ft. length of pipe. The circular joints between these lengths are usually riveted in the field. Thus far lock-bar pipe has been made from plates ranging from 1/4 to 7/16 in. in thickness.

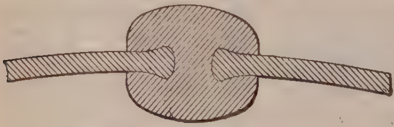


Fig. 13. Lock Bar for Steel Pipe

Welded Steel Pipe has been made possible by the progress in welding in recent years, and its use is extending. The greatest problem is to make a weld without changing the temper of the steel resulting in brittle material in the space between the heated metal in the weld and the cold metal a short distance from it.

Carrying Capacity of Steel Pipe. The steel sheets are much smoother than the interior surface of cast-iron pipe; and for this reason lock-bar steel pipe is found to carry appreciably more water than cast-iron pipe. Welded pipe is believed to be equivalent to lock-bar pipe. In riveted pipe the projecting rivets and rough edges at the joints increase the resistance. To make it comparable with other kinds of pipe the diameter must be increased by at least 4%.

The Diameter of Steel Pipe, either riveted or lock-bar, can be made of any required dimension. In this respect it differs from cast-iron pipe, which is commonly made only of the sizes for which the foundries have molds. The diameter should always be specified as the smallest diameter of the smallest ring, where the rings are not all of the same size.

Weight of Steel Pipe. The finished weight of steel pipe per lineal foot, either riveted or lock-bar, including the excess weight of plates rolled so that the thinnest points in the plate will be approximately of the nominal thickness, and including the laps, rivets and lock bars, material in the joints and coating, may be found approximately by the formula:

$$\text{Weight in pounds per foot} = (12.5 \times \text{diameter} \times \text{thickness}) + 10 \text{ lb.}$$

in which diameter and thickness are to be taken in inches. The weights of commonly used sizes are given in the table in this section. Welded pipe is not quite as heavy.

Thickness of Steel Pipe. The required thickness is computed by

$$\textbf{Thickness in inches} = \frac{\textbf{Diameter in inches} \times \textbf{pounds pressure}}{\mathbf{2 \times 16\,000} \times \textbf{efficiency of joint}}$$

To the thickness thus computed it is customary to add something as an allowance for weakening of pipe by corrosion. In other words, make the pipe heavier so that it may rust to a certain extent and still have needed strength. The strength of the metal with open-hearth steel under standard specifications is taken at 16 000 lb. per sq. in., which allows a factor of safety of about 3-1/2. The efficiency of the joint may be taken at 55% for single-riveted pipe, 72% for double-riveted pipe, with the best spacing of rivets, and at 90% for lock-bar pipe. Welded pipe should be as strong as lock-bar pipe.

In the steel-pipe table in this section are entered under lock-bar pipe and double-riveted pipe separately, first, the gross pressures, in pounds per square inch, that can be carried by pipe of various dimensions without making deduction for water ram and for deterioration, and second, the fair safe working pressures after making reasonable allowances for water ram and corrosion under average conditions.

Proper allowances for water ram and for corrosion necessarily vary with local conditions. Generally speaking, where great variations in rates of flow are to be expected and where a great deal depends upon the continuous service of a pipe, thicker pipe should be used, while under conditions of steady flow and where pipes are in duplicate or otherwise safeguarded thinner pipe may be used.

Depth of Cover for Steel Pipe must not be excessive or the weight of the earth will flatten and deform it. A slight flattening is **not** objectionable, as it does not cause the pipe to leak and does not greatly reduce its carrying capacity.

The table, in this section, shows the greatest depth that should be placed over various sizes and thicknesses of pipe under average conditions. In bad trenches and slippery material keep the depth of cover somewhat less than indicated. With firm material carefully placed around the pipe and well rammed on the sides a somewhat greater depth of cover for short distances may be permitted. If it is necessary to cover the thin pipe to a greater depth it may be stiffened by angle irons riveted to it at frequent intervals. A more substantial result is obtained by surrounding the pipe with concrete.

Cost of Steel Pipe. The relative cost of steel pipe laid complete under average conditions is shown in the table, p. 1505. The costs represent generally those in the northeastern states in the years 1925-27. They include pipe excavation and average amount of water in trench, and of rock excavation, anchorages, brook crossings, etc., but do not include river crossings or special and unusual obstacles or difficult conditions. Steel pipe has often been placed under contract at a price per foot laid, including the pipe and laying, but rock, valves, river crossings, and all other auxiliary works are usually paid for separately.

Bends in Steel Pipe are usually made by cutting off some of the plates at the joint. Both horizontal and vertical bends are made in the same way. It is easier to lay out the work if the horizontal and vertical bends are made in separate joints, but in case of need they can be combined.

The amount of bend that can be made in one joint depends upon the size and thickness of the plate. With 5/16-in. plates bends up to 5° in one joint are easily made; with 3/8-in. plates 4°, and with 7/16-in. plates 3°. Sharper bends are made when necessary but it is harder to calk them tight. With crooked lines the lengths of pipe may be cut, one bend made every 15 ft. or every 7-1/2 ft. With sharper bends special arrangements are made.

It is better to make all bends in steel pipe of steel plates riveted up, rather than of castings, and in case of sharp bends the pipe should be anchored on both sides, to carry the resultants of the temperature stresses.

Cement Lining in Steel Pipe. Steel pipe has been sometimes lined with cement mortar 2 in. thick or less to insure permanence. Where this is to be used the pipe has usually been surrounded with concrete on the outside for stiffness.

Coatings for Steel Pipe are more important than coatings for cast-iron pipe, because the steel is thinner and if unprotected may rust through more quickly. **Asphalt** was used for some of the earlier lines. Asphalt is entirely stable in a dry atmosphere, but it is quickly oxidized and becomes brittle when kept under water. Asphalt coating on steel pipe has actually become brittle and pulverized and has ceased to act as a protective coating in periods usually not exceeding ten years. Coatings made from the residues from the distillation of petroleum have frequently been sold as asphalts. **Redistilled coal tar** used on cast-iron pipe has usually been more durable than the asphalts and special preparations used on the earlier steel pipes. Part of the Rochester pipe line, 1893, was dipped in equal parts of coal tar and asphalt. At Springfield, the 42-in. pipe line, 1909, was dipped in coal tar alone.

The Life of steel pipe seems to be limited in general by perforations of the plate which start with imperfections in the steel and first show after a considerable number of years.

External Corrosion of Steel Pipe. Occasionally soil conditions are found that corrode steel pipe. Where sea water penetrates the soil in which the pipe is laid corrosion must always be anticipated. Corrosive conditions may not be apparent as the pipe is laid, but become known long afterward when the pipe begins to leak. They are not common and are usually localized in peaty areas or where there is corrosive mineral matter in the soil.

Leaks in Steel Pipe. When steel pipe is corroded either naturally or by electrolysis, the result is a small hole or perforation in the pipe. If the pipe is under moderate pressure there will be a small leakage through this hole which may go on unnoticed for a long time. If the pressure is high and the pipe is surrounded with soil containing gritty material, an eddy will be formed on the outside that will rapidly enlarge the hole after a material flow is established. Such perforations are usually stopped without shutting off the water pressure by first driving in a wooden plug and afterward covering with a steel cap with a gasket held by a hoop extending around the pipe and drawn up by a turnbuckle. Recently electric welding has replaced the hoop and turnbuckle. In a few cases it has been necessary to dig up lengths of old pipes in corrosive ground and repair them in this way or replace them. Where such conditions exist the pipe should be surrounded with concrete or otherwise adequately protected from external corrosion.

Concrete Pipe. Concrete pipe made by the Lock Joint Pipe Company coming into use in recent years is in many respects similar to cement pipe that became common about 1870. In the modern pipe a shell of steel welded to bells and spigots at its ends is lined and surrounded with reinforced concrete. The reinforcing in the concrete is planned to carry its share, often the largest part, of the pressure. The steel drum is depended on for tightness. The cement lining on the interior is relied on for durability and smoothness. Such pipe is usually made near the place where it is to be used, but cast in standard molds and with equipment furnished by the manufacturers. Such pipe is used only in large sizes, generally 24-in. and over. For very low heads the steel shell is generally omitted.

Wood-stave Pipe. Wood was one of the earliest materials used for water pipes. Old methods limited its usefulness, and other materials have largely superseded it. Recently methods of manufacture have been developed and it is again finding increased use. It has been largely used in cantonments for somewhat temporary service under war conditions. Decay is less rapid in pipe laid in impervious soil and under pressure that tends to keep the wood constantly saturated with moisture. If laid in loose ground, or in any situation which permits the material of the pipe to become dry at times, it will rot.

Wood pipe may be either " machine made," in sections running up to 20 ft. or more in length, shipped to the work to lay; or "continuous," built up of staves and banded in the trench. Tightness in the longitudinal joints is secured by grooves and beads on the edges of the staves, pinched into tight contact by the bands. Tightness in transverse joints is obtained by the use of metal plates set into the joints. Properly built and placed, wood-stave pipe gives good service at moderate pressure within the elastic limit of the bands.

For small sizes, up to 24 in. machine-made pipe is usually used. For larger sizes the pipe is built continuously in the trench.

Wood-stave pipe will carry as much water as cast-iron pipe when new and as it is not subject to tuberculation, when old, it will carry more than iron or steel pipe of equal age.

Pressure Tunnels. There comes a time in the growth of a large city when it is difficult to lay enough pipes in congested streets to meet growing demands. Under New York City a concrete lined pressure tunnel, varying from 15 ft. to 11 ft. in finished diameter, has been driven deep in the rock under the entire length of the city connected with the main pipes at intervals and taking the place of an enormous extension of the pipe system. A duplicate tunnel in sizes up to 21 ft. is being planned for early construction. Similar tunnels have been proposed for several other cities.

22. Gates and Gate Valves

Gate Valves are placed at intervals on all pipe lines of considerable length. In city streets four are placed at each intersection, one on each line of pipe. Outside the city valves are placed less frequently, and are best placed on summits where the pressure is least. The gates serve the purpose of facilitating tests of the pipe and shutting off portions of it for repairs in case of emergency.

Gates Smaller than the Pipe are often used on pipes 30 in. in diameter and over, connection being made by reducers on either side. The cost is less and the smaller gates are operated more quickly and easily. There is a little head lost, and the smaller the gate the more head is lost. This is controlling in determining how much smaller than the pipe it is best to make the gates.

Loss of Head in Gates with Taper Cone Connections

Diameter of gate, in.	Diameter of pipe in inches						
	30	36	42	48	54	60	72
20	1.57	3.47	6.59	......	......	......	......
24	0.65	1.57	3.07	5.40	8.72	......	......
30	0.15	0.52	1.14	2.09	3.47	5.40	11.45
36	......	0.15	0.44	0.91	1.57	2.51	5.40
42	......	......	0.15	0.40	0.75	1.25	2.83
48	......	......	......	0.15	0.35	0.65	1.57

The figures given are in velocity heads (or in feet, when the velocity in the main pipe is 8.03 ft. per sec.). They may also be taken as tenths of feet when the velocity in the main pipe is 2.54 ft. per sec.

Basis. Loss of head in a gate, 0.15 velocity head. Loss of head in cones, 0.20 of the amount that the velocity head at the throat is greater than the velocity head in the pipe.

The actual amount of head lost in a gate depends upon the form of gate, and considerable variations are to be anticipated with gates of different designs. The amount lost in the cones depends upon the taper design and smoothness of the surfaces, and considerable variations either way are to be anticipated.

Generally 24-in. gates may be used on 30-in. and 36-in. pipes, 30-in. gates on 42-in. and 48-in. pipes, and 36-in. gates on 60-in. and 72-in. pipes; but if head or elevation is very valuable the gate should be one size larger than above indicated.

The Cost of Standard Double Disk Gates with bronze working parts of the best and most permanent design varies with market conditions. The following are given as relative representative prices, including gears where needed, but not including by-passes. These prices are not very different from those which ruled in fair-sized contracts in the year 1927.

Connections and Accessories of Gates. Gates are furnished with either flange or bell ends at about the same cost. Bell ends are generally used in pipe lines in street work; flange connections are used in gatehouses, pumping stations, and about filter plants. **Gate Boxes** are metallic boxes covering the wrench connection and gate, extending to the surface of the ground, with an expansion joint to protect them from damage by frost and with a removable cover to allow the gate to be opened from the surface with a suitable wrench by removing the cover.

Manholes of masonry are often built about gates of special importance, large gates, and gates operated by gears, especially when located under pavements or in other places not easily accessible. It is not necessary to build

Cost of Double Disk Gates

Size in inches	20 lb. per sq. in. test pressure suitable for filter plants and for working pressures not exceeding 10 lb. per sq. in.	100 lb. per sq. in. test pressure for working pressures not exceeding 50 lb. per sq. in.	300 lb. per sq. in. test pressure for working pressures not exceeding 150 lb. per sq. in. suitable for nearly all water works purposes
6	$20	$21	$23
8	32	33	36
10	45	47	54
12	60	62	70
16	100	115	130
20	145	175	235*
24	190	260	350*
30	350	480*	680*
36	550*	750*	1050*
42	800*	1100*	1500*
48	1200*	1600*	2100*

* Geared.

such manholes about small gates nor about gates on supply mains outside the city, because such gates can be readily and cheaply dug up in the infrequent cases of access to them being necessary.

Gears are used on large gates and gates under heavy pressure. In general 36-in. gates, 10 lb. per sq. in. working pressure; 30-in. gates, 50 lb. per sq. in. working pressure; and 20-in. gates, 150 lb. per sq. in. working pressure, are the smallest gates to be geared. **Spur gears** are used on gates set vertically and opening upward, and **beveled gears** on gates set horizontally and opening sideways. The latter are to be used wherever the vertical space is not sufficient to put in the spur-geared gates.

By-passes are provided in many cases on large gates operating under heavy pressures. These are built into and form part of the main gate. A small gate on the by-pass is opened to equalize the pressures in the pipe on either side of the gate before the main gate is opened. This allows the main gate to be opened with less effort than would otherwise be required.

Hydraulically Operated Gates in which the screws of the ordinary gate are omitted, have hydraulic cylinders provided with plungers attached directly to the moving parts. A small control valve allows high-pressure water to act on one side or the other of the plunger, opening and closing the gate. The cost is about twice that of ordinary gates. Gates should not be placed where they cannot be inspected and tested and kept in good order. They are especially useful for gates that have to be opened and closed frequently in pumping stations and about filter plants. Oil may be used in place of water in such gates. The oil is pumped by small electrically operated pumps in a closed system and no outside storage is required. Pressures may be higher and the cylinders and working parts correspondingly smaller and less expensive than those designed for ordinary water pressure. Such equipment is not frozen.

Counter-weights provided for large gates placed vertically facilitate operation and prevent the danger of accidental closing of an open gate.

Electrically Operated Gates are furnished with electric motors, geared to the working parts. They must be provided with reliable equipment to shut off the current before the gate reaches the end of the run to prevent jamming. Good automatic equipment has been developed, and such gates have advantages for throttling when frequent small changes in position are required.

Butterfly Gates open like a damper turning on a shaft running through the middle of the pipe. They are made in large sizes and designs have been perfected that close almost watertight. There is great variety in the design of large gates and much special equipment has been built.

Gates for Throttling Rapid Flow. When a gate controls the flow of water into a masonry chamber, iron pipe or other enclosed passage, and when the velocities of discharge correspond to heads above atmospheric pressure, there develops a condition of chattering or making and breaking of vacuum with vibrations that rapidly destroy both the gate and the chamber walls below it. The best remedy is to secure ventilation by giving ample independent air passages terminating below the gate and extending upward to the open air above the highest water level.

Sluice Gates are of simpler construction, arranged for being built into masonry of reservoirs and other structures, and for holding water against moderate heads only. There is great variety in the design of sluice gates. They are usually cheaper than standard gates, and for the services to which they are adapted are fully as satisfactory.

23. Auxiliaries

Air Valves are small valves attached to pipes for the purpose of automatically letting out air. They are placed on summits only. Automatic air valves need only to be placed on summits of cast-iron pipe lines where the pressure is light and variable, that is, on summits nearly up to the hydraulic grade line. On all summits where the water is under considerable pressure it is sufficient to put on a petcock or a larger valve to be opened while the pipe is being filled and which can be closed at all other times. As air is more soluble in water under pressure there is no danger of the separation of air at summits under considerable pressure, and should air be accidentally introduced to them it would be slowly dissolved and removed by the passing water. As a general rule air valves with a diameter of one inch for each foot in diameter of the water pipe are sufficient. The air valves must be protected from frost by specially constructed housing to insure their being in readiness to act in winter.

For Steel Pipe air valves are also required to let air into the pipe rapidly in case of need, as the pipe is not strong enough to support itself against outside pressure with a vacuum in the inside. A break in a pipe at a low point, allowing the water to run out rapidly, would cause a vacuum in higher parts of the pipe, which would cause the pipe to collapse. Consideration of this feature has led to placing air valves for automatically admitting air on summits of steel pipe. Generally the air valve for this purpose should have a net area equal to one sixty-fourth the area of the pipe.

Air valves are to be insisted upon in all steel-pipe lines, but it must be remembered that they are called upon to act very rarely indeed, and for this reason a defective valve or arrangement may be used without the discovery being made that it is defective, and the fact that a simpler or cheaper type of air valve has been used in certain cases where there have been no breaks and consequently no demand that has taxed its capacity is not to be taken as an indication of the sufficiency of that particular design.

Blow-offs are small pipes attached at low points for the purpose of drawing off and wasting the water contained in the pipe during times of inspection and repair. Blow-offs are usually much smaller in diameter than the main pipe. The necessity of blow-offs depends upon the character of the water and the service of the pipe.

Manholes consisting of saddles attached to the pipe and removable covers capable of being bolted securely to the frame are placed on steel pipes at distances ranging from 1000 to 2000 ft. apart, to allow the pipe to be entered during construction and afterward for inspection and repair. In some cases manholes have been placed on cast-iron pipes, although most lines have been built without them.

Twin Lines of pipe are used in places of special danger. Either line will maintain at least a partial supply in case of break in the other. In case twin lines are long, there should be cross connections with gates so that in case of a break in either line a section only can be cut out, the flow at other points continuing through both lines. With this arrangement, the amount of water flowing through the system will be more than would flow through one line only.

The Cost of Twin Lines with cross connections is from 30 to 50% greater than the cost of a single line of pipe of the same strength and capacity. Where no other purpose than safety is secured by dividing the flow, it is generally better to spend the added money, or a part of it, in strengthening one line and making it secure beyond question rather than dividing it between two smaller lines. River crossings, lines over coal fields, where there are sure to be settlements, and other points of special hazard are best crossed with twin lines. Three lines of pipe cost from 60 to 80% more than one line of equal strength and capacity.

Tubercles in Cast-iron Pipe. The carrying capacity of cast-iron pipe is reduced in course of time by the growth of tubercles upon the interior of the pipe. In a general way the capacity of the pipe, other things being equal, is reduced from this cause by as much as one per cent per annum. In small pipes the deterioration is more rapid. Generally the deterioration is less rapid with clear lake waters and more rapid with turbid river waters, and especially waters carrying organic matter. Filtered river waters act more nearly like lake waters.

Tubercles can be removed by sending an instrument driven by the water pressure through the pipe. This instrument is called a "go-devil." Scraping off the tubercles in this way increases the carrying capacity of the pipe. After the pipe has been scraped tubercles grow more rapidly than before, so that the remedy is a temporary and not a permanent one. When the pipe is once scraped it is usually necessary to scrape it again, and the process becomes an annual one, or the period may be even shorter.

Effect of Cleaning upon the Quality of the Water. The corrosion and tuberculation of iron pipes always adds iron to the water, and this iron gives it a color, tends to deposit, and is objectionable. Scraping the pipes frequently increases the rate of tuberculation and increases whatever objection there may be to the iron in the water from this source.

Hydrants are attached to pipes in a distribution system to allow water to be drawn for fire purposes. Hydrants have 4-in. connections to allow them to be connected with steam fire engines and 2-1/2-in. connections for hose connections. A hydrant having one 4-in. and two 2-1/2-in. connections is a common arrangement. The water is commonly shut off from the hydrant by a 6-in. gate valve at the bottom; closing this shuts off water from all connections. Hydrants are also made in which the different connections can be shut off separately. The cost of hydrants varies from $50 to $125 according to size, type, and the way in which connection is made with the pipe. Hydrants are commonly placed at intervals of 200 ft. where buildings are large, close together, and inflammable, and further apart elsewhere, but in no case more than 500 ft. apart.

Domestic Meters are small instruments set on the service pipes of the takers to measure and record all water drawn through them. The most commonly used meters in the United States are disk meters, consisting of an oscillating disk in a case moved by the water and communicating the motion which represents a certain volume of water to a gear train which indicates the amount on a dial. Another type of meter called a Rotary meter has advantages for certain services and is sold at a somewhat higher price.

Water meters are least accurate with the smallest flows. It is comparatively easy to make a meter that is accurate with large flows. Meters that are accurate at such flows often fail to register very small flows, especially after they have been some time in use. In testing meters therefore it is necessary to pay particular attention to low rates of flow, as otherwise small leakages which when sufficiently numerous would sap the system may fail to be registered by the meters.

Current specifications require domestic meters to register 90 per cent of a flow of 1/4 gal. per minute. This is a relatively low standard and should be insisted upon. With reference to complete accounting for water it is desirable that meters should register much lower flows and more rigorous specifications should be used as rapidly as the advance in the art makes them practicable.

The Cost of Meters varies with market conditions. In a general way the following have been about representative costs in the year 1927.

Cost and Discharge of Domestic Meters

Nominal size inches	Approximate relative cost of disk meters	Manufacturers' rating, gallons per minute	Moderate rating of disk meters with loss of head not exceeding 5 lb.			
			Gallons per minute	Velocity in pipe of exact diameter	Million cubic feet per annum running all the time	Relative capacity
5/8	$ 10	20	7.11	7.44	0.50	1.0
3/4	15	34	12.09	8.78	0.85	1.7
1	22	53	21.33	8.72	1.50	3.0
1-1/2	40	100	42.67	7.45	3.00	6.0
2	80	160	71.1	7.26	5.00	10.0
3	120	315	142.2	6.46	10.0	20.0
4	200	500	213.3	5.45	15.00	30.0
6	400	1000	426.7	4.84	30.00	60.0

A capacity of 10 for a 2-in. meter is taken as the starting point for this table, and in case of revision, is a better starting point than the capacity of the 5/8-in. meter.

Disk Meters of different makes vary in frictional resistance. The figures given in the preceding table should be approximately reached by any standard make. **Rotary meters** differ much more in frictional resistance than disk meters. In some makes resistance would be from 50 to 100% greater than given in the table for disk meters. Other makes have less resistance than shown in the table. **Inferential** or **current meters** have considerably less frictional resistance than disk meters, the results differing widely for different designs.

The Load Factor of a meter is the percentage which the average sales from it are of the capacity shown in the next to the last column of the preceding table. Thus a 2-in. meter recording 40 000 cu. ft. per annum is said to have an 8% load factor $\left(\frac{40\,000}{5\,000\,000} = 0.08\right)$. Meters of the smallest size, 5/8 in., are large enough to give service under ordinary conditions to practically all private houses, and will normally be used on from 90 to 98% of all services. The average load factor on 5/8-in. meters in an average system will be between 1 and 2%. Much slippage or loss of water passing without registration is inevitable because of the low load factor. Smaller meters, if made, would not deliver water fast enough. The 3/4-in. meters used on large houses are in an intermediate position. In all other cases meters are best placed of the smallest size that will give the discharging capacity actually required by the taker. If rigidly held down by this rule an average load factor of all meters 1 in. and over of 15 or 20% may be realized. If the rule is less rigidly enforced lower load factors will result and more water will pass without registration. In general, any meter 1 in. and over with a load factor below 8% should be at once replaced with a smaller one. **Protector meters** and **compound meters** are used where large capacity is essential and where ordinary use results in a very low load factor.

Classification of Services was adopted by the New England and American Water Works Associations to secure uniformity in statistics and rates. Three classes are used with an optional fourth class.

1. Domestic, all using less than 820 gallons per day, equal to 10 000 cu. ft. per quarter or 300,000 gallons per annum.
2. Intermediate.
3. Wholesale or manufacturing, all using over 8200 gallons per day, equal to 100 000 cu. ft. per quarter, or 3 000 000 gallons per annum.
4. Large, optional, if used applying to takers using over 82 000 gallons per day, or 1 million cu. ft. per quarter, or 30 million gallons per annum.

Statistics for 33 Completely Metered Systems for 1915

	Per cent of number of takers	Average daily sales per service in gallons	Per cent of total sales in gallons
1. Domestic	95.17	159	38.4
2. Intermediate	4.37	1 750	19.6
3. Wholesale	0.46	35 700	42.0

Systems vary greatly in sales to takers of the several classes, and especially in sales to large takers.

The Resistance, in all meters, increases nearly as the square of the quantity of water passing. Resistance is added intentionally by makers of meters to prevent them from being used for flows so large that the meter would not be durable.

The use of meters on domestic services is the most efficient means of preventing useless waste of water through leaky plumbing fixtures. The growth in the use of meters during the last years has been very rapid. Many cities sell water only by the meter and the system promises to become as universal as in the sale of gas and electricity.

Water Not Accounted for. In completely metered systems the total quantity of water registered by meters of the takers is always much less than the total output. The loss is made up of: (1) leakage from the street mains; (2) leakage from service pipes and abandoned services; (3) under registration of meters; (4) loss by seepage and evaporation from service reservoirs; (5) water used for special purposes not metered, such as fires, sewers, flushing, etc. With present American methods there are but few systems so tight and well managed that 85% of the water is accounted for. The average is probably under 80% but there are some systems where no more than 60% or even 50% can be accounted for. The records of 27 completely metered systems for the year 1915 showed an average loss equal to 130 gallons per service daily.

The Venturi Meter, invented by Clemens Herschel, consists of contraction of the pipe through the throat of which the water flows at an increased velocity. As the vecloity increases the pressure decreases. A differential gage connected at its two sides with the water at the entrance and throat of the meter indicates the rate of flow. A mechanical integrating device shows the total amount of water which flows through the pipe. There are several devices for accomplishing this, but the principle of the meter in all cases is the same, see Sect. 13, Art. 20. With a good registering device the results are accurate within 1 or 2%. Attachments are made by which the rate of flow is shown by a pen on a chart, the paper being replaced daily or weekly, so that a permanent record is made not only of the rate of flow at all times but also of the total discharge since a specified date.

Venturi meters should be placed on all important supply lines to show the quantity of water used. In connection with the pumps they serve to show the amount of water actually pumped, and form a basis for computing the slip of the pumps and to show any falling off in the efficiency. They are especially useful on the outlets of distributing reservoirs and elsewhere, where they show at all times the actual rate of consumption, the fluctuation at different hours of the day, and the rate of flow during the early hours of the morning when there is but a small amount of use and the bulk of the flow usually represents leakage, and they also serve to show when fire drafts commence and end and how much water is used during fires and at what rates.

Service Pipes extend from the main in the street to the limits of the street. The same pipe usually extends within the taker's property, but he pays for and owns all beyond the street line or curb line. A majority of works furnish the service pipe to the property line without cost to the taker. A considerable variety of materials have been used and choice will frequently be controlled by local conditions and the quality of the water. With lead, copper or brass pipe the size of meter may correspond with the diameter of the pipe, but iron pipe should always be larger than the meter. The service cock, tapped into the street main, may be smaller. A flexible connection between the tap and the service pipe, most often a lead gooseneck, is necessary to prevent damage from temperature change and settlement.

Cement Lined Pipe has come into use rather recently, is economical, of excellent quality and general application. Galvanized pipe should be bought larger than would otherwise be used and the lining can be done with simple equipment by the men employed in laying pipe working on rainy days. The mortar is drawn into the pipe by a follower which regulates its thickness and leaves it smooth. Short nipples must be used to bring the lined barrels of the pipes close together at the ends. Such pipe is not corroded and will retain its carrying capacity indefinitely. Rarely where the soil conditions are bad must the pipe also be surrounded with cement mortar as it is laid.

Galvanized pipe of steel or, better, of genuine wrought iron, is widely used and gives good service where the water is hard and therefore quiet. With soft active waters such pipe deteriorates rapidly and should not be used.

Copper tubing has recently become available with convenient and reliable joints and has the advantage of strength and flexibility and of furnishing its own gooseneck. Such pipe is permanently free from corrosion and may be laid no larger than the size of meter to be served by it.

Brass pipe is made in wrought iron pipe sizes with screw joints and fittings and is not subject to corrosion.

Lead and lead-lined service pipes have been used widely in the past and there are great numbers of such services in use. Lead is permanent and its capacity is not reduced by corrosion but it is attacked slowly by soft active water, and because of lead poisoning, can be safely used only for waters demonstrated to be without action on it.

Goosenecks and Flexible Devices. Where stiff pipe is used, such as steel or iron either lead-lined or unlined or brass pipe, it is customary to use a gooseneck, i.e., a short piece of lead pipe, or to use two or more 1/4 bend fittings, to avoid fracture of the service pipe by temperature stresses. Some water companies using steel pipe have systematically omitted these provisions for many years without apparent ill effects. With waters which attack lead it is better not to use even these short goosenecks.

On an average from 5 to 7 people are served per service (more in very large cities) and there is one service for every 50 ft. of main pipe. For statistical purposes the number of services should correspond with the number of live accounts, abandoned or dead services and services placed in anticipation of business not being counted.

24. Electrolysis

Electrolysis in Cast-iron Pipes is caused by stray return currents of electricity from various sources especially from trolley cars. These stray currents find their way into water pipes through the soil or through service pipes or hydrant connections or gas pipes or telephone conduits or any other metallic structures coming in contact with the water pipes or the services connected with them. Such currents flow in the pipes, leaving them at points near the power stations, or go through other metallic conductors to the power station.

Destruction of a Pipe by electrolysis occurs in two ways: (1) By a current collected by the pipe, following it for a distance and then leaving it in moist soil, the electrolysis occurring at the point where the current leaves the pipe; (2) by the flow of electricity in the pipe, a part of which leaves the pipe at lead joints or other points of extra resistance, coming back into the next length of pipe. These two kinds of electrolysis, while having the same effect on the pipe, are to be sharply distinguished. Electrolysis of the first kind may be corrected in great measure by connecting the pipe system with the negative poles of the dynamos at all power stations. This has the effect of taking the return current out of the pipes directly through a copper wire and avoiding the necessity of currents leaving the pipe in moist ground on the return journey. This method of treating the electrolysis question was proposed in the early days of electrolysis and used to a considerable extent. The principal objection to it is that it produces electrolysis of the second kind. This system is openly followed in some works, and is actually followed by unknown and indirect connections in others.

In cast-iron pipe lines the lead joint is a point of high resistance. The temperature of the melted lead is not sufficient to burn off the tar coating, and actual metallic connection is not made in all cases. The electric current goes through the soil around the joint in sufficient quantity to produce electrolysis on one side of the joint. This takes place in wet soil only. Dry soil is a non-conductor. When electricity makes a passage it goes through the water contained in the pores of the soil, and not through the soil particles. Water is a non-conductor, but it becomes a conductor when mineral substances are dissolved in it.

Leadite and cement joints are non-conductors and cast-iron pipe laid with them cannot carry current. A single insulation joint will be jumped by current with the production of local electrolysis, but if the whole pipe system is insulated at every joint, it becomes non-conducting and there will not be enough flow of electricity to do damage.

Electrolysis of the interior of pipes is extremely rare, because the water used for public water supply is not sufficiently mineralized to act as a conductor. If the water in the soil outside the pipe were equally pure from an electrolytic standpoint there would presumably be little trouble from this kind of electrolysis.

Electrolysis occurs because the ground water contains mineral matter and salts which increase its conductivity. The mineral matters in ground water may result from many sources, among them: (1) From cesspools and similar sources, which are known to increase the chlorine contents of ground water to from 10 to 100 times the natural amounts, in villages and cities, and to less extent in rural districts; (2) urine of horses falling on public roads; (3) sea water brought by the rain, this being a matter of importance only when pipes are not very far from the ocean; (4) solution of mineral matters from the soil.

Insulation Joints are joints made of some non-conducting material to prevent the flow of electricity in pipes. Such joints have been made by driving wooden wedges between the spigot and bell of the cast-iron pipe in place of the lead. To be effective they must be repeated at short intervals, as the electric current will jump a number of such joints, passing through the surrounding moist soil and causing electrolysis at each of them.

Electrolysis of Steel Pipe. The riveted joints of steel pipe are almost perfect conductors of electricity. There is no evidence that a current flowing in a steel pipe injures it in any way as long as it does not leave the pipe. An electric current flowing in steel pipe and leaving it results in electrolysis at the point where it leaves the pipe.

Steel pipe is best protected by finding where the current tends to leave it and giving ample copper connections at those points to carry off the current. Sometimes the connections can be made to parts of the electric system that will absorb the stray current. In the absence of other opportunity old rails or any convenient form of old iron buried in the neighborhood of the pipe and in the direction of flow of current and connected to the pipe by liberal copper wires soldered or welded at both ends will take the major part of the corrosion and protect the pipe.

SECTION 15

DAMS, AQUEDUCTS, CANALS, SHAFTS, TUNNELS

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† Matter on Panama Canal written by Henry Goldmark.

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DAMS

1. Masonry Dams

Foundations of high masonry dams should be only on rock, which should be stripped and all loose or soft rock removed and the rock below thoroughly explored by core borings. If the rock is horizontally-bedded limestone or shale, mud seams may be a source of great danger by permitting sliding of the dam even when such seams are very thin. The igneous rocks frequently have small open seams which are not dangerous but permit leakage. This may usually be stopped by one or two rows of vertical holes drilled along the upstream toe and grouted with neat portland cement or by excavating a trench in the rock along the upstream toe and filling it with concrete. The whole foundation should be thoroughly washed; if it is smooth it may be roughened by chipping or blasting a fresh surface. (See Sect. 10, Art. 30.) If the rock is sound and so treated and the masonry begun by applying first a layer of rich mortar or grout to the clean fresh rock surface as good a joint is obtained as the joints between subsequent lifts of masonry above. Drains may be laid on the bed-rock and led out to the downstream toe to care for any slight seepage that otherwise might produce uplift, and a system of interior drains may relieve pressure both on joints and within the masonry above the base, but such drainage is seldom so sure to be effective that the internal stresses due to water pressure can be neglected.

Rubble Masonry. From the earliest times to the beginning of the present century rubble has been most frequently used for masonry dams. It is probably much better than the more expensive cut stone because of the better bond obtained. In the New Croton dam, one of the best examples of the rubble masonry type, the method of building was as follows: The derrick stones, limestone quarried near site, 15 to 50 cu. ft. in volume, were picked up by the derrick and, while suspended, fins, sharp corners, and edges were sledged off, and the stones thoroughly washed with a hose and then swung to place and lowered into a thick bed of mortar. Each stone was then picked up and examined for perfect bedding and at the same time spalls were packed into the mortar bed where it was thick enough. The stone was placed a second time and men with pinch bars worked it to a firm bed, being careful to move it only horizontally and not to rock it; sometimes the stone was removed and placed several times before the bedding was satisfactory. The vertical joints were then filled with hand rubble, consisting of spalls, well wetted and laid with soft mortar. On account of the irregular shape of the derrick stones the vertical joints averaged about 12 in. in width. The cut-stone facing was kept about one course above the rubble work, and shearing planes were prevented by allowing the hearting rubble to bond between the courses. The resulting proportions were: derrick stones 50%, spalls 26% and mortar 24%. In the Wachusett the proportions of the granite rubble were: derrick stones 54%, spawls 17%, and mortar 29%.

Cyclopean Masonry is rubble in which concrete is used in the place of mortar. Joints are correspondingly thicker. As portland cement became of better quality and cheaper, and skilled labor became dearer, cyclopean masonry began to replace rubble for masonry dams. In the case of several dams built between 1900 and 1910, the general method of constructing cyclopean masonry was as follows: One bucket or more of soft concrete was dumped, making a bed, say 1-1/2 ft. deep, on which large stones from 10 to 50 cu. ft. in volume were set by derrick as closely as their irregularities would permit. These settled into the bed, and were not usually picked up by the derrick again, as is the practice in bedding large rubble in mortar. Into the spaces between the derrick stones, averaging from 1 to 2 ft. in width, soft concrete was dumped and as many spalls as possible, from 1/4 to 1 cu. ft. in volume were forced into it. Care was always taken to prevent the formation of shearing planes by keeping a large part of the derrick stones projecting above the general level to form a bond with future work. In Ashokan dams 25% and in Kensico dam 27% of the total mass was composed of derrick stones.

Concrete Masonry. At present (1928) the use of concrete masonry for dam construction is almost universal. This is because of the development of machinery and methods which usually make a cubic yard of concrete cheaper than any other masonry and also shorten the time of building. However, the day of rubble and cyclopean masonry has not entirely passed because the design of a dam and materials of construction should always be the best adaptation to local conditions and there will still be masonry dams which can be more cheaply and more satisfactorily built of rubble and cyclopean masonry. It has also become almost standard practice to build concrete dams as a series of detached piers, each alternate block being brought up with forms surrounding its four sides and the intermediate blocks poured between upstream and downstream forms. Concreting is usually done in from 4-ft. to 8-ft. lifts. It is still the practice, wherever possible or convenient, to use plums in the concrete either in the form of large derrick stones or one-man stones well embedded in the concrete with special care that they project from each lift into the next above.

Temperature Cracks. It is observed that all straight dams and most curved dams in localities having considerable range of temperature have developed cracks which open in winter and close in summer. The question of temperature cracks has received much study, but the total knowledge derived is still meager. It seems to be fairly well established that temperature cracks in large masses of masonry are widest at the surface, becoming gradually narrower with penetration; that in the latitude of New York they disappear from 10 to 20-ft. depth depending on exposure, character of masonry, and whether the masonry was laid in warm or cold weather, so that if the dam is less than 20 or 30 ft. thick the cracks extend through. At the New Croton dam liquid dye was run into cracks and later the surrounding masonry was removed. All cracks were found to have the same general profile at their bottoms, as indicated in Fig. 1. In general the cracks formed a vertical plane located without regard to joints, passing, in some cases, through headers within two or three inches of their edges. Thermophones were placed in the masonry of Boonton and Kensico dams as they were built, and observations were made for a period of several years in each case. In the Boonton dam many instruments became unreliable so that complete laws could not be deduced; in Kensico dam the readings for nearly all positions were maintained for a period of four years. It was observed that the temperature at any point began to rise as soon as the concrete was placed and rose various amounts depending on the temperatures of the air and foundations, the mass of the concrete, the rate of placing the masonry, the distance to the nearest exposed face, the exposure of that face, and possibly upon the brand and content of cement used and the wetness of the mix. The time required to reach the

Fig. 1

maximum temperature depended on the same factors. The time required for dissipation of the heat generated by the reactions of the cement and water depended primarily upon the distance from an exposed face, the exposure of that face and the temperature difference inside and outside the dam.

The measurements at Boonton dam indicated the conclusion that a range of 130° F. in the atmosphere reduces to a range of about 70° F. 2 to 4 ft. below the surface of the masonry, with a gradual but slower decrease of range with greater depths, also a reduction in range of 10° F. per 1-3/4 ft. of depth.

The last measurements in Kensico dam, made in 1917 when the heat of setting practically had been dissipated, indicate that with a range in mean atmospheric temperature of 73° F. the range in temperature R in the masonry for depths in the masonry from 0 to 40 ft. may be approximately expressed by the formula $R = 48 - 12.3 \log_e D$ where D = distance from nearer face; which face had a sunny exposure. Measurements were taken at elevations where thickness of masonry ranged from 45 to 90 ft. and where D was always equal to or less than 1/2 such thickness.

Expansion Joints. It is now (1928) the universal custom to provide contraction joints at intervals of from 30 to 60 ft. normal to upstream face of gravity dams. The treatment of these joints is not standardized. Various devices which have been used for water stops are illustrated in Figs. 2 and 39.

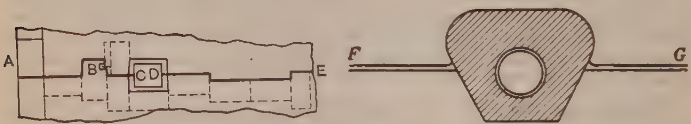


Fig. 2. Two Types of Expansion Joint

AE, Fig. 2, shows the joint in Kensico dam. $ABCDE$ is the trace of vertical planes of weak bond, made so by building first one side of precast concrete blocks, coating it with soft pitch, and later pouring the adjacent block against it. At B a sheet of copper 1/16 in. thick was placed across the joint with its two edges embedded in the concrete on either side in such a way that it will be bent but not ruptured by the movements of the concrete. At Ashokan the joint is the same except that there is no copper strip. CD is an inspection well. FG, Fig. 2, shows the water stop used at Jordan dam, Alabama. It consists of a block of high-melting-point pitch located across the construction joint. In the center of the pitch is a pipe running from top to bottom of the dam into which a second small steam pipe may be inserted to melt the pitch. The same devices as illustrated in Fig. 39 for water stops in aqueducts are used for dams

Waterproofing and Drainage Galleries. Watertightness in dams is desirable because water in penetrating increases the stress and may reduce the strength of the masonry. To construct a nearly vertical wall over a hundred feet high perfectly watertight is a matter of great difficulty. The method most commonly depended upon previous to 1905 was to make the water face of cut stone and point its joints very carefully with rich portland cement mortar, so that if the stone is impervious the leakage may be reduced and the downstream face will be practically dry. The following notes taken from published comments give the extent of leakage in some of the dams treated in this manner. The Lake Cheesman dam under a head of 212 ft. shows no sign of leakage and is entirely free from the appearance of seepage. The Sodom dam shows a few damp spots on humid days only. The Furens dam in France, under a head of 154 ft., showed only a few damp spots. The old Bear Valley dam, which for the greater part of its height is less than 8 ft. thick, showed only sweating under a head of about 50 ft. There is a growing practice of providing a drainage system in the masonry a short distance from the upstream face. In the Ashokan and Kensico dams there are drainage galleries 5 ft. by 7.5 ft., large enough for inspection, one near the top and the other near the base of the dam, connected by inclined drainage holes 16 in. in diameter. These drainage holes are formed in blocks of porous concrete 3 ft. by 3 ft. in plan set about 15 ft. from the upstream face and 12 ft. apart along the axis of the dam. The water face of the Mouche dam was given 3 coats of hot pitch and then whitewashed. The upstream face of the Urft dam was waterproofed with a plaster coat of cement 1 in. thick, on which a coat of asphalt was applied; against this, to hold it in place, a 3-ft. wall of masonry was built, backed by an earth embankment, and drainage pipes were placed in the masonry downstream from the asphalt coating. The upstream face of the Sand River dam, South Africa, was treated as follows: soap and alum washes were applied to the faces with a flat brush, the soap solution being at 212° F. and the alum at 60° to 70° F. The washes contained 1 lb. soap to 1.6 gal. water and 1 lb. alum to 9.6 gal. water. Three coats of each were used. Non-overflow dams in northern climates should have dry downstream faces because of frost action which will cause spalling of the face if water is present in the pores of the masonry. If for this reason or any other it is desirable to have a perfectly dry downstream face some method of waterproofing the upstream face is warranted. Usually the expedient of interior drainage is more satisfactory than attempting to waterproof the upstream face. If the dam is important or its failure would be accompanied by disaster, no reduction of cross-section should ever be made on account of a possible reduction of stresses on account of waterproofing for no waterproofing has been devised which is sure to be and always remain 100% perfect.

Essential details of notable precedents are given in the following condensed statements, where the numbers in parentheses refer to corresponding numbers within small circles on the cross-sections in the figures. Where height and width are given, the width is the base width corresponding to that height. Of all structures dams are probably most governed by precedents in their design and method of construction.

Straight Masonry Dams without Overflow (Fig. 3)

(1) Gros-Bois, France. 1830–38. 1805 ft. long, 21.3 ft. wide at top. Founded on soft. rock. When filled slid 2 in. Reinforced by 11 buttresses 37 ft. thick at bottom and 13 ft. thick at top. 96 ft. high; 52.5 ft. wide.

(2) Lozoyoa, Spain. About 1850. 238 ft. long; 105 ft. high; 128 ft. wide. Wall of cut stone backed by rubble masonry, partly covered on back face by sloping bank of gravel. Top width 21.98 ft.

(3) Habra, Algiers. 1856–71. Rubble masonry. 1082 ft. long. Top width 14.1 ft. Rock poor and porous. Mortar, natural hydraulic lime and fine clayey sand. When filled leaked badly, later leakage practically ceased. Failed 1881 during flood which overtopped dam; 300 ft. carried out down to base. About 400 lives lost. Repaired 1883–87. Profile was much changed and strengthened. 124.2 ft. high; 89.2 ft. wide.

(4) Cágliari, Sardinia. 1866. Granite, rubble, hydraulic mortar. 345 ft. long. Top width 16.4 ft. Height 70.5 ft. Width 52.4 ft.

(5) Boyd's Corner. New York City water supply. 1866–72. Cut-stone face, concrete heart (large plums in lower half). Concrete weighed 133.25 lb. per cu. ft. Length 670 ft. Top width 8.6 ft. Earth embankment upstream side 20 ft. wide on top, 4 : 1 slope. 27 000 cu. yd. masonry. Height 78 ft. Width 57 ft.

(6) Poona, India, 1868. Uncoursed rubble founded on rock. Length 5136 ft., of

which 1453 ft. is waste-weir, 11 ft. below top of dam. Alignment is several tangents. Reinforced by heavy buttresses at intersections. At first masonry showed signs of weakness. Reinforced by earth bank on lower face. Top width of bank 60 ft., height 30 ft. 360 000 cu. yd. masonry. Cost \$630 000. Height of masonry dam 100 ft. Width 60.7 ft.

(7) Tlelat, Algiers. 1869. 325 ft. long. Top width 13.12 ft. Height 68.9 ft. Width 40.3 ft.

(8) Djidionia, Algiers. 1873–75. Resultant said to be outside middle third. Top width 13.12 ft. Height 83.7 ft. Width 52.4 ft.

(9) Bouzey, France. 1878–81. 1700 ft. long. Red-sandstone foundation pervious. When filled dam slid 1 ft. Reinforcement at toe added 1888. Failed 1895, upper 33 ft. overturning for length of 558 ft. 150 lives lost. Height 84.5 ft. Width 57.3 ft.

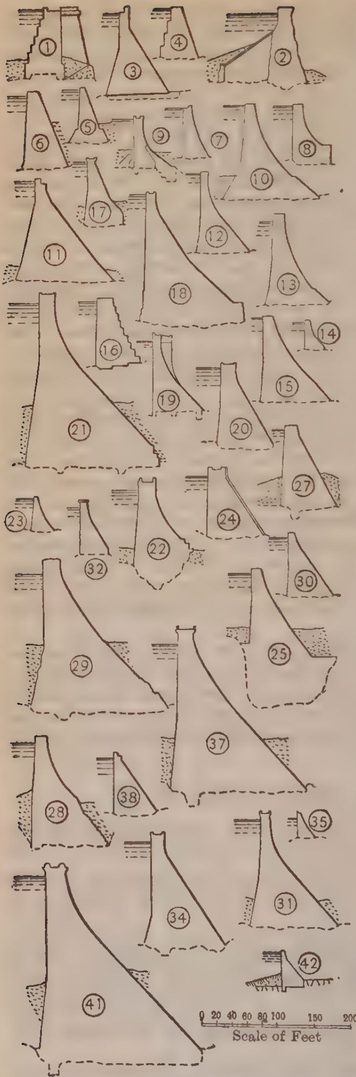
(10) Gran-Cheurfas, Algiers. 1882–84. Rubble. 508.4 ft. long. Top width 13.12 ft. Partially failed when filled 1885; immediately repaired. Height 131.2 ft. Width 134.5 ft.

(11) Lagolungo, Italy. Genoa water supply. 1883. Raised 10 ft. 1903. Top width 8.2 ft. Height 145.9 ft. Width 140 ft.

(12) Vingeanne, France 1885. Top width 11.48 ft. Height 113.8 ft. Width 80.1 ft.

(13) Hamiz, Algiers. 1885. Rubble. 532 ft. long. Top width 16.4 ft. Height 124.6 ft. Width 91.2 ft.

(14) Bridgeport, Connecticut. 1886. Gneiss rubble. Rosendale 1 : 2 mortar. 640 ft. long. Top width 8.0 ft. Base length 50 ft. Height 45 ft. Width 29.5 ft.



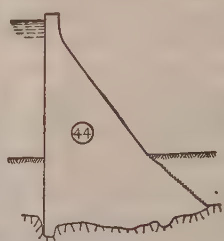


Fig. 3. Straight Masonry Dams without Overflow

(15) Tansa. Bombay water supply. 1886–91. Total length 8800 ft. 1650 ft. of dam is waste-weir, 3 ft. below top of dam. Uncoursed rubble masonry. Stones, hard

trap or greenstone. Cement, hydraulic lime burnt from kunkur nodules. Masonry 408 520 cu. yd. Reported to be watertight. Cost $988 000 by contract. Designed strong enough to permit height to be increased 17 ft. Height 118 ft. Width 99.8 ft.

(16) Tytam, China. 1887. Granite ashlar and concrete. Intended for 20 ft. higher. Present crest width 21 ft. Height 95 ft. Width 63.4 ft.

(17) Sodom. New York water supply. 1888–93. Coursed rubble on solid rock. Faces cut stone; beds perpendicular to faces. Mortar, 1 portland cement to 2 sand. 500 ft. long. Top width 12 ft. 35 887 cu. yd. Cost $366 499. Maximum progress 3000 cu. yd. per month. No leakage except slight sweating at joints. Height 94.7 ft. Width 53 ft.

(18) Periyar, India. 1888–97. Faces uncoursed syenite rubble; heart concrete; hydraulic lime. Situated in jungle. Unskilled labor. 1231 ft. long. Top width 12 ft. 185 000 cu. yd. masonry. Upstream face plastered with hydraulic lime and sand 1 : 1. Height 183 ft. Width 135.5 ft.

(19) Mouche, France. 1885–90. Rubble, 134.2 lb. per cu. ft. 1346 ft. long. Top width 11.5 ft. Given three coats hot pitch on upstream face, then whitewashed. Height 101.5 ft. Width 66.5 ft.

(20) Titicus. New York water supply. 1890–95. Rubble rough-coursed, cut-stone faces; beds perpendicular to faces. Mortar, 1 portland or natural cement to 2 sand. 534 ft. long. Top width 20.7 ft. Maximum progress 5700 cu. yd. monthly. Cost $933 065. Height 109 ft. Width 75.2 ft.

(21) New Croton. New York water supply. 1892–1907. Rubble faced with ashlar. Part cyclopean. Length 1200 ft. Top width 18 ft. Rock in greater part of dam 185 lb. per cu. ft. 855 000 cu. yd. masonry. Maximum progress 5700 cu. yd. monthly. Height 238 ft. Width 185 ft.

(22) Burrator, England. 1893–96. Granite blocks bedded in rich concrete. Cut-stone face. Joints pointed and caulked with neat cement mortar. Length 361 ft. Top width about 18 ft. Cost $495 700. Height 77 ft. Width 63 ft.

(23) Indian River, New York. 1898. Top width, 7 ft. Length 207 ft. Height 47 ft. Width 33 ft.

(24) Assuan, Egypt. 1898–1902. Granite rubble laid in 1 : 4 portland cement mortar. Length 6400 ft., 1800 ft. solid, remainder has 180 sluices each 6.56 ft. wide. Top width 17.8 ft. Masonry 704 000 cu. yd. Cost $11 907 000. Height 95 ft. Width 80.3 ft.

(25) Coolgardie, Australia. 1900–02. Rubble concrete. Length 755 ft. Top width 10 ft. Height 119.9 ft. Width 88 ft.

(26) Blackbrook, England. 1900–05. Length 525 ft. Top width 14 ft. Height 108 ft. Width 65 ft.

(27) Boonton, New Jersey. 1900–06. Cyclopean syenite masonry. Top width 17 ft. Length 2150 ft. Masonry 166 lb. per cu. ft. 255 000 cu. yd. Maximum progress 21 000 cu. yd. per month. Height 103 ft. Width 77 ft.

(28) Spier Falls, New York. 1900–05. Rubble. Length 552 ft. Top width 17 ft. 180 000 cu. yd. masonry. Height 150 ft. Width 107 ft.

(29) Wachusett, Massachusetts. 1900–06. Granite, rubble, cut-stone faces. Top width 22.5 ft. Length 971 ft. Three-quarters of heart is laid in natural cement. Volume 266 663 cu. yd. Cost $2 270 117. Height 205 ft. Width 187 ft.

(30) La Jalpa, Mexico. 1902. Body of dam rough limestone rubble laid in mortar of native hydraulic lime and sand. Clay embankment, slope 2 : 1 on upstream face. Length 1800 ft. Top width 9.1 ft. Contents 92 000 cu. yd. Cost $500 000 gold. Height 87.2 ft. Width 65.6 ft.

(31) Cataract, Australia. 1902–08. Cyclopean masonry. Sandstone blocks in cement mortar packed with concrete; blocks about 65% of mass. Faces, concrete downstream, concrete blocks upstream. Length 811 ft. Top width 16.5 ft. Rectangular conduits filled with broken stone and earthen pipes for drains. 146 242 cu. yd. Height 154 ft. Width 120 ft.

(32) Pedlar River, Virginia. 1904. Concrete with large stones embedded. Length 415 ft. Width at crest 10 ft. Concrete laid as large blocks about 10 × 10 × 15 ft. and dovetailed. Portland cement on face, natural in heart. Contract price $103 708. Height 73.5 ft. Width 39.2 ft.

(33) Swansea, Wales. 1905. Cyclopean masonry, brick facing. In heart concrete 1 : 2 : 5. Lower base and upper 6 ft. of water face 1 cement : 2 sand : 3.4 parts of

fine crushed rock. Length 1250 ft. Maximum height 144 ft. Thickness at base 107 ft. Maximum depth water 100 ft.

(34) Cross River, New York. 190–507. Cyclopean masonry, concrete block facing. Length 772 ft. Top width 23 ft. 158 000 cu. yd. Maximum progress 18 400 cu. yd. per month. Contract price $1 246 212. Height 155 ft. Width 125.3 ft.

(35) Connellsville, Pennsylvania. 1906. Concrete, boulders used as plums. Ashlar facing. Length 650 ft Top width 6 ft. Boulders about 25% of masonry. Height 39 ft. Width 26 ft.

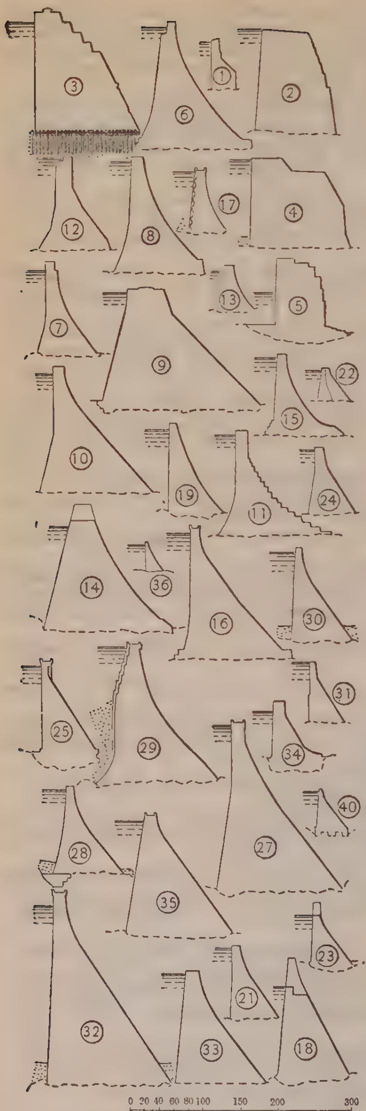
(36) Sand River, South Africa. 1906. Concrete 1 : 1.3 : 5.5 with large quartzite rocks, iron rods and rails embedded. Length 398 ft. Height 55 ft. Base width 38 ft. Foundation hard shale. Contents 9000 cu. yd. Cost about $140 000. Masonry 150 lb. per cu. ft.

(37) Ashokan. Olive Bridge dam, New York. 1908–13. Cyclopean masonry. Length 1000 ft. Top width 26.3 ft. Facing concrete blocks. Height 220 ft. Width 190.2 ft. Contraction joints, wells and galleries for inspection and drainage, and vertical drains provided. (See Fig. 2, p. 1522.)

(38) Blackwater, Scotland. Cyclopean concrete. Top width 10 ft. Height 84 ft. Width 58 ft.

(39) Esperanza, Mexico. Rubble masonry. Length 580 ft. Top width 19.7 ft. Base width 75.7 ft. Maximum height 137.5 ft. 55 000 cu. yd. Mortar of lime somewhat hydraulic. Leakage estimated 0.4 cu. ft. per sec.

(40) Barker, Colorado. 1909. Masonry 1 : 3 : 5 concrete with plums. Expansion joints in upper 145 ft.,



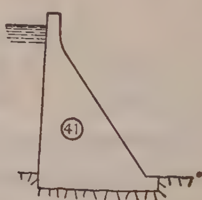


Fig. 4. Curved Masonry Dams with Gravity Sections without Overflow

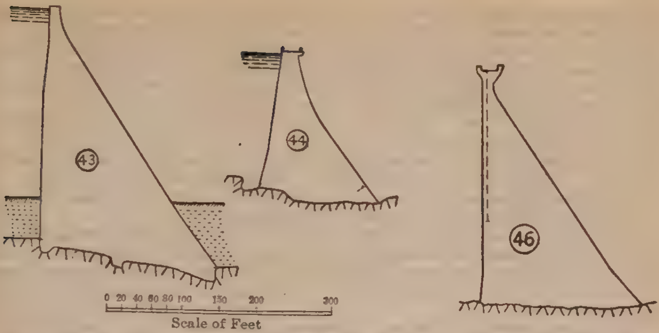


Fig. 4 (continued). Curved Masonry Dams with Gravity Sections without Overflow

48 ft. apart. Length 625 ft. Top width 16 ft. Height 175 ft. Maximum base width 124 ft.

(41) Kensico, New York. 1910–16. Cyclopean masonry; concrete blocks on upstream face, cut stone on downstream face. Length 1830 ft. Top width about 27.75 ft. Base width 227.7 maximum. Expansion joints, vertical drains and inspection galleries. Height 250 ft.

(42) Austin, Pennsylvania. 1909. Maximum height 50.5 ft. Width at base 30 ft. Top width 2.5 ft. Length 554 ft. Foundation, sandstone in layers 8 in. to 3 ft. thick with beds of shale and disintegrated sandstone between the layers. When filled in January, 1910, a section 90 ft. long slid 18 in. at bottom and 31 in. at top. Dam then repaired but not strengthened. In September, 1911, it failed suddenly with loss of 80 lives and $3 000 000 of property.

(43) Medina, Texas. 1911–12. 1 : 3-1/2 : 6-1/2 concrete with 9.9% plums. Length 1580 ft. Top width 25 ft. Maximum height 164 ft. Base width 128 ft. Total volume 300 000 cu. yd.

(44) Elephant, Butte, New Mexico. 1910–16. Cyclopean concrete. Length 1200 ft. Top width 18 ft. Maximum height 304.5 ft. Base width 212.58 ft. Volume 611 400 cu. yd. Drainage gallery, vertical drains and expansion joints.

(45) Brisbane, Australia. Completed 1916. Heart 1 : 2.5 : 6.5 concrete with 25% plums. Upper 30 ft. and faces to toe and heel 1 : 2 : 4 concrete. Length 580 ft. Top width 10 ft. Maximum height 125 ft. Volume 58 400 cu. yd.

Curved Masonry Dams with Gravity Sections without Overflow (Fig. 4)

(1) Almanza, Spain. Built before 1586. Radius 86 ft. Rubble cut-stone face. Oldest existing masonry dam. Top 9.84 ft. thick. Height 67.8 ft. Width 33 7 ft.

(2) Alicante, Spain. 1579–84. Rubble, cut-stone facing. Radius at top 351 ft. Height 134.5 ft. Width 110.6 ft.

(3) Puentes, Spain. 1785–91. Polygonal in plan, arched upstream. 925 ft. long. Rubble, cut-stone facing. Mostly founded on rock; on piles over deep pocket. Held 82 ft. depth for 10 years. 1802 water rose to 154 ft. depth and pile foundations washed out, leaving dam bridging deep pocket. 608 lives lost. Height 164.2 ft. Width 145.2 ft.

(4) Val de Infierno, Spain. 1785–91. Polygonal arch. Reservoir silted to top of dam. Built on rock. Height 110.3 ft. Width 136.4 ft.

(5) Nijar, Spain. 1843–50. Rubble, cut-stone facing. Height, 90.5 ft. Width 67.6 ft.

(6) Furens, France. 1862–66. Rubble of mica schist with cut-stone front face. Length about 330 ft. Top width 9.91 ft. Radius 828.4 ft. Built in narrow gorge. No leakage. Few damp spots. 52 300 cu. yd. Cost $318 000. Height 170.6 ft. Width 161 ft.

(7) Ternay, France. 1865–68. Granite rubble. Top width 13.12 ft. Radius 1312 ft. Height 124.6 ft. Width 81.7 ft.

(8) Ban, France. 1867–70. Rubble. Length 512 ft. Top width 16.4 ft. Radius 1325 ft. Built on rock. Cost $190 000. Height 156.8 ft. Width at base 126.9 ft.

(9) Gillepe, Belgium. 1870–75. Sandstone rubble. Top width 49.2 ft. Radius 1640 ft. Length top 771 ft. Height 156.5 ft. Width 215.89 ft. 325 000 cu. yd.

(10) Villar, Spain. 1870–78. Rubble. Top width 14.75 ft. Length 349 ft. Radius 440 ft. Height 170.3 ft. Width 154.6 ft. Cost about $390 000.

(11) Hijar, Spain. 1880. Top width 16.4 ft. Length 236 ft. Radius 210 ft. Height 141.1 ft. Width 146.9 ft.

(12) Gorzente, Italy. Genoa water supply. 1880–83. Serpentine rubble with casale lime mortar. 492 ft. long. Top width 22.96 ft. Height 126.3 ft. Width 99.4 ft.

(13) Thirlmere, England. 1886–93. Top width 18.5 ft. Plan is reverse curve to follow bedrock. Radius 100 ft. Height 63 ft. Width 51.7 ft.

(14) San Mateo, California. 1887–88. Concrete molded into dovetailed blocks on dam. Planned for maximum height 170 ft. Stopped at 146 ft. Top width 25 ft. and top length 680 ft. at 170 ft. height. Radius 637 ft. Volume 139 000 cu. yd. A few damp spots on face only sign of leakage. No cracks visible. Concrete 1 : 2 : 6.5. Width at base 176 ft.

(15) Beetaloo, Australia. 1888–90. Concrete. Top width 14 ft. Length 580 ft. Radius 1414 ft. 60 000 cu. yd. masonry. Founded on rock. Cost $573 300. Height 110 ft. Width 110 ft.

(16) Chartrain or Tache, France. 1888–92. Rubble or porphyritic rock and hydraulic mortar. Weight per cu. ft. 150 lb. Top width 13.12 ft. Radius 1312 ft. Upstream face coated to 33 ft. below coping with artificial cement of slaked lime, 1-1/4 in. thick, made of equal parts of cement and sand. Some leakage. Height 174.7 ft. Width 159.9 ft.

(17) Remscheid, Germany. 1889–92. Rubble linneite slate in trass mortar. Water face plastered with cement and asphalt, covered by brick wall. Top width 13.1 ft. Radius 410 ft. 22 886 cu. yd. Height 82 ft. Width 49.2 ft.

(18) Hemet, California. 1890–95. Granite rubble and concrete. Designed with maximum height 160 ft.; only carried to 122.5 ft. Top width 10 ft. Length 260 ft., including 50-ft. spillway in center. Radius 225.4 ft. 31 105 cu. yd. at present height. In time of flood spillway too small. Base width 100 ft.

(19) Bhatgur, India. About 1891. Uncoursed rubble. Upper portion concrete and rubble blocks. Faces coursed rubble. Top width 12 ft. Length 3257 ft. Irregular curve to follow outcrop of bedrock. Height 127 ft. Width 73.7 ft.

(20 Butte City, Montana. 1892. Concrete, granite facing. Top width 10 ft. Maximum height 120 ft. Bottom width 83 ft. Length of top 350 ft. Radius 350 ft.

(21) Lauchensee, Germany. 1892–95. Cyclopean masonry. Trass mortar, 1 lime, 1 trass, 2-1/2 sand made by crushing sandstone. Earth bank to protect from sun. Top width 13 ft. Length top 840 ft. Radius 2950 ft. Cu. yd. 37 400, of which 65% stone and 35% mortar. On sandstone. Total cost $243 750. Height 98 ft. Width 65.3 ft.

(22) Lennep, Germany. Old dam, 1893. Length 416 ft. Radius 460 ft. Dam was increased in height from 37.7 ft. to 48.4 ft. and buttresses built to take added thrust. Width at base 50.8 ft.

(23) Wigwam, Connecticut. 1893–96. Designed for 90-ft. height, built only 75 ft. high. Width 12 ft. at designed height, and length 600 ft. 14 887 cu. yd. in completed portion. Radius 600 ft. Width at base 62 ft. Completed to designed height in 1903.

(24) Einsiedel, Germany. 1894. Rubble. Top width 13.12 ft. Length 590 ft. Radius 1310 ft. 31 600 cu. yd. Height 93.6 ft. Width 65.5 ft.

(25) Echapre, France. 1894–98. Top width 17 ft. Length 541 ft. Radius 1148.2 ft. Hydraulic mortar coat on upstream face, on which coat of hydraulic cement was later placed. Maximum depth of water 121.4 ft. Width at base 88.6 ft.

(26) Ondenon, France. 1900–04. Top width 15.4 ft. Length 420 ft. Radius 984 ft. Maximum height 123 ft. Base width 94 ft.

(27) Lake Cheesman, Colorado. 1900–04. Top width 18 ft. Length 710 ft. on crest, 30 ft. at base. Granite rubble, coursed rough-pointed facing. Radius 400 ft. 103 000 cu. yd. Total cost about $1 000 000. Height 227 ft. Width 176 ft.

(28) Komotau, Austria. 1901–04. Cyclopean masonry. Gneiss in portland cement concrete. Top width 13 ft. Length 508.5 ft. at top, 170.6 ft. at bottom. Radius 820 ft. 53 600 cu. yd. Asphaltum and tar coat held in place by concrete dovetailed into upstream face. Height 118.9 ft. Width 98.4 ft.

(29) Urfa, Germany. 1901–04. Slate and trap masonry, earth embankment to 77 ft. below crest, slope 2 : 1, rock paving. Length 741 ft. Top width 18 ft. Radius 656 ft. Height 190 ft. Width 165.5 ft.

(30) Mercedes, Mexico. 1901–05. Rubble, cut-stone facing. Top width 11.48 ft. Length at crest 535 ft., also 98-ft. spillway; 13 ft. at base. Straight for half the length, rest 196.8 ft. radius. 28 000 cu. yd. Cost $200 000 Mexican currency. Height 135.6 ft. Width 84.5 ft.

(31) Granite Springs, Wyoming. 1903–04. Uncoursed rubble. Weight per cu. ft. 165 lb. Top width 10 ft. Length 410 ft. on top, 10 ft. at base. Radius 300 ft. 14 422 cu. yd. Cost $109 194. Height 96 ft. Width 56 ft.

(32) Roosevelt, Arizona. 1905–11. Cyclopean masonry, range rubble facing. Sandstone. Top width 16 ft. Length 680 ft. Radius 400 ft. About 340 000 cu. yd. Height 260 ft. Width 158 ft.

(33) Marklissa, Germany. 1905. Gneiss rubble. Upstream face has 2-in. mortar layer coated with siderosthen. Has interior drainage system. Length 427 ft. Radius 410 ft. 83 700 cu. yd. Height 147.7 ft. Width 124.8 ft.

(34) Marquina, Manila, Philippine Islands. 1906. Rubble, hard cyrstalline limestone marble. Length 400 ft. Radius 500 ft. Height 75 ft. Width 68.7 ft.

(35) Cher, France. Top width 15.4 ft. Radius 656 ft. Height 154 ft. Width 141 ft.

(36) Burraga, Australia. Cyclopean concrete in heart. T-rails bedded near top. Top width 2 ft. Length 285.6 ft. Radius 539.8 ft. Cost $44 500. Height 41 ft. Width 25.3 ft.

(37) San Jose, Mexico. Rubble. Top width 15 ft. Maximum height 151 ft. Base width 128 ft. Length 592 ft., including two spillways 95 ft. and 86 ft. Radius 6560 ft.

(38) Murrumbidgee River, Australia. Concrete, large stones embedded. Maximum height 232 ft. Length 910 ft. Radius 940.5 ft. Maximum base width 160.33 ft. Crest width 18 ft.

(39) Mochne, Germany. Length 1312 ft. Maximum height 98.4 ft. 353 160 cu. yd.

(40) Griffin, Pennsylvania. Concrete laid in 12-in. courses usually from end to end during one working day. Length 284 ft. Radius 400 ft. Top width 4 ft. Spillway near center 80 ft. long. Concrete 8000 cu. yd. At first leaks appeared near ends. Height 62 ft. Width at base 44 ft.

(41) Barren Jack, New South Wales. 1909–13. Cyclopean concrete. Length 784 ft. Top width 18 ft. Base width 163 ft. Height 240 ft. Radius 1200 ft.

(42) La Boquila, Chihuahua, Mexico. Cyclopean masonry with 10-ton blocks of limestone. Length 840 ft. Top width 19 ft. Base width 200 ft. Radius 866 ft.

(43) Arrowrock, Boise, Idaho. 1912–13. 1 : 2.5 : 5 concrete with large rock and cobblestone. Length 1050 ft. Top width 15.5 ft. Height 351 ft. Base width 238 ft. Radius 662 ft. Inspection gallery. Contraction joints every 150 ft.

(44) Mauer, Silesia, Germany. Completed in 1912. Stone masonry. Length 918 ft. Top width 23.6 ft. Height 203 ft. Base width 165 ft. 332 000 cu. yd. Radius 820 ft. Drainage and inspection tunnels.

(45) Camarasa. 80 miles from Barcelona, Spain. 1917–20. Length 460 ft. Base 250 ft. at height of 333 ft. Radius 1000 ft. Cyclopean concrete, 285 000 cu. yd. Top width 13 ft.

(46) Exchequer, California. 1923–26. Length 950 ft. Height 333 ft. Radius of upstream face 675 ft. Batter of downstream face .63.

Curved Masonry Dams Depending on Arch Action without Overflow (Fig. 5)

(1) Meer-Allum, India. About 1800. Large arch made up of 21 small arches with buttresses between. Spans between buttresses 70 to 147 ft. in clear. Spillway provided, but at times water flows few inches deep over entire dam. Length about 2640 ft.

(2) Zola Dam, France. 1843. Rubble masonry. Length 205 ft. Top width 19 ft. Radius at top 158 ft.

(3) Bear Valley dam, California. 1884. Radius 335 ft. Rough granite ashlar facing and rubble hearting. 300 ft. long. Thickness 2.5 ft. to 3 ft. at top and 8.5 ft. at 48 ft. below crest. Inaccessible location. Haulage of cement to site cost $10 per bbl. 3400 cu. yd. In 1911, in order to store more water, a new reinforced-concrete dam 30 ft. higher than the old was built just below this dam, submerging it.

(4) Sweetwater, California. 1887–88. Rubble. Weight per cu. ft. 164 lb. Length

about 380 ft. Radius 222 ft. 19 269 cu. yd. Top width 12 ft. Mortar used mostly 1 portland to 3 sand, but near upstream face mortar was 1 : 2. Cost about $234 000. 1895 dam was overtopped by 22 in. of water for 40 hours without damage. In 1911

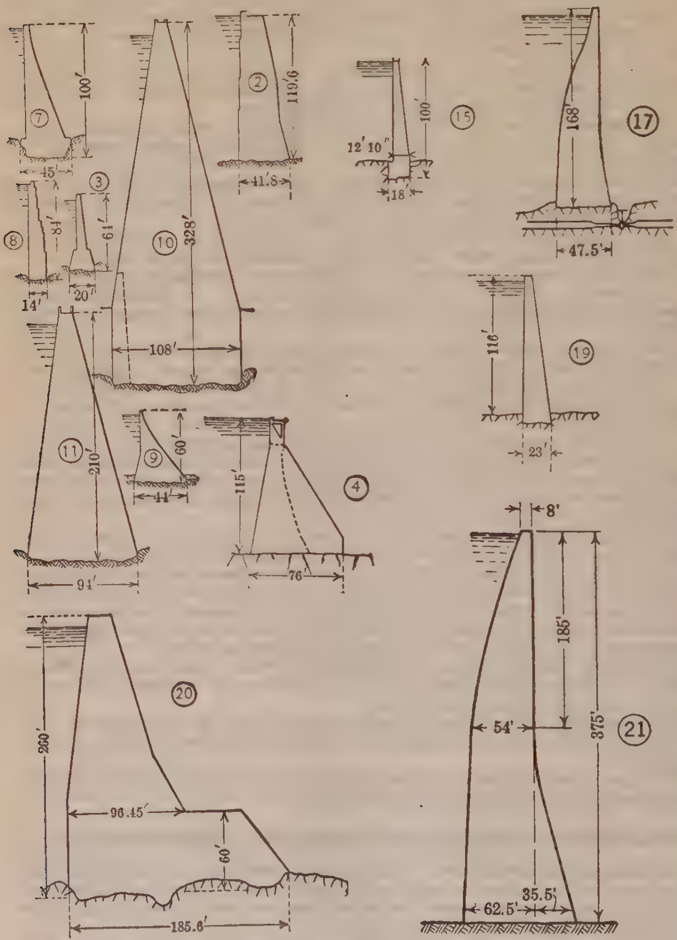


Fig. 5. Arched Masonry Dams

the dam was increased 20 ft. in height and a new section, reinforced with old rails, was built on the downstream side, 76 ft. wide at the base and reducing to nothing at the elevation of the top of the old dam. Care was taken to bond the old and the new concrete.

(5) Belubula, Australia. 1898. Concrete and brick. Lower part up to 23 ft. height

s concrete, above this brick arches and buttresses built 36.7 ft. higher. Series of buttresses 28 ft. c. to c., elliptical arches between, axes inclined 30 deg. to vertical. Arches 4 ft. thick at bottom, 1 ft. 7 in. at top. Buttresses 40 ft. long, 12 ft. wide at wall and 5 ft. at outer end. Each buttress forms a segment of circle 36 ft. 2 in. radius and diminishes in thickness from 8-1/2 ft. at center to 4 ft. at outer circumference.

(6) Johannesburg, South Africa. 1898. Rubble masonry, hard blue quartzite, concrete foundations reinforced with rails. Top width 7 ft. Length 585 ft. 30 000 cu. yd. Arch and tangent; part on tangent, gravity section. Radius of arch section 275 ft. at crest, decreasing to 206 ft. 75 ft. below to meet lines of gravity section.

(7) Barossa, South Australia. 1899–1903. Concrete with plums, reinforced at top with steel rails. Length 472 ft. Radius 200 ft. Cu. yd. 17 975, of which 12.3% large stones. Top width 4.6 ft. Range of temperature during construction was from 30° to 168° F. Total cost $827 000.

(8) Upper Otay, California. 1900. Masonry reinforced with steel plates and cables. Top width 4 ft. Length 350 ft. at crest, 20 ft. at base. Radius 359 ft. In a rock gorge.

(9) Geelong, Australia. Sandstone concrete. Top width 2.5 ft. Radius 300 ft.

(10) Shoshone, Wyoming. 1903–10. Concrete. Top width 10 ft. Length 200 ft. Radius 150 ft. 75 000 cu. yd. In narrow solid granite gorge. Contract price $515 730.

(11) Pathfinder, Wyoming. 1905–09. Cyclopean masonry. Radius 150 ft. 54 000 cu. yd. Length on top approximately 425 ft., at base 80 ft. In narrow granite gorge. Contract price $482 000.

(12) Sand River, Port Elizabeth waterworks, South Africa. 76 ft. high. Radius 300 ft. Length 360 ft. Cyclopean masonry faced with concrete blocks. Soap and alum washes on faces for waterproofing. Dam reinforced with steel rails.

(13) Dam at Hume, California. Concrete reinforced by railroad iron and cable. Concrete approximately 1 : 2 : 4, crusher run of granite being mixed with sand. Series of buttresses parallel to stream with arches between; 12 arches each of 50-ft. span. Water face and parts of downstream face plastered. Water face two coats cement mortar 1 to 1-1/2 and wash coat of neat cement on bases of middle arches. 2207 cu. yd.

(14) Salmon River dam, Idaho. Cyclopean masonry. 220 ft. high. 463 ft. long. Top width 15 ft. Base width 119 ft. Radius upstream face 225 ft.

(15) Huacal, Mexico. 1911–12. Crest length 140 ft. Radius 76 ft. Maximum height 100 ft. Base width 12 ft. 10 in., 80 ft. below top.

(16) Goodwin, California. 1911–12. Concrete two spans of 135-ft. radius abutting on central pier. Length 466 ft. Width at top 8 ft. and bottom 16 ft. Maximum height 78 ft.

(17) Salmon Creek, Juneau, Alaska. 1913–14. Concrete. Constant-angle arch type. 52 000 cu. yd. Height 168 ft. Width at top 6 ft. Base 47.5 ft. Radius at top 331 ft., at bottom 147.5 ft.

(18) Eagles Nest, New Mexico. 1916–18. Concrete. Length 300 ft. Top width 8 ft. Bottom width 46 ft. Maximum height 140 ft. Radius 155 ft.

(19) Corfina, Italy. 1915. Concrete. Top width 5 ft. Bottom width 23 ft. Maximum height 116 ft. Radius 75 ft. Built in 65 days.

(20) Spaulding, California Built to height of 225 ft. in 1913 and raised to 260 ft. in 1916. Estimated yardage 169 000. Width at 260-ft. level 14 ft. Planned ultimately to be 305 ft. high. 304 000 cu. yd. Vertical drains and inspection gallery. Lower 60 ft. of gravity section.

(21) Pacoima, Los Angeles, California. 1927. Variable-radius constant-angle arch type. Span across canyon at crest 550 ft. Height 375 ft. Thickness at crest 8 ft., and 98 ft. at 375 ft. below crest. Upstream radius 340 ft. at crest and 170 ft. at base. Downstream radius 332 ft. at top and 54 ft. at base. 150 000 cu. yd. concrete. Cost $1 750 000

Straight Masonry Overflow Dams (Fig. 6)

(1) Vir Weir, India. Uncoursed rubble. 2340 ft. long. 43.5 ft. high. Top width 9 ft.

(2) Henares Weir, Spain. Concrete with ashlar facing. 390 ft. long. Maximum height 23 ft. Base width 45.8 ft.

(3) Vyrnwy, England. 1882–89. Cyclopean masonry. Length 1164 ft.

(4) Lynx Creek, Arizona. Intended for ultimate height of 50 ft. Height was about 31 ft. In 1891 flood overtopped and destroyed dam.

(5) Folsom, California. 1886–91. Rough granite ashlar blocks. 48 590 cu. yd. Top width 24 ft.

(6) Betwa, India. 1888. Length 3296 ft., made in three tangents between islands. Top width 15 ft. Rubble masonry laid in native hydraulic lime. Ashlar coping 18 in. thick in portland cement mortar. Cost $160 000. Has had 16.4-ft. head on crest.

(7) Austin, Texas. 1891–92. Hard limestone rubble, granite facing and coping.

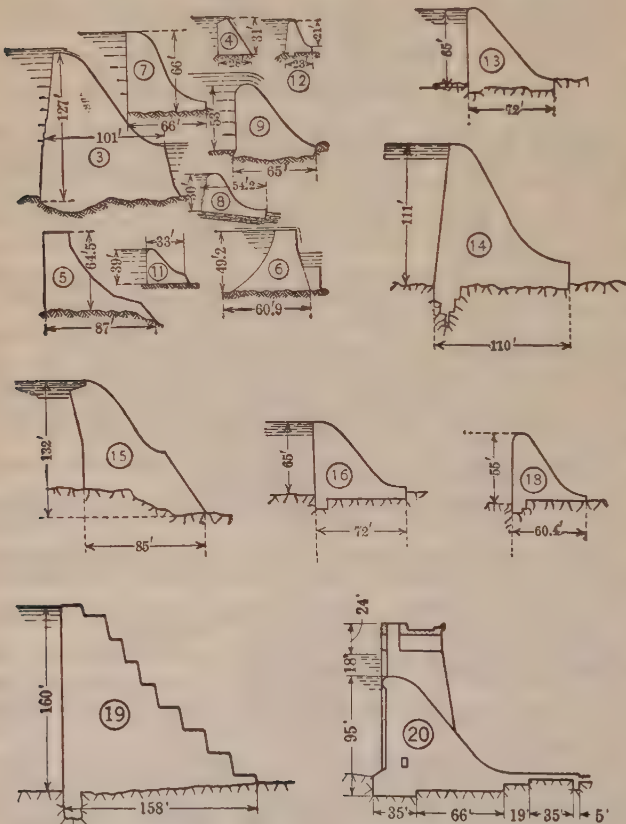


Fig. 6. Straight Overflow Dams

Length about 1100 ft. 88 000 cu. yd. Cost $608 000. Failed under head of 11.07 ft. on crest. Previously stood 9.8 ft. head. About 300 ft. slid forward. No overturning.

(8) New Holyoke, Massachusetts. 1897. Rubble with dressed stone cap and facings and concrete toe. Length 1020 ft. Upper part of downstream face is parabolic, lower part is cycloidal.

(9) Holtwood, McCall's Ferry, Pennsylvania. 1904. Concrete with plums. 2350 ft. long.

(10) Great Northern Power Co., Minnesota. 1907. Concrete. 365 ft. long, 38 ft. high. 42 ft. wide at base, curved crest.

(11) Connecticut River dam. Concrete with plums (1 : 2.5 : 5 except in deep foundations where 1 : 3 : 6 was used). Length 600 ft.

(12) Escanaba, Michigan. Concrete with earth banks at end. Length 810 ft. Concrete in heart 1 : 3 : 6 on sides and bottom 1 : 2 : 4. Facing 1 ft. thick upstream and 1.5 ft. downstream. Founded on stratified limestone. 8-in. drain pipes at 10-ft. intervals from cutoff to toe of dam.

(13) Hales Bar, Tennessee. 1905–13. Cyclopean concrete. Foundation limestone badly fissured. Fissures closed in part by grouting, and, in addition, 26 reinforced-concrete caissons 30 × 32 to 54 × 72 were sunk by pneumatic process through fissured rock to solid foundation. Height 65 ft. Length 1200 ft. Width 72 ft. Curved crest.

(14) Holter, Wolfcreek, Montana. 1910. Concrete with plums. Length 1362 ft., including 504 ft. overflow section. Height 111 ft. Width 110 ft. plus 70 ft. of apron. Cutoff trench 44 ft. deep.

(15) Hauserlake, Montana. Completed 1911. Replacing steel dam which failed by undermining. Length 490 ft. 85 000 cu. yd. Maximum height 132 ft. Width 85 ft. Ogee type. Part of foundations placed by pneumatic caissons.

(16) Coosa River, Lock 12, Alabama. 1912–14. Cyclopean masonry. Length of spillway 930 ft. Height 65 ft. Width 72 ft. Parabolic crest.

(17) Keokuk, Iowa. 1 : 3 : 5 concrete. Length 4278 ft. Height 32 ft. Width 42 ft. Ogee type.

(18) Youngstown, Ohio. 1916–18. Concrete. Length 638 ft. Height 55 ft. Width 60 ft. 5 in. Earth embankment at ends.

(19) Gilboa, New York. 1919–27. Stepped spillway section 1300 ft. long. 160 ft. high.

(20) Wilson, at Muscle Shoals, Alabama. 1918–26. Length comprises 200 ft. north abutment, 2668 ft. spillway section and 230 ft. sluiceway section. 1250 ft. power house section. Dam proper has height 98 ft. above river bed and 140 ft. from lowest point of foundation to operating bridge. 101 ft. thick at base. Inspection gallery 6 ft. wide and 9 ft. high. Dam divided into sections 46 ft. long by expansion joints.

Curved Masonry Overflow Dams (Fig. 7)

(1) Elche, Spain. 16th century. Rubble, cut-stone facing. Length 230 ft. Radius 205.38 ft. 1836 breach made during flood.

(2) Verdun, France. 1866–70. Rubble, cut-stone facing downstream. Concrete foundation. Riprap in front of wall. Length 131.23 ft. Radius 108.83 ft. Width of top 14.17 ft. Designed for depth 16.4 ft. water on crest.

(3) La Grange or Turlock, California. 1891–94. Rubble. Length 320 ft. on crest and 80 ft. at base. 39 500 cu. yd. Radius 300 ft. Has had about 15 ft. depth of water on crest.

(4) Cornell University dam, New York. 1897. Concrete of 4 parts argillaceous shale. 2 parts of sand, 1 part of cement. Length 153 ft. Radius 166.5 ft. Maximum height 30 ft.

(5) Walnut Canyon, Arizona. Rubble faced with ashlar. Length about 268 ft. Radius 400 ft. 6986 cu. yd.

(6) Seligman, Arizona. 1897–98. Concrete foundation. Rough rubble above with dressed ashlar facings. Top width 1.75 ft. Total length on crest 643 ft. The center section, 340 ft. long, is built as an overflow dam. Length at base 145 ft. Radius 800 ft.

(7) Espanola, Ontario. Concrete with plums. Built for 10-ft. overflow. Radius in plan 187 ft.

(8) Tallulah, Georgia. 1914. Crest length 444 ft. Radius 900 ft. Automatic flashboards of the Stauwerke type installed between piers on the 280-ft. spillway. Height 135 ft.

(9) Yadkin Narrows, North Carolina. 1917–19. Cyclopean concrete. 525 000 cu. yd. with 25% plums. Length 1400 ft. Radius 1678 ft. Height 200 ft. Designed for 10-ft. overflow. Taintor gates on crest. Drainage and inspection galleries.

(10) Martin Dam, Tallapoosa River, Alabama. 1923–26. Length straight abutment 300 ft. Headworks 224 ft. Curved spillway 720 ft. on arc of 530-ft. radius. Dam proper has height 138 ft. above river bed and 168 ft. from lowest foundation to operat-

ing bridge. Thickness at base including bucket, 160 ft. Dam designed for 21-ft. over crest.

(11) Baker River, Seattle, Washington. 1924–26. Upstream radius 250 ft. Spillway crest 493 ft. long provided with gates 12 ft. high and operating bridge 22 ft. above gate-sill. Gates at crest 10 ft. wide and 12 ft. high. Dam 24.5 ft. thick 13 ft. below

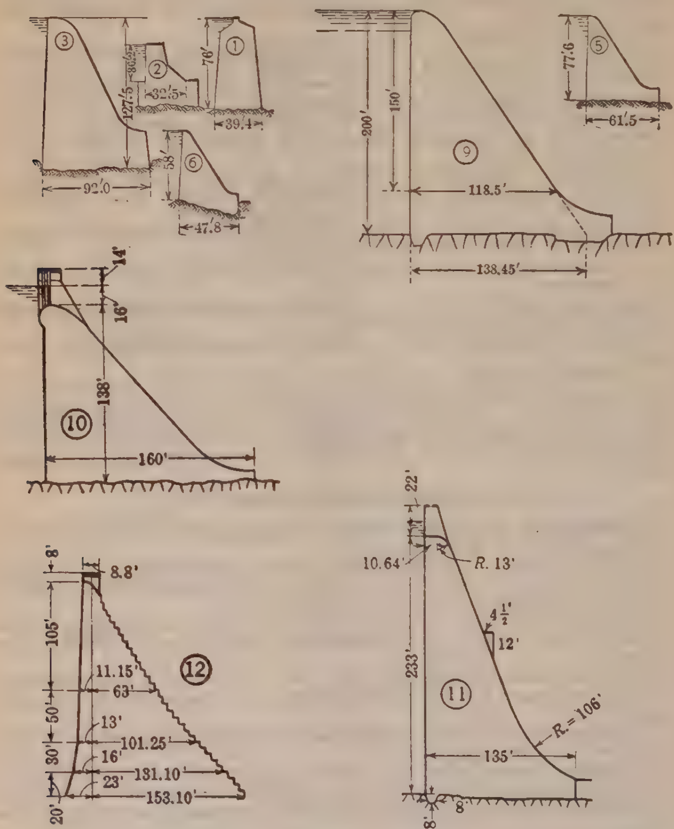


Fig. 7. Arched Overflow Dams

gate-sill. Below this point downstream face has batter of 3 on 8. Maximum height 263 ft., bedrock to bridge deck.

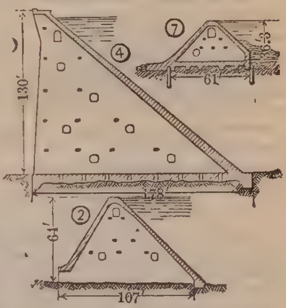
(12) St. Francis Dam, California. Maximum height 205 ft. 148 ft. thick at 193 ft. below top. Radius of upstream face 505 ft. Failed March 12, 1928, when the reservoir was first filled to within a few feet of the crest, suddenly releasing about two billion cu. ft. of water, causing a loss of between four and five hundred lives. Cause of failure: poor foundation.

2. Reinforced-Concrete Dams

Reinforced-concrete dams are usually of cellular construction, in some cases containing power houses. If properly designed they are admirably adapted to many situations. The general practice is to build a series of piers or buttresses 12 to 18 ft. apart and cover with a flat deck of concrete reinforced between the different bays as a beam. In the multiple-arch type, the buttresses are spaced 30 to 40 ft. apart and support a deck of concrete arches springing between the buttresses.

Unit stresses in such dams should be much lower than would be safe for ordinary building construction for an incipient failure in the latter case usually means only an unsightly crack, whereas for a dam it might mean a catastrophe. The following data refer to Fig. 8.

(1) Schuylerville, New York. 1904. Rollway 250 ft. long. Designed for 5-ft. head on crest. Founded on Hudson River shale. 5 × 3 cutoff wall. Passageway through dam from shore to shore. Average height 25 ft. with base width 52 ft.



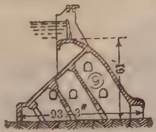


Fig. 8. Reinforced-Concrete Dams

(2) Ellsworth, Maine. 1907–08. Buttresses 15-ft. centers, 1 ft. thick at top, 2 ft. at bottom and braced by 12-in. × 18-in. reinforced concrete beams. Deck 1 ft. 2 in. thick at top, 3 ft. 1 in. at base. Total length 500 ft. Concrete, 8000 cu. yd. Flash-boards provided. Designed for flood 6.5 ft. on crest. Vents to prevent vacuum on apron.

(3) Pittsfield, Massachusetts. 1907–08. 42 ft. high. 465 ft. long. 3950 cu. yd. concrete. Founded on gravel underlain at depth of 12 ft. by dense yellow clay. Cutoffs extend 3 ft. into clay. Work carried on continuously when temperature was 12° below zero by heating materials and using salamanders. Concrete laid in winter and in moderate weather said to be uniform in quality.

(4) La Prele, Wyoming. 1908. Buttresses 18-ft. centers, 1 ft. thick at top, 4 ft. 2 in. at bottom, braced by 18 in. × 24-in. reinforced-concrete beams. Deck 1 ft. thick at top, 4 ft. 6 in. at base. About 15 000 cu. yd. concrete. Cutoffs 15 ft. to 18 ft. deep. Concrete in buttresses 1 : 3 : 6, deck 1 : 2 : 4.

(5) Dansville, New York. 368 ft. long including earth embankment. Gravel concrete used. Footing course on water bearing gravel and boulders. Bed of seamy blue stone under gravel. Cutoff around footing course. At height of 10-ft. base width 14 ft. Failed by undermining when water stood about 14 in. against flashboards. Deck slab snapped where reinforcing bars came together. Bars were not bent after failure.

(6) D. L. & W. R. R., Scranton, Pennsylvania. 262 ft. long. 16.5 ft. high. Base width 11.5 ft. Uniform slab thickness 10 in. Buttresses 7-1/2 ft. apart in center, increasing to 10-1/2 ft. at end. Soil under dam partly sand and partly yellow clay. 1/2-in. cup bars 12 in. c. to c. 1-1/2 in. from face, to prevent temperature cracks. Clay bank 8 ft. high at center to prevent percolation under dam.

(7) Uncas Power Co., Connecticut. Spillway 80 ft. long. Foundation very compact gravel. Buttresses 10 ft. centers, 1 ft. 6 in. thick at bottom, 1 ft. at top, stiffened by 12-in. × 15-in. beams. Upstream deck 1 ft. 6 in. to 1 ft. 8 in. thick, downstream 1 ft. 8 in.

(8) Bear Valley, California. 1910–11. Multiple-arch of Eastwood type. Ten arches between buttresses. 33 ft. 6 in. centers. Radius of extrados 16-ft. axis of arches vertical for 14. ft height at top, below which they are on batter of 4 on 3. Buttresses 1 ft. 6 in. thick at top increasing by .36 in. per ft. to bottom. Arch rings 12 in. thick at top increasing by .17 in. per ft. in radial thickness. Downstream edge of buttress 2 : 1

slope. Buttresses stiffened by reinforced-concrete struts. Maximum height 92 ft. Length 363 ft. 4684 cu. yd. concrete.

(9) Austin, Texas. 1911–15. 560 ft. long. Replaces portion of straight masonry overflow section which failed in 1900. Buttress 20-ft. centers. Height 61 ft. to crest, which is provided with automatic gates 14 ft. high.

(10) Stony River, West Virginia. 1912–13. Buttresses 15-ft. centers. 12 in. thick at top and 18 in. at bottom. Braced with reinforced-concrete struts and a curtain wall. Six months after erection, foundation for length of 7 spans was washed out and buttresses and deck dropped into the hole. Reconstructed 1914–15 by deepening cutoff wall, increasing the safety against sliding and adding 3-ft. flashboards. Maximum height 51.17 ft.

(11) Lake Pleasant, Arizona. 1926. Multiple-arch. 1850 ft. long. Maximum height 176 ft. above river bed and 256 ft. above deepest foundation. Has double-wall type of buttress. Buttresses 60-ft. centers. Clear span of arches 44 ft. Minimum thickness of arches and buttresses at crest 18 in. At 200-ft. depth arches 7 ft. thick and buttress walls 5.4 ft. thick. Concrete of arches 1 : 2 : 4. Buttresses 1 : 2-1/2 : 5, all heavily reinforced. Contains 75 000 cu. yd. concrete and 1200 tons steel.

(12) Anyox, British Columbia. 1923–24. Length including spillway 680 ft. Maximum height 156 ft. Buttresses 24-ft. centers with thickness varying from 1.2 ft. at crest to 10.08 ft. at 150 ft. below crest. Upstream slope variable. Thickness of arch concrete 12 in at crest and 3.17 ft. near base. Radius of extrados constant 15.66 ft. Central angle 100 deg. Arch concrete 1 : 2 : 4 reinforced with 3/4-in. square bars 24-in. centers at extrados and intrados with 1/2-in. square tie bars 6 ft. apart. Buttress concrete 1 : 2-1/2 : 5 not reinforced.

3. Steel Dams

Steel dams are not very numerous. In many cases they may be built much more cheaply than other types, but to keep them safe requires frequent and careful inspection, and the question of their maintenance and length of life should always be taken into account when considering their relative economy. The following data refer to Fig. 9.

Fig. 9. Three Types of Steel Dams

(1) Ash Fork, Arizona. 1898. Steel portion 184 ft. long and 46 ft. maximum height. 24 triangular bents, vertical downstream side and 1 : 1 upstream, 8 ft. c. to c., braced laterally in pairs. Batter posts 20-in. I-beams, 65 lb. per ft. and reinforced on under side with plate 1/2 in. thick and 18 in. wide. Series of 3/8-in. curved plates on upstream face concave upstream with radius 7-1/2 ft. No leakage. Total weight of steel 478 704 lb. Framed and erected at $55.78 per 2000 lb.

(2) Redbridge, Michigan. 1900–01. Steel portion 464 ft. long. Maximum height of dam 74 ft. Concrete base throughout. "A" frame bents 8 ft. apart, 3/8-in. face plates concave upstream radius 7 ft. 6 in., riveted to I-beams 15 to 24 in. deep. Batter upstream face 2 : 3. To cut off percolation under dam, line of drill holes 2 in. in diameter and 10 ft. long, 7 in. apart drilled and filled with grout under heavy air pressure. Rock floor covered with concrete and then by bank of puddle clay.

(3) Hauser Lake, Montana. 1907 630 ft. long. Maximum height 81 ft. Inclination upstream face 3 : 2. Portion of dam founded on solid rock. About 300 ft. founded on steel sheet piles in gravel. Bents 10 ft. apart connected in groups of four. Actual head to top of flashboards 69 ft. Working head about 66 ft. About 1700 tons of steel.

Blanket of volcanic ash above dam to prevent seepage. Downstream face had flat plates on slope 7 : 11, connecting with plank apron carried on stone-filled cribs. Dam undermined where founded on gravel. Concrete placed over top of steel piling, said to have been placed in depth of 10 ft. or more water, so quality may have been very poor. Part of dam on rock remained standing. Replaced by concrete dam in 1911.

4. Timber Dams

The Rafter and Strut-Framed Dam (Fig. 10) is a structure which may be proportioned with reference to its stresses, but requires careful design, sawed timber and skilled workmen and is proportionally more expensive.

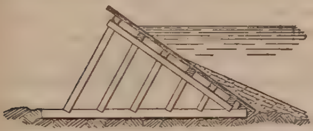


Fig. 10. Framed Timber Dam

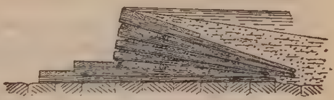


Fig. 12. Beaver Type Dam

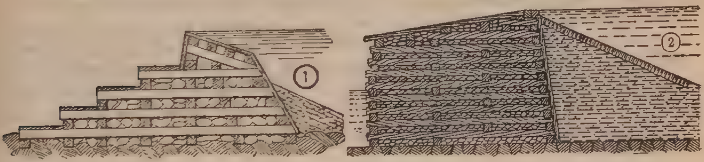


Fig. 11. Crib Dams

Crib Dams (Fig. 11) have been extensively used where timber was cheap and masonry dear. They must be designed largely by judgment and by following precedent. They are not subject to a very rigid analysis of stresses. The whole crib filled with loose rock may be considered as a unit adapted to resisting ordinary stresses of compression, tension and shear. The compressive stresses are resisted largely by the bearing of the timbers, one on the other; the tensile stresses are resisted by the bolting and sometimes dovetailing of the timbers and the shear largely by the friction in the rock fill. Considered from this angle the crib dam can be designed with some reference to stresses. Another way of looking at it is to give about the width required for a rock fill dam and consider the crib acting somewhat as a binder and mainly as an attachment for a watertight timber sheeting to cut off leakage on the upstream side and to take wear and impact of the overflow on the top and downstream sides. The width of the crib-work is seldom less than one and one-third times the height and may be two or three times where long aprons are provided for overflow as in (1) Fig. 11. The highest dam of this type which has not failed was the Butte dam, about 70 ft. high. No. (2) Fig. 11 is a cross-section of a crib dam in the Schuylkill River subject to floods 11 ft. high over crest. If crib dams are subject to heavy floods provision should be made for admitting air to the downstream side underneath the sheet of overflowing water.

The Beaver Type of timber dam is adapted to comparatively small heights. It is built as indicated in Fig. 12. The longitudinals are all laid with butts downstream. The crevices are filled with gravel, and the whole is covered with brush tightly packed and a bank of the most impervious earth obtainable.

A plank flooring is carried from the crest well under the earth fill. Most failures of timber dams have been due to leakage underneath or to undermining by the overfall.

5. Earth Dams

Foundations. One of the reasons for building a dam of earth is usually that a rock foundation cannot be had, though several earth dams have been built on the best of bedrock. The foundation, whether earth or rock, should be thoroughly examined to learn if it is impervious. If not impervious, cutoffs should be constructed, if possible down to an impervious stratum. Imperviousness is a relative condition. No earthy material is absolutely impervious, and if the width of the dam is so extended that the loss of head of water flowing through the shortest course to an exit is so great that the velocity is so slight that water will not move the earth or reduce its cohesion at the exit, then the cutoffs under the dam may be unnecessary. In any case the site is usually stripped and shallow trenches are dug across it. If the foundation is rock the creeping of water along its surface is guarded against usually by building a masonry wall entirely across the valley. This may be extended upward to the top of the dam as a core-wall, as in the Titicus dam, or, if the embankment is impervious, it is sometimes only 3 or 4 ft. high, as in the Borden Brook dam, Mass. Where the foundation is open sand or gravel, deep cutoff trenches have usually been excavated and filled with concrete or puddle clay. Sheet piling is sometimes used. The character of the ground and other local conditions must determine the kind of cutoff wall best to use. Springs which may be found within the area of the foundation may sometimes be dug out to their source outside the area of the dam, or intercepted outside that area and diverted.

Core-Walls. The dam itself usually consists of three parts: an interior impervious part with an embankment on each side to support it. The impervious part is a masonry, puddle or steel core-wall or a heart of selected impervious material. Occasionally the builders are fortunate enough to have at hand a natural mixture of gravel and clay of which the whole dam may be built, thus combining the three parts in one, but usually the impervious material available is either too expensive or is unstable at a practicable slope. What is the best type of core-wall is a mooted quested. Rubble, concrete, puddle clay, sheet steel and wood have been used. It is argued that masonry, whether rubble or concrete, must have a solid rock foundation, and that even then it is in danger of being cracked by settlement of the embankment or by temperature changes. On the other hand, it is argued that slight cracks will not be made larger by water but will be silted tight; that, if embankments are well built and the masonry core wall is 4 to 6 ft. thick at top with sides battered at about 1 to 10, it will not crack from settlement of the embankment; and also that, when outlets pass through or under the dam, they are much safer when masonry core-walls are used.

In regard to clay puddle it is argued that its integrity depends entirely upon great care and much intelligence and skill in mixing and placing, and that if once punctured its entire integrity is destroyed. On the other hand, it is argued that settlement does not rupture it, and that it makes a better union with the rest of the embankment.

At the Boston Water Works the endeavor was made to overcome all objectionable features and preserve the merits of both by building a masonry core-wall and placing a selected clayey material next the upstream side of the masonry.

Riveted steel plate cores coated with asphalt have been used. Timber sheeting should not be used, as it is likely to decay. Whether the core-wall should be placed in the middle or near the upstream side of the embankment will depend upon the material used and local conditions. If the core-wall is to be of masonry it should be vertical, and this usually requires it to be under the crest of the dam. If the core-wall is of clay it is usually placed vertically under the crest, but sometimes on the upstream face, in which case it should be protected by additional filling. The slope must be very flat or else there must be considerable filling outside it to prevent slipping; also provision must be made against burrowing animals, if the clay is near the surface. Small stones or furnace slag mixed with the puddle are frequently used for that purpose.

The Placing of Embankment when not done by the hydraulic process is usually done by carts or cars. It is usually specified that the earth shall be spread in from 4- to 8-in. layers, rolled with a heavy roller, and that, before placing the next layer, the last one shall be sprinkled and harrowed to insure bonding. Variations in details of construction on different dams have been

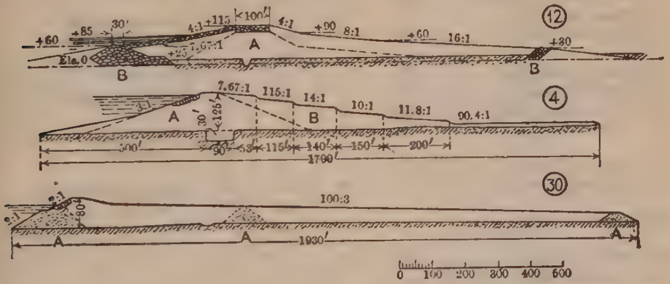


Fig. 13 Earth Dams

the use of a drove of horses and sometimes goats to trample the earth, also heavy four-horse teams for hauling, runways being avoided. Rollers having grooves or projections on their faces are used on many dams. The embankment is usually kept 4 or 5 ft. higher on the outer edge than in the middle.

Slopes must be determined by the engineer for each individual case. Precedents may be studied in the following table. If it were simply a question of the stability of the face the slope for the same material should be flatter on the upstream side than on the downstream side, for the angle of repose, especially of very fine material, is less in water than when not saturated but slightly damp, which is usually its condition on the downstream side. If the dam is without a core-wall or other impervious hearting and subject to seepage either under or through the embankment, it may be necessary to make the downstream slope very flat. The upstream face is pitched or otherwise protected against wash of waves. The downstream slope is sodded to protect it from wash of rains. Berms with paved drains are used frequently in addition to sodding.

Top Width and Height above Water. The formula $W = \frac{h}{5} + 5$ in which h = height of dam in feet, has been suggested as giving safe widths in feet, for tops of earth dams. The top should be so high and wide that no burrowing animals can reach the heart of the dam at water level. It should also be high enough above the highest water level to be beyond the reach of all waves. Stephenson's formula for height of waves is commonly used.

$$X = 1.5\sqrt{F} + (2.5 - \sqrt[4]{F})$$

in which X is the height in feet, F the fetch in miles or sweep of the wind in

Data on Compacted Earth Dams

Numbers in parentheses refer to Figs. 13 and 14 and to the following notes.

Number and name	Maximum height, ft.	Top width, ft.	Slope up-stream	Slope down-stream	Material	Core-wall
(1) Goose Creek, Idaho....	145	16	3:1	2:1	Sand, clay and gravel	C
(2) Idaho Irrigation Co....	135	40	3:1	5:2	Earth and rock	O
(3) Patillas, Porto Rico....	135	20	3:1	2:1	Gravel, boulders and clay	O
(4) San Leandro, Calif. ...	125	28	2:1	5:2		O
(5) Lahontan, Nev........	124	20	3:1	2:1	Gravel and silt	O
(6) Tabeaud, Calif........	123	20	5:2 3:1	5:2	Red gravelly clay	
(7) New Croton, N. Y.....	120	30	2:1	1:1		R
(8) Costilla, Colo.........	120	20	3:1	2.5:1	Clay and gravel	O
(9) Druid Lake, Md........	119	60	4:1	2:1		P
(10) Bell Fourche, S. Dak...	115	20	2:1 3:1	7:4 2:1	Heavy clay and gumbo	O
(11) Dodder, Ireland.......	115	22	7:2	3:1		
(12) Gatun, Panama........	115	80	7.67:1 10:1	8:1 16:1	Clayey sand and rock fill	
(13) Standley Lake, Colo..	113	20	2:1 3:1	2:1	Blue clay	P
(14) Titicus, N. Y..........	110	30	2.4:1	5:2		R
(15) Mudduk Tank, India..	108	......	3:1	5:2		
(16) Ashokan Dikes, N. Y..	....	34	2:1 5:2 11:4	2:1 11:4 3:1	Clay, sand, gravel and small stones	C
(17) Somerset, Vt..........	106	......	3:1	2.5:1	Clay, sand, gravel and small stones	O
(18) Temescal, Calif........	105	18	3:1	5:2		
	115	......	3:1	5:1		
(19) Carite, Porto Rico.....	105	20	2.75:1	2:1	Clay and rock	O
(20) Cummum Tank, India.	102	......	3:1	1:1		
(21) Yarrow, England......	100	24	3:1	2:1		P
(22) Morris, Conn.........	100	20	3:1	2:1		C
(23) Pilarcitos, Calif........	95	24	2:1	2:1		P
(24) Dale Dyke, England ..	95	12	5:2	5:2		P
(25) San Andreas, Calif.....	93	25	7:2	3:1		P
(26) South Haiwee, Calif....	91	20	2.5:1	2.5:1	Tufa, shale and clay	P
(27) Forest Park, Md.......	87	15	......	5:2	Rock and earth	O
(28) Dry River, N. Y.......	85	20	2.5:1	2:1	Earth	C
(29) Sherburne Lakes, Mont.	83	22	3:1	......	Clay, sand and gravel	
(30) Wachusett, N. Dike, Mass.............	82	......	2:1	100:3	Soil, sand, gravel	O
(31) Cold Springs, Ore.....	98-1/2	20	3:1	2:1	Gravel and earth	O
(32) Borden Brook, Mass...	....	24	3:2 2:1	3:2 2:1		P
(33) Talla, Scotland........	78	20	4:1	3:1	Clayey and open materials	P
(34) Throttle, N. M........	77	......	2.5:1	1.5:1	Earth and course rock	P

P = Puddle. C = Concrete. R = Rubble. O = No core-wall.

Data on Compacted Earth Dams—Continued

Numbers in parentheses refer to Figs. 13 and 14 and to the following notes.

Number and name	Maximum height, ft.	Top width, ft.	Slope up-stream	Slope down-stream	Material	Core-wall
(35) Seros, Spain..........	75	13	3:1	2:1	Earth, clay and gravel	O
(36) Las Vegas, N. M.......	75	20	3:1	2:1	Heavy clay, sand gravel	C
(37) Mammoth, Utah......	70	......	......	......	Clay and fine gravel	C
(38) Johnstown, Pa........	70	10	2:1	3:2	Selected earth with rock	O
(39) Keechelus Lake, Wash.	70	20	3:1	2:1	Gravel and earth	O
(40) Phelps Brook, Conn....	68	15	2:1	2.5:1	Gravelly material	C
(41) Youngstown, Ohio.....	66-1/2	20	2:1	2:1	Clay and gravel	O
(42) Kachess, Wash........	65	20	3:1	2:1	Earth and gravel	C
(43) Glenwild, N. Y........	63	13	2:1	5:2		R
(44) Hatchtown, Utah......	63	20	2:1	2.5:1	Loamy sand	P
(45) Bog Brook, N. Y......	60	25	2:1	5:2		R
	25	12	2:1	2:1		
(46) Ashti, India..........	58	6	3:2 3:1	3:2 2:1	Black and brown soil and "Murrum"	
(47) Hebron, N. M.........	56-1/2	12	3:1	1.5:1		O
(48) Horse Creek, Colo.....	55-1/2	16	1.5:1	1.5:1 2.5:1	Sandy loam and clay	O

P = Puddle. C = Concrete. R = Rubble. O = No core-wall.

the longest straight line which may be drawn from the dam on the water surface of the reservoir.

(1) Goose Creek, Idaho. 1910–13. Upstream face covered with 3 ft. of loose rock, and loose rock dumped at toes. Core-wall 3 ft. thick to 10 ft. above natural surface and 1 ft. thick from this point to top. Length 1050 ft.

(3) Patillas, Porto Rico. 1914. Length 1020 ft. Contains 970 000 cu. yd. First 50 ft. in height made by dumping from two trestles near either toe. Successive lifts then made by shifting track. Outer slopes of gravel and boulders. Middle of dam about 25% clay washed into position and allowed to settle in pool maintained by the side banks. Total settlement 0.56 ft. in three years.

(4) San Leandro, Calif. 1874–76. Portion "A" in cut built in 1874 of earth in layers 1 ft. thick. No rollers used, but material dumped and sprinkled and band of horses used to compact. In 1898 portion "B" was placed by the hydraulic process. Shaken by San Francisco earthquake 1906. No damage. Puddle trench 30 ft. below ground.

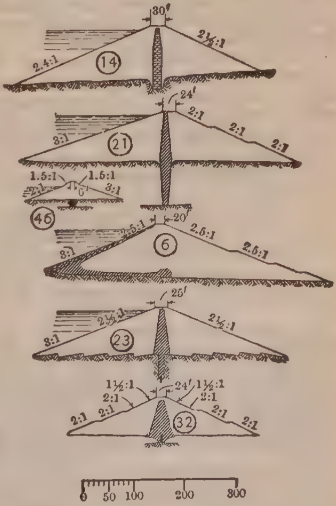


Fig. 14. Earth Dams

(5) Lahontan, Nev. 1913. Length 1359 ft. Volume 710 000 cu. yd. Upstream portion of gravel and silt, mechanically mixed and rolled in 4-in. layers and faced with riprap and gravel. Downstream portion of pit-run gravel. Foundations a red sandstone. Puddled cutoff wall. Grouted rock foundation.

(6) Tabeaud, Calif. 1900–02. Built in 6-in. layers first 60 ft. Not over 8-in. layers above. Top kept basin-shaped. Clay puddle on bottom and part of upstream face. 1 berm upstream, 3 downstream. Foundation hardpan and rock.

(7) New Croton, N. Y. 1901. Dam partly constructed, then replaced by masonry structure. 2 berms downstream.

(8) Costilla, Colorado. 1917. Length 600 ft. Upstream slope riprapped. Cutoff wall 30 ft. into clay and gravel foundation and 5 ft. above natural ground surface. Built by dumping earth in ridges about 5 ft. high and partly filling trench between with water, then filling the trenches by dumping from wagons, then building new ridge over former trench and repeating process.

(9) Druid Lake, Maryland. 1864–70. Upstream face covered with puddle. Stone wall in bottom of clay core. Embankment each side of puddle core well rolled. Part of the material in dam placed in basins filled with water. Surface soil stripped, and where foundation is sand, core carried to impervious stratum.

(10) Belle Fourche, South Dakota. 1910. 6-in. layers. Some sheet piling near upstream toe. $879 164 contract price.

(12) Gatun, Panama. 1913. Portion "A" in cut is hydraulic fill, "B" is rock fill and the remainder is dry fill. There is a layer of 3 ft. of rock paving 170 ft. wide on the upstream slope to resist wave action.

(13) Standley Lake, Colorado. 1908–12. Length 6630 ft. Material lumpy, blue clay which slaked very readily and became a slimy ooze on being saturated. Upstream face protected by loosely dumped riprap. In 1912 a small slide of the downstream face occurred and in 1914 there was a much larger slide of the same face. In 1916, following a rapid draining of the reservoir for irrigation purposes, about 88 000 cu. yd. slid from the extreme face.

(17) Somerset, Vermont. 1911–13. Length 2100 ft. Contains 1 000 000 cu. yd. Material was coarse gravel and sand with some clay. Excavated by steam shovel hauled to site and dumped from trestles on the upstream and downstream slopes. From there the material was sluiced down about a 2 : 1 slope toward the middle of the dam, the object being to leave coarse material on the faces and gradually grade to a fine impervious heart.

(18) Temescal, California. 1866–68. Originally built in layers with carts and scrapers, with top width 18 ft. at maximum height of 105 ft. and slopes 3 : 1 upstream and 5 : 2 downstream. In 1869 downstream slope flattened by sluicing. In 1886 height increased to 115 ft. by sluicing. As finished downstream slope 5 : 1.

(19) Carite, Porto Rico. 1914. Length 520 ft. Upstream face paved. Downstream slope, lower half paved. 200 000 cu. yd. Fill dumped from trestle and spread by scrapers and wheelbarrows. Very little sprinkling done. Settled 0.96 ft. in three years.

(21) Yarrow, England. Maximum depth excavation 97 ft. Puddle core-wall on concrete foundation in rock. 2 berms on downstream slope.

(22) Morris, Connecticut. 1910. Upstream slope 3 : 1 for a height of 30 ft. from toe, 2.5 : 1 above, faced with 1 ft. 6 in. of paving stones. Four berms 8 ft. wide on downstream slope, and slope sodded. Core-wall carried to rock 12 ft. wide at base and 2 ft. 5 in. at top. Length 1100 ft.

(23) Pilarcitos, California. 1864–66. Puddle core-wall 24 ft. thick, keyed to rock with concrete wall 3 ft. by 6 ft. Length of dam 640 ft. 1-3/4 miles from fault line of San Francisco earthquake of 1906. Dam uninjured by earthquake.

(24) Dale Dyke, England. Failed 1864. 95 ft. high, 1254 ft. long, 12 ft. wide on top. Upstream and downstream slopes 2.5:1. Puddle core. Failed; opinions as to cause varied. Said to be because outlet pipes laid naked or because dam above core-wall was a sort of rock fill, bringing full static pressure against core-wall.

(25) San Andreas, California. 1868–70. Dam raised in 1875. Puddle core-wall keyed to rock with concrete wall 3 ft. by 5 ft. Length 800 ft. Fault line of San Francisco earthquake of 1906 passes across east end. Crack 2 to 3 in. wide along axis of dam but it did not fail.

(26) South Haiwee, California. 1912. Length 1523 ft. 559 750 cu. yd. Upstream slope paved with 3 in. of concrete. Cutoff trench carried down to rock, in some cases to a depth of 120 ft.

(27) Forest Park, Maryland. Clay puddle on upstream face and carried to bed-rock in a trench near upstream toe.

(28) Dry River, Watervliet, N. Y. 1913–14. 1 berm 8 ft. wide, 40 ft. below top on both faces. Material rolled in 6-in. layers. Upstream face partly paved.

(29) Sherburne Lakes, Montana. 1918. Length 900 ft. 250 000 cu. yd. Upstream slope has 18-in. paving of field stone on a 12-in. layer of gravel, and concrete parapet wall 3 ft. high at top. Downstream slope covered with 12 in. of gravel. Unique feature was permeable core-wall of screened gravel 5 ft. thick at crest, increasing with 3.5 in 100 batter, with system of pipe drains running from base of core to downstream toe. Two cutoff trenches on upstream side, 1 on downstream. Material hauled in dump wagons spread in 6-in. layers rolled with 20-ton roller.

(30) Wachusett, N. Dike, Massachusetts. 1900–05. This dam is about 10 000 ft. long. 5 500 000 cu. yd. There are three embankments of sand and gravel shown at "A" in the cut. The remainder was built largely of fine loamy soil stripped from the reservoir site. Material spread in 1-ft. layers and rolled by steam road roller and harrowed before adding new layer. Foundation pervious sand and gravel. For a portion of the length a trench was dug 60 ft. deep in this gravel, and triple-lap tongued-and-grooved sheet piling built up of 2-in. plank was jetted down from 20 to 60 ft. On April 11, 1907, a portion of the upstream face 675 ft. long, of a thickness of 35 ft. normal to the slope, slid down the bank, causing little damage but indicating that 1 vertical to 2 horizontal was too steep for such fine material under water.

(31) Cold Springs, Oregon. No core-wall. Cutoff trench.

(33) Talla, Scotland. 1897–04. The clayey or adhesive material in middle third placed in 9-in. layers. Outside this is stony or open material in 18-in. layers.

(34) Throttle, New Mexico. 1914. Length 1060 ft. Concrete core wall 600 ft. long with galvanized corrugated iron extending from cutoff wall to crest. Upstream slope paved. Center portion of dam well puddled. Downstream slope faced with selected rock, hand laid.

(35) Seros, Spain. 1913–14. Dam No. 3 is the highest of 7 earth dams in hydro-electric development near Barcelona. Length 1312 ft. 497 250 cu. yd. Upstream face riprapped. Puddled cutoff trench. 1 berm downstream slope.

(36) Las Vegas, New Mexico. 1917. Upstream slope riprapped 9 in. thick. 1 berm on downstream face. Length 1400 ft. Material hauled by dump cars and sluiced into place with water jet.

(37) Mammoth, Utah. 1908–14. Proposed height 125 ft. Concrete core-wall with buttresses. Earth placed by wagons and scrapers and rolled. In June, 1917, dam was at a height of 70 ft. with water stored to nearly the same height. Washout occurred at flume carrying water across the top. One-half billion cu. ft. water released cutting dam to foundation causing great damage to structures below.

(38) Johnstown, Pennsylvania. Lower face entirely stone, 4 ft. thick at top, 20 ft. at bottom, and backed by slate rock 3 ft. thick at top and 30 ft. at bottom. Heart of selected earth. Upstream face of dry rubble 15 in. thick. Failed: flood over-topped dam.

(39) Keechelus Lake, Washington. 1914. Length 6500 ft. 522 000 cu. yd. Upstream slope faced with riprap. Concrete cutoff wall.

(40) Phelps Brook, Connecticut. 1917. Length 1200 ft. Upstream slope 3 : 1 below water surface, 2 : 1 above, faced with 12 in. of riprap. 1 berm on downstream slope. Concrete core-wall to rock.

(41) Youngstown, Ohio. 1913–16. Length 2202 ft. Concrete cutoff. Upstream slope faced with 9 in. to 18 in. of concrete.

(43) Glenwild, New York. 1902. Core-wall is broken boulders grouted with 1 : 3 cement mortar. Dam curved upstream with radius of 708.6 ft. Contract cost $47 360.

(44) Hatchtown, Utah. 1908. Failed in May, 1914, probably due to leakage along the outer surface of the outlet culvert and leakage through foundations of black clay sand and gravel. Length 780 ft. 126 000 cu. yd.

(45) Bog Brook, New York. Masonry core-wall 10 ft. thick at base and 6 ft. at top. Upstream face has 12-in. paving on 6-in. broken stone. Berm on downstream side.

(47) Hebron, New Mexico. 1914. Length 3700 ft. Puddled cutoff trench. In May, 1914, a section 200 ft. long washed out.

(48) Horse Creek, Colorado. 1911–12. Length 5150. ft. Volume 714 000 cu. yd. Downstream slope 1.5 : 1 for distance of 16 ft. from crest, and 2.5 : 1 for balance. Upstream slope 1.5 : 1 face paved with 5 in. of concrete. Earth placed by dump wagons and spread with scrapers. No sprinkling and no packing of material done. In January, 1914, 200 ft. of the dam above the concrete culvert outlet washed out. Immediately after reservoir emptied, 3600 ft. concrete paving slid down slope.

Failures of Earth Dams. In a record of 30 failures of earth dams, which may be assumed to be representative, 11 were due to overflowing, 10 to leaks in outlet pipes, 3 to undermining, and 1 was attributed to burrowing animals. The causes of the other five were somewhat doubtful, but several causes were assigned, such as poor foundations, slopes too steep, lax construction methods and carelessness in preparing the site.

6. Hydraulic Fill Dams

Sluicing is a method of building earth dams by which, especially in mountainous regions, the material often may be excavated, conveyed to the site and placed much more cheaply than by any other means. The ideal material is a mixture of sand, clay and gravel with plenty of small rock. The develop-

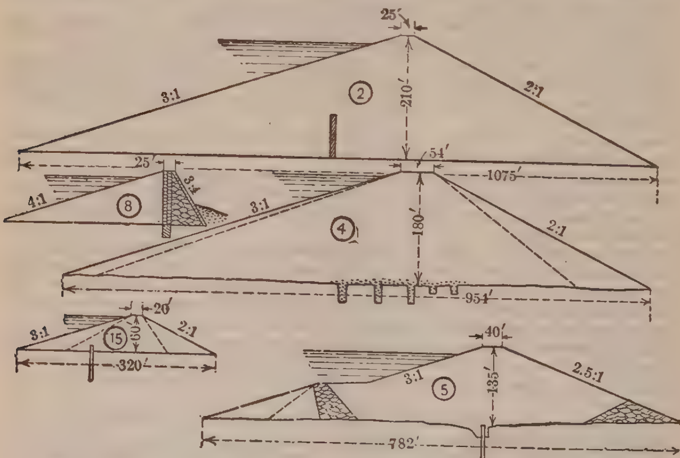


Fig. 15. Hydraulic Fill Dams

ment of the art has evolved a number of excellent properties of such dams, so many, in fact, that a number have been built by hauling dirt with cars to the site and sluicing it into place. The practice is for the sluices to discharge near the outside edges of the embankment. The coarse material drops there, and the fine flows with the water toward the middle of the dam, depositing, as it goes, the coarser first, the medium next, most of the clay going to the pond of still water in the center, where it is precipitated, forming a wide clayey core-wall. The coarse material remaining on the outer edge is most useful in maintaining the slopes due to its greater angle of repose. In practice, however, the danger is that the material will not be properly graded, either too fine, making the faces unstable, as in the case of the Calaveras Dam, or too coarse, making the core permeable. For that reason, continuous physical analysis should be made. Even if the material is properly graded the sluices may not deliver a uniform mixture, resulting in permeable layers across the dam. This danger is believed to be avoided by keeping men continuously spading and kneading the bottom of the pond in the middle of the dam. As the work progresses, a few men keep the edges built up just enough to keep the discharge from the sluices always flowing toward the center. The pond is sometimes allowed to drain by filtering through the coarser outside material, and sometimes by a vertical waste pipe built up in short lengths of from 6 to 15 in., changing its direction at the foot to a horizontal leading to the downstream face, or by pumping the surplus water out of the pond by a pump mounted on a raft and discharging over the top of the fill and never through it. There has been developed a modified hydraulic fill process in which the material is dumped on the outer edges of the dam and a stream of water is then played upon it which washes the finer parts into the central pond. This method was used on the Somerset, Davis Bridge, and Wanaque dams where the material was well graded from cobbles down to very fine.

Data of Hydraulic Fill Dams

Numbers in parentheses refer to following notes and to Fig. 15.

Number and Location	Date	L'gth, ft.	Maximum height, ft.	Top width, ft.	Slope upstream	Slope downstream	Total volume, cu. yd.	Water used, cu. ft. per sec.
(1) Calaveras, Calif.	1918	1260	240	25	3:1	2.5:1	....	......
(2) Terrace, Colo.	1905–9	605	210	25	3:1	2:1	500 000	26
(3) Little Bear Val., Calif.	......	......	200	20	5:2	2:1	....	......
(4) Necaxa, Mexico	1909	1220	180	54	3:1	2:1	2 000 000	15–30
(5) Magic Res., Idaho	......	......	135	40	3:1	5:2	....	......
(6) San Leandro, Calif.	1874–6	500	125	28	3:1	5:2	542 700	10–15
(7) Crane Valley, Calif.	......	720	100	20	2:1	3:2	....	15
(8) Waialua, Hawaii	1904–6	460	98	25	4:1	......	141 000	8
(9) Santa Maria, Colo.	1912	1300	95	20	3:1	2:1	310 000	......
(10) Quemahoning, Johnstown, Pa.	1913	950	95	20	4:1	4:1	600 000	......
					3:1	3:1		
(11) Lake Frances, Calif.	1899	992	50	16	3:1	2:3	80 265	......
	1901–2	1300	77				280 700	
(12) Snake Ravine, Calif.	......	294	64	12	Bet. 3.2	and 2.1	....	......
(13) Santo Amaro, Brazil	1907	5300	63	33	3:1	2:1	....	8
(14) Croton, Mich.	1906–7	200	60	......	5:2 to 6:1	2:1 and 4:1	104 000	......
(15) Conconully, Wash.	......	1030	60	20	3:1	2:1	351 000	......
(16) Silver Lake, Calif.	......	900	56	......	......	......	146 000	2.5
(17) Yorba, Calif.	1907	800	47	16	7:2	2:1	100 000	3–4
(18) Tyler, Texas	1894	575	32	......	3:1	2:1	24 000	1.4
(19) Dayton (5)	1920	......	75–125	......	......	......	....	......

(1) Calaveras, California. Material is clay sand and gravel with dry rock fill at toes. Material sluiced from borrow pits down open channel 5 to 7% grade into concrete sumps and raised to dam with 12-in. centrifugal pumps. Designed to contain

3 000 000 cu. yd. After 2 800 000 cu. yd. placed, upstream face for length of 700 ft. slid. Aggregate 20 to 50% clay.

(2) Terrace, Colorado. Material sluiced from a clayey bank to form heart of dam, and from a separate bank clean rock and gravel were sluiced to the two slopes in flume on 7% grade using 26 sec.-ft. of water. Concrete core-wall about 65 ft. high in bottom of canyon.

(3) Little Bear Valley, California. Disintegrated granite and clay deposited from spoil trains on toes of dam and finer materials were washed to the center. Concrete core-wall.

(4) Necaxa, Mexico. Broken limestone and yellow clay. Concrete core-wall, about 6.5 ft. above stripped surface. Sluicing flumes 20 in. in diameter, 4 ft. wide, rectangular section with V-shaped bottom, grades 5% and 8%. During construction a slide of the upstream face occurred which was attributed to the use of unsuitable material. Dam since completed and now in use.

(5) Magic Reservoir, Idaho. Cutoff walls in sides of canyon. Cutoff trench and sheet piling under dam. Constructed by combination of methods. Excavated by steam shovels; spoil trains delivered material close to site, thence it was sluiced to place.

(6) San Leandro, California. 160 000 cu. yd. sluiced. Sluicing flume grades 4–6%. See earth dams.

(7) Crane Valley, California. Sluicing flume 12 in. by 10 in. Flumes on 6% grade. Ground sluicing was used; that is, material was dumped into flume after loosening.

(8) Waialua, Hawaii. Rock fill portion: base width 80 ft., crest 11.5 ft.; downstream batter 3/4 to 1, upstream face vertical; volume 26 000 cu. yd. Wooden diaphragm embedded at bottom in concrete wall. Earth fill 141 000 cu. yd. Soil dumped into flowing stream of 8 sec.-ft. in ditch 1300 ft. long with bottom grade 4%. The soil was a decomposed lava of a cohesive and unctuous character, very free from grit. Earth fill 11 cents per cu. yd. Leaked during first year. 1905 material obtained by enlarging spillway sluiced against lower toe.

(9) Santa Maria, Colorado. Two cutoff trenches to hardpan sheeted and puddled. Dirt and boulders conveyed by rectangular sluices 3 × 3 ft.

(10) Quemahoning, Johnstown, Pennsylvania. Slopes 4 : 1 for lower half and 3 : 1 for the upper half. Concrete cutoff wall extends 10 to 27 ft. into the shale rock; also puddled cutoff. Rectangular flumes 24 in. wide and 18 in. deep, grade 6%. Material a mixture of clay and shale rock.

(11) Lake Frances, California. Old earth dam failed, about 20% being washed away. Repaired and enlarged by hydraulic process. 182 937 cu. yd. of new dam sluiced in. 1.76 sec.-ft. of water used at first; later 4.5 to 7 sec.-ft. Sluicing flume 22 in. in diameter, minimum grade 2.2%. Most of material sluiced was clay. Brush used to maintain slopes.

(12) Snake Ravine, California. Built of fine silt and clay; slid as a body 1000 ft. down ravine at rate 6 to 10 ft. per sec. Failure attributed to lack of coarse material on slopes and fact that fine material did not drain when building.

(13) Santo Amaro, Brazil. Clay and disintegrated granite. Slip of blanket over core-wall of section east of hydraulic fill occurred during construction. Excess of clay in blanket did not allow good drainage outward. Sluicing flume 2000 ft. long, grade 3%.

(14) Croton Michigan. Fine yellow sand. Sluicing flume 30 in. in diameter semicircular; grade 8–9%; length 800 ft. Cost 6.8 cents per cu. yd., including plant, materials, labor, power, etc.

(15) Conconully, Washington. Loose rock, gravel, sand and silt. Sluicing flume, sloping side 2-3/4 ft. apart at top. No. 10 steel curved bottom bent to 12-in. radius, clear depth being 2-1/4 ft. Lateral flumes trapezoidal 12 in. wide at bottom. Sheet piling across valley 70 ft. upstream from center line driven 33 ft. and projecting 3 ft. above valley bottom. Flume grade 4%.

(16) Silver Lake, California. Heavy sandy loam containing considerable clay which was carefully separated and placed in center. Drains to prevent slipping during construction. Concrete core-wall under dam and 3 to 6 ft. above original surface. Sluicing flume 8 in. in diameter. 2.5 sec.-ft. water used in later stages. Flume length 4000 ft.

(17) Yorba, California. About 80% by hydraulic ground sluicing process. Adobe clay soil, sand and gravel. No core-wall; puddle trench through top soil. Material supplied to parts too high for gravity flume by pumping through an 8-in. pipe a maxi-

mum distance of 800 ft. Cost 8 cents per cu. yd. for ground sluicing. Flume grade 4–7%.

(18) Tyler, Texas. Sluicing flume 13 in. in diameter. Material 65% sand and 35% clay. Cost 4-3/4 cents per yard, including everything.

(19) Miami Conservancy, Ohio. A group of five dams located at Taylorsville, Englewood, Huffman, Lockington and Germantown were built near Dayton in 1919–20 to control floods on Miami River. They range from 75 to 125 ft. high and in volume from 800 000 cu. yd. to 3 500 000 cu. yd., totalling 9 000 000 cu. yd. for the five dams. The material was a well graded mixture running from coarse gravel down to fine sand and clay. Most of this material was pumped to the dam by 14-in. centrifugal pumps which had their suctions in hog boxes that were supplied partly by trains hauling material excavated by dragline excavators from borrow pits and partly by material excavated by hydraulic monitors and sluiced into the hog boxes. By continuous mechanical analysis and control at the hog boxes it was possible to obtain a very uniform grading of material going to the dam.

7. Rock Fill Dams

Dams of Loose Rock were designed to meet the requirements of miners in western America, where the great cost of hauling cement made the use of that material prohibitive. Since most of these structures are located in deep rock canyons, it was a simple matter to quarry the material at a higher elevation than the dam and transport it by gravity to the site or even to throw the material directly to place by blasting. Dams of this type are made watertight by a variety of methods. In some cases the upstream face is covered with planking spiked to studding buried in the rock fill, or with steel plates fastened to I beams set in the face, or with rubble masonry walls or reinforced-concrete facing. Steel core-walls are also used. A later development is the combination earth and rock-fill dam in which the earth is either dumped or sluiced into place on the upstream face.

Most of the failures of rock-fill dams have been due to insufficient spillway, causing overtopping. Some have been caused by poor design in making the slopes steeper than the natural angle of repose of the loose fill and trusting to dry rubble facing walls to hold the mass in position.

The following numbers correspond to those on the cross-sections of Fig. 16:

(1) Escondido, California. 1895. Faced with redwood plank. Timber face backed with concrete. Contains 6000 cu. yd. dry rubble and 31 157 cu. yd. rock fill. Leakage when full 450 000 gal. per day. Cost $110 000. Stones laid by hand on inner face to form dry wall 15 ft. thick at bottom and 5 ft. at top. Bedrock trench at upper toe filled with rubble masonry, in which plank facing is embedded. 76 ft. high. 380 ft. long on top and 100 ft. on bottom. 140 ft. wide at base and 10 ft. on top. Slopes are 1 : 2 on water face; 1 : 1 on back for half the height and 5 : 4 from mid-height to base.

(2) Castlewood, Colorado. 1890. Lower slope covered in steps of 2 ft. with large blocks of stone in cement mortar. General slope 1 : 1. Upper face wall rough rubble masonry 4 ft. thick, on batter 1 : 10. Walls joined at top with coping. Crest width 8 ft. Dam founded on clay. Front face wall carried 6 ft. to 22 ft. into clay. Lower slope wall also founded on clay at depth 10 ft. below surface. Length 600 ft. Height above reservoir floor 70 ft.; above foundation of face wall 92 ft. Rock pavement 25 ft. wide, 200 ft. long, 3 to 6 ft. deep, heavily grouted at top with cement mortar to prevent scouring. Dam leaked. Earth embankment added to upstream face, 8 ft. wide at crest with 3 : 1 slope faced with 12 in. of riprap.

(3) Lake Avalon, New Mexico. Length 1050 ft. Earth slope covered with revetment of loose stone 2 to 3 ft. thick for wave protection. Failed 1893, water passing over top, washing out 300-ft. length to bedrock. Immediately repaired and built 5 ft. higher. Second failure 1904. Stated that water did not pass over top but forced way through the dam. Failure attributed to burrowing animals, or faulty construction where dam connected with bank. Reconstructed with top width 43 ft. Substantial

addition made to earth fill and core-wall built of concrete heavily reinforced. Part of core-wall on concrete base and part on sheet piling driven to rock.

(4) Walnut Grove, Arizona. 1887–88. Destroyed by flood 1890 with loss of 129 lives. Length 400 ft. Slopes much steeper than natural angle of repose of loose rock. Faces laid up as dry walls. With 70 ft. of water above bedrock, dam leaked 3.75 cu. ft. per sec.

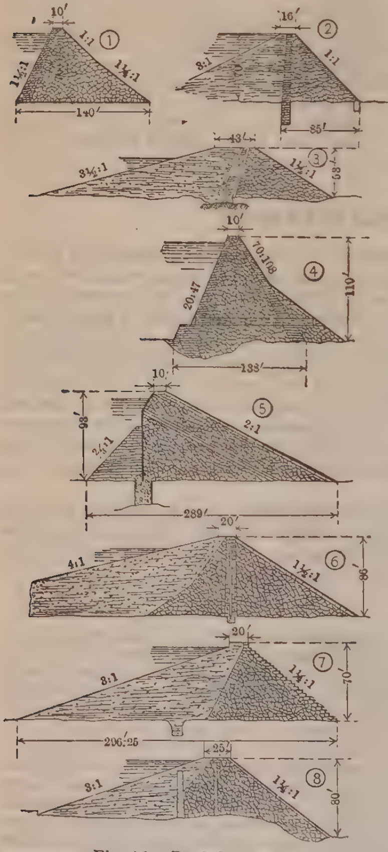
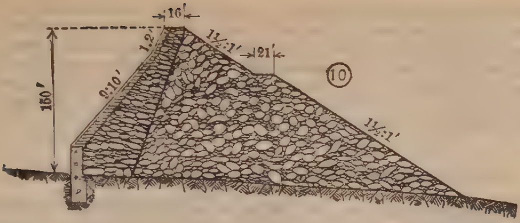


Fig. 16. Rock Fill Dams

(5) East Canyon Creek, Utah. 1899. Originally built 68 ft. high. Length 100 ft. Concrete wall 15 ft. thick, carried down through gravel to bedrock and steel plates for core anchored in center. Rock thrown on wall by blasting sides of canyon. Open cut made in pile of rocks thrown down and steel core-wall built from anchor plates. Asphalt concrete 4 in. each side of plate. Settlement of wall caused asphalt to draw away from steel an extreme distance of 5 ft. at top. Dam cost $60 200 and required 23 000 cu. yd. rock, 810 cu. yd. cement concrete, 183 cu. yd. asphalt concrete, 69 800 lb. of steel and 50 500 ft. b.m. of lumber. Enlarged 1900–01 by addition to downstream side. Maximum height 93 ft. Additional work required 16 000 cu. yd. rock, 410 cu. yd. of masonry and 370 cu. yd. concrete, 62 000 lb. steel and 20 000 ft. b.m. lumber. Length 173 ft. Crest width 10 ft. Steel core-wall continued to top, protected by concrete.

(6) Milner, Idaho. 1903–05. Three dams form one main structure. Rock fill with core of wood. Earth embankment sluiced into voids of rock above core. Main channel dam: Length 340 ft. Maximum height 86 ft. Crest width rock fill 10 ft. Downstream slope rock fill 3 : 2; upstream 3 : 4. Crest width earth fill 10 ft.; slope 4 : 1. Rock fill 39 650 cu. yd. Earth fill 58 000 cu. yd. Middle dam: 335 ft. long. 81 ft. maximum height. 42 800 cu. yd. rock, 62 850 cu. yd. earth. South dam: 560 ft. long. Maximum height 66 ft. 34 700 cu. yd. rock; 48 000 cu. yd. earth. Total length of 3 dams, spillway and regulating gates, considered as one structure, 2100 ft.

(7) Zuni, New Mexico. Combination rock and hydraulic fill. Rock fill built as dry wall. Rock fill 720 ft. long. Total volume dry wall 40 160 cu. yd. Indian labor. Cost rock work $2.50 per cu. yd. For sluicing, steam pumping plant with capacity 2.5 sec.-ft. at 90 to 95 lb. per sq. in. pressure. Total volume earth in dam 60 120 cu. yd., about 40 000 yd. sluiced in. Total cost for sluicing 12 cents per cu. yd. All earth, either very fine sand or clay, no attempt being made to separate them. Undermined 1909 by passage of water under cap of lava rock which flanked dam and extended



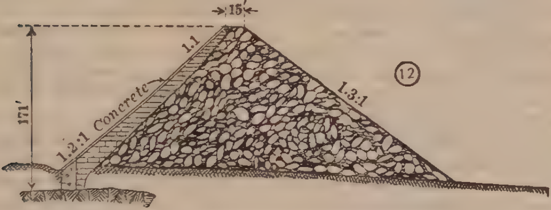


Fig. 16 (continued). Rock Fill Dams

beneath spillway. Spillway, south abutment and extreme south end of dam undermined. Considerable portion of spillway dropped 7 ft. and settlement of 9 ft. at junction of spillway and abutment. Small amount of fill at south end washed out and there was 5 ft. settlement about 30 ft. long in earth fill at north end. Main part of dam appears uninjured.

(8) Minidoka, Idaho. Rock fill, earth facing and concrete core-wall. Length 625 ft. Maximum height 80 ft. above bedrock and about 60 ft. above original bed of stream. Crest width 25 ft. Base averages about 300 ft. Total volume 191 000 cu. yd. Concrete core on solid rock. Top of wall 44 ft. below crest in central portions and about 11 ft. near end. Earth and rock-fill banks built up and core-wall built in trough between them. Fill then carried up on both sides over core-wall to required height. Cost $425 923.

(9) Lower Otay, California. 1897. Originally planned for masonry dam and masonry carried up 8 ft. above river bottom. Core of steel plates anchored to masonry and carried up to crest, protected with asphaltum and burlap and 1-ft. thickness of concrete either side. Failed January 27, 1916, by flood overtopping the dam. About 30 lives lost.

(10) Morena, California. 1896–1909. Length 520 ft. 306 000 cu. yd. Built in canyon about 130 ft. wide at stream bed with narrow fissure 4 to 16 ft. wide, extending 112 ft. below. In 1896 a rubble concrete wall 36 ft. thick at bottom and 12 ft. thick at top was built to seal fissure, and carried up 30 ft. above stream bed, when work was suspended in 1898. Construction resumed 1908. Upstream face for width of 7 ft. built of 6- to 10-ton granite blocks, set in cement mortar. Behind this for width of 50 ft. at bottom, reducing to 16 ft. at top, rock was placed by hand and derricks. Remainder of rock fill dumped by 2 cableways. Reinforced concrete slab 1 ft. thick on upstream face.

(11) Swift, Montana. 1914. Upstream face covered with a layer of hand-placed rock, 4 to 6 ft. thick, with face plastered with cement mortar. On top of this was placed a concrete slab, heavily reinforced, 6 in. thick at top of dam and 2 ft. at toe. Dam built on curve of 1276-ft. radius. Foundation of sand, gravel and boulders.

(12) Strawberry, California. 1914–16. Upstream slope faced with 9 in. to 18 in. of concrete carried down to concrete cutoff wall and backed by hand-laid rock. Main portion of rock dropped from cableways. River bed a cemented gravel. Dam arched in upstream direction. Radius 1880 ft. Contents 400 000 cu. yd. Length 612 ft.

(13) Dix River, Kentucky. 1923–25. Length 1000 ft. Height 275 ft. Contains 2 000 000 cu. yd. blasted rock. Upstream slope merges from 1 : 1.2 at river bed to 1 : 1 at crest. Downstream slope 1 : 1.4. Dam curved upstream on compound curve of 1250-ft. and 1700-ft. radii. Upstream slope of loose rock supports section of dry rubble masonry upon which is laid reinforced-concrete slab carried into sides and bed of canyon 20 to 30 ft. Spillway 2300 ft. long, 250 ft. wide and a maximum depth of excavation 107 ft., which supplied most of rock-fill.

8. Overflow Dams on Earth Foundation

Three Conditions influence the design of overflow dams on earth foundations. First, the character of the earth, ranging from loose fine sand to firm, well cemented hardpan; second, the height of the fall; and third, the volume of water to be passed. Designs for dams to meet various combinations of degrees of severity of these conditions may best be studied from precedents. The engineers of India have developed a type for their rivers in alluvial beds subject to great floods. The type has been adopted by the U. S. Bureau of Reclamation for similar conditions, the Laguna dam being a case in point. One form of the India dam is essentially a rock fill with very flat slopes downstream protected by heavy pitching or concrete-slab construction and containing one or more masonry core or cutoff walls. This type has never been used except for very moderate heights of from 6 to 15 or 20 ft. They have, however, been built on sand foundations and have passed great floods. The greatest trouble is usually experienced at the toe where the structure is likely to be undermined. The Avignonet dam is a precedent for a high dam on an earth foundation, but there the earth foundation is compact gravel, fine sand, and boulders. Following numbers in parentheses refer to Fig. 17.

(1) Narora Weir, India. Brick. Base 8 ft. thick. Founded on blocks or wells 10 ft. square and 18 in. thick sunk 7 ft. below river bed. Downstream from weir floor is 40 ft. wide, 5 ft. thick, consisting of 3 ft. 3 in. concrete, 9 in. brick and 12 in. ashlar. Beyond floor, talus is 100 ft. wide. Crest covered with ashlar. First crest covering destroyed and replaced with masonry set with wider joints. Head on crest 8 ft. End of talus 14 ft. below crest.

(2) Dauleshwiram, India. 1840. 16.8 ft. head on crest. Length 12 000 ft. Crest 11 ft. above normal bed level. Width 230 ft. Crest cover with ashlar masonry.

(3) Srivakantham, India. Length 1380 ft. Discharge over weir 123 000 sec.-ft. Weir wall and curtain walls on clay which lies under 3-ft. bed of sand.

(4) Pelandorai, India. Length 860 ft. Head on crest 13.3 ft. Crest 9 ft. above the river bed above weir.

(5) Burma Crib Weir, India. Boards were placed on top at first pending grouting of weir body with silt, and final settlement, with idea of replacing by rubble.

(6) Mahanuddee Weir. 6400 ft. long. Folding shutter 3 ft. high on crest. Calculated to discharge about 900 000 sec.-ft. Crest 13 ft. above average summer water level of river. Face of weir protected by rubble-stone packing with base varying from 20 to 40 ft. Breach 1886 at site of center sluices; about half sluices and portion of weir carried out and deep hole scoured on line of weir and below it. Cause never explained.

(7) Upper Colerum, India. 1836. Length 2929 ft. Head on crest 8.6 ft. Crest 7.4 ft. above river bed. River bed, fine sand. Founded on double row of wells sunk 6 ft. into river bed on upstream side and single row on downstream side. Masonry floor 27 ft. broad, 3 ft. thick, 2 ft. being brick masonry. Season after completion about 240 ft. of weir swept away. Caused by leakage undermining foundation. Weir repaired.

(8) Old Croton, New York. 1837–42. 180 ft. long. Mud and boulders cleaned from river bottom and crib cofferdams built enclosing space where masonry was to be laid. These cribs were left in the foundation. Space between cofferdams excavated to hardpan and filled with concrete and masonry. Earth bank 5 : 1 slope paved near top and with bottom width 275 ft. against upstream face. Dam has passed 8-ft. depth on crest.

(9) Kistna Weir, India. 1854–55. Length 3000 ft. for weir proper; nearly 4000 ft. including under sluices and piers. Crest 20 to 25 ft. above original river bed. Flood velocity over weir said to be 16 ft. per sec. and depth on crest 20 ft. Deep holes that had been scoured in river bed by floods were filled with sand. Wells sunk for foundation of weir.

(10) Avignonet, France. 1902. Concrete. Length 197 ft. Curved radius 656 ft. Height 75.5 ft. Built in narrow canyon on boulders, sand and gravel. Two cutoffs, 13 ft. deep and 8 ft. thick. Passes over 35 000 sec.-ft. during flood.

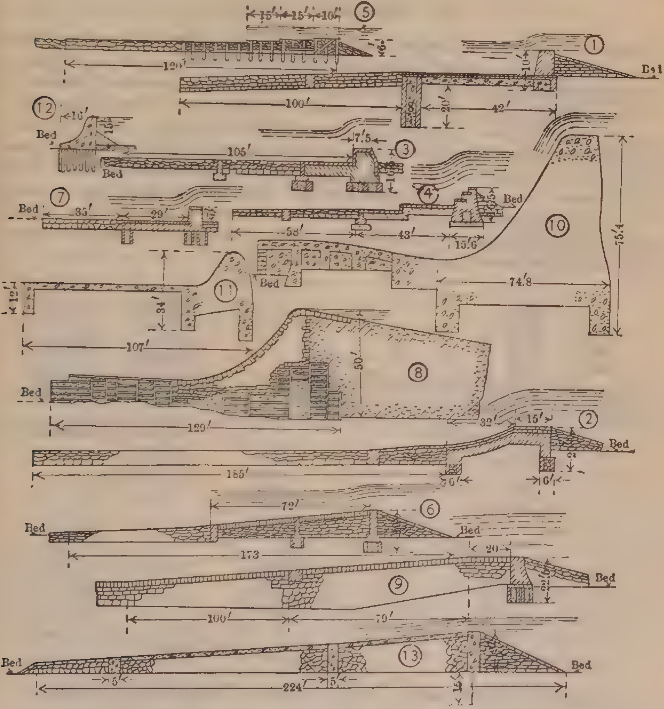


Fig. 17. Overflow Dams on Earth Foundations

(11) Granite Reef, Arizona. 1906–08. Partly founded on gravel and boulders. 1000 ft. long. Three curtain walls. Openings in the wall under lower toe 6 in. sq. 5 in. apart and about 6 ft. above bottom of wall to allow drainage of seepage. About 3-in. joints in apron to allow escape of water.

(12) Spooner, Wisconsin. Concrete structure 72 ft. long. Foundation sandy soil covered with thin layer of gravel. Pile foundation.

(13) Laguna, Arizona. Begun 1905. Crest 19 ft. above natural river bed. Three concrete walls 4800 ft. long and 57 and 93 ft. apart. Upper wall on row of sheet piling 12 to 20 ft. long. Between walls is rock fill. Apron of derrick stone extends 40 ft beyond lower wall. Rock fill between walls covered by 18 in. of concrete. Approximate quantities: rock fill 350 000 cu. yd.; concrete paving 37 000 cu. yd.; concrete core-walls 28 000 cu. yd.; sheet piling 90 000 lin. ft. Discharge from 4000 to 100 000 sec. ft.

(14) Sherman Island, Hudson River, New York. 1921–23. Length 590 ft. Height 80 ft. Founded on sand and boulders with interlocking steel piles to form cutoff. Lower half of upstream face slope 5 : 12. Upper portion 45 deg. There are 31 arches on buttresses 19 ft. on centers. Buttresses 3 ft. 6 in. thick stand on concrete base 108 ft. wide by 3 ft. thick. Arches are 24 in. thick in lower part and 18 in. in upper. To add to stability of dam interior of bays between buttresses partly filled with sand. Also 30 000 cu. yd. fine sand dumped upstream along dam to improve seal of cutoff piling.

9. Movable Dams

Needle, Wicket and Curtain Dams form a group of movable dams having many common and numerous interchangeable features. Common to all are upright pieces, the lower ends of which bear against a sill on the river bottom, the upper ends of which bear against some form of bridge. The bridge may be a stationary structure or a collapsible structure. In case needles are used (Fig. 18) they form the closure. In case wickets are used (Fig. 19) the closure is formed by sliding stop gates down grooves in the upright pieces. With curtain dams (Fig. 20) an overhead bridge is generally used, and a screen rolling on the upright pieces is let down from above. When a draw or fixed bridge is used, the upright pieces are hinged at the upper end and are swung up out of the water with suitable machinery.

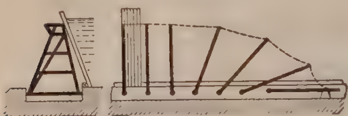


Fig. 18. Needle Dam

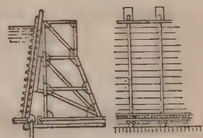


Fig. 19. Wicket Dam

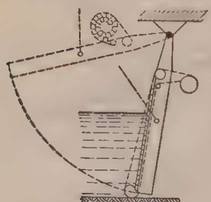


Fig. 20. Curtain Dam

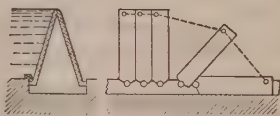


Fig. 21. Thomas A-Frame Dam

The Thomas A-Frame Dam (Fig. 21) consists of a series of A-frames set side by side and pivoted at the bottom to a sill. The plates which fasten the two legs of each section together at the top form a footpath. One of these dams 120 ft. long and 13 ft. high was built as part of Dam No. 6 in the Ohio River.

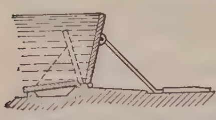


Fig. 22. Thénard Shutter Dam

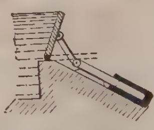


Fig. 23. Girard Shutter

The Thénard Shutter Dam (Fig. 22) consists of leaves hinged to a sill which extends across the channel. The leaves are raised and supported by struts hinged to the leaf and at the lower end fitting into a socket from which they may be tripped and the leaf lowered. Above is a counter shutter which is raised by the current as a temporary dam until the main dam can be raised by hand. A dam of this type, built in France and finished in 1843, had shutters 5.6 ft. high and 3.9 ft. wide. Others have been built in India to close flushing sluices, and some of them support heads of nearly 10 ft. The counter shutter is held in position by chains, which on account of the severe shocks occurring when the gate swings up, often break and are objectionable.

The Girard Shutter (Fig. 23), invented as a substitute for the Thénard, does away with the counter shutter, and the main dam is raised directly by hydraulic pressure. Seven of these were erected at Auxerre, the shutters of which were 11-1/2 ft. wide and 6-1/2 ft. high.

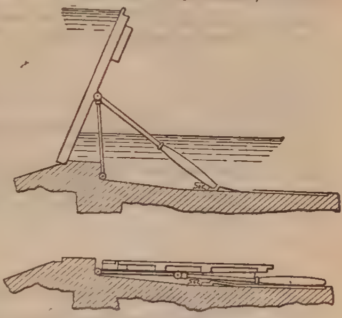


Fig. 24. Chanoine Wicket

The Chanoine Wicket (Fig. 24) was invented in 1852. The shutter is pivoted a little below its center point to a collapsible horse held in place by a long prop. It is adapted to large rivers where the flood rises are very sudden.

The Rolling Dam (Fig. 25) consists of a large steel cylinder placed across the current between piers or abutments and arranged to be rolled up entirely clear of the current on an inclined rack track. The rolling cylinder is operated by cables wrapped around one or both ends of the cylinder and passing to a winch. They have been built for openings up to 115 by 6-1/2 ft. and 60 by 14 ft., and are said to operate very quickly and pass ice readily.

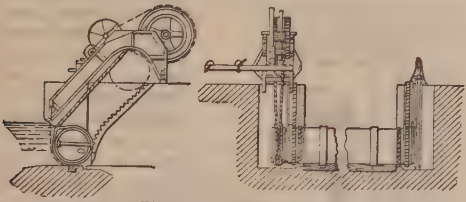


Fig. 25. Rolling Dam

Stoney Sluice Gates (Fig. 26) consist of a gate bearing at its end against a train of rollers. The gate is usually hung to chains at either end, which pass over sheaves to a counterweight. The roller train is also hung by a cable and counterweighted. They are usually operated by gearing on the supporting sheaves turned by hand cranks or traveling electric motors. These gates have given excellent satisfaction and are widely used. Power for operating has usually been underestimated. Fig. 26 is an example of Stoney sluice gate at the lake regulating works, Sault Ste. Marie, Canada. Fig. 27*a* shows the detail of the end bearing. Fig. 27*b* shows the end bearing of the Stoney gates on the Manchester Canal, England.

On the New York Barge Canal several movable dams of the bridge type have been used (Fig. 65*a*). Those dams have abutments, piers and superstructures like ordinary bridges; from the downstream sides hang steel frames resting against shoes in concrete sills that stretch across the stream between the abutment and pier. Against the upright frames are placed " Boule " gates forming the dam proper. Electric winches raise the gates and uprights by chains. The spans vary from 150 to 240 ft.

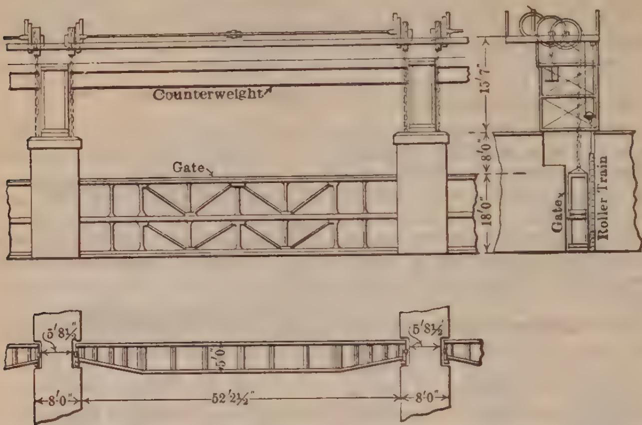
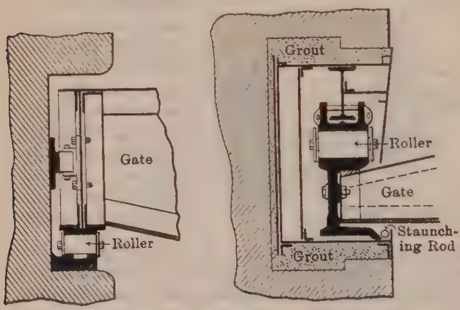


Fig. 26. Stoney Sluice Gate, Lake Superior Power Co.

The Bear Trap Dam (Fig. 28), of which there have been many modifications, consists of two rectangular leaves extending over the full width of the opening; and when in their lower position the downstream leaf overlaps the upstream leaf. The gate is raised by admitting water from the upper pool to a chamber under the leaves. A dam built at Davis Island on the Ohio River in 1889 was of this type. The gates were of wood and 52 ft. long. In 1905 this was replaced by a new bear trap with wooden leaves heavily bound with steel. The new leaves were much stronger than the old ones and were proportioned so that they would just float without the need of air forced to the under side. At the bear trap on the Marne in France a counter shutter was provided, as in the Thénard shutter dam, as an assistance to raising the main dam.



a *b*

Fig. 27. Ends of Stoney Gates

The Bear Trap on the Chicago Drainage Canal (Fig. 29) is of somewhat different design. The two leaves do not overlap, but are hinged together at the apex. The downstream edge of the dam is hinged to the foundation, while the upstream edge carries a roller and moves up and down on the masonry face. The gate is counterweighted and its movements up and down are controlled by hydraulic cylinders. The dam is 160 ft. long, the downstream leaf 35 ft. wide, and the upstream leaf 21 ft. wide.

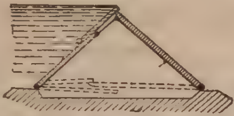


Fig. 28. Bear Trap Dam

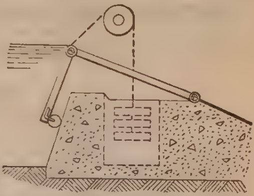


Fig. 29. With Hinged Leaves

The Parker Gate (Fig. 30) modifies the original bear trap by hinging the two leaves at the apex and introducing a hinge at a little above the midpoint of the upstream leaf; this does away with the sliding friction between the leaves. An auxiliary leaf, called the idler, is introduced above the upstream leaf to protect the moving parts from drift and ice. A dam for the Muscle Shoals Canal in Tennessee was built in 1892, 40 ft. wide and 8.5 ft. high.

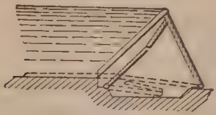


Fig. 30. Parker Gate

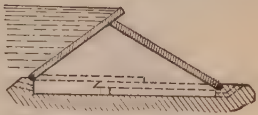


Fig. 31. Lang Gate

The Lang Gate (Fig. 31) is a modification of the Parker gate, in which the section of the upstream leaf above the hinge is removed and rods or chains take its place. The auxiliary leaf, or idler, is hinged to the downstream leaf, and its lower edge either slides or rolls on the upstream leaf. Several Lang gates, reaching the size of 80 ft. long and 14 ft. high, have been built in America.

The Chittenden-Drum Dam (Fig. 32) consists of a gate in the form of a sector of a circle, with a central angle of 67-1/2°, hinged at its center, and a watertight chamber into which it fits snugly when down. Water admitted to this chamber from the upper pool raises the dam. A dam of this type was constructed on the Osage River, Missouri, in 1901, with a difference in head of 16 ft. between the upper and lower pools.

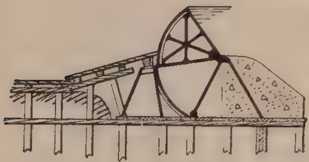


Fig. 32. Chittenden-Drum Dam

The Drum Dam of the Chicago Drainage Canal, Fig. 33, consists of a 45° sector of a cylinder 52 ft. in diameter, having its face covered with steel plates carried by radial frames pivoted at the center on the downstream side and a watertight chamber into which it fits when down. Dam raised or lowered by varying the head of water in the chamber, for which special valves are provided. This dam differs from earlier forms of sector dams in being pivoted on the downstream side. The vertical range of movement is 18 ft. A drum dam similar

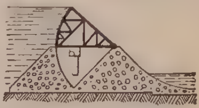


Fig. 33. Drum Dam, Chicago Drainage Canal

to that noted in Fig. 33 has been built across the Genesee River at Rochester with a vertical range of 9.4 ft., and each section 53 ft. 11 in. wide.

The Taintor Gate is similar to the drum dam (Fig. 32) except that the axis is higher and on the downstream side, and, to open it, it is usually lifted clear of the water by cables and winches. It is often used on power dams. The

Data of Needle Dams

Name or location	Date	Height trestle, ft.	Width of base, ft.	Distance c. to c. frames, ft.	Lift, ft.	Weight of each trestle, lb.	Weight of needles, lb.
Original Poiree....	1834	6.25	4.92	3.25	3.25	220	5
Belgian Meuse....	......	13.17	8.33	3.92	8.25	800	55
Louisa, Ky., Pass..	1896	15.17	9.85	4.00	13.00*	1145	263
Louisa, Ky., Weir..	1896	9.67	6.42	8.00	7.00	920	80
Klecan Moldau, Bohemia..........	1900	12.10 to 15.40	8.25	4.10	13.00	....	46 to 72

* = On Sill.

Stoney Sluice Gates

Name and location	Date	Number of gates	Height of gates, ft.	Clear width, ft.	Clear lift opening, ft.
Belleek, Ireland *....................	1883	4	14.50	29.17	9.00
Manchester, England †..............	1892–94	30	13–26	30.00 and 20.00	13–20 13–26
Richmond, England ‡................	1892–94	3	12.00	66.00	....
Glasgow, Scotland....................	....	3	12.00	80.00	....
Hagneck, Switzerland...............	1895–97	2	21.30	32.80	....
		1	9.80	41.00	....
Beznau..........................	1898–00	6	20.70	52.50	....
Chevres..........................	....	....	27.90	32.80	....
Rheinfelden.......................	1895–98	3	16.40	32.80	....
Lauffenburg.......................	1906	1	55.80	65.60	....
		2	41.00	47.60	
		1	41.00	65.60	
Electrical Works at Zurich............	1906	....	26.20	49.20	....
Assuan Dam.......................	1898–03	25	11.50	6.60	....
Assiout..........................	1898–03	111	16.00	16.40	....
Sault Ste. Marie Canal, Mich. §.......	....	4	26.79	48.00	....
Sault Ste. Marie Regulation Works ‖...	....	4	13.00	52.21	14.25
Chicago Drainage Canal ¶............	....	8	20.00	30.00	....
Minneapolis.......................	1906	....	18.00	16.30	....
La Gabelle, Quebec................	1924	2	43.00	50.00	....

* One man can easily operate. Distance between roller bearings 31 ft.
† Some of gates designed to withstand 26 ft. head. Rollers 7-1/2 in. diameter.
‡ Weight each gate 32 tons. Water pressure about 100 tons per gate. Can readily be raised to full height by 2 men in 7 min. Guides arranged so as to turn gates horizontally as raised so as to be out of sight under bridge. 15 rollers.
§ 32 rollers per train. 7 in. diameter. Medium steel.
‖ Rollers 5 in. diameter. 24 rollers in one train. Medium steel. 2 trains per gate.
¶ Rollers of pin steel with brass bushings. Diameter rollers 6 in. 30 rollers per train.

U. S. Government built at Sterling, Ill., gates 21 ft. wide and 13-1/2 ft. high. On the New York Barge Canal there are several Taintor gates, the largest having a lift of 14.3 ft. and a width of 48 ft.

AQUEDUCTS

10. Reservoir Outlets

Outlets for reservoirs are preferably put in the natural formation outside the dam whether earth or masonry. Fig. 37 shows an outlet through a natural hill. They may be carried through tunnels in the adjacent hillsides where practicable, otherwise through the dam. When in an earth dam, with masonry core-wall, the gatehouse may be economically built as an integral part of this core-wall. Instead of intake pipes, an intake channel may be provided by extending wing walls back into the reservoir, from the gatehouse. The outlet may be similarly constructed, or the pipe or conduit type of outlet may prove more advantageous in a given case. Fig. 34 shows an adaptation

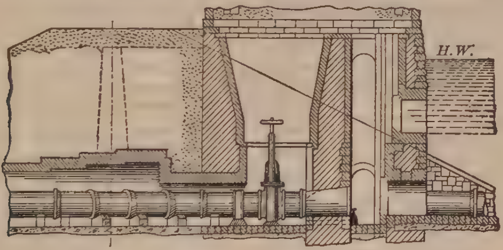


Fig. 34. Gatehouse, Jerome Park Reservoir

of this type. Outlet pipes in earth dams may be broken by the settlement of the embankment. If the dam is high, the outlets may be placed in the natural earth below, in a trench, where if properly built they are reasonably safe from dangers due to settlement. In low earth dams outlets are usually carried directly through the embankment. Leaky outlet pipes have caused many failures of earth dams. Water from the reservoir also is likely to force a passage along the outside surface of the pipe or conduit. A precaution is to place the outlet pipe within a masonry conduit where it may be inspected for leakage and repairs may be made. Transverse projections or cutoff collars are placed along such pipe or conduit at intervals, say of 20 ft. or less, to

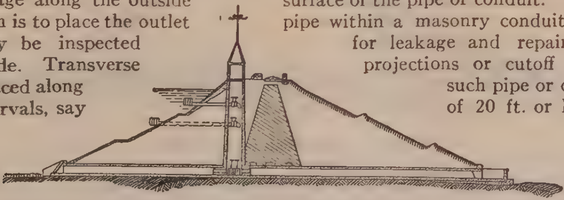


Fig. 35. Outlet Borden Brook Reservoir

prevent leakage along its outside. Fig. 36 shows metal collars for pipes embedded in masonry. Fig. 35 shows an outlet design where the conduit is placed wholly in the earth embankment. This is possible as the gatehouse is upstream of the core-wall, the head is small and the outlet below is without pressure. In masonry dams outlets weaken the section somewhat if built through them. The distribution of stresses in the mass of the masonry is changed and, if the opening is large, temperature and shrinkage cracks are concentrated. To compensate, the dam is strengthened locally, either by buttressing, or, as shown in Fig. 36, by extending the upper chamber back of the upstream face of the dam.

Gatehouses. The gatehouse is an essential feature for the control of water drawn from a reservoir through its outlet. Fig. 36 shows the gatehouse of the Boonton dam and is typical. Openings at *A* and stop-planks in grooves at *B* are the means by which water may be drawn from the top, mid-depth or bottom of the reservoir. Gratings in these openings prevent large objects from floating into the upper chamber. Fish screens could be placed in the line of stop-planks, although in the lower chamber, at *F*, there is provision for screening the water just prior to its admission into the service conduit. The sluice gates and upper stop-planks in the wall *C* provide the direct control of admission of water at the different elevations into the second well. By placing stop-planks in grooves *B* the inflow can be entirely interrupted in case it is necessary at any time to examine or repair any of these gates, access to which is obtained by means of the well above them, which in such emergency would be drained. Each half of the gate chamber is independent of the other, so that an inspection or repair would affect only half the flow at a time, the other half of the outlet continuing in operation. Water is thus admitted to each pair of steel pipes through these large openings with a small loss of head and to the control valves in the lower gate chamber at *D*. The upper sluice gates afford protection to these gate valves in an emergency. The overflow trough at *E* prevents dangerous heads from coming upon the grade conduit, should a

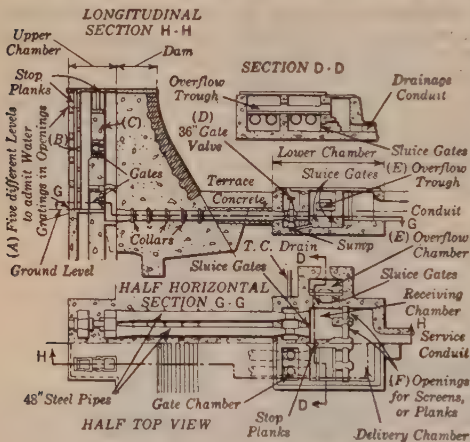


Fig. 36. Gatehouse, Boonton Dam

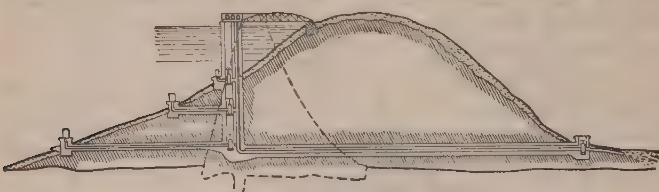


Fig. 37. Outlet San Mateo Dam

gate down the service line be closed without warning. In some systems a weir built along the side of the aqueduct serves this purpose. Screens are provided in the lower chamber at *F*, where they may be conveniently cleaned. Fish and trash screens are usually copper netting having about six meshes to the inch, with wire about 1/16 in. in diameter made up in frames which are slid into grooves, such as those for the stop-planks. Stop-planks of moderate size are constructed of wood but the larger sizes are preferably built of steel plates and shapes. The screen frames and those for the stop-planks are usually made to be used in the same grooves. The grooves are of cast iron, set into the concrete masonry. Smaller plants may have their chambers provided only with sluice gates, and stop-planks and screens upstream of them. A single chamber rather than an upper and lower chamber would be involved, although one chamber may be designed to accommodate racks, screens and stop-planks, sluice-gates and gate valves, in the order named, in the direction of flow.

The gatehouse for an earth dam may be put in the dam as in the case of the Jerome Park Reservoir (Fig. 34) and the Borden Brook Reservoir (Fig. 35). In the latter example, the gatehouse is of the tower type. This type is common for smaller reservoirs and the intake tower with bridge shown in Fig. 37 for the San Mateo dam is illustrated, except that the tower may be located at the toe of the slope and have screened inlets at different heights. The tower type, especially at toe of slope, is subject to ice thrusts which in colder latitudes may become serious. This objection was met in the case of the Borden Brook Reservoir by thickening the upper portion of the dam at that point.

At high water stages, ice formation in the reservoir may result in a thrust toward the dam, the ice between it and the embankment giving way by sliding up the embankment slope. This may be met by building a solid pier or stiff bridge between the gatehouse and the dam. Or the tower may be lifted vertically from its base by an ice sheet which rises with the water in the reservoir. Pressures due to ice are assumed at 30 000 lb. per square foot of ice one foot thick. (See Index for references on ice data.)

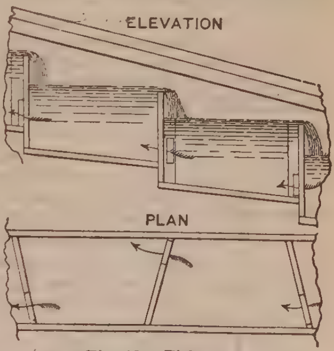


Fig. 38. Fishway

A 152-ft. concrete gate tower at the Magic reservoir on the Big Wood River near Shoshone, Idaho, was so ruptured at the base. The tower is octagonal, about 20 ft. in diameter, with a dividing wall in the center. It is situated at the toe of the embankment of an earth dam. In March, 1928, an unusual leakage of water into the normally dry operation chamber of the tower led to investigation. The tower appeared to have been lifted and then tilted several feet out of vertical; but afterwards settled back to plumb almost exactly in place. It had broken through the holding ice sheet on the side facing the dam, indicating a list towards the dam. (*Eng.-News-Rec.*, Aug. 30, 1928, p. 327.)

Fishways. The laws of many states require a fish ladder or fishway to be built at dams. Fig. 38 shows fish ladder recommended by the U. S. Commission of Fish and Fisheries. The cut shows two pools of the ladder which may be multiplied to surmount any desired height. Each pool is

1-1/2 ft. higher than the one below, 4 ft. wide, 6-1/2 ft. long on one side, and 4-1/2 ft. on the other. The opening in the bulkheads between pools should be about one foot square at the lower pool, increasing in size toward the upper level to insure a waterfall over the bulkhead; the amount of increase will depend upon the leakage.

11. Flow-line Masonry Aqueducts

Most of the flow-line aqueducts constructed since 1900 have been of concrete of horseshoe shape built in open cut and backfilled. Where the detour would be too great to carry the flow line around valleys, inverted siphons of steel pipe or pressure tunnels are introduced. In some cases for short distances the flow line with the horseshoe cross-section has been carried across depressions on filled ground. Masonry aqueducts constructed on embankments require great care in securing an unyielding foundation, since even a slight settlement may cause cracks and leaks. A crack below the water-line may erode the embankment materials and possibly give rise to destruction of a portion of the masonry with serious consequences. A small quantity of steel reinforcement in the invert of concrete sections on embankment will increase the ability of the structure to resist the ill-effects of slight settlement where the construction of the embankment type cannot reasonably be avoided. Elevated crossings of valleys and streams are seldom resorted to. The Assabet Bridge, 359 ft. long, on the Wachusett Aqueduct, completed in 1898, is the most notable case of a bridge crossing. The horseshoe shape is best adapted to resisting the pressure from the backfill and to ease of construction, cleaning and maintenance and at the same time has fairly good hydraulic properties.

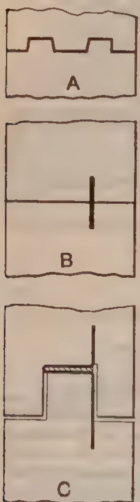


Fig. 39

Design. The intrados and extrados are made up of compound curves of circular arcs. The thickness at crown depends upon the earth and water loads to be expected; in the larger aqueducts the internal water pressure, varying with the depth, must be considered. Also where tests are prescribed before backfilling, the structure must be designed to meet this requirement of filling and emptying, as there is a critical depth of water in an aqueduct which may induce serious stresses at the crown, if unsupported by fill. The thickness of invert depends upon depth of ground water expected.

The dishing of invert need not be beyond that required for cleaning or draining, except to take excessive external upward pressure or side thrust from the walls, in which case the inverted-arch shape will serve. The methods of arch analysis for tunnels may be adapted to study of aqueduct cross-sections. The lines of pressure must be carried around from arch crown to invert, in this respect differing from arch bridge analysis.

The late F. F. Moore devised a graphic adaptation of the method of least work to these problems (Journal Association of Engineering Societies, Vol. 47, July, 1911). Fig. 14, p. 1562, shows the largest section of the Catskill Aqueduct in different kinds of material.

Contraction Joints. The practice in flow-line conduit construction is to place the concrete continuously in sections with a contraction joint, designed to prevent leakage between sections. The Catskill Aqueduct developed transverse cracks in some cases in those sections where the contraction joints were 75 ft. apart, but none in sections which were 60 ft. or less in length. Two types of contraction joints were tried, indicated by *A* and *B* in Fig. 39. Type *C* has been tried elsewhere with indifferent success, because of the construction difficulties of properly placing in the masonry of the curved sidewalls the thin bent strips of non-corrosive metal. In the Catskill Aqueduct, type *A* was found unsatisfactory because of the practical difficulty of making the tongues and grooves sufficiently exact for sliding, and the tongues broke off in some cases. Type *B*, a steel plate coated with asphaltum, gave in general good results, although the concrete was cracked by the plate in pulling in a relatively small number of joints, due to improper placing of the plate or to other defects of construction. However, all of the joints, of whatever type of construction and condition, were calked in the cold season, when open the maximum amount at the lower masonry temperatures. Several kinds of calking materials were used with more or less satisfactory results, but the most successful treatment was the complete filling of the joint with portland cement grout. Tests made subsequent to the grouting indicated that the outward leakage from the whole length of aqueduct, when carrying the maximum quantity, would not exceed one-quarter of one per cent of the capacity. For lesser quantities and aqueduct depths the leakage will, of course, be less, and moreover it has been the experience with this aqueduct, as well as with similar structures elsewhere, that the masonry tightens in service with consequent reduction in leakage.

Location and name	Date built	Length, miles	Height, ft.	Width, ft.	Capacity, sec.-ft.	Grade, ft. per 1000
(1) Boston, Mass., Cochituate..	1848	14.60	6.33	5.00	27.9	0.05
(2) Boston, Mass., Sudbury....	1878	15.90	7.67	9.00	155.0	0.19
(3) Boston, Mass., Wachusett..	1898	11.95	10.50	11.50	465.0	0.40
(4) Boston, Mass., Weston.....	1904	13.44	9.25	10.00	465.0	0.80
	......	....	12.17	13.17	....	0.20
(5) Brooklyn, N. Y............	1859	12.40	6.33	8.17	71.3	0.10
	......	....	8.67	10.00	....	......
(6) Brooklyn, N. Y............	1891	7.40	6.92	9.33	91.3	0.10
	......	....	5.92	7.33	62.8	......
(7) Birmingham, England, Elan.	......	73.37	8.00	7.66	116.0	......
(8) Glasgow, Scotland.........	1859	25.75	8.00	8.00	65.1	0.16
(9) Glasgow, Scotland.........	1894	23.50	9.00	12.00	77.5	0.18
(10) Los Angeles, Calif.*........	1913	233.76	8.25	7.50	430.0	2.00
	......	....	10.28	9.00	430.0	0.40
(11) Manchester, Eng., Thirlmere	1894	95.88	7.00	7.08	93.0	0.31
(12) New York, N.Y., Old Croton	1842	33.14	8.46	7.42	147.2	0.21
(13) New York, N.Y., N. Croton.	1890	33.25	13.53	13.60	465.0	0.13
(14) New York, N.Y., Catskill..	1916	110.00	17.00	17.50†	775.0	0.37
(15) Rochester Intake, N. Y.....	......	2.27	6.00	6.00	23.2	0.25
(16) Vienna—2d...............	1910	114.00	6.71	6.42	82.0	2.74
	......	....	8.21	6.42	....	.....
(17) Washington, D. C., Potomac	1863	11.00	9.00	9.00	118.6	0.15
(18) Apulian, Italy.............	1915	151.00	10.3	9.3	....	0.25
	......	....	8.7	5.4	....	.....
(19) Winnipeg, Canada.........	1918	97.00	9.0	10.75	131.0	0.60
	......	....	5.4	6.4	....	......

* Sections given are typical. † Standard for cut and cover type. Section 14*B* is for maximum size.

Data Regarding Flow-line Masonry Aqueducts. Numbers in parentheses in the table on page 1561 refer to Fig. 40 and to the accompanying notes.

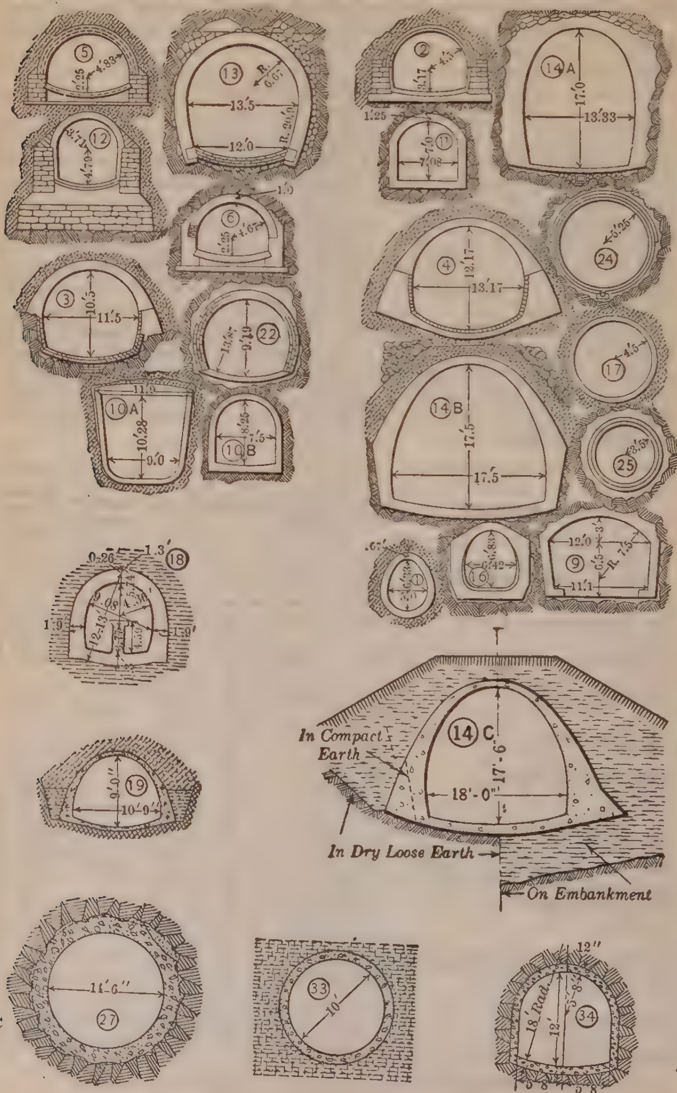


Fig. 40. Flow-Line Aqueducts

(1) Lining is brick throughout.

(2) Lining is brick and rubble. Cost per lin. ft., $23.90.

(3) Lining is concrete with brick-lined invert. 2 miles in tunnels, 6.9 miles in cut and cover or embankment, and last 3 miles in open channel. Cost per lin. ft. $22.90.

(4) Lining is concrete with brick-lined invert. 9.14 miles are in cut and cover or embankment, 1.02 miles are in open channel, 2.30 miles are in tunnel and 0.98 mile is in cast-iron pipe siphons. Cost per lin. ft. $32.60.

(5) Arch and invert are brick; walls and backing rubble. Invert on concrete cradle.

(6) Arch and invert are brick; walls and backing are rubble. Invert placed on concrete cradle. Average depth of cutting 16 ft. Maximum depth of cutting 23.9 ft.

(7) Lining is blue brick. About 11-1/2 miles are in tunnel, 25 miles in cut and cover and the remainder is in cast-iron pipe siphons.

(8) Pa.t of aqueduct was lined with masonry. About 10 miles are in open channel, 12 miles in tunnel and 4 miles in siphons.

(9) Lining is of concrete. 1.42 miles are in cut and cover or on embankment, 18.97 miles in tunnel and 3.10 miles in siphons.

(10) 164 tunnels with combined length of 42.7 miles, not including 9.2 miles of power tunnel. Open unlined canals 21.1 miles. Open lined canals 39.56 miles. Concrete covered conduit 97.72 miles. Siphons, steel and concrete 12.06 miles.

(11) The lining in tunnel and in cut and cover is of concrete. 36.75 miles are in cut and cover, 14.12 miles in tunnel and 45.01 miles are in 40-in. cast-iron siphons. Cost of whole work per lin. ft. $42.00.

(12) 37.1 miles are in cut and cover or on embankment, the lining being brick backed with rubble, 0.8 mile is in 4 36-in. cast-iron pipes, and 0.3 mile is in pipes carried on high bridge over the Harlem River. Average cost was about $59.00 per lin. ft.

(13) Lining is brick backed with rubble. 29.75 miles are in tunnel of which 7.17 miles are siphons, 1.12 miles are in cut and cover and the last 2.38 miles are in cast-iron pipe. Cost about $112.00 per lin. ft.

(14) Lining is concrete even in siphons. 54 miles of flow-line sections, 14*A* and 14*B* in cut and cover at grade of 21 ft. per thousand ; of which 14*C* type extends 17 miles. 0.5 mile is in embankment, 13.9 miles are in grade tunnel, 35 miles in pressure tunnel and 6.3 miles in siphons.

(15) Concrete. 1.42 miles are in tunnel and 0.8 mile is in cut and cover.

(16) Concrete is used quite generally. 44 miles are in tunnels, the longest being 17 630 ft.; there are also several iron pipe siphons, the longest of which is 6 miles, and a masonry arch bridge carrying a concrete aqueduct.

(17) Lining is brick and rubble masonry. 9.51 miles are in cut and cover or on embankment and 1.49 miles are in tunnel.

(18) Concrete. 99 tunnels of an aggregate length of 60 miles, the longest being 12 miles. 85 bridges.

(19) Concrete conduit in cut and cover or on embankment. Part on pile foundation. River crossing made by reinforced-concrete siphons. Supply line from reservoir to city, a reinforced-concrete pipe 5 ft. 6 in. in diameter and 10 miles in length.

(20) Water Supply, N. Y. City. Rock tunnel lined with brick, backed by rubble masonry. The face of the brick lining was washed with three coats of neat cement mortar.

(21) Milwaukee Water Works, Wisconsin. Hard clay and water-bearing sand and gravel. Compressed air used. No timbering used in hard clay. Progress averaged 6-2/3 ft. per day in clay. Drills and explosives used in hard clay. Circular section lined with 4 rings of brick.

(22) Water Supply, Washington, D. C. Rock tunnel lined with brick backed by rubble. Length of 500 ft. is iron-lined. Average cover is 150 ft.

(23) Water Supply, Jersey City, New Jersey. Steel plate pipe 5/16 in. to 7/16 in. thick except at crossing of Hackensack and Passaic Rivers where the thickness is 11/16 in.

(24) Water Supply, Torresdale, Philadelphia, Pennsylvania. Rock tunnel lined with 13-1/2 in. of brick in the rock and 9 in. in the invert, backed with a minimum of 6 in. of concrete. The maximum cover is 115 ft.

(25) Water Supply, Cincinnati, Ohio. Rock tunnel lined with two rings of brick backed by a minimum of 6 in. of concrete. Average cover over tunnel is 130 ft.

12. Siphons and Pressure Tunnels

Data regarding siphons and pressure tunnels are given below, the numbers in parentheses referring to the notes and to Fig. 40. The first pressure

Data on Siphons and Pressure Tunnels

Name and aqueduct	Date built	Diameter, ft.	Length, miles	Equivalent grade in ft. per 1000	Maximum internal head, ft.	Maximum, external head ft.	Capacity, sec.-ft.
(20) Harlem River, New Croton, N. Y........	1890	10.5	7.17	2.90	431	304	465
(21) Milwaukee, Wis.......	1895	7.5	0.60	....	115	115	......
(22) Washington Extension, D. C...............	1902	9.87	3.92	....	175	......	......
(23) Jersey City, N. J.......	1904	6.00	10.25	....	230	50	108.5
(24) Torresdale, Phila., Pa..	1904	10.50	2.61	0.75	......	99	465.0
(25) Cincinnati, Ohio......	1905	7.0	4.21	0.51	170	95	155.0
(26) Buffalo, N. Y.........	1910	......	1.95	....	50	50	......
(27) Rondout, Catskill, N.Y.*	1916	14.5	4.47	0.68	726	487	775
(28) Wallkill, Catskill, N.Y.	1916	14.5	4.43	0.66	544	265	775
(29) Moodna, Catskill, N.Y.	1916	14.2	5.68	0.80	630	240	775
(30) Hudson, Catskill, N.Y.	1916	14.0			1517	1100	775
(31) Yonkers, Catskill, N. Y.	1916	16.6	2.46	....	155	100	775
(32) City Tunnel, Catskill, N. Y..............	1916	11.0–15.0	18.11	....	1005	710	775
(33) Cleveland, Ohio.......	1917	10.0	3.00	....	99	99	......
(34) Chicago, Ill..........	1918	......	8.00	....	130	130	......

* Section shown for Rondout typical for all pressure tunnels on the Catskill system.

tunnel of note was the Harlem River crossing on the New Croton Aqueduct, and this was lined with brick.

(26) Water Supply, Buffalo, New York. Rock tunnel, concrete-lined 1 to 2 ft. thick. Extends 6000 ft. under Lake Erie; inside dimensions 12 ft. by 11 ft. 3 in. Land tunnel 4300 ft. long; inside dimensions 9 ft. by 8 ft.

(27) Rondout, Catskill, New York Water Supply. Rock tunnel driven from 8 shafts by top heading and bench method, known as American method. Maximum progress (full section) 488-1/2 ft. per month. Four 3-1/4-in. Ingersoll-Rand drills on 2 vertical columns and 2 on bench. 22 holes, 8 to 12 ft. deep per round in heading. 175 to 200 lb. of 60% dynamite per round. Concrete lining 1 : 2 : 4, 15 to 17 in. thick. Steel forms. Approximate cost to city per linear foot based on contract prices: Tunnel $180, shaft $285 in rock and $350 in earth. Minimum rock cover 180. Maximum depth 710 ft.

(28) Wallkill, Catskill, New York Water Supply. Rock tunnel driven from 6 shafts in Hudson River shale, by American method. Four air drills in heading and 2 on bench. 22 to 24 holes per round. Muck hauled by mules and electric locomotives. Maximum monthly advance one face 523 ft. Ventilation by blowers. Concrete lining 15 to 17 in. thick.

(29) Moodna, Catskill, New York Water Supply. Tunnel in shale and granite. 7 shafts. Concrete lining 13 to 15 in. thick.

(30) Hudson, Catskill, New York water supply. Tunnel in sound granite, 1100 ft. below surface of river. Hydraulic gradient 400 ft. above river. Shaft at either end. Concrete lining 13 to 15 in. thick. Water-bearing fissure sealed by building concrete

bulkhead 8 ft. thick heavily reinforced with drains and grout under pressures up to 1000 lb. per sq. in. forced through pipes set in bulkhead and leading to fissures.

(31) Yonkers, Catskill, New York. Tunnel in gneiss, driven from 4 shafts. Concrete lining 15 to 17 in. thick.

(32) City Tunnel, Catskill, New York. Tunnel under New York City in gneiss, schist and limestone from 200 to 750 ft. below surface. Driven from 25 shafts by American method. Electrically operated machinery generally used. Ventilation by blowers. Cost of excavation per linear foot of tunnel based on contract prices varied from $86 for 11-ft. diameter tunnel to $90 per 15-ft. diameter tunnel. Lining 11 to 19 in. thick of 1 : 1-1/2 : 3 concrete. Cost of lining based on bid prices varied from $51 for 11-ft. diameter tunnel to $52 for 15-ft. diameter.

(33) Cleveland, Ohio. Waterworks tunnel in stiff clay under Lake Erie, driven by hydraulic shield. Excavation for about half the length of tunnel done by rotary cutting machine which was abandoned and rest of work done by hand. Maximum monthly advance 886 ft. Considerable trouble with inflammable gas. Tunnel lined with concrete blocks 11 in. thick radially by 18 in. wide, six segment blocks and key to the ring, blocks weighing 1200 lb. each.

(34) Chicago, Illinois. Water works tunnel in rock, 3 miles under Lake Michigan and 5 miles under Wilson Avenue, Chicago 4 shafts. The top heading and bench method and bottom heading method were tried and the former proved more economical. Maximum monthly progress one heading 630 ft. Excavated rock crushed and screened and made into concrete, mixed in the tunnel and placed by pneumatic process back of steel forms. Tunnel of horseshoe section; 5883 lin. ft. of lake tunnel, 12 ft. 3 in. wide and 13 ft. high. Rest 11 ft. 4 in. wide and 12 ft. high. Lining 12 in. thick. Cost per linear foot, $61.48, done by city force account.

There are several cast-iron siphons not included in the above. The aqueduct for the Liverpool and Manchester Water Supply has siphons consisting of from three to five 39-in. and 42-in. cast-iron pipes laid side by side under heads of 300 to 480 ft. The new Croton aqueduct passes under the 125th Street valley in 8 cast-iron pipes 48 in. in diameter laid side by side. The Catskill aqueduct crossing under the Narrows is a 36-in. diameter cast-iron pipe with specially designed flexible joints, to facilitate sinking the connected pipes from the surface.

Siphons of steel plate, cement-mortar-lined, enveloped with concrete and covered with earth embankment were used on the Catskill Aqueduct to join the flow-line aqueduct across valleys where the rock was not sound or where for other reasons pressure tunnels were not feasible. There are 14 of these siphons aggregating 6 miles. Each consists of three separate pipes built of plates from 7/16 in. to 3/4 in. thick, riveted together. The diameters are from 9 ft. to 11 ft. and the lining is 2 in.

13. Reinforced-Concrete Conduits

Data regarding reinforced concrete conduits are given below, the numbers in parentheses referring to the following notes and to Fig. 41. In the Salt Lake City conduit the joints at the end of the sections were thoroughly grouted in the coldest weather and the concrete was thus thrown into compression, effectually cutting off leakage. In the Jersey City aqueduct fine cracks appeared at the joints, but the leakage was not serious, and eventually they silted up. The Cottonwood and Pinto pressure pipes were built without any provision being made for expansion joints, the ends of section being left rough for the purpose of affording a good bond with the next day's work. Numerous cracks developed which were from time to time repaired, with the result that, one year after their completion, tests showed no leakage in the two Cottonwood pipes; in the north Pinto pipes a leakage of 0.002 sec.-ft. and in the south Pinto pipe 0.03 sec.-ft., all tests being made under maximum head.

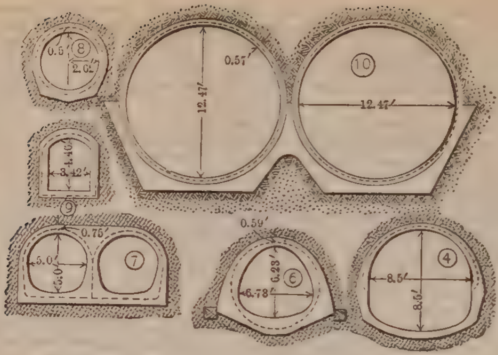


Fig. 41. Reinforced-Concrete Conduits

Name and location	Date built	Length, miles	Height, ft.	Width, ft.	Capacity, cu. ft. per sec.	Grade in ft. per 1000	Minimum thickness, in.
(1) Champ Isere, France...	.....	1.30	10.82	10.82	......	......	7.90
(2) Cottonwood, Arizona..	1906	0.20	5.25	5.25	250.0	1.05	7.00
(3) Eastern Colo. Power Co.	1909	12.00	3.00	3.00	......	4.92	3.00
(4) Jersey City, N. J......	1904	3.59	8.50	8.50	108.5	0.09	5.00
(5) Kensico By-Pass, N. Y.	1916	2.21	11.00	11.00	......	2.56	12.00
(6) City of Mexico, Mexico	1906	17.00	6.23	6.72	77.5	0.30	7.12
(7) Newark, N. J.........	.....	1.33	5.00	5.00	......	......	......
(8) Pinto, Arizona........	1906	0.92	5.25	5.25	250.0	1.23	6.00
(9) Salt Lake City, Utah..	1906	7.20	4.46	3.42	48.0	1.20 to 1.50	6.00
(10) Sosa & Ribabona, Spain	1909	0.61	12.50	12.50	1240.0	......	6.89
(11) Umatilla, Oregon......	.....	0.89	3.92	3.92	......	......	2.50
(12) Albelda, Spain........	.....	0.45	13.12	13.12	......	......	......

(1) Hydro-electric plant. Hoops and longitudinal rods forming a 4-in. mesh used for reinforcement. Maximum head sustained 65.6 ft.

(2) Power pipe lines. This is a double-line siphon with a maximum internal head of 75 ft. and an external head not exceeding 10 ft. Reinforced with 5/8-in. steel hoops spaced 3 in. c. to c. and 10 longitudinal rods.

(3) Pressure Pipe. In a trench 4 ft. wide and backfilled with a minimum cover of 30 in. Used for siphons up to a head of 105 ft. Reinforced with hoops of No. 5 steel wire spaced 12 in. c. to c. 7 tunnels on the line aggregate in length 1500 ft. Pipes were cast in lengths of 2 ft. before being placed in trench.

(4) Flow-line water supply aqueduct. Reinforced with transverse 3/8-in. twisted steel rods 12 in. c. to c. and longitudinal 1/4-in. twisted steel rods 24 in. c. to c. Portion of the 22.99 miles of aqueduct from Boonton to Jersey City.

(5) By-pass on Catskill Aqueduct at Kensico reservoir. Maximum depth of invert below flow-line of reservoir in which by-pass is located is about 53 ft.

(6) Flow-line water supply aqueduct. The reinforcement is expanded metal. Conduit entirely built in cut and cover.

(7) In Cedar Grove Reservoir. Maximum head inside 50 ft. and outside 50 ft., depending on whether pipe or reservoir is empty. Reinforcement is No. 10 gage expanded metal with 3-in. mesh and joints lapped. 4000 ft. are single conduit and 1500 ft. are double conduit.

(8) Power pipe lines. Double-line siphon with maximum internal head of 31 ft. and external head not exceeding 10 ft. Reinforcement is 5/8-in. steel hoops placed 6 in. c. to c. and 6 longitudinal rods.

(9) Big Cottonwood Water Supply, flow-line aqueduct. Part of conduit was built in cut and cover, part in fill and part in tunnel. The reinforcement is twisted steel bars 3/8 in. to 1/2 in. in diameter, spaced 6 in. to 9 in. c. to c. Cost was about $9.80 per linear foot.

(10) Consist of a steel tube 0.118 in. thick, embedded between an inner mortar coat 0.9 in. thick, plastered on wire mesh, and an outer concrete shell 5.9 in. thick reinforced by hoops of T iron, all in shallow trench, maximum head 85 ft.

(11) A siphon on the Umatilla irrigation system. The maximum head is 55 ft. The reinforcement is a coil of 5/16-in. wire.

(12) Same irrigation system as Sosa and Ribabona. Maximum pressure head 97 ft. Shell of pipe 7.28 in. of concrete faced with 0.59 in. of cement mortar. Reinforcement 124 longitudinal round bars 4 in. apart and circumferential T bars. Concrete: 1 portland cement : 1.28 sand : 2.56 gravel under 1-1/4 in. and 0.58 to 1.00 part of water, all by volume. Lining 1 cement : 1 sand. Seepage loss under full head 0.14 sec.-ft.

14. Wood-Stave Pipe

Stave Pipe is much used in Western United States on account of the cheapness of lumber and the high cost of steel and cement. It is built in place by assembling the staves for the lower half in a cradle, after which a pipe ring is set inside and the staves for the upper half assembled on it. The staves all break joints with those adjoining and are " driven home " on the end. Bands are placed but not finally tightened until the wood is thoroughly saturated. Junctions are made by cutting the last staves a little long and springing them into place.

The Staves vary between 1-1/4 in. and 2-1/2 in. in thickness and 6 to 8 in. in width and are uniform throughout the length of the pipe, the variation of head being met by variation in spacing of bands. The staves are shaped to true cylindrical surfaces on the inside and to true radial lines on edges. The outside is usually curved, but sometimes flat. A very shallow bead left on the edges crushes and results in greater watertightness. Thin steel tongues let into the ends of the staves make tight joints and insure a smooth interior surface. The timbers most used are California redwood, Oregon and Washington fir, yellow pine and several kinds of spruces. The timber should be clear, somewhat seasoned and protected from warping.

Bands are from 3/8 in. to 3/4 in. in diameter. They are proportioned for the initial tension on erection, plus the stress from water pressure plus the stress from swelling wood. It is generally agreed that a pressure sufficient to keep the wood saturated, or nearly so, prolongs the life of the staves, and that contact with earth charged with decaying vegetable matter and crushing of the exterior surface of the wood in adjusting bands shortens it. Examinations of pipes which have been in use for a number of years show that the interior surfaces wear smoother and do not become fouled.

Name and location	Diameter, in.	Total length of pipe line, miles	Length of wood stave pipe, miles	Capacity, cu. ft. per sec.	Thickness of staves, in.	Diameter of bands, in.	Band spacing, in.	Maximum head, ft.
(1) Astoria City, Ore.	18	11.6	7.4	6.4	1-3/8	7/16	2-1/4–12	175
(2) Butte, Mont.....	24	9.0	9.0	......	1-7/16	1/2	2-7/16–6	202
(3) West Los Angeles	30	7.5	7.5	......	1-1/2	1/2	3-1/2–12	105
(4) Vancouver, B. C..	30	4.3	4.1	......	2	1/2	1-3/4	210
(5) Pikes Peak Plant.	30	4.4	......	......	1-1/2	1/2	2-1/2–8	215
(6) Denver, Colo.....	30	20.3	16.4	13.0	1-5/8	1-1/2	2-3/16–12	185
(7) Johnstown, Pa...	36	6.4	0.6	......	1-1/2	1/2	2-3/4–12	176
(8) Atlantic City, N.J.	42	1.9	1.9	18.6	1-3/16	9/16	12	9
(9) Johnstown, Pa...	42	6.4	4.1	......	1-1/2	1/2	....	......
(10) Johnstown, Pa...	44	6.4	1.7	......	1-5/8	1/2	4-1/3–12	63
(11) Bear Valley, Calif.	52	0.4	0.4	......	2	5/8	12	28
					2.6	5/8	2	165
(12) Deep Canyon Pipe	52	......	0.2	120.0	2–2.6	......	2-1/8–12	307
(13) Morton Canyon Pipe..........	52	......	0.1	120.0	2–2.6	......	2-1/8–12	158
(14) Warm Spring Pipe	52	......	0.1	120.0	2	......	5-1/2–12	61
(15) Ogden, Utah.....	72	6.0	5.1	250.0	2	5/8	2-7/8–5-1/4	117
(16) Manchester, N. H.	72	0.1	0.1	100.7	4	......	....	38
(17) Tumwater, Wash.	102	2.2	2.1	500.0	......	......	....	170
(18) Floriston, Calif...	108	0.3	0.3	......	3-3/4	3/4	4-3/4–10	50
(19) Madison Canyon.	120	1.4	1.4	600.0	......	7/8	....	25
(20) Madison Canyon.	144	1.4	1.4	......	......	7/8	....	25

(1) 1895. Water Supply and Power Oregon yellow fir. Stave pipe cost city 90 cents per lin. ft. Grade 5.03 ft. per 1000 ft. Leakage 0.099 sec.-ft. At end of 10 years 11% of all staves were replaced and 5-1/2% of line was reconstructed.

(2) 1892. Redwood. After 14 years service was in very good condition; up to that time no staves were ever replaced on account of rot.

(3) 1896. Redwood.

(4) Water Supply. Mostly fir used, some cedar. Average spacing of bands 3-3/4 in.

(5) Beaver Creek, Pueblo. Redwood. One tunnel on line 1533 ft. long.

(6) 1890. Texas pine. Cost of stave pipe, including erecting, $1.365 per lin. ft. Trenching and back filling $0.483 additional.

(7) Little Conemaugh River Water Supply. Washington fir. Average cost of whole line $2.60 per lin. ft. exclusive of trench and supports. Grade 1 to 2 per 1000.

(8) 1903. Water Supply. Washington fir. Grade 0.16 ft. per 1000 ft. Cost $2.25 per lin. ft.

(9) See note under (7).

(10) See note under (7).

(11) 1893. Redwood.

(12) Santa Ana Canal. Cost of pipe $6.20 per lin. ft. Cost complete $11.29 per lin. ft.

(13) Santa Ana Canal. Cost of pipe $7.46 per lin. ft. Cost complete $10.04 per lin. ft.

(14) Santa Ana Canal. Cost of pipe $4.06 per lin. ft. Cost complete $6.51 per lin. ft.

(15) 1897. Pioneer Power Plant. Douglas fir. Pipe is laid in trench 8.5 ft. wide and covered to a depth of 3 ft. on top.

(16) 1874. Power Development. Southern pitch pine. Grade 43.33 ft. per 1000 ft. Said to be the first wood-stave pipe.

(17) Cascade Tunnel Power Plant. 1909. Washington fir. Mostly in rock trench.

(18) 1898. Power for paper mill. Redwood. Mainly laid above ground; bottom portion on earth and stone spalls. After 8 years service there was no evidence of rot.

(19) No. 2 plant of the Madison River Power Co., Norris, Mont. 120-in. pipe finished 1906, 144-in. pipe finished 1908. Redwood.

(20) See note under (19).

15. Flumes

Flumes are used where excavation for a canal is difficult, as on steep hill-sides, where the soil is so loose that an unlined canal is impossible, or where the canals cross rivers or deep depressions and the use of siphons is undesirable. Since much higher velocities can be used than in canals, and the frictional resistance is less, it is possible to use a much smaller area of cross-section. Concrete, iron and wood are the principal materials employed.

Reinforced-Concrete Flumes of varied design are used by the U. S. Bureau of Reclamation on its irrigation projects. No. 1, Fig. 42, shows the type con-

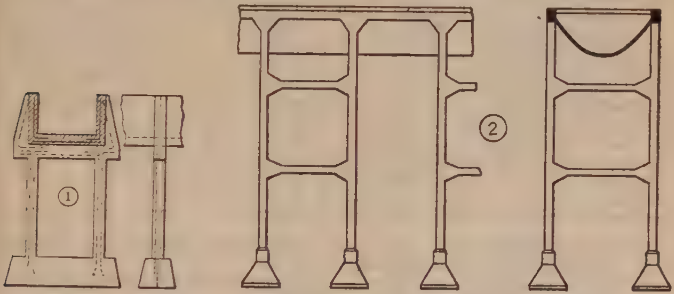


Fig. 42. Reinforced-Concrete Flumes

structed in 1908. This design is more or less standard for flumes up to 72-sq. ft. section, the spans between bents varying from 14 to 30 ft. No. 2, Fig. 42, shows the Brooks Aqueduct, Alberta, consisting of a curved shell of reinforced concrete suspended between two girders on a reinforced-concrete trestle, bents 20 ft. c. to c.

Wooden Flumes are most numerous on account of smaller first cost, although they have short life. Fig. 43 shows a 2 by 3-ft. wooden flume on trestle. The bents are 15 ft. between centers, 4 in. by 4 in. up to 7 ft. high and 6 in. by 6 in. up to 12 ft. high. The stringers are 4 in. by 10 in. by 16 ft. long. The sides and bottom are 2 in. by 12 in. with 5/8-in. by 1-in. splines set in edges and quarter-round molding nailed in corners; the sides are spiked to the bottom by

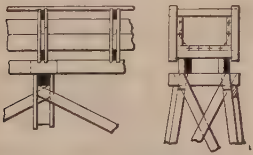


Fig. 43. Wooden Flume

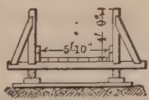


Fig. 44

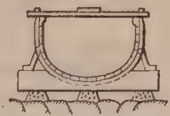


Fig. 45. Wooden Stave Flume

20d nails 1 ft. on centers; the yoke frames are spaced 4 ft. on centers. Fig. 44 is a cross-section of the San Diego flume, California. Fig. 45 shows the wood-stave flume. Flumes across valleys are frequently used as links in earth canals; in such cases it is most important that the junction between the flume and the natural earth bed of the canal be absolutely watertight.

Name and location	Length, miles	Height, feet	Width, feet	Thickness of walls, inches	Thickness of floor, inches	Grade, feet per 1000	Capacity, cu. ft. per sec.
(1) Bear River, Utah....	0.02	4.00	10.00	......	......	....	....
(2) Conconully, Wash...	3.03	2.00	3.00	......	......	1.50	15
(3) Dulgura............	0.94	3.83	4.50	2.0	2.0	0.76	60
(4) Henares, Spain......	0.01	6.20	10.17	......	......	....	177
(5) Illinois, Mississippi..	0.34	7.00	40.00	9.0	6.0	....	....
(6) Kern River, Calif....	0.50	7.17	8.00	3.0	3.0	1.50	470
(7) Nadrai, Ind........	0.20	7.00	130.00	......	......	....	....
(8) Nebraska-Wyoming.	0.04	12.50	34.00	11.5	24.0	....	....
(9) Northern Colorado..	0.50	7.00	25.00	......	......	1.00	1184
(10) Northern Colorado..	0.19	7.00	36.00	......	......	....	1184
(11) North Poudre, Col...	0.64	6.00	8.00	......	......	2.00	....
(12) North Poudre, Col...	0.14	4.50	12.00	......	......	2.00	....
(13) Pecos Val., New Mex.	0.09	18.00	20.00	24.00	48.0	....	1500
(14) Puget Sound, Wash..	10.20	4.50	8.00	2.50	2.75	1.36	280
(15) San Diego, Calif.....	36.00	4.00	5.83	2.00	2.00	....	....
(16) Santa Ana..........	2.16	5.00	5.83	1.75	1.75	....	240
(17) Tieton, Wash.......	....	5.83	8.30	4.00	4.00	....	....

(1) Bear River Canal, Utah. Plate girder bridge of three spans: 60 ft., 45 ft., and 25 ft. Flooring between lower flanges carries canal.

(2) Washington. Temporary flume for water supply used in hydraulic fill dam in 1909. Material used was timber.

(3) Sides and bottoms are redwood. Carried on trestle. Frames spaced 4 ft. c. to c.

(4) Canal in Spain. Steel bridge aqueduct formed by two box girders with iron floor between, attached to lower flanges. 62-ft. clear span.

(5) Canal. Reinforced-concrete bridge on concrete piers. Framework of floor reinforcement is 19 longitudinal I-beams 20 in. deep and 34 ft. 11 in. long laid abreast. Side wall reinforcement is two I-beams 20 -n. deep, 6 ft. apart vertically and trussed.

(6) Kern River Power Plant, Los Angeles. Redwood, seams beveled and calked on sides and flushed with asphalt on the bottom. 1-in. by 6-in. battens used. Carried on concrete foundations.

(7) Aqueduct of the Lower Ganges Canal. Masonry bridge of 15 spans, 60 ft. clear.

(8) Canal crossing Spring Canyon. Bridge carried on three arches of reinforced concrete. Walls are reinforced with 3/4-in. to 1-in. steel rods 6 in. c. to c. in two rows. Floor reinforcement is 1-in. steel rods 6 in. c. to c.

(9) Platte or Highline Canal. Timber.

(10) Platte or Highline Canal crossing Plum Creek.

(11) Colorado. Timber with joints calked with oakum. Constructed on rock bench.

(12) Colorado. Timber with joints calked with oakum. Constructed on trestle.

(13) New Mexico. Irrigation canal built in 1903. Concrete reinforced with T-irons spaced 4 ft. c. to c. Supported on four concrete arches of 100-ft. clear span and 25-ft. rise.

(14) Power canal in Washington built in 1903. Timber, carried on trestle from 6 to 80 ft. high. The maximum curve is 70 deg. and the total curvature is 10 280 deg.

(15) Redwood. Carried on timber stringers and mudsills.

(16) Wooden Stave. Redwood. Held by T-irons 2-1/4 in. by 2-1/4 in., 4 lb. per lin. ft., spaced 8 ft. c. to c. and two intermediate 5/8-in. steel binders. Supported on timber cradle and concrete piers.

(17) Washington. Semicircular reinforced concrete. Transverse reinforcement 3-8-in., steel rods 4 in.c. to c., longitudinal 18 steel rods 1/4 in. in diameter, 12 in. c. to c. Carried on reinforced-concrete trestle.

CANALS

16. Navigation Canals

Canal Prisms (Fig. 46). The cross-section varies with the material passed through, the value of adjacent property, and the facilities to be given to traffic. In earth the side slopes are usually from 2 : 1 to 3 : 1 but there are many examples of steeper and of flatter slopes. The Erie and Manchester canals have in places side slopes of 1 : 1, but these are protected by paving for the full height, and have failed in some cases. In cities it has often been found economical, as in the case of the Erie canal, to provide vertical side walls. In rock the sides are usually made vertical, or nearly so. The recent use of channeling machines has facilitated the formation of smooth vertical sides, with great benefit to vessels. Clearance between the keel of the vessel and the bottom of the canal varies with the form and area of cross-section of the vessel, the width of the channel and the speed desired, as the table shows.

Canal	Depth	Haul-age	Speed, miles per hour	Clear-ance	Authority
	ft. in.			ft. in.	
Erie (enlarged)	7 0	Horse	1.67	1 0	N. Y. S. Eng., 1877
Upper Escaut, St. Quentin, Lateral Oise	7 2-1/2	Horse	1.67	1 3-1/2	Lindley
Dortmund and Ems, Germany	8 2-1/2	Tug	3.10	1 7-1/2	Herman
Merwede, Holland	10 0	Tug	4.66	1 6	Int. Engr. Congress
Brussels to Rupel, Belgium	10 6	Tug	3.73	0 4	
Ghent-Terneuzen, Holland	20 8	Steam	5.40	1 5	Int. Engr. Congress
Amsterdam (before last enlargement)	28 3	Steam	5.59	2 0	Int. Engr. Congress
Suez (before last enlargement)	31 1	Steam	6.20	4 1	Panama Report, 1906

The width of the canal should be at least sufficient to permit two boats meeting to pass each other, but on account of the rapidly increasing resistance to the movement of a boat as the cross-section of the canal diminishes, the width of boat canals is generally greater, so that the area of the wet cross-section may be three to five times the area of the cross-section of the boat. On the New York State Barge Canal, the ratio is designed to vary from 4.76 in earth sections to 4.51 in rock sections.

Level Sections are required for a navigation canal, since strong currents are inadmissible. Difference of level between adjacent sections is overcome by locks, inclines or mechanical lifts. **Inclines** or high mechanical lifts are suitable only in the exceptional cases where the slope of the valley is very steep or where they can be so located as to concentrate a large descent within a short distance. They effect a saving of water as compared with locks. **Locks** are usually best adapted to meet topographical conditions; in general, lifts of locks vary from 5 or 6 ft. to 15 or 20 ft., but some locks have less than 5 ft. and a few have been built with lifts of from 25 to 30 ft. and upward. Larger lifts require more water for lockage and reduce the time required by a boat to overcome a given elevation, but tend to reduce the ultimate traffic capacity of the canal. Locks are used in most places where an incline or high

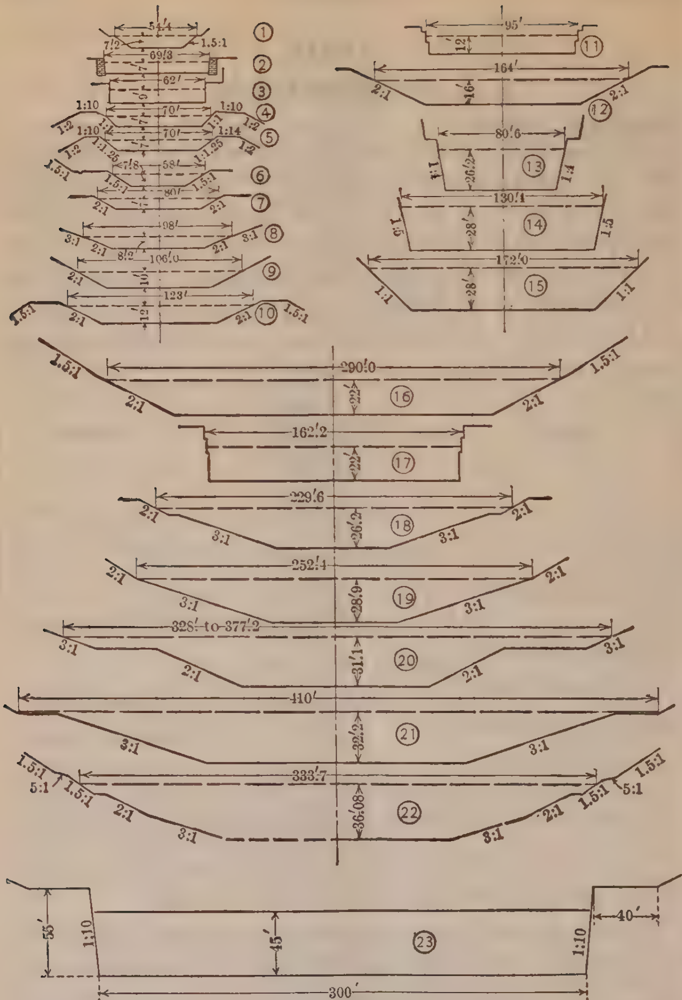


Fig. 46. Cross-sections of Navigation Canals

(1) Marne to Rhine
(2) Erie
(3) Erie
(4) Erie
(5) Erie
(6) Canal du Centre, Belgium
(7) Illinois and Mississippi
(8) Dortmund and Ems
(9) Merwede
(10) N. Y. Barge, Earth sect.
(11) N. Y. Barge, Rock sect.
(12) Soulanges
(13) Corinch
(14) Manchester, Rock
(15) Manchester, Earth
(16) Chicago Drainage, Earth
(17) Chicago Drainage, Rock
(18) Bruges-Maritime
(19) Ghent to Terneuzen
(20) Suez
(21) Amsterdam
(22) Kiel
(23) Panama

mechanical lift would be practicable. If the slope of the ground is not too steep, sections of canal are placed between adjacent locks which enables some saving of water to be effected; where the topography does not permit these intermediate canal sections the locks are connected into a succession of steps forming a "flight."

Widening Prism on Curves. It has not been usual in the United States to increase the width of canals on curves. In the plans of the New York Barge Canal no increase in width was proposed for curves of 5 deg. or less; for 6 deg. the widening proposed was 5 ft. On the Canal du Centre in Belgium the widening was $400/R$ in meters, R being the radius of the curve. At the International Congress for Inland Navigation held at Vienna in 1886, a rule was adopted that the amount of widening should be twice the versed sine of the arc whose chord is equal to the length of the largest boat traversing the canal. This was adopted for the Dortmund and Ems Canal, the length of the boat being taken at 220 ft.

On the Ship Canal from Ghent to Terneuzen, the widening on curves was deter-

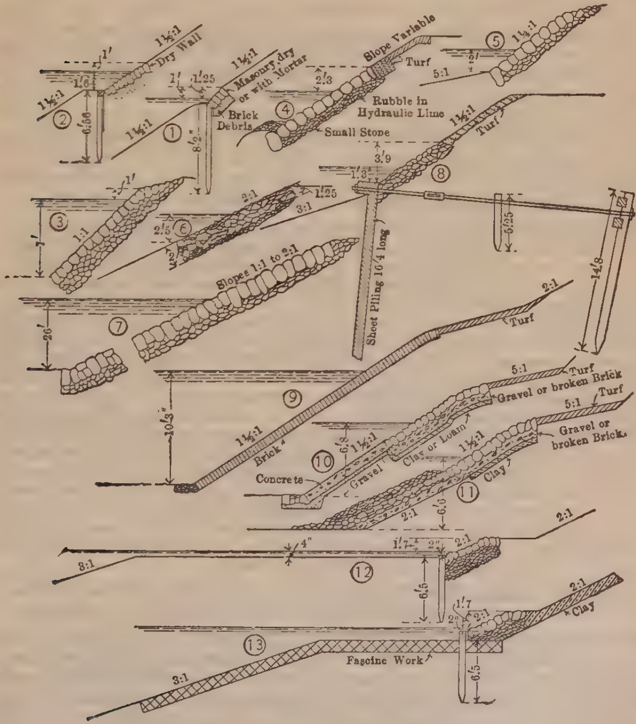


Fig. 47. Methods of Bank Protection

(1) Canals in the north of France. (2) Luik Maastrecht Canal. (3) Enlarged Erie Canal, Eastern Division. (4) Canal from Maastrecht to Bois le Duc. (5) Dortmund and Ems Canal. (6) New York Barge Canal. (7) Manchester Canal. (8) Ghent-Terneuzen Canal. (9) (10) (11) Kiel Canal. (12) (13) Amsterdam Canal.

mined by the formula $4\ (R - \sqrt{R^2 - 200^2})$ in feet, R being the radius of the curve. On the Kiel Canal, the widening was determined by the formula $26 - 0.01\ R$ in meters, or $85.3 - 0.01\ R$ in feet. On the Kiel Canal it has been found by experience that "it is not so much a large radius that is required as a proper widening of the cross-section." This view is abundantly confirmed in the United States regarding vessels moving under steam power.

Protection of Side Slopes (Fig. 47). In earth some form of protection is usually advisable, particularly in sandy soils, to protect the slopes against wave action caused by passing boats. The extent required depends upon the character of the traffic. At the Dortmund and Ems Canal, traversed by boats drawing 6 ft. 7 in. at a speed of 3.1 miles per hour, it has been found sufficient to extend the protection 2 ft. under water: one of the several forms used is shown in Fig. 47. On the New York Barge Canal, the protection is extended 2 ft. 6 in. or more under water. In larger canals, traversed by large ships and at higher speed, the protection is usually extended 5 to 8 ft. under water, and terminated at its lower edge on a berm. Of the three figures relating to the Kiel Canal, the last one is designed for use when the canal is filled with water, and the stones cannot be laid with regularity below water level without unwarranted expense. In some cases the protection has been designed to serve the further purpose of maintaining the bank at a steeper slope than it would otherwise sustain, as in the standard section of the Erie Canal and at places in the Manchester Canal. This is decidedly objectionable for propellers, as the slope walls are liable to break the wheels and it is better practice in nearly all cases to give the earth below the belt requiring protection from wave action a slope at which it will stand.

When the protection is of stone it usually consists of a paving of rubble masonry or selected stone laid dry with as close joints as practicable in order to prevent washing through the joints of the paving, the earth is first covered with a layer of gravel or of small broken stone. A cheaper form of revetment has been used successfully at the Soulanges Canal, and has been adopted for the New York Barge Canal, where specifications describe it as consisting of three grades of stone; first one-quarter of the whole volume to be quarry waste, stones ranging in size from 9 in. to 15 in. in longest dimension; second, one-quarter of stones from 4 in. to 9 in. in longest dimension; and, third, one-half of stones from 1 in. to 2 in. in diameter. The larger stones are bedded on smaller ones and rammed firmly in place; the remainder of the small stones are used to fill voids and form a smooth surface. No flat stones are permitted. On the eastern section of the Illinois and Mississippi (Hennepin) Canal a trench was cut in the face of the embankment, extending from 2 ft. above normal water level to 1 ft. below it, and filled with rubble laid roughly by hand. On other sections no protection was provided until after the canal was put in use and a berm had been formed by wave-wash; stone was then brought in on barges and deposited in the trench thus formed. Bricks have been used in various forms; also concrete blocks and reinforced concrete. Where water seeps into the canal, revetments of masonry or of concrete require to be back-drained by being laid on sand or gravel or by other means.

Control of Leakage from Canal (Fig. 48). Where canals are formed in sandy soil or other permeable material, the loss of water by percolation or seepage may be so great as to endanger the adequacy of the water supply. If the water supplied to the canal carries silt the seepage will become less as the voids in the sand gradually become filled with fine particles of silt carried by the water, or with silt, sawdust or similar material thrown into the water near the points where leakage is greatest; but in such material it is usually advisable to take means to prevent or reduce seepage during the construction of the canal by interposing less permeable material, and where the canal is carried on an embankment this is usually essential to insure the safety of the work. In such a case the canal should have an impermeable lining, which is usually of clay with sand or gravel well puddled or rammed. In order to prevent shrinkage cracks above the water line and in the sides and bottom when the water is drawn out, and also to protect the puddled material from abrasion by boats, the puddle is usually covered by a layer of ordinary soil.

In the canal from the Marne to the Rhine the watertight layer contained three parts of sand to two parts of clay, the mixture, after spreading, being sprinkled and rammed to two-thirds of its original volume. In the navigable feeder of the Arroux, Canal du Centre, France, the mixture contained 65 to 70% of clay; the material in the bottom was compacted by grooved rollers drawn by horses, and in the slopes was rammed; the material was spread in 4-in. layers and reduced in volume 40% by rolling or ram-

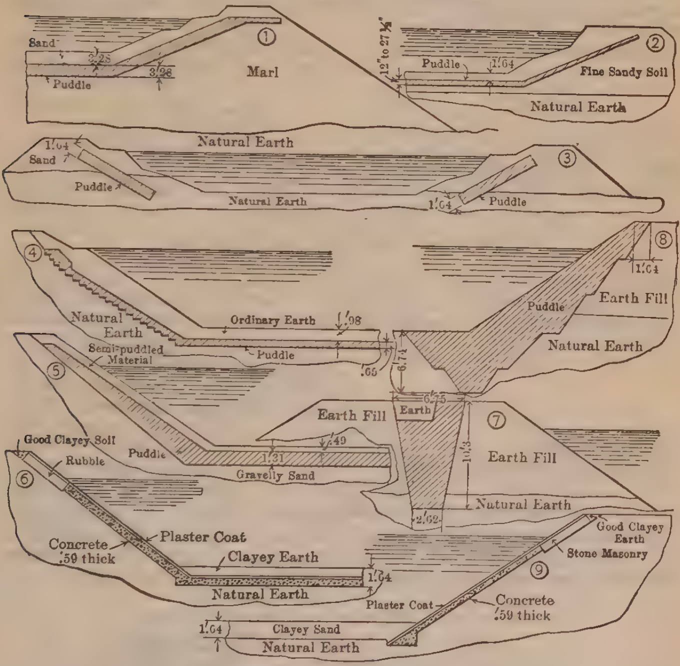


Fig. 48. Methods of Tightening Canal Prisms.

(1) (2) (3) Dortmund and Ems. (4) Marne to Rhine. (5) Arroux. (6) (7) (8) (9) Canals du Centre, Belgium

ming. In the Dortmund and Ems Canal, where the canal was in embankment, the material of the embankment was principally marl, and the watertight lining was of the form shown in the figure, the thickness of the puddled material increasing with the height of the embankment. In the Canal du Centre, Belgium, a lining of concrete has been used successfully, protected on the bottom of the canal by a fill of clayey earth. This is suitable only where the bank is of firm material not liable to irregular settlement, and is liable to injury by boats. In the canal from the Marne to the Rhine concrete in banks was also covered with earth.

Where the bottom of the canal is subjected to uplift pressure when the canal is laid dry, the watertight layer is likely to be breached; underdraining is sometimes practicable. The case may arise where the head of the inflowing water is above the bottom of the canal but below the water surface desired; each case of this kind must be studied as it arises. A watertight lining of the sides and a silt lining of the bottom may be suitable in such a case. Where the bottom of the canal is in excavation and the material sufficiently watertight, where the sides are embankments, seepage may be prevented by a puddle lining on the slope or by a puddle wall in the middle of the embankment. When placed on the slope the thickness should be great enough to permit considerable abrasion by boats without cutting through it. This is avoided by forming a puddle wall in the middle of the embankment, but the fill on the water side is likely to become saturated and then may contribute little to the stability of the embankment, or may even reduce it. Care must be taken to remove all loose or yielding material and to avoid forming a permeable joint between the original ground and the fill.

Drainage of Adjacent Territory. A canal in a river valley is so located that its banks may be above the highest flood level. Small streams, artificial

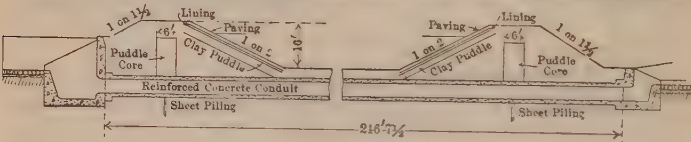


Fig. 49. Dive Culvert, New York Barge Canal

drains, or storm water may be received directly into the canal or passed under it through culverts (Fig. 49). When circumstances permit, the latter is preferable; the principal objections to draining into the canal are the resulting silting, which will involve continual expense, and the variations produced in the water level. Where practicable, water, if silt-bearing, should be made to pass through a settling basin before entering the canal. If the amount of water flowing into the canal is considerable, the resulting current is objectionable. The surplus water received into the canal is discharged over **waste-weirs** (Fig. 50*a*) or through **waste-gates.** Where the requisite control can be

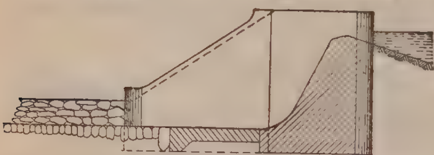


Fig. 50*a*. Waste-Weir, New York Barge Caaal

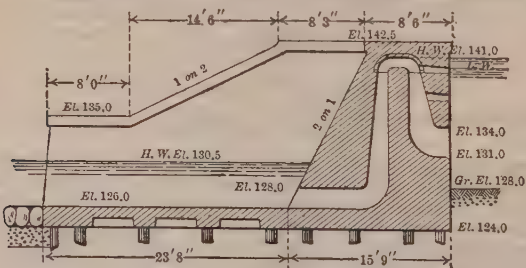


Fig. 50*b*. Siphon Spillway, New York Barge Canal

given by a weir it has the advantage of being automatic; but this may at times cause waste, since a high wind blowing in the direction of the canal, or a succession of boats moving in one direction, may cause a rise at one end of a canal level. Waste may also result from the difficulty of adjusting the supply to the summit level to the varying demand for lockages; to overflow from short levels; to unequal lift at locks, etc.

In the New York Barge Canal the crests of the waste weirs are at low water elevation. Channels are provided for placing flashboards to raise the crest of the waste weir if required. On this canal a siphon spillway shown by Fig. 50) 'has been used.

Waste-gates are usually placed with their sills at canal bottom and are useful for draining the level when desired for repairs.

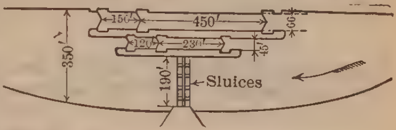


Fig. 51. Irlam Locks, Manchester Ship Canal

At the Manchester Canal where large amounts are received into the canal at times of floods, each level is controlled by Stoney sluice gates placed in a sluiceway adjacent to and parallel with the locks. The sills are at the same elevation as the sill at the upper end of the locks; the sluice gates are each of 30-ft. opening, and can be raised about 20 ft. The Irlam sluices (Fig. 51) can discharge 26 000 cu. ft. per second.

Large streams are usually crossed by aqueducts forming part of the canal, but usually reduced in width to the requirement for a single boat. The superstructure, or " trunk," may be of wood, masonry or iron. The crossing of the Erie Canal over the Mohawk River at Rexford Flats is a good example of a trunk of wood; of the same canal over the Genesee River at Rochester is a good example of a trunk of masonry; both of these structures were in use for many years. The Bridgewater Canal is carried over the Manchester Canal on a swing bridge, the trunk is of wrought iron, has a clear width of 19 ft., and a depth below ordinary water surface of 6 ft. and a free-board of 1 ft.

17. Water Required for Operation

Filling the Prism of the Canal. The prism of the canal may be emptied either for repairs or as a result of accident. In dealing with the question of water supply for the proposed New York Barge Canal, E. Kuichling assumed that the prism of the summit level would be laid dry once per year; also that, in addition to the volume of the prism, an equal amount would be required to saturate the adjacent soil to its normal condition, but this additional amount depends on the watertightness of the wetted contour of the canal prism, and, if this is not well provided for, by the porosity of the adjacent ground. In the case considered by Kuichling it was contemplated to line the canal with clay where formed in permeable ground.

Evaporation and Seepage. For the New York Barge Canal, Kuichling estimated that the evaporation from the water surface would be 0.30 in. per day, to which he added 10% as a liberal provision for consumption by aquatic plants, or a total of 0.33 in. evaporation per day from the surface of the canal.

In the summer of 1900 observations were made to determine the loss by percolation from various sections of the Erie and Champlain canals. In some cases where the canal had been deepened within a short time before the gagings were made the loss was large, running up to 773 cu. ft. per mile per minute; in other cases, even where percolation through the banks could be seen, the gagings indicated little loss, and, on the whole, the results were not of great value, but, rejecting the abnormal results, a loss from percolation of 100 to 150 cu. ft. per mile per minute was indicated.

Kuichling finally estimated that the loss from evaporation and seepage from the Barge Canal would appregate 4-1/2 in. in depth per day, equivalent to 169 cu. ft. per mile per minute, but this was based on the assumption that clay linings would be provided in permeable soil. The thickness of the puddled material was intended to be

3 to 4 ft. on the slopes on embankments and 1-1/2 ft. in excavation where the materials were of a nature to permit seepage. The thickness on the bed of the canal was to be 1-1/2 ft. in all cases where puddle was deemed necessary. The canal was to be 123 ft. wide at the water line, 75 ft. at the bottom and 12 ft. deep.

G. W. Rafter estimated the probable percolation from the summit level of the proposed deep waterway from the Great Lakes to tidewater at 330 to 440 cu. ft. per mile per minute. At the divide near Rome, N. Y., the water in the canal would be slightly below the level in the adjacent streams, and about 15 ft. above at the ends of the summit level. No puddle lining was intended for the canal.

Measurements of Loss of Water from American Canals by Evaporation and Seepage, Including, in Some Cases, Waste at Structures

Canal	When observed	Length of observed section, miles	Width at surface, ft.	Width at bottom, ft.	Depth, ft.	Loss per minute per mile, cu. ft.
Original Erie Canal:						
Lodi to Little Falls	1841	{61.8 20.7	40.0 70.0	28.0 52.5	4.0 7.0}	67.5*
Palmyra level	1841	8.3	40.0	28.0	4.0	108.8*
Clyde level	1841	27.7	40.0	28.0	4.0	35.3*
Lockport to Pittsford	1841	69.0	40.0	28.0	4.0	73.0*
Western Division	1841	122.0	40.0	28.0	4.0	74.5
Enlarged Erie Canal:						
Tonawanda to Clyde	1858	126.0	70.0	52.5	7.0	200.0*
Spencerport to Rochester	1877	12.0	72.5	53.7	7.5	190.0
Chenango Canal:						
Summit level	1838	22.0	40.0	28.0	4.0	65.5
Genesee Valley Canal:						
General	1846	....	40.0	28.0	4.0	19.0
General	1846	....	40.0	28.0	4.0	25.0
General	1846	....	40.0	28.0	4.0	30.0
Rochester to Mt. Morris	1854	36.0	40.0	28.0	4.0	37.0
Mt. Morris to Oramel	1854	42.0	40.0	28.0	4.0	153.0
Rochester to Mt. Morris	1855	36.0	40.0	28.0	4.0	21.9
Mt. Morris to Dansville	1855	16.0	40.0	28.0	4.0	23.8
Sonyea to Portage	1855	15.0	40.0	28.0	4.0	25.6
Portage to Oramel	1855	22.0	40.0	28.0	4.0	43.0
Chesapeake and Ohio Canal	1838	40.0	50.0	32.0	6.0	60.0
Chesapeake and Ohio Canal	1838	40.0	50.0	32.0	6.0	64.0
Sandy and Beaver Canal, O.	....	....	38.0	26.0	4.0	13.0
Ohio Canal:						
Fort Wayne summit	1839	54.0	50.0	36.0	5.0	52.0
Fort Wayne summit	1847	54.0	50.0	36.0	5.0	46.0
Various places	1882	....	50.0	36.0	5.0	33.0
Illinois and Mississippi Canal	1894	....	80.0	60.0	8.0	120.0
Illinois and Mississippi Canal	1895	....	80.0	60.0	8.0	60.0
Morris Canal, N. J.:						
Mountain View to Stonehouse Plains	1878	4.8	40.0	25.0	5.0	211.7
	1879	8.0	40.0	25.0	5.0	170.8
	1879	8.0	40.0	25.0	5.0	164.1
	1879	4.7	40.0	25.0	5.0	3.8

* For these cases evaporation, seepage, and waste were measured. For the other cases only evaporation and seepage were measured.

The excessive loss of 4144 cu. ft. per mile per minute from the Marne-Saone Canal is said to be due to fissures in the rock in which a section of the canal is excavated. A similar loss, and for the same reason, has been met in the Glen Falls feeder for the Champlain Canal. Such leaks are difficult to close after the water is once admitted to the canal. On the other hand, earth sections of canals become tighter with use. Some of the large losses shown in the table occurred when the canal was new or had been deepened within a short time.

Percolation, infiltration or seepage is usually much greater in amount than evaporation. The combined loss from evaporation, seepage and waste is usually measured together by current observations at two or more cross-sections of the canal, and simultaneous measurements made of waste. The loss by evaporation plus seepage is then given by subtraction and usually reported together. Evaporation can be estimated with reasonable correctness and seepage determined within a small range of uncertainty. The table on p. 1578 is condensed from Kuichling's report on Water Supply Canal for New York Barge Canal.

The loss by evaporation, seepage and leakage from the Illinois and Mississippi Canal is stated to be less than 60 cu. ft. per mile per minute.

In some of the European canals where the water supply is limited, and particularly where the summit level is maintained by pumping from the lower levels, greater expense is incurred to reduce seepage than is usual in the United States. In the Dortmund-Ems Canal the loss by percolation and evaporation amounted to 32 cu. ft. per mile per minute of which 9 cu. ft. was estimated to be due to evaporation and 23 cu. ft. to percolation. This was rather more than had been anticipated but it is expected that it will be reduced by silting. The watertightness of the prism was of much importance because the water on the upper levels was supplied in large part by pumping. Lindley states that the mean value of the loss by evaporation and percolation is estimated for the whole of France at about 40 cu. ft. per mile per minute. In a given permeable soil the amount of seepage varies with the width and depth of the canal, but no satisfactory expression of the relation has been advanced.

Leakage at Aqueducts, Culverts and Waste-Gates. The amount of leakage at these structures depends principally upon the care taken in design, construction, maintenance and operation, and can be limited to a small amount. In a report made in 1837 by W. H. Talcott the leakage of waste-weirs and aqueducts on 22 miles of the Chenango Canal (depth 4 ft.) was given at 220 cu. ft. per minute, or 10 cu. ft per mile per minute. Kuichling gives an estimate of 1.6 cu. ft. per mile per minute on certain old canals in France. The losses will obviously depend on the depth of water and may be assumed to vary with the square root of the depth as an approximation. Kuichling estimates for the New York Barge Canal (depth 12 ft.) 12 000 cu. ft. per day for each culvert and waste-gate, and 96 000 cu. ft. per day for each aqueduct, which appears to correspond approximately with the observations of Talcott, without allowance for increased depth. With modern methods the older class of masonry should be improved upon.

Leakage at Locks. This occurs at the valves for filling and emptying the lock and at the lock-gates. Seepage around the locks should be prevented by suitable construction or reduced to a negligible amount. From observations at the Chenango Canal (in 1839) W. H. Talcott estimated leakage at a canal lock of 11-ft. lift at 447 cu. ft. per minute or 643 680 cu. ft. per day. Kuichling gives an estimate as low as 14 100 cu. ft. per day on certain small French canals. Measurements of leakage around the filling valves of the Weitzel lock, St. Marys Falls Canal, made by J. Ripley in 1899, gave a coefficient of 0.49 for use in the formula $v = C\sqrt{2gh}$ or $v = 4\sqrt{h}$ nearly. The valves were of the butterfly type, 8 ft. by 10 ft., turning about the 10-ft. axis. The mean width of opening around the closed valves was about 3/8 in. The valves had been in use about 18 years, and saving of water was not important. This type of valve is wasteful and should not be used where saving of water is important, although there should be no difficulty in reducing the leakage to one-half the amount observed by Ripley; its advantages are in simplicity, low cost of maintenance, and facility of operating. Leakage at the gates results from imperfect fitting; in the case of the ordinary mitering gates, the gates may be too long, in which case one or both (usually one) will not be in contact with the sill when closed, resulting in an opening, approximately triangular, extending from the miter to or toward the quoin; or the gates may be too short (usually from wear), in which case an opening of similar form will exist between the ends of the gates, extending from the sill upward.

For the gates of boat canals, maintained in good condition, it will be sufficient to assume an opening between the sill and one gate 1/2 in. wide at the miter and gradually tapering to nothing at the quoin. For ship canals the maximum width of opening may be doubled. While locks are being operated the head on the opening through which water is being wasted may vary from nothing, when the filling or emptying of the locks begins, to the full lift of the lock when the operation is completed; but as a safe provision, which covers small leaks between the quions and the gates and elsewhere, the full head is usually taken.

Using the coefficient for leakage at valves found by J. Ripley, and computing the leakage at gates as just indicated, the leakage at the Weitzel lock (60 ft. wide at gates, 20-ft. lift) is estimated at 72 cu. ft. per second, but the valves are not close-fitting. The various commissions of the old and new French Panama Canal Company allowed 35 to 53 cu. ft. per second for each lock, the lifts being about 30 ft. Kuichling estimated the leakage at locks from the summit level of the Barge Canal at about 23 cu. ft. per second, the lock at one end having a lift of 20-1/2 ft. and the lock at the other end 16 ft. He followed the method above outlined, assuming 1/4-in. opening around each valve and an opening of 1 in. at the miter between the sill and one gate of a pair.

Loss over Waste-Weirs. It is impracticable to adjust the supply of feed-water to the varying requirements of traffic; therefore the crest of the waste-way must be fixed at such a height that the canal will always have the required depth of water, or the crest may be provided with flashboards, permitting a temporary surplus of water to be held until wanted, or the water must be wasted. Waste may also result from the action of the wind blowing toward the weir, from a succession of boats moving toward it, etc. It is customary to provide a limited amount of storage by fixing the elevation of the crest of the wasteway somewhat above the level required for traffic, but it is restricted by the cost of the corresponding increase in height of the canal banks, lock walls, etc. In general, the best form of wasteway for ordinary conditions is a simple overflow weir, because its action is automatic; its length must be fixed with reference to the allowable rise of water in the canal.

Data regarding existing or projected canals vary greatly. For the eight waste-weirs on a 22-mile level of the Chenango Canal W. H. Talcott observed (in 1839) a waste of 96 cu. ft. per minute; for the proposed deep waterway from the Great Lakes to tidewater, G. W. Rafter estimated the waste for the summit level at 150 to 200 cu. ft. per second = 9 000 to 12 000 cu. ft. per minute, but his large esimate resulted from the provision of a waste-weir long enough to pass the floods of the Mohawk which was to be taken into the summit level. For the summit level of the New York Barge Canal Kuichling assumed a waste-weir long enough to control the water level within about 5 in. with a water supply sufficient for normal amount of lockages flowing in but with no lockages, and estimated the discharge over the waste-weir from this and other minor causes of waste to average 46 cu. ft. per second (400 000 cu. ft. per day) from each end of the summit level.

Power for Operating Locks. This is usally developed by water from the higher level. Power may be required for (1) opening and closing valves; and (2) hauling boats into or out of the lock. (1) For opening and closing of gates and valves, occupying only a small part of the time, accumulators may be employed. At the Dortmund and Ems Canal, where the locks are

28-1/4 ft. wide with 9.84 ft. on the sills and maximum lifts of 20-1/3 ft., the lock-gates and sluice valves are moved by power; the greatest amount required for any single operation is 7.2 hp. At the Weitzel lock, St. Marys Falls Canal, there are two turbines of 25 hp. each; one is sufficient to accumulate power as quickly as necessary, the other is a duplication for safety. The largest gates are 39 ft. high, the lock 60 ft. wide at the gates; two valves for filling the lock and two for discharging, each valve 8 ft. by 10 ft. The power required being given or estimated, the corresponding water consumption can be approximated in any given case. (2) The power required for haulage at the locks of the Erie Canal was given at about 8 hp., the immersed cross-section of the boats being 17.5 ft. × 6 ft. = 105 sq. ft. For the Barge Canal and boats of 25 ft. × 10 ft. = 250 sq. ft. immersed cross-section, Kuichling proposed 20 hp. as sufficient, the power to move a boat at a given speed being assumed to vary with the immersed cross-section.

Power Required for Lighting. For the New York State Barge Canal from 10 to 14 arc lights each of 750 cp. have been used. The United States canal at Sault Ste. Marie has 13 lights, each of 1200 nominal cp. (300 actual) for the duplicate locks, and one similar light each 400 ft., approximately, on each side of the canal. The Canadian canal on the opposite side of the St. Marys River has 8 lights, each 1200 cp. nominal, at the locks, and one similar light on each side of the canal every 300 ft. The length of each of these canals is approximately 1-1/2 miles. The Kiel Canal is lighted by incandescent lights placed opposite each other on the canal banks, spaced on curves at distances $R/15$ between pairs up to a maximum of 820 ft., on tangents.

Practice is not well defined as to the extent of lighting desirable, and the only practicable course is to prepare a plan for lighting believed to be the best suited to the case in hand, from which the amount of water required can be computed. The fall at the locks is usually the most convenient source of power, in which case the amount of water consumed is an element of the quantity to be supplied to the upper level, but in many cases the electric current may be supplied from a distance, between the reservoir and the canal, requiring no additional water; in other cases it may be found economical to buy current.

Under present conditions arc lighting is estimated to require 0.4 hp. per 1000 cp. with efficient plant management, and incandescent lighting with carbon filament 4.2 hp. per 1000 cp. These are approximations, and for canal illuminations the power should probably be doubled. For the New York State Barge Canal, Kuichling estimated a consumption from the summit level of 21 cu. ft. of water per second or 6 hp. for four arc-lights, each of 1000 cp.

Water Consumed for Lockages. Let Q = area of lock multiplied by lift of lock, D = displacement of vessel, N = number of locks in flight. There are four possible conditions of lockage affecting the consumption of water. Case 1: A boat ascending after another has descended will draw from the upper level, past the upper gate, $V = NQ + D$. Case 2: A boat descending after another has ascended will force into the upper level, past the upper gate, $V = D$, the draft from the upper level being $- D$. Case 3: A boat ascending following another that has ascended, will draw from the upper level, past the upper gate, $V = Q + D$. Case 4: A boat descending following another which has descended will draw from the upper level, past the upper gate, $V = Q - D$. The ascending boats draw from the summit level their displacements, and the descending boats force their displacement back into the summit level. Ordinarily no material error will be introduced if the displacements be assumed equal, therefore D may be eliminated from the formulas, which become:

Case 1, $\mathbf{V} = \mathbf{NQ}$ (boat ascending); Case 2, $\mathbf{V} = \mathbf{Q}$ (boat descending);
Case 3, $\mathbf{V} = \mathbf{Q}$ (boat ascending); Case 4, $\mathbf{V} = \mathbf{Q}$ (boat descending).

If the summit level is reached by a single lock and the number of boats passing in one direction is equal to the number passing in the opposite direction the minimum consumption of water would be given by alternating ascending and descending boats, cases 1 and 2; in case 1, N becomes 1, and the two boats of a pair would consume $Q + O = Q$, or an average per boat of $1/2\ Q$. If the boats follow each other the average consumption per boat is Q. The actual consumption will be between these extremes $1/2\ Q$ and Q since it will frequently be impracticable to delay a boat which is awaiting another from the opposite direction.

If the summit level is reached by a flight of two locks, ascending and descending boats alternating require $2\ Q$ for the ascending boat and O for the descending boat, or an average of Q, and the same average amount if the boats follow in the same direction.

For a flight of three locks two boats alternating will require $3\ Q$ or an average for the two of $3/2\ Q$, and the average for all lockages will equal or exceed Q. When the number of locks in a flight exceeds two, economy in use of water for lockages results from duplicating the flights of locks, using one flight for up-bound and the other for down-bound boats, the consumption for each boat being Q.

Water for lockages may be economized by the use of side ponds receiving the water discharged from the lock chamber during the earlier part of the process of emptying and returning it to the lock chamber during the earlier part of the operation of filling At the Dortmund and Ems Canal two pairs of side ponds are used, at different elevations, two ponds at each side of the lock and about 50% of the lockage water is saved.

18. Speed of Canal Boats

Where steam power is used the speed is usually limited by regulations. The power required to move a boat increases rapidly as the ratio (wet cross-section of canal) ÷ (wet cross-section of boat) decreases. On boat canals the boats are usually of the largest dimensions permitted by the locks. On ship canals the tendency to uniformity is not well marked, and is affected or controlled by numerous conditions other than lock dimensions. The table gives data for various canals.

Canal	Cross-section of of canal			Cross-section of boat		Ratio of cross-section	Power	Speed, miles per hour
	Surface width, ft.	Bottom width, ft.	Depth, ft.	Beam, ft. in.	Draft, ft. in.			
Erie, Original...	40.0	28.0	4.0	{13 6 / 14 6}	2 6	{4.03 / 3.13}	Horse / 2 Horses	2.00* / 2.00*
Erie, Enlarged..	70.0	56.5	7.0	17 5	6 5	4.20	2 Horses	2.00*
Erie, Enlarged..	......	......	......	17 0	5 6	4.72	Steam	2.54†
Dortmund-Ems.	98.5	59.0	8.2	27 0	6 7	3.82	Steam	2.50‡
Dortmund-Ems.	98.5	59.0	8.2	27 0	5 9	4.41	Steam	3.10‡
Ghent-Ter., Belg.	183.7	55.8	21.3	....	......	3.92	Steam	6.21§
Ghent-Ter., Holl.	155.0	56.0	20.7	....	......	....	Steam	5.40‡
Suez..........	......	......	......	....	......	3.10	Steam	6.21‡
Kiel..........	......	......	......	....	......	....	Steam	9.32‖
Teltow.........	101.0	66.0	6.6	....	......	....	Electric	2.5–3.1*
Brussels-Rupel..	108.0	49.0	10.5	23 9	10 2	3.43	Steam	3.73
Merwede.......	107.0	66.0	10.2	34 3	8 6	3.06	Steam	4.75‡
Manchester.....	200.0	120.0	26.0	....	......	4.00	Steam	6.90¶

* Actual average is somewhat less. † By trial. ‡ Permitted by rules.

§ By regulations; actual speeds allowed on tangents 7.4 miles per hour. Average speed on sharp curves 3.7 miles per hour.

‖ Permitted by rules for small vessels. Large ships cannot attain 6.2 miles per hour.

¶ By regulation; enforced for large ships. Small vessels specially licensed to run 9.2 miles, but attain 11.5 to 15 miles per hour.

The New York Barge Canal with a wet cross-section of 1188 sq. ft. is intended for steam barges having a wet cross-section of 250 sq. ft. when loaded, giving a ratio of 4.75. In concluding a report on the probable speed in such a traffic, E. Sweet expressed the opinion that a higher speed than 3 miles per hour would be "economically inadmissible."

In France the speed limit on canals is considered to be 1.67 miles per hour; traction by horses. On the Northeastern canals where on account of congested traffic a concession for horse traction was instituted in 1875, the contractor is bound to maintain the speed of 1.24 miles per hour for ordinary service and 1.8 miles per hour for accelerated service. Interesting experiments have been made on the French waterways on the tractive force required by various types of boats at various speeds and in various channels. The types of boats were: " Peniche flammande,"—box bow and stern, only slightly rounded where the ends join the sides and bottom; " Flute," cut-away bow, rising in a curve from the keel; "Toue," pointed peaked bow and square box stern; " Prussian barge," pointed bow and stern; " Margotat," punt-shaped. The boats were towed in a river of large section (practically an unlimited cross-section) at a uniform speed of 3.35 miles per hour to determine the influence of the form of boat on tractive force required.

Type of boat	Length, ft.	Beam, ft.	Draft, ft.	Coefficient of displacement	Gross tonnage	Traction force, lb., at 3.35 miles per hour	
						On boat	Per gross ton
Peniche flamande....	126	16.4	5.25	0.99	305	1530	5.03
Flute..............	126	16.4	5.25	0.95	283	785	2.77
Flute..............	126	16.4	4.27	0.95	250	695	2.78
Toue...............	126	16.4	5.25	0.97	299	590	1.97
Prussian barge......	113	16.4	4.27	0.94	210	410	1.95
Margotat...........	72	16.4	4.27	0.82	116	310	2.67

Another set of experiments was made in various canals with the " flute " at 4.27 ft. draft at various speeds. The coefficient of resistance is the factor to be applied to the tractive force required in an unlimited cross-section in order to obtain the tractive force required in the canal named. The ratio of cross-section of channel to cross-section of boat is represented by r.

	Depth ft.	Area, sq. ft.	r	Coefficient of resistance for speed in miles per hour				
				0.56	1.12	1.67	2.23	2.79
Diversion of the Yonne at Joigny..................	8.20	450	6.39	.....	.....	1.38	.....	.....
Canal de Bourgogne........	7.95	320	4.54	.....	.....	1.98	.....	.....
Canal de la Curé (Yonne)...	6.85	250	3.57	1.92	2.13	2.38	2.75	3.17
Canal de St. Dizier at Vassy.	6.80	207	2.94	.....	.....	3.32	.....	.....
Canal du Nivernais.........	5.60	207	2.94	.....	.....	3.82	.....	.....

The resistance at 1.67 miles per hour in the unlimited section was 172 lb. and in the Diversion of the Yonne at Joigny with r = 6.39 it was 238 lb., or 38% greater; but with r reduced to 2.94 it increased 232% in one case and 282% in another, the greater resistance being in the shallower canal, a diminution of 1.2 ft. in depth or 48% in clearance under the boat, increasing the resistance 15%.

Experiments to determine influence of form of boat and cross-section of canal on cost of transportation were made in 1898 on the Dortmund and Ems Canal, and in 1906 on models in the government experimental tank in Berlin comparing cross-sections of equal area, one with a greater central depth, also cross-sections of different areas, and two forms of boats moving at different speeds with a view of determining the most economical conditions of transport for assumed amounts of traffic. The effects of draft and speed were found to be similar to those indicated by the French experiments and for the conditions assumed r = 4.2 to 5.3, and boats carrying 667 tons one way and 1/5 as much the other way, with mechanical haulage, a speed of 3.1 miles gave economical transport.

19. Locks and Lock Gates

Locks are usually employed to overcome differences of level. In the earlier canals in the United States they were frequently built of wood, but these were soon replaced with stone masonry, the face being cut accurately to true lines and surfaces, and until within the last 25 years practically all canal locks in the United States were of stone masonry. Now concrete made with portland cement has come into use. Brick masonry laid in cement or hydraulic lime has been used largely in European canals, either for the entire mass of masonry or for facing, except at quoins and other exposed places, where heavy stone blocks have been built into the brickwork. Bricks for facing are of specially good quality.

In the locks of the Kiel Canal the floors and lower portion of the side walls up to the floor level are of concrete; above the floor the side walls are of brick masonry built with large openings filled with concrete of 1 cement to 8 sand. Fender courses, lock-sills, angles of gate recesses, and guide grooves for the pontoon gates are of granite. The

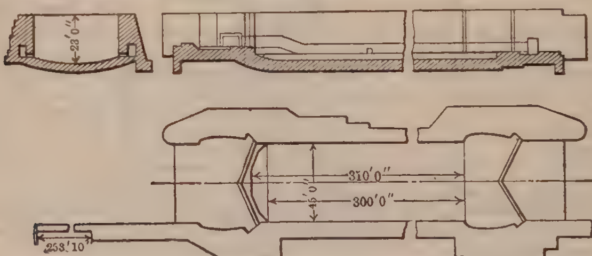


Fig. 52. Barge Canal Lock

locks of the Manchester Canal are of concrete, with brick facing above low-water and stone copings; miter sill masonry facing of sandstone, the coping of specially hard quality, but stones in contact with gates (no timber sill) are of granite; brick and stone are used at other exposed places.

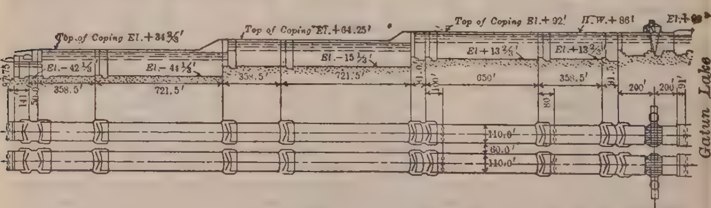


Fig. 53

The Poe and Weitzel locks at Sault Ste. Marie are of limestone masonry. The new locks are of concrete. Fig. 52 illustrates a lock of the New York Barge Canal. Fig. 53

shows the principal dimensions and Fig. 54 the cross-sections of the Gatun locks of the Panama Canal.

Foundations and Floors of Locks. Where rock foundation is not available, the lock is usually founded on piles, and the entire foundation enclosed within

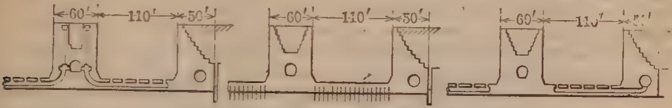
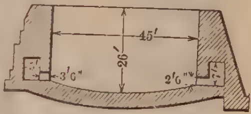


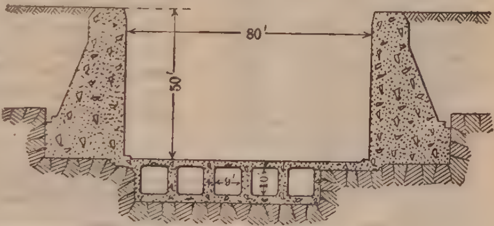
Fig. 54. Gatun Locks, Panama Canal

sheet piling and with lines of sheet piling under the sills. In American practice the floors of small locks have usually been built of wood, but with increased width the use of timber is not advantageous. In foreign canals, lock floors are usually of concrete in the form of an inverted arch. The thickness of the floor is usually sufficient to resist the full upward pressure of water when the lock is pumped out, but in some cases weep holes have been formed in the floor to relieve this pressure.



(a) Barge Canal

Where founded on rock, the thickness of the floor is more frequently reduced and may be, in great part, omitted if the rock is especially sound and free from fissures, but usually a covering is required, either to protect the rock or to give a smooth surface. In the St. Marys Falls Canal this protection is of timber and concrete; the floors are firmly bolted to the underlying rock. With the increasing price of timber, the reduced cost of portland cement, and the introduction of reinforced concrete, the economy formerly effected by the use of timber is reduced



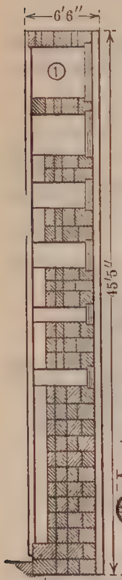
(b) St. Marys Falls Canal

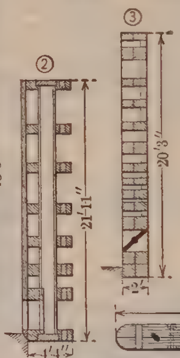
Fig. 55. Locks of Large Canal

Facilities for Filling and Emptying Lock. Small locks are usually filled and discharged through valves in the gates or through short culverts around the quoins. This causes the boat to surge, and in large locks this action prevents rapid operation. The large boat canals, and nearly all ship canals, have sluiceways or culverts formed in the side walls of the locks, provided with a valve at each end to open or close connection with either level and with openings leading into the lock chamber near the bottom of the side walls. At the St. Marys Falls Canal, the timber floors have facilitated forming culverts underneath, discharging upward through many small openings, affording a more uniform distribution of water supply and giving the least possible disturbance to ships.

Valves in gates are usually of the butterfly type turning on a central axis. In culverts or sluices they may be plain slide valves, either of wood or metal, but with large sluices friction makes the operation of such valves difficult or slow. Stoney sluices have been used, for example, at the Manchester Canal. At the St. Marys Falls Canal butterfly valves are used for the 8-ft. by 10-ft. openings, but are not economical of water. Valves moving on four large wheels have been adopted for the New York Barge Canal (Fig. 62). On this canal one lock at Oswego has no valves for filling and emptying the lock chamber, and it is filled and emptied by means of siphons in the lock wall.

Lock Gates. Mitering gates are in pairs, each of the two "leaves" turning about a vertical axis in a recess in the lock wall. When closed, the two leaves, usually of equal length, meet and support each other about 1/4 to 1/5 the width of the lock from a straight line connecting the two axes of rotation. In earlier practice, and particularly in small locks, the gates were made of timber, but steel is coming into more general use; at the Manchester Canal, and at the Liverpool docks, greenheart timber is still used for openings up to 80 ft. or more in width, and the engineers claim for it much longer life than for metal gates.





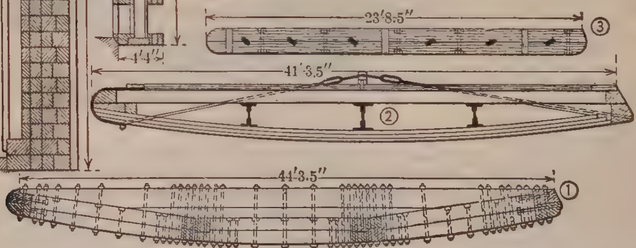


Fig. 56. Wooden Lock Gates
(1) Manchester (2) Kampsville (3) Kanawha

Timber gates (Fig. 56) consist of horizontal girders usually framed into strong end verticals called, respectively, "quoin post" or "heel post" and "miter post" or "toe post"; the gates are planked on the upstream face, where necessary for watertightness. On the Canadian canals the miter and quoin post and planking are omitted and the gates built solid.

Steel gates (Fig. 57) have a strong vertical girder at each end which is usually furnished with a timber-bearing face. The intermediate framing may consist of horizontal girders carrying stresses directly from the miter to the quoin, or of a heavy girder top and bottom to which the vertical girders carry the water-pressure loads. The upstream or both faces are covered with continuous plating, and in cases of heavy gates an airtight chamber is formed to facilitate moving the gate.

Fig. 58 shows one leaf of a gate for the New York Barge Canal. The gate is supported by a pivot under the quoin post and a steel strap or other device at the top strongly anchored to the side wall of the lock. In the United States and Canada, and in some foreign canals, these arrangements suffice to support the gates in position. In English practice, a heavy gate is usually supported on a roller at the miter end, which moves along a path laid in the lock floor.

Many forms of single-leaf gates are in use. On the Erie Canal the gate at the head of the lock is frequently of the form called "tumble" gate, which turns about the lower edge, falling upstream when opening and lying flat on the bottom of the lock, boats passing over it (Fig. 59). The span (width of lock) is 18 ft. and the depth of water 7 ft. Similar gates are in use at the head of locks on the Oder and Spree Canal, where the span is 29.52 ft. and depth of water 8.2 ft. These gates are of wood.

On boat canals, where a limited overhead clearance is admissible, gates lifting vertically have been used, as in the recently built Teltow Canal. A lifting gate on the Coevorder Canal, Holland, has a length of 21.3 ft. and a height of 5.6 ft., is of metal, and lifts vertically high enough for boats to pass under. On this canal one

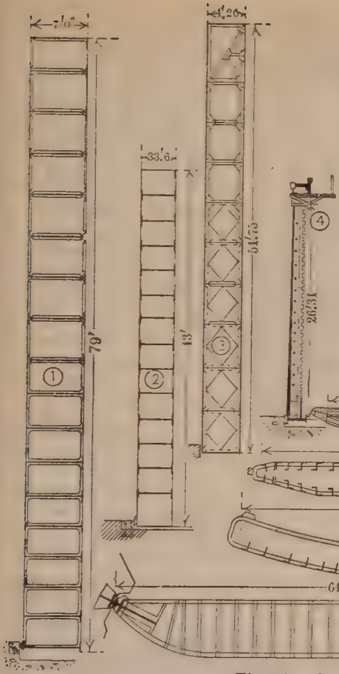
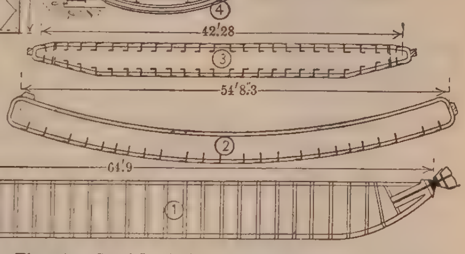



Fig. 57. Steel Lock Gates
(1) Panama (2) Poe (3) Kiel (4) Spree

lock with a lift of 40.5 ft. has a vertical lift gate at the lower end of the lock. Gates turning about a vertical axis at one end and closing the entire width of the lock are in use in some European canals. Following are examples: In the diversion of the Scarpe around Douai the single-leaf gates at the foot of the locks are 21.8 ft. long and 25.2 ft. high; those at the head of the locks are 21.6 ft. long and 11.7 ft. high; the depth of water over the sills is 8.2 ft. and the lift of the lock 13.4 ft.; gates are of iron with wood sheathing and contact

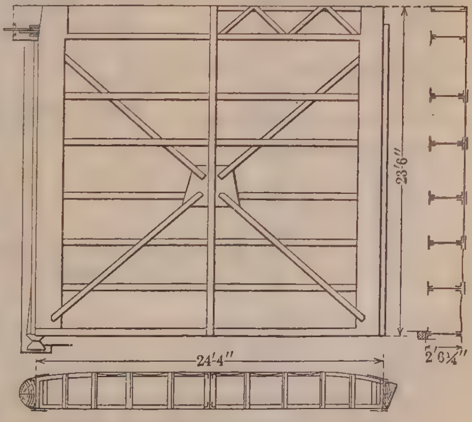


Fig. 58. Barge Canal, Steel Lock Gate

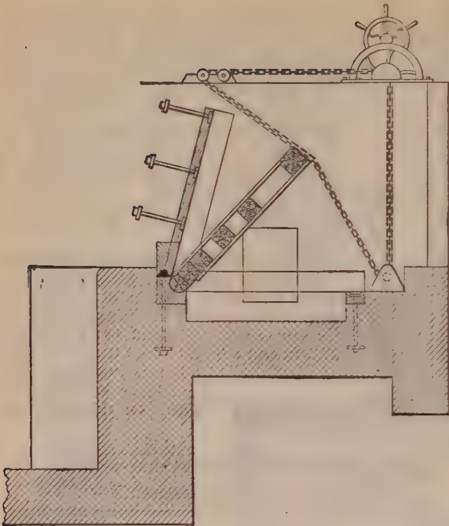


Fig. 59. Tumble Gate, Junction Lock, Erie Canal and New York Barge Canal

pieces. At Lichmis on the Dedemsvaart Canal, Holland, single-leaf gates at the head of the lock span 19.7 ft.; depth of water on sill 6.6 ft.; height of gate 7.3 ft.; built of iron. At the canal St. Denis a single-leaf gate is used at the foot of a lock where the lift is 32.5 ft. As a clear headroom of only 17.2 ft. is required, a masonry arch is extended across the lock, and the gate when closed is supported on all four sides; the gate is 28.7 ft. long and 33.6 ft. high; depth of water over sills about 10 ft. It has a frame of iron with wood sheathing and contact pieces.

Rolling gates moving transversely to the canal are in use in many places. In the locks for the Ohio River navigation at Davis Island and elsewhere (Fig. 60), which are 110 ft. wide, the gates at the head

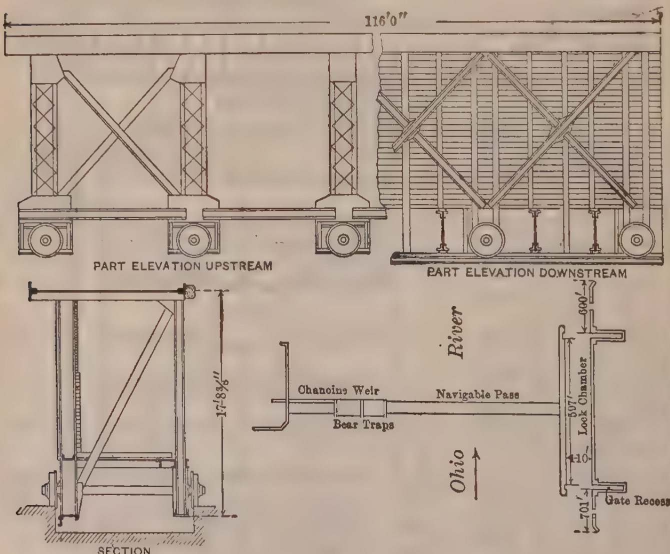


Fig. 60. Ohio River, Rolling Lock Gate

and foot of the lock are alike; the lift is limited to about 8 ft. and the maximum height of gate built up to this time is 18.5 ft. The gates have steel frames with wood sheathing, and are carried on wheels which run on tracks laid in the lock floor. The gates are opened and closed in 1-1/2 to 2 minutes with steam power. In the Canal François, Hungary, the locks (two in flight) near the R. Sisza have a width of 52.5 ft. and a depth of 8.2 ft. on sills; the upper and middle gates are of iron and of the roller type, suspended from trucks running on an overhead bridge. The middle gate is 55.6 ft. long, 30.1 ft. high, and weighs 150 tons mounted. It has air chambers which lift 44 tons, leaving the net weight to be moved 106 tons; operating machinery is electric. Time to open and close the gate 3 minutes.

Operating Machinery. In the smaller boat canals the gates and valve are operated by hand. The lock gates of the Erie Canal have " balance beams " attached to the top of the gate and extending back over the side walls, and the gate is moved by men pushing against the beam. In some foreign canals a segment of a toothed wheel is attached to the quoin post and extends back over the wall and is operated by means of a windlass with vertical axis carrying a pinion which acts upon the toothed segment. At the Dortmund and Ems Canal the piston rod of a hydraulic cylinder acts upon an arm extending back from the gate, replacing the toothed gear just mentioned. At the larger locks, hydraulic machinery is in most general use. As the machinery is operated for only a small part of the time, accumulators are employed, usually in the form of a vertical cylinder, into which water is pumped, lifting a heavily loaded plunger, and from which the water is drawn when wanted. At many places, as the Manchester Canal and the Liverpool docks, the gate is opened by means of a chain attached to one end of the gate and then led around guides and rove through two sets of pulleys, one attached to the lock wall and the other to the plunger of a hydraulic cylinder. The effect of this is to multiply the movement of the end attached to the gate when the plunger is operated. A similar arrangement effects the closing of the gate. At the Weitzel lock of the St. Marys Falls Canal a single cylinder is used to open and close each gate, a piston being substituted for the plunger, a long piston rod passing through both cylinder heads with a set of pulleys at each end (Fig. 61). At other places winding drums are operated by three-cylinder hydraulic engines, at the as Kiel Canal. A simple and efficient machine, consisting of a hydraulic cylinder with its piston rod guided and a connecting rod attached directly to the gate and opening or closing the gate by a single movement of the piston was introduced at the Barry dock at Bristol, England, and is used at several places.

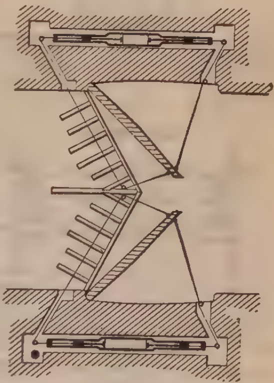


Fig. 61. Method of Operating Gates, St. Marys Falls Canal

In the smaller canals the valves for filling or emptying the locks are operated by hand power. The 8-ft. by 10-ft. butterfly valves at the locks of the United States canals at St. Marys Falls are opened or closed by a single stroke of a hydraulic engine, the connecting rod being attached directly to the valve—all below the lock floor. At the Canadian Canal, St. Marys Falls, the valves are operated by a long connecting rod attached to a crank arm of the valve stem. On the New York Barge Canal the valves which are counterweighted are operated by electricity (Fig. 62).

In foreign canals the water pressure used in hydraulic machinery at locks is usually 40 to 50 atmospheres. In recent years the use of electric current for operating lock machinery is increasing; two notable examples are the Canadian lock at the St. Marys Falls and the sea lock of the Amsterdam Canal.

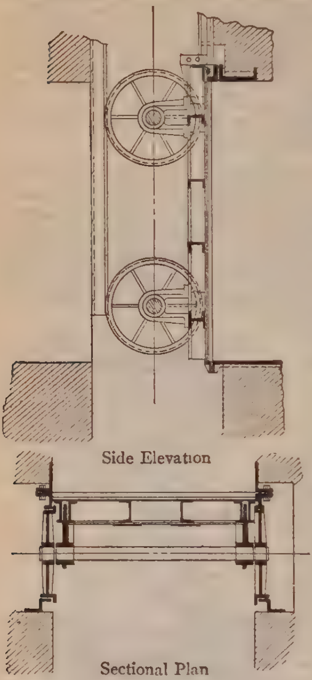


Fig. 62. Lock Valve, New York Barge Canal

Approaches to Locks. In canals where boats are moved by men or horses at low speed the change from canal section is effected abruptly at the ends of the lock walls but where vessels are moved by steam the danger of accidents to lock gates is greatly increased, and a long guide wall should be built at each end of every lock or flight of locks with posts strongly anchored placed at short intervals for use in checking speed by means of lines. Such arrangements exist at the St. Marys Falls canals, at the New York Barge Canal, and at the Panama Canal.

20. Lifts and Inclines

High Lifts. There are several examples in European and Canadian canals of so-called " hydraulic lifts." This type consists of two tanks, each large enough to receive a boat, and supported by a plunger working in a cylinder of sufficient length to give the desired lift by one stroke of the plunger. The cylinders of the two tanks are interconnected, so that the tanks balance each other. The boat to be lifted or lowered is placed in one of the tanks, displacing its own weight of water, and thus not affecting the balance. Movement is effected by admitting a small amount of excess water to the tank which it is desired to lower. The tank has a gate at each end; a gate is also placed at the end of each canal level. The table at top of next page gives some of the details of existing hydraulic lifts.

The Anderton lift, the first to be built, was operated with little trouble for several years, but finally the plunger became scored to such an extent that it was decided to remove the hydraulic appliances; each tank is now counterweighted separately, the counterweights being connected to the tank by wire cables carried over pulleys supported by an elevated frame; the operation is by electric machinery. This change was carried out in 1908.

The high lift at Henrichenburg, built in 1899 on the Dortmund and Ems Canal, consists of a single tank supported on five watertight air-filled cylinders immersed in wells which are interconnected and kept full of water. Principal data are:

	Ft.	In.		Ft.	In.
Depth of wells	98	6	Carrying capacity of boat, tons	950	
Diameter of wells	30	2	Length of largest boat	220	0
Diameter of cylinder floats	27	6	Width of largest boat	27	0
Length of cylinder floats	32	9-1/2	Draft of largest boat	6	7
Length of tank (useful)	225	0	Time to lift or lower tank	2-1/2 min.	
Width of tank (useful)	28	4	Time for one round trip, lifting one boat and lowering another	25 min.	
Depth of water in tank, min.	8	3	Power supplied	440 hp.	
Weight of lift	45	11	Cost including installation	$630 000	
Weight to be lifted, tons	3100				

Data	Anderton, England. Trent & Mersey Canal	Fontinettes, France. Neufosse Canal	La Louvieres, Belgium. Canal du Centre	Peterborough, Canada. Trent Canal	Kirkfield, Canada. Trent Canal
When built	1875	1888	1888	1904	1906
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
Tank, inside length	73 9	132 10	141 7	139 0	139 0
Tank, inside width	15 3	17 0	18 4	33 0	33 0
Tank, depth of water	4 5	6 6-3/4	8 6	8 0	8 0
Height of lift	50 2	43 0	50 6	65 0	48 5
Diameter of ram	2 11-1/4	6 6-3/4	6 6-3/4	7 6	7 6
Pressure of ram, lb. per sq. in.	532	....	500	600	600
Weight to be lifted, tons	250	770	1100	1700	1700
Displacement of boat, tons	120	300	400	800	800
Carrying capacity of boat, tons	80	....	....	....	....
Time to lift boat, minutes	....	....	2-1/4	1-1/2	....
Time complete operation, minutes	....	....	15	12*	....
Cost in dollars	230 000	380 000	320 000	500 000	....

* The record time of complete operation was 6-1/2 minutes.

The movement of the lift is controlled by guides, and uniformity is secured by four massive vertical screws, interconnected, and working through nuts fixed to the lift. It is stated that "a lift constructed on the open trough principle should be adopted only where conditions for it are favorable and a good foundation can either be found or can be prepared without any great trouble and expenditure. It should also be mentioned that when the further extension of the Dortmund and Ems Canal is taken in hand, it is contemplated to construct a flight of locks by the side of the canal lift."

Inclined planes have been used to surmount high lifts, both in the United States and Europe. The table gives details of several examples.

Canal	Number of inclines	Total lift, ft.	Largest boat: Length	Width	Draft	Load, tons	Built	In use or abandoned
			ft.	ft. in.	ft. in.			
Morris, U. S.	23	1449.0	89.0	10 6	4 0	70	1831	In use
Pennsylvania, U. S.	10	2007.0	85.0	13 6	......	75	1831	Abandoned
Ches. & Ohio, U. S.	1	39.0	90.0	14 5	5 0	115	1876	Abandoned
Monkland, Scotland	1	96.0	70.0	14 4	2 2	60	1850	Abandoned
Shropshire, England	1	213.0	20.0	6 0	2 9	.....	.....	In use
Shrewsbury, England	1	73.5	20.0	6 0	2 3	.....	.....	In use
Chard, England	3	27.5 to 86.0	26.0	......	......	60	.....	In use
Grand Junct., England	1	75.2	75.0	12 0	3 4	75	1900	In use
Ourcq, France	1	40.0	92.0	10 0	4 0	70	1888	In use
Oberland, Germany	5	66.0 to 80.3	....	......	......	70	1860	In use
Bude	1	....	20.0	5 6	1 8	.....	.....	In use

The small boats using the Bude incline have wheels built in, and are drawn up the incline in trains, counterbalanced to some extent by descending trains or boats. On the inclines of the Morris, Pennsylvania, Shropshire, Shrewsbury, Ourcq and Overland canals, boats are (or were) carried on trucks; on the inclines of the Chard and Monkland and Chesapeake and Ohio canals, the boats are hauled while water-borne, or nearly so, in tanks. In some cases the incline is double, the boats or tanks counterblancing each other. Boats are usually carried endwise, but on the Foxton incline on the Grand Junction Canal, the boat is carried sidewise and is water-borne; two tanks are used counterbalancing each other.

The incline of the Chesapeake and Ohio Canal (at Georgetown, D. C.) provided for larger boats than any other yet built. In connection with the projected Austrian system of canals to connect the Danube with the Ober, the Elbe and the Moldau, inclines have been proposed for boats of 600 to 800 tons capacity. It was announced in 1902 that the Government of Austria had decided to build an experimental incline for boats of 600 tons, investigations having indicated a saving in cost of transport by the use of inclines instead of locks, provided no unforeseen difficulties develop in actual use of inclines constructed on a scale so much exceeding any previously built.

21. Guard Gates, Safety Gates, Stop Gates

Guard Gates are arranged at the St. Marys Falls canals by extending the lock walls at each end sufficiently to receive an additional pair of gates, each opening outwardly. They could be used to protect the ordinary lock gates from approaching vessels, but are not so used, their purpose being to enable the lock to be closed quickly by unworn and perfectly fitting gates, for repairing the lock and its gates and valves. The summit level of the Charleroi Canal (supplied by pumping) is divided into short sections by **safety gates,** so that in case of a break in the canal bank the gates may be closed and the loss of water limited to about 20 000 000 cu. ft. **Stop gates** (Fig. 63) were introduced in the Dortmund and Ems Canal which in places is carried on heavy embankments. These act on the same principle as Taintor gates, and when opened permit the passage of boats underneath. In the New York Barge Canal guard gates (Fig. 64) are placed at least every 10 miles in canal sections; these are lifting gates and in duplicate, each affording a clear opening of 55 ft. and a clear head room of 15.5 ft. above the water surface at high navigable stage when opened. These gates are steel and carry wheels moving on vertical tracks fixed in the masonry. They are operated by hand power. The last three devices are designed to be operated in moving water.

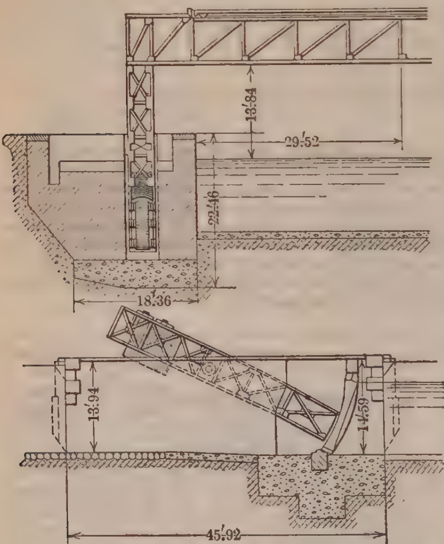


Fig. 63. Guard Gate, Dortmund and Ems Canal

These devices are impracticable for closing a ship canal where unlimited height is required, and for such canals the safety gates are some form of movable dam. As these are to be manipulated in powerful currents, a subdivision of the structure is considered necessary, and as the emergency requiring their use will seldom arise, and if not in sight they would be more liable to be neglected and unserviceable when needed, they should be above water and readily inspected when not in use. These conditions

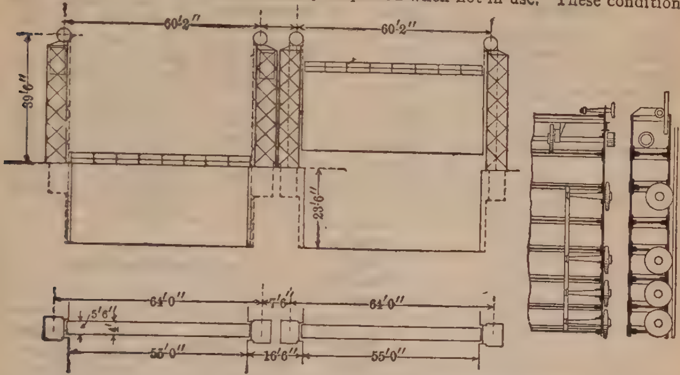


Fig. 64. New York Barge Canal Guard Gates

have been met at the St. Marys Falls canals by swing bridges carrying the sectional dams. Fig. 65 shows the structure in the canal on the Michigan side of the rapids. The canal, which is in rock excavation, is separated by an island into two channels, each 108 ft. wide and 25 ft. deep. Each channel is closed by 16 wickets which can be lowered separately. Each wicket will consist of two parts, a frame and a shutter mov-

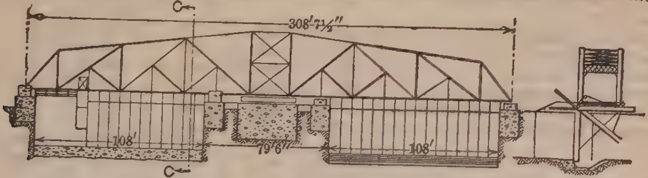


Fig. 65. Movable Dam. United States Ship Canal, Sault Ste. Marie

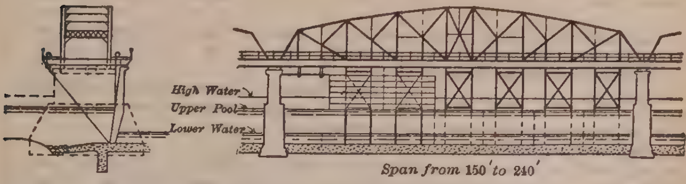


Fig. 65a. Movable Dam. New York Barge Canal

able in the frame. It is expected that the frames will be lowered first, offering only a partial obstruction to the current, and when all are properly placed, the shutters, guided in the frames, will be lowered one at a time. This structure is similar to the one at the Canadian Canal which was successfully operated when lock gates were carried away in June, 1909. On the Panama Canal a safety device is used similar to that shown in Fig. 65. It is described in more detail on p. 1600.

22. Statistics of Canals

United States. Total approximate mileage and cost of construction are given in the table.

Owned by	Total length, miles	Cost of construction
United States Government..	194.49	$42 443 357*
States †..................	1358.98	156 983 538
Private parties............	635.58	50 573 160
Total in operation........	2189.05	250 000 055*
Abandoned canals........	2444.26	81 171 374
Total..................	4633.31	$331 171 429

* To June 30, 1907. Includes expenditures by prior owners. Since above date the Cape Cod Canal has been transferred from private ownership to U. S. Government.

† Includes Chicago Sanitary and Ship Canal, built by Sanitary District of Chicago. Length 32.95 miles. Cost $52 697 495.

	Illinois and Mississippi	New York Barge Canal			
	Main Line	Main Line	Oswego Canal	Champlain Canal	Cayuga-Seneca Canal
Length, miles..................	75**	338.6*	23.8	62.6	92.7§
Width at bottom earth, ft........	52	75	75	75	75
Width at water surface earth, ft....	80	123–171	123–171	123–171	123–171
Width at bottom rock, ft..........	....	94	94	94	94
Width at water surface rock, ft....	....	96.4	96.4	96.4	96.4
Width, bottom, in rivers, ft......	....	100–200	100–200	100–200	100–200
Depth, ft......................	7	12	12	12	12
Number of locks................	33	35	7	11	4
Maximum lift, ft................	12	40.5	27	19.50	26
Total lift, ft....................	292	364.75	118.6	168.3	71
Length of lock chamber, ft........	170	339	339	339	339
Useful length of lock, full width, ft.	....	300	300	300	300
Width of lock, ft................	35	45	45	45	45
Culverts......................	In masonry at gates	In side walls	In side walls	In side walls	In side walls
Number of inlets to chamber......	....	16–28	18–28	16–28	16–28
Gates, type....................	Miter ††	Miter (†)	Miter	Miter	Miter wood
Gates, material................	Wood	Steel	Steel	Steel	Steel
Valves, type..................	Butterfly	¶	¶	¶	¶
Speed allowed, miles per hour.....	6	6‡	6	6	6
Motive power..................	Any	Steam or gasoline	Steam or gasoline	Steam or gasoline	Steam or gasoline
Cost of maintenance per year.....	$175 000	....	....	....	....
Time required for passage, hr......	20	....	....	....	....
Bank protection................	Riprap	Riprap	Riprap	Riprap	Riprap
Tightening....................	None	Steel piling concrete clay	Steel piling concrete clay	Steel piling concrete clay	Steel piling concrete clay

* Hudson River to Niagara River. † 1 lift gate Little Falls. ‡ Land lines.
§ Includes Cayuga Lake 31.2 and Seneca Lake 34.4.
¶ Rectangular steel frame valves on wheels, see Fig. 62.
** Not including 29.3 miles of navigable feeder. ††Tumble gates at head of 14 locks.

Nearly all the canals built by the United States Government are lateral canals, avoiding rapids in the natural waterways and providing for steamboats as well as barges. The most important one is the St. Marys Falls Canal, which will be described as one of the great ship canals (see p. 1598).

The Colbert Shoals Canal is typical of lateral canals along navigable rivers. It avoids Big Tree and Colbert Shoals in the Tennessee River; length 8 miles, depth at low water 7 ft. Width for lower 5-1/2 miles in earth excavation 112 ft. at bottom, 140 ft. at low-water surface, side slopes 2 : 1. Width for upper 2-1/2 miles, mainly in rock excavation, 175 ft. at bottom, about 190 ft. at water surface. Lock at lower end of canal: available length 350 ft., width 80 ft. Lift, maximum at extreme low water, 26 ft. Mitering gates steel, circular in plan. Culverts in side walls for filling and discharging, 8.6 ft. by 9 ft., with 11 openings into the chamber from each wall. Valves of the Stoney type. Banks 20 ft. wide at top extend 9.5 ft. above low-water line and are protected Bermuda by grass above that line. Side drainage received into the canal usually through culverts. Five waste-weirs, each 100 ft. long, are built into the bank on the river side, exclusive of a guide wall 7000 ft. long at head of canal serving also as a weir. Crests of weirs 0.5 ft. above low-water surface.

Only two canals with summit levels and intended for boat or barge navigation have been undertaken within recent years in the United States. One of these is the Illinois and Mississippi Canal (also called " Hennepin " Canal), and the other is the New York Barge Canal with its branches, the Oswego, Champlain, Cayuga and Seneca canals.

Traffic in the boat canals of the United States has been steadily decreasing with increasing competition of the railroads and motor vehicles, and this has not been checked materially by the removal of tolls and the assumption of fixed charges and operating expenses by the state. On the continent of Europe, where the canals are generally owned by the state and free from tolls, the assumption of construction, operation and maintenance by the state enables water transportation to compete with rail, and traffic on the waterways is increasing.

New York Barge Canal. The Barge Canal is the improvement of four existing canals, and consists of (1) the Erie, stretching across the state from east to west and joining the Hudson River and Lake Erie; (2) the Champlain, running north from the eastern Erie terminus to Lake Champlain; (3) the Oswego, starting north midway on the Erie and reaching Lake Ontario; (4) the Cayuga and Seneca, leaving the Erie a little west of the Oswego junction and extending south, first to Cayuga lake and then to Seneca lake.

The Barge Canal is largely a river canalization project, natural water-courses being available for much of its length. With intervening lakes and adjoining rivers there is a total length of a little more than 800 miles in the state's internal waterway system that is of Barge Canal dimensions.

The main water supply is taken from Lake Erie, from reservoirs in the Adirondack region and from the upper Hudson River. The supply is adequate not only for the 10 000 000 tons seasonal traffic for which the canal was designed, but also for the maximum traffic which the canal is capable of handling, namely, from 18 500 000 to 20 000 000 tons per season.

The locks on all branches of the canal are of standard dimensions. The lifts range from 6 ft. to 40-1/2 ft. They are built of concrete throughout and except on rock or hardpan have pile foundations. Within each side wall runs a culvert for filling and emptying the lock. The culverts are connected with ports that open into the chamber at the bottoms of the walls. With a few exceptions the lock gates are of the mitering, girder type, carrying the principal load as beams. They are built of steel, with single skin-plates, have white-oak quoin and toe posts, swing on cast-steel pivots set in the concrete, are held at the top by adjustable anchorage and bear against cast-iron quoin plates bolted into the side walls. In general a power station at each lock supplies the energy for operating and lighting the lock. A majority of these installations are hydroelectric, but at locks beside movable dams gasoline-electric stations are employed.

At about fifty cities and villages along the waterways the state has built terminals, which provide suitable dockage, mechanical devices for handling goods quickly and cheaply, a building for temporary storage and, where possible, railway connections—all available to any shipper or boatman.

France. Practically the whole of the canal system is owned and controlled by the state. In 1905 the total length of canals in use was 3010 miles. The more important canals have a bottom width of 33 ft. or more; depth 6.6 ft. to 7.25 ft.

Belgium. The total mileage of the waterways (canals and rivers) is 1345 miles. The total length of the more important canals is 334 miles. The total lift at 141 locks (excluding lifts) is 1445 ft. The total lift at 4 lifts (1 completed, 4 building) is 217 ft. When new work or enlargement is taken up the locks are given greater lifts, up to 14.8 ft., and water saving effected by side ponds.

Germany. The total length of the waterways on which the traffic is of importance is 6200 miles and the total lift at 359 locks (excluding lifts) is 3129 ft. In the canal connecting most directly with the French waterways the width at the water surface is about 52 ft., at the bottom 33 ft., depth 6.6 ft.; useful length of locks 126 ft., width 17 ft., adapted for use of boats on the French and Belgian canals.

Great Britain. The canals were built by independent companies, without assistance from the state. Nearly one-third of the total mileage has been acquired by railway companies. The total length is 3811 miles (Eng. News, Apr. 16, 1890). The annual traffic on the independent canals is about 33 000 000 tons; on the railway-owned canals, about 6 000 000. The greater part of the traffic is short distance, there being no uniformity of canal dimensions, making frequent transshipment necessary on through business. Excluding the ship canals, only 10% of the total mileage has a depth of 6 ft. or more, while 46% has a depth of less than 4 ft. On account of the diverse ownership a material extension or improvement of the boat canal system is not expected.

Holland. There are 265 canals, with a total length of 2100 miles, of which about 5% belongs to the state; the remaining canals are under control of provincial or municipal authorities, administrative corporations, or private companies. Many of these canals are designed primarily for drainage, and some of the branch canals can be navigated only by small boats poled or towed by manual labor. Power boats and tugs are used on the larger canals. On the Merwede Canal recently built, length 43-3/4 miles, 106 ft. wide at water surface, 66 ft. at bottom, 10.2 ft. minimum depth, the tugs tow trains of 12 boats. This canal cost $8 480 000 for construction; annual maintenance $103 000. All the canals owned by the state, and nearly all the others, are free from tolls.

23. Ship Canals

Suez Canal. Connects the Mediterranean Sea and the Red Sea. Shortens the sailing distance from northern Europe to India about 5000 miles; to Australia about 1000 miles. Total length of canal including harbor at Port Said and roadsteads at both ends 104.08 miles. Minimum depth 31.1 ft. Width at this depth (excluding harbor and roadsteads) 108.26 to 118.11 ft., widened on curves to 131 to 262 ft. There are 23 passing places where the bottom width is increased to 147.63 ft. for a length of 2460 ft., reducing at each end to standard width in a distance of 984 ft. Side slopes vary from 2 : 1 to 3 : 1 or flatter. The width and depth are being increased gradually. Cost of the work up to the end of 1908 was $122 275 000. Expenses during 1908 were about $8 668 000 and net profits about $12 850 000.

On account of the tides and effect of the wind in the Mediterranean and Red seas currents are produced in the canal reaching 1-1/3 miles per hour between the Bitter Lakes and Port Said, and 3 miles per hour between the Lakes and Suez. The speed of vessels is limited to 7-3/4 miles per hour on the 28-mile tangent beginning at Port Said, and to 6.2 miles per hour elsewhere in canal sections. A high speed is permitted in the lakes. The average time actually consumed in passing the canal is from 18 to 19 hours.

Corinth Canal. Connects Gulf of Corinth with Gulf of Ægina. Is at sea level. Shortens sailing distance from Adriatic ports about 175 miles, and from Mediterranean ports about 100 miles. The canal is 3.91 miles long. The great central cut is 2-1/2 miles long with a maximum depth of 286 ft. from surface of ground to bottom of canal. The canal is 26.64 ft. deep and has a bottom width of 68.9 ft. The sides have a batter or slope of 1 horizontal to 4 vertical (1 : 4). Unless vessels move very slowly they strike the sides of the cut frequently. The traffic is small.

Manchester Canal. Connects Manchester with the River Mersey at Eastham near Liverpool. It is 35-1/2 miles long; when opened for navigation (1894) its depth was 26 ft.; ruling bottom width 120 ft. Side slopes in earth from 1 : 1 to 3 : 1, in rock 1 : 5. For a distance of four miles from Manchester the bottom width was 170 ft. The depth of water on the lock sills was made 28 ft., and the canal was deepened to that depth.

The Manchester level is 70 ft. above mean tide at Eastham, and is reached in five lifts, varying from 9 ft. 6 in. to 16 ft. 6 in.; the smallest lift is at Eastham and overcomes the difference of level between mean tide and mean high water.

At the Eastham terminus there are three parallel locks having the length and width respectively, of 600 ft. by 80 ft., 350 ft. by 50 ft., and 150 ft. by 30 ft.; the lengths given are from quoin to quoin. At the other lock sites there are only two parallel locks, respectively, 600 ft. by 80 ft. and 350 ft. by 50 ft.

Each set of locks is paralleled by a sluiceway, with sluice gates of the Stoney type 20 to 30 ft. wide (30 ft. except at Eastham), and with sills at the same level as the upper sill of the adjacent lock. Each 30-ft. gate has a discharging capacity of about 6500 cu. ft. per second. There are three to five (excluding Eastham) such outlets in each sluiceway, so that in times of flood strong currents may be set up for short periods. The smaller sluiceway at Eastham is supplemented by extensive sluices in the bank of the canal in the level immediately above. The speed permitted is 6 miles per hour for the largest vessels, rising to 12 miles or more for smaller vessels and tugs. The time required for passing through the canal varies from 5 to 8 hours.

Kiel Canal. Connects Baltic and North seas. It is 61.31 miles long, 36.08 ft. deep, 144.32 ft. wide at bottom, 333.74 ft. and upward at water surface. The width is increased on curves at bottom. The canal has 11 passing places, four of which are 984 ft. wide at bottom to serve as turning places; the others are 439.52 ft. wide at bottom, length 1968 to 3608 ft. There is a lock at each end of the canal with ebb and flood gates at each end of the lock. Normal water level in the canal is mean tide in the Baltic. There is also in each an intermediate pair of ebb gates and one pair of flood gates, permitting use of a shorter lock chamber. The locks have a useful length of 984 ft., width 147 ft. 8 in., depth on sills 45 ft.

The speed allowed by regulations is 9.3 miles per hour, but the largest vessels cannot attain this. The time required to pass through varies with the draft, from about 8 hours for the smaller vessels to 13 or 14 hours for the larger.

Amsterdam Canal. Connects the North Sea at Ymuiden with Amsterdam and also with Zuyder Zee. Its principal purpose is to make Amsterdam a seaport. Its length from the North Sea locks at Ymuiden to the Zuyder Zee locks is 17.4 miles, and from the North Sea locks to the entrance to Ymuiden Harbor about 2 miles, making a total of 19.4 miles. Depth 32.14 ft. below "ordinary water-level"; width at bottom, 164 ft. on tangents increased to 196.8 ft. on curves. Side slopes generally 3 : 1 with a berm of varying width and depth.

The new great lock at Ymuiden is intended to accommodate vessels of the maximum length of 721.6 ft., width 82 ft., draft 30.18 ft. It has intermediate gates to permit the use of a shorter lock for small vessels, and ebb and flood gates, making 6 pairs in all. The operating machinery is electric throughout, and it is the second ship-canal lock to be so equipped.

The speed permitted varies with the draft of vessel; it is 6.52 miles per hour for vessels drawing 19.68 ft. or more, 7.45 miles per hour for vessels drawing 13.12 to 19.68 ft. and 9.32 miles per hour for vessels drawing less than 13.12 ft.

Leningrad and Cronstadt Canal. Connects deep water in the roadstead in the Gulf of Finland with one of the delta mouths of the River Neva at Leningrad, and it is intended to make Leningrad a seaport. It consists of a channel about 17-3/4 miles long dredged in the open water of the bay and across the Neva bar and protected by dikes for a distance of about 6 miles at the upper end. Its depth was made 28 ft. Bottom width from 213 to 275-1/2 ft. in the diked portion and 355-1/2 ft. in the portion not diked.

Cape Cod Canal. Sea level canal connecting Cape Cod Bay and Buzzards Bay; shortens the sailing distance from Boston to New York by 66 miles and avoids the dangerous Cape Cod coast. Highest part of ridge cut through was 29 ft. above mean sea level. Material chiefly sand. Length of canal proper 8 miles, 25 ft. deep at low water, 100 ft. wide at bottom, side slopes 1 on 2. There are 5 miles of approach channels having bottom widths of 250 ft. and 300 ft. and side slopes of 1 on 3. Mean range of tide at one end 8.9 at the other end 3.6 ft. This tidal difference produced velocities as great as 5 ft. per sec. Completed in 1915.

St. Marys Falls Canals. These are lateral canals avoiding St. Marys Rapids in St. Marys River between Lakes Superior and Huron. There are two canals, one on the American side of the river and the other on the Canadian side. The fall in the rapids varies from about 17 to 21 ft. with the varying stages of the lakes and is overcome by one lift in each of the canals.

The American Canal is about 1.6 miles along with four parallel and adjacent locks at the downstream end. The canal consists of two channels, the Poe and Weitzel locks serving the old south channel, and the new third and fourth locks serving the new channel to the north. The Weitzel lock is 515 ft. long from quoin to quoin, 60 ft. wide at gates, 80 ft. wide in chamber and has from 15 to 17 ft. of water on the sills. The Poe lock is 800 ft. long, 100 ft. wide, and has 20 to 22 ft. of water on the sills. The two new locks are each 1350 ft. long, 80 ft. wide, and have 24-1/2 to 26 ft. of water on the sills.

Upstream from the Poe and Weitzel locks is a narrow island on which is located the pivot pier of a swing bridge carrying the wickets and machinery of a movable dam, the purpose of which is to close this portion of the canal in case of a break at the locks permitting a free flow. (See Fig. 65, p. 1593.)

The Canadian canal is 1.4 miles long, 141 ft. 8 in. wide at bottom, 150 ft. at water surface, and about 22 ft. deep. The single lock is 900 ft. long from quoin to quoin, 60 ft. wide and has 20 to 22 ft. of water on the sills, depending on the varying stages of the lake.

Canadian Canals from Lake Erie to Tidewater at Montreal

Canal	Length, miles	Width, feet		Locks				Total lift, ft.
		Bottom	Water surface	Length, ft.	Width, ft.	Depth, sills, ft.	Number	
Welland	26.75	100.0	164.0	270.0	45.0	14.0	26*	326.75
New Welland	25.00	200.0	301.25	800.0	80.0	30.0	8*	325.5
Galops	7.33	80.0	144.0	800.0 270.0	45.0 45.0	14.0 14.0	1 2*	15.50
Rapide Plat	3.67	80.0	152.0	270.0	45.0	14.0	2	11.50
Farran's Point	1.25	90.0	154.0	800.0	45.0	14.0	1	3.50
Cornwall	11.00	100.0	164.0	270.0	45.0	14.0	6	48.00
Soulanges	14.00	100.0	164.0	280.0	45.0	15.0	5*	84.50
Lachine	8.50	150.0†	‡	270.0	45.0	14.18	5	45.00

* One is a guard lock. † Minimum. ‡ Variable.

The two canals actually constitute one system. No charges are made to vessels passing, which usually take the canal having the smaller number of waiting vessels. Hydraulic power is used at the American canal; the Canadian canal is operated electrically, and was the first ship canal to be provided with such operating machinery. A boat can be passed through the lock in either canal in 8 to 11 minutes, but on account mainly of fleet lockages, the average time for a lockage is about 33 minutes in the American canal. In the Canadian canal, where fleet lockages form a smaller proportion of the total time, the average is about 20 minutes.

The time necessary to pass through the canal depends largely on the amount of traffic and the congestion resulting, and has varied in the American canal from 4 hr. 58 min. in 1895, when only one lock was in operation, to 1 hr. 10 min. in 1900, when two locks were in use. With increased traffic since that date the time has increased to 2 hr. 40 min.

Panama Canal. Connects the Atlantic Ocean (Caribbean Sea) with the Pacific, crossing the Central American Isthmus in Lat. 9° N. and Long. 85° W. at approximately its narrowest and lowest point. It reduces the sailing distance from New York to San Francisco by 7873 miles (5262 miles as compared with 13 135 miles via Magellan Straits) with a similar saving in distance to many ports in South America and the Orient. The canal is 50 miles long between deep water in the two oceans. The average time of passage is 12 hours, the shortest recorded time is 6 hours 30 minutes.

Its construction was begun in 1882, by the French, whose rights were acquired by the United States for $40 000 000 in 1904, when a Canal Zone 10 miles wide was secured from the Republic of Panama, for $10 000 000 and a perpetual rental of $250 000 per annum. The canal was completed by the U. S. Government in 1914.

The canal is a lock canal with a summit level normally 85 ft. above mean tide in the two oceans, the maximum tidal range being 22 ft. at the Pacific and 2 ft. at the Atlantic end. Its most distinctive feature is Gatun lake, an artificial body of water, 164 square miles in area, which forms the summit level. The lake provides wide and deep channels which permit vessels to travel at high speed for over half the distance between the two oceans. Its formation involved the building of a huge earth dam for closing the valley of the Chagres at Gatun and of three expensive groups of locks, but on the other hand, the depth of the summit cut at Culebra was reduced decidedly, and a large amouunt of excavation eliminated so that there was a decided saving in the total time and cost of construction in comparison with a sea level canal. As practically no excavation was required in the deeper portion of Gatun lake the work of building the canal resolved itself essentially into the excavation of the sea level sections, and of the cut through the Continental Divide, the building of the Gatun and some smaller dams and the construction of the locks and spillways. Some details of the several parts of the work are given below:

Beginning at the Atlantic end, the successive sections of the canal are the following: (1) A sea-level channel 7 miles long and of 500 ft. bottom width, extending to the locks and dam at Gatun. (2) Gatun lock flight with 3 lifts of about 28 ft. each. (3) Gatun lake at Elev. + 85 forming the summit level which extends across the Continental Divide through the Culebra (Gaillard) Cut to the lock at Pedro Miguel. The total length of channel in the summit level is 32 miles, of which 16 miles is 1000 ft. wide, this section having a depth of 45 to 75 ft., 4-1/2 miles is 800 ft. wide, 4 miles is 500 ft. wide, and 8 miles (in Culebra Cut) is 300 ft. wide. (4) Pedro Miguel lock with a 30-ft. single lift. (5) Miraflores Lake (area 1.6 sq. miles) at Elev. + 55 with a channel 500 ft. wide, 1-1/2 miles long, and 45 ft. deep. (6) Miraflores Locks, a flight having two lifts of 22 to 33 ft. each, according to the tidal stage, and finally (7) A sea-level section 500 ft. wide and 7 miles long to deep water in the Pacific.

It will be noted that, except in the locks, the channel has a minimum width of

300 ft. and a minimum depth of 45 ft. Where there is a change in the direction of the canal axis, instead of connecting the tangents by curves, the channel was widened so as to allow vessels to turn readily.

Locks. The principal dimensions of the locks are shown in Figs. 53 and 54. They have a clear width of 110 ft., a usable length of 1000 ft., and a minimum depth on the sills of 41 ft. 4 in. There are twin chambers, with a central wall 60 ft. thick between them, which is extended 1000 to 1200 ft. beyond each end of the lock—to act as a guide wall for vessels.

For filling and emptying there is one longitudinal culvert 255 sq. ft. in cross-section (equal to a circle 18 ft. in diameter) in the middle wall and in each sidewall connecting with transverse culverts of elliptical section built in the lock floor, which are each 41 sq. ft. in area and discharge upward into the lock chamber through 5 circular outlets of 12 sq. ft. each. These laterals extend across the lock at intervals of about 35 ft., connecting alternately with the culverts in the side and middle walls. The main culverts are controlled by rising stem gate valves of the Stoney type, located close to the upper, intermediate and lower lock gates. They are 8 ft. wide and 18 ft. high and are placed in pairs separated by a dividing wall. As a safeguard, a duplicate set is provided close to each pair of valves. The valves are each opened and closed by a single valve stem, which extends through a horizontal cast-iron bulkhead, placed just above the valve, and fitted with a stuffing box. The stem connects with a guided cross head, which is raised and lowered by 2 non-rising screws driven through reducing gears by a 50-hp. electric motor. The time of operation is one minute. Each lateral culvert leading from the middle wall is fitted with a cylindrical valve 78 in. in diameter, placed vertically, which is opened or closed in 10 seconds by a 7-hp. motor.

Lock Gates. The lock gates are double-skin steel gates of the mitering type with air chambers. The sill angle is 26° 33′ 54″. There are duplicate operating gates at both ends of Pedro Miguel lock and at the upper chambers in the lock flights and an intermediate set in all the chambers except the lower ones at Miraflores. These subdivide the locks into usable lengths of 550 and 278 ft. A reverse guard gate is provided in the lower approach to each lock. There are in all 46 gates of 2 leaves each, weighing nearly 60 000 tons. The leaves are uniformly 65 ft. long and 7 ft. thick and vary in height from 47 to 82 ft. and in weight from 395 to 745 tons. The horizontal girders which are spaced from 3 ft. 8 in. to 5 ft. apart have straight parallel sides and curved ends. See Fig. 57. The bearings along the quoin and miter posts and in the hollow quoins are polished steel plates. Their joints have proved to be absolutely watertight. There is a wooden clapping sill with a rubber seal.

The gate-moving machinery is of a novel type developed and first used on the Panama Canal. A horizontal strut is used, hinged at one end to the top of gate, and at the other to a crank pin fixed in a horizontal wheel 19 ft. in diameter (the so-called "bull wheel"), which is supported on the lockwall. This is turned by a 25-hp. motor through a series of reducing gears, the time for opening or closing a leaf being 2 minutes. The special merit of the design is the fact, due to the kinematics of the mechanism, that it gives the leaf a relatively slow motion at both ends of its travel, where the hydraulic resistances are greatest.

Safeguards. The following safeguards are provided in the locks: (1) The duplicate lockgates mentioned above. (2) A novel system of chain fenders placed in the upper and lower approaches to all the locks and also just above the intermediate and lower gates in Pedro Miguel lock and in the upper chambers of the other two locks. They consist of 3-in. anchor chains stretched across the locks at the top, which are raised and lowered by vertical hydraulic cylinders in the walls. These chains serve to check vessels that may strike them, by paying out under strain, the amount of pull being regulated by relief valves on the cylinders. It has been shown by actual tests that a vessel of 18 000 tons displacement, moving at a speed of 2-1/2 miles per hour, can be readily brought to rest in a distance of 55 ft. without injury to the vessel or chain. (3) Electric towing locomotives, traveling on rack railways on the walls on both sides of the lock chambers, which are used in the case of all vessels for moving them during their passage through the entire length of the lock flight. They exert a pull of about 25 000 lb. and move at a speed of 1 or 2 miles per hour. (4) Emergency dams at the upper end of each lock for shutting off the flow of water in case of a serious accident to the lockgates, which has resulted in establishing a connection between the upper and lower levels. The dams are of the swing bridge type, similar in design to the movable dam at the St. Marys Falls Canal, shown in Fig. 65. At Panama, there is an independent dam (supported on the side wall) for each twin lock flight. The arm spanning the lock is 164 ft. 3 in. long, while the other arm, which is 98 ft. long, carries a heavy concrete counterweight and the operating machinery. The wicket girders are 60 ft. long, and spaced 9 ft. 2 in. apart. There are 5 wickets in each vertical bay, which roll down along the upstream flanges of the girders. In operation, the flow of water is to be gradually shut off by building up the wickets in horizontal tiers from the bottom. In still water, the dam has been swung, and all the girders and wickets put in place, in 26 minutes.

All the machinery in the locks is operated electrically. The current, which is 3-phase at 25 cycles, is generated at 2200 and 6600 volts in a hydroelectric station at the spillway in the Gatun dam operating under a head of 77 ft. Its present capacity is 13 140 kw., to be ultimately increased to 22 140 kw. There is also a reserve steam turbine plant of 7500 kw. capacity near Miraflores. The current is distributed at 6600 volts to transformer rooms in the lockwalls at Gatun, all motors operating at 220 volts. It is carried across the isthmus at 44 000 volts on an overhead transmission line supported on structural steel frames along the line of the Panama Railroad.

The machines for moving the lock gates, valves and chain fenders are located in small rooms in the lock walls just below the coping level, which are all connected by continuous longitudinal tunnels.

The principal machines in each of the three groups of locks are normally operated from a central control house on the middle wall. This contains a horizontal control board or table, which resembles a miniature ground plan of the lock, with models of the different gates, valves, etc., which open and close when the actual gates are being operated. Local control is also provided in the several machine rooms. The principal gates, lock gates and chain fenders are interlocked so as to guard against errors in operation.

In the operation of the locks, several novel and important hydraulic features have been observed:

(1) A very unequal discharge from the several orifices in a given lateral culvert, the flow being greatest from the outlet farthest from the main culvert. This results in forcing a vessel in the lock strongly towards the wall from which filling is taking place and has made it necessary to use both culverts throughout the process of filling, thus obtaining a very uniform distribution and permitting a maximum rate of changing levels of 7.5 ft. per minute with safety.

(2) An "over travel" of the water levels, due to the acceleration of the water in motion through the culverts. Owing to the great length and cross-section of the locks, this action, not taken into account in the ordinary formulas, is important, as it shortens the time of filling by about two minutes.

The coefficients of flow are approximately:

For filling—	using side and middle wall culverts,	$C = .65$
	using side wall only,	$C = .82$
For emptying—	using side and middle wall culverts,	$C = .67$
	using side wall only,	$C = .73$

The time for filling (for a length of lock of 900 ft.) is 7 to 8 minutes, using both culverts, and 12.5 to 13.5 minutes using the side-wall culvert only.

Dams. The Gatun dam is shown in cross-section in Fig. 13, No. 12. It is about 1-1/2 miles long and polygonal in ground plan. The maximum bottom width is 2300 ft. The central portion of the cross-section, except near the top, was hydraulically filled, by pumping a mixture of sand and clay from the bed of the Chagres by 20-in. suction pumps, working in two lifts. The hydraulic fill was confined on each side by rock-fill dams, built for the purpose, which form a part of the main dam. The remainder of the cross-section is dry fill, earth and rock, brought partly from the Culebra Cut, partly from borrow pits close to the dam. There is no core-wall. The total volume of Gatun Dam is 22 958 000 cu. yd., of which 12 229 000 cu. yd. was dry fill. Its cost, exclusive of the spillway, was $9 871 635. The much smaller dams, at the Pedro Miguel and Miraflores locks, are of a similar character.

Spillways. Lake Gatun is regulated entirely by a concrete spillway, built on a rocky ledge, which divides Gatun dam into two approximately equal parts. The total watershed drained is 1320 sq. miles, the annual rainfall 120 in., with a runoff of 62%. The yearly evaporation of the lake area is 60 in., while there is no appreciable seepage.

The maximum observed momentary discharge of the river was 175 000 cu. ft. per sec., the maximum for 33 hours, 137 500 cu. ft. per sec., and for 48 hours 120 000 cu. ft. per sec. The spillway has a capacity of 180 000 cu. ft. per sec., with the lake at + 87. It is circular in plan with 14 clear openings of 45 ft. controlled by electrically operated Stoney gates 19 ft. high. The crest is at +69.

A similar spillway but straight in plan, with 8 openings of 45 ft., is provided adjacent to Miraflores lock, as a safeguard in case of a serious accident to the Pedro Miguel lock. Its capacity is about 92 000 cu. ft. per sec.

Capacity of the Canal. The maximum number of complete lockages at both ends of the canal possible with an unlimited water supply is about 48 in 24 hours, corresponding to the passage through the canal of 60 commercial vessels of average size. With the present provisions for water storage, the practicable number of lockages is much lower, varying with the power demands and the rainfall in different years. In average years 21 lockages will be possible even when operating the power plant at its full projected capacity of 22 140 kw. In very dry years, the number would be still smaller, but the capacity of the canal can be largely increased by using the auxiliary steam plant for part of the year, or by providing additional storage by reservoirs on the upper Chagres.

Excavation. The earth and rock excavation formed the largest single item in the canal construction, its cost amounting to nearly half the total cost of the canal. To June 30, 1918, the total excavation was 267 834 201 cu. yd., of which 130 465 874 cu. yd. was dry excavation, the rest being removed by dredging and hydraulic methods. Apart from a portion of the lock excavation and a few short channel sections elsewhere, the dry excavation was confined almost wholly to the 8-3/4 miles of the Culebra Cut, which included all the most difficult work. The prism section is shown in Fig. 46. The excavation below the berm was wholly in rock as was fully 70% of all the excavation in the cut. The established slope above Elev. + 95 was 2 on 3, which was generally maintained except in the deepest portion close to the Continental Divide. This section, not over 7/8 mile long, was the scene of the serious slides which delayed the opening of the canal and subsequently stopped navigation for many months. These slides have been practically overcome by flattening the slopes to approximately 1 on 7 by dredging after the channel was filled with water in October, 1913. The material removed from the slides to June 30, 1918, was about 56 000 000 cu. yd.

The wet excavation, mainly in the sea-level sections, the lock pits and the slides, was carried on by an elaborate plant, comprising suction, ladder and dipper dredges of varied types and great capacity. Hydraulic sluicing was also resorted to on some parts of the work.

Aids to Navigation. In addition to the lighthouses at the entrances to the canal a complete system of range lights, beacons, and buoys marks the channel clearly both by day and at night. The tangents in the lake and sea-level channels are defined by range lights, vessels going in opposite directions using different ranges giving courses 200 to 250 ft. apart, a somewhat novel arrangement. In the Culebra Cut, beacons placed at frequent intervals on the banks take the place of range lights. There are also numerous gas buoys at the sides of the channel.

Terminals. Both terminals are protected by rock-fill breakwaters, giving ample anchorage space, and are equipped with concrete and steel piers for the transfer of freight. At the Pacific end, there is a concrete drydock with a clear width of 110 ft. and a usable length of 1000 ft., close to which large repair shops have been built. There are very complete coaling plants at each end of the canal, the one at the Atlantic end having a maximum storage capacity of 100 000 tons of submerged and 350 000 tons of dry coal. The corresponding figures for the Pacific Plant are 44 500 and 167 000 tons.

Total Cost of Canal. The total amount charged to Canal construction to June 30, 1918, is given on p. 1603; but a considerable additional sum since expended for dredging out the slides and for many other purposes should properly be charged to capital account. In the figures given, an overhead charge is included, covering general and administrative costs, expenses for hospital and sanitation, etc. An additional sum of $30 000 000 was appropriated for fortifications.

Cost of Construction of Panama Canal to June 30, 1918

Prism excavation	\$137 674 924.79
Locks and Dams	92 768 591.66
Breakwaters	9 098 018.41
Aids to navigation	650 139.87
Power plants and transmission system	5 417 309.72
Atlantic terminals	5 248 350.47
Pacific terminals	14 394 256.12
Permanent townsites	2 114 087.09
Buildings and Playgrounds	15 740 443.64
Sanitary fills, etc.	718 947.64
Water works and sewerage systems	3 285 831.85
Real Estate	2 776 121.35
Rebuilding Panama Railroad	9 800 626.46
Concessions from Republic of Panama	10 000 000.00
Purchase of canal from New Panama Canal Co.	38 728 484.05
Miscellaneous	154 649.28
Total	\$348 870 782.40

24. Power, Drainage, and Irrigation Canals

Cross-Sections. Unlined canals in earth have usually a large trapezoidal section, at least when first built. The side slope must be fixed with reference to the character of the soil in which the canal lies. It is usually assumed that the slope should be a little flatter than the angle of repose of the earth in water. In very light fine soil the slope may be as great as 3 or 4 horizontal to 1 vertical; in clayey loam or coarse soil or ordinary firm soil 1-1/2 or 2 horizontal to 1 vertical; in firm clayey gravel or hardpan or firm clay 1-1/4 or 1 horizontal to 1 vertical. Experiment with the earth of the locality may be necessary, to fix the best side slope.

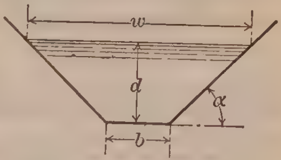


Fig. 66

Trapezoidal Sections. Having the side slope fixed, the relation of depth to breadth may be chosen. The efficiency of canals used as aqueducts, other things being equal, is directly proportional to the square root of the hydraulic radius. Let b, d and w be as in Fig. 66, A the area of cross-section, p the wetted perimeter and r the hydraulic radius. The values of b and d which will furnish a section of the maximum hydraulic radius for a given value of A, are given by the formulas:

$$b = \frac{A}{d} - d \cot\alpha \qquad d^2 = \frac{A \sin\alpha}{2 - \cos\alpha} \qquad p = b + \frac{2\,d}{\sin\alpha} \qquad r = \frac{A}{p}.$$

The table, p. 1604, computed by these formulas, gives the values of d, b, p and r in terms of A, which will furnish sections of maximum hydraulic radius, and consequently maximum efficiency, for various slopes in common use.

The Semicircular Section has the greatest hydraulic radius for a given area of cross-section, but it is only practicable when lined with concrete or steel. For a given A, the formulas are

$$d = 0.798\sqrt{A} \qquad p = 1.773\sqrt{A} \qquad r = 0.564\sqrt{A}.$$

Slope	α	d	b	p	r
1 on 3	18° 26′	$0.548\sqrt{A}$	$0.178\sqrt{A}$	$3.647\sqrt{A}$	$0.274\sqrt{A}$
1 on 2	26° 34′	$0.636\sqrt{A}$	$0.300\sqrt{A}$	$3.145\sqrt{A}$	$0.318\sqrt{A}$
4 on 7	29° 45′	$0.662\sqrt{A}$	$0.352\sqrt{A}$	$3.021\sqrt{A}$	$0.331\sqrt{A}$
2 on 3	33° 41′	$0.689\sqrt{A}$	$0.417\sqrt{A}$	$2.902\sqrt{A}$	$0.345\sqrt{A}$
4 on 5	38° 40′	$0.716\sqrt{A}$	$0.502\sqrt{A}$	$2.794\sqrt{A}$	$0.358\sqrt{A}$
1 on 1	45° 0′	$0.740\sqrt{A}$	$0.613\sqrt{A}$	$2.704\sqrt{A}$	$0.370\sqrt{A}$
3 on 2	56° 19′	$0.759\sqrt{A}$	$0.812\sqrt{A}$	$2.636\sqrt{A}$	$0.379\sqrt{A}$
7 on 4	60° 15′	$0.760\sqrt{A}$	$0.882\sqrt{A}$	$2.632\sqrt{A}$	$0.380\sqrt{A}$
2 on 1	63° 26′	$0.759\sqrt{A}$	$0.938\sqrt{A}$	$2.635\sqrt{A}$	$0.379\sqrt{A}$
Vertical	90° 0′	$0.707\sqrt{A}$	$1.414\sqrt{A}$	$2.828\sqrt{A}$	$0.354\sqrt{A}$

For small canals the equations above will often fix the best dimensions, but there are numerous considerations which may modify the section. If the canal is very large the excavation may be enough cheaper for a shallower canal, so that the necessary greater sectional area may be removed more economically than the smaller section of equal capacity. This will depend much upon the methods of excavation and the type of machinery used. Again, losses from seepage increase with depth, so that where such losses are liable to become important, or where the bottom of a deep canal would reach an open impervious stratum, it may be wise to adopt a shallower section than would otherwise be economical. On the other hand, for canals on side hills, a still deeper section of greater area may be more economical.

Small canals are subject to a reduction of cross-section by encroachment of vegetation on the banks, and by silting. Jeffreys, in Professional Papers on Indian Engineering, says: "Whatever slope is adopted in construction it is found that this cannot be maintained after the channel has been in use some time. A distributary at the close of an irrigating season invariably assumes the shape seen in Fig. 67. When the time for clearance comes around the engineer in charge, if he is wise, will not attempt to restore the original section. . . . . The custom in the Ganges Canal distributaries is to trim off the slope at 1/2 to 1, as shown by dotted lines *a–b* and *c–d*."

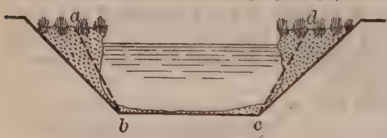


Fig. 67

The canal may be all in cutting, or all in embankment, or part in each. Each type has its advantages and disadvantages, depending on the local condition. In some localities canals in cutting may be cheaper in maintenance and suffer less loss from seepage; the drainage from the surrounding territory may be more easily handled. In other localities there may be danger of reaching an open pervious stratum in cutting which may require lining to prevent excessive loss.

Irrigation Canals all in embankment are sometimes necessary to command the surrounding land or for short lengths to avoid long detours. It is easier with this type, if for any reason it becomes necessary to avoid absorbing the drainage of the country, to pass that drainage across the line of the canal in culverts. The disadvantages are the danger that the banks may break, the higher cost of construction, inspection, and maintenance, and, usually, the greater loss by seepage, which in some cases is further objectionable by forming stagnant pools on the neighboring land. This type of canal generally costs more than either of the other two types.

The advantages of the type in which the section is part cutting and part embankment are that it is usually the cheapest and quickest constructed, that for irrigation it is high enough to command the neighboring land, and that the head of water against the embankments is usually small enough to reduce to little or nothing seepage through the banks.

Limiting Velocities. Loss of head is to be avoided as much as possible in power canals, whereas in irrigation and drainage canals the velocity will often be limited only by the ability of the earth of the banks and bed to resist erosion. The velocity should be great enough to prevent the growth of weeds and the deposit of silt, and not so great as to cause erosion. The experiments of Du Buat give velocities in feet per second next to the bed of the canal which will begin to erode various materials as follows: 0.25 for soft clay, 0.50 for fine sand, 0.70 for gravel as large as peas, 2.2 for gravel 1 in. in diameter, 3.3 for pebbles 1-1/2 in. in diameter, 4.0 for heavy shingle.

Seepage. Sometimes power canals, and usually irrigation canals, are located on hillsides well above ground-water level, so that seepage from unlined canals often amounts to 25%, and sometimes 50%, of the total water entering their headworks. For example, from 20 irrigation canals in operation, taken at random, including canals in France, Belgium, Italy and the United States, the average loss from seepage expressed in depth times area of the water surface of the canal was 3.8 ft. depth per day. Omitting two which were between 6 and 7 ft. per day, the average was 1.5 ft. depth per day. In gravel the loss was from 3 to 7 ft. depth per day; in sandy soils 1 to 2 ft.; in sandy loam and firm compact alluvial soil 0.2 ft. to 1 ft. depth per day. These canals varied in depth from 2 to 8 ft.

The common remedies for loss by seepage are silting, puddling, and lining with concrete. Excessive losses are likely to be in some short length. Good results are reported to have been obtained by producing stagnant water, which is then made turbid by dumping pulverized clay or other fine material. If the canal water is normally turbid, slack water may be formed by a temporary weir and silting encouraged at the leaky length. A more effective method, but more expensive, is to puddle the bottom and sides with clay when the canal is drained. It may be necessary to protect the puddle from erosion by a layer of coarser material. Where the saving will warrant the expense, the canal may be lined with concrete, as has been done in the case of numerous canals in California.

Canal Linings. Power canals in earth are frequently lined with either timber or concrete to reduce the friction and assure permanency of the cross-section. The timber lining consists of mud sills laid at right angles to the

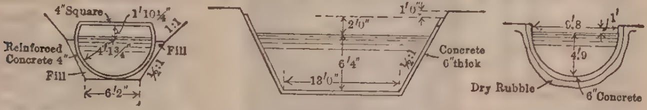


Fig. 68. Three Types of Concrete Lining

direction of flow and planed plank flooring spiked to the mud sills. The mud sills may rest upon and be held down by bearing piles or be held down by rods to masonry anchors. Fig. 68 shows linings used in the West.

If the ground is very firm, concrete may be laid upon it without other support; in very soft ground bearing piles may be necessary, care being taken, however, that the concrete is in contact with the ground. The concrete should be deposited back of very smooth forms in order to get a smooth surface. For irrigation canals the concrete-lined canal not only costs less to maintain, and prevents loss of water by seepage, but the first cost is often less than for an unlined canal, for the allowable velocities may be from two to four times as great and the section correspondingly smaller.

For the Tieton Canal (left-hand cut of Fig. 68) the reinforced-concrete lining was molded in steel forms in sections 2 ft. long which were later transported on cars to the canal and set in position by derricks; each section weighing about 1800 lb. was 4 in. thick, of gravel concrete reinforced by 3/8-in. corrugated rods 4 in. apart. Each section was stiffened by a 4-in. by 6-in. crossbar reinforced by two 3/8-in. rods. The molds were removed after about 3 days and the section allowed to harden for at least 30 days, during which time it was frequently sprinkled. The sections were set in place about 1-1/2 in. apart and immediately backfilled with earth which on the lower quarter was selected and tamped in from the ends with the greatest care; above this the backfilling was placed from above. The 1-1 2-in. space between sections was later filled with rich concrete of fine aggregate which would pass a 1/2-in. ring. The joint was rubbed smooth on the inside before the joining concrete was completely set. The canal has a fall of 8.71 ft. per mile and was designed to have a capacity of 300 sec.-ft. A test in the completed canal gave a value of .012 for the coefficient *n* in Kutter's formula. The gross cost, exclusive of excavation, of 13 000 ft. of the concrete lining was found to be $5.80 per foot.

SHAFTS AND BORINGS

25. Timbered Shafts

Timbered shafts are sunk vertically by mining much the same as a heading is driven horizontally. A frame of four waling timbers joined at their ends, forming a rectangle, is laid out on the ground. Outside are driven vertical polings or sheeting; inside, as the sheeting is being driven, the earth is excavated to a little above or a little below its lower end, depending upon whether the ground is soft or hard. As soon as the excavation is from 2 to 5 ft. below one frame a new frame is placed, and so on to the depth of the length of the sheeting; a new frame is then set with clearance for a new set of sheeting. If the shaft is to be shallow it may be started large enough at the top so that the sheeting may be vertical, and the shaft reduced in size by the thickness of the timbering at the bottom of each set of sheeting, which may be from 10 to 20 ft. long, as in Fig. 69*a*. If the shaft is to be deep, the sheeting will ordinarily be 5 or 6 ft. polings, as in Fig. 69*b*.

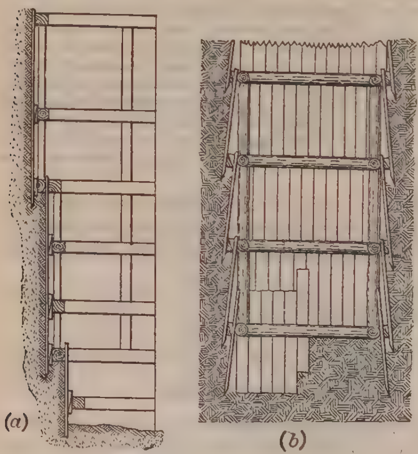

(*a*) (*b*)

Fig. 69. Timbered Shafts

The main thing to be avoided in shaft sinking is the starting of a general movement of the earth. The great danger is that cavities will be left or formed back of the timbering or that the timbering though not slack will have to deflect to develop resistance and a small progressive movement of the earth will be started which may extend to the surface. The precautions are, if possible, to allow no cavity to form, securely pack any accidental one, and to wedge all timbering tight against the sheeting so that it will develop resistance before the sheeting begins to move. Perhaps the greatest danger is in water-bearing earth, where leakage is liable to bring into the shaft earth in such small amounts that they are not noticeable but may form cavities back of the sheeting in time.

In rock shafts the principal use of timbering is to prevent the falling of loose pieces from the sides of the shaft, endangering the workmen below. If the rock is fairly good, the timbering is usually kept from 10 to 30 or 40 ft. behind the excavation. It is not an uncommon practice to place the permanent lining, usually of concrete, in short lengths 40 or 50 ft. above the bottom of the shaft as the work of excavation progresses.

The method of open excavation and timbering or otherwise lining the shaft as sinking progresses is no doubt cheaper, where practicable, than any other method that has been devised, but when quicksand or other soft material is encountered or where rock yielding too much water is encountered, the method may be either impossible or impracticable. For such cases various devices have been used, notably caissons sunk by the aid of compressed air, drop shaft, freezing process, and under-water boring.

Shafts by caissons are frequently used in tunnel construction for railroads. The depth of compressed air work is limited by the pressure at which work becomes either too expensive or too dangerous. After a depth of 70 ft. below water is reached the cost of labor in compressed air increases very rapidly on account of both greater wages and shorter hours. Very little caisson work has been done under heads of over 100 ft.

26. Drop Shafts

Drop shafts are sunk by building a heavy lining, usually circular, upon the surface, and then excavating the earth inside, allowing the structure to settle slowly into the ground. Drop shafts have been used in a great variety of places, from small wells a few feet deep to mine shafts 20 ft. in diameter and over 500 ft. deep. The shell or lining may be of timber, masonry, brick, reinforced concrete, cast iron or steel.

Several very large drop shafts of moderate depth in water-bearing soil have been built circular of brick masonry provided with a cast-iron shoe or cutting edge at the bottom. A large number of small drop shafts, having steel plate shells, have been sunk for foundation work. Most drop shafts for deep mines have had shells of cast-iron rings with, in some cases, steel shoes for cutting edges. The excavation in some cases has been by hand, but usually drop shafts are in water-bearing strata, often in quicksand, and the water is allowed to fill the interior while the earth is removed by under-water excavators, such as chain-bucket dredges, grab buckets, and sand pumps.

In moderate-depth shafts the weight of the lining is usually sufficient to overcome friction on the sides. In the case of deep shafts this is not the case, and the lining must be weighted with pig iron or sand or be forced down by jacks. Water or air jets which may be forced up along the exterior surface have been used to reduce friction on the exterior, but it has frequently been necessary to abandon the sinking of the first shell and proceed with a lining of a smaller diameter sunk inside the outer lining, which will be subject to friction only below the depth of the bottom of the exterior lining.

The cost of drop shafts is often less than sinking by compressed air, the freezing process, or the boring method. The method is, however, less sure than any of these. In the drop-shaft method the lining is very liable to be distorted, and in some cases ruptured, due to drawing in material from outside the lining, forming cavities, which results in unequal pressures.

27. Freezing Process for Shafts

About 60 shafts, mostly in northern Europe (France, Germany, and Belgium), were sunk up to 1910 by the aid of the freezing process. The general features of all cases are indicated by Fig. 70. Vertical pipes are sunk, usually forming a circle in plan, completely surrounding the site of the proposed shaft; inside each pipe is placed a smaller pipe, called a circulating tube, opening into the outer pipe at the lower end. These circulating tubes are all joined together at the top by a pipe known as the circulating ring. The freezing pipes are joined together at the top by a similar ring called the collector ring. Cold brine at a temperature of from − 10° to − 30° F. is then

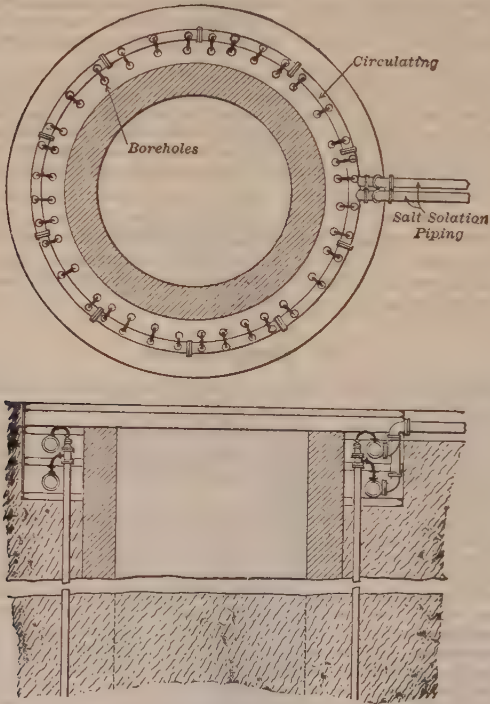


Fig. 70. Freezing Process for Shafts

pumped into the circulating ring and down the circulating tubes, back up the freezing tubes to the freezing machine.

The freezing pipes are put down by one of the well-known boring methods. Originally the casing of the bore hole was used for the freezing pipe. On account of the difficulty of closing the lower end of the pipe and making all joints of the casing watertight, in all later work the freezing pipes are put together and tested on the surface and then lowered into the bore hole, after which the casing is withdrawn. The freezing pipes are 4 to 6 in. in diameter, the circulating pipes 1 to 1-1/2 in.

In the center of the shaft an equilibrium pipe is usually sunk. This pipe is kept open, as a vent, for the escape of water forced out of the interior mass by the expansion of the freezing material; it is sometimes necessary to keep this open with a steam pipe, because rock or clay strata will freeze much faster than quicksand and possibly congeal to the equilibrium pipe and freeze it before the lower sand strata are frozen.

It is very difficult to keep deep bore holes for the freezing pipes from departing from the vertical. Long freezing pipes are very likely to spring a leak from contraction at low temperatures. Both these considerations have led to the attempt to do the shaft in shorter lengths, say 150 ft. in height. The time required for freezing a solid wall about the shaft site will vary with the character of the strata, size and distance apart of the freezing pipes, and the temperature at which the brine is maintained. The freezing usually continues from 4 to 10 months or longer. After an ice wall has been formed around the site of the shaft, the shaft may be sunk by hand methods and lined as the excavation progresses or it may be sunk as a drop shaft.

28. Shaft Sinking by Borings

The Kind-Chaudron System for sinking shafts in water-bearing rock strata is almost identically the same method as ordinary well drilling except that it is on a very large scale. In Fig. 71 a very large chopping bit or trepan is seen at *a*, which may be from 10 to 15 ft. in diameter, weighs from 30 000 to 50 000 lb., and cuts a hole the full diameter of the shaft; *b* are rods, usually of wood, each about 65 ft. long, connected by iron links; *c* is a bar in a swivel joint and screw connection by which the tool is rotated and lowered slightly between the blows; *d* is a walking beam which alternately lifts and drops the trepan; and *e* is a steam cylinder which operates the walking beam. The shaft is completely filled with water during sinking. A small trepan is used alternately with the large one, and bores a pilot hole, usually about 1/3 the diameter of the shaft, into which the choppings from the large trepan fall and are removed by a grab bucket.

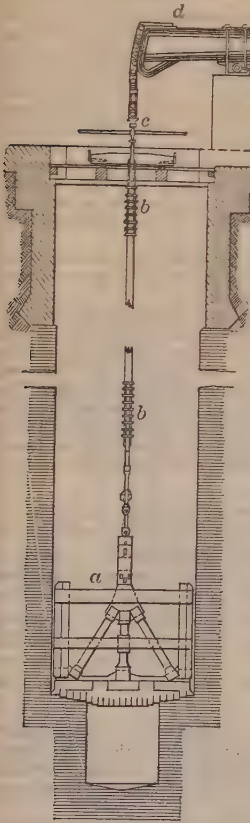


Fig. 71. Kind-Chaudron System

After the shaft is drilled, sometimes to a depth of 1300 ft., a watertight cast-iron casing or tubbing is then sunk inside the bore and sealed at the bottom. The sealing of the bottom is accomplished by building what amounts to a great stuffing-box. The packing used is moss.

In sinking the tubbing, which sometimes weighs several million pounds, the first rings are suspended in the top of the shaft from beams. Rings are added on top until the tubbing, which is provided with a bottom, reaches the water surface; the whole thing is then partially and later completely water-borne, after which it is sunk as the rings are added by admitting water to the interior. On reaching the bottom the great weight bearing upon the moss in the packing rings presses it against the sides of the rock and makes a watertight joint. There is then an annular space above the moss box between the tubbing and the rock, which must be filled with either grout or concrete. After this is placed and set, the shaft is pumped out, the false bottom removed, and sinking continued in the impervious strata below.

About 80 shafts from 300 to 1300 ft. in depth have been sunk by this method, nearly all in northern Europe (France, Belgium, and Germany), where, to reach the coal measures, water-bearing strata of marl, shale, rock salt, and limestone must be passed. The rate of sinking varies greatly. 25 ft. per month is a fair average. The cost varies for 15-ft. shafts from $500 to $1000 per foot of depth for boring and lining with cast iron.

29. Wash Borings

Apparatus for making wash borings can be obtained from companies which make rock-drilling machinery. However, a very efficient outfit, for holes 150 ft. deep or less, may be made by any mechanic, and is not only adapted to work in isolated localities but has often been used in cities. The following is an inventory of the outfit:

1 derrick. 1 hand force pump with suction hose and fittings. 200 ft. 1 7/8-in drill rods with couplings. 200 ft. 2-1/2-in. flush-joint casing. 1 bushing to 2-1/2-in. casing. 1 hoisting water swivel 1-7/8 in. 1 hoisting plug with coupling. 2 cross chopping bits 6 in. long to fit rods. 3 pairs Brown's pipe tongs; 2 No. 3, 1 No. 4, 1 pair No. 3 Brock's chain tongs. 2 Coe's monkey wrenches, 15 and 10-in. 3 Stillson wrenches, 10, 14, and 24-in. 1 15-lb. crowbar. 1 pair sister hooks for 1-1/4-in. rope. 1 16-in. iron hoisting sheave for 1-1/4-in. rope, with strap and hook. 50 ft. 1-1/4-in. manila rope. 40 ft. 3/4-in. three-ply water hose. 150 ft. 1-in. pipe with couplings. 1 blasting battery with 500 ft. No. 20 wire, and supply of exploders and 40% dynamite. 1 tool box. 1-1/2-gal. oil can. 1 squirt can. 1 pail and cup. 1 pick. 1 spade. 1 ax. 1 hatchet. 3 1-3/4-in. sand pumps. 2 1-3/4-in. earth augers to screw into rods. 4 sets reducers 1-1/2-in. to 1/2-in. 1 portable forge, small anvil, and small set blacksmith's tools.

The derrick is illustrated by Fig. 72. When set up, the drum amd attached wheel are used as a hoisting winch. When folded and brought horizontal, the wheels become a truck on which the whole apparatus is moved to the next hole.

The drill rods are usually 1-7/8 in. in exterior diameter, extra heavy pipe, in 5 to 10-ft. lengths, with square female threads on each end, and coupling of about 6 in. long of the same pipe with square male threads at each end. The casing (for shallow holes usually 2-1/2 in. external diameter pipe) may be either "flush joint" or "drive casing." The drive casing consists of heavy pipe with ordinary sleeve couplings. The flush-joint casing is of heavy pipe, each length of which screws into the next, making the joint flush, outside and in. The chopping bit may be screwed into the drill rods, and is sharpened like a rock drill, but is provided with holes to allow the water, forced down the drill rods, to pass out under the bit. A chopping bit should never be used unless a full force of water is passing through it.

The process of sinking holes with flush-joint casing is usually about as follows: The derrick is set up and a hole is made, usually with a crowbar, to admit a 5-ft. length of casing set a few inches into the ground; into this is inserted a 10-ft. length of rod with chopping bit on the lower end and a hoisting water swivel on the upper end, which is connected to the derrick rope and by hose to the force pump. Large pipe tongs are then clamped to both the casing and rod. Water is turned on, and as the material is washed from inside the casing the latter is turned by the pipe tongs and gradually settles into the ground. A new length is added to both casing and rods, and then the washing and turning are repeated.

In most localities a small portable gasoline engine equipped with a "nigger head" or other winding gear is substituted for the hand power portion of the above equipment. In this case the derrick may be a simple tripod made of saplings. If heavy beds of gravel are to be penetrated the common portable blast hole or well drill using 4-in. casing or larger can be used to advantage.

Careful measurement should always be kept so that the relative position of the bottom of the casing and drill rods is always accurately known. In soft clayey silts and sands, holes may be sunk to a hundred feet of more in depth in this way with very little trouble, but conditions are seldom so favorable. If hard sand and gravel are encountered, the drill rods are churned as well as turned, by which the material may be broken up, pulverized, and washed out with water and the casing allowed to descend. In general the drill rods work below the bottom of the casing. If loose sand or gravel is encountered, there is likely to be trouble by the bind- ing of the casing. If the sand is a thin stratum it may be possible to wash or even blast out a cavity in it which will allow the casing to pass through into the next possibly more favorable stratum. Another expedient is to withdraw the rods, cap the top of the casing and at- tach the water supply to the casing, then to twist or drive, if drive casing is being used, while the water, which is being forced down inside the casing and up the outside, reduces the friction between the sand and outside of the casing. If a casing becomes badly bound so that no fur- ther progress down- ward can be made, it may be necessary to abandon the hole, or having started with a large casing to put a smaller casing inside the first when the latter becomes fast, in which case the friction will only begin at the bottom of the first casing. If the required depth cannot be

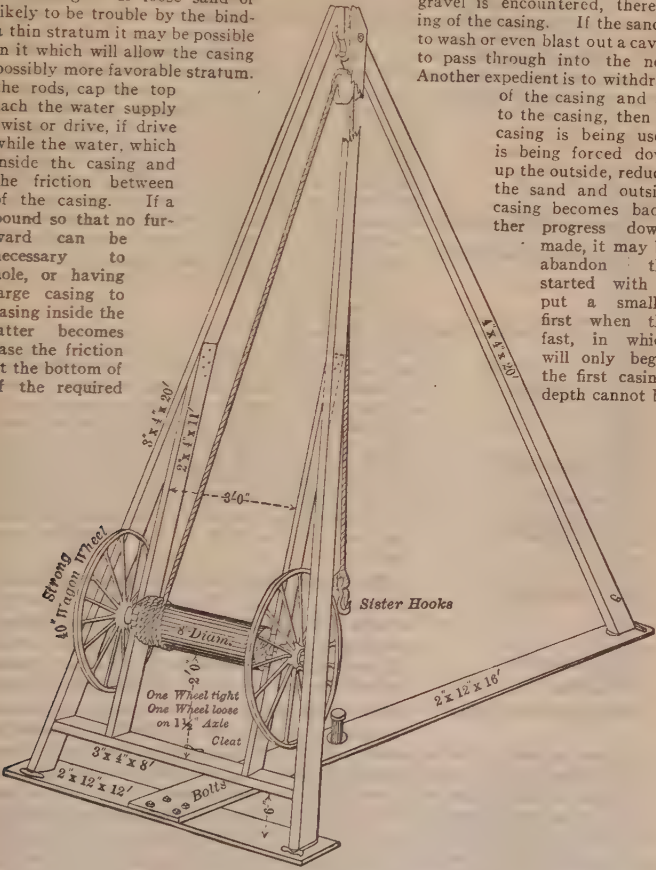


Fig. 72. Derrick for Wash Borings

reached with the second casing, a third and sometimes a fourth will be required. When a boulder or other obstacle is encountered a charge of dynamite is lowered to the bottom of the hole, the casing raised 5 ft. or more, depending somewhat on the size of the charge, a good rule being 5 ft. for one pound and 2 ft. for each additional pound of dynamite used, one-half to two cubic feet of sand is then poured down the hole, the charge exploded, the casing worked down again as quickly as possible, and if the boulder is a small one it may be found to be shattered so that the casing may be advanced through it. A single shot often makes no gain whereas a third or fourth will begin to break the boulder and show progress, but on bed-rock

7 or 8 shots will make practically no progress. If after blasting several times in this way no progress can be made, the conclusion is generally drawn that either bedrock or a very large boulder has been encountered, and if the purpose of the boring requires it, the hole is continued with a core-boring apparatus. A core boring is the only sure test to determine whether or not bedrock is reached.

For Drive Casing the bottom is provided with a heavy steel shoe or cutting edge, and instead of being simply twisted down as in the cash of the flush joint casing, it is driven by using the jar weight (Fig. 73); at the same time the chopping and washing are being done with the drill rods. Working with the drive casing is somewhat slower than with flush-joint casing, but in some cases such as where considerable depth of loose sand and gravel is encountered, it is preferable. In hard ground the process is to bore or chop with the drill rods a hole some distance ahead of the bottom casing, then withdraw the rods 10 or 12 ft. above the bottom and drive the casing down as far as it will go.

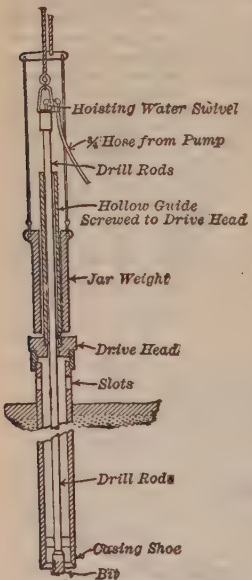


Fig. 73. Drive Casing

Fig. 73 shows one form of apparatus in place, where drive casing is used. There are many possible variations of this; the jar weights are of various sizes and shapes, a second drive head or collar is often screwed to the top of the hollow guides for use in pulling the casing. The jar weight hammers upward against this collar; at the same time the casing is turned with pipe tongs if possible. It is sometimes necessary, however, to clamp a yoke on to the casing itself and hoist on this yoke with jacks. When flush-joint casing is used, the drive head is seldom necessary, and the joints of the casing will not stand much driving.

To Obtain Samples of the material passed through by the drill the wash water is caught in a bucket after it rises to the surface through the casing, and the sediment is allowed to settle. Water should not be allowed to continue running into the bucket after it is once full, for it will wash out the clay and leave an unrepresentative sample. Such samples are often misleading as to the true nature of the material in place. A drive sample may be obtained by attaching to the bottom of the drill rods a sample barrel consisting of a pipe open at the bottom and provided with a small vent hole in the side near the upper end but plugged at the top to prevent the weight of water in the rods from forcing out the sample. This may be lowered and driven into the material which is removed from the sample barrel after being raised to the surface. Samples obtained in this way are much better than wash samples, but have the disadvantage that they have been considerably compacted in the taking, and unless the sample barrel is driven only a short distance into the material it may be doubtful from just what elevation the samples come. When the material is sand, samples may be taken or the casing cleaned out by using an ordinary sand pump, which is simply a length of pipe provided with a foot valve. This is churned up and down in the hole until filled with sand and water.

An auger may be screwed to the bottom of the drill rod and lowered into the hole and bored into the material at the bottom. If the material is not clean, open sand it will remain in the helix of the auger and come to the surface with it. In soft clayey materials it is often possible to use drive casing in connection with an auger instead of using water and the wash method. A hole is bored ahead of the bottom of the casing, which may then be driven down after the auger has been lifted well above the bottom.

The cost of wash borings is variable, depending on the character of the material penetrated. For holes up to 200 ft. deep in soft ground, such as silt and clayey sands the cost of well-managed work is $0.25 per foot and less under very favorable conditions. Where the ground is running sand or hard earth without boulders, the price runs from $0.50 to $1.00 per foot of depth. Where boulders or other obstructions are encountered the cost is from $1.00 in ordinary cases to $4.00 or $5.00 per foot. The above prices are for favorable conditions on the surface and enough work to perfect an organization. If the location is unfavorable, say in a swamp where staging will have to be built, or if water is not available, or if the conditions are otherwise difficult, the cost of overcoming the difficulties must be added. For jobs involving only a few holes, the cost of moving the apparatus to the ground and organizing the work may more than double the above prices. For deep holes, say between 800 and 1500 ft. deep, the cost is much increased. A larger plant with steam or other power must then be used.

Sub-aqueous borings are made upon the ice, or from a platform built on piles, or by use of a floating pile driver or from a catamaran.

30. Core Borings

The Diamond Drill consists of a short length of soft iron tube called a "bit," into the lower edge of which black diamonds are set. This bit is screwed into the bottom end of another tube called a "core barrel," which in turn is screwed to the hollow drill rod. A casing is sunk through the overlying earth to the rock surface by wash boring; inside this the drill rods with bit attached are placed and then rotated and pressed down by the drilling machine; at the same time water is forced down through the drill rods and bit, and comes to the surface outside the drill rods, carrying with it the rock, ground to a powder by the diamonds and bit. At intervals the drill rods are pulled out of the hole, the core removed from the barrel, the bit inspected, and the operation repeated. Diamond drilling, and especially the art of setting the diamonds in the bit, can be learned only by experience. All that can be done here is to call attention to some of the difficulties and methods of overcoming them. The first care should always be that an ample supply of water is being forced down the drill rods to the bit at all times; the second is to use every precaution to see that the space outside the drill rods does not become clogged by either borings or sand washing in around the top or bottom of the casing.

The difficulty most often encountered arises when seamy rock is met. Sometimes it happens that the water forced down the drill rods flows away through seams, carrying drillings with it. If drillings are carried into a seam sloping upward, they are likely to run back into the hole and pack around and bind the drill rods when the flow of water is shut off. Small pieces from the wall of the hole of seamy rock are likely to fall in and wedge the drill rods. A single bad seam may often be stopped by pouring sawdust or horse manure into the hole; the escaping water carries this into the seam and clogs it. The second method is to drop balls of soft neat cement mortar down the hole and after it has hardened proceed with the drilling. It is usually wise to treat all seamy rock in this way and drill carefully and remove drill rods relatively often. It is sometimes possible to blast or chop with a chopping bit and sink the casing through a loose seamy layer of rock. Frequently the hole is reamed out with a special tool and the casing driven down through the seamy rock.

It is a good rule to let the water run a little time after stopping the drill to clean out the hole and take careful measurement of the position of the bit. In lowering the rods again, if the bit does not reach the original position, the water should be turned on, and if then it will not sink to position without drilling, the bit should be raised and the hole cleaned out with a chopping bit to a smooth clean bottom.

Shot Drilling is done in much the same way as with diamond drills, and about the same rules apply; the difference is that instead of diamonds being set in the bottom of the bit, hardened steel balls are fed down with the water inside of the drill rods, and are rolled around under the edge of the rotating bit and wear out an annular ring the same as cut by diamonds. It has an advantage over diamond drilling because the apparatus is less expensive both to buy and to operate. It takes a much larger core, which is a great advantage in well drilling, and for prospecting usually gives a better sample because the large core is not so liable to break and crumble. On the other hand, shot drilling is usually slower than diamond drilling and it has been mostly used for vertical holes.

The cost of diamond drilling varies greatly with circumstances. In the softer rocks like limestone, for holes 200 ft. deep or less, taking 1-1/4-in. cores from 1-3/4-in. holes, the rate of actual drilling will vary from 1 to 3 ft. per hour. Delays will average from 50 to 200 per cent of time of actual drilling, and moving from hole to hole will usually take one or two days. Costs will vary from $2 to $5 per foot.

31. Test Pits

Test pits are sunk to determine the character of the earth as a preliminary to more expensive operations. If deep, the method of work is the same as already described for timbered shafts. One test pit is usually considerably more expensive than several wash borings, but it is worth much more in the reliability of the information which it gives. A combination of test pits and wash borings often yields the greatest amount of reliable information for a given outlay; the borings may locate the strata with sufficient exactness, and one test pit to every ten or twenty borings will show the character of the strata.

As the earth is excavated from the test pit, the different kinds of material should be placed in separate piles, and the boulders and smaller stones in each kind separated. It often happens that workmen in digging a test pit will pile the dirt in a single pile and cover up all the boulders. While the walls of a test pit generally show the character of the material, they do not give a correct idea of the amount of boulders likely to be encountered.

TUNNELS

32. Tunneling in Earth by Mining

The Elementary Operation of Mining is illustrated in Fig. 74. Starting with the face of earth which is to be mined, sharpened boards P, called polings, supported by the cross timber C, are driven into the earth. Under the protection of the polings the earth is excavated. The excavation is advanced beyond the end of the first polings by erecting another cross roof timber, C_2, and starting new polings and repeating the operation indefinitely. The mining of soft ground is largely a multiplication of this elementary operation. The sides of the excavation may be treated in the same way; small headings may be enlarged to full-sizes tunnels by opening the sides by the same process. In principle this elementary operation covers all conceivable requirements of protecting the sides and roof of a working where pressures are within the strength of the timbering. But there is always exposed a face F which must remain unprotected, at least for a short time, and here is where mining troubles begin.

The angle at which an exposed earth face will stand depends (1) upon the nature of the earth, in general, clay standing steeper than sand, and coarser and steeper than fine sand, and a mixture of clay with coarse sand or gravel being better than either alone. (2) Upon the amount of water present. Dampness will increase cohesion, but as complete saturation is approached the angle becomes flatter. If sand besides being completely saturated has an unbalanced head of water, that is, if the head or pressure of water in fine sand exposed to the air is greater than the air pressure, then it is quicksand and will take nearly a horizontal slope. Whatever the material, its thorough draining is of the greatest importance for successful mining. (3) The length of time exposed. Time is an important element in tunneling. Some of the most unstable materials will stand even vertical for a little while, so that a small vertical face may be exposed and a breast-board quickly placed. Clayey earth sometimes may move so slowly that the excavation may be made without the protection of the poling, which may later be simply put up against the roof and supported by timbering which will be thoroughly loaded later. (4) The area of the face exposed. A small area exposed will often stand vertical for a long time when a larger one would cave quickly. (5) The distance below the earth surface. By the ordinary accepted theory of earth pressures the horizontal thrust of earth is a function of the vertical pressure or distance below the surface. But where small areas are exposed, dry or only damp earth will arch across the area, so that depth does not usually appreciably affect the angle at which the exposed earth face will stand until some time has elapsed. The arching cannot be depended upon in wet running material. (6) The amount of agitation the earth has received. Earth which may be stable in its natural state may be very treacherous after it has been moved. The fear of starting a general movement of the earth makes miners thoroughly pack all voids back of supporting timbering and use the greatest care to prevent the starting of a run.

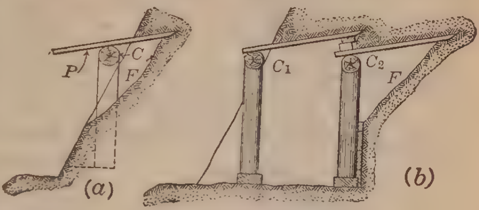


Fig. 74. Mining in Earth

Timbered Headings. In all methods of soft-ground mining the face is opened with a heading varying in cross-section according to the material penetrated, from a one-man burrow to full width and 7 or 8 ft. high, and any length (Fig. 75). The heading timbering consists of posts P, caps C, poling boards p, and breast-boards b. If the ground is stiff, the face of the heading is cut down straight at nearly the forward end of the polings and new posts and a cap are set. If the ground is soft a small excavation, say 2 ft. high, under one or two polings is made and a short board is quickly set vertical under the end of the polings and acts as a combined breast-board and prop for the poling. This is repeated under all polings. As soon as all short vertical breast-boards are placed a new cap is set supported by short posts. The remainder of the face is then worked down, usually with horizontal breast-boards, to the bottom of the heading.

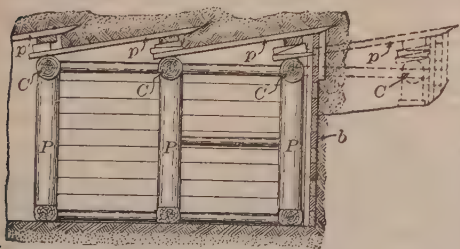


Fig. 75. Timbering Headings

33. Methods of Soft Ground Tunneling

Small bores are driven by simple headings. Large bores are worked in a variety of ways which have been classified in general according to the country in which the method originated, as English, Belgian, German, Austrian, and Italian. Obviously the classification though widely accepted is far from strict, because each method has many variations, nor has the use of any of the methods been confined to the country for which it is named.

English Method. The entire section is removed in short lengths, usually from 12 to 20 ft., in advance of the permanent lining already built, as shown in Fig. 76. The masons and miners alternate in the possession of the face, and the work of excavation and building the masonry is uninterrupted until

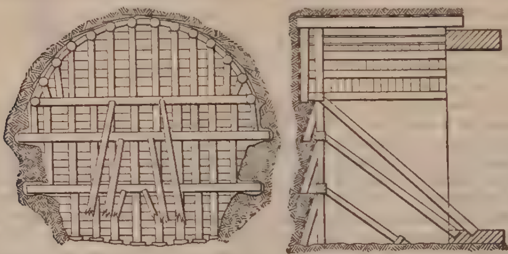


Fig. 76. English Method.

each is complete for the length. A small drift in wet material is commonly driven at the bottom of the tunnel prism from end to end of the tunnel, both for the purpose of furnishing good drainage facilities and for the establishment of the alignment below ground; also it allows the tunnel to be attacked at several points. The main attack on the face, however, always begins at the top of the section with a heading which in stiff material is often taken out without timbering but in softer material is timbered. Two roof bars are then placed in this heading with their forward ends resting on posts and their rear ends supported on the completed masonry lining. Transverse polings are then driven over these bars. The heading is then widened out under the transverse polings for the length of the roof bars, vertical breasting boards placed under the transverse polings and new side bars are placed; when the ground is stiff enough the polings are not driven over the side bars but placed against the earth roof after excavation as in Fig. 77. This operation is continued down the sides as far as required by the nature and pressure of the ground, sometimes to the bottom, while at the same time the face is securely breasted and back-strutted to the completed lining. The miners now give way to the masons, who construct the length of lining within the completed timbering, and the operation is repeated. If the roof bars are not too firmly gripped by the overlying material, the lagging is blocked up on the masonry and the roof bars are barred or pulled forward by jacks and used over again.

This method as a whole is best adapted to very firm material, but has been used with success where the ground was heavy and wet. The Saltwood tunnel on the Southeastern Railway between London and Dover was built through greensand, which when disturbed was a quicksand and not workable by the English method without being drained. A small bottom drift driven through from end to end drained off the water, care being taken to confine the sand, and the main construction proceeded by the usual English method. The advantages of the system are the large open area in which the masonry lining can be built up in one operation and the facility with which the spoil cars can be handled. The disadvantages are that the excavators miss alternate shifts while the masons are at work, thus delaying the work, and that the timbering is partly carried on fresh masonry. The fact that the full section is excavated at once limits the use of the method to fairly firm material.

Belgian Method. The upper half of the tunnel is excavated much the same as in the English method except that the excavation is frequently carried a considerable distance ahead of the masonry, and, as the method is most often used in firm ground, the transverse polings are frequently not driven over the roof bars, but instead are placed as in Fig. 77, against the earth roof after the excavation has been made for a side roof bar.

A cut is then excavated through the center half of the tunnel to the invert, leaving a berm on either side to support the arch of the tunnel lining. From this center trench narrow cuts are made at intervals to the side and the masonry arch is underpinned; the cuts are widened and the underpinning extended until a complete side wall is built, and finally the invert arch is turned. The main advantage of this method lies in the fact that it is possible in firm material to push the work from many points of attack, the successive developments of the tunnel, the heading, its enlargement, the masonry arch, the underpinning operations, etc., all being carried on simultaneously at different points along the tunnel. When the work is carried on with this system of successive development the timbering is arranged as in Fig. 77, to permit the ready access of the spoil cars to the furthest workings. This method is adapted to firm material, but may be used, with considerable risk, however, in the heavier soils by shortening the timbered section, in which case it loses its chief economy mentioned above. The disadvantage of this method is that the arch is first carried on earth and then on timbering, both liable to unequal settlement with the consequent risk of fracture in the key. If it becomes necessary in very stiff material to do blasting, danger to the temporarily supported arch is increased.

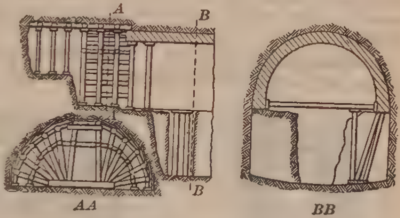


Fig. 77. Belgian Method

German Method (Fig. 78). The invert is put in last, but the rest of the lining is built from the bottom up, but without removing the center core of earth. In its most characteristic form the method consists in driving two bottom headings, one at the foot of each side wall. In these the side walls are built as high as the roof of the heading will allow. On top of these headings two others are driven and the side walls brought up to their roofs, and so on until the two side walls are joined in a center heading at the top. Practically the number of headings one on top of the other is limited to three, with

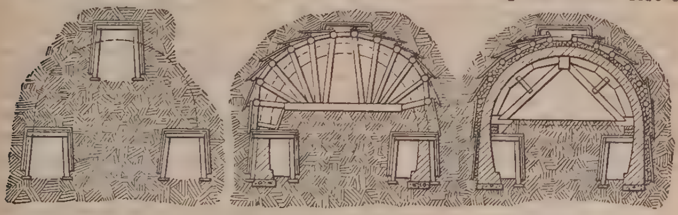


Fig. 78. German Method

a top center heading which is widened out to the upper side headings. Usually only two headings on each side have been used, which brings the sides up to the springing of the arch; above this the entire section is taken out by widening out a center top heading down to the side heading by the same plan as in the Belgian method.

The most notable example in America of a tunnel done by this method is the Baltimore Belt Line tunnel. There only one heading on each side was driven; the center top heading was widened out down to the spring line and pits sunk down from this large upper working to the side walls which had been built in the side bottom headings. The sequence of operations is shown in Fig. 78. The chief advantages of the German method are the saving in timber due to large cores of earth being used to brace against; the cheapness with which the central core, a relatively large mass of earth, can be removed; the fact that only small areas are opened, thus increasing safety in soft materials, and the fact that the masonry is built continuously from the bottom upward. The disadvantages are that the restricted area available for removing spoil from the small headings causes a great amount of inconvenience and interference with the masons and timbermen; ventilation in these small passageways is very difficult; and there may be danger in some cases of movement of the center core when used as an abutment for strutting.

Austrian System (Fig. 79). The essential feature is a center cut from top to bottom of the tunnel with timbering as in *G-G*. The cut is widened out from

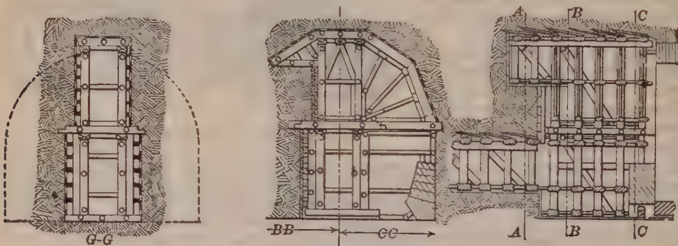


Fig. 79. Austrian System

the top as indicated in section *BB* and timbered with segmental arch timbering up to the time the sills are placed above the horizontal diameter and is supported by the central frame. The poling boards over the segmental arch timbers are all driven parallel to the axis of the tunnel, and the widening out is done in short lengths, working practically the full face. The variations in the method, depending largely on the character of the material, are practically all in the method of making the central cut. A top heading may be driven, which then may be cut down in short lengths to the invert of the tunnel, or a bottom heading may be driven, then the top heading, then the core between the two removed in short lengths, the short posts of the top heading being replaced by long ones resting on the cap timbers of the bottom heading.

Mr. Rziha, the eminent Austrian engineer, who did much to develop the system and who is its principal advocate, recommends the bottom heading to be first driven from end to end of the tunnel, then enlarging this to about half the height of the tunnel and placing in it all of the lower half of the timbering required in the center cut, section *CC*, Fig. 79. The upper half of the center cut is then taken out in a top heading and the timbers placed as in Fig. 79. The excavation of the remainder of the full section is then done, being kept 10 to 20 ft. ahead of the masonry. The masonry lining is begun at the bottom and carried up continuously, the reaction of the timbers being transferred to the masonry as they are reached. The advantages of the method are that the timbering is strong, that the timbers are in short lengths and easily handled, and that the work may be attacked at a number of points and each operation carried on continuously. A disadvantage is its expense.

The Italian Method (Fig. 80) was especially developed for very soft and treacherous ground and is strictly an emergency method. The success of the

Fig. 80. Italian Method

system depends on the fact that only small areas are opened and the material can thus be well controlled. A bottom center drift about one-eighth the height of the tunnel is started and driven a very short distance, 6 to 10 ft., after which it is enlarged to full width of the tunnel, very heavy and tight timbering being used. The invert and as much as possible of the side walls are constructed in the drift, and the open area is backfilled with earth. A center top heading is then driven and enlarged to full width and about one-half the height of the tunnel, leaving a bench the full width of the tunnel and something less than one-third the height. This upper section of the tunnel is very heavily timbered and braced; connection between the upper section and the masonry walls already built is made by trenching through the bench, after which the side walls are completed and the arch turned as one operation. The central bench and the backfilled earth are excavated inside the completed tunnel.

The chief advantage of this method is that the workings are so small as to be readily braced and maintained in very treacherous ground. The disadvantage is the excessive cost. Modern shield methods would now be used for all tunnels for which this method was designed, except possibly where only a short length of very treacherous ground is encountered.

American Method (Fig. 81). Here the segmental arch timbering and side posts only are used and no interior struts. It has been much used in all rock; it is especially adapted to fairly firm material. In rock a top heading is driven and timbered with posts and caps. A short length is widened out down to the springing line of the arch, the sills are placed and the segmental timber arches erected between the posts and caps, which may then be removed. In earth the face is opened by a top heading, which is poled and timbered in the usual way. In the space between two bents of the heading timber the crown segments of the timber arches are set in position and held in place temporarily by secondary posts or by strips of scantlings spiked to the

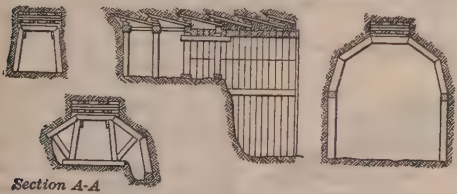


Fig. 81. American Method

main posts as shown in Fig. 81. A short length of from 2 to 6 ft., depending on the nature of the ground, section *A-A*, is then widened out without polings or other roof support and the segments adjacent to the crown segments are put in position and held by iron dowels and a short prop. If the segmental timbers are not set close together, lagging is inserted above the timbering and cavities between the lagging and the earth are packed. The widening out for the next timber is done in the same way down to the sill. After the two sills are placed the roof timber is completed and the bench is then removed.

During each operation the sill timbers are underpinned by any one of the several different methods. If the material is very firm a longitudinal cut is made in the bench, leaving a berm on which the sill rests, and the counterforts are excavated at intervals under the sill and posts placed. If the material is too soft for this, pits are sunk at intervals, and first short posts and later, if excavation has proceeded, long posts are placed to support the sills. If the material is firm enough, as for example rock, the sills or wall plates are set in niches at about the springing line and no posts are used. The number of segments in the arch varies from 3 to 7 or even more. It can be used successfully in firm material which will stand for a short time without support. This type of timbering has been used in American tunnel practice as a semi-permanent lining, but is usually replaced within 10 to 15 years by masonry. The chief advantages of this method are in the large open area within which the masonry lining can be built continuously from invert to crown and the saving of timber.

34. Tunneling in Rock

The methods employed to drive a rock tunnel should depend upon the character of the rock, the size and length of bore, the kind of plant available, the price of labor and the time required to complete.

Usually a small heading is driven within the area of a large cross-section of a rock tunnel and is then enlarged to full size. This heading may be long or as short as 10 ft. but generally it is kept to a size about 6 ft. high by 8 ft. wide, which is about the minimum size for economical driving. This minimum size is maintained because at best the cost of heading work is from two to four times the cost per cubic yard of the enlargement, which requires less drilling and explosives, the blasting being toward a free face.

Whether the heading is at the top, center, or bottom of the full-size cross-section, or is even to be of the full size, will depend on conditions of the particular job.

During the thirty to forty years previous to 1915 when labor was cheap and nearly all mucking was done by hand shoveling, the top heading, and bench method shown by Fig. 82 was the prevailing practice especially in America. The advantage of the method is that the roof is made safer by convenient scaling and timbering if necessary. Its disadvantages are that everything going into or out of the heading must go over the bench work at considerable expense for handling, and interference, which is one reason for keeping the heading very short so that the heading blast will throw most of its muck over the bench.

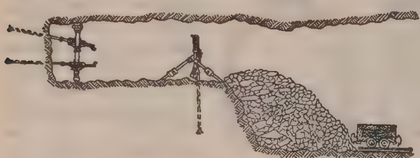


Fig. 82. Top Heading and Bench Method

For greater speed the bottom heading was sometimes used and the enlargement started at frequent intervals along the bottom heading permitting the full size work to be completed nearly as soon as the heading. The disadvantage is that the bottom heading had to be completely timbered to protect traffic through the intermediate working faces.

In the last fifteen years much faster cutting drills and especially faster mucking machines have been developed. Now (1928) there are available all sizes of mucking machines from the smallest adapted to working in a 6-ft. by 8-ft. heading up to power shovels of any desired capacity for large tunnels. These new drillings and mucking tools have made possible new methods and a greater flexibility in adjusting methods to the characteristics of a particular job and have resulted in a general cheapening of tunnel work.

In rock requiring timbering or other support for the roof during enlargement to full size, some form of top heading method is likely to continue to be used. Its disadvantage was partly overcome in the Moffat tunnel, recently completed, by the use of a belt conveyor carried between two plate girders 65 ft. long supported on a track on the bench and projecting into the heading and back over the bench. This conveyor passed muck from the heading to cars behind the power shovel working in the full-size enlargement. It also passed timber and tools into the heading, and the girders were used as temporary support for roof timbers just at the bench.

Another effect of the improved drilling and mucking plant is an increase in the advantages of the outside pioneer heading method for long tunnels which can be attacked from the ends only.

This method consists in driving a heading about 8 ft. by 8 ft., parallel to but 50 ft. or more from the main tunnel. At intervals of from 1500 to 2000 ft. cross drifts are driven to the main tunnel, which is there attacked by whatever method is best suited to existing conditions. Where the rock is sound and requires no timbering the favorite method is to drive a center heading which can proceed as fast as the pioneer heading for which records exceeding 1000 ft. per month have been made. From this center heading radial holes at right angles to the axis of the tunnel are then drilled to the limits of the full-size tunnel and blasted. The center heading between cross drifts is open at both ends therefore

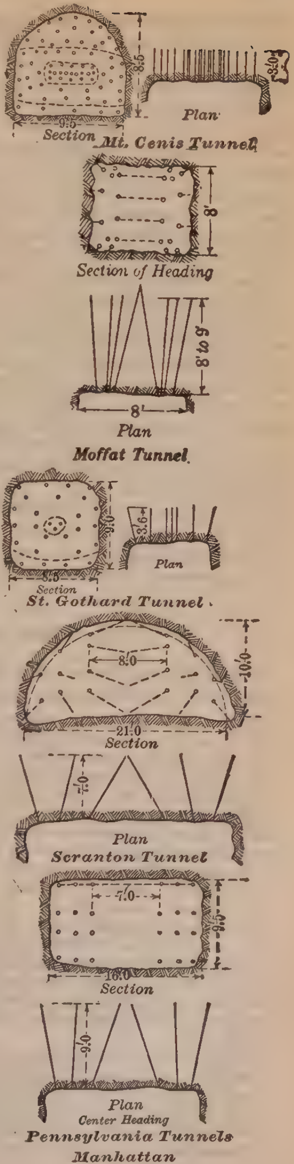


Fig. 83. Drilling Diagrams

the drilling operations can be carried on from one end while the mucking goes on at the other end without interference.

The pioneer tunnel was used for the Roger Pass, Moffat and New Cascade tunnels, 5, 6, and 7-3/4 miles along, respectively.

After the blast in a heading the face is usually nearly buried by muck which interferes with the next drilling. Speed is usually desired in heading work and much thought and ingenuity have been employed to lessen the loss of time caused by interference of the muck pile with drilling. A number of drill carriages have been developed differing in detail but all equipped with a long arm or boom to carry drills, drill columns and hose at its outer end. As soon after the blast as practicable the carriage is moved forward and passes the drilling equipment over the muck pile, and drilling is started on the portion of face exposed. The gain is the difference between the time carriage takes and the longer time required for men to handle the heavy columns and drills over the muck pile. The shorter the drill holes the smaller the muck pile and the shorter the time of setting up drills but the less advance per blast.

Prior to the inception of the plan of center cut holes on the Hoosac tunnel drill holes were placed normal to the face. This is still done frequently in European practice. The cut holes concentrate the explosive back of the center. These holes are fired before the side holes. This breaks a relieving cut so that fewer side holes and less explosive are required. Formerly, considerable delay was occasioned by the necessity of two blasts for first the cut holes and second the side holes. Now delay exploders are used and all holes are loaded and fired at once with faster exploders in the cut holes.

Fig. 83 shows the method of spacing drill holes in several notable tunnels. The drilling plan should be worked out to suit the characteristics of rock and the requirements of each individual heading.

35. Data Regarding Tunnels

In the following notes the numbers in parentheses refer to the tables on pages 1624 and 1625 and to the numbers within the circles in Fig. 84.

(1) Box, Great Britain. Oölite rock, forest-marble, and Lias marl. Lined with brick 27 in. thick at springing line.

(2) Blechingly, Great Britain. Blue Clay of Weald. 11 shafts. Maximum progress 792 ft. per month, 187 ft. per month average for completed work. Hand drills and gunpowder. Lined with brick varying from 22-1/2 to 27 in. thick.

(3) Saltwood, Great Britain. Greensand (quicksand before being drained). 12 shafts. Average progress completed work 238 ft. per month. Lined with brick in horseshoe section; thickness varies between 22-1/2 and 27 in.

(4) Moncreiffe, Great Britain. Poor rock. Widened and relined in 1902, 88% lined with brick arches 18 in. thick on 9-in. brick walls.

(5) Lydgate, Great Britain. One-fourth of length on curve. Clay, strong shale, rock with veins of shale, limestone, fire clay, and coal. Thickness of arch in shale 2 ft., in rock 1 ft. 6 in. 5 shafts. Progress for completed work 52 ft. per month. Hand drills used in rock and strong shale; clay and loose shale axed out.

(6) Netherton. Great Britain. Carries a canal. Marl, coarse sand, rock, hard shaly clay, coal, Lias ironstone and fireclay. 17 shafts. Average progress for completed work 294 ft. per month. Hand drills used. Lining 22-1/2 in. thick, in horseshoe section.

(7) Bergen No. 1, Erie Railroad, New Jersey. Dolerite (very hard trap). Hand drills used. 20% lined with brick arch and stone masonry side walls.

(8) Buckhorn Weston, Great Britain. Kimmeridge clay and veins of loose wet rubble, 5 shafts. Average progress 168 ft. per month. Lining 27 to 32 in. thick.

(9) No. 6 Union Pacific Railroad, California. Granite. Average progress 1.6 ft. of finished tunnel per working day, using 4 working faces.

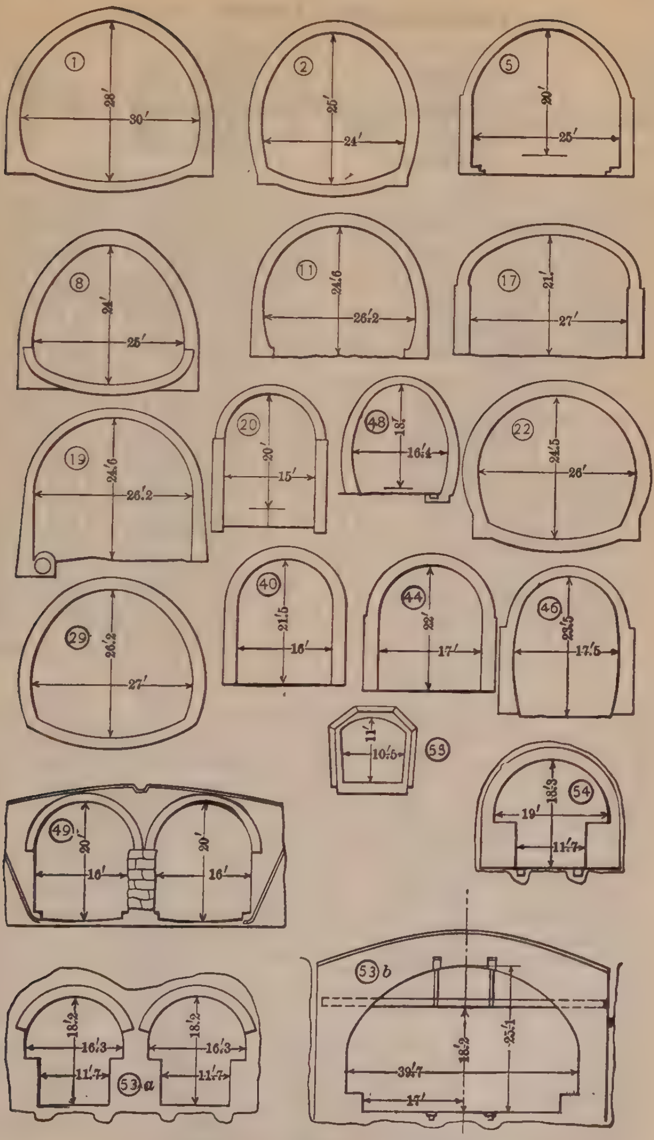


Fig. 84. Sections of Tunnels

35. Data Regarding Tunnels (Fig. 84)

Number and name	Single (S) or double (D) track and date	Length, miles	Clear Height, ft.	Clear width, feet. Straight (S) or curved (C)	Material, excavation method	Lining	Cost per linear foot
(1) Box, G. B.	D 1837–41	1.8	28.0	30.0	Rock E	Brick	$162
(2) Blechingly, G. B. . .	D 1840–42	0.8	25.0	24.0 S	Clay E	Brick	116
(3) Saltwood, G. B. . . .	D 1842–43	0.5	25.5	24.0 S	Sand E	Brick	191
(4) Moncreiffe, G. B. .	D 1845–48	0.7	19.0	26.5 C	Rock	Brick	. . . . .
(5) Lydgate, G. B.	D 1854–56	0.8	20.0*	25.0 Q	Mixed E	B & M	48†
(6) Netherton, G. B. . . .	. . 1856–58	1.7	24.3	27.0 S	Mixed E	Brick	63
(7) Bergen, Erie R.R. .	D 1855–61	0.8	21.0	28.0 S	Rock H	B & M	182
(8) Buckhorn Weston.	D 1859–63	0.4	24.0	25.0 S	Clay E	Brick	116
(9) No.6 U.P.R.R., Cal.	S 1866–70	0.3	20.2	16.0 S	Rock H	Timber	. . . . .
(10) Sand Patch, Pa. .	S D 1854–71	0.9	16.5 19.0	16.0 24.0 S	Rock H	Rubble	80
(11) Mont Cenis, F.-It.	D 1857–72	7.9	24.6	26.2 Q	Rock B	B & M	272
(12) Baltimore, B. & P.	D 1871–73	1.3	18.5	27.0 Q	Mixed V	B & M	142
(13) Clifton, G. B.	D 1871–74	1.0	20.8	26.0 S	Rock P	B & M	. . . . .
(14) Church Hill, Va. . .	D 1872–74	0.7	19.2	27.5 Q	Clay A	B & M	178
(15) Musconetcong. . . .	D 1872–75	0.9	21.0	26.0 S	Mixed W	B & M	. . . . .
(16) Hoosac, Mass.‡. . .	D 1854–76	4.7	19.8	25.0 S	Rock W	Brick	399
(17) Bergen, D.L.&W. .	D 1874–77	0.8	21.0	27.0 S	Rock H	B & M	. . . . .
(18) Osakayama, Japan.	S 1878–80	0.4	14.0*	14.0	Rock B	Brick	76
(19) St. Gothard.	D 1872–82	9.3	24.6	26.2 Q	Rock B	B & M	230
(20) Mullan, Mont.	S 1883	0.7	20.0*	15.0 S	Rock	B & M	. . . . .
(21) Arlberg, Austria. . .	D 1880–83	6.5	25.0*	26.3 S	Rock L	Rubble	180
(22) Severn, G.B	D 1879–86	4.4	24.5	26.0 Q	Mixed E	Brick	208
(23) Vosburg, Pa	D 1883–86	0.7	20.7	28.0 Q	Mixed K	B & M	180
(24) Stampede, N. P. R.	D 1886–88	1.9	22.0	16.5	Rock H	B & C	118‖
(25) Ronco, Italy.	D 1882–89	4.8	. . . .	. . . .	Rock X	B & M	. . . . .
(26) Balt. Belt Line. . . .	D 1890	1.6	22.0	27.0 S	Earth G	Brick	. . . . .
(27) Little Tom, Va. . . .	S 1888–90	0.4	20.1	14.0	Rock	Brick	. . . . .
(28) Cowburn, G. B.§. .	D 1888–92	2.1	24.5	27.0 S	Rock L	B & M	118
(29) Totley, G. B.§	D 1888–92	3.5	26.2	27.0 Q	Mixed R	B & M	. . . . .
(30) Niagara Falls Co. .	. . 1890–92	1.3	21.0	19.0 S	Rock K	Brick	. . . . .
(31) Busk, Colo.	S 1890	1.7	21.0	15.0	Rock H	Timber	83
(32) Panir, India.	D 1893	0.6	20.7*	29.5	Mixed B	. . . .	170
(33) East River Gas. . . .	. . 1891–94	. . . .	8.5	10.5 S	Mixed M	. . . .	. . . . .
(34) Tequixquiac, Mex.	. . 1866–95	6.2	14.0	14.0 S	Rock H	B & S	. . . . .
(35) Ampthill 2, G. B. .	D 1895	0.4	25.0	25.5	Clay E	Brick	. . . . .
(36) Cwm Cerwym, G.B.	S 1897	0.6	. . . .	Q	Mixed	Iron	. . . . .
(37) Catesby, G. B.	D 1895–97	1.7	25.5	27.0	Clay E	Brick	. . . . .
(38) Boulder, Mont. . . .	S	1.2	21.5	15.7 Q	Rock	B & M	. . . . .
(39) Palisades, N. J. . . .	D	. . . .	. . . .	18.0	Rock H	. . . .	. . . . .
(40) Cascade, G. N. R. .	S 1897–00	2.6	21.5	16.0 S	Rock K	Concr.	. . . . .
(41) Sherman, Wyo. . . .	D 1900–01	0.3	. . . .	. . . .	Rock K	. . . .	. . . . .
(42) Graveholtz, Nor'y.	S 1895	3.3	. . . .	. . . .	Rock	. . . .	. . . . .
(43) Barrientos, Mexico.	D 1903	0.1	28.2	37.0 S	Mixed H	Concr.	. . . . .
(44) Scranton, Pa.	S 1904–05	0.9	22.0	17.0 Q	Rock K	TC&M	90
(45) Chicago Subway . .	. . 1901–	65.0	7.5	6.0 Q	Clay M	Concr.	. . . . .
(46) Gallitzin Pa	S 1903–05	0.7	23.5	17.5 S	Rock K	C & M	139
(47) Alfreton 2, G. B. . .	D –1905	0.5	25.5	26.5	Rock E	Brick	. . . . .

* Above rails. † Contract price. ‡ Height = 18.5 to 21.0 ft.; width 24.0 to 26.0 ft.
‖ Excavation and timbering only. § Height = 23.2 to 25.8 ft.

Number and name	Single (S) or double (D) track and date	Length, miles	Clear Height, ft.	Clear width, feet. Straight (S) or curved (C)	Material, excavation method	Lining	Cost per linear foot
(48) Simplon, It.-Fr....	S 1895–06	12.4	18.0*	16.4 Q	Mixed L	Rubble	240
(49) Capitol Hill, Wash.	D 1904–07	0.8	20.0	16.0 Q	Earth Z	B C & M	.....
(50) Haversting, Nor'y.	..1902–08	1.4	20.0	15.0 S	Rock	....	.....
(51) Bergen, D. L. & W.	D 1906–09	0.8	26.9	30.0 S	Rock H	Concr.	.....
(52) St. Paul Pass.....	1907–	1.6	....	....	Rock H	Timber	.....
(53) a Penn. Twin, N.Y.	D 1905–09	1.8	18.2	16.3 Q	Rock H	B & C	.....
b Penn. 3Tr., N. Y.	..1907–09	0.2	18.2	39.7 S	Mixed Y	B & C	.....
(54) Bergen, P. R. R...	D 1905–09	2.2	18.3	19.0 S	Rock H	B & C	.....
(55) Gunnison, Colo....	..1905–09	5.8	11.4	10.5 S	Mixed H	Concr.	.....
(56) Arthurs Pass, N. Z.	S 1908–	5.31	16.75	15 S	Rock R	Concr.	.....
(57) Fu-Chin-ling, Mch.	S 1909–11	0.92	18.0	12.0	Rock P	B & M	75
(58) Loetschberg, Switz.	D 1906–13	9.03	22.3	25.0	Rock L	....	.....
(59) Necaxa, No. 1, Mex.	..1909–13	2.15	9.83	9.17	Mixed Z	Concr.	165
(60) Pirahy, Brazil.....	..1911–13	5.25	13.2	12.85	Rock	....	.....
(61) Laramie-Poudre, C.	..1909–	2.27	7.5	9.5	Rock M	....	.....
(62) Mt. D'Or. F. & Swz.	..1910–15	3.8	20.0	26.2	Rock L	Mas'ry	.....
(63) Astoria-Brx., N. Y.	..1910–15	0.88	18.0	16.75	Rock	Concr.	.....
(64) Rogers Pass., B. C.	D 1914–16	5.0	21.12	29.0 S	Rock Z	Concr.	.....
(65) Twin Peaks, Cal..	D 1914–17	2.27	15.0	25.0	Mixed	Concr.	.....
(66) Roosevelt, Colo...	..1907–	4	....	....	Rock M	....	.....
(67) Rove, France.....	....	4.0	40.0	72.0	Rock Z	Concr.	.....
(68) Mt. Royal, Mont'l.	D 1912–18	3.25	19.75	13.0	Rock L	Concr.	.....
(69) Moffat, Colo......	S 1924–27	6.2	24	16	Rock Z	Concr.	470
(70) New Cascade, Wash.	S 1926–28	7.75	24	16	Rock Z	Concr.	.....

* Above rails. For waterworks tunnels see Aqueducts.

S = Straight, C = Curve, Q = Straight and curved, B & M = Brick and stone masonry, B & C = Brick and concrete, B & S = Brick and artificial stone bricks, T C & M = Timber, concrete, and stone masonry, C & M = Concrete and stone masonry, B C & M = Brick, concrete, and stone masonry.

A = American method. B = Belgian method. E = English method. G = German method. H = Top heading and one bench. K = Top heading and two benches. L = Bottom headings, from which small shafts at intervals are driven to top of tunnel and full width top headings are excavated in both direction; the bench is removed between the top and bottom headings and the lower section of the tunnel is enlarged to full width. M = The full tunnel prism is removed as one operation. O = Cut and cover method. P = Top heading and enlargement, no other particulars are available. R = Bottom headings followed successively by their enlargement laterally and upward. V = Cut and cover and English method. W = Top heading and one bench and English method. X = Belgian and English method. Y = Top heading and one bench and cut and cover method. Z = See description of methods in the notes.

Costs are taken from published accounts in which the elements composing the figures are seldom recorded.

(10) Sand Patch, Pennsylvania. Red sandstone and shale varying from hard to soft. About 50% lined with sandstone masonry.

(11) Mont Cenis. From France to Italy. About 3/4 mile is on curve. Limestone, calcareous schist, quartz, carbonaceous schist, gneiss and schistose sandstone. Maximum depth below surface 5277 ft. Maximum rock temperature was 80° F. No shafts. Maximum progress one heading 297 ft. per month; average for both headings completed 231 ft. per month. Cost $272 per lin. ft. Hand drilling used

for first four years with holes 1.5 ft. deep, subsequently 3-in. cylinder compressed-air drills with holes 3.5 ft. deep were used. After first four years 70 to 80 drill holes 1.2 in. in diameter in heading averaged 67 lb. gunpowder per round, or about 3.8 per cu. yd. Mucking done by hand; horses for hauling. Gas for lighting. Ventilation at first by exhaust from drills, then blowers and bratticing were installed, and finally exhaust bells. 2 rounds were fired in 24 hours. Time of drilling 6 to 8 hr., loading 1.5 to 2 hr., mucking 3 to 5 hr. Lined with side walls of stone and arch which is stone for one-half length and brick for the rest; the thickness is 31-1/2 in.

(12) Baltimore, B. & P. R. R., Maryland, 20% on curve. Soft rock, clay, sand, and earth. 16% tunnel, 84% cut and cover. Hand drills and black powder used. Brick arch 22 in. thick; limestone side walls 54 in. thick.

(13) Clifton, Great Britain. Mountain limestone, conglomerate, and red sandstone. 2 shafts. Average progress of completed headings 196 ft. per month, using 6 faces. 92% by hand drilling, 8% diamond boring machine. Partly lined with 5-ring brick arch on stone masonry walls.

(14) Church Hill, Virginia. Miocene clay with infusorial shells. 22-in. brick arch on masonry side walls.

(15) Musconectong, New Jersey. Soft ground, limestone and syenitic gneiss. English method in soft ground; top heading and bench method used in rock. 3 shafts. Maximum progress in one heading 144 ft. per month, average for whole work 181 ft. per month, using 8 faces. 5-in. cylinder Ingersoll compressed air drills and dynamite used. 36 drill holes in heading, 10 in bench. Depth of holes 11 ft. 270 lb. explosive per round in heading, 107 in bench. 6 drills in heading, 2 on bench. Spoil loaded into barrows, wheeled to traveling platform at bench, and dumped into chutes over cars. A derrick on the platform handled large bench excavation. Average time of drilling and loading for one round 32 hr. 20% lined with 7 or 8-ring brick arch on masonry walls.

(16) Hoosac, Massachusetts. Hard granitic gneiss, conglomerate, and mica schist. Maximum cover 1800 ft. English method in soft ground. Top heading and bench method in other material. 1 shaft. Maximum progress for 1 heading 184 ft. per month, average per month for all faces 194 ft. Holes 2.1 ft. deep were drilled by hand during the first 12 years, using 16 lb. of explosive per linear foot of tunnel. During the last 10 years holes 11.5 ft. deep were drilled by 12 compressed-air drills at each face, and 33 lb. of powder per linear foot of tunnel were used. Spoil loaded into cars, wheeled to traveling platform at bench. Derrick on platform handled large bench excavation. Lined with brick in sections in poor rock.

(17) Bergen No. 1, D. L. & W. R. R., New Jersey. Dolerite (very hard trap). Rendrock powder used. Brick arch on masonry side walls.

(18) Osakayama, Japan. Soft rock with clay veins. Lining is 2, 4, or 6-ring brick arch on brick side walls. Hand drills. Fans for ventilation.

(19) St. Gothard, Switzerland. Straight except about 500 ft. Gneiss, mica schist, serpentine, and hornblende schist. Maximum depth below surface 5576 ft. Maximum rock temperature 88° F. No shafts. Average progress of single heading 225 ft. per month. Sommelier drills used for a few years, then Ferroux 2-1/4-in. cylinder and McKean 4-in.-cylinder drills with compressed-air. Enlargement by hand drills. 6 drills, mounted on a carriage, bored 24 to 26 holes in each heading 2.6 ft. to 4.2 ft. deep. 29 lb. 75% dynamite, explosive per round, or about 5 lb. per cu. yd., were used in each heading. Horses used in headings, compressed-air locomotives in full-size sections. Oil lamps for lighting. Exhaust bells for ventilation. Time of drilling 3 to 5 hr., mucking 3 to 4 hr. Brick arch on rubble masonry walls varying between 17.7 and 29.6 in. thick. Although in rock, was built by the Belgian method.

(20) Mullan, Montana. Treacherous rock. Relined with masonry in 1892. Costs, including engineering superintendence and interest: concrete $8 per cu. yd.; brick $17 per cu. yd. Total relining $50 per lin. ft., brick arch 20 in., and concrete side walls 30 in. thick.

(21) Arlberg, Austria. Mica schist and gneiss more or less rich in quartz. Maximum rock cover 2300 ft. Maximum rock temperature 64° F. No shafts. Maximum progress for 1 heading 641 ft. per month, average for single headings 408 ft. per month. Ferroux compressed-air and Brandt's rotary hydraulic drills used. With Ferroux drills 4.8 lb. of powder per cu. yd. and with Brandt 4.0 lb. Lining is mostly between 19.7 and 37.4 in. thick, 6% of length not lined.

(22) Severn, Great Britain. Conglomerate, limestone, carboniferous beds, marl, gravel, and sand. Lining of vitrified brick 27 to 36 in. thick.

(23) Vosburg, Pennsylvania. 138.5 ft. on curve, the rest is on tangent. Red, green, and black shales and red and blue sandstone. Short length earth roof. American system of timbering used. No shafts. Average progress single heading 137 ft. per month. In the heading 4 Rand or Ingersoll 3-1/8-in. cylinder drills bored 12 to 18 holes 9 to 10 ft. deep. 3.13 lb. rack-a-rock were used per cu. yd. excavated. 2 drills and Altas powder were used on the bench. 10% of all the drilling was done by hand. The average amount of explosive used was 1.71 lb. per cu. yd. of all excavation. No special provision was made for ventilation. Mostly arched with 3 rings of brick. About one-sixth arched with stone 18 to 24 in. thick. Side walls of stone 30 to 36 in. thick. Stone used is black limestone.

(24) Stampede, Washington. Soft basaltic rock. No shafts. Maximum progress of single beading 274 ft. per month, average for two headings 413 ft. per month. About 9% with hand drills, the rest with compressed-air drills. 31 lb. of Giant and Hercules 45% and 60% dynamite were used per linear foot of tunnel. 20 to 23 drill holes 12 ft. deep in heading, 18 ft. in bench. Spoil from heading hauled by mules to chutes at traveling platform at bench. Removed in cars from chutes by small locomotives. Electric lighting. Ventilation by exhaust fans. Concrete side walls and brick arch.

(25) Ronco, Italy. Argillaceous schist with considerable water. 6 shafts. Average progress one heading 231 ft. per month, average of completed tunnel 123 ft. per month. Brandt rotary hydraulic drills and Ferroux compressed-air drills. 0.48 to 0.68 lb. 75% to 78% dynamite used per cu. yd. Sheeles system of ventilation. 2 exhausts coupled.

(26) Belt Line, Baltimore. Howard Street Tunnel. Sand with seams of loam, clay, and gravel. 5-ring brick arch.

(27) Little Tom, Norfolk & Western R. R. Seamy gray sandstone disintegrating on contact with air. 1410 ft. originally lined with timber. Later relined with brick. Arch made of four rings of brick. Cost of relining complete was $33.50 per lin. ft.

(28) Cowburn, Great Britain. Material about one-third shale, two-thirds rock intermixed with thin beds of shale. Quite dry. Average progress for single heading, dry shale by hand drilling 299 ft. per month, rock by hand drilling 111 ft. per month, rock and shale by machine drilling 270 ft. per month, rock by machine drilling 199 ft. per month. One-third excavated with hand drills, rest with Larmuth compressed-air drills. 17 lb. gelignite used per lin. ft. 2 drills in heading. Horses used in headings and locomotives in finished sections. Ventilation by fan. Arch entirely brick. Side walls two-thirds length stone masonry and one-third length brick.

(29) Totley, Great Britain. Hard and soft black shale, coal, fire clay, sandstone, and grit rock. 4 shafts. Average progress machine-drilled heading 242.4 ft. per month. 3-1/4 and 3-1/2-in. Schram percussion drills and 3-in. Larmuth. Average number drill holes in heading 13.4. Holes 6.2 ft. deep. 38 lb. gelignite per round 2 drills in heading. For clearing heading of fumes and dust, a jet of compressed air and spray of water were used. The arch is entirely brick and the walls are brick for 70% of length and coursed masonry for 30% of length.

(30) Niagara Falls Power Co., New York. Limestone and shale with slaty seams extremely wet. Progress 304 ft. per month with 5 headings. Rand compressed-air drills, 4 to 6 per heading. Dynamite. Electric lighting. Ventilation by exhaust from drills. Lined with from 4 to 6 rings of brick.

(31) Busk, Colorado. Gray granite, disintegrated in places. Maximum progress of single heading 202 ft. per month, average completed tunnel 190 ft. per month. 3-1/2-in. cylinder Ingersoll compressed-air drill. Holes 12 ft. deep. Electric lighting. Blower for ventilation. 78% lined with timber.

(32) Panir, India. Limestone, clay, and soft sandstone. Average progress single heading 145 ft. per month, average whole work 95 ft. per month. 4-in. cylinder Climax compressed-air drill bored 25 holes in heading 3.8 ft. deep and 1-7/8 in. in diameter. Dynamite and gelignite were used. The time for drilling one round was 5 hr. 520 ft. is one-half lined and 2690 ft. is three-quarters lined.

(33) East River Gas, New York. Gneiss, mica schist, veins of decomposed feldspar, and black mud. Maximum progress 101 ft. per week on New York side in rock, average 69 ft. per week in good rock in each heading. Compressed air 15 to 48 lb. per sq.

in. used while in soft material. Good rock not lined; soft material lined with cast iron —see table of cast-iron lining in tunnels, p. 1641.

(34) Tequixquiac, Mexico. Sandstone with lime, soapstone, and conglomerate. Very wet. Progress 18 ft. per day in heading. Timbered Lined with 20-in. arch of brick and 18-in. walls of artificial stone blocks. Hand drills and dynamite. 6 drill holes in heading 6.5 ft. deep. 6 lb. of explosives per round in heading.

(35) Ampthill 2nd, Great Britain. Kimmeridge clay. Progress averaged 6 days to complete 1 length of 12 ft. 1-1/2 lb. tonite per lin. ft. Horseshoe section, walls, and arch lined with 33 in. of brick; invert 27 in. brick.

(36) Cwm Cerwym, Port Talbot R. R. Slight curves at end. Shale, hard clift, fire clay, thin veins of coal, and about 500 ft. of pennant rock. Originally lined with brick without invert, afterward relined without interrupting heavy traffic. Concrete blocks in invert and cast-iron segments above.

(37) Catesby, Great Britain. Clay. 9 shafts. Average progress for finished work 330 ft. per month. Lined with brick 22-1/2 to 31-1/2 in. thick.

(38) Boulder, Montana. 150 ft. curved at one end. Material penetrated 80% seamy-blue trap rock and 20% syenitic boulders and débris. Originally lined with timber. Later relined with 20-in. coarse granite rubble in walls and 4 rings of brick in arch.

(39) Palisades, New Jersey. Hard trap rock with many seams in places. 1 shaft. Average progress 186 ft. completed tunnel per month. 24 holes in heading drilled by four Ingersoll-Sergeant 2-1/2-in. compressed-air drills. 6 drills on bench. A derrick car was used to handle large stones from the bench. Horses were used at first, dummy locomotives later.

(40) Cascade, Washington. Medium hard gray granite, seamed and wet. Temporary 5-segment timbering used. Maximum progress using two headings 527 ft. per month. Maximum progress in single heading 301 ft. per month. Average two headings 350 ft. per month. Six 3-1/4-in. Rand compressed-air drills bored 24 to 28 holes 12 ft. deep in heading. 8 drills on bench. Spoil run out in wheelbarrows to a traveling platform at the bench and dumped through chutes into cars. A six-ton derrick mounted on platform handled large bench excavation. Ventilation by exhaust fan. Electric lighting and electric motor haulage.

(41) Sherman, Wyoming. Solid granite. Top heading and two benches. Heading progress 4.92 ft. per day. Drill holes 9 ft. deep. 6 drills on 3 columns in heading.

(42) Graveholtz, Norway. Quartzy granite or gneiss. Average progress of finished headings was 263 ft. per month. Hand drills used at one end for 2-1/2 years, after which Brandt boring machines and compressed-air drills used. Only 3% lined.

(43) Barrientos, Mexico. Hard granitic porphyry with clay seams. No shafts. Heading progress 367 ft. per month, completed excavation 245 ft. per month. 3 Ingersoll-Sergeant compressed-air drills bored 14 holes in heading 10 to 12 ft. deep. 29.3 lb. of 60% dynamite used per lin. ft. 3 drills on bench. Small cars were loaded and run to platform at bench, where they were dumped into larger cars. Lined with concrete blocks 28 to 31 in. by 60 in. deep by 7 ft. 8 in. long.

(44) Scranton, Pennsylvania. Sandstone and shale. 2 shafts. Maximum progress in single heading 261 ft. per month, average all headings 387 ft. per month. 3-1/4-in. cylinder compressed-air drills and 50% dynamite. 1305 ft. not lined, 2717 ft. 5-segment timber lining and 725 ft. concrete and stone masonry.

(45) Chicago Telephone and Express Subway. Stiff blue clay. Small-section tunnel. Full-size excavation. Excavated with spades and draw knives. Compressed air used, 9 lb. per sq. in. Floor 30 ft. below street. Lined with 10 in. of 1 : 3 : 5 concrete.

(46) Gallitzin, Pennsylvania. Sandstone and shale, also some limestone, fire clay, and slate. No shafts. Average progress of completed tunnel 164 ft. per month. 16 holes in the heading and 11 on the bench 10 ft. deep were drilled respectively by 4 and 2 Ingersoll-Sergeant compressed-air drills. 2.5 lb. 40% forcite were used per cu. yd. An air-operated steam shovel following up at the bench handled the excavated material. Horses were used in the headings and dinky locomotives in the full-size tunnel. Electric lighting. Time of drilling was 8 hr., two rounds being fired per day. Concrete arch 22 in. thick and concrete and rubble side walls increasing from 38 in. at spring line to 54 in. at bottom.

(47) Alfreton 2, Great Britain. Gray sandstone, rock, shales, and fire clay with

seams of coal. Worked from 5 shafts with no portal headings. Average time of mining a 15-ft. length was 19.7 days. Lined with 28 in. of brick.

(48) Simplon, Italy to France. Short curves at ends. Mostly very hard gneiss and calcareous rock, also short lengths in green cipolin, disintegrated slate, clay, and mica schist. Maximum depth below surface 9118 ft. No shafts. Maximum progress single heading 685 ft. per month, average both headings 440 ft. per month. In bottom headings Brandt rotary hydraulic drills, enlargement by hand drills. 83% dynamite and 64% blasting gelatine used in opposite ends of tunnel. 10 to 12 drills holes 4.5 ft. deep. 3-1/2 in. in diameter, in heading. 2.8 lb. explosive per cu. yd. 3 drills in the heading. Time of drilling one round 2.75 hr. and time of loading, firing, and mucking 6 hr. Steam and compressed-air locomotives for hauling. Blowers and water sprays used to keep tunnel air clean. Both gas and oil for lighting. Lined with coursed masonry of various thicknesses.

(49) Capitol Hill, Washington, D. C. About one-third on curve. Gravel and fine sand overlying hard blue clay. Very wet. 1600 ft. in open cut and 2400 ft. in tunnel. Separated by lining into twin tunnels. Each tube excavated separately. 2 side drifts at bottom and crown drift excavated in one tunnel. Crown bars 24 ft. long placed in top drift and supported by radial struts. As top heading was widened segmental timbering was placed under crown bars and carried down to posts on concrete footings in side drifts. The radial struts were removed and the core of earth between side and crown drifts was excavated with a steam shovel. The lining was placed in this portion of the tunnel and the excavation of the other tube was proceeded with, the timbering for it abutting against the crown bars of the first tube. The arch is brick backed by concrete of a total thickness of 36 in. The side walls are concrete 72 in. thick and the core-wall is stone 58 in. thick.

(50) Haversting, Norway. The cross-section is about 270 sq. ft. Gneiss with varying amounts of feldspar and quartz. Total progress both ends 114 ft. finished tunnel per month. Hand drills. Horses for hauling.

(51) Bergen No. 2, D. L. & W. R. R., New Jersey. Very hard trap rock. Ingersoll-Sergeant compressed-air drills. 29 holes in heading 7 to 8 ft. deep. 8 holes in bench. Electric lighting. Air-operated steam shovel for mucking. Lined with concrete 24 in. thick.

(52) St. Paul Pass, Idaho to Montana. Laminated quartzite with some talc layers. Average progress two headings for four months 576 ft. per month. 3-1/2-in. cylinder Ingersoll-Rand compressed-air drills, 8-in. heading and 5 on bench. An air-operated steam shovel used at bench. Excavated material hauled by electric locomotives. Fans for ventilation. Electric lighting. Lined with five-segment timber arch.

(53a) Pennsylvania R. R., twin tunnels under Manhattan from Sixth Avenue to East River. Hudson schist. Two sets of twin tunnels averaging about 4730 ft. long. At east end 350 ft. excavated as 4 single tunnels and the remainder was excavated as two tunnels from 42 to 52 ft. wide and separated by lining into four single tubes in sets of two. 2 shafts for each set of tunnels but no portals. Three methods of excavation used in wide tunnels: double heading, center heading, and full-width heading. Bench was usually 10 to 15 ft. behind the face. The maximum progress of a single heading was 206 ft. per month, the average for all six headings was 451 ft. The average length of tunnel completed per month was 257 ft. The number of holes drilled varied greatly with width and location of headings and averaged between 8 and 9 ft. in length. In single headings 4 Ingersoll-Rand 3-1/4-in. drills mounted on 2 columns and in full-width headings 10 drills mounted on 5 columns were used. 1.9 lb. 60% forcite were used per cu. yd. of all excavation. Air-operated steam shovel followed at the bench. Electric motors for haulage. Blowers for ventilation. Electric lighting. Lined mostly with concrete. Small sections in wet rock have brick arch and concrete side walls. Minimum thickness of lining 22 in.

(53b) Pennsylvania R. R., three-track tunnels under Manhattan. 218 ft. long on west end of one set of twin tunnels and 683 ft. long on the other. Hudson schist. Sand in the crown at places. Part done in open cut. Lining in open-cut sections concrete throughout with a minimum thickness of 59 in. and in tunnel brick arch 56-1/2 in. thick on concrete side walls.

(54) Pennsylvania R. R., Bergen Hill Tunnels. Mostly hard trap rock, also 940 ft. in decomposed sandstone, shale, feldspar, calcite, and sandstone. Maximum progress of a single heading was 145 ft. per month, average progress 3.13 ft. per day at each

face. 8 Rand 3-5/8-in drills on 4 columns drilled 32 holes 12 ft. deep in the headings. The holes tapered from 2-3/4 or 3 in. at the top to 1-3/4 or 2-1/4 in. at the bottom. 1 ft. of steel was used up for every 10 cu. yd. of excavation. An average of 5 lin. ft., of hole was drilled per cu. yd. at the average rate of 2.66 ft. drilled per hour of the machine at the face. An average of 2.9 lb. of 60% forcite was used per cu. yd. of all excavation. The average time of each attack was 36 hr. Air-operated steam shovels handled the muck. Fan for ventilation. Electric lighting. Lined with concrete with a minimum thickness of 22 in.

(55) Gunnison, Colorado. Irrigation Canal. Mostly black shale and metamorphic granite; other materials penetrated, clay, water-bearing gravel and sand, sandstone, limestone, coal, and marble. 1 working shaft. Maximum progress of single heading 824 ft. per month, average length of tunnel completed 566 ft. per month. Sullivan 3-in. and 2-1/2-in. and Leyner 2-1/2-in. cylinder compressed-air drills were used. Jeffrey coal augers were used in shale. 40% gelatine dynamite used. 18 to 22 drill holes 6 ft. to 7 ft. deep in the heading. 4 drills in the heading and one on the bench. Blowers for ventilation. It was necessary to drive an inclined shaft for ventilation. Electric lighting. Lined with concrete 16 to 26 in. thick.

(56) Arthurs Pass, New Zealand. Badly fissured sandstone and shale, 3% grade, not much timbering. 2 to 3 Ingersoll-Sergeant 3-1/4-in drills on a 4-1/2-in. cross-bar without arms in heading. 10 to 16 holes 4-1/2 ft. to 6-1/2 ft. deep per round and 4 to 6 lb. of Noble's gelignite per cu. yd. in heading. Electric haulage. Side walls of concrete poured in place, arch roof of concrete blocks 12 in. radial by 9 in. by 18 in. along tunnel, mortar joints. Over break of hand-packed stone.

(57) Fu-Chin-ling, Manchuria. Hard limestone. No timbering. Lined with cut lomestone on sides with a 3-ring brick semicircular arch of 8-ft. radius in roof. Cost $75.00 per ft. lined and ready for track.

(58) Loetschberg, Switzerland. Limestone, slate, gneiss and granite. Greatest thickness of overlying rock 4692 ft. No shafts. Temperatures 75° to 110° F. Maximum progress of single heading 1013 ft. per month. 4 to 5 Ingersoll-Rand 3-5/8-in drills on carriages in bottom heading and on columns in top heading. In the headings there were 12 to 14 holes 4 ft. to 5 ft. deep and 85% dynamite was used. Lighting by individual acetylene lamps. Due to an inrush of water, sand and gravel which filled 5900 ft. of the excavation and buried 25 men 0.85 mile of the old working was abandoned and the alignment changed.

(60) Pirahy, Brazil. To divert water from Pirahy River to power company's reservoir in adjacent valley. Mostly solid gneiss. Bottom heading 13 ft. wide by 8 ft. high. Best month 512 ft. Very little timbering. About 1000 ft. of concrete lining, the rest unlined. Five 3-1/4-in. Ingersoll-Rand drills at each face on an 8-in. horizontal bar operated by a Carter tunnel carriage. Total construction period less than 23 months working from portals and 4 shafts.

(61) Laramie-Poudre, Colorado. Irrigation tunnel. Granite syenite and quartzite. Leyner pneumatic hammer drills. 20 holes 7 ft. to 9 ft. deep per round. 100% blasting gelatine in the bottom of the cut holes and 50% and 60% dynamite for the rest of the blasting. Holes fired and fuse cut to proper length to discharge them in prearranged order. Muck hauled out by hoist and by mules. Average monthly advance 473.7 ft. for 19 months.

(62) Mount D'Or, France and Switzerland. Limestone and marl. Swiss heading averaged 566 ft. advance per month for 25 months. 4 Meyer type air drills mounted on a bar and 12 to 15 5-ft. holes per round in heading. Water up to 20 000 cu. ft. per min. encountered. Lining placed immediately behind the excavation. Compressed-air locomotives.

(63) Astoria-Bronx, New York. For gas mains under East River. Gneiss and dolomite. About 250 ft. below mean sea level. Much trouble with water. Rock seams grouted through pilot drill holes driven in advance of excavation, grouting under pressures up to 500 lb. per sq. in. and using a total of 118 000 bags of cement. Working once flooded by leak of 12 000 gal. per min. Tunnel then bulkheaded and whole heading grouted back of bulkhead. Work then advanced by small heading through grout with pilot drill holes for grouting as before.

(64) Rogers Pass, British Columbia. Quartzite and schists. Not much water. No shafts. Thickness of overlying rock 5690 ft. A pioneer tunnel 7 ft. by 8 ft. was driven from each end parallel to the main tunnel and about 50 ft. away. The central mile of this pioneer tunnel was not excavated. 3 and 4 light hammer drills on a hori-

gontal bar per heading and 21 to 28 holes per round. Maximum monthly advance for a single heading 932 ft. From the pioneer bore cross cuts to the main tunnel were made at intervals of 1500 ft. to 2000 ft. and headings 11 ft. wide by 9 ft. high carried each way along the tunnel center line. This central heading was then enlarged to full section at one operation, working in from each portal. Steam shovels and air locomotives used in mucking. The best monthly advance for one shovel was 1030 ft. Only partly lined with concrete.

(65) Twin Peaks, San Francisco. For 2-track street railway. Clay, sand and sandstone. Heading 8 ft. by 8 ft. 65 000 cu. yd. of reinforced-concrete lining placed by pneumatic process through a maximum length of 4,000 ft. of pipe. 8 lb. hydrated lime was used per 100 lb. cement.

(66) Roosevelt, Colorado. For underdraining Cripple Creek mining district. The tunnel at its inner end is over 1800 ft. below the surface and takes the ground water from the surrounding mines. Maximum discharge 17 000 gal. per min. Part in granite and remainder in hard volcanic rock. In granite it cost $27.27 per lin. ft. Section about 75 sq. ft. In volcanic rock cost $20 to $25 per ft. Section about 80 q. ft. Two Ingersoll-Leyner drills on a horizontal bar, about 30 holes per round. Muck hauled to shafts by mules.

(67) Rove, France. Rhone-Marseilles canal tunnel. Dolomite and limestone. 2 side drifts 9.8 ft. by 10.7 ft. are driven at bottom, enlarged to give room for track and drainage and small heading at the crown. Side headings enlarged upward and connect with crown heading. After this the lining is placed and the core removed.

(69) Moffat Tunnel. Single track, 25 ft. high, 16 ft. wide, six miles long, 2800 ft. below surface at divide, 8 ft. by 8 ft. pioneer tunnel, 75 ft. south of main tunnel with 20 cross cuts to it. Heading encountered 1500 gal. per min. flow of water. Much of roof of main tunnel required timbering.

Top heading and bench in timbered section center heading in good rock. Cost $15,000,000, which was over twice the amount estimated.

(70) New Cascade Tunnel, Washington. Single track 7-3/4 miles long nearly parallel to old tunnel but 500 ft. lower elevation, 16 ft. wide, 22 ft. high, 8 ft. by 9 ft. pioneer tunnel. Main tunnel enlargement from center heading.

36. Shield Tunneling

A Shield (Fig. 85) consists of a circular steel box or ring generally provided with a transverse diaphragm. The forward end of the ring, or cutting edge,

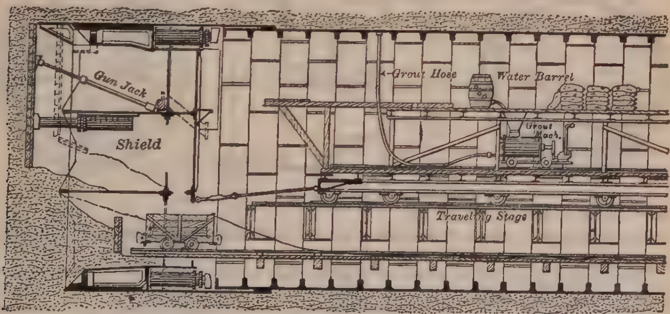


Fig. 85. Shield Tunneling

projects into the earth to be excavated, and the rear end or tail projects backward a little distance over the completed lining, which usually consists of rings of cast iron. To the shield are attached powerful hydraulic jacks which react against the completed lining and shove the shield forward as the earth is excavated. As soon as the shield is advanced a short distance the jack plungers are withdrawn, a ring of lining built, and the operation repeated.

Methods of Work. In clay or other firm material which will stand alone for a short time the full face is simply excavated to about the length of one shove in front of the cutting edge without poling or breasting, and the shield quickly shoved forward. If the earth is not hard enough to injure the cutting edge, only the center of the face is excavated for a depth equal to length of shove and the cutting edge is allowed to break in the rest. In softer clay the cutting edge and working floors may be always buried; only the back of the working chamber in front of the diaphragm is kept clear of earth. In still softer material the shield is sometimes shoved against the earth face, part of which is allowed to flow through the openings in the diaphragm of the shield and the rest is pushed aside. In sand or gravel the face is usually breasted with timber, which is held by struts against the shield. These struts must be collapsible so that the shield may be shoved up to the face. A favorite form is simply two pieces of pipe which telescope as the shield is shoved, the resistance being regulated by set screws in the larger pipe. Hydraulic rams attached to the shield have been used for this purpose.

Numerous London shields have been provided with sliding platforms operated by hydraulic rams. When driving the shield for the Waterloo and City tunnel, London, through sand and gravel, pot holes were raked out in front of the cutting edge and filled

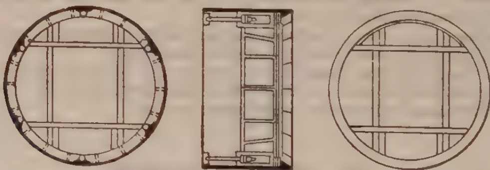


Fig. 86. Shield, Central London Tunnel

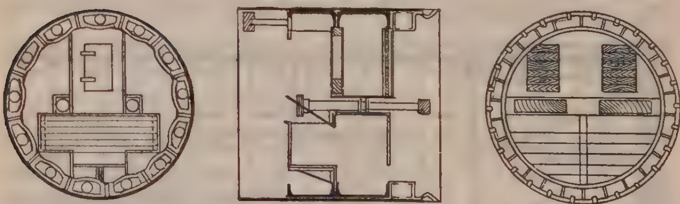


Fig. 87. Shield, Greenwich Tunnel

with soft tempered clay. This formed a continuous ring of soft clay into which the cutting edge was shoved. The full face was then excavated under cover of the shield.

Where the material to be tunneled is part earth and part rock the full-sized tunnel above the rock is usually poled and excavated and the face breasted as in ordinary mining for a length of one or two shoves in front of the cutting edge, the rock in the same length is then drilled, blasted and removed, and the rock bottom smoothed off to the form of the bottom of the shield with concrete in which steel rails are embedded; the shield is then advanced for that short length and the operation repeated.

The General Design of the different types of shields used for tube tunnels up to the year 1928 is shown in Figs. 86-89. The working chamber in front of the diaphragm has been small as in Fig. 86 or large as in Figs. 87, 88, 89, depending upon the character of the material and the methods of excavation. The diaphragm has openings for the passage of men and materials, and may be designed to be closed by locks or doors or be always open. Under either condition the diaphragm should be designed to lend stiffness to the skin, and since, when closed, it may take a part or even all of the water or earth pressure at the face, a system of transverse girders is introduced as reinforcement. Some of the later shields working in London clay had practically no diaphragm because the material was well known as being uniform, and not water-bearing. The shields for the Blackwall tunnel, Greenwich (Fig. 87), and Pennsylvania R. R. East River tunnels (Fig. 89) were built with a double diaphragm, the doors of which when closed formed air locks between the working chamber and the tail for the passage of men and materials. These locks were never used, and the lower plates in all cases were removed during the course of the work to give easy access from the tunnel to the face.

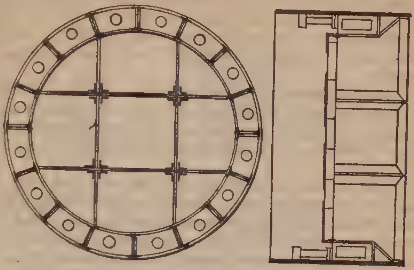


Fig. 88. Shield, Hudson Tunnel

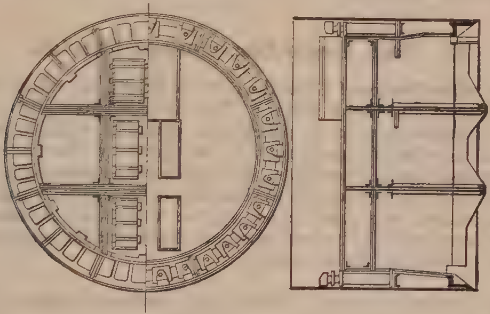


Fig. 89. Shield, East River Tunnel, P. R. R.

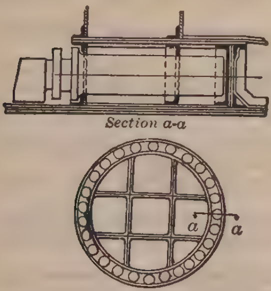


Fig. 90. Circular Box Girder

Fig. 91

The cutting edge at the extreme forward end of the shield in the earlier shields was the edge of the skin plates stiffened by brackets, but in later practice with few exceptions was a heavy ring made up of cast-steel segments beveled on the forward edge. Costly delays have resulted from damage to cutting edges caused by unexpected boulders and rock ledges. The reaction of the hydraulic jacks must be transmitted to skin, cutting edge, and partly to diaphragm. These requirements have developed the circular box girder, a form of which is shown in Fig. 90, attached to the skin, bearing on the cutting-edge brackets and carrying the connections of the transverse diaphragm girders. The box girder is provided with cells into which the jacks are placed, the location being convenient for substitution and repairs.

Shield jacks are all connected with a battery of valves under the hand of the operator so that he may advance or withdraw any one or any combination of jacks at his will. A number of spare jacks should be provided and they should be designed so that they may be easily and quickly removed as they are sure to need repairs and delays are very expensive.

Most shields for work in sand are provided with a hood or extension of the upper part of the cutting edge. The tail of the shield should be long enough to extend about 6 in. beyond the joint between the second and third rings before the shove, to facilitate repairs to jacks and shield, and, more particularly, so that, should a segment of the tunnel lining be broken during the movement of the shield, it may be replaced at the conclusion of the shove without exposing the earth. The inside of the tail at the extreme end is provided with a bead or narrow steel plate, Fig. 91, which more nearly closes the annular space between the tunnel lining and the skin of the shield and yet allows the shield to be pointed at an angle large enough to pass around curves or regain direction when off line or grade.

37. Data Regarding Tunnel Shields

Elements in the Design of a shield which are known are merely the depth below the ground and water surface. The meager information as to the character of the material to be penetrated, generally obtained from wash borings, and the absolute uncertainty of the nature of the forces involved make its rational design impossible; the only guide is precedent. The following tables give the principal dimensions and important facts about several shields which have been used under a great variety of conditions. The numbers in parentheses in the first column of the first table refer to the notes below where additional data are given.

(1) Thames, England. 1825-42. London clay. Maximum progress 2 ft., average 0.5 ft. per day.

(2) Tower, England. 1869. London clay. Maximum progress 9 ft. per day.

(4) City and South London, England. 1886. Mostly clay, also several lengths of water-bearing sand and gravel. The average progress of each shield was 333 ft. per month during 1/3 of work.

(5) Mersey, England. 1888. Clay, coarse ballast and running sand. Maximum progress was 57 ft. per week.

(6) St. Clair River, Michigan. 1889. Almost entirely in soft damp clay. Maximum progress per month was 382 ft., average 231 ft. per month.

(7) Hudson River, New York. 1879-1905. Mostly in silt; at New York side some sand and short length in rock. From May, 1890, to August, 1891, the average progress per month was 126 ft. in the first tunnel; in the second tunnel the average progress per day was 15.3 ft.

(8) Glasgow Harbor, Scotland. 1890. One-third in clay with boulders, the rest in sand and gravel. The average progress in sand was 2 ft. per day.

(9) Glasgow District Subway. 1891-95. Clay, sand, gravel, and quarry waste. The average progress was 100 ft. per month.

(11) Blackwall, London. 1892. London clay, sand, and one-third in open gravel. The maximum progress was 12-1/2 ft. per day. The average progress under the river was 98 ft. per month.

(12) Siphon de Clichy, France. 1892. Open water-bearing sand. The maximum progress was 10 ft. per day and the average 6.5 ft. per day.

(13) East River Gas, New York. 1892. Short lengths in decomposed schist and black mud.

Number and name of tunnel	Number of shields	External diameter		Skin., in. thick	Length		Number of jacks	Inside diameter of jacks, in.	Maximum thrust, tons c	Weight of shield, tons c	Distance driven, miles
		ft.	in.		ft.	in.					
(1) Thames, G. B.	1		a	......	9	0	10	......	.....	134	.....
(2) Tower, G. B.	...	....		1/2	4	9	6	b	.....	.....	.....
(4) City and South London	18	11	1	1/2	5	11	6	6-1/2	179	....	6.20
		11	4-1/2	1/2	6	6	6	6-1/2			
(5) Mersey, G. B.	1	10	3	3/4	11	7	10	7	770	....	0.25
(6) St. Clair R., Mich.	2	21	6	1	15	3	24	8	1795	92	1.14
(7) Hudson R., N. Y.	2	19	11	1-1/4	10	6	16	8	1540	92	1.60
(8) Glasgow Harbor	1	17	3	3/4	7	0	13	7	250	....	0.41
	2	17	3	1	8	6					
(9) Glasgow District	16	12	2-1/2	1/2	6	6	6	6-1/2	100	7	3.70
(11) Blackwall, G. B.	1	27	8	2-1/2	19	6	28	8	5785	224	0.59
							6	10			
(12) Clichy Siphon, Fr.	1	8	4-1/2	3/4	6	8	5	6-1/2	80	73	0.29
(13) East River Gas	2	11	0-3/4	7/8	7	2-1/4	12	5	595	12	d
(15) Waterloo and City, G. B.	2	13	2	1/2	7	0	7	7	135	....	.....
	2	13	9	1/2	9	6	7	7	135	....	2.84
	1	24	10	1	10	0	22	7	845	112	.....
(16) Concorde Siphon	1	6	9	3/4	6	8	4	6-1/2	65	....	0.15
(17) Central London	...	12	8	1/2	7	0	6	7	185	....	13.00
		22	10	1	6	10	22	7	680	....	.....
(19) L'Oise Siphon, Fr.	1	8	7-1/2	9/16	16	2	10	6	430	....	0.18
(20) Baker Street and Waterloo	1	13	0	1	9	8	14	6	475	33	6.28
(21) Greenwich, G. B.	1	13	0	1	14	1	13	7	840	84	0.23
(22) Lea, England	1	12	5	1	11	6	11	6-1/2	.....	.....	0.21
(23) Battery, East R.	6	16	11-1/4	1-1/8	9	6	14	8	1750	55	1.67
(24) E. River, P. R. R.	8	23	6-1/2	2-1/4	18	0	27	9	7730	240	2.87
(25) N. River, P. R. R.	4	23	6-1/4	2-1/8	17	3-3/8	24	8-1/2	3300	193	2.31
(26) Brackenagh, Ireland	1	6	2	1/2	6	9	4	6-1/2	.....	....	0.12
(27) Rotherhithe, England	2	30	8	2-1/4	18	0	40	9	5600	....	0.69
(28) Kingsway, England	1	16	2	1	8	9	16	8	.....	....	0.09
(29) River Dee, Scotland	1	8	8-5/8	1/2	6	9-3/4	7	6-1/2	.....	....	0.06
(30) Charing Cross, England	...	12	8	......	8	9	10	......	.....	.....	.....
(31) Great Northern Strand, England	...	12	8	1/2	6	11	8	......	.....	.....	.....
(32) 14th St. E. River	8	18	5-1/2	2	15	3-3/4 e	17	8	125 g	116 h	1.88
(33) Whitehall St. E. R.	4	18	6	2-1/4	16	4 f	17	8	125 g	104 h	1.60
(34) Lawrence Street, Brooklyn	2	18	6	2-1/4	16	4 f	17	8	125 g	104 h	1.00
(35) Old Slip, E. R.	6	18	0	2-1/4	16	4 f	17	8	125 g	98 h	2.31
(36) 60th St. E. R.	2	18	6-7/8	2-7/16	16	0-3/4 e	20	8	125 g	142 h	0.68
(37) Holland Vehicular	5	30	2	2-3/4	18	10	30	10	6000	400	2.46
	1	31	0	2-3/4	18	10	30	10	6000	....	.15

a Height 22 ft. 3 in ; width 37 ft. 6 in. *b* Hand operated screw jacks. *c* Tons of 2000 lb. *d* Driven a very short distance. *e* Includes 2 foot hood. *f* Includes 2 foot 2 in. hood. *g* Includes hydraulic equipment which weighs about 40 tons on 60th St. shield and 28 tons on other shields. *h* Per jack.

(15) Waterloo and City, England. 1893. Clay and ballast. 10 ft. per day was the average progress while the work was in full swing.

(16) Siphon de la Concorde, France. 1895. Clay and sand.

(17) Central London. 1896. London clay and silty sand.

(19) Siphon de l'Oise, France. 1896. Sand.

(20) Baker St. and Waterloo, England. 1898. About 350 ft. was in gravel under the Thames River, the rest was in London clay. The average progress per month was 117 ft. in gravel and 137 ft. in clay.

(21) Greenwich, England. 1898. Mostly in clay with the crown in close gray sand; about one-third length is in close gray sand and a small part in ballast. The maximum progress was 260 ft. per month and the average 148 ft.

(22) Lea, England. 1901. Mostly peaty clay and open ballast.

(23) Battery, East River, New York. 1901. 500 ft. almost entirely in rock; 1500 ft. in fine sand with some clay, the rest was coarse sand. Rates of progress follow: In normal air above water line in sand and gravel average 140 ft. per month; under river in rock, fine sand and clay, and sand and gravel, 91 ft. per month.

(24) East River, Pennsylvania R. R. 1903-09. Land portion in rock, soft top in places. Under river 18% in fine sand stretched with clay, stiff clay, sand and boulders; 35% sand and boulders, sand and clay and fine sand; 22% all in rock and 25% in rock with sand, gravel, and boulders in the crown. Rates of progress were: Best day's work 17.5 ft. Average for month in earth 176 ft., in rock and earth 56 ft., and in rock 61 ft.

(25) North River, Pennsylvania R. R. 1903-09. Mostly in silt, also short lengths in sand and in rock. The maximum month's progress was 545.2 ft. The average progress in rock was 58 ft. per month and in silt 2.39 ft.

(26) Brackenagh, Ireland. 1903. Water-bearing glacial deposit and running sand.

(27) Rotherhithe, England. 1904. Clay, conglomerate sand and gravel.

(28) Kingsway, England. 1904. Clay. Average progress 5.0 ft. per day.

(29) River Dee, Scotland. 1904. Alluvial clay and boulders. Average progress per day 4.5 ft.

(30) Charing Cross and Hampstead. 1904. London clay. Maximum progress 180 ft. per week.

(31) Great Northern and Strand. London clay.

(32) New York Subway. 1916. B. R. T. system from North 7th St., Brooklyn, to 14th St., Manhattan. Shaft at each side of river. 2 land shields, 2 river shields from each shaft (8 shields in all). Tunnel part in rock and part in sand and part in clay. Average progress per month river section, in earth 147 ft., in heart and rock 44 ft.; land section, in earth 168 ft.

(33) New York Subway. 1914-1919. B. R. T. system from Montague St. Brooklyn, to Whitehall St., Manhattan. Shaft on each side of river. From Brooklyn shaft two shields driven each way. River shields met rock tunnel driven without shields under river from Manhattan shaft. Material on Brooklyn side sand with some clay. Average progress per month river section, in earth 192 ft., in rock and earth 80 ft.; land section in earth 209 ft. Maximum progress per month, river section 345 ft.; land section 454 ft.

(34) New York Subway. 1914-1919. B. R. T. 2 shields driven under streets of Brooklyn at a depth of from 43 ft. to 75 ft. All above tide water. Extension of the Whitehall-Montague street tunnel. Material sand. Compressed air not used.

(35) New York Subway. 1914-1919. I. R. T. Shafts on each side of river. 2 shields in each direction from Brooklyn shaft; 2 shields toward river from Manhattan shaft. Material sand on Brooklyn side. Tunnels part in rock, part in sand on Manhattan side. Average progress for month, river section, in earth 184 ft., in earth and rock 74 ft. Land section in earth 167 ft.

(36) New York Subway. 1916-1919. B. R. T. from North Jane St., Queens to East 60th St., Manhattan. One shaft on each side of river and shaft in rock on Blackwell's Island. From Queen's Borough shaft, 2 shields were driven under east channel of East River, pushed through rock tunnels, previously excavated under Blackwell's Island and continued through earth under west channel to rock on Manhattan side. Average progress for month, river section in earth 188 ft., in rock 119 ft.

(37) Holland vehicular tunnel 1921-27 New York to Jersey City. One tunnel east-bound one west-bound each with 20-ft. roadway to accommodate automobile traffic only. There are four ventilating shafts equipped with powerful blowers, each shaft serving both tunnels. The blowers having a total capacity of 3,700,000 cu. ft.

per min., force fresh air into the ducts under the roadways from which it passes through small openings distributed along the curbs. Air is exhausted from the tunnel through the large duct above the roadway. There are 84 blowers and exhaust fans driven by electric motors totaling 6000 hp.

There were 13 795 lin. ft. of shield-driven subaqueous tubes. Of this, 2412 ft. was through mixed river mud, 1900 ft. mica schist rock, 7650 ft. Hudson river silt, and 1833 ft. through sand-clay and cinder fill. In addition, 2012 ft. of cut and cover and 649 ft. of open approach were built in New York and 1082 ft. cut and cover and 970 ft. of open approach in New Jersey. The New York power house from which two shields were operated furnished 28 000 cu. ft. free air per min. at a pressure of 50 lb. per sq. in. and 3550 at 125 lb. The shields were operated by three 25-gal. per min. pumps at 5000 lb. per sq. in. pressure. The New Jersey power house from which four shields were operated furnished 40 000 cu. ft. free air per min. at a pressure of 50 lb. and 2600 at 125 lb. The shields were operated by four 25-gal. per min. pumps at 5000 lb. pressure. All erectors operated at same pressure as jacks. The average progress was 295 ft. per month in silt, 142 ft. per month in mixed ground, and 57.6 ft. per month in rock. This progress included all delays due to starting shields, placing bulkheads and locks, and passing through shafts. In the Hudson River silt the shield could be pushed without taking any material into the tunnel but that procedure distorted grade and alignment of tunnel lining for a considerable distance back of the shield and therefore approximately one-half of the cross-section was taken through shield. This material was left in the tunnel until after driving was completed and then mucked out with a large power shovel.

The air pressure was seldom above 30 lb. per sq. in. All lining was cast iron calked with lead, except that for a short distance east and west of the New Jersey shaft the lining is of steel castings. Bolts in the lining are 1-3/4-in.-diameter high tensile steel of 110 000 lb. per sq. in. ultimate strength and 85 000 lb. per sq. in. yield point. This was a successful innovation as it reduced handling costs and waste due to stripped threads. All shafts were sunk as pneumatic caissons. Each river shaft on the New Jersey side rests on 42 24-in.-diameter steel pipe piles filled with reinforced concrete.

The cost of the 13 795 ft. of shield-driven tunnel was $17 050 000. The four shafts cost $3 100 000. The ventilating machinery cost $572 000. The cost of approaches, street changes, paving and housing for ventilating machinery and purchase of real estate, which included a number of city blocks, brought the cost of the whole project up to about $50 000 000.

38. Roof Shields

Roof shields for masonry-lined tunnels have been used with success principally in France and to some small extent in America. In use they are peculiarly adapted to the drier soils, usually under streets where open-cut methods are not permissible, although they have been used, as in the East Boston tunnel, in connection with compressed air for subaqueous tunnels. They are usually semielliptical or semicircular in form and consist of a skin of steel plates riveted to girders which at their lower extremities are tied together with a transverse girder and mounted on rollers. Hydraulic jacks supported by the ribs and bearing against the completed masonry or the arch centers move the shield forward. The two general methods of using roof shields are: first, to excavate the area of cross-section of the shield, build the masonry arch, and subsequently underpin with the side-wall construction; second, to drive timbered headings in advance and construct in them the side walls on which the shield is later supported or rolled.

(1) Commenced 1895. Contract price $55.45 per lin. ft. Semielliptical shield. Average progress 14 ft. 9 in. per day of 24 hr., maximum 29 ft. Material penetrated of loose sandy nature.

(2) Commenced 1897. Average progress 9 ft. per day of 24 hr. Material penetrated: 450 ft. compacted clay and sand, 100 ft. loose sand and gravel. Height = over all.

Number and name of tunnel	Number of shields	Width of shield		Height of shield*		Skin, in. thick	Length †		Number of jacks	Inside diameter of jacks, in.	Maximum thrust, short tons	Distance driven, miles
		ft.	in.	ft.	in.		ft.	in.				
(1) de Clichy..........	1	23	9	9	8	9/16	17	3	6	9-1/2	616	.78
(2) Boston, Tremont...	1	29	4	8	7-1/4	1	14	0	10	6	1250	.11
(3a) Orleans Railway	1	34	9	12	6	1	19	8	10 10	10 4-1/2	1120 168	.54
(3b) Extension......	1	32	0	11	8	3/4	23	0	8	9-1/4	1021	.22
Collector de Bievre:												
(4a) Chagnaud type....	1	16	2	6	6-3/4	3/4	18	0	6	6-3/4	228	.11
(4b) Chagnaud type....	1	16	2	7	6-3/4	3/4	18	0	6	6-3/4	228	.18
(4c) Dioudonnat type..	1	16	3	10	6*	9/16	13	4	5	6-1/4	224	.07
Paris Metrop. Ry.:												
(5a) Champigneul type.	4	28	3	8	9-1/2	7/10	23	2	8	9 1/4	923	.95
(5b) Dioudonnat type..	3	28	3	8	7	1	22	2	8	8-1/2	896	.33
(5c) Weber type.......	2	28	3	8	7	1	16	1	4 8	5-3/4 8-1/2	179 896	.90
(5d) Lamarre type.....	2	28	3	8	7	1	19	9	9	7-1/2	1149	.19
(6) Boston Harbor.....	2	28	10	14	5	1-1/4	13	0	16	3-1/4	75	.81

* Height = From springing line to crown unless noted. † Length = Distance over all.

(3a) Commenced 1898. Material penetrated mostly made ground, old masonry walls and foundations. Average progress 3.4 ft. in 24 hr., maximum 22 ft.

(3b) Material penetrated, see No. 3a. Average progress 10 ft. in 24 hr.

(4a) Commenced 1898. Employed on elliptical section. Material penetrated: made ground, decomposed limestone, marl, and foundation walls. Average progress 8.2 ft. per day, maximum 101.7 ft. per week.

(4b) Started 1898. Employed on circular section. Material penetrated, fine sand and gravel. Average progress 9 ft. per day.

(4c) Started 1898. Employed on circular section. Material penetrated, made ground, sand, foundation walls, etc. Average progress 7.2 ft. per day. Height = over all.

(5a) Started 1899. Two shields of this type employed on 1st section, 1 on 8th section, and 1 on 11th section. Material penetrated, made ground and soft sand, with some marl. Average advance 13 ft. per day on 1st section, 10 ft. on 8th, and 12 ft. on 11th.

(5b) Started 1899. One shield of this type employed on 2nd section, two on third section. Material penetrated, sand and gravel, some water-bearing. Maximum advance 29 ft. per week.

(5c) Started 1899. Two shields employed on 4th section. Material penetrated, sand and gravel. Shields proved unsatisfactory.

(5d) One shield of this type employed on 6th section, one on the 7th section. Material penetrated, sand, gravel, and marl. Average advance 6.5 ft. per day. Shield proved unsatisfactory.

(6) Started 1901. Material penetrated, blue clay and boulder clay, with some sandy and silty permeable clay. Air pressure, 18 to 20 lb. average, 25 lb. maximum; 1200 cu. ft. per man per hr. Average progress 4.8 ft. per day.

39. Cast-iron Tunnel Lining

The Lining for shield-driven tube tunnels has usually been cast iron. The reasons for its use are that it can be most quickly built; as soon as erected it furnishes a reaction for the shield jacks; it offers greater assurances of water-tightness, and being thin for its strength saves excavation. For subaqueous work done in compressed air all of these considerations are weighty. The number of segments forming a ring and the width of the ring depend on the economic weight that can be handled. Sometimes in small tunnels up to 13 or 14 ft. in diameter iron lining has been built by hand, but hydraulic erectors attached to the shields or to traveling platforms are generally used. The length of shield which can be conveniently used is another factor governing the width of segments, the length of the tail being affected correspondingly by increases in this width. Fig. 92 shows the cast-iron lining used in the Pennsylvania R. R. tunnels under the East River, New York. There has been built in compressed air at least one shield-driven tube tunnel lined with brick. In the construction of the 39th Street sewer in Chicago, segmental timbering formed a lining inside of which the brick was built. The timber was used to receive the reaction of the shield jacks.

The design of the cast-iron lining depends as much on the requirements of building, as upon the stresses involved, and the best guide is a study of precedents. The table gives principal dimensions and details of iron lining in existing tunnels.

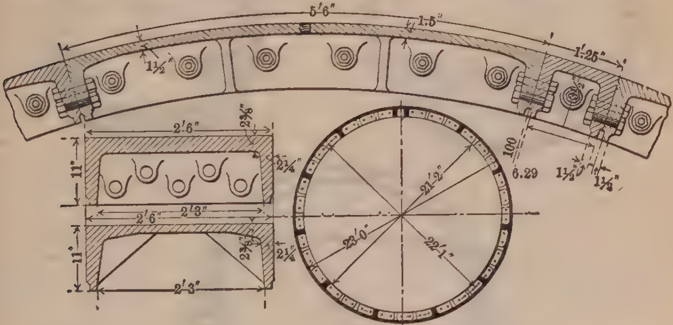


Fig. 92. Cast-Iron Lining, East River Tunnel

Stresses in tunnel lining can only be approximated by theoretical analysis, because they originate in earth pressures and the law of action of earth pressures is only imperfectly known. Although the designer should depend more on judgment and precedent than on theoretical analysis, such an analysis may be helpful by indicating the nature of the stresses.

Let C_1, C_2 (Fig. 94) be the line of centers of resistance (locus of centers of gravity if shell is homogeneous) of a unit length of tunnel shell which is taken circular but may have any other form. Let p_1, p_2, p_3, . . . to p_n be all forces acting upon the shell. These must satisfy the condition that the forces on opposite sides of any diameter must be equal and opposite, otherwise there would be movement. For example, the reactions and active forces, if any, on the bottom must balance the loads on top. It is assumed in Fig. 94 that the loading and the shell are symmetrical about the vertical axis $A-B$, which will usually be the case. The analysis would be similar in any case.

The load diagram a_1, a_2, a_3 . . . is constructed in the usual way by laying off the forces p_1, p_2, p_3 . . . to scale parallel to their direction. The position of the pole o is found by laying off on $a_1\ an$ the thrusts T_1 and T_n, which are found by taking moments of the horizontal components of p_1, p_2, p_3, . . . about C_1 or C_2. Then a_2, a_3, a_4, etc., are connected with the pole, and the line of thrust $T_1\ T_n$ is constructed in the usual way by drawing T_1, T_2, T_3, . . . parallel with oa_1, oa_2, oa_3, . . . But an infinite number of such lines of thrust may be drawn. The one to be used for computing stresses

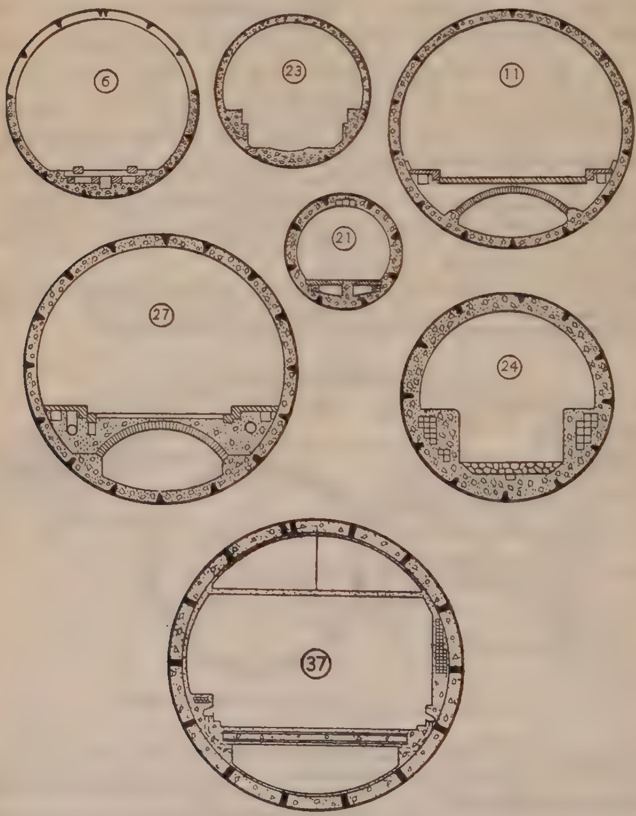


Fig. 93. Cast-Iron Lined Tunnels

in a ring of uniform cross-section may be found from the consideration that for any short length of bent member the bending moment is proportional to the angular deflection in that length, and, because in a closed ring the total change of angle of axis is zero, the sum of the bending moments integrated about the closed ring must be zero. The line of thrust to be used in computing the stresses in a tunnel ring may be found by choosing that one for which the positive and negative bending moments are zero. This may be done by trial. Divide the complete ring into small equal segments, multi-

Details of Iron Lining in Tunnels (Fig. 93)

Number and name of tunnel	External diameter of ring, ft.	External diameter of ring, in.	Length of Ring in.	Thickness of web, in.	Depth of flange, in.	Number of segments exclusive of key	Weight lb. per lin. ft.	Bolts per ring C	Bolts per ring H	Diameter of bolts, in.	High water to base, max., ft.
(2) Tower, London....	7	1-3/4	18	7/8	3	3	971	31	..	3/4	....
(3) Antwerp.........	*		19-3/4	....	4-6	4	......	14	4	.....	26
(4) City and South London......	10	10-3/4	19	1	4-3/8	6	1 880	47	..	1	75
	11	3	20	7/8	4-1/2	6	1 880	47	..	7/8	
	22	6	18	1-1/8	7-3/4	12	......	83	..	1-1/8	....
	32	0	18	1-1/2	12	16	......	113	..	1-1/4	....
(5) Mersey, G. B.....	10	0	18	1-11/16	6	10	3 061	28	22	.....	54
(6) St. Clair, Mich....	21	0	18-1/4	2	7	13	9 333	157	56	7/8	78
(7) Hudson, N. Y...	19	6	20	1-1/2	9	11	6 055	66	36	1-1/4	100
	18	9-1/4	20	2-1/2	8	....	......	66	36	.....	
(8) Glasgow Harbor....	17	0	18	1	6	13	4 559	66	28	.....	62
(9) Glasgow District...	12	0	18	3/4—1	6	9	2 241	46	20	1	70
(10) Kingston, N. Y....	9	0	18	7/8	4	6	1 580	37	14	1	....
(11) Blackwall, G. B.	....		......	1-1/2	10	14	9 408	...	..	.....	....
	27	0	30	2	12		13 265	70	75	1-1/2	80
(12) Clichy Siphon, Fr..	8	2-1/2	19-3/4	1	4	5	1 453	36	24	1	58
(13) East River Gas....	10	10	16	1-1/4	4	9	2 400	46	20	1	122
(14) Mound of Edinburgh..........	17	6	18	1-3/4	7	14	7 100	...	..	1-3/8	....
(15) Waterloo & City.	13	0	20	7/8	5-1/8	7	2 326	57	24	1	62
	13	7-1/4	20	7/8	5-1/8	7	2 431	57	24	1	
	24	6	18	1-1/4	9	13	......	92	..	.....	....
(16) Concorde Siphon..	6	6-1/2	19-3/4	7/8	3-5/8	4	895	21	15	7/8	38
(17) Central London.	12	6	20	7/8	4-7/8	....	......	...	..	7/8	....
	22	6	18	1-1/8	8	12	......	81	39	1-1/8	....
(18) Spree, Germany †.	13	1-1/2	25-5/8	3/8	4	9	1 270	72	36	.....	35
(20) Baker St. and Waterloo.......	12	9-3/4	18	1	4-7/8	6	2 643	53	14	7/8	70
(21) Greenwich, Eng...	12	9	20	1-1/4	6	8	3 080	47	..	1-3/8	70
(22) Lea, England.....	12	3-3/4	21	7/8	4-7/8	6	......	29	21	7/8	30
(23) Battery, East River	16	8-1/2	22	1-1/8	7-1/2	8	4 000 4 540 5 130	49	27	1	94
(24) East River, Pa. R. R. Land	23	0	30	1	8	11	5 166	67	60	1-1/4	....
Land	23	0	30	1-1/4	9	11	6 776	67	60	1-1/4	....
River	23	0	30	1-1/2	11	11	9 102	67	60	1-1/2	68
River	23	0	30	1-1/2	11	11	9 102	67	60	1-1/2	93
(25) No. River, P.R.R.	23	0	30	2	11	11	12 127	67	60	1-3/4	98
	23	0	30	1-1/2	11	11	9 273	67	60	1-1/2	....
(26) Hilsea Creek Portsmouth, Eng.	12	6	20	......	.....	6	2 240	...	..	.....	58
(27) Rotherhithe, G. B.	30	0	30	2	14	16	16 600	...	..	.....	....
				1-3/4	14	16	14 700	85	79	1-1/2	97
(28) River Dee, Scotland	8	6	18	1	5	5	1,960	42	18	1	50

* Height 5 ft. 7 in.; width 4 ft. 11 in. † Steel lining. C = Circumferential, H = Horizontal.

Details of Iron Lining in Tunnels—Continued

Name of tunnel	External diameter of ring		Length of ring, in.	Thickness of web, in.	Depth of flange, in.	Number of segments exclusive of key	Weight lb. per lin. ft.	Bolts per ring		Diameter of bolts, in.	High water to base, max., ft.
	ft.	in.						C	H		
(32) 14th St., East River, N. Y..	*18	0	26	1-3/8	9	9	6 323	55	50	1-1/4§	97
	†17	2	26	1	7	9	3 801	55	40	1	115
(33) Whitehall St., East River, N. Y.......	*18	0	26	1-3/8	9	9	6 323	55	50	1-1/4	87
	‡18	0	26	1	7	9	4 048	55	40	1	80
	†17	2	26	1	7	9	3 801	55	40	1	90
(35) Old Slip, East River, N. Y..	*17	6	26	1-3/8	9	9	6 166	55	50	1-1/4	89
	‡17	6	26	1	7	9	3 949	55	40	1	89
(36) 60th St., East River, N. Y..	*18	0	26	1-3/8	9	9	6 323	55	50	1-1/4	116
	‡18	0	26	1	7	9	4 048	55	40	1	85
(37) Holland Vehicular.........	29	6	30	1-5/8 to 2-3/4	14	14	17 300	85	75	1-3/4	101

* Earth or earth and rock. † Rock without shield. ‡ Rock with shield.
§ Bolts upset 1/8 in. for thread; also upset 1/8 in. near the head end.

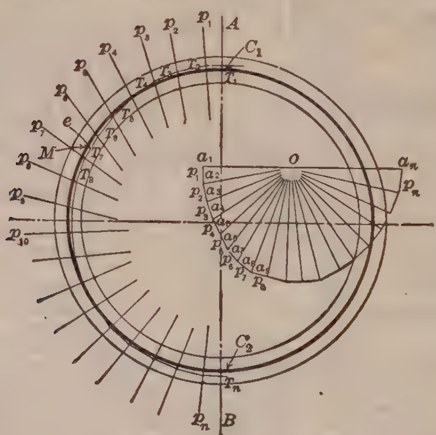


Fig. 94. Stresses in Tunnel Shell

ply the component of the thrust at right angles to the radius at the middle of each segment by e, the eccentricity of the thrust. The e is the distance from the center of gravity of the cross-section of the ring to the line of thrust measured on the radius. In this way find the line of thrust for which the sum of the positive bending moments equals the sum of the negative bending moments integrated about the complete ring.

40. Waterproofing and Grouting

Grummets. Bolt holes in cast-iron tunnel lining, being slightly larger than the bolts which are placed in them, permit water to seep into the tunnel unless a tight calking material is carried around the bolt hole or some form of grummet is placed under plate washers at the head and nut. Three principal types of grummets have been used. In the Baker Street and Waterloo, the North River Pennsylvania, the Battery, and for a time in the East River Pennsylvania tunnels rings of hemp saturated in red lead and oil under plate washers were used. In the East River Pennsylvania, where the bolts had rolled threads, the diameter of the threaded portion being larger than the diameter of the shank, greater success was attained by wrapping around the shank strands of hemp saturated in red lead and oil. In the Greenwich and Rotherhithe tunnels lead washers were used, and when the bolts were tightened the lead washers were forced into conical bolt holes.

Calking. The prevailing practice is to cast a small depression in the flanges at its inner edge which matches a like depression in the adjoining plate and forms a calking groove when the plates are bolted together; see Fig. 92. Into this is calked either soft lead or a rust mixture or sometimes portland cement. The grooves have usually been from 1/4 to 3/4 in. wide and about 1-1/2 in. deep. For lead it is desirable to make the groove much narrower, say 1/8 in. wide, but this is difficult to cast.

The rust mixture is usually one part by weight of sal ammoniac mixed with 400 parts of iron filings. The sal ammoniac is best dissolved in enough water to dampen the iron filings, which are then calked into the groove and well compacted either by hand tools or pneumatic hammer. The finer the iron filings the better. Rust mixture calking is not successful if there is water coming through the joint during calking. Even when the joints are made absolutely watertight slight movements of the flanges, such as are likely to occur from contraction in winter or in adjustment of bolts, are liable to cause a small amount of leakage. The table shows the practice in some of the more important tunnels:

Grouting. The shield being larger than the tunnel lining leaves an annular space which should be filled immediately after the shield has passed. This is especially necessary at the sides to prevent the lining from spreading. The material to fill this space must be ejected through holes in the lining. It must therefore be something that will flow and be permanent. A grout of equal parts of cement and sand is the most common material for the purpose, but simple pugged clay has been used as well as plain sand or gravel. These are probably all equally good so long as they are made to fill the space, although in rock the cement grout is preferred.

Waterproofing. Most subaqueous tunnels have had cast-iron or steel shells which are waterproof of themselves; the exceptions are those that are in nearly impervious clay, which prevents any quantity of water reaching the tunnel lining. There are a few such tunnels as the Severn, which has brick lining which is back drained to a sump which is kept dry by pumping. Numerous tunnels and subways in water-bearing ground are of masonry lining waterproofed by pitch in some form.

The most generally used form is a combination of pitch, usually applied hot, with sheets of some fibrous material used as binders; for this purpose burlap, building paper, and so-called felt have been used. These binder sheets are usually saturated with pitch by the manufacturer, and are applied to the outside of the wall in from 2 to 6 thicknesses, each layer being given a coat of hot pitch as it is placed. The specification for such work in the Pennsylvania R. R. land tunnels in New York call for "straight run

Name	Date	Maximum head water, ft.	Kind of calking
Antwerp, Belgium.....	1879	26	Groove in flange outside of bolts. Rope of tarred hemp.
City and South London.	1886	75	In joints between rings oakum in groove covered with cement. Longitudinal joints 2/3 filled with soft wood packing and the rest filled with cement.
Mersey, England......	1888	54	Face of flange beveled. Portland cement mortar, just damp, in groove.
St. Clair, Mich........	1889	78	Canvas coated with resinous compound between rings, creosoted oak packing 3/16 in. thick in longitudinal joints.
Blackwall, London.....	1892	80	Rust joint material—1 lb. sal ammoniac to 400 lb. iron borings. In wet joints lead first hammered into groove.
Baker St. and Waterloo.	1898	70	Pine packing between rings outside 1/2 of joint, rust joint material inner 1/2. Calked around bolts. Rust joint material in shallow groove on longitudinal joints.
Greenwich, England...	1898	70	Rust joint material in grooves into which soft lead had been hammered.
Battery, East River....	1901	94	Lead in grooves.
East River, P. R. R....	1903–09	93	20% calked with rust material—1 lb. sal ammoniac to 400 lb. iron borings. 80% calked with lead and pointed with portland cement.
North River, P. R. R..	1903–09	98	Sides and invert—2 lb. sal ammoniac, 1 lb. sulphur, and 250 lb. iron borings; arch, 4 lb. sal ammoniac, 3 lb. sulfur, and 125 lb. iron borings.
Rotherhithe, Thames R.	1904–09	97	Lead hammered into groove and covered with rust joint material.
East River, New York Public Service.......	1910–19	116	The calking for all the cast-iron-lined tunnels was done with leadwire over which a portland cement pointing was placed to fill the groove.
New York: 14th St.			
Whitehall St.			The calking groove was 1/4 in. wide and 1-1/4 in. deep.
Old Slip			
Lawrence St.			Lead wire elliptical in section (maximum diameter 1/2 in., minimum diameter 1/4 in.) used throughout the entire work.
60th St.			
Holland Vehicular, New York-Jersey City....	1922–27	101	Lead wire calked into groove 3/8 in. wide, 1-1/4 in. deep.

coal tar pitch which will soften at 60° F. and melt at 100° F., being a grade in which distillate oils, distilled therefrom, shall have a specific gravity of 1.05.'' Felt was coated and saturated with asphaltic products, and it was required that the wool in the unsaturated felt should be not less than 25% by weight and that the felt should weigh 0.05 to 0.06 lb. per sq. ft. unsaturated and 0.12 to 0.14 lb. saturated. Another type of waterproofing consists of an outside course of asphaltic brick laid in soft pitch or a mastic of soft pitch and fine sand.

41. Compressed Air

The Shield and Compressed Air generally go together, but there have been several exceptions. A portion of the Pennsylvania tunnels in Long Island City were mined, timbered, and lined in compressed air without use of shields, and the builders of the telephone tunnels in Chicago used compressed air but no shields. The amount of air required is very variable in different tunnels and at different times in the same tunnel. In the East River

Compressed Air Data

Tunnel	High water to invert, max. ft.	Minimum cover, ft.†	Max. air pressure lb. per sq. in.	Aver. air pressure lb. per sq. in.	Cubic feet of free air supplied
City and South London..	34	42	15	......	In water-bearing sand 1660 per min. per face; when grouted 1000 to 1300 per min. per face.
Blackwall, England......	80	5	37	35	10 000 per min. per face in open ballast for some time.
Baker St. and Waterloo..	70	18	35	28	In gravel 3300 per min. per face; parallel tunnel 1650 per min. per face.
Greenwich, England.....	70	30	28	20	Average 5000 per man per hr.; never less than 4000.
Battery, East River, N. Y.	94	12	42	26	Two working faces, maximum 32 000 in sand.
East River, P. R. R......	93	8	42	27	Maximum at one face 25 000 per min. for 24 hr. Capacity of plant for 8 faces 80 400 per min.
North River, P. R. R....	98	20	37	26	Maximum in gravel 10 000 per man per hr. Generally ranged between 1500 and 5000.
14th St., East River, N. 7th St. Tunnels.	97	4*	39-1/2	33	Maximum in sand 9000 in one heading for 24 hr.‡
Whitehall St., East River, Montague St. Tunnels.	87	8	34-1/2	30	Maximum in sand 12 000 in one heading for 24 hr.‡
Old Slip, East River, Clark St. Tunnels.	89	3*	37-1/2	30	Maximum in sand heading 10 000 one heading for 24 hr.‡
60th St., East River, N. Jane St. Tunnels.	116	9*	47-1/2	41	Maximum in sand 12 000 in one heading for 8 hr.‡

* The shield cut into the permanent clay blanket placed on the river bed. The clay blanket is protected from washing by a rock revetment on the sides and top.

† Does not include clay blankets.

‡ Requirements for all contracts was 10 000 cu. ft. of free air per min. per heading; except in one case where one power house supplied 8 headings, the requirement was reduced to 8000 cu. ft. of free air per min. per heading.

tunnels of the Pennsylvania R. R. in open sand with a clay blanket 10 to 20 ft. deep and 350 ft. wide dumped upon the river bed, the capacity of compressors required was about 80 400 cu. ft. free air per min. The average used was about 3500 cu. ft. per min. per heading. In the Rotherhithe tunnel under the Thames it was specified that the minimum amount of air pumped into the tunnel should be 8000 cu. ft. of free air per man per hour.

The great difficulty in water-bearing sand or gravel when compressed air is used arises from the impossibility of balancing the water pressure at more than one elevation, with the result that if the water pressure is balanced at the bottom of the tunnel there is an excessive escape of air at the top, which is expensive, and there is always danger that a " blow," or sudden outrush of air, will occur at the top, which endangers the workmen and may cause delay by flooding the tunnel. If the water pressure is balanced at the top, the bottom may be so wet that the ground is unstable and difficult to handle. As a compromise the water pressure in such materials is usually balanced by the air pressure at about the center of the shield, and blows are guarded against by breasting and plastering the face with clay and depositing a clay blanket on the river bed above.

42. Subaqueous Tunnels Built from Surface

In cases where the top of the tunnel is desired to be close to the bed of the river thus giving insufficient cover for shield work, or if the tunnel is short,

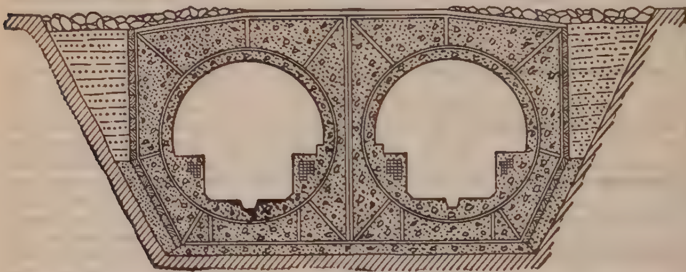


Fig. 95

making the plant installation required for shield work proportionately excessive, tunnels have been built successfully from the surface.

(1) Detroit River Tunnel, Detroit to Windsor. Trench was dredged to grade in the stiff clay of the river bottom. Twin steel tubes of 3/8-in. plate 23 ft. 4 in. in diam-

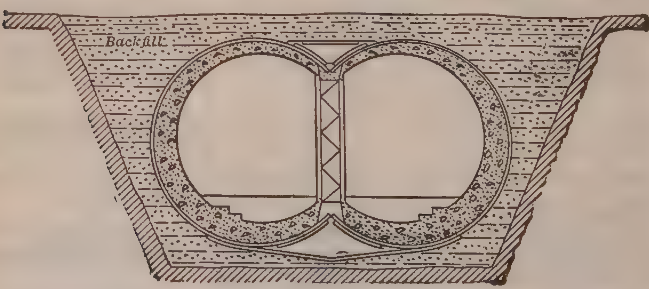


Fig. 96

eter and 262 ft. 6 in. long connected by structural steel diaphragms at 12 ft. c. to c. and provided with bulkheads were sunk in position on prepared foundations. Sections were connected up by divers with special bolting system. A timber form or sheathing was fastened to outer edges of the diaphragms. Concrete was placed by tremie between this sheathing and the tubes—completely encasing the tubes. This concrete was 3 ft. thick on the sides and 4 ft. 6 in. thick top and bottom. Reinforced-concrete lining having an interior diameter of 20 ft., was placed inside the steel tubes. (See Fig. 95.) Distance from water surface to top of rail in mid-channel 66 ft. Current in river over 2 miles per hr. Total length of river section 2667 ft. Built 1906-10.

(2) Chicago River Tunnel. Single length of two parallel steel cylinders with a longitudinal dividing wall; lined with reinforced concrete and sunk on prepared foundations in a trench dredged in the river bottom. Tubes of 3/8-in. steel plate, stiffened by plate and angle gussets above and below the central wall every 7-1/2 ft. Concrete lining 20 in to 40 in. thick. Central wall 3 ft. thick. Length of tubes 278 ft., width 41 ft., depth 24 ft. Weight when sunk 8000 tons. Backfill and cover sand. Connection to shore tunnels made by cofferdam. Built 1910-12. (Fig. 96.)

(3) Paris Subway under Seine, Pont Mirabeau crossing. The river portion is 644 ft. long and made up of 5 sections 116.8 ft. to 144.4 ft. long. Each section was a pneumatic caisson consisting of a structural steel frame covered inside and out with steel plates and filled with concrete, with a double-track tube of similar construction above the working chamber. Over-all width 29.9 ft., over-all height 30.1 ft. These sections were sunk in position 15-3/4 in. apart, this space filled with tremie-placed concrete through which an opening of required tunnel section was later excavated. A tunnel having a total caisson length of 1313 ft. was built under the Seine by a very similar method in 1906-7.

(4) Harlem River crossing, New York Subway, Lexington Avenue Line. Four-track tunnel built very much the same as the Detroit River tunnel noted above. The sections as assembled had an over-all width of 76 ft. and height of 24-1/2 ft. Four of the sections were 220 ft. long and the remaining one was 200 ft. long. Contract price $1500 per lin. ft. for four-track structure. Built 1911-12.

(5) Harlem River Tunnels, New York Subway, Lenox Avenue Branch. Twin tube sections 610 ft. long of cast-iron lining, inside diameter 15 ft. Across the river trench was excavated, bottom of which was a little below the middle of the finished tunnel. Two lines of heavy sheet piling were driven outside the line of the finished tunnel and cut off at the springing line of the arched roof. A floating timber box was then built in which the arched roof of cast-iron segments was erected and covered with concrete with proper internal bracing and bulkhead ends. The floating box was sunk from underneath and the roof left floating like a diving bell. This with steadying tackle was sunk and left resting on the sheet piling and bearing piles previously driven in the trench. After divers connected this to the section already in place, the lower half of the tunnel was built in compressed air. Built 1903-4.

(6) Oakland-Alameda vehicular tunnel, California, 1926-28. Single tube with 24-ft. roadway for two-way automobile traffic. 12 sections each 203 ft. long with temporary wooden bulkheads were built in drydock, floated to the site and sunk. The sections were reinforced concrete, 37 ft. outside and 32 ft. inside diameter. The arrangement of ventilating air ducts was like that in Holland tunnel.

43. City Subways

The Cut-and-Cover Method is usually cheaper than tunneling at depths less than 30 ft. and may be the more practicable method when the cover is shallow, as often with city subways for rapid transit. Attempts to excavate by shield within a few feet of street surfaces have not met with entire success because of disturbance to pavements and pipes. Where traffic will not permit an open cut, the pavement is taken up, if necessary at night, and replaced by a timber deck under cover of which the subway is built. Where wide cutting is objectionable, side-walls and interior supports are sometimes built in trench and roofed, after which the core is removed and the bottom placed. Where there is ground-water, its level frequently may be lowered by draining to sumps and pumping carefully. Where quicksand is met at subgrade, tight sheeting is driven to extra depth and it may be necessary also to excavate for and place the floor quickly in short sections. On the other hand, where the ground is dry and hard enough to stand with but little bracing and where decking is not required, steam shovels have been successfully used. Intercepted sewers may be depressed or under-siphoned, or the sewer system may be modified.

Adjacent foundations are underpinned, either temporarily or permanently, where required to prevent settlement. Finally, subsurface structures are restored or rebuilt and the street is backfilled and repaved.

Types of Sections are shown in Fig. 97. Earlier sections usually were of the single-arch type if headroom permitted; where close to the surface the roof was of beams with jack-arches between them. Later, beams came to be used also in sidewalls, forming bents, saving in width and facilitating construction. Reinforced concrete requires somewhat less and cheaper steel and permits the use of large forms, but bents continue in favor because better adapted to requirements of bracing, piecemeal construction, and support of subsurface structures. The single arch of concrete, plain or reinforced, is still used where conditions of head-room and lateral support are favorable. In soft or wet ground the floor may be formed as an invert or may be reinforced with beams or rods. Waterproofing is commonly felt or fabric saturated and laid with tar or asphalt, and in dry situations may be omitted from floor and sides. Stations are painted, plastered or tiled in white or light colors.

The New York Subway System, the largest, has 74 miles of underground structure in operation by two different companies. A new independent system now (1928) building by the city will add, it is estimated, 61 miles by 1933. Most of it is 2- or 4-track subway of bent type (3), Fig. 97. Portions first (1900-3) built are nearly all of bent type (1), and some later (1902-12) built are of reinforced-concrete type (2) or similar. Type (3) provides a partition between the middle tracks to promote ventilation by train movement. The roof and where necessary the sides and floor are waterproofed generally with felt or burlap and pitch. The inside tracks are for express, and the outside for local trains. At express stations the platforms are between local and express tracks, serving both, and at local stations at the sides. In some parts the four tracks are built in two tiers. Excavation was partly in rock. Groundwater and very fine quicksand were frequently met. Methods varied widely. The system includes several miles of arch tunnels in rock, cast-iron-lined tubes and steel viaduct extensions.

The Boston Subway System, the first in America, with 28 miles underground trackage, radiates to Cambridge, Dorchester, East Boston and Brookline. It is mostly 2-track subway and principally of bent type (4) and arch type (5) in earlier (1895-1908) portions and of reinforced concrete type (6) in a later portion. Where traffic was dense a short portion somewhat like (5) was tunneled with roof shield. The Cambridge portion (1909-12) is of arched, reinforced type (11), but where cover is shallow the roof is made flat. Excavation was nearly all in sand, clay and gravel. The system is for both surface cars and trains. It includes two shield-driven subaqueous tunnels and steel viaduct extensions or connections.

The Philadelphia Subway east and west in Market Street, built 1903-07, 2-1/4 miles long, is in about equal parts a 4-track (7) and a 2-track (8) structure. The walls of (7) and the roof and walls of (8) are rod-reinforced. The roof is waterproofed with asphaltic mastic and the sides with asphalted burlap. The outer tracks of (7) are for street cars and the inner tracks, con-

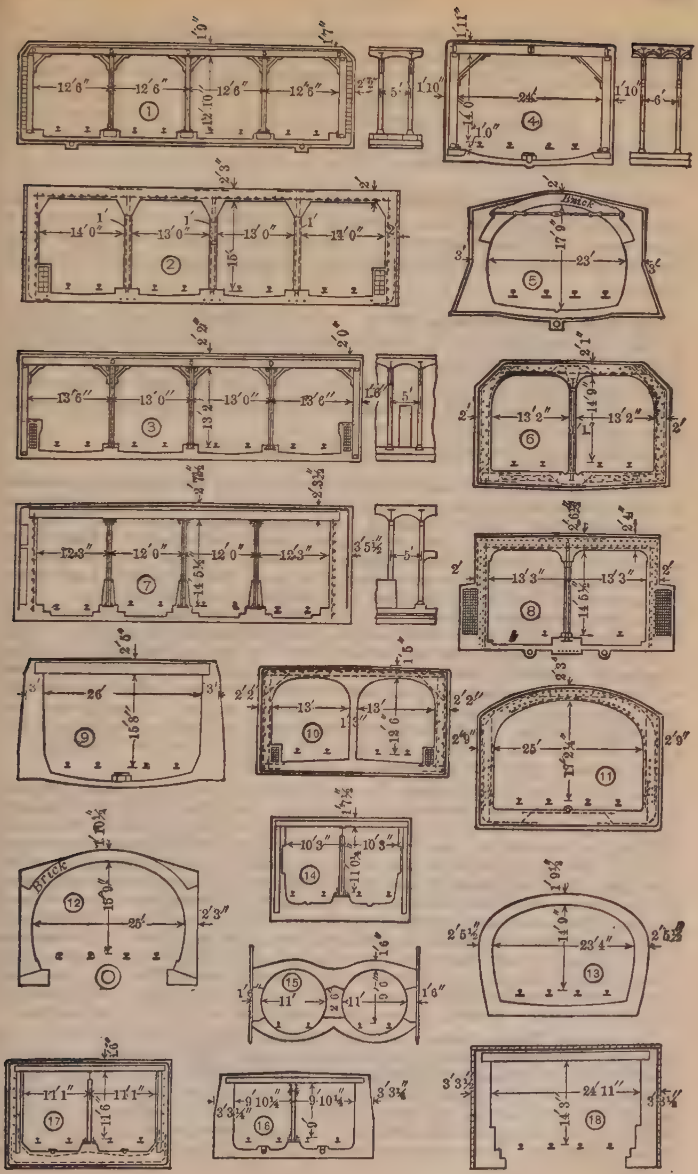


Fig. 97. Subway Sections

tinuous with those of (8), for trains. The columns of (7) are protected against derailment by low reinforced-concrete bulkheads. In 1924 the city began a 4-track subway 6-1/2 miles long north and south under Broad Street, to extend by a 2-track branch to the southwest. The new subway is of the bent type. Waterproofing is generally limited to the roof; the floor and walls are underdrained. Excavation is largely in gravelly soil. The system includes steel viaduct extensions.

The Atlantic Avenue Subway, Brooklyn (9), built 1901-7, 2 miles long, is for electric trains of the Long Island R. R. While not a city subway in a strict sense, it is structually such. The sides are retaining walls and with the roof are waterproofed with tarred felt. Excavation was mostly in sand and gravel.

The Sixth Avenue Subway N. Y. (10), built 1906-11, 1 mile long, is for Hudson and Manhattan R. R. trains entering the city by tunnel under the Hudson. Roof and sides and, where in earth, the floor, are reinforced. Waterproofing was mostly of asphalted burlap but in a short portion a waterproofing compound was mixed with the concrete. Excavation was partly in rock. Some quicksand was met.

Foreign subways generally are of smaller cross-section.

The London Metropolitan and District Subway, (12), an irregular oval line termed the Inner Circle, was the first city subway for rapid transit. It was built 1860-84 and later extended from its original length of 13 miles, part of which is open cut. The arch and walls are of brickwork. The extrados is waterproofed with asphalt. Where cover is shallow, cast- or wrought-iron roof girders were used. With soft or wet foundation, an invert was added. Excavation was mostly in gravel, sand and clay. Later extensive underground lines are in deep-level tubes, shield driven in London clay.

The Paris underground railway, (13), a network of belt and intersecting lines, begun 1898 and opened 1900, has in 1928 a length of about 60 miles, mostly subway. A link south of the Seine, begun 1912 and halted by the war, was resumed 1923. The plan in progress is to provide about 80 miles of double-track line within the city proper, in addition to a large mileage of extensions on viaduct or surface. Short portions first built were tunneled with roof shield or by the Belgian method. Watertightness was secured by grouting. Excavation was mostly in sandy soil and in some parts in soft rock. There are two operating companies—the " Metro " and the " Nord-Sud "—which exchange passengers by transfer at intersections.

The Berlin underground railway, (14), an original east-west trunk, begun 1896 and opened 1902, with later extensions or connections to suburbs, has about 25 miles of 2-track line, of which about half is subway. A new north-south trunk, 7.7 miles long, built by the municipality, was opened for operation by the existing company early in 1923. The section shown is that of the Schoneberg branch, opened 1910, of the bent type. The longitudinal girders are discontinuous, each supporting 5 roof beams, and rest on two columns over which they cantilever. A feature was the use of small I-beams, driven as piles at each side every few feet, to the inner flanges of which were clipped breast boards, retaining the earth and serving as back forms for the concrete of the sidewalls, in such manner as to permit the final withdrawal of the beams. The beam-piles were braced apart only at the top, leaving a free working cut. An earlier type has thicker walls without beams. A late type has no columns. Waterproofing is asphalted paper. Excavation was mostly sandy and much of it water-bearing.

The Glasgow District Subway, (15), an irregular oval line, built 1891-97, is 6-1/2 miles long, comprising subway, tunnel and open cut. For the subway, a heavily sheeted trench was excavated deep enough to build the roof, which was next waterproofed with layers of asphalt and covered with the restored pavement. The roof, supported by the sheeting, was then undermined with drifts in which were built the walls and floor, completing the structure. A short section in soft ground was built under air pressure. Excavation was in sand, clay, mud and rock. Traction is by cable.

The Budapest Subway, (16), for surface cars, a line 2 miles long, built 1894-6, was the first underground electric railway. The I-beams of the roof, which is close under the pavement, are supported at the middle on a continuous girder of two I-beams resting on columns spaced about 13 ft. apart. Waterproofing consists of sheets laid in pitch.

The Hamburg Subway, (17), begun 1907, forms 5 miles of a ring with external branches opened in part in 1912 and since extended. Excavation was in varied soil, in parts water-bearing. In dry ground, openings are left in the floor under the tracks and only the roof and sides were waterproofed. Beam piles were used at the sides, as in Berlin. Where depth permitted, a single arch section with invert was used.

The Buenos Aires Subway, (18), for surface cars, was begun 1911 and opened 1913 for a length of two miles, with extensions in progress or planned. It is of the earlier German type without steel in side walls. Waterproofing is tar paper and asphalt. Excavation was in hard clay. Electric shovels were used.

The Madrid Subway, opened in part in 1919 and extended by 1926 to a length of 9.3 miles, is roughly cross-formed in plan. The double-track arch and the rectangular section are both used. For the latter, the side-walls were first built in separate trenches, the pavement was then removed, the roof built and the pavement replaced, after which the core was excavated, and the floor and middle support placed.

The Sydney, Australia, Subway, a 2-track loop 3.6 miles long, from the Central Terminal station through the business district, with extensions, was opened in part late in 1926. Some viaduct and some open cut are included. Steel framing is used where headroom is restricted.

The Tokyo Subway was opened for a length of 1-1/2 miles late in 1927, and will connect the thickly populated districts of Uyeno and Asakusa. Other portions of the system are in progress or planned. The structure is 2-track and of the bent type.

The cost of subway work varies widely. The kind and depth of excavation, proportion of temporary decking required for traffic, size and number of sewers, pipes, etc., to be supported or changed, and the extent of underpinning to protect adjacent buildings, greatly limit methods and thereby influence cost. The accompanying table gives costs, per lineal foot of equivalent single track, of several New York subway sections put under contract between 1925 and 1928, all of a type resembling (3) of Fig. 97. The figures are based on contract prices which may be unbalanced, and preliminary estimates which may be large. They include nothing for station finish, track, equipment, engineering or inspection, and are not safe for use in estimates without fuller presentation of conditions than is possible here.

Route No. / Section No.	78 / 4	109 / 2A	109 / 4	78 / 2	78 / 1A	78 / 3	102 / 2	78 / 2A	109 / 1	107 / 11	102 / 6	102 / 1
Length, miles	.5	.6	.7	.6	.5	.6	.5	.6	.4	.8	.5	.3
Tracks, number	6	4	4	4	4	4	4	4	4	2	4	4
Average depth, ft.	25	31	30	42	39	23	25	36	38	30	36	35
Rock, per cent	45	....	....	59	51	15	36	50	....	....	46	8
Stations, number	1	1	1	1	2	1	1	2	1/2	2	1	1
Date contract	1925	1928	1928	1925	1925	1925	1925	1925	1927	1927	1925	1926
Excavation	$114	$ 84	$ 90	$170	$185	$195	$237	$255	$208	$165	$298	$293
Structure	72	110	124	102	114	108	109	122	175	153	153	193
Sewers, pipes, etc.	17	38	24	17	13	34	65	33	11	41	32	33
Underpinning	5	2	23	8	7	44	14	17	17	54	45	22
Repaving and miscellaneous	23	50	42	24	29	43	35	37	79	77	47	50
Total (per lin. ft. of track)	$231	$284	$303	$321	$348	$424	$460	$464	$490	$490	$575	$591

Section No.

78–4. St. Nicholas Ave., 132nd to 140th St.—Dry, shallow cut; all decked. (See cut and cover method).

109–2A. Smith St., Brooklyn; Bergen St. to 4th Pl.—Tracks in two tiers; partly dry; part open under private property.

109–4. 9th St., Brooklyn, 4th Ave. to Prospect Park.—Dry, shallow, earth cut; mostly decked.

78–2. Central Park West, 88th to 101st St.—Tracks in two tiers; dry; all decked.

78–1A. Central Park West, 79th to 88th St.—Tracks in two tiers; dry; all decked.

78–3. 8th Ave., 111th to 123rd St.—Dry, filled ground; underpinning elevated railway; all decked.

102–2. 8th Ave., 18th to 28th St.—Shallow cut, partly below water; extensive subsurface restoration; all decked.

78–2A. Central Park West, 101st to 111th St.—Tracks mostly in two tiers; dry; all decked.

109–1 Jay St., Brooklyn, Nassau St. to Willoughby St.—Deep cut; mostly dry; all decked.

107–11. Manhattan Ave., Boooklyn, Nassau Ave. to Freeman St.—Partly below water; much underpinning; extensive subsurface restoration; all decked.

102–6. 8th Ave., 58th to 68th St.—Rather deep cut; dry; expensive underpinning; all decked.

102–1. Greenwich St. and 8th Ave., Bank St. to 17th St.—Mostly soft material; partly below water; all decked.

SECTION 16

IRRIGATION AND DRAINAGE

BY

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IRRIGATION

SWAMP AND OVERFLOWED LANDS

FARM DRAINAGE

LEGAL AND ECONOMIC FEATURES

IRRIGATION

1. Fundamental Considerations

Definition. The practice of supplying water to crops by artificial means is called **irrigation.** Irrigation is essential in arid districts and may be profitable in semi-humid regions where the rainfall is occasionally insufficient to mature crops and for certain purposes in humid regions as an assurance against drouths. Irrigation water may be obtained by gravity flow by the diversion of natural streams or by pumping from surface or sub-surface supplies.

The Development of the Irrigation Resources of a country usually begins with the bottom lands lying close to the stream which provides the water supply. In the early days of irrigation, ditches were built to serve a single farm or a small group of farms. Later, community ditches were built and irrigation companies were formed which irrigated larger areas under a single system. In the United States and other countries whose irrigation resources have been extensively developed, the more accessible lands are under irrigation and the flow of streams during the irrigation period has been largely appropriated. There thus remains for development only the more inaccessible lands and those which must be supplied by the storage of flood flows and water not available during the irrigation season.

Operations of an Irrigation Project. In general, irrigation may be divided into four operations: the storage of water, the diversion of water, the conveyance of water, and the application of water to the land.

The Soils of arid districts extend to a greater depth and are commonly more fertile than those of humid localities. In the irrigated sections of the United States the soils are generally deep and of uniform texture. Hardpans are found near the surface of the ground in some localities, but they are usually dissolved by irrigation. Care should be taken to determine the crops to which an irrigated area is best adapted. This may depend upon the length of growing season as well as the soil formation. A strong growth of sage brush, bunch grass or other vegetation indicates fertile soil. Salt grass and grease wood are indicative of alkali. In general, the soils of arid districts are well supplied with lime, potash, and nitrogen, but are low in humus content.

2. Disposal of Irrigation Water

Soil Moisture occurs in three forms, usually termed hygroscopic, capillary, and gravity water. **Hygroscopic water** is the moisture which perfectly dry soil will absorb from the water vapor in the air. It either may form an extremely thin coating of the soil particles or may be held in loose chemical combination. It can not be removed without artificial heat and has no value in sustaining plant life. **Capillary water** exists as a thin coating of the soil particles. It moves from a wet soil to a drier soil in any direction. Capillary water is of chief value in supplying moisture for vegetation. **Gravity water** is the water contained by soil beyond its capacity to contain capillary water. It moves downward by gravity and is not retained by the soil, as such, if there is satisfactory drainage. After rains or if water is applied by irrigation, gravity water near the surface of the ground in passing downward may become capillary water in the lower strata.

The amount of hygroscopic water in the soil depends upon the temperature, humidity of the air, texture of the soil, and other less important factors. It varies from a minimum of about 0.5% of the weight of the soil for coarse sand to maximums of 8% and 17% respectively for silt and clay. The limit of soil to hold capillary water is usually reached when it contains 15% to 20% of its weight of moisture. Water will rise 0.5 to 2 ft. in one day and 0.7 to 4 ft. in one week by capillary action. The rate of capillary movement increases with the fineness of the soil particles. The distance which moisture may be drawn by capillary varies from about 2 ft. for sand to 5 ft. for fine silt or clay. Cultivated soils contain 30% to 50% of voids which marks the limit of their capacity to hold water.

Ultimate Destination. Irrigation water applied to land disappears in several ways: (1) **plant transpiration,** (2) **evaporation** from the ground, (3) **percolation** to strata beyond the reach of plant roots, and (4) **surface waste.** Only the water used in plant transpiration serves a useful purpose, and in successful irrigation disposal by the other three methods should be reduced to a minimum.

Plant Transpiration is the process by which growing plants obtain nutriment from the soil. Carbon, which constitutes about one-half of the dry plant, is taken by leaf action from the carbon-dioxide in the air. Plants contain 2% to 10% of mineral matter. The remainder of the plant is composed of the elements of water combined with the mineral matter and carbon. There is a continual upward current of water through the plant. The soil moisture, containing mineral matter in solution, enters the hair roots of the plant, the water being dispelled by the leaves into the air in the form of water vapor. The rate of movement of the water usually varies from 1 to 6 ft. per hour, but may be much greater than this.

The Amount of Transpiration, other conditions being the same, increases with the moisture in the soil, the amount of sunlight, and the temperature. Transpiration is practically zero for temperatures less than 42° F. The amount of water required to produce a given quantity of dry matter decreases as the humidity of the air increases. In general, for a given locality, the amount of water used in plant transpiration varies approximately as the quantity of dry substance produced. The average plant consumption and distribution of consumption on the Boise Project, Idaho, as determined by J. C. Stevens from statistical reports of the U. S. Bureau of Reclamation are given in the tables at top of page 1656.

The Amount of Evaporation from Soil depends primarily upon the available moisture. For saturated soil the evaporation is equal approximately to that from a free water surface. As the soil moisture is depleted the rate of evaporation decreases. The amount of evaporation is also affected by the character of soil. Fine soil has a greater capacity for capillary water and permits greater capillary movement and therefore affords a greater opportunity for evaporation than coarse soil. Cultivation reduces the amount of evaporation. A mulch of 6 in. will conserve 80% of the soil moisture. Other conditions being the same, the rate of evaporation increases with the wind velocity, the temperature, and the saturation deficit of the air.

Percolation. If the quantity of irrigation water applied to a tract of land is just sufficient to supply plant needs, there is no percolation to the deeper strata. With the irrigating methods ordinarily employed, however, percolation losses are 20% to 75% of the water applied. They increase with the porosity of the soil.

Return Seepage. Waters which percolate from canals and irrigated areas to underlying strata raise the ground-water level and eventually return to natural channels. Return waters may in some instances be used for irrigating other areas but they often become so heavily impregnated with mineral salts (see Art. 28) as to render them unfit to use for irrigation. In the San Joaquin valley of California large areas are irrigated by irrigation waters returned to the Stanislaus, Tuolumne, and Merced rivers. In some instances 40% or more of the water diverted is returned to streams through underground seepage.

Surface Waste is the irrigation water which runs off the surface of the ground and does not soak into the land being irrigated. It is not practicable to eliminate this waste entirely. An allowance for surface waste of 8% to 15% of the total water applied to the land is conservative. Without care in irrigating these percentages may be greatly exceeded. In general, the waste increases with the slope of the land to which the water is applied.

Average Plant Consumption, Boise Project

Crop	Acres	Yield per acre	Plant consumption	
			Depth in feet	Acre-feet
Alfalfa	39 090	3.9 tons	1.66	56 800
Barley	3 220	20.4 bu.	0.50	1 610
Clover	7 680	2.0 tons	0.84	6 430
Corn	1 410	37.9 bu.	0.97	1 370
Mixed hay	510	1.4 tons	0.62	320
Oats	3 210	25.5 bu.	0.48	1 540
Pasture	6 550	....	0.50	3 280
Wheat	25 680	21.2 bu.	0.63	16 100
Total	83 350	....	1.06	87 450

Distribution of Mean Plant Consumption

Month	Mean monthly temperature, deg. F = T	T − 42	Percentage of total	Inches
March	42	......	....	......
April	50	8	6	0.8
May	58	16	13	1.6
June	66	24	17	2.2
July	73	31	22	2.8
August	72	30	22	2.8
September	62	20	14	1.8
October	50	8	6	0.7
Total	....	137	100	12.7

3. Duty of Water

Units. The unit of volume commonly used in irrigated districts to express storage or the quantity of water required to irrigate a given area of land is the **acre-foot.** It is the amount of water required to cover an acre of land 1 ft. in depth, or 43 560 cu. ft. An **acre-inch** is 1/12 of an acre-foot. The unit used to express flowing water is the **cubic foot per second.** One **second-foot** (one cubic foot per second) flowing for 24 hours is equal to 1.9835 acre-ft. A unit of measure for flowing water that is becoming obsolete is the miners' inch.

Duty of Water refers to the relation between the quantity of water applied to the land and the area irrigated. It is commonly expressed as: (a) the area which can be served by 1 cu. ft. per sec. of water, (b) the volume in acre-feet used during one irrigation season to irrigate 1 acre, or (c) the depth of water in inches per irrigation season that is applied to the area irrigated. The term is ambiguous and could well be dispensed with, but because of its extensive use and common acceptance by engineers it must be recognized. The **net duty** of water for an irrigation project is based upon the average use per acre measured at the borders of farms. The **gross duty** is based upon the total diversion at the intake of the main canal, thus including conveyance losses in main canals and laterals. In using the term, duty of water, it should be clearly understood at what point the water is measured.

The Factors Entering into Duty of Water, exclusive of conveyance losses (see Art. 4) together with quantities of water ordinarily chargeable to each during one irrigation season, are listed below. Extreme values may be greater or less than those given.

Plant transpiration..........	0.50 acre-foot to 1.75 acre-feet
Evaporation................	0.75 acre-foot to 2.00 acre-feet
Percolation................	0.25 acre-foot to 6.00 acre-feet
Surface waste..............	0.25 acre-foot to 0.50 acre-foot

Only water used in transpiration serves a useful purpose. Losses resulting from evaporation, percolation, and surface waste should be prevented insofar as practicable. Evaporation is reduced by cultivation but cultivation is impracticable for hay and grain crops. Percolation losses result from applying too much water to the land and permitting seepage to depths beyond the reach of plant roots. In general, percolation losses increase with the coarseness of the soil. Percolation losses are reduced by irrigating only a small tract at a time and using a large head of water so that the area can be covered quickly. Surface waste can be reduced or even entirely eliminated by having the ground well prepared to receive water and by care in irrigating.

The Kind of Crop has an important bearing on the duty of water. Excepting fruits, the water required will vary with the length of the growing period. Alfalfa which, with an adequate water supply, produces successive crops during the growing season, requires more water than other crops.

Water Requirements for Different Crops

Crop	Soil	Water requirements in acre-feet per acre		
		Average maximum used	Average minimum used	Average for most economic use of water
Alfalfa (three cuttings)	Clay and sandy loam.	4.0	1.5	2.5
	Porous sandy soil....	10.0	4.0	6.0
Grain...............	Clay and sandy loam.	3.0	1.0	1.5
	Porous sandy soil....	5.0	1.5	2.5
Potatoes............	Clay and sandy loam.	3.0	1.0	1.5
	Porous sandy soil....	5.0	2.0	2.5
Sugar beets..........	Clay and sandy loam.	3.0	0.5	1.2
	Porous sandy soil....	5.0	1.2	2.0
Deciduous fruits......	Clay and sandy loam.	2.0	0.7	1.0
	Porous sandy soil....	4.0	1.5	2.0
Citrus fruits.........	Clay and sandy loam.	3.0	1.2	1.7
	Porous sandy soil....	5.0	2.0	3.0

The Times to Irrigate to secure the best results from a given water supply will be the times which come nearest to fulfilling the ideal condition that the soil shall have an adequate and uniform moisture content throughout the growing period. Since transpiration from large plants is greater than from smaller ones, and also because maximum growth is obtained late in the summer when evaporation is greatest, more water should be applied during the latter part of the growing season. After vegetation has attained its full growth and becomes well matured, but little water is required. When water is obtained directly from streams this ideal condition is not usually attainable, since the water supply, plentiful during the early part of the growing season, usually decreases rapidly as the summer advances. If the source of supply is from storage reservoirs water is available when needed. An important consideration is that the ground should be thoroughly moist at seeding time or when plant growth begins. To accomplish this, fall, winter or early spring irrigation has been practiced with excellent results. If the fall and winter rains are heavy enough to supply adequate spring moisture such irrigation is unnecessary. In localities where water is not available during the growing period, short-season crops such as grains and early fruits have been grown successfully with fall and winter irrigation only.

If the soil is moist at the time plant growth begins in the spring, the first irrigation of the crop should be deferred until the need for more water is apparent. This permits the roots to obtain a more vigorous growth. After the first irrigation successive irrigation should follow as required. As a rule frequent irrigations using comparatively small quantities of water are advisable. The usual practice, for fine soils, is to apply 3 to 5 in. of water at each irrigation. A smaller quantity of water should be used for coarse soils, if practicable, but irrigations should be more frequent.

The Number of Irrigations varies with the crop and the character of the soil. For fine soils, alfalfa should be given one or two irrigations for each cutting; potatoes, sugar beets, corn and similar crops are usually irrigated 3 to 5 times, the greater part of the water being applied from July 1 to August 15 in the northwestern states; grains are given 1 to 3 waterings near the time of seed formation; deciduous fruits require a fairly uniform distribution of moisture, with rather more at the time fruit is forming; citrus fruits require irrigation throughout the year, with the maximum requirement in the fall. Coarse soils require more frequent irrigations than the finer soils.

The Economical Use of Water has been investigated by the United States Department of Agriculture and by many of the western states. It has been found that while the use of water up to a certain amount is beneficial, beyond this amount little if any apparent increase in the productivity of the land results. It has also been found that where the supply of water is insufficient to irrigate all of the available land lying under it, there is an economical limit to the amount of water which should be applied. In other words, a greater profit from a given quantity of water may, in some cases, be obtained by spreading it over a larger area. This point is illustrated in the following example by Widstoe (Principles of Irrigation Practice, p. 337), assuming that sugar beets are worth $5 per ton and the cost of producing them, including interest on investment, is $60 per acre:

30 acre-inches spread over	Inches of water on each acre	Yield of beets per acre (tons)	Total yield of beets (tons)	Price paid for ton of beets	Gross income from beets	Cost per acre	Total cost	Net income from beets
1 acre........	30.0	21.0	21	$5	$105	$60	$60	$45
2 acres........	15.0	19.5	39	5	195	60	120	75
3 acres........	10.0	18.6	56	5	280	60	180	100
4 acres........	7.5	16.3	65	5	325	60	240	85

The above table shows that the greatest yield per acre is obtained by applying the 30 acre-inches of water to 1 acre of land, but the greatest net income for this quantity of water is obtained by spreading it over 3 acres.

4. Conveyance Losses

Seepage and Evaporation losses from canals are usually considered together under the term conveyance losses. In earth canals, seepage losses are much greater than evaporation losses. The coarser the soil the greater the seepage. If canals carry much silt, seepage may in some instances be gradually reduced by sedimentation but extensive investigations by the United States Bureau of Reclamation on many projects have failed to indicate that canals after long periods of use show any decrease in rates of seepage losses. Inseepage of ground-water, such as may occur when there are irrigated areas at higher elevations, may reduce apparent seepage losses. Conveyance losses are usually considered to vary directly with the area of the wetted surface.

Canal Linings of various types have been used to reduce seepage losses. The best type to use depends largely upon the materials available and other local conditions. Canal linings are expensive and are used most extensively

Irrigation Water Losses

Irrigation Water Lost in Canal Seepage, Delivery, and Waste in Canal Waste-ways and Spillways on Federal Reclamation Projects, 1916 to 1926

Compiled by E. B. Debler; see Engineering News-Record, January 19, 1928

* Project	Irrigated area, acres	Area commanded by canals in 1926, acres	Canals and laterals operated		Delivered to farms, acre-feet per acre	Per cent of total diversion		
			Total miles	Lined miles		Seepage loss in canals	Wasted	Delivered to farms
1	45 164	74 569	547	58	1.11	38	15	47
2	145 616	167 780	1004	37	3.60	24	......	......
3	22 535	25 056	45	11	2.36	48	6	46
4	10 139	30 350	180	......	3.61	43	22	35
5	19 406	32 540	232	......	1.26	39	30	31
6	6 441	16 888	96	......	6.41	34	15	51
7	43 325	64 983	240	2	1.24	46	9	45
8	16 900	16 276	196	......	1.28	45	20	35
9	16 793	73 252	275	......	.65	36	19	45
10	44 945	48 172	275	......	2.54	39	3	58
11	38 808	65 277	319	......	2.88	41	14	45
12	107 694	253 094	1154	......	2.23	43	8	49
13	6 210	7 300	75	39	2.55	25	......	75
14	14 554	20 754	135	89	3.17	27	9	64
15	90 387	150 000	412	10	3.38	34	37	29
16	7 963	20 063	166	......	2.19	42	21	37
17	32 380	41 923	279	4	2.38	38	7	55
18	7 650	13 902	99	......	1.54	36	26	38
19	9 867	41 975	190	......	1.28	31	22	47
20	10 970	24 587	173	157	5.02	32	18	50
21	61 178	95 202	470	11	5.76	13	10	77
22	91 726	102 464	602	125	3.29	23	7	70
23	27 607	32 000	335	86	2.51	24	2	74
24	51 950	64 865	336	......	3.01	15	40 +	......

* The following are names of projects and numbers which correspond to the numbers in column 1, together with soil classification for canals: (1) Belle Fourche, clay and sandy soil; (2) Boise, volcanic ash and sandy loam; (3) Carlsbad, sandy loam and gravel; (4) Grand Valley, pervious—puddle lined; (5) Huntley; (6) King Hill, volcanic ash and sandy soil; (7) Klamath, sandy loam; (8) Lower Yellowstone; (9) Milk River, river bottom; (10) Minidoka—South Side Pumping Div., sandy loam; (11) Newlands, 60% sandy, 40% clay; (12) North Platte, sandy loam; (13) Okanogan, gravelly soil; (14) Orland, clay and loam; (15) Rio Grande, silt; (16) Shoshone, Frannie Div., sand and clay loam; (17) Shoshone, Garland Div., gravel and 'oam; (18) Sun River, Ft. Shaw Div.; (19) Sun River, Greenfields Div., gravel-clay; (20) Umatilla, sandy loam; (21) Uncompahgre, 1/2 adobe-shale, 1/2 sand and gravel; (22) Yakima, Sunnyside Div., sand and volcanic ash; (23) Yakima, Tieton Div., sandy loam and gravel; (24) Yuma, fine sandy loam.

where water is valuable. In California many canal systems are lined throughout, but in other localities only those portions of canals are lined in which seepage losses are greatest. Canal linings are beneficial in reducing operation costs and also in reducing friction losses.

Concrete Linings range in thickness from 1/2 to 5 in. Fine aggregate should be used for the thinner linings. Proportions of cement to aggregate vary from about 1 to 8 for the thicker linings to 1 to 4 for the thinner linings. Concrete linings should reduce seepage 75 to 95%, the thicker linings being more effective. The slopes of banks should not be less than the angle of repose of the material in which the canal is situated. Canal linings of cement mortar 1/2 to 1 in. thick in western United States have given excellent service. By using a comparatively dry mixture, concrete can be placed without forms on banks having a slope of 1 to 1. For steeper slopes, forms are necessary.

Losses Per Day in Canals of the United States Bureau of Reclamation

Depths in feet per square foot of wetted area

Soil	Number of observations	Maximum	Minimum	Mean
Gravel and sand	2	3.62	1.44	2.53
Gravel	38	7.05	.22	1.70
Gravel and rock meal	9	2.84	.88	1.56
Sand	5	1.79	.81	1.27
Rock meal	18	1.73	.38	1.24
Sand and loose rock	7	.86	.62	.72
Sandy loam	60	3.70	.01	1.01
Loam	5	1.44	.84	1.07
Sand and volcanic ash	21	1.66	.22	.90
Gravel and volcanic ash	5	1.54	.38	.88
Volcanic ash	3	1.16	.56	.82
Brule clay	4	1.11	.72	.91
Clay and gravel	18	1.34	.16	.79
Clay and sand	12	1.43	.50	.76
Adobe	9	.93	.46	.65
Hardpan and loose rock	2	1.12	.24	.68
Clay and shale	7	1.02	.30	.59
Volcanic ash, clay, and hardpan	4	.77	.46	.60
Cemented gravel and sandy loam	2	.47	.40	.44
Caliche	2	.49	.38	.43
Clay	12	.77	.11	.34
Clay loam	12	1.12	.06	.30
Gumbo and sandy loam	11	.77	.12	.29
Concrete lining	9	1.07	.06	.33
All soils	277	1.66	.43	.87

Approximate Costs of Concrete Canal Linings in Western United States

Thickness of lining, inches	Cost, cents per sq. ft.	Thickness of lining, inches	Cost, cents per sq. ft.	Thickness of lining, inches	Cost, cents per sq. ft.
1/2	3.1 to 4.4	1-1/2	5.3 to 7.5	3	8.3 to 12.1
3/4	3.8 to 5.1	2	6.3 to 9.0	4	10.5 to 15.4
1	4.3 to 6.0	2-1/2	7.3 to 10.8	5	12.6 to 18.5

Gunite Lining, 1 to 1-1/2 in. thick, has proved to be very effective in preventing seepage. Wire-mesh reinforcing is usually placed near the center of the lining. Satisfactory proportions of cement to aggregate are 1 to 4 or 5. All stone in the aggregate should pass a 1/2-in. screen. The cost of such lining, including reinforcing but not including grading, varies from 7 to 11 cents per sq. ft.

Oil Linings are made by sprinkling 1 to 3 gal. of crude oil per square yard on the sides and bottom of the canal. Several light applications are used, and in some cases the surface is worked with a rake or harrow after each application. Good results have been obtained, however, by leaving the surface undisturbed after applying the oil. The oiled surface is generally rolled or otherwise compacted. Experiments indicate that oil linings when new will reduce seepage losses in canals 50 to 75%. They deteriorate, however, and their effectiveness diminishes with age. The average cost of preparing oil linings in the United States, not including the cost of the oil, has been about 1.5 cents per gallon of oil used.

Clay Puddle Linings have been used in localities where suitable material is available within a reasonable distance. The clay is deposited in a layer 3 to 6 in. thick along the bottom and sides of the canal, after which water is turned into the canal and the material is puddled by harrows or the tramping of animals. A good clay puddle lining will reduce the seepage loss 60 to 80%. It does not deteriorate, but becomes more effective with age. Fine surface soil thrown into a canal containing water and thoroughly stirred up by teams dragging heavy chains or otherwise agitated has in some cases been quite effective in reducing seepage.

Hydraulic Sluicing has been used to stop bad leaks in canals. Fine material for sluicing must be available. The sluiced material should be deposited directly on the porous area so as to cover it with a thin blanket. Very bad leaks in shale, gravel, and seamy rock have been successfully treated by this process.

5. Water Supply

Total Water Supply. One of the first considerations in the investigation of an irrigation project is the determination of the amount of water available for the project. The discharge of the stream or streams forming the source of supply should be carefully investigated. A knowledge of the distribution of the flow as well as the total annual discharge for a period of years is essential. The best data on which to base an estimate of future discharge are continuous discharge records covering a period of years. Such records, for many streams in the United States, are published in the Water Supply and Irrigation Papers of the U. S. Geological Survey. They may also in some instances be obtained from state engineers' offices or from private sources. On many streams, however, available discharge records cover only a comparatively short period or they may be entirely lacking. It also may be found that discharge measurements on a stream have been made at a place quite remote from the proposed point of diversion. Precipitation records are valuable for supplementing incomplete stream discharge data. Such records, published by the U. S. Weather Bureau, are available at one or more points for practically every drainage basin in the United States.

The safest data for estimating future discharges are actual discharge measurements, near the place of diversion, covering a period of not less than ten years. A record for this period will probably include ordinary high- and low-water stages of the stream, and show the general distribution of flow throughout the year. If a project is under investigation and discharge records near the place of diversion are not available, a gaging station should be established at the first opportunity. If there is any doubt as to the adequacy of the water supply for a proposed project, construction work should be delayed until supplementary data sufficient to provide a satisfactory estimate of discharge have been obtained. In general, at least two years of continuous discharge records should be available near the place of diversion and these should be supplemented with such additional data that a reasonably reliable record for a period of at least ten years may be estimated.

Prior Rights to Water. In estimating the water available for irrigation from a given stream, all prior rights should be investigated. In western United States the streams are controlled by the states and the laws regulating the use of the streams are not uniform and are continually being changed by new statutes and court decisions. The state laws governing the stream should therefore be understood. The waters of some streams have been adjudicated by the states and in such cases records of prior rights are obtainable at state engineers' offices. In many instances it will be found that the records of water rights are in such a condition that an examination of each canal diverting from the stream must be made to determine the amount of water actually used and the area of land under it that is entitled to water.

Water Supply Available. The difference between the total water supply and the quantity of water represented by prior appropriations gives the water supply available. The estimate of available water supply is usually prepared by months by deducting from the discharges for each month of the period covered by measured or estimated discharges, the water represented by prior rights for corresponding months. These differences give the estimated amount of water that would have been available for each month during the period considered and if the period is long enough (preferably at least ten years) they may be taken to represent the available future supply.

The Water Supply Required for a given area will be that necessary to provide for the irrigation of crops and in addition all seepage and evaporation losses in canals and reservoirs. (Arts. 3 and 4.) An estimate of the water supply required should be prepared for each month by adding the estimated seepage, and evaporation losses. By comparing the water available with the water required, the deficiency for any month of the period covered by records may be determined. If storage is to be provided, such deficiencies will give the amount of storage required. About 20% of an irrigable area will be occupied by roads and buildings or for other reasons will not be irrigated. This area should be deducted for purposes of estimating water requirements for an irrigation project.

6. Conduits

The Conveyance System which delivered water from the place of diversion to farm laterals may include canals, flumes, pipes, and tunnels as well as many auxiliary structures such as drops, wasteways, and turnouts.

Canals (See Sect. 15, Art. 24). The canal section of a given area which will have the greatest capacity, other things being equal, is the one having the greatest hydraulic radius. Such sections are usually not practicable for earth canals. As commonly constructed (Figs. 1 and 2), $b = 3d$ to $4d$, $a =$

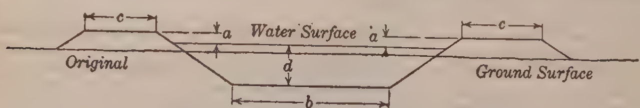


Fig. 1. Canal Section for Level Ground

$0.4d$ to $0.6d$, $c = 1d$ to $2d$. Canal banks in level ground and on the downhill side of sloping ground are usually given a slope of 1 on 2 or 1 on 1-1/2. The uphill slope may be made as steep as 1 on 1 to reduce excavation. There should be an allowance of 10% for waste and shrinkage in embankments.

Velocities in Earth Canals. Canals should ordinarily be designed to carry water at the highest velocity that can be maintained without erosion. This is the most economical velocity since it gives the smallest canal section and therefore the lowest construction cost practicable, and permits a minimum deposition of silt. The velocity at which erosion begins varies with the size of soil particles and the cohesive properties of the material forming the canal surfaces. Colloidal matter either in the material in which the canal is situated or in the water carried by the canal tends to cement soil particles in such a way as to resist erosion.

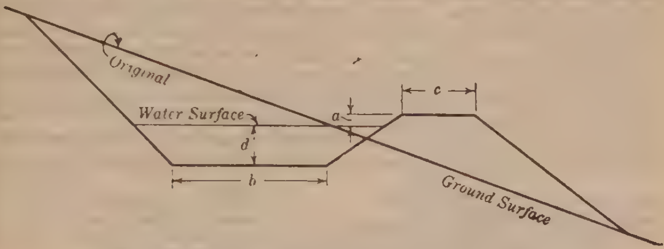


Fig. 2. Canal Section for Sloping Ground

The Age of a Canal has an important bearing on the velocity at which erosion occurs. The erosive velocity may be twice as high in a canal after several years of use as it was when the canal was new. Seasoned canal beds contain particles of different sizes and when the interstices are filled by the smaller particles the mass becomes more dense and stable, and less subject to erosion.

Permissible Canal Velocities after Aging

Recommended in 1926 by Special Committee on Irrigation Research, American Society of Civil Engineers

Velocities in feet per second

Original material excavated	Clear water, no detritus	Water transporting colloidal silts	Water transporting non-colloidal silts, sands, gravels, or rock fragments
Fine sand, non-colloidal	1.50	2.50	1.50
Sandy loam, non-colloidal	1.75	2.50	2.00
Silt loam, non-colloidal	2.00	3.00	2.00
Alluvial silts, non-colloidal	2.00	3.50	2.00
Ordinary firm loam	2.50	3.50	2.25
Volcanic ash	2.50	3.50	2.00
Fine gravel	2.50	5.00	3.75
Stiff clay, very colloidal	3.75	5.00	3.00
Graded, loam to cobbles, non-colloidal	3.75	5.00	5.00
Alluvial silts, colloidal	3.75	5.00	3.00
Graded, silt to cobbles, collodial	4.00	5.50	5.00
Coarse gravel, non-colloidal	4.00	6.00	6.50
Cobbles and shingles	5.00	5.50	6.50
Shales and hardpans	6.00	6.00	5.00

Canals in Rock are excavated with steep side slopes, commonly 1 on 1/4, and with bottom widths not more than two times the depth. They are also sometimes built with a semicircular cross-section. Canals excavated in rock are usually lined with concrete. For canals in hardpan or other indurated material the conditions are intermediate between those for earth and rock.

Flumes (see Sect. 15, Art. 15) are used to convey water across streams, ravines or other depressions of the ground, and also around hillsides to avoid difficult excavation. They are built of wood, steel and concrete, or of combinations of these materials. The substructure may consist of piers or posts of concrete or of wooden or steel posts or trestles. Trestles should be braced longitudinally and posts should be well tied together with both transverse and longitudinal bracing. Concrete bases or mud sills may be used for supports. Flume channels usually have rectangular cross-sections when constructed of wood or concrete and semicircular cross-sections when constructed of steel. Fig. 3 shows a steel flume with wooden substructure and concrete pedestals. Short flumes have been built more than 100 ft. high, but as a rule flumes are

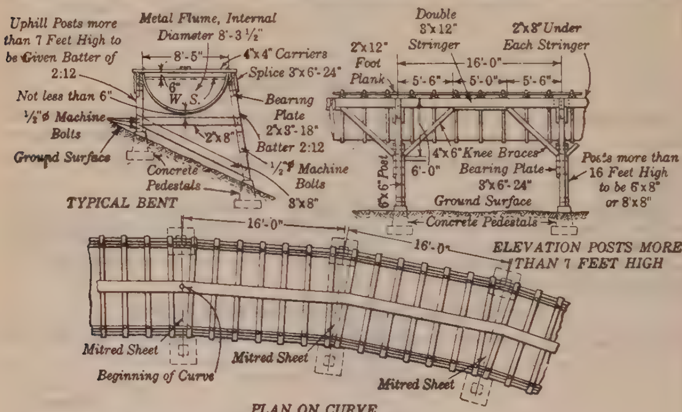


Fig. 3. Steel Flume with Wooden Substructure

better adapted to conditions where the height is not much over 20 ft. For greater heights, especially for long depressions, pressure pipes will usually be more economical.

Pressure Pipes (see Sect. 14, Art. 21 and Sect. 15, Arts. 13 and 14) for irrigation purposes are usually built of wood, steel, or reinforced concrete. For small quantities of water, if the pressure head does not exceed 20 ft., vitrified sewer pipe and plain concrete pipe have proved satisfactory. Pressure pipes are used to carry water across depressions or around steep hillsides; small pressure pipes may be used as laterals for distribution systems.

Reinforced-concrete pipes are best adapted to pressure heads of less than 100 ft. Wood-stave pipes have been used for heads of 300 ft. or more, though 250 ft. is probably very near to the economical limit of such construction. For the higher heads, steel is used exclusively. The average life of steel pipe is greater than that of wooden pipe, but the life of each is dependent upon the care with which it is maintained. (Art. 32.) Under ordinary conditions of use, concrete pipe should last indefinitely. Wooden pipe being smoother than concrete or steel has a greater capacity for a given diameter. Deposits will form on the inner surface of a steel pipe, which may reduce its capacity as much as 25% after a few years of service. In determining the most economical pipe for a given locality, where continuous service is required, the first cost, life, and carrying capacity of each kind of pipe should be considered. Fig. 4 is an example of a wood-stave pipe, as manufactured by the Washington Pipe and Foundry Company.

Tunnels (see Sect. 15, Arts. 32 to 41) may be constructed across points to shorten canal lines, or through divides to divert water to a different drainage basin. They are also built in the sides of rocky canyons where canals, flumes, or pressure pipes would be expensive to maintain. In difficult side-hill location, the water may be carried successively in canals, flumes, tunnels, or pipes, following each other in any order.

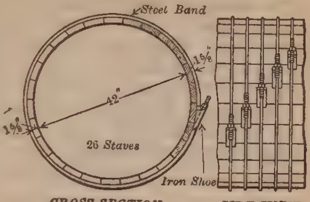


Fig. 4. Wood-stave Pipe

Tunnels for irrigation purposes are usually lined with concrete, though short tunnels in firm rock are sometimes left unlined. Concrete linings serve the double purpose of preventing caving of tunnels and of reducing resistance to the movement of water, thus increasing their capacities. The minimum dimension of the cross-section of a tunnel, except for the very short tunnels, should not be less than 6 ft. and preferably 7 ft. in order to allow free movement of the workmen. Beyond this limit they should be kept as small as practicable in order to reduce the cost. Velocities of 10 to 12 ft. per second in concrete-lined tunnels are permissible. Fig. 5 shows a cross-section of the Strawberry Tunnel, Utah.

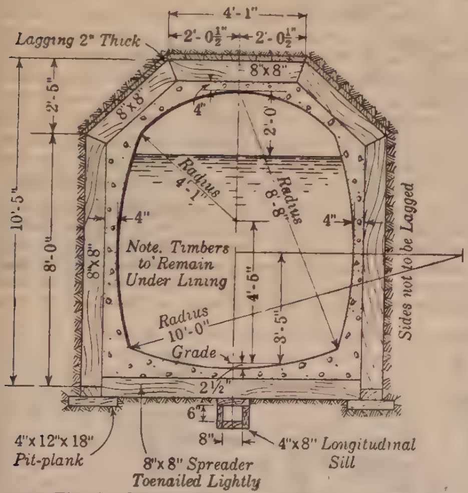


Fig. 5. Strawberry Tunnel, Shale Section

The Grade and elevation of water surface at the controlling points of a canal system should be thoroughly investigated. Special consideration is required wherever a change in the velocity of the water occurs. When water from a canal enters a pipe or culvert, changing to a higher velocity, allowance should be made for velocity head and head lost at entrance. A similar allowance should be made where a canal is changed to a smaller section. In this case the difference in elevation of water surfaces above and below the change in section represents head allowance for entrance conditions and change of velocity. If change from a higher to a lower velocity is made gradually some of the velocity head will be converted into static head.

In designing a canal or other structure for the passage of water the engineer should make liberal estimates of capacities. Uncertainties as to the proper values of coefficients necessarily exist and in general it will be safer to design a structure with a capacity slightly greater rather than less that required.

7. Drops and Chutes

The Grade in Earth Canals should be ordinarily that which will give the greatest velocity that may be carried without eroding the sides or bottom of the channel. When, therefore, it is desired to drop an earth canal to a lower elevation, the excess grade must be taken up by a structure designed for the purpose. The function of such a structure is to maintain the normal velocity of the water in the upper canal and deliver it at the normal velocity to the canal at the lower elevation. Two types of structures, commonly called Drops and Chutes, have been used for this purpose.

Drops are structures which concentrate the fall at one or more points. They usually are built of concrete or wood, the latter being satisfactory for temporary construction. The essential parts of a drop are the brest-wall, which extends across the channel and connects the upper and lower canals, the water cushion or platform on which the water falls, the various wing walls which connect the structure to the sides of the canal, and the canal linings, which prevent erosion immediately above and below the structure. The brest-wall extends from the elevation of the highest water surface in the upper canal to the bottom of the lowest part of the structure. Sufficient opening should be left in the portion of the brest-wall which extends above the bottom of the upper canal to discharge just the required volume of water while maintaining the normal depth of water in the canal. The opening in the brest-wall may be either a weir or one or more notches. Trapezoidal notches, having the narrower of the two parallel sides flush with the bottom of the canal, are frequently used. Such notches may have semicircular lips projecting from the base which cause the water to spread out and reduce the impact of the fall.

The notched form of fall crest with water cushion below has been used with excellent results. It appears to reduce the difficulties from erosion above and below the fall. Steel rails placed under the fall have not proved very satisfactory. Cutoff walls should be built into the backfilling, which should be moistened and thoroughly tamped to prevent the water from breaking through. Reinforced concrete has largely replaced other materials for the construction of drops. Fig. 6 shows a section of a 10-ft. drop

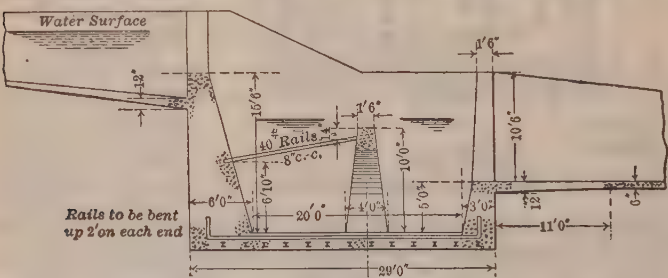


Fig. 6. Drop with Grating and Water Cushion

with grating and water cushion at Uncompahgre, Colorado. Ordinarily, drops are not built more than about 15 ft. high. For greater heights two or more drops in a series may be used. The amount of excavation for the canal leading from the drop increases as the slope of the ground decreases. In gently sloping ground a series of low drops may therefore be more economical than one higher structure.

Chutes are either lined canals or flumes, constructed on steep grades which connect two canals of different elevations. In addition to the conduit a chute must have a structure at the entrance to connect it to the upper canal and a structure at the exit to retard the velocity of the water before discharging it into the lower canal. The opening at the entrance to the chute should be just large enough to discharge the required volume of water while maintaining the normal depth of water in the canal. The conduit usually discharges into a water cushion in which baffles may be built to help break up the high velocity of the water. Canals should be paved for a short distance above and below the chute. The conduit may be of any length, and chutes several hundred feet long are not uncommon. Chutes with short conduits built on steep inclines are used in place of drops Fig. 7 shows a reinforced concrete chute with a short conduit, of the Okanogan project, Washington.

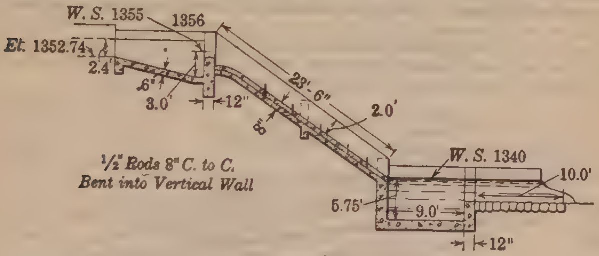


Fig. 7. Reinforced Concrete Chute

The velocity of water in a chute will accelerate until it is just great enough to overcome the frictional resistance. To determine the required dimensions of the conduit it should be divided into short reaches and the hydraulic computations made for each reach. The rate of reduction in area of the cross-section of the conduit will be more rapid at the upper end.

8. Minor Structures

Canal Crossings for Natural Waterways. At every place where a canal crosses a natural drainage channel, whether such channel carries water continuously or at infrequent intervals, provision must be made for conveying the natural drainage to the lower side of the canal. This may be done in either of the following ways: (1) by carrying the water under the canal; (2) by carrying the water over the canal; (3) in the case of small flows which may be expected only at rare intervals, by allowing the water to flow into the canal with provisions for it to escape over the opposite embankment; (4) by intercepting ditches above the canal which collect the drainage from small channels and convey it to some other channel for which a crossing is provided. Before designing a structure for this purpose a careful estimate of the maximum flow that the channel may be expected to carry should be made, and a liberal waterway should be provided in all cases.

If the bottom of the canal is high enough natural drainage waters in small quantities may be carried under the canal in culverts or pipes. For crossing streams where large flows may be expected, it will generally be safer to convey the canal water over the stream in a flume, preferably supported by a truss, or under it in a pressure pipe. Short flumes or overchutes are sometimes used to carry small quantities of water over canals. Where drainage water is allowed to enter a canal a depression should be left in the upper embankment which, together with a short section of the canal, is usually lined with concrete. The objection to this method is that sediment is washed into the canal.

Wasteways are structures which may, in an emergency, divert the entire flow of a canal into a natural drainage channel. They are intended primarily as a safeguard to the canal and should preferably be located above the weaker portions where breaks are most likely to occur. In case of a break or other trouble, if there is a wasteway above, the water may be turned out of the canal more promptly than if the headgate alone must be relied upon for this purpose and the resulting damage will be less.

The essential features of a wasteway are a system of gates in the lower embankment of the canal and provision for diverting the water through them. Many different types of wasteways have been built. Usually the gate sills are placed at an elevation lower than the bottom of the canal in order to increase the head on the gates. A low submerged weir is sometimes placed across the canal below the gates to help divert the water through them. A similar weir upstream from the gates will retard the velocity of the water in the canal when the gates are open and reduce the erosion above the structure. If built in earth, the canal for a short distance above the wasteway and the channel leading from it should be concrete lined or paved. Fig. 8 shows a wasteway of the Umatilla project, Oregon, with a tunnel outlet.

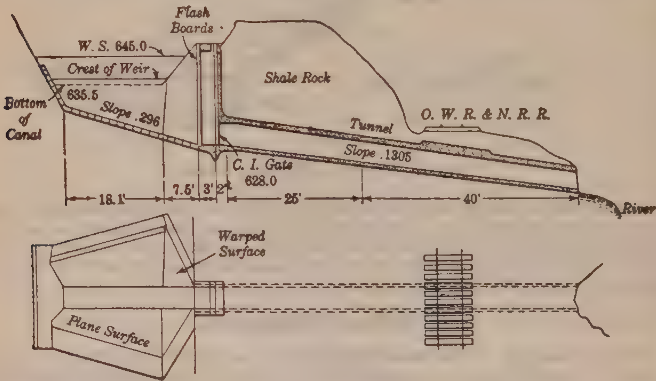


Fig. 8. Wasteway from Canal

Spillways are constructed by depressing the lower embankment of a canal to the elevation of the highest water surface which the canal is designed to carry. A spillway in an earth canal should have its crest and both slopes of the embankment lined with concrete. Spillways protect a canal against overflow, when drainage water enters it, when the water is backed up by an obstruction in the canal, or when a sudden rise in the stream from which the diversion is made allows too great a volume of water to enter the headgates.

Bridges must be provided where a canal is crossed by road or railway. Highway bridges may be built of wood or steel, or concrete arches may be used. Wooden bridges resting on concrete piers and abutments are in very common use. For earth canals, where the velocity of the water is low, piers or posts in the channel are not objectionable except that they create a slight tendency to erosion in their vicinity.

At Railway Crossings if the roadbed is too low to allow clearance for the water under a bridge, short pressure conduits are used. Fig. 9 shows a crossing of the feed canal of the Umatilla Project under the O. W. R. & N. Railway. To secure minimum loss of head in changing from one cross-section to another the transition should be made gradually and all corners should be rounded.

Where large changes in velocity are involved, the practice of the United States Bureau of Reclamation is to make the length of transition structures (particularly outlets) such that a straight line joining the flow lines at the two ends will make an angle of about 12-1 2 degrees with its axis; and to so design the structure that the computed water-surface profile through the transition shall be a smooth, continuous curve, approximately tangent to the water-surface curves in the channels above and below.

See: Julian Hinds, The Hydraulic Design of Flume and Siphon Transitions, Proc. Am. Soc. C. E., October, 1927.

Sand Gates and Sand Traps. Streams used for irrigation frequently carry large quantities of silt, especially during flood stages. Diversion works on such streams may be designed to exclude a portion of the silt, but some of it

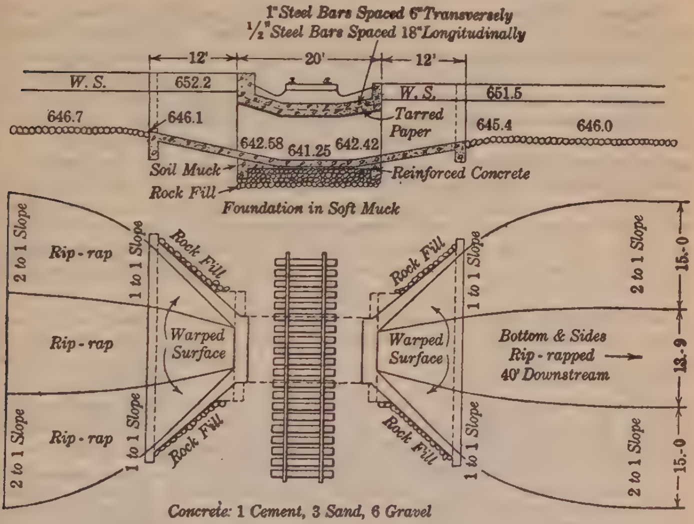


Fig. 9. Canal Crossing under Railroad

will usually enter the canal. The finest sediment is carried in suspension through the canals to the land being irrigated whereas the coarser material is rolled along the bottom, finally depositing in the canal and reducing its capacity. The water in canals acting as feeders to reservoirs should be made as free from silt as practicable in order to reduce to a minimum the deposition of silt in the reservoir.

Different types of structures have been designed to remove the coarser material carried by canals. In general such structures consist of an enlarged canal section which terminates in a low weir placed across the channel or a basin having its bottom depressed below the subgrade of the canal, with sluice gates at the lower end. Owing to the retarded velocity of the water at this place the coarser sediment will move closer to the bottom or be deposited and a portion of it may be scoured out or allowed to pass out through the sluice gates. Frequently sand traps are built a short distance below headgates to remove the sediment as soon as possible after it enters the canal. In such cases, when there is more water in the river than is required for the canal, the sluice gates may be left open allowing a continuous discharge through them. If there is no surplus water, the gates are opened only occasionally for sluicing out the sediment which has been deposited. Sand traps built on this plan may also serve as wasteways, or regulating works.

Telephone Systems are essential for the proper operation of large irrigation projects. Telephone booths should be placed at intervals along main canals and at other critical points and should be connected to the operator's house at the headgate. This enables the water to be shut off promptly in case of a break in the canal or other trouble and also assists in the economical distribution and delivery of water.

9. Diversion Works

Purpose. Wherever water is diverted from a natural stream into an artificial conduit some form of diversion works is necessary. This usually consists of a diversion dam or weir, which controls the elevation of the water surface in the stream, and a headgate, just above the diversion dam, which regulates the amount of water allowed to enter the canal. Sluice gates through the dam and adjacent to the headgate or other devices to prevent the entrance of silt into the canal may be provided. Structures, called *regulating works*, are sometimes constructed a short distance below headgates for the purpose of diverting surplus water back into the stream. With such a structure the amount of water entering the canal may be more effectively regulated than when the headgate alone is relied upon for the purpose. A sand gate is frequently combined with the regulating works. The laws of most states require that fish ladders must be constructed in dams across natural streams. In some cases logways must also be provided.

Low Diversion Dams. In general, when the diversion is made at a place where the stream lies in a flat sedimentary bottom, the natural regimen of the stream should be changed as little as possible. Under such conditions a very low dam or weir is preferable. If the stream has a flood plain subject to inundation, the diversion dam should extend across the natural channel of the stream and earth embankments several feet higher than the highest flood stage should connect the structure to the higher land on either side of the stream.

Low diversion dams are commonly built of concrete or wood or a combination of the two. Concrete dams are usually constructed on rock formations. For earth or gravel foundations, a substructure consisting of a rock and timber grillage or bearing piles tied together with walings and caps covered with a deck of heavy planking, will be satisfactory provided it is built low enough to be continually submerged. On this substructure either a wooden or concrete weir may be built. Fig. 10 shows a plan of the diversion works and a cross-section of diversion weir of the Umatilla project, Oregon.

High Diversion Dams may be built where a suitable site is available. Any type of dam may be used for this purpose, the problem being similar to that encountered in constructing a dam for other purposes. High diversion dams may be built in rock canyons to avoid expensive canal construction, the proper height being that which will result in the lowest combined cost of canal and dam.

Between the highest and lowest types of diversion weirs any intermediate height may be used. Each diversion presents special features and the height and type of dam must be chosen to conform to them. Collapsible weirs may be used when it is required to reduce the flood elevation of the water above the dam. Temporary brush and rock dams are sometimes used for diverting water into small canals.

Headgates control the entrance of water into the canal. They are preferably built on the line of the bank of the stream at right angles to the diversion dam. They should be joined to the diversion weir so as to form part of the same structure with it. Headgates usually consist of a series of gates separated from each other by piers. The piers rest upon a floor, of the same elevation as the bottom of the canal, which should be of sufficient thickness to give the required strength and stability. Cutoff walls should be provided to prevent erosion and excessive seepage beneath and around the structure. The

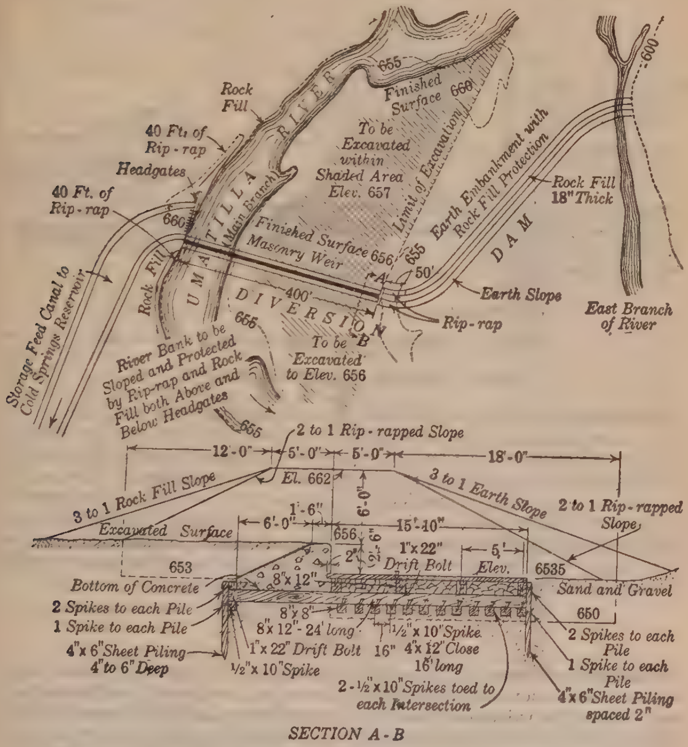


Fig. 10. Low Diversion Weir with Earth and Gravel Foundation

width of gate opening will be the distance between piers and the height of opening is regulated by curtain walls between the piers. A walk is usually built across the top of the structure and hoisting devices to operate the gates rest upon the walk. Gates should preferably be of such a size that they can be operated by one man. The total gate opening should be sufficient to discharge the maximum capacity of the canal in the contingency that one gate cannot be operated.

Headgates are commonly built of concrete, but wood may be used for temporary construction. Gates may be made of cast iron, steel, or wood. They are usually set in steel or iron guides embedded in the concrete. The bottom and sides of an earth canal should be protected by riprap or concrete lining for some distance below the headgate to prevent erosion. Headgates diverting from streams which carry drift during flood stages should have the noses of piers protected by fenders to prevent clogging of the gate openings. Fig. 11 is a cross-section of the headgate shown in Fig. 10.

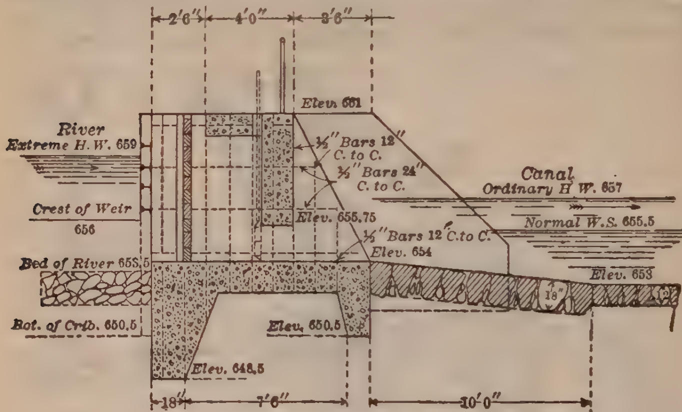


Fig. 11. Headgate for Low Diversion Weir

Sluiceways are openings in a diversion weir, usually adjacent to the headgate. They may consist of open panels in the weir or gates near the base of the weir. The latter are especially adapted to the higher dams. Sluiceways are necessary only on streams that carry considerable sediment. They serve to keep an open channel immediately in front of the headgates and to prevent the heavier sediment from entering the canal. They may also be used to regulate the elevation of the water surface above the dam. Sluiceways must be kept closed when it is desired to divert all of the water in the stream, but with a surplus of water they may be left partially or entirely open.

10. Storage

Necessity for Storage. Storage is essential to the complete utilization of a stream for irrigation purposes. Only those waters are directly available which flow during the irrigation period and are not in excess of the immediate requirements of the area which they serve. Without storage all water in excess of that which is directly available must be wasted. On many streams, further development for irrigation purposes is restricted to the use of stored waters.

Storage Reservoirs. The possibility of economical storage is dependent upon the availability of suitable reservoir sites. Natural lakes or basins may be used for storage when a suitable dam site exists at the outlet. A reservoir may be located on a stream from which water is diverted, or on a tributary of this stream. The water supply from a tributary may not be sufficient to fill a reservoir located upon it, in which case the flow may be supplemented by a feeder canal diverting from some other stream. If conditions are favorable water may be taken through a canal or tunnel from an adjoining watershed.

An investigation of the feasibility of storage for a stream should include a complete reconnoissance of the drainage basin. Each possible site should be first roughly investigated to determine approximate data relative to capacity, character of dam site, geological formation of the area to be flooded, availability of construction materials, value of rights of way and other information relative to the cost and value of storage. From these data the sites appearing to possess the greatest merit may be selected and a more thorough investigation may be made of them. Topographic maps of reservoir sites are indispensable in computing capacities.

Seepage and Evaporation from Reservoirs. One of the first considerations in investigating the feasibility of a proposed reservoir should be to make a careful study of the geological formation of the area to be flooded with a view to estimating possible seepage losses. Many instances can be cited where, after construction, reservoirs have proved worthless because of their inability to hold water. Such a condition results in great financial loss and humiliation to the responsible parties. The problem requires the mature judgment of the most experienced engineer. No rules can be laid down which will be safe guides in all cases. In general, it may be stated that seepage losses from basins lying in rock (not volcanic rock) or clay will not be excessive. Reservoirs built in formations of fine loam or volcanic soil have generally given satisfaction, although seepage losses from such reservoirs have been greater than from those lying in rock or clay. If any portion of the banks or bottom of the reservoir is formed of sand or gravel even though it may be overlain with a blanket of fine soil, and such sand or gravel might form an underground channel leading around or under the impounding dam or to some other drainage basin, there is grave danger of excessive seepage loss. On account of the large areas involved, efforts to reduce seepage in reservoirs by clay puddle or other methods have usually been unsatisfactory. If the water flowing into the reservoir carries much sediment, there will be a tendency for the reservoir to silt up and reduce seepage losses. Seepage losses from reservoirs in rock or clay formations should not be more than 5% of the inflow, and in fine loam or other soil probably not more than 10 or 20%. In coarse sand or gravel they may be anything up to the entire inflow. In determining the net supply of storage water evaporation losses should be deducted. These may be obtained from tables of evaporation losses from free water surfaces. (See Sect. 3, Art. 19.) The apparent capacity of a reservoir may be considerably increased by **bank storage.** (See Sect. 13, Art. 43.)

The following rules of caution are given by A. P. Davis (Engineering News-Record, April 4, 1918, p. 665):

1. Avoid reservoirs adjacent to gypsum deposits and to limestone deposits which show evidence of caves. 2. Examine critically reservoirs in volcanic rock, as a few have failed in such locations. Coarse-grained sandstone seems to be an object of suspicion and should be critically examined. 3. Natural depressions are treacherous and should be examined with care, and if they are near deep canyons or underlain with coarse material where water might readily escape, no superficial tightness will avail to make them effective.

Storage Problems commonly encountered in irrigation work are the determination of reservoir capacity to supply a given quantity of water at specified rates of use, and determination of the amount of water available for irrigation from a reservoir of given capacity. Where a reservoir receives all or a part of its water through a feeder canal it will also be necessary to determine the capacity of canal required to supply the storage. The solution of such problems may be readily accomplished by applying the principles of the mass diagram. (See Sect. 13, Art. 43.) In all cases irrigation water will be drawn from the reservoir at a variable rate and the use line will be curved.

11. Distribution Systems

The Function of a distribution system is to convey water from the main canal to each parcel of land to be irrigated. Water is usually diverted first into main distributing canals, then into main laterials and sub-laterals, and finally into the farmers' ditches, from which it is applied directly to the land. Because it is desirable to have as few openings as possible in the banks of main canals in order to reduce the danger from washouts, farm ditches should divert from laterals and sub-laterals rather than from main canals. The distribution is usually made through a system of earth canals, but where water has a high intrinsic value, or where seepage losses would otherwise be excessive, water may be delivered in flumes, pipes, or lined canals.

Location. Water should be drawn at proper intervals from main canals into moderate sizes branches so located as to command the greatest area and serve the land in the most direct manner practicable. The distribution is most economically affected when the main distributing canals are located along the tops of ridges so that they can supply water to laterals on either side. The laterals likewise should conform to the dividing lines between water-courses. It may not be found practicable, however, to conform rigidly to this plan. On flat areas distributing canals may be run along or parallel to property lines. Fig. 12 illustrates distributing canals and laterals diverting from a main canal.

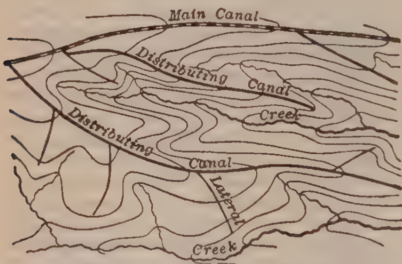


Fig. 12. Distributing System.

A Topographical Map of the area to be irrigated, with a contour interval not more than 5 ft., should be prepared before attempting the design of a distribution system. In addition to topographical features, this map should show non-irrigable lands and other areas to be excluded from the project. Special features, such as vegetation, property lines, improvements and soil formation, are in some instances required to be shown.

Design of Distribution System. In general, the entire distribution system should be planned, and the scheme for irrigating each part of the project should be worked out before any construction work is begun. This should be done even though the construction for a portion of the system is to be deferred for some time. As the plan is developed the position of all canals, flumes, pipes, and other structures should be projected on the topographic map, and the area to be irrigated under each should be determined.

Capacities of Canals should be based upon the water requirements for the period when the greatest amount of irrigating will be done. It is customary to determine the number of acres that will be served by 1 cu. ft. per sec. during the period of maximum requirements. The required capacity of main canals and distributaries may vary from 1 cu. ft. per sec. for each 120 to 150 acres irrigated, for fruit lands where the water supply is fairly uniform and maximum economy in the use of water is practiced, to 1 cu. ft. per sec. for each 50 acres irrigated, where the supply of water is available for comparatively short periods and conditions are otherwise unfavorable. For diversified crops where the water supply is regular a capacity of 1 cu. ft. per sec. for each 70 to

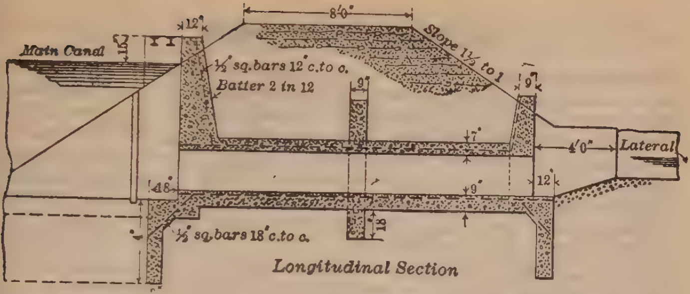


Fig. 13. Reinforced Concrete Turnout to Lateral

90 acres irrigated is commonly used. The capacity of farm laterals should not be less than the maximum head to be used in irrigating. This will vary from 1 to 2 cu. ft. per sec. where furrow irrigation is practiced to 5 to 10 cu. ft. per sec. for flooding methods. If rotation irrigation is to be practiced the capacity per unit area served should be greater for laterals than for main canals in order to provide flexibility in operation. In general, an allowance for seepage and evaporation should be added to the above figures.

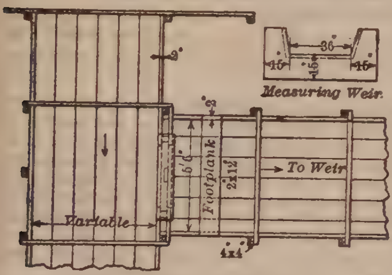


Fig. 14. Wooden Head to Lateral

Structures for Distribution Systems are similar to those for main canals. Turnouts must be provided where water is diverted from main canals to distributaries and at the heads of all laterals. Flumes or pressure pipes are constructed for carrying water across depressions. Drops are required in earth canals where the slope of the ground is greater than the allowable grade. These structures may be built of wood or concrete. Wood is more commonly used for the small turnouts and lateral heads. Measuring devices, generally weirs, are placed at the heads of farm laterals. Figs. 13, 14, and 15 are examples of structures for distribution systems.

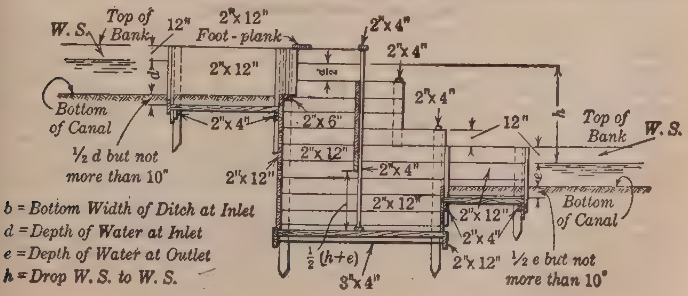


Fig. 15. Wooden Drop for Lateral

12. Irrigation by Pumping

Surface Water Supply. Water may be pumped from a natural stream or a canal. Areas lying above the main canals of gravity projects are sometimes irrigated from these canals by pumping. In some instances turbines are installed at drops in canals and either direct-connected to pumps or connected to generators from which power is transmitted to pumping plants. Such installations provide very cheap power for pumping. Often pumps may be considered an alternative to an expensive diversion and conveyance system. The comparative costs of irrigation by pumping and by gravity should be estimated on the basis of annual costs.

In 1923, about 150 pumping plants costing $2,360,000 were being operated for irrigation and drainage on projects of the United States Bureau of Reclamation. A statement of the cost of operation and maintenance of these plants, not including interest and depreciation (Berry Dibble; Engineering News-Record, July 26, 1923) is contained in the table on page 1677.

Wells. The essentials of a satisfactory pumping plant using well water are: (*a*) an ample supply of water of quality suitable for irrigation and near enough to the surface for economical pumping; (*b*) a pump adapted to the conditions as to lift and quantity to be pumped; (*c*) an engine or motor that will furnish the required power with the maximum efficiency and the minimum cost; (*d*) a plant carefully planned and installed.

The well casing should be of a standard type such as screw, riveted, or stovepipe casing, and the portion in contact with water-bearing strata should be perforated so as to admit water in sufficient quantities and to screen out gravel. Casing 6 to 24 in. in diameter are used, 16 in. being a common diameter for irrigation wells. Before pumping equipment is purchased the well should be thoroughly tested to determine its capacity and drawdown. Types of pumps commonly used for irrigation are the vertical and horizontal centrifugal, the deep-well turbine centrifugal, the plunger, and the air-lift. The height of lift is the chief factor governing the type of pump. The power may be supplied by an electric motor or by an internal-combustion engine. (See Farmer's Bulletin 1404, U. S. Dept. of Agriculture, by P. A. Ewing.)

The Cost of Pumping from Wells 100 to 200 ft. deep having capacities of 0.5 to 2 cu. ft. per sec., including 15% for interest, taxes, and depreciation in addition to all operating costs, varies from $2 to $16 per acre-foot of water pumped. Among the factors affecting cost of pumping are: cost of power, lift, capacity, and efficiency of plant. The effect of the last two factors on cost is shown in the table below.

Cost, for Power Only, of Raising Water per Acre-foot with Power at 1 Cent per Horsepower-hour

Pump efficiency	Head in feet									
	10	20	30	40	50	60	70	80	90	100
100	$0.14	$0.17	$0.41	$0.55	$0.69	$0.82	$0.96	$1.10	$1.24	$1.37
80	.17	.34	.51	.69	.86	1.03	1.20	1.37	1.54	1.72
70	.20	.39	.59	.78	.98	1.18	1.37	1.57	1.77	1.96
60	.23	.46	.69	.92	1.14	1.37	1.60	1.83	2.06	2.29
50	.27	.54	.82	1.10	1.37	1.65	1.92	2.20	2.47	2.75
40	.34	.69	1.03	1.37	1.72	2.06	2.40	2.75	3.09	3.43
30	.46	.91	1.37	1.83	2.29	2.75	3.20	3.66	4.12	4.58
20	.69	1.37	2.06	2.75	3.43	4.12	4.80	5.49	6.18	6.86

Operation and Maintenance Cost of Pumping Plants Of United States Bureau of Reclamation During Fiscal Year 1921-22

Project	Type of pumping unit*	Number of units	Average net lift, feet	Acre-feet pumped	Cost per acre-foot per foot of lift
Grand Valley, Colo.	1	1	31	6 635	$0.0058
Huntley, Mont.	4	2	45	10 200	0.0065
	6	2	45	1 870	0.0345
Minidoka, Idaho	1	5	29.2	186 283	
	1	4	30.2	153 805	0.0072
	1	3	29.9	92 923	
	1	2	19.8	12 951	
	7	1	3.5	2 950	
	7	1	2.5	2 601	
	2	1	7	432	0.0182
	2	1	4	2 258	
	7	1	4.8	1 087	
	2	2	21.2	6 604	
North Dakota	3	2	56	1 004	0.1286
	2	2	26.6	958	0.2474
	2	3	22	2 386	0.2427
	2	1	26	628	0.2392
North Platte, Neb.	1	1	41		
	1	1	30	1 235	0.1353
	1	1	49		
Okanogan, Wash.	6	1	55	....	....
	1	1	55	370	0.1617
	1	2	35	78	0.0986
	2	2	188	1 372	0.0571
	6	1	10	808	0.5410
Salt River, Ariz.	1 2	11	33.6	17 451	0.0140
	2	4	47	33 927	0.0089
	1	16	20.7	13 472	0.0227
	1	14	54.6	12 041	0.0280
	1	21	29.6	12 727	0.0128
	2	1	30	293	0.0617
	1	21	28.4	25 138	0.0165
	4	1	103	400	0.0073
	5	1	50	....	....
	4	2	110	16 429	0.0021
	5	1	105	3 304	0.0032
	4	2	200	6 511.6	0.0015
	5	1	90	3 672	0.0041
Yuma, Ariz.	6	2	5.6	2 000	0.2460
	6	2	10	31 969	0.0460
	6	1	7	102.5	0.8614

* Type 1—Vertical motor-driven centrifugal pump.
2—Horizontal motor-driven centrifugal pump.
3—Steam-turbine-driven centrifugal pump.
4—Vertical hydraulic-turbine-driven centrifugal pump.
5—Horizontal hydraulic-turbine-driven centrifugal pump.
6—Gas-engine-driven centrifugal pump.
7—Scoop wheel.

13. Preparation of Land

Clearing. Arid lands are usually covered to some extent with native vegetation. This may consist of grasses and small brush which can be plowed under, or heavy brush or scrub trees. All surface vegetation and roots that will interfere with the cultivation of the land should be removed and burned.

Sagebrush is the most widely distributed of all desert plants. It commonly attains a height of 2 to 5 ft. A substantial growth of sagebrush usually indicates fertile soil. Sagebrush may be removed by grubbing, or broken off by a team dragging a steel rail over it, or by other special devices. The cost of clearing land of sagebrush usually varies from $5 to $9 per acre. The cost of removing large brush and trees may be $30 to $90 per acre. The cost varies with the size of plant, thickness of growth, and labor conditions.

Grading. An accurate topographic map of the area to be graded should be obtained and the method of irrigation (see Art. 14) to be adopted should be given careful consideration. A study should be made of all factors affecting economy and efficiency in the use of water. A detailed plan for grading the tract should then be prepared. The cost of grading smooth areas may be less than $10 per acre. For rough or rolling areas the cost may be $20 to $80 or more per acre.

In many irrigated districts the soil is of uniform texture for some distance below the surface. Other localities have a comparatively thin surface soil which is underlain by a less fertile subsoil. Where the latter condition obtains, too extensive excavation may seriously affect the productivity of the soil, and it may be advisable to modify the plan of grading and system of irrigation from that which would otherwise be desirable. Where land is sufficiently valuable, it may be practicable to move to one side the surface soil from the areas to be excavated or filled, and after grading to recover them with the surface soil. Grading is usually done with scrapers. Buck scrapers and Fresno scrapers may be used to advantage if the haul is short, while wheeled scrapers are preferable for long hauls. Roads graders and various types of home-made levelers are used for smoothing the surface of the ground.

The First Cultivation of raw land is frequently difficult and expensive and the first crop returns will usually be less than later ones. Light sandy soils are especially difficult to subjugate, as they are apt to drift and allow the seeds to be removed by wind action. Such soils must usually be cleared and planted in small areas, and protection from wind provided as soon as possible. Straw or manure placed over the exposed surface may provide a satisfactory protection. Sometimes rows of sagebrush, about 50 ft. apart, at right angles to the direction of the wind, are left temporarily on the ground to act as wind breaks. A hardy grain, such as rye, is frequently used for the first crop.

14. Application of Water to Land

Methods. Water may be applied to land by surface or subsurface methods. Subsurface irrigation, or subirrigation, is usually more expensive than surface irrigation and is not commonly practiced. Surface irrigation may be accomplished by lateral percolation or by flooding.

Farm Conduits. Earth ditches are commonly used to convey water to the different parts of a farm. Where water is sufficiently valuable to justify the expense, flumes or underground pressure pipes may be used. Both flumes and pipes reduce seepage losses and prevent erosion, but flumes like ditches cut up a farm and interfere with cultivation and harvesting. Water is delivered from pipes through standpipes or "stands" as illustrated in Fig. 16. The permissible head is limited only by the strength of the pipe. Where open conduits are used, the water must be delivered to the highest point in the area to be irrigated. Fig. 17 shows a field lateral with distributing ditches. Farm ditches should preferably run parallel to property lines but often they must be curved the better to conform to topographic features.

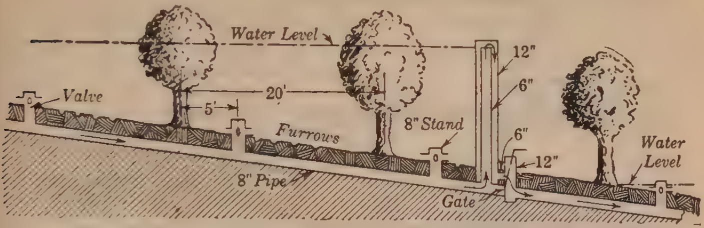


Fig. 16. Use of Underground Pipe in Furrow Irrigation, Showing Riser and Stand

Concrete and Clay Pipe for Irrigation

Prices quoted by California firms

Clay pipe				Concrete pipe			
Size, in.	Weight per ft., lb.	Length, ft.	Price per ft.	Size, in.	Weight per ft., lb.	Price per ft.	Price per ft. installed
3	6	2	$0.12	8	27	$0.25	$0.33
4	8	2	.15	10	37.6	.30	.40
5	12	2	.18	12	46.2	.38	.50
6	16	2	.21	14	56.4	.48	.63
8	22	2.5	.30	16	75.2	.65	.88
10	31	2.5	.42	18	98.8	.82	1.05
12	41	2.5	.54				

Uncontrolled Flooding is a crude method of applying irrigation water. Parallel ditches are constructed across a field at intervals of 20 rods or less. The area between any two ditches is irrigated by opening the upper ditch at a number of places and controlling the flow of water to some extent by throwing up small dykes across depressions or by making small temporary ditches along the higher places.

The Border Method is best adapted to fields having a moderate slope. Small levees, or borders, are built 2 to 4 rods apart. The land should be so graded that the ground surface on any cross-section between borders will be level. The land is irrigated by letting water flow over the land between borders. This method of irrigating has the objection that the upper portion of the area receives more water than the lower portion.

The Contour Check Method. Distributing ditches lead from the supply canal and pass directly down the slope (Fig. 19). Check levees along contours and cross levees divide the land into areas of 1 to 3 acres. The size of checks that is desirable decreases with the porosity of the soil. Levees should be about 9 in. high and at least 6 ft. wide at the base. The land between levees should be graded to a level surface. Checks are filled with water from the ditch adjacent to them. On steep slopes the ground is graded to a series of level benches and terraces with a levee at the top of each terrace. The cost of grading becomes proportionally higher as the slope and roughness of ground increases.

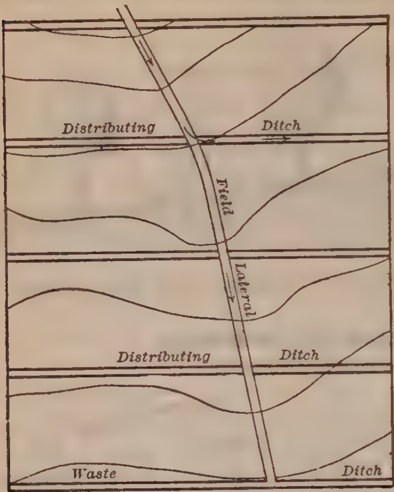


Fig. 17. System of Farm Ditches

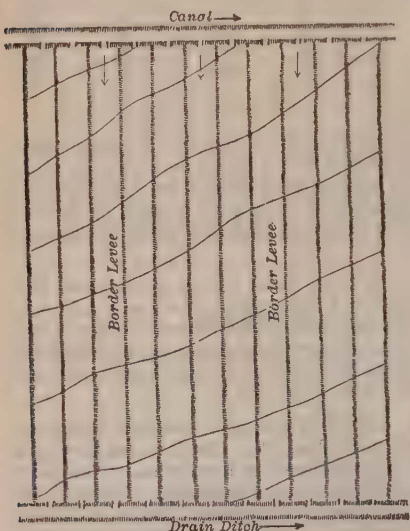


Fig. 18. Flooding by Border Method

The Square Check Method is suited only to gently sloping ground. It is similar to the contour check method except that the field is laid out in square checks. The best size of checks is 1 to 3 acres.

The Basin Method is best adapted to irrigating orchards. Levees are built around each tree, the land near trees being left high enough to prevent its submergence. The basins may be filled from ditches between alternate rows or water may be allowed to flow diagonally across basins as indicated in Fig. 21. Jointed pipes extending from the supply canal are sometimes used to fill basins. The lowest basin is filled first, then joints of pipe are removed and the basin next above is filled and the process is repeated until all of the basins in the row have been irrigated. This method is economical of water and is well adapted to the use of a small irrigation head.

The Furrow Method of irrigating by lateral percolation may be used for irrigating hay or grain (Fig. 22) and is used almost exclusively for crops planted in rows. Furrows, 3 to 6 in. deep and spaced 2 to 4 ft. apart, are made with a plow or special marking machine. Very little grading is necessary and the cost of preparing land is generally less than for the methods of irrigating by flooding. Furrows are made on slopes as steep as 10 ft. per 100 ft. but light soils may be eroded at this slope. The grade of furrows can be reduced by running them diagonally down a hill. The irrigation is accomplished by turning the water into several furrows at one time and allowing it to flow to the end of each furrow.

Subsurface Irrigation or subirrigation is the name applied to all those methods in which the water is applied below the surface of the ground. The

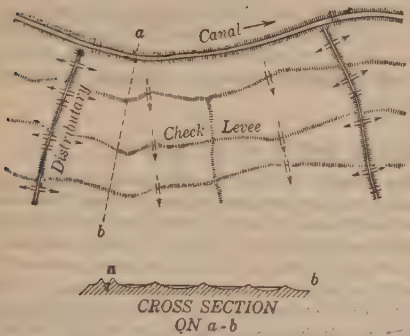


Fig. 19. System of Check Levees

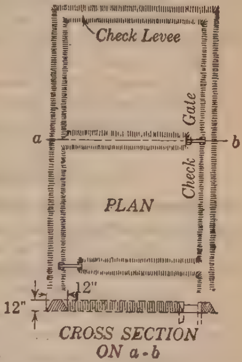


Fig. 20. Flooding by System of Squares

water is conveyed in underground channels containing openings, beneath the area to be irrigated, and through these openings the water percolates and is distributed through the soil by capillary action. The underground channels may consist of ordinary tile drains or perforated pipes. The cost of installation of such systems is high and in general the advantage obtained by them has

Fig. 21. Basin System of Irrigation

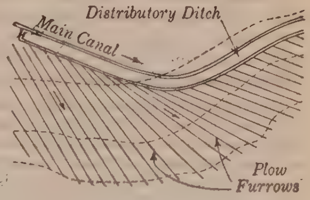


Fig. 22. Furrow Irrigation of Grain

not warranted the additional expense. In most localities surface irrigation is to be preferred to subirrigation.

Some lands are naturally subirrigated. Generally these are bottom lands underlain by gravel where the surface of the ground is but a few feet above the normal water surface of the stream. Water is drawn from this subsurface supply by capillary atraction to or nearly to the surface of the ground. Deep-rooted crops such as alfalfa are especially adapted to naturally subirrigated lands. Porous soils underlain by an impervious stratum quite near the surface may sometimes be subirrigated by simply digging ditches at intervals of 1/4 to 1/2 mile. Water flows down the slope between the two strata, and, if near enough to the surface, is available for plant growth.

Spray Irrigation has long been used in cities for watering lawns and small gardens. In certain humid districts it is used for supplementing the rainfall on larger areas devoted to the production of truck and small fruits. The essentials for this method of irrigating are a pumping plant and distributing pipe system. Hydrants may be located at suitable intervals over the field from which the water is applied to the land through hose, or parallel pipe lines 50 to 60 ft. apart, elevated a few feet above the ground and provided with nozzles at suitable intervals may be used to distribute the water over the field. An irrigation system of this kind is expensive, but the increased returns will often more than justify the expense.

15. Distribution and Use of Water

Distribution of Water among Users. At certain periods of the irrigation season the water requirements of crops are greatest and the demand for water may be excessive. The use of water cannot exceed the capacity of the irrigation system, and some adjustment in the use of water is usually necessary during certain portions of the irrigation season. On many large projects the farmer is required to give written notice 1 to 6 days in advance, stating when he will want water and the head that he desires. When it is not possible to deliver water to all users on the date requested without exceeding the supply of water, or the capacity of the irrigation system, the distribution is adjusted so as to accord as closely as possible with the requests of the various consumers.

Rotation in Irrigation. The consumer's right to the use of water is frequently expressed as a certain continuous flow, usually in cubic feet per second, throughout the irrigation season. As it is not economical to irrigate with a small head, it is customary for farmers whose rights to water are thus expressed, to rotate in its use, each taking a suitable head for such a time that the total water used will not exceed the allotment for the season. In many irrigated districts the right to water is expressed in some unit of volume, such as acre-feet per acre, or inches depth for the area to be irrigated. Such units are preferable for districts with an assured water supply.

Measurement of Irrigation Water is usually required at each farmer's intake. The ideal method of measurement would be with a meter that records the volume of water in cubic feet or acre-feet, but no practicable device for such measurement is now available. The present practice is to measure the rate and time of flow and to compute the volume.

Weirs of various types are used to measure the rate of flow of irrigation water. Headgates may be rated with a current meter to determine coefficients of discharge for different gate openings and heads. Discharge curves may be obtained from current-meter measurements for flumes or permanent canal sections. The Venturi flume may may be used where the head available is not sufficient for installing a weir. The control meter, reported in 1926 by the Special Committee on Irrigation Research of the American Society of Civil Engineers, is a modification of the Venturi flume that has the advantage of requiring only one head measurement.

Economy in Use of Water. In most arid districts the area of fertile land is much in excess of that which can ever be irrigated by the most effective use of all available waters. The limit of irrigation is therefore fixed by the water supply and not by the land area. Any water that is wasted or used in excess of that which may be applied to the land beneficially reduces the irrigable area. On the other hand, any improvement in methods of irrigation which reduces the water requirements, and all waste prevention, increase the area of land which may be reclaimed. Investigation of the use of water that will yield the greatest net return is therefore an important branch of irrigation engineering.

The Method of Payment for water has much to do with the economy of its use. Commonly the farmer's contract calls for a maximum quantity of water and there is no incentive for him to use a smaller amount. A better method of payment is for the farmer to pay for just what water he uses, or, if his contract calls for a fixed quantity of water, to allow a bonus for any smaller amount that he may use. Under such conditions the farmers will readily become interested in practicing economy in the use of water and in developing more efficient methods of irrigation.

SWAMP AND OVERFLOWED LANDS

16. Swamp Lands as Drainage Projects

Classification. Those areas which are covered with water or the soils of which contain an excess of water either continually or to an extent that renders them unfit for cultivation are commonly spoken of as swamp or overflowed lands. There are three general divisions: inland marshes, river-bottom lands, and tide lands. Most of these areas are susceptible of reclamation, and in many instances their development is of great economic importance.

Location and Distribution. Swamp and overflowed lands are distributed quite generally throughout the world and many such areas have been successfully reclaimed. The largest enterprises of this kind have been in Europe, where land values are high. Notable examples are the English Fens, where 700,000 acres of tide lands have been reclaimed, and Haarlem Lake, Holland, where a shallow lake 43 000 acres in extent has been converted into a fertile farming district. Large drainage enterprises have also been successfully completed in France, Italy, and other European countries. Of the large areas of swamp and overflowed lands in the United States only a small fraction have been reclaimed.

A compilation by the United States Department of Agriculture in 1922 gives approximately 57 000 000 acres of swamp lands, 31 000 000 acres of overflowed lands, and 8 000 000 acres of tidal marsh lands east of the 103rd meridian. Of this total of 96 000 000 acres, 18 000 000 acres are in Florida. In each of the following states there are: Louisiana, Georgia, and Minnesota, 6 000 000 to 7 500 000 acres; Michigan, North Carolina, Texas, South Carolina, Mississippi, Wisconsin, Arkansas, and Missouri, 3 000 000 to 5 000 000 acres; Alabama, Indiana, Illinois, Virginia, Kentucky, North Dakota, Tennessee, Kansas, Ohio, and New York, 1 000 000 to 2 000 000 acres.

Soil. Marsh lands are commonly covered with decayed vegetable matter and are usually very fertile. In many cases, however, some element of plant food is deficient and certain crops will not grow satisfactorily without artificial fertilization. This is especially true of peat and muck lands, which are usually lacking in potash. The subsoil of inland marshes is ordinarily similar to that of the surrounding country. Overflowed lands along rivers are composed largely of sedimentary deposits, silt and fine sand usually predominating. Tidal marshes may be formed of clay, sand and silt in any proportions. The soil formation of swamp and overflowed lands should always be carefully investigated with a view to determining is fertility, and its influence on construction methods and plan of drainage.

Subsidence of Muck and Peat Soils. Soils containing large percentages of vegetable matter subside when drained and cultivated. In the English Fens, at a place where in 1848 the peat was 18 ft. thick the ground surface has gradually lowered 10 ft. A shrinkage of 40% of the original thickness of peat soils is not uncommon. In the muck lands of Louisiana and Florida where the peat is impregnated with varying quantities of silt, a settlement of 2 ft. or more in five years has occurred. The amount of shrinkage decreases as the percentage of silt increases. In designing ditches and pumping plants for areas of deep muck land some provision should be made for the gradual but certain subsidence of the ground. For full discussion of this subject see Trans. Am. Soc. C. E., Vol. 82, The Subsidence of Muck and Peat Soils in Southern Lousiana and Florida, by C. W. Okey.

Natural Vegetation. Inland marshes frequently contain heavy growths of timber, or where the timber has been cleared they may be covered with dense underbrush, stumps, and fallen trees. Other marshes contain growths of coarse grasses, reeds, or moss. In all cases the expense of clearing must be included in the cost of reclamation.

Topographic Map. The general characteristics of a drainage project can best be studied from a topographic map, which should be prepared early in the investigation. Contours of the area to be reclaimed should ordinarily be shown for 1-ft. intervals, and natural drainage channels should be carefully indicated. Information as to soil formation, property lines, and source and amount of water to be disposed of may be recorded on the map. Existing maps may sometimes be of value in determining tributary areas which naturally drain into the area to be reclaimed.

17. Runoff

Surface Drainage. The channels and outlets of a drainage system should be of adequate capacity to provide for the removal of water at such a rate that the ground will be ready for early cultivation in the spring, and that the growth and harvesting of crops will not be seriously retarded. It is therefore the maximum or flood flows which must be considered in designing a surface drainage system, and the same principles apply regardless of the scheme of reclamation. Excessive runoff may result either from heavy rains, or the sudden melting of snows, but the most severe floods are caused by the former.

The Maximum Runoff per unit area that may be expected from any drainage basin decreases as the area increases. In general, the law of maximum discharge from flat areas, similar to those encountered in large drainage projects, may be represented by the equation

$$Q = KM^{0.75}$$

in which Q is the maximum discharge in cubic feet per second, M the drainage area above the point at which Q is required, in square miles, and K an empirical coefficient. If A is the area in acres, the formula may be written

$$Q = 0.00786\, KA^{0.75}.$$

Values of K obtained from measurements of maximum runoff of a number of drainage projects in the Mississippi valley range from 20 to 60. They correspond to rainfalls of 3 to 7 in. in 24 hours. The measurements were made on areas varying from approximately 2 to 200 square miles. The value of K depends primarily upon the maximum rainfall, but is influenced to a large extent by several other factors. It increases with the slope of the ground and the degree of imperviousness of the soil. The condition of the ground also influences the value of K. A heavy rain falling on wet ground will cause a greater runoff than the same rain falling on dry ground. If the ground is frozen or covered with ice, conditions will be favorable for a large surface runoff. Precipitation records covering as long a period as possible should be consulted to determine the maximum rainfall to be provided for.

The Total Annual Runoff from a given area is the difference between the annual precipitation and the water returned to the atmosphere through evaporation and plant transpiration, subject to modifications from ground-water conditions and possible deep seepage losses. The effect of the two latter considerations is commonly neglected in approximate calculations. Many factors affect the amounts of evaporation and transpiration, the more important being the mean annual temperature, and the amount and distribution of rainfall. The humidity of the air, velocity of the wind, character of vegetation, degree of cultivation, and soil formation are also important influences.

An accurate estimate of transpiration and evaporation losses for a given area involves a thorough investigation of the various factors involved. Rough estimates may be made by considering only the temperature and precipitation. Mean values of combined transpiration and evaporation losses, based upon observations in the United States, are given in the following table. The yearly runoff in inches is obtained by subtracting the proper tabulated value from the annual precipitation. Observed values may be expected to differ by as much as 5 in. from those given in the table.

Average Annual Transpiration and Evaporation Losses in Inches

Annual precipitation, in.	Mean annual temperature (Fahr.)								
	35°	40°	45°	50°	55°	60°	65°	70°	75°
25	16	17	18	19	20	21	22	23	24
30	17	18	19	20	21	22	23	24	25
35	18	19	20	21	22	23	24	25	26
40	19	20	21	22	23	24	25	26	27
45	20	21	22	23	24	25	26	27	28
50	21	22	23	24	25	26	27	28	29
55	22	23	24	25	26	27	28	29	30
60	23	24	25	26	27	28	29	30	31
65	24	25	26	27	28	29	30	31	32
70	25	26	27	28	29	30	31	32	33

18. Pumping Plants

Capacity. The removal of water may be accomplished by gravity or by pumping or by a combination of the two methods. A pumping plant should be of adequate capacity to remove water at a rate that will prevent injury to crops up to the point where additional capacity will cost more than the damage to crops that will result if the additional capacity is not provided. The time for which land may be flooded without serious damage differs for different crops, truck crops being more susceptible of damage than corn, sugar cane, grains and similar crops, but in general the land should not be submerged for more than 24 hours. The capacity of pumping plant must be based upon a consideration of maximum runoff (Art. 17) and also upon the storage provided by canals, the maximum length of time that crops may be submerged without damage, and the slope of the reclaimed area. Areas with considerable slope require greater pump capacity than more level areas, because of the tendency of water to collect and flood crops at the lower elevations.

In southern Louisiana pumping capacities sufficient to remove 1 to 1.5 in. per 24 hours have proved sufficient for areas of 5000 to 10 000 acres. One inch of runoff in 24 hours is equivalent to approximately 27 cu. ft. per sec. per square mile. In the upper Mississippi valley a pump capacity of 0.25 to 0.50 in. for similar areas has been found satisfactory.

The Period of Operation. Ordinarily pumps will be operated early in the spring to get the ground ready for cultivation, and after heavy rains during the growing period. In the lower Mississippi valley, where plant growth continues throughout the year, the pumping plant must be ready to operate at all times. In northern United States pumping usually is not required for more than 20 days, whereas in the extreme south it may be required 40 or more days in the year.

Location. Pumping plants should be located near the natural drainage outlet or, if the area is approximately level, so that the water will travel the minimum length of canal. Unless electric power is supplied from a central station the facilities for transporting fuel must be considered in selecting a location for the plant. The ground should also be explored with a view to obtaining a favorable foundation.

Buildings. The footings under machinery and buildings should be of concrete, and if built on earth they should be supported by piling. The pump should be mounted on the same block of concrete as the engine that operates it so that subsequent settlement will not throw them out of line. Buildings should be durable, of fireproof construction, and of sufficient stability to withstand the heaviest winds.

Power. The kind of power to be used in a particular case must depend upon local conditions, but each possible source of power should be investigated. The power finally selected should be that which requires the smallest operating expense, based upon interest charges, attendance, fuel, maintenance and depreciation. **Electric Power** requires a minimum expense for attendance. It is convenient and adapted to intermittent service and if the cost is low enough its use is to be preferred for plants of all capacities. **Crude-oil engines** provide a most satisfactory source of power and have been used extensively on many drainage projects of the southern states. **Steam-power plants** are quite generally used on large projects, but the large initial cost, the expense of keeping them continually ready for service, and the high grade of operators which they require are objectionable features.

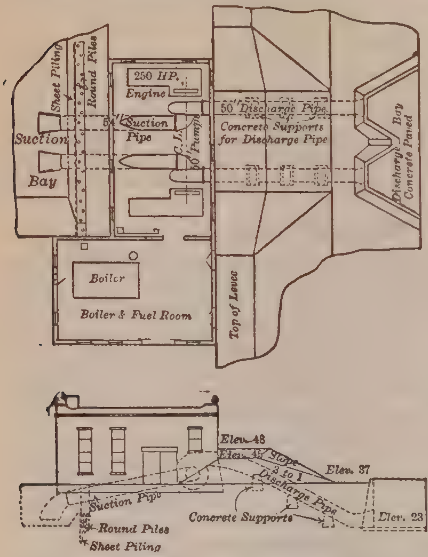


Fig. 23. Pumping Plant for Drainage System

Gasoline engines have been used to operate small plants. **Producer-gas plants** have not proved satisfactory for intermittent service.

Pumps should be selected and operated so as to secure the maximum efficiency practicable. The maximum lift will not usually exceed 10 ft. and the bulk of the water will not be lifted more than 3 or 4 ft. It is at the lower lifts also that the greatest pumping capacity is required. It is desirable, therefore, that the pumps should operate at very nearly their maximum efficiency for

Average Loss of Head in Feet per 100 ft. Length of Suction or Discharge Pipe of Pumps

Velocity in pipe, ft. per sec.	Diameter of pipe in inches													Add for each 90° bend
	6	9	12	15	18	21	24	30	36	42	48	54	60	
8	4.5	2.7	1.9	1.4	1.1	0.9	0.8	0.6	0.5	0.4	0.3	0.3	0.3	0.2
9	5.7	3.4	2.4	1.8	1.4	1.2	1.0	0.8	0.6	0.5	0.4	0.4	0.3	0.3
10	6.9	4.2	2.9	2.2	1.7	1.4	1.2	0.9	0.7	0.6	0.5	0.4	0.4	0.4
11	8.3	5.0	3.5	2.6	2.1	1.7	1.5	1.1	0.9	0.7	0.6	0.5	0.5	0.5
12	9.8	5.9	4.1	3.1	2.5	2.0	1.7	1.3	1.0	0.9	0.7	0.6	0.5	0.6
13	11.5	6.9	4.8	3.6	2.9	2.4	2.0	1.5	1.2	1.0	0.8	0.7	0.6	0.8
14	13.2	7.9	5.5	4.2	3.3	2.7	2.3	1.7	1.4	1.1	1.0	0.8	0.7	0.9
15	15.1	9.0	6.3	4.8	3.8	3.1	2.6	2.0	1.6	1.3	1.1	1.0	0.8	1.0

Costs of Pumping Plants in Southern United States

From Bulletin 1067, United States Department of Agriculture, by W. B. Gregory

Name of plant	Acres	Cost	Cost per Acre	Year constructed
Phillips Land Co.	2 500	$15 000	$6.00	1911
Subdistrict No. 1, Lafourche drainage district No. 6	1 880	10 000	5.32	1912
Subdistrict No. 3, Lafourche drainage district No. 12	2 250	13 500	6.00	1910
Jefferson drainage district No. 3	5 000	28 000	5.60	1912
Subdistrict No. 1, Gueydan drainage district	7 500	40 000	5.32	1912
Subdistrict No. 2, Avoca drainage district	4 350	34 000	7.81	1911
Subdistrict No. 3, Avoca drainage district	11 250	73 000	6.48	1913
Subdistrict No. 4, Jefferson drainage district No. 4	1 800	18 000	10.00	1915
Fayport subdistrict No. 1, Lafourche drainage district No. 9	2 000	14 000	7.00	....
Dalcour drainage district	650	10 000	15.40*	1913
Subdistrict No. 1, Lafourche drainage district No. 12	835	10 500	12.57	1915
Subdistrict No. 2, Lafourche drainage district 12	940	12 500	13.30	1915
Subdistrict No. 4, Lafourche drainage district No. 12	4 240	31 500	7.42	1913
Little Woods tract	6 943	37 500	5.39	1913
Port Arthur	5 720	54 290	9.50†	1918
Ferre relift	....	20 000	....	1918
Richard relift	....	15 000	....	1918
Hine's relift	....	8 000	....	1918

* Acreage will be considerably increased later.

† Plant costs more than it would for an agricultural proposition, as two units must lift water 11 ft. if occasion demands it.

the lower lifts and operate at a satisfactory efficiency through the entire range of lift. Centrifugal pumps have been quite generally used for drainage reclamation work. Fig. 23 shows an outline of the plan and elevation of the pumping plant for Louisa-Des Moines Drainage District No. 4, Iowa.

The rated capacity of ordinary low-head centrifugal pumps is based upon a velocity through the discharge opening of 10 to 12 ft. per sec. The loss of head due to friction may therefore equal or exceed the static lift. For this reason the suction pipe and discharge pipe should be made as short as possible and sharp bends should be avoided. The outlet should be submerged so as to induce siphon action in the pipes, and the discharge pipe should be flared at the lower end so that the water will discharge at a velocity of not over 5 ft. per sec. The suction pipe must be air-tight and the water should be drawn from a concrete-lined intake several feet deeper than the minimum elevation of water surface to be maintained. Centrifugal pumps if properly designed and installed should give efficiencies of 60 to 70% throughout the range of lift ordinarily encountered in drainage work, but improperly selected pumps may give very much lower efficiencies. These figures refer only to the pumps, and friction losses in pipes and velocity head at exit must be added to the static lift to obtain the actual head against which the pumps must operate.

Cost. The cost of drainage pumping plants varies widely according to the type of machinery, expense of transportation, character of foundation and difficulities of erection.

Relative Fuel Costs of Several Types of Pumping Plants

For 1000 acres, 6-ft. difference in levels, 8-ft. total head. Amount pumped = 1 acre-foot per acre per season, at rate of approximately 1 acre-inch per acre in 24 hours or 42 cu. ft per sec.

Water horse-power	Type of plant	I.H.P. of steam engine B.H.P. of gas engine	Total cost of fuel oil per hr. at $1.25 per barrel	Total cost of gasoline per hr. at 12 cents per gallon	Total fuel cost per year of removing total 12 in. at pump efficiencies named					
					100%	70%	60%	50%	40%	30%
41	Simple slide-valve engine, belted.......	H.P. 50.5	$0.88	....	$238	$340	$397	$476	$595	$795
41	Simple slide-valve engine, direct-connected...........	45.5	.80	....	218	311	364	436	545	727
41	Gasoline engine, belted	45.5	....	$0.68	184	263	307	368	460	613
41	Gasoline engine, direct-connected....	41.0	....	.61	165	236	275	330	412	550
41	Simple Corliss engine, belted............	49.5	.50	....	135	193	225	270	338	450
41	Simple Corliss engine, direct-connected...	44.5	.45	....	122	174	203	244	305	407
41	Compound condensed slide-valve, direct-connected........	45.5	.36	....	98	140	163	196	245	326
41	Compound condensed Corliss, direct-connected...........	44.5	.31	....	83	119	138	166	208	276

19. Levees and Ditches

Levees are required to prevent the entrance of outside water on lands subject to overflow. If subjected to current and wave action they should be designed on liberal lines with ample freeboard and they should be protected by rock riprap or other construction to prevent erosion. For the design and construction of river levees see Sect. 8, Art. 25, and Sect. 19, Art. 10.

Levees in Still Water may have a somewhat smaller cross-section than those subjected to current or wave action. They are commonly located on property lines without regard to the topography. This results in regularly shaped districts and minimum length of levee, but frequently the cost of construction and maintenance may be reduced by following the higher and firmer land and deviating somewhat from the more direct line.

Common dimensions for this type of levee (Fig. 24) are 1 on 2 or 1 on 3 for embankment slopes, a top width of embankment of 6 ft., a berm of 10 ft. and a freeboard above the maximum high water elevation of 3 ft. To reduce seepage, the borrow pit

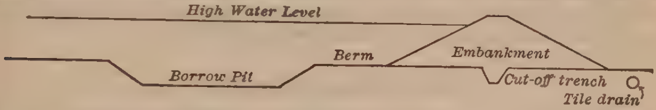


Fig. 24. Cross-section of Levee

should be on the outer side of the levee, but it is sometimes placed within the tract and thus serves as a drainage canal. All vegetable matter and humus should be removed from the strip of land on which the levee is to be built and as far as practicable only the better grade of material should be placed in the embankment. Carelessness in these regards may result in excessive seepage through the levee. Levees built of the mud commonly found in swamps will shrink 20 to 50% of its original volume, and allowance for this shrinkage should be made when the material is placed. The shrinkage will be less for sandy soils than for those in which clay predominates.

Ditches of various sizes must be constructed and so located as thoroughly to drain the area to be reclaimed. In pumping projects the larger ditches also serve for storing water after heavy rains and thus give greater flexibility to the pumping plant. In many instances the latter function is fully as important as the former, and quite commonly some of the main ditches are made much larger and deeper than would be necessary for drainage alone in order to provide more storage.

The size of each drainage ditch must be carefully determined from the duty which it has to perform. The general features are shown in Fig. 25. The side slopes are usually made about 1 on 1-1/2. When excavated with dredges it may be more con-

Fig. 25. Cross-section of Drainage Ditch

venient to leave steeper banks and allow additional width with the idea that the banks will cave in and eventually conform approximately to 1 on 1-1/2 slopes. Excavated material should be deposited in spoil banks on either side of the ditch, leaving suitable berms between the ditch and spoil banks.

Excavating Machinery. Either teams and scrapers or machinery can be used to construct levees and ditches on dry land but on wet land dredges are used exclusively. The use of power machinery for all drainage work has become general in the United States. New types of excavators have been put on the market in recent years and older types have been much improved. (See Bulletin 300 of the U. S. Dept. of Agriculture, by D. L. Yarnell.) The following excavating machines are used on drainage work: floating dipper dredge, floating grab-bucket dredge, dragline scraper excavator, dry-land dipper dredge, dry-land grab-bucket excavator, wheel excavator, hydraulic dredge, and various machines for cleaning old ditches.

The Cost of dredging drainage ditches in the United States is usually 12 to 16 cents per cu. yd. The cost of levees is somewhat higher depending upon the expense of selecting and placing material and the distance of the embankment from the borrow pit. The total cost of excavation for drainage projects where both levees and interior ditches are required varies from $20 to $60 per acre.

Maintenace of Levees and Ditches. Bushes and other vegetation of rank growth should not be allowed on levee banks since water will follow the roots and increase seepage. After the soft material in a levee has dried sufficiently it should be smoothed off and preferably seeded to Bermuda grass. It may then be mowed with a mowing machine, or the grass may be kept down by allowing stock to graze upon it. A heavy sod will assist in preventing erosion and in keeping out weeds and other objectionable vegetation. Grass and weeds should be cut out of ditches 1 to 3 times each year. At least once in 2 years the ditches should be cleaned of the accumulated mud and silt.

20. Inland Marshes

Source of Water. The first step in the investigation, preliminary to deciding upon the plan of reclamation, is to determine the source of the water or the reason why the land is too wet. The water may come from direct rainfall, from visible streams, from the runoff from neighboring hills, or from springs.

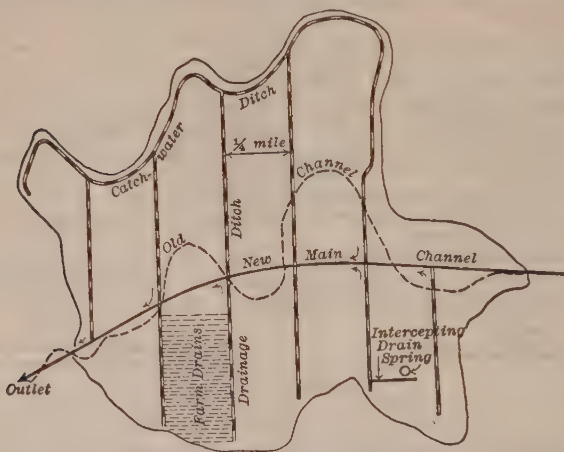


Fig. 26. Reclamation of Inland Marsh

The condition may be due largely to the fact that natural outlet channel is obstructed or of insufficient depth or capacity to drain the area properly.

Main Drainage Channel and Outlet. In order that a swamp area may be properly drained it is necessary that the outlet and main drainage channel should be deep enough and of sufficient capacity to carry the drainage of the swamp and all tributary areas while maintaining an elevation of water surface low enough to provide for effectively draining the area to be reclaimed. The natural drainage channel through an inland marsh may not be clearly marked or it may be very winding and obstructed with fallen timber and debris. One of the first requirements in such cases is to excavate or straighten and enlarge and deepen such channels and carry them to a natural outlet. If no satisfactory outlet exists, pumping may be resorted to, but inland marshes are usually drained entirely by gravity.

Auxiliary Drainage. The amount of drainage required in addition to that provided by the main channel will depend largely upon the size of the marsh, the natural slope of the ground, the soil formation, and the source of underground water. If the subsoil is sandy and especially if the ground has a fair slope toward the main channel no additional drainage may be necessary. Also if the marsh is small no additional ditches may be required. If the subsoil is of impervious formation auxiliary drainage ditches across the tract may be required at 1/4- or 1/2-mile intervals. When springs occur on the tract they should be cut off by intercepting drains, that is, drains which tap the source of supply before it reaches the surface of the ground. This result may usually be accomplished by means of an underground tile drain. If there is much flow on to the tract from surrounding hills a catchwater open ditch at the junction of the high and low land may be advisable. After the main drainage channel and principal auxiliary ditches have been completed, if further drainage is required it may be accomplished by open farm ditches or tile drains. Fig. 26 indicates the different methods of drainage that may be used in reclaiming inland marshes. In general the drainage should be adequate to assure a minimum depth to ground-water of 3 to 4 ft. after the subsidence following drainage has occurred.

21. Overflowed Lands

Classification. There are two general classes of overflowed lands: tidal marshes which are periodically submerged and uncovered by the rise and fall of tides, and river bottom lands which are temporarily inundated during high stages of the streams. The lower lying marshes in either case may be permanently submerged. Some lands, like those in southern Louisiana, may be subject to overflow from both causes. As regards topography, the area to be reclaimed may either adjoin high land on one side, or be entirely surrounded by overflowed land. In general, a similar plan may be adopted for reclaiming all overflowed lands, the details being changed to suit the particular conditions.

Protection from Overflow is secured by constructing levees around the area to be reclaimed. (Art. 19.) If the tract joins high land on one side, levees will be required on but three sides. In general the area of such lands that may be reclaimed by one levee system is limited to that which is included between the main stream or body of water and two of its tributaries. A levee runs along the bank of the main water and return levees follow the tributaries to the high land. Diversion ditches may be constructed near the foot of the slope where the hills join the flat land to deflect the hill waters and prevent their entering the tract. If the area to be rclaimed is surrounded by overflowed lands, levees must be built on all sides. Fig. 27 shows the system of levees and ditches of Area No. 7, Gueydan, Louisiana. Fig. 28 shows the method of reclaiming a tidal marsh near Dorchester, New Jersey.

Drainage. The drainage system consists of one or more main ditches with auxiliary ditches spaced as required, but usually 1/4 to 1/2 mile apart. Open farm ditches or drain tile may then be used to provide whatever additional drainage may be necessary. Open ditches should preferably be located on property lines or parallel to them, in order that they may interfere as little as possible with farming operations.

The Depth of Ditches should be such that the entire tract may be satisfactorily drained to a depth of 3 to 4 ft. The practice in the lower Mississippi valley, Fig. 27, is to dig farm ditches 4 ft. deep. The main ditches must have dimensions and slopes which will provide satisfactory outlets for the smaller ditches. Marsh lands will frequently settle 1 to 4 ft. after reclamation, and this settlement may continue for several years. The probable settlement of the ground should be considered in designing the drainage system.

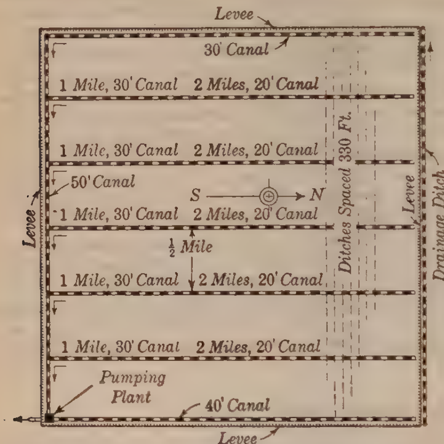


Fig. 27. Reclamation of Overflowed Land

The Outlet for the drainage system may be provided by gravity or pumping or by a combination of the two. If a natural gravity outlet is relied upon a sluice gate must be provided where the water passes through the levee. Sluice gates should act automatically and allow the water to pass out when the water surface is higher within than without the tract and close when the opposite condition obtains. A flap valve, hinged at the top, is ordinarily used for this purpose. A hand-operated gate should be installed in conjunction with the automatic gate to be used in case the latter

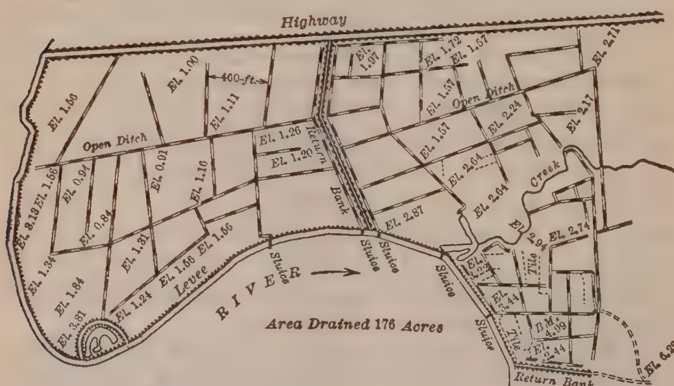


Fig. 28. Reclamation of Tidal Marsh

gets out of order. River bottom lands when subject to overflow for only short periods may be drained by gravity outlets if the water surface in the river is normally low enough, otherwise auxiliary pumping will be necessary to drain the tract satisfactorily. Tidal lands may be drained through automatic sluice gates, the gates being opened during low tides and closed during high tides. In localities where the tidal range is small and the elevation of the ground is low, reclamation can be accomplished only by pumping. In many

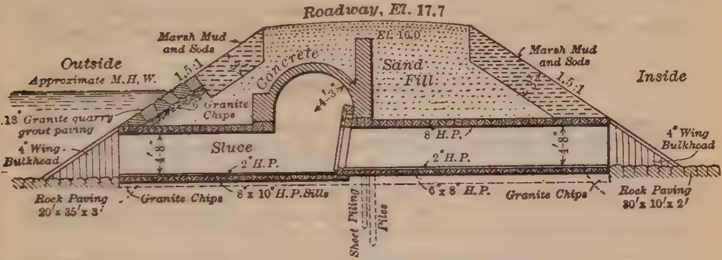


Fig. 29. Longitudinal Section of Levee through Sluice

instances where tidal lands are drained by gravity the expense of auxiliary pumping would be more than justified by increased returns from the land.

In general, tide lands drained only by gravity lying 1 ft. or less above mean low water have no value for agricultural purposes, those between 1 and 2 ft. above mean low water will produce hay, and those between 2 and 3 ft. above mean low water will produce rye, potatoes, corn and other crops. The best results are obtained from land 3 ft. or more above mean low water elevation.

FARM DRAINAGE

22. Drainage Possibilities

Soil Moisture. All water in excess of that required by vegetation is harmful (Art. 2). Air within the soil is essential to healthy plant growth and this is excluded by excess moisture. Air is completely excluded from saturated soil, and the land becomes cold, sour, and entirely unproductive.

Three Classes of Lands may be considered: (*a*) those that are entirely unproductive or that cannot be profitably farmed without drainage; (*b*) those that can be profitably farmed without drainage but that with drainage will give increased returns sufficient to justify the cost; (*c*) those that cannot be sufficiently improved to justify the cost of drainage. All lands not having a porous subsoil and especially clay lands can be improved by drainage.

Benefits of Drainage. Drainage benefits farm lands in the following ways: (*a*) By removing the surplus water, which is harmful to plants, and aerating the soil; (*b*) by warming the soil and thus decreasing danger from frosts, causing the seeds to germinate earlier, and grow more promptly, permitting earlier cultivation of the land, and increasing the length of the growing season; (*c*) by making the soil more granulated, porous, and friable, and thus increasing its ability to absorb rainfall. In general, the results of farm drainage are to make certain the production of crops each year and to increase the yields and profits.

Preliminary Investigation. Before laying out a plan of drainage for a given area a careful preliminary study should be made and a topographic map should be prepared showing contours and intermediate elevations at controlling points. It is important that natural drainage lines should be distinctly shown.

A soil survey of the area should be made and the character of the soil and subsoil at different points may be conveniently marked on the map, supplemented by additional field notes if necessary. The ground-water also should be studied, especially if there is evidence that it comes from some source other than precipitation upon the area to be drained. The position of the outlet should be determined from the field investigation and marked upon the map. The map and other data may then be used as a basis for the plan of drainage.

23. Drains and Drain Tile

Principles of Drainage. There are two kinds of drainage: Surface drainage, by which water is conveyed over the surface of the ground; and subsurface drainage, by which water is conveyed beneath the surface of the ground. Surface drainage removes the water that cannot percolate into the ground as fast as it is supplied by precipitation or melting snow. Subsurface drainage removes the water which cannot be retained as soil moisture.

Farm lands require surface drainage to remove excess precipitation but if slopes are too steep much of the water from heavy showers may flow off the surface before the ground has been wet as deep as is desirable. Surface drainage may be retarded by cultivating the land across slopes. Farm drainage is primarily a problem of supplementing natural subsurface drainage.

Open Ditches receive water from the surface of the ground and also water that passes through the soil. They therefore assist in surface drainage, which may be an advantage on very flat areas. In the lower Mississippi, where the rainfall is heavy and the land practically level, open farm ditches are used almost exclusively (Fig. 27). Farm ditches occupy land which might otherwise be cultivated and make it difficult to move farm machinery from one part of the farm to another. They are expensive to maintain and in clay soils they are apt to become puddled and prevent the free entrance of ground-water.

Subsurface Drains are conduits below the surface of the ground with openings of a size that will permit the ground-water to enter freely and will exclude the surrounding earth. Box drains have been used to a limited extent. They are made of four boards nailed together to form a rectangular cross-section with cleats between the top and sides to leave narrow openings for the entrance of water. Blind ditches partially filled with stones or poles were formerly used, but they were not permanent and have been abandoned. Tile drains have been found to provide the most satisfactory drainage, and they are used almost exclusively.

Kinds of Tile. There are two kinds of drain tile in common use: Clay tile and cement tile. Clay tile are sometimes made hexagonal or octagonal, but more commonly they are round. They range from 2 to 36 in. in diameter and from 12 in. in length for the smaller sizes to 30 in. for the largest. Cement tile under 12 in. in diameter are more expensive than clay tile, but in the larger sizes they are cheaper. The following comments on clay and cement tile are contained in Farmers' Bulletin 524 of the U. S. Department of Agriculture.

Clay Tile. Soft, medium, and hard-burned or vitrified clay tile are made. It costs less to make the soft-burned than the hard-burned tile and the selling price is lower, but the quality is not so good. Soft-burned tile have done good service, however, and when put under ground below the frost line have lasted indefinitely. The best tile are burned to a cherry red and when struck by a piece of steel give a sharp metallic ring. In the north, where they are laid above the frost line, only the hard-burned tile should be used. Hard-burned, or vitrified, tile are practically nonporous; thus, they absorb little moisture and unless water stands in them they are not injured by freezing temperatures. A tile that cracks and shatters in winter when lying in the yards unprotected from the weather is not fit to use above the frost line and is not the best under any conditions. Thick tile make the best joints. Those with thin sides, especially in the smaller sizes, are likely to slip out of place and leave openings. Some manufacturers who ship tile make them with thin sides to reduce the weight and the consequent cost of transportation. A 4-in. tile should weigh at least 6 lb., a 5-in. 8 lb., and a 6-in. 11 lb. Some factories make them heavier than this, which is better so far as utility is concerned.

Cement Tile. There are a number of machines on the market for making cement (or concrete) tile that cost from $50 to $100. One part of cement to about 3 parts of aggregate should be used. The largest pieces in the aggregate should not be more than one-half the thickness of the wall. The cement and aggregate should be well mixed and sufficient water should be added to make the mixture moist but not wet. After the tile are taken from the machine they should be placed in the shade. The longer the mold is allowed to remain around the tile the better. After the mold is removed the tile should be sprinkled twice a day for 3 or 4 days, or until thoroughly cured. A well-made and well-cured cement tile, like a good clay tile, when struck with a piece of iron will have a sharp metallic ring. Large cement factories are equipped with steam driers, with which the curing can be done to the best advantage.

24. Location of Drains

The General Plan of draining a given area should be determined before any drains are constructed, in order that all parts of the system when completed will work together effectively even though the construction is to extend over a long period. Details not pertaining to the general scheme of drainage may be worked out as required. When more than one farm will be benefited by the construction of main drains, each party interested should contribute in the investigation in so far as the work affects his property. State laws usually cover the legal points involved.

The Outlet is usually the first consideration. On rolling or hilly lands a natural outlet ordinarily exists. On low level land an artificial outlet is usually necessary. The elevation of the water surface in the outlet should be low enough to provide for the effective drainage of the area which it serves. It should be of sufficient capacity to take care of the requirements of surface and subsurface drainage. The expense should be assessed against all of the farms using the outlet in proportion to the benefits accruing to each. Outlets may be either open ditches or covered pipes. Tile up to 36 in. in diameter have been used for this purpose.

Drainage Systems. There are two general systems of drainage: Partial and complete. Partial drainage systems are used on rolling land when it is desired to drain only isolated portions of a tract, as illustrated in Fig. 30. The complete system is used when all of the land of a comparatively large area is to be drained. Examples of complete drainage systems are shown in Figs. 31 and 32. It will be noticed that certain areas adjacent to the main laterals are double drained, and for maximum economy this double drained area should be as small as practicable. For this reason long parallel field drains are preferable. The system indicated in Fig. 32 requires a smaller amount of tile than that indicated in Fig. 31.

Field drains should be laid in the direction of the greatest slope. This insures the greatest velocity and capacity. High velocities of water assist in scouring out the drains and in keeping them clean. It is very important that drains in sandy land should have velocities great enough to scour out the sand which filters into the drains between the tile. A 4-in. tile drain in sandy land should not be laid on a grade of less than 0.3 ft. to 100 ft. The laterals, submains and mains should preferably be located approximately in the lines of natural drainage, since the slope of the ground leads the water in this direction. The drains should be laid as straight as practicable and bends should be made by smooth curves. It usually will be found necessary to deviate somewhat from the natural drainage channel in order to improve the alignment.

The Depth and Frequency of Drains that will produce the most satisfactory results depends largely upon the character of soil. Where deep drainage can be used the drains may be spaced farther apart. The more porous the soil the deeper the drains can be placed. In heavy clay soils of close textures, drains placed 2 to 3 ft. beneath the ground surface have produced good results when deeper drains have failed. In other cases clay soils have been satisfactorily drained at depths of 3 to 4 ft. A depth of about 4 ft. appears to be the best for

Fig. 30. System of Partial Drainage

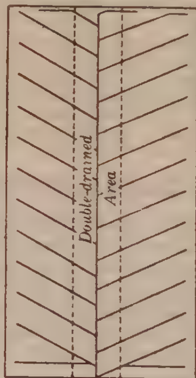


Fig. 31. System of Complete Drainage

Fig. 32. System of Complete Drainage

sandy soils. Practice also varies widely regarding the proper spacing of drains. In dense clay soils they should probably be placed 30 to 40 ft. apart, and in sand a spacing of 200 ft. may be used. For intermediate grades of material distances of 50 to 150 ft. may be selected to suit the conditions.

Before deciding definitely as to the proper depth and frequency of drains for a given area a careful study should be made of all factors likely to affect the problem. Conditions vary widely and results obtained in one locality may not apply to another locality, even though conditions appear quite similar. Local conditions especially should be studied, and if time is available and other conditions seem to warrant it, experiments with different depths and spacings of drains should be made before the completion of the work as a whole is undertaken.

25. Capacity and Size of Drains

The Required Capacity of a subsurface drain depends upon the rate at whch water will pass from the surface of the ground to the drain. The two principal factors affecting the rate at which water will enter a drain are the porosity of soil and the amount and rate of precipitation.

Drainage Coefficient. The inches depth of water which must be removed in 24 hours to drain any area satisfactorily is called the drainage coefficient of that area. Its value for different soils must be obtained from experiments. An investigation by the U. S. Department of Agriculture of a number of under ground drains in Illinois and Iowa that were doing satisfactory service showed maximum discharges in 24 hours of 0.11 to 0.27 in., most of the results varying from 0.15 to 0.20 in. These results were obtained on areas having fairly porous subsoils. Heavy clay soils and coarse sand soils will have respectively smaller and larger drainage coefficients than the above. It also appears that the drainage coefficient of rolling land is about 20% greater than for flat land. The following may be considered average values of drainage coefficients for different soils:

Compact clay soil, flat, 0.12; rolling, 0.14.
Medium porous soil, flat, 0.18; rolling, 0.22.
Porous sandy soil, flat, 0.30; rolling, 0.36.

With the drainage coefficient decided upon, the capacity of drain in cubic feet per second required to drain a given are can be obtained by multiplying the area in square miles or acres by the proper value in the accompanying tables:

Decimals of an Inch of Runoff per Twenty-four Hours Expressed as Cubic Feet per Second per Square Mile

Inch	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	......	.27	.54	.81	1.08	1.34	1.61	1.88	2.15	2.42
.1	2.69	2.96	3.23	3.50	3.76	4.03	4.30	4.57	4.84	5.11
.2	5.38	5.65	5.92	6.18	6.45	6.72	6.99	7.26	7.53	7.80
.3	8.07	8.34	8.60	8.87	9.14	9.41	9.68	9.95	10.22	10.49
.4	10.76	11.02	11.29	11.56	11.83	12.10	12.37	12.64	12.91	13.18
.5	13.44	13.71	13.98	14.25	14.52	14.79	15.06	15.33	15.60	15.86
.6	16.13	16.40	16.67	16.94	17.21	17.48	17.75	18.02	18.28	18.55
.7	18.82	19.09	19.36	19.63	19.90	20.17	20.44	20.70	20.97	21.24
.8	21.51	21.78	22.05	22.32	22.59	22.86	23.12	23.39	23.66	23.93
.9	24.20	24.47	24.74	25.01	25.28	25.54	25.81	26.08	26.35	26.62
1.0	26.89									

Decimals of an Inch of Runoff per Twenty-four Hours Expressed as Cubic Feet per Second per Acre

Inch	.00	.01	.02	.03	.04	.05	.06	.06	.08	.09
.0	......	.00042	.00084	.00126	.00168	.00210	.00252	.00294	.00336	.00378
.1	.00420	.00462	.00504	.00546	.00588	.00630	.00672	.00714	.00756	.00798
.2	.00840	.00882	.00924	.00966	.01008	.01050	.01092	.01134	.01176	.01218
.3	.01260	.01302	.01344	.01386	.01428	.01470	.01512	.01555	.01597	.01639
.4	.01681	.01723	.01765	.01807	.01849	.01891	.01933	.01975	.02017	.02059
.5	.02101	.02143	.02185	.02227	.02269	.02311	.02353	.02395	.02437	.02479
.6	.02521	.02563	.02605	.02647	.02689	.02731	.02773	.02815	.02857	.02899
.7	.02941	.02983	.03025	.03067	.03109	.03151	.03193	.03235	.03277	.03319
.8	.03361	.03403	.03445	.03487	.03529	.03571	.03613	.03655	.03697	.03739
.9	.03781	.03823	.03865	.03907	.03949	.03991	.04033	.04075	.04117	.04159
1.0	.04201									

Size of Tile. If the required capacity has been determined, the Manning formula written in the form

$$d_i = \left(\frac{1630\ Qn}{s^{1/2}}\right)^{3/8}$$

Diameter in Inches of Drain Tile

Computed from the Manning Formula with n = 0.011

Discharge, cu. ft. per sec.	s = fall per foot .0005	.001	.002	.004	.006	.008	.01	.02	.03	.04	.05	.1
.01	2.2	1.9	1.7	1.5	1.4	1.3	1.2	1.1	1.0	1.0	0.9	0.8
.02	2.8	2.5	2.2	1.9	1.8	1.7	1.6	1.4	1.3	1.2	1.2	1.0
.04	3.7	3.2	2.8	2.5	2.3	2.2	2.1	1.8	1.7	1.6	1.5	1.4
.06	4.3	3.8	3.3	2.9	2.7	2.5	2.4	2.1	2.0	1.9	1.8	1.6
.08	4.8	4.2	3.6	3.2	3.0	2.8	2.7	2.4	2.2	2.1	2.0	1.8
.1	5.2	4.5	4.0	3.5	3.2	3.1	3.0	2.6	2.4	2.3	2.2	1.9
.2	6.7	5.9	5.2	4.5	4.2	4.0	3.8	3.4	3.1	3.0	2.8	2.5
.3	7.8	6.9	6.0	5.3	4.9	4.7	4.5	3.9	3.7	3.4	3.3	2.9
.4	8.7	7.7	6.7	5.9	5.5	5.2	5.0	4.4	4.0	3.8	3.7	3.2
.5	9.5	8.3	7.3	6.4	5.9	5.6	5.4	4.7	4.4	4.2	4.0	3.5
.6	10.1	8.9	7.8	6.9	6.3	6.0	5.8	5.1	4.7	4.5	4.3	3.8
.8	11.3	10.0	8.7	7.7	7.1	6.7	6.4	5.7	5.2	5.0	4.8	4.2
1.0	12.3	10.8	9.5	8.3	7.7	7.3	7.0	6.2	5.7	5.4	5.2	4.5
1.2	13.1	11.5	10.1	8.9	8.2	7.8	7.5	6.6	6.1	5.8	5.5	4.9
1.4	13.9	12.2	10.7	9.4	8.7	8.3	7.9	7.0	6.5	6.1	5.9	5.2
1.5	14.3	12.5	11.0	9.6	8.9	8.5	8.1	7.2	6.7	6.3	6.0	5.3
2.0	15.9	14.0	12.3	10.8	10.0	9.5	9.1	8.0	7.4	7.0	6.7	5.9
2.5	17.3	15.2	13.3	11.7	10.8	10.3	9.9	8.7	8.0	7.6	7.3	6.4
3.0	18.5	16.3	14.3	12.5	11.6	11.0	10.6	9.3	8.6	8.1	7.8	6.9
3.5	19.6	17.2	15.2	13.3	12.3	11.7	11.2	9.8	9.1	8.6	8.3	7.3
4	20.6	18.1	15.9	14.0	13.0	12.3	11.8	10.3	9.6	9.1	8.7	7.6
5	22.4	19.7	17.3	15.2	14.1	13.3	12.8	11.2	10.4	9.9	9.5	8.3
6	24.0	21.1	18.5	16.3	15.1	14.3	13.7	12.0	11.2	10.6	10.1	8.9
7	25.4	22.4	19.6	17.2	16.0	15.1	14.5	12.8	11.8	11.2	10.7	9.4
8	26.8	23.5	20.6	18.1	16.8	15.9	15.3	13.4	12.4	11.8	11.3	9.9
10	29.1	25.6	22.4	19.7	18.3	17.3	16.6	14.6	13.5	12.8	12.3	10.8
12	31.2	27.4	24.0	21.1	19.6	18.5	17.8	15.6	14.5	13.7	13.1	11.5
14	33.0	29.0	25.5	22.4	20.7	19.6	18.8	16.5	15.3	14.5	13.9	12.2
16	34.7	30.5	26.8	23.5	21.8	20.6	19.8	17.4	16.1	15.3	14.6	12.9
18	36.3	31.9	28.0	24.6	22.8	21.6	20.7	18.2	16.8	16.0	15.3	13.4
20	37.8	33.1	29.1	25.6	23.7	22.4	21.5	18.9	17.5	16.6	15.9	14.0
25	41.0	36.0	31.7	27.8	25.8	24.4	23.4	20.6	19.0	18.0	17.3	15.2
30	44.0	38.6	33.9	29.8	27.6	26.1	25.1	22.0	20.4	19.3	18.5	16.3
35	46.6	40.9	35.9	31.5	29.2	27.7	26.6	23.3	21.6	20.5	19.6	17.2
40	49.0	43.0	37.7	33.1	30.7	29.1	27.9	24.5	22.7	21.5	20.6	18.1
45	51.2	44.9	39.5	34.6	32.1	30.4	29.2	25.6	23.7	22.5	21.6	19.0
50	53.2	46.7	41.0	36.0	33.4	31.6	30.4	26.7	24.7	23.4	22.5	19.7
55	55.2	48.4	42.5	37.4	34.6	32.8	31.5	27.6	25.6	24.3	23.3	20.4
60	57.0	50.0	44.0	38.6	35.8	33.9	32.5	28.5	26.5	25.1	24.0	21.1
65	58.7	51.6	45.3	39.8	36.9	34.9	33.5	29.4	27.3	25.8	24.8	21.8
70	60.4	53.0	46.6	40.9	37.9	35.9	34.4	30.2	28.0	26.6	25.6	22.4
80	63.5	55.8	49.0	43.0	39.8	37.7	36.2	31.8	29.5	27.9	26.8	23.5
90	66.4	58.3	51.2	44.9	41.6	39.5	37.8	33.2	30.8	29.2	28.0	24.6
100	69.0	60.6	53.2	46.7	43.3	41.0	39.3	34.6	32.0	30.4	29.1	25.6
110	71.5	62.8	55.2	48.4	44.9	42.5	40.8	35.8	33.2	31.5	30.2	26.5

can be used for computing the diameter. d_i is the diameter in inches, Q is the discharge in cubic feet per second, s is the slope or fall per foot, and n is the coefficient of roughness which has approximately the same value as n in the Kutter formula.

From a compilation of extensive experiments by D. L. Yarnell (published in 1922 as Bulletin 300, U. S. Department of Agriculture), the value of n for well-laid clay or concrete tile flowing full is approximately 0.011. This should be considered as the minimum value and should be used only under most favorable conditions. For poorly laid tile n may be 0.012 or even 0.014 or more. Values of d computed from the above formula with $n = 0.011$ are given in the accompanying table. For other values of n multiply d as given in the table by the following factors:

For n equal to........	**.012**	**.013**	**.014**	**.015**	**.016**	**.017**
Multiply d in table by .	**1.03**	**1.06**	**1.09**	**1.12**	**1.15**	**1.18**

Formerly 2-in. tile were used for farm drains, but modern practice is to use no tile smaller than 3 in. in diameter. The size of tile selected for a given case should ordinarily be the commercial size nearest to and larger than the computed size, excepting that no tile smaller than 3 in. should be used. The following are diameters in inches of common commercial tile: 2, 2-1/2, 3, 4, 5, 6, 7, 8, 10, 12, 15, 18, 20, 21, 22, 24, 27, 30, 33, and 36. For main drains above 15 in. in diameter vitrified sewer pipe is commonly used.

26. Construction of Tile Drains

Staking Out. Tile drains should be set true to line and accurately to grade. When curves are necessary they should be smooth and regular. Before beginning excavation stakes should be set 50 ft. apart parallel to and about 2 or 3 ft. from the center line of the drain on the side opposite to that on which the earth will be thrown. Elevations of tops of stakes should be taken with a level and the cut marked on each. These stakes will serve as guides for the rough excavation. After the excavation is approximately to grade, batter boards should be placed across the trench opposite each stake at the same distance, preferably about 6.5 or 7 ft. above grade. The center line is then marked on the batter boards and a string connecting these points will be directly above and parallel to the grade line. The center line at any point may then be obtained by dropping a plumb bob from the string, and the grade by measuring down from it with a pole of proper length.

Excavation should be begun at the outlet. The trench may be dug either by hand or with a trenching machine. If much work is to be done the trenching machine will prove more economical. For 6-in. tile or smaller the trench may be made 12 in. wide at the top and 7 in. at the bottom.

Fig. 33. Tile Hook

Trenching Tools and Machines. The special tools commonly used are tile spades, a shovel, pick, drain scoop, and tile hook. There are two kinds of tile spades, solid and open, the latter being used for wet sticky soils. Tile spades are 16 to 22 in. long and 5 to 6 in. wide. A drain scoop is used for finishing the bottom of the trench preparatory to laying the tile. It should be curved to a radius approximately equal to the outer radius of the tile. A tile hook, Fig. 33, consists of a bent 1/2-in. rod attached to a wooden handle. There are a number of types of trenching machines on the market. They vary from different styles of plows, which loosen the earth so that it may be more readily shoveled, to large machines of the wheel bucket and endless chain type, which do all the work of excavation. The larger trenching machines vary in cost from $2200 to $7500. Trenching plows may be obtained at $50 to $300.

Laying of Tile like the excavation of trenches should begin at the outlet. If the bottom of the trench has been properly prepared the laying of the tile should proceed rapidly. The smaller sizs of tile are usually laid with a tile hook by a man standing at the top of the trench. Care should be taken to see that the tile joints fits tightly, especially at the top. Crooked and imperfect tile should be thrown out and if used at all they should be placed together near the upper end of the drain. Curves are usually made by using crooked tile or by chipping off one edge of the tile. It is easier and better, however, to have special curved shapes, either drain or sewer tile, available for the purpose. Y-junctions should be placed where needed, the open end being plugged up until the connection is made. Before the tile are covered the work should be inspected to see that all joints are properly made and that the tile are laid true to line and grade.

Backfilling. Before the trench is filled the tile may be surrounded by coarse hay, twigs, burlap, small stones, or pieces of brick. This provides for a freer entrance of water and helps to exclude fine sand from the drain. The first earth should be put into the trench carefully so as not to movc the tile. The remainder of the backfilling may be done with shovels, scrapers, or a turn plow.

Outlets. Where a tile drain discharges into an open waterway, Fig. 34, the outlet should be protected by building a substantial retaining wall around it. The foundation should extend well below the bottom of the channel so that the wall will not be undermined by the current in the waterway or the water discharging from the drain. Only vitrified clay, concrete, or metal pipe should be used near the outlet, as soft clay tile may disintegrate under the alternate conditions of freezing and thawing to which they will be subjected.

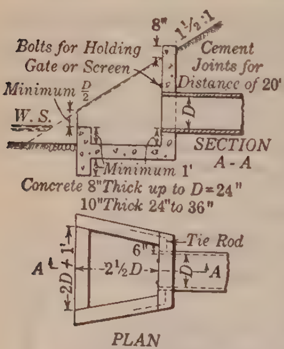


Fig. 34. Outlet Structures for Closed Drains

Surface-Inlets are sometimes constructed at low places to supplement surface drainage. They are commonly made of wood, concrete, or sewer pipe. They consist essentially of a small basin that extends to or below the bottom of the drain to which they are connected, with an opening at the top for the entrance of surface water. The surface opening should be covered with an iron grate which also should be protected by a covering of loose rock to prevent the entrance of dirt and drift.

Silt-Basins or Sand Traps are basins placed in drains or at the junctions of drains for the collection of silt and sand. They also afford an opportunity for the inspection of drains. The bottoms of the basins should be 2 or 3 ft. below the tile to provide a receptacle for the silt, which may be cleaned out as often as is necessary. These structures are not required for clay or loam soils, but are useful where drains are laid in sandy soils.

27. Costs and Profits

Cost of Round Drain Tile f.o.b. Factory

Prices quoted by manufacturers, subject to small discounts.

Size, in.	Length of one tile, ft.	Weight per linear ft., lb.	Price per 1000 ft.	Price of Y's, T's, ells and curves, each	Linear ft. per carload of 30 000 lb.
3	1	4	$30	$0.15	7500
4	1	6	45	.23	5000
5	1	8	60	.30	3750
6	1-1/2	11	82	.41	2760
8	2	16	120	.60	1875
10	2	23	172	.86	1300
12	2	31	232	1.16	970
14	2	38	312	1.56	790
15	2	43	350	1.75	700
16	2	45	375	1.88	670
18	2	60	500	2.50	500
20	2	71	600	3.00	420
22	2	100	850	4.25	300
24	2	110	950	4.75	280
27	2	140	1200	6.00	220

Amount of Tile per Acre. Where, over a considerable area, field drains are placed parallel and equal distances apart, the number of linear feet of tile required per acre may be obtained by dividing 43 560 by the distance between drains in feet. The following are linear feet for distances commonly used. An allowance of 4% should be made for breakage and imperfect tile.

Distance apart	25	30	33	40	50	66	80	100	150	200
Lin. ft. per acre	1742	1452	1320	1089	872	660	545	436	291	218

Cost of Drains. With labor at 40 cents per hour, for 6-in. tile or smaller, the cost of digging trenches 3 ft. deep by hand will be 3 to 5 cents per linear foot. On large contracts, the cost will usually be about 25% less with trenching machines than with hand labor. Trenches 4 ft. deep cost about 50% more than trenches 3 ft. deep. Laying tile and backfilling costs 1/2 to 3/4 cent per linear foot. If the ground is rocky or contains many roots the expense of excavation will be greatly increased. Very wet soil and especially quicksand may add to the cost of the work. The cost of laying a 12-in. drain will be about twice as great as the cost of laying a small drain to the same depth.

Profits from Drainage. The profits to be derived from farm drainage depend largely on the character of the land. The greatest profits usually result from the drainage of lands too wet for cultivation. The following examples are given in the Yearbook of the U. S. Department of Agriculture for 1914 as typical of results obtained from properly draining farm lands in the humid region of the United States.

In the Coastal plain of North Carolina about 25 acres that were producing nothing were tile drained for perhaps $250, probably not including costs of teaming and of supervision, and since then have produced a bale of cotton per acre. A field of 6 acres was drained for about $160, and the owner makes good crops on soil worthless without drainage. In the black prairie belt of Alabama, a field that had not been cultivated in years because too wet was drained with tile; then it produced one bale of cotton per acre and repaid the entire cost of drainage the first year. The following year the field yielded 50 bushels of corn per acre, twice the rate from other parts of the farm. Another drained field produced one bale of cotton per acre, whereas the undrained land produced only half a bale. A 10-acre field that yielded practically nothing in 1912 was tile drained, and in 1913 produced 60 bushels of oats per acre; in 1914 the rate was again 60 bushels of oats, in contrast to 10 bushels per acre from the adjoining 15-acre field planted to the same grain. The cost of most of the tile drainage in Alabama has been about \$25 per acre, some of it as high as \$30 to \$35, but increases of 50 to 200% in yields and the assurance of good crops every year instead of only in very favorable seasons are very satisfactory returns. The cost of drainage there has usually been repaid in 2 to 3 years by improved crops. In Iowa, a field of 40 acres too wet for planting was tile drained at a cost of \$24 per acre, after which it produced 60 bushels of corn per acre. Another field was drained for \$23 per acre, thereby increasing the yield from 15 bushels to 40 and 50 bushels of corn per acre. In Arkansas, on one of the state farms, 1 bale of cotton per acre was secured in favorable years, and nothing at all when the early part of the season was wet; the year following the installation of tile the yield was 1-1/2 bales per acre. In Nebraska a tract of more than 700 acres was tile drained at \$24.25 per acre, the pumping plant cost \$2 per acre, and as part of a larger district the cost of levees to protect from overflow was \$9 per acre. The improvement, at a total cost of \$35 per acre, immediately increased the crop on about 80 acres of corn 22 bushels, and on another part the increase in 2 years was from nothing to more than 30 bushels of wheat per acre.

28. Drainage of Irrigated Land

Necessity. More than 10% of the area in the United States, which has been under irrigation for a considerable period, has become partially or entirely unproductive through waterlogging or becoming impregnated with harmful mineral salts. These conditions are brought about by over-irrigation and inadequate natural drainage, and the remedy lies in supplementary artificial drainage. The necessity for artificial drainage or the system of drainage to be employed is not apparent usually until after several seasons of irrigation, but from a study of the topography and soil strata it should be possible to foretell approximately the future drainage requirements of a newly irrigated tract. Some irrigation projects do not require artificial drainage and others require it only to a limited extent, but in all cases the matter should be thoroughly studied in connection with the investigation and design of an irrigation system.

Alkali is the popular name applied to a number of mineral salts, harmful to vegetation, which are found in the soils of arid districts. The more common of these are sodium chloride, sodium sulphate, magnesium sulphate, calcium chloride, calcium sulphate and sodium carbonate. All of these except the last appear on the ground as a white crust and are called white alkali. Sodium carbonate, or black alkali, is the most injurious to plants. It is indicated by dark spots on the surface of the ground due to the dissolution of humus and vegetable matter which it causes. These salts are readily soluble in water and irrigation waters after passing through the soil become more or less impregnated with them. When the ground water reaches a sufficiently high elevation, moisture is drawn from it by capillary attraction to the surface of the ground, where the water is evaporated and the alkali is left as a thin deposit. After this action continues long enough sufficient alkali collects on the ground to injure or destroy vegetation. The height to which water will rise by capillarity varies from about 2 ft. for coarse sand to 5 ft. for fine clay.

Removal of Alkali. Lands which contain an excess of alkali either from natural causes or from irrigation usually may be cleansed by irrigation after adequate drainage has been provided. The water must be lowered below the limit of capillary action. and then by applying water the salts are dissolved and carried through the soil to the ground water. Usually two good irrigations will reduce the alkali on the surface to a harmless quantity, though in rare instances two years or more may be required for reclamation.

Objects of Drainage. The specific results to be accomplished by the drainage of irrigated land may be one or more of the following: (*a*) The lowering of the ground water to a level where it will not interfere with the penetration of plant roots to the required depth; (*b*) the removal of alkali from the surface of the ground; (*c*) the removal of excess soil moisture and the aëration of the soil. The problem differs from that encountered in humid regions in that alkaline soils are peculiar to arid districts and the amount of water which the land receives under irrigation may in a large measure be controlled.

Source of Water. A drainage system cannot be designed successfully until the source of the damaging water is known and its movement understood. The source of the water may be (*a*) irrigation water applied to the tract itself, (*b*) irrigation water applied to an area at a higher elevation, (*c*) a canal or reservoir at a higher elevation. In the first case the movement of the water will be downward through the soil. In the last two cases the water will move laterally either in porous strata near the surface or under pressure beneath an impervious stratum of hardpan or clay.

Methods of Drainage. The drainage of irrgated lands is accomplished by: (a) tile drains, (b) open ditches, or (c) pumping from wells.

Intercepting Drains are used to cut off the flow of water from higher areas. They are usually placed near the foot of a hill or at a change in slope. A secondary intercepting drain is sometimes placed below and approximately parallel to the main drain to collect water that passes the upper drain.

Parallel Field Drains, either tile drains or open ditches, are used where the water does not come from some clearly defined outside source. They should be designed to keep the ground-water level below the depth from which water will be drawn to the surface by capillary action. For fine soils the depth to ground-water should always be more than 4 ft. Drains are excavated to depths of 6 to 10 ft., and spaced 300 to 3000 ft. or more apart. The deeper the drains the greater the spacing that is permissible. The outlet may be either a natural channel or an irrigation ditch.

On the Rio Grande Project of the U. S. Bureau of Reclamation comprising 200 000 acres, about 1916, 3 years after beginning irrigation, 70% of the area was becoming unproductive through rise in ground-water. The soil consists of clays, fine sands and silt. The thread of the valley slopes 4 to 5 ft. per mile. This area was reclaimed by open ditches 1/2 to 3/4 mile apart and 10 ft. deep. After reclamation the ground-water sloped 1 in 1000 toward the drains, with an abrupt loss of head of about 1.5 ft. close to ditches. This loss of head must be considered in estimating the depth of drains. The discharge of the drainage ditches during the irrigation season is about 1 cu. ft. per sec. per linear mile. The cost per cubic yard of excavating by machinery over 10 000 000 cu. yd. of ditches was 8-1/2 cents. The cost of structures, engineering, overhead, and right of way including rebuilding of irrigation laterals was 7 cents per cubic yard of earth excavated. (See Engineering News-Record, Sept. 18, 1919, and Oct. 5, 1922.)

Fig. 35 shows a tract of land near Garland, Utah, that was reclaimed by a combined system of intercepting drains and field drains. Where water is carried from a higher area through a porous stratum the intercepting drain should extend down to the impervious material, as indicated in Fig. 36, otherwise some of the water will pass below the drain. Fig. 37 shows a method of intercepting water under pressure beneath an impervious stratum. This method may be used when the water is at a great depth.

Capacity of Drains. Where the water comes entirely from the irrigation of the tract being drained it is usual to provide for the removal of 0.05 to 0.1 in. of water per 24 hours. This is equivalent to 9 to 18 in. for a growing period of 6 months or a flow per acre of 0.0021 to 0.0042 cu. ft. per sec. If all or part of the water comes from other irrigated lands, their areas and the amount of water coming from them should be determined as accurately as possible. If it comes from irrigation canals the seepage losses in such canals

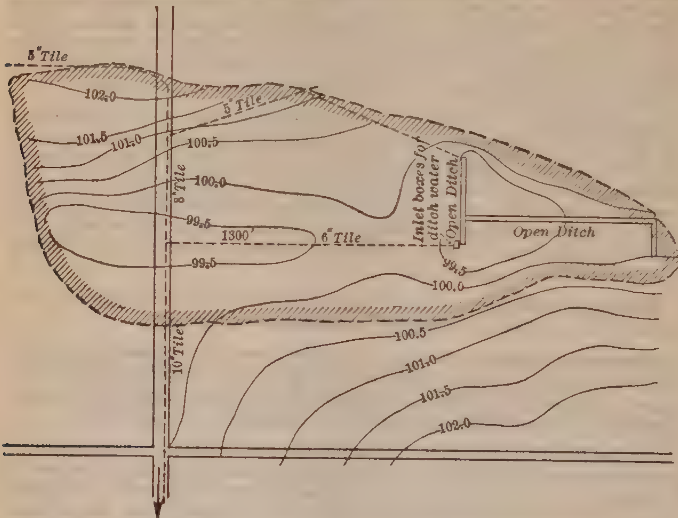


Fig. 35. Drainage of Irrigated Land by Intercepting and Field Drains

should be investigated. Nothing smaller than 4-in. drain tile should be used on irrigated lands.

The Cost of Trenching for the drains in irrigated districts will be relatively higher than in humid regions because of the greater depth of drains. If the banks cave, tim-

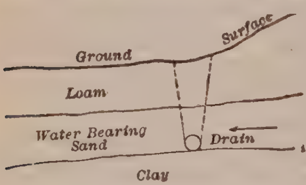


Fig. 36. Intercepting Drain

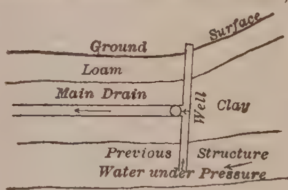


Fig. 37. Intercepting Drain for Water under Pressure

bering must be resorted to. In general the cost will be comparable to that of trenching for sewers and water mains at similar depths.

Drainage by Pumping from Wells has been in successful opration on the Salt River project to Arizona since 1922 and the success of this enterprise has influenced other districts to adopt the method. The method is particularly adapted to areas underlain by coarse water-bearing formations from which water may be pumped and to districts where cheap power is available. (See Bulletin 1456 of the U. S. Department of Agriculture, by J. C. Marr.)

In 1918, 64 000 acres of the 203 000 acres comprising the Salt River project were considered to be in immediate need of drainage. On this area 124 wells were driven to an average depth of 226 ft. The wells are 12 to 24 in. in diameter, most of them being 18 in. The total capacity of all wells is over 300 cu. ft. per sec., individual wells ranging in capacity from 1 to 10 cu. ft. per sec. The total cost of the drainage project was \$1 056 000. Of this amount about 90% was for pumping plants and power distribution system. During the year ending Sept. 30, 1923, 149 600 acre-feet of water were pumped. The cost of pumping was: Power, \$30 000; labor and materials, \$25 500; depreciation and overhead, \$56 500. The following are average data for the period: Lift, 33 ft.; power cost, 0.25 cent per kw.-hr.; total cost of pumping per acre-foot per foot of lift, \$0.023. A little more than half of the water pumped was used for irrigation. The remainder was wasted into natural streams, either because of the cost of conveying it to lands upon which it could be used or because of its unsuitability for irrigation. It is estimated that on the Salt River project the value for irrigation of the water pumped is as great as the cost of pumping. Economic conditions, especially the cheap power, were particularly favorable on this project.

Drainage Water for Irrigation. The outlets may be so situated that drainage waters can be used for irrigation. In many instances the value of such waters offsets to a considerable extent the cost of drainage works. All drainage waters are not suitable for irrigation because of the varying quantities of alkali salts which they dissolve from the soil. There is a diversity of opinion regarding the quantity of such salts that is permissible in irrigation water. The nature of the chemicals undoubtedly has an important bearing. It is generally conceded that water containing 0.1% by weight of solid material can be used safely. In some sections water containing twice this percentage of salts has been used without apparent injury to crops. Drainage waters containing salts in harmful quantities are often used for irrigation after mixing them with surface water.

Over-Irrigation. If water is applied to land only within the capacity of the land to contain capillary water the needs of vegetation are provided and no water is lost through seepage. The attainment of this ideal condition is not practicable but in general the use of less water will be beneficial in the following ways: (*a*) In saving irrigation water; (*b*) in reducing the amount of drainage required, and (*c*) in reducing the harmful effects of soil leaching. Whenever over-irrigation is practiced this process of leaching is in progress, and humus and nitrogen and various mineral elements of the soil which are valuable plant foods are washed away.

LEGAL AND ECONOMIC FEATURES

29. Water Rights

The Doctrine of Riparian Rights as commonly understood and interpreted is: "All persons owning land abutting on a natural stream have the right to demand that the waters of that stream shall pass their lands undefiled in quality and undiminished in quantity."

In the early history of the United States, the Common Law of England was generally accepted by the states and with it the doctrine of riparian rights. This doctrine is still in force in eastern United States. In western United States it is recognized only by California, Nebraska, Oregon, Texas, and Washington. In these states appropriation rights are also recognized and it is under the latter that most of the irrigating is done.

The subject of water rights is fully discussed by R. P. Teele in Bulletin 913 of the U. S. Department of Agriculture. The following is extracted from this bulletin:

Appropriation Rights. There has grown up in western United States a system of laws and customs controlling the use of water under which a farmer secures a " water right," that assures in greater or less degree his future water supply.

The use of streams for irrigation and like purposes is subject to control by the states, and must be in accordance with state laws. Actual use of water is necessary to hold a right and when the use ceases the right is forfeited. This is known as the doctrine of "beneficial use." Among users from the same stream the "first in time is the first in right." When there is not enough water for all, the rights are supplied, to the extent of the supply available, in the order of the dates on which they were acquired. This is known as the doctrine of "priority." Exceptions to this rule exist in a few of the states where, in cases of unusual shortage, the available water is apportioned among users either by state officers or by the courts. The date of a water right is fixed by the time of taking the first step to acquire it rather than by the time of putting the water to use. This is known as the doctrine of "relation."

Acquirement of Water Rights. In each of the western states except Kansas and Montana the acquirement of water rights direct from streams is under the control of state officials and any one wishing to get such a right must follow the procedure prescribed by law.

The Procedure is much the same in all the states and consists (*a*) in making application to some state official or board on forms supplied by the state, giving full information as to plans for irrigation works and use of water, (*b*) in carrying out of the plans as approved by the state, (*c*) in submitting proof of completion of work and use of water, and (*d*) in granting of certificates of license by the state, defining the right as to quantity of water, use to be made of water, and time during which it may be used. In Kansas and Montana it is required that a person wishing to acquire a water right shall post at the point of diversion and record with the County Clerk a notice showing the intention to take water, the amount to be taken, and the use to be made of it. The proposed work must begin within a reasonable time and must be prosecuted diligently to completion, and the water must be put to a "beneficial use." Anyone proposing to obtain a right to divert from a stream should consult the law of the state in which the land to be irrigated is located. The title to water is fully as important as title to land and it should receive the same careful attention.

Evidence of Title. Rights to water direct from streams are represented by the following evidences of title (*a*) filings in the county records; (*b*) filings in state engineers' offices; (*c*) certificates from courts, state engineers or boards; and (*d*) permits from state engineers or boards.

Rights to Underground Waters. Underground waters are divided into four classes: (*a*) underground streams flowing in known and defined channels; (*b*) underground streams flowing in unknown and undefined channels; (*c*) artesian waters; and (*d*) percolating waters. Although these classes are distinct in law, it is not always easy to tell to which class a particular supply belongs. Water which has long been considered in one class may later be found to be in another class and thus subject to a different law.

Subterranean streams flowing in known and defined channels are subject to the same laws as surface streams and one may not take water from such streams by means of wells or other means if it interferes with the rights of prior appropriators. Ownership of the land on which a well is located does nor give any right if the water is, in fact, a part of the stream. If a well draws water from an underground stream of which the channel is unknown and undefined, the ownership of land carries with it the right to take water. It is clear that the channel of such a stream may become known as a result of investigation, in which case a stream will become subject to the law of appropriation, and the prior user may stop use by later appropriators. Artesian water is held to belong to all the land overlying the artesian basin and each owner of such land is permitted to make any reasonable use of the water which will not interfere with a like use by all other land owners. In this respect, rights to artesian waters are similar to riparian rights. Percolating water, that is, water moving through the soil but not under pressure and not confined to a known and defined channel, belongs to the overlying land and the owner of the land may withdraw all he can get for use on his land. It is difficult to tell in which class the water found under any tract of land falls, but under the law all underground water is presumed to be percolating water unless it is proved otherwise.

Distribution of Water from Streams. Water from streams is distributed in canals in accordance with their right by public officials usually called water commissioners. Each commissioner has charge of the water within a certain district. He has a list of the rights showing amounts, dates, and locations, and distributes the water accordingly. In most states commissioners control diversions only when called upon by water users. When there is water enough for all, each takes it as he pleases. In the more highly developed communities, commissioners are on duty most of the time. Interference with the work of a water commissioner, by changing gates set by him, is a misdemeanor in most states.

30. Reclamation Laws

State and National Laws have been enacted for the purpose of encouraging or assisting irrigation development in the United States. Drainage statutes have been enacted by most of the states but there has been no national drainage legislation. There is, however, need for such legislation, particularly in connection with the reclamation of bottom lands along interstate streams where drainage is incidental to flood prevention and the problem is of national as well as local importance.

Federal and state policies with reference to land reclamation are discussed by R. P. Teele in Bulletin 1257 of the U. S. Department of Agriculture. Much of the material that follows has been taken from this bulletin.

Government Legislation for aiding irrigation began with the act of 1866. Difficulties had arisen regarding the right to divert water and the right to construct ditches over public land. This act recognizes rights acquired under local customs, laws, and court decisions and acknowledges and confirms rights of way for ditches over public lands. It stipulates that nothing in the act shall be " construed as affecting or intended to affect " state laws providing for the control of water used in irrigation.

The Desert Land Act as approved in 1877 provided for the procuring of title to 640 acres of arid land by conducting water upon it and the payment of $1.25 per acre. The entryman was required to spend at least $3.00 per acre in improvements and actually to reclaim at least one-eighth of the land. The area that may be taken by one person was limited by the act of 1890 to 320 acres. Under this act, a person may provide his own water supply or obtain a water supply from a system supplying many farms. In the latter case the entryman purchases a water right from the parties who build the irrigation works, and submits evidence of such purchase as proof of reclamation.

The Carey Act of 1894 grants to each of the states containing arid lands an area limited to 1 000 000 acres on condition that the states provide for its reclamation. The details are left to state legislation. The plan of operation under the Carey Act is for the states to contract with construction companies for building the works to reclaim specific areas of public lands. These contracts provide that the construction companies may sell " water rights " to reimburse themselves for the cost of construction, while the states sell the land to parties who have contracted for the purchase of water rights.

The United States Reclamation Act was authorized by Congress in 1902. It provides that all revenues derived from the sale and rental of public lands shall be employed in the construction of irrigation works for the reclamation of arid lands. Settlers acquire lands in areas that are estimated to support a family comfortably. Under some projects the farm areas are as small as forty acres. These are entered under the Homestead Act and the settler is given twenty years to complete his payments toward a proportionate interest in the irrigation works. **Water Users' Associations** are formed under each project and as the settlers make their payments they become active participants in the management of the irrigation works. Finally, the government withdraws and leaves all responsibility in the hands of the local organizations.

The Earliest State Legislation for aiding irrigation, as was the case with early federal legislation, was designed to remove obstacles to development rather than to provide direct public aid. State aid to irrigation development, with a few important exceptions, has been extended through irrigation district laws which provide means for making lands liable for the cost of their own reclamation.

Irrigation District Laws are similar in all of the arid states. These laws provide that a district may be organized only upon the petition of at least a majority of owners of land in the proposed district, who must also represent a majority of the acreage included; and upon a favorable vote (from a majority to two-thirds) of these land owners. If these conditions are fulfilled, land can be included in a district against the will of its owner and obligated for its share of the cost of providing a water supply.

California State Land Settlement represents a policy new in this country. The state buys land, provides irrigation and drainage works, subdivides the land, and sells the land on easy payments to actual settlers. The law authorizes the land settlement board, which administers the law, to prepare the land for cultivation, seed, plant and fence the land, and put up buildings or make other improvements. Settlers are required to pay at the time of purchase 5% of the price of the land and 40% of the cost of improvements. The balance is to be paid in amortized payments running not more than 40 years.

Federal Legislation Relating to Swamp Lands is limited in the main to the swamp land acts of 1849, 1850, and 1860. Under these acts the public swamp lands within their borders were granted to Alabama, Arkansas, California, Florida, Illinois, Indiana, Iowa, Louisiana, Michigan, Minnesota, Mississippi, Missouri, Ohio, Oregon, and Wisconsin. The total area conveyed to the states up to June 30, 1922, was about 64 000 000 acres. This granting of the swamp lands to the states explains the absence of any other federal legislation relating to the reclamation of swamp lands.

Drainage District Laws as enacted in various states are based upon similar principles as follows: (*a*) The consent of the majority of the interested land holders is necessary before a drainage district may be organized; (*b*) the drainage district may may not be organized unless it is clearly shown that the benefits to be derived will exceed the cost; (*c*) no drainage district may be organized unless it is shown that the work will be conducive to public health or general welfare. In general the state laws require the presentation of a petition by interested land owners, accompanied by a bond, setting forth the necessity and nature of the improvement. The drainage laws of some states provide for the appointment of county drainage commissioners and engineers.

31. Cost and Value of Project

The Cost of a Reclamation Project may be divided into five general divisions: (*a*) Preliminary costs, (*b*) the cost of construction, (*c*) the cost of financing, (*d*) the cost of colonization, and (*e*) the cost of preparation of land.

Preliminary Costs includes costs of surveys and investigations, rights of way, damages, legal fees, and all expenses incurred in perfecting rights and titles to water and land. In making preliminary estimates each of these items should be investigated with particular care.

The Cost of Construction includes the cost of labor, materials, supplies, equipment, supervision, overhead and all other expense required to complete a system of canals and structures that will deliver water to each farm in such quantities and at such times as is required. The cost of engineering is usually 5 to 10% of the construction cost. In preparing cost estimates provision should be made for unforeseen contingencies, particularly in connection with structures along streams which are liable to damage by floods. Unexpected difficulties are often encountered in excavating for subaqueous foundations.

The Cost of Financing. Interest on investment begins with the first expenditures and increases in amount as work progresses until the project is completed. As each tract of land is settled it carries its proportionate share of the interest charge. In order to reduce this expense to a minimum the construction work should be carried on without undue delay. If projects are financed by bonds the bond discount is a charge against the cost of the project.

The Colonization of a reclaimed area is expedited through the medium of an independent advertising and selling organization. The advantages of a newly reclaimed project must be advertised and opportunities for showing the land must be provided. Frequently free transportation from long distances is furnished by colonizing agencies to prospective settlers. It is important that a project shall be settled as promptly as possible in order to reduce interest and secure early returns on the investment. A vigorous colonizing campaign is therefore generally advisable even though a larger expenditure will be required than if settlement is extended over a longer period. The cost of colonization may vary from $5 to $25 per acre. It is usually included in the purchase price of the land and is thus paid by the settler. In any event it should be considered as a charge against the cost of the project.

The above comments refer more particularly to projects developed by private capital, but in general the same principles apply to government reclamation work. Under the terms of the U. S. Reclamation Act, the reclamation fund is available without interest and the expenses resulting from delays in construction and colonization, though just as real, are not so readily apparent. The government has provided no method of effectively advertising and colonizing its projects, and much of the available reclaimed area has not been settled.

The Cost of Preparation of Land for cultivation (Art. 13) includes the expense of clearing, grading, construction of farm ditches and drains, and all other work incidental bringing the raw land to a productive state. Interest on the investment during the non-productive period is an important item of the cost.

Value of Project. Unless the value of a project after the land has been colonized and brought to a productive state is greater than the total expense involved in the development the project is not a sound business venture.

Failure of a Reclamation Project may be caused by mistakes in engineering such as faulty designs, underestimating construction costs, or, for irrigation projects, over-estimating the available water supply. Failure may also result from poor business management, inadequate arrangements for financing, colonization difficulties, or the short-sighted policy of putting an exorbitant price on land. Another and perhaps the most common cause of failure has been the inexperience of the settlers and their inability to finance themselves through to the time when the revenue from their farm equals or exceeds expenditures.

Dr. Elwood Mead is of the opinion (Trans. Am. Soc. C. E., 1927, Vol. 90, p. 733) that any development scheme will break down that is based on the idea that settlers can be secured who have sufficient capital of their own. The records of the U. S. Bureau of Reclamation show that 70% of the applicants for farms have less than $1500 capital and only about 10% have more than $2500. If the settler with $2000 capital takes a farm that will require an expenditure of $4000 to make it a going concern, the additional $2000 that is needed should be provided from some source on terms entirely different from those which local banks will give. Money for changing the raw land into farms must be advanced on long-time payments at a low rate of interest.

In Australia preparing land for irrigation is regarded as a part of construction and the settlers take land that is ready for immediate cultivation. Payments are amortized. A small yearly payment keeps up interest and eventually pays off the debt. A few private projects in the United States and the state projects at Delhi and Durham, California, have been developed under this plan and have been successfully colonized.

32. Operation and Maintenance

Organization. The right to participate in the benefits of an irrigation or drainage system is inherent with every tract of land which it embraces. This implies that every land owner within a project is entitled to a voice in its management. In some instances reclamation works are owned and operated by companies who contract with land owners to provide the benefits of the system at a fixed annual rate. More commonly the ownership of reclamation works ultimately passes to an organization composed of the land owners each of whom participates in the operation of the system proportionately to the area of land which he owns. This may be accomplished by forming a stock company in which one share of stock is issued for each acre of land. Such an organization has general direction of the operation of the system.

Water Users' Associations are formed by the water users of the projects of the U. S. Bureau of Reclamation as soon as it appears probable that a project is to be constructed. The object of these associations is to facilitate dealings with the Department of the Interior and, after the construction work is completed, to participate in the operation of the project. The ownership, and operation of storage works remains in the hands of the Government while the distribution of water is left to the management of the Water Users' Associations.

The Operation of a reclamation system may be divided into three general divisions: (*a*) The operation proper, which includes the operation of pumps and other machinery, and for irrigation projects the delivery and distribution of water; (*b*) maintenance; and (*c*) clerical work. The force necessary will depend upon the size of the project. The organization should be such as to provide for effective and efficient service from the system, to keep the project maintained in a satisfactory operating condition, and to keep all records and accounts in a proper manner. General supervision will rest with the board of directors, or its equivalent, who determine the general policies of the system. Direct supervision is under a general manager, who is the responsible head.

The Head of the Operating Department of a drainage project may be the chief engineer of the pumping plant. On irrigation projects he is the water superintendent and has charge of the distribution of the irrigation water. Under the superintendent are ditch riders who apportion the water among the various users. One ditch rider can oversee the distribution of water for 2000 to 3000 acres, where water is controlled at the farmers' headgate. If delivered to lateral headgates he serves from 5000 to 10 000 acres. The water superintendent should be thoroughly versed in the principles of water measurement and the ditch riders should be sufficiently skilled to be able to make an equitable distribution of the water.

Maintenance as applied to reclamation projects includes the repairs and labor necessary to keep the system in successful operation. Repairs to canals or structures damaged by floors, cleaning canals, replacement of broken parts of machines and similar work belong properly to maintenance, and it may also include minor extensions and betterments and the replacement of inexpensive structures. A regular maintenance force should be employed to keep the system in satisfactory operating condition, except in case of unusual accidents, in order that the expense of maintenance for different years may, as nearly as practicable, be uniformly distributed.

Depreciation. There is a natural limit in the life of many of the parts of a reclamation system. Canals, levees, earth and rock dams and similar structures if properly maintained, and not accidentally destroyed, should last indefinitely. Also it cannot be said that there is any limit to the life of many properly constructed concrete or masonry structures. Wooden and steel structures, however, will eventually deteriorate to such a state that they can no longer be maintained in a satisfactory working condition. The depreciation of the different parts of a reclamation system should be estimated in advance and provisions for replacements should be made by an amortization fund created by equal annual contributions, which are included as a part of the operating expense.

The Average Life, in years, of various parts of reclamation systems is approximately as follows:

Wood flumes, redwood	15 to 20
Wood flumes, fir	12 to 15
Wood flumes, pine	8 to 10
Wood-stave pipe, fir, uncoated	12 to 20
Wood-stave pipe, fir, well-coated	20 to 25
Wood-stave pipe, redwood, uncoated	20 to 25
Wood-stave pipe, redwood, well-coated	25 to 30
Miscellaneous small wood structures	10 to 20
Steel riveted pipes, uncoated	20 to 30
Steel riveted pipes, well-coated	30 to 40
Centrifugal pumps	12 to 20
Steam engines	15 to 25
Boilers	12 to 20
Crude-oil engines	10 to 15
Pole lines for telephones or transmission	10 to 15
Electric motors	15 to 20
Buildings and improvements	25 to 75

The life of a structure depends largely upon the care that is taken to preserve it and the conditions of use to which it is subjected. Wood that is alternately wet and dry will deteriorate more rapidly than if kept continually wet. Structures above water may be preserved by keeping them well painted. Wood or steel buried in the ground or submerged or in contact with water for considerable periods will last longer if well coated with asphaltum or tar. The following relative to the life of wood-stave pipe (by D. C. Henry, Reclamation Record, August, 1915) was collected from a large number of installations:

(*a*) Under favorable conditions of complete saturation, fir well coated may have the same life as redwood uncoated.

(*b*) Either kind of pipe will have a longer life if well buried in tight soil than if exposed to the atmosphere. Such life may be very long, 30 years or over, if a steady pressure is maintained.

(*c*) Either kind of pipe will have a longer life if exposed to the atmosphere than if buried in open soil, such as sand and gravel and volcanic ash, provided in a hot or dry climate the pipe is shaded from the sun.

(*d*) Under questionable conditions, such as light pressure or partially filled pipe, fir, even if well coated, may have only one-third to one-half the life of redwood.

(*e*) Under light pressure the use of bastard staves should be avoided.

(*f*) The use of wooden sleeves in connection with wire wound pipe is objectionable and has caused endless trouble and expense.

(*g*) If wooden sleeves are employed they should be provided, at least for sizes from 10 in. up, with individual bands to permit taking up leaks.

Tar Paint has been found by engineers of the U. S. Bureau of Reclamation to be superior to any other kind of paint for all submerged metal work. It is also used exclusively for wood-stave pipe and the interior surfaces of metal flumes to prevent both erosion and corrosion. It is considered very satisfactory for use on wooden structures, or in any place where a black paint with a slightly sticky surface is not objectionable.

Specifications of Bureau of Reclamation for Tar Painting. All metal work, except as herein provided, shall be thoroughly cleaned of all loose scale and given one coat of water-gas tar followed by two coats of coal-gas tar. All coats shall be applied when the temperature of the air and metal is not less than 60° F. The water-gas tar shall have a specific gravity of not less than 1.05 nor more than 1.10 at 60° F., and shall be of such consistency that it can be applied with brushes. The melting point of the coal-gas tar, as determined by the cube method, shall lie between 105° and 110° F. Both the water-gas tar and the coal-gas tar shall be freed from moisture, and all fat shall extracted from the water-gas tar. If the water-gas tar is too thick to spread, the contractor may use suitable light oil, satisfactory to the engineer, for thinning. The water-gas tar may be applied without heating, but the first coat of coal-gas tar, which may be applied a few hours after the application of the water-gas tar, shall be applied hot and brushed out as thin as possible so that the coating will not run or peel after it is dry. The second coat of coal-gas tar shall not be applied until after the first coat has set. This tar paint shall be well worked into all joints and open spaces. Pin-holes, screw threads, and all machine-finished surfaces shall not be painted, but shall be coated with white lead and tallow as soon as they are finished. (Reclamation Record, Jan., 1921.)

Annual Cost of Operation includes all expenditures required to secure the benefits from the reclamation works and maintain them in working condition. There is a wide variation in costs for different projects. The following represent the annual range of costs on a large number of projects:

Cost per acre of distributing water..........	\$0.40 to \$1.00
Cost per acre of maintenance..............	.50 to 1.20
Cost per mile of canal maintenance.........	50.00 to 125.00
Cost per acre of general expense............	.40 to .80
Cost of pumping water. (See Art. 18.)	
Cost of depreciation; each part of system must be considered separately.	
Total cost per acre of operation.............	2.00 to 4.00

Other costs are taxes, insurance, interest on investment and sinking fund, any or all of which may be included as a part of the annual expense.

33. Engineering Reports

Purpose. A report on a land reclamation project should set forth, in clear and concise form, the essential characteristics of the enterprise. All data on which conclusions are based should be given, in order that the report will withstand a critical inspection by other engineers, but the controlling features, on which the feasibility of the project depends, should be intelligible to any good business man. Voluminous details, such as runoff, temperature, and precipitation records may be included in an appendix. Reports are always required in connection with the financing of reclamation projects, and their value in this regard may depend, in a large measure, on the standing and reputation of the engineer who makes them. In order to bring a project properly before investors, the promoters of the enterprise should have the report of a reputable engineer. Financial concerns, before investing, ordinarily require an independent report from their own engineer. Frequently several reports are made before a project is financed. Reports may be made at intervals during construction to determine whether work is progressing satisfactorily and in case of failure the creditors may require a report to determine the status of their securities.

An investigation must precede the preparation of a report. This includes a field examination and a search for all data bearing on the development or success of the project. Ordinarily all surveys and other data of the field engineers will be available and these should be scrutinized with sufficient care to justify conclusions as to their reliability. Information relative to land values, crops, market conditions and similar matters may frequently be obtained from interviews with reliable persons, but data obtained from such sources should be accepted with caution. Government reports should be used if possible for securing temperature, precipitation, and stream flow data. Information obtained from investigating other projects in the locality is always valuable. Frequently additional surveys, test pits or other work will be required, but these should not be of extensive character. If data sufficient for an intelligent report are not available, this fact may be stated in the report, or the investigation may be postponed until full and complete data are secured. The field investigation ordinarily should not require more than one or at the most two weeks.

The Contents of a Report should be such as will set forth clearly the feasibility of the project from both engineering and business standpoints. In general, the main points to be shown are: (*a*) the efficiency of the reclamation system, (*b*) the cost of reclamation, and (*c*) the value of the project after reclamation or the net profit to result from the development of the project. The material should be presented in logical order and each subject should be discussed under an appropriate heading. The following are headings, with outlines of subject matter to be treated under each. They will have to be modified to suit the particular conditions of different projects:

(1) **Introduction.** There should be a few introductory statements leading up to the main body of the report. These may include the authority for making the report, the data on which the report is based, the engineer's opinion of the reliability of such data, the period covered by the investigation, and other general information of a similar nature.

(2) **Location.** It is important that the general location of the project should be described. Principal boundaries, railway and water transportation facilities, and distances to the more important markets should be given. A small scale general map showing this and other information will be valuable.

(3) **General Description.** A general description of the project should be given before entering into details. This may include important topographical features, the area of the project and a comprehensive outline of the scheme of reclamation. A map of the project showing the general plan of reclamation should be included with the report and referred to in the discussion.

(4) **Climate.** Since the growth and time of marketing crops depends in a large measure on climate, this subject may be discussed in considerable detail. Tables may be prepared from government records showing maximum, minimum, and mean temperatures and amounts of precipitation for the different months of the year. Dates of last killing frosts in the spring and first killing frosts in fall should be given.

(5) **Soil.** The soil formation of the project from the standpoints of texture, fertility, adaptability to particular crops, and drainage conditions should be discussed. If the soil formation varies greatly a soil map may be included. Government and state publications give valuable data relative to soils for many localities.

(6) **Crops.** Information as to the kinds, yields, and values of crops to which the lands of the project are particularly adapted should be included in the report. The costs of growing, the times of marketing, and the net returns to be expected from producing different crops may also be given. Information of this kind may frequently be shown to advantage in tables.

(7) **Markets.** The market for crops is usually an important consideration, and this should be discussed. Transportation facilities and freight rates should be given consideration in this connection.

(8) **Water Supply.** The success of an irrigation project depends in a large measure upon the adequacy of its water supply and this subject should be treated with considerable detail. The two things to be determined are the water supply available and the water supply required, and the analysis of the problem should show the data used and

the steps taken in arriving at conclusions in regard to these matters. The discussion may be incorporated under the following headings:

(8*a*) **Period of Irrigation** showing the time when different crops will ordinarily be irrigated and the number of irrigations for different crops, with conclusions as to the approximate distribution in the use of water.

(8*b*) **Water Requirements** for crops or duty of water based upon a study of the needs for irrigation of the crops likely to be grown on the project.

(8*c*) **Water Supply Necessary** for the project determined from considerations of plant requirements, actual area to be irrigated, and seepage and evaporation losses in canals and reservoirs.

(8*d*) **Water Supply Available**, determined from a study of stream flow and other hydrological data, with proper deductions for prior rights to water. This phase of the problem will require careful investigation and all conclusions should be supported by as many records as practicable. If such records are too voluminous to be incorporated in the body of the report they should be put in the appendix.

(9) **Runoff.** The runoff from an area to be reclaimed by drainage should be discussed in detail and all precipitation records and other data necessary to support conclusions should be given. If pumps are to be used at the outlet the reasons for selecting a pumping plant of a certain capacity should be fully explained.

(10) **Cost of Project.** An estimate of the cost of developing a project, up to the point when it will become a going concern, should be included in the report and the main elements entering into the cost should be discussed. Estimates of this class should be made on a liberal basis so that ample allowance will be made for all contingencies.

(11) **Annual Cost.** An estimate of the annual cost of operating the project should be submitted. This, like the estimate of cost, should be on a liberal basis.

(12) **Value of Land.** The various conditions affecting the value of lands after reclamation should be discussed and an estimate of values should be submitted. It is important to distinguish between the value of raw lands after reclamation and the value of cultivated lands.

(13) **Net Returns.** An estimate of net returns on the investment may be submitted. This will be obtained by deducting the total expense of reclamation from the estimated value of land.

(14) **General Conclusions.** A few short, clear-cut paragraphs summarizing the main points brought out in the investigation should be given at the beginning or at the end of the report. The conclusions should contain definite statements of opinion regarding the value of crops, the adequacy of water supply for irrigation projects, the estimated cost of reclamation, the cost of operation, the value of land, estimated returns from the investment, and other important features. In some instances the report may be concluded with the engineer's recommendations.

SECTION 17

SEWERAGE AND SEWAGE DISPOSAL

BY
METCALF & EDDY
ENGINEERS

SEWERAGE SYSTEMS

DISPOSAL OF SEWAGE

SEWERAGE SYSTEMS

1. Definitions and Purposes

Sewage is a combination of the liquid wastes conducted away from residences, business buildings and institutions, together with those from industrial establishments, and with such ground- and storm-water as may be present. **Domestic Sewage** is that from residences, business buildings or institutions. **Industrial Wastes** are the liquid wastes resulting from the processes employed in industrial establishments. **Storm-Water** is that portion of the precipitation which runs off over the surface during and for a short time following a storm. **Ground-water** is that which is standing in or passing through the ground.

A Sewer is a conduit for carrying off sewage; a **Drain**, one for storm or ground water. A **Combined Sewer** is one intended to receive domestic sewage, industrial wastes and storm-water. A **Separate Sewer** is one from which storm-water is intended to be excluded. **Underdrains** are sometimes provided beneath separate or combined sewers, for the purpose of removing ground-water and reducing the amount of infiltration into the sewers.

A **Building Connection** is a pipe leading from the plumbing system of a building to a sewer. A **Lateral Sewer** is one receiving sewage from building connections only; a **Submain** or **Branch Sewer**, one receiving the discharge of two or more laterals; a **Main** or **Trunk Sewer**, one receiving the discharge of two or more submains.

An **Outfall Sewer** is one extending from a collecting system of conduits to a point of discharge or treatment. An **Intercepting Sewer** is a sewer provided to intercept the sewage from a number of outfall sewers, or the dry-weather flow of sewage from one or more combined sewers, and convey it to a suitable point for treatment or disposal. A **Force Main** is a conduit in which sewage is forced under pressure from a pump. An **Inverted Siphon** is a section of a conduit which is depressed below the hydraulic grade line, and which is always full and under pressure. Although the term is generally used, it is a misnomer, since siphonic action does not occur in an inverted siphon.

A **Sewerage System** is made up of a system of sewers with all necessary appurtenances, connected and in working order. If composed of separate sewers it is a **Separate System;** otherwise it is a **Combined System.** The separate system may have some advantage when it is necessary to treat the sewage before disposal. It is also likely to be desirable for small communities in which storm-water can be adequately cared for by street gutters and natural water courses, and storm-water drains are not required, or, at most, are needed for only part of the area served. Separate systems are subject to misuse from the connection of storm-water inlets and roof-water drains to separate sewers, resulting in their surcharge and the flooding of basements. This sometimes happens as the result of carelessness when storm-water drains are available. This is less likely to occur if it is made standard practice to use two sizes of pipe for connections, as for instance 5-in. for building connections to the sewer, and 6-in. to the drain. A similar result may be obtained by requiring all pipes of one system—for instance, the building connections for sewage—to have the socket painted with some conspicuous color, and insisting that only pipes so marked may be connected to the separate sewers.

2. Quantity of Sewage and Storm-Water Runoff

The quantity of domestic sewage and industrial wastes is generally related rather definitely to the population, and will increase substantially in direct ratio to the increase in population. It is necessary therefore to make a careful study of statistics of population and a forecast of the future population as a basis of estimating the total quantity of sewage. In the majority of cases the total daily volume of domestic sewage and industrial wastes approximates the daily consumption of water from the public supply. Some water used for sprinkling lawns, making steam, and other purposes is not discharged into the sewers after use; but ground-water and water from private wells and secondary supplies is likely to reach the sewers in sufficient quantity to make up for it. In some places the amount of water obtained from other sources than the public supply may be so great that the quantity of sewage may largely exceed the recorded water consumption.

The maximum rate of sewage flow is generally from two to three times the annual average rate of water consumption. In the smaller communities the per capita water consumption is generally less and the range between average and maximum rate is greater than in the larger cities. With water consumptions from 60 to 125 gal. per capita daily, the **Maximum Rate of Domestic Sewage Flow** may be expected to be from 200 to 250 gal. per capita daily.

The **Quantity of Industrial Wastes** and the rate of discharge may vary so widely in different communities and with different classes of industries that no general statement can be made. Data must be obtained locally for the conditions to be met. Frequently the total quantity of sewage other than ground- and storm-water from a community is not materially affected by the industrial wastes, and the maximum rate of flow may be estimated as a definite number of gallons per capita daily.

The **Amount of Ground-Water** entering the sewers varies widely, depending upon the extent of the sewers below ordinary ground-water elevation, and the type and excellence of construction; it also fluctuates with the season of the year and the conditions regarding precipitation. In general it is not practicable to keep out all ground-water, and allowance must be made for such a maximum rate of ground-water infiltration as is likely to occur under the circumstances. Data relating to actual infiltration are fragmentary, but indicate that maximum rates of 50 000 gal. per day per mile of sewer are not uncommon in systems supposed to be well built and maintained. In design it is best to allow for some quantity, such, for example, as 1000 or 2000 gal. per day per acre served by the sewer.

The **Maximum Rate of Sewage Flow** to be used as the basis of design of separate sewers is obtained by adding the rates for domestic sewage, industrial wastes, and ground-water. Assuming an area of 1000 acres, a density of population of 40 persons per acre, a rate of 200 gal. per day per person for domestic sewage, and 2000 gal. per day per acre for ground-water, the maximum rate of sewage flow would be 10 000 000 gal. per day, exclusive of industrial wastes. The conditions as to density of population to be expected will depend upon local conditions, zoning control and the like, and must be analyzed for each community. The probable average density of population is greater in small districts than in large ones, and therefore lateral sewers and submains should generally be designed for greater capacity in proportion to the areas served than the trunk sewers and intercepters. The design of separate sewers in a portion of Louisville, Ky., was based upon future densities of population as follows:

Extent of district, acres	10	100	500	1000	5000	10 000
Density of population, persons per acre	100	63	46	40	29	25

Storm-Water. The rate of storm-water runoff is best estimated by the **rational method**, expressed by the formula $Q = CiA$ where Q is the rate of runoff in cubic feet per second; A is the drainage area in acres; i is the average intensity of precipitation in cubic feet per second per acre (practically, the rate in inches per hour); and C is the "coefficient of runoff." The rate of precipitation i should be that which may occur in the period of time, called **"time of concentration,"** required for water to flow from the most distant point of the drainage area over the surface to the nearest inlet ("inlet time") and thence through the drains to the point for which the rate of discharge is required ("time of flow"). The inlet time can be estimated only on the basis of experience and local conditions; it will seldom be less than 3 or more than 20 minutes. The time of flow can be computed with sufficient accuracy by dividing the lengths of the several sections of drain by the estimated velocities of flow. It is seldom advisable to construct drains large enough for the maximum storm-water flows which may occur and it is assumed that the drains may be surcharged occasionally, say at average intervals of five, ten or fifteen years.

For the satisfactory determination of the rate of rainfall to be expected in any period of time and for a certain time of recurrence it is necessary to have at least 20 years' records of a recording rain-gage. Analysis of such records will show for any given duration of rainfall the rate of precipitation which has been equaled or exceeded, 1, 2, 3, and 4 times in the period covered by the record. If this is 20 years, the rates correspond approximately to those which have occurred at average intervals of 20, 10, 6-2/3 and 5 years. A smooth curve drawn to represent the points for various durations and a single

Meyer's formulas for value of i are as follows: (t = time in minutes):

District	Average Frequency, 25 years	Average Frequency, 10 years	Average Frequency, 5 years
1. Gulf Coast	$\frac{355}{t+40}$	$\frac{276}{t+32}$	$\frac{220}{t+27}$
2. Atlantic Coast from Narragansett Bay to Savannah; most of Georgia, Alabama, Mississippi, Arkansas, Missouri, Kansas, Oklahoma, and central Texas	$\frac{252}{t+28}$	$\frac{214}{t+26}$	$\frac{171}{t+23.5}$
3. Central New England and New York; region between District 2 and Great Lakes, as far west as 100th meridian	$\frac{181}{t+21}$	$\frac{150}{t+19.5}$	$\frac{122}{t+18}$
4. Northern New England and New York	$\frac{160}{t+20}$	$\frac{132}{t+19}$	$\frac{108}{t+17.5}$
5. Eastern Montana and Wyoming, Western Dakotas and Nebraska, Northeastern Colorado	$\frac{126}{t+14}$	$\frac{105}{t+13}$	$\frac{90}{t+13}$

frequency of recurrence may be taken as the curve of time-intensity relation for that frequency. If precipitation records are not available, recourse must be had to those of some neighboring locality, or general formulas like those of Meyer must be used.

The determination of the coefficient of runoff requires the application of judgment to a greater degree than any other element which enters into the estimation of storm-water runoff. No exact rules for computing it can be formulated. If an area were entirely impervious, and rain fell at a uniform rate, there would be a time when water was running off as fast as it fell, except for slight evaporation, and the coefficient would be 1.00; but comparatively few surfaces are absolutely impervious, and considerable water is required to wet surfaces, fill depressions, build up sufficient depth to provide for flow, and the like, so that the coefficient seldom reaches unity for so-called impervious surfaces. Pervious surfaces vary in character so widely that their coefficients may be almost zero, or nearly as high as for impervious surfaces. In estimating the rate of runoff for design of storm-water drains, it is necessary to allow for conditions as they are likely to exist some time in the future, and fix the runoff coefficients for such conditions, including the effects of varying proportions of impervious and pervious surfaces, the variation of precipitation from an uniform rate, the difference due to varying times of concentration, and all other influences which can affect the rate.

Values for runoff coefficients have been commonly assumed similar to the following:

Kind of surface	Duration of heavy rainfall, minutes						
	10	20	30	45	60	90	120
Impervious..........	0.60	0.80	0.85	0.90	0.95	0.95	0.95
Pervious.............	0.20	0.35	0.40	0.45	0.50	0.55	0.60

Values for pervious surfaces might vary considerably, depending upon kind of soil and character of cover. The coefficient for any given area would be

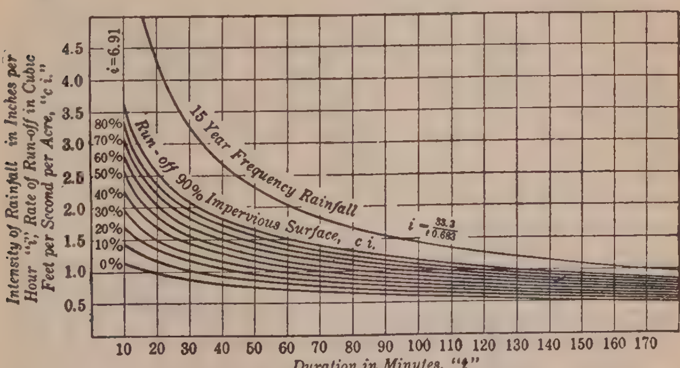


Fig. 1. Runoff Curve for Rainfall of 15-year Frequency at Louisville, Ky., corresponding to Various Proportions of Impervious Surface

obtained by combining the coefficients for impervious and pervious areas in accordance with the ratios of such areas, and further modified for the effect of any other factors requiring consideration.

In the application of the rational method to the estimation of storm-water runoff, it is convenient to construct curves similar to those in the following figure. The rainfall curve for 15-year frequency was adopted, and the run-off coefficients for pervious and impervious surfaces given in the table above were taken as basic coefficients, but were modified for the effect of time of flow—that is, allowance was made for the fact that at the end of any time of concentration the flow would be made up of water which fell at the beginning of the period upon the most distant parts of the area and has since been flowing to the outlet, and of water which fell in the last few minutes of the period on the portion near the outlet, so that various zones would be contributing water to the outlet at differing rates.

3. Sizes and Slopes of Sewers and Drains

Having estimated the maximum rate of sewage or storm-water flow, the sizes of sewers should be fixed to provide capacity for that flow. The practice

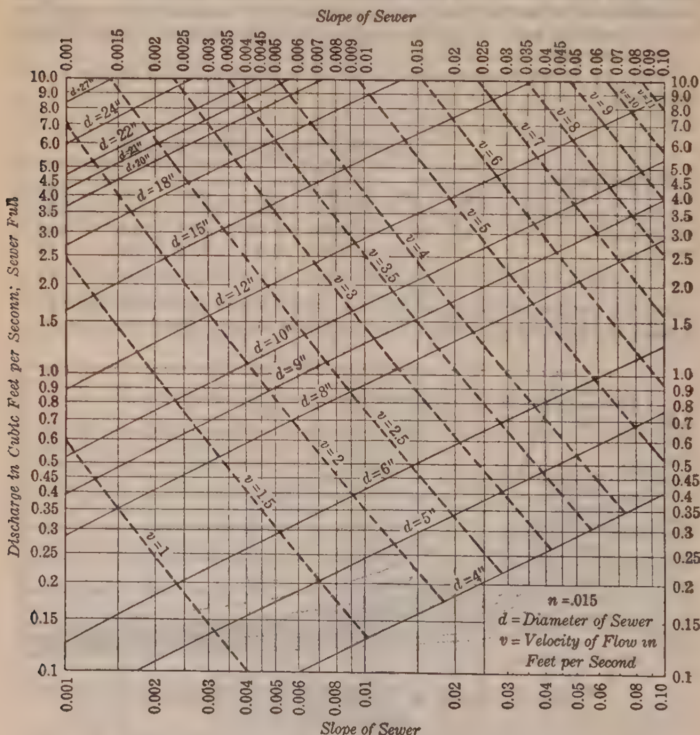


Fig. 2. Discharge of Circular Sewers for $n = 0.015$ and Steep Slopes

of providing sewers large enough to carry the maximum estimated flow when half full, especially in the case of separate sewers, was formerly widespread. This provides a 100% margin as a "factor of ignorance." While there are many elements of uncertainty in the estimation of capacity to be provided, it is now recognized as good practice to make all allowances in the estimate of quantity of sewage, and provide sewers large enough to carry these quantities when flowing full but not under pressure. In the case of separate sewers

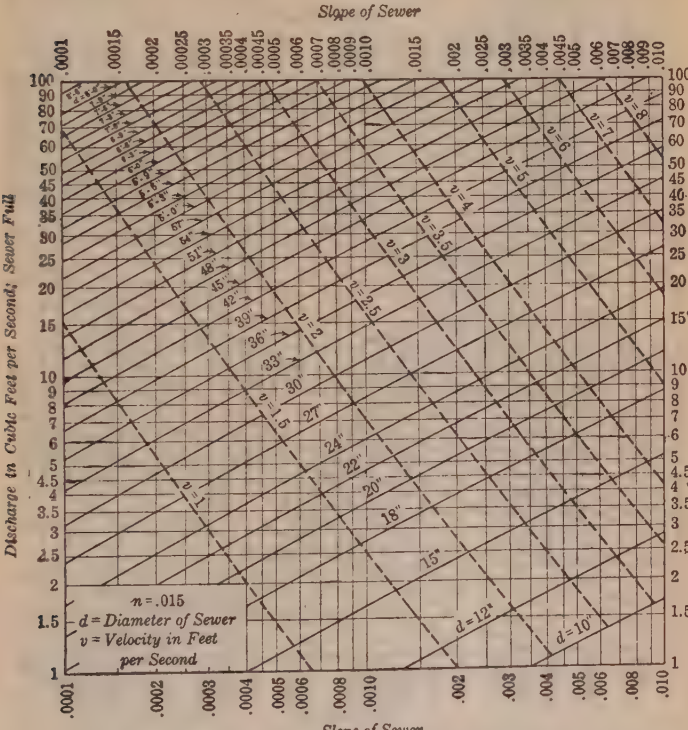


Fig. 3. Discharge of Circular Sewers for $n = 0.015$ and Flat Slopes

it may sometimes be advisable to make an allowance beyond the quantity of sewage and ground-water estimated as described above, for roof water which may be admitted by mistake or surreptitiously. In the case of combined sewers, the maximum rate of storm-water flow is usually taken as the basis of design, since the quantities of domestic sewage and industrial waste are so small in comparison that the size of sewer required is unlikely to be affected by their omission, except that in cases of unusual amounts of industrial wastes, the maximum flow should be taken as the sum of the maximum storm-water and the maximum industrial wastes.

Before determining the sizes of sewers, it is necessary to fix tentatively the slopes to which they shall be laid. In general, the topography will limit the possible slopes of the sewers. The depth at which sewers are to be laid is also an important consideration; they must be far enough below the surface to serve the lowest basements, unless the latter are so very deep as to make this impracticable, in which case the use of ejectors in deep basements may be necessary.

Having determined the capacity to be provided, and the slope, the size of

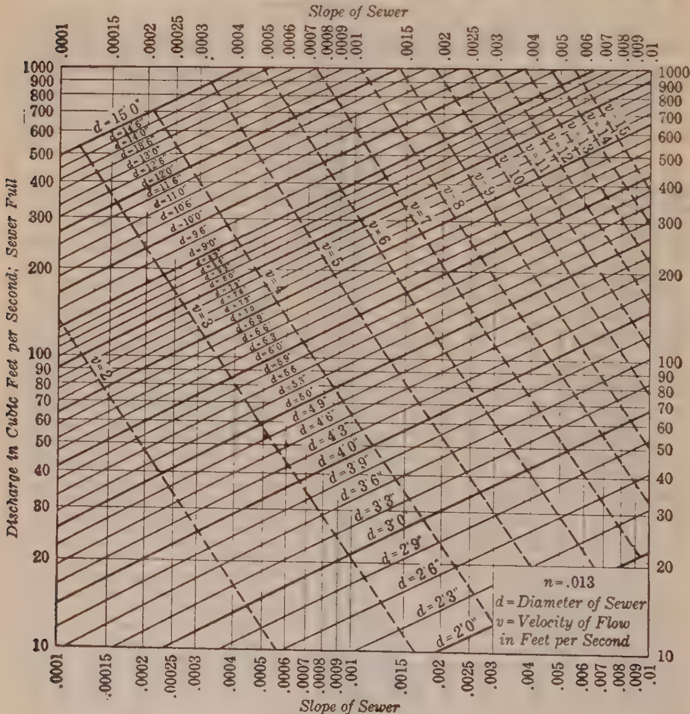


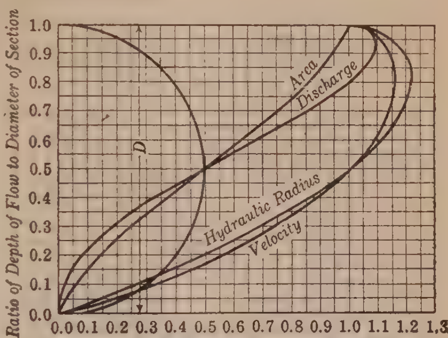
Fig. 4. Discharge of Circular Sewers for $n = 0.013$

conduit required may be computed by any of the accepted formulas for flow in pipes, as the Hazen and Williams formula (*q.v.*).

It must not be forgotten that in hydraulic computations, the slope is the hydraulic gradient, which may not be the same as that of the invert of the sewer. This condition is particularly noticeable in the case of inverted siphons and submerged outlets.

The Kutter formula (*q.v.*) is most commonly used in estimating the flow in sewers and in fixing the size required for a given capacity. Since analysis of flow conditions when the sewers are partly filled is generally necessary, a formula for flow in open channels must be employed; and the applicability of the

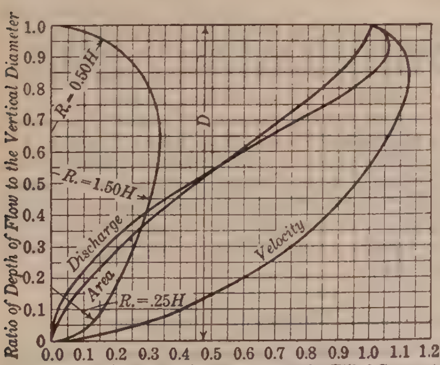
Kutter formula to full-sewer conditions is sufficiently accurate for practical use. Recommended values of n in this formula are, for pipe sewers 24 in. and less in diameter, 0.015, and for monolithic concrete of best workmanship 27 in. or larger in size or reinforced-concrete pipe of the same sizes, so jointed as to provide equally smooth interior surfaces, 0.013. Under test conditions with conduits clean and free from deposits and with no disturbances in flow due to the entrance of branch streams, enlargements, curvature and the like, the coefficients would be lower; but the values recommended are applicable to actual conditions likely to exist in sewers reasonably well cared for.



Ratio of the Hydraulic Elements of the Filled Segment to those of the Entire Section.

Fig. 5

In fixing the size of sewer required, the Kutter formula is most readily used by means of diagrams like those here given, from which the size of sewer and the velocity corresponding to a given discharge and slope can be read.



Ratio of the Hydraulic Elements of the Filled Segment to those of the Entire Section.

Fig. 6

For conditions other than full sewers, and for other shapes than circular, as well as for other values of n, the Kutter formula may be solved arithmetically by the aid of the tables in Sect. 9, Art. 15, interpolating for the proper values of R and n, or by the use of diagrams such as those in Metcalf & Eddy's American Sewerage Practice, Vol. I; Creager & Justin's Hydro-Electric Handbook, or Bulletin 852, U. S. Dept. of Agriculture. Hydraulic conditions for various depths of flow are compared most conveniently by means of diagrams showing the relations of the hydraulic elements, similar to those in Figs. 5, 6 and 7. The preparation of such a diagram is somewhat laborious but, once made, it can be used with sufficient accuracy for other sections of the same shape through a considerable range of sizes and slopes, although not for other values of n. This formula is only approximately applicable to part-full conditions, but no better method is available.

The **Minimum Size** of lateral sewers is generally at least 6 in.; many engineers consider 8 in. as the minimum. It is essential that the pipes shall be large enough not to be readily clogged by scrubbing brushes, mops and similar articles which sometimes enter the sewers through building connections.

The **Minimum Velocity** of flow which will prevent the formation of deposits in sewers is about 2.5 ft. per sec. in combined and 2.0 ft. per sec. in separate sewers. Higher velocities are desirable if the sewers are to be kept clean without frequent flushing. It is important to study the conditions which will exist at times of minimum flow; should it be found that the velocity at such times would be as low as 1.5 ft. per sec., it may be advisable to change the slope, dimensions, or form of section so as to ensure a more adequate velocity.

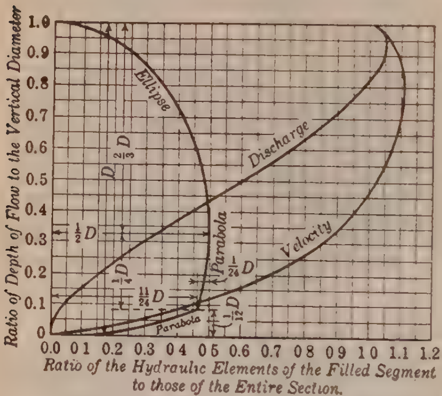


Fig. 7

Flushing may be resorted to in extreme cases, where it is impossible to provide sufficient slope to obtain self-cleaning velocities. This is frequently done by means of **Automatic Flush Tanks** located at the upper end of the laterals, and so arranged as to discharge a considerable quantity of water at least once a day.

Sewage Relift Stations or **Ejectors** are sometimes required to lift the sewage to a higher elevation, when the continuance of the sewers at reasonable slopes would involve excessive depth of trench, or to raise the sewage from areas too low to drain into available sewers. Such a station usually consists of a storage well or tank large enough to retain the sewage flow for a period of one or two hours, and a vault or steel tank containing one or better two vertical centrifugal pumps driven by electric motor and operated by float controlled switch. Such stations are so nearly automatic that they require attention not oftener than once a day.

4. Materials and Shapes of Sewers

Vitrified Clay or **Cement Concrete Pipes** are almost universally employed for sewers up to about 24 in. diameter. They are usually in lengths of 2-1/2 or 3 ft. and provided with sockets for jointing. The American Society for Testing Materials has adopted specifications for sewer pipes which are followed by the manufacturers of both clay and concrete pipe. Standard sizes are 4, 5, 6, 8, 10, 12, 15, 18, 21, 24, 27, 30, 33 and 36 in. **Joints** in such pipes are made by forcing a jute gasket into the socket, followed by either cement mortar, molten sulphur and sand, or a bituminous compound. **Cast-Iron Pipes** with lead or composition joints as in water works practice, are sometimes used for sewers when ground-water conditions are unusually bad and it is especially important to limit the infiltration of ground water.

Reinforced-Concrete Pipes are used to a considerable extent, in sizes from 24 in. to 72 in. or larger.

The **Load** to be carried by pipes depends not only on the depth of trench and the character of the refill, but also upon the width of the trench. If the width is kept as small as possible, a substantial part of the weight of the backfill may be carried by the sides of the trench.

The **Maximum Loads** on pipes imposed by backfill of saturated clay weighing 130 lb. per cu. ft., according to Marston and Anderson, may be as great as the figures given below (lb. per lin. ft.):

Width of trench at top of pipe	Depth of fill above pipe			
	8 ft.	12 ft.	16 ft.	20 ft.
2 ft.	1380	1730	1940	2090
4 ft.	3360	4560	5510	6280

When the pipe is laid substantially at the original ground level and an embankment is built over it, the loads to be carried would be very much greater than the above figures. In the case of a conduit 4 ft. wide outside and with a height of fill of 16 ft. over the top, and with saturated earth weighing 130 lb. per cu. ft., the load on the conduit might approximate 15 000 lb. per lin. ft., or nearly three times as great as that resulting from the backfill in a trench of the same width and depth.

The **Strength** of sewer pipe to withstand loads is dependent upon several factors in addition to the specific strength of the pipe itself—especially upon the manner in which the trench bottom is shaped to receive the pipe and the backfill is replaced at the sides and over the haunches. The provision of a concrete cradle to sustain the pipe enables it to develop much greater strength than if supported in any other way.

The specifications of the American Society for Testing Materials require that sewer pipes of either vitrified clay or concrete shall have a strength in sand-bearing tests at least as great as shown in the following tabulation (lb. per lin. ft. of pipe):

Size, in.	8	12	18	24	30	36	42
Strength	1430	1710	2200	3070	3690	4400	5030

The specifications of the American Concrete Institute for reinforced-concrete sewer pipe require the following minimum dimensions and strengths;

Size of pipe, in.	Shop-made pipe		Field-made pipe		Strength in sand-bearing tests, lb. per lin. ft.
	Thickness of shell, in.	Area of steel in longitudinal section 1 ft. long, sq. in.	Thickness of shell, in.	Area of steel in longitudinal section 1 ft. long, sq. in.	
24	2-1/2	0.15	3	0.065	4500
36	3	0.36	4	0.125	6300
48	3-3/4	0.46	5	0.210	7650
60	4-1/2	0.54	6	0.290	8700
72	5	0.62	7	0.360	9300

Investigations by Prof. Anson Marston have shown that pipe as laid in the ground, with the bottom shaped to fit the pipe for 60 to 90 deg., and with backfill loosely placed, may be expected to develop a strength about equal to that shown by sand-bearing tests: if the bottom is well bedded for at least 90 deg. and the backfill thoroughly tamped as high as the top of the pipe, about 20%

greater strength may be developed; and if the pipe is supported in a concrete cradle extending at least as high as 1/4 the diameter of the pipe, the resulting strength will be from 50 to 100% greater than that shown by sand-bearing tests. The depths of earth weighing 130 lb. per cu. ft. which can be carried by pipes having the strength stipulated in the American Society for Testing Materials and American Concrete Institute specifications, without any factor of safety or allowance for superficial loads, are very roughly as follows:

	Vitrified or concrete pipe		Reinforced concrete pipe	
	Without cradle	With cradle	Without cradle	With cradle
In narrow * trench.............	8–12 ft.	20–25 ft.	10–40 ft.	20 ft. or more
In embankment, filled over pipe.	4–6 ft.	6–9 ft.	9–10 ft.	10–12 ft.

* Assumed to be as narrow as practicable for any size of sewer.

The larger sizes of sewers may be built of brick, or of concrete with or without reinforcement. Reinforced concrete is generally employed. Many forms of section have been used; the circular, egg-shaped, and horseshoe or semi-elliptical are in most frequent use. The section must be designed of proper thickness and with sufficient steel to have suitable strength for the load to which it may be subjected. Where velocities are unusually high and erosion may be severe, or where the sewage is acid or otherwise corrosive, it may be necessary to line a concrete sewer partly or wholly with vitrified brick or liner plates.

5. Manholes, Catchbasins and Inlets; Building Connections

A **Manhole** is a shaft extending from the surface of the ground to the sewer, and large enough to provide access for inspection, cleaning, or repairs of the sewer. Manholes may be built of brick or concrete. It has been common practice to provide them on pipe sewers at each junction or change of direction or slope, so that the sewer may be straight between manholes; and not so far apart that jointed rods for cleaning cannot be effectually utilized: in general they have been placed at intervals of 300 to 400 ft. The wear of pavements adjacent to manholes and of the manhole frames and covers and the annoyance caused by them are so serious that care should be exercised to avoid unnecessary manholes. They should be large enough at the top so that a man can readily enter—not less than 20 in. in diameter; and at the bottom for handling rods, pails, and the like—about 4 ft. across. A channel or channels should be shaped in the bottom of the manhole to continue the sewer across it, and to provide for any change of direction or branch. The platform or bench upon which men may stand should be higher than the center of the sewer, preferably at the elevation of the crown, to minimize disturbance of flow conditions in the manhole when the sewer is full. **Manhole Frames and Covers** must be heavy enough to stand a great deal of wear and not to be displaced by traffic; the cover must be ribbed or covered with bosses to reduce slipperiness. In the past it has been common to perforate manhole covers to provide for ventilation of the sewers; this is generally unnecessary, and the holes have admitted much dirt and occasionally considerable storm-water; they are now frequently omitted, except one or two for the insertion of the point of a pick in raising the cover. **Manhole Steps** built into the masonry may be of cast or wrought iron; if the latter they should be heavily galvanized or well coated with asphalt or other coating to protect them against rust. **Inlets** are openings in the curb or in the street surface for admitting storm-water. They may include direct connections to the sewer or drain, or they may be built upon or connected with **Catchbasins,** which are brick or concrete vaults of considerable size, having sumps below the level of the outlet pipe for the purpose of retaining street detritus and trash and preventing their being washed into the sewers. If they are to serve this purpose efficiently the accumulated material must be removed regularly. **Building Connections** are usually of 5-in. or 6-in. pipe, and should enter a pipe sewer through a Y-branch set when the sewer was laid, or a masonry sewer through a slant built into the side. The direction of flow should be such as to cause the least possible disturbance to flow in the sewer.

6. Maintenance of Sewerage Systems

Sewers should be inspected regularly in order to have definite knowledge of their condition and to remove causes of obstruction before they result in stoppage. Many sewers require periodic cleaning for the removel of sediment, grease, rags, sticks, tree roots, and the like, all of which have effect in reducing the capacity of a sewer if no more serious consequences result. Tree roots may enter through defective joints; they have been known to form mats 20 ft. or more in length and practically filling 6-in. and 8-in. pipes. Grease is most likely to be troublesome in the neighborhood of hotels, restaurants and the like. It may be retained in large measure by adequate grease traps: ordinary commercial grease traps are likely to be much too small. Sediment may be of organic matter settling out of the sewage, or of mineral matter resulting largely from street wash entering the sewer through inlets, catchbasins, and perforated manhole covers. Rags, sticks, and other articles, sometimes of considerable size, occasionally enter the sewer, sometimes through building connections, sometimes thrown into manholes. Locked manhole covers afford some protection against this abuse. **Cleaning** of sewers is frequently accomplished by flushing, generally by a stream of water from a fire hose introduced at a manhole. If this is not sufficient, a rope may be put through a section between manholes, and a piece of heavy chain attached to it and drawn back and forth for the the purpose of scraping off the sediment; or a stiff brush or steel scraper may be necessary, particularly if accumulations of grease are to be removed. It is frequently necessary to use rods, which are made in sections short enough to be handled in a manhole, with joints for ready connection and yet providing some degree of flexibility.

Catchbasins must be cleaned frequently or they will either become stopped or the water and dirt will go through them into the drain. This should be done after each considerable rain, and in any event often enough to leave sufficient capacity for the detritus which may be brought in by a heavy storm. The usual method is for a man to enter the basin and shovel the dirt into pails which are hoisted to the surface; sometimes a specially designed grab bucket handled by a derrick mounted on a truck is utilized. Mechanical appliances such as regulators, backwater or tide gates, and the like, require very frequent inspection and cleaning in order to keep them in operation. Manhole frames and covers must be occasionally renewed or reset, from being broken, worn smooth, worn so that covers rattle, or sometimes because the roadway is worn down and the casting projects so much as to constitute a menace.

Explosions in sewers due to illuminating gas or gasoline have sometimes occurred. The probability of trouble from gasoline can be greatly reduced by requiring all garages and places using gasoline to install suitable traps. Even if gasoline or illuminating gas is present, there will be little danger of explosion if lanterns are not used and men are not allowed to carry matches into sewers. The use of electric torches where wired lights are not available will be an adequate safeguard in most cases.

DISPOSAL OF SEWAGE

7. Sewage Characteristics

Physical, Chemical and Biological Characteristics. Sewage consists of about 1 part of solid matter to 999 parts of water. Of the solid matter, about a third is in suspension and the remainder in solution. About half the solid matter is organic and half inorganic. Sewage contains large numbers of bacteria, some of which may cause disease. The harmful bacteria and the putrescible organic matter are the constituents which are responsible mainly for the sewage problem.

Constituents. The constituents of sewage vary with the character of water supply, habits of the population and kind of industrial wastes, and the concentration is affected by the per capita use of water and by ground-water leakage. Sewage from combined sewers varies greatly according to the presence or absence of storm water. Hourly, daily and seasonal variations in constituents also occur. Furthermore, the character of the constituents may be materially modified by the age of the sewage which tends to undergo decomposition while being carried or retained in the sewers.

Analyses of sewage and of sewage effluents are made according to "Standard Methods for the Examination of Water and Sewage" 6th edition, 1925, as adopted by the committees of the American Public Health Association, American Chemical Society and American Water Works Association.

Data relative to the analyses of sewage from 16 typical American cities are given in the following table:

Constituent	Parts per million *			Grams per capita daily		
	Average	Maximum	Minimum	Average	Maximum	Minimum
Organic nitrogen	15.3	28.0	6.6	5.78	10.6	2.5
Ammonia nitrogen	21.6	37.8	11.6	8.17	14.3	4.4
Oxygen consumed	134	154	25	50.7	58.3	9.5
Residue on evaporation:						
Total	686	1790	280	260	678	106
Loss on ignition	332	753	166	126	285	63
Fixed residue	354	1037	114	134	393	43
Suspended solids:						
Total	246	781	77	93	296	29
Loss on ignition	180	626	64	68	237	24
Fixed	66	155	13	25	59	5
Chlorides as chlorine	40.5	116.1	17.2	15.3	44.0	6.5
Biochemical oxygen demand † (5-day)	135	180	119	51	68	45

* Based on flow of 100 gal. per capita per day.
† From data in U. S. Public Health Bull. 132.

European sewages are usually much stronger than American due to the fact that the per capita water consumption in Europe is much smaller than in America.

The effect of industrial wastes is shown by the following estimates of the increase in certain constituents brought about by the discharge of wastes into the sewers of several cities:

City, and kind of wastes	Per cent increase, per capita basis	
	Suspended solids	Oxygen consumed
Akron, Ohio: rubber reclaiming plant wastes	120	140
Chicago, Ill. (packing house district only): packing house wastes	46	225
Dayton, Ohio: paper mill wastes	60	25
Fort Worth, Tex.: packing house wastes	65	....
Gloversville, N.Y.: tannery wastes	155	115
Milwaukee, Wis. (portion of city): packing house and tannery wastes	70	10

Suspended Solids. Solids carried in suspension may be roughly divided into three classes: First, the grosser solids, such as fecal matter, garbage, paper, rags, sticks, and leaves, which are capable of being removed by mechanical means, such as screens. Some of these matters should not reach the sewers but do so as a result of misuse of house plumbing systems and of the sewers. Second, the more finely divided solids, the specific identity of which is difficult to establish, but which are capable of being removed by sedimentation. Third, the grease and finely divided solids which do not settle out within any reasonable period of time. Among the grosser solids are those which produce visibly offensive conditions at the outlets of sewers and which are often difficult to handle and to dispose of in treatment plants. The solids capable of settling are those which may form offensive deposits in streams or other bodies of water and which in treatment plants produce the sludge which is so often the most troublesome feature in connection with sewage treatment. The finely divided and colloidal solids tend to produce discoloration and turbidity in the streams receiving the sewage and also effect a demand upon the oxygen content thereof. The organic solids in all three classes of suspended solids exert an oxygen demand.

Oxygen Demand. Inasmuch as the purification of sewage, both artificial and natural, is essentially an oxidation process, the demand for oxygen exerted by sewage is a characteristic desirable to measure. The biochemical oxygen demand (B.O.D.) of sewage, effluent, polluted water or industrial wastes is the oxygen required for the oxidation through biological or chemical agencies of the organic matter and oxidizable inorganic matter contained therein. For detailed information relative to the subject of oxygen demand and methods for determining it, reference should be made to Public Health Bulletin 175, "The Oxygen Demand of Polluted Waters," from which the following table relative to oxygen demand of the sewage of several American cities is compiled.

Putrescibility is the susceptibility of sewage or other polluted water to undergo putrefaction. This is measured by determining the time required for the sewage or other polluted water to exhaust the oxygen present therein. The ratio of oxygen available to oxygen required to satisfy the oxygen demand

Total Oxygen Demand of American Sewage

Pounds per capita daily

Alliance, Ohio	0.15	Dayton, Ohio	0.20	Peoria, Ill.	0.25
Baltimore, Md.	0.24	Detroit, Mich.	0.17	Pipestone, Minn.	0.38
Brooklyn, N. Y.	0.19	Edina, Minn.	0.26	Procter, Minn.	0.14
Buhl, Minn.	0.19	Fitchburg, Mass.	0.17	Reading, Pa.	0.15
Canton, Ohio	0.17	Kenyon, Minn.	0.11	Rochester, N. Y.	0.17
Chicago, Ill.	0.27	Lexington, Ky.	0.16	Schenectady, N.Y.	0.20
Cincinnati, Ohio	0.22	Litchfield, Minn.	0.16	Syracuse, N. Y.	0.21
Columbus, Ohio	0.25	Marshall, Minn.	0.07	Washington, D. C.	0.25
				Average	0.197

is designated as relative stability. The common test for this ratio is made by adding a small quantity of dye, methylene blue, to the sample and incubating the mixture at a constant temperature for a certain number of days or until the color of the dye is discharged which occurs when the oxygen is exhausted. The relative stability of the sample is determined by the number of days elapsing prior to decolorization according to the following table:

Relative Stability Numbers

("Standard Methods of Water Analysis," American Public Health Association)

t_{20}	t_{37}	S	t_{20}	t_{37}	S	t_{20}	t_{37}	S	t_{20}	t_{37}	S
0.5	.....	11	3.0	1.5	50	8.0	4.0	84	13.0	6.5	95
1.0	0.5	21	4.0	2.0	60	9.0	4.5	87	14.0	7.0	96
1.5	.....	30	5.0	2.5	68	10.0	5.0	90	16.0	8.0	97
2.0	1.0	37	6.0	3.0	75	11.0	5.5	92	18.0	9.0	98
2.5	.....	44	7.0	3.5	80	12.0	6.0	94	20.0	10.0	99

S = Relative stability or ratio of available oxygen to oxygen required for equilibrium; expressed in percentages.
t_{20} = Time in days to decolorize methylene blue at 20° C.
t_{37} = Time to decolorize at 37° C.

8. Disposal by Dilution

Pollution by sewage has several serious aspects in the following order of importance: contamination and infection with disease germs of public water supplies; contamination and infection of shellfish areas; contamination of bathing waters and beaches; contamination of water supplies for cattle; the destruction of fish life; and the production of generally objectionable conditions as regards appearance and offensive odors.

Few cities now take their water supplies from sewage-polluted streams without purification, but there are many streams so grossly polluted that water purification plants treating their waters can barely provide safe supplies. The contamination of shellfish areas is of significance only in coastal waters, but the matter has become one of considerable economic importance. Bathing in materially sewage-polluted waters is objectionable both from the esthetic standpoint and the public health standpoint. The use of sewage-polluted water for the watering of stock may not always be dangerous, but in some cases, such as pollution by sewage containing tannery wastes, there may be danger of the spread of anthrax among cattle. As regards fish life the effect of sewage is probably less than that of industrial wastes, except when the sewage pollution becomes so concentrated as to reduce the amount of oxygen below the limit required for fish. Locally objectionable conditions frequently occur at the point of discharge of sewers due to the presence of floating sewage solids, oil slick, the formation of banks of decomposing sludge, and the production of unsightly scum and offensive odors. Generally objectionable conditions with respect to decomposition and odors do not result until there occurs complete depletion of the dissolved oxygen normally present in the stream or other body of water receiving the sewage.

Natural Agencies of Purification. When sewage is discharged into a body of water, natural processes immediately begin its purification. Microscopic and other living organisms in the presence of the oxygen held in solution in the water tend to oxidize the organic matter and unfavorable environmental conditions as regards temperature and food supplies bring about the destruction of large numbers of harmful bacteria. If the dissolved oxygen supply is sufficient, the oxidation of the organic matter proceeds under aerobic conditions without creating offensive odors. If the oxygen supply is inadequate and becomes exhausted, putrefaction or anaerobic decomposition sets in and putrid odors result. Reaeration, fostered by turbulence and sometimes by oxygen-producing organisms, tends to prevent putrefaction.

Where the discharge of sewage is into a quiescent body of water or into a stream of low velocity, the suspended matters settle, forming deposits which in the absence of oxygen, and particularly in warm weather, undergo decomposition. In time, these deposits may become converted to a stable humus-like material and under certain conditions of flood they may be scoured out and carried down to the sea.

Dissolved Oxygen. At any temperature and barometric pressure a specific quantity of atmospheric oxygen can be dissolved in water, then said to be saturated. The saturation point in salt water is less than that in fresh and the quantity capable of being dissolved in warm water is less than that in cold. The solubility of oxygen in fresh water at a barometric pressure of 760 mm. (approximately sea level) and at different temperatures is expressed in parts per million in the following table:

Temp., C.	Oxygen	Temp., C.	Oxygen	Temp., C.	Oxygen
0	14.70	10	11.31	20	9.19
1	14.28	11	11.05	21	9.01
2	13.88	12	10.80	22	8.84
3	13.50	13	10.57	23	8.67
4	13.14	14	10.35	24	8.51
5	12.80	15	10.14	25	8.35
6	12.47	16	9.94	26	8.19
7	12.16	17	9.75	27	8.03
8	11.86	18	9.56	28	7.88
9	11.58	19	9.37	29	7.74

Deduct 1% for each 270 ft. of elevation above sea level. For sea water deduct 20%, and for mixtures of fresh and sea water in direct proportion to the amount of sea water.

Dilution Required to Avoid Objectionable Conditions. To maintain satisfactory conditions where sewage is discharged into a stream, lake or ocean the diluting water in proportion to the quantity of sewage must be at least sufficient to permit purification to take place under aerobic conditions. In temperate climates, self-purification and depletion of oxygen proceed rapidly in summer, but slowly in winter; consequently, the volume of diluting water required for a given quantity of sewage is much greater in summer than in winter.

The quantity of diluting water required to prevent offensive stream conditions estimated by various authorities has varied from a minimum of 2-1/2 cu. ft. per sec. to a maximum of 7 cu. ft. per sec. per thousand persons contributing sewage. The lower limit is applicable only in the case of rapidly flowing turbulent streams, and the upper limit to sluggish streams with mill dams or quiet reaches in which there are opportunities for the deposition of solids and the formation of sludge banks. As experience and knowledge have increased, there has been a tendency to set the quantity of diluting water required nearer the upper limit.

Prompt and thorough mixing of the sewage with the diluting water is essential. This can be accomplished by use of multiple outlets discharging into the main current of streams or into deep water of lakes or oceans. Care must be taken to select points of discharge to avoid possibility of tidal currents carrying sewage matters to bathing beaches.

The most notable example of sewage disposal by dilution is that of the Chicago Sanitary District whose drainage canal since 1900 has diverted sewage of the district, together with large quantities of diluting water, from Lake Michigan to the Illinois River and thence to the Mississippi. The growth of tributary population and the limitations of the volume of diverted water established by the Federal authorities, and the limited capacity of the canal, have now rendered it necessary for the District to undertake the construction of treatment plants on a large scale.

For exhaustive studies into the matter of pollution and self-purification of streams, reference should be made to U. S. Public Health Bulletins 139, 146 and 171, and to the Appendix on Sewage Disposal accompanying the report of the Engineering Board of Review of the Sanitary District of Chicago.

9. Screening

In cases where disposal by dilution may be practiced, but where the removal of floating solids is desirable, screens alone are used. Screening is also employed to protect pumps and in connection with more elaborate methods of sewage treatment. There are many types of screens, from the simple hand-cleaned, coarse, bar rack to the elaborate mechanically operated and cleaned screen of small apertures.

Racks and Coarse Screens. There are several types of racks and coarse screens used to protect pumps and treatment plant features. The type most frequently used is the inclined bar rack with spacing ranging from 1/2 in. to 6 in. These racks are usually placed across the channel, the individual bars being inclined to the horizontal at various angles, such as 30 to 60 deg. Cleaning is usually done by hand, the screenings being raked to the top of the rack. In a few instances, mechanical equipment has been provided for cleaning.

Another type of rack is composed of movable segments, in the form of an endless belt, which in operation convey the screenings out of the sewage to a point where they are removed by mechanical means.

At certain places, notably the Boston Metropolitan District and Milwaukee, screens in the form of movable cages have been employed for the coarse screening of large flows of sewage from combined sewers. These cages are usually installed in duplicate so that one screen can always be in place while the other is raised for cleaning. In the Ward Street station at Boston, the cage screen is about 9 ft. high and wide and 3-1/2 ft. deep, and is composed of vertical 3/4-in. round bars with 1-in. clear opening.

Screenings Removed by Racks and Cage Screens

Municipality	Date	Type of screen	Clear spacing, in.	Screenings, cu. ft. per m. g.
Boston, Mass.: Deer Island	1926	Cage	1	1.78
Nut Island	1926	Cage	3/4	4.77
Ward St.	1926	Cage	1	3.50
Fitchburg, Mass.	1918	Rack	1-3/4	0.62
Gloversville, N. Y.	1916	Rack	1-1/8	1.45
Schenectady, N. Y.	1927	Rack	1-1/2	1.94
Toronto, Ont.	....	Rack	1/2	2.8–3.7
Washington, D. C.	1918	Rack	5/8 and 1/4	12.5

The quantity of screenings removed by coarse racks varies with the spacing of the bars, and with the degree of comminution of solids. The table above gives data relative to the removals by different installations of various clear spacing.

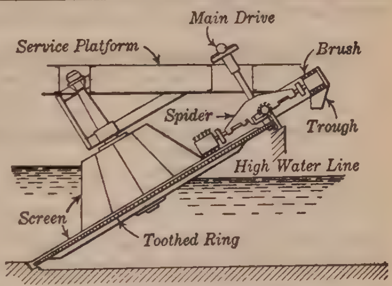


Fig. 8. General Arrangement of Riensch-Wurl Screen

Fine Screens. In recent years there has been a marked development in mechanically operated fine screens. At present the four leading types of fine screen in use in the United States are the Riensch-Wurl disk screen, the Dorrco and the Link-Belt drum screens, and the Rex band type. Other types are in use in Europe.

The **Riensch-Wurl** type of screen consists essentially of a slotted metal disk 8 ft. or more in diameter, revolving slowly around a shaft inclined slightly from the vertical in such a way that any particular portion of the disk passes alternately in and out of the sewage flow. Floating and suspended solids too large to pass through the slots are held back and are removed by revolving brushes which clean that portion of the screen out of the sewage.

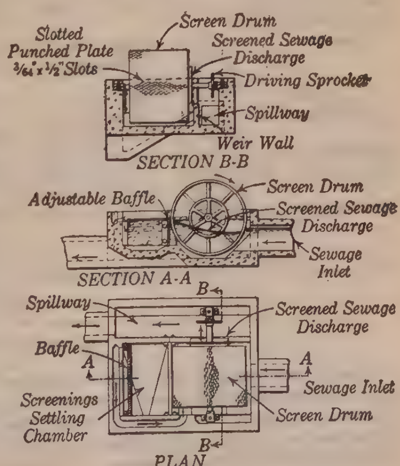


Fig. 9. Dorrco Screen

In the case of the **Dorrco** and **Link-Belt** screens, the slotted plates are fastened to a cylindrical framework rotated around a central horizontal shaft. The sewage flow passes from the outside of the cylinder to the inside and thence through an outlet at the end of the cylinder. The Dorrco screen is cleaned by washing the screenings off the drum by the outward flow of the sewage, which is carried slightly above the general water level by centrifugal action. The solids thus washed off, together with some which had never become attached to the drum, settle in the screen pit and are removed by means of a perforated bucket elevator. The Link-Belt screen is cleaned by revolving brushes travelling along the top of the drum.

The **Rex** screen consists of an endless band of slotted metal plates, moving in and out of the sewage flow, the screenings being brushed off just before the plates turn over the upper end pulley on their way back to the sewage.

The following table gives information relative to the quantity of suspended solids removed by several installations of fine screens. Under ordinary conditions a removal of 5 to 10 per cent is all that can be expected.

Removal of Suspended Solids from Sewage by Fine Screens

Location	Removal of suspended solids, per cent	Type of screen	Size of opening, in.	Date	Remarks
Cleveland, Ohio...	3.5	Sanitation disk	1/32×2	1916	Experimental 100 days test *
Daytona, Fla.....	8–14	Sanitation disk	3/32×2	1917	....
Long Beach, Calif..	12.0	Sanitation disk	1/32×2	....	Not now in regular use.
Milwaukee, Wis...	3.4	Chain-belt	1/8×2	1918–19	Experimental testing station
	3.9	Chain-belt	3/32×2	1918–19	
New Britain, Conn	16.3	Dorrco—drum	1/16×1/2	1922	1-week test
New York, N. Y.:					
Brooklyn.......	14.0	Sanitation disk	5/64×2	1916	Experimental, official test April–May
	18.0	Sanitation disk	1/16×2	1916	
Canal Street....	9.3	Rex—band	3/64×2	9126	8-day test
Dyckman Street.	16.4	Sanitation disk	1/16×2	1919	24-hour test
	26.3	Sanitation disk	3/64×2	1919	24-hour test
Plainfield, N. J....	4.6	Sanitation disk	1/16×2	1927	Year
Pleasantville, N. J.	20.5	Link-belt—plate	3/64×2	1921	7-hour test
	27.5	Link-belt—wire	1/32	1921	6-hour test
Reading, Pa.......	20.0	Weand—drum	0.015 in. square	1909–10	Not now in use
Rochester, N. Y...	2.8	Sanitation disk	1/8×2 (one); 1/16×2 (two)	1918	Year

* Not now in use.

Sewage screenings are wet, odorous, putrescible and generally objectionable. Prompt removal and disposal are essential. In some cases excess moisture is extracted by presses or centrifuges and the partially dried material burned under boilers or in special incinerators. In other cases the screenings are buried or plowed into the ground. Unless precautions are taken, buried screenings are likely to attract rats.

10. Sedimentation

The removal from sewage of the settleable suspended solids is accomplished by sedimentation in suitable tanks. The degree of removal generally depends chiefly upon the period of detention, and but slightly upon the velocities prevailing in the tanks. Slight checking of such velocities as are common in sewers brings about the settling of some of the heavier solids, such as sand and grit, while in the case of solids of low specific gravity in a finely divided condition long detention at very low velocities is required. Sedimentation alone may constitute adequate treatment, in cases where the volume of diluting water is relatively large and where no public water supply is taken from the stream which receives the effluent. In most cases where a greater degree of purification is required, sedimentation is a valuable preliminary or preparatory treatment. Sedimentation is also utilized for the final clarification of effluents from trickling filters, and from aeration tanks in the activated sludge process. There are several types of sedimentation tanks differing from each other in function, design, and methods of operation.

Grit Chambers are small basins, shallow and narrow in proportion to their length, usually arranged in parallel units of two or more. Their function is to check the velocity of flow only sufficiently to allow the deposition of the heavier solids. The velocity usually provided for is approximately 1 ft. per sec. Grit chambers are cleaned by hand, by hose flushing or by grab bucket. A tank with mechanical equipment has been developed for continuous removal of deposits and separation of the lighter organic solids from the heavier inorganic solids.

Grit chambers are more necessary with combined than with separate systems of sewers, but they are commonly provided with the latter because exprience has shown that considerable grit such as cinders and sand finds its way into such sewers. The quantity and composition of the grit collected depends upon the composition of the sewage, the velocities through the chamber and the frequency and method of cleaning.

The following table gives data relative to the collection of grit in certain typical installations:

Volume of Grit Removed by Grit Chambers

Location	Character of sewer system	Velocity of flow (theoretical), ft. per sec.	Period to which data apply	Volume of grit removed, cu. ft. per m. g.
Cleveland, Ohio:				
Easterly works......	Combined	0.5–1.0	Feb., 1923–Aug., 1925	2.1
Westerly works......	Combined	0.5–1.0	Jan., 1924–Aug., 1925	2.0
Rochester, N. Y.:				
Irondequoit.........	Combined	....	1917–25	2.6
Fitchburg, Mass.:				
Water Street........	Combined	1.0–1.6	1915–26	4.1
Worcester, Mass.......	Combined	0.5	1913	2.4

Skimming Detritus Tanks are sometimes required to remove oils and grease which form unsightly scum on settling tanks or clog filters. The installation at Akron, Ohio, is typical. Two small circular tanks afford detention periods of about fifteen minutes. In these tanks the velocity is checked sufficiently to bring about the flotation of oil, grease and other substances lighter than water, together with the settling of a portion of the heavier suspended solids. The sewage flows out through two submerged openings, one a short distance below the water level and the other at the bottom. The portion discharged near the bottom, amounting to about one-fifth the flow, is somewhat concentrated by settling and passes through grit chambers and fine screens before reaching the Imhoff tanks, while the portion drawn through the upper opening goes directly to the tanks. Oil and floating solids are removed by skimming with a revolving scum board.

Plain Settling Tanks are usually rectangular in plan and section with the inlets so arranged as to permit fairly uniform distribution of flow through the cross-section of the tank. Detention periods range from 1 hour to 6 hours and baffle and scum boards are usually provided to aid in regulating flow and to retain scum. Cleaning is accomplished by drawing off the supernatant liquid and then removing the sludge by gravity or by means of pumps. The more common type of settling tank utilizes horizontal flow. Care must be taken to secure proper inlets and outlets. In some instances (Dortmund tanks) vertical flow is employed with the inlet for the sewage at the center of a cylindrical tank and the outlets arranged around the periphery. This type of tank permits the use of a hopper bottom from which the sludge can be drawn under the pressure of the supernatant water without interrupting operation of the tank. Plain settling tanks are cleaned at frequent intervals. The quantity and character of the sludge depend upon the characteristics of the sewage and the design and efficiency of operation of the tank.

Clarifier Tanks have been adapted for the settling of sewage. These are comparatively shallow circular or square tanks, equipped with mechanism for continuous removal of the deposited solids. The mechanism consists primarily of revolving arms just above the bottom of the tank to which are attached plates or plows so directed as to move the sludge slowly into the center as the arms revolve. From the center the sludge can be pumped to the point of disposal.

Septic Tanks are essentially sedimentation tanks so designed and operated as to permit the anaerobic decomposition of the deposited solids. It was originally claimed that this decomposition would proceed so far that the sludge would be completely gasified and liquefied, but experience has shown that this cannot be accomplished. The disadvantages of the tank are such that few are now installed for municipal treatment plants, although their use for residences is common. The septic tank is operated for much longer periods without the removal of sludge than plain sedimentation tanks. This has the disadvantage of producing a floating mat or scum of considerable thickness which becomes difficult to remove. The effluent of a septic tank is often offensive to both sight and smell.

Septic tanks are usually similar in construction to the ordinary type of plain rectangular settling tank. The detention period varies according to the wish of the designer, as from 18 to 24 hours or longer, and the velocities are correspondingly low. Many septic tanks are covered, but this is not essential to proper biological action, and may be a source of danger due to the collection of explosive gases. Sludge is allowed to remain in the tank long enough to secure the benefits arising from the digestion of a large proportion of the accumulated organic solids. Under favorable conditions the sludge when drawn off is generally denser and less odorous than that from plain settling tanks.

Two-Story Tanks. In order to secure in a single tank the advantages of plain sedimentation and the digestion of sludge by biological action and to avoid some of the disadvantages of the septic tank, two-story tanks have been quite commonly used. This type of tank, usually known as the **Emscher or Imhoff Tank,** consists essentially of an upper compartment for sedimentation and a lower one for sludge storage and digestion. The compartments are separated from each other by a sloping bottom provided with slots so arranged that solids settling in the upper chamber can pass into the lower chamber while sludge or gas cannot easily rise through these slots into the

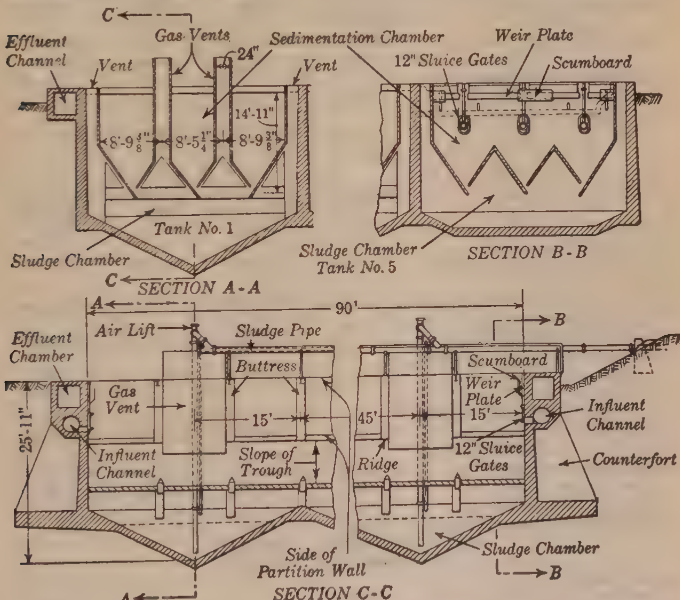


Fig. 10. Sections through Imhoff Tanks

settling compartment. This type of tank permits the sewage to pass through it without becoming seriously affected by the septic action taking place in the sludge.

In the sludge compartment the rate of digestion of organic matter varies according to the temperature, being very rapid at from 65° to 80° and practically ceasing around 40° Fahrenheit. Consequently the accumulated sludge is generally found to be substantially free from odor and in good condition for drying in warm weather, when the sludge should be drawn off frequently and the tanks left in the late fall with only enough sludge in them to provide for seeding. During periods of low temperature the sludge must be stored in the digestion compartments for decomposition during the next warm season. The storage period has often been taken at six months in the northern part of this country. Methane gas is produced in large quantities during normal digestion and gas vents are provided for its escape into the atmosphere

Item	Schenectady, N. Y.	Plainfield, N. J.	Fitchburg, Mass.	Rochester, N. Y.
Tributary population (1922)....	65 000	40 000	38 000	260 000
Character of sewage:				
Flow, in million gallons per day	6	3.4	3.4	32
Separate or combined sewers.	Separate	Separate	Combined	Combined
Strength (suspended solids), in parts per million.......	163	166	219	163
Freshness..................	Fresh uncomminuted	Stale	Fresh	Stale
Industrial wastes...........	Practically none	Practically none	Small amount	Small amount
Hardness of water supply, in parts per million..........	130	88	10	65
Temperature, in degrees, Fahrenheit..................	Av (1922) 56.1 max. 64.7	46–70	No data	Av. 54
Preliminary treatment:				
Screening..................	Coarse rack	Fine screens, 1/16-in. slot	Coarse racks	Fine screens, 1/8-in. slots
Grit-chambers..............	None	None	Yes	Yes
Imhoff Tanks:				
Sedimentation period, in hours	3.3	3.4	6.4	1.1
Depth of Tanks:				
Total maximum water depth.	13 ft. 9 in.	19 ft. 9 in.	24 ft. 8 in.	33 ft. 10 in.
Sludge Compartment:				
Depth below plane of slots...	6 ft. 1 in.	8 ft. 9 in.	11 ft. 0 in.	22 ft. 6 in.
Depth below 18-in. neutral zone....................	4 ft. 7 in.	7 ft. 3 in.	9 ft. 6 in.	21 ft. 0 in.
Distance of lowest point of overflow below 18-in. neutral zone.................	2 ft. 6 in.	2 ft. 0 in.	2 ft. 0 in.	13 ft. 0 in.
Depth below overflow to adjacent compartment........	2 ft. 1 in.	5 ft. 3 in.	7 ft. 6 in.	8 ft. 0 in.
Cubic feet per capita........	1.40	1.49	1.88	2.40
Number of hoppers.........	8	5	3	3
Ease of intercommunication..	Poor (2-ft. square opening)	Poor (opening 20 by 24 in.)	Good	Good
Scum Compartment:				
Cubic feet per capita........	0.76	0.72	1.59	0.55
Area, gas vents; percentage of tank area................	14.8	14.3	15	26.8
Area, gas vents; square feet per cubic foot sludge capacity	0.034	0.023	0.031	0.0133
Loading, Deposited Solids:				
Pounds per year per cubic foot sludge and scum space......	14.2	10.4	13.1	9.2
Pounds per year per cubic foot sludge space.........	21.9	15.5	24.1	11.3
Pounds per year per cubic foot sludge space after deducting 18-in. neutral zone........	30.0	18.9	30.9	12.3
Pounds per year per cubic foot scum space..............	40.4	32.1	28.5	49.7
Pounds per year per square foot gas-vent area........	644	674	777	849

or in some cases for its collection and use. The sludge produced in these tanks is usually distinctly less offensive than that from plain settling tanks or septic tanks, with a lower moisture content and of such a nature as to dry more easily.

There have been operating difficulties with this type of tank due to the tendency for the gas-filled solids to rise in the form of scum and in some cases to form a foam, due in part to rapidity of decomposition, which has overflowed the gas vents. In the laboratory and in practice it has been found possible to control the proper digestion of sludge by the use of lime, a hydrogen-ion concentration (pH) of 7.2 or 7.3 being found most advantageous.

Data relating to four typical American Imhoff tank installations are given in the table on p. 1738 from "Imhoff Tanks—Reasons for Differences in Behavior," p. 465, vol. LXXXVIII (1925) Trans. Am. Soc. C. E.

Of these four installations those at Fitchburg and Rochester have been operated practically without difficulty, while those at Plainfield and Schenectady have experienced considerable trouble.

Imhoff tanks are usually from 20 to 35 ft. deep, relatively deep ones appearing to function better than shallow ones. Sludge is ordinarily removed by hydrostatic pressure through pipes extending to the bottom of the sludge compartments. Sometimes water piping is placed at the bottom of the sludge compartment to permit stirring of the sludge by water jets to facilitate removal and to improve biological conditions by mixing the new with the older sludge. The bottom may be arranged either as a long trough or as a series of hoppers. The detention period in the sedimentation compartment varies, according to design, from one hour to six hours, depending upon the degree of removal desired; a period of two hours is commonly adopted. The slope of the bottom is usually at least 1.5 on 1, and often steeper, to prevent retention of solids. The clear width of these slots may be 6 in. or more. Influent and effluent systems are arranged so that the direction of flow through the sedimentation compartment may be reversed at frequent intervals, except in small tanks. Scum boards may be useful in front of the outlets. Inlets and outlets should be designed to provide as uniform distribution of flow through the sedimentation compartment as possible.

In some installations, provision is made for collecting and utilizing the gases given off in the gas vents. The principal gases given off and their approximate proportions are as follows: Methane 75-85%, Carbon dioxide 5-15%, Nitrogen and miscellaneous gases 5-10%. The mixture of gases has a calorific value between 600 and 900 B. t. u. per cu. ft. It has been estimated that on the average the volume of gas produced by the digestion of sludge from sedimentation tanks is from 0.5 to 1.0 cu. ft. per capita daily.

Partial oxidation of colloidal and dissolved organic matter has been accomplished by placing brush or lath filters in the sedimentation compartments of Imhoff tanks, and introducing air underneath these filters. It is claimed that this method produces an effluent intermediate between that of a plain Imhoff tank and that of a trickling filter.

Humus Tanks or secondary settling tanks are used to improve the effluents from trickling filters by sedimentation of the suspended matters which periodically are flushed out from the filters. They may be of the ordinary horizontal-flow type, or of the vertical-flow type, and may or may not be provided with mechanical appliances for the removal of sludge.

Chemical Precipitation may be employed to increase the quantity of solids removed by sedimentation. It requires the addition of chemicals to the sewage prior to its entrance into settling tanks. Lime and sulphate of iron are

Removal of Suspended Solids from Sewage by Imhoff Tanks

(Yearly average)

Location	Year	Sewage flow, m.g.d.	Tributary population	Detention period, hr.	Sedimentation capacity, cu. ft. per cap.	Suspended solids: In influent, p.p.m.	In effluent, p.p.m.	Removed: p.p.m.	Per cent
Albany, N. Y.	1920	14.3	113 433	4.6	3.23	130	58	72	55
Atlanta, Ga.:									
Intrenchment Cr.	1915	5*	50 000	3	1.67	140	44	96	69
Peachtree Cr.	1915	8*	80 000	3	1.67	145	59	86	59
Proctor Cr.	1915	3*	40 000	3	1.25	328	46	282	86
Columbus, Ohio	1926	30.3	285 000	4.0	2.36†	214	94	120	56
Fitchburg, Mass.	1926	3.1	40 000	6.5	2.80†	205	47	158	77
Plainfield, N. J.	1922	3.4	40 000	3.4	1.57	166	77	89	54
Rochester, N. Y.:									
Brighton	1923	1.1	10 000	3.5	2.14	158	71	87	55
Charlotte	1923	0.4	4 000	5.4	3.01	549	56	493	90
Irondequoit	1923	27.9	260 000	1.2	0.72	185	107	78	42
Schenectady, N. Y.	1922	6	65 000	3.3	1.70	163	48	115	71
Worcester, Mass.	1926	22	185 000	2.0	1.34	278	108	170	61

* Capacity. † Based on the number of tanks in use.

most frequently used, producing a flocculent precipitate which upon settling carries with it much suspended and some colloidal matter. This method is capable of bringing about a more complete removal of suspended solids than plain sedimentation. The sludge produced, however, is considerably greater in bulk, not only as the result of a more complete removal of sewage solids, but also because of the precipitation of the added materials. For treatment of sewage, chemical precipitation is now used by but a few cities in this country, although for treatment of certain industrial wastes it is a valuable process.

Tanks for chemical precipitation are usually similar to those used for plain sedimentation. Provision must be made for the addition of the chemicals and their thorough mixing with the sewage. In the old plant at Worcester, Mass., baffles of the fish-ladder type in the inlet channel were employed for mixing. The quantity of precipitants required varies with the character of the sewage and may range from 500 to 2000 lb. of lime per million gallons, and about one-half these quantities of ferrous sulphate. In some cases the sewage contains enough iron salts from pickling liquors to make the addition of ferrous sulphate unnecessary.

11. Sludge

Sludge is the material settled from sewage, exclusive of heavy inorganic matter or grit. Its characteristics depend largely upon the manner in which it is produced and the changes occurring during its retention. In general each type of settling tank produces a distinctly characteristic sludge. Sludge from plain settling tanks is grayish brown in color, coarse in texture and ill-smelling. Such sludge contains a large proportion of undigested sewage solids. Chemical precipitation sludge is fairly heavy, gray, brown or black in color, and more or less offensive in odor, depending upon its stage of decomposition. The proportion of chemicals used affects its characteristics considerably.

Imhoff tank sludge or **Digested Sludge,** if digestion has proceeded properly, is black. moderately uniform but granular in texture, full of entrained gases and has a comparatively unobjectionable tarry odor. The material has been so changed by the digestion process as to give little indication of its original constituents. The sludge from the activated sludge process is very bulky, high in moisture content, brown in color and with an earthy odor. Humus sludge, as its name implies, resembles humus and is composed of coagulated sewage solids and of bodies of organisms which are unloaded from the filter medium of trickling filters. It is brownish black in color, smooth in texture and of an odor which varies according to the condition of the sludge. At times when such sludge contains large numbers of decaying worms, the odor is distinctly offensive.

The quantity of sludge produced, per million gallons of sewage treated, varies with the kind of treatment, the strength and character of the sewage and the efficiency of the settling tank. The following table ndicates typical quantities and densities of the several kinds of sludge produced from sewage of the same characteristics.

Typical Volumes of Sludge Produced by Different Processes of Sewage Treatment

Treatment process	Nominal volume of sludge, gal. per mil. gal. of sewage treated	Proportion of solids in sludge, per cent	Specific gravity of sludge	Weight of dry solids in sludge, lb. per mil. gal. of sewage treated
Activated sludge	10 000	2.00	1.005	1675
Chemical precipitation	5 000	7.50	1.040	3250
Sedimentation	2 500	5.00	1.020	1060
Septic tank	500	5.00	1.040	220
Imhoff tank	500	15.00	1.070	670
Trickling filter—Humus tank	500	7.50	1.025	320

Separate Digestion of Sludge. In order to secure the benefit resulting from storage and digestion, while avoiding the difficulties sometimes experienced with Imhoff tanks, the sludge may be settled in plain settling tanks and removed to separate tanks for storage and digestion. Such separate digestion tanks are sometimes equipped with mechanical devices for agitation to mix the sludge and reduce scum formation. In some cases the sludge is heated to promote digestion, and sometimes the gases given off from the digesting sludge are collected and utilized for heating. The sludge from such tanks resembles Imhoff tank sludge.

Sludge Disposal. In all processes of sewage treatment, disposal of the sludge is a problem of primary importance. Except where sludge is lagooned or barged to sea some form of dewatering is necessary. The most important methods are: draining on sand beds, filter pressing, and complete drying for sale as fertilizer. The dewatered sludge from the first two methods is usually disposed of as fill, although in certain cities, notably Rochester and Schenectady, N. Y., dewatered Imhoff tank sludge is sold to local farmers at a nominal price for use as fertilizer.

Sludge drying beds consist of level areas of underdrained sand upon which the wet sludge is applied to a depth depending upon the kind and character of sludge. The applied sludge drains and dries with a rapidity depending upon weather conditions and character of sludge. The dried sludge reaches a moisture content of about 50-60% and shrinks about 65% in bulk. Ten days to three weeks may be required for the sludge to reach a spadable condition. To facilitate drying, particularly during adverse weather conditions, sludge beds are sometimes covered with glass roofs of greenhouse construction. The area of sand beds required may vary from about 0.5 to 2.0 sq. ft. per capita, depending on production of sludge, climatic conditions and whether covers are provided.

Several types of filter presses have been used for dewatering sludge. Usually conditioning of the sludge by acid, alum or lime is required to facilitate pressing. The resulting sludge cake contains about 75% water. Filter pressing is employed in relatively few places.

Lagooning consist in discharging wet sludge into relatively deep natural or artificial earth basins and allowing the material to compact as the result of drainage and evaporation. This method is applicable only when ample areas of isolated land permit. Lagooning of excess activated sludge is practiced on a large scale at Houston and Indianapolis.

The large volume and high moisture content of activated sludge cause a difficult dewatering problem where utilization as commercial fertilizer is attempted as at Milwaukee. Experience has demonstrated that before attempting to dewater the sludge it is essential to condition it by means of chemicals, such as sulphuric acid, aluminum sulphate and ferric chloride. At Milwaukee three chemicals and combinations of the three are used. After conditioning, the moisture content can be reduced by means of filter presses or by vacuum filters, the latter being used on a large scale at Milwaukee. The vacuum filters produce a thin cake with a moisture content of about 85%. Further reduction, to less than 10%, can then be accomplished by means of heat drying. The dried material is ground, screened and sold as fertilizer or fertilizer base.

Recent experiments have shown that activated sludge when mixed with fresh sludge from plain sedimentation or Imhoff tanks can be satisfactorily digested either in separate tanks or in the digestion compartments of Imhoff tanks. The mixed sludge after digestion may be dewatered on sand beds. At Chicago it is now planned (1928) to pump the excess sludge from the North-Side activated sludge plant to the West-Side Imhoff tank plant for digestion in the sludge compartments of the latter. At Fitchburg humus tank sludge is pumped to the sludge compartments of the Imhoff tanks for digestion with the primary sludge.

Fertilizer value of sewage sludge is not great as the most valuable fertilizing constituents of sewage are soluble. The following table gives the results of analyses of typical sludges.

Typical Analyses of Sludges

Per cent—Dry basis

	Nitrogen as ammonia		Potash	Phosphoric acid (P_2O_5)
	Total	Available		
Fine screenings*	2.5	....	0.5	0.5
Fresh sludge from plain sedimentation	4.0	....	....	1.5
Septic tank sludge	2.0	....	....	1.4
Imhoff tank sludge	2.5	1.3	trace	1.2
Chemical precipitation sludge	2.5	....	....	0.5
Humus tank sludge	3.5	2.5	trace	3.5
Activated sludge	5.5	4.0	0.2	2.5

* Included for comparison with sludges.

In addition to the chemical constituents noted, sludge contains humus material of some slight value.

Screenings and sludge from plain sedimentation are likely to be so ill-smelling and to contain so much valueless trash as to prevent farmers from using such material. Digested sludge is less open to these objections but there has been little demand for it, the sales at Rochester and Schenectady being exceptional. Humus sludge alone is hard to dewater, but it has some fertilizer value. Sludges dewatered by drainage and evaporation and as ordinarily handled contain so much moisture as to render them less valuable than stable manure. Activated sludge contains a relatively high per cent of nitrogen and for this reason is more valuable than digested sludge which has lost some nitrogen. A composite analysis of Milorganite, the trade name under which the product of the Milwaukee activated sludge plant is sold, is as follows:

	Per cent
Moisture	4.08
Total nitrogen	5.42
Equivalent to ammonia	6.58
Water-soluble organic nitrogen	0.30
Water-insoluble organic nitrogen	5.12
Nitrogen insoluble in neutral permanganate	0.71
Availability of water-insoluble organic nitrogen by neutral permanganate method	86.13
Active nitrogen by alkaline permanganate method	3.22
Availability of water-insoluble organic nitrogen by alkaline permanganate method	62.89
Total phosphoric acid	3.08
Insoluble phosphoric acid	0.65
Available phosphoric acid	2.43

This material is being produced (1929) at the rate of about 100 tons per day and sells for approximately $16 per ton f.o.b. cars Milwaukee.

12. Oxidation Processes

A large portion of the putrescible organic solids of sewage are in solution or in the form of colloids not removable by sedimentation. To change these solids to inert organic and mineral substances is the object of oxidation processes such as filtration through fine or coarse material and the activated sludge process. These oxidation methods are comparable with the action of the lungs in which the impurities of the blood are oxidized by bio-chemical processes.

Broad Irrigation was the earliest method adopted on a large scale for the treatment of sewage in which oxidation is involved. This method consists of applying sewage to relatively level areas of land and allowing the soil to absorb it. In some cases farming of land thus treated was attempted, to utilize the manurial value of sewage. Large areas of land are required, approximately an acre to each 10 000 gal. of sewage daily, and objectionable conditions with respect to flies and odors are often brought about. The use of this method in this country has been practically abandoned except in arid or semi-arid regions where the water content of sewage is valuable for irrigation purposes.

Intermittent Sand Filtration is a distinct advance over broad irrigation. The filters consist of beds of sand or fine gravel 2 to 6 ft. in depth, usually underdrained by tile pipe laid with open joints at intervals of about 40 ft. The effective size of the sand should preferably be between 0.20 and 0.35 mm. and the uniformity coefficient as near unity as practicable. Sewage is applied fairly evenly to the surface of the beds in doses from twice a day to once every

3 or 4 days. Application should be at a rate of about 1 cu. ft. per sec. per 5000 sq. ft. and the depth of dose should be about 3 in. The sewage per-

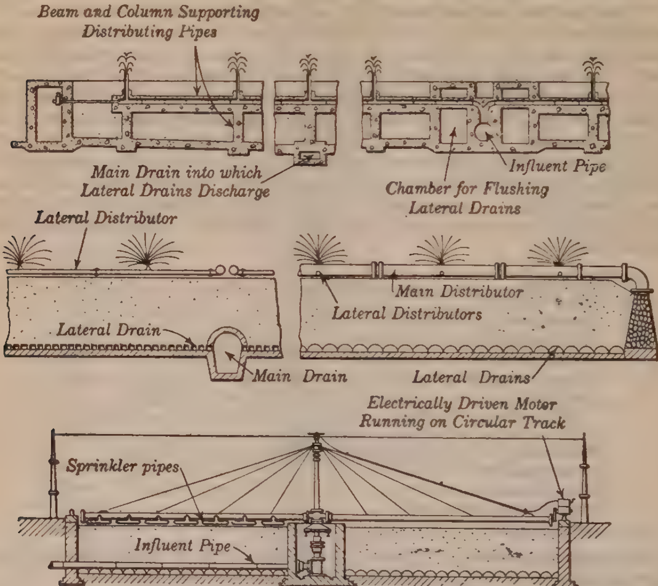


Fig. 11. Three Types of Trickling Filters

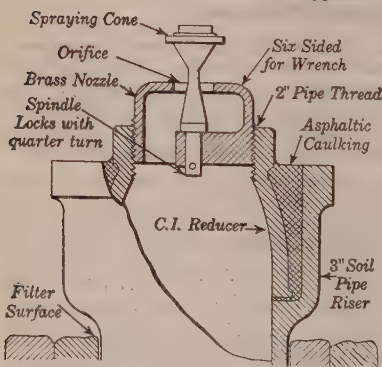


Fig. 12. Riser and Worcester Nozzle

colates through the sand to the underdrains. Application of the sewage may be through trough distributors or at the corners of the beds.

The action of this type of filter is partly to strain out suspended matters upon the surface and partly to oxidize the dissolved organic solids by the micro-organisms living upon the surface of the sand grains. At intervals weeds and the mat of solids accumulating on the surface of the filters must be removed. After cleaning, the beds should be harrowed and leveled by brush drag. Intermittent application of sewage and reasonably clean surfaces are essential in order that air may be admitted into the voids of the sand. For winter operation in cold climates the beds should be furrowed to permit the formation of a protective ice roof.

Effluents from properly operated filters are practically clear and colorless, non-putrescible and of low bacterial content. An acre of filters will treat about 30 000 gallons of sewage daily. Preliminary sedimentation increases the capacity of such filters to about 50 000 gal. daily.

Contact Beds are tanks filled with broken stone to a depth of 4 to 6 ft., the stone being from about 1/2 to 1-1/2 in. in size with 40 to 50% voids. The beds are filled with settled sewage, the effluent promptly drawn off and the beds allowed to rest for some hours with the voids full of air. In some cases it is necessary to apply the sewage to two or more beds in series. Automatic filling and emptying can be accomplished by timed siphons or similar devices.

In time the voids of the filter stone become clogged and the material has to be removed and cleaned, or replacement may be necessary as a result of disintegration. Contact beds can be operated at rates as high as about 100 000 gal. per acre-ft. or 500 000 gal. per acre per day with 5 ft. of depth. Effluents from carefully operated contact beds are usually non-putrescible, but rather turbid and of high bacterial content.

Trickling Filters are primarily beds of broken stone, 1 to 2-1/2 in. in size, 4 to 10 ft. in depth, laid on concrete floors covered with systems of underdrains for rapid discharge of effluent and for ventilation. Sewage is applied to the surface of these filters in the form of fine spray through spray nozzles or with mechanical traveling distributors. Traveling distributors may follow a circular or a rectilinear course. In the case of spray nozzles the sewage is discharged through them from automatic dosing apparatus under varying pressure, so as to secure uniform application of the sewage. This application takes place intermittently with a few minutes for dosing and a somewhat longer period for resting. The sewage sprayed through the air absorbs oxygen and as it passes down through the filter medium the film of micro-organisms on it utilizes this oxygen to oxidize and mineralize the organic matter.

Due to the practically continuous application of sewage and the abundance of oxygen in the voids of the filter media and sometimes in the applied sewage, this type of filter may be operated at high rates, 200 000 to 500 000 gal. per day per acre-ft. being typical. Usually the effluent is turbid, brown, non-putrescible and contains materially less bacteria than the applied sewage. Sedimentation of the effluent improves its quality by the removal of readily settleable suspended solids.

Data with respect to filter media may be found in "Progress Report of the Committee of the Sanitary Engineering Division on Filtering Materials for Water and Sewage Works," Proceedings Am. Soc. C. E., 1928, p. 243.

13. Activated Sludge Process

In this process large quantities of air are forced in fine bubbles through sewage as it passes through a tank having a detention period of 4 to 8 hours. After aeration the sewage is settled for the deposition of sludge. The sludge thus formed contains numerous micro-organisms similar to the biological film on the media of trickling filters, and is said to be activated. Part of the sludge deposited is returned to and mingled with the entering sewage. As the sewage is agitated by the air the activated sludge mingling with the sewage acts both to absorb and coagulate suspended solids and oxidize the organic matter.

The plant requires a comparatively small area of land and construction cost is relatively low. More elaborate mechanical equipment is necessary than in other processes and very careful operation is essential. The disposal of the bulky excess sludge is attended with some difficulty and considerable expense. The effluent is clear, sparkling, somewhat yellowish-brown in color and with relatively few bacteria.

Tanks are usually 10 to 15 ft. deep. The air is forced through porous plates set in the bottom of the tanks. Inclined deflector baffles placed longitudinally near the top and along one wall of an aeration tank produce a spiral motion in the sewage flow with a resultant improvement in mixing and a decrease in the quantity of air required for agitation. Washing and filtering of the air is necessary to prevent clogging of the pores of the diffuser plates.

The amount of air required varies with the character of the sewage, but for ordinary domestic sewage from 1 to 1-1/2 cu. ft. of air per gallon of sewage is required. For supplying air at a rate of 1.5 cu. ft. per gallon in a 10-ft. tank, approximately 30 hp. per hour are required per m.g.d. of sewage. The volume of activated sludge returned to the tanks may amount to from 20 to 25% of that of the sewage. The quantity of excess sludge removed is equivalent to from 1 to 1-1/2% of the volume of sewage treated.

Sedimentation after activation is generally accomplished by the use of tanks equipped with mechanical scrapers for continuous removal of the deposited solids. In the design of these tanks an allowance of 1 sq. ft. of tank is made for each 1600 gal. daily of flow at maximum rate.

Instead of compressed air for agitation of the sewage, surface stirrers have been devised consisting essentially of paddle wheels which when revolving agitate and aerate the sewage.

The economy of the activated sludge process depends mainly upon the net cost of disposal of sludge. Recent (1928) results at Milwaukee are encouraging.

14. Chlorination

Disinfection. None of the foregoing methods of treatment brings about the complete destruction of bacteria. Under certain conditions where water supplies or shellfish may be affected by the discharge of effluents, it is important to obtain practical freedom from disease germs. To accomplish this result, resort is frequently had to disinfection by means of hypochlorite of lime or liquid chlorine, the latter being most frequently used. The equipment used for application of chlorine is similar to that used for chlorination of water supplies. The quantity of chlorine required to effect satisfactory disinfection varies from about 10 to 15 p.p.m. for settled or screened sewage to 1 to 3 p.p.m. for the effluents from intermittent sand filters. A short period of contact, usually not over 15 minutes, is necessary to bring about proper disinfection.

Other Applications. Attempts have been made, apparently with some success, to minimize odors at treatment plants by chlorination of the sewage as it arrives. Attempts have also been made to reduce by chlorination the number of small moth flies frequently prevalent around trickling filters, but conclusive results have not been obtained. Chlorine is also used to deodorize the vapors given off by heat driers for activated sludge. Ferric chloride used for conditioning activated sludge is produced by chlorination of solutions of ferrous sulphate. Chlorine has also been found useful in reducing excessive growths of micro-organisms upon trickling filter media.

15. Small Sewage Disposal Systems

In the absence of a municipal sewerage system, the simplest method for meeting the sanitary needs of a single residence is the privy, in some cases fitted with removable containers or pails. Chemical closets are also used in which deodorization and disinfection are accomplished by caustic chemicals. Such methods are safe if suitable care is used, but are not applicable for houses provided with water supply and plumbing fixtures, for which a common means for disposal of sewage is the **Cesspool.** There are two types, the tight cesspool and the leaching cesspool. The former has the serious disadvantage of filling up rapidly, necessitating frequent emptying if overflow and objectionable conditions are to be avoided. The leaching cesspool permits the seepage of excess liquids into the surrounding soil and constitutes a satisfactory method of disposal where the soil is porous and the ground-water level below the bottom of the cesspool. Usually the earth around a leaching cesspool becomes clogged in course of time and emptying becomes necessary as in the case of a tight cesspool. There is also danger of the contamination of ground-water supplies derived from wells in the vicinity of such cesspools.

Where cesspools alone cannot be used, tight settling tanks followed by secondary means for the disposal of the excess liquid can be adopted.

The commonest form of settling tank for small disposal plants is the so-called **Septic Tank.** Such a tank is usually constructed of reinforced concrete, of capacity sufficient to provide a detention period of from 12 to 60 hours, the usual allowance being about 24 hours. The effluent from these tanks is discharged intermittently to the secondary treatment plant, by means of a dosing tank equipped with an automatic siphon. Ready-built commercial septic tanks of tile or iron should be used only if of adequate capacity and suitably arranged, particularly if no provision is made for intermittent application of the effluent to subsurface filters or trenches. The following secondary means of treatment are applicable: leaching cesspools, filter trenches, subsurface sand filters and open sand filters.

Filter trenches contain a series of pipes laid from 1 to 2 ft. below the surface of the ground, with open joints so that the settled sewage may pass out into the gravel with which the trenches are refilled, and thence into the surrounding soil. The length of pipe necessary is estimated at 20 to 100 ft. per person, depending upon the porosity of the soil. Where the soil is so dense and impervious as to make the use of filter trenches or leaching cesspools impracticable, subsurface filter beds may be used. Such beds are constructed by excavating a suitable area to a depth of about 4 ft. and backfilling with sand and graded level to a depth of about 3 ft. covering the top with about a foot of loam. The settled sewage is discharged through tile pipe laid with open joints at the surface of the sand, and the whole bed is underdrained. The size of bed should be sufficient to provide from 1 to 2 sq. ft. of surface area for each gallon of sewage per day, depending upon the strength of the sewage and the fineness of the sand. For residences, the area should be between 25 and 90 sq. ft. per capita and for day schools the requirement will be generally between 12 and 20 sq. ft. per capita.

Open intermittent filters can be adopted for large residential and institutional plants providing proper isolation can be secured. These filters are similar to the intermittent sand filters employed for the treatment of municipal sewage. The loading adopted for residential filters is usually less per acre than in a municipal plant because of the greater percentage of sediment carried upon the bed and the likelihood that it will receive less careful operating attention. The usual allowance for rate of treatment is in the neighborhood of 40 000 gal. per acre per day, the population served ranging from about 650 to 1000 persons per acre.

16. Industrial Wastes

There are almost as many kinds of wastes as there are industries and each industry has its characteristic wastes varying in quality and in quantity. In some industries the wastes are relatively unobjectionable or small in quantity. In other industries the wastes may be large in volume or extremely objectionable, or both. A convenient classification of industrial wastes is according to the nature of their principal constituents, namely animal, vegetable or mineral. The putrefactive quality of the several kinds of constituents are approximately in the order given.

As a matter of fact, many wastes are combinations of more than one class. Silk mill wastes contain animal gums from the silkworm cocoons, vegetable oils from the soaps and tin salts from the weighting process. Tannery wastes contain animal fleshings, tan bark extractives and spent lime.

The effects of pollution by industrial wastes are similar to those of sewage. Generally speaking, danger of infection from industrial wastes is probably less than from sewage, because many industrial processes involve treatment by heat or by chemicals which are germ-destroying. Furthermore, apart from the sewage from the industrial buildings, the materials in wastes are not such as are likely to carry bacterial infection. On the other hand, some wastes, like tannery wastes, may constitute a greater hazard to cattle than does sewage, due to the possible presence of anthrax germs.

Some industrial wastes contain quantities of froth-forming constituents, which when discharged into streams produce unsightly and persisting scum, objectionable to bathers and to boat owners. Other wastes high in coloring matter bring about unsightly discoloration. Wastes containing highly putrescible organic matter, animal or vegetable, decompose and exhaust the dissolved oxygen content of the stream, with resulting production of offensive odors. Wastes with large quantities of settleable solids form sludge deposits, which if organic in character undergo decomposition, and as this decomposition proceeds masses of offensive slimy material rise to the surface. Chemical wastes may have a direct destructive action upon fish life, either as a result of poisoning or by clogging of gills. Oil wastes form unsightly sleek and scum and hinder the natural reaeration which exposed water surfaces normally undergo. Phenol wastes from gas plants or from coke by-product recovery plants, reaching water supplies even in minute quantities, have frequently been the cause of medicinal tastes, particularly where the water is chlorinated. This type of pollution has been of great annoyance in the case of the Great Lakes cities of Milwaukee, Chicago and Cleveland.

Treatment of Industrial Wastes requires much the same methods as for sewage, although with some particular types of wastes special adaptations are required. Chemical precipitation, although now practically abandoned for the treatment of sewage, remains a very valuable method for the treatment of certain wastes, such as those from tanneries and paper mills. Screening and plain sedimentation are also employed for the removal of solids. Where a high degree of purification is required intermittent sand filters are used, and trickling filters, also, are of value. The activated sludge method has been tried experimentally and found capable of satisfactorily treating certain wastes such as tannery, packing house and beet sugar wastes. The acid crackling process for the treatment of wool scouring liquors has been employed successfully for many years. Chlorination has been practiced with tannery wastes for the destruction of anthrax germs. Controlled dilution, using storage ponds for conserving flood flows, has been employed in some cases, with a view of reducing the extent of treatment otherwise required.

The disposal of the sludge produced in the treatment of industrial wastes constitutes a serious problem. It is sometimes found feasible to utilize some of the sludge, as for instance cellulose fibres from paper mill wastes or from rubber reclaiming plants. In certain cases the treatment of industrial wastes has resulted in the recovery of materials of value, and the possibility of such recovery should always be considered in connection with wastes treatment problems. It is important, however, to avoid overstressing the profit possibility of wastes treatment, to the extent of overlooking the fact that the primary purpose is to prevent stream pollution, not to make profits.

SECTION 18

REFUSE COLLECTION AND DISPOSAL

BY
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ENGINEERS

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1. Municipal Refuse

Municipal refuse consists of the following classes of waste material: garbage, rubbish, ashes, street sweepings, stable refuse, offal or market refuse, dead animals, and industrial refuse (including waste building materials). The first three classes are those with which householders are concerned chiefly and they constitute the principal part of municipal refuse.

The municipal refuse of North America and Europe differ so greatly in composition that data from one continent should be used in design problems on the other only after making certain that conditions are comparable. This is due to differences in climate, habits of diet, methods of house heating, wrapping of merchandise and thrift of the inhabitants of the two continents. In England, for example, all three types of household refuse are stored in the same receptacle and collected in the same vehicle. This mixed refuse is known as "dust" and the collectors as "dust men." The contrast between this material and wet American refuse in the watermelon and corn season in the middle West is marked. In order to avoid misleading American engineers, no data regarding European practice are given herein.

Garbage is defined by the Committee on Collection and Disposal of Garbage (Sanitary Section, Boston Society of Civil Engineers) as the solid animal and vegetable waste matter produced wherever food is handled raw or prepared for consumption. The report* of this committee defines the principal terms used in dealing with collection and disposal of garbage. Frequently, as collected in municipalities, garbage includes more or less rubbish. There is a marked variation in the character of garbage produced in different cities, owing to differences in local conditions. It is difficult to secure representative samples for analysis because of the heterogeneous character of the material. Few available analyses cover periods of more than a few weeks and the results, therefore, show the composition only under stated conditions, and, due to the wide variation in character with the seasons and according to the localities collected, such analyses may differ widely. Garbage reaches the municipal collection vehicle in varying degrees of decomposition depending upon the frequency of collection, the state of the weather and the degree of cleanliness of the household. **The Moisture Content** in general lies between 68 and 80% by weight. The quantity of **Grease** in garbage varies approximately from 1 to 7% of the weight of raw garbage. As grease is derived principally from animal matter and as less meat and more vegetables are ordinarily eaten in warm weather, the proportion of grease in the garbage varies with the seasons, climate and habits of eating. The **unit weight** of garbage varies widely, due to the differences in habits of the population, the season, rainfall and the character of the collection service. In general the weight of a cubic yard of garbage lies between 900 and 1200 lb. An average unit weight frequently assumed is 40 lb. per cu. ft. (1080 lb. per cu. yd.).

Rubbish includes many kinds of waste material, such as newspaper, wrapping paper, boxes, cans, crockery, rags, rubber, leather, wood, brick, glassware, tinware, lawn and brush clippings, much of which material is combustible. The bulk of individual pieces varies greatly, from dust to articles as large as bedsteads and automobile fenders. Frequently, as collected in municipalities, rubbish includes more or less garbage and ashes. Much of the

* Jour. Boston Soc. C. E., 1927; **14**, 305.

material is of light weight and is easily blown about by the wind. The **Unit Weight** of rubbish is much less than that of either garbage or ashes, generally ranging from 125 to 250 lb. per cu. yd. An average unit weight of 200 lb. per cu. yd. is frequently assumed.

Mixed Refuse is a term used in this country to include garbage and rubbish but not ashes.

Ashes, as collected in municipalities, commonly have mixed with them more or less rubbish, especially tins and bottles, and frequently some garbage, but the major portion of this material consists of household ashes. The character varies with the geographical location, type of fuel used in houses, the seasons, the degree of separation and method of storage of household refuse and the frequency of collection.

Ashes as collected are generally dry and very dusty. Where stored in the open in wet weather the moisture content is greatly increased. The **Unit Weight** is usually slightly greater than that of garbage, lying between 980 and 1350 lb. per cu. yd.

Street Sweepings consist of the refuse which accumulates on the surfaces of streets and sidewalks, including the droppings from horses and other animals, miscellaneous litter thrown upon the streets, material carried by wheels from unpaved to paved streets, leaves, and sweepings from buildings and sidewalks. Street sweepings may be used as fertilizer due to their content of nitrogen, phosphoric acid and potash, but their value is not great, and they are most commonly utilized in covering over the surface of dumping grounds.

Dead Animals, Offal and Market Refuse must be promptly collected and disposed of to avoid their giving rise to offensive odors. Offal from fish markets is offensive even when handled promptly. Frequently the parties producing offal and market waste can be required to attend to its collection and disposal.

Industrial Refuse is of many kinds; some partly combustible and some non-combustible. Examples are boxes and package materials, steam cinders, blast furnace slag and waste excelsior. Little of this type of refuse enters into the problem of municipal collection and disposal because it is commonly considered the duty of the manufacturer to dispose of such refuse. The same is usually true of stable refuse, which, is of some value and generally can be sold.

2. Quantity of Refuse

Garbage. The quantity of refuse produced varies widely in different cities and from day to day and from season to season in a single city. The table at top of next page gives the annual per capita production of garbage, rubbish and ashes in several municipalities.

A reasonable figure for the average production of garbage in the larger cities in the northern portion of the United States which are not affected by peculiar local conditions, is about 180 lb. per capita per year or 0.5 lb. per capita per day.

Other things being equal, it is evident that the quantity of garbage in southern cities where fresh vegetables and fruit are cheap and abundant will be greater than in northern cities. The wealth of the population has a marked influence on the production. The following table which shows the difference in this respect between three manufacturing cities and three residential cities in Massachusetts is taken from the 1926 report of the Massachusetts Department of Public Health.

Quantity of Municipal Refuse in Pounds per Capita per Year

Municipality	Year	Garbage	Ashes	Rubbish
Akron, Ohio	1926	171	....	....
Arlington, Mass.	1924	125	745	32
Belmont, Mass.	1926	438	....	....
Brookline, Mass.	1924	337	1920	132
Buffalo, N. Y.	1919–24	126	....	....
	1922–24	....	....	117
Cambridge, Mass.	1924	112	1800	55*
Columbus, Ohio	1926	207	....	....
Dayton, Ohio	1925	166	....	....
Lynn, Mass.	1924	183	1340	....
Medford, Mass.	1924	201	1055	79
Milwaukee, Wis.	1924	164	....	....
New Bedford, Mass.	1926	188	....	....
Newton, Mass.	1926	290	....	....
Pawtucket, R. I.	1926	285	....	....
Rochester, N. Y.	1926	218	1660	101
Somerville, Mass.	1924	180	1445	102
Springfield, Mass.	1924	260	1420	125
Washington, D. C.	1923	342	....	....
Watertown, Mass.	1924	276	303	....
Worcester, Mass.	1926	182	....	....

* Light refuse.

Garbage Production in Manufacturing and Residential Cities in 1924

Manufacturing Cities	Pounds per capita per year	Residential cities	Pounds per capita per year
Fall River	185	Brookline	337
Lowell	123	Newton	278
New Bedford	206	Watertown	276
Average	171	Average	297

Economic conditions have a marked influence, more garbage being produced in good than in bad times. Nationality may also affect the garbage production, since most foreign-born inhabitants are less wasteful of food than native born citizens. In stormy weather uncovered receptacles collect rain and snow, thereby increasing the weight of material to be collected. In rural communities a portion of the garbage is sometimes disposed of on the individual premises by burial or feeding to hens or pigs, thus reducing the amount to be collected. With efficient service and frequent collections, a greater quantity of garbage is collected than where service is inefficient and collections infrequent. When garbage is wrapped in paper in the household a greater volume is collected due to the wrappers which are included. On the other hand a substantial amount of moisture is drained off before wrapping which tends to reduce the moisture content and weight.

There is a great **seasonal variation** in production due to the consumption of fruits and vegetables, particularly melons and green corn. This seasonal

Monthly Variation in Quantity of Garbage Collected, Expressed in Per Cent of Average Monthly Weight Collected in 1926

Month	Akron	Brookline *	Rochester
January	67	102	82
February	60	87	66
March	69	108	83
April	72	103	86
May	84	103	84
June	102	98	99
July	124	75	107
August	144	71	113
September	167	97	140
October	134	120	141
November	97	123	104
December	80	104	95
Estimated population	209 000	42 700	320 000

* Residential suburb of Boston.

variation is illustrated by the table above, in which minima and maxima of 60% and 167% occur. The range differs materially in different cities. The variation in the quantity of garbage collected in short periods is illustrated by the next table.

Variation in Quantity of Garbage Collected in Short Periods
(In per cent by weight)

	Buffalo	Rochester	Akron
Daily average for year	100	100	100
Maximum four weeks	135	155	171
Maximum week	150	162	181
Maximum day	169	167	197

Rubbish. The quantity of rubbish collected varies substantially in different cities, as is illustrated by the table on page 1752. The per capita production of rubbish is greater now than it was a generation ago, due to changing customs such as the use of package goods and the great increase in size and circulation of newspapers and magazines. Furthermore, the average American family is probably more wasteful and discards more materials now than formerly. Other things being equal, a well-to-do residential city will produce more rubbish per capita than a manufacturing city. The amount of rubbish to be disposed of by municipalities increases with the efficiency of the collection service. The seasonal variation is not great. It is shown for several cities in the table at top of page 1754.

Ashes vary in quantity with the climate and the season, although practically every city will produce a considerable quantity of ashes even in summer. The type of fuel used for heating and cooking has a marked influence on the ash production; for example the production is high in New England where anthracite commonly is burned, while it is low where bituminous coal is used and even lower in the natural gas districts.

The number of small **dead animals** (mainly cats and dogs) to be disposed of

Monthly Variation in Quantity of Rubbish Collected in Per Cent of Average Monthly Quantity (by weight)

Month	Buffalo 1921	Cambridge 1924	Lynn 1924	Rochester 1926	Somerville 1924	Springfield 1924
January.....	80	95.4	112.0	82	112.2	99.7
February....	73	95.4	94.3	76	90.5	86.5
March......	91	97.9	98.5	92	112.2	90.1
April........	102	102.0	106.8	107	91.9	113.8
May........	116	104.1	94.2	93	108.4	95.7
June........	110	92.7	96.4	110	90.5	96.3
July........	100	105.7	106.8	100	115.9	120.4
August......	93	103.6	93.5	105	112.2	94.6
September...	109	106.3	104.6	107	90.5	93.9
October.....	121	111.8	97.5	124	97.1	117.0
November...	100	90.3	91.0	99	92.8	92.4
December...	105	95.6	106.5	105	95.6	99.7

varies widely in different cities, as well as from month to month in the same city. In general, there will be more dead animals during the summer months, owing chiefly to the detrimental effect of hot weather upon dogs. Humane societies frequently deliver large numbers to the city for disposal. The following table gives the number of small dead animals collected by the municipal forces in three cities:

Number of Small Dead Animals Collected

City	Year	Population (estimated)	Number of dead animals	
			Total	Per thousand population
Akron, Ohio..................	1926	215 000	3 101	14
Milwaukee, Wis..............	1919	450 000	3 536	8
Rochester, N. Y..............	1926	320 000	20 865	65

The following table gives some data concerning the quantity of **street sweepings** collected and the area of streets cleaned:

Quantity of Street Sweepings Collected per Year

City	Year	Estimated population	Total quantity of street sweepings, cu. yd.	Per capita quantity of street sweepings, cu. ft.	Million square yards cleaned
Columbus, O.......	1926	285 000	37 604	3.56	203
Boston............	1926	790 140	77 350*	2.64*	243*
Chicago...........	1926	3 074 382	354 953†	3.12†	2092†
Washington, D. C...	1927‡	500 000	....	....	2736

* Figures refer to paved streets and macadam gutters only.
† Figures refer to streets cleaned by hand only.
‡ Year ending June 30.

3. Household Handling and Storage

It is essential to the maintenance of sanitary conditions for each householder to fulfill his obligations with respect to the care of garbage, rubbish, and ashes. Of primary importance is the separate storage of the different classes of refuse. Their mixing may give rise to offensive, insanitary conditions and to serious fire hazards. Wrapping of garbage in paper is desirable from the standpoint of household sanitation.

Containers for each class of refuse should provide sufficient capacity to store the accumulation of that class of material between collections under conditions of maximum production. Garbage pails should vary in size according to the number of persons in the family, but, as a rule, no pail should be used having capacity in excess of about 12 gal. Where greater capacity is required, additional pails should be provided. For ashes, no container should have a capacity exceeding about 4 cu. ft., as this represents the maximum weight that can readily be lifted by two men into the collection vehicle. It is generally advisable for each householder to have not less than two ash cans. Galvanized iron is probably the most suitable material for garbage pails and rubbish and ash cans, since it can be kept clean with comparatively little difficulty. Garbage pails should be provided with tight covers. On account of the serious risk of fire, ashes should never be stored in wooden barrels or other combustible containers.

It is usually advisable to store garbage pails on the rear porch or in the back yard, where they are readily accessible to both householder and collector, and where sunlight and fresh air have access to the surroundings. Ashes and rubbish, on the other hand, are generally stored in the basement, but combustible rubbish should not be kept near the heating plant on account of the fire hazard.

It is not wise to place ashes and combustible refuse in the same container, because of the danger of fire. Neither is it desirable to mix garbage and rubbish, because of the increased volume of foul material stored and the increased number of containers to be kept clean. The admixture of hot ashes with garbage may result in offensive odors; moreover, if the mixture of ashes and garbage is disposed of by filling waste land, the dumps may become infested with flies, rats and other vermin.

Regulations. In order to promote cooperation between householders and collectors, simple regulations for the household care of garbage, rubbish and ashes should be printed on cards and delivered to every household. If such regulations set forth clearly the conditions under which refuse collections are to be made, the number of needless complaints on the part of the householders will be greatly reduced, and the disposal of refuse will be simplified. All complaints should be investigated immediately, and if they prove valid, prompt remedial measures should be taken.

4. Collection

Separate vs. Mixed Collection. Collection of municipal refuse may be classified either as separate or mixed collection. Separate collection necessitates independent storage in separate containers by the householder and independent collection of each of the different kinds of refuse. Mixed collection does not require household separation, although it can be adopted even if the different classes of refuse are separately stored by the householder. Which collection method should be adopted by the municipality depends upon several factors. Each method has both advantages and disadvantages. Separate collection is essential if attempt is made to obtain profit by salvage of material and recovery of by-products, or if disposal of the three principal classes of refuse involves use of ashes and non-combustible materials for fill, the incineration of combustible rubbish and the reduction or feeding to hogs of garbage. Mixed collection brings about a thorough mixture of garbage and rubbish, which may be of some advantage where disposal is by incineration, although the whole mass of material may become moist and difficult to burn. In some cities, particularly those with large foreign populations, it is difficult to secure proper separation of refuse by householders.

Separate collection has the advantage that a much smaller bulk of offensive material accumulates in periods of interrupted service. After such interruptions concentrated effort upon the collection of garbage alone would be much more effective as a measure of household sanitation than if a much larger accumulation of mixed refuse had to be collected.

With separate collection the methods of handling by the city as well as the householder and the equipment used in collection must be adapted to the kinds of refuse. Collection vehicles can be made of such material and so designed as to be easily and quickly cleaned and covered. In the case of mixed refuse the collecting vehicle must conform with the sanitary requirements incidental to the collection and transportation of the wet offensive garbage, whereas with separate collection the vehicles for rubbish and ashes may be of a different type suited to the much greater bulk of light rubbish,

Number of Collections of Garbage per Week in Twenty-six New England Municipalities

Municipality	Residences		Restaurants and hotels		Markets	
	Summer	Winter	Summer	Winter	Summer	Winter
Belmont	2	1	....	....	6	6
Brockton	1	1	6	6	6	6
Brookline	3	3	6	6	6	6
Cambridge	3	2	6	6	3	3
Concord, N. H.	1	1	7	7	1	1
Everett	3	2	7	7	7	7
Fitchburg	2	2	7	7	7	7
Framingham	2	1	7	7	7	7
Gloucester	3	2	7	7	0	0
Laconia, N. H.	1	1	1	1	0	0
Lynn	1	1	6	6	6	6
New Bedford	3	2	7	7	7	7
Newton	2	2	7	7	7	7
North Andover	2	1	....	....	2	1
Norwood	2	2	3	3	3	3
Pawtucket, R. I.	3	2	3	2	3	2
Pittsfield	2	1	6	6	6	6
Providence, R. I.	3	2	7	7	7	7
Salem	5	5	7	7	7	7
Somerville	2	1	2	1	2	1
Springfield	2	2	7	7	3	3
Stoughton	2	1	6	3	6	3
Swampscott	2	2	7	2	7	2
Watertown	2	2	6	4	6	4
Westfield	2	2	7	7	7	7
Worcester	2	2	7	7	7	7

and with less rigorous requirements as regards ease of cleaning, watertightness and covers.

Frequency of garbage collection should be governed by considerations of sanitation, householders' convenience and cost. In winter it depends more upon conditions of storage on the householder's premises than upon decomposition and odors. In cold weather frozen garbage often occupies a portion of the outdoor containers, thus reducing their effective capacity. In disagreeable weather the collector occasionally skips some houses, and during very severe storms or heavy snows the collection service may be interrupted. In many municipalities daily collections of garbage are made from restaurants, hotels and markets, because of the large quantities produced. Such collections are frequently made by crews and equipment specifically assigned to that duty. The general practice as regards frequency of garbage collection in New England municipalities is indicated in the table on page 1756.

Practice in regard to rubbish and ash collection is not nearly as uniform as in the case of garbage collection. While many cities have frequent and regular collections of rubbish and ashes, both in winter and summer, many others do not collect either of these types of refuse. In many cases periodic collections are made from once a year, generally in the spring or the fall, to three or four times per year. The table below shows the frequency with which ashes and rubbish are collected in the residential districts of various cities.

Frequency of Collection of Ashes and Rubbish in Residential Districts

Municipality	Number of collections per month		
	Ashes		Rubbish
	Summer	Winter	
Buffalo, N. Y.	4	8	4*
Indianapolis, Ind.	2	4	....
Milwaukee, Wis.	1	1	....
Montclair, N. J.	4	4	4
Richmond, Va.	4	4	4
Rochester, N. Y.	2	4	4
Salt Lake City, Utah	4	4	4
Seattle, Wash.	4	4	4

* 8 Collections per month in summer.

Frequently ashes and rubbish are collected daily in business districts.

Collection Districts. In devising a system of refuse collection for a given municipality, it is generally wise to divide the territory from which collections are to be made into a number of districts, each one of which shall represent the area to be covered by one collection crew in the course of say a week. If ashes, rubbish and garbage are to be collected separately, it may be necessary to have different numbers of districts for the several classes of refuse. Moreover, if the frequency of collection of any class of refuse is to vary with the seasons, it will be necessary to have, for such refuse, a different number of districts in summer than in winter. The total number of districts for one class of refuse will depend largely upon the total production of refuse, the frequency of collection, the type and size of collecting vehicle, the number of men per vehicle, and the average length of haul to transfer station or point of disposal.

The relative sizes of districts will depend upon local conditions, involving such factors as density and character of population, number of miles of streets, grades of streets, and distance from transfer station or disposal site.

Each district may be subdivided into a number of routes, representing the territory to be covered by one collection crew on each day of the week. If each collection crew works regularly in the same district and follows the same routes, the men soon become familiar with the location of all refuse containers, which promotes speed and thoroughness.

Transfer Stations. Except in small municipalities or where refuse is disposed of at centrally located sites, it is necessary to provide transfer or loading stations, or to relay the refuse as by the tractor-trailer system. Such stations should be as centrally situated as is consistent with suitable isolation. Structures should be of durable, easily cleaned, material. Long storage of garbage at such stations should be avoided and cleanliness should be enforced. Grounds should be maintained in neat and attractive condition.

Vehicles. There are in general use throughout the country at the present time three types of vehicles for refuse collection; namely, horse-drawn wagons, motor trucks, and trailers drawn by horses during collection and picked up, when loaded, by motor tractor-trucks, for hauling in trains to the point of disposal. Local conditions determine in each case which type is best adapted to the collection of the class of refuse under consideration.

Consideration should be given to such factors as the time spent in actual collection, the time spent in transportation, the cost of motor trucks while operating at low speed with house to house stops and while operating at high speed fully loaded, expense of chauffeurs, whether or not chauffeurs help in collections, the number of men per vehicle, the size of vehicle, the traffic problem presented by trains of trailers, the necessity of brakemen or flagmen on trailer trains, criticisms by the public of parking of empty trailers and trailers loaded with refuse throughout the city. In some cases it may be desirable and economical to use different types of vehicles for the different services, and different types of vehicles for the same service in different portions of the same city.

Experience shows that vehicles of many sizes are used, varying from one cubic yard capacity up to as much as twenty-six cubic yards capacity in the case of rubbish wagons.

Express Service for isolated markets and lunch rooms requiring daily garbage collection and for the collection of small dead animals is often satisfactorily provided by one-ton trucks.

Organization. A comprehensive organization for the collection of refuse includes the following subdivisions under one or more superintendents: collection gangs respectively for garbage, rubbish and ashes, inspectors, clerks, repair gang, and either stable or garage force, or perhaps both. The collection gangs are made up of teamsters or chauffeurs and helpers; the repair gang includes mechanics and helpers; the garage force includes men who wash the vehicles, and the stable force consists of veterinarians, barn men, harness makers, blacksmiths and watchmen. The majority of the personnel is actively engaged in collection.

The principal duties of the clerks are those of receiving and tabulating complaints, time-keeping and record keeping. If such records are kept in considerable detail, they are valuable in checking the efficiency of the individual collectors and in studying ways to improve the collection system as a whole.

Fig. 1 shows a summer organization chart prepared in 1927 for Akron.

The following table from the report of the Boston Society of Civil Engineers Committee shows the equipment and personnel comprising the garbage collection gangs of 16 New England municipalities:

Equipment for Collection of Garbage in Certain New England Municipalities, 1926

Municipality	One-horse wagons			Two-horse wagons		
	Number	Capacity, cu. yd.	Men per unit	Number	Capacity, cu. yd.	Men per unit
Belmont	......	......	......	5	4.4	......
Brockton	16	1.5	1	1	3	2
Brookline	......	......	......	......	......	......
Cambridge	22	2	2	......	......	......
Concord, N. H.	......	......	......	......	......	......
Fitchburg	2	2	2	1	3.5	2
Framingham	......	......	......	......	......	......
Lynn	13	2.4	2	......	......	......
New Bedford	1	2	1	12	3.3	2
Newton	17	2.4	1	1	3.6	1
Pawtucket, R. I.	......	......	......	7	2*	......
Salem	13	1.2	2	......	......	......
Somerville	......	......	......	10	4.7	2
Springfield	2	3.3	2	10	4	2–3
Watertown	......	......	......	4	3	2
Worcester	......	......	......	15	2.5*	2

Municipality	Motor trucks			Collection capacity per 1000 population, cu. yd.	Covers
	Number	Capacity, cu. yd.	Men per unit		
Belmont	1	5.2	......	1.65	......
Brockton	2	3	2	0.50	Canvas, trucks; Wood, wagons
Brookline	4 1	3.5* 5*	3	0.41*	Canvas
Cambridge	2	5.5	4	0.46	Canvas and wood
Concord, N. H.	2	1.5	2	0.13	Steel
Fitchburg	2	5.5	3	0.42	Canvas
Framingham	2	4.7	2	0.43	......
Lynn	......	......	......	0.30	Wood
New Bedford	......	......	......	0.32	Canvas
Newton	1 1	5* 1*	2 1	......	Wood and canvas
Pawtucket, R. I.	......	......	......	0.22*	Wood
Salem	1	......	......	......	Wood and iron
Somerville	......	......	......	0.47	Canvas
Springfield	......	......	......	0.31	Canvas
Watertown	1	4.7	4	0.66	Canvas
Worcester	8	3*	3	0.32*	Canvas

* Net tons (2000 lb.)

Cost of Collection. Records indicate a wide variation in the annual per capita cost of collection of refuse throughout the country, due to differences in methods of collection, frequency of collection, degree of thoroughness with

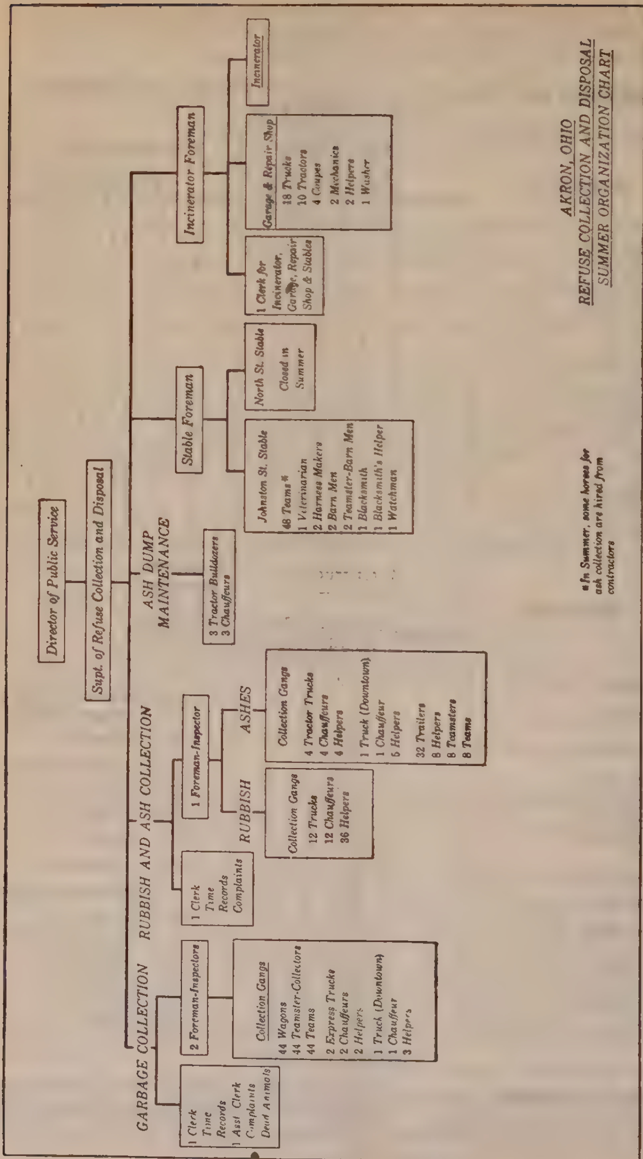
AKRON, OHIO
REFUSE COLLECTION AND DISPOSAL
SUMMER ORGANIZATION CHART
Director of Public Service
Supt. of Refuse Collection and Disposal
GARBAGE COLLECTION
1 Clerk
Time
Records
1 Asst Clerk
Complaints
Dead Animals
2 Foreman-Inspectors
Collection Gangs
44 Wagons
44 Teamster-Collectors
44 Teams
2 Express Trucks
2 Chauffeurs
2 Helpers
1 Truck (Downtown)
1 Chauffeur
3 Helpers
RUBBISH AND ASH COLLECTION
1 Clerk
Time
Records
Complaints
1 Foreman-Inspector
RUBBISH
Collection Gangs
12 Trucks
12 Chauffeurs
36 Helpers
ASHES
Collection Gangs
4 Tractor Trucks
4 Chauffeurs
4 Helpers
1 Truck (Downtown)
1 Chauffeur
5 Helpers
32 Trailers
8 Helpers
8 Teamsters
8 Teams
ASH DUMP MAINTENANCE
3 Tractor Bulldozers
3 Chauffeurs
Stable Foreman
Johnston St. Stable
48 Teams *
1 Veterinarian
2 Harness Makers
2 Barn Men
2 Teamster-Barn Men
1 Blacksmith
1 Blacksmith's Helper
1 Watchman
North St. Stable
Closed in Summer
Incinerator Foreman
1 Clerk for Incinerator, Garage, Repair Shop & Stables
Garage & Repair Shop
18 Trucks
10 Tractors
4 Coupes
2 Mechanics
2 Helpers
1 Washer
Incinerator
* In Summer, some horses for ash collection are hired from contractors

Fig. 1

which the work is performed and methods of municipal accounting. In the more densely populated cities garbage collection costs are likely to range from $0.30 to $1.50 per capita under present wage scales (1928). Costs of ash collection are likely to have about the same range and costs of rubbish collection may be slightly lower per capita.

Costs of garbage collection per ton, except for unusual cases, range from $3.00 to $9.00, depending upon efficiency, length of haul, type of collecting vehicles and other local conditions. Per ton costs of rubbish collection are likely to run slightly higher and costs of ash collection may run considerably lower than those for garbage.

5. Miscellaneous Methods of Disposal

The **burial of garbage** by the individual householder is common practice in small municipalities in which no organized municipal collection and disposal service exists. If disposal by burial is made a municipal practice, the garbage should be placed in shallow trenches and covered with earth. In winter in the north this is difficult to accomplish. Isolated location for such practice is essential if conducted on a large scale. In order to keep the rate of application to land sufficiently low to bring about rapid and inoffensive decomposition of organic matter, large areas are required.

Although, if properly carried out, burial is a reasonably safe and sanitary method for small communities, it is an expensive method for large cities. It is also extremely difficult to carry out in such manner as to avoid nuisances from odors, rats, cockroaches and crows.

Filling. This method consists of the deposition of refuse on low lying ground. Filling differs from burial in that the depth of fill is limited by topographical considerations only, whereas with burial the depth is restricted to a few inches in order to foster active decomposition. This method has the advantage that several different sites may be utilized simultaneously in a municipality, thus reducing the cost of haul without greatly increasing the operating cost of disposal. It is the method most commonly used for ashes, and where ashes are collected separately it is a satisfactory method for their disposal. It is also frequently used for the disposal of rubbish and sometimes for garbage.

For sanitary reasons, garbage alone should be disposed of by filling only under exceptionally favorable conditions. If mixed with ashes, earth or other suitable materials, the use of garbage for filling may be less objectionable, providing proper operation of the dumps is secured, but even under the best operation, objections are likely to be raised due to the breeding of rats and other vermin in the filled material.

Dumps frequently catch fire, particularly where combustible refuse is present, thus providing a fire hazard to nearby buildings. The odor and smoke from burning dumps are highly objectionable.

Blowing papers and clouds of dust may cause annoyance to adjacent property owners and traffic upon nearby highways.

Some sea coast cities dispose of their efuse by **dumping at sea.** The chief objection to this method is the stranding of garbage or rubbish upon beaches which are used for recreational purposes. This method is used to a large extent by New York City. Recently a large number of bottles were discharged at the regular dumping ground containing addressed post cards, asking for information regarding the location where the bottles are picked up. The result of this investigation gave interesting data on the distance traveled by floating objects. A few of the bottles apparently floated southerly into the Gulf Stream and thence across the Atlantic to Portugal, France, England and Ireland.

Salvage. There is a substantial amount of material of value in municipal refuse, as for example, paper, rags, bottles and cans. In rubbish dumps some of this material is commonly salvaged by individuals, occasionally operating under concessions from the municipality. Several municipalities have practiced salvaging saleable material from rubbish prior t odumping or burning the unsaleable portion. In such cases the sorting is generally done on a traveling belt conveyor. Many cities have abandoned this practice but a few still carry it out, as for example, Washington, Baltimore and Rochester. Under recent market conditions salvage by American cities is of doubtful economy.

6. Feeding Garbage to Hogs

If properly carried out under suitable conditions, the feeding of garbage to hogs is a satisfactory method of partial disposal, particularly for the smaller communities. Under proper management and favorable market conditions, there may be an economic advantage arising from the conversion of garbage into pork.

The selection of a suitable site for a piggery is of prime importance. It should be at such a distance from the nearest dwellings that there will be no ground for complaint from aerial nuisances. It should be situated on soil which drains readily at all seasons, preferably sand or gravel. There should be plenty of water available for washing and flushing, and good sewerage facilities. The location of piggeries is usually comparatively remote from the municipality in which the garbage is collected. On this account, long haul transportation problems are involved, and legal difficulties may be encountered in transporting materials through municipalities other than that from which the garbage is obtained. It is important that the route traveled to the farm should be over good roads, but preferably ones without too much traffic.

The buildings where hogs are kept should be of substantial construction, well lighted and ventilated, and they should preferably be located on a southern slope.

The economical operation of a piggery requires that the number of hogs be varied with the seasonal production of garbage so that as large a proportion as possible of the garbage shall be eaten. This can be accomplished by regulating the farrowing season, or by purchasing new stock from dealers who in some localities specialize in raising and selling suckling pigs. The following table gives data regarding the relation between quantity of garbage collected and number of hogs fed with garbage.

Relation of Quantity of Garbage Collected to Number of Hogs Fed

City	Year	Average quantity collected per hog, lb. per working day	New stock obtained by
Akron	1926	27	Breeding
Buffalo	1922	49	Purchase
Los Angeles	1924	22	Breeding
Newton, Mass	1924	25	Breeding
Worcester, Mass	1926	23	Breeding

The ratio of the number of men employed in operating piggeries to the number of hogs fed varies with the degree of cleanliness maintained. At Worcester one caretaker is employed to feed, bed and clean the pens for each 250 to 300 hogs.

Data in regard to the costs of construction and operation of piggeries are meager. In the great majority of cases such cost figures are probably lower than they should be because the structures are too cheaply built and the premises are seldom maintained in a thoroughly clean and sanitary manner.

The feeding of garbage to hogs is not a process of complete disposal. There are three products of a hog farm, the pork, the uneaten garbage, and the manure. Experience indicates that the quantity of uneaten garbage and manure or piggery refuse may be equivalent to from 30 to 50% of the quantity of garbage received.

Disposal of Piggery Refuse is perhaps the most important sanitary problem connected with this method of garbage disposal. If not frequently and thoroughly removed from pens and buildings and properly treated it decomposes rapidly and produces offensive odors. Generally it is composted and then used on nearby farms. Compost heaps, unless properly cared for, give off offensive odors and are prolific breeders of flies. Furthermore, the material when spread upon land as a fertilizer disseminates odors broadly, at least until plowed under. Other methods of disposal of piggery refuse are burial, incineration and drying. With either of the latter two methods the total investment in the hog farm is greatly increased. Following is an analysis of dried piggery refuse from a test by the authors in a commercial indirect steam dryer at Buffalo in March, 1922:

Analysis of Dried Piggery Refuse

	Per cent
Moisture	7.70
Ammonia as NH_3	3.14
Calcium phosphate as P_2O_5	2.28
Potash as K_2O	1.15
Grease (ether-soluble matter)	12.33

Should epidemics of hoof and mouth disease, hog cholera or tuberculosis occur the whole process of garbage disposal will be interrupted for an indefinite period of time. Protective inoculations, however, will aid considerably in reducing the hazard of interrupted service from the common hog diseases. Foreign matter in garbage, such as phonograph needles, safety razor blades, nails and glass cause some casualties.

If objectionable conditions are to be avoided, every precaution must be taken in design, in operation and in disposal of piggery refuse to prevent the breeding of rats, flies and crows, which abound at garbage piggeries where preventive measures are not taken. Where nuisances from piggeries have become sufficiently obnoxious, injunction proceedings by aggrieved property owners have seriously embarrassed the continuance of operation.

7. Reduction of Garbage

In general the process of reduction consists of cooking the garbage under steam pressure for a number of hours, after which the water and grease are removed and the solid matter dried. The grease is separated and used generally for soap making, and the dry material, called "tankage," is used as a base for fertilizers. Recently in some plants some of the finer tankage has

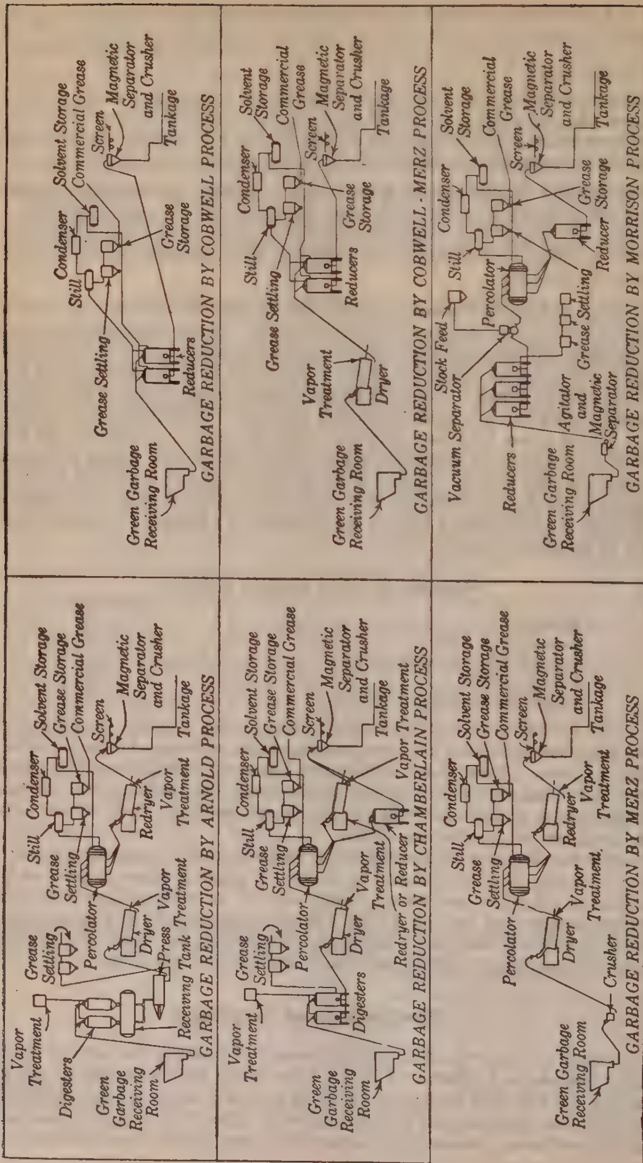
GARBAGE REDUCTION BY ARNOLD PROCESS
Vapor Treatment
Digesters
Green Garbage Receiving Room
Grease Settling
Percolator
Press
Receiving Tank
Dryer
Vapor Treatment
Still
Grease Settling
Condenser
Solvent Storage
Grease Storage
Commercial Grease
Screen
Magnetic Separator and Crusher
Redryer
Vapor Treatment
Tankage
GARBAGE REDUCTION BY CHAMBERLAIN PROCESS
Vapor Treatment
Green Garbage Receiving Room
Grease Settling
Digesters
Percolator
Dryer
Vapor Treatment
Redryer or Reducer
Still
Grease Settling
Condenser
Solvent Storage
Grease Storage
Commercial Grease
Screen
Magnetic Separator and Crusher
Vapor Treatment
Tankage
GARBAGE REDUCTION BY MERZ PROCESS
Green Garbage Receiving Room
Crusher
Percolator
Dryer
Vapor Treatment
Grease Settling
Still
Condenser
Solvent Storage
Grease Storage
Commercial Grease
Redryer
Vapor Treatment
Screen
Magnetic Separator and Crusher
Tankage
GARBAGE REDUCTION BY COBWELL PROCESS
Green Garbage Receiving Room
Grease Settling
Reducers
Still
Condenser
Solvent Storage
Commercial Grease
Grease Storage
Screen
Magnetic Separator and Crusher
Tankage
GARBAGE REDUCTION BY COBWELL - MERZ PROCESS
Green Garbage Receiving Room
Vapor Treatment
Dryer
Grease Settling
Still
Reducers
Condenser
Solvent Storage
Commercial Grease
Grease Storage
Screen
Magnetic Separator and Crusher
Tankage
GARBAGE REDUCTION BY MORRISON PROCESS
Green Garbage Receiving Room
Reducers
Vacuum Separator
Stock Feed
Agitator and Magnetic Separator
Grease Settling
Still
Percolator
Condenser
Solvent Storage
Commercial Grease
Reducer
Grease Storage
Screen
Magnetic Separator and Crusher
Tankage

Fig. 2

been separated and sold for stock feed. Considerable grease remains in the tankage after cooking and usually is extracted by a solvent such as gasoline. In some processes the material is not cooked but is dried by direct heat or steam in revolving steel drums and the grease is recovered by extraction. Fig. 2 is a diagramatic outline of various reduction processes.

In the **Arnold process** the garbage is cooked in steel tanks for several hours, after which it is passed through a roller press to remove a portion of the grease and moisture. The liquor pressed out is known as "stick liquor." It contains a substantial amount of fine solids and grease and has a foul odor. The grease and stick liquor then flow through a series of tanks in which the grease rises to the top and is separated. The stick liquor is then discharged into a sewer or adjacent water course, or in some cases is evaporated and the residue added to the tankage. The solid matter, after the stick liquor has been pressed out, is passed through a direct heat dryer to remove moisture. Additional grease is then extracted by gasoline in a percolator. After extraction, the solvent remaining in the material is blown out with steam to avoid subsequent explosion. In a second direct heat dryer the tankage is given a final drying. The grease and solvent are separated by distillation and the solvent used again. Tankage is finished for market by screening and crushing and by the removal of metallic objects. The Arnold process has been in use for many years at Columbus and elsewhere.

The **Chamberlain process** is essentially identical with the Arnold, the chief difference being that in the Chamberlain process the stick liquor is pressed out of the cooked tankage through openings in the bottom of the cooking tank by steam admitted through the top of the tank, while in the Arnold process this is performed in a rotary press. The Chamberlain process has been in use for many years at Cleveland and elsewhere.

The **Morrison process** is similar to the Chamberlain process, the chief difference being that the steam for cooking is in a steam jacket on the outside of the tank, while in the Chamberlain process it is admitted directly to the tank. Some of the grease is removed by skimming from the top of the tank after cooking, some is drained off with the stick liquor and some extracted with solvent. Subsequent procedure is much the same in both processes. This method has been installed recently at Dayton and Indianapolis and a few other places.

In the **Cobwell process** the drying and grease extraction are carried on in the same tank in which the cooking has been done. The moisture is vaporized in the tank by heat from an external steam jacket and is removed by vacuum pumps. All the grease is obtained by extraction with solvent and separation of the solvent and the grease is by distillation. This method of removal of water takes a longer time than that required by the processes already discussed, and while two or sometimes three charges can be treated in a day in a single tank in the other processes, the Cobwell reducers can handle only one charge per day. The solvent loss with this process is considerably greater than with the other methods. The grease and tankage recoveries are, however, somewhat greater than in any of the other processes. The Cobwell process has been in use for many years at Rochester.

A modification of the Cobwell process which has been employed in some cities consists in removing a portion of the moisture in the raw garbage by direct heat drying in horizontal revolving drums prior to cooking in the Cobwell reducers. This method of moisture removal is more economical than that employed in the Cobwell process itself. This method is now in use at Syracuse, and is known as the **Cobwell-Merz process.**

Drying Processes. Many methods of drying garbage without cooking have been practiced from time to time. In Chicago there is a large plant using the **Merz process,** which consists merely of drying the garbage in direct heat dryers and extracting the grease with a solvent in percolators. In direct heat dryers a portion of the material may be burned or scorched, in which case very foul odors are created. If a moderate quantity of moisture is retained, scorching may not be extensive, but in any event the vapors from this process must be deodorized before discharge to the atmosphere if complaints from odors are to be avoided. This deodorization is effected by scrubbing the vapors by water spray, followed by chlorination.

Drying can be effected also by indirect heat dryers in which the heat is provided by steam, either in a jacket around the outside of the tank containing the garbage or in a series of tubes within a horizontal revolving steel drum through which the material passes. Vapors from indirect heat drying also must be deodorized to prevent the escape of objectionable odors.

Tankage is finished for market generally by screening, grinding or crushing and the removal of metallic objects by magnetic separators. In some cases, as at Indianapolis and Rochester, a portion of the finer material in the tankage is separated by suction and sold for stock feed. In Rochester the grease is extracted with solvent from that portion of the tankage sold as stock feed, but in other cities all the original grease remains in the stock feed except that which is removed by skimming or draining in the digestor. In cases where the grease is not removed by solvent extraction from the stock feed all revenue derivable directly from that portion of the grease is lost; consequently, in such cases the relatively high price obtained for the stock feed does not necessarily indicate that production of feed brings a greater revenue than the production of ordinary tankage and a larger quantity of grease.

Patents and Royalties. The older processes of reduction have been carried on for many years. Fundamentally many of the cooking processes do not differ greatly from one another, and they are similar in many respects to the process of rendering animal matter which has been used for years in the packing industry.

The Cobwell process is patented and royalties have been and are now being collected from municipalities for the use of this method. This royalty in some cases has amounted to about 25 to 30 cents per ton of garbage. Some items of equipment used in other processes have been patented, but in few cases is any attempt made to charge a royalty for the use of the equipment, patentees probably preferring to encourage the use of their devices by making a single charge for purchase. Patents and royalties do not offer a serious problem.

General Sanitation and Odors. Delay in unloading, with its consequent line of standing garbage vehicles, is one of the chief causes for complaints at some reduction plants. Therefore, provision should be made in design for prompt unloading.

During the reduction process the material should be handled with dispatch in closed equipment in so far as is practicable in order to minimize the liberation of objectionable odors. In design careful consideration should be given to questions of ventilation, deodorization of foul air, the effective control of dust in the tankage finishing process, and to the treatment of stick liquor and condensate.

In most cases reduction plants have been built in outlying locations away from dwellings with a view to avoiding complaints from objectionable conditions and odors. At Rochester, however, the reduction plant is located within a few hundred feet of an important business street and within three-quarters of a mile of the geographical and business center of the city, and it is said that no complaint has ever been received. Remote location is not essential, provided a plant is properly designed, is operated along sound engineering lines with the maintenance of rigid discipline and extreme cleanliness and provided adequate appropriations are made when required for operation and maintenance.

Explosion and Fire Hazard. Some reduction plants have been partially or completely destroyed by fires or explosions, in some cases accompanied by loss of life. Such accidents generally have been caused by solvent vapors coming in contact with flames in boiler plants or direct heat dryers. Careful design can reduce the probability of such accidents to a low point, but, even with a sound design, if maintenance is not taken care of and pipe lines and equipment containing solvent develop leaks, it may be possible for these fumes to form explosive mixtures with the air and become ignited. The explosion and fire hazard alone, however, need not be sufficiently serious to make it unwise for a city to adopt the reduction process.

By-Products. As previously stated, the principal by-products recovered in the reduction process are grease and tankage. In some cases an attempt is made to salvage bones and foreign materials but the revenue derived from such sources is comparatively small. Where large carcasses are handled, a slight additional revenue can be obtained from the sale of hides. As stated elsewhere, some of the tankage can be made into stock feed.

Recoveries of grease vary widely between the limits of 25 to 90 lb. per ton of garbage. With a high grease market and a low solvent market, it will be economical to extract more grease than will be the case where market conditions are reversed. It is seldom, however, that more than 60 to 65 lb. of grease are recovered.

The recovery of tankage also varies widely between limits of about 175 to 450 lb. per ton of garbage, depending upon the process employed and the effort made to obtain high recoveries.

Cost of Construction. So few reduction plants have been built in recent years that there is little available information on construction costs, and many of the existing plants apparently were designed with a view to minimizing the construction cost without adequate provision for sanitation.

The Rochester plant which was put in operation in 1921 has a maximum rated capacity of 180 tons per day. Its cost as reported by Mr. John V. Lewis, now Director of the Bureau of Maintenance and Operation, in his paper before the Sanitary Engineering Division of the American Society of Civil Engineers (Proceedings, October, 1926) was as follows:

Item	Cost	Cost per ton of maximum capacity
Land	\$47 140	\$262
Buildings	138 090	767
Equipment	568 960	3160

Actual bids received by the city of Buffalo in 1925 on detailed specifications were \$4940 and \$4150 per ton for buildings and equipment for the Cobwell-Merz and Morrison processes respectively, to which additional items, to provide a complete plant with all safeguards against fire and explosion hazards and with every provision for cleanliness and sanitation, amounted to 20 and 25%, bringing the total figures up to \$6060 and \$5170 respectively.

The Indianapolis plant is reported to have cost in the neighborhood of $2300 per ton of capacity, not including equipment for grease extraction.

Cost of Operation. The net cost of operation, or net profit from operation if there should be any, is dependent primarily upon the price received for grease and to a lesser extent on that for tankage, and consequently fluctuates widely. Not only are there uncertainties as to the trend of the tankage and grease markets, but there is doubt as to the permanence of the tankage market due to the advances being made in the chemical industry in the fixation of atmospheric nitrogen and the development of other sources of nitrogen. Activated sludge from the treatment of sewage also possibly may become a serious competitor. At several plants it has at times been impossible to sell the tankage at any price and huge tankage dumps have been built up.

The following table gives the cost of operating the Dayton plant:

Year	1916	1917	1918	1919	1920
Tons of garbage.....	16 330	15 930	15 380	18 720	13 800
Operation and maintenance..........	$21 719.48	35 429.87	50 039.18	58 833.32	71 790.40
Bond and interest...	4 710.26	8 809.77	9 266.68	9 266.88	9 266.88
	$26 429.64	44 239.64	59 270.06	68 100.20	81 057.28
Sale of products.....	31 272.41	43 421.94	65 145.89	46 852.98	70 826.52
Net cost............	$4 842.77*	817.70	5 875.83*	21 247.22	10 220.76

Year	1921	1922	1923	1924	New plant	
					1925	1926
Tons of garbage.....	17 100	18 750	17 550	16 100	16 270	19 858
Operation and maintenance..........	$49 763.79	58 429.22	61 685.38	61 658.58	66 800	74 500
Bond and interest...	9 266.88	9 266.88	9 266.88	9 266.88	20 000†	20 000†
	$59 030.67	67 696.10	70 952.26	70 925.46	86 800	94 500
Sale of products.....	30 596.09	46 828.69	61 774.97	61 793.86	88 600	102 700
Net cost..........	$28 434.58	20 867.41	9 177.29	9 131.60	1 800*	8 200*

Total tons reduced in all years..................	185 988
Total net cost of reduction in eleven years..........	$79 177.96
Net cost per ton, average eleven years.............	42.5 cents

* Profit. † Fixed charges.

8. Incineration

In most American cities mixed refuse, that is, rubbish and garbage, has sufficient heating value to evaporate its moisture and to actively and continuously support combustion at a suitably high temperature to avoid objectionable odors. This generally is not practicable with garbage alone. If garbage is burned on a slow fire the odors developed are disagreeable. On this account, incineration cannot be considered as a satisfactory means of disposal of garbage unless fuel is provided to help burn the garbage; municipal rubbish is the only economically available fuel for general use although coal or oil may be advantageously used in small quantities to meet emergencies.

The cost per ton of garbage of operating the plants at Columbus and Rochester has been as follows:

	Columbus, 1911 to 1926 inclusive	Rochester				
		1924	1925	1926	1927	1928
Operating cost.....	$3.62*	$8.80	$6.60	$7.31	$7.21	$6.19
Revenue..........	3.78	5.00	5.05	5.49	4.98	4.96
Net operating cost	$0.16†	$3.80	$1.55	$1.82	$2.23	$1.23
Fixed charges......	.55‡	2.50	2.40	2.41	2.42	2.40
Total net cost....	$0.39	$6.30	$3.95	$4.23	$4.65	$3.63

* Includes water furnished by city.
† Net operating revenue.
‡ Interest on bonds only.

The Columbus record covers the war period when prices received were abnormally inflated, and does not provide for adequate maintenance.

Methods of Incineration. Municipal refuse incinerators have generally been developed and exploited by manufacturers. Most makers have patents upon methods or devices, but the validity of many of them is doubtful because similar devices have been used in steam engineering practice and otherwise for many years. In most cases the purchase of the equipment carries the right to use the patents, and the payment of royalties is not required. The patent situation offers no obstacle to the adoption of incineration.

Most furnaces are charged from overhead by dumping through a hole in the arch. The refuse falls upon a grate or hearth on which it is burned or upon a drying hearth intended to evaporate some of its moisture before incineration begins.

Some furnaces are arranged with one to four grates in series, placed side by side so that the path of the flame is across all the grates into a combustion chamber at the end whence the products of combustion pass through the flue to the stack. Each grate is fed through an individual charging opening in the arch above and has an individual ash pit. In this manner the charging and draft conditions of each grate or cell, as they are commonly called, can be controlled independently. In such a furnace, when slow combustion is taking place in one cell due to the recent admission of a charge of cold, wet material, the heat and flame from the other cells and the radiant heat from the combustion chamber walls (provided the chamber is kept at a sufficiently high temperature) assist in raising the products of incomplete combustion from that cell to a deodorizing temperature.

In order to facilitate combustion of wet material and produce higher temperatures some furnaces are designed to operate under forced draft. A further aid to combustion is the use of preheated air. The air heaters are devices for transferring some of the heat in the flue gases to the cold air prior to its delivery under the grates, and generally consist of a series of tubes with hot gases on one side and cold air on the other. Maintenance charges will be reduced if air heaters are shut off when not required to raise the products of combustion to adequate temperatures.

Differences also exist in methods of charging. In some cases the entire contents of the collecting vehicle are dumped directly into the furnace, while in other cases small charges are admitted at frequent intervals. Under the latter system it is usually possible to select the material, varying the relative quantities of wet and dry material. Where large volumes of wet refuse collected in rainy weather are dumped directly upon the fire, the cooling effect is marked, and if the temperature of the products of combustion is reduced to too low a point, odors may be created.

There are furnaces having water-jacketed steel walls and a drying grate built in the form of a basket of tubes through which water circulates. The material in the basket becomes partially dried and is raked out or falls upon the fire on a second grate below. Such furnaces of necessity generate steam, which is available for power about the plant if desired. Where it is not economical to use the steam, it is discharged into the atmosphere.

In Germany, England and formerly in Switzerland a few vertical furnaces somewhat similar to the blast furnaces of the steel industry are in use. In such furnaces the refuse is admitted at the top, air is supplied through tuyères near the base of the furnace, and the residue is cut off by an hydraulic knife and discharged through the base. Waste heat from the top of these furnaces can be passed through waste heat boilers for the generation of steam. This is known as the Didier system.

Design of Incinerators. In the general design of incineration plants sanitary engineering principles should be rigidly followed, but the detailed design of the furnaces and auxiliary equipment are problems of mechanical and heat engineering. Draft provision, grate area, furnace volumes and combustion chambers should be adequate for proper combustion. Flue and stack areas should be proportioned to give reasonable velocities for the deposition of dust, the prevention of erosion of brick work and in order to assure reasonable draft losses. It is particularly important that expansion and contraction joints in the furnaces be properly located and designed due to the extreme range in temperatures encountered. There are of course many other details which should be carefully designed.

Specifications frequently have required average temperatures of 1400° F. and minimum temperatures of 1250° F. but in working out the designs provision should be made for the maximum temperatures which will be reached.

The design should be based on the material to be burned, for example, if it is intended to burn rubbish without garbage the calorific value and moisture content will be so different from those of mixed refuse that a simpler design may be justified. In the table below are given a few results of calorimeter tests and proximate analyses of refuse.

Average Results of Calorimeter Tests and Proximate Analysis of Refuse

(From "Municipal Refuse Disposal: An Investigation," by J. T. Fetherston *)

	Garbage	Rubbish	Ashes
Calorific values, B.t.u. per lb.:			
Original sample	2 233	6832	8 396
Dry sample	8 351	7251	8 510
Combustible portion	10 338	8503	14 296
Proximate analyses, per cent:			
Moisture	73.26	5.78	1.34
Volatile matter	16.89	65.66	3.73
Fixed carbon	4.71	14.69	55.00
Ash	5.14	13.87	39.93

* Trans. Am. Soc. C. E., 1908; **60,** 345.

General Sanitation, Odors and Smoke. The primary consideration in the sanitary disposal of refuse by incineration is the continuous maintenance of sufficiently high temperatures to prevent any odor from incomplete combustion. If a furnace is so designed and operated that a sufficiently high temperature is maintained at all times, there will be no smoke or objectionable odor given off with the stack gases, which, under such conditions, will have a faint bluish-white color and be noticeable only a short distance. If, however, the temperature drops too low, objectionable odors, smoke and bits of charred paper may be given off through the stack.

In some cases the ashes from incinerators have contained partly burned garbage which when placed on dumps has given rise to objectionable odors of decomposition, and attracted rats and flies. This condition will not obtain if the plant is properly designed and operated.

As in the case of reduction plants, provision should be made at incinerators for the immediate dumping of garbage collection vehicles. Properly designed and operated incinerators can be built in thickly settled neighborhoods, but objections often result in injunctions which forbid their location where planned. Experience indicates that the temporary storage of mixed refuse prior to incineration need not be objectionable, provided due care is exercised in thoroughly cleaning the storage facilities at frequent intervals.

Utilization of By-Products. With incineration the only possible by-products are heat, ashes and flue dust.

Considerable dust is carried over the bridge wall with the products of combustion and is subsequently deposited in the combustion chamber, flues and at the base of the stack. This material is somewhat similar to wood ashes and may contain sufficient potash to have some fertilizing value, although its utilization on a commercial scale is not practicable.

The ashes produced by incineration in North America have no value except for filling low land. In Europe on account of the quantity of household ashes contained in the refuse the residue from incineration contains considerable hard clinker. Sometimes this is ground and used as an aggregate for making concrete tile or for paving walks. The sale of ground clinker produces a slight revenue. Under recent conditions, however, this was not a profitable operation even in England. In this country ashes are seldom burned with mixed refuse, and consequently little hard clinker is obtained; and where ashes are burned there is no market for ground clinker.

In some furnaces sufficiently high temperatures are maintained so that it would be possible, were boilers installed, to generate steam in considerable quantity. In water-jacketed furnaces generation of steam is of necessity a part of the process. In other furnaces, lower temperatures render it inadvisable to attempt steam production.

If there is a market for steam at the site, as for pumping sewage or for operating a garbage reduction plant, or a municipal asphalt plant, it may be economical to install steam generating equipment at an incinerator. At some plants where steam is generated, it has been found impossible to synchronize the production of and demand for steam and the installation and daily use of coal fired boilers have been required. If a reduction plant is located outside the city, the additional cost of hauling the rubbish to an incinerator at such a site may exceed the value of the rubbish as fuel.

Steam can be used for heating the incinerator building in cold weather or for the generation of electricity for general use about the plant. Before purchasing equipment for this purpose, however, it is prudent to compare the additional fixed and operating charges due to such use, as for example, the employment of licensed firemen and engineers instead of unskilled labor, with the cost of purchased electricity or coal for heating. It is wise to consider also that some or all of the furnaces will be shut down over holidays, Sundays and frequently for longer periods due to necessity for repairs, interruption of refuse deliveries, and for other reasons.

Bids for incinerators with and without steam generating equipment received in Buffalo in 1925 indicated that in this case it would have been more expensive to generate electric current by the steam produced than to purchase the same current from the local power company.

It is probably true with some types of incinerators that if steam generating equipment is installed the depreciation of the flue and stack linings and the air heaters will be less rapid than if the hot gases do not receive the cooling thus afforded. On the other hand, the cost of the air heater will be greatly increased due to the much larger heating surface required on account of the lower temperature of the gases, and as stated above, more expensive labor may be required for operation.

Construction Cost. Bid prices for a few incinerators are given below. To estimate actual costs additions should be made for contingencies, such as paving at site, and water and sewerage facilities, and for engineering and other overhead costs.

Cost of operation of mixed refuse incinerators varies widely. In large plants a low figure may be taken at $1.00 per ton and an average figure at about $1.50 per ton of refuse burned.

Location	Date	Make	Capacity tons per day		Total cost	Cost per ton of average capacity
			Average	Maximum		
Buffalo..........	1925	Heenan	400	500	Equipment. $265 000 Building.... 194 000 Total....$459 000	$665 485 $1150
Buffalo..........	1925	Sterling	400	500	Equipment. $360 000 Building.... 282 200 Total....$642 200	$900 705 $1605
Buffalo..........	1925	DeCarie	400	500	Equipment. $410 200 Building.... 228 500 Total....$638 700	1030 570 $1600
Toronto: Wellington St.	1924	Sterling	320	400	$550 000	$1560*
New Bedford....	1925	DeCarie	......	......	....	1880
		Sterling	......	......	....	1246
		Heenan	......	......	....	1214
Toledo..........	1927	Sterling	100		$119 700	1197

* J. A. Burnett in Canadian Engineer, August 11, 1925.

SECTION 19

HARBOR AND RIVER WORKS

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HARBORS

1. General Information

A Harbor is a refuge for ships, a port or haven. From the very earliest understanding of the term, a harbor was a sheltered arm of the sea in which vessels could be built and launched, and taken for repair, or seek refuge in time of storm. One of the requisites was a good anchorage or holding ground. Such cargo as was carried by the vessel was transported to the shore by smaller craft or carried to the vessel by hand. Natural harbors are those which, in themselves, are usable without resort to engineering works of improvement. Primitive harbors were almost always of this character. With the increase in size and draft of vessels most of the so-called natural harbors have required works of extension and improvement. As a distinctive term, a natural harbor may now be considered as one located on an estuary or an enclosed bay. Artificial harbors, in a more restricted sense, are those requiring the construction of extensive engineering works, such as breakwaters for protection of the shipping from wave action, or harbors in which shallow or otherwise unsuitable areas have been excavated and deepened and other improvements made in order to make them available for the purpose intended.

Prevailing usage fails to distinguish clearly between the term "harbor" and "port." When to a harbor are added terminal facilities, the harbor becomes a port. A haven of refuge for shipping from storm and the elements would not be a port unless to the usage made of this shelter there are also added the facilities for loading and unloading the vessels and re-shipping or supplying the necessary cargo by land-drawn and water-borne supply lines and feeders. R. S. MacElwee suggests as a definition, "The harbor may be defined as consisting of the waterways and channels as far as the pier head lines, the port to include everything on the landward side of those lines, that is, the piers, slips, wharves, sheds, tracks, handling equipment, etc.," and suggests that this distinction, while technical and novel, tends to mark off distinctly the lines of the jurisdiction of the federal government as against that of the state and municipal authorities. This technically restricted definition would appear to have advantages, but it is believed that the term port should be considered the general one and includes the harbor whereas the harbor does not include the port.

Design. Harbors are usually located by force of circumstance over which the engineer has little or no control. This is especially so with natural harbors where the local geographical advantages and growth of population have resulted in the need for, and location of, harbors irrespective of the existence of better natural harbor facilities at points not far distant. The best natural harbors are probably, by reason of the very facilities they offer, often centers about which important interests and activities grow. In cases where the engineer has some voice in the selection, a thorough survey should be made of the neighborhood, including the foreshore and the depths of water in the vicinity. Borings and soundings should be taken to ascertain the character of the ground, both as to anchorage and as regards the readiness with which it will lend itself to economical dredging or deepening operations, should such be necessary. In an important harbor the depth of water should be sufficient to meet the requirements of the maximum draft of vessel likely to use the harbor, allowance being made for over-depth for the pitching and surging of vessels under wave action and the drag or set of vessels when underway. Borings on land should also be made to indicate probable subsurface conditions with a view to future location of the necessary harbor works. Observations should be made as to the meteorological phenomena, such as prevailing winds, frequency of storms, height and force of waves, establishment of mean high and low water and range of tide, direction and velocity of prevailing currents, evidence of silting and of littoral drift.

The large ports and harbors of the world, especially the older ones, were undoubtedly located to afford shelter and protection not only from the elements but from the enemy, and to protect the shipping, while laid up, from destruction and ravages of raids which were frequent in ancient, medieval and comparatively recent times. This condition was common in the Mediterranean and in the North Sea; consequently many old ports are located some distance up rivers, probably as far up as the vessels of those times could be navigated. The facility with which the river and its tributary branches could be used as a water highway by light draft craft for transporting cargo undoubtedly had an important influence, as water transportation had many advantages over the possible land transportation conveniences available in those times. As a consequence, certain large European ports, such as London, Antwerp and Hamburg, were river ports and to them later, with the change in times and the increase in size of vessels, the sea had to be brought by continuous expensive harbor and river work in order that they might retain their supremacy as ports; and there were also others which have disappeared or are no longer ports, as for example Paris.

On account of the facility with which land transportation, especially rail, can now be handled, the newer ports, especially of the United States, are located on the seacoast or on estuaries. This same condition has caused the development and construction in European practice of wet docks together with those reasons given in Art. 24 of this Section.

Tides. The tidal day is always of greater length than the solar day, two tides occurring, generally, in each day, high tide being on an average 50 minutes later every day. Although the tides on the coast line are produced by the sun and moon, this is not due to their immediate effect on the waters of the sea adjacent to the coast line, but is due to secondary tidal waves, resulting from primary waves in the wide expanse of the oceans. It is calculated that the primary effect of the moon on rise of tide is 1.34 ft. and of the sun 0.61 ft.; so that with the two together it is 1.95 ft. or against each other 0.73 ft., the velocity of the primary tidal wave being from 50 to 60 miles an hour. The momentum of this mass moving around the earth, taken in connection with the shoaling of the ocean and configuration of the coast line, explains the very great variation in range of tide found at different localities. The atmospheric condition, by reason of the pressure on the surface of the water, affects the tidal range inversely as the height of the mercury in the barometer. Sudden variations of the height of mercury in the barometer will give a difference of about 0.35 in. for each foot of tidal range for each inch of variation in height of barometer. Tidal range is affected considerably by wind, dependent upon location, and characteristics of the coast line, and the force, direction, and continuation of wind. In the open sea the tide is never slack, constantly changing its direction in a rotary movement, with varying velocity. When plotted the curve will be of elliptical shape. The fact that the directional change occurs clockwise in the Northern Hemisphere and counter clockwise in the Southern Hemisphere, shows very clearly the effect of the rotation of the earth.

Tidal Changes. Engineering works of improvement in harbors or rivers may result in quite surprising changes in tidal conditions. In the location and construction of harbors and harbor works, it is of the greatest importance to ascertain definitely the data of high and low water and range of tide, for which purpose automatic tide gages should be used. Elevation of mean high and mean low water at the various important parts of the United States can be readily secured from local authorities, or from the U. S. Coast and Geodetic Survey, but in important new harbor work it is advisable to establish these independently. Although the height to which the surface of the sea rises and falls at different points along the coast varies considerably, yet there is a mean level to which, at a certain stage of the tide, the water returns. This is known as mean sea level. Where possible, it is advisable to refer to this datum.

Tidal Prism. This is the volume of water represented by the area of the bay, river or harbor affected by tidal changes multiplied by the range of tide which would generally be the volume of water entering at the flood and leaving at the ebb. Harbor engineers are loath to permit the construction of any works that will serve to change materially this tidal prism, as a decrease in the tidal prism has a direct effect upon the quantity and velocity of the water of the ebb and flood tide and a consequent effect upon the channel depth and direction by reason of the resulting tidal scour.

2. Waves

Wave Height. In the location of harbors and the design of harbor works, it is necessary to ascertain the probable maximum wave action and forces likely to be encountered. The height of wave to which an exposed entrance may be subjected will depend upon the greatest fetch or reach of open sea from the windward shore; for open exposed locations

$$H = 1.5\sqrt{d}$$

where H is the height of the wave in feet, and d is the length of fetch in miles. For shorter lengths of exposure in bays, or harbors, this height is represented by

$$H = 1.5\sqrt{d} + (2.5 - \sqrt[4]{d})$$

Extensive observations closely check the results obtained by the application of these two formulas, first proposed by T. Stevenson, and it is to be borne in mind that in the open sea the fetch is confined to the distance over which the wind acts continuously in one direction, the rotary character of ocean storms restricting the applicable fetch even in the broadest expanses of the Pacific Ocean to a distance of not over 1000 miles.

Formation of Waves. The direction of the wave crest in deep water is at right angles to that of the wind, but on approaching the shore it tends to become parallel to the shore line, due to the lagging effect, on the wave, of the rising bottom. Most waves are caused by the action of the wind, due to the wind blowing over the surface with varying velocity, acting by friction on the surface and by direct pressure on the rear of the wave crest. A confused sea existing during a heavy storm usually changes to a regular swell afterward, resulting in trochoidal seas. Waves formed in this manner in deep water are generally understood to be oscillatory waves, in which the particles of water forming the wave move through circular or elliptical orbits. This is not strictly correct, as, undoubtedly, all storm waves are to some extent waves of translation. Deep-water storm waves on reaching shallow water become waves of translation, the friction of the rising bottom on the lower portion of the wave tending to retard this portion to the point where the front of the crest of the wave becomes steeper and steeper, and finally breaks and becomes a breaker or comber.

Wave Velocity in deep water is dependent upon the velocity of the wind, taken together with its continuity, and the extent of the surface of the sea over which it acts, which in turn affects the height of the wave and the wave length. The velocity may be given by

$$V = \sqrt{5.123\ L} \text{ for deep-water waves}$$

$$V = C\sqrt{5.123\ L} \text{ for shallow-water waves}$$

in which the coefficient C is the square root of the ratio of the axes of surface orbits b/a, as it is recalled that the shoaling depth changes the orbits from circular to elliptical, V being the velocity in feet per second, and L length of wave in feet. It is necessary to use the coefficient C only in cases where the depth of water is less than one-half the wave length. The following are values of C:

d/L = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
C = 0.552	0.746	0.858	0.922	0.958	0.977	0.988	0.994	0.997

in which d = depth of water, or distance from the center of the surface orbit to the bottom, and L = distance from a wave crest to the next.

The relation between wave height and length is dependent upon wind, sea and other local complex influences; in general, the proportion between length and height may be taken as: 33 for moderate wind and light sea, 20 for strong wind and rough sea, 18 for storm and heavy sea.

The wave pressure varies greatly; pressures as high as 7840 lb. per sq. ft. have been recorded. A series of observations at Skeeryvore Rocks and at Tyree gave these results: 6-ft. swell, 3041 lb.; 10-ft. ground swell, 3041 lb.; 20-ft. heavy sea, 4562 lb.; strong gale, heavy sea, 6083 lb. per sq. ft.

Wave Energy of a wave of one unit width, with length L and height H is

In deep water, $E = 8\ L\ H^2\ (1 - 4.935\ H^2/L^2)$

In shallow water, $E' = 8\ L\ H^2\ (1 + 19.74\ a^2/L^2)$,

in which a is the semi-major axis of surface orbits, H and L are in feet, and E is in foot-pounds. This energy is for salt-water waves; for fresh-water waves reduce amount by 2-1/2%.

Values of a in Terms of Wave Height H

d/L = 0.10	0.15	0.20	0.25	0.30	0.35	0.40
a = 0.91 H	0.68 H	0.59 H	0.55 H	0.52 H	0.51 H	0.504 H

When a wave enters a bay, or arm of the sea, between natural or artificial barriers its height tends to decrease on account of friction of the shoaling bottom and the retardation due to barriers; and on emerging into open, interior water, it further decreases in height due to distribution and the tendency of the wave crests to diffuse parallel to the shore line.

Wave Action. The percentage of wave height above water level is important as indicating the height to which harbor structures may be subjected to direct wave action. Observations indicate that, in general, about two-thirds of height of the wave is above mean water level, and about one-third below. Due to the action of high winds, opposing currents and further unknown causes, deep sea waves may break partially in water of sufficient depth for their free propagation, so that a barrier opposed to them in water of great depth may at times be subjected to the direct action of breaking waves. It is known, however, that waves invariably break on reaching water of insufficient depth. Considering the height of the maximum wave as determined by actual observation, or by application of Stevenson's formula, the ratio of depth of water to wave height in which waves may break will vary from 1 to 2.71; or 1.67 is a mean ratio taken from 134 observations. Where wave action is arrested by barriers, a portion, at least, of the wave energy will be exerted against this barrier. It is important in designing harbor works that such artificial barriers shall be made strong enough to resist the successive attacks of maximum sized storm waves. Wave force is exerted and transmitted against such barriers in divers ways, some of which are: (a) The force may be static pressure due to the height of the column of water. (b) It may result from the effect of rapidly moving particles of the liquid. (c) It may be due to the impact of floating bodies on the surface of the water hurled by the wave against the barrier. (d) It may result from the rapid subsidence of masses of water thrown against the structure, producing a partial vacuum and causing sudden pressure to be exerted from within. (e) Destruction of the structure may result from the falling on it from above of large and heavy masses of water thrown up by the wave action.

The interior of such barrier structures is also often affected by forces transmitted through joints or cracks, by hydraulic or pneumatic pressure. The most destructive effects of waves are exerted at or about sea level. The result of wave action may be apparent at considerable depths, although it diminishes considerably in force and extent. In the design of structures subjected to wave forces it should be considered that the material entering into the structure is at least partially submerged and that, therefore, the weight of the mass submerged must be considered. Consideration should also be given to the presence and movement of ice where harbor works are located in colder climates.

3. Breakwaters

Uses of Breakwaters. A breakwater is a work or barrier constructed around or in connection with artificially sheltered harbors in order to protect the interior water areas from the effect of heavy seas, and make it possible for this area to be used for the safe mooring, operating, handling, loading, and unloading of shipping. Breakwaters are almost invariably a most prominent and essential feature of artificial harbors, and are employed to convert an open area or roadstead into a harbor or to render more secure and usable harbors that are enclosed and protected, excepting against the sweep of the sea with prevailing winds coming from certain directions. Breakwaters on a smaller and less important scale are sometimes constructed in the interior areas of large natural harbors to protect shipping from wave action under conditions of heavy storms from an unfavorable direction. A breakwater the protected side of which is used as a quay for wharfage is also known as a **Mole.**

Design. There are three general types of breakwaters: 1, that in which the exposed face is vertical; 2, partially vertical and partially inclined; 3, inclined. The type to be selected is dependent upon local conditions. Claims in favor of the vertical type of breakwater are that because the action of deep sea waves is purely oscillatory, the wave force encountered is due to water head pressure, dependent upon the height of waves. As has already been stated, nearly all waves are to some degree waves of translation, and since breakwaters are usually located and constructed in comparatively shallow waters, the wave forces to be encountered are more complicated than those due purely to hydrostatic pressure; therefore, vertical face breakwaters are undoubtedly subject to severe wave impact. The combination of the inclined and vertical face often results in the throwing up, vertically, of large masses of water which, in turn, fall on the top of the breakwater directly behind the parapet. In crib breakwaters, with heavy timber decks, structures have often been seriously damaged by such action. In inclined breakwaters there is danger, if they are not carried high enough and surmounted by a vertical wall, with heavy storm waves, that the waves will sweep entirely over the breakwater, resulting in a disturbed effect and an appreciable secondary wave action behind the breakwater. When waves impinge obliquely in the direction of the breakwater, they often accumulate size and energy as they travel along its front. In the design of harbor breakwaters, information should first be obtained concerning the direction and force of prevailing winds, character of coastal currents, probable maximum height of waves, character of bottom or foundation and cost and availability of materials to be employed in construction.

Rubble-Mound Breakwaters are a more or less heterogeneous assemblage of rubble, riprap, or cobble, deposited without any particular regard to bond or bedding. This is the simplest form of breakwater and is usually constructed by the dumping of material into the sea from barges or cars run out on trestles or staging. This operation is carried on until the mound or heap emerges from the water and is carried up a distance above the same, the action of sea and waves being depended upon to give the sides a natural stable slope. It is usual in constructing extensive works of this character to make some attempt at grading materials and to control the slopes, the interior being composed of cobble or smaller riprap and the sea side of the breakwater of heavier and more massive stone. As the force of the wave decreases with the depth, in deep water the slopes are often steeper and smaller stones are used. Such a structure is often supplemented by placing heavy massive stone or concrete blocks on the sea side, whereas the slopes and faces above the sea level are roughly paved with blocks of stone or concrete.

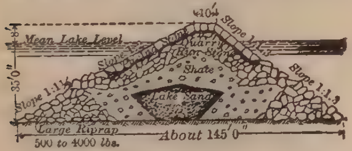


Fig. 1

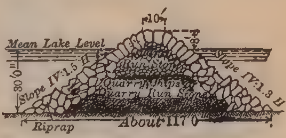


Fig. 2

Rubble-Mound Breakwaters

Figs. 1 and 2 show typical sections of rubble-mound breakwaters, Fig. 1 illustrating the first modification made in the Cleveland, Ohio, breakwater. The earlier breakwater had a complete sand core or hearting; during heavy storms this hearting was washed out, resulting in the subsidence of the rubble top and eventual disruption. The work was finally built as shown in Fig. 2 with all sand omitted from the core and with rough instead of smooth stone covering.

Timber Breakwaters may consist of timber cribs floated out to place and loaded and sunk by being filled with rubble, or may be built of close round piles or sheet-piles driven into the bottom, framed, braced and backed up with rock or concrete blocks. Timber cribs of this character are sometimes superimposed on rubble-mound breakwaters. Breakwaters of timber are more or less temporary structures and are principally used in shallow water for unimportant barriers. They have, however, been extensively used on the Great Lakes.

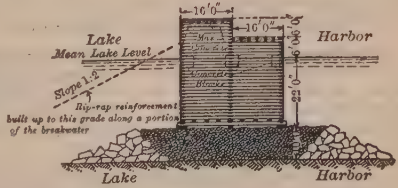


Fig. 3. Timber Crib Breakwater

Fig. 3 shows a typical timber crib breakwater built at Cleveland, Ohio. This work was damaged by storms, masses of water projected upward by storm waves fell on the top timber work, breaking the cross members and planking. The work was reinforced by backing the structure with riprap.

Fig. 4 shows a low-coursed stone wall breakwater surmounting a rubble-mound, at Dog Bar, Gloucester, Mass.; a somewhat similar breakwater at Sandy Bay, Cape Ann, Mass., was considerably strengthened by preventing the large blocks from sliding by inserting steel dowels into the stones on the harbor side against the toe of the course directly above.

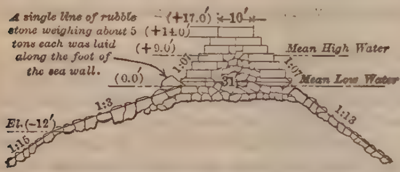


Fig. 4. Mound Breakwater with Wall

Masonry Wall Breakwaters are, in whole, or part, constructed in a regular and systematic manner of coursed stone masonry or concrete with vertical or inclined faces. Such walls are constructed in cofferdam in the dry or under water by divers or diving bells. This character of breakwater is often built from the tide level up surmounting a rubble-mound substructure or capping a timber crib.

Figs. 5 and 6 show typical sections of the mole breakwater at Harbor Beach, Mich., consisting of a timber crib, capped with concrete blocks. The section shown in Fig. 6

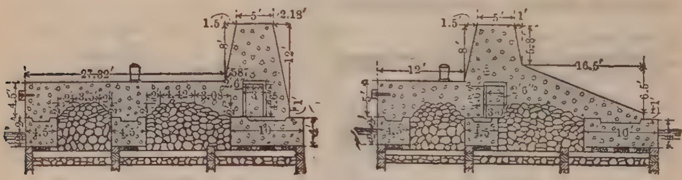


Fig. 5 Fig. 6

Timber Breakwaters Capped with Concrete

was damaged by storm by having the concrete capping lifted and cracked. The cost of wall and composite wall breakwaters varies widely.

Reinforced-concrete Breakwaters may be built by means of caissons built on shore, or in floating structures, launched or lowered into the sea and sunk to place, and settled upon a prepared foundation of rubble or piles, the breakwater being formed by filling the compartments with stone or sand; or they may consist of reinforced-concrete piles and sheet-piling banked or filled with sand, riprap or rubble. See Sect. 11, Art. 57, p. 1108. In the following examples of breakwaters the numbers refer to Fig. 7.

(1) Zeebrügge, Belgium, 1905. Breakwater or mole 6560 ft. long, lower course built of monoliths, set in floating caissons of structural steel and plating with concrete lining arches. Lower sections 82 ft. long, 24-1/2 to 29-1/2 ft. wide, 28 to 36 ft. deep. Built in graving dock and floated out. Sills for setting 20 in. deep. Sections sunk at low water by flooding, and then filled with concrete. Blocks weigh 1500 to 1600 long tons light and 4000 to 5000 long tons when filled.

(2) Barcelona, Spain, 1906. Section of floating caisson for monoliths for extending the breakwater. Blocks 39 ft. long divided into five pockets, which were filled with pre-molded blocks.

(3) Touapsé, Russia, on Black Sea, breakwater 1400 ft. long, largest caissons 56 ft. long 21 ft. wide, and 21 ft. deep, divided into 21 divisions, 7 sets longitudinally. Caissons of reinforced concrete. Large boxes weighed 268 long tons when launched and 1600 long tons when filled.

(4) and (5). Talcahuano, Chili, 1908. Breakwater 1750 ft. long, section (5), dam (4), each caisson 33 ft. long, built of reinforced concrete and filled with sand and stone.

Breakwaters have been constructed employing floating caissons at Bilboa, Bizerta, Algoma, Wis., and Welland Canal, Canada. (See Sect. 11, Art. 57.)

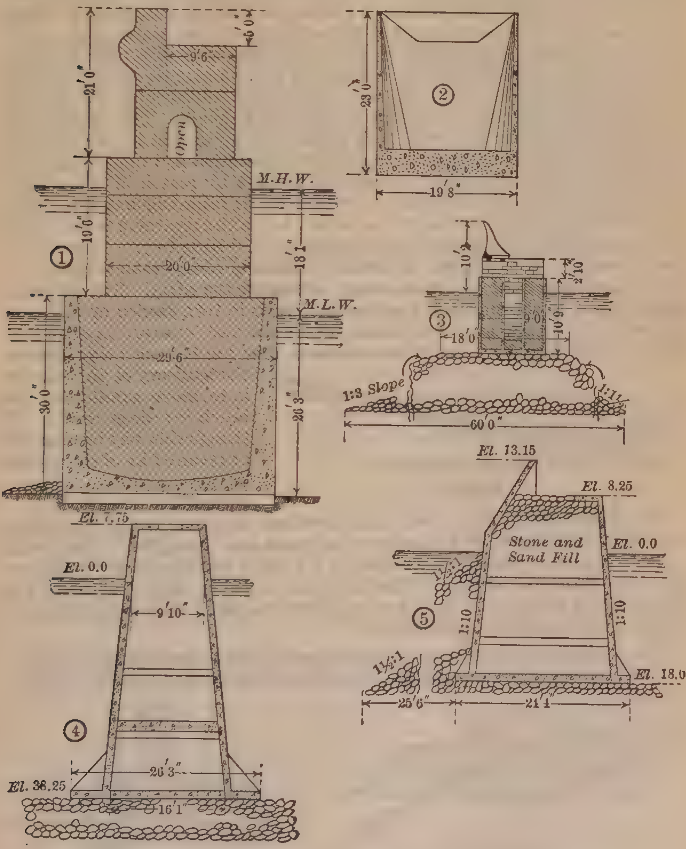


Fig. 7. Reinforced-concrete Floating Caisson Breakwaters

4. Bars. Sea Encroachment

Littoral Drift. The coast line of all exposed foreshores indicates a gradual but continual change, in some locations in the direction of retrogression or washing away, and in other locations in deposition and accretion, the carrying away and depositing of materials being due to wave action in connection with current flow. The breaking of waves on a beach or foreshore serves to stir up loose material and also to break up solid material by direct erosion or by the impact or wearing away, due to the carriage by the water of particles of sand and shingle; the lighter particles of such material remain in suspension long enough for them to be projected some distance along the shore by the resultant of combined wave action and littoral currents. The heavier particles are rolled along the beach and partake of a zigzag movement, the principal action being generally confined to the reach between high and low water mark. Littoral drift is generally attributable to the prevailing wind direction, although it is modified by onshore currents. Breaking waves acting on a beach or foreshore by reason of their velocity are carried some distance up the sloping beach, mixing with, and stirring up, the sand and shingle. The wave having spent itself, the water then runs back by gravity, but is met by the next oncoming wave, the returning water carrying in suspension and rolling down the beach some sand and shingle, the oncoming wave by reason of its greatest velocity being at the top or crest is carried over the returning water, building up a hydraulic head at the beach line which forces the returning water down and outward thus setting up what is known as the "undertow." The undertow in some instances becomes an outshore current of considerable importance, owing to peculiar local conditions which accentuate the effect. The material carried down, as the undertow loses its velocity, is in turn carried up again on the beach by returning waves moving backward and forward, as shown in Fig. 8. The quantity of sand in suspension near the bottom being greater the tendency of wave action is to move sand from the beach to deep water, although this may be retarded or reversed by offshore gales setting up surface offshore currents.

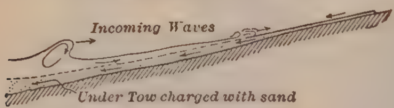


Fig. 8

The general effect of the wind on the foreshore, as shown in Fig. 9, tends to carry this drift of sand and shingle up the beach in a zigzag line. The actual action, as can be readily understood, is far more complex, but the general principle illustrated obtains. The drift varies, being dependent upon the direction and force of the wind, but in general the result follows the direction of the prevailing winds of the locality.

Bars. Bars at the entrances of harbors are of four general classes: (a) Natural bars, consisting of hard material not affected by scour or current; (b) those

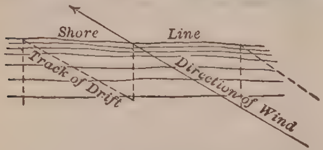


Fig. 9

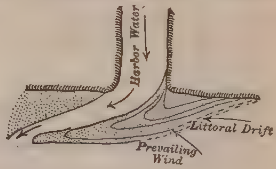


Fig. 10

due to the deposition of alluvial material brought down by river drainage; (c) casual bars occasionally and irregularly heaped up by storm wave action and afterwards dispersed by similar action, or by one or more currents; (d) bars consisting of sand or shingle of certain general features and permanent direction, but constantly subjected to alteration by action of winds, waves and varying currents. It is generally concluded that the last-given type of bar is that most usually encountered at harbor entrances, and is due, principally, to wave action and the littoral drift engaged in depositing material across the mouth of the outfall channel and the constant tendency of the ebb and flood currents to remove and disperse the drift. The result of such drift action and harbor ebb current is shown in Fig. 10, which is typical of the conditions that have affected the entrances to interior bodies of water on sandy coast lines.

Influence of Coast Line on Formation of Bars. Where the bed of the sea decreases in depth rapidly, the drift moving along the coast line is less readily carried into the channel by flood tide and is more readily transported by the ebb tide to deep water. Under such conditions bar formation is unlikely. Harbor entrances on precipitous foreshores of rock materials are not liable to bar formations. Prominent projections of the coast line on the side from which the flood tide sets in cause the current to run around it with velocity sufficient often to prevent depositing of drift and formation of bars at harbor entrances.

5. Beach Erosion and Protection

Sea Walls, Dikes or Bulkheads are constructed along the shore line to prevent encroachment of the sea by direct wave action, and, as in the case of breakwaters, may consist of loose rubble-mounds or heaps, masonry wall work, usually, however, supplemented with timber, steel, or reinforced-concrete sheet-piling driven into the beach and strengthened by wales, guide and brace piles, fascines and mattress work held in place by piles and loaded with rock. The character or massiveness depends on the location and wave forces to which the work will be subject. In following notes of sea walls the numbers refer to Fig. 11.

(1), (2), (3), (4), Coney Island, N. Y. Section of sea wall.

(5) Governor's Island, N. Y. Sea wall, 1901–1911. Average settlement 0.8 ft., maximum settlement 2.4 ft.

(6) New York, Brooklyn Parkway, 1913. Masonry wall.

(7) Atlantic City, N. J.

(8) New Orleans, La., 2650-ft. wall, fill behind wall 8 to 12 ft. deep, piles 50 ft. long, expansion joint every 50 ft.

(9) Ocean Beach Esplanade, San Francisco, California. Length 2066 ft.; width 30 ft. 5 in.

Effectiveness of Sea Walls. In many instances large sums of money have been expended on sea wall protection with entirely unsatisfactory results. In exposed locations subjected to high wave action, with a sea current along the beach, on which the sea wall is constructed, the action of the waves, together with the current, will gradually, but continually, carry the beach material away until the foundation is exposed and undermined and the work destroyed. The most carefully constructed massive works of this character will not endure under such conditions.

The Board of Commerce and Navigation of New Jersey, through its Erosion Committee and Engineering Advisory Board, after a painstaking study of the New Jersey coast conditions, concluded:

(1) Neither bulkheads nor groins can be recommended for use separately except in special cases.

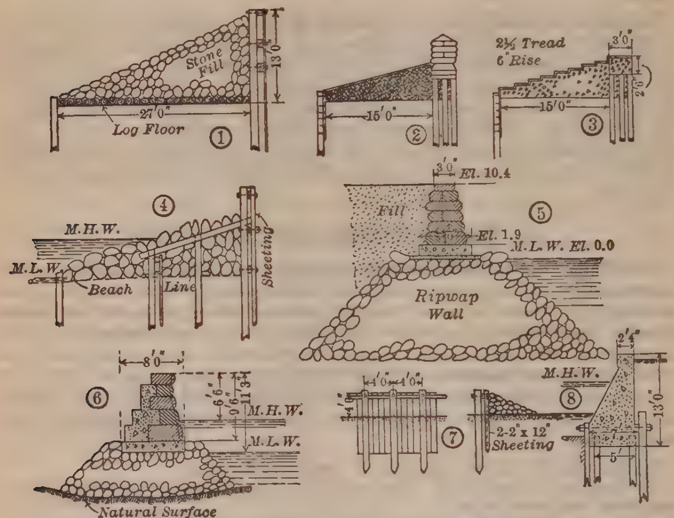
(2) The use of concrete for bulkheads is not recommended except for those heavier and more permanent types which may be classed as sea walls. Its use is highly hazardous where any serious settlement may be expected.

(3) Timber bulkheads should always be provided with rock protection. The rock should be of large size.

(4) The construction of shore bulkheads and sea walls seaward of the ordinary high-water line is to be permitted only when absolutely necessary for the immediate protec-

tion of existing structures such as buildings, roadways, etc. When the construction of such bulkheads and sea walls is unavoidable they should be supplemented by suitable groins.

(5) The distance between groins requires, for its determination, due consideration of many factors including the slope of the beach, height of waves, distance to offshore bar, proximity of ocean inlets, etc. If the space between groins is too great, the front-



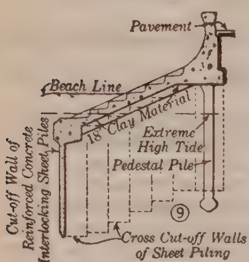


Fig. 11. Sea Walls

age to the leeward of the groins will be left without protection. If this space is insufficient, scour may result from back-wash and interference of the two groins. In general, groins should be spaced from one to one-and-one-half times their length.

(6) Groins must always be securely and tightly connected to a bulkhead or must extend well inshore to points beyond the reach of the highest waves. Disaster will almost certainly follow if the water is permitted to pass the inshore end of the groins. Any breach or poor connection between a groin and a bulkhead on a sandy beach is almost certain to nullify the effect of the structures.

(7) In groins sheet piles should have a minimum penetration of 10 ft. Even this will be inadequate in some situations. The groins must be of tight construction, preferably of tongued and grooved sheet piling well lined and braced by wales and piles. The piles must have a penetration of at least 25 ft., and the timber should generally be treated with creosote. All bolts, rods, spikes, etc., should be galvanized.

The rock should be of large size. Stone less than 100 lb. should not be used. Even rock of such small units as 100 lb. is of very slight value. Where the exposure is severe, nothing less than one-half ton should be considered, and approximately 75% of the rock should be of units weighing two to six tons or more. In general, rock should have a least dimension at least equal to one-third its largest dimension.

The incorporation of a sandtight core in groins is essential. This should be ordinarily of timber piles and sheet piling. It may be of rock where sufficient material is used, in the class of large structures such as in the Longport Eleventh Avenue jetty, which contains a very deep, wide foundation of rock and foundry slag. The core must be a substantial structure. If of timber it must be supported by piling or by rock or by both.

(8) Groins or other structures designed to gather large areas of beach should be permitted only for special reasons and after careful study of the probable effect upon adjacent communities. What is to be sought is coast protection and not coast extension. Extension at one place is likely to be accomplished by recession at points in the vicinity.

(9) The height and slope of the groin with respect to the profile of the beach depend upon many factors. Structures of the type found to be necessary on the steep beaches of northern New Jersey would be needlessly expensive in many sections of the southern New Jersey beaches where the profile is much flatter and where greater fluctuation of the shore line may be permitted.

If the beach has been seriously lowered and the slope permits, it may be most economical to build a low structure, that is, some 3 ft. above the normal profile of the beach, but so design the core as to permit the addition of wales and sheeting to obtain higher level of the beach when the low jetties attain their maximum effect.

(10) The construction of curved groins should be permitted only after thorough investigation. There is no evidence that curved groins offer any greater promise of success than straight groins.

(11) Coast protective works must be adequately maintained. The tendency has been to pay little or no attention to maintenance until serious damage occurs. Where state aid is given to protect the shores, provision should be made for the maintenance of such works in good repair. Municipalities fronting on the ocean should adopt and maintain general programs for the adequate protection of their ocean front. Perhaps the best means to accomplish this would be the establishment of a fund into which moneys would be paid every year, the expenditures to be made therefrom as required. The almost universal tendency is to view with indifference the inroads of the ocean, making no effort to hold the beach until valuable structures are actually attacked.

(12) The growth of sand dunes along the coast should be encouraged and promoted. Unfortunately, in New Jersey the tendency has been to level the sand dunes as construction of roads and dwellings progresses. Public authorities should discourage the removal of sand from the beaches where this sand is used for construction purposes or for filling of low lands. The artificial removal of sand from the New Jersey beaches is a serious evil.

The best means of encouraging the growth of sand dunes is to plant beach-building vegetation, such as sea oats, sea wheat and other beach plants. In some places, particularly low spits, fences may be employed to gather the drifting sand.

(13) There is no evidence to justify the contention that the protection of the beaches can be well attained by means of heavy key works of great extent and massiveness, such as heavy outshore breakwaters or current deflectors.

(14) To control the location and navigable depth of inlets, tight jetties are necessary, and in the case of many inlets two parallel jetties are needed to carry the current into deep water.

(15) Twin jetties at inlets possess a superiority over a single jetty, whether straight or curved, and may be used with more confidence of results.

(16) In length jetties should always extend from high-water line, or form a proper bulkhead outward to such distances as may be required by the problem at hand. For navigation purposes they must necessarily extend out sufficiently far to obtain the low-water depth sought. For fixation of location of inlets and for beach protection purposes an extension to or only slightly beyond low-water mark is necessary. It should be borne in mind in all cases that long jetties may temporarily cut off from the leeward beach the supply of drifting sand upon which its maintenance in normal conditions depends.

(17) The northern beaches of New Jersey, that is from the vicinity of Long Branch northward, are relatively steep. This is a situation which requires the application of substantial structures, that is, heavy rock breakwater jetties. Light wooden structures, not adequately supported with heavy rock, will not serve on this frontage.

The southern beaches are less steep, and here the construction of such heavy works is not ordinarily justified from the standpoint of economy, except in the vicinity of inlets.

(18) Every section of beach presents a problem with its own peculiarities. It is therefore impossible to give precise rules that can be applied offhand to particular localities, but the types of structures discussed and recommended in this report have proved their effectiveness as shown by the results obtained during the past few years, and consequently much of the confusion and uncertainty on this subject has been removed.

(19 For breakwater jetties of magnitude the tight sheet pile core type surrounded by large rock is recommended. The dimensions of the jetty should be suitable for the location.

(20) In extensive, or key construction of magnitude, as distinguished from local protection on the beaches, no small stone should be used. Contracts should call for large stone, say 85% of three tons and upwards, with no stone of less than one ton. In general, rock should have least dimension at least equal to one-third of its major dimension.

(21) Preparations for jetty contracts where rock is to be used should include, in addition to soundings, boring to locate a hard bottom, if one exists.

(22) Where hard or firm bottom exists, and a core wall is to be used, plans should call for piles and sheet piles to extend well into it. Where no hard bottom is found, then the longest piles and sheet piles should be used within reasonable expense.

Groins are built out from the shore line perpendicular to, or making an angle with, the shore line. They are generally constructed perpendicular to

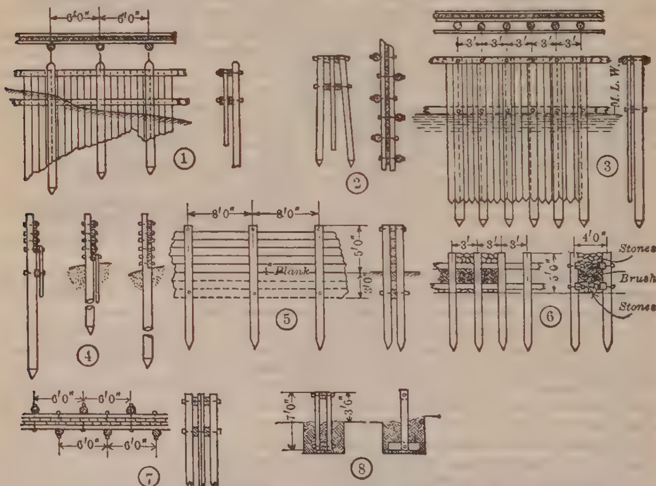


Fig. 12. Groins and Spur Dikes

the direction of the current or drift. Their object is to cut off and prevent the carrying of beach materials along the foreshore. They need not be of massive construction, and usually consist of close piling, or sheet-piling, driven between guide wales and piles, or horizontal courses of plank held between vertical timbers or piles driven into the beach. They should be built so as to offer the least resistance to wave action, follow closely at a few feet height the general slope of the beach, and extend from high to low water. When of more massive and stronger construction on a steep beach they should, if built out below low water, extend out into water of depth so that wave action will not seriously disturb bottom conditions, such as would result in scour and undermining and the destruction of the outshore end. When built in this way they are also known as **Spur Dikes** or **Jetties.** Groins should cut off the lateral sweep of waves. The distance between groins should equal their length and may be one and one-half times the length. In following notes, the numbers refer to Fig. 12.

(1) New York, N. Y., foot of Bay 22nd, Brooklyn. Wales 5 × 10, sheeting 4 in.

(2) Bath Beach, N. Y. Sheeting 3 in., 11 ft. long. Top wales, 6 × 6; lower wales, 6 × 8 in.

(3) Coney Island, N. Y. Wales 4 × 10, 4-in. sheeting 22 ft. long.

(4) Telewana Park, N. Y. Horizontal planking 3 × 10, wales 4 × 8, sheeting 3 in., 6 ft. long, piles 15 to 20 ft. long.

(5) Atlantic City, N. J. Planking was put in, sand as littoral deposit, raised beach, all lumber creosoted.

(6) Atlantic City, N. J.

(7) Asbury Park, N. J. Placed 1908; successful up to 1914. Beach protected and extended.

(8) Type of low groin adopted by E. Case on English coast with entire success.

Groins of reinforced concrete have been extensively used on the English coast, consisting of grooved reinforced-concrete piles and horizontal slabs added as the beach rose in height. On the Holland coast spur dikes consisting of brush piles covered by pavement of stone have been successfully used, length about 350 ft.

Principle of Action of Groins. The object of groin construction is to confine the action of the breaking waves on the beach to moving the sand and shingle composing the beach up and down locally in the confined space, and at the same time prevent the current along the beach from carrying this sand away to other locations, the beach current being compelled to seek a path outshore of the extremity of the groins at a depth where the wave action will be insufficient to stir up a sandy bottom, so that it may be carried away by the beach current. On the other hand, this current will carry, in suspension and along the bottom, sand or shingle from more distant points not protected by groin construction, and deposit this in the pockets formed by the groins resulting in beach accretion, as indicated in Fig. 13.

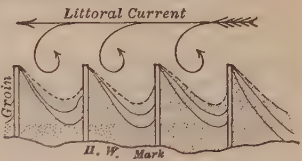


Fig. 13

The windward accumulation will be more pronounced, the deposit on the leeward side being materially reduced by eddies caused by the lee side of the outer end of the groins; as the pockets between become filled up, it becomes necessary to raise the height and extend the groin seaward. In most instances it is possible by this method to add considerably to the foreshore where, previous to the work, it was being steadily and continuously carried away. Groins are advantageously employed in connection with sea-wall protection.

Offshore Breakwaters. In certain instances breakwaters constructed parallel with the beach and outshore have been effective in stopping beach erosion and have resulted in accretion behind the breakwater. See Transactions Am. Soc. C. E., Paper 1541 by H. C. Ripley, and discussion.

Dunes. Besides the retrogression or deposition of materials by direct water action of breaking waves, coastal currents and littoral drift, the air currents or the winds frequently have a direct effect of considerable importance, as is evidenced by the erosion, carrying away and deposition of sandy beach materials in the building up and constant movement of dunes. Such action may often become of so important a character in connection with foreshore protection as to require stabilizing.

On the Holland coast this is accomplished by resorting to planting of coarse grass and the placing of brush to protect the sand surface from wind action and also to catch the flying particles of sand and artificially build up dunes or embankments at predetermined places.

The fixing of sand by the planting of grass, weeds, shrubbery, although an effective permanent method where applicable, requires time. For more immediate results and as a preliminary, fences are frequently employed; if they become buried in the sand deposit they may be built up and the height increased until protection is afforded by permanent artificially built-up dunes. It is important in channel construction between lagoons and the open sea to bear in mind the sand drift, inasmuch as frequently in the case of jetties built to the open water where the bottom is beyond the influence of wave action and where the littoral drift is not properly provided for, the wind-borne sand is carried over the jetty into the channel, obstructing it.

6. Channel Regulation

Variation in Channels. In bay or estuary harbors on coast lines subject to appreciable tidal ranges, it is quite frequently the case that the ebb current in finding its way to the sea has a tendency to scour out a channel for itself in the line of least resistance, water from the sea entering the harbor at flood tide, eroding for itself a different channel-way, sometimes across the channel-way of the ebb tide, with a consequent inclination to fill up this channel. The influence of littoral current and drift at the entrance of such harbors, together with that of gales and storms at intervals, often seriously complicates the problem of maintaining navigable channels. However, there is a constant tendency toward an equalization of forces looking to the establishment of a more or less fixed channel-way. The navigable channel need not necessarily follow the natural channel eroded by the ebb or the flood tide, but may be established by crossing a series of deep pockets, availing itself, in whole or part, of either or both tidal channels. For methods of marking channels, see Art. 11.

Dredging. Such navigable channels are often established and maintained by excavating shoal reaches between natural channel-ways and deep holes, or by straightening out and making more direct the natural channels, by cuts through bends, and cross-bars. A description of the different processes of excavating under water will be found in Sect. 8, Art. 15. Dredging operations of this character are frequently sufficient in themselves although, to some extent necessary as a continuous operation for maintenance of channels. In other cases, dredging operations are necessary in conjunction with other works of channel control and regulation.

Jetties. In cases where there is considerable littoral drift, resulting in the formation of bars across harbor entrances, or formation of bars from deposit of sediment, jetties are built out from the harbor entrance into deep water. In the construction of jetties, attention must be given to the natural conditions or inclination of the channel to establish itself in a certain direction, with a view to not unnecessarily disturbing the equilibrium of forces.

Single Curved Jetty. Where a heavy littoral drift is constantly forcing to leeward, the outflow of water should be directed into the sea by a curved channel having its convex side presented to the direction from which gales are prevailing, a single jetty of curved section being used on the windward side of the channel. In such construction it will often be necessary to extend the jetty seaward from time to time, as the foreshore is built up on the windward side of the jetty; under some conditions such a jetty would be likely to carry the drift beyond the entrance into deep water, and not require seaward extension.

Single Straight Jetties have been constructed but, unless designed for a special purpose to fit in with definite geographical conditions, are not likely to be efficient.

Twin Curved Jetties. The jetties built by Eads at the South Pass of the Mississippi River are an example of this type.

Twin Straight Jetties were built at the mouth of the Rhône, France. These were built so slowly that the bar advanced more rapidly than the jetty extension. The work was abandoned as a failure. Similar jetties were built at Tampico, Mexico. In this case, the extension seaward was rapidly performea and the work was successful.

Jetties Open at the Shore End were built at Charleston, S. C., but were ineffective.

High and Low Jetties. These two types have their advantages and disadvantages. High jetties are carried up several feet above high tide, and cut off the sand drift. They are the type now generally adopted in American and English practice.

Converging Jetties run parallel with the general direction of the ebb tide and are carried out from the land and converge, leaving a comparatively narrow opening. Jetties of this type tend to guide and restrict the outflowing tide, controlling and directing the scouring effect by increasing the velocity. The windward jetty serves to cut off the littoral drift, the leeward jetty assisting the windward jetty in directing the force of the ebb tide to scouring out the drift brought into the channel-way by flood tide. Jetties of this character often, from time to time, require outshore extension into deep water or the assistance of dredging.

Diverging jetties bring the entrance to a more natural condition, and afford a better approach for navigation, but have a tendency to scour out a deep hole in the inshore entrance-way.

Design and Construction. Jetties are usually constructed of mounds or heaps of large rubble in a manner similar to that of the construction of mound or heap breakwaters. The material is usually carried up slightly above the elevation of high tide. In estimating the quantity of material to be used, it is to be borne in mind that considerable is lost through settlement in soft bottom, besides which an excess of material must be allowed as the action of the waves will tend to wash down the stone to flatter slopes than would ordinarily obtain. From time to time, after completion, stone will have to be added to insure high enough elevation to keep cross currents at high tide from sweeping beach drift over the jetty into the channel. Fascine and mattress work with rubble loading is employed in jetty construction. Jetties are also built of wall work and also of reinforced concrete.

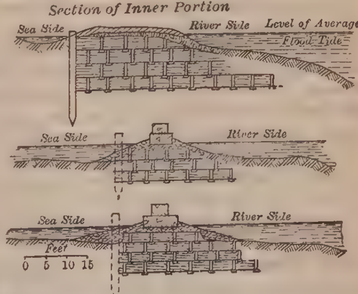


Fig. 14. South Pass, Mississippi River Jetties

A jetty built in 1913, at Shark River Inlet, N. J., consists of two practically parallel walls curved with concave side toward the south, the beach sands drifting toward the north. The jetties consist of a row of 16-in. reinforced-concrete sheet-piling, with counterforts at 10-ft. intervals, and with a braced reinforced-concrete superstructure; the pockets are filled with sand, the toe or apron is protected with riprap. Fig. 14

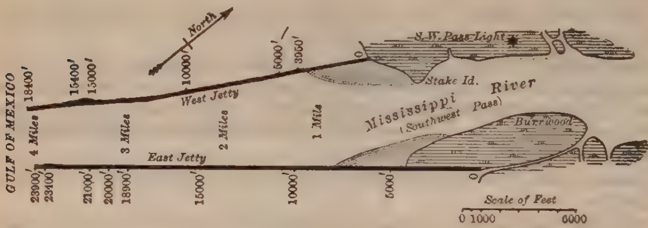


Fig. 15. South West Pass, Mississippi River

shows typical sections of jetties at the South Pass of the Mississippi River, and Fig. 15 a general plan of the jetties at South West Pass of this river.

RIVERS

7. Character and Description

Origin. Rivers owe their origin to the natural drainage or flow of water from the land to the sea, the development of their beds and their direction being due to the character of the soil, natural obstructions, inclination to follow the path of least resistance, and to certain modification in these characteristics due to influences, in some cases no longer existing, traceable to the breaking up of the Glacial Period. Their final formation and direction are the results of erosive action of the water, as balanced by soil resistance, resulting, in the cases of most older rivers, of an established equilibrium within certain limitations between the contending forces and the regimen as existing.

Sources of River Waters are in all cases, derived from tidal or rain water, the tidal water entering at the lower end, or mouth, and being directly due to the great primary or secondary ocean tidal waves, which, at high tide, pass up the estuary and the river, raising the level of the water and causing a flow of water up the river, and during the period of low water the process is reversed. The quantity of water passing up the river, due to such tidal influences, is, of course, on the average the same as the quantity flowing out on the reverse tide. The supply of fresh or rain water coming into the river at its source and constantly augmented by smaller supplies from branches and rivulets and the drainage from the banks along its course, travels solely in one direction from the source to the mouth and is subject to considerable variation in quantity and duration.

Tidal Rivers. Nearly all rivers emptying into a tidal sea are affected by tidal influences for some distance up from the mouth, depending principally upon the range of tide and the slope or fall of the river. At the crest of the tide, the salt water of the sea, by action of gravity, flows up the river; as the sea water is denser than the fresh river water, the flow has a tendency to take place by moving under the less dense fresh water and lifting this water up. On the trough of the tidal wave the flow is reversed, causing a continual oscillation in and out, the quantity of the ebb tide being augmented by the fresh water of the river. The volume of the ebb is under normal conditions always in excess of the volume of the flood, this condition being reversed at rare intervals by the occurrence of heavy onshore gales. The method of action of the tide is that, as the flood tide begins to make up the river, the current is at first slowed up, then entirely checked, and then reversed, banking up and preventing the flow and escape of the fresh river water into the sea. On the turn of the tide, the tidal water and the stored accumulated river water flow out into the sea, the greatest velocity occurring at about half flood or half ebb. At the periods of reversal, or of slack water, a short time elapses with no current existing. Contrary to the conditions generally prevailing directly on the coast line, the duration of ebb tide in rivers is longer than that of the flood tide, the difference depending upon the character of the river and the quantity of fresh water flowing down the river as compared with the volume of tidal water entering the stream.

Non-Tidal Rivers are rivers emptying into tideless seas or lakes or the upper reaches of tidal rivers beyond the effect of tidal action. The rise and fall of such rivers are entirely dependent upon the precipitation of rain on the areas drained by such rivers and their tributaries, the melting of snow and ice in such areas or as modified by the characteristics of the surface of the ground, growth of vegetation, and the peculiarities of the stream itself, such as depth, width, and slope.

8. Flow of Water in Rivers

Velocity of Current. The direction of flow and velocity of river water is due solely to the effect of gravity dependent upon the level of the surface of the water in the river. The difference in such level in any given length is termed "slope" or "fall." The particles of water at high level exert a pressure on those below them; these, being free to act in any direction, are pressed downward, forward, and upward toward the lower level, the whole mass being thus set in motion generally in the manner indicated in Fig. 16.

Fig. 16

The particles which come in contact with the bottom and banks of the stream are retarded by friction and do not move with the same freedom as the particles in the center of the stream section. Particles in contact with the bottom and sides are also deflected from their true course and cause disturbances in the stream action. As the stream moves forward, the particles describe orbits, varying in dimension with the section and depth of the stream; in large deep streams, the orbits being larger, the disturbing elements are less potent, the mean velocity being that obtaining in open channels as given in Sect. 13, Art. 15, and may be obtained more directly by

$$V = C\sqrt{2R \cdot F}$$

in which F equals the fall per mile in feet, R the hydraulic mean depth in feet, and C a coefficient of varying value, as follows:

For small streams of about	50 cu. ft. per sec.	$C = 0.65$
For larger streams of from 200 to	300 cu. ft. per sec.	$C = 0.75$
For tidal rivers of	1 000 cu. ft. per sec.	$C = 0.85$
For tidal rivers of	10 000 cu. ft. per sec.	$C = 0.95$
For tidal rivers of	100 000 cu. ft. per sec.	$C = 1.00$
For tidal rivers of	1 000 000 cu. ft. per sec.	$C = 1.50$

Transporting Capacity of Water. All rivers are charged with alluvial matter carried by them in suspension, the turbid conditions of many rivers during periods of heavy flow illustrating the extent of the work being carried on by them in the transporting of material. In addition to the materials actually carried in suspension, heavier and larger particles are transported by being rolled along the bottom of the stream. The sources of materials carried in suspension are the disintegrating effect of frost, breaking up and loosening of soil materials, the wash and erosion of rain, and the scour or eroding effect of the stream upon its banks. Under normal conditions the sectional area of the river is sufficient to provide for a velocity which will not cause undue erosion, the natural bed remaining in a state of equilibrium. However, any agencies which tend to increase such velocity tend to cause erosion with increase of sectional area and eventual decrease of velocity to the point of equilibrium. With a constant variation of volume of water flowing in rivers, detritus brought down at one time and deposited, is again taken up and transported farther downstream and so on. See discussion of Experiments by Grove K. Gilbert in Professional Paper 86 of U. S. Geological Survey, 1914; abstract in Eng. News, Sept. 10, 1914.

In a tidal river, the soil material carried by the stream is carried back and forth by the tide; the volume of ebb being greater than the flood, it is eventually slowly carried out to sea, or is deposited on the shores of the river mouth or estuary forming salt marshes. In non-tidal rivers, as the velocity of the current slackens on approaching and emptying into the sea, the material carried in suspension is deposited at the mouth of the river, forming deltas. The delta of the Mississippi River is a prominent example.

If a stream is loaded to its full carrying capacity it will tend to flow within the confines of its bank and the bed without further erosion; in some rivers more than 2% in weight of the total volume of water passing along the channel is solid matter. The ability of a stream to carry material is dependent upon the sixth power of the velocity as modified by the depth. Refer to Sect. 13, Art. 18.

Erosion and Scour. A non-tidal river with its flow constant in a downstream direction has an inclination to flow in a sinuous or meandering course. The particles of water moving in circular orbits or paths tend to impinge against one bank or the other; and on account of the lack of uniformity of the character of soil materials forming the banks, there is a constant tendency to erode or scour on one side or the other. A curved direction having once been assumed, the tendency is to increase the curvature, as illustrated in Fig. 17.

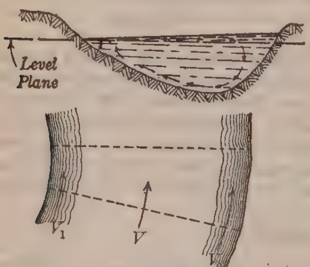


Fig. 17

The stream velocity is V. On account of the curved direction of the stream, the centrifugal force will tend to increase the velocity near the concave bank to V_2, and to decrease the velocity on the convex bank to V_1; the increased velocity on the concave bank, together with the dynamic pressure, due to the centrifugal force, will tend to deepen the stream bed at this side of the stream and erode and scour out the bank, these same forces tending to broaden a river at the bend by decreasing the mean hydraulic depth. The same force will tend to raise the elevation of the water on the concave bank and decrease it on the convex bank. The deflective force of the earth's rotation has a similar effect on both elevation and erosion. The right-hand bank in the Northern Hemisphere and the left-hand bank in the Southern Hemisphere clearly indicating by the bluff lines the small but continuous force directed against it. See Article in "The Military Engineer," Vol. XIX, No. 108, by G. T. Rude. Also Trans. Am. Soc. C. E., Paper 1599, by H. C. Ripley. The usual condition of a stream is illustrated in Figs. 18 and 19, the first showing a slight reverse bend, and the second a more considerable one.

The current, in each case, being close to the concave bank, has a tendency to scour out on this side deep channel-ways with consequent steep banks. Where material is

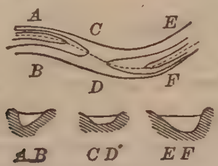


Fig. 18

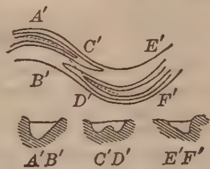


Fig. 19

carried in suspension the decrease in velocity on the convex side permits the deposition of detritus and the formation of flats, as illustrated in Fig. 20, and, eventually, the growth of the meandering effect of marsh land.

In some streams the bends become so great that the bights approach each other and a natural cut-off occurs, the bends soon taking the form of crescent lakes. The intersection having once occurred, the new channel is rapidly eroded as the shortest path and the one of least frictional resistance. This meandering effect is more prevalent in rivers with low velocity than in torrential rivers in which the increment due to centrifugal force is proportionately less. As the navigable distance is often naturally shortened by such cut-offs, these have been made by dredging; this should be done only after careful investigation as to water height and the consequent velocity through the cut-off; experience has shown that the shortening of the river course by this means often results in scour and bank caving.

Fig. 20

Channels and Bars. In non-tidal streams, the flow being constantly in one direction, the regimen of the stream flow is closely fixed and the flow, quantity and velocity vary with the precipitation of rain so that, within reasonable limitations of time, the location and direction of channels and flats are likely not to vary greatly. In tidal streams the conditions are different and more complex, the variation in fresh water flow being materially complicated by tidal flow as regulated and controlled by the extent and direction of storms. Fig. 21 illustrates the condition often obtaining at or near the mouth of tidal streams, the reversal of current due to the ebb and flood resulting in two or more variable channels with consequent formation of bars, very often shifting in character and extent.

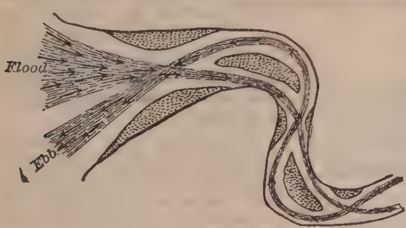


Fig. 21

The width and sectional area of channels in non-tidal rivers and in the upper reaches of tidal rivers bear a certain fixed relation to the drainage area and to the consequent quantity of water discharged. The width and depth of the tidal reaches of a tidal river do not appear to be so fixed, although the channels once fixed are usually maintained by the tidal flow. Any cause that tends to obstruct the free flow of tidal water and the propagation of the tidal wave is detrimental to the maintenance of the channel condition and frequently leads to shoaling. It is important to observe this effect in the design and construction of works of proposed improvement.

9. Training Works

Dredging. As with harbor work, the improvement of rivers, though often attempted by dredging, is not always effective as a measure of permanent improvement and usually entails continuous maintenance operations. Channels may be straightened out by cuts across bars and flats and the navigable distances along rivers decreased by excavating between the bends of meandering rivers. Dredging for the improvement of tidal or non-tidal rivers is principally useful in connection with other improvement works for stream regulation and control, and as a regular operation to maintain adequate depth and width of channels, slips, and basins.

Training Walls, also called dikes, are employed to direct the flow of the current in rivers with a view to the establishment of more favorable and fixed channels and often, also, to prevent scour and the erosion and carrying away of river banks. In the navigation of rivers it is important that the curvature of the channel should be of such radius as to permit the ready handling and movement of ships, this depending upon the depth of water and the size of ship to be considered. For large ships, if possible, it should be not less than 2500 ft. Single or double walls may be employed, the selection depending on the object to be accomplished. Two parallel walls may be used in a river flowing through a broad alluvial valley, in a wide bed and with an inclination to meander, the walls confining and directing the current flow, keeping the space between them open by sluicing and scour. The slack water area inshore of the training walls will tend to shoal and fill up. Such wall work in rivers is similar to and in many respects merely an interior continuation of jetty work at river mouths or harbor entrances.

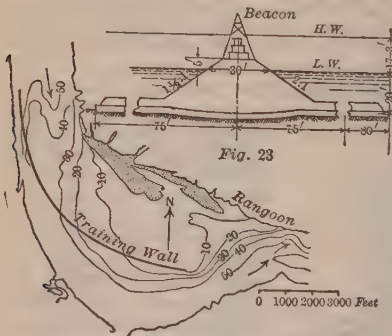


Fig. 23

Fig. 22. Training Wall, Rangoon River

At Rangoon, India, 1907, the Rangoon River (Figs. 22 and 23) was cutting out the concave river bank just above the city, causing formation of sand bars and shoaling of water on the opposite side, and on being deflected by a prominent point opposite the city was scouring out and undermining the foundations of wharves and piers. The training wall built was 2 miles in length and consisted of brushwood mattress 3 ft. thick, covered with a weighting layer of rubble, a rubble-mound wall surmounted by a concrete wall filled with rubble. Range of tide 20 ft. with 5 to 7 mile river current. River has scoured out new channel along wall and area behind wall has filled up.

In the case of the Mississippi River at Natchez, Miss., slides on the river banks were undermining the railroad tracks above. To relieve the situation, sedimentation was brought about by the use of a mud cell dike and when this was secured, the banks were permanently protected with concrete slab paving. See Trans. Am. Soc. C. E., Paper 1468, by W. C. Curd.

Construction of Training Walls or Dikes, as with jetties, may be of loose rubble-mound construction, with or without a surmounting masonry wall, or may be of timber, timber close piling, timber sheet-piling, steel sheet-piling, or reinforced concrete. In the employment of rubble-mound walls, considerable material will have to be used that will afterward be buried in the bottom, for as the wall is built and extended the scour of the stream at its end will tend to deepen the channel at this point, leaving deep holes to be filled with stone as the wall work is carried forward. Training walls should be carried up to the elevation of high tide, and when curved should be given easy curves so as not to cause sudden changes in the direction of channel currents. The area in back of the training wall eventually becomes filled up with sediment deposited by the stream in the slack water. A typical section of a loose, rubble-mound dike in use on Arkansas River is shown in Fig. 24, the rubble being deposited on fascines or brushwood mattress. Fig. 25 shows a typical solid, pile brushwood and stone dike in use on Arkansas River.

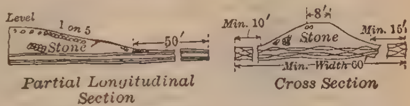


Fig. 24. Stone Dikes, Arkansas River

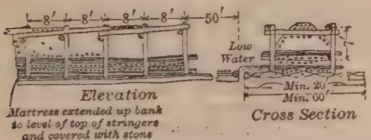


Fig. 25. Solid Pile Dike, Arkansas River

Spur Dikes, also called Spur Jetties, may be employed to regulate and direct the current of a river by contracting flow area and causing scour and lateral deposition of material behind or in between dikes. The influence of the spur dikes is similar to that of the groins employed in foreshore protection work, as described in Art. 5. Spur dikes are sometimes employed in connection with longitudinal dikes or training walls, or such longitudinal walls or bulkheads may be constructed connecting the end of the spur dikes after the space between them has become filled up with deposit.

Fig. 26 illustrates the improvement of the Upper Mississippi River by the employment of spur dikes which are built out perpendicular to or making an oblique angle with the direction of the current.

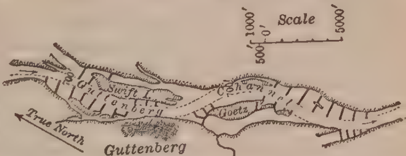


Fig. 26. Spur Dikes, Upper Mississippi

Permeable Dikes, instead of entirely cutting off or diverting the current flow, merely slacken and retard it. They consist of open-work construction, usually of timber piles and cribbing with rubble weighted brushwood mattress work hurdles. The velocity of the current being retarded behind or between such dikes, material in suspension is deposited and the area is filled up, the dikes themselves eventually become buried in a hydraulic fill dam of their own creation. On account of the initial increase in velocity at the contracted area of the dike itself, the fascine or mattress work is built with an apron on each side of the work to prevent scouring and undermining.

Fig. 27 illustrates the employment by the Chicago and Northwestern Ry. Co. of permeable dikes on the Missouri River at Blair Crossing. The river, as shown in (1), was following a cut-off and threatening the Blair bridge. The work was done in 1913. Permeable dikes were constructed across this cut-off, forcing the river into the old channel. (2) shows a section of the 2-pile dike; the main dike employed a 5-pile section.

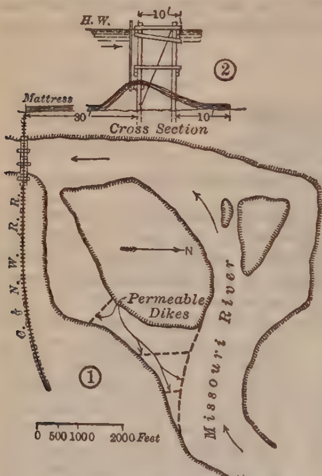


Fig. 27. Permeable Dikes, Missouri River

Bank Revetment is effected, when practical, by sloping the banks to stable slopes, protecting them from scour by brush mattresses, riprap, or block paving, timber planking, piling and sheet-piling. Such work is necessary, not only on rivers to protect banks from erosion due to the flow and velocity of the current, but at tide level to protect against wave action and wash of passing vessels. See Sect. 15, Art. 16.

In Fig. 28, (1), (2), (3), (4), (5), (6), (7) show typical methods of revetment on the Arkansas River. (8) and (9), typical methods, Upper Mississippi River. (10), typical methods, the Missouri River.

The standard revetment consists now of grading the bank on a slope of 1 : 3, laying a woven willow mattress 86 ft. wide from 3 ft. above low water and over the river bed, ballasting and sinking it, and paving the bank from the inshore edge of mattress to the top of bank. If the paving (riprap) fails by erosion, a substantial reinforced-concrete revetment is placed extending from the top to almost low water and from there a system of connected concrete flexible blocks about 24 in. square, extending beyond low water. This protects the bank and weak point of willow mattress.

10. Flood Control

Overflow. The causes which contribute to a river overflowing its banks are as a rule natural and seasonal in origin. In smaller rivers of limited drainage area these causes are found in exceptionally heavy precipitations of rain and in the melting of snow and ice in the early spring over the watershed; heavy precipitations of warm rain accelerate such melting. In a large river, with several tributaries discharging water from large drainage areas remote from each other and under different climatic, precipitation and other influences, the annual variation of flood in the main river is liable to be very great. The floods in the various tributaries might readily occur in such succession as not to tax the capacity of the main river, but the possibility is always present of these flood waters all reaching the main river at the same time or when it is, itself, under maximum flood condition. The Mississippi River is an outstanding example of such a possibility.

In most rivers draining a large alluvial valley, under normal conditions floods are to be expected annually. In the natural condition of the river, the river valley acts as a storage reservoir, the river overflowing its banks, the flat lands on either side being flooded and retaining the water, which is drained off and discharged at a later time. These annual floods have, in such cases as the Mississippi and the Nile, helped to enrich the flat agricultural lands along the stream by bringing down rich alluvial matter and depositing it as the water recedes after the flood period. This natural regimen has in some cases been upset by changes in these natural conditions through reclamation of the low lands on each side, by levees or dikes. Slowly but surely as this

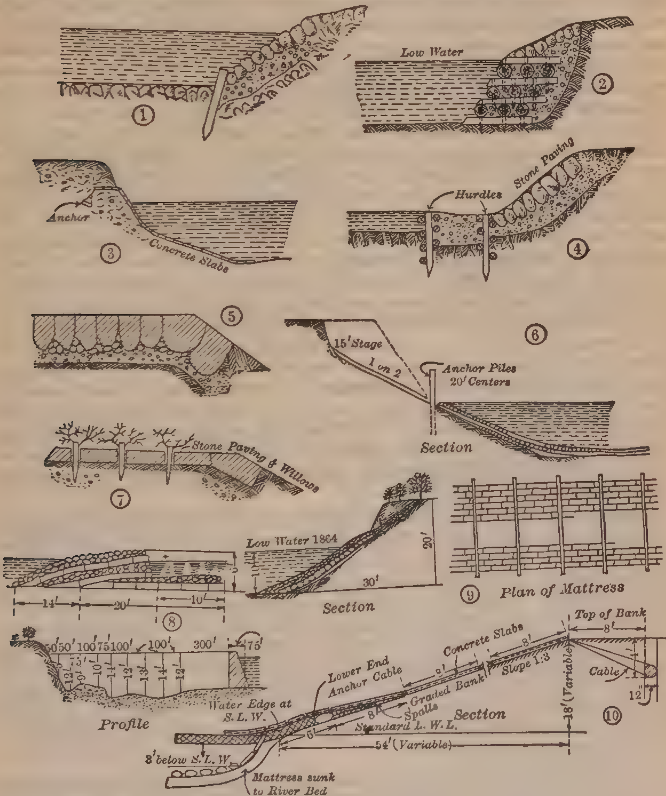


Fig. 28. Revetments

character of work advances, it confines the flooded river between these artificial barriers with resulting reduction in cross-sectional area, increased velocity and raised elevation of the river. This in turn requires increased height of embankments to prevent their being overtopped.

In addition to this cause of overflow or flood, it is frequently claimed that human inhabitation of the river valley results in a change in the character of the vegetation and various growth over the drainage area; the cutting of timber, removal of forest underbrush, and the placing of land under cultivation all serving to increase the rapidity of the runoff, with the result that periods of rainfall which would not ordinarily have caused serious floods will, under the change in the natural condition of the drainage area, have a tendency to hasten the drainage and outflow of rain water into the river.

An understanding of the causes of overflow of any important river is necessarily difficult and complex and can come only after thorough study of the special conditions under consideration, such study involving a complete topographical survey of the entire areas, rainfall and evaporation records over a long period of time, observation and gaging of stream velocity, cross-sectional areas, volume of discharge and various other features. The very voluminous records and reports in connection with the question of flood control of the Mississippi River indicate the difficulty of ready solution of the problem.

The conservation of forest growth, especially on the high lands where river tributaries have their source, has been advocated as a measure to reduce the rapidity of runoff and consequent flood, and at the same time by the retarding or storage effect to increase the flow at low stages. Forest growth undoubtedly does exert some regulating effect on flow and resultant floods, but such effects are materially changed and even reversed by altitude and frost, especially by snow and ice. The effect of forestation on flood would appear to be a local problem, and it can hardly be asserted, as a general proposition, that changes in such forest areas produce noticeable effects on river floods.

The control of floods by storage reservoirs has been found effective in certain cases hereafter referred to. In a river draining a large area this method involves such complex questions as the cost of the work and value of the land taken over for storage reservoir purposes. Another means of flood control is through relief afforded by diverting part of the flood flow through artificial or partially artificial channels. This frequently requires the construction of expensive controllable diversion spillways. It is often the case that no one of the means of relief mentioned herein can be effectively resorted to, but that a combination of all may be necessary.

Levees are embankments of earth built up on the sides of rivers to prevent the overflow of the banks. They should be built up 2 or 3 feet higher than the highest water recorded in the vicinity. In their design and construction, consideration should be given to the fact that the confining of the stream between the side levees will serve to decrease the stream area during flood flow, and probably entail increase in velocity and in elevation of the highest water over and above that recorded before the construction of the levees. The side slopes are dependent upon the character of the soil and vary from 2 : 1 to 4 : 1. The top width of the levee should be sufficient so that in case of settlement or unusually high water, the crest of the levee may be raised by additional embankment or by placing of bag extensions. Refer to Sect. 8, Art. 25, Levee Construction, and Sect. 16, Art. 19.

Fig. 29 shows typical sections for various heights of levees employed on the Mississippi River up to 1927; these are being increased in height and area as a result of the experiences of the 1927 flood. Revetment work is employed to protect levee slopes where river conditions are such as to indicate the necessity for such work.

The dikes of Holland perform a duty somewhat similar to that imposed on river levees, protecting the low lands from inundation by the sea. This system of dikes consists of the natural sand dunes, artificially built up sand dunes, and embankments. The rivers outflowing along the coast line have levees constructed along their banks. These sand dunes are protected against wind erosion by a growth of coarse grass and reed, which also serves to retard and hold the blowing sand and build up the dunes, fresh grass being planted on top of this. Where subject to wave action, the slopes are protected by fascine and mattress work, weighted with basalt or concrete blocks; short timber piles are driven to pin and hold down the work and to help break the force of the waves. In localities where lateral currents occur, spur dikes are built out perpendicular to the main dike.

Flood protection walls may be employed in connection with, or to replace, levee work. Such walls, consisting of two reinforced-concrete walls backfilled, were employed at Portsmouth, Ohio. The walls were fitted with flood gates which are closed when the elevation of the river reaches a dangerous height; the sewage discharge from the city must as a consequence be pumped instead of flowing through the wall.

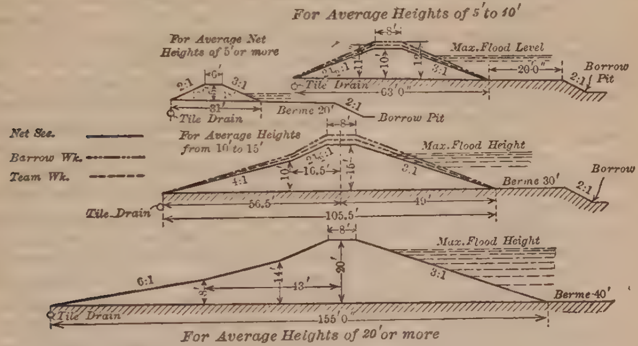


Fig. 29. Mississippi River Levees

Navigation. Most rivers at high stages usually afford the necessary facilities for navigation restricted within the known and expected local requirements. The low stages of rivers often make navigation impossible without the assistance of engineering works. At such low stages with the decrease in stream velocity and in the quantity of water discharged, bars extend diagonally across the river, especially at bends, blocking the fairway and making the passage of vessels difficult or impossible. With sudden rises, new channels are eroded through such bars, varying much in direction and location. Formerly in the Mississippi, when this river carried a large local river traffic, it was customary at New Orleans to hold frequent conferences between pilots of ascending and descending steamboats in order to ascertain the latest changes of channel. Rivers, especially during freshets, bring down large quantities of drift logs, developing snags or sawyers, which tend to obstruct navigation, raising barriers and sand bars and changing the channel conditions; it is important that such snags shall be removed as soon as possible.

Reserve Basins or Reservoirs. Under favorable conditions, river floods may be somewhat controlled by constructing in the upper reaches basins or reservoirs consisting of large areas with impounding dams as it is contemplated to hold in reserve in these reservoirs the excess flood volume which is to be released during the periods of low stages, and thus to increase artificially such low stage flow with a view to establishing an average flow. The fundamental principle involved is a sound one, but natural difficulties are generally encountered that render impracticable the obtaining of the desired results. The problem involved, for successful application for such works for flood control, requires special and careful study of local conditions, the areas available for reservoir sites, value of land likely to be submerged, and the existence of suitable sites for dam construction. The plan has been tried on the upper Mississippi River, the region being remarkable for the number of small lakes. A large Indian reservation presented conditions unusually favorable. Five reservoir lakes were immediately available and additional ones possible and in prospect. The watersheds of the five reservoirs constituted 11.8% of the whole area above St. Paul and 21.5% of that above Minneapolis. The actual result to navigation on the Upper Mississippi system was an average increase in low water depth at St. Paul of about 14 in. When the release of water was specially regulated, 22-in. increase was obtained; these figures are based on a record of 20 years, during which period the total outlay, including construction and operation, was about $1 500 000. From this it is concluded that, as a general proposition, such reserve reservoir work, though instrumental in controlling flood discharge and assisting navigation, is of use principally in connection with other engineering work of improvement. The difficulty of regulating the discharge from such storage reservoirs can not be overestimated. The release of reservoir water may be necessary and beneficial to one section, and have quite the opposite effect on other sections served by tributaries or branches which, due to local severe precipitation, are experiencing local floods.

Canalization or slackwatering is a method employed to afford a sufficient depth for navigation at all seasons of the year, and also to overcome rapids or falls that would otherwise make navigation impracticable. In this method a series of pools is formed by the construction of dams, and connected by locks. This method has been extensively employed in Europe on such rivers as the Fulda, Main, Oder, Elbe-Moldau and the Seine, and has been carried on extensively in the United States since 1840 on the Allegheny and Monongahela Rivers in Pennsylvania, the Ohio River, the Black Warrior and Tombigbee Rivers in Alabama, the Columbia River in Oregon, the Cumberland, Green and Kentucky Rivers in Kentucky, the Fox River in Wisconsin, the Hudson and Mohawk Rivers in New York, and the Kanawha River in West Virginia.

Its application to the Ohio River comprises a series of 51 locks and dams, overcoming a total fall of 424 ft. from Pittsburgh to Cairo, of which 26 ft. are at the Falls at Louisville. This system of locks and dams planned for completion in 1929 provides a minimum depth of 9 ft. for a distance of nearly 1000 miles during all seasons of the year.

11. Signals and Lighthouses

Buoys. Buoys are employed for demarcation of entrances, interior channelways, and for the location of dangerous shoals. They are placed at intervals at either side of the channel-way, the port and starboard boundaries being indicated by the shape and color of the buoy. Buoys must be moored to heavy anchors, either cast iron, such as the mushroom, or frequently large blocks of concrete are used with heavy iron eyes cast therein. Dependent upon the range of tide, the buoy is secured to an anchor by a length of cable two or three times the maximum depth in which it is to float. In channel demarcation the distance between buoys varies with the character of the channel as to width and straightness. In the interior of harbors mooring buoys are frequently provided at various fixed locations to enable vessels to moor without resort to ship's anchors. In harbors of restricted area, at times two buoys, one for bow and one for stern, are provided for mooring. Buoys are used for location of wrecks in outer harbors or in the open sea, also to indicate cable crossings.

Beacons are fixed signals or structures used for means of alignment or for indicating changes of direction. Frequently prominent objects or landmarks, natural topographical features, or prominent structures, are used as beacons.

Channel Lighting. The location of channels is often fixed for night use by luminous buoys lighted by means of oil or gas, or the range is fixed by two illuminated points, afloat on buoys or fixed on shore, or one afloat and the other ashore, the change in direction being shown by a similar line or range.

Sound Signals. Light being impracticable and useless during heavy mists or fogs, resource is had to sound or warning signals, a bell or whistling buoy being used for this purpose. In deep water where there is always some wave motion, bell buoys depend on this motion to actuate balls or pendent clappers. Whistling buoys depend on the taking in of air and its expulsion as the buoy rises and falls, or the bell may be struck or the whistle sounded by compressed air or gas carried in a tank that is regularly recharged. Sound transmitted through the air often gives a misleading idea of locality; resort is therefore had to submarine sound signals which can be heard a great distance and the direction of which can be identified.

Lightships. At locations where, on account of the natural conditions, it is impracticable to locate and maintain lighthouses, or at locations where luminous buoys would not be striking enough in character, necessary light for the assistance of navigation is placed on lightships. Signals of this character are more certain and reliable than are light buoys exposed to strong currents in heavy seas. The principal requirements are staunch construction and steadiness under severe storm conditions.

Lighthouses. (Lighting and Design.) Dangerous promontories, points, and bars are marked by lighthouses, especially at entrance to bays and harbors. The lights are fixed, revolving, or flashing, for purposes of identification. When a navigator has identified a light and knows its height above sea level, the distance at which it first becomes visible at sea is known. Let H be the height of light, h the height of the observer above sea level, both in feet, and D the distance from the vessel to the light in statute miles. When the light is seen exactly on the horizon, then

$$D = 1.32\left(\sqrt{H} + \sqrt{h}\right)$$

If the distance in nautical miles is required, 1.32 is to be replaced by 1.15. For example, if the height of the light is 100 ft. and the observer is 25 ft. above the water, he sees the light on the horizon when he is 19.8 statute miles or 17.2 nautical miles from the lighthouse.

The following statements regarding the lights of lighthouses and the structures themselves have been kindly furnished by J. S. Conway, Deputy Commissioner of the U. S. Lighthouse Service.

(1) **Intensity.** Lights are of classes heretofore known as "orders," based on the focal diameter of the lens, as "first-order," "second-order," etc., and miscellaneous smaller lights. Recent practice substitutes the candlepower of the light in place of the now more or less misleading term "order."

(2) **Ranges of visibility,** two classes: the geographical range of the light due to its height above sea level, and the luminous range due to the intensity of the light emitted by the illuminating apparatus.

(3) **Character** of the illuminating apparatus, three classes: catoptric or reflecting, dioptric or refracting, and catadioptric, in which the "lens" is provided with both reflecting and refracting media. The latter type is the most in use.

(4) **Characteristics** of the light, such as Fixed, Flashing, Occulting, Fixed and Flashing, or other combinations for distinguishing purposes. Fixed lights are liable to be confused with neighboring private lights and should be avoided if practicable. Fixed, combined with flashing, and lights with prolonged flashes should be avoided for the reason that the "fixed" and "prolonged" rays emitted by the lens are weaker than those of the "flashes," giving a misleading characteristic at a distance. Multiple lights, except for minor lights, are now considered obsolete, because they largely

increase the cost and are less distinctive than flashing lights. Lights of alternating colors should be so designed that the white and colored rays are of equal intensity.

(5) **Arcs of visibility,** two classes: single lights illuminating a large arc of the horizon and two lights in range illuminating a limited arc for purpose of marking narrow channels and passages. Often plates of red glass are introduced in the lantern so that the sector of rays passing through them will cover an outlying danger such as a reef, rock, or shoal, or to indicate the edge of a channel.

(6) **Attendance,** two classes: Watched and Unwatched lights.

(7) **Fog-signals,** four classes: the ordinary bell struck by hand or machinery and submarine bells struck by machinery, also whistles, reed horns, and sirens blown by compressed air. Some of the whistles and sirens are sounded by steam.

(8) **Materials.** Almost all the usual materials of construction have been used in building lighthouses in the United States Lighthouse Service: stone masonry, brick-work, reinforced concrete, framed timber, structural cast iron, structural steel, cast-iron plates, steel plates and piping.

(9) **Arrangement.** Light stations situated on land sites usually consist of the light tower, oil house, fog-signal building, keepers' dwellings, workshop, water supply and drainage systems, boathouse and ways, barn, and the usual outbuildings, roads and walks, although, owing to the restricted area of some sites, one or more or all of the buildings may be combined in one. On submarine sites the whole station is practically confined to one structure.

(10) **Foundations.** For land sites, the foundation for a closed tower of masonry, or metal work, is usually a single block of concrete resting upon the foundation soil. Occasionally these blocks are placed upon a timber grillage supported by piles for sites upon low or marshy land. In all cases the block is extended so as to bring the unit pressures within the bearing power of the soil. Occasionally a skeleton structure is placed upon a single foundation block, but usually each column or leg of the tower has its individual block. Where the site is subject to overflow, the buildings are sometimes grouped together, raised upon braced columns and connected by a system of galleries.

For submarine sites, the foundation may consist of a cylindrical cast-iron or sheet-steel caisson filled with concrete, or a masonry pier. Cast-iron caissons are in general use on the Atlantic Coast. When placed upon a ledge of rock, the latter is usually leveled up with concrete bags if below low water, or with tools if exposed, and the ledge or rock is then heavily ragbolted to the concrete filling. For soft bottom, the best method of procedure is to float the caisson out and sink it by the pneumatic process. Both timber and metal working chambers have been used and the depths from the cutting edge to high water have varied from 19 to 85 ft. Other foundations, consisting of timber cribs and concrete blocks, used in fresh water, have been placed either directly on good existing bottom, or upon a layer of small rock usually 3 or 4 ft. thick deposited upon the soft or dredged bottom prior to floating the crib out. These timber cribs are usually filled with stone and terminate about 2 ft. below low water level; the concrete blocks are then placed to the height of the deck.

(11) **Forces.** Structures upon land sites are exposed to wind pressures, and occasionally to waves in addition. Those upon submarine sites are exposed to wind, wave, currents, and ice. The usual procedure in determining the stability of tower is to locate the common center of effort of all the forces acting upon the structure to overturn it and so proportion the weights of the entire structure (the buoyancy of the water must be taken into account for submarine structures) that the resultant of the active forces and the net weight falls so far within the outer edge of the base that there is compression over its whole surface if the foundation soil is compressible. In seeking for this result it is proper to include the lateral resistance of the soil when the structure penetrates it for some distance for the reason that it is often heavily compressed by a large deposit of riprap and offers good support. The maximum unit pressures, both vertical and lateral, must not exceed the bearing power of the soil. In case the foundation is rock, the resultant must fall so far within the outer edge of the base that the maximum unit pressure does not exceed the compressive strength of the materials in contact.

The wind, wave, current, and ice pressures assumed should be the maximum in each instance, as lighthouses are commonly exposed to severe gales and flows of ice. It has been the practice to assume wind pressure for flat surfaces at 60 lb. per sq. ft., allowing two-thirds of this figure for rotundity. Maximum wave pressures of 6000 lb. per sq. ft. on flat surfaces are assumed, based upon Stevenson's experiments, the force of the wave being greatest at its crest and diminishing to zero at its base. The pressures exerted by currents vary with their rate of speed. The pressures due to ice have been assumed at 30 000 lb. per sq. ft. for a field of melting ice, 1 ft. thick, striking the pier and crushing its way past.

(12) **Superstructures.** The superstructures of all towers, whether separate or combined with other buildings, have certain features in common. There is a main entrance door at the base, a flight of winding stairs, broken by landings in high structures leading to the service room, which in the larger lights is usually separated by an airlock from the watchroom above, the latter supporting the lantern. Occasionally in large lights and usually in small ones the service and watchrooms are combined. The pedestal of the illuminating apparatus usually rests upon the watchroom floor, but in the small lights the lantern floor supports it. The clear glazed opening of the lantern is just sufficient to pass the horizontal rays from the illuminating apparatus, and if the latter is of a revolving type, showing flashes or occultations, the newel post of the tower serves as a weight shaft for the clock. There are railed galleries outside both lantern and watchroom. The tower should be thoroughly fireproof and isolated in this respect from the other buildings. For calculating the strength of closed and skeleton towers the manner prescribed for chimneys and viaduct bents is employed with the exception that great stiffness and rigidity must be provided, as vibrations are detrimental to the proper working of the lamps and clocks of the illuminating apparatus.

QUAYS

12. Design

Definition. Wharves are landing places or platforms built out into, or on to the water for the berthing of vessels. Wharves parallel to the shore, marginal wharves, are generally known as quays, and their protection walls as quay walls; wharves built into the stream or fairway perpendicular or oblique to the shore are generally known as piers.

Quay Walls, or bulkheads, are used for wharfage for retaining and protecting embankments or retaining filling. Their proper design and cost are largely dependent upon local conditions and the use to which they are to be put. The character of the foundation and the depth of water to be provided are important factors in the cost and design. Some data on design are given in Sect. 10, Art. 28.

Data on Design. It is important to observe the character of the foundation on which the wall is placed, whether the wall may be built directly on rock or hard-pan, or on a floating foundation on timber piling or grillage work. In the latter case, information must be secured as to the prevalence of marine wood-borers, and, if present and active, precautions must be taken to protect the exposed timber work by creosoting or other preservative or protective methods. Quay walls are designed in a similar manner to retaining walls with the difference that on the water side they are subject to the water pressure varying with the height of the tide, and on the land side to the earth and contained water pressure with a proper allowance for surcharge. The contained water on the land side of the wall will usually stand higher than the tide water, especially at low tide. A rapidly falling tide may result in a considerable earth and water pressure tending to push out or overturn the wall.

The Combined Pressure of water and earth upon a wall must be known in making a design. Let w_1 be the weight of a cubic foot of water, w_2 the weight per cubic foot of the submerged earth, ϕ the angle of repose of that earth, and h the height of the vertical wall. Then, for one foot in length of the wall,

$$\text{Water pressure} = 1/2\, h^2 w_1$$
$$\text{Earth pressure} = 1/2\, h^2 w_2 \tan^2 (45° - 1/2\, \phi)$$
$$\text{Combined pressure} = 1/2\, h^2 (w_1 + w_2 \tan^2 (45° - 1/2\, \phi))$$
$$\text{or, Combined pressure} = 1/2\, h^2\, W$$

To find W the weight per cubic foot, w_1 of sea water is taken as 64 lb. and w_2 may be found experimentally by taking samples of the earth as near as possible in its natural compacted state, and ascertaining its weight submerged in sea water, or may be fixed theoretically in approximate terms by taking the weight of the original material, such as limestone rock, granite, slate, and other materials less the weight per cubic foot of water multiplied by the percentage of solid material per cubic foot, that is, 100% less the percentage of voids in the material.

Combined Weight W of Sea Water and Earth, Lb. per Cu. Ft.

(Equivalent Liquid Pressure)

Slope of repose of earth	Weight w_2 of submerged earth, lb per cu. ft							
	40	44	48	52	56	60	64	68
1/2 : 1	66.2	66.4	66.7	66.9	67.1	67.3	67.6	67.8
3/4 : 1	68.4	68.9	69.3	69.8	70.2	70.7	71.1	71.6
1 : 1	70.9	71.6	72.2	72.9	73.6	74.3	75.0	75.7
1-1/4 : 1	73.2	74.2	75.1	76.0	76.9	77.9	78.8	79.7
1-1/2 : 1	75.4	76.6	77.7	78.9	80.0	81.2	82.3	83.5
1-3/4 : 1	77.5	78.8	80.2	81.5	82.9	84.2	85.6	86.9
2 : 1	79.3	80.8	82.3	83.9	85.4	86.9	88.4	90.0
2-1/2 : 1	82.3	84.2	86.0	87.8	89.7	91.5	93.3	95.2
3 : 1	84.8	86.9	88.9	91.0	93.1	95.2	97.2	99.3
3-1/2 : 1	86.8	89.0	91.3	93.6	95.9	98.2	100	102
4 : 1	88.4	90.8	93.3	95.7	98.1	101	103	105
5 : 1	90.9	93.6	96.3	99.0	102	104	107	109
6 : 1	104	108	112	116	120	124	128	132

This table gives the combined weight W, or the values of $w_1 + w_2 \tan^2 (45° - 1/2\, \phi)$, the weight w_1 being taken as 64 lb. per cu. ft. for sea water. It should be noted that the submerged weight of the earth will be dependent both on character of original soil rock and on the extent to which the soil is compacted, the angle of repose becoming greater as the soil is more compacted. See table A, p. 914, for values of angles of repose corresponding to various slopes.

It will be unusual in practice to find conditions where a combined weight W in excess of 90 lb. per cu. ft. should be used, although where a structure is built on soft mud, or is to be filled with, or retain, mud, especially if it is placed as a hydraulic fill, the combined pressure may reach or exceed 120 lb. per cu. ft. The total pressure in pounds against a linear foot of a vertical wall **h** feet in height is found by multiplying **W** by $1/2\, h^2$.

Some observations have been made at various depths on the weight "in situ" of soils immersed in sea water, and, as should be expected, they show wide variation.

The Overturning Moment is the product of the total equivalent liquid pressure on the rear side of the wall, and one-third the height, less the product of the water pressure on the sea side of the wall, by one-third its depth. In drydocks, canal locks, masonry quaywalls or bulkheads, the height to be taken would be the distance from the top of the fill down, whereas in walls with timber or other relieving platforms the height would be from the bottom of the platform. When the bulkhead or wall is founded on viscous or plastic soil, the depth or height to be considered will not necessarily be the height of the wall, but may extend some distance below the depth of water at the sea face.

Weight of Soils Immersed in Sea Water

Material	Pounds per cubic foot		
	Maximum	Minimum	Average
Gravel and marl	....	42.0	62.9
Gravel and sand	73	42.0	62.4
Sand	66	42.0	58.3
Gravel, sand and clay	80.9	51.2	70.0
Stiff clay	64.8	38.4	47.8
Stiff clay and gravel	70.3	44.8	52.6

Practice and Custom. It is too frequently the case in the design and construction of wharfage work that the designer is guided entirely by the accepted custom of practical wharfbuilders. It should be remembered that the same fundamental principles apply here as to other engineering structures. If the wall is a gravity structure, the safe bearing capacity of the foundation soil must be considered, and it must be remembered that the submerged part of the structure loses dead weight to the extent of the buoyancy effect of the water displaced by it.

Where piles are employed, their safe bearing capacity should be obtained by driving test piles and either applying a test load or making observations of weight and drop of hammer or penetration when bringing up, and these data are to be used in connection with one of the well-known piledriving formulas, the pile structure being examined as a column, and, if of timber, due allowance made for character of material used, the creosoting of timber reducing the strength and safe carrying capacity of the pile. If capped with timber, consideration should be given to the safe bearing power across the grain of the timber used. In wharf work under conditions generally obtaining, timber piles 14 in. in diameter at butts, with the usual framing, should not be counted on to give more than 18 tons carrying capacity per pile. If creosoted timber, not more than 15 tons. The safe bearing capacity of reinforced-concrete piles is frequently taken at twice this amount but is, of course, dependent on the pile section and the driving of the pile. In reinforced-concrete piling the bearing capacity at the top of the pile is reduced by the weight of the pile. In a reinforced-concrete pier built in Havana, Cuba, piles 20 in. square, and up to 85 ft. long, were used, and gross bearing capacity of 40 tons per pile allowed, or net capacity of 25 tons, 15 tons being allowed as the weight of pile. When employing batter, brace, or spur piles, this bearing capacity should be somewhat reduced, as the driving effect of the piledriver hammer will be less effective on account of the inclination and consequent friction on the leaders or guides.

Expansion Joints. In long quay or sea walls or bulkheads, expansion joints at intervals of about 50 ft. are necessary in order to avoid unsightly shrinkage and settlement cracks. It is desirable to design these so as to permit irregular vertical movement in adjoining sections, but to lock these sections so as to prevent a horizontal break. A simple and effective joint of this character is made by casting one or more V's or grooves in the end of one section, and coating this with pitch or tar, or the end of two adjoining sections may be built in this

Fig. 30. Expansion Joints

manner a few feet apart and the space filled up. The two methods are illustrated in Fig. 30.

For joint *A* tar is applied with brush to side first concreted. For joints *B* and *C* tar is applied with brush to body concrete after forms are removed and before key concrete is placed.

13. Foundations

Sheet Piling. When sheet piling is employed in quay walls or bulkheads in connection with or without relieving platforms, the sheet piling will act as a simple or partially fixed beam supported where driven into the bottom and at its anchorage or fastening at the platform. Due precautions should be taken to see that the sheet piling is driven deep enough into the bottom to secure a proper hold and to guard against being pushed out, for which purpose it must be driven to a depth so that the passive resistance of earth will take the bottom reaction. In reinforced-concrete sheet piling, steel can be placed accordingly, but in any case sufficient reinforcing steel must be placed on both faces so that the pile will be strong enough to be lifted and handled while being placed in the work. It is usually advisable to lift the piling at two points so that the negative and positive bending moments are equal, and to fix these points by casting gas pipe section or eyes in the piling.

A too frequent cause of failure of walls or bulkheads is the pushing out of the bottom or toe of the sheet piling, or, in very soft material where sheet piling is not long enough, the running out or mudwave effect underneath the bottom of the sheet piling.

Steel sheet piling is frequently used in place of timber or reinforced concrete, as it can be driven into and through materials that cannot be penetrated by either timber or reinforced-concrete sheet piling. Steel sheet piling has been driven into soft rock. The steel, although it rusts away where exposed to the air, or between tides, will nevertheless last a long time under water or where cut off from free oxygen. Steel sheet piling is now available with a small copper content in the steel. The effectiveness of this alloy in preventing corrosion is questioned.

Methods of Computing Depth to which Sheet Piling should be Driven, size of piles, struts or tension of tie backs.

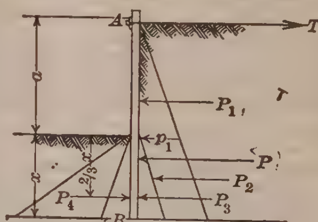


Fig. 31

In Fig. 31 the problem is shown in its simplest form.

AB is a sheet pile driven to depth x into the soil and a the known height above the soil to which it is retaining earth, assume it is held at *A* by a strut or tie *T*. P_1 is the known active pressure above the outside soil. p_1 the intensity at the level of the soil, p the increment of intensity per unit of depth for active pressure above or below outside soil. kp the increment of intensity per unit of depth for the passive pressure below the outside soil. $P_2 = p_1x$; $P_3 = 1/2\ px^2$; $P_4 = 1/2\ kpx^2$; $P_1 = 1/2\ pa^2$ or $P = P_1 + P_2 + P_3 = 1/2\ p\,(a + x)^2$.

Taking moments around A, $P \times 2/3\,(a + x) = P_4\,(a + 2/3\ x)$.

This is an equation with one unknown quantity x. Also $P_4 + T = P$ or T tension in tie or compression in strut $= P - P_4$.

Fig. 31a shows the usual trench sheeting, the two struts at D and C take the place of the tie, T, in Fig. 31 and under these conditions $P_4 + S_1 + S_2 = P$ and from this the size of sheeting and depth to which sheet piling is to be driven can be determined, and also the dimensions of struts and wales. Sheet pile

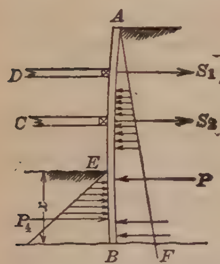


Fig. 31a. Length of Sheeting and Dimensions of Braces

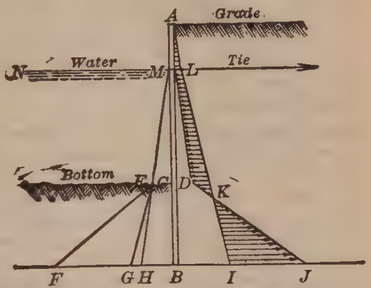


Fig. 31b. Bending Moment and Shear on Sheet Pile

AB is considered as a beam loaded as shown in ABF. The same analysis is applied to the general type of sheet pile bulkhead, Fig. 31b, where AB is the sheet pile, MN the elevation of water. The triangle ALM indicates the earth pressure above tide level and $MLIB$ indicates the total active pressure of submerged earth and water below tide level on the land side. MHB indicates the total water pressure on the water side, and EGH the total active soil pressure on the water side and EHF the total passive pressure. Superimposing the active and passive pressure on the water side on the active pressure on the land side $AMDKL$ represents the intensity of the forces outward. KIJ represents the intensity of the active and passive forces inward which with T equals the outward forces. From this may be constructed the moment diagram and the size and details of the sheet piling actually desired.

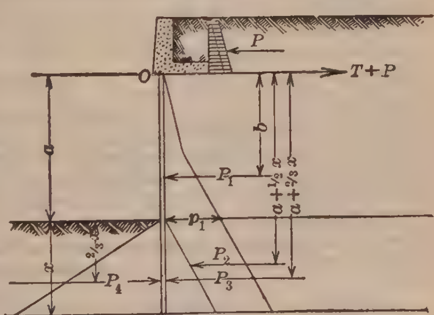


Fig. 31c

Fig. 31c is the general case of a platform wall with surcharge. The length and size of the sheet piling may be determined as follows:

Find the known active force P_1 extending from the relieving platform to the level of the outside soil. Find the intensity p_1 at the level of the outside soil. Find p the increment of intensity per unit of depth for the active pressure below the outside soil. Find kp the increment of intensity per unit of depth for the passive pressure below the outside soil. Then $P_2 = p_1x$, $P_3 = 1/2px^2$

and $P_4 = 1/2kpx^2$. Take the moment of all the forces below the platform about the point O, giving

$$P_4 (a + 2/3x) - P_3 (a + 2/3x) - P_2 (a + 1/2x) - P_1 b = O;$$
$$1/2px^2 (a + 2/3x) (k - 1) - p_1 x (a + 1/2x) - P_1 b = O;$$
$$1/3p (k - 1) x^3 + 1/2ap (k - 1) x^2 - 1/2p_1 x^2 - ap_1 x - P_1 b = O.$$

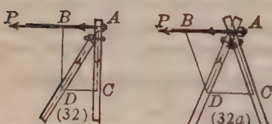
The last is an equation with one unknown x. Solve for x. Then determine P_2, P_3 and P_4 and $T = P_1 + P_2 + P_3 - P_4$.

In applying this method the table of equivalent liquid pressure in Art. 12 may be employed. kp the increment of intensity of passive pressure is dependent upon the character of soil bottom and must be obtained by experiment or experienced inspector.

Brace Piles, when driven with opposite inclinations, are subjected to a pull in one case, and a push in the other, and connection should be made accordingly. It is too frequently the case that brace piles, though provided for in ample number and of sufficient bearing capacity, have weakened and improper connections and that the full lateral effect is not secured from them.

A quay wall or bulkhead can be anchored against the thrust or horizontal pressure by deadmen or anchors buried in the embankment some distance back and below the plane of rupture made by the angle of repose, or by anchors of brace piles. Having determined the amount of this thrust, the anchorage can be readily designed.

Such anchorages are shown in Figs. 32, 32*a* and 33. In Figs. 32 and 32*a*, the amount of horizontal thrust is represented by $A\ B$, $A\ D$ would be the push or



Figs. 32, 32*a*. Pile Anchorages

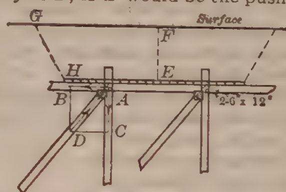


Fig. 33. Pile Anchorage with Platform

bearing on the brace pile, and $A\ C$ the uplifting effect or pull on the vertical pile in Fig. 32, and inclined pile in Fig. 32*a*; Fig. 33 is an anchorage in connection with a relieving platform, or might be used separately as anchorage for a bulkhead or wall. $A\ C$ is the uplifting effect, but the actual pull or uplift on the vertical pile shown is decreased by the weight of the soil or backfill surcharge on the platform, represented by $E\ F\ G\ H$.

Effective and Ineffective Anchorage details of brace or anchorage connections are shown in Figs. 34 and 35. Fig. 34 shows five typical brace pile anchors

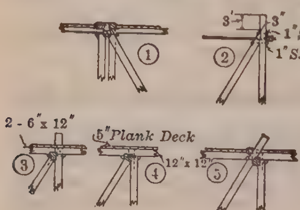


Fig. 34. Effective Pile Anchorage

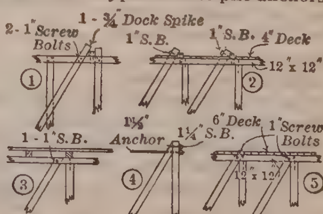


Fig. 35. Ineffective Pile Anchorage

for a quay wall or bulkhead. Fig. 35 also shows five typical cases of improper brace or spur, pile details or connections. Some of these have been taken from actual work, Fig. 35 (1) and (2) having been discovered and sketched only after the failure of the work by the outward movement and overturning of the wall. It will be noted that in these two cases the bearing or thrust in the brace pile is entirely taken by the bending in two 1-in. screw bolts and one 3/4-in. dock spike. These, apparently, held for some time, but, being submerged, even this support was greatly weakened by the corrosion of fastenings. In Fig. 35 (3) and (4) depend entirely upon the strength in bending of one screw bolt, as does (5), except that the deck planking directly above the piles furnishes an additional bearing. In designing work that will be submerged, or kept partially so, if the work is to be considered permanent, it is important, in so far as possible, to avoid entire dependence upon iron or steel fastenings. Galvanizing has been attempted with indifferent success, and, in one or two instances, extruded brass has been used for fastenings. As far as possible, work of this character should be detailed for dependence upon the fitting of timber joints and the use of wedges, dowels, and treenails. The general practice in engineering design should be applied to obtain dimension of caps and stringers, thickness of planking or decking, bearing surface in contact, and other details. Liberal allowance must be made, however, for the weakening of aged water-logged timber or the deterioration of timber, by rot, when exposed to successive wetting and drying out; and in reinforced-concrete construction to the possible disintegrating effect of concrete when submerged in salt water, especially when subject to the action of frost.

Safe Bearing Capacity of Groups of Piles. As stated in Sect. 8, Art. 10, the bearing capacity of piles is dependent upon the support at the bottom of the pile; if the pile penetrates a short distance to hardpan or rock, the strength of the pile as a column must be considered; when the pile penetrates some distance into soft materials, the bearing capacity depends upon the skin frictional resistance. It would be well, however, to note that the skin frictional resistance is the means by which the load on the pile is transmitted through the pile and through a vertical cone of earth, of which the pile is the center, to the subsoil at the plane of extreme penetration of the pile, the bearing capacity of the soil also being increased by being compacted by the driving of the pile. For this reason the safe bearing capacity of one pile is no criterion for fixing the safe bearing capacity of a number of piles driven closely together, as in the latter case the cones of resistance overlap and the bases of these cones at the bottom of the piles will, in cases of a group of piles driven closely together, each be required to carry the load, at least in part, of several piles. Whereas the bearing capacity of the subsoil at some distance beneath the surface, as at the foot of long piles compacted by pile driving, will safely carry considerably higher loads than the same character of subsoil at or near the surface not compacted, there is, nevertheless, a limitation which can be readily exceeded by placing piles too close together, expecting them to carry considerable loads. It is inadvisable in pile foundations, for harbor work, to place piles closer together than 2-1/2 ft. to 3 ft. It will be seen that at 2-1/2-ft. centers, in carrying loads of 18 tons each the interior piles of a large group will subject the subsoil to a load of approximately 3 tons per sq. ft. The same principle applies with equal force to any surface or subsurface loading, such as by means of spread footings.

Anchorage or Pull on Piles. Where brace piles are used for anchorage of quay walls or bulkheads, or in connection with pier work, as previously stated the bringing into play of these brace piles to resist lateral movements frequently results in an upward pull on some inclined or vertical piles. As a general rule, piles driven some distance into the soil and dependent upon skin friction for their carrying or bearing capacity can be safely counted on as giving an anchorage effect or resistance to pulling equal to the estimated bearing or carrying capacity. Experience in pulling up or attempting to pull up piles driven in harbor work fully confirms this statement. Observations made on piles 55 to 75 ft. in length, driven in 23 to 35 ft. of water, indicate that a pull of from 20 to 45 tons was necessary, in many instances tops of piles pulling off before the piles were moved. This resistance to up-pull is dependent upon the skin friction of the pile or actual mechanical anchorage in the case of types of concrete piles with enlarged bases, which, in turn, is dependent upon the weight submerged in water, of the cones or pyramids of soil engaged by the pile and having their small ends or apex at the bottom or foot of the pile and the enlarged bases at the river or sea bed. Hence the anchorage effect in a group of piles driven closely together will be less than the sum of the anchorage effect of a like number of single piles, because the anchorage is entirely dependent upon the weight of the mass of earth engaged by the group of piling and in no case should be expected to exceed the weight of the affected inverted prism of soil to the bottom of the piles of the group. The effective weight is, when below tide level, of course, the submerged weight of the soil, for which refer to Art. 12.

Relieving Platform Type of Quay Walls are those in which a platform is constructed at about the water level, and takes the weight of the fill and the surcharge as vertical loads carried in turn by timber or reinforced-concrete foundation piles. Structures of this type may be built having a platform of timber or reinforced concrete, a wall above the platform of reinforced concrete or gravity section and with timber, reinforced-concrete or steel, sheet piling in front or in rear of the platform, or with no sheet piling if the wall is carried back far enough.

The two types in general use are shown diagrammatically in Figs. 35*a* and 35*b*. Fig. 35*a* is the type generally employed on the sea coast in waters in

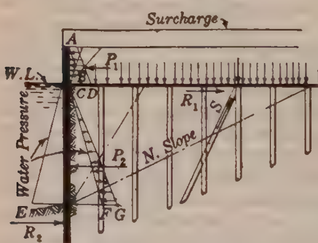


Fig. 35*a*

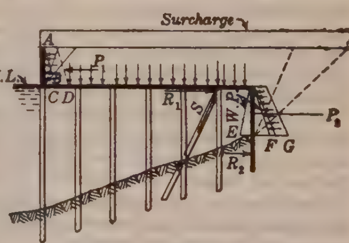


Fig. 35*b*

which the marine wood-borers are prevalent, the earth fill around the foundation piles protecting them. P_1 represents the horizontal thrust of earth fill and the surcharge on top of the platform. P_2 represents the horizontal thrust of the fill below the platform. The earth being submerged loses a portion of its weight due to the buoyancy effect of the water. The water pressure on the outside is counterbalanced by the water pressure on the inside and P_2 is represented by the triangle *BFG*. The two horizontal thrusts, P_1 and P_2, are resisted by R_1, platform tie, and R_2, the passive resistance of the earth at the bottom. R_1 equals the horizontal component of S, the bearing capacity of the inclined brace pile.

In Fig. 35*b*, the sheet piling is at the rear. The outward thrust is represented by P_1 above the platform, and P_2 at the rear, below the platform level. P_1, representing the earth thrust and surcharge, P_2, the earth thrust from the platform level down and the surcharge of the earth, etc., above the platform. These outward thrusts are resisted by R_1 and R_2. R_1 is the platform tie and is the horizontal component of the bearing on the inclined brace pile S, R_2, the passive resistance of the earth. It is evident that in Fig. 35*b*, if the natural slope of the bottom were steeper or the platform carried back further, no sheet piling at the rear would be required.

14. Timber Quays or Bulkheads

Inexpensive and Temporary Character. When work is to be of a more or less temporary character and of inexpensive type, timber is largely employed, often, where necessary, treated by one of the preservative processes. In harbors where marine wood-borers do not exist or are not active, timber work below mean tide level, where always kept wet, will last indefinitely and can be employed in the foundation and other parts of permanent harbor work.

Creosoted Timber. Available information shows that the life of creosoted timber, submerged in water where marine wood-borers are prevalent, varies a great deal and this is due to differences in quantity and quality of the oil used and to the method of treatment. A variation in quantity of 10 to 24 lb. per cu. ft. is frequently employed. Short leaf pine and fir, on account of their structure, readily take more oil per cubic foot than pitch pine, and while not possessing the same strength, are in many instances better materials to use where creosoted lumber is required.

Yellow pine piles treated with 16 lb. of creosote per cubic foot, after being in place in Charleston harbor 12 years, showed no evidences of having been attacked by marine wood-borers. In some of the other South Atlantic and Gulf ports, where the marine wood-borer is very active, there are examples of creosoted timber piles that have not been seriously damaged by the marine wood-borers, and have been in place over 20 years.

Examples of various timber quays are shown in Fig. 36, the numbers referring to the brief descriptions given below.

(1) Garrison Ave., Bronx River, New York City. Front close piling filled with riprap.

(2) Harlem River, New York City. Three-inch sheet-piling backed with riprap Anchor bents 10-ft. centers.

(3) Echo Bay, New Rochelle, New York. Close piling backed with riprap. Bents 6 ft. 6 in. on centers.

(4) Oak Point, New York.

(5) Hudson River, New York. Riprap embankment. Bents of platform 10 ft. on centers.

(6) Astoria, L. I., New York, Astoria Light and Power Co. Crib filled with rubble, pockets 8 ft. square formed by logs.

(7) Hudson River, New York, Interborough Rapid Transit Co. Slope paved with riprap. Bents 5-ft. centers.

(8) Harlem River, 171st Street, New York. Three-inch sheet piling, backfilled with riprap. Guide piles 5-ft. centers, two anchor piles every 10 ft.

(9) Rockaway Inlet, New York, Barren Island. Bents 10-ft. centers, one brace pile to each.

(10) San Pedro, Port of Los Angeles, California. Live load allowed 500 lb. per sq. ft. Piles creosoted, bents 15-ft. centers.

(11) Missouri River, Kansas City, Mo. Timber and piles creosoted. 526 ft. long.
(12) Chicago, Ill. Standard sheet pile bulkhead. Sanitary District.

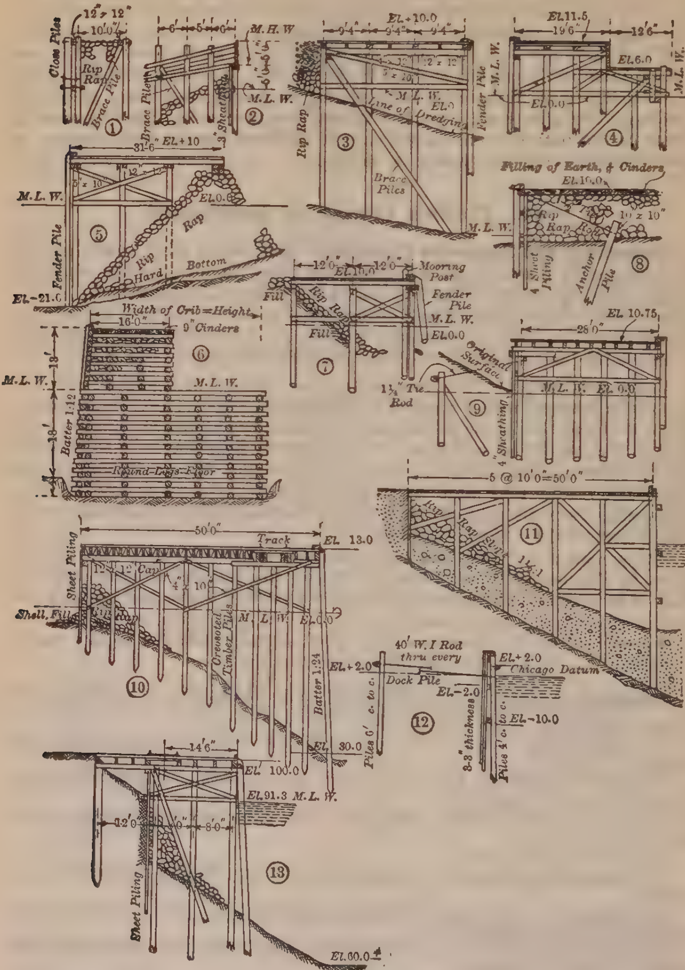


Fig. 36. Timber Quays

(13) Navy Yard, New York, 1912. 200 ft. long. Platform with 6-in. sheet piling. Bents 10-ft. centers.

15. Masonry, Concrete and Timber Quays

Foundations. In the construction of quays or bulkheads of more permanent character where rock of hard unyielding foundation can not be readily secured, timber piling and grillage work are often employed for the foundations or may be carried up to mean tide level, the work being surmounted by a masonry or reinforced concrete structure.

Riprap in locations where it can be secured in quantity and at a low cost is extremely valuable as a quay wall material, for the foundations of the wall, the protection of slopes, filling behind the wall, and for the underwater part of the wall itself. On account of the voids the material in mass submerged does not weigh more than earth fill and stands at a much steeper slope, less than 1 on 1.

Essential Details of various precedents are given in the following statements, the numbers referring to Fig. 37.

(1) Savannah, Ga. Standard Fuel Supply Co. 1912. Twelve-inch reinforced-concrete wall on timber piles, 15 ft. 6 in. on centers, reinforced-concrete deck slab. Live load 600 lb.

(2) Brooklyn, N. Y. Gowanus Canal. Timber platform, with concrete wall on piles and riprap.

(3) Chicago, Ill. Proposed plan for bulkhead wall alongside long pier.

(4) San Diego, California. 1912. 2675 ft. long. Concrete incased piles on 7-ft. centers.

(5) Iloilo, P. I. 1912. Cylinders on 12-ft. centers. Reinforced concrete on timber piles.

(6) Chicago, Ill. Long pier bulkhead.

(7) Charleston, S. C. 1911. 4000 ft. long. Untreated timber piles, sheet piling and concrete wall. Reinforced-concrete sheet piling, 3 ft. wide, to protect timber work from marine borers.

(9) New York (South Brooklyn), N. Y. One of several types of wall used by Department of Docks and Ferries.

(10) Boston, Mass. Northern Avenue. 1911. At Commonwealth Pier. Length 645 ft.

(11) Schenectady, N. Y. N. Y. State Barge Canal. (12) Amsterdam, N. Y. N. Y. State Barge Canal. (13) Utica, N. Y. N. Y. State Barge Canal.

(14) Providence, R. I. Fields Point. Bents 4 ft. centers, two brace piles to each bent. Sheet piling 6 in. and 8 in.

(15) New York, N. Y. Central R. R. of New Jersey, Bronx Terminal. Bents 8-ft. centers, two brace piles to a bent, sheeting 6-in.

(16) Boston, Mass. U. S. Navy Yard. (17) Norfolk, Va. U. S. Navy Yard. 1910. Timber platform, surmounted by concrete wall on reinforced-concrete sheet piling, 55 ft. long. Similar wall faced with granite ashlar, built at New York Navy Yard, 1912. Bents, 5-ft. centers.

(18) Berlin, Germany. Spree Canal. (19) Neufahrwasser, Commercial Railroad, Germany.

16. Masonry and Reinforced-concrete Quays

Foundations. Where the foundation is of rock or unyielding material, quay walls built up of large stone blocks, concrete blocks, or concrete monoliths are permanent and effective. In localities where granite is readily procurable at a reasonable price, this type of wall can not be improved upon both as to permanency and cost. With less satisfactory foundation conditions, especially where marine wood-borers are prevalent, reinforced-concrete piles or columns are effectively used. In masonry walls the underwater work, if of blocks, is usually laid by divers to elevation of low water. Where monolithic concrete

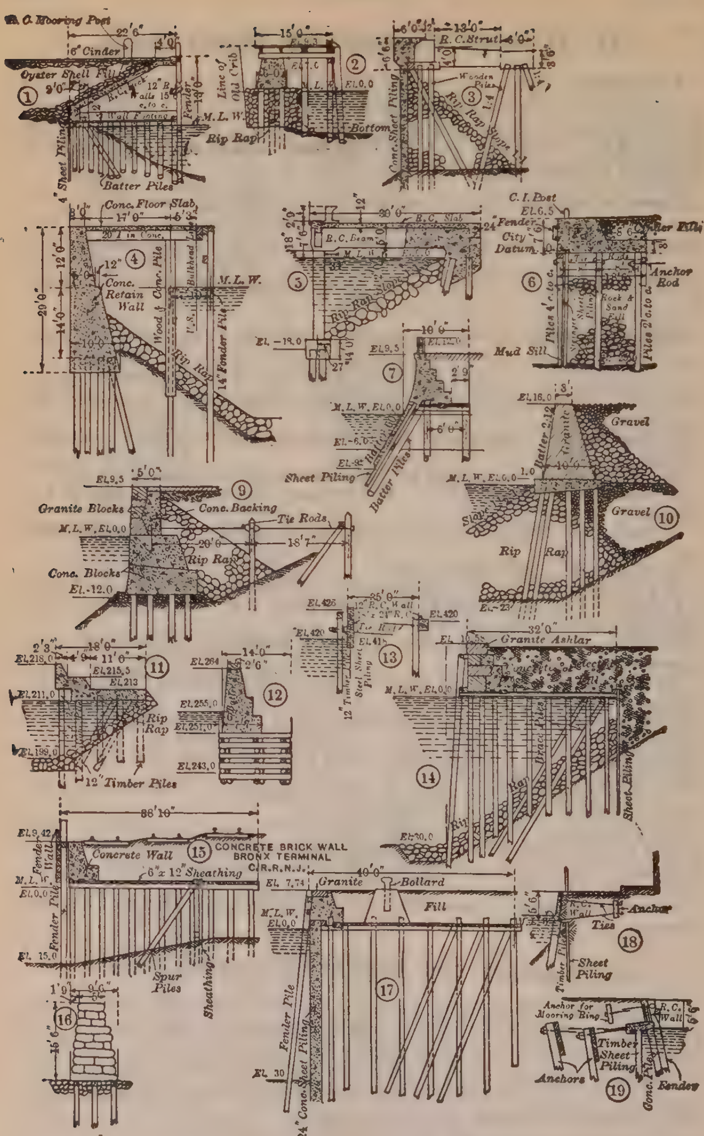


Fig. 37. Masonry, Concrete, and Timber Quays

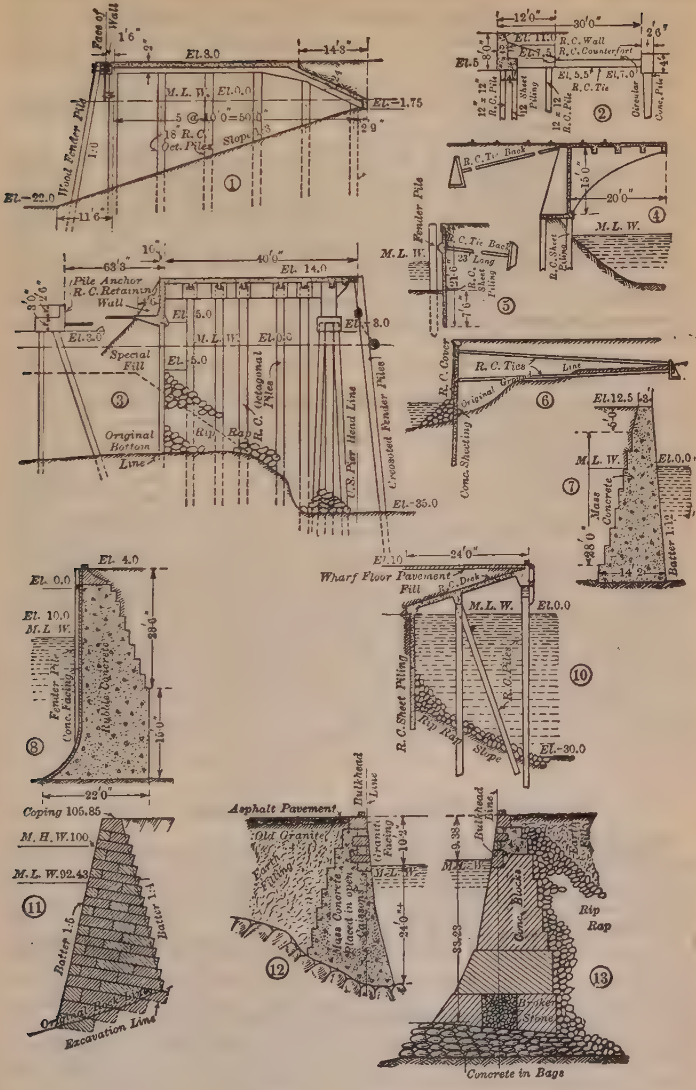


Fig. 38. Masonry and Reinforced-concrete Quays

blocks are used these may be premolded and lowered to place by heavy derricks, the foundation being prepared by the use of riprap and broken stone or concrete bag work. Concrete or stone masonry walls are also built in the dry in cofferdams, or use is made of reinforced-concrete or timber floating caissons or boxes, built on shore, launched, and sunk in place by filling.

Essential Details of various precedents are given in the following statement, the numbers referring to Fig. 38.

(1) Key West, Fla. U. S. Navy Yard. Reinforced-concrete piles and deck. Bents 10-ft. centers. (2) Albany, N. Y. State Barge Canal. (3) Los Angeles, California, 1914. Reinforced-concrete piles and deck. Bents 20-ft. centers; anchors, 4 vertical, 4 brace timber piles. (4) Nantes, on Loire, France. Reinforced concrete. (5) Spandau, Germany. Municipal Quay. (6) Nantes, France. (7) New York, N. Y. Port Morris, Bronx, N. Y. C. & H. R. R. (8) Oakland, California. 2005 ft. long. (9) Boston, Mass. Fish Pier. Wall around Pier 6. (10) Baltimore, Md. Proposed design. (11) Portsmouth, N. H. Navy Yard. Loose coursed granite laid by diver. A type of wall used along New England Coast. 1907–1912. 320 ft. (12) New York, N. Y. 116th Street, Department of Docks and Ferries. (13) New York, N. Y. Cedar Street. Department of Docks and Ferries.

The Department of Docks and Ferries of New York City has for many years employed large premolded concrete blocks for quay wall construction. With rock foundation within 40 ft. of the surface, if material over rock is soft, it is dredged off and rock cleaned, and the wall based on bag concrete brought up to level by divers. Upon this successive tiers of concrete blocks are laid up to low tide level; above this, granite ashlar, backed with mass concrete, is used. The blocks weigh from 25 to 95 tons. Where the foundation is such that piles can be driven to clay, sand or other firm bottom, soft material is excavated to 30-ft. depth, piles driven and cut off 15 ft. below low water and given lateral resistance by an embankment of riprap and gravel brought up to tops of piles. Upon these piles a single tier of concrete blocks is laid, each weighing 80 tons. A granite ashlar wall facing with concrete backing surmounts the blocks. Where the bottom is soft, the wall is provided with a relieving platform. In some walls in soft bottom, settlements as great as 4 ft. have occurred, which were remedied by adding courses of granite. The blocks are handled by scows and set in place by a 100-ton or a 40-ton crane. Cost has varied greatly from \$100 to \$750 per lin. ft.

Quay walls, in which monoliths are employed consisting of floating caissons, in a similar manner to their use in breakwaters (see Art. 3), have been constructed at Zeebrugge, Belgium; Rotterdam, Holland; Norresundby, Denmark; Talcahuano, Chili; and hollow reinforced-concrete boxes at Passau on the Danube. At Antwerp, Belgium, a quay wall was constructed using caissons launched and floated to place and then sunk by fixing over their upper part a pneumatic caisson or diving-bell handled by two floats, the section being then filled with concrete.

17. Failure of Quay Walls

Movement and Settlement of quay walls and their eventual failure in whole or part are not uncommon, even in well-designed walls or bulkheads where ample provision has, apparently, been made for the outward thrust; it is not unusual to find after work has been started evidences of settlement at the toe and irregular outward movement at the top. This may, in some cases, be accounted for by the slight settlement of the foundation under the front of the wall on account of the concentration of load and the outward thrust at that place and also in walls where timber platforms or anchors, employing brace piles, are used, to the slight movement allowed in the bringing up of bearings and connections. In most instances, this will not continue to any great extent, and may be corrected in so far as appearance goes, by resetting the coping. On walls or bulkheads having ornamental copings or balustrades, it is often advisable to postpone the placing of these until a short time after the wall or bulkhead has been built and has had an opportunity to take its set.

In gravity masonry walls, a not infrequent form of failure, or partial failure, is the moving or sliding out, in mass, of the wall on its base. It is customary, where possible, to lay the foundation and upper courses of masonry at an inclination to overcome this.

Examples of Failures and the corrective measures successfully applied in some instances:

(a) An outward movement was observed during the construction of the Fish Pier wall in Boston Harbor, the wall resting on an inclined riprap bed, laid on clay overlaying hardpan and rock. The movement was arrested by driving piles directly in front of and at the toe of the wall and constructing in the rear of the wall a relieving platform. 400 ft. of wall were involved.

(b) A section about 300 ft. long of a masonry and timber platform wall built at the New York Navy Yard failed by outward movement and overturning, the cause being, undoubtedly, improper brace pile connections, and as referred to in Art. 12.

(c) A somewhat similar wall in Washington, D. C., built on a very narrow platform without brace pile ties, moved outward 10 ft. when the backfilling had been carried only to mean tide elevation. When repaired, by constructing a timber relieving platform with adequate brace piles, backfilling was carried to grade and no further movement of importance found.

(d) A wall in Charleston, S. C., harbor, 4000 ft. long, was located on a salt marsh; the river silt of increasing compactness and density extended to a depth in excess of 90 ft. below tide. The depth of water at face of wall varied from 0 to approximately 8 ft. The support for the wall consisted of a timber platform and brace piles with sheet piling driven at an inclination. Filling of river silt was placed behind the wall by hydraulic dredging. In two locations the filling material blew out underneath the sheet piling, pushing out the bottom of the sheet piling. This was corrected by loading the toe of the wall with riprap and, in some instances, by driving heavy vertical sheet piling inside and back of the wall platform.

(e) A platform bulkhead built in New York Harbor, consisting of a timber platform with sheet piling on the inside with brace piles at 5-ft. intervals, with 15 ft. depth of water at the sheet piling, and 30 ft. depth along the face, gave evidences of outward movement when the depth of water was increased by dredging to 34 ft. This was corrected by placing 2-in. round steel anchors every 20 ft. and carrying these back 40 ft. inshore of the sheet piling to anchors of timbers, dead-men and piles, and placing riprap along the slope, decreasing the depth of water at sheet piling to 12 ft.

(f) A wall built at the Navy Yard in New York, about 1885, of gravity type, consisted of heavy granite blocks, with massive counterforts, laid by divers on a timber pile foundation, and of granite ashlar work above tide level. It was designed at a time when the draft of the vessels did not exceed 20 ft. With the increase in draft of vessel, the basin directly in front of this wall was dredged out, and, finally, to a depth slightly in excess of 30 ft. A somewhat similar wall in another locality failed by outward movement and overturning. The wall in question moved outward and gave every indication of early failure. It was repaired by the method indicated in Fig. 39, the ashlar work being stripped off to low water, sheet piling driven directly in front of the wall, and this tied back in turn by timber ties to a platform in back of the wall provided with vertical and brace piles, the space between the face of the wall and the sheet piling filled with riprap, and a new wall built directly on top of the sheet piling and the work backfilled.

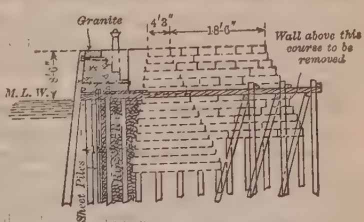


Fig. 39. Quay Wall Repair, New York

(g) A crib in D., L. & W. R. R. yard, Hoboken, N. J., the top of which had given way and needed renewal, was repaired as shown in Fig. 40.

(h) A seawall built on a timber platform, the protective face of which under water consisted of 3-lap, 2-in. yellow pine treated with creosote, began to fail on account of the destruction of the wood by the teredo. Eight-inch grooved and tongued reinforced-concrete sheet piles were driven and a reinforced-concrete face put on top of this and the structure tied back by tie rods.

Fig. 40. Quay Crib Repair, Hoboken, N. J.

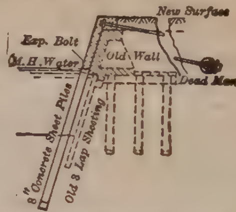


Fig. 40a. Wall Repair, Hampton Roads, Va.

There are numerous other examples, especially of walls or bulkheads constructed to retain filling that have failed by the outward movement of sheet piling at the toe or bottom, or by outward movement at the top, and overturning on account of insecure and insufficient anchorage.

The increase in the depth of harbors, due to the increased draft of vessels, has in some instances rendered insecure wall structures designed with sufficient strength and stability for the conditions existing when they were built.

LANDING PIERS

18. General Information

Harbor Lines. In the United States the Secretary of War is authorized to fix harbor lines for the protection of the harbor and its navigation, beyond which lines no wharves, quays or piers are allowed to extend. See laws for the protection and preservation of the Navigable Waters of the United States. In the interior of harbors and on navigable rivers, two lines are usually established by the government, or, outside of this jurisdiction, by local port authorities. The bulkhead line is the line to which solid or solid filled structures may be built. The pier-head line is the line fixing the boundaries of the fairway to which wharf or pier structures may be built, but when so built, they must be of open construction, permitting the flow through them of the full sweep of the current or tide, such open contruction being in the nature of pile or column bents. It is usually permissible, if desired, to make the structure from mean-low-tide up continuous or solid.

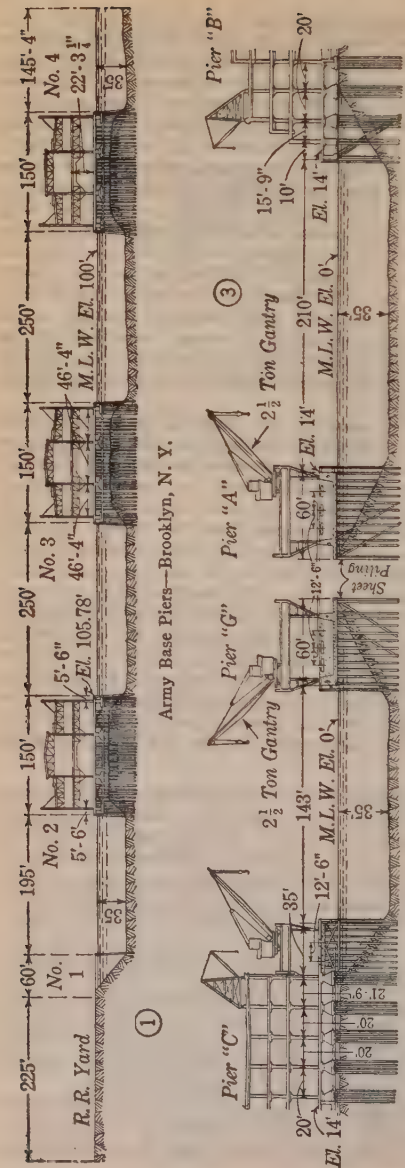
Length and Width of Piers. Piers should be of at least sufficient length to berth the longest ship likely to use them, and hence should project out into the stream a short distance beyond the bow or stern of the ship to protect the hull of the vessel from collision by passing navigation or floating ice. Piers are often built long enough for berthing two or more vessels on either side. Allowance should be made in determining the length of piers for the growth in dimension of shipping. The width of piers is largely dependent upon the use to which they are to be put, although a safe minimum width, in order to secure lateral stability by bracing, would be fixed by the depth of water and the length of the pier and the character and size of the vessels to which it will be required to afford wharfage. Piers to be used for the berthing of vessels, but not to any extent for the discharge, handling, or storing of cargoes, should be not less than 40 to 50 ft. wide. For piers in shipbuilding and repair yards, carrying standard-gage railroad track, and for heavy crane track service, requiring the handling of bulky material, the width is a local problem. For commercial pier work and in an important port where the largest ships are likely to be berthed, and cargo handled and rehandled, the width should, where possible, bear a relation to the volume of cargo to be handled and the time allowed for this. For a freighter pier or quay handling 10 000-ton ships, about 150-ft. width should be allowed, or for two ships one on each side, twice this, or 300 ft. Where four ships are to be berthed two on each side, an additional width of 50 ft. should be allowed for entrance and egress of cargo to the outer ships. No fixed rule can be given for the length of piers. Local conditions and the value of property must control. If piers are designed long enough for two or more vessels to lie alongside, the traffic congestion incident to handling and rehandling cargo at the entrance to the pier must be considered. As the width of piers, or transit shed erected on the pier, should afford storage space for at least one cargo outgoing or incoming, that is, considering whether a one- or two-story shed is used, the cargo capacity of the vessel at the pier and the necessity for aisles or passage ways through the storage space, and for roadways and railroad tracks to take in and bring out cargo, must be considered. There has been a tendency to increase the width of piers at cargo handling terminals.

The width of piers for handling incoming and outgoing cargo is directly related to the quantity of cargo to be accumulated on the pier for loading or the quantity to be landed in unloading while awaiting transfer and distribution. At a properly laid-out terminal vessels should be promptly loaded or unloaded. In determining the proper width the cargo carrying capacity and length of the largest vessel likely to use the pier should be taken. For man handling 5 ft. high would appear to be the highest economical height of stowage; while 60 cu. ft. can be allowed for a cargo ton, or with passageway, etc., allow 15 sq. ft. of pier or wharf per cargo ton. With mechanical stacking and handling this could be decreased or the width of piers can be decreased by using two-decked transit sheds, although in that case one deck is generally employed for incoming and the other for accumulating outgoing. On account of the increase in sizes of vessels and the importance of space for accumulating cargo in advance and discharging without delaying vessels, the tendency is to construct wider piers. The New York City pier and waterfront congestion is undoubtedly due largely to too narrow piers for the size of vessels using them.

Width of Slips. The berthing space or slip between two piers should be wide enough to accommodate two vessels readily, one lying at each pier, and also give room for the entry or departure of either. Sufficient space should also be provided for loading and unloading by barges or lighters on the off pier side of the vessel. The greatest beam of the largest ships is now about 100 ft., so that in slips where the largest vessels are to be accommodated the width should not be less than 300 ft., and it should preferably be more than this.

Depth of Water at Wharves. In former years the amidship, hull section of wooden ships was such that the maximum draft required was not necessary directly in front of quays or piers, but with the advent of steel hulls of practically rectangular amidship section, with bilge keels, the maximum draft is required directly in front of the wharf to be occupied, unless the vessel is kept off by spur shores, or floats, in which case there is a loss of space in the slip and inconvenience in the placing of gangways and the handling of cargo.

At the first-class piers on the Atlantic Coast 40 ft. appears to have been recently fixed as the desired depth for port approach channels. Several notable examples exist of transatlantic liners which have a depth of approximately



Army Base Piers—Brooklyn, N. Y.

Army Base Piers—Philadelphia, Pa.

Fig. 40b. Width of Slips

40 ft. at full load. It is doubtful if the tendency to increase full-load draft design of ocean-going vessels will continue beyond this 40-ft. limitation.

On the Atlantic Coast, in the important harbors, where possible, a usable depth of not less than 35 ft. should be provided, as some of the larger vessels in commercial use, when fully loaded, have a draft in excess of this. Provision should be made for this usable depth at extreme low water by allowing additional depth for silting up, and to provide for movement on account of wave action.

Naval vessels seldom have a maximum displacement draft in excess of 33 ft. An allowance, however, must be made for over-draft, especially in case of injury, such as the flooding of compartments.

Direction of Piers. In the prevailing American practice, piers are built out perpendicular to the shore line and to the direction of the channel. Some advocate placing piers obliquely, urging that the task of bringing vessels into and out of slips is thereby rendered easier. This is, of course, dependent upon tidal and current conditions. In a tidal harbor with the ebb and flood of the tides, this would be so under one of the two conditions of the tide, but it would be more difficult, or impossible, to bring vessels in or out with the reverse condition of current.

Laying Out of Wharf Work. The location of piles or columns in bents is usually fixed by batten ranges placed on shore, the spacing being laid off on two bases, a short distance apart, parallel with the bents, and the distance out to bents determined by similar ranges on adjoining structures, or by steel or cloth tape lines measuring out from a fixed base, the piles being held in place by stay-lathing or temporary braces below the cutoff or finished grade. The elevation of cutoff is given by level and carried from the given point by straight-edge and checked by instrument. Piles are cut off below water, by divers using a hand or power saw, or by using a horizontal circular saw operated from the surface.

Piers in Deep Water. When piers are laid out so that for the entire length or at the outer extremity they are in water of considerable depth, resort is sometimes had to raising the bottom, into which the piles are to be driven by making embankments of riprap. In pile piers, if placed after the piles are driven, there is likely to be a settlement in the structure due to the weight of the riprap or cobble. This method of construction has been employed along the New York City waterfront, and in the case of the New York Central pier, at Watt Street, resulted in a settlement of nearly 2 ft. Riprap embankments, under water, are also used to give the pier lateral strength, where the piles can not be driven deep enough into the bottom to insure good holding. The raising of the bottom by embankments of riprap or cobble is also resorted to to strengthen the pier laterally and especially to support the piling, which would otherwise involve consideration of the carrying capacity of such piles as an extremely long unsupported column.

19. Design

The Live Load carrying capacity of piers and quays depends largely on the use to which they are to be put, and ranges from 250 to 1200 lb. per sq. ft. The minimum should be used for piers intended solely for berthing light traffic and the support of oil and water mains, and the maximum for especially heavy work, such as the handling and storage of ordnance, armor plate, and heavy machinery. In commercial wharves, 500 lb. per sq. ft. should be used for piers, and 750 lb. for quays. The foundations are to provide for this live load and dead load of the structure itself. The foundation for standard-gage railroad track, or heavy crane tracks, shed or building foundations, require special treatment as in other engineering structures.

Prevailing Practice. In pier design it will be found that details frequently have to conform with established rules fixed by local authorities. Where this is not the case, the deck or slab should be designed to carry full live load and dead load, the stringers, rangers or beams to carry full live load and dead load; caps or girders, 80% of live load, and full dead load. Piles or columns 75% of live load and full dead load. For other structures such as sheds, railroad tracks, and special work, design in conformity with general building practice.

Timber Pier Framing is largely done by the methods used in ship framing. Pile caps, if not in one piece, are usually scarfed at such locations as to break joints in adjoining bents. In substantial work piles are usually tenoned and caps mortised. Frequently clamps are substituted for caps, these consisting of two timbers, the pile being shouldered out on each side to receive them. String pieces or rangers are either butted over caps and drifted, or spiked, to the cap, or if the joint occurs in the middle of the bay, the timbers being fish-plated. In heavy and important timber piers, the deck system usually consists of heavy planking with somewhat lighter wearing sheathing laid diagonally to the direction of traffic over it. Outshore corners on piers are usually rounded and heavily reinforced. In harbors where floating ice is prevalent, especially in tidal rivers, the bays at the outer end of the pier are made of double width, and the pile or column bents at either side protected between tides with steel or iron plates. The fastenings used are, in general, similar to those employed in timber, ship, and bridge work, and are frequently galvanized. Piers are often built with a slight crown at the center to facilitate drainage.

Fender System. Wharves, when used for berthing, are protected by fenders fitted so as to be readily removed for repair and replacement. The fender system is made up of vertical fenders or fender spring piles and horizontal wearing ribbons. The outer ends or corners of piers are usually protected by heavy clusters of fender piles, or by separate pile clusters or dolphins. In reinforced-concrete and masonry piers the fender system is often arranged on an elastic system with sufficient spring to save the main structure from the shock of vessels coming alongside. For this purpose the spring of the vertical and horizontal timbers is employed, or groups of heavy car springs are used. Frequently such fender systems are assisted or replaced by floating fenders, rolling logs or spars of single or made-up timbers sometimes served with old hemp line and fastened at the ends with lines or chains fixed to the side of the piers or quays.

In addition to the fenders employed it is sometimes customary to construct dolphins or clusters of piles along the side of a pier to breast off vessels with. Such dolphins are also used without the pier for mooring purposes while vessels are awaiting for pier or berthing space or being loaded or unloaded by lighters.

Moorings. Piers are provided with mooring posts or bollards, bitts, and cleats, one for each 50 ft. length of berth, the inshore and outshore moorings for bow and stern lines to be heavy bollards or posts fitted with horns or pins. Typical details of mooring fittings are shown in Fig. 41 When vessels are berthed alongside of quays, it often becomes necessary to spur them off so that at low tide the bilge keel will not rest on bottom slopes. Spur shores of heavy spars are used for this purpose, one end resting against the vessel's side, and being supported from it by lines or chains, the other end resting on wheels or rollers on the wharf platform, and made fast by lines or chains to bitts or mooring posts. The same object is accomplished by placing floats in between the vessel and the wharf.

Accessibility of Wharf Platforms. In harbors where the range of tide is only moderate in extent, the platform elevation of main piers and of second floors of pier sheds is usually so fixed that access may be had to cargo openings in sides of ships, for placing and transfer of cargo, by portable gangways. Where there is considerable range in tide, piers are sometimes built on two levels, the low one used at low tide, and the upper one at high tide, or inclines or ramps are built into the sides of the wharf, and sometimes these are provided with drop or adjustable gangways. A detail of such a gangway, in use at Philadelphia, Pa., is shown in Fig. 42.

Ocean Piers are generally built for passenger service, landings or recreation purposes. Ocean piers have been constructed for commercial purposes in locations where natural harbors were not available or convenient. Piers of the recreation type are found at resorts near populous cities, and are very often constructed of steel or iron piles or columns with steel and timber superstructure. The piles or columns are made up of pipe, or structural shapes, iron or steel disks or sections of screws being fixed to the lower end. The piles are put into position with the aid of water jets, an attempt being made to place them sufficiently deep so that the sand surrounding them will not be disturbed and washed away by wave action. These iron or steel piles or columns, when

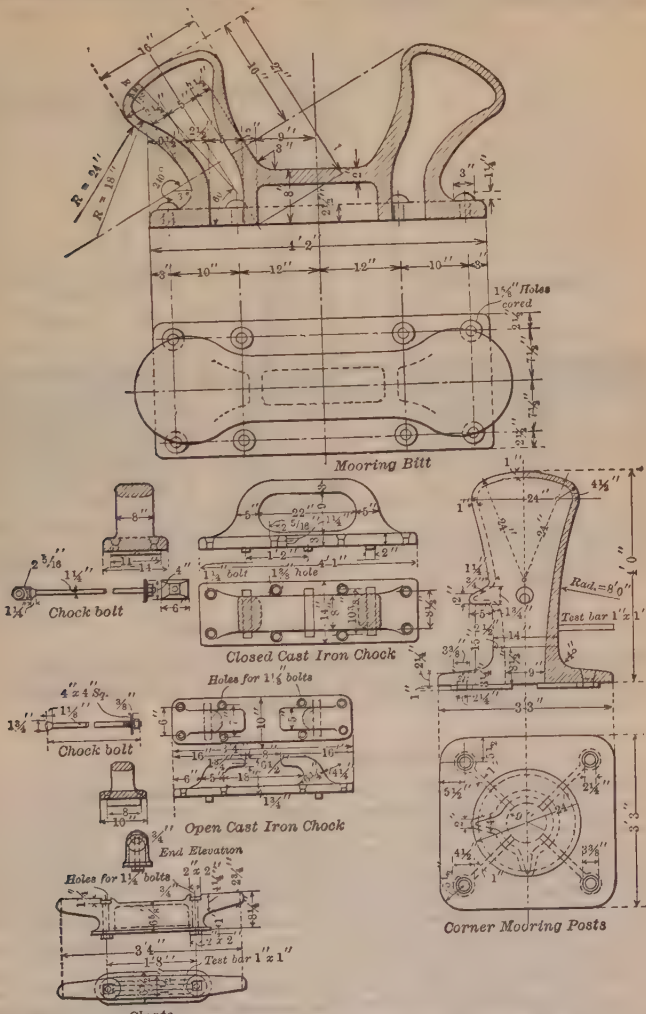


Fig. 41. Mooring Fittings

made of pipe, offer a minimum resistance to wave action; the requisite lateral stiffness is usually secured by cross bracing of steel rods. Piers of this character have also been built employing reinforced-concrete piles surmounted by a reinforced-concrete deck system. Ocean piers of the commercial type have been constructed on the southern coast of Cuba and in South America; in

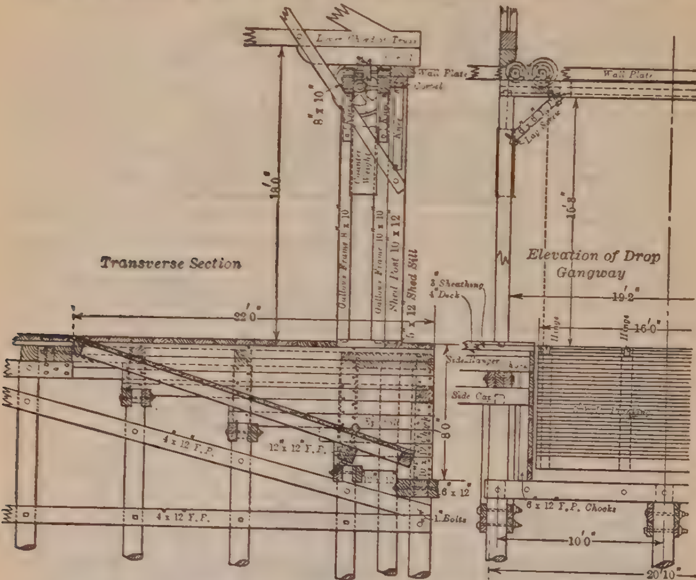


Fig. 42. Drop Gangway, Philadelphia

some cases, instead of being continuous piers built out into the sea, they consist of a detached platform or pier in deep water, the cargo being handled to and from the shore by means of overhead cableways.

20. Life and Cost of Piers

The Life of Piers is entirely dependent upon the location of the work and character of construction employed. Timber piers built in harbors where marine wood-borers are not active will last indefinitely below mean tide, but above this are subject to rapid deterioration and decay. Under such circumstances the life of a timber pier, in its entirety, could be probably fixed at 20 years. There are instances of work of this character requiring extensive repair, amounting to replacement, in 15 years, while in other cases piers after a life of 25 to 30 years are still in use. C. W. Staniford gives the following facts in Trans. Am. Soc. C. E., Vol. 77, 1914:

From the above it is derived that the average life of the entire pier is 21.6 years. The timber piles, however, below mean tide elevation, which are still in good condition, can be made use of by building a new superstructure above and resting on them. Climatic conditions, the care with which the construction work is performed, and the character of timber employed will materially modify length of life. Timber piers, with creosoted piles, in ports where

Description	Percentage of total original cost	Renewal required
Sheathing	12	Every 6 years
Backing log	1.8	Every 8 years
Fender chocks, including vertical sheathing	4	Every 10 years
Fender piles	4.7	Every 12 years
Decking	11.3	Every 15 years
Bracing	7.1	50% in every 20 years
Rangers and caps	24.4	50% in every 20 years
Piles, above mean low water	34.7	33-1/3% in every 20 years

marine wood-borers are active, may have a length of life between 15 and 25 years. In timber piers with reinforced-concrete decks, the length of life is dependent on the length of life of rangers and caps, and, when of substantial construction in a locality such as New York Harbor, can be taken as 30 years. Masonry, reinforced-concrete, masonry and reinforced-concrete on timber foundations or platforms where marine wood-borers are not prevalent or active, have practically an indefinite length of life, although under some conditions there is a possibility of deterioration of concrete work immersed in sea water requiring extensive repair and renewal. It seems safe, however, to fix the length of life of this character of structures at 60 years. Untreated, unprotected timber in harbors where marine wood-borers are prevalent and active, is destroyed rapidly, in some instances the piles being eaten away and destroyed in two or three years.

Costs of Piers. The data available as to the cost of piers are likely to prove extremely misleading. These costs are, of course, dependent upon details of construction, character of foundations, live load carrying capacity for which designed, the character and cost in the particular location of the material employed, availability and cost of labor and appliances. For general estimating purposes, for the cost per superficial foot of different types of pier work, in American ports, the following may be taken, the range between the maximum and minimum being dependent on location, foundations, and detail of design. These costs do not include auxiliary structures, shed foundations, railroad track foundations, subways, galleries, or other structures:

	Cost per sq. ft.
Untreated pine timber piers	$1.25 to $2.00
Creosoted pile untreated timber superstructure piers	$1.50 to $2.50
Untreated timber piers with reinforced-concrete deck	$1.30 to $2.25
Creosoted timber with reinforced-concrete deck	$1.65 to $2.75
Untreated timber platform with concrete cross-walls or reinforced-concrete columns and deck	$1.75 to $3.00
Creosoted timber piles with reinforced-concrete columns and deck	$2.00 to $3.25
Reinforced-concrete piles and timber system and deck	$2.00 to $3.50
Reinforced-concrete piles and deck and deck system	$2.40 to $3.75
Timber platform to low water and solid fill above	$2.00 to $4.00

On reinforced-concrete deck piers there should be added to the above figures, if asphalt wearing surface is to be placed, from 10 to 16 cents per sq. ft.; and if creosoted wood block wearing surface is to be placed, 20 to 28 cents per sq. ft. On filled piers, in addition to this, add 8 to 12 cents per sq. ft. for concrete base. It is to be noted that on the last type of filled platform pier, the enclosing wall structure representing the principal cost of the structure, the width of the pier would be a controlling feature in the cost. No cost can be given for solid filled piers, that is, piers enclosed by one of the various types of quay walls, as this cost will depend entirely on the cost of the wall and the width of the pier.

21. Timber and Timber Composite Piers

The Temporary Character of timber piers which, apparently, serves as a reason against their extensive use, in many instances loses much of its force in view of their low first cost, simplicity in design, and short length of time in which they can be built. Furthermore, in localities or harbors where a comprehensive and well-planned port improvement project has not been developed, the wharf work is often laid out and constructed to serve the immediate purpose, to be later torn down and rebuilt on new lines. The growth in importance of many American ports has been so rapid, and unforeseen, that no attempt at port planning in the past could have adequately provided for the future. The unprecedented and unforeseen growth in the size of vessels has also made necessary far-reaching and sweeping changes in port facilities. For these reasons timber piers of temporary character were as serviceable as more permanent piers, as it was easier and less expensive to remove them to make way for other more extensive work. The timber pier has played a not unimportant part in the rapid growth and expansion of American harbors. The engineer in planning pier work, when considering cost, is hardly justified in employing a permanent structure of a probable life of 50 or 60 years, when it is apparent that the wharf work will long before that time require extensions and improvements amounting to rebuilding.

Although timber wharf construction in American ports has played the part indicated in the development of such ports, and has made it possible to rearrange and rebuild without excessive cost, this very condition has brought about a more haphazard construction with very little serious and comprehensive planning for port problems and construction of port facilities. It is to be regretted that pier or transit sheds are frequently used in American practice for long storage periods and cargo is not expeditiously brought to and taken away from vessels. Vessels frequently lie at and monopolize wharfage space when they might as well be at anchor in the stream or be laid up at dolphins and thus make the wharves available for other vessels awaiting for loading or unloading cargo. As an illustration the area devoted to wharves at the port of New York is several times that devoted to similar purposes at ports such as Hamburg, Liverpool and London, all of which ports have in recent years handled about the same quantity and value of incoming and outgoing cargo.

Composite Piers, in localities where the under-water timber work will not be attacked by marine wood-borers, are practically permanent structures and, as a rule, are economical in first cost.

Crib Work may be employed for pier construction on a yielding or rock foundation. The crib, consisting of round or square timbers, is built in courses with pockets filled with riprap or rubble, in soft ground the crib being sunk to place on a prepared foundation of piles cut off to grade, on a level bed of broken stone or gravel, or directly on the bottom. On rock the crib is built up to rest and fit on the ledge, the contours of which have been previously determined by soundigs.

Essential Details of various precedents are given in the following statements, where the numbers refer to Fig. 43.

(1) Philadelphia, Pa. 1912. Pier 5. 657 ft. long; bents 10-ft. centers. Live load 500 lb. inner 400 ft., 300 lb. outer. Supports 40-ton crane track and standard-gage track. Reinforced-concrete top on timber piles.

(2) New York, N. Y. Navy Yard, Pier 3. 1905. 520 ft. long. Timber piles below water; 35 ft. depth of water.

(3) New York, N. Y. Department of Docks and Ferries, West 77th St. Timber piles and framing to mean tide elevation. Reinforced-concrete walls, 12 in. thick; 10-in. reinforced-concrete deck.

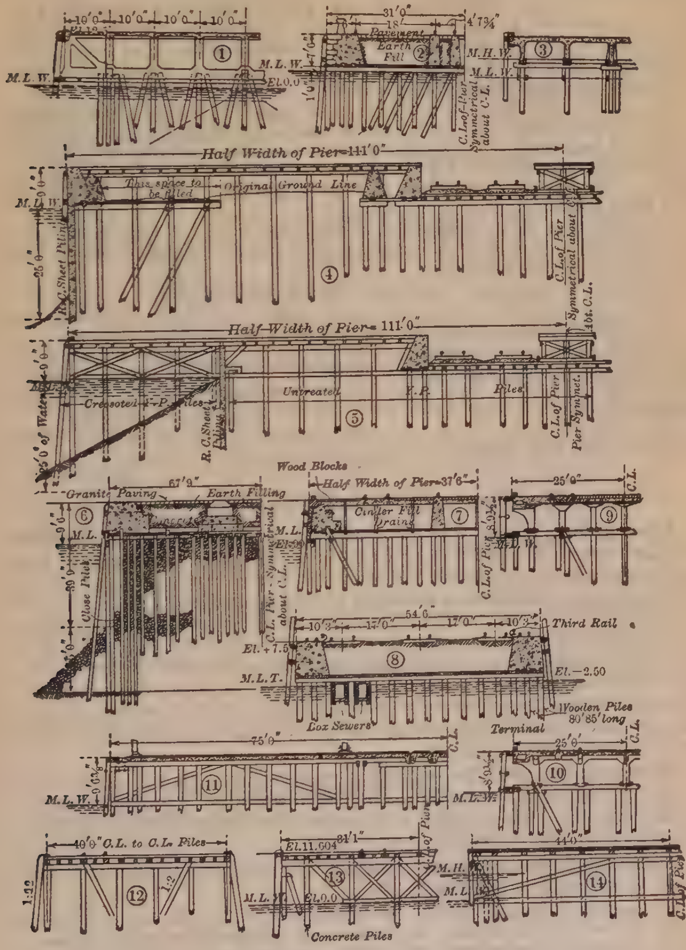


Fig. 43. Timber and Timber Composite Piers

(4) Norfolk, Va. Proposed Old Dominion Terminal. Untreated timber piles. Surrounding wall with reinforced-concrete sheet piling 16-in. 45 ft. long. Wall bents, 4-ft. centers; pier bents, 8-ft. centers. This type of construction to bulkhead line; open creosoted timber pile pier to pier-head line.

(5) Norfolk, Va. Proposed alternate plan, Old Dominion Terminal.

(6) New York, N. Y. Navy Yard, Pier 2. 1901. Timber low water platform, filled and banked with riprap, supports coaling plant. Length of pier 500 ft.

(7) New York, N. Y. Navy Yard, Pier D. 1912. Timber pile platform to low water; 34 ft. of water. Piles 55 to 60 ft. long. Granite ashlar faced surrounding wall. Paved creosoted W. B. pavement. Two standard-gage and one 50-ton crane track entire length of pier 440 ft. long. Live load, 600 lb. per sq. ft.

(8) Hoboken, N. J. D., L. & W. R. R. Terminal, Pier 1. 1909. 958 ft. long. Bents 10-ft. centers; 12-in. caps; 6-in. deck.

(9) and (10) New York, N. Y. Navy Yard. Alternate designs. (10) adopted. Two piers, each 500 ft. long. 1913. Timber piles and grillage below mean tide level. Piles 85 ft. long, depth of water 35 ft. Reinforced-concrete columns and deck system, columns premolded and set. Piers carry 2 lines of standard-gage track, subways, and piping. Paved in center, creosoted W. B. pavement. (9) Mushroom system, and (10) Beam system.

(11) New York, N. Y. Department of Docks and Ferries. Typical construction at present in use. Untreated timber structure with 10-1/2-in. reinforced-concrete deck with 2-1/4-in. asphalt pavement.

(12) Puget Sound, Wash. Navy Yard. Timber approach wharf.

(13) Charleston, S. C. Navy Yard, Pier 4. 1905. 650 ft. long. Reinforced-concrete piles, 16 in. square. Timber braces, clamps and deck system, 8 × 16 top clamp; 6 × 10 low water clamps; 3-in. deck; bents 10-ft. centers. Live load 500 lb. per sq. ft. Creosoted timber pile fenders. Similar approach wharf 45 × 920. 18-in. reinforced-concrete piles 40 to 50 ft. long; 10-in. reinforced-concrete deck. Similar construction employed in Railroad terminal pier, Brunswick, Ga.

(14) Philadelphia, Pa. Clyde Line. 1900. Pier 556 ft. long; bents 10 ft. Timber piling, bracing, deck system.

22. Masonry and Reinforced-concrete Piers

Filled-in Piers consist of walls similar to quay walls of masonry, or of the platform type, the interior being filled with earth; when properly built they are permanent and unyielding and require little or no repair work, other than that due to the settlement of the filling usually occurring only during the first few years of use. Such settlement prevents the pier from being paved or kept level during the period of settlement. They can only be employed inshore of the bulkhead line. On account of the cost of the surrounding wall work, narrow filled piers are not economical, except possibly at times when pier work is located on flats, in shallow water, or on high land, and the approach channels, slips or berths alongside require excavation or dredging, so that this dredged material can be used for filling.

Reinforced-concrete Piles and Cylinders are employed in pier construction where marine wood-borers are active, for economy in locations where the cost of timber piles is too high, or where the character of the pier foundation is such as to require the use of cylinders.

Essential Details of various precedents are given in the following statement. The numbers refer to Fig. 44.

(1) Balboa, Canal Zone. 1911. Lumber pier; live load, 400 lb. per sq. ft.; length 706.6 ft.; bents 30-ft. centers. Hollow concrete cylinders built in 6-ft. sections of 1 : 2 : 4 concrete. provided with steel cutting edge shoe; sunk to rock by jetting and interior excavation, filled with 1 : 3 : 5 concrete.

(2) Olongapo, P. I. U. S. Navy Yard, Pier B, 332 ft. long; bents 12-ft. centers; reinforced-concrete cylinders, 6 ft.; reinforced-concrete deck.

(3) Boston, Mass. Pier 6. 1912. 1200 ft. long; 300 ft. wide. Surrounding wall of courses of split granite 2 to 3 ft. high, laid by diver.

(4) Puget Sound, Wash. U. S. Navy Yard, Pier 8. 397 ft. long; bents 16-ft.

centers. Live load 400 lb. per sq. ft. Cylinders 3 ft. in diameter at top, 6 ft. at bottom, shell 4 in. thick, 23 to 52 ft. long; sunk to hardpan by interior excavation. Two standard-gage tracks full length of pier. In the same location a similar designed pier, No. 4,

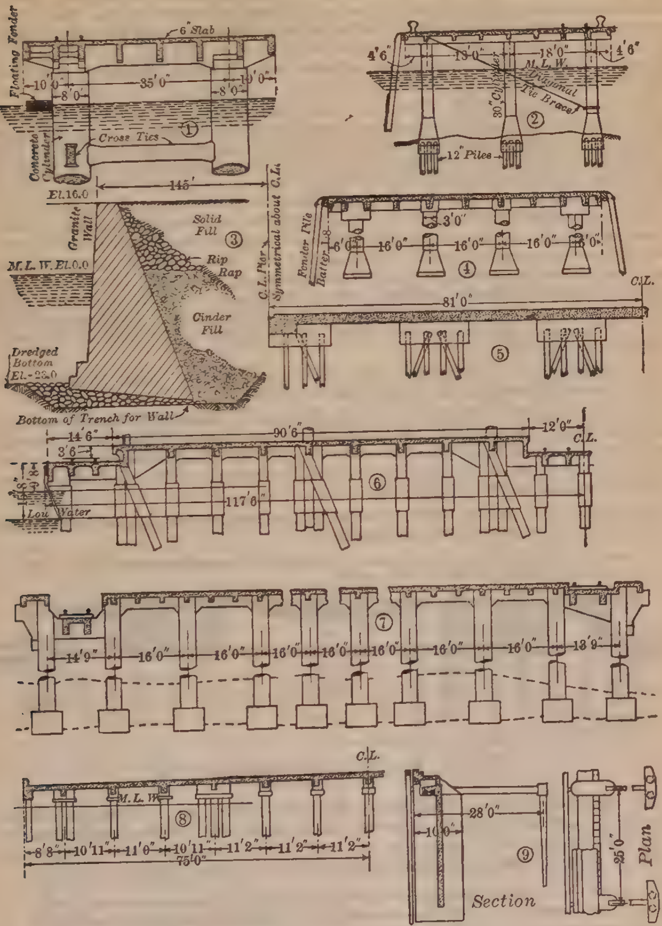


Fig. 44. Masonry and Reinforced-Concrete Piers

490 ft. long; cylinders 42 in. in diameter, 11 ft. in diameter at bottom, 43 ft. long. This pier was constructed by placing wooden shells, sinking these by interior excavation, sealing bottom with tremie concrete and filling with concrete.

(5) Havana, Cuba. Pier No. 1, 660 ft. long by 162 ft. wide, bents 20-ft. centers. A similar pier No. 2, 680 ft. long, was built, and a quay 800 ft. × 80 ft. Reinforced-con-

crete piles 16 in., 18 in., and 20 in. square. Water 20 to 40 ft. deep. Soft bottom. 12-in. reinforced-concrete deck, deck live load 250 lb. per sq. ft. and shed load.

(6) Halifax, N. S. Pier No. 2. 686 ft. long. 24-in. square reinforced-concrete piles, 45 to 77 ft. long; reinforced-concrete brace piles cambered 2 to 5-1/2 in. Special cement employed with minimum of aluminum to resist salt-water action. Bents 18-ft. centers; rock foundation 68 to 52 ft. below surface; area under pier filled by dredging to increase lateral stability. Piles loaded 80 to 90 tons each test load 120, 1 : 2 : 4 concrete. Piles protected against ice between tides by timber sheathing. Live load 1000 lb. per sq. ft.

(7) San Francisco, Calif. Pier 28, 677 ft. long; bents 15-ft. centers. Creosoted W. B. pavement 31 ft. wide down center, remainder of pier asphalt. Car springs fender employed. Types of construction general in this harbor, which is best given by brief abstract of specification: "The excavation for the cylinders shall be made inside of steel caissons which shall be driven into the hard bottom and sealed. The steel caisson shall be of such strength and rigidity as to resist external pressure, and of such size that they may be withdrawn without injuring cylinder bases. The forms shall be wood-stave forms of the diameter indicated. After the steel caisson have been set in place and the excavation made, as hereinbefore specified, the wood stave forms and reinforcement shall be set in place in the exact location shown on the plans. The concrete shall then be deposited through a tube or bottom-dump bucket. Under no circumstances will the placing of any concrete be allowed in or under water." Cylinders are sunk 12 to 14 ft. below dredge line, and where bottom is soft, timber piles are driven to support the concrete.

(8) Baltimore, Md. Broadway pier. Reinforced-concrete piles, 16 in. square; 8-in. reinforced-concrete deck. Bents 10-ft. centers.

(9) Baltimore, Md. Piers 4, 5, and 6; filled piers. Surrounding wall consists of flattened steel cylinders 25 ft. on centers, sunk, excavated and filled with concrete. Reinforced-concrete sheet piles between, achored by reinforced-concrete anchors and piles. Cylinders 27 ft. in length, built up in sections of 4 ft. Cylinders driven, excavation made by jet and pumping.

Two examples of piers of widely different types constructed during the period of the war in 1918 and 1919 are herewith shown.

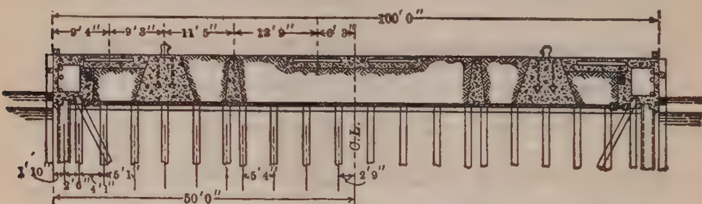


Fig. 44*a*. Filling-out Pier, Portsmouth, Va.

Fig. 44*a* shows Pier 4 at Portsmouth, Va., 1919, 1000 ft. long. Interior untreated pine piling, 4-ft. centers, live load 900 to 1200 lb. per sq. ft. 350-long ton fixed tower, revolving hammer head crane (see Fig. 91 *f*), inclosed space by 60-ft. reinforced-concrete sheet piles; 40-ft. depth of water at mean low water, range of tide 3 ft.

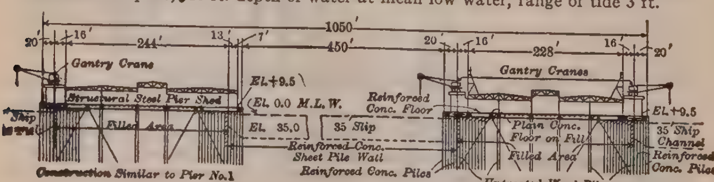


Fig. 44*b*. Freight Terminal Piers, Norfolk, Va.

Fig. 44*b* shows two piers for general loading and unloading of cargo of miscellaneous character, built 1919, 1328 ft. long. Filled center surrounded with reinforced-concrete sheet piles, interior untreated piles, exterior concrete piles.

23. Pier or Transit Sheds and Warehouses

Shelter from the Weather is provided on piers or quays used for the loading and unloading of cargoes, by the construction on the piers of sheds, one or two stories in height, the floor area covered by the sheds being devoted to the handling and distribution of incoming and outgoing cargo requiring protection and used for the storage of this cargo for a short time. Warehouses are usually provided on shore or directly behind transit sheds for goods to be stored for periods of more or less lengthy duration. When such warehouses are for the purpose of storing dutiable articles, remaining under custom seal until called for, and the custom imposed paid, they are known as Bonded Warehouses. Sheds are constructed of timber framing and siding with roofing covering of appropriate roofing material; timber framing with corrugated sheet metal siding and roofing; structural steel framing with corrugated sheet metal siding and roofing; of reinforced-concrete and other combinations of materials. On open pier construction, lightness of construction within safe limits is essential and the tendency is to make such structures fire retarding or resisting, especially where quantities of valuable cargo are handled. The width of the shed is, in most cases, fixed by the pier or quay width, allowing space in front for driveway or railroad tracks and a similar space in the center of piers, or rear of quays. An ideal arrangement on piers in general would be 25 ft. for driveway and trackage, 60 ft. for transit shed and 50 ft. for uncovered central space. A storehouse in the center for storage longer than 48 hours would require additional space and a greater pier width. Space must also be provided for open storage for cargo requiring no shelter such as steel rails, lumber, etc. The outshore end of piers is usually left open to permit ready handling of the bow and stern lines of incoming and outgoing ships.

Pier or Transit Sheds are of one or two stories dependent upon local conditions and practice and the availability and value of waterfront. On account of the conditions, pier or transit sheds should be capable of affording floor space for the storage of the cargo of incoming vessel or the accumulation of cargo ready to be placed in an outgoing vessel. Two-story sheds permit a narrow pier width, or the second story is utilized for passenger traffic.

Railroad Tracks, when carried out on piers, are as a rule run either directly through the center of the pier, at the sides of the pier, or in both places. Tracks are often laid in a depression, so that the level of the car platform will be at or about the level of the pier deck, or are laid on the deck level. When railroad tracks are run on the side, they are not usually covered by the main pier shed.

Details of Construction of Sheds are generally the same as those followed in building construction and it is often customary to give the sides of the shed a slight inclination inwards or "tumble-home." The second floor in two-story sheds is sometimes given a slight crown and the lower chords of roof trusses a camber. Openings should be provided at frequent intervals in the sides used for berthing, or should be made continuous so as to afford ready access to the various ship hatchways and side openings. Doors may either be swung from the top with counterweights employed to facilitate ready and rapid opening and closing, or slide horizontally on overhead trackways. (Various folding cargo doors and metal rolling shutters are also used.) When continuous openings are provided, several lines of overhead tracks are placed, so that the door leaves may be rolled by one another. When the siding of the shed is of light sheet metal, on either structural steel or timber framing, it is advisable to protect this on the inside to a height of about 6 ft. by cargo battens placed at a slight distance from the inside of the siding.

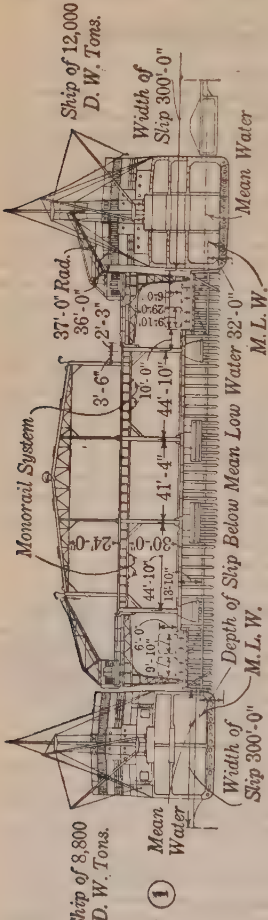


Fig. 45 Staten Island Piers

Lighting and Ventilation. In wide sheds the lighting arrangements are of considerable importance; natural light is provided by a continuous skylight or series of skylights, or by monitors constructed in the roof, or by both methods. The employment of monitors also affords opportunity for ventilation, but is likely to permit leakage in heavy driving rain storms. Circular sheet-metal ventilators of various types may be employed on the roof ridge spaced 50 to 100 ft. apart. As occasions frequently arise where the loading and unloading of vessels and handling of cargo must be done at night, proper artificial lighting is important, for which the practice is similar to that obtaining in general building lighting.

Miscellaneous Appliances. There have been many disastrous pier fires, therefore adequate fire protection and service systems are important adjuncts to properly constructed pier sheds. Piers, if of timber or timber composite, being highly inflammable, the fire insurance rate on the structure itself and on cargo stored is an item that can properly be fixed in the annual charge and capitalized cost. Important commercial structures should have installed modern fire alarm and indicator systems, automatic sprinkler systems, preferably of the dry pipe type, and fire service mains, laterals, riders, connections and fire hose on reels or racks readily accessible. If local fresh or salt water pressure lines are not available, elevated storage tanks, with pumps and overboard suction connections, must be included in the equipment of the pier shed. If water mains are carried out on piers, precautions should be taken to prevent freezing by means of proper insulation, or by draining the lines when not in use.

Essential Details of various precedents are given in the following statements, where the numbers refer to Figs. 45, 45*a*, 45*b* and 45*c*.

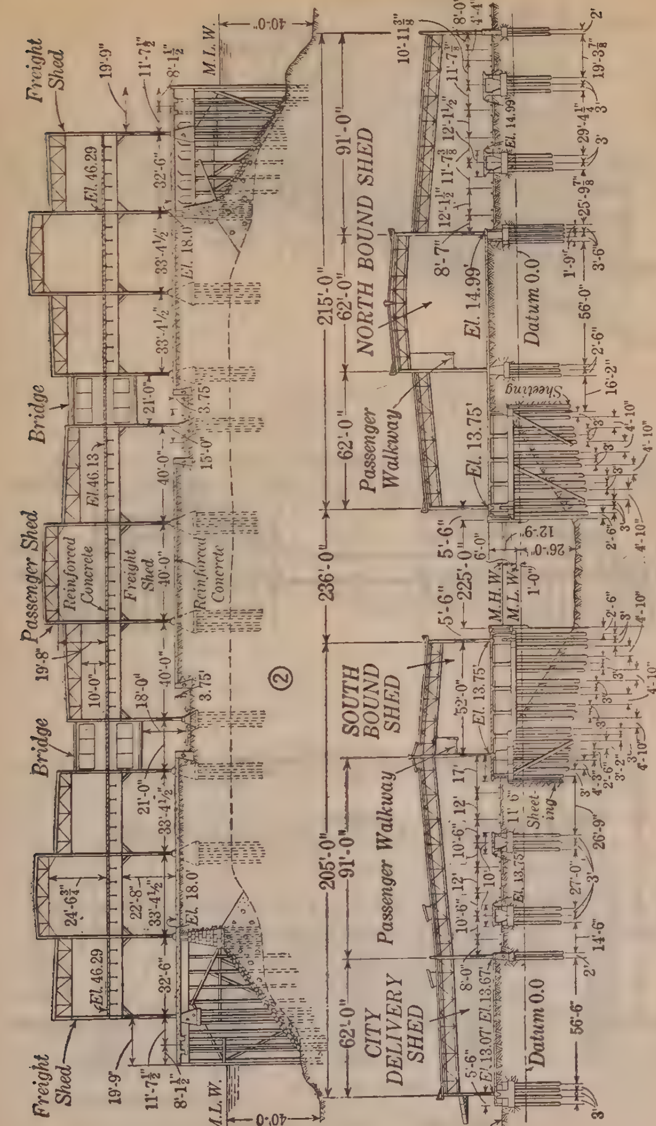
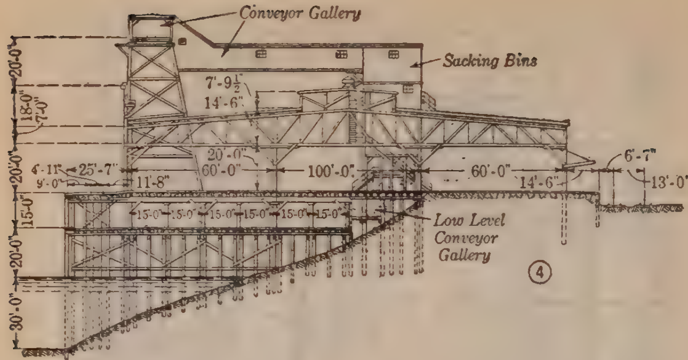
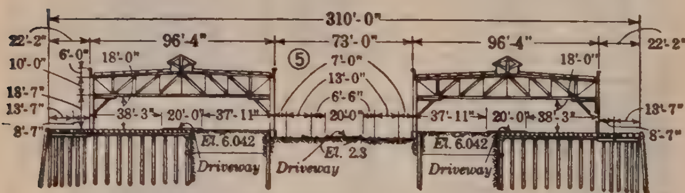


Fig. 45a—Pier Sheds—Continued





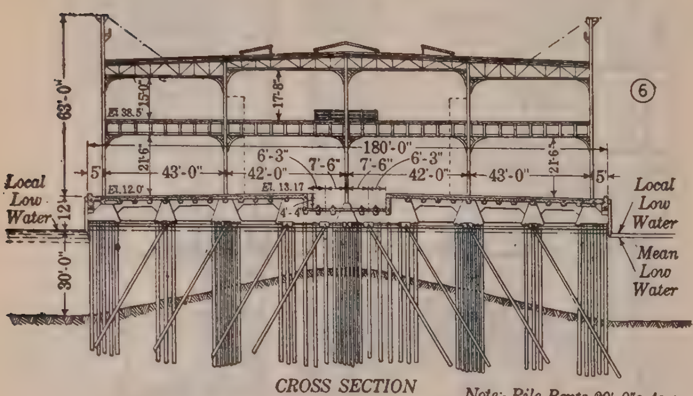
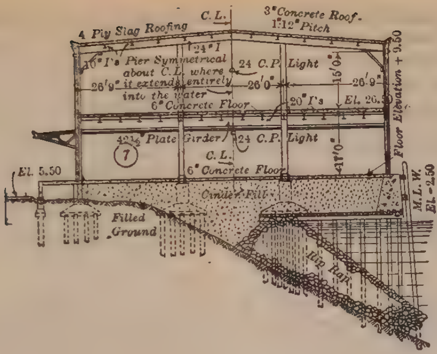


Fig. 45b—Pier Sheds—Continued



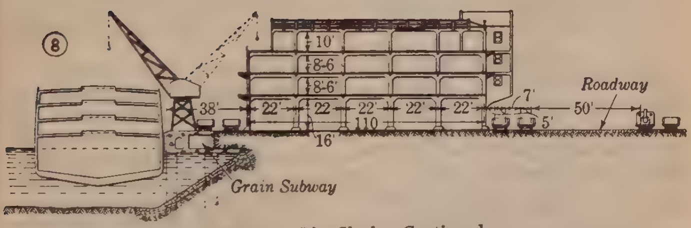


Fig. 45*c*. Pier Sheds—Continued

(1) Staten Island, N. Y. Pier 13.
(2) Boston, Mass., Pier 5. Open aprons both sides; ground floor, by means of bridges, is made available for freight in transit.
(3) Savannah, Ga. Construction is a combination of timber, reinforced-concrete and steel. Slip is a little too narrow for its length for loading and unloading large vessels.
(4) Portland, Ore. Timber construction.
(5) Seattle, Wash., Pier A, Smith Cove. Timber construction, combining open construction for the apron on each side of the fill confined behind sheet piling bulkheads, for the center portion of the pier.
(6) Philadelphia, Pa. Southwark Piers 38 and 40. Timber pile supports below permanent wet line. The two-story shed is of steel.
(7) Jersey City, N. J., Pier 9, D., L. & W. R.R. Steel frame. Reinforced-concrete roof, siding and floor.
(8) Manchester, England. Note grain subway.

WET AND DRY DOCKS

24. Wet Docks

Definition. Docks may be divided into two general classes—harbor docks and repair docks. The former, sometimes known as Wet Docks, are enclosed, or partially enclosed basins, usually, in European practice, provided with locks and entrance gate, similar to canal locks. They are sheltered basins for the receiving of shipping, so arranged that the water in the dock may be kept at more or less constant level to facilitate the loading and unloading of cargo.

The general term Dock is applied to such basins whether closed or open, and through a misunderstanding of the word is often, in general parlance, applied to the surrounding or bordering structure such as the quay or pier.

Use of Wet Docks. Their usefulness is apparent in harbors where there exists a considerable range of tide, and especially where under low tide conditions the approach to the harbor itself is not navigable for deep-draft vessels. They also have a special field of usefulness in harbors or rivers where considerable silting up takes place, the dock area being kept free of such silting by keeping out turbid river or harbor water and often by supplying water to the dock basin from other sources. This type of structure, in its restricted sense, is rarely found in American practice, although it is employed in some South American ports.

The reasons given for the use of wet docks are undoubtedly not the only ones that have occasioned their appearance in European practice, as otherwise they would undoubtedly have before this been employed in American ports where a considerable range of tide is involved. They have undoubtedly been resorted to in order to attain the maritime commerce of river ports, where the increase in draft of vessels has become greater than the normal navigable depth of the river. The wet dock has been developed as the instrument by means of which Old World ports have retained maritime supremacy.

Design of Wet Docks. It is apparent that in wet docks, where entirely enclosed and provided with entrance gates, as the surface of the water is to be kept at approximately a constant elevation, the sides and bottom of such a basin must be practically impervious, or arrangements must be made to replenish the water lost by leakage. This is especially so in all seas having a considerable variation in tide, and a consequent appreciable difference at times in the height of water inside and outside of the wet dock. Where the basin is located in a practically impervious soil, such as clay or clay soils, and many kinds of rock, these conditions are generally provided for. For this reason, a thorough exploration of the subsoils in locations where wet docks are projected is of the greatest importance. In locations where the subsoil is readily water bearing, the bottom must be blanketed with clay puddle or a masonry lining placed. The dock walls can be treated in a manner similar to that described under Quay Walls, in Art. 12; the walls, however, of dock basins are at times subjected to external hydrostatic pressure in excess of that to which they would be subjected if located on open basins. This is, usually, not a serious matter, as the combined earth and contained water pressure, together with the passive resistance of the earth backfill, will be found sufficient to provide for this condition.

Locks and Lock Gates. The condition governing the design and construction of these is, in all respects, similar to that found in canal work. Information on this subject will be found under Canals, in Arts. 17 and 19, of Sect. 15.

25. Marine Railways

Definition and Description. Repair docks may be marine railways, lift docks, graving docks, or floating docks. Marine railways are inclined slips or ways, extending out some distance into and under the water, and running up on the foreshore a sufficient distance so that the vessel when hauled out will be entirely clear of the water. A platform or cradle moves on these ways, usually on nests of rollers, and is hauled out of the water by means of chain or wire rope cable. Racking and pawls are usually provided, so that if the hauling device should break, the cradle and vessel will not slip back into the water.

It is apparent that even without considering the feature of range of tide, the application of this type of repair dock is limited to vessels of comparatively small size, as in long vessels of deep draft the ways would have to be carried out a great distance under water, approximately twice the length of the vessel to be docked, making the entire length of the ways over 3 or 4 times the length of the vessel to be docked. This condition is sometimes materially improved by use of a collapsible cradle. Although there is no reason why sufficient capacity of hauling device can not be designed, nevertheless important considerations would tend to limit this. This type of repair dock is, undoubtedly, valuable and economical for the docking of smaller vessels. Its application, however, appears to be limited. No marine railways have been built in excess of 5000 tons lifting capacity. Fig. 46 shows a marine railway of 2000 tons capacity. Estimated cost, $100,000.00 in 1919.

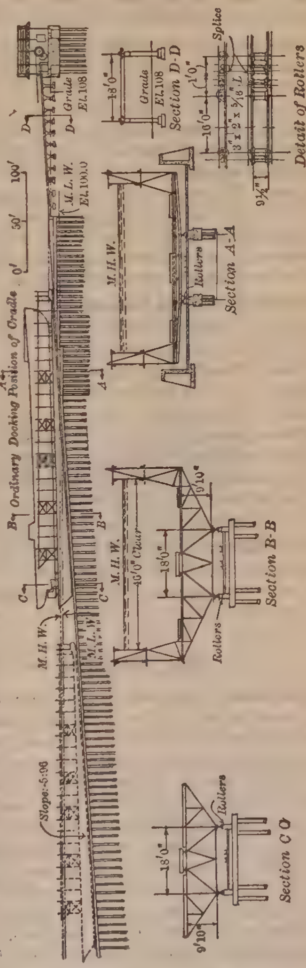


Fig. 46. Marine Railway

Marine Railways were designed in 1918-1919 of 3100 tons capacity; the estimated cost $300,000 or nearly twice the cost per ton capacity as that given hereinbefore.

The Foundation should, if possible, be incompressible. In many larger and more important marine railways the ways are built on piling. The intensity of pressure, though not great, is, nevertheless, in the form of a moving load, becoming greater as the vessel hauled out loses its buoyancy. The ways are in many instances similar to the ways employed in shipbuilding work, which, however, do not go out under water to the same extent, and besides this, the weight of a vessel when launched is usually considerably less than would be the docking weight of the same vessel.

Ways. The ways are laid at a gradient of between 1 on 13 and 1 on 25, 1 on 20 being the usual slope for large marine railways. The ways consist of 2 to 4 rails of iron or steel securely fastened to longitudinal bearers which in turn rest on cross-ties bearing on the foundation. The racking to receive the cradle safety pawls is usually placed in the center of the ways between the two central rails, if 4 rails are used. The under-water work is either performed in the dry by the construction of a cofferdam around the site, or by divers. If no foundation piles are used, the grading must be carefully done, working from over-water battens. If piles are used the under-water portion of the ways is built in sections, floated to place, lowered onto the foundation by weighting and fastened securely in place.

The Cradle or Platform is usually constructed of timber or structural steel, and is provided with keel and bilge blocks. The cradle is constructed on longitudinal sleepers, placed directly over the longitudinal ways, and provided on the underside with an iron track. The cradle is moved on nests of rollers placed in between the upper and lower lines of track. The rollers are usually of cast iron, double flanged, and held in place by flat bars or links; sometimes the rollers are fixed on the cradle and run on tracks on the longitudinal way-sleepers. In large marine railways, the cradle is built with side frame work, which is used for working platforms. Frequently, cradles, instead of having the same inclination as the ways, are built with the platform horizontal, or nearly so, in order that the ship being docked will immediately come to a bearing and not be lifted first by the bow.

Broadside Marine Railways. With a view to overcoming the difficulty as to length of marine railway, they are sometimes built so as to withdraw vessels from the water broadside on.

Power Absorbed in Friction. The power required to raise a vessel is dependent upon the inclination of the ways and the force required to overcome friction. Experiments made on the power absorbed in friction indicates a variation from 3.3% to 7-1/2% of the weight lifted. On larger and well-designed marine railways, the percentage is in the neighborhood of 4%, to which should be added 5% for initial friction in starting from rest. Thus in a marine railway having an inclination of 1 to 20, the hauling machinery would have to exert a pull of 14% of the weight of the vessel lifted.

Lift Docks are platforms lowered into and raised from the water. This type of structure is of limited capacity and has fallen into disuse. Hydraulic power was generally employed, applied through a series of cylinders placed on both sides of the platform and connected together transversely by girders.

26. Graving or Basin Dry Docks

Definition. Basin dry docks, often called graving docks, are basins generally made by excavation in the foreshore of the harbor, having entrance-ways closed with gates or portable caissons, and are usually of such dimensions as to be capable of receiving, with sufficient clearance, the maximum sized ship to be docked therein. After the ship is floated into the dry dock, the opening is closed by means of a gate or caisson and the water in the interior is removed by pumping, the ships settling on blocks provided for that purpose. In some graving docks advantage is taken of the fall of tide to reduce pumping or eliminate it entirely. In nearly all cases the graving dock must be pumped out twice for the docking of the vessel. The dock is usually left unwatered so that when word is received that a certain vessel is to use the dock, the blocking may be arranged to conform with the profile of the keel and the character of the vessel. The dock is then flooded and the vessel is floated in and centered and held in the proper position, the opening closed and the water removed. On completion of the work on the ship, the dock is flooded and the ship removed, the opening again closed, and the water removed from the dock.

Foundations. In large graving docks, water at considerable head must be excluded, and for that reason the character of the subsoil and foundations are of considerable importance in locating structures of this type. A graving dock can, although at times at a prohibitive cost, be constructed in almost any location, but, where possible, it should be located at a site where sound bedrock will be encountered at some elevation not below the subgrade of the structure or in locations where marl, stiff clay, or mixtures of marl or stiff clay, sand and gravel are encountered. Suitable foundation conditions, other than rock, sometimes result in less expensive and more expeditious construction. Fig. 47 shows a half transverse section at the entrance sill of a graving dock on pile foundations.

Graving docks have been constructed in unstable water-bearing soils, but at high cost. With a view to securing a suitable site, or to enable proper precautions to be taken in design and in method of constructions, if selection of a site

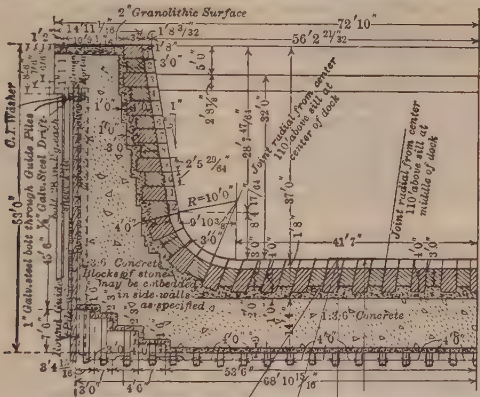


Fig. 47. Section of Graving Dock with Pile Foundation

is restricted, an important preliminary step is the taking of exploration borings, operations for which are described in Sect. 15. Arts. 29 and 30. It is advisable, where possible to do so, to secure core borings showing the condition of the soil " in situ " and, in special cases, to supplement these with test pits. See Sect. 15, Art. 31. In water-bearing soils such test pits may have to be made by employing the pneumatic process.

Masonry Graving Docks. In the design and construction of such docks, provision must be made for counterbalancing the hydrostatic pressure by sufficient weight of masonry or other anchorage effect. In porous water-bearing soils, the hydrostatic pressure to be considered will be at least the full hydrostatic pressure due to maximum tide elevation, and under certain conditions, on account of the presence of ground, surface or artesian waters, a pressure in excess of this. Where the dock is located on a solid rock ledge without water-bearing fissures, this condition will not obtain, although ideal foundation conditions of this character are seldom, or never, encountered in practice, and it is therefore customary on this character of foundation to provide for relieving the water pressure by under-draining the floor of the structure. In some soils, such as tenacious marls or clays, or hardpan, the same precautions may be taken, the walls of the graving dock being designed as retaining or quay walls. In graving docks built in water-bearing soils, full hydrostatic pressure should be expected and provided for. A number of graving docks have been constructed in which the weight of the masonry mass is somewhat less than the uplifting force due to full hydrostatic pressure, their integrity depending upon the friction of the backfill on the back of the side-walls, and, in some cases, to anchorage or pull on foundation piles. In stable water-bearing soils where the uplifting forces are clearly defined, dependence can be placed on the anchorage effect of the foundation piles, where such exist. This anchorage effect, however, should not exceed the submerged weight of the mass of subsoil engaged by piling, as stated in this section, Art. 12. The friction of the backfill on the rear of side-walls, being indeterminate at best, should not be given a definite value, and should be considered merely as a factor of safety. In a dry dock completed at the New York Navy Yard in 1912 the anchorage effect against hydrostatic uplift was secured by deep piers with flared-out basis, secured into the floor by a system of anchor rods, the anchorage provided in each being in the neighborhood of 1 000 000 lb., these piers being sunk to position by the pneumatic process.

A masonry graving dock is subjected to three following conditions:

(a) When empty: The bottom or floor is subjected to hydrostatic pressure, usually considerably in excess of the weight of the floor itself. This excess is transmitted to the heavy side-walls by actual or virtual inverted arch action, the abutments of the arch being furnished by the weight of the side-walls taken together with the thrust of the earth and water on the rear of the wall and the passive resistance of the earth itself. When this abutment action is insufficient, reinforcing steel should be provided in the floor.

(b) The dry dock empty with a ship of maximum tonnage in the dock: The conditions are the same as (a), except that there is concentrated at the center line on keel blocks, and on the side blocking, the weight of the ship, for which concentration, where the foundation is somewhat yielding, special provision must be made by distribution of this load by reinforcing the masonry with steel, or the foundation directly under the blocking designed accordingly.

The loading per running foot of dry dock is dependent on the length, depth, and width of the structure and the character of ships to be docked. For war vessels of the largest size weights as high as 90 long tons per running foot should be allowed, as the concentration under machinery and gun turret locations is liable to run this high. For commercial steamers the loads will be less. Under ordinary docking conditions 5/8 of this load will come on the keel blocks and 3/16 on each side, docking keel or bilge block. In the work of bottom repair, as blocking is removed the concentrations are shifted, this, however, only occurring locally over small areas.

(c) Dry dock full of water. Under this condition, the foundation, considered in its entirety, is subjected to the greatest load, consisting of the submerged weight of masonry to tide level, plus the weight of masonry above tide level, the conditions being somewhat altered with the presence of surface or artesian ground-water in the soil. Under this condition the thrust on the rear of the wall is due to submerged earth pressure to ground or tide water level, tide or ground water pressure and surcharge of earth pressure above tide or ground water level, and surcharge on wall, reduced by water pressure on the interior of the dry dock. It is often customary to use combined liquid pressure to the coping of the structure and neglect surcharge (Art. 12). Under this condition, very often little or no virtual or actual arch action can be counted on, and the foundations under the side-walls should provide for the weight directly above them, or provisions for distribution made by employing reinforcing steel. In yielding foundations, if this is not done, settlement at the side-walls will be in excess of the settlement under the center of the floor, causing cracks at the toe of the walls, and along the center line of the dry dock. It should be noted, as between Conditions (a) and (c), that there is likely to be a complete reversal of stresses in the floor.

To these three conditions a fourth may be added, that is, of the dry dock while under construction. This should be given careful consideration and analysis, the omission of which has, in some instances, resulted in damage to the uncompleted structure. Many older masonry graving dry docks have been constructed of rubble or ashlar stonework, faced with ashlar work. Nearly all the the more modern graving decks are constructed of concrete, unlined, or lined with granite ashlar work, or, as in the case of the graving dock built at the New York Navy Yard in 1912, vitrified brick, in either case caisson or gate seats consisting of granite stonework with radial joints for the bottom and lower parts of the side steps, the contact faces of which are fine cut work, or, at times rubbed. The beds and joints of caisson-seat stonework should not exceed 1/4 in., and, in some cases, are limited to 1/8 in.

Examples of recent graving docks are shown as follows:

Figs. 47*a* and 47*b* show a plan and a transverse section of a group of four dry docks at the Norfolk, Va., Navy Yard. Dry Dock 3 was built between 1903 and 1911 while

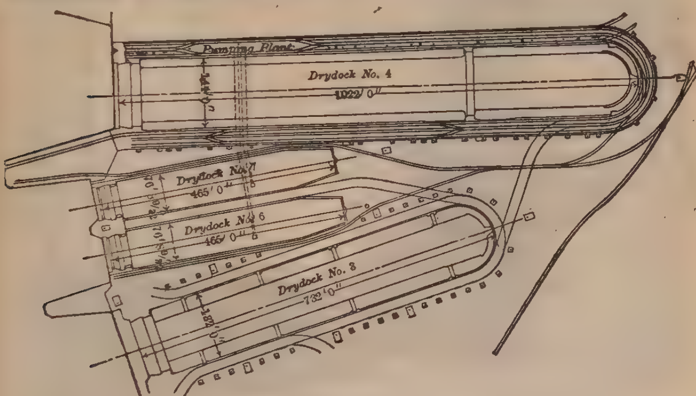


Fig. 47*a*. Group of Four Dry Docks for Norfolk, Va.

Dry Docks 4, 6 and 7 were completed in 1919, the larger for the Navy and the two smaller ones for the United States Shipping Board Emergency Fleet Corporation.

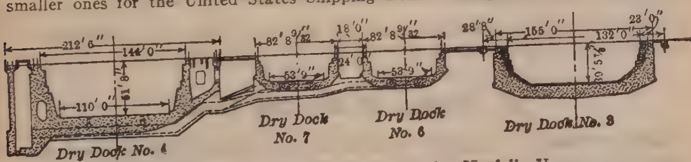


Fig. 47*b*. Transverse Section of Docks, Norfolk, Va.

Details of all four dry docks as to dimensions and cost can be found in the tabulation in Art. 30. One pumping plant consisting of three 54-in. vertical volute pumps serves all three of these docks. Dry Dock 4 is provided with an intermediate caisson gate seat so that the inner end of the dry dock can be used for smaller vessels on long-time repair work and the outer portion continue available for docking larger vessels. This arrangement practically provides two docks in one. A similar arrangement is provided in the Philadelphia Navy Yard Dry Dock and in the Commonwealth Dry Dock at Boston, Mass. These arrangements entail the additional cost of the interior seat, two dry dock caisson gates and independent pumping for the two sections of the dock. The older dry dock of the group, No. 3, as will be noted in Fig. 47*b*, is lined with granite ashlar masonry, but the three new docks are of concrete throughout without other facing or lining. A section of Dock 4 shown in Fig. 47*b* is at the pump well. The half-section to the right of the center line is so designed that the dock could be constructed on the natural stable slope of the excavation which was made by drag line in the dry. The floor invert is 20 ft. in thickness. The floor of the two smaller docks is 10-1/2 ft. thick. Excavation for these docks was also made in the dry. Water was kept out of the excavation site by a cofferdam of steel sheet piling, wooden sheet piling and earth embankment. Dry Dock 4, the large dock, was completed in about 2 years, Dry Docks 6 and 7 in 1 year, indicating that this type of dry dock can, under favorable conditions, be constructed as expeditiously as floating docks of similar size.

Fig. 47*c* is a half typical section of a graving dock constructed at the Philadelphia Navy Yard, and is similar in dimensions to that at the Norfolk Navy Yard. The

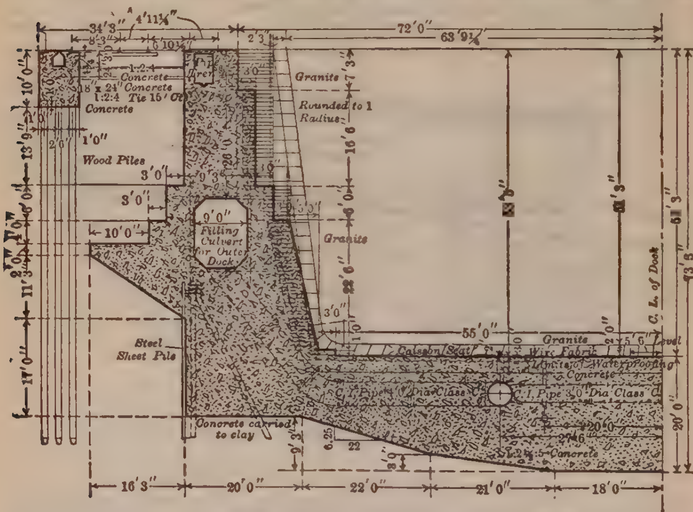


Fig. 47*c*. Half Section Graving Dock, Philadelphia, Pa.

steel sheet piling shown surrounds the entire site at the bottom and was placed to cut off the flow of water in the sand and gravel above the clay subsoil.

27. Design of Graving Docks

Data on Design. A graving dock should be designed having in mind the three conditions enumerated in the paragraph above, and consideration should also be given to the possible fourth condition. The necessary information having been obtained as to the character of the soil, its probable submerged weight and angle of repose and allowance made for the safe weight of masonry per cubic foot, stress diagrams should be drawn for all conditions. Fig. 48 is such a diagram for condition (a). In this case the hydrostatic pressure is just counterbalanced by the weight of masonry. The line of pressure does not pass through the middle-third at the base of the side-wall or in the center. If on analysis the maximum unit pressure in the virtual arch is not too high, the masonry at the center above the virtual arch may be considered inert, although it may be desirable to employ reinforcing metal to avoid unsightly longitudinal center line cracks.

Under these conditions the vertical components of the downward forces would be in excess of the vertical components of the upward forces of hydrostatic pressure, this excess being counterbalanced by the soil pressure or passive resistance and would appear as a vertical closure in the polygon of forces

(Fig. 48). In analyzing the floor arch thrust in the same way it may be necessary to consider not only the active earth and water forces on the rear of the retaining wall but also the soil pressure or passive resistance that would be afforded to this wall acting as an abutment for the inverted floor arch thrust, from which it will be seen that it is entirely probable in the preparation of the design and in the analysis of the structure and in the making of the stress diagram to insert exterior soil resistance or passive pressure forces.

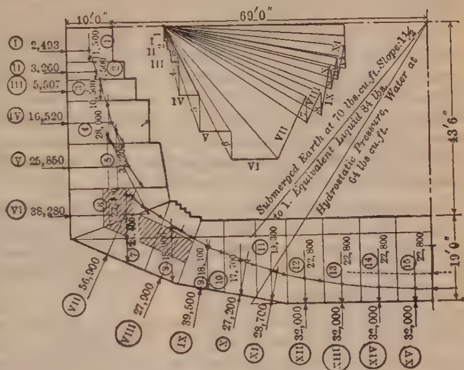


Fig. 48. Stress Diagram

When the weight of the section is in excess of the hydrostatic pressure, the diagram would have to take into consideration the bearing of the structure on the foundation, or soil pressure, which could be considered as equally distributed over the entire area of the foundation. However, in a case such as that shown where the line of pressure was outside of the middle-third at the center, in order to develop longitudinal cracks on the tension side, the floor at the center would have to lift slightly, relieving the soil directly beneath it of pressure and transferring the entire soil pressure to the sides, resulting in raising the line of pressure to within the middle-third as indicated by the dotted line.

Masonry Graving Docks on Rock Foundations. The same considerations govern the design, excepting that provision can probably be made for relieving hydrostatic pressure in whole or part; under the most ideal conditions the rock can be excavated out to the rough dimensions of the graving dock, and then lined with masonry, with or without relieving drains. A graving dock built at Hunter's Point, San Francisco, Calif., was built in this way, the excavation being made in impervious serpentine rock, and lined with a comparatively thin layer of concrete. No provision was made for draining. In graving docks founded on ledge rock, the necessary stress diagram should be made for all conditions. Fig. 49 shows a half transverse section of a graving dock on rock foundation.

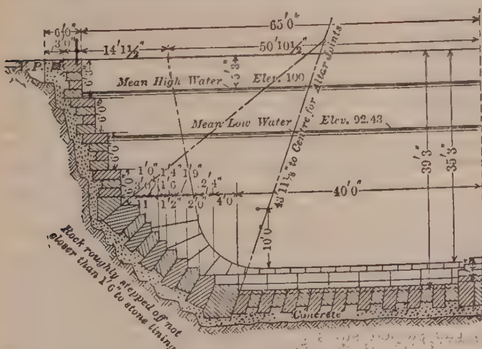


Fig. 49. Half Section Graving Dock, Rock Foundation

Altars are provided in the side-walls at convenient heights, the first at elevation of about mean high tide and additional altars at intervals below this. They consist of steps or platforms 2 to 3 ft. wide, and are used for the placing of shores to hold upright the vessel being docked, and as a means of access from and to plank stages or working platforms placed alongside the hull.

Stairways and Timber Slides. Stairways are provided for going into and coming out of the graving dock, two sets on either side of the entrance and at or on either side of the head with intermediate stairways to suit convenience. These are built directly into the altars or through the side-walls. Timber slides are used for letting down and hauling up material and are placed at convenient intervals in the side-walls on both sides of the structure. In order to simplify design and construction, portable stairways may be provided, leaving the altars clear and continuous.

Drainage. The floor is drained by means of either open gutters at the sides or in the center of the floor, or by drainage culverts with drain connections, located at intervals. Where gutters or drains are used the floor must be graded transversely to these, the gutters or drains being given gradients to insure self-cleaning.

Head. The inshore end of the graving dock is known as the Head, as distinguished from the outshore end, or Entrance. Heads are built rectangular, trapezoidal or circular or elliptical, correspondingly, the walls are designed and figured as retaining walls, or are built as simple or compound, circular or elliptical arches.

Timber Graving Docks have in the past been extensively employed in American practice, when vessels of smaller size with less draft and of different hull section were constructed for the merchant and naval service. They had the advantage of lower first cost and more expeditious construction and were especially suited for location in stable water-bearing subsoils. The principle of design and construction is different from that obtaining with respect to masonry graving docks and is based upon the partial, or entire, reduction of hydrostatic pressure on the bottom by drainage and the building of the side-walls at a slope approximating the natural slope of the soil, relieving this portion of the structure from the earth thrust. The flow of ground or tide water through these walls is cut off by a continuous and enclosing wall of sheet piling driven at the coping or some distance back from it, the timber altars of the side-wall being laid on inclined stringers supported on piles, a sloped wall of puddled clay being laid between the soil and the timber altars. The flow of water onto the floor is cut off or reduced by a continuous enclosure of sheet piling driven at the foot or toe of the wall or slope and 1 to 3 rows of timber sheet piling are driven across the entrance, the floor itself consisting of heavy timber planking, fastened to dimensioned caps bolted to the foundation piling, with which the bottom is studded at 3 to 4-ft. intervals, additional piles being driven in the center, where necessary, under the keel blocks, and under the probable position of bilge blocks. Three to 5 ft. of concrete is placed under the timber floor in between timber caps and piling. In this type of structure a certain amount of leakage or self-draining is expected and allowed for. Among the advantages claimed are that the sloped sides give better light and ventilation. Although this argument, undoubtedly, is of importance as applied to the older types of vessels, it loses its force when steel vessels of rectangular amidship sections, with practically flat bottoms, are considered, as, in both masonry and timber graving docks, artificial light must be employed, under such hulls. The disadvantage of this character of construction is that in order to secure necessary width of graving dock at the height of the blocks, the width at the coping line becomes excessive. Thus, with a vessel of 100-ft. beam, and 30-ft.

draft, with allowance for clearance, the width of the timber graving dock at the coping line would have to be nearly 200 ft. as against 140 ft. for a masonry graving dock, entailing the requirement of a considerably larger area of land, and difficulty in handling materials for building or repair by side-wall cranes. Timber graving docks have a further disadvantage of not being a permanent structure and requiring continuous and expensive repair and replacement. The surface of the ground around the structure is in a continuous process of settlement due to the percolation of water through it and to the washing out by this water of the finer particles of soil. On account of the difficulty of obtaining complete and intact sheet-pile water cutoff walls, frequently extensive washouts and undermining occur in this character of structure.

Blocking, Shores, and Fittings. Keel blocks are provided on the center line of the floor and consist of heavy dimensioned hardwood timbers. They should be so laid out as to afford sufficient bearing to avoid undue crushing under the maximum weight likely to be brought upon them. In American practice they usually consist of three or more tiers, placed transversely of the dock on 4 to 2 ft. centers longitudinally. Heights of blocks above floor should not be less than 3-1/2 to 4-1/2 ft. to give clearance for working under a ship.

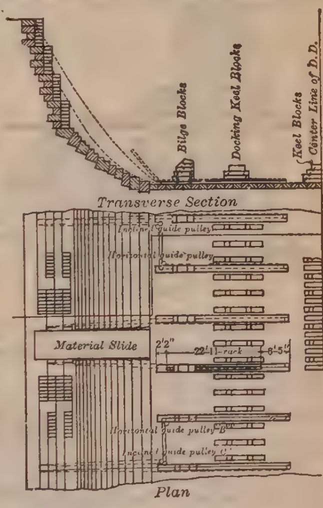


Fig. 50. Arrangement of Blocking

In European practice the lower blocks are often of cast iron, sometimes wedge-shaped, to permit ready removal. The lower course is fastened to the floor by anchor bolts, and the upper course, if of timber, by iron dogs. In addition to these blocks, bilge blocks are provided on both sides of the keel blocks, but placed at longer intervals apart. Some of the larger modern ships, especially in the naval service, are provided with four docking keels, two on either side of the center line, so that in being docked the ship will come to a horizontal bearing on central keel and side docking keel blocks, avoiding the expense and delay of holding the graving dock while blocking is being rearranged to fit the hull of the particular ship to be docked. When docking, keel blocks are not employed. bilge blocks are used, consisting of several tiers of blocks, the upper one slightly inclined or hinged to the block below and provided with a wedge. These blocks slide on bilge block bearers placed transversely of the dock. They can be drawn in under the ship and against the bilge by means of hauling chains, cast-iron racking being placed on the bearers and adjustable automatic pawls attached to the lower bilge block, holding the block in place when it has been drawn under the ship. These pawls are lifted and the block released when the hauling-back chain is employed. The docking keel and bilge-block bearers are sometimes made of reinforced-concrete, or entirely done away with, the cast-iron racking being fastened directly to the floor of the structure. Fig. 50 shows a half cross-section and floor plan giving position and arrangement of blocking.

Shores are used with, or to replace bilge blocks, to hold a vessel upright when the graving dock is unwatered. They bear against the vessel's side at frames and are wedged against the walls, rest on the altars, or are suspended from the coping by lines, for which purpose small cleats are placed at convenient distances. In order to assist in bringing and holding the vessel to alignment in position over the blocking, brest lines are employed, final adjustment often being made by block and tackle, for which small hand winches are installed at convenient intervals along the top of the dock wall.

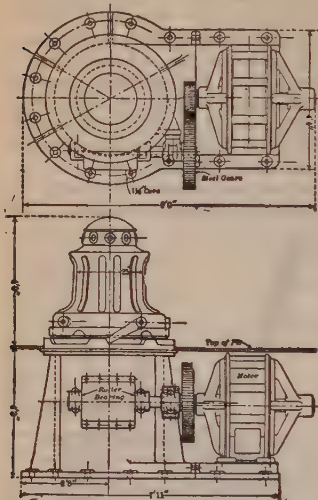


Fig. 51. Electric Capstan

Capstans and Bollards. As it is impracticable to place ships in graving docks under their main engine power or by tugs, power capstans are very often provided for this purpose. In smaller docks hand capstans are employed, assisted by windlasses on the ship being docked. On large graving docks, power capstans should be provided at the head end of the dock on either side of the entrance and in long structures midway of the length. These should have two speeds, the maximum for slow hauling from 15 000 to 35 000 lb. at the capstan barrel, the minimum is used for overhauling lines so as not to permit too much slack and possible snarling. Bollards or mooring posts are placed on both sides at intervals of about 50 ft.

Figs. 51 and 51*a* show an electric-motor-driven capstan of the latest type. Fig. 51*a* is designed with barrel height so as to permit the passage over it of the heavy dry dock traveling cranes.

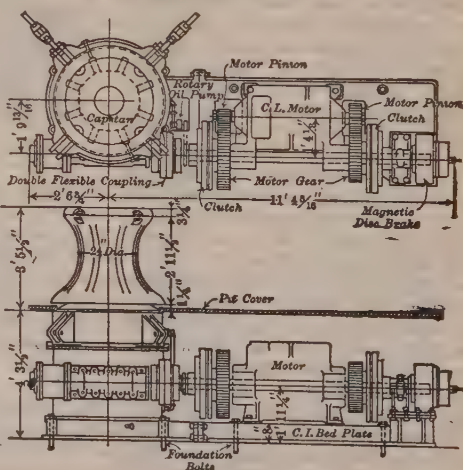


Fig. 51*a*. Electric Capstan

28. Construction of Graving Docks

As graving docks are always located on the foreshore of harbors, the problem of construction has an important influence on design. With the increase in draft of vessels and the necessity for providing for overdraft in case of accident, the clearance depth over entrance sills of graving docks has increased greatly and with it the head of ground or sea water to be taken care of during the construction period.

In certain locations it becomes impossible or impracticable, within the range of ready execution, to construct graving docks by any method involving the unwatering of the site. When such cases occur, recourse must be had to a method of construction that will make the unwatering of the site unnecessary. As the method of construction is an important item in graving dock design, it is well worth while to illustrate various methods that have from time to time been employed.

The usual course followed in the construction of timber graving docks is illustrated in Fig. 52 where (*a*) indicates the original conditions of the site. The entire site is surrounded by a wall of sheet piling, a portion of the round piling being driven previous to excavation; (*b*) shows the excavation, in part completed; (*c*) illustrates the driving of the round piling on the side slopes and in the bottom, and of the bottom enclosing sheet piling; and (*d*) shows the work completed. This method was generally followed in the construction of two timber dry docks built in the New York Navy Yard, and two similar structures built in South Brooklyn, N. Y.

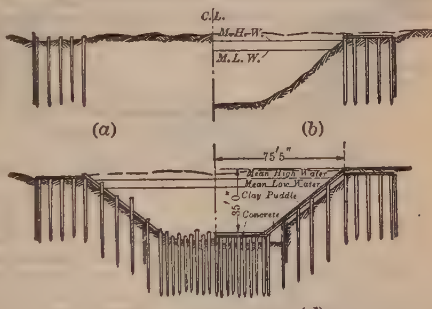


Fig. 52. Timber Graving Dock

Dry dock for Port of Boston: The method of construction for this dock is illustrated in Fig. 53. The structure is located on ledge rock, covered with a blanket of soil under water. On either side and around the head of the dock double timber bulkheads were placed, filled in with excavated material. The steps of construction (*a*), (*b*), (*c*), (*d*), are shown, (*b*) with the space between bulkheads filled and excavation made to ledgerock; (*c*) excavation in ledge rock; (*d*) completed structure backfilled. In this design the side-walls are retaining walls, the floor or bottom is shallow, and is underdrained, so that the underside of the floor is not subjected to hydrostatic pressure.

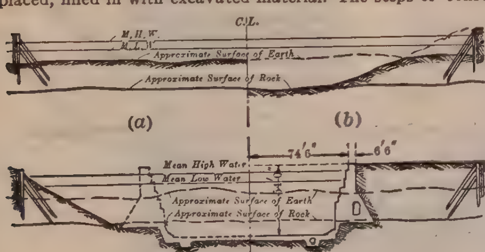


Fig. 53. Graving Dock, Boston, Mass.

Naval Dry Dock at Charleston, S. C., was located on a salt marsh, with underlying marl foundation. The entire site was surrounded with heavy sheet piling, held in place by timber trestle work, brace piles, and shoring, as illustrated in Fig. 54. The final excavation into the marl is shown in (*c*), and completed structure in (*d*). The method employed in making the excavation for the structure was: the first 10 ft. were taken out by hydraulic dredging, the remainder of the excavation in the marl being taken out by clamshell buckets operated by cableways and by pick and shovel and blasting. Cofferdam around the entrance to the work is illustrated in (*e*).

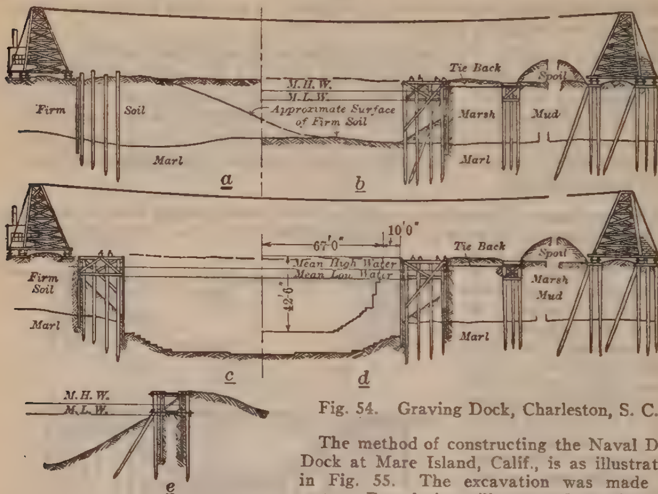


Fig. 54. Graving Dock, Charleston, S. C.

The method of constructing the Naval Dry Dock at Mare Island, Calif., is as illustrated in Fig. 55. The excavation was made in water. Foundation piling was then driven in the bottom and the timber crib of the general dimensions of the structure, built up on the surface of the water, loaded, and lowered until it rested on certain of the foundation piling, as shown in (*b*). Timber sheet piling was then driven around the

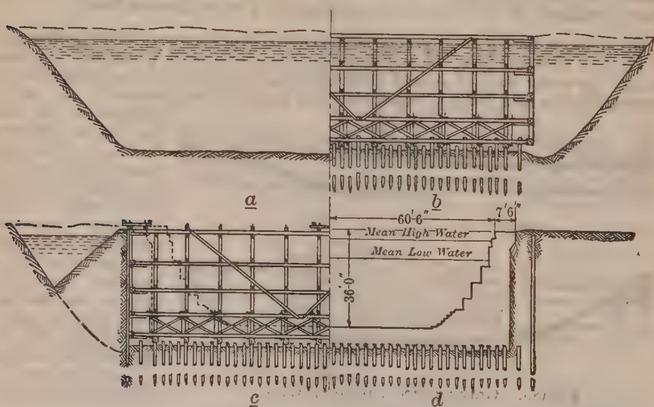


Fig. 55. Graving Dock, Mare Island, Calif.

crib work and then backed up with earth, as shown in (*c*). The interior of the crib was then pumped dry and the structure completed as illustrated in (*d*).

The side-wall, or English plan, is illustrated in Fig. 56. This method has been used with success, in the construction of some European graving docks, (*a*) illustrating the preliminary step of driving sheathing and excavating a trench for the side-walls; (*b*) completion of this trench and driving of the side-wall foundation piles; (*c*) the completion of the side-wall and the excavation of the interior or "dumpling," and the driving of the interior foundation piles; (*d*) completion of the work. In this method of construction there is a tendency for the side-walls to be thrust or move inward when the interior of the "dumpling" is removed. The floor work should be constructed in sections, or heavy bracing placed between the wall work pending completion of the bottom. A modification of this method was employed in the preliminary attempts at construction of Dry Dock 4, Navy Yard, New York, but failed, the uncompleted side-walls being thrust in.

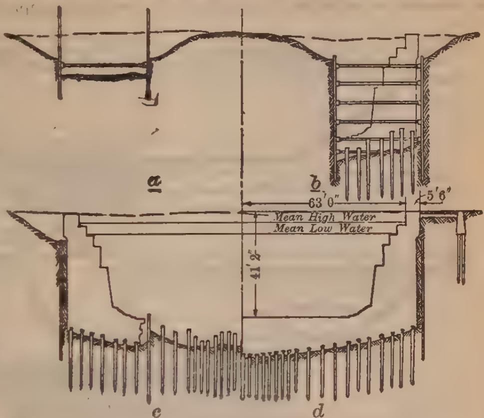


Fig. 56. Graving Dock Construction, English Trench Method

The dry dock at the New York Navy Yard was constructed by the method illustrated in Fig. 57. This dry dock was located in water-bearing quicksand soils. Open excava-

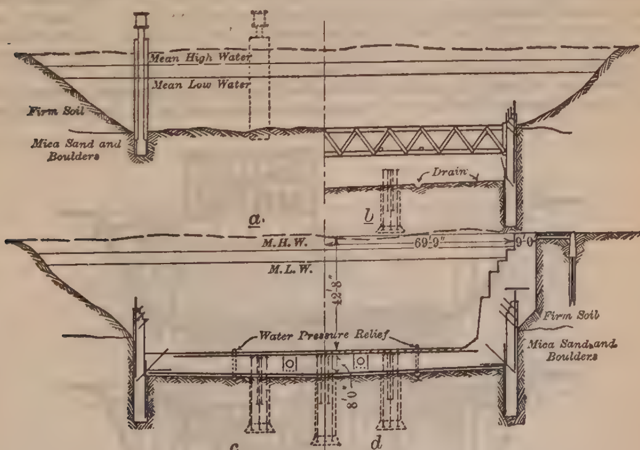


Fig. 57. Graving Dock, New York

tion was made by steam shovel to the quicksand. The entire site was then surrounded by a reinforced-concrete cutoff wall, placed by pneumatic process. The excavation of the interior was then made, bracing being placed between cutoff walls, as illustrated in (*b*). Interior rectangular piers or anchors were placed by pneumatic process. The floor was then built in 20-ft. sections, entirely across the structure, as shown in (c). Side-walls were then built up and the structure backfilled.

At Kobe, Japan, the character of the site was such that after an unsuccessful attempt to construct in cofferdam in the dry, it was decided to employ a method not requiring

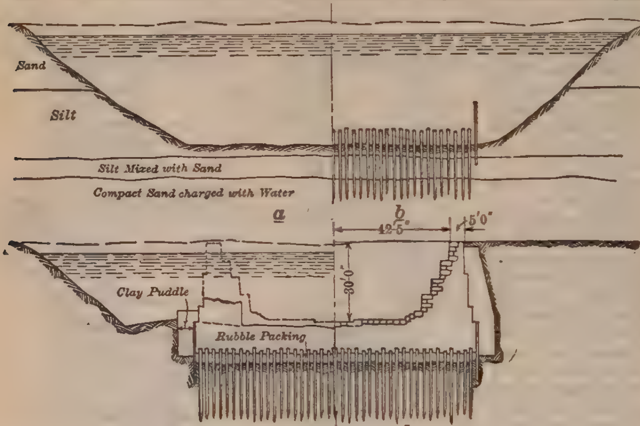


Fig. 58. Graving Dock, Kobe, Japan

the unwatering of the site. The plan followed is illustrated in Fig. 58. Excavation was made in water, as shown in (a), and piling then driven as shown in (b). Concrete was then deposited by skips and buckets under water, as illustrated in (c). Upon completion of placing of the underwater concrete up and above elevation of high water, the structure was pumped out and interior lining placed, as shown in (d).

A somewhat different method of construction proposed in connection with another dry dock is illustrated in Fig. 59. Excavation made under water, foundation piling

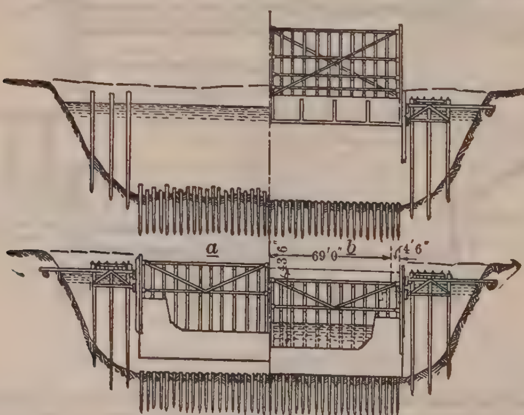


Fig. 59. Construction Employing Floating Caissons

driven as shown in (*a*), the foundation piling then cut off under water to grade, and the graving dock structure built in sections by means of caissons lowered to place, as illustrated in (*c*) and (*d*), in a somewhat similar manner to the method employed for breakwater and quay wall construction. See Arts. 3 and 15.

The above method was proposed for the construction of the Pearl Harbor, Hawaii, Navy dry dock. In the construction of this dock the general principle was used, but the 16 sections were constructed on a floating dock, and then by lowering the floating dock a steel tank cofferdam was floated over a section and bolted to it. The floating dock was then lowered and the whole section floated to place, the tank cofferdam flooded and the section lowered, the side-walls and bottom of the section built complete in the cofferdam, the tank cofferdam then floated by removing water from the tank, and then floated away and placed on the next section and so on. Space between sections was closed by concrete deposited in tremie.

Tremie Concrete. In view of the difficulty, at times, encountered in unwatering to permit construction in the dry, it is sometimes desirable to place concrete under water. This may be accomplished by the use of bottom-dump buckets or tremies; care must be exercised that the cement is not washed away from the fine and coarse aggregate. This is best accomplished by confining the concrete within forms and depositing it within the form in one continuous operation. In tremies, 12-in. pipes are found satisfactory. The operation is started by inserting a plug (usually a sack of straw) inside the tremie pipe and filling the tremie hopper above with concrete, the lower end of the tremie pipe resting on the bottom, being raised slowly as the concrete flows out. Care must be taken to exclude the outside water from entering or breaking through the flowing concrete. More than one pipe may be used in placing, but they should not be spaced more than 20 ft. apart, allowing a 10-ft. radius for the flow of the concrete. See Sect. 11, Arts. 9 and 10.

29. Gates and Pumping Plant

Caissons or Gates. Graving dock gates are, in many respects, similar to lock gates, description of which will be found in Sect. 15, Art. 19. In American practice caissons are more often employed for closing entrance to graving docks, and are usually box- or ship-shaped structures which are flooded and sunk in place at the seat, the removal of water from the interior of the dock

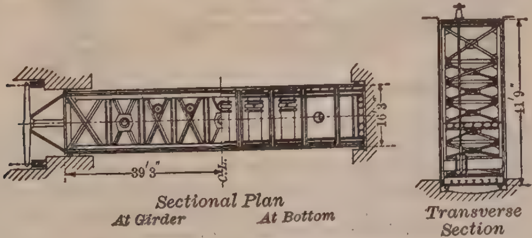


Fig. 60. Entrance Caisson, Sliding Type

drawing them up against the seat. When to be removed after the graving dock has been flooded, they are pumped out and hauled or towed to one side. The sliding types of caisson for which in connection with the dry dock structure a recess is provided at the entrance, is moved across the entrance by a hauling device, and returned to its recess in a similar manner, an illustration of which is shown in Fig. 60. This is the favorite type in English graving dock practice and it is employed for the entrance of the Quebec graving dock. The removable type of caisson is rectangular or ship-shaped, as shown in Fig. 61, or hydrometer-shaped, as shown in Fig. 62, the second type having a contracted

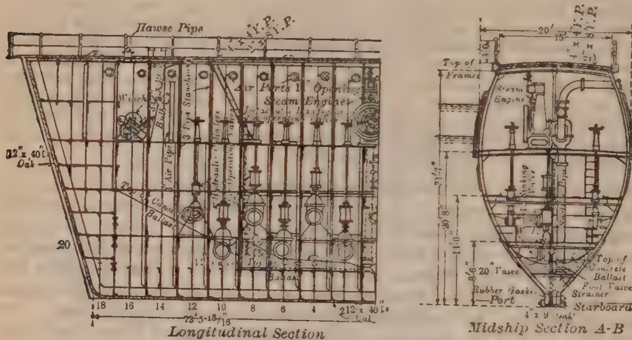


Fig. 61. Ship-Shape Caisson Gate

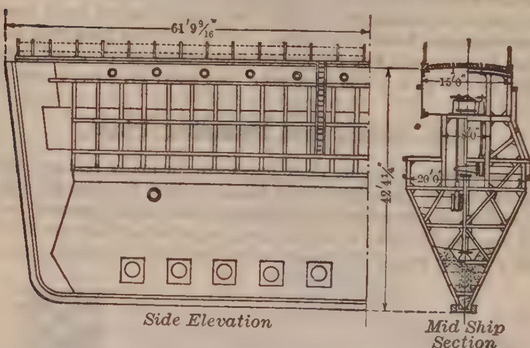


Fig. 62. Hydrometer-shape Caisson Gate

section for a depth of 6 to 12 ft. at the water line, to permit more rapid raising and lowering when being pumped out or flooded. Or the caisson may be of the rectangular or fin keel type. Where the caisson bears on the sill, water is excluded by either depending on the close fit of a timber meeting piece, or on rubber or hemp gaskets. To resist water pressure the caissons are built and designed with a series of interior horizontal girders or a main horizontal girder at about low water elevation, with the frames acting as a vertical beam. When one of the types of ship-shaped caissons is in place and the graving dock pumped out, the lifting effect of the exterior water is only active on the surface of the caisson in contact with water, so that the buoyancy at such time is somewhat over half of that of the caisson when floating. However, in nearly all cases the dead weight without the water ballast is not sufficient to hold the caisson in place under these conditions. Where the proportion of the net uplift or buoyancy to the total horizontal water pressure on the gate does not exceed 20%, the gate will not need special anchors. Hand- or power-driven capstans are usually installed on the caisson, at both ends, besides the pumping plant for raising the caisson. Caissons of this type being floating structures, care must be taken in the design to assure proper stability under all conditions, and for that purpose it is necessary to make the calculations and curves transverse and longitudinal showing displacement, center of gravity, center of buoyancy, contained water, and metacentric heights under various conditions, especial attention being given to the effect of the contained free water during flooding and lowering. On completion of the caisson these calculations should be checked by inclining experiments.

Modern practice in building caisson gates is departing somewhat from orthodox ship construction and is instead more and more following bridge practice. This tends toward economy, inasmuch as the trusses within the caisson are made to withstand primarily the water pressure, when the gate is in position, and the dry dock empty, whereas the stresses encountered in the ship construction become secondary in the case of the caisson gate, as its seaworthiness is a secondary consideration. Rivets in the shell plating need not be countersunk except on the outside to facilitate calking, and ordinary bridge practice of button heads is followed.

Fig. 62*a* shows a caisson gate of rectangular box type.

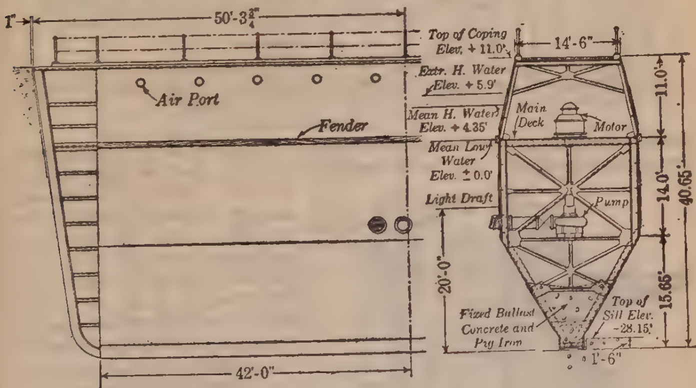


Fig. 62*a*

Pumping Plant. A graving dock is freed of water by a power pumping plant, electricity or steam being generally used, the selection depending upon the special conditions obtaining in the locality. The plant should be divided into two or more units, so that in case of a breakdown it would be still possible to operate the graving dock. The units employed have usually been vertical or horizontal centrifugal pumps and should be of such combined capacity and design that the dock without a ship in it will be unwatered by the entire pumping plant in from 1-1/2 to 2-1/2 hours. More recent practice in dry dock pump design, has employed pumps of screw or spiral type. The water head at the beginning of the pumping will be zero, the power being used up in friction and in velocity head, so that the pumping plant must be designed for the especial conditions in order to obtain overall efficiencies of over 50%. It is usually impossible to remove the last 2 ft. of water over the floor by these main pumps, the blocking and block bearers holding back the water so as to result in a surging effect in the drainage forebay. Where possible, main pumps should be

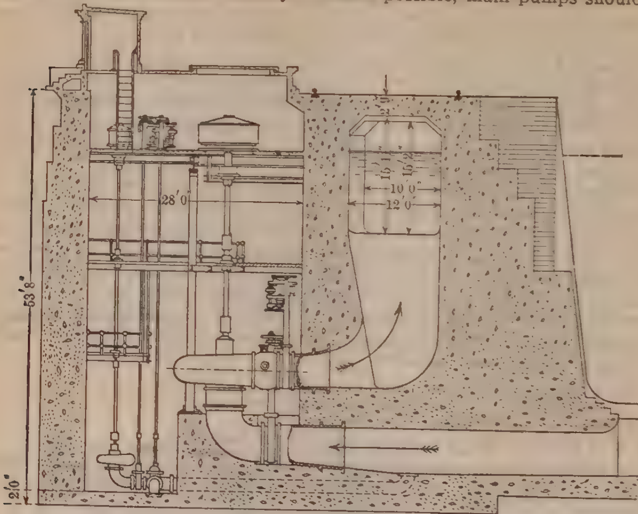


Fig. 63. Pumping Plant

placed low enough and with intakes so arranged as to postpone as long as possible this surging effect. For finally freeing the graving dock of water and for regular drainage purposes, smaller drainage pumps are provided. Fig. 63 shows a sectional elevation of the graving dock pumping plant for Dry Dock 4, at the New York Navy Yard; there are 3 main pumps and 2 drainage pumps.

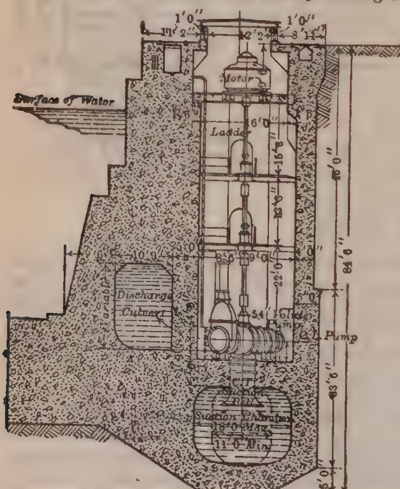
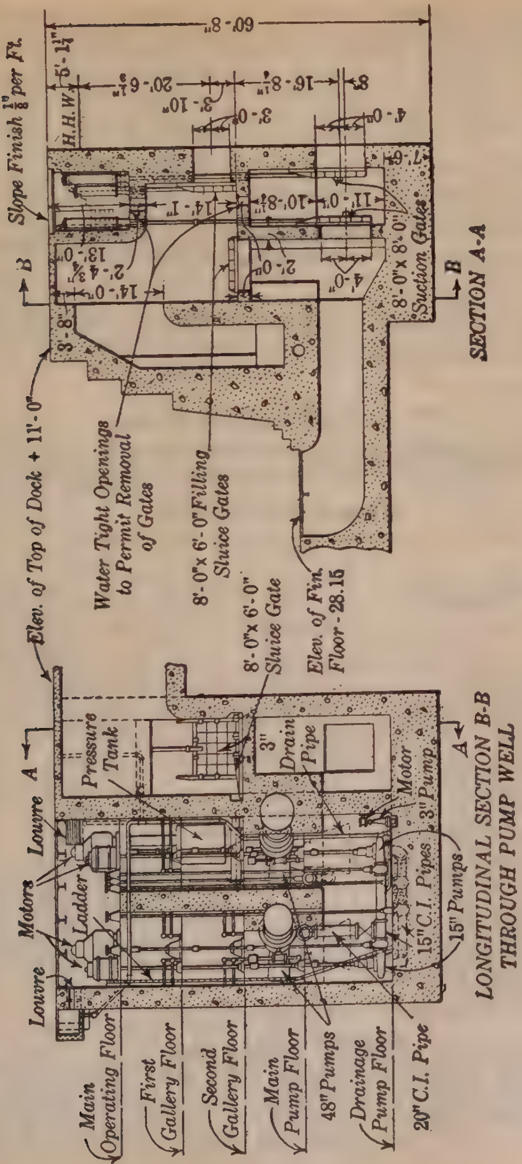


Fig. 63*a*. Cross-section of Pumping Plant, Philadelphia, Pa., and Norfolk, Va.

Fig. 63*a* shows a sectional view of the graving dock pumping plant for the dry dock at the Navy Yard, Philadelphia, Pa. There are three 54-in. electric-motor-driven pumps. The same general type of pumping plant is employed at the Navy Dry Dock in the Navy Yard, Norfolk, at the State Dry Dock at Boston, Mass., and the Balboa Dry Dock at Panama.

Fig. 63*b* shows a transverse and longitudinal cross-section of a pump well and plant. The well was built and placed as an open caisson. The main pumps are screw or spiral in type.

In the design of the main pumping machinery, the work to



SECTION A-A

LONGITUDINAL SECTION B-B THROUGH PUMP WELL

Fig. 63b

be done by the entire plant is obtained by the total volume and weight of water contained in the graving dock at high tide; the product of this weight by the distance between its center of gravity and the elevation of water surface at high tide will give the work done. The required capacity of the pumping plant may then be obtained by dividing the product by the number of minutes in length of time allowed for pumping out the dock, and in the same way the required motor horsepower obtained by assuming a 50% efficiency. It is customary to employ constant-speed motors.

Flooding of the structure is usually accomplished by flooding valves installed in the entrance caisson or by sluiceways and gates making connection with the sea or by both. These should be designed so that the structure can be flooded within 1 hour. On account of the location of the flooding gates in the lower part of gate caissons, frequently under certain conditions considerable sediment is carried into the dock by this method of flooding, so that sluiceways and gates, connecting to tide water at a higher elevation, are often more desirable. When of large area, they also make more rapid the final leveling of water in the interior and exterior of the graving dock, and make possible a more expeditious removal of the entrance caissons.

30. Cost of Graving Docks

The Cost of Graving Docks varies between wide limits, depending, as it does, upon the character of foundation, dimensions of the graving dock, partic-

Masonry Graving Docks

No.	Location	Built		Length, ft.	Cost		
		Begun	Finished		Total	Per running foot	Per ton, max. ship docked
1	Boston, Mass., N. Y. # 2	1899	1905	748	$1 100 000	$1470	$28.80
2	Charleston, S. C., N. Y. # 1	1902	1908	575	1 250 000	2170	36.80
3	Mare Island, Cal., N. Y. # 2	1899	1910	752	1 680 000	2210	44.20
4	New York, N. Y., N. Y. # 4	1905	1912	703	2 450 000	3470	49.70
5	Norfolk, Va., N. Y. # 3.	1903 1910	1908 1911	732	1 730 000	2360	38.50
6	Philadelphia, Pa., N. Y. # 2	1899	1908	754	1 470 000	1950	38.50
7	Portsmouth, N. H., # 2	1899	1906	751	1 125 000	1500	29.70
8	Puget Sound, # 2	1908	1913	838	2 100 000	2500	32.70
9	Boston, Mass.	1914	1919	1188	3 231 000	2500	28.09
10	Quebec, P. Q., Canada	1914	1918	1190	3 000 000	2500	30.30
11	Norfolk, Va., N. Y. # 4	1917	1919	1022	4 356 000	4250	46.85
12	Norfolk, Va., N. Y. # 6	1918	1919	471	765 000	1625	63.75
13	Norfolk, Va., N. Y. # 7	1918	1919	471	714 000	1515	70.00
14	Balboa, Panama	1911	1915	1100	2 795 000	2540	34.10
15	Pearl Harbor, T. H.	1910	1919	1008	5 356 675	5314	73.58
16	Philadelphia, Pa., N. Y. # 3	1917	1921	1005	6 300 000	6268	67.41
17	Hunter's Point, San Fran.	1916	1918	1013	....	......	....
	Timber Graving Docks						
18	New York, N. Y. # 3	1873	1897	668	555 000	835	23.30
19	Norfolk, Va., N. Y. # 2	1887	1889	500	505 000	1010	38.30
20	Philadelphia, Pa., N. Y. # 1	1889	1891	500	549 000	1100	45.00
21	Puget Sound, W. N. Y. # 1	1892	1896	650	633 000	970	27.20

ularly the usable depth and the kind of material employed in its construction. In masonry dry docks the lining of the structure with stonework, particularly granite, will add considerably to the cost. Excluding land value, timber graving docks have cost between $700 and $1200 per running foot of dock, and masonry graving docks between $1500 and $7000 per running foot of dock.

The cost per ton of maximum size ship that can be docked was obtained by taking the product in feet of the usable length, allowing 5-ft. clearance; the width of entrance at 1 ft. above blocks, less 2-ft. clearance; and the depth to top of blocks or 6 in. over sill at high water, dividing this by 35 to obtain long tons and taking 0.7 of this as a probable block coefficient. The following numbers refer to the table just given:

(1) Founded on blue clay and gravel and is built of concrete lined with granite.

(2) Founded on stiff marl, is of concrete lined with granite.

(3) Founded on piles driven in soft clay. Built of concrete with granite sills and entrance.

(4) Founded on reinforced-concrete surrounding walls and interior pedestals placed by pneumatic caissons in fine running mica sand and built of reinforced concrete lined with vitrified brick, granite sills and entrance.

(5) Founded on piles in clay and marl. Built of concrete, granite lined. This dock, after completion, was extended in length.

(6) Founded on piles driven in sand and gravel. Built of concrete, granite sills and entrance.

(7) Founded on ledge rock, concrete lined with granite.

(8) Founded on sand, gravel, clay; concrete and granite.

(9) Founded on ledge rock, concrete, granite, entrance sills, coping and steps, intermediate seat dividing dock into two sections when desirable, side-walls lined with granite. Actual cost in excess of that given by perhaps $500 000 on account of war conditions.

(10) Founded on rock, concrete with granite altars and entrance, sills, sliding caissons at entrance.

(11), (12), (13) These docks founded on so-called marl containing much shell detritus at entrance and blue clay. Concrete laid directly on marl and clay except at entrance to Docks 6 and 7, founded on timber piling.

(15) Cost and delay due to collapse of earlier work and fact structure was built in 16 sections as floating caissons. Foundation, coral rock, mud lava.

(16) Cost due in part to war construction, and also heavy water flow through open gravel on which the structure rested.

(17) Replaced older dock on same site. Cut out of soft serpentine rock by hydraulic dredge and rock lined with concrete.

The cost of graving dock appurtenances, although independent of foundation conditions, bears some relation to the character and size of the structure. Electric power capstans, including foundations, depending upon size, have varied from $7000 to $9000 each, and the pumping plant from $40 to $70 per motor or engine horsepower. Caisson gate costs will be a function of the width and depth of the entrance and, in general, will vary as the product of the square of the width and the square of the depth.

The upkeep and maintenance cost of the body of masonry graving docks is small and should not exceed 0.25% per annum. The upkeep cost of graving dock appurtenances varies with the character of the same, the cost of repair and replacing of blocking being high and, to some extent, dependent upon the frequency of use, and may be taken at from 10 to 15% of first cost, per annum. The upkeep cost of caisson gates is, in general, similar to that of the hulls of steel vessels and may be taken at from 3 to 5% per annum.

31. Floating Dry Docks

Definition. A floating dry dock is a structure of wood, wood and steel, steel, or reinforced concrete, capable of being submerged by the admission of water to its interior compartments, at which stage, if desired, a ship is floated into position; the structure is then raised by the removal of water from its interior compartments by pumping, or other means.

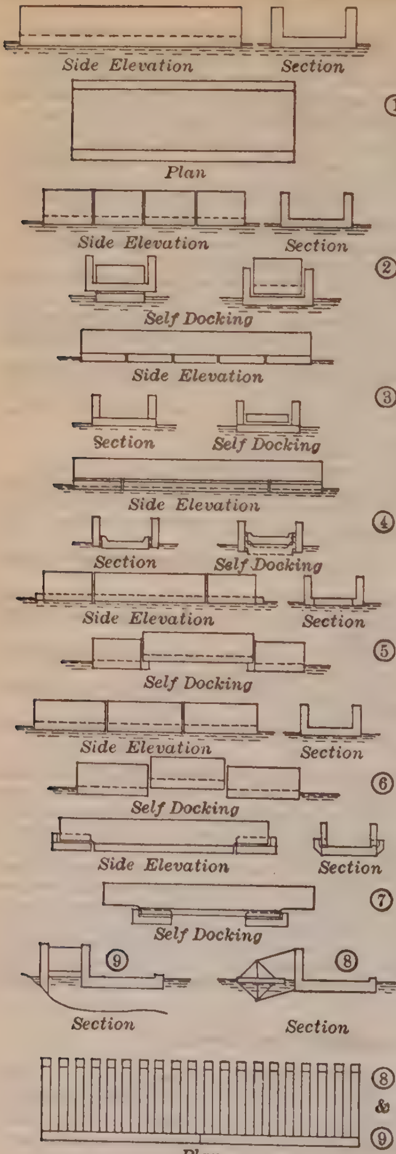


Fig. 64. Types of Floating Docks

Relative Advantages. Lower first cost, less time to complete, less operating expense in pumping, and the fact that it is a movable property, are claimed as advantages for floating docks as compared to graving docks.

The comparative first cost is entirely dependent upon the type of the floating dock and the location of the site available for the graving dock. Under the most favorable conditions for graving dock construction, the time consumed in the construction of a floating dock is somewhat less than would be the case with a graving dock. A graving dock has the advantage of permanence, more rigidity, less danger of accident, less expensive maintenance and repair, and less annual charges on account of its greater length of life. The selection of the type of structure is entirely dependent upon local conditions and requirements.

Types of Floating Docks are shown in Fig. 64: (1) The simplest type of floating dock is the solid trough type of timber or steel. It has the advantage of simplicity, of longitudinal rigidity, with the disadvantage that it must be built in its entirety intact, and can not be self-docked for repairing, cleaning, and painting.

(2) Sectional docks consist of a number of solid trough sections or slices of Type (1), and are constructed of such length that any one section can be docked, as shown, by one or two other sections. The sections are connected together by locking logs which hold them in alinement, but are not intended to furnish longitudinal rigidity for the entire structure. The operation of the dock usually requires skill and care in order that each section may take the load directly above it, the buoyancy of sec-

tions being adjusted by the amount of water pumped from them. This adjustment of weights, however, is not always necessary. With ocean-going vessels, the vessel itself is designed with sufficient longitudinal girder strength to distribute inequalities of load, unless the hull structure has been seriously injured, in which case the vessel should not be docked on a sectional type of floating dock. This type of structure is a favorite type in timber, especially for commercial use; on account of its low first cost and its flexibility, additional sections can be added, increasing the length of the dock.

(3) The Rennie type consists of continuous side-walls forming girders for longitudinal rigidity. These side walls rest on, and are fastened to, a series of pontoons of such dimensions that one or more pontoons may be removed from under the side-walls and lifted for repair by the remaining structure. Docks of this character are built with steel walls and timber pontoons, or entirely of steel.

(4) In the Clark and Stanfield type the side-walls are continuous and extend to practically the full depth of the structure, the pontoons being fastened between the side-walls, the fastening consisting of fish-plates and bolts. Pontoons can be self-docked; repair, cleaning and painting of side-walls being performed by careening of the entire structure. The side-walls being deeper than in the Rennie type, more rigidity is secured. Disconnection of pontoons for self-docking and repair is complicated. This type of dry dock is usually constructed of steel. The Havana dock now in commercial use in New York harbor, and the Naval dock at Algiers, La., are constructed on this principle.

(5) The Pola type is a sectional dock consisting of three sections each a solid trough. The central section is of about the same size as the two end sections together. The sections are connected together for longitudinal rigidity, which connection, however, is complicated. The central section is locked by the two end sections by means of a projection on each. The end sections are docked by the center section.

(6) The Cunningham Sectional type of dock is, in many respects, similar to Type (1) Sectional dock, but consists of larger sections, provision being made for joining the sections together by fish-plates and bolts. Self-docking is accomplished by raising one section with two other sections.

(7) The Maryland Steel Company type consists of a main pontoon of solid trough section with side-walls extending beyond its length and carried on two shorter end pontoons having low independent side-walls for stability in self-docking operations. In self-docking the two end pontoons raise the central section and the two end pontoons are in turn raised by the center section. This type is constructed in steel. The Naval dock "Dewey" at Olongapo, P. I., is of this type.

(8) The Single Side Wall type consists of one rectangular side-wall and pontoon or a series of sectional pontoons. On the other side of the wall is provided a shallow pontoon with vertical members and series of two parallel members pin-connected to the vertical members and to the side-wall of the dock. The purpose of this pontoon is to afford stability and hold the structure upright when raising or lowering.

(9) The Off-Shore type is similar to the Single Side Wall dock, but the vertical member is fixed directly on shore, the pontoon being done away with.

Types (8) and (9) can both be constructed with openings between pontoons so that the vessel after being lifted can be deposited on a separate floating structure or fixed platform, so that while the ship is undergoing repair the dock is ready to lift another vessel and is then known as a Depositing Dock. Neither of these types is in extensive use.

In the Burgess sectional dock the side-walls are replaced by an open framework in which slide ballast pontoons, which always remain on the surface and give stability to the structure. A structure of this character was built of timber in New York Harbor, but is not now in use.

The Camel type, as shown in Fig. 64*a*, consists of a one-piece rigid trough, closed at one end and having a gate similar to lock gates at the other end. The ship enters the dock, is partially lifted by removing the interior water, the gates are then closed, and the water inside the trough removed by pumping. This dock has the advantage of longitudinal rigidity, but it cannot be self-docked for repairing, cleaning and painting.

Location. Floating docks require considerable depth of water for their operation, this depth being fixed by the draft of the maximum ship to be docked, height of blocks, clearance of blocks to keel of ship, depth of floating dock pontoons, and the allowance under the dock for clearance and for filling up. The requisite depth must often be secured by dredging and the expense of dredging operations at such great depth is considerable. In many harbors, the excavation for deep floating docks will rapidly fill up. Provision must also be made for mooring the structure, which is usually done by placing it alongside a pier built for that purpose. When this is not done, approach piers to the floating dock must be constructed. A pier or quay is also necessary outside of the entrance to the floating dock to moor vessels while awaiting docking, or to assist in guiding the vessel into the floating dock. Such a structure is also necessary in connection with the use of graving docks. Sheltered locations are desirable for floating docks, this being more necessary on account of the difficulty of handling the dock in a high wind.

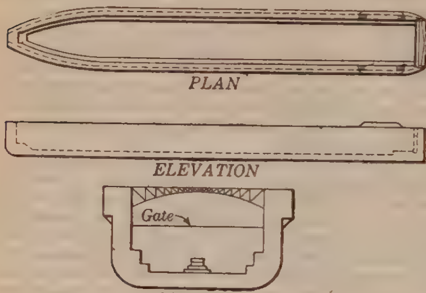


Fig. 64a

Data on Design. A floating dock must have sufficient transverse strength so that the excess buoyancy of the side-walls and the pontoons at the sides will be enabled to carry the concentrated weight of that portion of the ship supported on the central keel and side bilge-blocking, and also when floated light to carry the excess weight at the sides of the side-walls and the pumping machinery. The floating dry dock, when fully sunk, ready to receive a ship, is usually flooded with water to within a small distance from the position of the outside water. Especially with timber floating docks is this done. To guard against accidental over-flooding in the sinking of the dock, safety decks are often provided in the side-walls, above which the dock can not be flooded. When a ship has been placed in the dock and the operation of raising is commenced, the contained water is, as a rule, first removed from the side-walls. As the displacement of these side-walls per foot of lift is from necessity considerably less than the weight per foot of lift of the maximum-sized ship to be docked, the water is entirely removed from the side-walls when the ship has been lifted only a few feet; thus the maximum water pressure due to difference between external and internal water head occurs at the period when the side-walls have been emptied of contained water. The exterior sheathing or plating of the structure, beams, frames, and trussing must be analyzed for this condition of external pressure, which is illustrated in Fig. 65. In most floating dry docks this will not exceed 20 ft. of water head. This condition may be avoided by pumping from compartments of the pontoon directly under the ship, before removing all the water contained in the wing walls. It is to be borne in mind, however, that with a floating dry dock it is possible to raise partially out of the water a ship of greater displacement than could be lifted clear of the water, and for this reason it is often advisable to design the structure to withstand water pressure equivalent to the maximum submersion of the pontoon deck. In timber structures, as the timber is more or less constantly immersed, a reduction should be made in the allowable strength in tension, compression, and bearing. In steel floating docks, the practice of structural steel and steel ship design should be followed, the plating being considered as a continuous beam and not a catenary.

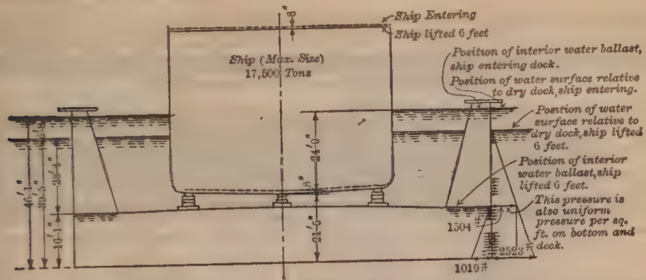
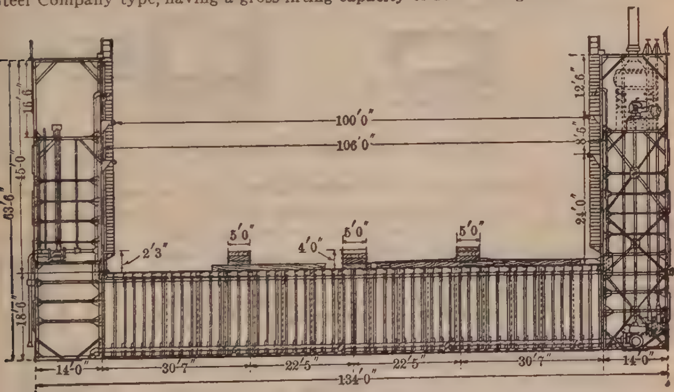


Fig. 65. Maximum Water Pressure on Dock

Fig. 66 shows a typical cross-section of a self-docking steel floating dock, Maryland Steel Company type, having a gross lifting capacity of 18 000 long tons.



Transverse Section

Fig. 66. Steel Floating Dock

Fig. 66*a* gives a plan, longitudinal section and cross-section, side and end elevation of a 45 000-ton steel floating dock of the bolted, three-section trough type (Cunningham) designed but not built.

Fig. 67 shows a half cross-section through one of the sections of a sectional timber dry dock, a type favored in commercial use on account of its flexibility, that is, the possibility of increasing its length and sizes by adding additional sections. The floating dock in question consists of 6 sections, each 80 ft. long, having a combined maximum lifting capacity of 18 000 long tons.

Fig. 67*a* gives a plan, longitudinal and cross-section and half end elevation of a 12 000-ton timber sectional dry dock.

Loading. The loading conditions to which a floating dry dock is subjected are dependent upon the method employed in removing the contained water from the structure. It is desirable, to some extent, to pump water from the various compartments so that the buoyancy or lifting effect will bear some relation to the load or weight of that part of the vessel directly above this

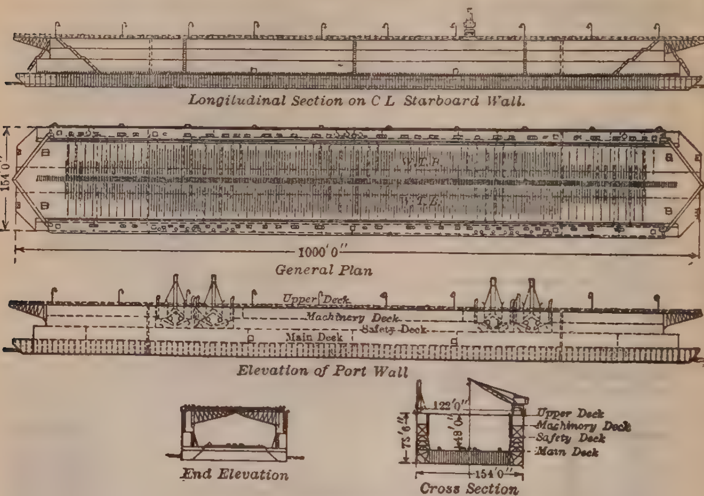


Fig. 66a. Cunningham 45 000-ton Dock

section of the floating dry dock. In order to obtain the maximum lifting effect of a floating dry dock, it is necessary to remove all of the contained water from the structure, with the result that the bending strain must be taken by the dock structure or the ship being docked. Consequently the manner of regulating the pumping and lifting dock and ship is of greatest importance, as it might readily be so operated as seriously to strain both ship and dock.

The Bending Moment produced in the ship and dock is shown graphically by Fig. 68: (a) showing the effect when lifting a long ship, (b) when lifting a short heavy ship, and (c) the method employed to decrease the bending moment by removing only a portion of the water from the end compartments. Fig. 69 illustrates the transverse loads to which a section of a floating timber dry dock is subjected.

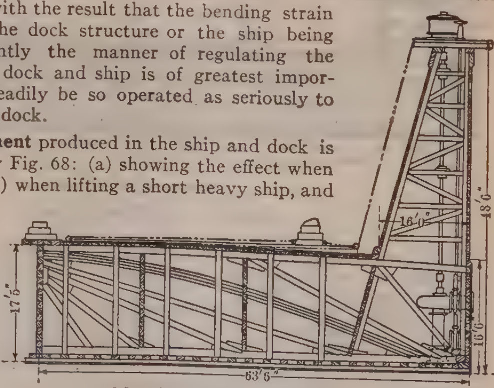


Fig. 67. Timber Floating Dock

Effective Capacity. The effective lifting capacity of a properly designed floating dry dock, whether of timber or steel, or a combination of them, will vary between 60 and 70% of the displacement of the pontoons. In general, it may be taken that one-third of this displacement will represent the dead-weight of the structure and the contained water that can not be re-

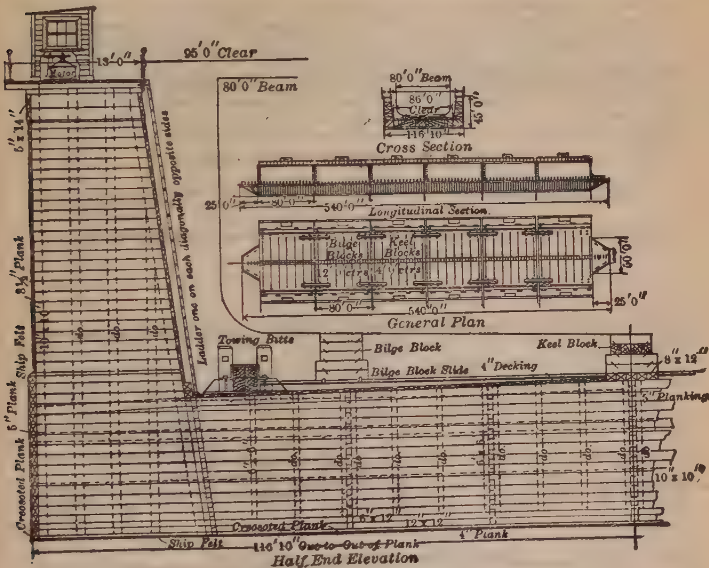


Fig. 67a. 12 000-ton Timber Dock

moved, and two-thirds will represent the effective maximum lifting capacity. In timber floating dry docks the buoyancy effect of the timber must be coun-

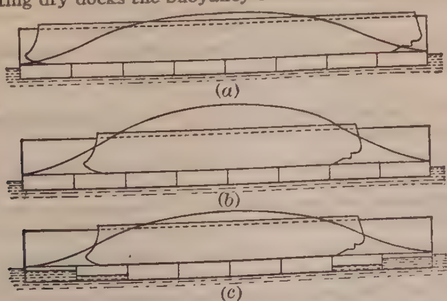
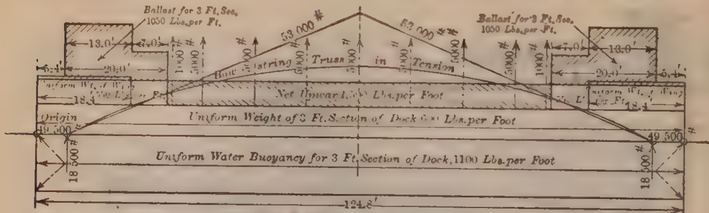


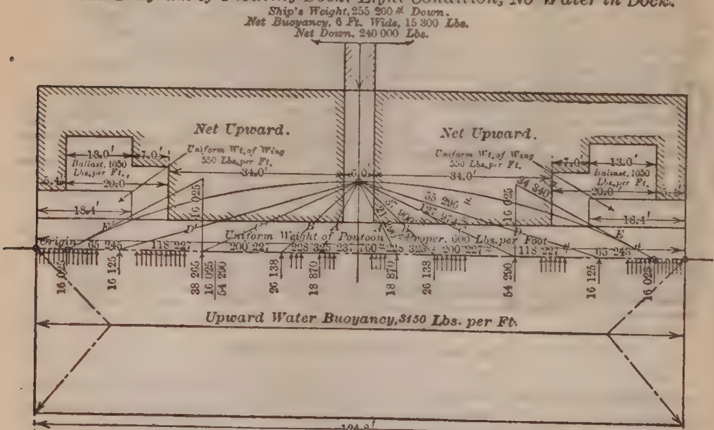
Fig. 68. Longitudinal Bending Moment

teracted by ballast, in which case proper allowance must be made for the reduction in weight of this ballast by reason of its submersion. Ballast is not needed in steel floating dry docks and usually is not necessary in floating dry docks of steel and timber.

Costs of Floating Dry Docks of different types can be given only approximately, as these are influenced by the varying costs of materials, size of struc-



Load on 3 Ft. Section. (One Frame)
Load Diagram of Floating Dock, Light Condition, No Water in Dock.



Load on 3 Ft. Section. (One Frame)
Load Diagram of Floating Dock, Full Load, Ship on Dock.

Fig. 69. Transverse Loads

ture, and location in which it is to be built. Such approximate relative costs may be taken as follows:

Timber solid trough type....................	$50 to $75 per lift long ton
Timber sectional type.	$48 to $70 per lift long ton
Timber pontoons, steel wing walls.........	$60 to $80 per lift long ton
Steel, sectional or solid trough..............	$70 to $100 per lift long ton

These costs are for the floating dry dock complete, but take no account of the cost of the necessary work required in connection with the operation of the structure, such as piers, approaches, dredging and other work. The variation given allows for differences in size and details and for location at which the floating dry dock is built, the higher figures for the steel structures being based on costs on the Pacific Coast of the United States.

The upkeep and maintenance costs of floating dry docks depend on the location and the materials employed. All timber floating dry docks have been in continuous use for 60 years, with only slight repair to them. In waters in which marine borers are prevalent, the exterior of timber pontoons must be protected by yellow metal, copper, tar felt covered with creosoted sheathing plank; or all planking must be creosoted. The interior of compartments will not require protection. The annual charge for upkeep of nine steel docks, as given by L. E. Clark, was 1.12% of first cost. Experience with such docks in warm climates indicates that annual upkeep cost will, in instances, exceed 3% of first cost. Steel structures in sea water require periodic painting to prevent undue corrosion.

32. Stability of Floating Docks

Stability. In order to secure the requisite stability against overturning when light or when lifting a ship, provision must be made for longitudinal and transverse watertight compartments. In a rectangular floating structure the general stability varies directly as the square of the number of watertight divisions. In timber floating dry docks, in most cases, the structure is divided into two transverse watertight compartments, additional longitudinal timber bulkheads being provided on either side of the central watertight bulkhead, but these bulkheads merely act as strengthening and swash bulkheads and do not materially affect the transverse stability. In steel floating dry docks four to six transverse watertight divisions are usually provided. The metacentric height is represented by

$$\mathbf{GM} = \frac{\mathbf{I - G\,B \times V - \Sigma i}}{\mathbf{V}}$$

in which I is the moment of inertia of the water-plane of flotation, $G\ B$ the distance between center of buoyancy and center of gravity, V the volume of displacement in cubic feet, and $\Sigma\ i$ is the summation of moment of inertias of interior contained water surfaces. Or the stability moment is

$$\mathbf{M_1} = \frac{\sin\theta}{35}(\mathbf{I - GB \times V - \Sigma i})$$

This calculation should be made for both transverse and longitudinal conditions, and it should be remembered that with a ship on the dry dock the stability of the ship must be taken into consideration, I, G, B, V and $\Sigma\ i$ of the formulas being the combined expressions for ship and floating dock.

In order to apply the necessary formula, diagrams, or curves should be available showing the characteristics of the floating dry dock under various conditions.

Fig. 70 gives an example which shows the curves of displacement, center of buoyancy, center of gravity, weight of contained water, and moment of inertia of exterior and interior water planes. Employing this and information as to similar characteristics of the maximum sized ship to be dry docked, curves can be drawn showing the metacentric heights at various drafts of ship and dock as is shown in Fig. 71, which is a typical case for a 12 000 long ton capacity Timber Sectional Floating Dock.

Imprisoned Air. When sinking a floating dry dock it is necessary to afford means for the escape of the imprisoned air. This is done by providing piping for air escape with connections at the upper parts of compartments, or in timber dry docks by providing air escapes directly through the deck.

Wind Effect. Sufficient stability should be provided so that with a maximum sized ship on the dry dock there will be no undue heeling of the structure due to wind action. The angle or heel of inclination θ is obtained from

$$\cot \theta = \frac{D \times G\,M}{A \times C\,L}$$

in which A is total pressure of wind in pounds, D is the displacement in pounds,

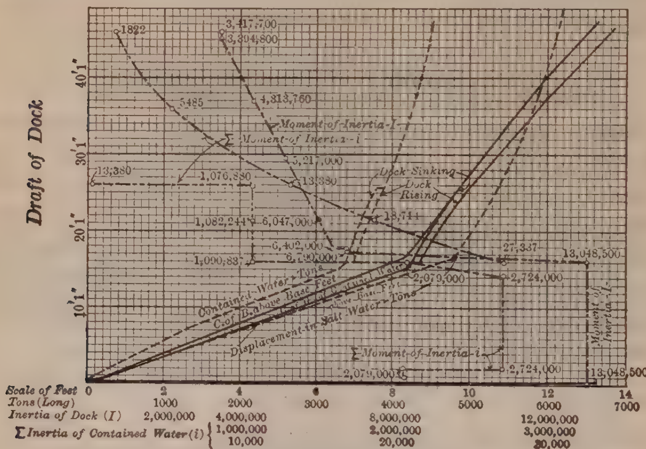


Fig. 70. Displacement and Other Curves

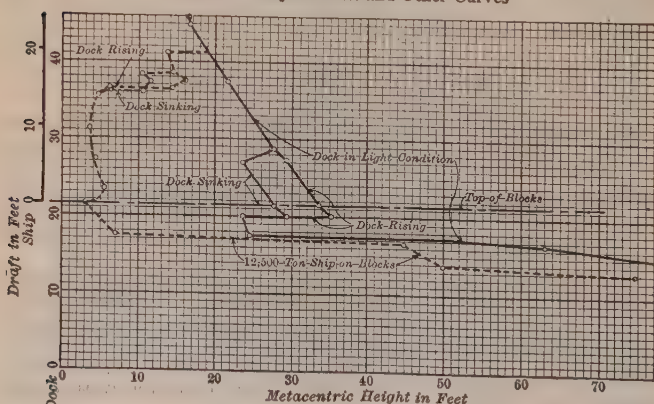


Fig. 71. Transverse Stability Curves

$G\,M$ is the metacentric height, and $C\,L$ is the distance between center of pressure and center of lateral resistance. A list in excess of 10 degrees is dangerous.

Pumping and Flooding Plant is so arranged that each watertight compartment can be controlled in pumping or flooding, independently of other compartments. In a solid trough dock, one or two main pumping plants are usually provided, on one or both sides of the dock, and connected to a pumping main, having a lateral connection to each compartment. Fig. 72 shows the general arrangement of such a pumping plant. Each section connection is controlled by direct-acting wedge-shaped valves, the valve rod being brought to one station by a system of bell crank levers, the dockmaster located at this station controlling the entire pumping of the dock. In sectional floating dry docks it is necessary to provide a separate pumping plant in each section. These may be driven individually or by means of continuous sectional shafting connected between sections by universal couplings, the operating power being either steam or electric. The most satisfactory system at present in use is electrically operated vertical shaft centrifugal pumps. In sectional docks, with two pumps in each section, one on each side of the structure, a cross-connection is often provided so that in case one pump should break down the section could still be operated by the other pump. This involves a longer time for pumping out and also serves to decrease the stability of the entire structure by decreasing the stability of one of its sections. The sinking by flooding is carried out by flooding valves or gates, leading to the various compartments, the dockmaster controlling this entire operation to insure even settlement without undue straining of dock or ship. Floating docks are often fitted with an indicator system to show in a central control station the depth of water in all compartments. An entirely satisfactory system has not as yet been successfully applied.

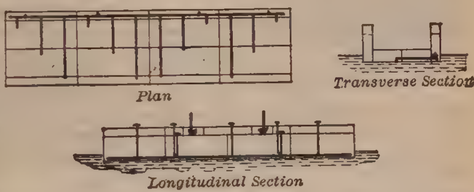


Fig. 72. Arrangement of Pumping Plant

It has been proposed to raise floating docks by compressed air, by removing the water from compartments by forcing air into them, and in a somewhat similar manner sinking by allowing this air to escape. While this has been tried experimentally on a small scale, it has not been successfully used in practice. The general idea has had some success when applied to raising sunken vessels by pumping air into the hull by forcing it into submerged cylindrical pontoons, made fast to the hull by chains or wire cables.

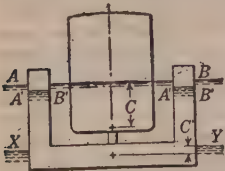


Fig. 73

Power Required in Operating Floating Dry Dock. The work done in raising a floating dry dock with a ship is directly proportional to the weight of the dock light and weight or displacement of the ship:

$$E = D \times c - D' \times c'$$

in which, as shown in Fig. 73, D is the displacement in pounds of ship and floating dock ready to lift, c in depth in ft. below surface of water of center of buoyancy of D, D' is displacement in pounds of dock or ship and dock wholly or partially raised, c' is depth in feet below surface of water of center of buoyancy of D', and E is the work in foot-pounds performed in raising the dock from position as shown by water line at $A - B$ with respect to dock to position as shown by water line at $X - Y$ with respect to the dock.

The necessary data for application of the formulas can be obtained from curves showing displacement and position of center of buoyancy of the ship and displacement of dock, weight of contained water and positions of center of gravity and center of buoyancy. The characteristics of the pumping plant can then be obtained by fixing on the time in which it is desired to raise the floating dry dock with a ship of maximum weight, usually taken at from 1 to 1-1/2 hours. The overall efficiency of an electrically driven centrifugal pump of the type herein described should be between 50 and 60%.

Fig. 74. Blocking

Blocking. The arrangement of blocking on a floating dock is in many respects similar to that employed in graving docks, and is shown in Fig. 74.

FERRIES AND SPECIAL WORKS

33. Ferry Racks

Ferry Racks. The landings or terminals for ferries consist of a slip with piles and fenders on each side called racks, transfer bridge, platform, and ferry shed. The racks are usually built diverging outshore, the inshore ends closely fitting the guard rails of ferry-boats. The racks consist of one or more rows or banks of piles with upper, lower, and one or more intermediate string pieces or wales on which are fastened vertical fender planking of oak, yellow pine, or some other hard wood. Where the ferry-boats are small or light, and the traffic infrequent, a single row of piles is used, the piles being spaced about 3 ft. on centers. The rack is stiffened at the inshore end by additional piles or clusters of piles, as this portion of the rack receives the principal wear and shock. The height of the rack is dependent upon the range of tide, as it is evident that the fender system must be of sufficient height, or depth, so that the guard rail of the ferry-boat, when loaded or unloaded, will never be below or above the rack. The wales are made up of several thicknesses of heavy planking or dimensioned timber, so as to be readily fitted to the required curve and are bolted to the round piles. The vertical fender planking is fastened to the wales by drift bolts or dock spikes at the upper and lower extremity. Ribbon pieces are fastened to the wales and through the vertical fenders by countersunk-head screw bolts. The upper ends of fender planking are sawed off at a bevel to shed water, and the tops of piles are rounded for the same purpose and often covered with canvas or zinc for protection against the entry of water and consequent rot. Frequently the fenders at the inner end of ferry racks are coated with heavy oil, or grease. In heavy ferry rack construction, two, three, or four rows of piles are employed, the piles in the rear rows being staggered, and the tops of rear rows are cut off at a lower elevation. In two-bank racks two or more wales are fastened to the back of the front high row and one or two heavy wales to the front of the second pile row, so as to transmit the shock of the incoming ferry-boat to the second row of piles, and if three rows of piles are used, the third row is also provided with wales of this type. The outshore ends or entrances of racks are usually constructed with a cluster of piles, driven a slight distance apart, and drawn together at the top, and lashed with wire cable fastened with staples, these dolphins or clusters frequently receiving the first impact of the entering ferry-boat, guiding the ferry-boat into the slip. Fig. 75 shows general arrangement of various types.

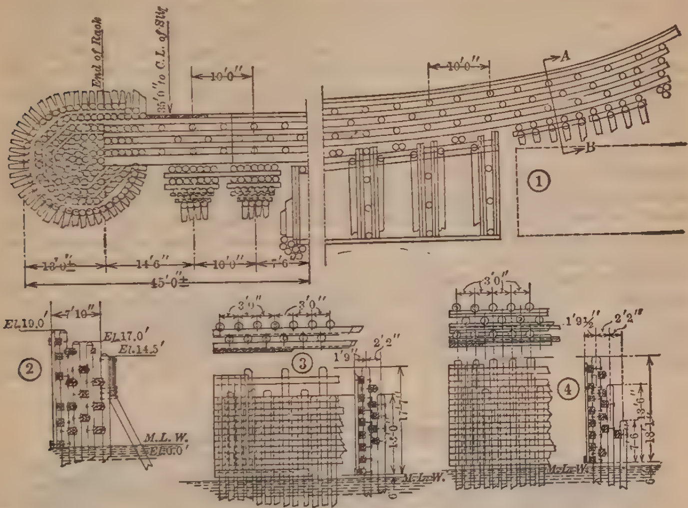


Fig. 75. Ferry Racks

(1) Plan of three- and four-bank rack for Municipal Ferry, New York City, 39th Street, Brooklyn.

(2) Typical section of (1). Front wales, two 6 × 12-in. Rear wales, three 4 × 12-in. Fenders, 5 × 12-in. white oak.

(3) Typical elevation, section, and plan of two-bank rack, D., L. & W. R. R. Passenger service.

(4) Elevation, section, and plan of three-bank rack, D., L. & W. R. R.

The work can be laid out by securing plans of the guard rails of various ferry-boats that are to use the slip, and from these making a plan showing maximum dimensions. Lay out the slip with racks fitting this plan and measure ordinates from and perpendicular to the center line to pile locations. Construct a light temporary platform along the slip and on this locate points on which batten ranges can be fastened, giving the transverse range of piles, the piles being located and driven by measuring out the proper distance on this range. All piles need not be so located, but piles every 10 or 12 ft. having been accurately located, intermediate piles can be driven in general conformity with the desired curve. The work can be laid out from a temporary platform or existing adjoining pier or structure alongside of the site of the ferry slip. A ferry-boat can be moored in place and governing dimensions obtained by driving piles at intervals against the guard rail or the templet used.

34. Transfer Bridges

Ferry Transfer Bridges should be of such length that at extreme tides, with maximum and minimum loading of ferry-boat, the gradient will be possible for traffic, the range being from low tide with heavily loaded boat having little freeboard to high tide with light load and greater freeboard. The outshore end of the bridge, when light, is usually carried by a pontoon, so that it will, at all times, float to about the elevation of the ferry-boat deck. For this adjustment of height the buoyancy of the pontoon is employed, sometimes with the assistance of counterweights fastened to the bridge by wire or chain cables, and carried up over sheaves mounted on an overhead frame. The final adjustment of height is made by hand or power-operated windlass fastened to the end of the bridge. The outshore end of the bridge is usually curved to fit the bow or stern of the ferry-boat, and is provided with heavy hardwood or iron toggles which can be moved out to rest on the deck of the ferry-boat, so that when the weight of traffic leaves the ferry-boat and moves on to the bridge, this weight, in excess of the buoyancy effect of the pontoon, is carried by the ferry-boat and not by the overhead gear. The inshore end of the bridge is supported on rollers, in roller chocks, supported on the platform, or is carried on heavy hinges on a rolling or sliding bearing fitted with car springs or rubber buffers to assist in taking up the shock of the incoming boat. The ferry-boat is secured to the bridge by means of two lines made fast to eyes or chocks on the deck of the ferry-boat, the slack in the lines being taken up by windlasses. The bridge itself usually consists of a platform suspended from two or four structural steel or timber trusses, a timber bowstring truss being frequently used. The overhead gear with counterweight should be of sufficient strength to support the outshore end of the bridge when unloaded, so that the pontoon may be removed when necessary for calking and repair. Fig. 76 shows a ferry

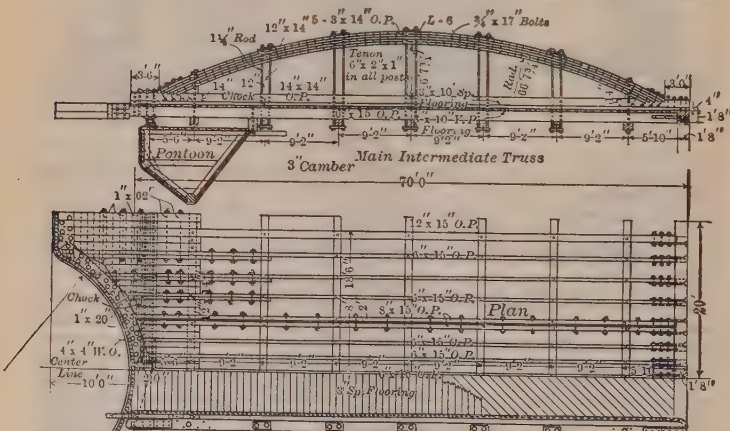


Fig 76. General Traffic Transfer Bridge

bridge and pontoon and is typical of this character of structure in general use. Where double-deck ferry-boats are employed, elevated adjustable gangways or bridges are provided for the upper-deck traffic.

Ferry Houses or terminal sheds are, as regards materials employed and details, similar to pier sheds, structural steel framing with metal siding and sheathing being largely employed. The shed is carried out over the transfer bridge and often projects out a distance over the slip to afford protection against the weather.

Railroad Car Transfer slips are more rectangular than ferry slips for general traffic, and the racks are consequently built straighter or no rack used.

Fig. 77 is a section of a typical three-bank rack; the fender planking is horizontal, and this appears to be standard practice for car transfer racks. The transfer bridges must be of sufficient length to insure a reasonably flat slope for variations in tide and freeboard of the car float. Where possible grades should not exceed 5%. It is not desirable to run locomotives on the float, and the transfer of cars is effected by using two or more pushers or empties to couple up with. When operating on a steep grade the locomotive remains on the level platform. The outer end of the bridge is supported on a pontoon or by means of a counterweight.

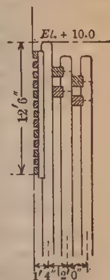


Fig. 77

Fig. 78 shows a transfer bridge with pontoon and overhead gear. The adjustment of the height of track is made by removing water from the compartments of the pontoon by means of a small power pump, or by filling it, the final adjustment being made by the overhead gear with hand-operated winches.

Fig. 79 is a two-track transfer bridge, employed by the N. Y. Central R. R. in Weehawken, N. J. In this the supporting pontoon is replaced by two counterweights, each weighing 46 short tons, the adjustment of height being made by a power-operated vertical screw shaft. In this type the weight of cars is almost entirely carried by the overhead gear. The estimated weight of structural steel for one transfer is 740 000 lb.

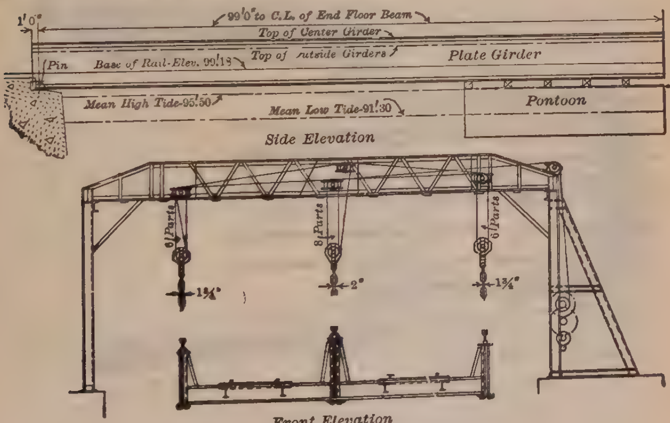


Fig. 78. Railroad Car Transfer Bridge

The general type of railroad and car transfer in use often consists of a bridge made up of two or more timber Howe trusses hinged, or supported, on a rolling log inshore, with the outshore extremity resting on a float or pontoon. Directly over the outer end, and supported on pile foundations, on either side, an overhead or gallows frame is provided; the bridge is connected to this by chain or wire cable tackle, and the final adjustment of height made by this means.

River Car Transfers are employed in connection with ferrying railroad cars across rivers. When the range of tide is not very great, such transfers are similar to those used in harbors. However, when the elevation of tide fluctuates to a considerable extent, as in the great interior rivers of the United States, it is customary to use a method of transfer similar to that shown in Fig. 80.

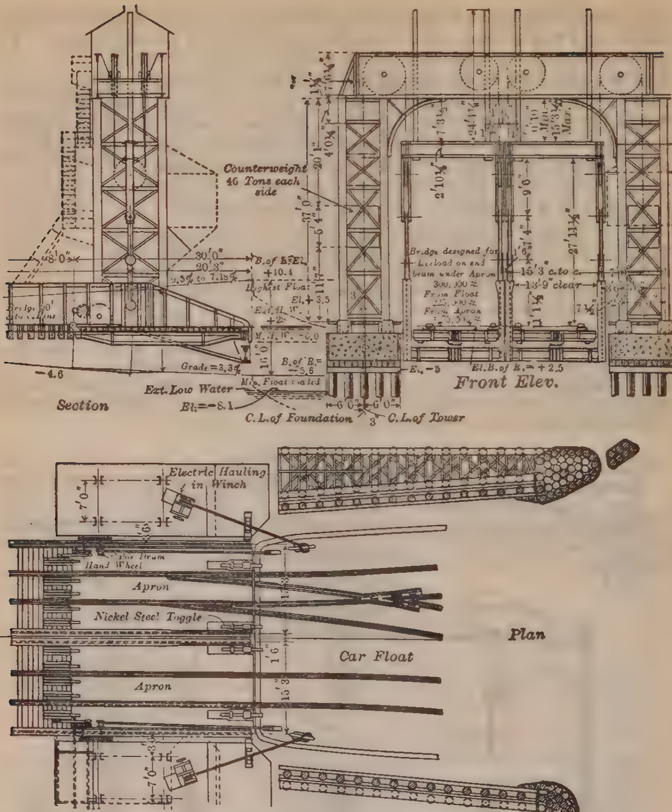


Fig. 79. Transfer Bridge, N. Y. C. & H. R. R. R.

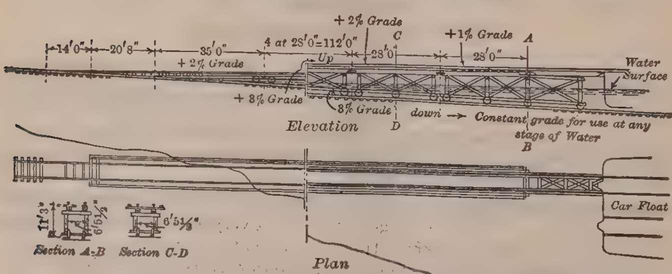


Fig. 80. Cradle, Illinois Central R. R., Mississippi River

This system has been in successful use for many years for transferring railroad cars from or to tracks from car floats. The arrangement is known as a "cradle." Standard-gage railroad track is laid on an incline, usually diagonally, along the bank or river levee slope. The cradle running on this track is hauled up and down the incline according to the stage of tide. Fig. 80 shows a cradle in use at Harahan and Baton Rouge by the Ill. Cent. R. R. for transferring cars across the Mississippi River.

35. Coal, Ore, and Grain Plants

Coal Handling Plants are usually of two general types: (a) The wharf storage type, in which the coal is received by rail or otherwise from shore, by ship or barge from the water, and stored on the wharf itself for distribution and use; (b) the type where the coal is received by ship or barge cargo, and transferred to, and stored on, shore, for shore consumption, or to be again distributed for ships' use; or delivered to the pier from the shore by rail or otherwise, and distributed directly to ships or barges. The last-mentioned type is often provided with some pier storage as an auxiliary feature.

Type (a) is, manifestly, somewhat limited in capacity, depending on the dimensions of the pier and the consequent storage-bin area, as it is inadvisable to store coal in bins to a greater depth than 20 ft., on account of the possibility of spontaneous combustion, and the consequent necessity for re-handling coal. The bins are usually given a bottom slope to permit ready self-discharging, with a minimum of man-handling, the discharging being effected by portable chutes, collapsible or fixed, for discharge into ships' or barges' holds, or for discharge on shore under the bins into carts or railroad cars, the loading of hoppers being effected by fixed hoists with collapsible cantilever arms, on which the bucket or unloading device travels transversely on trolleys or carriages, the distribution in the length of the bin being performed by bucket or belt distribution; or the entire loading device may travel longitudinally of bins for the purpose of distribution. Fig. 81 shows such a plant built in 1901, at the Navy Yard, New York.

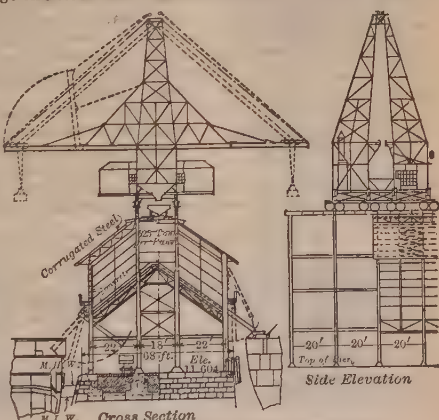
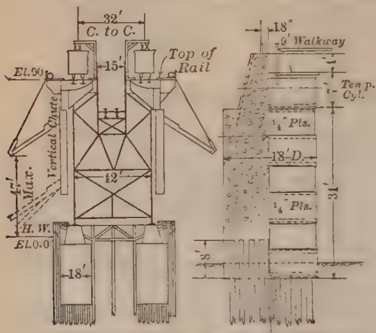


Fig. 81. Coaling Plant, New York

The arrangement of coal handling plants of Type (b) is multitudinous in character, and no attempt will be made to cover this feature in this section, other than to state that for small plants the stationary or movable coal tipple is employed with great economy. In larger plants, a very satisfactory method appears to be a loading apparatus, cable-operated railroad dump cars, discharging to shore storage bins or piles. When coal is required for ship or barge cargo, cars are run out on the pier and discharged into smaller bins or hopper chutes convenient to ship or barge hatchways, cars being discharged directly over the bins by bottom or side-dump cars, or, in some cases, by tipping the car.

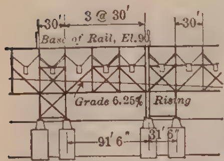
Fig. 82 shows a recent coal pier and plant of Norfolk & Western R. R., Lambert's Point, Va. The method of operation is: Coal from mine arrives at yard and travels by gravity over automatic weighers, and is hauled up incline to pier by power hauling cable to tipping platform, where the car is inverted and contents of car dumped, the capacity being 30 cars per hour. Empty car, on being righted, returns to yard by gravity. Special electric trolley pier cars are raised to the top of the pier by elevators, discharging coal through hoppers and chutes. Empty pier cars are returned for new load. Capacity of this coal pier ranges between 3000 and 5400 tons per hour.



Trans. Section   Detail of Cylinders

Fig. 83 shows the layout of a somewhat similar plant in use by the D., L. & W. R. R., Jersey City, N. J.

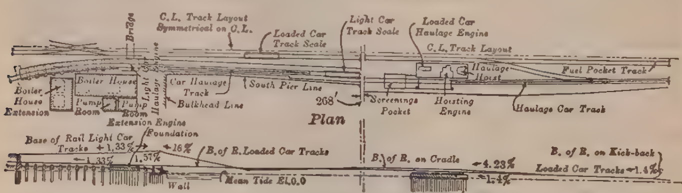
Where a considerable amount of coal is to be stored, it may be placed in piles on shore in close proximity to the coaling pier by being dumped by cars from overhead tracks or trestles, or directly from grabs or buckets operating on travelers. When required, coal is re-loaded by means of grabs or buckets operated by travelers or locomotive cranes and placed in cars. Vessels are loaded with coal from coal barges placed alongside by manhandling in bags, and hoisting by ship's gear and winches. This method of loading is often found most economical, as the ship's crew is employed in the operation, and has other advantages where coal bunkers are small, widely distributed, and access to them is difficult, such conditions especially applying to naval vessels. Floating mechanical coal conveyors are also employed for loading coal on steam vessels.



Side Elevation

Fig. 82. Coaling Plant, Lambert's Point, Va.

Coal is sometimes stored under water in compartments or enclosures for that purpose, as there is no deterioration due to atmospheric action, no losses due to dust blowing away, and the possibility of spontaneous combustion and consequent expensive rehandling is avoided. Coal stored under water 14 years shows a deterioration of only 1-1/2 %.



Elevation

Fig. 83. Coaling Plant, Jersey City, N. J.

Ore Handling Devices for loading and unloading are generally similar in layout to Type (b) Coal Handling Plants, except that the material is seldom moved back and forth, but is usually moved in one direction either from the source to be loaded or from the ship to be unloaded. An important feature in the design is the considerably greater weight per cubic foot of the ore.

Fig. 84 is a section showing an ore trough and pier at Cleveland, Ohio. Ore can be delivered from the vessels directly to freight cars, to stock piles, or placed in the ore trough to be picked up for distribution by conveyors. The ore pier is provided with loading and unloading traveler operating longitudinally of the pier. Fig. 85 shows an ore pier at Two Harbors, Minn.

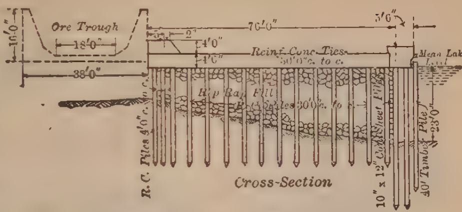


Fig. 84. Ore Pier, Cleveland, Ohio

Grain Handling Plants. When cargoes of grain are to be handled in bulk, the grain is usually loaded and unloaded by means of grain elevators, the material being stored in bins. Grain elevators may be portable, mounted on barges, or constructed on piers or quays. The material is handled by endless belts, by a succession of buckets on a continuous chain, by pneumatic power, or in lesser bulk by buckets or grabs. The pneumatic system of unloading has the advantage of extreme flexibility and can be applied in any position, saving the cost of hand trimming. Grain unloading plants for handling large cargoes are found on the Great Lakes, but not in other American ports where the plants are solely for loading into vessels. For unloading, floating equipment is most often employed, unloading the cargo of the vessel and loading barges for distribution and trans-shipment in the same operation.

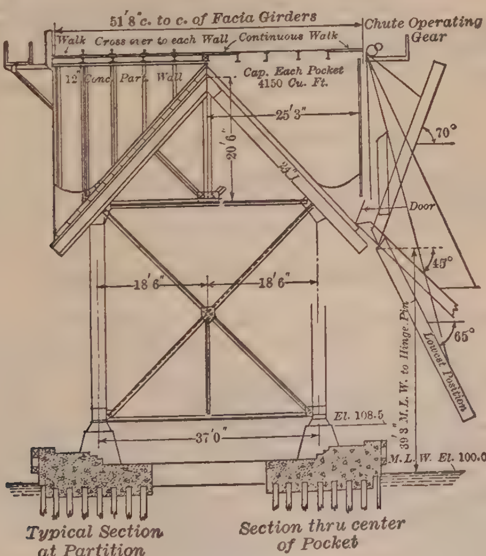


Fig. 85. Ore Pier, Two Harbors, Minn.

Fig. 86 shows the latest type of grain elevator constructed at Girard Point, Philadelphia, Pa., Pennsylvania R. R., having a total capacity of over 1 000 000 bushels.

Grain comes in on cars on a descending grade, and is hauled to and from the unloading shed by cable. Unloading is performed by power shovels into receiving hoppers, discharging therefrom on receiving belts, and is raised to receiving garners and weighed,

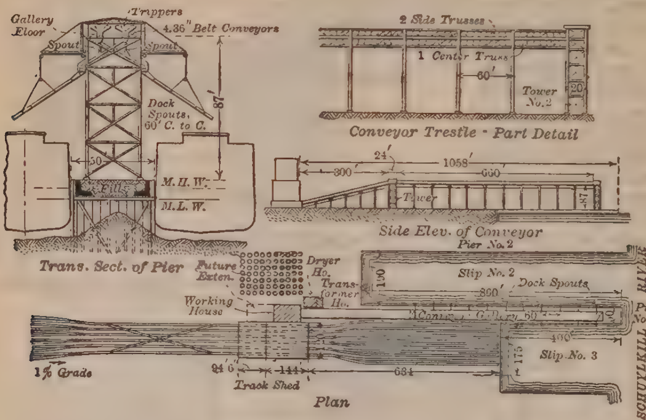


Fig. 86. Grain-handling Pier, Philadelphia

and thence by means of spouts distributed to the storage bins or to the pier shipping conveyor consisting of four 36-in. belts, from whence it is distributed by trippers through side pier spouts.

36. Dumps and Landing Stages

Dumps are constructed and set aside for the handling of refuse material. On account of securing and retaining ample depth for the purpose of navigation, in most ports it is against the law to dump material within the navigable water area, whether the material is of such character as to sink to the bottom or float on the surface of the water and interfere with navigation or be unsanitary or obnoxious. In some of the larger cities public dumps are provided where excavated ma-

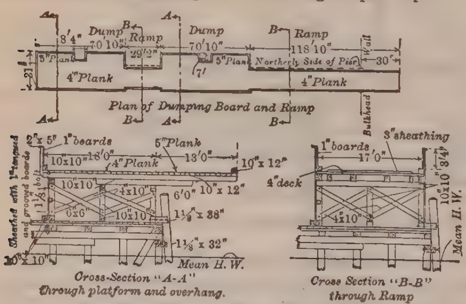


Fig. 87. City Dump

terial or refuse may be dumped into barges for transhipment for local filling, or disposal at sea. Fig. 87 shows a dumping board at Pier 33, New York City. Fig. 88 is a typical example of a dumping board for distribution of excavated material.

Floating Landing Stages are provided where there is considerable range of tide and it is not desirable to incur the expense for the construction of a wet dock, or where, for a similar reason, it is inadvisable to construct a fixed pier or quay with two or more platforms to be used at various stages of tide. Floating landing stages on a small scale are employed for small boat landings and consist of a float or pontoon with a gangway from the pontoon to the pier or quay platform. When constructed, the deck is supported by its own buoyancy or on hollow pontoons, the buoyancy of which must be sufficient to carry the weight of passengers or cargo. The pontoon must be divided into two or more longitudinal and two or more transverse watertight compartments in order to insure stability against overturning when the compartments are partially full of water. The formula to be applied is given in Art. 32 under Stability of Floating Docks.

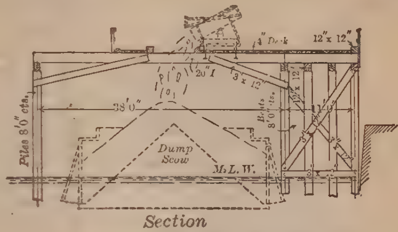


Fig. 88. Railroad Car Dump

The floating landing stage at Liverpool, England, consists of a deck supported on pontoons, and is 2500 ft. long by 80 ft. wide, and has eight gangways or bridges connecting it with the shore. The maximum range of tide at this port is 30 ft.

On many of the interior rivers of the United States, there is considerable freight and passenger traffic, loading and unloading taking place at various places along the bank where no regular landings are provided, the light-draft, stern-wheel steamboats employed running close to the bank and transferring traffic by platforms or gangways. At many of the more important ports where the elevation of the river varies greatly, and ordinary wharves are not available or advantageous, floats are employed. Figs. 89 and 89*a* show how the problem is solved at Memphis, Tenn.

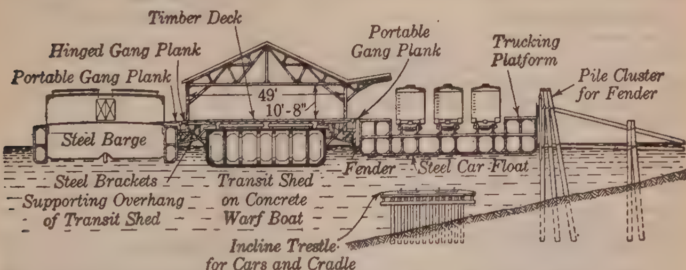


Fig. 89. Freight Transfer between Cars and River Barges, Memphis, Tenn.

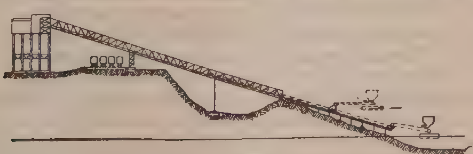


Fig. 89*a*. Coal Handling Equipment, Memphis, Tenn.

Where the river bank is steep, and bulky cargo is to be handled, landing stages or floats are employed in connection with inclines, as shown in Fig. 90. This type of incline is usually double-tracked, with a platform on each track, the two platforms being connected together by a cable running over a drum of a hoisting engine which operates the mechanism. This particular type is in use on the Tennessee, Mississippi, and Ohio Rivers, used in connection with the handling of iron and steel products, coal, and cotton. The telferage systems are also extensively employed for river landings, a typical example of which is shown in Fig. 90*a*, employed by the Illinois Central R. R., South Memphis, Tenn.

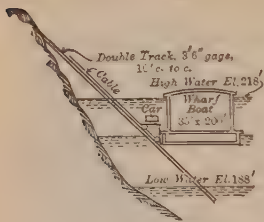


Fig. 90. Car Handling

37. Shipbuilding Ways

Although the construction and launching of ships come within the province of the naval architect, the design and construction of the platform or ways on which the vessel is built is a question of foundation design and one that will be dealt with briefly here.

Shipbuilding Ways are in many respects similar to marine railways, and are, primarily, inclined slips on which a vessel is built, and which project far enough out into and under the water so that when the vessel is ready to be launched it may pass down from the land to the water. As with marine railways, building ways may be of the lengthwise or broadside type. The broadside type is resorted to where the basin or stream into which the vessel is launched is narrow and does not afford the requisite width for lengthwise launching. Broadside ways are common on the Clyde River. Building ways consist of the fixed ways, which is the incline or platform fastened to the foundation, and the cradle or sliding ways. The vessel itself is built on the fixed ways or on blocking, and the fixed ways are frequently not placed until shortly before the vessel is ready for launching. Then the weight of the vessel is transferred to the cradle or sliding ways and in turn to the fixed ways by means of wedges. A lubricant is placed between the sliding and the fixed ways, usually a mixture of tallow and soap, and at the proper time, the vessel is released by sawing or by a trigger device. The gradient or inclination of the ways is fixed dependent on the type and launching weight of the vessel so that with the release, by whatever method secured, the sliding ways and vessel will move down to the water and

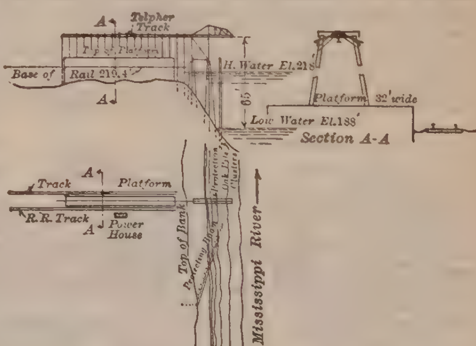


Fig. 90*a*. Telferage Landing

be launched into it. It is customary as a matter of precaution to place horizontal jacks to give the sliding ways and vessel a push or "kick."

The fixed ways consists of two longitudinal strakes, one on each side of the center line of keel, and of sufficient width to carry the maximum moving load of the vessel and sliding ways, the safe bearing capacity of the timber used across the grain being considered.

Inclination of Ways. Inclination or the gradient of the ways is dependent upon the length, type and probable launching weight of the ship to be built and is seldom greater than 7/8 in. to the foot run. Ways for moderate sized steel vessels with a launching weight of from 2000 to 4000 tons have a gradient of 5/8 to 3/4 in.; on larger heavier vessels 5/8 to 1/2 in. is used, and on very large vessels of great length such as the largest transatlantic liners and battle cruisers 1/2 or 7/16 in. is employed. For the last two classes of ships the ways are often built with a varying gradient, that is, with a very flat circular profile, so that the ship when launched and moving down the ways moves on the arc of a great circle. This arrangement has the advantage of bringing less strain on the trigger or launching device, and the vessel once released and moving down the ways receives an increasing acceleration. With a vessel of considerable length this has the further advantage of decreasing the required length of the outboard or under-water portion of the ways; that is, the requisite depth of water at the end can be obtained in a shorter run.

Foundations. The foundations of the blocking, or fixed ways, must, of course, be designed to carry the maximum weight of that section of the vessel built directly above it, and the lower or outboard portion must be designed with a view to the fact that as the vessel is launched the maximum weight section will move down, superimposing its load over the entire lower ways as it passes over it.

As the vessel reaches the water and becomes submerged therein it is immediately in part water-borne, and the foundation directly under the stern becomes relieved in increments of the superimposed weight, while the load under the bow or upper end of the vessel is increased by the upward force of the outboard vertical buoyancy effect. This change in condition continues until the stern or outboard end of the vessel is entirely water-borne, and about to be lifted from the ways when a pivoting action about the upper end of the bow occurs, and that portion of the weight of the vessel representing the upper end reaction is concentrated at the bow or upper end. This pivoting pressure is the maximum pressure brought to bear upon the ways foundation and has a decreasing effect continuing down the ways from the initial point of pivot until the vessel leaves the ways entirely. With larger vessels a pivot cradle is constructed under the bows detached from the sliding ways or cradle, the hull of the vessel at the bow end resting in a trunnion saddle (a) which sets in the pivoting cradle (b), all shown in Fig. 91. In designing ways it is imperative that all of the load and maximum weight pressure conditions be obtained. Fig. 91*a* shows the buoyancy pivoting hogging curves and maximum way pressure curves for a 3500-ton dead weight cargo coal barge. The ways must be extended outboard sufficiently so that the depth of water at the end will be approximately the draft of the vessel at the bow, where the sliding ways leave the fixed ways when the vessel is launched; otherwise the vessel would drop through a vertical distance represented by the excess in draft at this point. In practice, it is quite usual to launch expecting and providing for some drop,

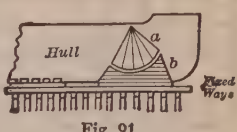


Fig. 91

the opening between the fixed ways at the outboard end being cleared sufficiently so that the hull of the vessel at the bow will not strike any obstruction and cause damage to the hull or the ways.

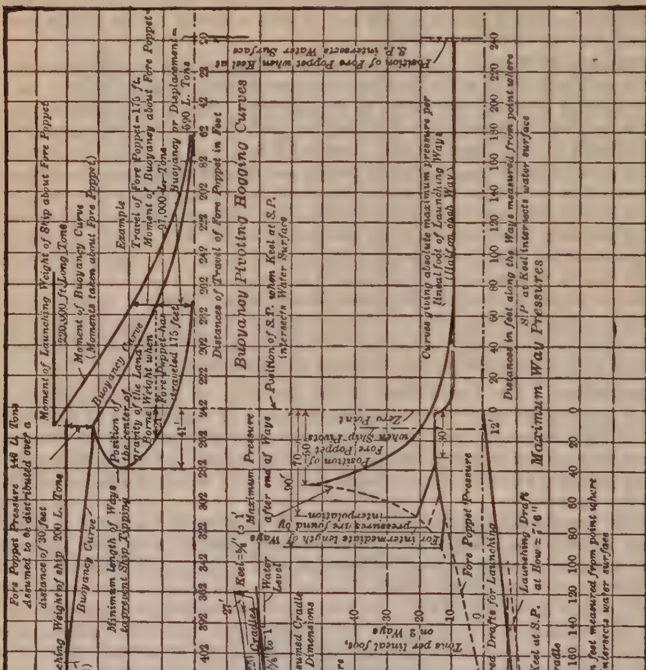


Fig. 91a. Hogging Curves and Way Pressures

The type of foundation required is dependent entirely upon the location and can be treated similarly to other foundation conditions. However, due to the necessary restrictions of being located on some foreshore, timber or concrete piling is the general rule. When timber piling is employed it is, of course, subject to rot and decay above the wet line. Sometimes to avoid the necessity of constantly repairing this portion of the ways, the piling is cut off at mean tide or wet line and concrete or masonry cross-walls or concrete piles are built on the timber piles, or the necessity of such construction is avoided by the use of reinforced-concrete piles of the precast or cast-in-place type, the relative advantage of the two types of construction being, as in other foundations, a question of the particular conditions and restrictions involved. It is impossible to lay down a rule as to the relative desirability or permanency in ways foundations as it so frequently happens that for reasons of development and change in ship dimensions and design or for other reasons it becomes necessary to change materially the carrying capacity of a ways if it is utilized during the life of that particular shipbuilding plant. Experience has rather demonstrated that changes in the foundation, usually with a view to increasing its carrying capacity, are called for whenever a new ship is to be laid down.

The outboard end of the ways, as with the marine railway, must be constructed below water level, and must be heavily braced longitudinally and transversely. A portion of the ways can be built by sawing off the foundation piles by an under-water saw arbor, or by employing divers, or the entire site can be enclosed by a cofferdam and the water then removed and the ways constructed in the dry. The last method is frequently resorted to in new ways, but as the under-water part of the ways is so subject to damage in the course of launching or damage by passing vessels, or floating objects, it is customary before launching to have divers carefully inspect this portion of the ways and to rebuild it also when needed by this method. Fig. 91*b* shows the plan and elevation of a typical building and launching ways for a 9000-ton dead weight cargo ship, the launching weight of which is assumed at 2850 tons. Attention is invited to the piling plan necessary at the outboard end where the foundation at *CC* and *DD* is greatly reinforced. This is to take care of the pivotal pressure hereinbefore referred to.

In construction of merchant vessels frequently the launching weight is always the weight or displacement of the bare or light ship, the boilers, machinery and most of the accessories having been placed while the vessel is on the ways under construction. It is often desirable either for the purpose of rapidly clearing the ways and getting it ready for another ship, or by reason of the fact that the particular plant does not construct and install the machinery and other ship's gear, to launch the hull and place the machinery, etc., afterward, in which case the launching weight is to that extent so much less. In the construction of naval vessels, especially battleships and cruisers having heavy armor and armament, the exterior armor, turret armor and often turrets, together with much of the machinery and all of the guns, are placed after the vessel is launched, so that the launching weight is usually only 30% to 40% of the full load displacement.

Types of Building Ways. The ways shown in Fig. 91*b* is of the simplest type, and that most generally used, and has the advantage of economy of first cost. As previously stated, a more permanent foundation is obtained by the use of concrete cross-walls or pillars. Ways of this character have the disadvantage that the outboard end is under water, not subject to visual inspection, and when preparing to launch there is, of course, some difficulty in providing the proper lubricants under water. Such ways must always be extended some distance out to insure a depth for clearance.

A twin set of ways was constructed by the Newport News Shipbuilding Co., a plan, section and side elevation of which are shown in Fig. 91*d*. In these ways the water is excluded from the lower end by means of floating caisson gates. These ways have the advantage of permanency, shorter length and less height required for overhead crane runway structure. They undoubtedly involve the element of a much higher first cost and entail drainage of the lower portion of the site during the process of building.

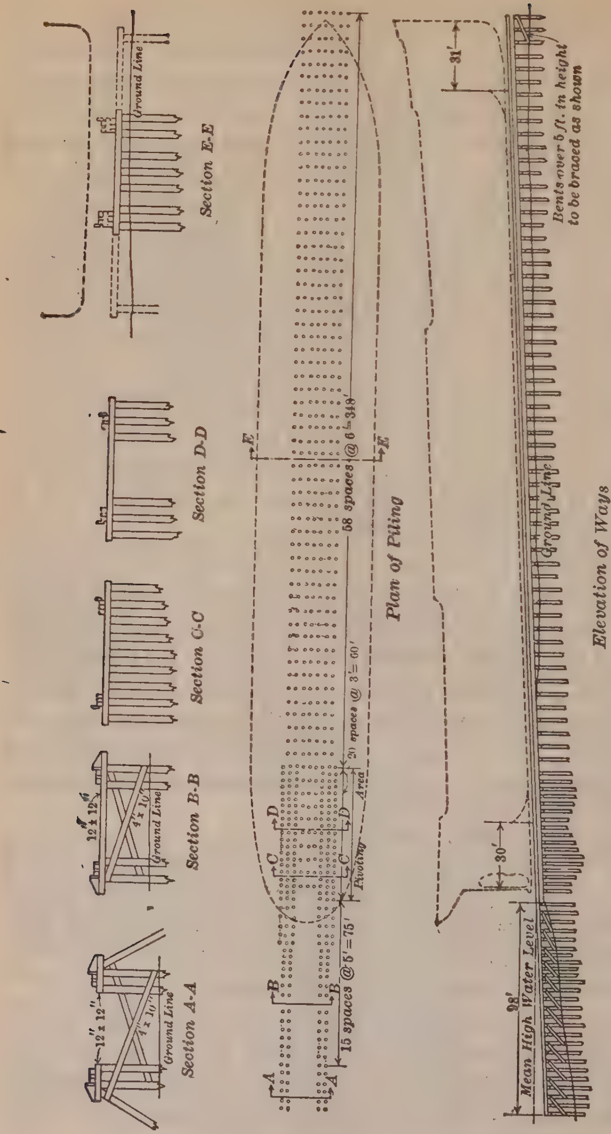


Fig. 91b. Cross-section Showing Transverse Bracing

Description of these types of ways naturally lead to the consideration of the use of a shallow graving dock for shipbuilding purposes. Ships have in the past been built in a basin or graving dock, which results, of course, in tying up this important part of the repair plant and preventing its use for ship docking and repair work; such use would not be resorted to in ordinary circumstances unless there was assurance that the dock could not be profitably used for ordinary docking purposes. A shallow dock has been built at the Puget Sound Navy Yard especially for shipbuilding purposes. A shipbuilding ways of this type is particularly adaptable to this location on account of the extreme range of tide, nearly 20 ft., which would have made the length and cost of an ordinary building ways and the maintenance of the under-water portion very expensive as compared to other locations.

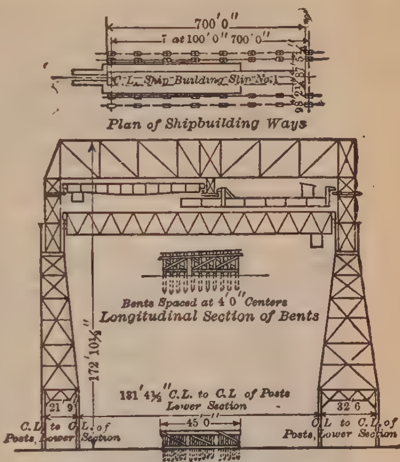


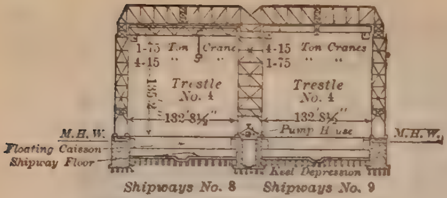
Fig. 91*c*. Timber Building Ways

Shipbuilding Cranes. Because this matter begins to come within the scope of the shipbuilding plant layout and design, it will be very briefly dealt with as a matter of general information. A simple and very effective method of handling the frames, shapes, plates, built-up bulkheads, etc., for a ship is the employment of two or more aerial cable ways, usually arranged longitudinally, picking up material at the head of the slip and moving it longitudinally and transversely to the desired location. Such handling has been effectually employed in merchant shipbuilding, and is especially adaptable to certain types of work.

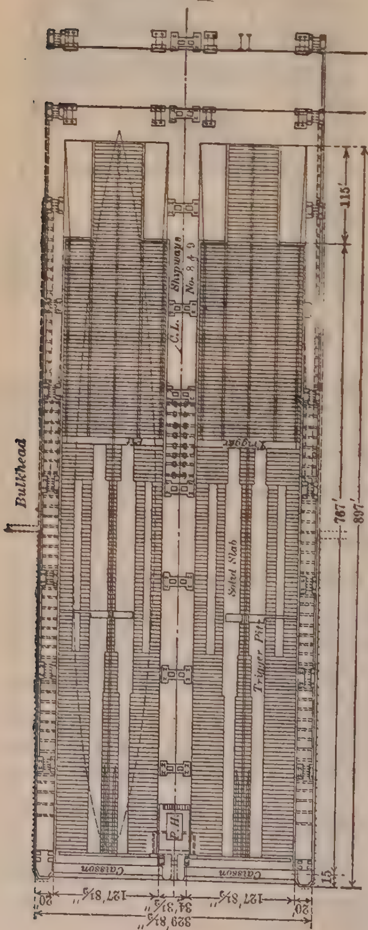
An effective type of crane is the light, fixed-hammer-head tower crane. This usually has a lifting capacity of 5 to 15 tons, and must be so arranged as to cover the entire area of the vessel and be able to handle material from storage and assembly spaces alongside of the ways. The cranes are usually grouped on either side of the ways, three, five or seven being employed. The arms must be put at varying elevations in order to avoid interference and give the requisite clearance for the depth of hull, etc.

A similar type crane is sometimes employed, on one or both, sides of the ways, but instead of being fixed travels on a standard-gage or special-gage track parallel with the ways and at one side, or cranes of this character may be of the jib or luffing type.

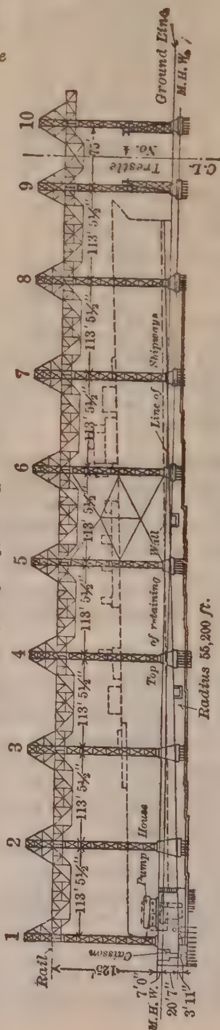
When a large complicated passenger vessel such as the transatlantic liners or a battleship or battle cruiser is to be built it has often been considered desirable to resort to more complete and extensive weight-handling devices than can be afforded by the fixed or movable tower crane service. The building ways of several of the important shipbuilding plants are served by longitudinally traveling cranes; it will be noted in Fig. 91*d* that the twin ways of the New-



Transverse Section Looking Outshore



Plan View of Shipways No. 8 & 9



Longitudinal Section Looking North Showing Battle Cruiser on Ways

Fig. 91d. Timber Ways

port News Shipbuilding Co. are each served with one 75-ton and four 15-ton longitudinal cranes, which pick up material from the assembly and storage space at the head of the ways. Fig. 91*c* is a plan and cross-section of typical building ways and overhead crane structure employed by the Navy for the shipbuilding ways at the New York, Philadelphia and Norfolk yards. The upper cranes each span one-half of the width of the ways, are of 15 tons capac-

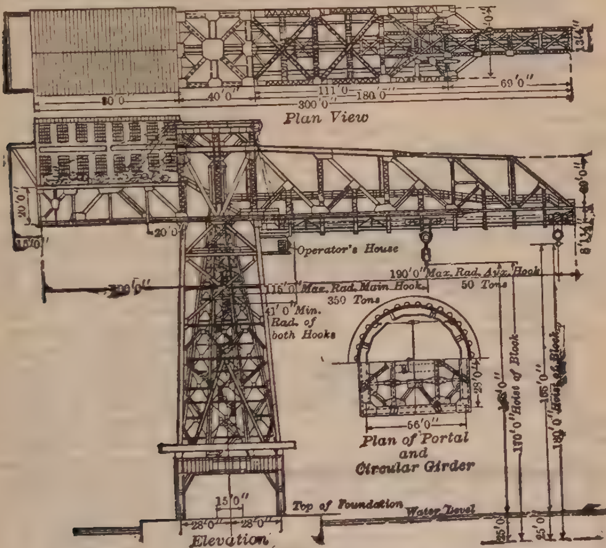


Fig. 91*e*. **350-ton Fitting-out Crane**

ity; the lower, larger crane is of 40 tons capacity. They handle material from the head of the ways and also are so arranged as to pick up material from cars running parallel with the ways. Fig. 91*e* is a side elevation and plan of a 350-ton fixed-hammer-head crane which is to be employed for placing of heavy weights on vessels after they are launched. It is designed to be placed on a special fitting-out pier and vessels are brought to the pier and traversed longitudinally along its side so as to come within the reach of this appliance.

38. Cargo Handling Cranes

Cargo Handling Devices and Appliances are utilized dependent upon the character of merchandise and the special conditions affecting availability of ships' gear and crew. Cargo ships are provided with masts, cargo booms, winches, etc. Modern freighters are usually provided with two to eight cargo booms per hatch. To facilitate or replace such ship gear, piers or quays are provided with various apparatus, cargo hoists, cranes of various types—gantry or portal or semi-portal cranes. elevators, conveyors, revolving cranes, straight line roof cranes, telfers. Besides this, various crawler types of cranes, electric motor trucks, sectional elevators or conveyors.

Numbers in the following examples refer to Fig. 92.

(1) and (2) Antwerp, Belgium, on the Scheldt. Quay wall runs entire length of the city. Principal wharfage capacity is derived from interior wet docks. Range of tide is 16 ft. Open sheds constructed some distance from quay wall. Portable crane spans outside railroad track; cranes handle cargo from the ship to cars, or to the face of the shed, in which it is handled by hand trucks for storage or transfer to cars in the rear for re-shipment. Shed is for transfer or temporary storage. Warehouses for permanent storage are usually found with railroad connections close by.

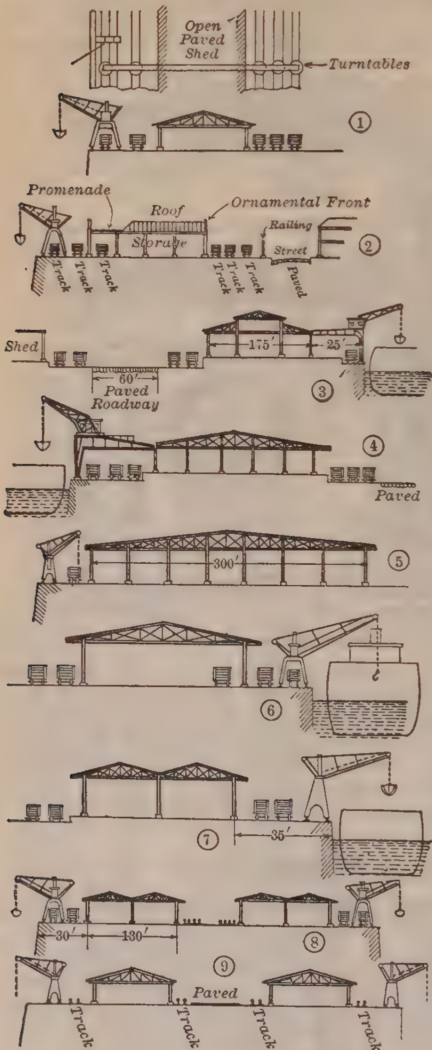


Fig. 92. Cargo Handling Methods

(3) Hamburg, Germany, on the River Elbe. A greater amount of wharfage room is secured by interior basins. The plant of the Hamburg-American Line is the one illustrated. The sheds are about 2500 ft. long and 175 ft. wide, and are from 30 to 35 ft. from the face of the quay wall. In this space are operated electric cranes which span an outside railroad track which handles the cargo from steamer to railroad track or to side of ship. In interior of shed cargo is handled by hand trucks through the shed for local storage or to carts or railroad tracks in the rear for direct delivery, or delivery to warehouses.

(4) Bremerhaven, mouth of the Weser on the North Sea. Tide has range of 8 ft., but on account of insufficient depth of water wet docks are employed. Shows typical section of the North German Lloyd terminal. Sheds are about 200 ft. wide, of corrugated iron siding, with wooden roof trusses, tar and gravel roofs. Electric cranes are provided for loading and unloading; cranes operate by picking up cargo from the holds of steamers by circular movement, swinging onto railroad cars on three lines of track, or to the sides of the shed. Loading and

unloading are accomplished by this system on the shore side, and by barges and ship's boom and tackle on the waterside.

(5) Havre, France, at the mouth of the Seine. Range of tide 25 ft. Wet docks employed. Coffee and cotton landings. Sheds are 300 ft. wide and 2600 ft. long. The sides are continuous rolling doors, roof of tar and gravel, with glass skylights. Cargo is handled by crane from ship to side of shed or to railroad track.

(6) Southampton, England, American Line terminal.

(7) London, England, shows system of handling at Albert Dock. Sheds are of light corrugated iron construction and run entire length of quays.

(8) London, England, Tilbury docks.

(9) Glasgow, Scotland. Typical arrangement of cargo handling.

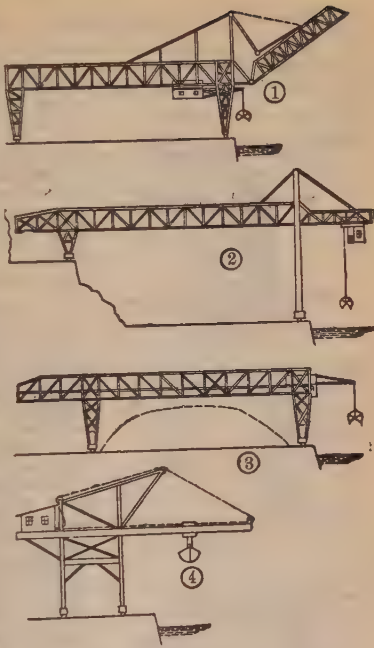


Fig. 93. Cargo Handling Travelers

Movable Shore Cranes for miscellaneous cargo handling, though not generally employed in American practice, are usually employed in connection with handling of homogeneous cargo in bulk, various types of standard-gage locomotive cranes being used for that purpose. Special cranes or traveling transporters are extensively used, the following types being shown in Fig. 93.

(1) One of four loading and unloading bridges at Rotterdam, Holland, 8 long tons capacity.

(2) One of two transporters, 10 long tons capacity.

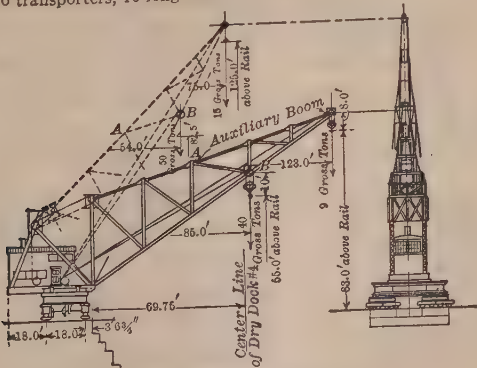


Fig. 94. Traveling Dry Dock Crane

(3) Coal loading and unloading bridge. (4) Loading appliances.

Fig. 94 shows type of heavy locomotive crane employed in connection with ship repair and dry dock work. The crane is of 50 long tons capacity at 68-ft. radius, of lesser capacities at greater reach and of 15 long tons capacity at 123-ft. reach, and travels 120 ft. per minute, with a maximum load. Cranes of this type are in extensive use in various U. S. Navy Yards.

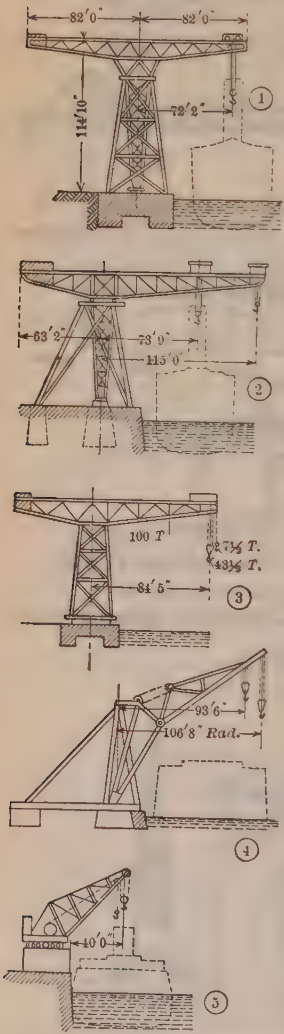


Fig. 95. Heavy Fixed Cranes

Stationary Cranes. Fig. 95 shows types of fixed shore cranes for heavy lifting and for building or repair purposes.

(1) 150-ton hammer-head crane at Bremen, Germany. Interior spindle is supported on base step bearing, the crane revolving in the 4-leg tower.

(2) 150-ton revolving crane at Kiel, Germany. Crane of the hammer-head revolving type. Interior spindle, stepped at the base, operating within the 3-leg support.

(3) Bremen, Germany, capacity 100 tons, hammer-head type, revolving on circular track at the base.

(4) Hamburg, Germany, 100 tons, stiffleg derrick type of crane.

(5) Barrow-in-Furness, England, 120-ton jib crane, revolving on pedestal base.

Cranes of the hammer-head type have been constructed of 250 long tons lifting capacity. Fig. 96 illustrates English practice, and shows a crane at Portsmouth Dock Yard.

Fig. 97 illustrates German practice, and shows a 250-long ton capacity crane constructed for Blohm and Voss at Hamburg, Germany, in 1912. The crane is supported at a bearing on the top of the fixed spindle or tower; the enclosing structure revolves with the crane and bears against a track at the base.

Sheer legs are often used for heavy lifting, Fig. 98 showing a 100-long ton sheer leg with luffing jib device, the reach of the sheer leg being regulated by power-operated horizontal screw, connected to the foot of the rear inclined member. This horizontal screw is sometimes replaced by a vertical quadrant with worm screw drive or pinion and racking. Sheer legs are also used for floating cranes.

Floating Cranes are employed in connection with shipbuilding and repair, for loading and unloading heavy weights, handling lock gates, and for wrecking purposes. Fig. 99 shows the A-frame boom type in extensive use in American practice. The A-frame may be held in position by back stays of structural material or by wire cables; or in smaller derricks the A-frame may be replaced by a mast. Cranes of this kind from 10 tons to 250 tons capacity are in use, the larger capacity being obtained in one crane by a lift directly over the bow, employing water ballast at the other end.

Fig. 100 shows a revolving tower floating crane of 75 long tons capacity. A crane of this type is employed in New York by the Department of Docks and Ferries, for placing quay wall concrete blocks. A similar crane is in use at the Navy Yard, Boston. The load moment is taken care of by fixed ballast, water ballast, and the stability of the structure.

The bridge type of floating crane is shown in Fig. 101, the bridge or trolley running through the uprights so that the load may be transported from shore or vessel to the deck of the crane, or through the crane to be handled on barge or shore at the other end. This crane was constructed in 1901, and rebuilt in 1910, and is at the Navy Yard, New York. A similar crane is at the Navy Yard at Bremerton, Wash. Two bridge cranes of 150 tons capacity at the Navy Yards, Boston, and Pearl Harbor in Hawaii, are similar. The load moment is counterbalanced by power, automatically operated counterweights or by water ballast.

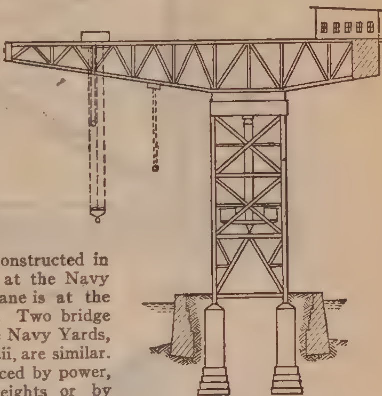


Fig. 96. 250-ton Crane, Portsmouth, England

An Incline Bridge Crane is illustrated in Fig. 102, and is in use by Schneider & Company, Chalons, France, built in 1911, of 25 tons capacity. The revolving jib type crane is shown in Fig. 103. One of 250 long tons capacity is self-propelling and is in use at Wilhelmshaven, Germany. Two similar cranes of the same capacity, with less reach, are in use in the Isthmian Canal Zone, where they are to be employed for removing and placing lock gates, and for other purposes. These cranes can lift 672 000 lbs. The pontoons are 150 ft. long, 88 ft. wide, and 15 ft. 9 in. deep.

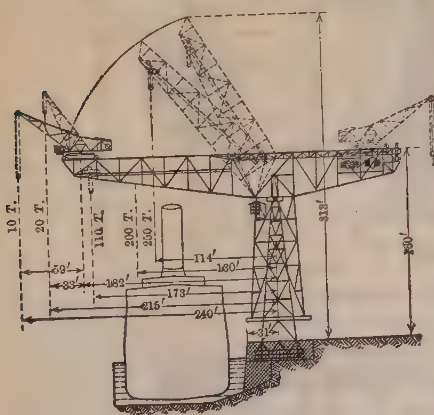


Fig. 97. 250-ton Crane, Hamburg, Germany

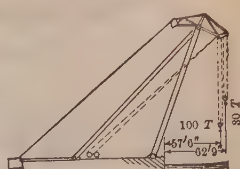


Fig. 98. Sheer Legs

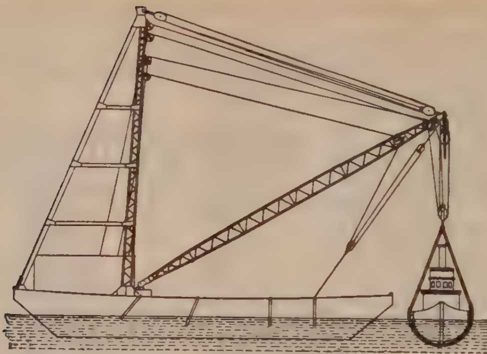


Fig. 99. A-Frame Boom Crane

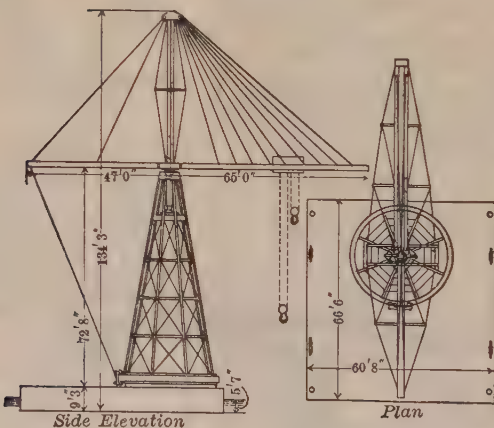


Fig. 100. Revolving Tower Crane

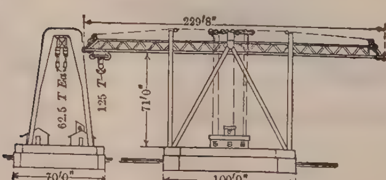


Fig. 101. Bridge Type Floating Crane

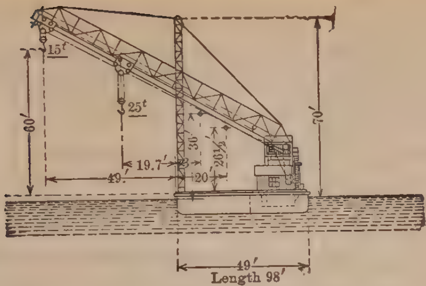


Fig. 102. Incline Bridge Crane

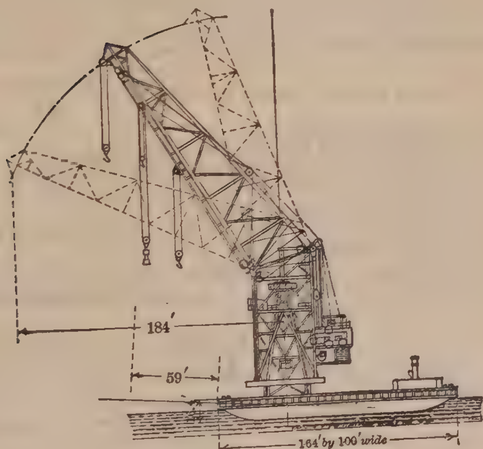


Fig. 103. Revolving Jib Type

39. Design of Cranes

The Design of cranes involves the field of mechanical, electrical, and civil engineering, and for floating cranes, also naval architecture. For stationary or movable shore cranes, the structural features are analyzed and designed in accordance with prevailing structural steel practice, the operating machinery in accordance with electrical and mechanical machine design. Having decided on the required lifting capacity and the speed of lift, in order to obtain the horsepower of the power unit, allowance must be made for the loss in friction in hoisting blocks, gearing, bearings, and operating mechanism. Three to five per cent loss in friction should be allowed for each block sheave. The diameter of sheaves should preferably be 50 times the diameter of the wire cable and should not be less than 30 diameters. Steam power is generally used, although on larger stationary or floating cranes electric motor operation has come into general use, generated by central plants or by isolated plants installed on the crane.

With the application of the maximum load moment, structural steel work must be designed to carry this and also the wind load, taken at 30 lb. per square foot of exposed surface, or without load, 40 lb. per sq. ft. For floating cranes the longitudinal and transverse stability is obtained, applying the formulas.

$$\mathrm{G\,M} = \frac{\mathrm{I - B\,GV - \Sigma i}}{\mathrm{V}} = \text{Metacentric Height}$$

$$\mathrm{R\,M} = \frac{\sin\theta}{35}\,(\mathrm{I - B\,GV - \Sigma i}) = \text{Righting Moment}$$

for which purpose the position of the center of gravity for the entire structure is secured. This calculation is made for the crane when light and when lifting the maximum load at maximum reach. The point of application of the lifted load will be at the center of the sheave pin on the jib or boom of the main structure. The overturning moment due to the lifting of this load is the product of the load by the horizontal distance from the center of the load to the center of the float or pontoon, and the change of trim due to the application of such load may be obtained in approximate terms by dividing this overturning moment by the moment due to inch change of trim.

$$\text{Moment due to change of trim of 1 in.} = \frac{\mathrm{D \times G\,M}}{Wl}$$

where GM = metacentric height in feet;
I = moment of inertia in feet of water plane of flotation;
BG = vertical distance in feet from center of buoyancy to center of gravity;
V = volume of displacement in cubic feet;
D = displacement in tons or pounds;
Σi = summation of moment of inertia in feet of interior water planes;
Wl = length of waterline in inches.

Half of the total change of trim will represent the reduction in freeboard at one end of a rectangular pontoon and the increase in freeboard at the other end. The heel or inclination due to wind pressure is given by

$$\cot\theta = \frac{\mathrm{D \times G\,M}}{\mathrm{A \times CL}}$$

in which A = total wind pressure in tons or pounds; and
CL = distance in feet from center of pressure to center of lateral resistance.

40. Port Development

A port was mentioned in Art. 1 as one definition of a **harbor.** In the commercial sense a port is a harbor, improved or developed for commerce. Many improvements necessary for this purpose have been discussed in this and other sections. Breakwaters, channels, anchorages, buoys, lights, moorings, slips, basins, docks, wharves, piers, coal- and oil-burning plants, lighters, derricks and derrick barges, railroads, roads, transit or wharf sheds, warehouses for long-time storage, ship repair plants, inclusive of dry dock, cranes and trucks all have their place in the equipment of ports for water-borne cargo-carrying vessels. These are physical attributes of the port involving engineering design and construction. There are other necessary requirements of the first-class port.

A port affords the means of interchange between land-borne and water-borne commerce. Ships are useful only in transporting the freight and passengers between ports. On an average only one-half of their total life is spent in transit and in performing this useful purpose. The other half is spent in ports, loading and discharging cargo, repairing and fitting. It is therefore important that all of these port facilities shall be arranged to reduce the stay of the ship in port and consequently increase the time the ship is at sea performing her useful functions. This consideration is the all-important one to civil engineers in charge of port development and operation, and of equal importance to the naval architect whose profession is concerned with the design, construction and operation of ships. Too frequently this is ignored and these two separate branches of the engineering profession work independently. In port development there should be more cooperation between these branches.

Port Planning requires information other than physical characteristics of the harbor. Not only does the first-class port imply a neighboring city and the facilities above named but to plan and design successfully the engineer must first ascertain the nature and quantities of the commodities to be handled, which, in turn, involve the matters of bulk or package, nature and size of packages, etc. The seasonal movements should also have consideration.

Port Hinterland. To design the port, the territory back of the port city must be studied as to production of goods for export and absorption of imported commodities. This in turn requires consideration of railroad and road access.

Port Details. Given the above-mentioned details, the port engineer is prepared to design the commercial port. Sea access must be provided for the deepest draft ships likely to use the port. No port today can be considered first-class which will not accommodate ships drawing 30 ft. of water and some cargo vessels, notably oil tankers and ore carriers, have a draft approaching 35 ft. Some Atlantic liners draw over 40 ft. There must be several feet greater depth of channel and berth for safe ship operation, but when there is a considerable tidal variation this may be utilized to cheapen channels, though access at high tide only is a handicap to a port.

Wharves and Piers may be narrow and of a temporary nature for the successful handling of certain commodities such as oil in bulk, which requires only a series of pile dolphins 50 ft. on centers for securing the ship, backed by a trestle or runway, only wide enough to support the oil pipes and furnish a footing for handling lines; or they may be of permanent construction and wide enough to handle and classify miscellaneous cargo from a large ocean vessel. Where practicable, the design should fit the commodity. An ore " dock " at the head of the Great Lakes has filled a 12 000 ton Lake carrier in sixteen minutes. The time of the ship is the most important element. One extra day in port may cost the ship operator from $1000 to $3000, when direct and indirect charges are considered.

For Miscellaneous Cargo, the modern wharf, except for special instances, should have an apron wide enough to accommodate a standing railroad track and a running track in front of the transit shed, and the latter should be wide enough for the **transit storage** of the ship's cargo in the space opposite to the ship for classification and storage as to marks of destination. This, for large ships, requires wide sheds. The width can be figured mathematically for a ship of given capacity and size. The Dock Commission of New Orleans, La., builds its sheds 200 ft. wider for ships berths on one side only. Manchester, England, has a quay warehouse 134 ft. wide and five stories high, with two railroad tracks on the apron and two at the rear, all served by traveling cranes. This is for transit business only and was built after experience with its

Dock 9, which is lined with quay warehouses 110 ft. wide and four stories high. The normal movement of miscellaneous cargo requiring classification is from the ship, into the shed, through the shed to car or truck at the rear. A standing track and a running track at the rear platform are thus required. The platform should be the height of the car floor.

Coordination of the water carrier with the land carrier is a prime object of proper terminal construction and for miscellaneous package, freight in particular, is best attained through parallelism, the transit shed serving for classification and temporary storage, the time limit being fixed generally by port regulations to clear the sheds for a new set-up for the next ship. Whether by ship's gear exclusively, by a combination of ship and shore hoisting equipment or by means of cranes, fixed or traveling upon the apron, miscellaneous cargo can be lifted in and out of the ship much faster than it can be broken down in the hold for removal or stored therein for shipment. Rate of break-down or storage of miscellaneous freight naturally varies between wide limits, due not alone to the skill of the particular gang of stevedores but also to the uniformity or the lack of uniformity of the packages. A fair average figure is 20 tons per hatch per hour.

Necessary Berths. Even the small ports expecting to do a miscellaneous business should have berths for several ships, with sheds and equipment for loading and discharging them. Wharf front of 450 ft. should be provided for each 10 000-ton ship. In the absence of better data upon which to plan, 120 tons of miscellaneous cargo may be figured per lineal foot of wharf front per annum, but capacity should be figured for the individual case and for the commodities of the particular port.

Belt Lines. Port cities of the United States are in general served by several different railroad trunk lines. Proper service and proper coordination demand that every wharf shall be accessible to every trunk line. This is best accomplished through local terminal belt line railroads which intersect all of the trunk lines, take the freights of each at a terminal yard outside the congested district of the port and serve the water front and all industries. In populous port districts, there may be an inner belt line serving the industries and shipping berths and an outer belt for general interchange between railways. For the port district of New York an inner belt, an intermediate belt and an outer belt are contemplated. At present the lighterage system of the port constitutes the inner belt. It is expensive as it involves an extra handling of goods for each operation or shipment.

Business of the Port will, for the first-class port, comprise **exports** and **imports,** but in planning a water terminal all classes of freight interchange must be studied. Coastwise business forms the largest percentage of the total of some of the most important communities. Others have a large river and intraport traffic. The individual community must be studied as to river, rail, highway, and ocean possibilities and probabilities, and the terminal must be made convenient and capacious for the coordination of these agencies of transfer.

Mechanical Applicances of standard or adapted design may be had for handling either bulk or package cargo. They should be provided whenever an engineering study indicates they can be used with economy.

The Free Port has not been resorted to in American port policy. The nearest approach is in the bonded warehouse referred to in Art. 23. A free port is an area on the seacoast or on navigable water within the boundaries of which imports or foreign cargoes may be brought, unloaded and stored without being required to pay the legal tax impost of the country in which a port is located. The advantages of a free port are that importers and foreign merchants can store and retain dutiable goods within the area without the additional investment of the duty involved, and can retain such goods within the area of the free port until it is brought inland, when duty is collected at the boundaries of the port, or it may be transhipped without being subjected to the collection of duty or expense other than that of handling and storage. Raw materials may be worked into manufactured articles within the confines of a free port, and such articles transhipped without payment of custom fees or if taken over the boundary of a free port are subject to the impost on these manufactured articles. The various advantages of the free port have greatly stimulated the growth of such ports, as is witnessed in the case of Hamburg, and this method of port legislation and regulation might with profit be followed in American port economics.

41. Shipping Terms

Tonnage. When displacement of a vessel or docking capacity of a dry dock is referred to the tons are always long tons of 2240 lb., and in all ship and dock computations 35 cu. ft. of sea water or 36 cu. ft. of fresh water are taken as weighing one ship ton.

The gross tonnage of a vessel is the measure of her cubic capacity in terms of 100 cu. ft. and is the volume in cubic feet of all enclosed spaces inside of frames divided by 100 cu. ft.

The net registered tonnage is in the same way the gross tonnage less the volume in 100 cu. ft. of certain machinery and other spaces not accessible to cargo, such as crew space, steering-gear house, chain lockers, stores, etc.

The term "registered," as applied to net tonnage, refers to the measurements for that particular vessel as measured and registered by one of the national registry agencies, such as British Lloyds, Bureau of Veritas of France, or the American Bureau of Shipping. The displacement of a vessel is the weight of the vessel or water displaced by the vessel under any particular conditions and varies with the draft of the vessel at any particular time.

The dead weight tonnage of a vessel is the gross carrying capacity in tons and is the displacement of the vessel fully loaded, full draft to the Plimsoll mark, less the displacement of the vessel light and bare without coal, crew and ship's or crews' supplies. The cargo-carrying capacity of a vessel in tons is something less than her dead weight tonnage, being this dead weight tonnage less the weight of bunker coal, crew, ship and crew supplies, etc.

The actual cargo-carrying capacity of any vessel is dependent on the spaces or cubage available for cargo loading and dead-weight tonnage exclusive of fuel. The ideal arrangements for which stevedores contend is to take advantage of full weight carrying capacity and completely fill volume of cargo-carrying holds. In general and for approximation purposes cargo-carrying steamers in the ocean trade will carry about two-thirds of the tons of freight represented by dead-weight tonnage or about the number of tons weight represented by the numerals in the term "gross tonnage." Forty cubic feet are generally considered as representing a cargo ton.

In the design and construction of dry docks the weight to be lifted or the draft considered is that of the vessel light with no cargo and little coal. Dockage and other tolls are usually based upon the registered gross tonnage; an entra charge is made for cargo or excess fuel, etc. The **Plimsoll Mark** is the mark made on the sides of the hull amidship to indicate the depth to which the vessel may be safely loaded.

The Block Coefficient is the ratio of the under-water volume of the ship to the volume of the circumscribing rectangular parallelopiped, namely, the rectangular solid of the same length as the load water line and with width equal to the beam of ship and depth equal to the ship's draft.

The Load Water Line Coefficient is the ratio of the area of the load water line to that of the circumscribing rectangle.

SECTION 20

HIGHWAY ENGINEERING

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GENERAL DATA

1. Preliminary Investigations

Relocation of Existing Highways. A report should be made, accompanied by maps and plans, covering the following: Climate, topography, soil, location, population, census of nearby cities, past and present traffic census, local materials of construction, labor supply, local industries, and a forecast of probable future growth of both population and traffic in the territory.

Location of Proposed New Highways. A reconnaissance survey, similar to that for a railway, is necessary. In addition, the items above should be supplied insofar as is possible. All trustworthy information concerning stream flow, drainage and soil conditions must be furnished. The traffic forecast is a matter of uncertainty, and may be wholly a guess, but the guess should be toward a maximum rather than a minimum.

Alignment. Changes in alignment of existing streets in the built-up sections of cities and towns are costly. Except in extreme cases such changes are not attempted. Dangerous jogs at street intersections may often be improved or eliminated by suitable changes in the arrangement of the sidewalks without making any expensive changes in the abutting structures.

Sharp curves are usually not as dangerous as long tangents, if the curves are properly marked. Landscape architects should be restrained from making needless, and sometimes foolish, curves in suburban developments. These are always sources of danger. Curves that exist because of topographic conditions are generally satisfactory, and cause no trouble. However, long, sharp curves on steep grades should be avoided wherever possible, as frequently there is danger of cars and trucks getting out of control.

For highways it is not necessary to lay out curves with the long radii usual in railway location. Yet, with speeds on the highways of 50 m.p.h. and more, it is undoubtedly necessary to keep the curves as flat as is economically possible. Various state and county departments have adopted standards to which they try to adhere as far as may be. During 1928 the minimum radius usually allowed on federal-aid projects was about 500 ft.

The reprehensible practice of designing rural pavements with no superelevation of the outer edge of curves should be prohibited. In 1929, nearly all the new designs of roads called for superelevation. The formula for superelevation recommended by the American Association of State Highway Officials is:

$$\mathbf{E} = 0.067\,\frac{\mathbf{V}^2}{\mathbf{R}}$$

where $\mathbf{E}$ = maximum elevation in feet per foot of width;
$\mathbf{V}$ = velocity in m.p.h. (35 recommended);
$\mathbf{R}$ = radius of curve in feet.

Provision should be made wherever possible to give a line of sight at least 400 ft. ahead. This means that on sharp curves in cut, where buildings do not interfere, the embankment on the inside must be removed so as to give the needed vision. This is not always possible in mountainous country, but much can be done by simple relocation of the existing roadway. On new roads there is scant excuse for not providing sufficient sight ahead.

With the slower speeds of yesterday it was found that simple curves were sufficient, but with speeds of over 30 m.p.h., curves on first-class highways should be spiraled. This is specially necessary where the radius is less than

400 ft. Any method of figuring spirals for railway alignment will be satisfactory for highway curves. Reverse curves are dangerous and should be avoided. Even a very short tangent introduced between two curves will make the road safer than none at all. If spirals are used their ends may be brought together instead of introducing a tangent.

Grades. Horse-drawn traffic restricted grades to a greater extent than does motor traffic. The horse now may be eliminated from future consideration. The limiting factor is the heavy motor truck. Contrary to prevalent opinion, it is not possible for loaded 5- and 7-ton trucks to climb any and all grades. Many hills that may be fairly easy for a passenger car are all but impossible for the heavy truck. Existing steep grades should be investigated, not alone from the standpoint of theory, but also from the standpoint of the truckman who operates over such grades. It is not possible to keep trucks in perfect condition the year round; it is not possible to keep the highway itself in perfect condition continuously; and the weather is another changeable factor. Hence in location or relocation these facts must be kept in mind. Highway engineers will learn much that will be of benefit to the users of the highways if they will talk and drive with motor truckmen.

Arbitrary grade limits are unsatisfactory. A short steep grade occasionally may be preferable to a longer route over a light grade. The average truck, when not overloaded, has no difficulty with 12% grades if such hills are not too long. It must not be forgotten that these trucks and even passenger cars frequently encounter more difficulty in descending than in ascending. A long, steep grade must be avoided because of danger from runaways. Burned-out brake bands and stripped gears may result in disastrous accidents. If possible, a short up-grade should follow a long down-grade, so that runaways may have a chance to recover themselves. In spite of legislation to the contrary it is a matter of frequent occurrence to see motor trucks descending at 40 m.p.h. It is possible to so design a highway that even a coasting truck may not exceed some safe limit.

In general, the grade should be kept under 6%, not so much for economics as for safety. The grade established may determine the type of pavement to be selected. If a pavement is kept clean, then all types are reasonably safe on grades up to 7%. On steeper grades the following should be used: Stone block, hillside brick, concrete, bituminous macadam, or bituminous concrete with a cover of 3/4-in. stone chips rolled into the surface. Brick and the bituminous types may be unsafe on frosty mornings.

Widths. For city, town and village streets the following widths are recommended by Harland Bartholomew:

On country highways of the first class the width should never be less than 18 ft., increased an additional 2 ft. when approaching cities. This permits safe passing at high speeds. Traffic lanes should be at least 8 ft. wide on roads wider than 18 ft. Thus, three-lane traffic calls for a 24-ft. width, four-lane for 32-ft. and so on. These widths are minimum. A far better arrangement is the 10-ft. lane, which permits trucks to pass each other in safety.

Traffic lanes should be identified by painted lines upon the surface of the roadway.

On curves the width should be increased. The increase should be on the inside of the curve. Motor vehicles require a wider lane on curves than on tangents, but how to figure this amount exactly is a difficult problem. The longer the wheelbase the greater is the width required. If a transition curve is used the widening should begin at the point of curve of the spiral. If no

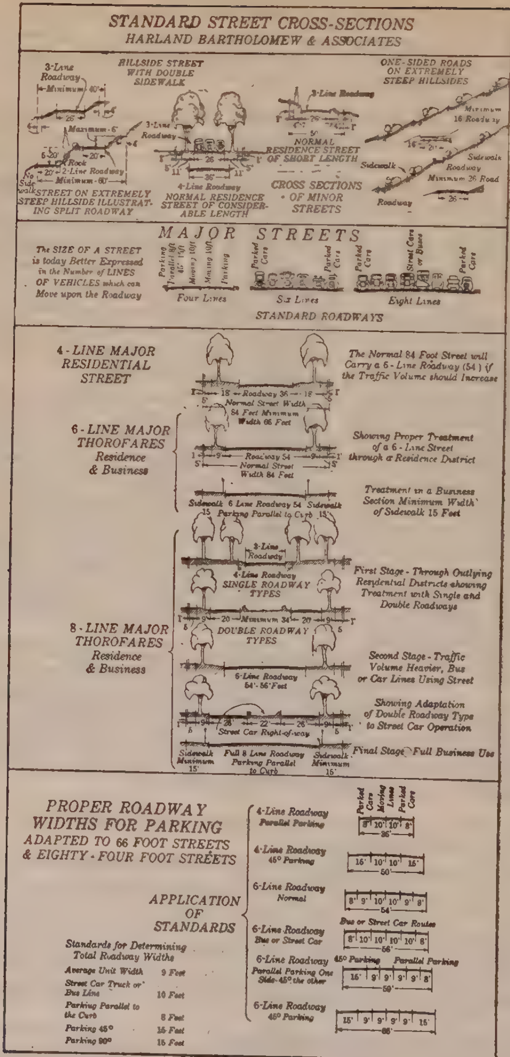

STANDARD STREET CROSS-SECTIONS
HARLAND BARTHOLOMEW & ASSOCIATES
HILLSIDE STREET WITH DOUBLE SIDEWALK
ONE-SIDED ROADS ON EXTREMELY STEEP HILLSIDES
STREET ON EXTREMELY STEEP HILLSIDE ILLUSTRATING SPLIT ROADWAY
NORMAL RESIDENCE STREET OF CONSIDERABLE LENGTH
NORMAL RESIDENCE STREET OF SHORT LENGTH
CROSS SECTIONS OF MINOR STREETS
MAJOR STREETS
The SIZE OF A STREET is today Better Expressed in the Number of LINES OF VEHICLES which can Move upon the Roadway
Four Lines
Six Lines
Eight Lines
STANDARD ROADWAYS
4-LINE MAJOR RESIDENTIAL STREET
The Normal 84 Foot Street will Carry a 6-Line Roadway (54) if the Traffic Volume should Increase
6-LINE MAJOR THOROFARES Residence & Business
Showing Proper Treatment of a 6-Line Street through a Residence District
Treatment in a Business Section Minimum Width of Sidewalk 15 Feet
8-LINE MAJOR THOROFARES Residence & Business
SINGLE ROADWAY TYPES
DOUBLE ROADWAY TYPES
First Stage - Through Outlying Residential Districts showing Treatment with Single and Double Roadways
Second Stage - Traffic Volume Heavier, Bus or Car Lines Using Street
Showing Adaptation of Double Roadway Type to Street Car Operation
Final Stage Full Business Use
PROPER ROADWAY WIDTHS FOR PARKING ADAPTED TO 66 FOOT STREETS & EIGHTY-FOUR FOOT STREETS
APPLICATION OF STANDARDS
4-Line Roadway Parallel Parking
4-Line Roadway 45° Parking
6-Line Roadway Normal
6-Line Roadway Bus or Street Car
6-Line Roadway Parallel Parking One Side-45° the other
6-Line Roadway 45° Parking
Standards for Determining Total Roadway Widths
Average Unit Width 9 Feet
Street Car Truck or Bus Line 10 Feet
Parking Parallel to the Curb 8 Feet
Parking 45° 16 Feet
Parking 90° 16 Feet

Fig. 1

spiral is used the pavement should begin to increase in width at a point not less than 30 ft. before the P. C.

The Extra Width. The extra width should be such that two motor buses may pass each other safely at a reasonable speed. The only way to ascertain the exact width required is actually to measure the path of moving motor vehicles of long wheelbase when rounding curves at high speed. In addition to extra width really needed to take care of the path of the car, there is an additional amount needed to make the road *apparently* safe. The average motorist finds it difficult to judge distances while driving on curves, so that an additional width must be supplied to aid him in his judgment.

The **width required,** irrespective of the psychological effect on the driver, may be figured from the following formula: $\mathbf{W} = l^2/2\,\mathbf{R}$, where **W** is the extra width in feet of the traffic lane, l is the length of the vehicle in feet, from the rear axle to the extreme front (usually the bumper), and **R** is the radius of the curve in feet. To this must be added whatever may be considered necessary in order to make the curve *appear* safe to the motorists who are either careless or not too sure of their driving abilities.

Local Materials. All sources of information concerning the location and possible use of suitable local materials should be tapped. The state geologist can frequently be of great help and should be consulted at the outset. Government reports, and other reliable data should be searched out and used. Contractors and engineers interested in such work should be interviewed, and in this way much time may be saved, while a more intelligent estimate may be prepared. A most important matter that is often overlooked is that of freight rates and local transportation charges. Local statutes and ordinances should be looked up. Labor rates and union rules must be carefully investigated. The Bureau of Public Roads, Washington, D. C., and most state highway departments will test road-making materials free of charge, and report upon their suitability for use. For the foundation of a road or pavement it is frequently possible to use local materials that might be wholly unsuited for use in the wearing course. The use of local materials often modifies the original scheme, and if intelligence is mixed with the materials a much cheaper form of construction may be possible.

Railroad Grade Crossing Eliminations. Railroad grade crossings may be eliminated by (1) relocating the highway, (2) relocating the railroad, (3) separating the grades. All are expensive. Where grades are separated extreme care must be taken in the design so that no new element of danger is introduced, such as sharp turns on cuts and fills, restriction of sight distance, columns, piers or bents in subways, poor drainage, too little headroom, narrow highway bridges, no sidewalks on highway bridges or through highway subways, and the like.

Headroom. For highway under railroad, allow 14-ft. minimum clearance. For highway over railroad, allow 22-ft. clearance unless railroad permits less. This is governed by law in some states. Highway grades on approach should not be greater than 6%, using 200-ft. vertical curves at changes of grade. The width of pavement should be the same as that of the highway itself and allowance must be made for ditches and sidewalks. Structures should be designed to take care of future traffic growth by subsequent additions as may be necessary.

2. Traffic Census

"The fundamental purpose of traffic or highway transportation research is to provide for the highway executive an accurate and reliable analysis of the demand for highway service. The demand for highway service can be measured only by an accurate and comprehensive study of present traffic, its volume and type, leading to an estimate of the future volume and character of traffic on the highway system and the individual routes."

Data may cover the following items: Count of passenger cars, motor trucks, buses, motor cycles, bicycles, horse-drawn vehicles, pedestrians, and parked vehicles. Detailed information may be obtained for each of these items, and may assist in forming a complete picture. Passenger car data should show state of registration, place of ownership, purpose of trip, origin, destination and number of passengers carried. Motor truck data should show weight on front and rear axles, tire equipment, weight of truck, load capacity of truck, state of registration, place of ownership, origin of trip, destination, commodity on board, probable return routing.

These data should be collected at such periods as will give the student of them a broad view of each case. In city districts data should be collected during 24-hour observation periods. This is true also of important through state routes which carry much night traffic. On the majority of roads, however, a 10-hour period is ample. Special observations must be taken to learn what hours this 10-hour period should cover, as in some localities heavy traffic begins at daybreak and is over early in the afternoon, whereas in others traffic may not begin until the middle of the morning. Observations must be made at different seasons, under various weather conditions, week days, Sundays, and holidays. The past history of the roads or streets should be looked up, and any pertinent facts recorded. United States census figures, school statistics, election returns, and any other available sources of information as to the contiguous population should be examined. A list of industries in nearby towns together with the number of employees should be compiled. All factors that make use of the highways should be studied.

In most cases it is possible to predict with a fair degree of accuracy what the traffic may be in five years on any particular route. Occasionally something upsets all calculations, but always it will be found that the calculation involved showed too small an increase. There is no decreased traffic. Prediction of future traffic is based primarily on prediction of future registration of motor vehicles, and this is fairly simple. The United States Bureau of Public Roads has prepared curves which agree well with actual figures. For full information see Report of a Study of Highway Traffic and the Highway System of Cook County, Illinois, Report of a Survey of Transportation of the State Highway System of Connecticut, and Report of a Survey of Transportation on the State Highway System of Ohio.

The writer suggests that the engineer or city official who undertakes to make a survey for the first time, unaided by any who have had experience, will look over the area carefully, and will make up several different types of forms on which to tabulate data. These forms may be tried out and the one that is easiest to follow should be selected. In city work it is possible to place only one man at each intersection where traffic does not exceed 4000 vehicles per 10-hour day. The information obtainable would include type of vehicle, approximate load, direction traveling (including turns), state of registration, hour, and weather. On such a busy intersection one man cannot count pedestrians. If traffic is greater than 4000 vehicles per 10-hour day it will be necessary to place two observers at such intersections on opposite sides of the street. The form as illustrated in Fig. 2, on next page, was found to work well in practice.

3. Finance

Money for road building is obtained by taxation. The forms of taxes are: **Direct taxes,** such as property taxes, poll tax, license fees, fuel tax, income taxes, special assessments, and tolls; and **indirect taxes,** such as import duties, excise taxes, and internal revenue. Direct taxation furnishes the revenues used by state and local units, whereas the federal government secures its funds largely from indirect taxes.

Motor vehicle license fees are usually levied on the horsepower of passenger cars, and on the weight capacities of trucks. The S. A. E. or the N. A. C. C. formula for

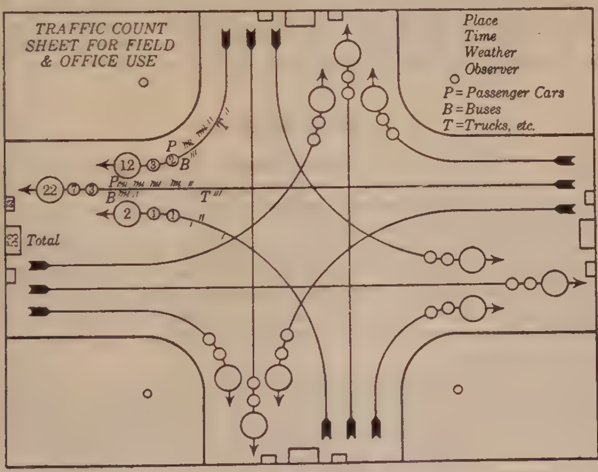


Fig. 2

the former is: **Hp. = 0.4 ND²,** where **N** = number of cylinders, and **D** = diameter of bore in inches. The minimum fees and taxes in 1928 for a touring car varied in the United States from $5 to $15.

A wheel tax is legal in some states. It is a license fee levied by a city.

A gas tax or a fuel tax was levied in 1929 in all states. The rate varied from 2 to 5 cents per gallon. Motor vehicle fees and gasoline revenues are divided between the construction and the maintenance of roads.

There are two methods of **financing** a highway construction program: The **pay-as-you-go** method, and the plan of capitalizing annual taxation by the **issuance of bonds.** The former inflicts no direct financial burden on posterity, and the program may be changed at will to meet current conditions. Bond issues require that a definite program must be followed. This method assures a speedily constructed system and should be used when the present system is small and incomplete. When the system approaches completion the pay-as-you-go method may be successfully adopted.

There are three types of bonds: Term, annuity, and serial. Of these the serial bond is the cheapest. Bonds should be sold in conformity with the progress of the construction program.

E. W. James points out that one detail is essential in determining a choice between the methods. The total cost of the same road varies according to the method of financing adopted. If the sum of $100 000 is to be spent in building 10 miles of highway, the cost is $10 000 per mile if the funds are produced from annual revenues. But the total cost of a $100 000 bond issue, serial type, may be $142 000, and the total cost of a mile of road paid for by this method would then be $14 200.

Local assessments are frequently necessary in financing municipal improvements. The assessments may be spread by: (a) Frontage Method, in which the assessment on any lot is found by multiplying its frontage by the rate per front foot; (b) Area Method, where each lot is assessed in proportion to its area; (c) Area-Distance Method, in which the assessment is made directly proportional to the area and inversely proportional to the distance from the improvement; (d) Zone-area Method, in which zones of uniform depth parallel to the street lines are laid out, and to each zone a rate of assessment is arbitrarily assigned, and (e) Benefit-factor Method, where the assessment decreases as the distance from the improvement increases.

4. Federal Aid

The policy of federal aid for highway construction was initiated with the Federal Aid Road Act of July 11, 1916; under the provisions of this act and subsequent amendments a total of $840,000,000 (through the fiscal year 1929) has been apportioned to the states.

All funds have been apportioned to the states in the following manner: One-third in the ratio which the area of each state bears to the area of all the states; one-third in the ratio which the population of each state bears to the total population of all the states; and one-third in the ratio which the mileage of rural delivery routes and star routes in each state bears to the total of such roads in all the states.

Since the passage of the Federal Highway Act of November 9, 1921, amending the original federal-aid act, federal-aid funds have been available for expenditure only on a system which has been selected and is known as the federal-aid highway system, consisting of approximately 187 000 miles of interstate and intercounty highways. Except in states where more than 5% of the area is unappropriated public land the amount of federal participation is limited to 50% of the cost. In those states having more than 5% of their area in unappropriated public land, the participation at present is limited as follows:

Arizona	72.34	Nevada	87.72	South Dakota	55.62
California	60.05	New Mexico	63.43	Utah	78.90
Colorado	56.08	Oklahoma	55.47	Washington	54.38
Idaho	59.75	Oregon	62.25	Wyoming	64.20
Montana	56.46				

Expenditures from appropriations of recent years have been limited to $15 000 per mile exclusive of bridges over 20 ft. in span, except in the public land states, where the amount is increased in the same proportion as the allowable federal participation.

In the construction of federal-aid roads, all plans and specifications are prepared by the state highway departments, subject to the approval of the Secretary of Agriculture, who acts through the Bureau of Public Roads. Each state is permitted to select designs suitable for the particular conditions existing, and it is required only that such designs shall be adequate for the conditions. A minimum width of 18 ft. is required unless particular conditions justify the approval of a lesser width. Contracts are let by the state highway departments and construction supervised by them. As a condition to receiving federal aid each state agrees to maintain the roads constructed with such funds and should a state fail to comply with this agreement the federal government is authorized to perform such maintenance, paying the cost from

States	Mileage in federal-aid highway system	Total federal-aid apportionment 1917-1929	Apportionment for fiscal year 1929
Alabama	3 884.00	$17 441 158.00	$1 547 220.00
Arizona	1 498.00	11 733 324.00	1 056 994.00
Arkansas	5 021.13	14 165 485.00	1 277 896.00
California	4 771.50	27 042 667.00	2 483 437.00
Colorado	3 332.00	15 085 733.00	1 376 520.00
Connecticut	835.43	5 280 579.00	472 685.00
Delaware	415.81*	3 205 308.00	365 625.00
Florida	1 926.00	9 885 716.00	899 451.00
Georgia	5 560.40	22 391 177.00	1 979 209.00
Idaho	2 770.00	10 427 782.00	935 193.00
Illinois	6 616.78	36 121 852.00	3 154 429.00
Indiana	4 701.50	22 052 590.00	1 926 772.00
Iowa	7 212.00	23 565 853.00	2 044 999.00
Kansas	7 922.00	23 595 139.00	2 068 532.00
Kentucky	3 702.45	16 051 785.00	1 417 947.00
Louisiana	2 712.90	11 304 998.00	1 013 308.00
Maine	1 393.46	7 827 053.00	680 794.00
Maryland	1 503.72*	7 195 082.00	635 119.00
Massachusetts	1 308.00	12 286 634.00	1 089 100.00
Michigan	5 235.00	24 766 652.00	2 214 691.00
Minnesota	6 849.60	23 825 116.00	2 120 741.00
Mississippi	3 604.00	14 745 626.00	1 307 879.00
Missouri	7 530.00	27 595 958.00	2 405 175.00
Montana	4 665.00	16 528 960.00	1 551 499.00
Nebraska	5 576.55	17 805 354.00	1 585 138.00
Nevada	1 398.00	10 701 720.00	948 510.00
New Hampshire	980.91	3 900 742.00	365 625.00
New Jersey	1 179.80	10 337 153.00	934 611.00
New Mexico	3 298.00	13 348 510.00	1 186 763.00
New York	5 451.00	41 310 291.00	3 635 217.00
North Carolina	3 860.80	19 146 472.00	1 713 356.00
North Dakota	7 194.00	13 137 050.00	1 194 951.00
Ohio	5 899.30	31 251 969.00	2 762 209.00
Oklahoma	5 528.00	19 560 744.00	1 751 891.00
Oregon	2 840.50	13 251 256.00	1 182 202.00
Pennsylvania	4 871.22	38 008 302.00	3 335 735.00
Rhode Island	356.86*	3 398 819.00	365 625.00
South Carolina	3 230.00	11 916 045.00	1 054 988.00
South Dakota	5 767.00	13 610 835.00	1 220 064.00
Tennessee	3 252.80	18 507 369.00	1 614 766.00
Texas	11 685.00	49 606 279.00	4 497 272.00
Utah	1 677.33	9 513 778.00	846 906.00
Vermont	1 043.00	3 999 757.00	365 625.00
Virginia	3 233.00	16 381 776.00	1 442 714.00
Washington	2 927.50	12 420 534.00	1 131 532.00
West Virginia	2 048.31	8 939 943.00	793 636.00
Wisconsin	5 493.36	21 173 482.00	1 870 455.00
Wyoming	3 097.00	10 440 190.00	934 369.00
Hawaii	174.60	1 831 403.00	365 625.00
Totals	187 034.52	$817 625 000.00	$73 125 000.00

* These states have completed or provided for completion of the 7% system and additions in excess of 7% have been authorized.

funds due to the particular state, and to withhold further federal-aid funds from the state until reimbursement has been made for such payments. In the administration of federal aid, the federal government deals only with state highway departments.

In addition to the funds which have been provided for federal-aid road construction, a total of $77 000 000 has been provided for the construction of roads and trails in the national forests. This work is done either in cooperation with state or local authorities or independently by the federal government.

5. Soils

Classification. Soils are formed by the decomposition of mineral, animal, and vegetable matter. They may be designated as sedentary or transported. Sedentary soils are those which remain near their source of formation, while transported soils have been carried by some geological agency from the place where they were first formed to some other. Soil as far as its composition and properties are concerned is extremely variable. The principal constituents of any soil, however, whatever its source, are nearly always silica, with varying amounts of alumina, oxides of iron, lime, magnesia, and the alkalies. A small amount of organic matter is usually present also. Soils are generally classified as gravel, sand, clay, loam, marl, peat, and muck.

Gravel consists of small pieces of rock, worn smooth by abrasive action, mixed with sands and clays in varying proportions. Gravels occur throughout the United States and Canada in those districts which were at one time covered with the glacier. Gravel should contain enough binding material, mixed with the stone particles, to bind the whole into a solid mass. Clay is the most common form of binder found in gravel in its natural state. If present in too large quantities, however, it is detrimental. If gravel does not have sufficient binding material, this can be remedied by adding some cementing material such as clay, shale, marl, loam, or stone screenings. Some gravels contain so much earthy material that it is necessary to screen them before they are suitable for road-making purposes. An indication of the binding qualities for pit gravel may be obtained by noticing the gravel in the pit. If the bank faces are vertical and the gravel breaks off into chunks, good binding qualities may be expected. River gravel generally contains less clay and more silica than the pit gravel of the same locality. Soil classified as hardpan may mean either a very compact clay, or a gravel which is cemented with clay or an iron oxide.

Sand is largely the result of the breaking-down of sandstone rocks, and a sandy soil so-called probably contains over 80% of pure sand. A sandy soil is hard to compact, and possesses little binding power unless wet or mixed with some cementing material such as clay. Quicksand is sand saturated with water, and possesses no stability.

Clay results from the decomposition of feldspar, oligoclase, and micaceous rocks. A clayey soil might be termed such when containing at least 60% of clay. Clay when wet will swell and puddle with water, becoming very plastic. When mixed in the proper proportions with sand or gravel, it makes a very satisfactory road material in localities where stone is unobtainable. Clays used for fire clays in the manufacture of brick, besides being plastic, must possess certain refractory qualities which enable them to withstand long periods of heat at high temperatures without fusing. **Shales** have chemically the same composition as clays, but they have a laminated structure, and are similar in appearance to slates. Shales, however, will rapidly disintegrate on exposure to the atmosphere. In the southwestern states a clay of which mud bricks might be made is called " adobe."

Loams may be any soil between sand and clay. They contain more or less of each of these two materials. They may be classified as heavy clay loams, clay loams, sandy loams, and light sandy loams, depending upon the quantity of the sand or clay content. In the Middle West a black loam which contains so much clay as to be sticky when wet is known as " gumbo."

Marl is a term which applies to all calcareous clays containing as a minimum 15% of carbonate of lime, and as a maximum 75% of clay.

Peat and Muck are generally distinguished from other soils by the presence of humus or vegetable matter. They are formed by the decomposition of vegetable matter under water, and are undesirable materials for road building.

DRAINAGE AND FOUNDATIONS

6. Drainage

Subdrainage

Object. It is of the utmost importance that the natural foundation of a road should be kept dry in order to provide a firm and unyielding support. This can in certain instances be accomplished by subdrainage only. If the subdrainage system is correctly designed it may serve to lower the level of the ground-water and thus allow the ground to dry out; to remove water which is prevented from flowing off by an impervious stratum which underlies the road surface; to remove water which is always present in a road when the latter is thawing; to reduce the injurious action of frost by removing the moisture from the road; and to intercept water before it reaches the roadbed. Tile drains do not prevent nor remove capillary moisture. But capillary moisture itself may not be harmful unless it leads to saturation of the soil. Some soils do not respond to any method of drainage.

Methods in use for improvement of drainage conditions are: Placing a thick layer of sand, cinders, or similar materials on top of the soil and working this layer in by harrowing, or else leaving it in place without mixing. Tile drains, deep side ditches, and blind drains are advocated for certain conditions.

Porous Tile and Vitrified Pipe. The pipes are cylindrical in shape and are manufactured in 1-ft., 2-ft., or 2-1/2-ft. lengths with varying diameters. The porous tile pipe is made with plain ends, whereas the vitrified pipe is provided with a bell end. It is good practice not to use a size smaller than 4 or 5 in. in diameter. Where a large quantity of water is expected, if its amount is known, the size of pipe may be determined by one of the well-known formulas for flow of water through pipes.

The pipes should be laid in a properly constructed trench. The bottom of the trench should be covered with a layer of small-sized gravel or broken stone passing a 1/2-in. screen. The pipe is laid in the trench with open joints, or with the joints protected with small strips of burlap, and is then covered for a depth of about 1 ft. with a material of a size similar to that used in the bottom. The rest of the trench is then filled with large size broken stone. The filling should be carefully tamped around the pipe. Where a pipe with a bell end is used, the bell is placed toward the high end of the trench. On macadam roads it is customary to lay the side drains at a distance of from 1 to 2 ft. beyond the edge of the stone. The pipes are generally placed at a depth of from 2-1/2 to 4 ft. and are usually laid to the grade of the road, but with a minimum of 2 in. in 100 ft. The outlet end of the pipe should be protected by a small headwall of boulders to prevent the washing out of the pipe at this end. Whether a line of pipe is needed on one or both sides of the road is a matter of judgment. On side-hill work one line of pipe on the uphill side will generally serve. In constructing a road through broad, flat, wet places, one line of pipe at the side of the road may not be sufficient, and a line on each side of the road will be necessary. Through cuts, two lines are sometimes specified. In streets, tile drains should be laid under grass parkings, or, if such do not exist, under the gutters. For sizes of tile pipe under street pavements see Folwell's Sewerage, pp. 147–171.

Capacity of Tile Drains in Cubic Feet per Minute

From Spalding's Roads and Pavements, p. 36

Slope per 100 ft.		Diameter of pipe in inches				
Inches	Feet	4	6	8	10	12
2	0.17	4.0	12.0	27.0	49.5	81
4	0.33	5.5	16.5	38.0	70.0	114
6	0.50	6.5	21.0	46.5	86.5	143
9	0.75	8.0	25.5	57.5	106.5	176
12	1.00	9.5	29.5	66.0	122.5	204
24	2.00	13.5	41.5	92.5	173.0	288
36	3.00	16.5	51.0	114.0	212.0	353
48	4.00	19.0	59.0	132.0	245.0	408
60	5.00	21.0	66.0	148.0	275.0	456

Box Drains may be used in place of pipe in localities where stone is available. They should never be built of wood.

Blind Drains. Subdrainage is sometimes accomplished by digging trenches either across or alongside the road, afterward filling them with stone. The depth and distance apart of the trenches will depend upon the conditions encountered.

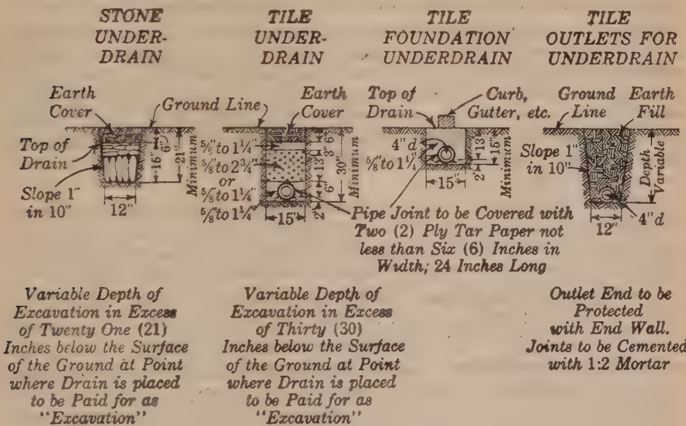
The V-Drain Foundation of the Massachusetts Highway Commission has given very good satisfaction on poor subsoils. It is built by excavating the full width of the surfaced roadway from 6 to 8 in. deeper at the sides and from 12 to 18 in. deeper at the center than usual, thus producing a flattened V-shaped trough. This trench is filled with stone varying in size from 1/2 in. to 12 in. in longest dimensions. The large stones are placed at the bottom of the trench. The grade of the trench is parallel to the finished grade of the road. The water follows the trench to the low points, where it is led to the sides of the road by a culvert across the road.

Soil Treatment. In places where the soil is very poor and is a hindrance to drainage, an improvement can be made by excavating the soil for a certain depth, depending upon conditions, and refilling with field stone, broken stone, or a good gravel.

Surface Drainage

Surface drainage is accomplished by giving the road or pavement surface a crown or transverse slope, which sheds the water to the side ditches or gutters. The ditches or gutters have a longitudinal grade which generally corresponds to the grade of the center of the road and the water is carried by them to the point where it is discharged. In this manner the flow of water is confined to a small area.

The Crown of the road or pavement is formed by the intersection of two planes or as a parabolic curve. At street intersections, the crown will have to be modified to fit the grades of the intersecting streets. On curves of main trunk highways the crown should consist of one plane sloping up from the inside edge of the curve. On streets that are bordered with curbs the elevations



UNDERDRAINS

Fig. 3

of which cannot be changed, a uniform slope both ways from the center of the road can be obtained in some cases by making one gutter deeper than the other. Frequently, however, it is necessary to use a different slope each side of the center.

Amount of Crown. The Special Committee on "Materials for Road Construction," Am. Soc. C. E., in 1918, recommended the following crowns:

Kind of roadway	Inch to the foot		Kind of roadway	Inch to the foot	
	Maximum	Minimum		Maximum	Minimum
Asphalt block	1/4	1/8	Cement-concrete	3/8	1/4
Bituminous surfaces	1/2	1/4	Gravel	1	1/2
Bituminous concrete	1/2	1/4	Sheet-asphalt	1/4	1/8
Bituminous macadam	1/2	1/4	Stone block	1/2	1/4
Brick	3/8	1/8	Wood block	1/4	1/8
Broken stone	3/4	1/2			

Crown Formulas. The following formulas, originated by Mr. Andrew Rosewater, M. Am. Soc. C. E., give the crown for brick, stone-block and sheet asphalt pavements. Let C = crown of pavement in feet, W = distance between curbs in feet, and f = grade of street in feet per 100. For brick, stone-block, wood-block, and compressed European rock-asphalt, $C = \frac{W(100 - 4f)}{6000}$.

For American sheet asphalt composed of sand and asphalt, or of compressed natural sand rock, $C = \frac{W(100 - 4f)}{5000}$. The formulas are based on the laws of the parabola, and the crown C is the total rise at the center above the pavement at the curb. The crown can be found for any other point by deducting from C an amount equal to C times the square of the distance, expressed as a fractional part of the half width, from the center to the point in question.

Ditches for country roads are made by either cutting out a trapezoidal-shaped section at the edge of the shoulder, or by a more gradual rounding off of the road at this point, giving a flatter and shallower ditch. The latter is preferable particularly if a road-scraper is used to any extent in the maintenance of the road. Ditches to be effective should be given a good bottom slope and should be kept clean. The slope of the ditch is made the same as the grade of the road surface, except in some cases where the grade of the road surface is very slight.

Gutters. Cobblestone, brick, concrete and stone-block gutters are laid on steep grades where water may wash out the sides of the road. Gutters are made from 1-1/2 to 6 ft. in width, and have the same slope as the road surface. It is better, however, to drop the center so as to form a trough section.

The Minimum Longitudinal Grade recommended by some engineers is 0.5%. If the road surface is kept in good repair and the water has a chance to run off at the sides. a flatter grade than 0.5% can be used.

Methods of Construction. During construction the shape of the surface may be obtained in four ways: by measuring from a string at grade which is stretched across and along the road at intervals; by blocks placed on the subgrade and each course; by using a board templet corresponding to the curvature of the cross-section; and by a series of stakes set at intervals across the roadway. Where the cross-section is composed of two planes the string method is rapid and accurate. Where the cross-section is a parabolic arc, the board templet or stakes are preferable.

Pipe Culverts

Capacity. The size of opening may be determined by formulas or an estimate of the runoff of water may be made and a size of pipe designed to take care of this amount. If the required area calls for a size of pipe that either is too large to be used, or cannot be obtained, two or three lines of smaller pipes may be substituted for it. It is, however, inadvisable to use a pipe less than 12 in. in diameter because a smaller pipe than this is liable to become choked.

Construction. Pipe culverts of all types should be laid on a firm bedding. If the soil furnishes a poor support, the pipes should be bedded in a layer of concrete or broken stone. This is more essential for culverts of vitrified pipe than for those made of the other materials. The trench should be excavated to the grade of the pipe. After the pipe is laid in the trench, good earth or small stone should be filled in around it and carefully tamped so that the pipe will be supported throughout its length. Concrete makes the best headwalls, since it is cheap, durable, and can be molded in any form desired. The headwalls for small culverts are generally built parallel to the center line of the road, with a thickness of at least 12 in. The bottom of the headwall should be 18 in. or more below the bottom of the pipe to prevent the water from flowing around the outside of the pipe and thus washing it out. The headwalls must be made long enough to keep the earth away from the pipe.

Vitrified Pipes used for culverts should be the best quality salt-glazed sewer pipe of the double strength type, with socket joints. This pipe is made in

2-1/2-ft. lengths with diameters from 12 to 36 in. The pipes are laid in the trench with the socket end toward the inlet, so as to have at least 15 in. of material over the top of the pipe. The joints are sometimes filled with cement. Under conditions ordinarily encountered in highway work vitrified pipe makes a very satisfactory as well as a cheap culvert.

Cast-Iron Water Pipe with bell and spigot joints has been used in culvert construction for a long time. It is manufactured in 6-ft. and 12-ft. lengths, and hence is not so easily adaptable for use. It is very strong and will last for many years. This kind of pipe can be placed within 6 in. of the road surface without danger of breaking. The principal objection to cast-iron pipe, outside of its cost, is its weight, which makes it expensive to handle. The weight per foot of heavy-weight pipe for some of the sizes from 12 to 72 in. in diameter is: 12-in., 85 lb.; 18-in., 200 lb.; 24-in., 300 lb.; 36-in. 500 lb.; 48-in., 850 lb.; 60-in., 1250 lb.; 72-in., 1750 lb.

Corrugated Metal Pipe is made in any length desired, ranging by multiples of 2 ft. up to 36 ft., or is made in nest sections that are later bolted together in the field. The pipe may be laid to within 6 in. of the road surface. Since it weighs about one-twentieth as much as cast-iron it is much more easily transported. The nest sections are of a particular advantage in this respect. Care should be taken to select pipes made of the proper kind of metal. Wrought iron is superior to steel as far as its non-corrosive properties are concerned, and hence pipes made of iron generally have a longer life. The non-uniformity of results obtained with metal culverts of different makes has been due almost entirely to the different kinds of material used in the manufacture.

Concrete Pipes may be cast and laid as any other form of pipe. The joints are made tapering, or with some form of socket. The pipes are built in lengths of from 2-1/2 to 8 ft., with thicknesses varying from 2 to 6 in., depending upon the diameter. Concrete pipes weigh more than cast-iron pipes, but may be constructed so as to cost about one-fourth as much. In the case of concrete pipes constructed in place, the use of reinforcement will not be economical until they exceed 4 ft. in diameter.

7. Foundations

The loads due to traffic are transmitted by the wearing surface to the foundation. If the foundation fails, the pavement or surface above it will fail. The kind of foundation and its thickness depend upon the amount of traffic and the nature of the underlying subsoil.

The Different Materials Used as a foundation are the subsoil encountered, gravel, broken stone, slag, broken brick, hydraulic cement concrete, bituminous macadam and bituminous concrete, and old pavements. When a poor subsoil is encountered, the poor material should be excavated for a depth of several inches and refilled with a good gravel. Sandy subsoils may be improved by the addition of clay, and clayey soils by the addition of sand.

Broken Stone. The lower course of a macadam road, when the latter is built in two courses, is the foundation for the upper or wearing course. Ordinarily this course is about 6 in. in depth after compaction. Where the subsoil is poor or the road is subjected to heavy traffic, or it is desired to aid the subdrainage of the road, the lower course should be increased in thickness, or an additional layer of broken stone should be used. This extra layer varies from 3 to 10 in. in thickness, and is composed of the larger-sized products of the crusher. Instead of this layer of broken stone, a telford base is sometimes constructed, which consists of placing by hand stones broken into sizes 6 to 8 in. deep, 3 to 8 wide, and 6 to 15 long. The stones are placed on edge with their longest dimensions at right angles to the axis of the road. The spaces between the stones are filled with spalls, after which the whole surface is thoroughly rolled. Large stones laid flat instead of on edge are sometimes substituted for telford. The cost of either a broken stone or a telford base depends upon the amount of stone used. Slag and brickbats, where available, can be used in place of broken stone. Gravel foundations are usually laid 6 to 8 in. in thickness.

Cement-Concrete foundations should ordinarily be used under all types of stone block, wood block, brick, sheet asphalt, and bituminous concrete pavements. The thickness of the concrete foundation varies from 4 to 11-in., 6 in. usually being employed. An 11-in. foundation is necessary when the traffic is exceptionally heavy. The usual proportions of the cement, fine aggregate and coarse aggregate vary from 1 : 2 : 5 to 1 : 3-1/2 : 7. Whenever a concrete foundation is constructed, traffic should be kept from it for 7 to 10 days in order to allow it to set up thoroughly. The processes of proportioning, mixing and placing are identical with those of the concrete pavement. Center joints are rarely used. Transverse joints are seldom necessary and are liable to cause difficulties with the wearing course.

Black Base. Bituminous-bound foundations are often termed "Black Bases." Many have been in use for a half century. The old black bases in Washington, D. C., were built of gravel bound with coal tar. Later, crushed stone was used instead of gravel, and either may be used. Either a mixed base or a penetration base may be laid. Coal tar or asphalt may be the binder; both have been successful. If the penetration method is used, the manner of construction is similar to that for the bituminous macadam except that the seal coat is omitted together with the cover. If a mixed base is desired then the method is that of bituminous concrete.

For details of construction and materials see Asphalt Base Pavements, Brochure No. 12, published by The Asphalt Association. Also, "Black Base," by Hugh W. Skidmore, published by The Asphalt Association.

New Tops for Old Pavements. Many old pavements are admirably suited for new tops, and such pavements should not be torn up until it is certain that they will not serve.

Slag. Crushed slag filled with granulated slag makes an excellent foundation. In the course of time such a base sets up like concrete. It should be about 8 in. thick, or more. It is a suitable base for bituminous tops, brick, and other block pavements.

MATERIALS

8. Non-Bituminous Materials: Stone, Brick, Wood

Rocks used for road-building purposes are trap, granite, limestone, sandstone, chert, slate, and field stones.

Diabases and Basalts, which are dark-colored igneous rocks, are commonly known as **trap.** Trap is extremely hard and tough, and its excellent wearing qualities have caused its widespread use throughout those sections of the country where it is found. When used in the construction of broken-stone roads subjected to a light traffic, the wear on the stones will not be sufficient to make enough binder to hold the stones together. To prevent the surface from raveling, more binder of a bituminous material must be applied.

Granite is made up chiefly of quartz and feldspar. In trade, gneiss, syenite and porphyry are commonly known as granite, although not so in a geological sense. If the structure is close, even and granular, these stones make excellent road material for broken-stone roads. If of a coarse structure, they are not so desirable. Granite and syenite are more largely employed for paving stones than other materials, the latter being one of the best materials for this purpose. Stone blocks made of porphyry have been found to wear slippery, and in Europe their use is being discontinued.

Limestone possesses excellent binding qualities, but much of it is neither hard nor tough, and such stone is therefore only suitable for roads taking a light traffic. Some limestones are as hard and tough as trap.

Sandstone, due to the fact that it easily breaks up under the action of traffic, and is lacking in binding qualities, is generally considered as only a fair material for broken-stone roads. Stone blocks, however, made of Medina, Potsdam, and Colorado sandstone have been used to a considerable extent with excellent results.

Quartzites, which are metamorphosed sandstones, give better results when used for broken-stone roads than sandstones, as they are harder.

Chert is a variety of quartz of a flinty structure. It occurs in certain parts of the country either as a solid mass or in a broken-up state, mixed with clay similar to gravel.

Field Stones are boulders which have been carried along by the glacier and are found mainly in those districts which were covered by the glacier. They are composed of a variety of different kinds of rocks, some of which make good road-building materials. Those stones which show signs of weathering and decomposition should not be used. Cobblestones used for paving gutters and streets are small boulders which have been selected from field stones.

Slate, an indurated or hardened clay, is of little value as a road-building stone on account of its fracture and low cementing value.

Chats is a term used to denote the tailings of lead mines. It is a dolomitic limestone, and considered good for use in road construction.

Tests for Broken Stone. By making petrographic and chemical analyses, a rock can be identified and its constituents ascertained. For the purpose of determining the value of a rock as a road-building material, several tests have been devised. The usual tests made of broken stone are for abrasion, toughness, absorption of water, specific gravity, hardness, and cementing power.

The Abrasion Test as adopted by the American Society for Testing Materials, is made by means of the Deval machine, and consists of placing 50 pieces of stone approximately the same size and having a total weight of 5000 g, $\pm$ 10, in one of the cylinders, the cylinder being rotated for 10 000 revolutions, at the rate of between 30 and 33 to the minute. The weight of the detritus that will pass a sieve of 1/16-in. mesh is determined, and the percentage of wear computed. The coefficient of wear, also known as the French coefficient, equals $400/W$, in which W is the weight in grams of the detritus passing the above sieve, obtained per kilogram of rock used. In interpreting results of this test a coefficient of wear below 8 is called low; from 8 to 12, medium; over 12, high. (See A. S. T. M. D2-26.)

The Test for Toughness, as adopted by the American Society for Testing Materials, is made on rock cylinders, 25 mm. in diameter by 25 mm. in height, which have been bored out of the rock by a diamond core drill. These cylinders are placed in an impact machine underneath a plunger 1 kg. in weight, and the latter is subjected to the blow of a 2-kg. hammer. The hammer falls 1 cm. for the first blow, and the fall is increased 1 cm. for each succeeding blow. The number of blows required to destroy the test piece is used to represent the toughness. The average of three specimens is used. Results of this test are interpreted so that rocks which run below 8 are called low; from 8 to 12, medium; and above 12, high.

Common Commercial Sizes of broken stone are screenings, 3/8-in chips, 1/2, 3/4, 1, 1-1/4, 1-1/2, 2, 2-1/4, 2-1/2, and 3 in. The size of the crushed stone depends upon the kind of stone, the crusher, and the screen, and the details of operation. The inclination of the screen and, if a rotary screen, the weather, the speed at which it is run, will make a variation in the separation of the sizes of the screened product.

Specifications for Stone Blocks (A. S. T. M. D59-26) should call for a close, medium-grained, homogeneous material, durable, sound and uniform, with no outcrop, soft, brittle or laminated stone. The stone is tested for crushing, abrasion, and toughness. Cobble should be sound, durable and uniform stone, 4 to 8 in. in diameter.

Clays for Making Vitrified Bricks are not often found in a natural state. A clay for this purpose should be fusible, plastic, and be able to be heated to a high temperature without losing its shape. Vitrification is obtained by subjecting the clay to heat, which changes the chemical properties of its constituents, making them coalesce with each other into a new and homogeneous solid. All clays are composed mainly of silica and alumina and certain impurities such as quartz, lime, magnesia, potash, and soda. The impurities, with the exception of quartz, act mainly as fluxes. An excess of silica will cause a weak and brittle specimen, and an excess of alumina will cause shrinkage, cracking, and warping. An excess of lime and magnesia hastens disintegration upon exposure. Shales are also used for the manufacture of paving-bricks. They produce a harder and more brittle brick than one of fire-clay. Paving-bricks are manufactured by crushing and screening the properly mixed clays or shales. This material is then mixed with water in a pug-mill to the right state of consistency, and is then pushed through a mold, the clay being fed to the mold by means of an augur. The bar of clay as it comes from the mold is cut by machines into the size of brick desired. Paving-bricks are made with plane faces and also with some projections on the faces so that, when laid, there will always be a space between the faces which will later be filled with the joint filler. Some machines are designed so as to cut the brick with lugs on one side and grooves on the other. Other bricks are repressed after being cut, and lugs or grooves, together with the curved edges, are formed by the die used in repressing. The raised letters on the brick serve this same purpose. After being cut or repressed, the bricks are dried and then burned for from seven to ten days at temperatures varying from 1500 to 2300 deg. F. The bricks are slowly cooled in the kiln after the fire is withdrawn, which serves to anneal and toughen them.

The recognized sizes and varieties of paving brick manufactured in 1928 are: Plain wire cut (no lugs), 2-1/2 × 4 × 8-1/2 in., 3 × 4 × 8-1/2 in., 3-1/2 × 4 × 8-1/2 in. Repressed lug, 3-1/2 × 4 × 8-1/2 in. Wire cut lug, 3-1/2 × 4 × 8-1/2 in.

Tests for Paving Brick (A. S. T. M., C7-15). Visual inspection may result in rejection of bricks for being broken, chipped, off size, misshapen, bent, twisted, or badly kiln marked; obviously too soft or too poorly vitrified to endure street wear; so badly off color as to give the pavement a mottled appearance. The rattler test shows the percentage of loss of weight due to abrasion and wear in the rattler. The following scale of average losses represents what may be expected:

	General average loss, per cent	Maximum permissible loss, per cent
For brick suitable for heavy traffic	22	24
For brick suitable for medium traffic	24	26
For brick suitable for light traffic	26	28

Which of these grades should be specified in any given district and for any given purpose is a matter wholly within the province of the buyer.

Wood Blocks that have not been treated by some preservative process have given unsatisfactory results. Rectangular-faced blocks of Southern long-leafed yellow pine, Norway pine, and tamarack are usually specified, only one kind of wood, however, to be used on any one contract. Black gum and short-leaf pine may be satisfactory under certain conditions, but require a different treatment than the woods first mentioned. The Australian hard woods, Jarri and Karrah, are slippery, and are considered too costly for use in this country.

The common method of preserving wood blocks is to treat them with some preservative fluid, the most common being a pure creosote oil or a water-gas or a coal-tar product. In the United States the yellow pine blocks are impregnated with the oil under a pressure of from 50 to 150 lb. per sq. in. The amount of oil absorbed by the wood varies from 10 to 22 lb. per cu. ft.

Timber (A. S. T. M., D52-20). The wood which shall be treated shall be Southern yellow pine, Douglas fir, tamarack, Norway pine, hemlock, or black gum. Only one kind of wood shall be used in any one contract. The blocks shall be sound, well manufactured, square butted, square edged, free from unsound, loose, or hollow knots, knot holes, worm holes, and other defects such as shakes, checks, etc., that would be detrimental to the blocks. No one block shall be accepted that contains less than 50% of heart wood.

It is recommended that blocks 4 in. in depth, shall be used for streets with very heavy traffic, and blocks 3-1/2 in. in depth for streets with moderate traffic. Wood blocks should not be used on light traffic streets.

9. Bituminous Materials

The Bituminous Materials used in the United States may be classified as follows: asphalts, asphaltic and semi-asphaltic oils, coke oven tars, coal-gas tars, water-gas tars, combinations of coal-gas and water-gas tars, combinations of asphaltic materials and tars, rock asphalts.

Nomenclature of Bituminous Materials and Their Uses. Definitions adopted by the Special Committee on "Materials for Road Construction" of Am. Soc. C. E. are noted thus, †; others adopted by the A. S. T. M. are designated thus, * and others proposed by the Committee on "Standard Tests for Road Materials" (Committee D-4) of the A. S. T. M. have been indicated thus, ‡.

Asphalt.†* Solid or semi-solid native bitumens, solid or semi-solid bitumens obtained by refining petroleum, or solid or semi-solid bitumens which are combinations of the bitumens mentioned with petroleums or derivatives thereof, which melt on the application of heat, and which consist of a mixture of hydrocarbons and their derivatives of complex structure, largely cyclic and bridge compounds.

Asphalt Block Pavement.† One having a wearing course of previously prepared blocks of asphaltic concrete.

Asphalt Cement.† A fluxed or unfluxed asphalt, especially prepared as to quality and consistency, for direct use in the manufacture of asphaltic pavements, and having a penetration at 25° C. under a load of 100 g. applied for 5 sec. of between 5 and 250.

Asphaltenes.†* The components of the bitumen in petroleum, petroleum products, malthas, asphalt cements and solid native bitumens, which are soluble in carbon disulphide, but insoluble in paraffin naphthas.

Bitumens.*† Mixtures of native or pyrogenous hydrocarbons and their non-metallic derivatives, which may be gases, liquids, viscous liquids, or solids, and which are soluble in carbon disulphide.

Bituminous Concrete Pavement.† One composed of broken stone, broken slag, gravel, or shell, with or without sand, portland cement, fine inert material, or combinations thereof, and a bituminous cement incorporated together by a mixing method.

Bituminous Macadam Pavement.† One having a wearing course of macadam with the interstices filled by a penetration method with a bituminous binder.

Bituminous Material.† Material containing bitumen as an essential constituent.

Liquid Bituminous Material.† Bituminous material showing a penetration at 25° C. under a load of 50 g. applied for 1 sec. of more than 350.

Semi-solid Bituminous Material.† Bituminous material showing a penetration at 25° C. under a load of 100 g. applied for 5 sec. of more than 10, and under a load of 50 g. applied for 1 sec. of not more than 350.

Solid Bituminous Material.† Bituminous material showing a penetration at 25° C. under a load of 100 g. applied for 5 sec. of not more than 10.

Bituminous Pavement.† One composed of broken stone, broken slag, gravel, shell, sand or fine inert material, or combinations thereof, and bituminous cement incorporated together.

Bituminous Surface.† A superficial coat of bituminous material with or without the addition of stone or slag chips, gravel, sand, or material of similar character.

Blown Petroleums.* Semi-solid or solid products produced primarily by the action of air upon liquid native bitumen which are heated during the blowing process.

Carbenes.†* The components of the bitumen in petroleums, petroleum products, malthas, asphalt cements, and solid native bitumens, which are soluble in carbon disulphide, but insoluble in carbon tetrachloride.

Coal-tar.†* The mixture of hydrocarbon distillates, mostly unsaturated ring compounds, produced in the destructive distillation of coal.

Coke Oven Tar.†* Coal-tar produced in by-product coke ovens in the manufacture of coke from bituminous coal.

Consistency. The degree of solidity or fluidity of bituminous materials.

Cut-Back Products.* Petroleum, or tar residuums, which have been fluxed with distillates.

Dead Oils.†* Oils, with a density greater than water, which are distilled from tars.

Dehydrated Tars.†* Tars from which all water has been removed.

Emulsion.† A combination of water and oily material made miscible with water through the action of a saponifying or other agent.

Fixed Carbon.†* The organic matter of the residual coke obtained upon burning hydrocarbon products in a covered vessel in the absence of free oxygen.

Flux.†* Bitumens, generally liquid, used in combination with harder bitumens for the purpose of softening the latter.

Free Carbon.†* In tars, organic matter which is insoluble in carbon disulphide.

Gas-House Coal-Tar.†* Coal-tar produced in gas-house retorts in the manufacture of illuminating gas from bituminous coal.

Native Asphalt.†* Asphalt occurring as such in nature.

Normal Temperature.†‡ As applied to laboratory observations of the physical characteristics of bituminous materials, is 25° C. (77° F.).

Oil-Gas Tars.†* Tars produced by cracking oil vapors at high temperatures in the manufacture of oil-gas.

Penetration. See penetration test.

Petroleum.‡ Liquid bitumen occurring as such in nature.

Topped Petroleum. Petroleum deprived of its more volatile constituents.

Pitch.†* Solid residue produced in the evaporation or distillation of bitumens, the term being usually applied to residue obtained from tar.

Hard Pitch.† Pitch showing a penetration of not more than ten.

Soft Pitch.† Pitch showing a penetration of more than ten.

Refined Tar.†* A tar freed from water by evaporation or distillation which is continued until the residue is of desired consistency, or a product produced by fluxing tar residuum with tar distillate.

Rock Asphalt.†‡ Sandstone or limestone naturally impregnated with asphalt.

Rock Asphalt Pavement.† A wearing course composed of broken or pulverized rock asphalt with or without the addition of other bituminous materials.

Sheet Asphalt Pavement.† One having a wearing course composed of asphalt cement and sand of predetermined grading, with or without the addition of fine material, incorporated together by a mixing method.

Straight-Run Pitch.* A pitch run to the consistency desired, in the initial process of distillation, without subsequent fluxing.

Tar.†* Bitumen which yields pitch upon fractional distillation and which is produced as a distillate by the destructive distillation of bitumens, pyro-bitumens, or organic material.

Viscosity. The measure of the resistance to flow of a bituminous material, usually stated as the time of flow of a given amount of the material through a given orifice.

Water-Gas Tars.†* Tars produced by cracking oil vapors at high temperatures in the manufacture of carburetted water-gas.

Tests and Specifications for Physical and Chemical Properties. Various tests have been devised in order to determine the physical and chemical properties of bituminous materials. Tests are made for control of the manufacture of bituminous materials, to obtain a record of the properties of materials used, and are employed in specifications to secure the materials desired for use in the construction and maintenance of roads and pavements.

Asphalt Cements. Specific gravity at 25° C.; flash and fire points; solubility in CS_2; solubility of bitumen in CCl_4; solubility of bitumen in petroleum naphtha; penetration at 0° C., 200 g., 60 sec.; penetration at 25° C., 100 g., 5 sec.; penetration at 46° C., 50 g., 5 sec.; float test; melting point by ring and ball method; ductility at 25° C.; fixed carbon content; paraffin content; loss on evaporation at 163° C., 5 hours; penetration of residue (same as for asphalt cement); melting point of residue, by ring and ball method, float test on residue; ductility of residue at 4° C.; ductility of residue at 25° C.

Road Oils. Specific gravity; specific viscosity; total bitumen; proportion of total bitumen insoluble in paraffin naphtha; flash point; residue at 100 penetration.

Tar Cements. Water; specific gravity at 25° C.; solubility in CS_2; specific viscosity, Engler; melting point, by cube method; softening point, ring and ball; float test; distillation by weight and by volume; specific gravity of total distillate at 25° C.; softening point of residue by ring and ball method.

The tests used in a given specification depend upon the kind of bituminous material employed and the method used.

For the purposes of this section it is not necessary to describe tests of which the name itself gives an indication of the method of performing the test. Such tests include specific gravity, solubility in carbon disulphide, carbon tetrachloride, petroleum naphtha, evaporation, and distillation. For detailed descriptions of methods of conducting all of the tests for tars and asphaltic materials, see A. S. T. M. standards and tentative standards.

Flash and Fire Points (**open-cup**). The material is placed in a Cleveland open cup apparatus and the temperature is raised at approximately 10 deg. F. per minute. A test flame of 5/32-in. diameter is passed across the center of the cup as the temperature read on a thermometer inserted in the material reaches each successive 5° F. mark. The **flash point** is taken as the temperature read when a flash appears at any point on the surface of the oil. The **fire point** is taken as the temperature at which the oil ignites and continues to burn for at least 5 seconds.

Softening Point (**ring and ball**). The material is melted and run into a brass ring 5/8 in. in inside diameter and 1/4 in. deep. It is allowed to cool in the air, and is then suspended in a beaker of water maintained at 5° C. for 15 minutes. A 3/8-in. steel ball is immersed at the same time. The ball is then placed in the center of the upper surface of the bitumen in the ring and heat is applied in such manner that the temperature of the water is raised 5° C. each minute. The temperature of the water recorded at the instant that the material touches the bottom of the beaker is known as the **softening point.**

Consistency. The consistency of bituminous materials is determined by the Engler viscosimeter, the New York Testing Laboratory flat apparatus, or the penetrometer.

With the **Engler Viscosimeter** the viscosity of liquid bituminous materials is determined by noting the time which is required for a given amount of the material, having a given temperature, to flow through a very small orifice. The result of the test should be expressed as specific viscosity, which equals the ratio of the number of seconds required for the passage of 50 cc. of the bituminous material at the temperature used divided by the number of seconds required for the passage of the same volume of water at 25° C.

The New York Testing Laboratory Float Apparatus consists of an aluminum float and a brass collar. The collar is filled with bituminous material and screwed into the bottom of the aluminum float and the apparatus placed on the surface of a water bath. As the plug of bituminous material in the collar becomes warm and fluid, due to the heat from the water bath which is maintained at any temperature desired for the test, it is gradually forced upward and out of the collar until water gains entrance to the saucer and causes it to sink. The time in seconds between placing the apparatus on the water and when the water breaks through the material is taken as the measure of consistence.

The Penetration Test is made by measuring the distance a weighted standard needle will penetrate into the material at a given temperature in a given period of time. The temperatures, weights, and periods of time which are employed to a considerable extent are as follows: Penetration at 0° C. with a weight of 200 g. for 1 minute; penetration at 25° C. with a weight of 100 g. for 5 seconds; penetration at 46° C. with a weight of 50 g. for 5 seconds. When the penetration of a material is mentioned without reference to temperature, weight of the load, or time, it is understood that reference is made to the penetration at temperature of 25° C. with a weight of 100 g. for 5 seconds. The unit of penetration is 0.1 mm. In literature and specifications the penetration is referred to in terms of the above unit either as a penetration of 6.4 mm. or 64.

Ductility. In the ductility test a briquet of the material is formed in a standard briquet mold. The briquet with clips attached is placed in a ductility testing machine filled with water at a temperature of 4° C. or 25° C. The briquet is then pulled apart at a uniform rate and the distance in centimeters registered at the time of rupture of the thread of bituminous material is taken as the measure of ductility.

Fixed Carbon. Fixed carbon is the organic matter of the residual coke obtained upon burning hydrocarbon products in a covered vessel in the absence of free oxygen.

Paraffin. One hundred grams of the material is distilled rapidly in a retort to a dry coke. Five grams of the distillate is then thoroughly mixed in a 60-cc. flask with 25 cc. of Squibbs' absolute either. Then 25 cc. of Squibbs' absolute alcohol is added, and the flask packed closely in a freezing mixture of finely crushed ice and salt for at least 30 minutes. The precipitate is filtered out quickly with a suction pump, using a No. 575 C. S. and S. 9-cm. hardened filter paper. The flask and precipitate is then rinsed and washed with a mixture of equal parts of Squibbs' alcohol and either cooled to − 17° C. (1° F.) until free from oil. When sucked dry, the filter paper is removed and the waxy precipitate transferred to a small glass disk and evaporated on a steam bath. The residue (paraffin) remaining on the disk is weighed, and from this weight the percentage on the original 5-g. sample is calculated.

Extraction of Bitumen from Bituminous Aggregates. The aggregate is prepared for analysis by heating it in an enamel-ware pan on a hot plate until it is sufficiently soft to be thoroughly disintegrated by means of a large spoon. The disintegrated aggregate is then allowed to cool, after which a sufficient amount is taken to yield on extraction from 50 to 60 g. of bitumen. It is then placed in a mechanical extractor and carbon disulphide is poured into the receptacle containing the aggregate. After allowing the material to digest for a few minutes, the machine is started, slowly at first in order to permit the aggregate to distribute uniformly. The speed is then increased sufficiently to cause the dissolved bitumen to flow from the receptacle. When the first charge has drained, the machine is stopped and a fresh portion of disulphide is added. This operation is repeated from four to six times until the liquid flowing from the receptacle is clear. After the aggregate is thoroughly dried, it is weighed. The difference between this weight and the original weight taken shows the amount of bitumen extracted.

ROADS

10. Earth Roads

Since earth roads may be constructed out of any kind of soil it is difficult to compare their characteristics. Soils act differently under different conditions. A sand road is good in wet weather whereas one of clay is impassable. A clay road is good in dry weather whereas sand makes hard traveling. If these two materials can be so mixed that their good characteristics predominate, then a road good in any weather is possible. Earth roads are cheap to construct, and, if traffic is not unduly heavy, their maintenance cost is low.

Of the 3 000 000 miles of road in the United States (1928) 85% are unsurfaced, 10% are a low type surfacing such as sand-clay, gravel or untreated stone; 4% are an intermediate type, and less than 2% are high type. Hence the earth road for years to come must be the predominating type. With this in mind many experiments have been tried for the purpose of improving earth roads at low cost. Surface treatments with a number of different materials have been tried with varying degrees of success. On the whole the bituminous materials have been the most successful. C. N. Conner cites three general methods of treatment: (1) The skin surface treatment or penetration method, hot or cold; (2) the mixed-in-place treatment, cold; (3) the premixed method, hot or cold. Treatments (1) and (2) are discussed under "**Gravel Roads,**" p. 1925, and (3) is discussed under "**Asphalt,**" p. 1942.

One point of utmost importance is often lost sight of by those who are new to the practice of treating low type roads, and that is that each road, and even certain stretches of each road, may be a problem in itself. A material or a method that may be satisfactory on one stretch of road may be unsatisfactory on the next stretch. Soils may change in character within a few feet, and all materials are not equally suited for use on all soils. The man who wants to be successful must experiment for himself, and he must watch developments with an all-seeing eye. Failure to realize this has retarded the art in many localities. Tars, asphalts, and road oils are not identical materials. Maintenance men have wasted great sums because they have used incorrect methods and wrong materials.

Earth roads may be improved sometimes by the admixture of small amounts of such materials as pea-gravel, granulated slag, portland cement, stamp sand, lime, steam coal ashes, chats, and similar hard particles. A layer is usually spread over the earth road and then is harrowed into the surface; or traffic is supposed to work in the materials. As a rule a layer an inch thick is about as much as can be handled easily. Sometimes no harrowing should be attempted, and traffic will compact the surface. Again, it may be necessary to blade or drag the road, and some roads respond well to rolling, while others do not. If you have no experience you must experiment; if you have experience you *will* experiment. Experiments on earth roads in Iowa seemed for a while to indicate that admixtures of portland cement might be of great value; later, these roads so treated were not as good as untreated sections, and the cost was considerable. Yet it is possible that the same method might be eminently successful with some other earth road. A light tar may be useful with some soil roads, but if there is much clay present it will surely be a failure. In general, the maintenance engineer should buy two or three barrels each of several different bituminous materials and try them out on the same road where conditions and soil are the same. After a year's trial much should have been learned.

The bituminous materials suitable for earth road treatments are road oils, cut-back asphalt, water-gas tar, and coal tar. As a rule the road oils are cheaper than the tars, and the tars are cheaper than cut-back asphalt.

See specifications Road Oil, Illinois, and Pennsylvania specifications for cold application (cut back).

In some instances it has been found that earth roads should be given a prime coat of light tar (Pennsylvania Specification C2) followed by a seal coat of cut-back asphalt. A cover of pea-gravel, stone chips, or coarse sand often helps make a tougher surface that is capable of carrying more traffic.

A method of making soft earth roads passable is to spread three layers of ordinary 1-1/2-in. chicken wire for wheel tracks. Heavy trucks will not cut through a road so reinforced. Another device is a longitudinal section of a corrugated metal pipe laid across mud holes, the pipe thus forming a rut for the wheels. The device has been trade-marked under the name of "Metal-rut."

Construction of Earth Roads. Good drainage is an absolute necessity. Proper subdrainage and surface drainage must be provided. In providing for surface drainage the slopes from the center to the sides should not be made too steep; a slope of from 3/4 in. to 1 in. per foot is used, the latter being a maximum. The surface of the road may be made to conform to the arc of a circle or be made up of two planes meeting at the center and sloping to the ditches. Care should be taken in constructing the ditches to see that they have sufficient fall to carry the water away. In places where the country is flat and the work involved is simply that of giving a crown to the road, earth roads may be constructed most economically by means of road scrapers or graders. If the soil is not too compact, the use of the plow could be done away with in using the road grader. The latter is usually worked by beginning at the sides, going down one side and back the other, gradually approaching the center, the blade being so adjusted as to plow to the proper depth, and move the earth up from the sides toward the center. The material deposited by the grader is sometimes harrowed and then rolled; when rolling is not resorted to, the material is left to be packed down by traffic. Care must be taken to spread the material evenly and in layers usually recommended not to be over 6 in. deep. Where much grading is involved, the bulk of the work will have to be done by the common methods of earth excavation, the road grader being used to finish the road to the desired surface.

Sand-Clay Roads are constructed somewhat differently, depending upon whether the subsoil is of sand or of clay. The amount of clay necessary is that amount which will just fill the voids in the sand. It will also depend upon the character of both the sand and the clay. It may be approximately determined by finding the quantity of water which is contained in a known volume of sand, the amount of water representing the percentage of voids. Proper drainage must always be provided. The construction of a sand-clay road is a slow process, and the best results can be obtained only by giving the road constant attention for some time after it is first finished.

The Maintenance of Earth Roads consists principally in keeping the surface shaped up and the ditches cleaned out so that the water will not have any chance to stand either in the road or at the sides. Water, if it has a chance to soak into the road, soon softens it to such an extent that it is easily cut up by the action of traffic and is soon destroyed. Two of the most useful tools for the maintenance of earth roads are the road scraper and road drag. There are many different types of these machines. The road drag, which is the simplest, consists of two blades about 7 to 9 ft. long set parallel to each other about 30 in. apart. The blades may be made of steel, of plank, or of split logs. This device is so hitched to the team that it may be dragged along the road at an angle at about 45 degrees with the axis of the road. Work with the drag scraper is done in a similar manner as with the road scraper; namely, starting in at the side of the road, and working up toward the center. Dragging may be done at all seasons, but should be carried on only after a rain when the road is in a moist condition. If the road is properly drained it will be found that the surface can be kept in excellent condition by the use of the road drag or road scraper at frequent intervals.

Summary of Specifications for Road Oil of the Illinois Division of Highways

E1 is a light asphaltic oil.
E2 is a light semi-asphaltic oil.
E3 is a heavy semi-asphaltic oil.
E4 is an extra heavy semi-asphaltic oil.
E5 is a heavy asphaltic oil.

The road oil shall be homogeneous and shall contain not to exceed 1% by volume of water. It shall meet the following requirements:

	E1	E2	E3	E4	E5
Specific gravity 15.5°/15.5° C...	0.94–0.97	min. 0.91	min. 0.92	min. 0.93	0.95–0.98
Specific viscosity, 60° C.......	8–16	8–16	16–28	28–42	16–28
Total bitumen, per cent, minimum....................	99.5	99.5	99.5	99.5	99.5
Bitumen insoluble in 86° Bé. paraffin naphtha, per cent....	min. 11	max. 8	max. 9	max. 10	min. 12
Flash point, deg. C., minimum.	80	80	80	80	80
When 50 g. of the oil are evaporated at 250-260° C. until the residue has a penetration at 25° C., 100 g., 5 sec., of 90-110, the residue shall have a ductility at 25° C. of not less than cm...........	50	......	....	....	50

Asphalt Cut-Back. Asphalt for Cold Patching

Specification Pennsylvania Highway Department

This material shall be a cut-back asphalt prepared by compounding a suitable volatile naphtha with an asphalt meeting the requirements of the Department's Specifications for Penetration Asphalt.

Specific viscosity, 25° C.	80 to 120
Residue at 100 penetration, per cent by weight	65 min.
Loss at 163° C., 20 g., 5 hr., per cent	25 min.
Penetration of residue, 100 g., 25° C., 5 sec.	30 to 75
Ductility of residue, 25° C., cm.	30 min.
Bitumen insoluble in 86° Bé. paraffin naphtha, per cent	12 min.

11. Gravel Roads

Dust Palliatives, Surface Treatments

Gravel roads are easy to construct, repair and maintain, offer ease of traction and resilience, are cheap in first cost if the material is at hand, and maintenance charges may be kept at a figure less than the interest on investment in a high-class pavement. Many more miles of gravel roadway would be built if its worth were realized. Every state has gravel that is available for road use. Only a relatively small percentage of the highways of the country can ever be built with high cost roads. The gravel road is the happy mean between the earth road and the expensive pavement. When mileage in a district is large with small assessed valuation, gravel is the material to be used rather than the more expensive crushed stone. But freight rates are the same on gravel as on stone, therefore if the material must be brought from a distance it may be cheaper in the long run to pay more for stone, if it is of good quality.

Many roads carry scarcely any traffic except in summer. Such roads should be built and maintained cheaply. Usually an earth road is unsatisfactory in these cases, but a gravel road serves to advantage. Where traffic is local and is fairly heavy throughout the year, a more expensive form of construction should be used, or, if it can be satisfactorily done, the gravel road should be surface treated with a bitumen.

Gravel consists of small pebbles, stones, or fragments of stone intermixed with powdered rock, sand, loam, clay, marl, flints, iron oxide, silicon, etc., formed by the action of water upon disintegrated portions of rock.

Good results in road work will be obtained with a graded gravel that consists of 80% small pebbles or fragments of stone that pass a 1-1/2-in. ring and are retained on a 1/4-in. mesh sieve, and 20% of sand and clay (or similar materials) in such proportions that they act as a binder which does not become objectionably dusty when dry nor unduly muddy when wet. A gravel that will stand for some time with a vertical face in the pit or bank will ordinarily be satisfactory.

Angular pebbles are better than those that are round; the pebbles of seabeach and shore gravels are generally too round. By careful grading, a gravel may be produced of which the particles key well, thus reducing voids to a minimum, and the quantity of binder needed. Since the wearing surface depends on the hardness and toughness of the stone, the less binder necessary the better the surface will be, and the less mud and dust will be formed.

The best **binder** is that formed from the dust of rocks high in cementing value. Trap rock gravels are therefore better than granite gravels, since they possess both durability and fair cementing qualities. Limestone and sandstone gravels are not so durable. If the binder does not naturally possess enough cementing value, then clay must be added.

Bank-run, or pit gravel, is often too earthy, and should be screened. The larger pebbles may be crushed, or may be raked into the foundation course.

Construction

1. **Drainage.** Thoroughly drain the subgrade. Gravel free from excess binder may be used to advantage for filling draining ditches.

2. **Subgrade.** Shoulders on each side of the road may be constructed (a) as shown, and gravel deposited in the trench. (See Fig. 4).

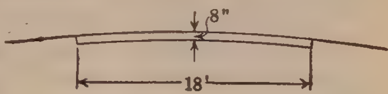


Fig. 4

(b) Or, on a flat subgrade the gravel is spread, deeper in the center than at the sides. (See Fig. 5.) The center takes most of the traffic and a gravel road always flattens under traffic. The depth of gravel may vary at the center from 8 to 15 in.

3. **Grading of Gravel.** Grade gravel so as to key well; it will then require less binder, and give a more durable wearing course. Where sea-beach or shore gravel is used it should be run through a crusher to give angularity to the pebbles; otherwise they do not key well and the road ravels.

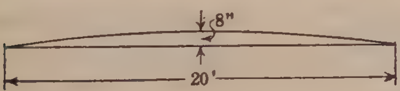


Fig. 5

4. **Bottom Course.** Deposit gravel, and any binder needed in excess, on dumping boards; mix, and rake into place. Rake large pebbles to form foundation of bottom course.

Character. Pebbles retained on 1/4-in. and passing 2-1/2-in. screens.

5. **Depth of Course.** Do not spread more than 4 in. deep for any one course at a time. Satisfactory rolling is impossible with greater thicknesses.

6. **Rolling.** Roll each course with a roller weighing at least 2 tons. A grooved roller is better than a smooth one as it gives more uniform compaction.

7. **Second Course.** Not more than 4 in. deep.

Character. Retained on a 1/4-in. and passing a 1-1/4-in. screen.

8. **Rolling.** Roll so as to obtain a crown of 3/4 in. to the foot, and until the road is thoroughly compacted. If water is to be obtained cheaply, the gravel should be moderately sprinkled while it is being rolled. Too much water will float off the binder. Inasmuch as a gravel road is built for the sake of economy, watering may be too expensive, and rain must be depended on to make the materials bind.

If no roller is available, then traffic must compact the gravel *after each course is put on*, in which event a patrolman should be retained to fill up the ruts for at least 30 days after the apparent completion of the road. He should shape up the road with a blader or a drag, and endeavor to keep traffic spread out over the entire graveled width.

Gravel roads are sometimes constructed in two courses, leaving off the topping of fine material.

Repairing

1. If badly rutted and worn, break up the surface to make the new material bond with the old, raking the large pebbles to the bottom.

2. Remove all vegetation from the roadbed.

3. If necessary, scarify and harrow the old surface. Then reshape with a grader and hand rakes. If a grader is not available use a log drag.

4. Apply sufficient graded gravel to make a good wearing course. A road in very bad condition may require up to 6 in. of new material. Apply new gravel in *thin layers*, adding a little clay, if necessary, to make the old and the new materials bond. Or, if the old road has an excess of sand and binder, use clean gravel free from fines.

5. If water is handy sprinkle moderately while rolling in dry weather.

6. Roll, always beginning at the side of the road and working toward the center.

7. Form a crown as in a new road of not more than 3/4 in to the foot. A higher crown will force vehicles to keep to the center of the road, soon forming ruts.

Maintenance

1. **Install a Patrol System,** and furnish the patrol man with a complete kit of tools.

2. Place piles of gravel at convenient intervals along the road. They should be at least as close as 1/8 of a mile.

3. Rake *new* gravel into the ruts, raking the large pebbles from the surface.

4. Use a grader, or a drag, or both, to keep the road in shape. Proper dragging helps preserve the crown.

Gravel Roads Treated with Bituminous Materials

All gravel roads are not susceptible to successful treatment with bituminous materials. In most cases, however, careful study of the materials at hand will indicate what method should be tried. Sometimes it is wise to try out more than one method on any given road or in a definite locality, and then select that one which promises the best results over a period of time. Some gravels will respond well to any of the usual treatments, whereas others require a particular method. It is sometimes impossible to tell from a laboratory examination of the gravel what the results will be, and in such cases only experiments with the gravel in place on the road will tell the story.

There are two general methods of treatment, but these may be modified to suit individual cases. These methods are (a) surface treatment by the superficial spraying of tar or asphaltic oil over the finished gravel road, and (b) the mulch method. In general, a well bound gravel road, that is free from loose particles, that does not contain over 20% of clay, and that presents a firm surface to traffic will take a surface treatment of bitumen satisfactorily. If the gravel in the road is loosely bound, is dusty, and does not tend to set up well after a rain, then the mulch method will probably be the better one to try.

Highway engineers and the general public must realize that a gravel road is a cheap type, and that although bituminous treatments may improve such roads to an almost unbelievable extent, still, a gravel road is a gravel road. Too much must not be expected of it. It will not carry an unlimited tonnage of heavy, fast truck traffic, although it will carry an endless procession of passenger cars with no apparent damage the year round unless such cars use chains in the winter months. A treated gravel road is rapidly destroyed by chains in a surprisingly brief time. Pot holes and long ruts are formed in such profusion that economical repairs are impossible.

Having determined upon the kind of treatment, it then becomes necessary to continue the maintenance of these roads by successive applications year after year. No gravel road should be treated unless provision is made certain to continue until such time as a better type of road is built. A treated gravel road that is allowed to deteriorate soon becomes almost impassable. It is far worse than an untreated surface, which, though it may become corrugated and full of holes, is still serviceable.

Maintenance of Gravel Roads with Tar or Asphaltic Oil

Method (a)

1. Put the surface of the road in good condition some time before the bitumen is to be applied, by blading, dragging, rolling, and thus thoroughly filling all ruts and depressions and removing " wash board."
2. Immediately before spraying sweep off all loose dust and dirt.
3. Do not treat a wet road.
4. Apply two coats of bitumen (cold application) of 1/4 gal. per square yard. Applications should be at least 24 hours apart.
5. Let first application sink into the road without applying cover. (If possible keep traffic off that section of the road.)
6. After the second application apply sufficient fine gravel or clean sand to prevent traffic from picking up the surface.
7. In case a thicker, heavier crust is desired omit the second coat of cold application and substitute a hot treatment (tar, p. 1929, or asphalt, p. 1928) at the rate of 1/4 to 1/3 gal. per square yard, and cover with 3/4-in. screened gravel, rolled in.

Note. There are many variations of the above outline, but this is the basis of them all.

Method (b)

The second method has been variously known as the mulch method, the Wisconsin method, the Minnesota method, and several others. It resembles the old Cape Cod method of treatment of sand roads, and under proper conditions it is highly successful. Though many twists and kinks are put in it, essentially it is as follows:

1. Clean off manure and other filth, but do not sweep the road.
2. Apply a cold surface treatment of tar or asphaltic oil over the entire road surface at the rate of 1/4 gal. per square yard.
3. With a heavy grader drawn by a tractor push the upper inch to inch and a half of treated gravel from one-half the width of the road onto the other half.
4. Apply 0.2 gal. per square yard to the fresh surface thus exposed.
5. With the grader push all the loose material from the other half of the road onto the freshly treated side, making a cut with the blade such that the second half resembles the first, after operation (3) above.
6. Apply 0.2 gal. per square yard to the fresh surface of the other half, identical with operation (4) above.
7. With the grader push one-half the loose material back onto the surface just treated. The whole road has now had two treatments, of which the original has been pushed back and forth so that the bitumen has intimately mixed with the gravel. The loose gravel on top is then ready for the next operation.

8. Work the road with bladers, scrapers or drags until a smooth, waveless, rippleless, non-corrugated surface is obtained.

9. When traffic has thoroughly compacted the road, apply from 0.15 to 0.2 gal. per square yard of bitumen.

10. After 2 hours apply a cover of fine gravel sufficient to take up any excess of bitumen. In some localities the cover is applied immediately after the distributor has applied the bitumen.

The tar to be used should conform to specification D-104 or D-105, 27-T. The asphalt should be a cut-back conforming to specification for cold surface treatment.

Note. Either tar or asphaltic oil may be used, but in this work tar has been more successful, on the whole, than asphaltic oil.

Maintenance

1. Clean out ruts and holes that may have formed.

2. Make up a batch of sand and gravel mixed with tar (for cold application) or asphalt cut-back in the proportion of 17 gal. of bitumen to one cubic yard of aggregate. The gravel used should contain no pebbles over 3/4 in. Sand should be free from earth.

3. Fill the ruts and holes with the mix.

4. Let traffic consolidate the patches. It is sometimes advantageous to spread the mix freely over the old surface and drag until the mix has set up in the low spots.

5. Apply cold application of bitumen, 1/4 gal. or less per square yard.

6. Sand the surface after 24 hours. Some states require sanding within two hours, but this does not allow sufficient time for the bitumen to penetrate. By treating only half the width at a time traffic will not be unduly inconvenienced.

A word of caution should be given here concerning **hot** surface treatments with either tar or asphalt. Experience has demonstrated that there is great danger of these causing rolls and waves in the road surface if they are persisted in year after year. The **cold** treatments are far less liable to wave. Some of the rolling can be avoided if the covering material is of large size, that is, over 1/2 in. and less than 1-1/4 in. If gravel of such sizes is used it must be rolled into the surface to prevent its being thrown off by traffic. Crushed stone is not so liable to be whisked out of place.

Sometimes, after repeated treatments, a gravel road presents a surface that resembles a huge washboard. When this is such that cars are disagreeably shaken, there is no way to make the road smooth again short of scarifying it, shaping, rolling and the application of a light surface treatment.

In some sections of the country the existing gravel roads break up badly in springtime. When this is the case it is usually cheaper to scarify the entire area that is broken up, let it dry out, and then retreat it as in the beginning.

It will be noted above that initial treatments consist of a total of 1/2 gal. per square yard. Subsequent treatments, for the most part, must be reduced to 1/4 gal. or less. It is a great mistake to continue to treat these roads with 1/2 gal. applications yearly, as a wavy surface is almost certain to form.

Cost of Treatments with Tar or Asphalt

Costs vary with location, labor rates, freight charges, and similar items. The initial cost for treating a gravel road that is in fair condition will be from \$1500 to \$2000 per mile of 18-ft. road or, 14 to 20 cents per square yard. Subsequent treatments including maintenance should not average more than

$1000 per mile per year, and with care may be somewhat less (1927 prices). Maintenance throughout the year is absolutely essential. By repairing small breaks promptly it is possible to keep the annual maintenance charges low.

Dust Palliatives

Many materials have been tried to alleviate the dust nuisance on gravel roads. Light asphaltic oils have been moderately successful in some localities and unsuccessful in others. On the whole, calcium chloride has proved to be a better dust layer than other materials. It is customary to make two applications per season, the first being at the rate of about 1-1/2 lb. per square yard, followed by 1/2 lb. to 1 lb. Spreading machines are used to insure uniform distribution.

Calcium chloride is not successful in climates where there is either too much or not enough moisture. As it is a hydroscopic and deliquescent salt it requires some moisture in the atmosphere, but an excess will make a muddy, dirty road that is more objectionable than a dusty gravel, whereas a deficiency will make it necessary to sprinkle the road occasionally.

Sea water has been tried where available, but the salt thrown upon the vehicles ruins the finish, and it injures horses' hoofs.

Roads in the Southwest

The attention of engineers and contractors unfamiliar with the arid area of the United States is drawn to the fact that it is cheaper in many instances to build a bituminous-bound road than one of the water-bound type. Water in Oklahoma, Arizona, New Mexico, and parts of Texas, southern California, Colorado, and Nevada, is more expensive to use than tar or asphalt. Hence certain types of bituminous-bound or treated roads are particularly suited to these localities.

Gravel, of one kind or another, is to be found in most of the inhabited portions of these states, and often rock of good quality as well. Adobe gravel and caliche are best built and maintained by a double bituminous treatment, as follows:

Adobe and Caliche. 1. Shape up, and roll to cross-section.

2. Spread 1/2 gal. of tar or asphaltic oil per square yard. (Spec. tar for cold application for surface treatment and oil for cold application.)

3. Let this dry in for 12 hours or more and follow with:

4. A seal coat of 1/2 gal. of tar applied hot (spec. for hot surface treatment), or a seal coat of 1/2 gal. of asphaltic oil (spec. for hot application).

5. Cover with stone chips, or pea gravel.

As a rule a tar priming coat is superior to one of asphalt. This priming coat alone will not suffice. It *must* be followed by the seal coat.

Maintenance must be most thorough, or the road will rapidly go to pieces. Six barrels of seal coat should be kept along each mile of road for patching purposes, and the road must be carefully watched for any breaks in the surface. Eternal vigilance will accomplish wonders with this simply constructed road. Many of the southwest gravels can be turned to good account by the following:

1. Spread gravel evenly 6 in. deep.
2. Apply 1-1/2 to 2 gal. of hot asphaltic oil in several applications, harrowing the road with a disc harrow after each application.
3. When the oil is thoroughly mixed with the gravel, shape up the road and roll it.
4. Apply a seal coat of 1/4 gal.

Eastern and northern contractors should remember that gasoline rollers and tractors must be used in this waterless country.

Asphaltic Surfacing Material for Gravel Roads

1. Free from water.
2. The gravity at a temperature of 60° F. shall be between 15 and 19° Baumé (0.94 to 0.966 specific).

3. The asphalt contents of 100 penetration using a No. 2 needle for 5 sec. at 77° F after evaporation in the open air at a temperature not exceeding 500° F. shall be not less than 55%.
4. Specific viscosity (Engler) at 50° C. for 50 cc. from 17 to 29.
5. 20 g. upon being maintained at uniform temperature of 325° F. for 5 hours in a cylindrical vessel 2-3/8 in. in diameter by 1 in. high shall lose not less than 15% by weight.
6. Soluble in CS_2 = 99.5%.
7. It shall not flash below 130° F. (Cleveland tester).

Specifications
Pennsylvania Department of Highways

Asphalts. A hot and a cold application asphalt are specified.

The cold application material must be prepared by fluxing an asphalt cement of 120 to 150 penetration (100 g., 5 sec., 25° C.) with naphtha. The naphtha used when distilled in accordance with the A.S.T.M. tentative method, D 86-24T, shall have an over point of not greater than 250° F. and the dry point shall not exceed 450° F., and the naphtha shall show a continuous distillate between these points.

The mixture must conform to the specification listed in the table below, in which the viscosity will be subject to variation within the limits designated.

The hot application must be the properly prepared residuum from an asphaltic base petroleum and must meet the requirements for it listed in the table below.

Summary Specification Requirements for Cold and Hot Application Asphalts for Surface Treatment

Specifications of Pennsylvania Highway Department

	Cl Cold Application	Hl Hot Application
1. Specific viscosity, 40° C.	34–50	......
2. Float test, 50° C., sec., minimum	......	125
3. Residue at 100 penetration, per cent	68 min.	88–95
4. Ductility of residue, 25° C., cm.	30 min.	25 min.
5. Evaporation loss, per cent:		
(a) 20 g., 5 hr., 100° C.	22 min.	......
(b) 20 g., 5 hr., 163° C.	25 min.	5 max.
6. Penetration of residue (5b) 100 gr., 5 sec., 25° C., per cent	60–100	......
7. Total bitumen, per cent, minimum	99	99

The following notes refer to the table on page 1929.

[1] All tests shall be made on the sample as received including distillation test, and the results reported on a dry basis.

[2] Within the viscosity limits designated, a material should be chosen to meet local conditions of temperature, road conditions and climate. It is recommended that materials be called for under the following range and headings: Light 8-13, medium 13-18, heavy 18-25, extra heavy 25-35.

[3] It is recommended that materials be called for under the following range and headings: Light 35-60, heavy 60-80. The heavy material may require heating before use and care should be taken to avoid foaming on account of the possible water content.

[4] The specification range for total bitumen covers a wide variety of materials. If products from vertical retort or low carbon coke-oven tars are desired, a range 88-95% should be specified. If high-carbon coke-oven or mixtures of coke-oven and gas-house tars are desired, a range of 78 to 88% should be specified.

[5] If desired, a float test may be substituted for the softening point test, in which case the requirements shall be as follows: Float test at 50° C. (122° F.) 100 to 220 sec.

Specifications for Various Kinds of Tars

American Society for Testing Materials[1]

The Tar shall conform to the following requirements:

	Cold application				Hot application		Tar cement for bituminous macadam		Coal tar pitch, stone block filler	Test specifications
	Surface treatment		Cut-back for repair work		Surface treatment					
	H. C.	L. C.	H. C.	L. C.	H. C.	L. C.	H. C.	L. C.		
A. S. T. M. serial designation	D 104-27T	D 105-27T	D 106-27T	D 107-27T	D 108-27T	D 109-27T	D 110-27T	D 111-27T	D 112-27T	A. S. T. M.
Water per cent, maximum	2	2	2	2	0	0	0	0	.0	D 95-24T
Specific viscosity, Engl 50 cc 40° C.	8-35[2]	8-35[2]	35-80[3]	35-80[3]	....	....	....	....	....	Bull 1216, U. S. D. A.
Float test at 32 sec.	....	....	....	....	65-180	65-150	....	....	....	D 139-23T
Softening point (ring and ball) ° C	....	....	....	....	....	....	30-40[5]	30-40[5]	....	D 36-26
Softening point (cube in water) ° C	....	....	....	....	....	....	....	....	46-57[6]	D 61
Total distillate by weight, per cent, maximum:										D 20-26T
0-170° C	7	5	3-10	3-10	1	1	1	1	....	D 20-26T
0-235° C	20	20	8-20	8-20	10	4	2	2	....	D 20-26T
0-270° C	30	35	18-30	18-30	15	13	10	10	....	D 20-26T
0-300° C	35	45	35	38	25	26	20	20	14	D 20-26T
Residue by weight, per cent maximum	65	55	65	62	75	74	80	80	86	D 20-26T
Specific gravity 25°/25° C., total distillate to 300° C.	min. 1.01	....	....	....	min. 1.02	....	min. 1.02	....	min. 1.02	
Softening point residue (ring and ball), maximum	60° C.	60° C.	65° C.	65° C.	65° C.	65° C.	65° C.	65° C.	65° C.	D 36-26
Total bitumen, per cent[4]	88-97	95 min.	78-95	95 min.	78-95	95 min.	78-95	95 min.	65-80	D 4-26T
Softening point residue cube in water, maximum	....	....	....	....	....	....	....	....	75 C.	D 61
Ductility at 50-100 penetration at 25° C., maximum	....	....	....	....	....	....	....	....	50 min.	D 113-22T

The specification range for softening point, within the above limits, should be 5° C. for any given locality, for example, 30 to 35° C. for cold climates, equivalent to a float test at 50° C. of 100 to 160 sec.; 35 to 40° C. for warm climates, equivalent to a float test at 50° C. of 160 to 220 sec.

6 The softening point (cube-in-water method) specified should have a range of not over 10° F. within the above limits. The range, within the limits of 115 to 135° F., should vary with the use of the material, for example, if used in a mixture with sand, in a northern locality or a southern locality. The softening point range, within the above limits, should also vary according to the character of the paving.

Summary Specification Requirements for Tars for Surface Treatment

Specifications of Pennsylvania Highway Department

			C 2 Coal tar base	C 3 Low carbon base	D 2 Coal tar base	D 3 Low carbon base
Specific viscosity.	60° C.	Base	3.6–13.6	....	3.6–13.6	....
	40° C.	Flux	1.1–3.6	1.0–3.6	1.2–3.6	1.0–3.6
		Mixture	8.6–35.0	13.6–35.0	4.5–8.6	5.4–13.6
Total bitumen, per cent....		Base	88–97	95 min.	89–97	95 min.
		Flux	95 min.	97 min.	95 min.	97 min.
		Mixture	89–97	95 min.	89–97	95 min.
Float test, 50° C., sec......		Base	....	80–110	....	80–110
		Flux	....	....	....	....
		Mixture	....	....	....	....
Distillation, per cent by weight, maximum....	0–170° C.	Base	4.5	1	7	1
		Flux	7	10	7	10
		Mixture	5	3	7	5
	0–270° C.	Base	....	12	....	12
		Flux	....	....	....	....
		Mixture	....	33	....	37
	0–300° C.	Base	35	25	37	25
		Flux	25–87	40–85	25–87	40–85
		Mixture	40	43	45	50
Softening point, residue from distillation test, ring and ball,° C., maximum......		Base	65	65	65	65
		Flux	....	....	....	....
		Mixture	65	65	65	65

Refined Tar for Use in Repair Work Cold-Patch

General Requirements. The refined tar shall be homogeneous.

Properties. The refined tar shall meet the following requirements:

1. Specific gravity at 25° C. (77° F.) 1.140 to 1.190.
2. Specific viscosity, Engler, 50 cc. at 40° C. (104° F.) 35.0 to 60.0.
3. Total distillate, per cent by weight,

    To 170° C. (338° F.) 2.0 to 8.0.
    To 235° C. (455° F.) 10.0 to 18.0.
    To 270° C. (518° F.) not more than 26.0.
    To 300° C. (572° F.) not more than 30.0.

(a) Softening point of residue (ring and ball), not more than 65° C. (149° F.).

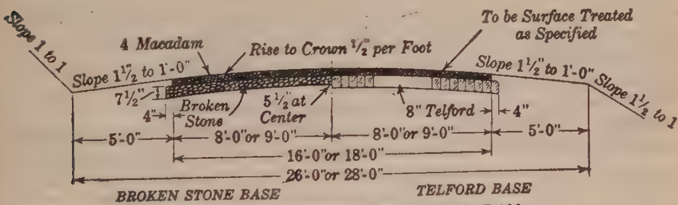
4. Bitumen (soluble in carbon disulphide), per cent by weight, 80.0 to 88.0.
5. Water, per cent by volume, not more than 2.0.

Methods of Testing. Tests of the properties of the refined tar shall be made in accordance with the following methods:

1. Specific gravity: A. S. T. M. Tentative Method D 70–26 T; Proc. A. S. T. M., 1926, Part I, p. 869.
2. Specific viscosity: U. S. Department of Agriculture Bulletin 1216, p. 59.
3. Distillation test: A. S. T. M. Tentative Method D 20–26 T; Proc. A. S. T. M., 1926, Part I, p. 881.
   (a) Softening point: A. S. T. M. Standard Method D 36–26; A. S. T. M. Standards adopted in 1926, p. 93.
4. Bitumen (soluble in carbon disulphide): A. S. T. M. Tentative Method D 4–26 T; Proc. A. S. T. M. 1926, Part I, p. 876.
5. Water: A. S. T. M. Standard Method D 95–24; A. S. T. M. Standards, 1924, p. 901.

12. Broken Stone Roads

Characteristics. An ordinary macadam road, if properly built of the right kind of stone, is a very economical and satisfactory surface for light traffic. It affords an excellent foothold, is noiseless, does not offer much resistance to traffic, and is comfortable to use. In dry weather, however, a macadam surface is extremely dusty unless the surface is treated with a palliative or coated with bituminous material.



BITUMINOUS SURFACE TREATED MACADAM

Fig. 6

Foundation and Subgrade. The lower or foundation course of a macadam road may be strengthened by a telford base, a V-drain foundation, or by increasing its thickness. The construction of telford and V-drain foundations has been described in Art. 7. When traffic and soil conditions are favorable, however, it is customary to build the foundation course directly upon the subgrade. The subgrade is a shallow trench composed of two or more planes or a curved surface sloping from the center to the sides of the road. The trench is the same width as the broken stone surface. The sides of the trench are formed by the earth shoulders, generally 3 to 5 ft. wide. The subgrade should be brought to true line and grade and be thoroughly compacted with a steam roller. Any low spots which appear during compaction should be brought up to grade with good material and rerolled.

Size of Stone. Broken stone roads are ordinarily built in three courses. The sizes of stone and the materials used vary in different specifications. For the first or foundation course the size of the stone is often from 1 to 3 in. in longest dimensions, while sometimes it is specified as from 1-1/4 to 2-1/2 in. Gravel and slag are sometimes substituted for broken stone in the foundation course. The second course is composed of stone ranging from 1 to 2 in., or from 1/2 to 1-1/4 in. in longest dimensions. The top course consists of screenings varying from 1/2 in. down to dust. Since nearly all of the broken stone used for road construction is screened through a rotary screen, it should be noted that the speed at which the screen is revolved, the pitch, the length and the size of holes in the screen all influence the grading of the stone into different sizes.

Sizes of Crushed Stone for Water-Bound Macadam

	Stone for base	Top course	Screenings (large)	Screenings (small)
Product of soft rock...	2-1/2–3/1/2 in.	1-1/4–2-1/2 in.	0–3/4 in.	0–1/4 in.
Product of hard rock..	1-1/4–2-1/2 in.	1-1/4–2-1/2 in.	....	0–1/4 in.

Laying the Stone. Some engineers advocate laying the larger stone on top of the smaller in cases where the stone is low in hardness and toughness and is liable to be crushed during rolling. Large stones in the surface wear longer than smaller ones, but more rolling is required to secure a smooth surface. To gage the thickness of a layer, wood cubes of a depth equal to the thickness of the layer are sometimes placed at intervals in the roadway and the stone is filled in to the tops of them. Another method of obtaining the same result is to set longitudinal strings at the proper elevation at the sides and center of the roadway. Stone for the foundation course, if brought in bottom-dump wagons, may be dumped directly upon the subgrade and spread with stone forks or with a stone spreading machine. Stone brought to the work for the second course should be dumped on boards and shoveled from there to the road to prevent the segregation of sizes which might occur if the stone was dumped directly upon the road from the wagons.

Rolling. The first and second courses are each laid to the required thickness and separately rolled, being sprinkled with water to aid compaction if necessary before the next course is placed. The roller used in compacting this material should be at least 10 short tons in weight. The process of rolling should be to begin at one edge of the roadway, roll longitudinally, and work toward the center. After reaching the middle of the road, the roller should pass to the other side, again roll longitudinally, and work toward the center This manner of rolling keeps the road in shape and prevents either pushing the crown out of line or flattening it. Careful rolling is absolutely necessary in order to obtain a good shape to the road surface. Steam or gasoline rollers always should be used as horse-driven rollers are not sufficiently heavy to properly compact broken stone. Usually 12 to 18-ton rollers should be used for hard, tough rock, and 10 to 12-ton rollers for soft stone.

Applying Screenings. Having finished and rolled the second course as above described, the surface is covered with a layer of stone screenings and thoroughly sprinkled with water in order to fill the voids in the stone with the screenings. More screenings are added as desired and rolling is continued, the surface being sprinkled in front of the roller. When the proper amount of water and screenings have been used, a wave of grout will be pushed along in front of the roller A coating of screenings should be left over the entire surface, no more being used than is necessary to cover the stone. The stone screenings resulting from the crushing of the rock with which the first two courses are built are generally used for the binder. When this material is unsuitable for this purpose, clay, loam, sand, or screenings of a different rock are substituted for it.

The Thickness of the Courses varies in different specifications, and is governed by the amount of traffic which the road is to receive and the condition of the subgrade. Common values are 4, 6, and 8 in the total thickness after rolling, where the subgrade furnishes a good support. The upper course is from 2 to 3 in. in thickness after rolling. In some states the stone surfacing is the same thickness throughout the width, while in others the thickness is reduced at the sides from 1 to 2 in.

Maintenance. Water, if allowed to stand on a macadam surface, will soften the macadam and cause it to wear out rapidly. Also when the frost is coming out of the ground the surface will be in a soft condition and require attention. Thorough surface and under drainage is just as essential as it is in the case of an earth cr gravel surface.

Slag Roads. Blast furnace slags are produced in the manufacture of iron and steel, and, in some cases, are very similar in appearance to close-grained igneous rocks. In some cases, blast furnace slag may be excavated from slag banks by means of a steam shovel, which serves to break of the material sufficiently so that it may be screened. The slag from the open-hearth process is generally run into molds. Usually it is broken up in a rock crusher into sizes suitable for road work. The methods used in building slag roads are similar to those described for the construction of broken stone roads.

Shell Roads. The state of Maryland has built many miles of oyster shell roads along the eastern shore of Chesapeake Bay.

Maryland State Roads Commission Specifications stipulate that the subgrade shall be firm and well rolled. The depth of the first course of shells is either 5 in. or 5 in. at the center and 3 in. at the sides. The depth of the second course is either 3 in., or 5 in. at the center and 3 in. at the sides. The shells are spread upon the roadbed with shovels from piles along the road or from a dumping board. They are rolled with an 8-ton roller and are sprinkled with water or bound with sand during the process of rolling until the surface is firmly compacted. The third course is composed of clean, sharp sand, spread just thick enough to cover the second course after the latter has been thoroughly compacted.

Surface Treatments

A macadam properly built of good materials and well maintained with bituminous surface treatments will last indefinitely under medium traffic, and the cost of maintenance is far less than the interest at 6% on a pavement of the first class. It may be safely said that a bituminous treated macadam will stand up for many years under unlimited pleasure car traffic. Heavy truck traffic causes internal wear.

Surface treatments may be divided into: (a) cold surface, and (b) hot surface treatments. On the whole the former have been the more successful. Hot treatments have resulted all too frequently in wavy or "wash-board" surfaces. Once such surfaces form it is almost impossible to eradicate them short of scarifying the entire road. Cold treatments rarely wave unless too much bitumen is applied. Hot treatments suit the traveling public better than cold treatments because as soon as the bitumen is cool it sets up, whereas the cold treatments splash traffic until the bitumens harden, often taking nearly a week. Asphaltic oils and tars are used. Location of the job, materials of construction, weather conditions, and price are factors to be considered. Materials with high viscosity are used in hot weather or in the southern states, and the lighter materials are used in cool weather or in the northern states and Canada. As a rule the asphaltic oils are cheaper than the tars. In general tars have been more successful for cold surface treatments, and asphaltic oils have been successful for hot treatments. Asphalt cut-backs have been the

Bituminous Materials for Cold and Hot Surface Applications Suggested by the U. S. Bureau of Public Roads

Material	Specific gravity 25°/25° C.	Flash point	Specific viscosity	Float test
Asphaltic road oil for cold application	0.935 to 0.970....	Not more than 50° C.	At 25° C., 80 to 120	....
Oil for hot application	Not less than 0.980	Not less than 80° C.	Not more than 60 at 100° C.	At 32° C., not less than 60 sec.
Tar for cold application	1.10 to 1.14......	....	At 40° C., 25 to 35	....
Tar for hot application	Not less than 1.13	....	....	At 32° C., 60 to 150

Material	Loss at 163° C. in 5 hours	Float test of residue	Total bitumen soluble in CS_2	Per cent of bitumen insoluble in 85° B naphtha
Asphaltic road oil for cold application	Not more than 30%	Not less than 90 sec. at 50° C.	Not less than 99.5%	Not less than 6%
Oil for hot application	Not more than 15%	Not less than 110 sec. at 50° C.	Not less than 99.5%	Not less than 6%
Tar for cold application	....	....	95 to 100%...	....
Tar for hot application	....	....	Not less than 85%	....

Material	Total distillate by weight	Temperature when applied	Quantity per sq. yd.	Cover per sq. yd. in lb.
Asphaltic road oil for cold application	....	Air..........	1/3 to 1/2 gal.	28 to 35
Oil for hot application	....	200° F. to 250° F.	1/3 to 1/2 gal.	30 to 40
Tar for cold application	Not more than To 170° C.—2% 270° C.—25% 300° C.—35%	At least 50° F.	1/3 gal.......	25 to 30
Tar for hot application	To 170° C.—1% 270° C.—15% 300° C.—25%	200° F. to 250° F.	1/3 to 1/2 gal.	30 to 40

most successful asphaltic materials used for cold applications. They set up quickly but their cost is greater than for the road oils.

If it is desired to lay the dust only, and not to form a wearing surface, then a cheap road oil may be spread which will waterproof the surface for a season. Initial treatments will approximate 1/2 gal. per square yard. Such oil can be bought for about 6 cents per gallon delivered at siding. No cover is put on such treatments, but the oil is permitted to penetrate the surface. It is a mistake to waste money on oils which merely lay the dust.

Method for Surface Treatment

1. Sweep macadam thoroughly, removing all loose material, and make sure that manure and other filth are wholly removed.

2. Patch holes, using either hot-patches or cold-patches. Be sure that no dust pockets are left. If at this time it is observed that the old macadam is rough and will not be helped much by patching, then it must be scarified and rolled. In this case use new stone where necessary.

3. When macadam is dry apply bitumen. For an initial treatment with cold material use about 1/2 gal. per square yard. Puddles of bitumen must be broomed out or fat spots will result. If the road has been treated before, a treatment of from 0.1 to 0.3 gal. is all that is required. As a rule, if as much as 1/2 gal. is necessary it will be best to spread it in two applications of 1/4 gal. each.

4. Two hours or more after the last application cover the bitumen with pea-gravel, clean stone chips, or slag chips. It is a mistake to use sand for a cover. With tars the amount of cover for initial treatment will be about 25 lb. per square yard. With asphaltic oils use about 35 lb. Use less cover on subsequent treatments. If a hot treatment is given the bitumen may be immediately covered.

On hills it will be found advisable to use 3/4-in. stone for cover, rolling the stone into the treatment. This will prevent skidding, and will also keep the treatment from pushing or waving.

If holes form no time should be lost in repairing them. Chains on motor cars form ruts during the winter. These must be repaired at the first opportunity.

The important point to remember is that in all surface treatment work on whatever type of road or pavement the amount of bitumen used must be kept at a minimum. Too much is worse than none at all.

For specifications see A. S. T. M. specifications for tars and asphalts and U. S. B. P. R. specifications for cold and hot applications.

13. Retread

There are many milesof old roads that are not satisfactory for modern traffic and yet are too good to tear up and replace with something else. Such roads are old gravel, shale, badly worn macadam, shell, caliche, and similar types. A method known as **Retread** has been devised to salvage these roads and to make them into good roads at low cost. The method is also called **mixing-in-place.** The operations necessary are extremely simple; the machinery actually called for is only a blade grader, although a roller is desirable; the cost is very low; the method seems to be wrong in principle, but the results are surprisingly good. Any road other than an earth road may be retreaded. Materials used are clean 3/4 in. crushed stone, and either tar or asphalt of the proper consistency. To date (1929) the tars have been more successful for this type of work.

* Trade mark registered, The Barrett Co.

Construction

1. Prepare old road by filling up ruts, removing bumps, and making the surface smooth and firm. This work can all be done with the blader, letting traffic compact the old road. It is essential that waves and depressions shall be taken out before the new top is put on. It is sometimes wise to wait a week or so to see if a smooth riding surface has been produced by the grading operations.
2. Spread clean 3/4-in. stone over the road to a depth of about 2 in. Blade it.
3. Apply tar (spec. "**Retread**") at the rate of 3/4 gal. per square yard.
4. Use road grader, blading until the tarred stones show a tendency to ball up under the grader. This may take three to four days. Ruts and waves must be bladed out before the tar sets up.
5. If a roller is available, roll until solid.
6. When road has set up, apply 1/4 gal. per square yard of the same grade of tar.
7. Cover with stone chips or pea gravel. Roll.

Traffic is not kept off during construction. In fact, if no roller is to be had, traffic will satisfactorily compact this type of road.

Maintenance

A retreaded road is maintained either by surface treatments, usually annually, or by an additional retread top which, of course, will add additional depth and strength to the road. For surface treatments the amount of bituminous material applied will vary from 0.2 to 0.5 gal. per square yard. These treatments should be covered with pea-gravel or stone chips, although it is a wise plan to let the bitumen soak into the road for a few hours to liven up the old material. Then apply the cover.

Refined Tar for Retread

General Requirements. The refined tar shall be homogeneous.

Properties. The refined tar shall meet the following requirements:

1. Specific gravity at 25° C. (77° F.), not less than 1.120.
2. Specific viscosity, Engler, 50 cc. at 40° C. (104° F.), 25.0 to 40.0.
3. Total distillate, per cent by weight:

   To 170° C. (338° F.), not more than 3.0.
   To 270° C. (518° F.), not more than 30.0.
   To 300° C. (572° F.), not more than 40.0.

   (a) Softening point of residue (ring and ball), not more than 60° C. (140° F.).

4. Bitumen (soluble in carbon disulphide), per cent by weight, 88.0 to 97.0.
5. Water, per cent by volume, not more than 3.0.

PAVEMENTS

14. Bituminous Macadam Pavements

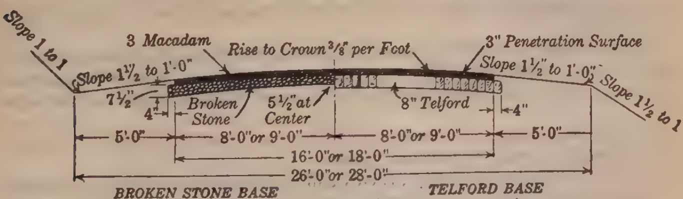
A bituminous macadam differs from other types of bituminous pavements in that the bitumen is added to the aggregate after the latter is in place on the road. All the materials for sheet asphalt, asphaltic concrete, tar concrete, and asphalt block are mixed in a central plant or roadside plant. But a bituminous macadam is constructed in place by placing a layer of aggregate on the foundation and, after rolling, pouring or spraying binder over it. This is often called the "penetration method."

When built with the same care and attention to detail that is accorded higher types of construction the penetration pavement is an excellent one, and it will stand up under all but the heaviest truck traffic. Failures of this type can always be traced to one or more of the following: insufficient foundation, poor aggregate, carelessness in construction and inspection, and bad weather at the time of construction.

The equipment required by the contractor is less than that for any other bituminous type except asphalt block. Ordinary day labor is sufficient for the most part. Yardage possible per day is large. Cost is low. Results are good. Usually the cost should be about 20% lower than bituminous concrete.

A number of variations from the general method of construction have been devised. It is doubtful if any of these show any real points of superiority whereas every one of them increases the cost. These will be briefly commented on after the general method has been explained.

Foundations. It is assumed that the subgrade has been drained and shaped to correct cross-section, and that ditches and shoulders have been built. Time for the subgrade to settle must elapse before the foundation is placed. Satisfactory types of foundation are concrete, black base, broken stone or slag, and



BITUMINOUS PENETRATION SURFACE COURSE

Fig. 7

gravel. Broken stone bases should be at least 6 in. deep, and may be 12 in. where heavy traffic is expected or the character of the soil demands such a depth. Gravel should be at least 9 in. deep.

Existing macadam or gravel roads make good bases for new tops if they are really thick enough. There is but one way to determine this and that is to dig test holes here and there, indiscriminately, and thus learn exactly how much stone is in the road. If less than 6-in. is found more stone must be added or the old road must be rebuilt as a foundation for the new top. One item of the utmost importance is: All holes, waves and irregularities in the old road must be removed before the new top is built, otherwise these will show up in time in the top.

[illegible], unprotected [illegible] are frequently used [illegible]. These must be [illegible] will [illegible] through the surface.

The [illegible] as follows: Base, wearing course, [illegible] course, cover. The stone must be clean, hard, tough, and durable.

Construction

Wearing Course. 1. On the prepared base lay wearing course stone at [illegible]. This stone [illegible] should pass a [illegible] ring and be retained on a [illegible] ring.

2. Remove all fines [illegible] dirty stone, and replace with clean stone.

3. Roll with a [illegible] roller until stone is keyed in place. [illegible] being rolled to prevent [illegible] of the stone. Roll enough to [illegible] but do not [illegible] voids.

4. Spray or pour binder over the [illegible] stone. As a rule use [illegible] gal. per square yard [illegible]. The binder may be asphalt or tar for [illegible]. [illegible] of binder may be mechanical or by hand from [illegible]. [illegible] uniform distribution.

5. Immediately spread a layer of [illegible] stone chips (choking course) in [illegible] the [illegible] of the wearing course.

6. Roll with [illegible] roller. Continue rolling until the surface is perfect [illegible]. [illegible] it may be necessary [illegible] and wait until the sun is off the pavement. [illegible] to develop waves in the pavement [illegible] it is almost impossible to roll them out. Diagonal rolling is advantageous.

7. Sweep off all loose material that is not held by the binder.

8. Apply a second or seal coat of binder at the rate of from 1/2 to 3/4 gal. per square yard.

9. Immediately spread the cover. This may be stone chips, pea gravel, or coarse sand. There should be an excess over that which will be held by the seal coat. This excess tends to find its way into voids and helps make a tight [illegible].

10. Roll [illegible]. A tandem roller may be used here, but roll slowly. Fast rolling is certain to develop waves.

11. If [illegible] within 60 days a second seal coat should be applied using about [illegible] gal. per square yard. Use a cold application tar.

Repairs

1. Dig out area [illegible] and examine base carefully for failure. If base has failed it must be rebuilt with good materials.

2. On repaired base build up the macadam patch in identically the same way as in original construction.

Maintenance

From time to time penetration pavements require surface treatments with some bituminous material. There is no rule for determining this. The only guide is the condition of the pavement. If the mosaic surface of the aggregate is plainly visible then it is time to treat without delay. As a rule tar [illegible] require treatment sooner and more frequently than where asphalt is used. There are numerous instances of asphalt macadams that have not needed treatment over a period of 8 to 10 years. The essential thing is to watch the pavement and treat it before it begins to go to pieces.

Treatments may be either hot or cold. The hot treatments have the advantage of drying immediately so that they do not cause traffic to be spattered with bitumen, although they have the disadvantage of sometimes causing waves to form. This disagreeable feature can be largely overcome by covering the treatment with 3/4-in. stone chips which are rolled into the road surface. A sand cover should never be used with a hot treatment because waves are almost certain.

Cold treatments take longer to dry, but they have the decided advantage of finding the tiny cracks and filling them, thus waterproofing the pavement far better than the hot treatments. They rarely cause pushing or waving of the wearing course. The cover for the cold treatments should be either chips or washed pea gravel, rolled in.

Do not form a thick mat. Keep the amount of surface treating material to the minimum, otherwise a bump will form, followed by other bumps, and soon the entire surface will be a succession of bumps or waves. These may sometimes be removed by cutting them off with a hot shovel.

It is possible to treat an asphalt macadam satisfactorily with a tar surface treatment, and vice versa. Some engineers prefer to surface treat tar macadam with a hot asphalt, and it is probably true that such a treatment lasts longer than a like application of tar.

So long as maintenance is continued and truck traffic does not become too intense, there is no reason why a bituminous macadam, properly built, should not last indefinitely.

Maintenance Methods

1. After repairs have been made, thoroughly sweep and clean the pavement. If a fire hose or a water sprinkler wagon is available the pavement may be scrubbed.

2. Apply a treatment of asphalt (specifications for hot application, surface treatment, or cut-back), or tar (specifications for cold surface treatment, or hot surface treatment) at the rate of from 0.1 to 0.5 gal. per square yard, depending on the condition of the surface.

3. (a) If hot application or cut-back asphalt is used, cover immediately with 1/2-in. stone chips or pea-gravel, using from 20% to 60 lb. per square yard, depending on amount of bitumen used. Tars require 35% less cover than asphalts.

   (b) If cold surface treatment of tar is used allow at least two hours to elapse before cover is applied. In some cases, where the pavement shows signs of being brittle it is best to close the road, applying no cover, and let the old surface absorb the fresh tar.

4. Roll the cover into the surface.

One of the most successful variations of the method outlined above is that known as "pitch grouting," or "asphalt grouting." It is similar in most respects to the usual method except that, instead of using asphalt cement or tar binder alone, these materials, in either case, are mixed with an equal volume of hot sand, and the resulting mastic is flushed over the stone. Results have been excellent, but the cost is about that of asphaltic concrete.

Another successful variation is to lay a mastic course directly on the base and spread the wearing course stone therein. This forms a waterproof bed and securely binds the wearing course stone in place, and it also prevents the escape to the base of any binder from the wearing course.

Another scheme is to spray the base with binder, spread a layer of stone and roll it into the binder; then spray more binder and cover with stone, and roll; and so on until the desired depth is obtained. This is an inverted type. A number of other variations have been tried with some success. Some have been patented.

Specification for Asphalt

Suitable for asphaltic macadam

The bituminous material, which shall be oil asphalt, or fluxed Bermudez asphalt, shall meet the following requirements:

	Oil asphalt	Bermudez
1. Specific gravity at 25° C., not less than..........	1.000	1.040
2. Flash point, not less than........................	175° C.	175° C.
3. Penetration at 25° C., 100 g., 5 sec..............	100 to 120	100 to 120
4. Ductility at 25° C., not less than................	30	30
5. Per cent of total bitumen soluble in carbon tetrachloride, not less than........................	99	99
6. Per cent loss at 163° C., 50 g., 5 hr., not more than.	1	3
Penetration of residue at 25° C., 100 g., 5 sec., as per cent of original penetration, not less than...	60	50
7. Total bitumen percentage, soluble in carbon disulfide, not less than..........................	99.5	94

Specification for Refined Tar for Tar Macadam

General Requirements. The refined tar shall be homogeneous and free from water.

Properties. The refined tar shall meet the following requirements:

1. Specific gravity 25° C. (77° F.), 1.220 to 1.280.
2. Float test at 50° C. (122° F.), seconds, 130 to 190.
3. Total distillate, per cent by weight:
   To 170° C. (338° F.), not more than 1.0.
   To 270° C. (518° F.), not more than 10.0.
   To 300° C. (572° F.), not more than 20.0.
   (a) Softening point of residue (ring and ball), not more than 65° C. (149° F.)
4. Bitumen (soluble in carbon disulphide) per cent by weight, 75.0 to 85.0.

15. Bituminous Concrete Pavements

A bituminous concrete pavement is one composed of broken stone, broken slag, gravel, or shell, with or without sand, portland cement, fine inert material, or combinations thereof, and a bituminous cement incorporated together by a mixing method. The reader's attention is called here to the difference between bituminous concrete and bituminous macadam pavements. In the former the ingredients are combined in a mixing plant from which they are then taken and spread on the foundation. In the latter the aggregate is spread over the foundation to the required depth and it is then coated in place with bitumen which is either poured or sprayed. Bituminous concrete is frequently spoken of as a "mixed job," while bituminous macadam is known as a "penetration job."

Bituminous concrete pavements occupy a place between sheet asphalt and bituminous macadam. Materials that may be unsuitable for either of these types may often be used to advantage. Thus, a sand which would not produce a good sheet asphalt pavement might be incorporated with other aggregate to form a good concrete. It is possible to use gravel and obtain satisfactory results. The combinations that are possible are numerous. Some of these have been patented, but the basic patent for the best of these combinations, namely Warren Brothers' Bitulithic pavement, has expired. The basic patents of Amiestite pavement have expired.

The fundamental idea of all concrete mixtures is that of stability. If, therefore, a mix can be made of which the particles are so formed and the percentages of each size so proportioned that a minimum of cement is required to bind the whole into a solid, the result will be good concrete. Following this reasoning it will be seen that stability will be arrived at if the individual pieces of aggregate interlock. And those bituminous concretes which are made of a graded aggregate such that the percentage of voids is a minimum while the stability, due to interlocking, is greatest, are the best.

Due to the fact that it is not always possible to secure ideal materials several types of bituminous concrete have been developed. Under careful supervision these have proved themselves to be satisfactory. In general, asphaltic concrete is a very serviceable pavement. Tar concrete, as a rule, is not as good, although the early Bitulithic pavements were for the most part built with tar binder and many of them are in excellent condition at the present date (1929).

The Asphalt Association's Brochure 10 divides asphaltic concrete into three classes: Class 1 — Mixtures in which practically all of the mineral aggregate is retained on the 10-mesh sieve. Class 2 — Mixtures containing substantial quantities of both coarse and fine aggregate with a preponderance of particles retained on the 10-mesh sieve. Class 3 — Mixtures containing substantial quantities of both coarse and fine aggregate with a preponderance of particles passing the 10-mesh sieve.

The advantages of asphalticconcrete are: Resistance to impact and abrasion of traffic, low tractive resistance, freedom from producing dust, easy cleaning, the fact that it may be opened to traffic as soon as it has cooled after final rolling, maintenance cost low, easy repair, and the fact that local materials possibly unsuitable for other types may be used. The disadvantages are that: A mixing plant is required, experienced inspectors are necessary, the utmost care must be used in proportioning and mixing to secure uniformity, frequent tests must be run on the materials and the mix, the pavement sometimes becomes more slippery than other types after use, and the surface may become unduly wavy until it resembles a washboard if the proportions of aggregate and consistency of the asphaltic concrete are not correct.

Foundations. For all classes of bituminous concrete pavements, foundations of gravel, broken stone, slag, old macadam, bituminous concrete, bituminous macadam, concrete, old brick, and stone block pavements have been used. The foundation must be strong enough to carry the traffic to which the pavement is to be subjected. Weak foundations mean failure.

Mixing Plants. There are three types: Permanent, railway, and portable or semiportable. A permanent plant is economical only in cities whose paving programs call for considerable amounts of asphaltic concrete over a period of years. The other types are suitable for occasional city work and for rural road work. For detailed description of paving plants and their operation see Brochure 13 of The Asphalt Association, and booklets of the various manufacturers.

Mixtures. *Class* 1 — Broken stone, or slag, or crushed gravel may be used. The aggregate should be hard, tough, and clean. All fragments should pass a 1-1/4-in. screen, and at least 85% should be retained on a 1/4-in.

screen. Stone should have a percentage of wear of not less than 5, and a toughness of not less than 8. Usually this class is referred to as one-size stone asphaltic cement. Since there is practically nothing passing the 10-mesh sieve the resulting mix will be full of voids. It has been found in practice that from 5 to 7% of asphaltic cement produces a good mix. It sometimes happens that an inexperienced inspector, noting the open character of the mix, endeavors to fill the voids by adding asphaltic cement. This will insure failure, as the pavement will be so " fat " that it will shove and become wavy. If too soft an asphaltic cement is used the same effect is produced. If too hard, the pavement cracks and disintegrates. The surface voids are sealed against moisture by the application of a seal coat and cover. The proper penetration limits of asphaltic cement for Class 1 mixtures are tabulated herewith.

Traffic	Temperature		
	Low	Moderate	High
Light	100–120	100–120	60–70
Moderate	85–100	85–100	60–70
Heavy	85–100	60–70	60–70

Class 2. If to the aggregate of Class 1 is added enough fine aggregate and filler to make a dense, almost voidless mix when the proper amount of asphaltic cement is introduced, the resulting mixture is Class 2. It frequently happens that crusher run makes an ideal mix. The fine aggregate, if not the product of the crusher, should be a clean, hard sand. Not too much attention need be paid to the grading. As a matter of fact it will be found that a combination of coarse and fine aggregates that is satisfactory for the production of a portland cement concrete will be satisfactory for asphaltic concrete. What is of prime importance is that the aggregate shall be of such size, shape, and proportion that perfect interlocking takes place during the rolling of the pavement. When this is attained and a suitable grade of asphalt cement is added in sufficient quantity to bind all particles together then an excellent pavement is certain. The penetration limits of asphaltic cement for Class 2 mixtures are

Traffic	Temperature		
	Low	Moderate	High
Light	50–60	50–60	40–50
Moderate	50–60	50–60	40–50
Heavy	40–50	40–50	30–40

Grading of Aggregate for Class 2

	Per cent
Passing 1/4-in. screen	0– 15
Passing 1/2-in. screen	25– 75
Passing 3/4-in. screen	95–100

A seal coat is advocated. It should be applied after the final rolling.

Class 3. " Topeka " is the representative of this class. As this mix was evolved to evade the Bitulithic patent, and as this patent has now expired there is no good reason why anyone should lay Topeka. It is an inferior mix. The grading of the aggregate and penetration limits for asphaltic cement are:

Traffic	Temperature		
	Low	Moderate	High
Light	50–60	50–60	40–50
Moderate	50–60	50–60	40–50
Heavy	40–50	40–50	30–40

	Per cent
Mineral aggregate passing 200-mesh screen	5–11
Mineral aggregate passing 40-mesh screen	18–30
Mineral aggregate passing 10-mesh screen	25–55
Mineral aggregate passing 4-mesh screen	8–22
Mineral aggregate passing 2-mesh screen	less than 10
Bitumen	7–11

The tendency of this mix is to roll, push and form waves. In order to give stability to such a mix there should be added a quantity of crushed stone that will pass a 1-1/4-in. ring and be retained on a 3/4-in. ring. The exact amount should be determined by laboratory experiment.

Analysis of a Coarse Graded Aggregate for Bituminous Concrete

	Per cent
Coarse Aggregate	
Passing 1-in. screen, not less than	95
Total passing 3/4-in. screen	25
Retained on 1/4-in. screen, not less than	85
Chips for Seal Coat	
Passing 1/2-in. screen, not less than	95
Retained on 1/4-in. screen, not less than	85
Fine Aggregate	
Passing 1/4-in. screen	100
Total passing 40-mesh sieve	30–70
Retained on 200-mesh sieve, not less than	90
Penetration of Bituminous Material	60–80
Composition of the Mixture	
Coarse aggregate	45–60
Fine aggregate	25–40
Mineral filler	3– 5
Asphaltic cement	6– 8

Analysis of Typical Bitulithic Pavement

Aggregate	
Passing 1-1/4-in. retained on 1/2-in. screen	30–60
Passing 1/4-in. retained on 4-mesh screen	15–25
Passing 4-mesh, retained on 10-mesh screen	5–15
Passing 10-mesh	20–35
Analysis of that part passing the 10-mesh sieve:	
Passing 10-mesh, retained on 40-mesh sieve	15–40
Passing 40-mesh, retained on 80-mesh sieve	22–53
Passing 80-mesh, retained on 200-mesh sieve	15–40
Passing 200 mesh	10–15
Asphaltic Cement	8–12

Hot Mix, Laid Cold. There is a large field for a bituminous concrete that can be laid cold. Several have been developed; to date Amiesite and Tarvialithic are most representative. The methods of manufacture in all are similar. Briefly, the aggregate is cleaned, heated and dried. It is mixed with a suitable bitumen, loaded into cars or trucks and shipped to its destination. Some trouble has been experienced in unloading these mixes from railroad cars, but if a clamshell bucket is used it is fairly easy. In both of these pavements two mixes are used, a coarse mix for the lower course of the pavement and a fine mix for the top. The coarse mix is laid about 2 in., or more, thick and spread uniformly and rolled with a 10 to 12-ton roller. The top is then spread from 1/2 in. to 1 in. thick, and rolled until a smooth pavement results. In cold weather it is customary with Amiesite to follow with spreading a thin layer of sand which has been treated with asphaltic oil and to roll this into the pavement, thus sealing it. In hot weather these pavements seal themselves, but in the late fall they are liable to remain unsealed unless some such method is employed. When carefully made and laid either of these pavements will carry heavy trunk line traffic.

The hot-mix-laid-cold type is specially suited to the small town which has no asphalt plant, and where an occasional bituminous job is indicated. This type has been successfully laid over old, worn-out pavements such as brick, concrete, water-bound macadam, stone block, and even wood block. There are a few tricks to be learned by the novice who has never laid this type but who may have had experience with hot mixes. He should consult the manufacturers before he attempts to lay his first job.

It is a mistake to attempt to lay one of these pavements with a total thickness of less than 2 in.

Analysis of Amiesite

Coarse Aggregate in Lower Course	Per cent
Passing 1/4-in. sieve	0– 5
Passing 5/8-in. screen	0– 25
Passing 1-1/2-in. screen	75– 95
Passing 2-in. screen	100

The top course contains coarse aggregate and filler. The filler consists of clean rock screenings, all passing a 1/8-in. screen. The coarse aggregate for the top is:

	Per cent
Passing 1/4-in. screen	0– 20
Passing 5/8-in. screen	95–100

Lime is added after the asphaltic cement has been mixed with the aggregate. The percentage of the materials used is:

	Lower course	Top course
Mineral aggregate	86 –90	85 –89
Filler	4 – 6	5 – 8
Asphaltic content	4 – 6	5 – 7
Lime	.5– 1	.5– 1

The aggregate is supposed to be washed, before mixing, with petroleum naphtha.

The Amiesite aggregate is mixed with asphaltic cement. The Tarvialithic aggregate is mixed with a special grade of Tarvia.

16. Asphalt

Sheet Asphalt

Characteristics. Sheet asphalt pavements are those in which the wearing course is composed of a mechanical mixture of asphalt cement with a carefully graded sand, passing a 10-mesh sieve, and mineral filler. A sheet asphalt pavement is waterproof, smooth, noiseless, easily cleaned, and non-productive of dust. It is *not* slippery when wet unless it is dirty, and that is not the fault of the pavement. Repairs and maintenance are simple and inexpensive, with no inconvenience to the traveling public. It wears away uniformly. It will carry the heaviest traffic of the largest cities, and yet it is well suited to residential districts. As it is a black pavement it absorbs heat. Unless carefully constructed of proper materials it may become wavy or rutted in hot

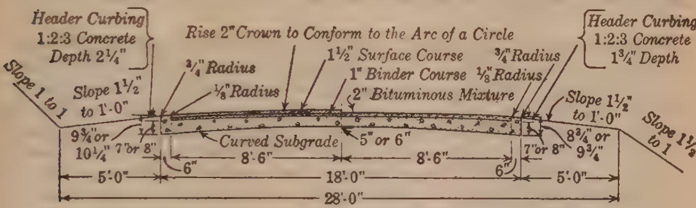
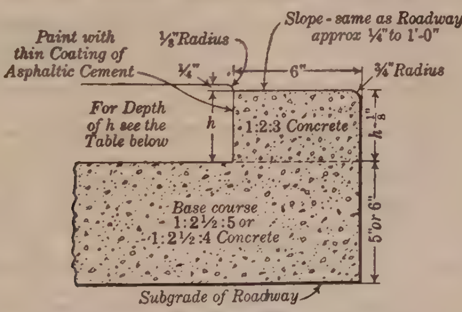


Fig. 8

summer weather. Gas leaks may ruin it. Lubricating oil dropped from motor cars tends to cut back, or soften, the asphalt cement.

The wearing course mixture is usually laid over a binder course which is laid directly on the foundation.

Foundations. Sheet asphalt pavements have been successfully laid and maintained over the following foundations: cement concrete, black base;



h	*TYPE OF PAVEMENT SURFACE*
1¾"	*Bituminous Pavement Spec. D-E*
2¼"	*Bituminous Pavement Spec. A*
3¾"	*3" Vitrified Brick or Wood Block*
4¼"	*3½" Wood Block Pavement*
4¾"	*4" Vitrified Brick or Wood Block*
6¼"	*5" Stone Block Pavement - Sand Bed*
6"	*5" Stone Block Pavement - Cem. Sand Bed*

HEADER CURBING

Fig. 9

old brick, stone block and concrete pavements; old macadam; and gravel roads. If an old pavement is to be used, run a heavy roller over it to ascertain the weak spots. Repair such areas. Loose blocks will roll even after the sheet pavement has been laid, and this will cause a failure at such a point.

Depressions in the old pavements must first be filled with some binder course material and the entire surface of the old pavement should be made fairly true. If the old road is widened, then sufficient time must be allowed to let the new stone or gravel consolidate and pack. Six to eight weeks is not too much.

Binder Course. The binder is an intermediate course between the foundation and the wearing course. It is composed of a mixture of asphalt cement, or coal tar binder, with broken stone, slag, or crushed gravel. Whatever the material it should be strong and tough. A soft stone will cause a failure of the wearing course. Binder stone should all pass a 1 in. to 1-1/2 in. ring, depending on the depth of the binder stone. There are two classes of binder course: open binder, and close binder. Open binder consists of stone or slag aggregate from 1/4 in. to 1-1/2 in. in size, with practically no fine material. There should be enough variation in sizes between these limits that the mixture will interlock under rolling. No attempt is made to fill the voids as the sheet mixture serves this purpose. Close binder is really an asphaltic concrete the aggregate of which consists of broken stone and sand. The grading of these materials is as follows:

Total Mineral Aggregate	Per cent
Passing largest screen specified and retained on 1/2-in. screen....	15–65
Passing 1/2-in. screen and retained on 10-mesh sieve...........	20–50
Passing 10-mesh sieve...................................	15–35

Of asphalt cement 4.5% to 6% is added, the exact percentage being found by trial. Each particle should be thoroughly and uniformly coated with asphaltic cement. Too much asphaltic cement causes fat spots to develop in the wearing course. Too little makes for a brittle binder course. The mix should be delivered on the job at between 200° F. and 350° F., and if it is between these limits it should crawl when dumped from the truck.

Wearing Course. The aggregate consists of a hard, durable, finely graded sand and a fine filler, such as limestone dust, slate dust, or portland cement. The amount of asphaltic cement varies from 9% to 13%. These materials are mixed in a central plant and brought to the job in trucks. The temperature of the mix should be between 250° F. and 350° F. Stiff mixes require the higher range. Also, when hauling long distances the higher temperatures are necessary, and the truck load should be covered to keep in the heat. When the mix is dumped, it is raked over the binder course to a depth (loose measurement) of about an inch greater than that desired for the finished depth. The rakes are hot, the men work rapidly, and the greatest care must be exercised to see that the spreading is done uniformly, so that no high or low spots may develop under rolling. Absolute uniformity of mix is essential. Variations that are noticed should be reported immediately.

Sand for the wearing course should be hard, tough, clean, and fairly sharp. The Asphalt Association recommends the gradings shown in table at top of next page.

The mineral filler aids in filling voids, and it also seems to increase the stability of the mix. The most satisfactory materials have been found to be limestone dust and portland cement. Both of these are very finely ground and yet they do not tend to make the mix ball up. Other materials are sometimes used. The amount of filler varies from 10% to 20% of the total mineral aggregate.

Asphalt Cement. Satisfactory asphalt cements, as shown by service tests, have been made from California, Mexican and Texas asphaltic oils, gilsonite and asphaltic oil, and "Bermudez" and "Trinidad" refined asphalts.

Standard Sand Gradings

Sieves	Heavy traffic, per cent		Light traffic, per cent	
Passing 10-mesh, retained on 20-mesh	5	23	10	35
Passing 20-mesh, retained on 30-mesh	8		10	
Passing 30-mesh, retained on 40-mesh	10		15	
Passing 40-mesh, retained on 50-mesh	13	43	15	45
Passing 50-mesh, retained on 80-mesh	30		30	
Passing 80-mesh, retained on 100-mesh	17	34	10	20
Passing 100-mesh, retained on 200-mesh	17		10	
Passing 200-mesh	0		0	
	100		100	

The following is from Brochure 9 of the Asphalt Association:

The most desirable consistency of asphalt cement for sheet asphalt construction will be governed largely by climatic and traffic conditions. Stability of the topping mix in particular depends to a considerable extent upon the consistency of the asphalt which binds the sand grains together, as the most carefully graded and compacted sand exhibits but little resistance to displacement by itself. A very soft asphalt cement will not hold the sand grains together in their original position with sufficient tenacity and its use will therefore result in undue deformation of the surface under traffic. On the other hand, too hard an asphalt cement will produce a brittle mixture in cold weather which is not sufficiently malleable to prevent the formation of contraction cracks in the pavement. The following table will serve as a general guide in selecting suitable ranges of penetration for asphalt cements to be used under various climatic and traffic conditions:

Penetration Limits of Asphalt Cement

Traffic	Temperatures		
	Low	Moderate	High
Light	50–60	50–60	40–50
Moderate	50–60	50–60	40–50
Heavy	40–50	40–50	30–40

Inspection of Sheet Asphalt. Competent inspection of both plant operation and street work are very necessary to successful sheet asphalt construction. This will ordinarily necessitate the services of two inspectors who have acquired sufficient experience to suggest modifications in proportions or methods of manipulation, within specification limits, which from time to time may be desirable. These plant and street inspectors should work in harmony and keep in close touch with one another. The former should occasionally visit the street in order to observe how the mixtures behave during laying.

The plant inspector should carefully observe all details of plant operation, particularly in connection with the heating of individual constituents, their measuring and proportioning, the period of mixing, and the condition of the mix upon leaving the plant. He should keep a record of the temperatures to which the asphalt, sand and stone are heated prior to mixing and the temperature of the finished mixture when discharged. He should also record the number of loads sent out. He should test and record the consistency of the asphalt cement particularly if fluxing is carried on at the plant, in which case he should see that the proper proportions of refined asphalt and flux are used and that a uniform product is obtained. He should make grading tests of the mineral aggregate constituents and see that at all times proper proportions of asphalt cement and aggregate are used. He should check measurements from time to time and should make occasional pat tests of the surface mix as it is being prepared. He should be provided with the necessary equipment for plant testing and a room in which such tests may be made. In addition to his own tests he should, however, forward to a testing laboratory samples of the various constituents and of the mix itself for check and control tests. The testing equipment of the plant inspector will ordinarily include the following:

An armored thermometer of suitable range for recording temperatures of the hot aggregates and mixes.

A set of standard screens and sieves, as may be called for in specification requirements.

A stiff brush for cleaning sieves.

A large balance of 10 lb. pan capacity and a set of decimal weights for broken stone weighings.

A small sand balance with pan capacity of 200 g. for weighing sand.

A complete penetration test outfit for determining the consistency of asphalt cement.

A pat test outfit.

A supply of report forms.

The room assigned to plant testing should contain a short workbench for testing apparatus, shelves for samples, a chair and a table. For sampling, the inspector should be provided with suitable cans, gum labels, pans and a long-handled metal dipper.

The street inspector should carefully observe all details of the laying of the pavement. He should take and record the temperature of the various loads of hot mix received on the work and for this purpose should be provided with an armored thermometer. He should see that the loads are so distributed as to give the specified thickness of pavement and that they are properly raked and compacted. He should take occasional check samples of the mix as received and should forward these to the testing laboratory and in some cases it may be advisable for him to take samples of the finished pavement for laboratory analysis and density determinations.

Construction

(a) **Binder Course.** 1. Thoroughly clean base. Remove any standing water. Let base dry.

2. Deliver binder at a temperature of at least 225° F. and not more than 350° F.

3. Dump binder outside area where it is to be spread, although it may be dumped on the base in advance of the actual spreading area.

4. Use hot rakes and spread binder course in a uniform layer of such depth that when compressed it will be of correct thickness.

5. Roll. Use a roller of at least 10 tons weight. Roll longitudinally, beginning at sides and rolling gradually toward center. *Roll slowly.*

6. Do not stop the roller except at the beginning and end of area to be rolled. Once the roller starts let it continue until the end is reached.

7. Use hot tamper at places that the roller cannot reach.

8. If fat spots develop, cut them out and fill in with fresh, hot binder.

9. Do not roll over the unprotected end of the binder course unless no more is to be laid for the present. In that event the rolled-out ends are trimmed off when new binder is placed.

(b) **Wearing Course.** 1. Be certain that the binder course is clean.

2. Deliver wearing course at a temperature of at least 250° F. and not over 350° F. Do not attempt to work when the air temperature is below 45° F.

3. Dump mix outside of area on which it is to be spread. Spread with hot shovels, and rake with hot rakes. Rake to uniform depth such that after compression the wearing course will be of correct thickness.

4. Paint contact surfaces of gutters, manholes, and curbing with a thin coat of hot asphaltic concrete or cut-back asphaltic concrete.

5. Work rapidly, and have no delays of any kind. Keep moving.

6. Roll. Use a tandem roller 10 tons or more in weight. Do not use a light roller for the initial rolling. A 15- to 18-ton roller is best. Always start rolling along the sides and gradually work to the center. Then roll diagonally. On wide streets roll perpendicular to first rolling. Keep on rolling until certain that no more compression is possible. If mix sticks to roller wheels moisten them with water or oil.

7. Correct for high or low spots before final rolling by raking or addition of fresh mix. Test surface with 10-ft. straight-edge.

8. Do not let roller man reverse except at ends of fresh mix when rolling longitudinally on initial compression. Waves are almost certain to be formed if he does.

9. Use hot tampers where roller cannot compact the mix.

10. Before roller finishes, spread a light coating of limestone dust or portland cement over the surface of the fresh mix, sweeping it and then rolling it. A light tandem roller can be used for finishing.

11. The roller should stop just before it reaches the unprotected end of the fresh mix. On the end where the fresh mix joins that part of the wearing course that has already been rolled the roller should continue onto the compacted area. If there is a delay or work is stopped for the day the roller may run over the unprotected end, but that portion must be cut away and the edge trimmed up vertical and painted with hot asphaltic concrete before the next load of mix is spread.

12. Keep traffic off until pavement has cooled to air temperature. A minimum of six hours is desirable; 24 hours is better.

Repairs

1. Cut out damaged area, making sides of cut vertical.
2. Examine base thoroughly to see if it has failed. If a gas leak is suspected, use your nose. If that fails, a lighted match may reveal the leak.
3. If base is at fault cut out the poor section and replace with new material. If it is a concrete base and traffic on the street is heavy the substitution of black base for the concrete will speed up repairs.
4. On the base, spread binder course; tamp or roll it thoroughly.
5. Paint the vertical edges of the old pavement with hot asphaltic concrete.
6. Spread wearing course and proceed as under "Construction."

Maintenance

The essential item needed to keep a correctly designed sheet asphalt pavement in first-class condition (assuming it has been properly mixed and laid) is: traffic. All bituminous pavements need traffic to "iron them out" and keep them fresh and lively. If the sheet pavement is right in the first place then the more traffic it gets the better it is. And traffic is its best source of maintenance.

Failures

There have been numerous failures of sheep asphalt, but these are attributable to faulty materials, poor construction methods, poor design, weak bases, underground gas leaks, and so on, and they are not attributable to the type of pavement itself.

Heavy motor truck traffic has caused the failure of many old sheet pavements which were built long before such traffic was anticipated. Similar failures are to be found for every type of pavement. There will be no failures of the wearing course if asphaltic cement of the proper penetration is used and if sand of the standard grading only is permitted.

A sheet asphalt pavement is a high-class pavement; it requires high-class workmanship and expert supervision, together with first-class materials. Sloppy, careless work will positively show up later. If those responsible for a pavement achieve a failure the fault is theirs, and not the pavement's.

South American oils are replacing to some extent the oils from the Mexican field. The asphalts obtained from these South American oils may differ from those which have been used, and it is possible that considerable experimentation must be carried on to insure satisfactory results.

A start has been made toward producing mechanical spreaders and rakes. Since sheet asphalt pavements are usually laid in city areas, it seems unlikely that machines similar to concrete finishing machines will be used to any great extent, since such machines are seldom used within city limits for finishing concrete pavements.

A Typical Sheet Asphalt

	Per cent
Binder Course:	
Passing 1-in. and retained on 1/2-in. screen	15–65
Passing 1/2-in. and retained on 10-mesh screen	20–50
Passing 10-mesh sieve	15–35
Sand for Wearing Course	
Total passing 10-mesh sieve	100
Total passing 10-mesh sieve and retained on 40-mesh	12–50
Passing 10-mesh, retained on 20-mesh	2–15
Passing 20-mesh, retained on 30-mesh	5–15
Passing 30-mesh, retained on 40-mesh	5–25
Passing 40-mesh, retained on 50-mesh	5–30
Passing 50-mesh, retained on 80 mesh	5–40
Total passing 80-mesh, retained on 200-mesh	20–40
Passing 80-mesh, retained on 100-mesh	6–20
Passing 100-mesh, retained on 200-mesh	10–25
Total passing 200-mesh	0– 5
Mineral filler	6–20
Asphaltic cement	9-1/2–13-1/2

Specifications Various Kinds of Asphalt

American Society for Testing Materials

The asphalt cement shall be homogeneous and free from water. It shall meet the following requirements:

Type of pavement	Asphalt macadam			Manufacture asphalt block		Methods of test
A. S. T. M. serial designation	D102 24T	D103 24T	D135 23T	D133 23T	D134 23T	
Penetration 25° C., 100 g., 5 sec.	85–100	100–120	120–150	10–15	15–25	D5
Flash point° C., open cup, over	175	175	175	200	200	D92–23T
Loss at 163° C., 50 g., 5 hr., maximum	2	2	2	1	1	D6
Penetration of residue loss at 163° C., per cent original	60	60	60	50	50	D5
Ductility centimeters not less than	30	30	30	5–15	5–20	D43–22T
Bitumen soluble in carbon tetrachloride	99	99	99	99	99	D165–23T

Type of pavement	Sheet asphalt and asphaltic concrete					Methods of test
A. S. T. M. serial designation	D163 23T	D164 23T	D99 23T	D100 23T	D101 23T	
Penetration 25° C., 100 g., 5 sec.	25–30	30–40	40–50	50–60	60–70	D5
Flash point ° C., open cup, over	175	175	175	175	175	D92–23T
Loss at 163° C., 50 g., 5 hr., maximum.	2	2	2	2	2	D6
Penetration of residue loss at 163° C. Per cent original	60	60	60	60	60	D5
Ductility centimeter not less than	15	25	30	30	30	D43–22T
Bitumen soluble in carbon tetrachloride	99	99	99	99	99	D165–23T

Note. When less than 99% of asphalt cement is soluble tetrachloride, the percentage of bitumen (solubility in carbon disulfide) shall be separated.

Asphalt Block Pavement

"An asphalt block pavement is one having a wearing surface composed of a properly proportioned mixture of asphalt cement, crushed rock and inorganic dust, heated to a proper temperature and molded into uniform blocks under heavy pressure." Blocks are made in the following sizes:

Length, in.	Width, in.	Depth, in.	Weight, lb.
12	5	2	11*
12	5	2-1/2	13-1/2
12	5	3	16
8	4	1-1/4	3-1/2†

* This size is commonly used.
† For special work requiring light weight.

Penetration limits for the asphalt cement are as follows:

Traffic	Temperature		
	Low	Moderate	High
Light	15–45	15–45	10–35
Moderate	15–45	15–40	10–30
Heavy	10–35	10–35	10–25

A large yardage of this type of pavement has been laid in and around some of the large cities. Some of these pavements have been very successful; others have been disappointing. Undoubtedly the failures can be traced to inferior blocks. If the asphaltic concrete is too soft the blocks get out of shape, causing the pavement to wave. If too hard, then the blocks break off at the edges and a rough surface develops. On grades of over 3% considerable trouble has been experienced from the blocks dragging under traffic. The cost of an asphalt block pavement is almost always higher than that of a sheet asphalt pavement in those localities where there is a sheet plant or where there is enough business to warrant setting up a portable plant. The logical place for asphalt block pavements is in those towns where a yardage is laid that does not attract a contractor who owns a sheet asphalt plant. If the blocks are honestly made, of the proper materials, and well laid, the resulting pavement is excellent and will carry very heavy city traffic. Cuts in such a pavement are easily made and repair accomplished. Maintenance consists in surface treating the blocks with a hot asphaltic oil, and covering with coarse sand. Screenings passing a 3/8-in. mesh are sometimes used for cover, but if they are of limestone the pavement is liable to become dusty for a while.

Foundations. Best results are obtained where a new cement concrete foundation is laid. It is not economy to use old macadam, brick, or other old roads, as this pavement is expensive and a good base is required. In Florida the natural sand base has been used with varying degrees of success. Using a concrete base the blocks are laid in a mortar cushion or bedding course about an inch or less thick. Since a variation in depth of block is allowed of from 1/8 in. to 1/4 in. it is not possible to lay them directly on a smoothly finished foundation as is done with wood block. The mortar bed is made of sand and cement in the proportion of one part cement to four parts sand, and enough water to make a workable mortar. Care must be taken to strike this bedding course to a true and even surface, otherwise unevenness in the resulting pavement will be most apparent. As in brick and stone block construction, the pavers stand on the blocks already laid and must not step into the bedding. Joints are broken, and the blocks are laid at right angles with the curb.

It is customary in most cases to sweep sand into the joints. A far better method would be to give the blocks a surface treatment with a hot asphaltic oil which will fill every joint. A waterproof pavement is then secured immediately. A sand cover should be applied over the surface treatment.

Rock Asphalt

Rock asphalt is sandstone or limestone naturally impregnated with asphalt.

There are a number of deposits throughout the world. In the United States rock asphalt is found in Kentucky, Alabama, Missouri, Oklahoma, Texas, California, and in unimportant deposits elsewhere. The materials found in the various deposits vary greatly in character, and but few of them, as found, are suitable for road work. There is no uniformity of asphalt content. In 1928 most specifications required that there be about 6-1/2% of asphalt present. This was based on the assumption that because the Kentucky rock asphalts that had given good service contained approximately that amount, therefore all rock asphalts should contain a similar amount. It is reasonable, however, to suppose that the size of sand particles will have much to do with the amount of asphalt necessary. Investigation of this was begun in 1928.

Rock asphalt is mined, loaded into cars and shipped by rail to a convenient unloading point. Difficulty is frequently experienced in unloading, especially if the rail haul has been far and long, and if the weather is cold when unloading is attempted. Sometimes it has been necessary to use dynamite to loosen the material. Steam pipes are sometimes introduced into the mass to liven it. The cost of unloading may be a serious item and must be included in all estimates. While pick and shovel unloading is customary, it is sometimes possible to use a clamshell shovel. In this case the men in the car may have to hang onto the bucket to give enough weight to force it into the rock asphalt.

Under favorable conditions the material is fairly easy to handle. It is unloaded from the car and loaded into trucks and taken to the job where it is spread cold on the foundation, which may be a concrete base, or an old pavement. It is spread to such depth that when finished the rock asphalt will be 2 in. thick, minimum. It is a great mistake to try to skimp.

Rolling is very essential. Best results are obtained with a corrugated roller, but a regular roller will do. Much of the European work is hand-rolled.

If the material is of uniform consistency, is carefully spread and well rolled, the resulting pavement is good. Most failures have been due to non-uniform asphalt content, resulting in fat spots and lean spots, poor foundations, and faulty rolling.

The cost will vary with the length of haul, and the difficulty experienced in unloading. Cost at the mines in Kentucky is about $6.50 per ton, f.o.b. cars.

Rock asphalt has been successfully used as a cold patching material in other types of pavement. It is also a good material to use at highway-railway crossings. It may be laid in the rain.

It has been found advantageous to give the foundations a prime coat of asphaltic oil about 24 hours before the rock asphalt is laid. Some of the rock asphalts work better when heated, notably those from Texas and Alabama.

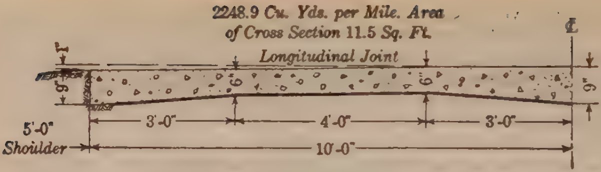
17. Concrete Pavements

Characteristics. A portland cement concrete pavement furnishes a smooth surface, is easy to clean, and produces practically no dust. The resistance to traffic is low. When clean it is not slippery. The white glare is tiring on the eyes. Under horse-drawn traffic concrete paving is noisy. Grades of 8 to 10% are common in a number of instances from 15 to 18% grades have been used. Cracks can be easily repaired, but depressions and disintegrated spots are difficult to repair satisfactorily without cutting away the old concrete. Worn-out concrete pavements are often topped with bituminous wearing course or, occasionally, with new concrete or brick. With well built and carefully designed pavements there is almost no cost for repairs for the first five years. After that the costs may become somewhat heavy.

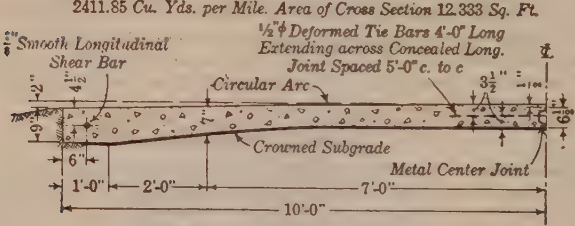
Subgrade. A concrete pavement should be laid on a well compacted and well drained subgrade. If the subgrade is of clay or other heavy soil, it should be replaced with clinker, broken stone, cinders, gravel, or some other suitable material. Although this is desirable it is not always possible. The general practice is to require that the subgrade be of uniform density. Rolling should be reduced to a minimum on heavy soils. Prior to placing the concrete the subgrade should be thoroughly wet, otherwise the subgrade will absorb water from the concrete. It should not be so wet as to be muddy.

Ingredients and Proportioning. The materials used for the fine aggregate of a cement concrete pavement are generally sand or stone screenings, and for the coarse aggregate either broken stone or gravel. The broken stone employed should be obtained by crushing hard, tough rock. Preferably the stone should be composed of naturally graded sizes and free from dust or dirt. What has been said relative to broken stone applies as well to gravel. The sand used should be clean, sharp and coarse, free from loam, clay, and vegetable or organic matter. The cement should be a first-class portland cement that will meet the standard specifications of the A. S. T. M. Care should be taken to use clean water. The proportions which are used in connection with the construction of concrete foundations are not rich enough in either cement or mortar to make satisfactory concrete which is to be subjected to the abrasive and impact forces of highway and street traffic. The proportions used for the best class of one-course concrete pavements are about 1 : 1-1/2 : 2. For two-course pavements the base may be 1 : 2-1/2 : 4, and the top, 1 : 1 : 1-1/2.

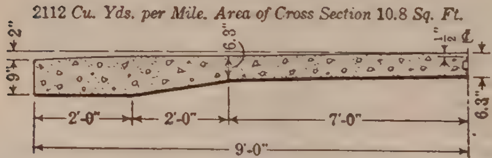
Materials may be proportioned either by volume measurement or by weighing. When volume measurement is used it is important to use devices



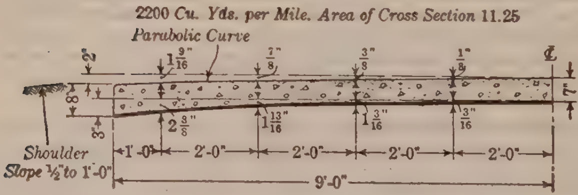
TYPICAL CROSS-SECTION – CALIFORNIA



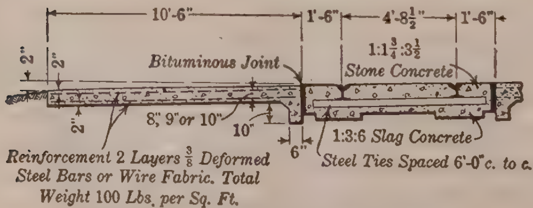
TYPICAL CROSS-SECTION – ILLINOIS



TYPICAL CROSS-SECTION – A. A. S. H. O.



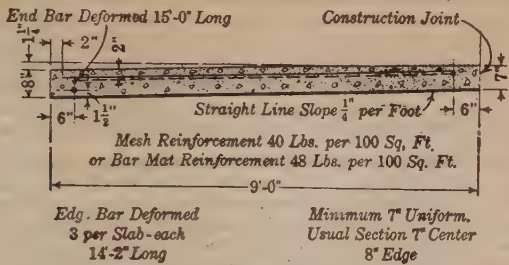
TYPICAL CROSS-SECTION – NORTH CAROLINA



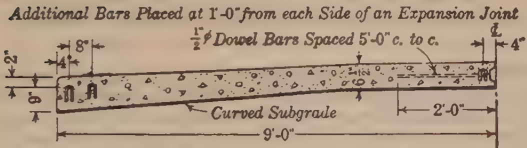
TYPICAL CROSS-SECTION – NEW JERSEY

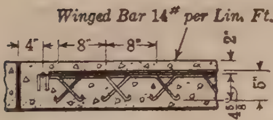
Fig. 10

2200 Cu. Yds. per Mile. Area of Cross Section 11.25 Sq. Ft.



TYPICAL CROSS-SECTION–NEW YORK

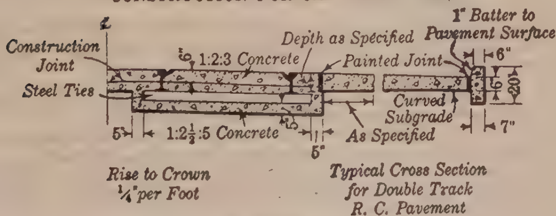




		Cu. Yds. Conc. per Mile	Area of Cross Section
18' Roadway	$6\frac{1}{2}$" x 9"	2152.44 Cu. Yds.	11.01 Cu. Ft.
	6" x $8\frac{1}{2}$"	2005 77 " "	10.25 " "
	6" x 8"	1956.10 " "	10.00 " "
	$5\frac{1}{2}$" x $7\frac{1}{2}$"	1809.44 " "	9.26 " "
16' Roadway	6" x 8"	1739.86 " "	8.90 " "
	$5\frac{1}{2}$" x $7\frac{1}{2}$"	1608.97 " "	8.23 " "
	5" x 7"	1478.5 " "	7.56 " "

TYPICAL CROSS-SECTION–PENNSYLVANIA

CONSTRUCTION FOR CAR TRACK ROADS



TYPICAL CROSS-SECTION–PENNSYLVANIA

Fig. 10—Continued

that will give fairly accurate results. Measurement by shovels or wheelbarrows is poor practice. In volume measurement compensation should be made for the bulking of the aggregate due to moisture. Proportioning by weight is probably the most satisfactory method. It automatically compensates for aggregate bulking, and in addition furnishes a close check on the quantity of materials used.

Design. In 1929 there were as many designs for concrete highways as there were states; and the cities and towns within the individual states had, in many cases, designs of their own which differed from the state designs. Fig. 11 shows the 1927 cross-sections for the several states noted. There was no uniformity of opinion of highway engineers as to standard cross-sections, transverse joints, longitudinal joints, joint fillers, methods of curing, reinforcement, crown, proportioning the mix, materials, nor waterproofing. In general the slab with the thickened edge was more popular than the slab of uniform thickness. For theory of plain and reinforced concrete see Sect. 11.

Reinforced Pavements. Reinforcing is usually placed about 2 in. below the surface of the pavement. The concrete is first struck off to the approximate depth, the reinforcing is placed, and then the top 2 in. of concrete is placed and finished as in unreinforced construction. The materials and their proportions in the base are usually the same as in the top. The top course should be placed as soon as possible after the base course is laid to secure a monolithic slab. The reinforcement usually consists of woven wire or expanded metal, although a mesh work of small round bars is sometimes used. The economic value of reinforcing has not been definitely determined. It is valuable from an esthetic standpoint, since it reduces cracking.*

Construction. Mixers should be equipped with timing and locking devices so that the mixing time is accurately controlled. The length of time for proper mixing should not be less than one minute after all of the materials including the water are in the drum. Concrete should be delivered from the mixer to the subgrade by a boom and bucket rather than by a chute. The amount of water used in the mix should be reduced to the minimum consistent with workability. The tendency is always to use too much water, which results in loss of strength, more work to get a smooth surface, and sometimes in a weak wearing surface. Scaling of the surface is probably due in part to excess water.

Concrete is deposited in one or two layers. The finished thickness varies from 5 in. to 10 in. The usual crown for highways is 1/4 in. per foot. The use of templets to strike the surface of the concrete, and of bridges which span it, thus enabling the laborers to work over the surface without standing on it, should be insisted upon.

Finishing the pavement is most important. When a finishing machine is used, proper methods will produce a smooth surface. The following is recommended by the Portland Cement Association:

1. Strike off and compact the newly placed concrete with a mechanical finisher. Under ordinary conditions three times over the surface will suffice.
2. Belt the surface. The belt should be applied by working it backward and forward transversely and at the same time pulling it forward longitudinally.
3. Straight-edge the surface for high and low places. Use a 10-ft. straight-edge. All such places should be removed by adding or taking away concrete. Use a wooden float on all such disturbed places.
4. Allow the concrete to stand for an appreciable time to let the excess water come to the surface. (10 to 20 minutes). With a wooden float or a wooden straight-edge remove the excess water, laitance, or inert matter by lightly scraping the surface. These tools should have a working edge of from 6 ft. to 10 ft., and long handles.
5. Again test the surface for irregularities and remove them with the float as in operation 3.
6. After the surface water has evaporated the final belt is applied. Apply it at such time and in such manner as to produce a coarse, gritty surface texture.

It should be noted that finishing machines are adapted to pavements of constant widths, and are not suitable for many locations in city work.

* The Portland Cement Association says: " Reinforcement is put in concrete pavements for one purpose only — to hold together any cracks that form."

When hand finishing methods are used the procedure is precisely the same as in the machine method, except that a heavy hand templet should be used to strike off the concrete to the proper contour. It should then be tamped, using a heavy tool resting on the forms. The tamping face should be about 4 in. wide. After tamping, the entire surface should be floated with a wooden float. From this point on the procedure is the same as given in operations 2 to 6 above.

Joints. Longitudinal joints should be used when the width of pavement is greater than 14 in. This central joint should be of the tongue and groove type. Tie bars should be used to prevent the adjoining parts of the pavement from separating one from the other, thus destroying the ability of the joint to transmit loads. In city construction only one joint will usually be necessary up to widths of 40 ft. For greater widths two or more joints will be necessary. In rural roads the number of joints will depend on the method of construction, the total width of the pavement, and the relation of pavement width to traffic lanes. In general, the width between any two joints or any joint and the edges of the pavement should not be less than 9 ft., nor more than 20 ft. Each longitudinal joint should be built to coincide with a traffic lane marker.

Transverse joints always should be provided. There will be more or less contraction and expansion of the concrete due to changes of temperature, variation in the moisture content of concrete, and variation in the condition and character of the subgrade. If expansion joints are not present, when the concrete contracts, the tensile strength will be exceeded and the pavement will crack; when it expands it will tend to crush, spall or bulge. The edges of the joints will need protection from the abrasive action of traffic; a bituminous filler should be used. The width of the transverse joints depends on the distance between them. This distance varied in 1928 specifications from 15 ft. to 202 ft., with 30 ft. as an average.

Curing. Cement concrete pavements must be cured. If the water present at the time of placing the concrete can be retained in it, then that amount is sufficient to produce maximum strength. Therefore it is possible to cure concrete pavements in one of several ways and secure satisfactory results. All of the following methods are in use:

Earth Cover. As soon as the concrete has attained its final set it is covered with earth to a thickness of about 2 in. This earth is immediately sprinkled and must be kept moist until curing is effected, usually from 14 to 28 days. The earth is removed before traffic is admitted. Instead of earth, straw or hay may be used. In cold weather manure is sometimes used.

Ponding. On level roads the slabs may be kept wet by throwing up earth dams or dikes along the edges and across the pavement at suitable intervals, and keeping the areas so formed covered with water. This is rather expensive, especially if water is scarce.

Burlap. As soon as possible after the concrete has set it is sprinkled and covered with two layers of burlap which is then kept wet by frequent sprinklings. After 14 days it may be removed.

Calcium Chloride. Soon after the final set of the pavement $CaCl_2$ is scattered over the surface at the rate of 1-1/2 to 2-1/2 lb. per square yard. This method has been satisfactory, and it has the advantage over those preceding that there is no cleaning up to be done. After the curing period traffic may be admitted.

Water Glass. Sodium silicate may be sprinkled over the pavement as soon as the initial or the final set has taken place.

Bitumens. Tars and asphalts are used. To be effective they must be applied immediately after the final set. Cold or hot application materials have been satisfactory, applied at the rate of from 1/4 to 1/2 gal. per square yard. They may be applied successfully on any grade, and are especially useful in hot, dry climates.

Water. Contractor and engineer alike must make certain that the water supply is adequate. Water supplies that fail in midsummer may cause contractors to fail in the fall. Inquiry should be made locally as to the certainty of supply. If ponding is attempted in hot weather then large gallonage is necessary.

Early Strength. It is often inconvenient to keep traffic off a finished road for from 14 to 28 days. Numerous schemes have been tried, other than using special cements, to attain a strength at 3 days equal to that normally produced at 28 days. The Universal Portland Cement Co. suggests the following:

1. Decrease the amount of mixing water.
2. Increase the mixing time up to 5 minutes.
3. Increase the amount of cement.
4. Place the concrete at a temperature of at least 70° F.
5. Keep concrete at 70° F. for at least 3 days.
6. Keep concrete damp for 3 days.
7. Use $CaCl_2$ where tests show it increases strength. (**Note:** Some cements do not show increased strength when $CaCl_2$ is added.)

The temperature of materials and of the weather plays a very important part in attaining strength. Pavements built in hot July and August weather of 1 : 1-1/2 : 3 mix, standard cement, properly cured, should be of such strength as to be usable after three or four days.

The amount of $CaCl_2$ is 2 lb. per sack of cement. It is customary to mix 100 lb. of it with 50 gal. of water, and then substitute 1 gal. of the solution for 1 gal. of mixing water.

Early Strength Cements. There are a number of such materials on the market. Some are excellent and some are simply standard cement with $CaCl_2$ mixed in them. Lumnite cement attains normal 28-day strength in 24 hours or less. Its manipulation is different from portland cement, and the user is cautioned to observe most carefully the directions for its use. Such brands as Incor and Velow are used in the ordinary manner and attain 28-day strength, or better, in about 3 days.

Hydrated lime, infusorial earth, and similar materials are sometimes used to insure a more workable mix. Their value is as yet undetermined.

Waterproofing. Changes in moisture content cause serious deformation, and stresses that lead to disintegration. Experiments are under way in which the endeavor has been made to waterproof concrete road slabs on all sides. The subgrade has been covered with roofing paper which is then mopped with coal tar pitch. The slab is cast thereon. Tar is sprayed over the top surface, and along the edges, immediately after the final set. Experiments which the author has conducted lead him to believe that if the concrete is cured with water, is then allowed to dry for a week, and is then treated to refusal with a very light water-gas tar, such slabs will be made waterproof on the upper surface, and that abrasion and wear will be greatly reduced. Rattler tests showed 9.2% wear on treated cubes of concrete, and 14.4% on untreated. The bottom of the slabs must be waterproofed by some such means as is noted above. The same grade of water-gas tar saves concrete from the disintegrating effect of seawater, alkali water, and water containing other deleterious substances.

Reinforcement. Reinforcement consists of steel mesh or steel bars. Mesh and light bars probably do not add to the strength of the concrete, but if cracks appear then such reinforcement holds the slab together and prevents the cracks from opening. Bar reinforcement, when properly placed, and if the bars are 1/2 in. or more in thickness, will increase the load capacity. The cost of a concrete pavement is measurably increased when steel is introduced. There is a wide variation of ideas on the subject. For weights and areas of steel bars see the table on page 680. For gages of iron wire and sheets see the tables on pages 688 and 699.

Reinforcement in city street pavements is very undesirable if there is reason to believe that there will be many service cuts made. It is extremely costly to cut through heavy steel, and much damage to the contiguous concrete may result. It is much better in such locations to add an inch depth of concrete and leave out the reinforcement.

Doweling. In order to prevent slabs from rising or sinking to elevations different from their neighbors, recourse has been made to steel dowels which are so inserted that they extend from one slab into the next. This method is apparently efficacious in some cases and disastrous in others. Experience shows that the steel must be of such size that bending is eliminated, and that the ends of the dowels must be free. Since the slabs will expand and contract there must be room in the dowel hole to take care of this. Various forms of caps and tubes have been tried. One simple method is to insert the dowel in a length of gas pipe about 2 in. longer than the dowel, and grease the dowel. Tin, paper and cardboard tubes have been tried. There is no standard.

If the dowels are 3/4-in. bars, or larger, and the spacing is 3 ft. or less, it is probable that part of the moving load will be carried across the joint to the adjoining slab. But if the dowel is placed in a large dowel hole this is not true.

Blow-ups. Doweling may help to prevent blow-ups. But, if the blow-up does occur and is bad, the dowels may help shatter the ends of the slabs.

Warping. There is often a great difference in temperature between the top and the bottom of a slab. This causes the slab to change shape. With a cool subgrade and a hot sun the slab will arch along the center line. At night the opposite effect will be observed. Therefore, at times the edges are supporting the load, whereas at other times the edges are clear of the subgrade. Corners of the slab are sometimes unsupported; corner breaks are commonest. In order to compute the thickness necessary for a concrete road slab, Clifford Older assumed that a load is applied at an unsupported corner, and that rupture takes place in a plane at right angles to the bisector of the corner.

Let S = unit stress in tension in pounds per square inch, which should not exceed one-half of the modulus of rupture of the concrete.

W = load.

t = thickness of slab at edge in inches.

Then $t = \sqrt{3\ W/S}$.

Let t_c = thickness of slab at center $= 0.7t = 0.7\sqrt{3\ W/S} = \sqrt{1.5\ W/S}$.

If a continuous dowel is used, then half the load is carried across the joint, and $t_2 = \sqrt{1.5\ W/S}$.

If four corners carry the load, then $t_3 = \sqrt{0.75\ W/S}$.

Scaling. Scaling is caused by too much rolling, tamping, belting, or other manipulation which brings the fines to the surface. Frost may also cause scaling. This layer of material seems to have, frequently, no real bond with the aggregate below it, and in time a peeling or scaling begins which sometimes continues until from 1/4 in. to 3/4 in. of the surface has peeled off. The large size aggregate is then exposed to the elements and to traffic, and unless a bituminous surface treatment is applied to seal the surface there may be serious disintegration. A bituminous surface treatment should not be attemped until the pavement has stopped scaling, as the treatment will peel off with the scale.

Joint Filler. Numerous fillers are on the market. There are two classes, premolded and poured. The former is made of roofing felt impregnated with asphalt. It is easy to handle, is made in convenient lengths, and of various thicknesses. The 1/2-in. thickness is common. At transverse joints the practice is to let the material extend 1/2 in. above the concrete, and it must extend through the slab to the subgrade. If a poured filler is used the form is removed at the joint, the joint is cleaned and the filler is poured so as to fil the joint completely. A bitumen which will positively adhere to the concrete should be used. A new form of filler is applied from a pressure gun, similar to grease guns for automobile lubrication.

Repair

Corner breaks, shattered edges, and blow-ups are the most frequent causes of trouble. If the broken corner or edge seems to have settled firmly into the subgrade it is often possible to make a satisfactory bituminous patch that sets up quickly and lasts as long as the pavement itself. But if the broken portions have no firm foundation they must be removed and fresh concrete patches must be built. For this purpose an early strength cement is imperative. Curing must be effected by $CaCl_2$, bituminous cover, or water glass. Barricades must be put around the patches and must be lighted at night. If the original slab was reinforced then it is well to cut away the old concrete in the broken area, leaving the reinforcement in place in the new patch.

Trenches and service cuts should have the edges of the old concrete beveled so that the patch will be supported if the backfill settles. It is well to place extra steel reinforcement in such areas. It is fairly easy to introduce heavy steel reinforcement across such cuts.

Maintenance

Cracks form in the best built concrete pavements. The most satisfactory material yet found (1929) for filling them, to keep out water and dirt, is coal tar (specification D-110–27-T).

1. Clean cut crack with an iron hook or poker.
2. Sweep out dirt.
3. Be sure the crack is *dry*.
4. Fill it with coal tar which has not been overheated.
5. Wait for tar to settle and shrink.
6. Repour joint until it is full to overflowing. It is not desirable nor is it economical to smear the concrete surrounding the crack.

The pavement should be examined twice a year at least, and all cracks should be poured. If wide cracks are to be filled a mastic of sand and tar is better than tar alone.

Concrete pavements that show signs of disintegration can often be saved by surface treatments with bituminous materials. Both tars and asphalts are suitable. Frequently it will be found that the material applied does not adhere well to the concrete. In such cases a priming coat of water-gas tar applied at the rate of 1/6 gal. per square yard will tooth into the concrete and the surface treating material will then adhere firmly. On the whole, hot surface treatments are superior to cold. Emulsions of asphalt have been satisfactory, but are somewhat more expensive. It is most necessary to clean the concrete thoroughly before treatment. A cool day is preferable to hot weather as the hot treatment will then chill and remain in place on the concrete. Too heavy a treatment must be avoided, or a wavy surface will be produced. Tar (specification D-108 or D-109), or asphalt (specification Pennsylvania H1), are suitable.

1. Thoroughly clean the concrete with push brooms and water.
2. Let pavement dry thoroughly.
3. Apply tar or asphalt at the rate of 0.3 to 0.5 gal. per square yard.
4. Immediately cover with clean 3/4-in. stone chips, or washed pea-gravel.
5. Roll the cover into the bitumen with a light roller.
6. Touch up any spots that failed to be covered.

Some engineers claim that they find that such a treatment fills all cracks and that it is unnecessary to pour open cracks before treatment. A treatment should last for about two years under ordinary traffic. The subsequent treatments should be lighter. As little as 1/8 gal. per square yard has been satisfactory for second treatments.

In the autumn all holes should be patched with coal tar.

From time to time specialties in concrete pavement design and construction have appeared. The most successful have been Hassam, and Vibrolithic. Hassam is, in reality, what may be termed a penetration type. Crushed stone is laid on the prepared subgrade, shaped and lightly rolled or brought to crown by any suitable means. Cement grout is then poured over the stone, and the pavement is then finished as a regular concrete pavement except that it is rolled.

Vibrolithic is a concrete pavement that has been pounded or vibrated with a machine which consists of a gasoline engine whose flywheel is slightly off center. After the concrete has been poured and struck, a layer of hard, tough, dense, clean stone is spread, and then platforms are placed over it and the machine is run back and forth over the platforms. The pounding effect pushes the new stone into the mix and tends to make the concrete denser. The platforms are removed and the surface is belted or floated as usual. If care is used in the selection of the top layer of stone so that only excellent material is allowed, then the resulting pavement is superior in wearing qualities to ordinary concrete. This method practically assures a drier mix, and a denser and more durable pavement. The only question is whether or not the extra expense is justified. On rural highways it probably is not; on city streets it probably is justified.

18. Brick Pavements

Vitrified bricks are used for pavements. They are made in a few sizes only. There are two styles: wire-cut, and repressed. Both styles may or may not be furnished with lugs, according to the preference of the buyer. Wire-cut, either plain or with lugs, have largely supplanted repressed, though the latter are used for special forms. "Vertical fiber brick" are plain wire-cut brick. They are no better than other wire-cut brick.

A brick pavement is durable, easy to clean, offers slight resistance to traction, is not slippery, does not create dust, is expensive in first cost but may be cheap over a term of years, and properly laid it will carry the heaviest modern traffic successfully.

Paving bricks must be hard, tough, evenly burned, thoroughly annealed, and uniform in texture. They are made uniformly 8-1/2 in. long; their permissible variation from this may not exceed 1/2 in. Transverse dimensions vary, but are usually 2-1/2 × 4 in., or 3 × 4 in., or 3-1/2 × 4 in. A brick 2 in. in depth has been tried but it has not sufficient strength to stand up under heavy truck traffic. Brick are sold as No. 1 pavers, No. 2 pavers, and culls.

Foundations. If the character of the soil is sandy and firm, and the traffic that is expected on the road is not excessively heavy, it may be possible to lay a good pavement directly on such soil. Many brick pavements of this kind have been laid in Florida and elsewhere where similar conditions exist. Such pavements last for years with little maintenance cost. The soil, in any case, ultimately carries the load, and if the soil is firm and well drained there is no reason why a brick pavement should not be laid directly on it, provided that heavy truck traffic, such as is met with close to large cities, is not encountered. In all cases where the soil is the foundation it is necessary to provide a curb flush with the top of the pavement. This should be of either stone or concrete. The old method of using a wooden form or curb is worthless now, if it is expected that such a form will hold the brick in place.

If the soil does not drain rapidly and thoroughly, then some other form of foundation must be used.

Rolled Stone Base. A brick pavement laid on a firm macadam base with a 1-in. bedding course is a good form of construction where drainage is good and the soil is firm. The macadam should be at least 6-in. deep, but a greater depth is desirable. Such a base is usually much cheaper than one of concrete.

Black Base. This type of foundation has been used but little, although it has many points of excellence.

Slag Base. A crushed air-cooled blast furnace slag foundation is good. The particles of slag should all pass a 2-1/2-in. ring and be retained on a 1/4-in. screen. The filler should be granulated slag. If water is used, as in macadam construction, it will be found that these slag bases set up in a few months so that they resemble concrete. A slag base should be at least 6 in. in depth, and somewhat thicker if very heavy truck traffic is expected. Vitrified slag does *not* set up, and this material does not make a first-class base.

Cement Concrete Base. This is at once the most popular and the most expensive foundation for brick. It has been used so extensively that many people are not aware that a brick pavement may be laid on any other type. It has one distinct advantage over other forms of base which is that it bridges over soft spots in a poorly drained subsoil. Another merit is that its greater supporting strength, inch for inch of depth, makes it possible to construct a thinner concrete foundation than any other type with consequent saving in materials and excavations. Concrete bases should be mesh reinforced, since it is imperative to keep at their minimum width such cracks as may form in it. Cracks in a base permit a sand bedding course to find its way into them with consequent damage to the brick top. Further, every crack in a concrete base sooner of later tends to show up in the brick. Hence the necessity for reducing the possible number of cracks, and keeping those that form at their least width. Old concrete pavements are sometimes resurfaced with brick.

Bedding Course, or Cushion. No matter what type of foundation is used it is necessary to overlay it with a bedding course on which to lay the brick. The bedding course, or cushion, as it is often called, takes up any inequalities in the surface of the foundation. Many materials have been used, but the most successful have been sand, granulated slag, sand-tar, sand-asphalt and sand-cement. Of these sand is the poorest and sand-tar the best.

Sand. Sand has the property of bulking when moist, and therefore, if moisture finds its way to the bedding course, the sand may tend to push out of place. When dry, sand will run, so that if there is a crack in the foundation the sand cushion may run through the crack and thus leave an empty space under the brick. Sand may shift its position and so may cause a similar shift to take place in the brick surface above it. Yet, in the tests of U. S. Bureau of Public Roads at Arlington, Va., those sections of brick pavement that were laid on a sand bedding course withstood the tests better than those on sand-cement.

Granulated Slag. This material has been very satisfactory. In the course of time it sets up; this may take a number of months, but in the interim between laying the brick and the setting of the cushion the former have had ample opportunity to find a perfect bed which thereafter does not change. It is good practice to sprinkle the slag before the brick are laid in order to assist slightly the setting up tendency. The depth of the bedding course when laid is usually 3/4 in. to 1 in.

Sand-Cement. This has not been uniformly successful. The idea of using these two materials, mixed in the proportion of 1 part cement to 4 parts sand, is that the mix will overcome the deficiencies of plain sand. Usually the materials are mixed dry in a concrete mixer and spread just ahead of the pavers. Unfortunately there is a tendency to skimp on the quantity of cement, and to mix the materials inadequately. In theory this bedding course should set up, but in practice there are numerous instances where it never does so. In such cases it is no better than sand.

Bitumen-Sand. The best type of bedding course is one composed of sand mixed with either tar or asphalt. Light tar, or a cut-back asphalt, is mixed cold with clean sand in the proportion of about 7% of bitumen to 93% of sand, by weight. This mix is spread in advance of the pavers and struck true to cross-section and grade. The brick thus are embedded in a waterproof cushion that will not shift. If, then, an asphalt filler is used the pavement will be quiet, easy riding, and repairs are simple.

If an old pavement is used as the base for a new brick top it will be found expedient to use a bitumen-sand bedding course.

Fillers. A filler is used in block pavements (1) to prevent movement of the blocks, (2) to waterproof the joints, (3) to facilitate easy cleaning of the pavement, (4) to prevent entrance of filth into the joints.

Sand, as a filler, fulfils none of these functions.

Cement Grout Filler. This is a mixture of 1 part cement and 1 part sand, and enough water to make a paint-like consistency. Grout fulfils all of the requirements, and in addition it protects the edges of the bricks from spalling, provided it is well made grout and properly applied. A curb is not necessary when grout is the filler except to hold the cushion in place. A 1 : 1 grout is liable to cause cracks to form; experience has shown that a 1 : 4 grout preserves the pavement better, although there is difficulty in making such a mix because segregation tends to take place. The disadvantages of using grout are: (1) The pavement may not be used until the filler has set sufficiently, usually 10 days to 2 weeks; (2) the pavement may not be cut to gain access to pipes and underground wires without breaking the bricks so badly that they cannot be used again; (3) after the pavement has been cut and repaired, time must be given for the grout to set in the patched area, thus shutting off the patch to traffic; (4) if the brick are not of uniform quality, or the grouting is not good, the surface of the brick is liable to shatter, the shattered areas later becoming holes and ruts; (5) unless extreme care is used longitudinal and transverse cracks will be formed; (6) arching of the pavement occurs in hot weather and the pavement becomes noisy; (7) power sprinklers will wash out poor grout, thus opening the joints; and, (8) under extremely hot weather conditions grouted pavements may buckle, heave, or blow up. When this occurs the shattered bricks should be removed and replaced with new ones of the same make and size and color. In cutting out the shattered areas care should be taken to tooth out the old bricks, leaving only whole bricks in place, so that there will be an adequate bonding area for the new ones. Pneumatic chisels for this work greatly reduce labor costs.

Blow-ups usually are caused by the grout having been so poorly distributed that only the top portions of the joints are filled. Hot weather causes expansion, and as the bottom of the joints is partially or wholly empty, the pavement bulges upward. If a sufficient number of joints are in this condition, and the expansion is great enough, there is an explosion. Sometimes the bricks are thrown a considerable distance. If only a few joints are poorly filled a shattering or spalling of the contiguous bricks may result.

Asphalt Filler. A suitable asphalt filler fulfils all of the requirements, and has none of the disadvantages of grout. The National Paving Brick Manufacturers Association advocates its use in preference to any other. For successful work the brick must be clean, dry and warm, and the application of the filler should be made on a warm day if possible. Care must be taken not to overheat the asphalt before application. A suitable asphalt for filler purposes is given under specification **"Oil Asphalt Filler — Squeegee Method."**

Coal Tar Paving Pitch. Many of the most successful brick pavements ever laid were filled with coal tar pitch. This material is easier to use than asphalt, but its drawbacks are that it is liable to run in summer and be brittle in winter. Asphalt has largely supplanted it.

Protection of Edges. Truck traffic is liable to damage the edges of brick pavements unless they are protected by curbs. In an endeavor to reduce costs, engineers often omit curbs, but this, in the end, is an expensive omission. On rural highways the curb should be set flush with the surface of the brick, and it should preferably be of hard, tough stone. If concrete is used it should be heavily reinforced. A concrete curb may well be built integral with the base, making the curb of a somewhat richer mix than the base, to resist wear.

Shoulders. Gravel, stone, or slag shoulders should be built along the edges or curbs. These may be water-bound, but preferably should be bituminous-bound, using either hot or cold binder.

Construction: On Natural Base

1. Prepare base by removing all objectionable material.
2. Drain.
3. Set curbs.
4. Grade to proper line and cross-section.
5. Roll soil with three-wheel roller until thoroughly compacted. Water may be used on some soils to assist compaction. Don't over-compact clay soils.
6. Test line and cross-section with straight-edge and templet. Fill in low

spots and trim off high spots. Roll again. The natural base should be the depth of the brick below curbs and headers.

7. Lay brick with better face uppermost. Lugs, if any, must be turned in one direction only. Begin alternate courses with a half brick. Bricklayers must stay on the brick already laid, and in no case step onto the prepared base.

8. Inspect for culls and remove them from the work.

9. Roll brick with a light tandem roller (3 tons maximum), beginning at edges and working to center. Then roll diagonally.

10. Test surface with straight-edge and templet. Correct depressions of over 1/4 in. by removing bricks and adding sand to the cushion. Replace bricks and re-roll.

11. Apply filler. If bituminous filler is used on a very cold day the surface of the bricks should be heated first with a blow torch.

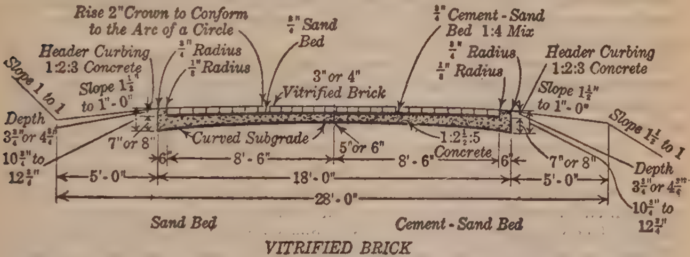


Fig. 11. Pennsylvania Highway Department Standard Brick Pavement

Construction on Old Concrete Base or Pavement

1. Replace shattered areas of concrete with new concrete.
2. Clean out cracks and fill with tar or tar mastic (specification D112-27T).
3. Spread bedding course 1 in. thick.
4. Roll with light tandem roller.
5. Test rolled bedding course with templet and straight-edge. Add enough cushion to level up any depressions. Roll again, and test again. Then repeat these operations until the bedding course is true to line and cross-section.
6. Lay brick as noted in the following section.

Construction on New Base

1. On the prepared base spread a 1-in. layer of bedding course, and spread to line and cross-section. Bedding course should not be spread too far in advance of paving.

2. Roll bedding course with a light tandem roller, and test frequently with straight-edge and templet, adding extra material where necessary.

3. On the bedding course lay the brick with the better face or wire-cut side upward. Lugs must be kept in one direction. Begin alternate courses with a half brick, turning fractured end inward. Joints across the course should be broken by at least 3 in. Each course is laid at right angles to traffic, except that it is possible to lay the brick longitudinally on curves that are not too sharp. Brick layers must keep on the brick, and must not step on the bedding course.

4. Inspect the brick laid, and cull defective ones, substituting good bricks.

5. Roll with a light tandem roller. Begin rolling at edges and work toward center. Then roll diagonally.

6. Test surface with straight-edge and templet. Correct depressions of over 1/4 in. in 10 ft. by removing bricks and adding extra bedding course. Replace bricks and re-roll. Test and retest until a smooth surface is assured.

7. Apply filler: (a) cement grout, (b) asphalt, (c) coal tar paving pitch.

(a) Mix 1 part portland cement with 1 part of sand, and then add enough water to make a thin mortar, about the consistency of paint. Do not mix too much at one time. Of the mix 2 cu. ft. will be enough. Mix the ingredients very thoroughly. When the water has been added, sweep the grout into the joints until they are full. A settling soon takes place, after which make a second application of grout, this time using a thicker paste. Push grout into joints with rubber-edged squeegees. On hot, dry days it is advantageous to sprinkle the surface of the bricks before beginning grouting. After the grout has been applied close the pavement to traffic for at least 10 days. Keep wet and cure like concrete.

(b) Use an asphalt filler which meets specification for Oil Asphalt Filler. Heat the asphalt in a suitable kettle, preferably of over 2 bbl. capacity. Do not heat over 400° F., nor try to apply it at less than 350° F. While asphalt is heating, clean surface of brick. Do not apply filler to a wet or damp pavement. Do not attempt to apply filler if temperature of brick and air is less than 55° F., as will not penetrate the joints. Quickly pour hot filler onto the surface of the brick and push it into the joints with squeegees. Best results are obtained by pushing always in the same direction and at an angle with the courses. Keep applying filler and using squeegee until certain that the joints will hold no more. A thin asphalt surface now covers the brick. Cover this with a light application of sand, pea-gravel, granulated slag, or fine stone chips. Roll with a light roller. The street may be immediately thrown open to traffic.

(c) Use a coal tar pitch which meets specification D-112. The method is identical with (b) above, except that the pitch must not be heated more than 300° F., nor less than 225° F.

Repair

1. Cut out and remove all defective, shattered, or soft bricks, removing each damaged brick completely.

2. Add enough bedding course to assure correct bedding of new brick.

3. Lay new brick as above; fill joints.

If an area of a grouted pavement has shattered it will be best to use an asphalt filler for the new bricks, as an expansion joint will thereby be placed about each individual brick.

Maintenance

Clean out thoroughly every crack and fill them when they are *dry* and *warm* with either asphalt or tar pitch.

If the pavement is badly cobbled, with numerous rough spots, it may sometimes be maintained in a reasonable state of repair by bituminous surface treatments given at yearly or bi-yearly intervals. The method is as follows:

1. Thoroughly clean the old pavement by flushing and scrubbing. All dirt must be removed, as bituminous materials will not stick to the bricks unless they are absolutely clean.

2. Patch all depressions. Hot patch is best for this work.

3. When patches have set up and the bricks are dry apply a hot surface treatment of tar II, or asphalt at the rate of 1/3 to 1/2 gal. per square yard.

4. Cover immediately with a heavy application of 1/4 in. to 3/4 in. hard, tough stone chips. Use about 60 lb. per square yard.

5. Roll with a light tandem roller.

6. Keep a close watch on these treatments and if a break appears it must be patched immediately. A uniform surface is then assured.

Specification

Oil Asphalt Filler — Squeegee Method

Description: Asphalt filler shall be homogeneous, free from water, and shall not foam when heated to 200° C. (392° F.). It shall meet the following requirements:

(a) Flash point: Not less than 200° C. (392° F.).

(b) Melting point: Ring and ball. 65° C. (149° F.) to 110° C. (230° F.).

(c) Penetration: At 0° C. (32° F.) 200 gms., 1 min., not less than 10.
At 25° C. (77° F.) 100 gms., 5 sec., 30 to 50.
At 46° C. (115° F.) 50 gms., 5 sec., not more than 110.

(d) Loss on evaporation: 163° C. (325° F.) 5 hr., less than 1.0%.

(e) Ductility: Not less than 3.

(f) Total bitumen: (Soluble in carbon disulphide) not less than 99%.

(g) Per cent of total bitumen: (Soluble in carbon tetrachloride) not less than 99%.

(h) Reduction in penetration: At 25° C. (77° F.), due to heating specified under Loss on Evaporation, not more than 50%.

19. Stone Block Pavements

In the United States, stone block for pavements usually are made from granites and sandstones. Limestone and trap blocks are used to a limited extent. Durax, a special type of stone pavement developed in Europe, was first used in America in 1913. Square blocks and rectangular blocks of a smaller size than is common in this country are also used in some parts of Europe.

Characteristics. The chief advantage of a stone block pavement is its durability. Motor traffic on rubber tires has eliminated the noise, and the granite block manufacturers are cutting improved blocks, such as the "Manhattan," so that the modern granite block pavement is no longer rough. It is adaptable to use on grades up to 15% if the joints are filled with bituminous materials, and on grades up to 10% if cement grouted joints are used. Some blocks become slippery from wear. Unless properly constructed and maintained, a block pavement is unsanitary.

Belgian Block pavements were constructed with blocks approximately 6 in. square on the upper face, with a bottom face smaller than the top, and of variable depths. Confusion as to the meaning of the name Belgian block is caused by the fact that many rectangular block pavements constructed with trap rock are now so named. A square yard of pavement required about 27 blocks.

Size of Blocks. Pavements are built usually of blocks having rectangular faces. From 1910 to 1918, the depth was reduced from 8 to 5 in. in many localities, thus saving in the cost of the blocks. The blocks should be dressed so as to be rectangular on the faces, having parallel sides and ends, with right-angle corners. Some specifications do not allow bunches or depressions on a face exceeding 1/4 in. If the faces are not free from bulges and hollows, it is impossible to get close and even joints in laying the blocks. Blocks are from 4-3/4 to 5-1/4 in. deep, 3-1/2 to 4-1/2 in. or 4-1/2 to 5-1/2 in. wide, and from

8 to 12 in. long. The blocks shall be so dressed that the joints shall not exceed 1/2 in. in width at the top and for 1 in. downward and not more than 1 in. in width at any other part of the joint.

Physical properties:

	For heavy traffic	For moderately heavy traffic
Percentage of wear, not more than	3.6	5.0
French coefficient of wear, not less than	11.0	8.0
Toughness, not less than	9.0	7.0

Laying the Blocks. The subgrade should be rolled hard with a 10-ton roller, and should be shaped with the same crown as the finished surface. The concrete foundation is then laid as described in Art. 17. The thickness of the foundation is variable, depending upon the traffic conditions, 6 in. being an average value. After the concrete has set, it is covered with a layer of sand 2 to 2-1/2 in. deep, which forms a cushion for the blocks to rest on and allows for any variation in the depths of the blocks. The blocks are placed by hand as closely together as possible, usually in straight parallel courses with the long dimensions perpendicular to the curbs. The blocks in one row should break joints with the blocks in another row by at least 3 in. At street intersections, in order to prevent the traffic of the cross streets from traveling parallel to the long joints and thus forming grooves, the blocks are laid with their long dimensions parallel to one or both of the diagonals of the square formed by the intersecting streets. The work of laying the blocks is started from the curb lines at either side and progresses toward the center of the street. The crown of a stone block pavement is usually parabolic in shape, and the amount of crown can be obtained by formula. The blocks may be laid to the desired crown by the use of strings, cross-section stakes or a board templet. After placing the blocks to the proper lines, they are thoroughly tamped with a tamper weighing from 60 to 70 lb. until no further settlement occurs. The blocks should all lie perpendicular to the sand bed. If any blocks settle more than others, they should be taken out and relaid. The joints should be filled with a suitable material.

Tar-Gravel Filler. The gravel should be of such a size that it will all pass a sieve of 1/2-in. mesh and be retained on a 1/4-in. mesh. A 3/4-in. joint is necessary for this type of filler. The gravel should be heated and swept into the joints until they are filled to within 3 in. of the top. Enough bituminous material is poured to fill the joints to the top of the gravel. The joints are then filled to the top with gravel and poured again with the bituminous material until entirely filled.

Specifications for Asphalt Filler for Stone Block Pavements

(To be applied as grout when mixed with sand)

General

The asphalt filler shall be homogeneous, free from water and shall not foam when heated in 175° C. (347° F.). It shall meet the following requirements:

Physical and Chemical Properties

1. Specific gravity 25°/25° C. (77°/77° F.).........not less than......... 1.000
2. Flash point.........not less than 175° C. (347° F.).
3. Penetration at 25° C. (77° F.), 100 g., 5 sec......*......to......*......

4. Ductility at 25° C. (77° F.) not less than.........30.
5. Loss at 163° C. (325° F.), 5 hours, not more than.........3%.

Penetration of residue at 25° C. (77° F.) 100 g., 5 sec. as per cent of original penetration, not less than.........50%.

6. Per cent of total bitumen soluble in carbon tetrachloride.........not less than ....99%.

* **Note.** The exact penetration limits should be inserted by the engineer with a 20-point limit. It is suggested that for low temperature conditions a penetration of from 70 to 100 be specified and for high temperature conditions a penetration of from 50 to 70 be specified.

Methods of Testing

Tests of the physical and chemical characteristics of the asphalt filler shall be made in accordance with the following methods:

1. Specific gravity, U. S. Dept. of Agriculture Bulletin 314, p. 5.
2. Flash point (open cup), U. S. Dept. of Agriculture Bulletin 314, p. 17.
3. Penetration, A. S. T. M. Standard Test D-5-16.
4. Ductility, Trans. A. S. C. E., Vol. LXXXII, 1918, p. 1460.
5. Volatilization Test, U. S. Dept. of Agriculture Bulletin 314, p. 19 using 50 gram sample.
6. Bitumen soluble in carbon tetrachloride, U. S. Dept. of Agriculture Bulletin 314, p. 30.

The asphalt grout or mastic is prepared by mixing the melted asphalt filler with as much of the hot dry sand as it will carry without making a mixture that is too stiff to flow into the joints. In no case, however, should the volume of sand exceed that of the asphalt and in general volume proportions approaching 3 parts of asphalt to 2 parts of sand will be found most satisfactory. Both asphalt and sand should be heated to between 300° F. and 400° F. but the latter temperature should never be exceeded for fear of injuring the asphalt. Portable kettles and sand heaters are used for this purpose and the mixture is usually made in large steel wheelbarrows or concrete carrier push carts. A measured quantity of the hot asphalt should first be run into the mixing receptacle and a measured quantity of hot sand then added. The contents should be thoroughly mixed with a rake or hoe and immediately flooded over the surface and pushed or squeegeed into the joints. Once the proper proportions are fixed they should be rigidly adhered to throughout the work.

It is very important that the asphalt grout be worked into the joints as fast as possible after the mixture has been made. A second application is almost always required over a considerable proportion of the area flooded by each batch in order to fill the joints flush with the surface. Bringing the joints flush by the addition of sand or other material is very bad practice and will ultimately result in a surface which is rough and cobbly in places.

Approximately 2.5 gal. of asphalt is required for filling each square yard of standard 5-in. granite block pavement, this amount being, of course, subject to variation with the width of joints.

Cement Grout composed of one part of portland cement and one part of clean sand is also used as a filler. The blocks are laid with thin joints and poured with grout. Traffic should be kept off the pavement for a period of 7 days, or until the cement has set up. This type of filler makes a smooth and durable pavement. The surface, however, will be more slippery than where the other types of fillers are used. Repair work will also be more costly, since in removing the blocks more will be broken due to the bond furnished by the cement.

The Maintenance of a stone block pavement, if properly constructed, is practically nothing for the first few years. The life of a granite block pavement constructed on a concrete foundation and with joints well maintained is almost limitless.

The Number of Blocks used per square yard will depend upon the size of the block and the width of the joints. Using the ordinary-sized blocks laid with 1/2-in. joints, the number per square yard will vary from 25 to 32.

Durax

A form of stone block pavement called Durax, or Kleinplfaster, has been laid to considerable extent in Europe, and to lesser extent in the Americas. Small cubes of granite cut by hand or machine, are laid on a dry mortar bed over a concrete base. The joints are filled with grout, or an asphalt filler. The cubes may be laid in straight courses or in concentric interlocking segments, with close joints. The dimensions of the blocks are 3 to 4 in. for length, width and depth. Owing to their small size these blocks should make for a cheaper pavement in first cost than that laid with regular granite blocks. Durax is suitable for medium-traffic streets. When grout filled it is noisy. A straight coal tar pitch is unsuitable as a filler. An asphaltic filler suitable for brick gives good results. A bituminous mastic has been satisfactory. In an 8-hour day a man can lay from 25 to 30 sq. yd.

20. Wood Block Pavement

There are three types of wood blocks in general use: the plain rectangular, the lug, and the hexagonal. By far the greatest yardage has been laid with the plain block. The lug block, being a patented product, has had a more limited sale, although it possesses several advantages over the plain block.

Long ago it was recognized that one of the causes of the bleeding of blocks was the pressure on them due to expansion. This pressure forces the preservative out of the blocks and onto the surface. If lugs take up the pressure, instead of the body of the block, then there will be less bleeding. The lug also forms a space around the blocks so that the filler which is applied readily finds its way to the bottom of the joints. A strip of corrugated cardboard inserted between the rows of blocks accomplishes all that lugs do.

Hexagonal blocks are cut from small trees, and the center of the tree is the center of the block. The blocks are not truly hexagonal and hence, when laid in a pavement, they do not fit close to each other, thus permitting space around the sides for the filler.

In 1928 there was but little wood block pavement laid in highway work as compared with factory floor construction.

Construction: Method A

For all types of blocks

1. Prepare, drain, and roll the subgrade.
2. Lay a concrete base, not leaner than 1 : 3 : 6, minimum thickness of 6 in.
3. Bring concrete to true and fairly smooth, even surface.
4. Let concrete cure for at least 7 days.
5. Spread a dry mortar cushion 1 in. thick on the base; 1 : 2.
6. Just before laying the blocks, sprinkle the cushion with water from a watering can. Do not use a hose. Do not soak, but just dampen the cushion.
7. Lay the blocks on the mortar cushion as soon as possible, being sure the joints are broken, and permitting no one to step on the cushion. Keep all workmen on the paved area.
8. Roll, with light tandem roller.

9. Cull inferior blocks.
10. Apply filler.
11. If sand or bituminous fillers are used the street may be immediately opened to traffic.

Method B

For plain rectangular blocks

1. As above.
2. As above.
3. Finish the concrete carefully, giving it a very smooth, true surface, free from any irregularities, depressions, or protuberances.
4. Let concrete cure for at least 7 days.
5. Spread a mop coat of coal tar pitch on the base using about 10 lb. of 150° F. melting point (cube in water) coal tar pitch per square yard. Spread this directly in advance of paving. Pitch may be sprayed on.
6. Dip one end and one side of each block in hot paving pitch and lay them on the pitch coated base. No defective blocks should be laid, as it is difficult to remove culls.
7. Fill any unfilled joints by pouring paving pitch into them.
8. Cover pavement with sand.
9. Open street to traffic.

Just as good results may be obtained by laying the blocks at right angles with the curb as at any other angle, and at far less trouble and expense.

Fillers

The question of fillers for creosoted wood blocks is a troublesome one. Cresote softens pitch, and ultimately destroys asphalt. It is for this reason that a hard pitch should always be specified, since a pitch of 150° F. melting point is softened in a year's time to 125°, or less. The cutting-back action of the creosote after this time is slight. Asphalt, under like conditions, often becomes a greasy mass that cleaves from the blocks and may be swept up by the street cleaners. Asphalt frequently does not waterproof the joints. It is undoubtedly true that pitch-filled wood block pavements sometimes give trouble and annoyance by bleeding through two summer seasons, yet if sand is strewn on the surface at the first sign of bleeding most of the annoyance will be avoided.

Cement grout has been used as a filler to some extent, but it is not recommended. Better results for far less money may be obtained with clean sand.

Application of Filler. The proper procedure for Method A is to flush pitch onto the blocks and force it into the joints with squeegees. A coating of pitch remains on top, and if the work is done in the summer it becomes a nuisance for the rest of the season. Plenty of sand should be used to take up this excess of pitch. If the joints are poured instead of flushed, the nuisance will be just as great and the expense of pouring will be much in excess of the flushing, while the joints will not be so well filled. If the paving is done in the late fall no trouble from this source will be experienced as the pitch on the surface will chip off during cold weather.

Method of Application of Pitch Filler

1. Heat pitch to not more than 300° F.
2. Pour or flush on the pavement only as much as the squeegee men can push into the joints with a minimum on the surface.
3. Use squeegees made of two pieces of wood about 5/8 in. thick, 4 in. wide,

and 16 in. long, fastened together with screws, and with a piece of old belting between them and projecting about 3/4 in. from the lower edge. Do not use **rubber as the hot pitch will destroy it.**

4. Work rapidly with small quantities.
5. Spread hot sand immediately over the tarred surface.
6. If the work is done in the late fall or the early spring when the weather and the blocks are cold, fill the joints only three-quarters full. Expansion will bring pitch to top of blocks.
7. If lug blocks are being used it is advisable to brush a little sand into the joints before applying pitch. This forms a dam around the bottom of the blocks and prevents the pitch from running underneath them in hot weather. If the blocks seem to have a tendency to float this should always be done.

Dried Out Old Pavements

Often trouble is experienced with old pavements that swell badly after rainy or damp weather. It will be found that the oil in the blocks has evaporated, leaving the pores open. Treat the pavement with a cold application of light tar, using from 1/4 to 1/2 gal. per square yard. After the pavement has absorbed all of the tar that it will, cover it with sand. The tar seals the pores and is absorbed by the wood. Of course this should be undertaken only after a long hot, dry spell when the pavement is in condition to receive the tar.

Bridge Floors

On bridges of the lift type wood block floors have been successfully used, though some means must be employed to keep the blocks in place. On some bridges this has been accomplished by placing metal fasteners every few rows. Probably the best method is to use the hexagonal block, and spike each block down with a heavy nail driven through the center. On other types of bridges the plain or the lug block has given excellent service. Care must be taken, however, to provide adequate expansion joints longitudinally.

Street Car Track Area

Much trouble has been experienced with wood blocks laid in the car track area. This has been due almost entirely to the faulty methods of laying the blocks. Often the blocks are laid as close as possible, so that if they swell they press unduly against the rails. Either the rails must spread, or the blocks must give. In modern track construction there is little chance for the rails to spread, so the blocks bulge or arch over the foundation. Under heavy truck traffic the arch is soon broken down, the blocks splintering off where the break occurs, and the area has to be relaid. It is very important that the blocks be suitably treated and that the joints be filled with coal tar pitch so that they may be made as nearly waterproof as possible.

1. Lay the blocks with sufficient space on all sides to permit expansion. Lug blocks, or plain blocks with corrugated cardboard spacer should be used.
2. Thoroughly fill all joints, especially along the rail, with pitch.

Factory Floors

Wood block makes the ideal factory floor. But common sense must be used to obtain good results. Pitch or asphalt on top of blocks in a factory is objectionable. A sand filler is unsatisfactory in many factories because the floors are cleaned by flushing with water, and the sand is washed out. Where edged tools are being made or used, or where painting is being done, sand is objectionable. Pitch is useless where heat is high and continuous, as in paint-drying and finishing rooms. If untreated or kyanized blocks are used, a low penetration asphalt may be successfully applied for the filler. If cement grout is used the blocks are certain to shrink, leaving a thin partition of grout between each block, which soon breaks down into sand and cement, and the joints are not as well filled as if sand had been used in the first place. The author suggests that responsible representatives of wood block manufacturing companies be consulted in every case before the plans and specifications are approved.

Space should be left around columns, walls, and fixtures so that if the blocks swell from moisture such swelling will not damage the structure. Such space should be filled with pitch or asphalt filler.

21. Sidewalks, Curbs and Gutters

Sidewalks in business districts usually extend from property line to curb but in residential districts it is common to find a grass plat and a row of trees placed between the walk and the curb. The widths of the walk vary from 4 to 6 ft. in residential sections, 8 to 20 ft. in business districts. The usual width for each walk is from one-fifth to one-sixth of the total width between property lines. The business district sidewalk is built with a slope toward the curb of from 1/8 to 1/2 in. per foot, depending on the material of which the sidewalk is made. The residential district walk is sometimes built with a crown, like that of the roadway. This is objectionable if a driveway to a garage is built across the walk. Sidewalks are built of planks, cinders, gravel, crushed stone, brick, flagstones, tar concrete, asphaltic concrete, and portland cement. Where the grade is over 15 to 20% it is advisable to substitute steps with 7-in. risers and at least 5 ft. wide. Short sections of walk may be introduced between the steps. On steep grades the pavement should be made somewhat rough to afford good foothold. At intersections the streets' names may be stamped into concrete walks, or tile or metal letters may be inserted They should be placed so as to be easily read by the pedestrian as he approaches the curb before he crosses the street. Names may be similarly placed on the curbs.

Concrete Sidewalks may be laid directly on the soil, if it is not continually wet. If it is, then a 6-in. layer of cinders, gravel, broken stone or slag must be used for a foundation. In this event a tile drain must be used as well in order to lead away any water that would ordinarily be retained in such foundation. The total thickness of the concrete is usually 5 in. in residential districts (with an extra inch or two at driveways) and 6 to 8 in. in business districts. If the walk extends to the curb, a 1-1/2 to 2-in. expansion joint must be left to be filled with bitumen, so that neither the sidewalk nor the curb may be damaged. Transverse joints which are filled with bitumen must be placed about 50 ft. apart; these may be 1 to 1-1/2 in. wide, and must extend downward through the concrete to the base.

Either a one-course or a two-course pavement may be laid. The one-course costs no more, and, if it is laid with care, is better than and just as attractive looking as the two-course. The one-course mix is 1 : 2 : 3. The two-course walk consists of the lower course 4-1/4 in. thick, of 1 : 2-1/2 : 5 mix, topped with a 3/4-in. course of 1 : 2 mortar. Coarse aggregate for the one-course walk may have a maximum size of 1 in.; for the two-course up to 1-1/2 in. The joints that mark off the slabs should extend through the concrete, otherwise unsightly cracks may develop. The wearing surface is finished by troweling or with a wood float.

Brick Sidewalks are laid either on a concrete base or on a bedding course of sand or gravel over the soil. The joints are usually filled with sand, which later gives vegetation a chance to grow therein. It would be better to fill the joints with cement grout or with a bituminous filler. But few brick pavements are now laid except where the decorative scheme calls for them.

Flags of Blue Sandstone or Granite are laid on a 3-in. to 4-in. foundation of sand or other suitable material. The joints are filled with cement mortar. Granite walks wear smooth and become dangerous due to their slipperiness. Limestone walks are not as good as sandstone.

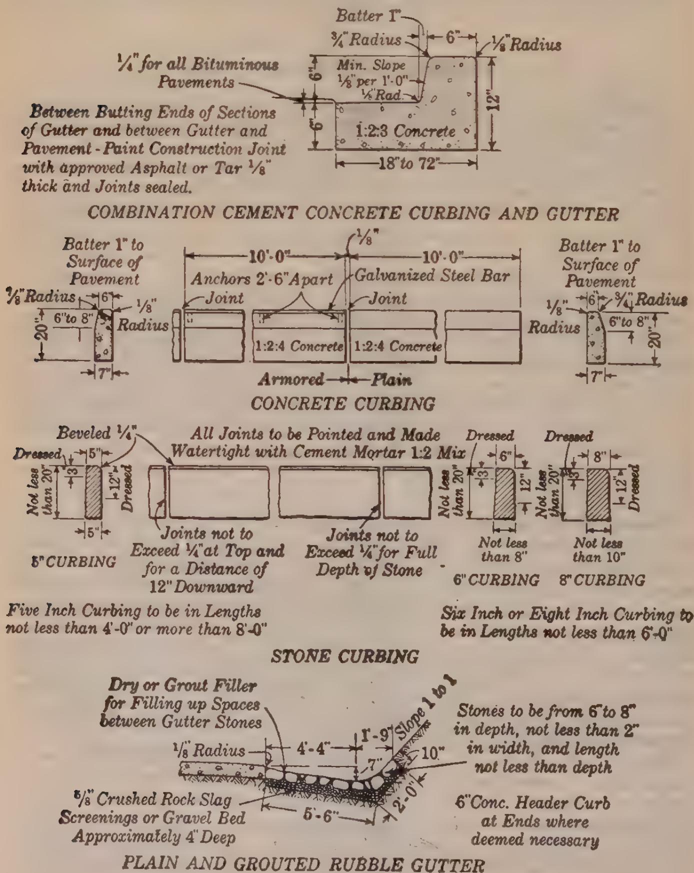
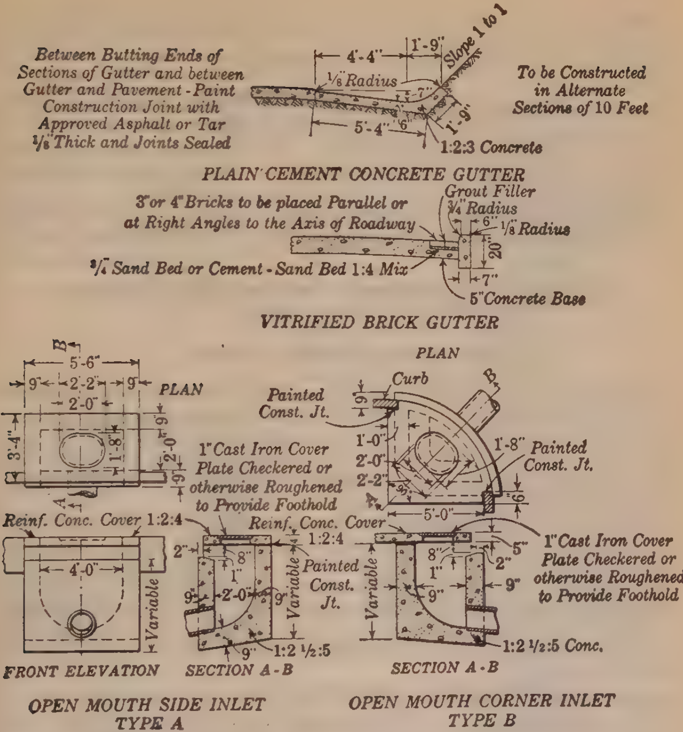


Fig. 12



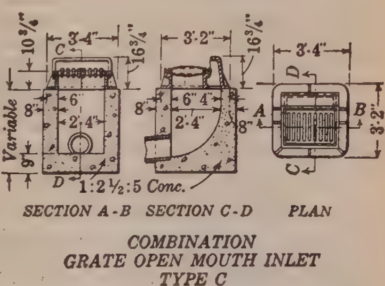


Fig. 12—Continued

Cinders are laid 6 in. deep and are then flooded and tamped or rolled.

Tar Concrete Walks are built on a coarse gravel foundation. Sometimes this is sprayed with hot coal tar before the binder course is laid. The binder course is composed of gravel, the pebbles of which will pass a 1-in. ring, and hot coal tar, in the proportion of 1 gal. of tar to 1 cu. ft. of gravel. These are mixed before the course is laid. This is laid to a depth of about 2 in. and thoroughly rolled. A topping or wearing course composed of 25% of coal tar and 75% of coarse sand is then spread and rolled, so that a total thickness of the wearing course is about 3/4 in. Coarse sand is then strewn over the entire walk.

Some very successful walks have been laid in parks, cemeteries, playgrounds, and similar localities, composed of either tar cut-back, or asphalt cut-back and 3/4-in. stone chips. Such walks are easy to build, and their cost is extremely low.

Curbs and Gutters. Stone curbs are usually composed of granite, bluestone, limestone or sandstone. Plain concrete curbs are of a 1 : 2 : 3 or 1 : 2 : 3-1/2 mix with a 1 : 2 mortar face. In size curbstones are 4 to 12 in. wide, 16 to 24 in. deep, and 4 to 12 ft. long. Concrete curbs should have expansion joints corresponding with those of the pavement.

The stone and the plain concrete curbs are usually known as separate curbs. There are two other types: integral, and combined curb and gutter.

SIGNALS, MARKING, PLANTING, SNOW REMOVAL, ETC.

22. Street Traffic Control

Accidents. Nearly 27 000 people were killed in automobile accidents in 1928 in the United States. This means that one out of four accidental deaths was caused by the automobile. The accompanying charts indicate clearly the types of accidents, direction of vehicle, action of driver, action of pedestrian, condition of vehicle, and condition of pedestrian at the time of the accident. (From Accident Facts, 1928.)

An analysis of fatalities at railroad and highway grade crossings shows that in 1926 31% occurred in urban areas, and 69% in rural areas. Also, rural grade crossings were responsible for 16% of all rural highway fatalities. Pedestrians are hit by automobiles at street intersections more frequently than at other locations. And collisions of cars occur more often at intersections than elsewhere. Unfavorable weather and light conditions do not play a very important part, since 75% of all accidents occurred on dry roads in clear weather.

Intersections. Control of traffic at intersections has been attempted by installing at such points: (a) STOP signs, (b) traffic officers, (c) signals. The Boulevard STOP sign is very useful, works well, and causes little annoyance to the driving public. It is installed on the cross streets where they intersect a Through or Boulevard route. Cross street traffic stops before entering the Through street or Boulevard. But little policing is necessary as drivers readily obey the sign.

At rush hours it is often wisest to place traffic officers at the busiest corners to direct traffic. This method is often far better than that of installing traffic lights, as in most cities under 100 000 population traffic can take care of itself very well without signals or policemen save at rush hours.

The Committee on Street Traffic Signs, Signals and Markings, of the American Engineering Council, says in its 1929 report that the reason " why signals to control street traffic should be installed are: (a) To increase the safety of pedestrians and vehicles at congested intersections; (b) to facilitate the movement of traffic with a minimum of delay at congested intersections; (c) to provide for the continuous movement of traffic throughout a heavy route; (d) to interrupt a heavy traffic stream at intervals so as to afford an opportunity for cross traffic to move."

A Signal is defined as comprising all signal lights that are operated together to control traffic at an intersection, whether the signal is mounted in one unit or in more. Traffic control systems are classified according to the character

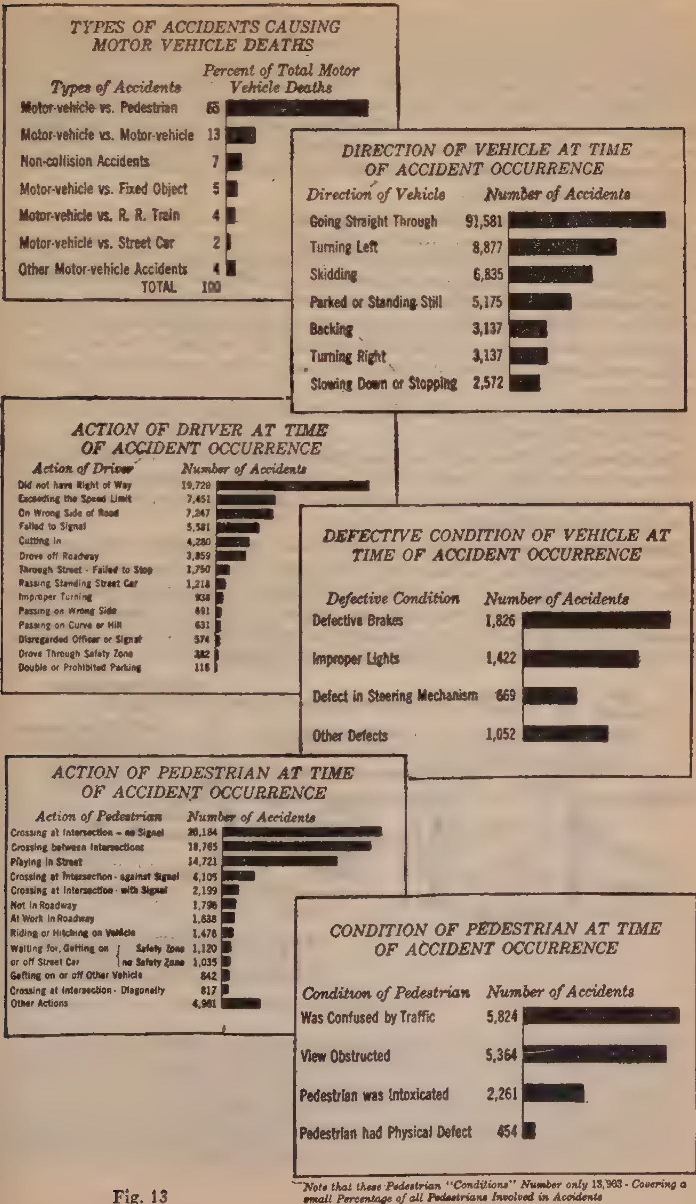

TYPES OF ACCIDENTS CAUSING MOTOR VEHICLE DEATHS
Types of Accidents
Percent of Total Motor Vehicle Deaths
Motor-vehicle vs. Pedestrian 65
Motor-vehicle vs. Motor-vehicle 13
Non-collision Accidents 7
Motor-vehicle vs. Fixed Object 5
Motor-vehicle vs. R. R. Train 4
Motor-vehicle vs. Street Car 2
Other Motor-vehicle Accidents 4
TOTAL 100
DIRECTION OF VEHICLE AT TIME OF ACCIDENT OCCURRENCE
Direction of Vehicle
Number of Accidents
Going Straight Through 91,581
Turning Left 8,877
Skidding 6,835
Parked or Standing Still 5,175
Backing 3,137
Turning Right 3,137
Slowing Down or Stopping 2,572
ACTION OF DRIVER AT TIME OF ACCIDENT OCCURRENCE
Action of Driver
Number of Accidents
Did not have Right of Way 19,728
Exceeding the Speed Limit 7,451
On Wrong Side of Road 7,247
Failed to Signal 5,381
Cutting In 4,280
Drove off Roadway 3,859
Through Street - Failed to Stop 1,750
Passing Standing Street Car 1,218
Improper Turning 938
Passing on Wrong Side 691
Passing on Curve or Hill 631
Disregarded Officer or Signal 574
Drove Through Safety Zone 382
Double or Prohibited Parking 116
DEFECTIVE CONDITION OF VEHICLE AT TIME OF ACCIDENT OCCURRENCE
Defective Condition
Number of Accidents
Defective Brakes 1,826
Improper Lights 1,422
Defect in Steering Mechanism 669
Other Defects 1,052
ACTION OF PEDESTRIAN AT TIME OF ACCIDENT OCCURRENCE
Action of Pedestrian
Number of Accidents
Crossing at Intersection – no Signal 20,184
Crossing between Intersections 18,765
Playing in Street 14,721
Crossing at Intersection - against Signal 4,105
Crossing at Intersection - with Signal 2,199
Not in Roadway 1,798
At Work in Roadway 1,638
Riding or Hitching on Vehicle 1,476
Waiting for, Getting on or off Street Car Safety Zone 1,120
no Safety Zone 1,035
Getting on or off Other Vehicle 842
Crossing at Intersection - Diagonally 817
Other Actions 4,981
CONDITION OF PEDESTRIAN AT TIME OF ACCIDENT OCCURRENCE
Condition of Pedestrian
Number of Accidents
Was Confused by Traffic 5,824
View Obstructed 5,364
Pedestrian was Intoxicated 2,261
Pedestrian had Physical Defect 454
Note that these Pedestrian "Conditions" Number only 13,903 - Covering a small Percentage of all Pedestrians Involved in Accidents

Fig. 13

of the traffic movement resulting from the system. The types of systems are: (a) Independent; the operation of the signal is not related to the operation of any other. (b) Synchronized; all signals show the same color in the same direction simultaneously. (c) Limited progressive; the signals are grouped, the alternate groups showing opposite colors in a given direction and all signals changing simultaneously. (d) Flexible Progressive; a form of progressive system in which the operation of each signal is determined by the traffic requirements of the intersection and which, in addition, provides for the continuous movement of traffic.

Use of Colors. Three colors are recommended in the Model Municipal Traffic Ordinance, but many cities use only two. If three colors are used the order in which they should be displayed is *red, green, yellow*. Yellow is not recommended to be used to govern the turning of vehicles nor the movement of pedestrians. If a two-color system is used the colors are red and green, and the red is displayed simultaneously in all directions for the change period.

Experience shows that, in general, right and left turns should be made only while *green* is shown, although some of the large cities permit right turns on red and green.

Signals should not be operated at times when the volume of traffic is too light to justify their operation.

Cycle Lengths. The length of time that one color should be shown must be determined by trial. Usually this lies between 40 to 80 seconds. At certain periods of the day it may be necessary to change the length of cycle to meet the demands of traffic.

Guard Fences may be stone walls, timber fences, concrete fences, wire cables, or wire fabric. Cables and fabric have been the most satisfactory types. The visibility of these should be increased by suitable painting of them. In snow country they should be painted yellow, with black spots for contrast. The posts may be wood or reinforced concrete. The steel-wire fabric type is most in favor in many parts of the country.

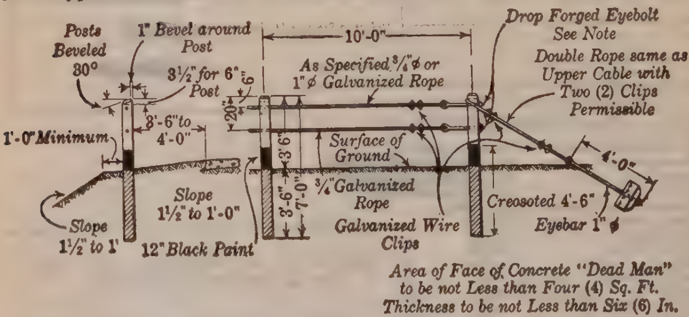


Fig. 14. Pennsylvania State Highway Type

Parking of vehicles in municipalities is controlled by local ordinances.

Vehicles may be parked parallel with the curb, or at angles of 30, 45 and 90 degrees. A lane at least 7 ft. wide must be provided for parallel parking, but motor cars parked at an angle require a lane at least 14 ft. and often 18 ft. wide. Theoretically the number of cars that may be conveniently parked on a block 250 ft. long on each side of the street is: parallel, 14; 30 deg., 17;

45 deg., 24; 90 deg., 32. Practically, however, this is impossible, due to location of hydrants, alley intersections, and "no parking" areas. It must be noted that whereas 32 cars may be parked at 90 deg. in the same curb length that 14 cars may be parked parallel, yet a lane 18 ft. wide is required for the former as against a 7-ft. lane for the latter. In general only parallel parking should be allowed, except on very wide streets.

Parking areas supplied by the municipality are gaining in favor.

23. Road Markers and Signs

The American Association of State Highway Officials adopted a system of standardized signs and markers which was developed by the Bureau of Public Roads. This set of designs is based on definite principles calculated to produce uniformity of significance in the signs, and make familiarity with them easy to acquire on the part of the most casual driver.

Shape. The **octagonal** sign is used to indicate STOP. The **diamond** signs indicate SLOW and CAUTION. The **circular** sign is used as an advance warning at railroad grade crossings only. The **square** signs are used to indicate any condition requiring **caution** that is not inherent in the road itself, but which is due to contiguous or adjacent conditions which also are often intermittent. **Rectangular** shaped signs of various dimensions are used to carry directions, information and restrictions of use or benefit to the driver. The **arrow** direction sign may be substituted for the rectangular direction sign. **Route markers** are of various distinctive designs.

Color. All signs of a precautionary character, including the circular railroad sign, the octagonal STOP, the diamond SLOW, and the square caution signs have black designs on a yellow background. All **direction, information** and **restriction** signs are black on a white background, except that the Rest Station sign is white on a green background.

Erection and Display of Markers and Signs

The standard markers are erected to direct traffic over a specified route. They should be so located as to be conspicuously visible day and night. They should be set facing and on the right-hand side of approaching traffic. Care should be taken to avoid placing them on the inside of curves, in sags and behind objects.

Type of Support. Markers should be mounted on standardized posts. Bolt holes in signs are punched on a multiple of 6 in. If posts are punched or bored similarly there will be no need for drilling in the field. Wooden, metal, and concrete posts are used. Leather or fiber washers should be used against the face and back of all metal signs. Where signs are described as being set at right angles to traffic, the angle should not be exact, but in order to avoid glare reflected back to the driver the sign should be turned away slightly from the road.

Height of Markers. Under ordinary conditions the center of the marker should be placed approximately 3-1/2 ft. above the crown of the pavement or traveled way. On ascending or descending grades this may be varied so that the rays from the headlights may properly illuminate the marker. On city streets where the standard marker is used and where street lights furnish adequate illumination, markers should be placed with the lower edge not less than 7 ft. above the gutter grade in order to clear parked cars.

Lateral Distance. Markers should be erected not less than 5 ft. nor more than 7 ft. from the edge of surfacing on improved roads, except that where a raised curb exists they may be set as close as 3 ft. to the edge of the curb. On unimproved roads there is no rule.

Location. At important cross roads and side roads, markers should be set from 50 to 125 ft. beyond or around the intersection in order to "pull" traffic past the intersection. At intersections or forks where the marked route turns, the marker should be erected 200 to 300 ft. in advance of the fork or turn with proper directional letter "R" or "L" immediately under the marker.

Overlapping Routes. Where two or more routes follow the same road, markers for all routes should generally be erected and carried on the same support. Two or more Route Markers should not be carried on the same post immediately in advance of a turn when only one route turns. If two or more overlapping routes turn at the same intersection, each individual marker should be accompanied by its own directional letters just beneath it.

Markers in Cities. Through towns and cities markers should be erected or placed at relatively short intervals so that one or more will be in view at all times. Where a stenciled marker is used on poles, a white band 20 in. high should first be painted around the pole at the required height from the ground. If a marker and a directional letter are to be stenciled the white band should be 30 in. high.

Standard Detour Marking. A detour should be made fool proof, therefore it is advisable to erect markers at *EVERY* intersecting road. Advance warning of a detour should be provided. Similarly, as the end of the detour, and consequently the main route, is approached the traveler should be warned.

Caution, Slow and Warning Signs

The color scheme for these signs is yellow background with a black design. The yellow shade is known as Federal Yellow. Two signs for different purposes should not be placed closer than 100 ft. if possible. The warning sign should precede the marker.

Longitudinal Location

Stop Sign. This sign is for use at places on a highway where traffic is required to stop. The STOP sign should not be set back from the point of danger in an effort to save a SLOW sign, because the stop should be made at a point where the cause of danger is visible and the driver can see and understand the reason why he has been made to stop. A SLOW sign should be set about 350 ft. in advance of STOP if there is reason to believe that traffic should be warned, in order to slow down for the stop.

Railroad. The railroad crossing signs should be erected 350 to 450 ft. from the crossing. If a stop sign is required as well it should be placed from 25 to 50 ft. from the tracks.

Turn and Curve Signs. A TURN sign is used where the curve has a radius of less than 200 ft. A CURVE sign is used where the curve has a radius of between 200 and 600 ft. The DOUBLE TURN sign is used in advance of two turns less than 350 ft. apart.

HILL Sign. A HILL sign should ordinarily be used only on descending grades steeper than 7% and longer than 200 ft., and on 6% grades if longer than 300 ft. A HILL SHIFT GEAR sign should be used on descending grades under the following conditions:

On grade, per cent	Hill shift gear sign used if descending grade is more than
6	2000 ft. long
7	1000 ft. long
8	750 ft. long
9	500 ft. long
10	400 ft. long
13	300 ft. long
15	200 ft. long
16	Any length

This sign should be used where the per cent or length of grade is less than above indicated, if the grade is also on a sharp curve.

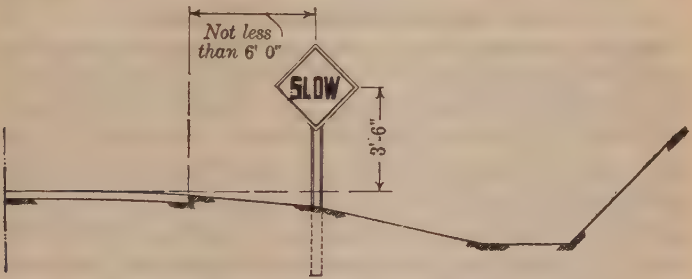


Fig. 15. Height and Location of Signs

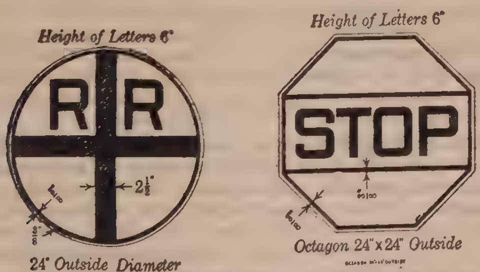


Fig. 16

SLOW Sign. The Slow sign should be used only where for safety, careful driving at a reduced speed is necessary. The use of the SLOW sign in conjunction with other signs is not recommended. It may be used as a prewarning for a STOP sign or to call attention to RAILROAD signs. The SLOW sign is a warning sign only; its use as an attempt to regulate the speed of traffic entering towns is not appropriate.

Direction and Information Signs

This series is intended to furnish the traveler with directional and general information, but involves nothing of a precautionary nature. The signs are generally rectangular, and the color scheme is black on a white background.

The list includes City and Village signs, Physical Feature, Camp Sites, Potable Water Signs.

Rest Station Signs. A REST STATION sign is provided for the use of garages, filling stations, merchants and others desiring them for appropriate display.

General Directional Signs. For indicating direction and distance to a place two types of signs are provided; a solid design and a slat design. A sign is clearer to read if not over four place names are on it. Directional signs should be placed at cross roads, forks and other points where signs of the numbering system are unavailable. They should be so placed that they may be read at night with the aid of headlights.

24. City and Roadside Planting

Shade Trees in Cities. Great care should be taken to select trees that will do well under the conditions in which they are expected to grow and thrive. The following points should be kept in mind in selecting the best trees for the prevailing conditions: (a) Form; (b) shade; (c) hardiness; (d) freedom from insect attack; (e) climate; (f) cleanliness; (g) ease of transplanting; (h) longevity. The following are recommended for northern cities in the United States: Elms, oaks, American and European lindens, Norway maples, black walnut, red gum, tulip (on wide streets only), oriental plane, and white ash (near smoky factories). The following are to be avoided: Poplars, willows, silver maples, birches, beeches, and American plane. The following are recommended for southern states: Pecan, laurel, magnolia, live oak, water oak, willow oak, laurel oak, camphor and palmetto trees.

Trees should be planted from 30 to 50 ft. apart. They should not be placed next the curb, but rather inside the sidewalk line and with turf or sod around them. When planted between the sidewalk and the curb they do not thrive as well, and they may also move the curb out of line. Trees should not be located within 25 ft. of a corner, and they should not be close to lamp-posts nor hydrants. Gas mains should be kept as far away as possible from rows of trees, as the leakage of gas will kill the trees in time.

Shade Trees on Rural Highways. Some trees that are undesirable in city streets may be permitted along country highways. The Lombardy poplar and the cottonwood are such. It is difficult, sometimes, to say where trees should be planted. If the pavement is 20 ft. wide, with two 8-ft. shoulders, and two ditches each of 8 ft. width, then, if the right-of-way is 66 ft., there would be a 7-ft. strip between the ditch line and the property line in which to plant roadside trees, but this may cause undue shade to be cast on the adjoining farm land. If the right-of-way is wider than 66 ft. no trouble from the farmers along the way should be experienced.

Erosion on cuts and fills may often be prevented by proper planting of shrubs and grass; see Bulletin 816, U. S. Department of Agriculture. Dorothy Perkins roses have done well in many localities on cuts and fills.

25. Snow Removal

Economical snow removal is dependent on prompt action. This is true alike of rural and urban districts. Action must start with the snowstorm and must continue without interruption, if the highways are to be kept open continuously. Experience has shown that fast moving trucks equipped with snow plows (stubble plow type) are, in general, better than tractors. The latter, either equipped with stubble plows, rotary plows, or blades, are necessary in deep drifts.

Snow Removal in Urban Districts. There are several methods: (1) Plow the snow to either the center or the sides of the street, after which it is loaded in trucks and hauled to a suitable dump. (2) Push the snow into sewer manholes. (3) Flush the snow into inlets and manholes, using fire hose and street flushing machines.

Snow Removal in Rural Districts. When the storm breaks the plows should be sent out. The equipment should consist of 3-ton or heavier trucks with the plows securely attached, and wing wideners to assist the plows. The trucks work best when equipped with pneumatic tires and very heavy tire chains. The chains should be as heavy as logging chains. If there is a very high wind blowing it is usually useless to begin to plow. Wait until the wind drops. The trucks should be able to travel at the rate of 25 miles per hour. There should be caterpillar tractors equipped with rotary plows to take care of heavy drifts. These may accompany the trucks. Trucks built specially for this work should have a differential lock so that one driving wheel will not spin while the other is stuck fast. A high speed truck equipped with a rotary plow is sometimes better than the caterpillar rotary as it can reach drifts more speedily.

It is important to widen cuts to accommodate snow of succeeding storms. The caterpillar rotary is frequently needed for this work. As much as possible the tractors should be kept off of bituminous pavements as the cleats are liable to cut into the surface and thus start disintegration.

Equipment. Powerful trucks in first-class condition are essential. Plows, blades and scrapers should be stocked to suit local conditions. As night work is common it is important that the trucks be equipped with special head-lights which can be turned, and it is well to also provide colored lights similar to those on a boat so that oncoming traffic will know that snow-fighting equipment is on the road and will not run into it. The tire chains should be examined frequently, and new ones substituted for old ones before trouble happens. In severe storms when the operators are working 24 or more hours at a stretch provision must be made for feeding them.

Costs. V-plow attachments complete cost about $2000. Rotary plows, complete for mounting, cost about $3000. Suitable 3-ton trucks cost $4000, whereas 5-ton trucks may cost as much as $6000. The cost of removing snow from rural highways is indeterminate. The average cost per mile in 1926-7 was $43.50 in the United States. No reliable data are to be found for cost per inch of snowfall, since much depends on the wind.

Snow Fences. The wooden fence of the collapsible type is satisfactory but it is too expensive. Lighter fences are on the market which are quite as effective and one-tenth as expensive. These light fences may be rolled into bundles and stored in convenient barns along the highways during the summer.

26. Distributors

A motor truck for distributing road oils, asphalts and tars must be substantially built of the best materials; it must have adequate power to propel the vehicle and work the pumps; it should be simple in operation and in construction, so that ordinary repairs can be readily made at any garage; it should have the gears arranged so that constant uniform speed may be maintained; its weight when loaded should not exceed the legal limits of the localities where it is to operate.

Trucks may be separated into two classes: those for hauling light loads for short distances over nearly level ground, and those for hauling heavy loads for long distances over hilly territory.

The type of bituminous material to be applied makes considerable difference in the equipment to be placed on the truck. For light oils and light tars to be applied cold, the truck needs simply a tank of suitable size connected to a pump or a compressed air supply which forces the material onto the road through a distributor, manifold, or spray bar equipped with suitable nozzles. For heavy oils and binders, applied hot, the tank should be lagged with a heat non-conductor, and fitted with an oil-burning heater which can rapidly raise the temperature of the material to the required degree.

Nozzles are made to throw sprays of different capacities. Many nozzles are made to throw a fish-tail spray. Two jets impinge and thus form the spray. The size of the holes that the jets emerge from determines the capacity of the nozzle. Nozzles are known by the size of these holes, which are regular standard steel drill sizes. The larger the size number, the smaller is the jet or spray.

Nozzles are liable to clog. When this occurs the truck must be stopped and the nozzle cleaned out before proceeding.

In place of the fish-tail nozzle a plain slotted nozzle may be used. Light materials have been distributed satisfactorily from a pipe having a slot sawed its entire length on the under side. The ends of the pipe are capped. Another type of nozzle throws a cone-shaped spray. This nozzle is very simple in construction and operation.

A motor truck usually spreads the material at some other speed than high. It is possible to control the speed closely and therefore the amount of material applied. Although the speed of the pump is supposed to vary with the speed of the truck this is not always so, and there is a limit to the capacity of the pump which cannot be exceeded. If the speed of the truck is known and the amount the pump delivers a minute, the amount spread per square yard may be found. In high, it is very difficult to control the speed of the truck, but most trucks are made to travel at a certain speed in each gear when the engine is running normally. No truck should spread binder at a greater speed than a good walk, or about 4 miles per hour. Good work of this class cannot be done at greater speed. Furthermore, if anything goes wrong with the nozzles it should be possible for the rear man who may be trailing the truck to be close enough to signal the chauffeur to stop.

On a grade the spreading should be done coming down the hill, thus giving sufficient power to the pumps by taking the load off the engine. *But the truck should never coast.* There are many trucks on the market that have not enough reserve power to operate and pump at the same time when going up hill. There are also trucks that cannot ascend a 10% grade when fully loaded. The prospective purchaser should study carefully the actual demonstrations of the trucks he may be interested in. Some trucks that give the best of service in some parts of the country are worthless in other localities where the topography is different, or the service demanded is dissimilar.

Some distributors are equipped with air pressure instead of pumps. Air pressure is handy, and has some advantages over pumps and some decided disadvantages. Of the latter, two are outstanding. First, it is necessary to have an extra engine on the truck to supply pressure, and this adds to the weight of the outfit, besides making just that much more machinery to keep in order. Second, since the air equipment is entirely independent of the truck's motor there is no definite control of the amount of material applied per square yard. Thus, a truck may run fast down a hill and labor slowly up hill, while

Data on Kinney Road Oil Spraying Trucks

Giving gallons per square yard at speed of 1 mile per hour. To get actual delivery divide by speed of truck spray 8 ft. wide × 5280 = 42 240 sq. ft. = 4683 sq. yd. per hour or 78.2 sq. yd. per minute.

Size of noz-zle	Gallons per nozzle at			No. of noz-zles	Total gallons			Gallons per square yard at			Grade of oil
	25 lb.	40 lb.	50 lb.		25 lb.	40 lb.	50 lb.	25 lb.	40 lb.	50 lb.	
#48	.825	1.22	.....	16	13.20	19.50	......	.169	.250	.....	Light
#40	1.71	2.24	.....	16	27.40	35.80	......	.350	.487	.....	Light
#37	2.01	2.70	.....	16	32.10	43.20	......	.410	.551	.....	Light
#34	2.37	3.10	.....	16	38.00	49.60	......	.487	.635	.....	Light
#30	3.23	4.61	.....	16	51.70	73.40	......	.660	.940	.....	Light
#26	4.35	5.80	.....	16	69.50	92.70	......	.890	1.08	.....	Light
#22	4.77	6.45	.....	16	76.20	103.00	......	.990	1.32	.....	Light
#16	6.00	7.85	.....	16	96.00	125.50	......	1.23	1.61	.....	Light
#16	3.20	5.75	6.95	16	51.20	92.00	111.00	.65	1.17	1.41	Heavy
#12	4.15	7.08	8.40	16	66.50	113.00	134.20	.85	1.45	1.72	Heavy
# 1	6.16	10.01	11.30	16	98.50	160.00	181.00	1.26	2.05	2.32	Heavy
5/16	8.55	16.15	20.50	16	137.00	258.00	328.00	1.75	3.30	4.20	Heavy
#16	3.20	5.75	6.95	32	102.4	184.0	222.0	1.30	2.34	2.82	Heavy
#12	4.15	7.08	8.40	32	133.0	226.0	268.4	1.70	2.90	3.44	Heavy
# 1	6.16	10.01	11.30	32	197.0	320.0	362.0	2.52	4.10	4.64	Heavy
5/16	8.55	16.15	20.50	32	274.0	516.0	656.0	3.50	6.60	8.40	Heavy

Spraying Data
Light Application—Speed, Six Miles per Hour

Gal-lons per square yard	Amount for 6 miles	No. 16 nozzle		No. 12 nozzle		No. 1 nozzle		No. 05 nozzle		Remarks Standard Oil No. 4
		No.	Pres-sure, lb.	No.	Pres-sure, lb.	No.	Pres-sure, lb.	No.	Pres-sure, lb.	
1/8	3 520	16	27	16	23	.....	.....	.....	.....	Width of spray 8 ft.
1/4	7 040	.....	.....	16	43	16	34	.....	.....	
1/3	9 387	.....	.....	16	55	16	45	.....	.....	
						32	26			
1/2	14 080	.....	.....	.....	.....	32	34	32	22app.	Average temperature 80° F.
3/4	21 120	.....	.....	.....	.....	32	53	32	29	
1	28 160	.....	.....	.....	.....	.....	.....	32	37	
1-1/4	35 200	.....	.....	.....	.....	.....	.....	32	45	
1-1/3	37 547	.....	.....	.....	.....	.....	.....	32	47-1/2	
1-1/2	42 240	.....	.....	.....	.....	.....	.....	32	53	

the air pressure distributor is applying material at very different rates per square yard. The total amount distributed over a stretch of up-and-down hill road may be correct, while the actual amount applied per square yard may vary greatly, according to the speed of the truck. Air equipment is handy for blowing out pipe lines and for use in emptying tank cars. With experienced drivers it is equally as good as pumps.

There are but few makes of distributors on the market that are fully equipped. The smaller sized communities would do well to purchase one of

Heavy Application—Speed, Two Miles per Hour

Gallons per square yard	Amount for 2 miles	Nozzle required		Pressure, lb.	Temperature of test	Remarks Standard Oil No. 4
		Size	No.			
1/2	4 693	16	16	35	85°	Width of spray 8 ft.
3/4	7 046	16	16	53	85°	
1	9 386	12	32	25	85°	
1-1/3	12 516	12	32	37	85°	
1-1/2	14 080	12	32	42	85°	
2	18 774	1	32	45	85°	Above pump capacity at this speed.

these at the outset, and then, after a year or two a chassis may be bought and equipped to meet the particular needs of the purchaser. But to try to do this at the start means unwarranted expense, delays, and constant trouble. It is often advantageous to buy a chassis, and then purchase the tank and parts complete on skids. The chassis may be used then for other purposes during the year, while the distributor equipment may be slid onto the chassis in a few moments' time.

Purchasers should look carefully into the matter of total weight. Not only is it desirable to have a distributor that is within the legal weight limit, but it is often impossible to send very heavy distributors away from home because the bridges encountered will not support them safely. It may be a better bargain to buy a smaller outfit that can be used everywhere than to have a heavy machine on hand and not be able to use it except at home.

Quantity of Bitumen Required per Mile at Various Rates of Application

(From Tarvia Road Book)

Width ft.	1/8 gal.	1/7 gal.	1/6 gal.	1/5 gal.	1/4 gal.	3/10 gal.	1/3 gal.	4/10 gal.	1/2 gal.	3/4 gal.
18	1320	1509	1760	2112	2640	3168	3520	4224	5280	7920
17	1247	1425	1662	1995	2493	2992	3324	3989	4987	7479
16	1173	1341	1564	1877	2347	2816	3129	3754	4693	7041
15	1100	1257	1466	1760	2200	2640	2933	3520	4400	6600
14	1027	1173	1369	1643	2053	2464	2738	3285	4107	6159
13	953	1089	1271	1525	1907	2288	2542	3050	3813	5721
12	880	1006	1173	1408	1760	2112	2346	2816	3520	5280
11	807	922	1076	1291	1613	1936	2151	2581	3227	4839
10	734	838	978	1173	1467	1760	1955	2346	2933	4401
9	660	754	880	1056	1320	1584	1760	2112	2640	3960

Width ft.	1 gal.	1-1/4 gal.	1-1/2 gal.	1-3/4 gal.	2 gal.	2-1/4 gal.	2-1/2 gal.
18	10 560	13 200	15 840	18 480	21 120	23 760	26 400
17	9 974	12 465	14 958	17 453	19 948	22 440	24 935
16	9 387	11 735	14 082	16 429	18 774	21 120	23 467
15	8 800	11 000	13 200	15 400	17 600	19 800	22 000
14	8 213	10 265	12 318	14 371	16 426	18 480	20 533
13	7 627	9 535	11 442	13 349	15 254	17 160	19 067
12	7 040	8 800	10 560	12 320	14 080	15 840	17 600
11	6 454	8 065	9 678	11 293	12 908	14 521	16 133
10	5 867	7 335	8 802	10 269	11 734	13 200	14 667
9	5 280	6 600	7 920	9 240	10 560	11 880	13 200

Distance in Linear Feet Covered by Various Tank Capacities at Specified Quantities per Square Yard

9 Feet Wide

(From Tarvia Road Book)

Tank capacity	2/10 gal.	1/4 gal.	3/10 gal.	1/3 gal.	4/10 gal.	1/2 gal.	3/4 gal.	1-1/4 gal.	1-1/2 gal.	1-3/4 gal.
1200	6000	4800	4000	3600	3000	2400	1600	960	800	685
1100	5500	4400	3666	3300	2750	2200	1466	880	733	628
1000	5000	4000	3333	3000	2500	2000	1333	800	666	571
900	4500	3600	3000	2700	2250	1800	1200	720	600	514
800	4000	3200	2666	2400	2000	1600	1066	640	533	457
700	3500	2800	2333	2100	1750	1400	933	560	466	400
650	3250	2600	2166	1950	1625	1300	866	520	433	371
600	3000	2400	2000	1800	1500	1200	800	480	400	343
550	2750	2200	1833	1650	1375	1100	733	440	366	314
500	2500	2000	1666	1500	1250	1000	666	400	333	286

8 Feet Wide

Tank capacity	2/10 gal.	1/4 gal.	3/10 gal.	1/3 gal.	4/10 gal.	1/2 gal.	3/4 gal.	1-1/4 gal.	1-1/2 gal.	1-3/4 gal.
1200	6750	5400	4500	4050	3375	2700	1800	1080	900	772
1100	6185	4950	4123	3713	3093	2475	1650	990	825	708
1000	5624	4500	3750	3375	2812	2250	1500	900	750	644
900	5062	4050	3373	3037	2531	2025	1350	810	675	577
800	4500	3600	3000	2700	2250	1800	1200	720	600	514
700	3936	3150	2623	2362	1968	1575	1050	630	525	448
650	3656	2925	2433	2194	1828	1463	975	585	487	416
600	3375	2700	2250	2025	1687	1350	900	540	450	384
550	3095	2475	2060	1856	1547	1237	825	495	412	352
500	2812	2250	1873	1687	1406	1125	749	450	375	321

Pounds of Covering Material per Square Yard Necessary per Rate of Application

(From Tarvia Road Book)

	Gals. per sq. yd.			Construction
	1/4	1/3	1/2	
Tar for cold surface treatment.				
Covering material:				
Stone............	15–17	18–20	20–22	Water-bound and bituminous macadam
Sand.............	10–12	12–14	14–16	
Tar for hot surface treatment.				
Covering material:				
Slag..............	15	18–20	22–25	Macadam.
Stone.............	16–18	22–24	25–28	Macadam.
Stone.............	16–18	18	28–30	Bituminous macadam.
Stone.............	16–18	18–20	28–30	Concrete.

27. Tables

Table of Highway Quantities

Width of road in feet	Per	Square yards of surface	Cubic yards per various depths loose spread			
			2 in.	3 in.	4 in.	5 in.
10	100 ft.	111.1	6.17	9.26	12.35	15.43
	Mile	5 867.	325.9	488.9	651.9	814.8
12	100 ft.	133.3	7.41	11.11	14.81	18.52
	Mile	7 040.	391.1	586.7	782.2	977.8
14	100 ft.	155.5	8.64	12.96	17.28	21.60
	Mile	8 213.	456.3	684.4	912.6	1140.7
15	100 ft.	166.6	9.26	13.89	18.52	23.15
	Mile	8 800.	488.9	733.3	977.8	1222.2
16	100 ft.	177.7	9.88	14.81	19.75	24.69
	Mile	9 387.	521.5	782.2	1043.	1303.7
18	100 ft.	200.	11.11	16.67	22.22	27.78
	Mile	10 560.	586.7	880.	1173.3	1466.7
20	100 ft.	222.2	12.35	18.52	24.69	30.86
	Mile	11 734.	651.9	977.8	1303.7	1629.6

Width of road in feet	Per	Square yards of surface	Cubic yards per various depths loose spread			
			6 in.	7 in.	8 in.	9 in.
10	100 ft.	111.1	18.52	21.60	24.69	27.78
	Mile	5 867.	977.8	1140.7	1303.7	1466.7
12	100 ft.	133.3	22.22	25.93	29.63	33.33
	Mile	7 040.	1173.3	1368.9	1564.4	1760.
14	100 ft.	155.5	25.93	30.25	34.57	38.89
	Mile	8 213.	1368.9	1597.	1825.2	2053.3
15	100 ft.	166.6	27.78	32.41	37.04	41.67
	Mile	8 800.	1466.7	1711.1	1955.6	2200.
16	100 ft.	177.7	29.63	34.57	39.51	44.45
	Mile	9 387.	1564.4	1825.2	2085.9	2346.7
18	100 ft.	200.	33.33	38.89	44.45	50.00
	Mile	10 560.	1760.	2053.3	2346.7	2640.
20	100 ft.	222.2	37.04	43.21	49.38	55.55
	Mile	11 734.	1955.6	2281.5	2607.4	2933.3

Proper rolling will reduce loose spread about 25%.
For 1000 ft. quantities multiply 100 ft. quantities by 10.
For road 5 ft. wide take half of 10 ft. quantities.
For road 6 ft. wide take half of 12 ft. quantities.
For road 8 ft. wide take half of 16 ft. quantities.
For road 22 ft. wide take 10 ft. plus 12 ft. quantities.
For road 24 ft. wide take twice 12 ft. quantities.
For road 26 ft. wide take 12 ft. plus 14 ft. quantities.
For road 28 ft. wide take twice 14 ft. quantities.
For road 30 ft. wide take twice 15 ft. quantities.

Approximate Weights per Cubic Yard Loose Measurement

	Pounds
Slag	2000
Limestone	2400
Trap rock	2600
Sand	2700
Pea gravel	2750
Washed gravel	2800

Tons of Covering Material per Mile

Width of road in feet	Pounds per square yard												
	8	10	12	14	16	18	20	22	25	28	30	32	35
8	18.8	23.5	28.1	32.9	37.6	42.2	47	51.6	58.8	65.8	70.4	75.1	82.1
9	21.1	26.4	31.7	36.9	42.2	47.5	52.8	58.1	66.	73.9	79.2	84.5	92.4
10	23.5	29.4	35.2	41.1	46.9	52.9	58.8	64.6	73.3	82.2	88.2	93.8	102.7
11	25.8	32.3	38.7	45.2	51.6	58.1	64.6	71.	80.7	90.4	96.8	103.2	112.9
12	28.2	35.2	42.2	49.3	56.3	63.4	70.4	77.4	88.	98.6	105.6	112.6	123.2
13	30.5	38.1	45.7	53.4	61.	68.6	76.2	83.9	95.3	106.8	114.3	122.	133.5
14	32.9	41.1	49.2	57.5	65.7	73.9	82.2	90.3	102.7	114.9	123.3	131.4	143.7
15	35.2	44.	52.7	61.6	70.4	79.2	88.	96.8	110.	123.2	132.	140.8	154.
16	37.6	46.9	56.3	65.7	75.1	84.4	93.8	103.2	117.3	131.4	140.7	150.2	164.2
18	42.3	52.8	63.4	73.9	84.5	95.	105.6	116.2	132.	147.8	158.4	169.	184.8
20	46.9	58.8	70.4	82.2	93.8	105.6	117.6	129.2	146.6	164.4	176.	187.6	205.4
22	51.6	64.6	77.4	90.4	103.2	116.2	129.2	142.	161.4	180.8	193.6	206.4	225.8
24	56.3	70.4	84.4	98.6	112.6	126.7	140.8	154.8	176.	197.2	211.2	225.2	246.4

Tons of Stone per Mile Required to Build Bituminous Macadam

Finished depths and widths based on stone weighing 2500 lb. per cu. yd.

Depth, in.	Width, 10 ft.		Width, 12 ft.		Width, 14 ft.	
	Tons, coarse	Tons, chips	Tons, coarse	Tons, chips	Tons, coarse	Tons, chips
2-1/2	587	176	704	210	821	246
3	704	176	844	210	986	246
3-1/2	821	176	985	210	1150	246

Depth, in.	Width, 16 ft.		Width, 18 ft.		Width, 20 ft.	
	Tons, coarse	Tons, chips	Tons, coarse	Tons, chips	Tons, coarse	Tons, chips
2-1/2	938	280	1056	316	1173	352
3	1126	280	1266	316	1408	352
3-1/2	1314	280	1478	316	1642	352

Table of Ready Computation of Stone Costs in Road Building

Weight of cubic yards, pounds	Cost per ton (2000 lbs.), $	Cost per cubic yard, $	One cubic yard will lay number of square yards of the thickness (loose) indicated, and at the cost per square yard stated							
			2 in. deep		3 in. deep		4 in. deep		5 in. deep	
			Number of square yards	Cost per square yard, $	Number of square yards	Cost per square yard, $	Number of square yards	Cost per square yard, $	Number of square yards	Cost per square yard, $
2800	3.00	4.20	18	.233	12	.350	9	.467	7-1/5	.583
2800	2.75	3.85	18	.214	12	.321	9	.428	7-1/5	.535
2800	2.50	3.50	18	.194	12	.292	9	.389	7-1/5	.486
2800	2.25	3.15	18	.175	12	.263	9	.350	7-1/5	.438
2800	2.00	2.80	18	.156	12	.233	9	.311	7-1/5	.389
2800	1.75	2.45	18	.136	12	.204	9	.272	7-1/5	.340
2800	1.50	2.10	18	.117	12	.175	9	.233	7-1/5	.292
2800	1.25	1.75	18	.097	12	.146	9	.194	7-1/5	.243
2800	1.00	1.40	18	.078	12	.117	9	.156	7-1/5	.194
2800	.75	1.05	18	.058	12	.088	9	.117	7-1/5	.146
2800	.50	.70	18	.039	12	.058	9	.078	7-1/5	.097
2600	3.00	3.90	18	.217	12	.325	9	.433	7-1/5	.542
2600	2.75	3.58	18	.199	12	.298	9	.398	7-1/5	.497
2600	2.50	3.25	18	.181	12	.271	8	.361	7-1/5	.451
2600	2.25	2.93	18	.163	12	.244	9	.326	7-1/5	.407
2600	2.00	2.60	18	.144	12	.217	9	.289	7-1/5	.361
2600	1.75	2.28	18	.127	12	.190	9	.253	7-1/5	.317
2600	1.50	1.95	18	.108	12	.163	9	.217	7-1/5	.271
2600	1.25	1.63	18	.091	12	.136	9	.181	7-1/5	.226
2600	1.00	1.30	18	.072	12	.108	9	.144	7-1/5	.181
2600	.75	.98	18	.054	12	.082	9	.109	7-1/5	.136
2600	.50	.65	18	.036	12	.054	9	.072	7-1/5	.090
2400	3.00	3.60	18	.200	12	.300	9	.400	7-1/5	.500
2400	2.75	3.30	18	.183	12	.275	9	.367	7-1/5	.458
2400	2.50	3.00	18	.167	12	.250	9	.333	7-1/5	.417
2400	2.25	2.70	18	.150	12	.225	9	.300	7-1/5	.375
2400	2.00	2.40	18	.133	12	.200	9	.267	7-1/5	.333
2400	1.75	2.10	18	.117	12	.175	9	.233	7-1/2	.292
2400	1.50	1.80	18	.100	12	.150	9	.200	7-1/5	.250
2400	1.25	1.50	18	.083	12	.125	9	.167	7-1/5	.208
2400	1.00	1.20	18	.067	12	.100	9	.133	7-1/5	.167
2400	.75	.90	18	.050	12	.075	9	.100	7-1/5	.125
2400	.50	.60	18	.033	12	.050	9	.067	7-1/5	.083

Table of Ready Computation of Stone Costs in Road Building—Cont.

Weight of cubic yards, pounds	Cost per ton (2000 lbs.), $	Cost per cubic yard, $	One cubic yard will lay number of square yards of the thickness (loose) indicated, and at the cost per square yard stated							
			6 in. deep		7 in. deep		8 in. deep		9 in. deep	
			Number of square yards	Cost per square yard, $	Number of square yards	Cost per square yard, $	Number of square yards	Cost per square yard, $	Number of square yards	Cost per square yard, $
2800	3.00	4.20	6	.700	5-1/7	.817	4-1/2	.933	4	1.050
2800	2.75	3.85	6	.642	5-1/7	.749	4-1/2	.856	4	.963
2800	2.50	3.50	6	.583	5-1/7	.681	4-1/2	.778	4	.875
2800	2.25	3.15	6	.525	5-1/7	.613	4-1/2	.700	4	.788
2800	2.00	2.80	6	.467	5-1/7	.544	4-1/2	.622	4	.700
2800	1.75	2.45	6	.408	5-1/7	.476	4-1/2	.544	4	.613
2800	1.50	2.10	6	.350	5-1/7	.408	4-1/2	.467	4	.525
2800	1.25	1.75	6	.292	5-1/7	.340	4-1/2	.389	4	.438
2800	1.00	1.40	6	.233	5-1/7	.272	4-1/2	.311	4	.350
2800	.75	1.05	6	.175	5-1/7	.204	4-1/2	.233	4	.263
2800	.50	.70	6	.117	5-1/7	.136	4-1/2	.156	4	.175
2600	3.00	3.90	6	.650	5-1/7	.758	4-1/2	.867	4	.975
2600	2.75	3.58	6	.597	5-1/7	.696	4-1/2	.796	4	.895
2600	2.50	3.25	6	.542	5-1/7	.632	4-1/2	.722	4	.813
2600	2.25	2.93	6	.488	5-1/7	.570	4-1/2	.651	4	.733
2600	2.00	2.60	6	.433	5-1/7	.506	4-1/2	.578	4	.650
2600	1.75	2.28	6	.380	5-1/7	.443	4-1/2	.507	4	.570
2600	1.50	1.95	6	.325	5-1/7	.379	4-1/2	.433	4	.488
2600	1.25	1.63	6	.272	5-1/7	.317	4-1/2	.362	4	.408
2600	1.00	1.30	6	.217	5-1/7	.253	4-1/2	.289	4	.325
2600	.75	.98	6	.163	5-1/7	.191	4-1/2	.218	4	.245
2600	.50	.65	6	.108	5-1/7	.126	4-1/2	.144	4	.163
2400	3.00	3.60	6	.600	5-1/7	.700	4-1/2	.800	4	.900
2400	2.75	3.30	6	.550	5-1/7	.642	4-1/2	.733	4	.825
2400	2.50	3.00	6	.500	5-1/7	.583	4-1/2	.667	4	.750
2400	2.25	2.70	6	.450	5-1/7	.525	4-1/2	.600	4	.675
2400	2.00	2.40	6	.400	5-1/7	.467	4-1/2	.533	4	.600
2400	1.75	2.10	6	.350	5-1/7	.408	4-1/2	.467	4	.525
2400	1.50	1.80	6	.300	5-1/7	.350	4-1/2	.400	4	.450
2400	1.25	1.50	6	.250	5-1/7	.292	4-1/2	.333	4	.375
2400	1.00	1.20	6	.200	5-1/7	.233	4-1/2	.267	4	.300
2400	.75	.90	6	.150	5-1/7	.175	4-1/2	.200	4	.225
2400	.50	.60	6	.100	5-1/7	.117	4-1/2	.133	4	.150

Length of Road in Linear Feet Which a Load of Stone of Given Size will Cover to the Given Loose Depths

(From Report of Wisconsin Highway Commission)

Width of road	Loose depth	Size of load in cubic yards								
		1	1-1/4	1-1/2	1-3/4	2	2-1/4	2-1/2	2-3/4	3
Ft.	In.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.
8	3	13.5	16.9	20.2	23.6	27	30.4	33.7	37.1	40.5
	4	10.1	12.6	15.2	17.7	20.2	22.8	25.3	27.8	30.3
	5	8.1	10.1	12.1	14.1	16.2	18.2	20.2	22.3	24.3
	6	6.75	8.4	10.1	11.8	13.5	15.2	16.9	18.5	20.3
9	3	12	15	18	21	24	27	30	33	36
	4	9	11.25	13.5	15.75	18	20.25	22.5	24.75	27
	5	7.2	9	10.8	12.6	14.4	16.2	18	19.8	21.6
	6	6	7.5	9	10.5	12	13.5	15	16.5	18
10	3	10.8	13.5	16.2	18.9	21.6	24.3	27	29.7	32.4
	4	8.1	10.1	12.2	14.2	16.2	18.2	20.2	22.3	24.2
	5	6.5	8.1	9.7	11.3	13	14.6	16.2	17.8	19.4
	6	5.4	6.75	8.1	9.45	10.8	12.1	13.5	14.8	16.2
12	3	9	11.2	13.5	15.8	18	20.2	22.5	24.7	27.2
	4	6.7	8.4	10.1	11.8	13.5	15.2	16.9	18.5	20.2
	5	5.4	6.7	8.1	9.5	10.8	12.1	13.5	14.8	16
	6	4.5	5.6	6.7	7.8	9	10.1	11.2	12.3	13.5

Square Yards per 100 Feet and per Mile for Different Widths of Surface

Width in feet	Number of square yards		Width in feet	Number of square yards	
	Per 100 ft.	Per mile		Per 100 ft.	Per mile
8	88.889	4 693	26	288.889	15 253
10	111.111	5 867	28	311.111	16 427
12	133.333	7 040	30	333.333	17 600
14	155.556	8 213	32	355.556	18 773
15	166.667	8 800	34	377.778	19 947
16	177.778	9 387	36	400.000	21 120
18	200.000	10 560	38	422.222	22 293
20	222.222	11 733	40	444.444	23 466
22	244.444	12 907	42	466.667	24 640
24	266.667	14 080	44	488.889	25 813

Costs per Mile at Price per Square Yard

(From Report of Commissioner of Public Roads of New Jersey)

Width	Sq. yds.	25	30	35	40	45	50	55	60
8	4 693	1173	1408	1642	1877	2112	2346	2581	2816
10	5 867	1466	1760	2053	2346	2640	2933	3226	3520
12	7 040	1760	2112	2464	2816	3168	3520	3872	4224
14	8 213	2053	2464	2874	3285	3696	4106	4517	4928
16	9 386	2346	2816	3285	3754	4224	4693	5162	5632
18	10 560	2640	3168	3696	4224	4752	5280	5808	6336

Width	Sq. yds.	65	70	75	80	85	90	95
8	4 693	3050	3285	3520	3754	3989	4224	4 458
10	5 867	3813	4106	4400	4693	4986	5280	5 573
12	7 040	4576	4928	5280	5632	5984	6336	6 688
14	8 213	5338	5749	6160	6570	6981	7392	7 802
16	9 386	6101	6570	7040	7509	7978	8448	8 917
18	10 560	6864	7392	7920	8448	8976	9504	10 032

To arrive at any price per yard higher than that given, add to price under cents column the sum or multiple of the sum in the yard column, i.e., 16 ft. at $1.65 would be 9386 + 6101 = 15 487.

Cubic Yards of Loose Gravel Required to Make One Mile of Road of Different Widths and Thicknesses

(From Report of Commissioner of Public Roads of New Jersey)

Width, ft.	Thickness of road after consolidation, inches						
	6	7	8	9	10	11	12
6	880	1027	1173	1320	1467	1613	1760
7	1027	1198	1369	1540	1711	1882	2054
8	1173	1369	1564	1760	1956	2151	2346
9	1320	1540	1760	1980	2200	2420	2640
10	1467	1711	1956	2200	2444	2689	2934
11	1613	1882	2151	2420	2689	2958	3226
12	1760	2053	2346	2640	2933	3227	3520
13	1907	2224	2542	2860	3178	3496	3814
14	2054	2396	2738	3080	3422	3764	4107
15	2200	2567	2933	3300	3667	4033	4400
16	2346	2738	3128	3520	3912	4302	4692
17	2493	2909	3324	3740	4156	4571	4986
18	2640	3080	3520	3960	4400	4840	5280
19	2787	3250	3716	4180	4644	5109	5574
20	2933	3422	3911	4400	4888	5378	5866

Number of Tons of Stone per Mile Required to Build the Following Finished Depths and Widths, Based on Stone Weighing 2650 Lb. per Cu. Yd.

(From Report of Commissioner of Public Roads of New Jersey)

Width, 12 ft.		Width, 14 ft.		Width, 16 ft.		Width, 18 ft.		Width, 20 ft.	
Deptht	Tons	Depth	Tons	Depth	Tons	Depth	Tons	Depth	Tons
4	1312	4	1531	4	1750	4	1968	4	2187
6	1969	6	2296	6	2625	6	2953	6	3281
8	2625	8	3062	8	3500	8	3937	8	4375
10	3281	10	3828	10	4375	10	4921	10	5468
12	3937	12	4593	12	5250	12	5906	12	6562

Gage of Cylindrical Tanks

(From Tarvia Road Book)

Per cent depth filled	Per cent capacity	Per cent depth filled	Per cent capacity	Per cent depth filled	Per cent capacity	Per cent depth filled	Per cent capacity
50	50.00	63	66.34	76	81.50	89	94.02
51	51.27	64	67.56	77	82.60	90	94.80
52	52.55	65	68.81	78	83.68	91	95.55
53	53.81	66	69.97	79	84.74	92	96.26
54	55.08	67	71.16	80	85.77	93	96.93
55	56.34	68	72.34	81	86.77	94	97.55
56	57.60	69	73.52	82	87.76	95	98.13
57	58.86	70	74.69	83	88.73	96	98.66
58	60.11	71	75.93	84	89.68	97	99.10
59	61.36	72	77.00	85	90.60	98	99.50
60	62.61	73	78.14	86	91.50	99	99.80
61	63.86	74	79.27	87	92.36	100	100.00
62	65.10	75	80.39	88	93.20		

Areas

Square Yards per Mile

Width, ft.	Sq. yd.	Width, ft.	Sq. yd.	Width, ft.	Sq. yd.	Width, ft.	Sq. yd.	Width, ft.	Sq. yd.
8	4693	12	7040	16	9 387	22	12 907	28	16 427
9	5280	14	8213	18	10 560	24	14 080	30	17 600
10	5867	15	8800	20	11 733	26	15 253	40	23 467

Volumes

Cubic Yards per Mile

Width, ft.	Thickness, in.	Cu. yd.	Width, ft.	Thickness, in.	Cu. yd.
9	2	293	16	2	522
9	2-1/2	367	16	2-1/2	652
9	3	440	16	3	782
9	4	587	16	4	1043
9	6	880	16	6	1564
9	7	1027	16	7	1825
9	8	1173	16	8	2086
9	9	1320	16	9	2347
15	2	489	20	2	652
15	2-1/2	611	20	2-1/2	815
15	3	733	20	3	978
15	4	978	20	4	1304
15	6	1467	20	6	1956
15	7	1711	20	7	2281
15	8	1956	20	8	2607
15	9	2200	20	9	2933

Weights of Materials

Broken Stone

Pounds per Cubic Yard

Kind	Specific gravity	Loose spread 45% voids	Compacted 30% voids
Trap	2.8	2590	3300
	2.9	2680	3420
	3.0	2770	3540
	3.1	2870	3650
Granite	2.6	2400	3060
	2.7	2500	3180
	2.8	2590	3300
Limestone	2.6	2400	3060
	2.7	2500	3180
	2.8	2590	3300
Sandstone	2.4	2220	2830
	2.5	2310	2940
	2.6	2400	3060
	2.7	2500	3180

Gravel and Sand

Approximate Number of Pounds per Cubic Yard

Voids	Weight	Voids	Weight
50%..................	2240	35%..................	2910
45%..................	2460	30%..................	3130
40%..................	2680	25%..................	3350

Asphalt

Pounds per Gallon

Specific gravity	Weight	Specific gravity	Weight
1.00..................	8.33	1.04..................	8.66
1.01..................	8.41	1.05..................	8.75
1.02..................	8.50	1.06..................	8.83
1.03..................	8.58		

Approximate Quantities of Materials Required for Asphalt Wearing Courses and Foundations

Asphalt Macadam Wearing Course

2-1/2 In. Thick

Materials	Pounds per sq. yd.	Tons per mile 1 ft. wide
Coarse stone..............................	227	66.6
Intermediate stone..............................	45	13.2
Fine stone..............................	25	7.3
Asphalt..............................	19	5.6

Coarse Graded Aggregate Asphaltic Concrete

2 In. Thick, Excluding Seal Coat

Materials	Pounds per sq. yd.	Tons per mile 1 ft. wide
Coarse stone..............................	147	43.1
Sand..............................	58	17.0
Mineral filler..............................	9	2.6
Seal coat stone..............................	25	7.3
Asphalt..............................	18	5.3

Fine Graded Aggregate Asphaltic Concrete

2 In. Thick

Materials	Pounds per sq. yd.	Tons per mile 1 ft. wide
Stone chips	53	15.5
Sand	123	36.1
Mineral filler	18	5.3
Asphalt	18	5.3

Sheet Asphalt

Binder 1-1/2 In. Thick, Wearing Course 1-1/2 In. Thick

Materials	Pounds per sq. yd.	Tons per mile 1 ft. wide
Binder stone	116	34.0
Sand, Top	115	33.8
Sand, Binder	45	13.2
Mineral filler	22	6.5
Asphalt, Top	16	4.7
Asphalt, Binder	9	2.6

Asphalt Macadam Base

6 In. Thick

Materials	Pounds per sq. yd.	Tons per mile 1 ft. wide
Coarse stone	545	159.8
Intermediate stone (optional)	45	13.2
Asphalt	22	6.5

Asphaltic Concrete Base

3 In. Thick

Materials	Pounds per sq. yd.	Tons per mile 1 ft. wide
Coarse stone	216	63.4
Sand	100	29.3
Asphalt	18	5.3

Conversion Table, Linear Feet to Miles

1 to 9		10 to 90		100 to 900		1000 to 9000		10 000 to 90 000	
Feet	Mile	Feet	Mile	Feet	Mile	Feet	Miles	Feet	Miles
1	0.00019	10	0.00189	100	0.01894	1000	0.18939	10 000	1.8939
2	0.00038	20	0.00379	200	0.03788	2000	0.37879	20 000	3.7879
3	0.00057	30	0.00568	300	0.05682	3000	0.56818	30 000	5.6818
4	0.00076	40	0.00758	400	0.07576	4000	0.75758	40 000	7.5758
5	0.00095	50	0.00947	500	0.09470	5000	0.94697	50 000	9.4697
6	0.00114	60	0.01136	600	0.11364	6000	1.13636	60 000	11.3636
7	0.00133	70	0.01326	700	0.13258	7000	1.32576	70 000	13.2576
8	0.00152	80	0.01515	800	0.15152	8000	1.51515	80 000	15.1515
9	0.00170	90	0.01705	900	0.17046	9000	1.70455	90 000	17.0455

Specific Gravities, Degrees Baumé, Weight in Pounds per Gallon and Volume in Gallons per Pound of Materials at 60° F. Having Specific Gravities Exceeding 1.00

Specific gravity	Degrees Baumé	Lb. per gal.	Gal. per lb.	Specific gravity	Degrees Baumé	Lb. per gal.	Gal. per lb.
1.00	0.00	8.328	0.1201	1.15	18.91	9.577	0.1044
1.01	1.44	8.411	0.1189	1.16	20.00	9.660	0.1035
1.02	2.84	8.495	0.1177	1.17	21.07	9.744	0.1026
1.03	4.22	8.578	0.1165	1.18	22.12	9.827	0.1018
1.04	5.58	8.661	0.1154	1.19	23.15	9.910	0.1009
1.05	6.91	8.744	0.1144	1.20	24.17	9.994	0.1001
1.06	8.21	8.828	0.1132	1.21	25.16	10.077	0.0993
1.07	9.49	8.911	0.1122	1.22	26.15	10.160	0.0984
1.08	10.74	8.994	0.1112	1.23	27.11	10.243	0.0976
1.09	11.97	9.078	0.1102	1.24	28.06	10.327	0.0967
1.10	13.18	9.161	0.1091	1.25	29.00	10.410	0.0960
1.11	14.37	9.244	0.1082	1.26	29.92	10.494	0.0953
1.12	15.54	9.327	0.1072	1.27	30.83	10.577	0.0946
1.13	16.68	9.411	0.1062	1.28	31.72	10.660	0.0930
1.14	17.81	9.494	0.1053	1.29	32.60	10.743	0.0931

SECTION 21

STEAM RAILROADS

ASSOCIATE EDITOR

FRED ASA BARNES *

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* Walter Loring Webb was the author of this section for the first edition and made the revisions for the second and third editions. He was asked to make the revisions for the fourth edition and commenced the work but service in the U. S. Army in France prevented, and the work was then taken up by Prof. Fred Asa Barnes.

STEAM RAILROAD ROADWAY

1. Right-of-Way

For single-track the usual width of right-of-way for the older eastern railroads is 4 rods or 66 ft., whereas western roads have generally taken a width of 100 ft. The necessary width on ground level transversely is: **w = b + 2 sd,** in which *b* is the width of roadbed, *s* the side slope (horizontal to vertical) and *d* the depth of fill, or cut. For ground sloping transversely the total width is greater and more land must be taken on the lower side than on the upper if the total is to be kept at a minimum. These are for minimum widths to accommodate the fills. In cuts, additional width is required for the side ditches, and possibly also an intercepting ditch at the top of the bank to keep surface water out of the cut; and some allowance should be made in all cases to give room for fencing, etc. For a Class A roadbed of the minimum width, 20 ft., a 25-ft. fill with side slopes of 1.5 to 1 requires a width of: $w = 20 + 2 \times 1.5 \times 25 = 95$ ft. Areas of land for different widths are given in the following table:

Area of Right-of-Way

Width in Feet	Acres per Mile	Acres per 100 Ft.	Width in Feet	Acres per Mile	Acres per 100 Ft.	Width in Feet	Acres per Mile	Acres per 100 Ft.
16.5 } 1 rod	2.00	0.038	27	3.27	0.062	40	4.85	0.092
			28	3.39	.064	41	4.97	.094
17	2.06	.039	29	3.52	.067	41.25 } 2.5 rods	5.00	.095
18	2.18	.041	30	3.64	.069			
19	2.30	.044	31	3.76	.071	42	5.09	.096
20	2.42	.046	32	3.88	.073	43	5.21	.099
21	2.55	.048	33 } 2 rods	4.00	.076	44	5.33	.101
22	2.67	.051				45	5.45	.103
23	2.79	.053	34	4.12	.078	46	5.58	.106
24	2.91	.055	35	4.24	.080	47	5.70	.108
24.75 } 1.5 rods	3.00	.057	36	4.36	.083	48	5.82	.110
			37	4.48	.085	49	5.94	.112
25	3.03	.057	38	4.61	.087	49.5 } 3 rods	6.00	.114
26	3.15	.060	39	4.73	.090			

For greater widths use sums or multiples; thus, for 87 ft. take the sum of the acres for 43 and 44 ft.

2. Cross-Sections of Railroads

For Railroad Roadbeds the specifications adopted by the American Railway Engineering Association call for flat subgrades having minimum widths of 20 ft., 16 ft., and 14 ft., Classes A, B, and C, respectively, see Art. 7. These are for depths of ballast shown in Fig. 14 for Classes B and C, Art. 9, and must be increased for the deeper ballast recommended for Class A as shown in Fig. 13. They apply to both cut and fill, but in cuts the total width of subgrade must be increased by the width of the two ditches.

Side Slopes of Excavations should ordinarily be 1.5 to 1. Firm earth will usually retain its slope permanently at 1 : 1. Loose rock will stand at 0.5 to 1, and solid rock at 0.25 to 1. Very soft earth, such as quicksand, may require a slope as flat as 4 to 1. Such earth as is proper to use for embankments will stand permanently when made with 1.5 to 1 slope. Loose-rock embankments will stand with 1 to 1 slopes.

Sodding the side slopes of embankments and excavations was formerly considered extravagant landscape gardening, but it is now realized that the protection from erosion makes the practice worth while. The shoulders and toes of embankments should be rounded off to circles of about 4-ft. radius rather than left with sharp edges, which are not easily maintained.

3. Clearing, Grubbing and Grading

Clearing is defined by the American Railway Engineering Association (Manual, 1929 Edition), as " Removing natural and artificial perishable obstructions to grading," and **Grubbing** as, " Removing the stumps and roots."

The specifications adopted by the association require the clearing of right-of-way and station grounds, except portions that may be reserved, and require that the clearing shall be kept at least 1000 ft. in advance of the grading. Grubbing is required wherever excavations occur, including borrow pits, etc., and also between slope stakes of all embankments that are to be less than 2-1/2 ft. high. It must be kept at least 300 ft. in advance of the grading.

On power shovel work it is more economical to allow the stumps to be undermined in the process of excavation, but they must not be placed in shallow fills and the specifications should require the tops of stumps under embankments to be at least 2-1/2 ft. below subgrade.

Grading includes all excavations and embankments for the formation of the roadbed, ditching, diversions of roads and streams, foundation pits and all similar works pertaining to the construction of the railway, its sidetracks and station grounds.

The specifications of the association require the *Classification* of all excavated material as " Solid Rock," " Loose Rock," " Common Excavation," and such other classes as may be established before the award of the contract.

Solid Rock is defined as rock in solid beds or masses in its original position which may best be removed by blasting, and boulders or detached rock measuring 1 cu. yd. or over.

Loose Rock comprises all detached masses of rock or stone of more than 1 cu. ft. and less than 1 cu. yd., and all other rock which can be properly removed by pick and bar and without blasting; although power shovel or blasting may be resorted to in order to facilitate the work.

Common Excavation includes all materials that do not come under the classifications " Solid Rock," " Loose Rock," or such other classes as may be established before the award of the contract.

For methods of clearing, grubbing, and grading and discussion of shrinkage and settlement, see Sect. 8

4. Fences

Wire Fences with wood, metal or concrete posts are recommended by the A.R.E.A. for right-of-way fences. The height is generally about 4 ft. 6 in., but it and other features must conform to the statutory requirements, if any. Four classes are provided, with the longitudinal wires spaced as follows:

Class A: *5 in. above ground, 4, 4-1/2, 5, 5-1/2, 6, 7, 8 and 9 in.
Class B: 7 in. above ground, 6-1/2, 7, 7-1/2, 8, 8-1/2 and 9 in.

* A barbed wire is stretched in the middle of the space to make a fence " hog-tight."

Class C: 2 in. above ground, 3, 3-1/2, 4, 4-1/2, 5, 5-1/2 for woven wire (25-1/2 in.) with 3 strands of barbed wire above spaced 4-1/2, 10 and 12 in.
Class D: 10 in. above ground, 10, 10, 12 and 12 in.

Smooth, round wire of No. 9 gage is specified for Classes A, B and the woven part of C, except that for the top and bottom strands of a Class A fence No. 7 wire is required. Ribbon, smooth, round or barbed wire may be used for Class D fences, the smooth wire being preferred, although barbed wire is still quite generally used, Vertical stay wires of No. 9 gage are specified for classes A, B and C, spaced 12 in. apart. These must be attached to the longitudinal wires with a mechanical lock or fastening which will prevent slipping either longitudinally or vertically, or they may be electrically welded. Woven wire fencing, giving the same effect or any combination of spacing, both longitidinal and vertical, or a combination of materials, is increasingly used. It is preferably fabricated in the shop if the ground is fairly level, but if desired may be woven in the field to fit rough ground better. In any case. all longitudinal wires should be provided with tension curves or " spring coiled " to take up expansion and contraction. Wires should be placed on the side of the post away from the track. Staples should be of No. 9 wire, 1 in. long for hardwood and 1-1/2 in. for soft wood. They should be set diagonally with the grain of the wood and driven home. The top wire should be double stapled.

Wire for fencing and staples should be steel galvanized with an even coating of zinc, and the fence should be so fabricated as not to remove the galvanizing or impair the tensile strength of the wire. The minimum ultimate tensile strength should be as follows: Line wire No. 7, 2200 lb.; line wire No. 9, 1500 lb.; stay wire No. 9, 1100 lb. If the fencing is electrically welded the galvanizing should be applied after fabrication. The galvanizing must stand one-minute immersion tests in a solution of commercial sulphate of copper crystals and water, the specific gravity of which is 1.185 and temperature between 60° and 70° F. The sample should be washed with water and wiped dry after each immersion, and if the zinc is removed or a copper-colored deposit formed after the fourth immersion, the lot from which the sample was taken is rejected.

Wooden Posts should be straight and free from splits, rot or other defects. They may be made of cedar, locust, chestnut, Bois d'Arc, white oak, mulberry, catalpa, or other durable wood native to the locality, or of treated timber. The dimensions of sawed or split posts should at least equal those given below for round ones. End, corner, anchor and gate posts should be at least 8 ft. long, 8 in. in diameter at small end, and set 3 ft. 4 in. in the ground; intermediate posts should be at least 7 ft. long, not less than 4 in. in diameter at small end, and set 2 ft. 4 in. in the ground. The posts should be set with the large end down. The spacing of posts should be not more than 20 ft. and the first line post from any corner, anchor or gate post should be only 10 ft. from it, center to center. Holes should be dug to full depth, even if blasting is necessary. To avoid blasting holes in solid rock, intermediate posts may be set on 6 by 6-in. sills, 4 ft. long, braced on both sides by 2 by 6-in. braces, 3 ft. long. Not more than two such posts should be placed consecutively. After the fence is erected, the tops of the posts should be sawed off on a one-fourth pitch, the high side being that on which the wire is fastened and 2 in. above it. Intermediate posts in hollows or sags should be anchored down by gaining and spiking two cleats near the bottom. End and corner posts should be anchored by cleats set one near the bottom and the other just below the ground surface, and also braced by braces to adjacent posts. The cleats should be made of 2 by 6-in. common lumber, 3 ft. long. The braces should be 4 by 4-in. common lumber. Wire braces made of a double strand of No. 9 wire should be used where bracing by tension is needed.

Concrete Posts are recommended as practical, economical, and a suitable substitute for wood. The cross-section is preferably a square or a circle, the latter having the most extended use, whereas square or nearly square sections are slightly more efficient in resisting the forces that ordinarily cause failure. This may be offset by greater resistance to deterioration and better methods of manufacture. Corners should be rounded off to a radius of not less than 5/8 in., and posts should taper from base to top. End, corner, anchor and gate posts should be at least 8 ft. long, 8 in. at base and 6 in. at top, set 3 ft. 4 in. in the ground and reinforced with four 3/8-in. rods. Intermediate posts should be at least 7 ft. long, 5 in. at base and 3-1/2 in. at top for round posts, and 4-1/2 in. at base and 3 in. at top for square ones.

Reinforcement must be held rigidly 5/8 in. from surface by at least four spacers per post, and consists of a No. 7 wire or 3/16-in. bar in each corner of the square posts and of six No. 8 wires or 3/16-in. bars, evenly spaced, for the round ones. Bars are preferably deformed, and all reinforcement should be of hard steel. Before being placed in the mold, it should be thoroughly cleaned of mill and rust scale, and of coatings that will destroy or reduce the bond. The proportions of cement, water and aggregate should be such as to produce a concrete having a compressive strength at 28 days of 2500 lb. per sq. in.; using a screen analysis as a guide in the proportioning of fine and coarse aggregates and the slump test to govern the consistency. The maximum slump should not exceed 5 in. The fine aggregate should consist of clean, hard sand, or stone screenings, or a mixture of the two, of which not less than 90% shall pass a No. 4 sieve nor more than 30% a No. 50 sieve, and not more than 3% by weight shall be removed by decantation. Sand should be tested for organic impurities and not used if it fails to meet the standard unless actual tests show that the concrete made with it is of the desired strength. The coarse aggregate should consist of crushed stone, or gravel, or a mixture of the two, ranging in size from fine to coarse within the following limits:

For line posts:

Passing 1/2-in. screen not less than 95%.
Passing No. 4 screen not more than 15%.
Passing No. 8 screen not more than 5%.

For corner and end posts:

Passing 3/4-in screen not less than 95%.
Passing No. 4 screen not more than 15%.
Passing No. 8 screen not more than 5%.

Clean, hard bank gravel, a natural mixture of fine and coarse aggregate, may be used if of suitable screen analysis, and if it meets the test for organic matter; or it may be modified by the addition of fine or coarse aggregate to produce the desired result. Concrete should be mixed in a batch mixer of approved type for at least 2 minutes, and no retempering of concrete should be permitted. Metal molds are preferred as more substantial and rigid than wooden ones, and all molds should be thoroughly cleaned and coated with a non-staining mineral oil or approved substitute just before being filled. Concrete should be thoroughly compacted into the molds and around the reinforcing, preferably by jogging or vibrating the molds during filling. Posts should remain in the molds at least 24 hours after being poured and should be carefully handled and protected from shock until thoroughly cured. They are preferably stacked in a vertical position where they are protected from direct sunlight. They should be kept thoroughly wet for 8 to 10 days and cured for 90 days before being shipped or set if cured naturally. Posts should not be cured out of doors during freezing weather.

Fencing may be attached to the posts by means of Western Union ties, molded holes through post, loops of galvanized wire projecting from the post, or other suitable device.

Steel Posts are increasingly used, the higher first cost being more than offset by the reduced labor costs since they may be driven, either by hand or the one-man post driver. Special types of fences from picket to solid board, sometimes surmounted by one or two strands of barbed wire, in wooden construction and from ornamental iron to woven wire on steel posts, also with barbed wire at the top, in metal construction, are used around yards and shops.

Farm Gates should be of all-metal construction and from 12 to 16 ft. wide, depending on the width of agricultural implements used in the vicinity or on legal requirements. The height should be at least 4 ft. 6 in. They should swing away from the track and, if hinged as is desirable, should shut by gravity and overlap the post so as not to swing toward track.

The First Cost of fencing depends on prices of material and labor and efficiency of the latter and should be estimated in each case from quotations on material and estimated output of labor. The **Annual Cost** involves also the estimated life and interest rate and should be used in making comparisons. Data collected in 1915 by a Committee of the A. R. E. A. showed the average life of wood posts to be as follows: Bois d'Arc, 26 years; catalpa, 12; cedar, 15; chestnut, 11; cypress, 11; locust, 17; oak, 9; and pine, 8. Pine posts treated with cresote are now widely used, having a life of 15 to 20 years and presenting a better appearance than many of the other kinds of wood. They are also much less subject to damage by fire.

A rough comparison of costs may be illustrated as follows, the quotations on materials and estimates of output of labor should be used for actual comparisons:

Material	Life, years	Cost, cents	Labor, cents	Total, cents	Annual costs		
					Interest	Amortization	Total
Oak	9	35	75	110	6.6	4.0	10.6
Pine (treated).	15	40	75	115	6.9	2.7	9.6
Steel.........	25	60	25	85	5.1	2.4	7.5
Concrete.....	50	55	85	140	8.4	1.3	9.7

Materials Required for Board Fences

Per Mile of Fence One Board High

Spacing of Posts	Number of Posts	Nails in pounds		Lumber in feet B. M.				
		8d	10d	1×4 in.	1×6 in.	1×8 in.	1×10 in.	1×12 in.
8 ft.	660	20.7	31.5	1760	2640	3520	4400	5280
10 ft.	528	22.0	33.5	1760	2640	3520	4400	5280
12 ft.	440	18.3	28.0	1760	2640	3520	4400	5280
14 ft.	378	15.7	24.0	1760	2640	3520	4400	5280
16 ft.	330	13.7	21.0	1760	2640	3530	4400	5280

The quantity of nails is figured on the basis of two nails to a board to each post and an allowance of 5% is made for loss. Where posts are 8 ft. apart it figured that 16-ft. boards will be used.

For Wire Fences the 16-1/2-ft. spacing, if used, will require 320 posts per mile. Barbed wire varies in weight with the type, but may be estimated roughly at 1 lb. per rod, or 320 lb. per mile, whereas No. 7 and No. 9 wire weigh 439 and 306 lb. per mile, respectively.

Materials, Except Posts, Required for Wire Fences. A.R.E.A. Standards

Per Mile of Fence

Class	Wire					Staples		Locks
	Longitudinals			Verticals No. 9	Total	1-in.	1-1/4-in.	
	No. 7	No. 9	Barbed					
	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	No.
A	878	2142	....	1300	4320	31.1	46.4	47 500
B	....	2142	....	1236	3378	24.9	37.1	37 000
C	....	2142	960	700	2842	34.4	30.8	37 000
D	....	....	1600	....	1600	18.0	26.8	....

One inch extra was allowed at each end of vertical or stay wires and 5% was allowed for loss of staples. Nothing was allowed for loss of locks or ties, as, if fabricated in the field, some of the stay wires would be omitted at posts.

Staples are put up in 100-lb. kegs, the number per keg if of No. 9 wire being approximately as follows: 1-in., 10 800; 1-1/4-in., 8700; 1-1/2-in., 7200 and 2-in., 5800. For posts 20 ft. apart use 83% of weights given in table for staples. Locks are of various types, the weight may be estimated at 25 lb. per 1000.

Prices of both materials and labor are high (1929) and subject to wide variations in different sections of the country. Barbed wire costs from 5 to 7 cents per pound and smooth from 8 to 10. Staples cost about the same per pound as the wire and locks 50 to 75% more.

The Output of Labor is affected by topography of site, presence of trees or brush, and kind of soil as well as experience and efficiency of workmen. Assuming that one man will dig the holes and carry and set 35 posts per day, Camp estimates the labor per mile for various types of fences as follows:

Four-board fence, 16-ft. boards, posts 8 ft. apart, without battens......	29-1/2 days
Same, with battens..	36 days
Barbed wire fence, 4 strands, posts 16 ft. apart......................	13 days
Same, with posts 12 ft. apart..	16 days
Same, with top board and three wires, posts 12 ft. apart...............	17 days
Same, with top board and four wires, posts 12 ft. apart................	18 days

He allows about one day's labor, more or less, for each strand. Other data indicate that inexperienced men, particularly on short jobs, will take from 25 to 50% more time than stated above, and cost of foreman, whose time may be estimated at one-sixth that of men, and of clearing brush and distributing material must also be considered. Holes may be dug with an auger or digger at about half the cost with bar and shovel or 100 holes may be dug by one man in a day instead of about fifty.

Due to the shorter working day and the decreased efficiency of labor it is not likely that the outputs estimated by Camp could be duplicated at the present time; and the following estimates of labor in man-days per mile for the standard fences of the A.R.E.A. are based on more recent data. Values are based on a fence with posts 20 ft. apart.

Labor in Man-Days per Mile of Fence

Class	Wooden posts		Concrete posts		Steel posts Driven
	Auger	Bar and shovel	Auger	Bar and shovel	
A	42	48	44	50	31
B	35	41	37	43	28
C	40	46	42	48	30
D	26	28	28	34	21

Snow Fences. A snow fence is a structure erected for the purpose of accumulating drifting snow. Snow is carried by the wind close to the surface of the ground and is deposited in railway cuts on account of the eddies and diminution of wind velocities caused by these cuts. A snow fence is so placed that artificial eddies will form on the windward side of a cut at a sufficient distance to cause the snow to be deposited between the snow fence and the cut. A tight fence of sufficient height will cause snow to accumulate on its windward side. An open fence causes snow to accumulate principally on the leeward side. Frequently the best type of fence and its proper location may be determined only by experiment, therefore portable snow fences are used. When practicable a permanent snow fence located on the right-of-way line is the most economical. Where this line is 50 ft. or less from the center of the track, the boards should be close. For greater distances space may be left between the boards. When the distance is 100 ft. 50% of the fence should be open space. The height of the fence should not exceed 10 ft. Hedges of California privet, Armour Barberry, evergreens, spruce, and locust have been planted in recent years for this purpose. Their economical efficiency is still doubtful. Stone walls have been used effectually where field stone are plentiful.

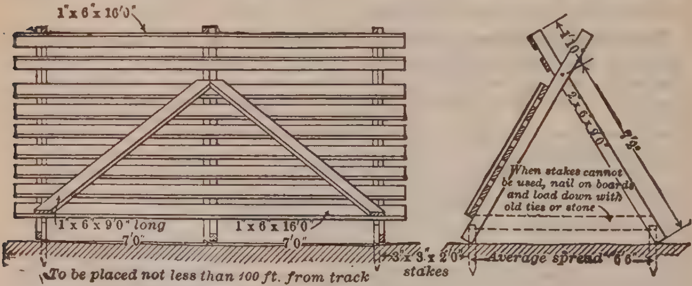


Fig. 1. Portable Snow Fence

A specification, quoted from Orrock's "Railroad Structures and Estimates," calls for cedar posts 8 in. in diameter, 12 ft. long set 3 ft. 6 in. in the ground and spaced 8 ft. on centers. The boards are 7/8 in. × 6 in. × 16 ft., and are nailed on with 6-in. clear spaces. This requires eight boards, four breaking joints at each post. A **portable fence** is made in panels 16 ft. long. Six boards 1 in. × 6 in. × 16 ft., spaced 6 in. clear, are fastened onto three pieces 2 in. × 6 in. × 9 ft., spaced 7 ft. apart. Two pieces 1 in. × 6 in. × 10 ft. act as stiffeners. Three other pieces, also 2 in × 6 in. × 9 ft., are bolted, using 1/2 in. × 4-3/4 in. carriage bolts, with plate washers, scissor-wise to the other three similar pieces. Two more boards are fastened to the tops of the second set of pieces. These sections may be folded up flat for storage. When in use, the scissors are opened until the lower ends are spread about 6 ft. 6 in. apart, and are fastened to stakes, 3 in. × 3 in. × 2 ft., driven into the ground. On hard, rocky ground a tie-board, 2 in. × 6 in × 7 ft., may be nailed to the lower ends of the scissors and weighted down with old ties, stones, etc. This construction is estimated to cost $22 per panel of 16 ft.

Another type of portable snow fence is made by weaving wooden laths 4 to 6 ft. long with five to seven double strands of smooth galvanized wire, making a sort of picket fence. By varying the widths of the lath and the number of turns of wire between them considerable difference in the percentage of area shut off may be obtained. This fence is made in rolls up to one

Fig. 2. Slide Fall, Northern Pacific Ry.

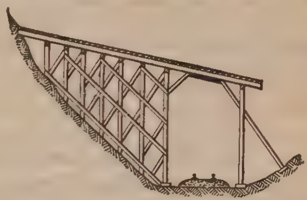


Fig. 3. Slide Fall, Northern Pacific Ry.

hundred feet long and is set up by fastening it at intervals to posts set or driven into the ground. By keeping the laths well painted and using wire heavily galvanized the fence may be made quite durable; and it is easier to handle than the other type.

Snowsheds are expensive to build and to maintain. Their probable necessity and the possibility of avoiding them should have its influence in deciding on the location of the road. There are two types of construction: (1) those to protect the track against slides, the track being on a side hill or at the base of a hill, and (2) those to prevent the filling of cuts or to protect the track from an excessive level fall. Although the detail varies somewhat with nearly every location, typical designs for timber sheds are shown in Figs. 2 to 5.

Sheds cut off the view considerably and are disagreeable to operate; hence a "summer track" is sometimes laid outside. To partially open up level fall sheds and to localize fires, movable sections 100 ft. long are provided which

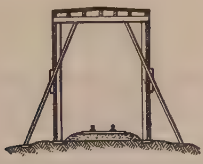


Fig. 4. Level Fall, Southern Pacific Ry.

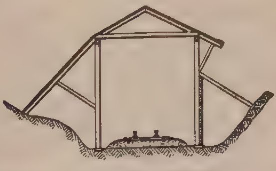


Fig. 5. Lower Fall, Canadian Pacific Ry.

are mounted on wheels and telescope into enlarged adjacent sections. The cost of maintaining the extremely heavy avalanche sheds, particularly those for double track, and the great fire risk have led some roads to build combination concrete and timber sheds and reinforced-concrete sheds. See Eng. News, Dec. 15, 1910.

Highway Grade Crossings. Three-inch planks, spiked to blocks on the ties on either side of each rail, the outer planks tight against the heads of the rails and the inner allowing a flangeway of 2-1/4* in., are commonly used for unimportant country roads and farm crossings. The space between the inside planks should be filled in flush with ballast. An old rail laid on its side with its head against the web of the track rail, or standing with separators between it and the track rail, is often used for the flangeway. The ends of such rails should be bent inward to give an opening of at least 4 in. On more important crossings the planking should entirely fill the space between the rails and the ends of the planking should be beveled both between and outside the rails to a thickness of 1 in., commencing at a point 10 in. from the end. On improved roads and paved streets the macadam or pavement is continued between the rails leaving flangeways as above. The width of the crossing should be not less than 18 ft. The width of a farm or private road crossing should not be less than 12 ft.

For specifications for bituminous grade crossings see Manual of American Railway Engineering Association, 1929 Edition.

5. Drainage and Waterway for Culverts

Longitudinal Drains are of three kinds: (a) intercepting ditches which protect the slopes of cuts, and intercepting ditches or tile drains which similarly protect embankments on saturated soil; (b) side ditches in cuts; (c) subsurface drains, which are usually made of 6-in. tile pipes and are located about 2 ft. underneath those of class (b). The **slope** must ordinarily conform to the natural slope of the ground for class (a) and of the roadbed for class (b) Since tile pipes will carry water on a flatter slope than mere earth ditches, drains of

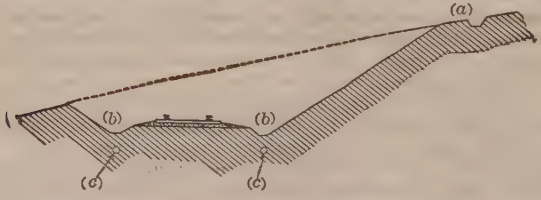


Fig. 6. Longitudinal Drains

type (c) are particularly necessary when the grade of the road is level or very flat, and especially if the soil is retentive and impermeable. Care should be taken that all such drains have a free and unobstructed outfall.

Cross Drains are used only when it is necessary, on account of the lack of a proper outfall, to conduct the water through the roadbed. When lumber is cheap, they may be made of boards so as to give an interior cross-section of 6 by 8 in. 6-in. cast-iron pipes are far better.

The Area of Waterway, or size of opening, of a culvert should be sufficient to care for the flow resulting from all ordinary floods but, unless the probability of loss of life or damages from interruption of traffic are very great, it is not considered necessary to provide for those phenomenal floods which occur at

* Increase to 2-1/2 in. on curves of 8 deg. and over.

very long intervals, because it is more economical to repair such damages once in fifty to seventy-five years than to tie up additional capital in construction for such long periods. It should be noted, however, that cost does not increase in proportion to capacity, this fact together with the impossibility of accurately determining required sizes and the advantages of standards in estimating and construction, have led most roads to adopt a few standard sizes for small openings rather than try to fit each case accurately.

The Maximum Rate of Flow or Runoff at a given point depends upon the following factors:

(a) Rate of rainfall, not necessarily the maximum, which is likely to be of such short duration as not to give time for the water to reach the culvert from any considerable portion of the area unless the watershed is very small.

(b) Area of the watershed.

(c) Permeability of surface and vegetation. Saturation by long continued rains makes most soils more or less impervious, also freezing. Vegetation retards flow.

(d) Shape and slope of the watershed, which affect the time required for the water to reach the culvert from different points. See (a).

The Form of Entrance Slope and Smoothness of a culvert affect somewhat its carrying capacity, although these factors are not usually considered in determining sizes but are simply made as favorable as local conditions and type of structure permit.

A Theoretical Determination of the proper area of waterway being impossible on account of the unknown influence of the factors affecting the runoff, recourse is had to observation and the use of empirical formulas. Consensus of opinion seems to favor "careful field observation and the use of intelligent judgment," particularly in settled country where other culverts and bridges exist over the same stream, thus giving opportunity for a study of their sufficiency at times of high water. In new country, high water marks, preferably at points where the stream has a well-defined channel, will furnish valuable evidence. Where there are no such guides the safest plan is to build a temporary trestle, which will not only provide an ample waterway for all floods during its life, but which will permit the construction of a culvert of proper dimensions under the trestle. The life of such a trestle should be sufficient to determine the area with all-sufficient accuracy.

Empirical Formulas are useful as a guide where only general information is available, but their use in other cases is warranted only to the extent that the formulas and the constants used therein are known, from long usage and observation of results, to fit local conditions. All of the above conditions should be considered in estimating the value of the constant to be used in a particular case in applying any empirical formula, of which some are quoted below. Let $\mathbf{a}$ = area of waterway in square feet and $\mathbf{A}$ = drainage area in acres.

E. T. D. Myer's formula, $\mathbf{a} = \mathbf{C}\sqrt{\mathbf{A}}$, in which $\mathbf{C}$ is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground.

A. N. Talbot's formula, $\mathbf{a} = \mathbf{C}\sqrt[4]{\mathbf{A}^3}$. "For steep and rocky ground C varies from 2/3 to 1. For rolling agricultural country, subject to floods at times of melting snow, and with the length of the valley three or four times its width, C is about 1/3; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, 1/5 or 1/6 or even less may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert."

J. T. Fanning's formula, $\mathbf{a} = \mathbf{0.23}\sqrt[6]{\mathbf{A}^5}$, in which no allowance is made for variations in conditions which affect the flow.

R. McMath's formula, $\mathbf{a} = 0.5908 \sqrt[5]{A^4}$, which was designed primarily for application to sewers. Like Fanning's, it makes no allowance for variations in conditions.

C. B. & Q. formula, $\mathbf{a} = 0.46875\ A/(3 + 0.079 \sqrt{A})$.

Fanning's formula and the C. B. & Q. formula agree very closely, and also agree with Talbot's for a mean value of C. But Talbot's has the advantage

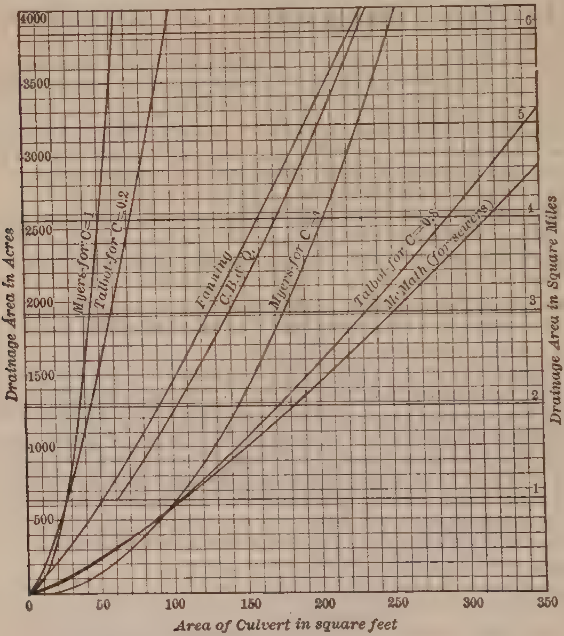


Fig. 7. Required Culvert Areas by Different Formulas

Area of Full Semicircular Arches with Vertical Abutment Walls

Height of abutment walls, ft.	Span of arch, ft.											
	5	6	7	8	9	10	11	12	13	14	15	16
1	14.8	20.1	26.2	33.1	40.8	49.3	58.5	68.5	.....	.....	.....	.....
2	19.8	26.1	33.2	41.1	49.8	59.3	69.5	80.5	92.4	.....	.....	.....
3	24.8	32.1	40.2	49.1	58.8	69.3	80.5	92.5	105.4	119.0	.....	.....
4	29.8	38.1	47.2	57.1	67.8	79.3	91.5	104.5	118.4	133.0	148.4	.....
5	34.8	44.1	54.2	65.1	76.8	89.3	102.5	116.5	131.4	147.0	163.4	180.5
6	....	50.1	61.2	73.1	85.8	99.3	113.5	128.5	144.4	161.0	178.4	196.5
7	....	....	68.2	81.1	94.8	109.3	124.5	140.5	157.4	175.0	193.4	212.5
8	....	....	....	89.1	103.8	119.3	135.5	152.5	170.4	189.0	208.4	228.5
9	....	....	....	....	112.8	129.3	146.5	164.5	183.4	203.0	223.4	244.5
10	....	....	....	....	.....	139.3	157.5	176.5	196.4	217.0	238.4	260.5

of flexibility. Curves for these formulas are plotted in Fig. 7, from which values of a may be readily found.

As a check on the waterway area found by the above methods, the rules for the proper size of a sewer to carry the storm flow of a watershed may be used. See Sect. 17, Art. 2.

The table on p. 2010 will be useful in selecting an arch culvert of the desired area.

Pipes of standard sizes are extensively used for small openings, and the following table, based on Talbot's formula, probably more used than any other given above, will be found convenient. For example, if the drainage area is 14 acres, a 16-in. pipe culvert might answer under the most favorable conditions, but the size should be increased, up to 36 in., according to the coefficient deemed proper for the locality.

Drainage Area in Acres Drained by Standard Culvert Pipes, for Various Coefficients in Talbot's Formula

Formula: Drainage area = $(\text{culvert area})^{4/3}/C^{4/3}$

Diameter of pipe, in.	Area, sq. ft.	Talbot's formula: C =					
		0.2	0.3	0.4	0.5	0.75	1.0
8	0.35	2	1	*	*	*	*
10	0.55	4	2	2	1	*	*
12	0.79	6	4	3	2	1	*
16	1.40	13	8	5	4	2	2
18	1.77	18	11	7	5	3	2
20	2.18	24	14	10	7	4	3
24	3.14	39	23	16	12	7	5
30	4.91	71	42	28	21	12	8
36	7.07	116	57	46	34	20	14
48	12.57	250	146	99	74	43	29

* Indicates less than one acre.

Circular Pipes for Culvert Openings

Diameter......	12 in.	16 in.	18 in.	20 in.	24 in.	30 in.	3 ft.	4 ft.	5 ft.	6 ft.
Area, sq. ft....	0.79	1.40	1.77	2.18	3.14	4.9	7.1	12.6	19.6	28.3
Weight of 12-ft. sections of cast-iron pipe.....	899	1322	1510	1798	2458	3325	4862	8038	12 500	18 500
Thickness of cast-iron pipe, inches........	9/16	5/8	5/8	11/16	3/4	13/16	1-1/16	1-1/4	1-5/8	2

Cast-iron pipe of other thicknesses and weights can also be had in the market. See Sect. 14, Art. 20, p. 1499. Sewer, corrugated iron and concrete, both plain and reinforced, pipes are also used.

6. Designs for Culverts

The Length of the barrel of a culvert between the inside faces of the head walls is **L = 2 sd + b**, in which **s** = slope ratio of the embankment (horizontal to vertical), **d** = depth to top of culvert, **b** = width of roadbed. (Fig. 8.)

End or Wing Walls. These act as retaining walls for the lower part of the embankment and are of three types (see Fig. 9):

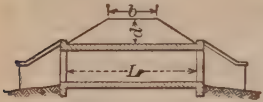


Fig. 8

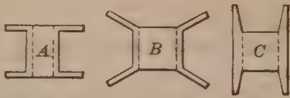


Fig. 9. Types of End Walls

Type A is the simplest form, but the volume of masonry is usually greater than that required by the other types except in narrow gorges. It is generally used for pipe and other low and small culverts.

Type B requires less masonry and is more commonly used for large culverts.

Type C is liable to the objection that the water may work its way behind the wing walls, scour them out and endanger the embankment and culvert. When this can be unquestionably prevented, the type has the advantage of simplicity and economy of construction.

The Length of an end wall of type A should be a little more than sufficient to keep the earth of the embankment spilling around its ends from reaching the opening or more than twice the slope ratio times the height under coping plus the width of opening minus twice the thickness of the wall. Type B is tapered down to a height of from 2 to 4 ft. and type C to practically nothing. The types are often combined for culverts on a skew in order better to guide the stream into the culvert.

Standard Designs. Many railroads have prepared sets of standard plans for culverts and other structures, track work, signs, etc. These usually give quantities and are convenient for both estimating and construction as the men become accustomed to their use. Some examples are given below.

Pipe Culverts should be well bedded in firm earth or have plank, concrete or even piling foundation in soft soils. If slope is light they should be laid with a small camber, and ends should usually be protected with end walls. Sewer pipe should have a cover of at least 3 ft. of earth and no stones should be allowed in contact with it. Joints should be at least partially filled to smooth surface by centering spigots in hubs and prevent scour.

Stone Box Culverts. The thickness of the roof slabs for stone culverts usually varies from 10 in. for a 2-ft. span to 15 in. for a 4-ft. span. Fig. 10

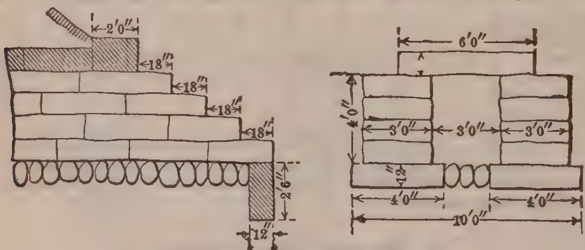


Fig. 10. Typical Stone Box Culvert

represents what may be considered a typical design. Such a culvert requires 7.0 cu. yd. of masonry for each of the end walls and 1.41 cu. yd. of masonry for each foot of length between the inside faces of the coping stones. This estimate does not include the volume of the paving. Dry masonry culverts should not be used on soft soil due to the danger of being undermined; for these conditions concrete is practically always used at the present time.

Wooden Box Culverts similar in design to the stone box, are used to save time and expense during construction, where timber is cheap. They are made large enough to allow a pipe culvert to be put through them before the timber gives out. The sides are of 12 by 12-in. timbers, laid up solid, and the top varies in thickness from 8 in. for 3-ft. span to 12 in. for 5 ft. The bottom should be planked and the ends protected from undercutting.

Masonry Arch Culverts. These usually are built as full semicircular arches unless the span is very great and the rise would be objectionably high. Designs are various, with resulting variations in quantities. Those for stone masonry and plain concrete are similar; Fig. 11 gives data for Erie Railroad standards for concrete. The subscripts to the dimensions (A_1, A_2, etc.) indicate that all of the dimensions are of the same general character. The plans illustrate a culvert of type B for the upstream end and of type C for the downstream end. If there are two or more tracks, let **Z** = distance between track centers of the outer tracks and other notation as in Fig. 11; then

For single track $L = 2\,(s_1y + T + T_1 + T_2)$.
For two or more tracks $L = Z + 2\,(s_1y + T + T_1 + T_2)$.

The neat line is supposed to be **U** inches below the surface of the ground. Carry foundations as deep as conditions may require. Minimum depth **U** inches except on rock bottom.

The volume of one of these culverts may be easily computed by the use of the quantities found in the accompanying table. These tables can also be used to compute the volumes of stone culverts if the quality of masonry is fairly good and joints thin. For rough rubble the thickness of walls and arch should be increased.

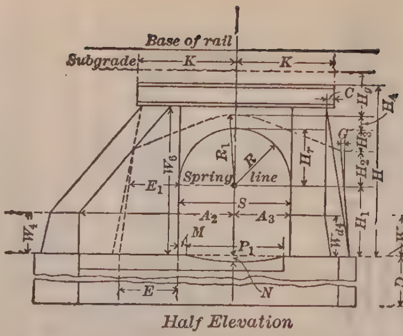
Volume of Standard Plain Concrete Arch Culverts, Erie Railroad

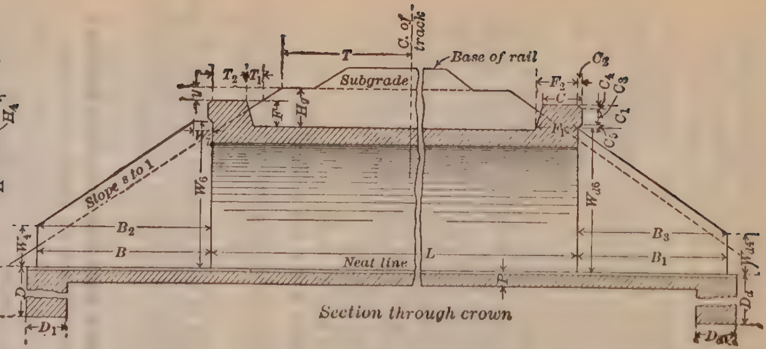
Condensed from several tables in Bulletin 105, Am. Ry. Eng. & M. W. Assoc.

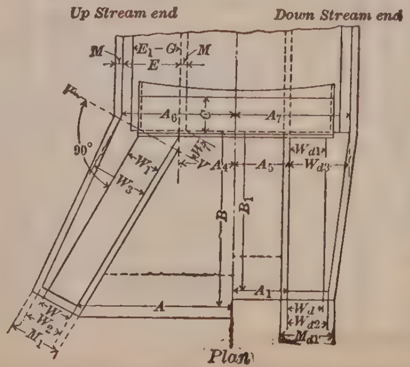
Span	Per lin. ft.		Paving between wing walls	Curtain walls, 1 ft. deep	Two portals, wing walls and parapet
	Barrel	Paving under barrel			
3	0.873	0.055	0.60	0.30	11.0
4	1.034	0.083	1.10	0.44	13.2
5	1.117	0.111	1.31	0.59	13.8
6	1.979	0.185	7.50	1.48	28.7
8	2.784	0.260	12.7	2.0	48.2
10	3.703	0.333	20.38	2.63	74.5
12	4.792	0.408	29.23	3.04	105.2
14	5.998	0.482	54.68	5.48	158.3
16	6.703	0.556	65.11	6.00	186.7
18	7.598	0.630	76.29	6.7	212.0
20	8.087	0.704	88.86	7.3	242.6

For example, if a fill for a single-rack road is 30 ft. deep and an 8-ft. culvert is to be used, since H = 12 ft. 1 in., v = 17 ft. 11 in. = 17.92,

L = 2 (1.5 × 17 ft. 11 in. + 9 ft. 6 in. + 10-1/2 in. + 2 ft. 6 in.) = 79 ft. 6 in.
Vol. = 79.5 (2.784 + 0.260) + 12.7 + 2.0 + 48.2 = 304.9 cu. yd.







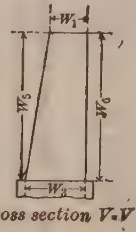


Fig. 11. Standard Plain Concrete Arch Culverts, Erie Railroad
From Bull. 105, Am. Ry. Eng. & M. W. Assoc.

In these plans A_4 and A_5 always equal the half-span, C is constant at 2 ft. 10 in., C_1 = 1 ft. 6 in., C_2 = 0 ft. 4 in., C_3 = 1 ft. 2 in., C_4 = C_5 = 0 ft. 2 in., D and Dd are variable, D_1 = Dd_1 = 2 ft. 0 in., F = 2 ft. 0 in., H_g = 3 ft. 0 in., M = 0 ft. 6 in., P_1 is one foot less than the span, S is the slope ratio 1.5 : 1, T = 9 ft. 6 in., T_1 = 0 ft. 9 in. for 3, 4 and 5 ft. spans and 10-1/2 in. for longer spans when wings are at 30° with the axis, U = 1 ft. 0 in. Wd and Wd_1 are the same as W, also W_6 and Wd_6 are the same as W_5.

Dimensions of Standard Plain Concrete Arch Culverts, Erie Railroad

Condensed from Bulletin 105 (1908) of Am. Ry. Eng. & M. W. Assoc.

Dimensions	Span in feet										
	3	4	5	6	8	10	12	14	16	18	20
	ft. in.	ft. in.	ft in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
A A_2	1 6	2 0	2 6	8 6	11 3	14 1	16 10	19 9	21 8	23 7	25 6
A_1 A_3	1 6	2 0	2 6	3 0	4 0	5 0	6 0	19 9	21 8	23 7	25 6
A_6 A_7	4 0	4 9	5 6	6 10	8 3	9 8	11 2	12 7	13 11	15 4	16 8
B B_2	4 11	5 6	5 5	9 6	12 7	15 9	18 10	22 1	23 9	25 3	26 11
B_1 B_3	4 11	5 6	5 5	8 3	11 4	14 6	17 7	22 1	23 9	25 3	26 11
E	2 6	2 9	3 0	3 10	4 3	4 8	5 2	5 7	5 11	6 4	6 8
E_1	2 2	2 3	2 6	3 6	3 10	4 2	4 7	4 11	5 3	5 8	6 0
F_1	2 0	2 0	2 0	← 2 ft. 10-1/2 in. →							
F_2	2 0	2 0	2 0	← 3 ft. 0 in. →							
G	...	...	...	0 2	0 3	0 3	0 4	0 4	0 5	0 5	0 6
H	7 5	7 10	7 9	10 0	12 1	14 2	16 3	18 5	19 6	20 6	21 7
H_1	4 0	4 0	3 6	4 0	5 0	6 0	7 0	← 8 ft. 0 in. →			
H_2	...	...	...	2 0	2 6	3 1	3 8	4 3	4 9	5 3	5 10
H_3	0 8	1 0	1 4	1 0	1 6	1 11	2 4	2 9	3 3	3 9	4 2
H_4	0 9	0 10	0 11	1 0	1 1	1 2	1 3	1 5	1 6	1 6	1 7
H_g	0 8	1 0	1 4	3 0	4 0	5 0	6 0	7 0	8 0	9 0	10 0
K	4 0	4 9	5 6	5 10-1/2	6 10-1/2	7 10-1/2	8 10-1/2	9 10-1/2	10 10-1/2	11 10-1/2	12 10-1/2
M_1	3 6	3 9	4 0	3 10	4 0	4 2	4 3	4 3	4 4	4 4	4 4
N	...	...	...	← 0 ft. 6 in. →							
P	0 9	0 9	0 9	← 1 ft. 0 in. →							
R	2 0	2 6	3 0	3 0	4 0	5 0	6 0	7 0	8 0	9 0	10 0
R_1	2 9	3 4	3 11	4 0	5 1	6 2	7 3	8 5	9 6	10 6	11 7
T_2	2 0	2 0	2 0	← 2 ft. 6 in. →							
W W_1	2 0	2 0	2 0	2 6	2 6	2 6	2 10	2 10	2 10	2 10	2 10
W_2 W_{d2}	2 6	2 9	3 0	2 10	3 0	3 2	3 3	3 3	3 4	3 4	3 4
W_3 W_{d3}	2 6	2 9	3 0	3 4	4 3	5 2	6 0	7 0	7 5	7 10	8 4
W_4	6 1	6 6	6 5	← 3 ft. 0 in. →							
W_{d4}	2 10	2 10	2 10	← 3 ft. 0 in. →							
W_5 W_{d5}	6 1	6 6	6 5	8 6	10 7	12 8	14 9	16 11	18 0	19 0	20 1
W_7	...	...	...	← 1 ft. 5-1/2 in. →							
W_{c7}	...	...	...	...	...	...	← 1 ft. 5-1/2 in. →				
Area	13.3	18.7	22.1	39.8	67.5	102.3	144.2	193.3	233.5	276.9	323.4

The last line gives area of waterway in square feet.

Reinforced-Concrete Culverts. The ability to resist stress, which permits the use of flat slabs, both in top and bottom, makes this construction of especial value. In general, any soil which is firm enough to support an embankment will support a reinforced-concrete box culvert, with a slab bottom, with almost any span, without resorting to piling. Spread footings, unless very narrow, are less economical and less effective than the slab bottom. The design of culverts for railroad work permits the adoption of standard designs,

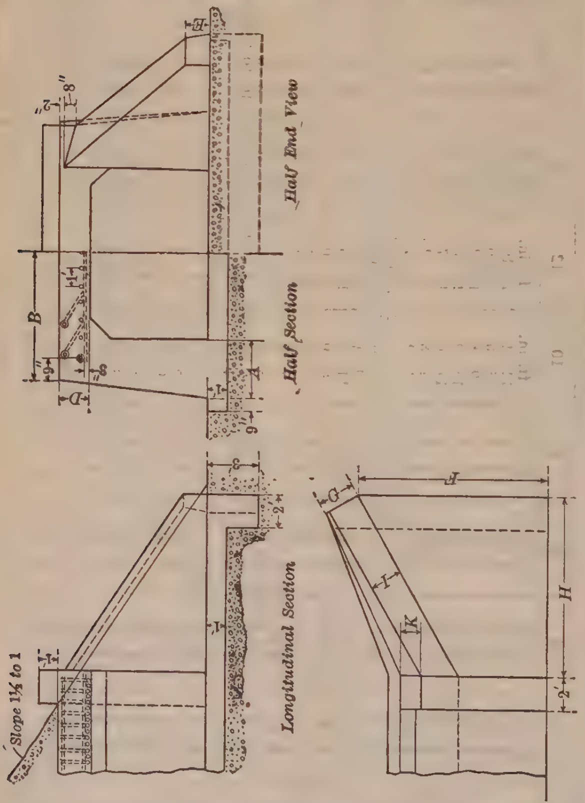


Fig. 12. Standard Reinforced Concrete Culvert

using standard forms which may be removed and used several times with great economy. The forms may be standardized regardless of the use of a slab bottom, which in any case must be laid first. Excavation in the gully or stream bed will frequently expose a rocky bottom into which trenches may be dug which will not only give a sufficient footing but will also furnish sufficient resistance against any lateral thrust of the side walls. In each case it becomes a question of judgment whether to use a slab bottom or perhaps to dig a little deeper and make the side walls a little higher in order to set the side walls in firm soil. In either case the upper corners should be filled out and suitably reinforced in order to resist any unbalanced thrust perpendicular to the barrel of the culvert. When a bottom slab is used the thrust at the bottom of the side walls may be taken up by shallow trenches in the top of the slab, immediately under the side walls, but it is cheaper and sufficiently effective to place greased plugs in the slab at the proper places for the vertical bars of the side walls. These plugs are afterward removed and the bars grouted in. Short pieces of scrap reinforcement may also be used as dowels by inserting them for half their length in the slab while still soft. Typical plans and data for them are here given. Some roads use longer spans, but reinforced-concrete girders are commonly used for spans of 15 to 30 ft. and arches for greater spans.

The longitudinal reinforcement is of 1/2-in. bars, the lower spaced 12 in. on centers for all spans, the upper used on 8- and 10-ft. spans only, and spaced 10 and 12 in., respectively. The transverse reinforcement varies with span from 9/16 to 19/16 in. and is spaced 3 in. on centers for the center section and 8 in. for the end sections. One-third the bars are straight, one-third bent up near the side walls, and one-third nearer the center as shown on the half section. Other designs carry reinforcing in the side walls and bottom as shown in Sect. 11, which see also for retaining walls and other structures.

Data for Standard Reinforced-Concrete Culverts, Nashville, Chattanooga and St. Louis Railroad

Condensed from Vol. 10, Proc. Amer. Ry. Eng. & M. W. Assoc. (1909)

Letter	Span by height						
	4×3	4×5	6×4	6×6	8×6	8×8	10×10
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
A	1 9	1 9	2 6	2 6	3 0	3 0	3 6
B	3 9	3 9	5 0	5 0	6 0	6 0	7 6
D	0 10	0 10	0 11	0 11	1 2	1 2	1 4
E	1 0	1 0	1 6	1 6	1 6	1 6	1 6
F	2 0	2 0	5 9	7 6	8 8	10 4	13 3
G	1 9	1 9	2 3-3/4	2 2-1/2	2 3	2 2-1/4	2 1-3/4
H	5 0	9 0	4 10	7 10	8 3	11 3	14 6
I	1 9	1 9	2 0	2 0	2 0	2 0	2 0
K	0 0	0 0	0 11	0 11	1 2	1 2	1 2
Area	12	20	24	36	48	64	99
Barrel (*a*)	0.963	1.222	1.497	1.829	2.258	2.628	3.790
Barrel (*b*)	21	21	66	66	100	100	190
Portals	8.11	16.0	16.4	28.4	31.5	47.1	71.9

Barrel (*a*) gives cubic yards of concrete and (*b*) pounds of steel per lineal foot of barrel.

STEAM RAILROAD TRACK

7. Classification

The following classification of railways, based on tonnage and on maximum speed of passenger trains, is that used by the American Railway Engineering Association as the basis for recommended practice in the construction of roadbed, dimensions and quality of ballast, cross-sections, etc.

Class "A" shall include all districts of a railway having more than one main track, or those districts of a railway having a single main track with a traffic that equals or exceeds the following: Freight car mileage passing over district per year per mile, 150,000; or, Passenger car mileage per year per mile of district, 10,000; with maximum speed of passenger trains of 50 miles per hour.

Class "B" shall include all districts of a railway having a single main track with a traffic that is less than the minimum prescribed for Class "A," and that equals or exceeds the following: Freight car mileage passing over district per year per mile, 50,000; or, Passenger car mileage per year per mile of district, 5000; with maximum speed of passenger trains of 40 miles per hour.

Class "C" shall include all districts of a railway not meeting the traffic requirements of Classes "A" or "B."

8. Ballast

The following definitions and specifications have been officially adopted by the American Railway Engineering Association, and are therefore quoted as desirable standards for the classification and selection of ballast.

Ballast. Selected material placed on the roadbed for the purpose of holding the track in line and surface. **Broken or Crushed Stone:** Stone broken by artificial means into small fragments of specified sizes. **Chats:** Tailings from mills in which zinc, lead, silver and other ores are separated from the rocks in which they occur. **Gravel: (a) Pit Run.** — Worn fragments of rock and sand occurring in natural deposits. **(b) Screen.** — Worn fragments of rock occurring in natural deposits that will pass through a 2-1/2-in. ring and be retained on a No. 10 screen. **(c) Washed.** — A gravel from which foreign matter has been washed and the relative proportions of gravel and sand have been determined. **Sand:** Any hard, granular, comminuted rock which will pass through a No. 10 screen and be retained upon a No. 50 screen. **Chert:** An impure flint or hornstone, occurring in natural deposits. **Cinders:** The residue from the coal used in locomotives and other furnaces. **Slag:** The waste product, in a more or less vitrified form, of furnaces for the reduction of ore; usually the product of a blast furnace. **Burned Clay:** A clay or gumbo which has been burned into material for ballast. **Gumbo:** A term commonly denoting a peculiarly tenacious clay, containing no sand. **Disintegrated Granite:** A natural deposit of granite formation, which, on removal from its bed by blasting or otherwise, breaks into particles of size suitable for ballast.

Stone Ballast. Stone shall be durable enough to resist the disintegrating influences of the climate where it is used; it shall be hard enough to resist pulverizing under the treatment to which it is subjected; it shall break in angular pieces when crushed. The maximum size of ballast shall not exceed pieces which will pass in any position through a 2-1/2-in. ring; the minimum size shall not pass through a 1/2-in. ring and the gradations between shall be fairly uniform. It should be free from dirt, dust or rubbish.

Gravel Ballast. At least 15% of sand is essential in gravel ballast to prevent the stones from shifting under their load. For Class A roads bank gravel containing more than 2% of dust (material finer than sand) or 40% of sand must be washed or screened and the product should contain between 15 and 20% of sand. For Class B roads bank gravel containing more than 3% of dust or 60% of sand should be washed or screened and the product should contain from 25 to 50% of sand. For Class C roads any material which makes better track than the natural soil may be used.

Cinders. The use of cinders as ballast is recommended for the following situations: On branch lines with a light traffic; on sidings and yard tracks near point of production; as sub-ballast in wet, spongy places; as sub-ballast on new work where embankments are settling, and at places where the track heaves from frost. It is recommended that provision be made for wetting down cinders immediately after being drawn.

Burned Clay. The material should be black gumbo or other suitable clay free from sand or silt. The suitability of the material should be determined by thorough testing in a small test kiln before establishing a ballast kiln. The material should be burned hard and thoroughly. The fuel used should be fresh and clean enough to burn with a clean fire. It is important that a sufficient supply be kept on hand to prevent interruption of the process of burning. Burning should be done under the supervision of an experienced and competent burner. Ballast should be allowed to cool before it is loaded out of the pit. Absorption of water should not exceed 15% by weight.

9. Standard Ballast Sections

The Ballast Sections in Figs. 13*a* and *b* are those officially adopted and recommended as good practice by the American Railway Engineering Associa-

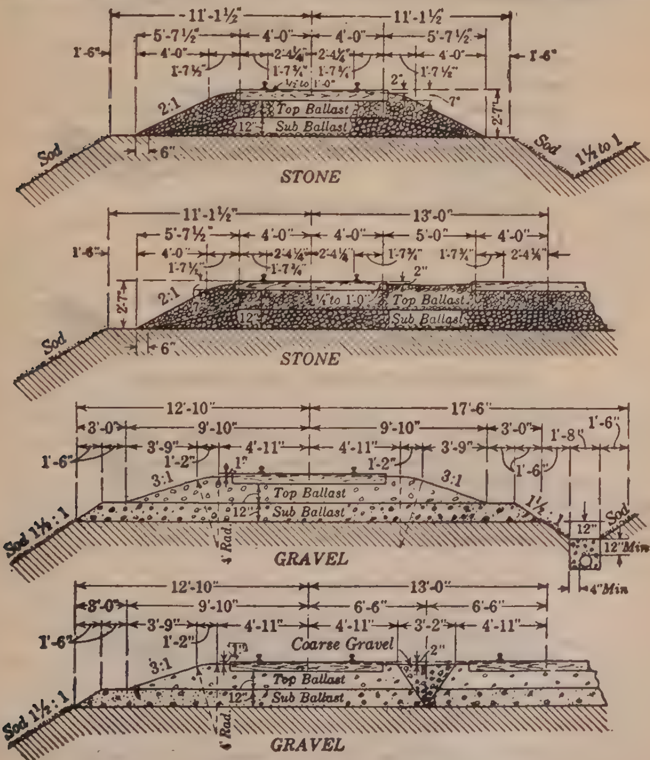
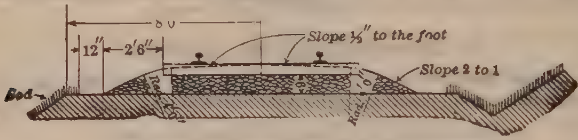


Fig. 13*a*. Standard Ballast Sections, Class A Roadbeds

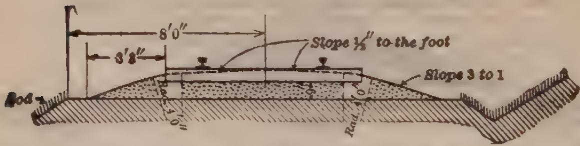
tion; and, on account of the care used to obtain the consensus of opinion of the best officials of the country, they may be considered as the most authoritative designs obtainable.

The Quantity of Ballast required in cubic yards per mile for the standard sections shown in Fig. 13*a* is as follows:

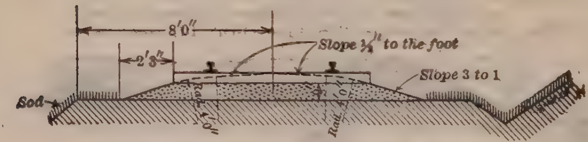
Kind of ballast	Single track		Double track	
	Sub-ballast	Top-ballast	Sub-ballast	Top-ballast
Stone	2664	3995	5206	7320
Gravel	4726	4144	7268	7626



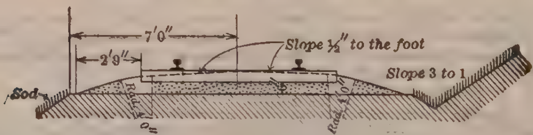
Stone and slag, Class B Railways



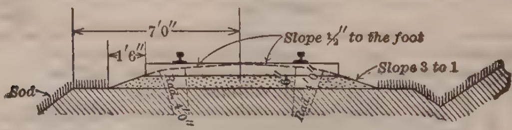
Gravel, cinders, chats, etc., Class B Railways



Cementing gravel and chert, Class B Railways



Gravels, cinders, chats, etc., Class C Railways



Cementing gravel and chats, Class C Railways

Fig. 13*b*. Standard Ballast Sections, Class B and C Roadbeds

Note. Slag is broken and similar in character to stone. For granulated slag the same section as for gravel, cinders, etc., should be used.

The Quantity of Ballast required in cubic yards per mile for the standard sections shown in Fig. 136 is as follows:

Kind of Ballast	Class B	Class C
Stone and slag	2195	....
Gravel, cinders, chats, etc.	2284	1695
Cementing gravel and chert	1882	1309

These computed volumes are based on the use of 6 in. by 8 in. by 8-ft. cross-ties, spaced 24 in. on centers, for both classifications. The use of wider ties (same depth) would slightly decrease the required volume of ballast; the use of deeper ties (same width) would increase the volume; wider spacing would increase the volume; the use of longer ties would decrease the volume slightly, except for cementing gravel and chert.

Shrinkage of Ballast. "Shrinkage of ballast" is the decrease in volume from that as loaded at the pit and measured as dumped loosely in the car to that as measured in the track after the ballast has been tamped and compressed by the passage of trains. This "shrinkage" is not a loss of ballast; and the "pay quantities," usually measured as dumped loosely in the car at the pit, must be increased sufficiently to allow for the shrinkage.

The following table from Bulletin 277, A.R.E.A., p. 84, gives results of actual measurements of this shrinkage:

Shrinkage for Various Kinds of Ballast

Kind of ballast	Number of projects	Yards of material		Per cent of shrinkage
		Purchased	Measured in track	
Stone	8	2 113 117	1 812 020	16.62
Chats	2	222 936	205 450	8.50
Gravel	3	381 586	313 295	21.80
Sand	1	74 910	65 484	14.39
Slag	1	195 159	157 234	24.12
Cinders	1	Altoona test		27.00

Cleaning Ballast. Ballast becomes **fouled** or "dead," in from 2 to 7 years, depending largely on its hardness and durability but also on account of cinders and other dirt dropped into it. It then does not drain readily and must be cleaned. The hand method, still much used, is to throw it out from between the ties with shovels and return it with ballast forks, thus leaving the dirt outside the track. This method is slow, laborious and costly; but no very satisfactory machine has been developed for getting the ballast out of the "cribs." Machines of the vacuum-cleaner type have been tried, and on mountain roads where the amount of cinders is large a machine similar to a street sweeper, but with pan and conveyor, to dispose of the dirt, is sometimes used.

For dirty ballast alongside single track or in the intertrack spaces on multiple-track roads, several types of machine are used. The first was the locomotive crane rigged with a clamshell bucket, which deposited the dirty ballast on screens. They are most effective if operated in pairs, or multiples, and the screens should be of the vibrating type. The same principle is followed in a self-contained ballast-cleaning machine recently put on the market. Another type of machine is the mechanical mole which digs its way through the material cleaning the ballast as it goes. Cleaning ballast by machine is faster and cheaper than by hand, but conditions vary so widely that cost data are conflicting. One large eastern road reports savings of about $1500 per mile of double track using four cranes and only about $870 per mile if only one is used.

Cost. The total cost is made up of three items: the cost of the material on the cars, the cost of hauling to the place of distribution, and the cost of placing and tamping under the track. The cost of **loading gravel** at a gravel pit is quoted at 7 cents per cu. yd. Another quotation, which included "washing" the gravel, was 18 cents; still other quotations ranged from 5.62 cents to 13.9 cents. The cost of crushed limestone at the crusher is quoted at 53.5 cents; other quotations for "crushed stone" were 48 cents, 59 cents, and 61.5 cents. The cost of **hauling** 50 miles is quoted as 5.5 cents; on another road, hauling 50 miles and unloading, 10.7 cents; on another road the cost of hauling combined with unloading and placing in track was given as 40 cents. The cost of **placing in track** is about 12 to 15 cents for cinders, a little more for gravel, and up to 30 cents for broken stone; the cost of placing stone ballast is given in one case as 31.9 cents, and the cost of placing gravel under the same conditions was 11.8 cents. The cost of digging out all worn-out gravel ballast was given as 15 cents per cu. yd. The tie renewals in stone ballast cost more than in gravel ballast; the relative cost (to quote one case) was 16.8 cents per tie for stone and 10.3 cents per tie for gravel.

These costs were all obtained before the present era of high prices and should be increased from 50 to 100% for the same methods. The increasing use of labor-saving machinery on many operations serves to partially neutralize these increases, but conditions are so variable that it is impossible to cover them.

10. Ties

Wooden Ties. For heavy traffic the minimum thickness should be 7 in., the width 9 to 12 in. and the length at least 8 ft. 6 in.

The following sizes are listed by the A.R.E.A., but the first two are not accepted for standard-gage track.

Size	Sawed or hewed on top, bottom and sides		Sawed or hewed on top, and bottom	
	Inches thick	Inches wide on top	Inches thick	Inches wide on top
0	5	5	5	5
1	6	6	6	6
2	6	7	6	7
3	6	8	6	8
4	7	8	7	7
5	7	9	7	8
6	7	10	7	9
			7	10

Durability. Durability depends on the weight and amount of the traffic, the character and drainage of subsoil and ballast, the use of tie plates, the climate, the time of year of cutting the timber, the age of the timber, and the amount of its seasoning before being placed in the track, and chiefly on the kind of wood. Therefore, any figures are necessarily subject to wide variations. A quoted consensus of opinion as to the life of untreated ties, which may be considered very reliable, is as follows: white oak, 7 to 12 years; pine, 5 to 8 years; chestnut, 8-1/2 years; cypress, 7 to 9 years; cedar, 15 years;

tamarack, 5 to 6 years; hemlock, 5 years. From 75 to 98% of white oak ties fail from decay, the remainder failing from rail cutting and spike cutting. For pine, chestnut, cedar, cypress, gum and similar timber, the corresponding figures for failures from decay are from 25 to 80%. The softer the wood, the greater is its liability to fail from rail cutting or spike cutting. Failures from these causes are practically eliminated by the use of tie plates. The durability of chemically treated ties depends on the kind of chemical treatment and on the protection of the ties against mechanical injury, such as rail cutting and spike cutting. It is possible to treat ties chemically so that they will resist decay for 30 years, but it is useless to do so unless they are adequately protected against mechanical destruction by the use of tie plates. Even with tie plates ties sometimes fail from wear under them, and it is now considered good practice to fasten the plates to the tie.

Spacing. The usual maximum spacing is 2 ft. on centers. An open space of 10 in. between ties is sufficient for tamping, and maximum bearing area on ballast is obtained by the use of the wider and longer ties with this spacing. The number of ties per mile for various spacings is as follows:

Number of ties per 39-ft rail.........	26	25	24	23	22	21	20	19.5	19	18	17	16
Average spacing, inches...........	18.0	18.7	19.5	20.3	21.3	22.3	23.4	24.0	24.6	26.0	27.5	29.2
Number of ties per mile.............	3520	3385	3250	3114	2980	2843	2708	2640	2573	2437	2302	2166

The Effect of Chemical Treatment on the strength of cross-tie woods has been especially investigated by the U. S. Government, and the final conclusions are: (a) A high degree of steaming is injurious; (b) the presence of zinc chloride does not decrease the static strength, but it renders the wood brittle and therefore more liable to fracture under impact; (c) cresote is not injurious.

Cost of Treatment. Neglecting the variable items of royalties on patents, profit, interest or depreciation, the cost of three of the common types of treatment will average about as follows, for a tie containing 3 cu. ft.: Zinc chloride, 16 cents; zinc chloride and creosote, 27 cents; creosote, 10 lb. to the cubic foot, 55 cents. The cost of the full creosote process is so great that many roads are now using a mixture in various proportions of creosote and petroleum as shown by the statistics in the following table. Approximately three-fourths of all the ties now used in the United States are treated.

Economics of Treated Ties. The durability of ties treated by various methods and with various amounts of chemicals is approximately proportional to the expense incurred in treating them. It is possible to waste money in extra cresote so that the tie will be worn out mechanically long before any appreciable decay has set in. The method of treatment should correspond to the other elements of deterioration. The true relative value of two ties, one of which is long-lived but costly, and the other is cheaper but less durable, may be readily determined from the table showing annual charge against a tie when the cost of each in the track and the expected life of each in years is known. For example, assume that a cheap untreated hemlock tie costs 35 cents, that the cost of placing it in the track is 20 cents, and that the tie will last 5 years. Assume that a yellow-pine tie is bought for 35 cents, is treated thoroughly with cresote at a cost of 60 cents, and that, as before, the cost of placing it in the track is 20 cents. The cheap tie costs 55 cents in the track

Number of Treated Ties — 1927

Kinds of wood	Preservative					Total	Per cent
	Creosote *	Creosote *-petroleum	Zinc-creosote *	Zinc-chloride	Miscellaneous		
Oak	21 467 239	1 476 678	1 954 721	2 559 934	......	27 458 572	37.0
Yellow pine	14 020 828	5 457 271	430 404	1 057 006	......	20 965 509	28.3
Douglas fir	80 834	4 442 015	53 602	1 281 168	30 000	5 887 619	7.9
Gum	3 790 881	1 123 996	194 297	290 025	......	5 399 199	7.2
Beech	1 494 203	320 741	244 945	748 366	......	2 808 255	3.8
Maple	1 419 133	477 092	105 608	686 982	......	2 688 815	3.6
Lodgepole pine	95 000	236 184	....	2 185 909	......	2 517 093	3.4
Birch	730 331	1 027 434	200 608	392 188	......	2 350 651	3.2
Western yellow pine	340	653 321	431 239	367 558	......	1 452 458	2.0
Hemlock	997	18 592	....	722 828	......	742 417	1.0
Tamarack	9 712	464 879	....	211 189	......	685 780	0.9
Elm	319 442	40 610	677	119 224	......	479 953	0.6
Others	384 957	52 334	100 985	257 423	......	795 609	1.1
Totals	43 813 897	15 791 147	3 716 996	10 879 800	30 000	74 231 840	100.0
Per cent	59.02	21.27	5.01	14.66	0.04	100.00	......

* Creosote includes distillate coal-tar creosote, creosote coal-tar solution, refined water-gas tar, and water-gas tar solution. Creosote-tar mixtures are more used somewhat than mixtures of creosote and petroleum.

Annual Charge against a Tie, Based on Original Cost and Assumed Life of Tie; Interest Compounded at 5%

From Webb's Railroad Construction

Life of tie in years	Original cost of tie in cents									
	20	30	40	50	60	70	80	90	100	Each 5 cents
3	7.34	11.02	14.69	18.36	22.03	25.70	29.38	33.05	36.72	1.836
4	5.64	8.46	11.28	14.10	16.92	19.74	22.56	25.38	28.20	1.410
5	4.62	6.93	9.24	11.55	13.86	16.17	18.48	20.79	23.10	1.155
6	3.94	5.91	7.88	9.85	11.82	13.79	15.76	17.73	19.70	0.985
7	3.46	5.18	6.91	8.64	10.37	12.10	13.83	15.55	17.28	0.864
8	3.09	4.64	6.19	7.74	9.28	10.83	12.38	13.92	15.47	0.774
9	2.81	4.22	5.63	7.03	8.44	9.85	11.25	12.66	14.07	0.703
10	2.59	3.89	5.18	6.48	7.77	9.07	10.36	11.66	12.95	0.648
11	2.41	3.61	4.81	6.02	7.22	8.43	9.63	10.84	12.04	0.602
12	2.26	3.38	4.51	5.64	6.67	7.90	9.03	10.15	11.28	0.564
13	2.13	3.19	4.26	5.32	6.39	7.45	8.52	9.58	10.65	0.532
14	2.02	3.03	4.04	5.05	6.06	7.07	8.08	9.09	10.10	0.505
15	1.93	2.89	3.85	4.82	5.78	6.74	7.71	8.67	9.63	0.482
16	1.85	2.77	3.79	4.61	5.54	6.46	7.38	8.30	9.23	0.461
17	1.77	2.66	3.55	4.43	5.32	6.21	7.10	7.98	8.87	0.443
18	1.71	2.57	3.42	4.28	5.13	5.99	6.84	7.70	8.55	0.428
19	1.65	2.48	3.31	4.14	4.96	5.79	6.62	7.45	8.27	0.414
20	1.60	2.41	3.21	4.01	4.81	5.62	6.42	7.22	8.02	0.401

and will last 5 years. By the table, the annual charge against it is 11.55 + 1.15 = 12.70 cents. The other tie costs $1.15, and is assumed to last 20 years, hence from the table the annual charge against it is $(8.02 + 3 \times 0.401) = 9.223$ cents. Even if the treated tie lasted only 15 years, the annual charge would be only 11.076 cents. Reducing the rate of interest from 5% to, say, 4% of course reduces the annual charge against any tie; it also makes the longer-lived ties relatively more valuable, or with a smaller relative annual charge.

11. Rails

Standard Length. The standard length is 39 ft. Shorter lengths, varying by single feet down to 25 ft. are permissible up to 11% of the entire order.

Expansion. The allowance for expansion should be 0.000 006 5 of the length per degree Fahrenheit change in temperature. For a 39-ft. rail and a change of temperature of 25 deg., the expansion would be 0.00624 ft. = 0.0075 in. nearly. The following allowances are recommended by the American Railway Engineering Association.

Temperature Expansion for Laying Rails

Rail temperature, deg. F.	160 joints per mile, 33-ft. rails, Inch	135 joints per mile, 39-ft. rails, Inch	117 joints per mile, 45-ft. rails, Inch
−20 to 0	5/16	3/8	7/16
0 to 25	1/4	9/32	11/32
25 to 50	3/16	7/32	1/4
50 to 75	1/8	1/8	5/32
75 to 100	1/16	1/16	1/16
Over 100	laid close	laid close	laid close

Weight of Rail. The old rule of the Baldwin Locomotive Works requiring 10 lb. per yard for each 3000 lb. of wheel load is now generally exceeded by the practice of first-class roads, partly because of higher speeds, although these have been made less destructive by improved counterbalancing, but largely because the use of heavier rails is economical, reducing both maintenance and operating expenses. The following table shows the trend toward the use of heavier sections.

Production of Rails by Weight per Yard

Year	Under 50 lb.	50 to 85 lb.	85 to 100 lb.	100 to 120 lb.	120 lb. and over	Total, gross tons
1914	238 423	309 865	868 104	528 703		1 945 095
1915	254 101	518 292	742 816	688 995		2 204 203
1916	295 535	566 791	1 225 341	766 851		2 854 518
1917	308 258	882 673	989 704	763 526		2 944 161
1918	395 124	665 165	888 141	592 462		2 540 892
1919	263 803	495 577	965 571	478 892		2 203 843
1920	489 043	433 333	952 622	729 118		2 604 116
1921	211 568	214 936	902 748	849 566		2 178 818
1922	265 541	274 731	728 604	902 900		2 171 776
1923	272 794	300 907	864 965	1 465 850		2 904 516
1924	191 046	213 274	853 431	1 175 581		2 433 332
1925	163 607	219 648	765 371	1 636 631		2 785 257
1926	197 260	256 287	797 662	1 966 440		3 217 649
1927	161 740	173 257	539 465	1 314 424	617 524	2 806 390

The **gross tons per mile of rails** of any weight is readily found by multiplying **the weight per yard by 11/7.** For example $70 \times 11/7 = 110$, the gross tons of 70-lb. rails per mile of track. The rule is theoretically exact, but in ordering an allowance of about 2% should be made for rail cutting.

Curving. An approximate but practical rule for the middle ordinate **m** (Fig. 14) is that it is equal to the square of the length divided by 8 times the radius of the curve. Applying the approximate rule that radius of curve = $5730/D$, where D is the degree of the curve, the rule becomes, for a standard 39-ft. rail, m (in inches) = 0.395 D; or for any length rail, m (in inches) = $0.00026\,L^2D$, where L = length in feet.

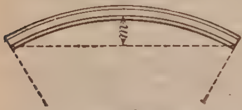


Fig. 14

For ordinary curvature and lengths up to 33 ft. there is no practical error in this rule, the maximum error below for a 30-deg. curve being 1/8 in. and that for even a 50-deg. curve being only 1/2 in. The following table was computed by its use. For greater lengths the error varies practically as the square of the distance. For curves of less than 8 to 10 degrees it is not generally considered necessary to curve the rails before laying.

Middle Ordinates for Curving Rails, in Inches

D	Length of rail in feet												
	10	15	20	24	25	26	27	28	29	30	31	32	33
2	....	1/8	1/4	1/4	3/8	3/8	3/8	3/8	1/2	1/2	1/2	1/2	5/8
3	1/8	1/8	3/8	1/2	1/2	1/2	5/8	5/8	5/8	3/4	3/4	3/4	7/8
4	1/8	1/4	3/8	5/8	5/8	3/4	3/4	7/8	7/8	1	1	1-1/8	1-1/8
5	1/8	1/4	1/2	3/4	7/8	7/8	1	1	1-1/8	1-1/8	1-1/4	1-3/8	1-3/8
6	1/8	3/8	5/8	7/8	1	1	1-1/8	1-1/4	1-3/8	1-3/8	1-1/2	1-5/8	1-3/4
7	1/8	3/8	3/4	1	1-1/8	1-1/4	1-3/8	1-3/8	1-1/2	1-5/8	1-3/4	1-7/8	2
8	1/4	1/2	7/8	1-1/4	1-1/4	1-3/8	1-1/2	1-5/8	1-3/4	1-7/8	2	2-1/8	2-1/4
9	1/4	1/2	1	1-3/8	1-1/2	1-5/8	1-3/4	1-7/8	2	2-1/8	2-1/4	2-3/8	2-5/8
10	1/4	5/8	1	1-1/2	1-5/8	1-3/4	1-7/8	2	2-1/4	2-3/8	2-1/2	2-5/8	2-7/8
12	3/8	3/4	1-1/4	1-3/4	1-3/4	2-1/8	2-1/4	2-1/2	2-5/8	2-7/8	3	3-1/4	3-3/8
14	3/8	7/8	1-1/2	2	2-1/4	2-1/2	2-5/8	2-7/8	3-1/8	3-1/4	3-1/2	3-3/4	4
16	3/8	1	1-5/8	2-3/8	2-5/8	2-7/8	3	3-1/4	3-1/2	3-3/4	4	4-1/4	4-1/2
18	1/2	1	1-7/8	2-3/4	2-7/8	3-1/8	3-3/8	3-3/4	4	4-1/4	4-1/2	4-7/8	5-1/8
20	1/2	1-1/8	2	3	3-1/4	3-1/2	3-7/8	4-1/8	4-3/8	4-3/4	5	5-3/8	5-3/4
25	5/8	1-1/2	2-5/8	3-3/4	4-1/8	4-3/8	4-3/4	5-1/8	5-1/2	5-7/8	6-1/4	6-3/4	7-1/8
30	3/4	1-3/4	3-1/8	4-1/2	4-7/8	5-1/4	5-3/4	6-1/8	6-5/8	7-1/8	7-1/2	8	8-1/2
35	7/8	2	3-5/8	5-1/4	5-3/4	6-1/4	6-5/8	7-1/8	7-3/4	8-1/4	8-3/4	9-3/8	10
40	1	2-3/8	4-1/8	6	6-1/2	7-1/8	7-5/8	8-1/4	8-3/4	9-3/8	10	10-3/4	11-3/8
45	1-1/8	2-5/8	4-3/4	6-3/4	7-3/8	8	8-5/8	9-1/4	9-7/8	10-5-8	11-5/8	12-1/8	12-7/8
50	1-1/4	3	5-1/4	7-1/2	8-1/8	8-7/8	9-1/2	10-1/4	11	11-3/4	12-3/8	13-3/8	14-1/4

Failures are classified by the American Railway Engineering Association under the following heads:

1. Broken Rail. (A) Transverse Fissure. (B) Oridinary Break.
2. Flowed Head.
3. Crushed Head.
4. Split Head.
5. Split Web.
6. Broken Base.
7. Damaged.

Complete reports are required of all failures classified as above, and must give complete data regarding characteristics, manufacture, location and condition of rail, ties and ballast, alignment, weather, etc. These reports are collected and summarized for study on various bases such as length of service, kind of steel, mills, etc.

The total average failures per 100 track-miles (32 000 rails, 33 ft.) classified on basis of length of service up to five years are given in the table below.

The steady improvement up to 1916 indicated by these statistics was due principally to the improvement in the metal used for rails, the adoption of heavier rails better adapted to the increased loads and the gradual replacement of Bessemer by open-hearth rails. Disturbance of manufacturing conditions and under-maintenance during the war are blamed for the poorer and more erratic record after 1916 or 1917, but there are doubtless other factors contributing to the failure to return to the best record, such factors being increased speed and weight of rolling stock and especially increased density of traffic, as well as increased trouble from transverse fissures in recent years. It should be noted, however, that less than one rail per thousand fails each year on the average for the first five years and that the failures per year increase with the life of the rail, which is as it should be and just the reverse of common experience some twenty-five to thirty years ago when the inferior quality of rails resulted in very heavy replacements from defects during the first two or three years of use.

Number of Rail Failures per 100 Track-Miles

From Bull. 315, March, 1929, American Railway Engineering Association.

Year rolled	Years of service				
	1	2	3	4	5
1908	....	....	....	....	398.1
1909	....	....	....	224.1	277.8
1910	....	....	124.0	152.7	198.5
1911	....	77.0	104.4	133.3	176.3
1912	28.9	32.1	49.3	78.9	107.1
1913	12.5	25.8	44.8	69.5	91.9
1914	8.2	19.8	32.9	50.9	74.0
1915	8.9	19.0	34.2	53.0	82.4
1916	11.8	29.2	47.7	70.6	105.4
1917	21.6	38.9	66.0	110.5	137.0
1918	8.9	27.6	54.0	92.8	125.4
1919	14.8	39.4	73.7	104.8	115.7
1920	14.2	32.4	63.1	84.5	119.6
1921	10.9	34.9	56.9	70.9	98.9
1922	15.9	34.8	55.2	80.4	110.0
1923	14.3	33.2	57.6	86.0	....
1924	14.0	33.4	58.3	....	....
1925	15.5	36.6	....	....	....
1926	17.1	....	....	....	....

Life. Rails not replaced on account of failure wear out in service unless replaced by heavier sections and the life depends on the rate of wear and the total wear permitted before renewal, each road fixing its limit of wear to suit its own conditions. Wear is usually considered proportional to tonnage, although heavy axle loads and high speeds undoubtedly increase the destructive effect of traffic. Grade also increases wear 50 to 100% due to the slipping of drivers and use of sand. The excess wear on curves may be taken at 1/6 that on tangents per degree of curve, or, the wear on a 6-deg. curve will be double that on tangents. The normal wear of rails may be estimated at 1/16 in. per 25 000 000 to 35 000 000 tons of traffic. Assuming the allowable depth of wear for main track at 3/8 in. gives, say, 200 000 000 tons as its total tonnage life in main track. It may then be used on branch lines or sidings, with or without rerolling. Since the quality of the metal inside the rail is inferior to that on the surface, some roads remove rail from main track early and reroll it, thus again securing a better wearing surface but, of course, a reduced section. The tonnage treated in this way and the practical displacement of Bessemer steel rails by open-hearth steel rails are shown by the table given below.

Sections. The section in common use originated in the so-called " flat bottom " pattern designed by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad, in 1830. It was reinvented in England in 1836 by Charles Vignoles and is used as the " Vignoles " rail in Europe and to some extent in England, although the standard in the latter country is the double-headed or " bull head " rail which requires chairs for its support. The early rails had pear-shaped heads, being designed for the use of iron, on account of the danger of the side of the head breaking down with that material. After the introduction of steel in 1865, many modifications were made by the various roads using rails until in 1891 nearly 300 different patterns were in use with 27 different weights per yard practically all 80 lb. and less. This caused needless expense and higher prices due to the large stock of rolls which the eleven Bessemer steel rail mills had to carry on hand; and in 1893 the American Society of Civil Engineers, having studied the subject exhaustively through

Production of Rails by Processes, in Gross Tons

Year	Open hearth	Bessemer	Rerolled	Electric	Iron	Total
1902	6 029	2 935 392	....	....	6 512	2 947 933
1903	45 054	2 946 756	....	....	667	2 992 477
1904	145 883	2 137 957	....	....	871	2 284 711
1905	183 264	3 192 347	....	....	318	3 375 929
1906	186 413	3 791 459	....	....	15	3 977 887
1907	252 704	3 380 025	....	....	925	3 633 654
1908	571 791	1 349 153	....	....	71	1 921 015
1909	1 256 674	1 767 171	....	*	....	3 023 845
1910	1 751 359	1 884 442	....	*	230	3 636 031
1911	1 676 923	1 053 420	91 751	462	234	2 822 790
1912	2 105 144	1 099 926	119 390	3 455	....	3 327 915
1913	2 527 710	817 951	155 043	2 436	....	3 502 780
1914	1 525 851	323 897	95 169	178	....	1 945 095
1915	1 775 168	326 952	102 083	....	....	2 204 203
1916	2 269 600	440 092	144 826	....	....	2 854 518
1917	2 292 197	533 325	118 639	....	....	2 944 161
1918	1 945 443	494 193	101 256	....	....	2 540 892
1919	1 893 250	214 121	96 422	50	....	2 203 843
1920	2 334 222	142 899	126 698	297	....	2 604 116
1921	2 027 215	55 559	96 039	5	....	2 178 818
1922	2 033 000	22 317	116 459	....	....	2 171 776
1923	2 738 779	25 877	139 742	118	....	2 904 516
1924	2 307 533	16 069	109 730	....	....	2 433 332
1925	2 691 823	9 687	83 747	....	....	2 785 527
1926	3 107 992	12 533	97 124	....	....	3 217 649
1927	2 717 865	1 566	86 959	....	....	2 806 390

* Small tonnages rolled but included with Bessemer and open-hearth. Rerolled rails were also included 1902 to 1910. There were also rerolled from new, second quality, and defective rails 22 010 tons in 1911; 42 586 in 1912; 43 793 in 1913; 26 772 in 1914; 9 129 in 1915; 3 860 in 1916 and 9 007 in 1917. The totals show that the practice of rerolling has decreased markedly since 1913, but total production has also decreased somewhat.

In addition to the above tonnages, alloy-treated steel rails have been produced by the open-hearth and electric processes in the quantities given in gross tons in the table on the next page.

Production of Alloy-Treated Steel Rails, in Gross Tons

Year	Alloys: Titanium	Alloys: Other alloys	Weights per yard: Under 50 lb.	Weights per yard: 50 to 85 lb.	Weights per yard: 85 to 100 lb.	Weights per yard: 100 to 120 lb.	Weights per yard: 120 lb. and over	Total
1918	2 891	220	....	47	2 640	424		3 111
1919	6 207	269	....	....	3 920	2 556		6 476
1920	11 652	1 257	....	514	5 069	7 326		12 909
1921	2 804	3 472	....	71	4 277	1 928		6 276
1922	2 493	670	....	321	835	2 007		3 163
1923	346	1 796	....	56	317	1 769		2 142
1924	1 696	3 471	....	....	847	4 320		5 167
1925	1 616	2 393	70	47	....	3 892		4 009
1926	1 099	3 117	....	42	1 027	3 147		4 216
1927	....	1 265	....	....	374	391	500	1 265

various committees since 1873, adopted standard sections the use of which was recommended. These sections were extensively adopted, as at one time about 75% of the output conformed to those standards, but when the heavier rails came into common use much difficulty was experienced. Although much of this was due to poor quality of metal and poor methods of manufacture, a change in section was also necessary and the American Railway Association in 1905 appointed a committee to consider the matter. This committee made its final report in 1909, recommending the adoption of two types of sections, A and B, the first with shallower head and greater height than the second. The association concurred and referred the sections to the American Railway Engineering Association whose Committee on Rail reported in 1915 in favor of the "A" section for a single type for the 90-lb. rail and in favor of sections of their own design for 100-, 110- and 120-lb. rails, thus leaving the Am. Soc. C. E. sections standard for weights under 90 lb. per yard. These recommendations were adopted. Considerable tonnage of both sections A and B has been rolled, however, and they are likely to continue in use to some extent, as the A section satisfies those who believe that the head should be thin and the moment of inertia as great as possible and the B section satisfies those who believe that the head should be narrow and deep and that the moment of inertia is comparatively unimportant. The A section is used on the comparatively straight prairie roads of the Central West, whereas the B section has been adopted by some of the crooked, heavy-traffic, coal roads through the mountains in the East. Standards for heavier sections have been adopted by the American Railway Engineering Association as follows: 130- and 140-lb. in 1920 and 150-lb. in 1924.

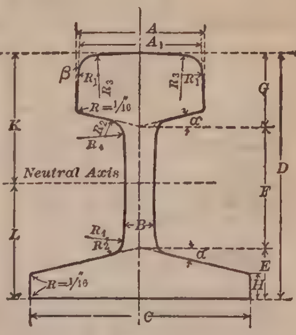


Fig. 15

In Fig. 15 is given a diagram showing the dimensions which are common to all designs and indicating by letters the dimensions which vary. The Am. Soc. designs included thirteen different weights of rails, but these were later reduced to five designs varying by 10 lb. increments from 60 to 100 lb.

Angles and Dimensions of Standard

Standard	Radii, in.				Angles		Weight of rail, lb. per yd.
	Upper corner of head R_1	Fillet corners* R_2	Top of head R_3	Side of web R_4	Bottom of head and top of flange α	Side of head β	
Am. Soc. C. E.	5/16	1/4	12	12	13°	Vert.	60 70 80 90 100
A. R. A. A	3/8	3/8	14	14	4 : 1 14° 02′	1/16 : 1 3° 35′	60 70 80 90 100
A. R. A. B	3/8	5/16	12	12	13°	3°	60 70 80 90 100
A. R. E. A.	3/8 1/2 9/16	3/8 5/8 3/4	14	14 20	4 : 1 14° 02′	1/16 : 1 3° 35′	90 100 110 120 130 140 150
Penn. R. R.	7/16	5/16 1/2	10 12	10 16	15° 18°	1 : 9.65 Vert.	85 100 130
L. V. R. R.	7/16	1/2	10	14	4 : 1	4°	136
Dudley	5/16	1/2	14	14 18	4 : 1	1/16 · 1	105 115 127

* Fillet at bottom of web differs from that at top, R_2, in following sections: A.R.E.A. 100-lb., 110-lb., and 120-lb., 5/8 in.; 130-lb., 140-lb., and 150-lb., 3/4 in.; Penn. R. R. 130-lb., 1/2 in.; L. V. R. R. 136-lb., 3/4 in.; Dudley 105-lb., 1 in., 115-lb.

per yard. One dimension only is common to all standards and all weights of rails, the radii of the upper and lower corners of the flanges and the lower corners of the head being almost invariably 1/16 in. Some other dimensions and angles are constant for a set of standards as shown in the table given above.

Specifications. The following are the standard specifications of the American Railway Engineering Association for open-hearth carbon-steel rails:

Inspection. 1. Inspection and tests shall be made at the works of the manufacturer before shipment, and the works management shall afford all reasonable facilities for determining the satisfactory quality of the rails accepted.

Designs for Rails (see Fig. 15)

Dimensions, in.									
A	A_1	B	C	D	E	F	G	H	K
2-3/8	....	31/64	4-1/4	4-1/4	49/64	2-17/64	1-7/32	......	....
2-7/16	....	33/64	4-5/8	4-5/8	13/16	2-15/32	1-11/32	......	....
2-1/2	....	35/64	5	5	7/8	2-5/8	1-1/2	......	....
2-5/8	....	9/16	5-3/8	5-3/8	59/64	2-55/64	1-19/32	......	2-7/8
2-3/4	....	9/16	5-3/4	5-3/4	31/32	3-5/64	1-45/64	......	3-1/32
2-1/4	....	15/32	4	4-1/2	13/16	2 29/64	1-15/64	5/16	2.37
2-3/8	....	1/2	4-1/4	4-3/4	29/32	2-1/2	1-11/32	3/8	2.55
2-1/2	....	33/64	4-5/8	5-1/8	31/32	2-23/32	1-7/16	3/8	2.81
2-9/16	....	9/16	5-1/8	5-5/8	1	3-5/32	1-15/32	3/8	3.08
2-3/4	....	9/16	5-1/2	6	1-1/16	3-3/8	1-9/16	3/8	3.25
2-1/8	2-1/16	31/64	3-11/16	4-3/16	7/8	2-1/16	1-1/4	29/64	2-15/64
2-3/8	2-5/16	33/64	4-3/64	4-35/64	59/64	2-17/64	1-23/64	29/64	2-25/64
2-7/16	2-11/32	35/64	4-7/16	4-15/16	1	2-15/32	1-15/32	31/64	2-85/128
2-9/16	2-15/32	9/16	4-49/64	5-17/64	1-1/32	2-5/8	1-39/64	31/64	2-105/128
2-21/32	2-9/16	9/16	5-9/64	5-41/64	1-5/64	2-55/64	1-45/64	31/64	3-1/64
2-9/16	....	9/16	5-1/8	5-5/8	1	3-5/32	1-15/32	......	3.08
2-11/16	....	9/16	5-3/8	6	1-1/16	3-9/32	1-21/32	......	3.25
2-25/32	....	19/32	5-1/2	6-1/4	1-1/8	3-13/32	1-23/32	......	3.42
2-7/8	....	5/8	5-3/4	6-1/2	1-3/16	3-17/32	1-25/32	......	3.51
2-15/16	....	21/32	6	6-3/4	1-7/32	3-11/16	1-27/32	......	3.72
3	....	11/16	6-1/4	7	1-1/4	3-27/32	1-29/32	......	3.86
3-1/8	....	11/16	6-3/4	7-3/4	1-9/32	4-9/16	1/29/32	......	4.30
2-3/16	....	17/32	5	5	7/8	2-3/8	1-3/4	......	2-9/16
2-43/64	....	9/16	5	5-11/16	1-3/32	2-25/32	1-13/16	......	3-7/128
3	....	11/16	5-1/2	6-5/8	1-7/32	3-13/32	2	......	3-3/8
2-15/16	....	21/32	6-1/2	7	1-1/4	3-7/8	1-7/8	7/16	3.94
3	....	5/8	5-1/2	6	31/32	3-13/32	1-5/8	......	3-5/32
3	....	5/8	5-1/2	6-1/2	1-1/16	3-3/4	1-11/16	......	3-1/2
3	....	21/32	6-1/4	7	1-5/32	4-5/32	1-11/16	......	3-115/128

and 127-lb., 3/4 in. Another variation is that the radii of the lower corners of the head of the L. V. R. R. standard are 1/8 in. instead of 1/16 as given in Fig. 15.

Chemical Composition. 2. The chemical composition of the steel, determined as prescribed hereafter, shall be within the following limits:

Constituents	Weight in pounds per yard			
	70–84	85–100	101–120	121–140
Carbon....................	0.53–0.70	0.62–0.77	0.67–0.83	0.72–0.89
Manganese.................	0.60–0.90	0.60–0.90	0.50–0.90	0.50–0.90
Phosphorus, not to exceed....	0.04	0.04	0.04	0.04
Silicon, minimum...........	0.15	0.15	0.15	0.15

Average Carbon. 3. (a) In any rolling, it is desired that the number of heats above the mean carbon percentage of the specified range shall be at least equal to the number of heats below the mean, and that the average carbon shall be as high as the mean.

(b) For information only, the manufacturer shall furnish the carbon and manganese analysis on drillings taken from both the " O " and " M " positions of the head at the top end of the " A " rail of the last full ingot rolled on each tenth heat.

Analyses. 4. Separate analyses shall be made from drillings taken from test ingots representing the second and one of the last full ingots of the heat to determine the percentage of carbon and manganese. The percentage of phosphorus, sulphur, and silicon shall be determined on equally mixed drillings from the test ingots. The average analysis of the ladle test ingots shall conform to the chemical requirements. A portion of the drillings shall be furnished to the inspector upon request for check analysis.

Physical Requirements. 5. Ductility and resistance to impact will be determined by the standard A.R.E.A. drop testing machine, with test specimens from 4 ft. to 6 ft. long cut from the top of the "A" rails from the second, middle, and last full ingots of each heat. The distance between supports shall be 3 ft. for sections under 106 lb. For sections 106 and over it shall be 4 ft.

Temperature of the test pieces must not exceed 100° F.

Drop Test. 6. The test specimens shall be placed base upward on the supports and subjected to one blow from the tup, falling free from the following heights:

For 81–90-lb. rail	18 ft.
91–100-lb. rail	19 ft.
101–120-lb. rail	20 ft.
121–140-lb. rail	22 ft.

If all these specimens endure the above tests without fracture, all the rails of the heat will be accepted, subject to final inspection for surface, section and finish.

If one of the three specimens fail, all the "A" rails of the heat will be rejected. Specimens shall then be cut from the bottom end of the same "A" rails or the top end of the " B " rails and tested. If any of these tests fail, the " B " rails of the heat will be rejected. Three additional specimens shall then be taken from the bottom end of the " B " rails or the top end of the " C " rails. If all these tests stand, the balance of the heat will be accepted. If any of these tests fail, the entire heat shall be rejected.

Elongation and Permanent Set. 7. (a) One of the three test specimens shall be given a sufficient number of blows to determine, for information only, the exhausted ductility, reported inch by inch over the entire 6 in. gaged. No ductility readings will be taken between blows. The other specimens shall be nicked and broken. The fracture of each specimen shall be examined to determine the requirements of Section 8.

(b) For information only, the permanent set measured by middle ordinate in inches in a length of 3 ft. shall be recorded after the first blow on all test specimens.

(c) For information only, one of the three test specimens shall be tested by Brinell indentation upon the head of the rail, and the diameter of the indentation shall be entered up on the test record. The ball shall be 19 millimeters in diameter and the pressure 100,000 lb.

Interior Condition. 8. If the fracture on any test specimen exhibits seams, laminations, cavities, interposed foreign matter, or a distinctly bright or fine-grained structure, all top rails represented shall be classified as " X-Rayls."

Classification. 9. No. 1 rails. No. 1 rails shall be free from injurious defects and flaws of all kinds.

X-Rayls. Rails as described in Sections 8 and 15 (c).

No. 2 Rails. Rails which conform to the following requirements will be accepted as No. 2 rails.

(a) Rails which do not contain surface imperfections in such number or of such character as will in the judgment of the inspector render them unfit for recognized uses.

(b) Rails arriving at the straightening presses with sharp kinks or greater camber than that indicated by a middle ordinate of 6 in. in 39 ft.

Discard. 10. Sufficient discard shall be taken from the ingot to insure freedom from injurious segregation and pipe.

Lengths. 11. Standard length of rails shall be 39 ft. at a temperature of 60° F. Eleven per cent of the entire order will be accepted in shorter lengths varying by 1 ft. from 38 ft. to 25 ft. A variation of 3/8 in. from the specified length will be allowed, except that on 15% of the order a variation of 7/16 in. will be allowed.

Section. 12. Section of rails shall conform as accurately as possible to the templets furnished by the purchaser. A variation of 1/64 in. less or 1/32 in. greater than the specified height will be permitted. A variation of 1/16 in. in the length of either flange will be permitted, but the variation in total width of base must not exceed 1/16 in. No variation will be allowed in dimensions affecting the fit of the joint bars, except that the fishing templet approved by the purchaser may stand out not to exceed 1/16 in. laterally.

Weight. 13. A variation of one-half of 1% from the calculated weight of section as applied to the entire order will be allowed.

Drilling. 14. Circular holes for joint bolts shall be drilled to conform to the drawings and dimensions furnished by the purchaser. A variation of 1-32 in. in the size and location of bolt holes will be allowed.

Finishing. 15. (a) All rails shall be smooth on the heads, straight in line and without twists, waves or kinks. The supports for rail in the straightening presses shall have flat surfaces and be free from hollow places, bends or crooks, and shall be spaced not less than 60 in. When placed head up on a horizontal surface, rails that are slightly higher at the ends than the middle will be accepted, provided they contain a uniform sweep, the middle ordinate of which does not exceed 1-1/4 in. in 39 ft. They shall be sawed square at the ends, a variation of not more than 1/32 in. being allowed and burrs shall be entirely removed.

(b) Rails presented for inspection which do not conform to the requirements of Section 14 or Section 15 (a) may be reconditioned by the mill, provided they can be made to meet the requirements fully.

(c) When any finished rail shows conditions as described in Section 8 at either end or at any drilled hole, it shall be cut back to sound metal, and accepted as an " X-Rayl."

Branding. 16. Brands made so plain and sharp that they may be read as long as the rails are in service shall be rolled on or hot-stamped into the side of the web of each rail in accordance with the following requirements and to indicate:

(a) Name of the manufacturer, month and year of manufacture, and weight and type of section of rail as rolled.

(b) The heat number and the ingot number as rolled shall be stamped in the web of each rail where it will not be covered by the joint bars.

(c) The top rails shall normally be lettered "A" and succeeding ones " B," " C," " D," " E," etc., consecutively, but in case top discard is greater than normal, the rail lettering shall conform to the amount of discard, the top rail becoming " B " or other succeeding letter to suit the condition.

(d) All rails shall be branded " O-H " in addition to other marks.

Classification Markings. 17. (a) Rails accepted as No. 2 rails shall have the ends painted white and shall be stamped with the figure 2 on both end faces.

(b) Rails accepted as " X-Rayls " shall have the ends painted brown and shall be stamped with the letter " X " on both end faces.

(c) "A" rails shall have both ends painted yellow.

(d) No. 1 rails less than 39 ft. long shall have both ends painted green.

(e) All rails of heats whose carbon content exceeds the mean carbon percentage of specific range shall have both ends painted blue.

Individual rails shall be painted only one color, according to the order of precedence listed above.

Loading. 18. Rails shall be carefully handled in such manner as to avoid injury, and shall be loaded as follows:

(a) No. 1 low-carbon rails shall be loaded in separate cars.

(b) No. 1 high-carbon rails shall be loaded in separate cars.

(c) No. 2 rails shall be loaded in separate cars.

(d) " X-Rayls " shall be loaded in separate cars.

(e) No. 1 "A" rails shall be loaded in separate cars.

(f) No. 1 short rails shall be loaded in separate cars.

No subdivision by classification markings other than listed above is necessary for separate loading.

Milling Practices. 19. The entire process of manufacture shall be in accordance with the best current state of the art. It is expected that thoroughly deoxidized dead-setting steel will be furnished and that in every stage of manufacture strict adherence to the standards of best practice of the individual mill will be observed.

Shall anything occur in the process of manufacture which, in the judgment of the inspector, varies detrimentally from regular care and practice, the inspector will immediately notify the chief inspector and the general superintendent of the mill, confirming such report in writing. If after full investigation the inspector is not satisfied as to the good quality of the rails in question, and the rails, complying in other respects to this specification, be loaded for shipment, the inspector will immediately forward a copy of this report to the purchaser.

Acceptance and Payment. 20. (a) In order to be accepted, the rails offered must fulfill all the requirements of this specification.

(b) No. 2 rails to the extent of 8% of the whole order will be accepted.

(c) Rails accepted will be paid for according to actual weights.

12. Elevation of Outer Rail on Curves

(See also Sect. 6, Art. 61.) The theoretic formula is $e = dV^2/gR$, where d = distance between centers of rails, V = speed of train, g = acceleration of gravity, and R = radius of curve, or, very closely for standard gage, e = 0.00069 DV^2, where D is degree of curve, V is speed in miles per hour, and e is elevation of outer rail in inches. The elevation to be chosen evidently depends on the speed, which should be that of the fastest passenger trains unless their speed is reduced on curves. The Lehigh Valley R. R. has successfully used a superelevation of 10 in.; 8 in. is generally considered the limit, while some roads limit it to 6 in. Whatever the limit, the speed of trains on sharp curves should be reduced, if necessary, to that corresponding to the limiting elevation. A common rule for level grade is one inch for each degree of curve, which agrees with the above formula for a speed of 38 miles per hour. An extension of this rule is that if the grade is over 2%, the elevation is 3/4 in.

Superelevation of Outer Rail in Inches, Standard Gage

(Elevation of center of outer rail above center of inner rail)

Deg. of curve	Velocity in miles per hour											
	15	20	25	30	35	40	45	50	55	60	65	70
1	1/8	1/4	3/8	5/8	7/8	1-1/8	1-3/8	1-3/4	2-1/8	2-1/2	2-7/8	3-3/8
2	1/4	1/2	7/8	1-1/4	1-3/4	2-1/4	2-3/4	3-1/2	4-1/8	5	5-7/8	6-3/4
3	1/2	7/8	1-1/4	1-7/8	2-1/2	3-1/4	4-1/4	5-1/8	6-1/4	7-1/2	8-3/4	10-1/8
4	5/8	1-1/8	1-3/4	2-1/2	3-3/8	4-3/8	5-5/8	6-7/8	8-3/8	10	.....	.....
5	3/4	1-3/8	2-1/8	3-1/8	4-1/4	5-1/2	7	8-5/8	.....	.....	.....	.....
6	7/8	1-5/8	2-5/8	3-3/4	5-1/8	6-5/8	8-3/8	.....	.....	.....	.....	.....
7	1-1/8	1-7/8	3	4-3/8	5-7/8	7-3/4	.....	.....	.....	.....	.....	.....
8	1-1/4	2-1/4	3-1/2	5	6-3/4	8-7/8	.....	.....	.....	.....	.....	.....
9	1-3/8	2-1/2	3-7/8	5-5/8	7-5/8	.....	.....	.....	.....	.....	.....	.....
10	1-1/2	2-3/4	4-1/4	6-1/4	8-1/2	.....	.....	.....	.....	.....	.....	.....
11	1-3/4	3	4-3/4	6-7/8								
12	1-7/8	3-1/4	5-1/8	7-1/2	.....	.....	.....	.....	.....	.....	.....	.....
13	2	3-5/8	5-5/8	8-1/8	.....	.....	.....	.....	.....	.....	.....	.....
14	2-1/8	3-7/8	6	8-3/4	.....	.....	.....	.....	.....	.....	.....	.....
15	2-3/8	4-1/8	6-1/2	9-1/4	.....	.....	.....	.....	.....	.....	.....	.....

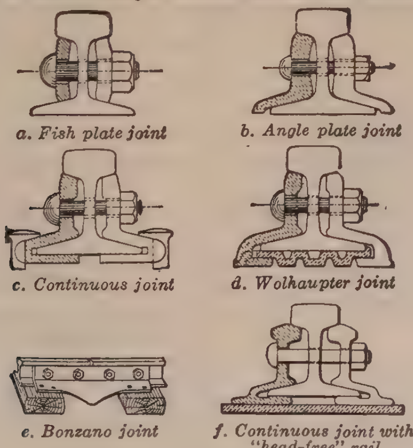
for each degree, which is the equivalent of saying that on the 2% grade the speed up the grade would not be greater than 33 m.p.h. and that the speed down the grade should be limited to that.

Standard Gage of Track is 4 ft. 8-1/2 in. on tangents and on curves of 8 deg. and less. For sharper curves gage is widened 1/8 in. for each 2 deg. of curve up to a maximum of 4 ft. 9-1/4 in. for tracks of standard gage. Gage should not be widened unless the locomotive wheels bind in going around the curve. The gage of a track is the distance between the heads of the rails, measured at right angles thereto at a point 5/8 in. below the top of the rail. Gage, including widening due to wear, should never exceed 4 ft. 9-1/2 in. Gage of track at a frog should be standard even when on a curve, although the flangeway may be widened to compensate for the increase in gage in this case.

13. Rail Fastenings

General Requirements. The ideal rail joint should have the same strength, stiffness and elasticity, no more and no less, both laterally and vertically, as the rails which it joins. Such a joint therefore would not be deformed or take a permanent set under any load which the rail can support without yielding. The joint must also permit longitudinal slipping of the rail to allow for expansion and contraction. It is impossible that any form of scarfed joint should have, at the same time, the same strength, stiffness and elasticity as the original rail. Only a perfect butt welding would accomplish this, but welding would not permit expansion and contraction. Therefore the above conflicting requirements must be filled as perfectly as possible by a compromise design whose cost is not prohibitive.

Designs. The old-fashioned fish-plate design (see Fig. 16*a*) proved so inefficient, especially in lateral strength, that its use has been abandoned except in



a. Fish plate joint *b. Angle plate joint* *c. Continuous joint* *d. Wolhaupter joint* *e. Bonzano joint* *f. Continuous joint with "head-free" rail*

Fig. 16. Various Types of Rail Joints

connection with the very light rails used in industrial railways or the light temporary track used by contractors. A typical modern design for the ordinary angle plate is given in Fig. 16*b*. Among the multitidinous designs which have been brought out the following have come into more common use: the Continuous, Fig. 16*c*; and the Wolhaupter, Fig. 16*d*. The last two designs furnish a plate which runs under the rail between the two joint ties and which keeps the ends of the rails absolutely in line vertically as well as laterally. The Bonzano rail joint, illustrated in Fig. 16*e*, is a development of the angle-plate joint with an extended flange which is bent down between the two joint, ties thus giving a stiffer joint. This can be used only in suspended joints or those in which the rail gap is midway between two ties and the two base joints shown above are also intended for such use. The supported joint, with the rail gap directly over a tie, is very little used and then generally in the so-called three-tie joint. The Bonzano bars are usually heavier and therefore more expensive than the common angle bars and the base joints are not only subject to this disadvantage but also require adzing of the joint ties or setting them on an angle, thus being more expensive to install. The consensus of opinion seems to be that their advantages over the angle bar are not commensurate with the extra cost and many roads have returned to the use of angle bars, sometimes heavily ribbed to furnish additional stiffness. On the other hand, some roads are using the continuous joint with the "head-free" rail shown in Fig. 16*f*. This is the standard RE 130-lb. rail with lower corners of the head chamfered off and the metal thus saved added to the top of the head, giving a stiffer rail and additional metal for wear. The Reading Railway has adopted this as standard and, after several years' experience with it, claims less rail battering and longer life of rail.

Specifications. Bessemer steel is no longer specified for joint bars, high-carbon steel or quenched carbon steel (both open-hearth) and quenched alloy steel being the materials covered by the present (1924) specifications of the American Railway Engineering Association. The principal requirements are:

	High-carbon	Quenched	
		Carbon	Alloy
Carbon, per cent	....	0.42 to 0.55	*
Manganese, per cent, maximum	....	0.80	....
Phosphorus, per cent, maximum	0.04	0.04	0.04
Tensile strength, lb. per sq. in.	85 000	100 000	110 000
Elastic limit, lb. per sq. in.	....	70 000	85 000
Elongation, per cent, in 2-in. length	....	$\frac{1\,600\,000}{\text{Tensile strength}}$	
Minimum, per cent	16	12	
Reduction in area, per cent	....	$\frac{3\,500\,000}{\text{Tensile strength}}$	
Minimum, per cent	....	25	
Bending	Cold bending without sign of fracture on outside of bent portion through arc of 90 deg. with diameter 3 times the thickness of the test specimen.		

* Nickel and chromium to the extent of 1.0% and 0.35%, respectively, are considered equivalent to 0.07% carbon.

All test specimens must be cut from finished bars and all bars must be punched, slotted and shaped at a temperature of not less than 800° C. (1470° F.).

"Quenched" bars must be quenched in oil, or water if so specified, from a temperature of about 810° C. (1490° F.) and kept in the bath until cold enough to handle.

Material requiring quenching in water but otherwise meeting specifications may be accepted at the option of the purchaser. All bars must be finished smooth and true with no variation in fishing angle and height, which factors affect the fit of the bar to the rail. Branding indicating manufacturer, year, design, material and treatment, if any, is required and the number of the melt must be stenciled on the bars.

Splice Bars, Bolts and Spikes for Various Weights of Rails

Rail lb. per yd.	Length of bar, in.	Pounds per foot	Pounds per pair	Proper size of track bolt, in.	Proper size of spikes, in.
30	17	4.0	13.6	2-1/2 × 5/8	4 × 1/2
35	17	4.6	14.1	2-3/4 × 5/8	4-1/2 × 1/2
40	20	5.0	16.1	3 × 5/8	5 × 1/2
45	20	5.8	18.7	3 × 5/8	5-1/2 × 9/16
50	24	6.6	25.5	3-1/2 × 3/4	5-1/2 × 9/16
55	24	7.5	28.9	3-3/4 × 3/4	5-1/2 × 9/16
60	24	8.4	32.4	3-3/4 × 3/4	5-1/2 × 9/16
65	24	9.2	35.5	4 × 3/4	5-1/2 × 9/16
70	34	10.0	54.6	4 × 3/4	5-1/2 × 9/16
75	34	10.7	58.0	4 × 3/4	5-1/2 × 9/16
80	34	11.5	62.6	4-1/2 × 7/8	5-1/2 × 9/16
85	34	12.4	68.2	4-1/2 × 7/8	5-1/2 × 9/16 or 5/8
90	34	16.6	91.3	4-3/4 × 1	5-1/2 × 9/16 or 5/8
100	34	18.8	103.3	4-3/4 × 1	5-1/2 × 5/8
110	34	20.8	114.4	5-1/2 × 1	5-1/2 × 5/8
120	34	22.6	124.3	5-1/2 × 1-1/8	6 × 5/8
130	34	24.4	134.3	6 × 1-1/8	6 × 5/8
140	34	26.2	143.0	6 × 1-1/8	6 × 5/8
150	34	27.9	153.5	6-1/4 × 1-1/8	6 × 5/8

Two hundred and seventy pairs of splice bars will be required per mile of single track, using 39-ft. rails. The maximum number permitted by the rail specification governing short-length rails is 278. Allowing individually for switches and sidings, the number per mile should not exceed 275.

Bolt Holes. The standard spacing for bolt holes recommended by the American Railway Engineering Association is 5-1/2 in., giving lengths of about 24 in. and 34 in., respectively, for the 4-bolt and 6-bolt splices. The hole in the rail should be 1/16 in. larger than the bolt used and the centers of the end holes should be 2-11/16 in. from the end of the rail to allow for expansion and contraction. This combination of measurements permits a total relative motion of the rail ends of 1/8 in. The holes in the bars are made oval, fitting the bolt and preventing it from turning.

Dimensions of Angle Bars. Although the dimensions and weight per linear foot of angle bars, even for the same weight of rail, vary with different manufacturers, the above table gives average figures. The net weight of the bars is from 2.5% to 4% less than their weight unpunched.

Spikes. It has been demonstrated that of all the various forms of driven spikes which have been proposed, the plain spike of uniform section has the greatest holding power. Making the spike jagged, twisting the spike, or even swelling the spike at about the middle of its length, actually reduces its holding power. The holding power, however, is increased by having a cutting edge at the lower end which cuts rather than crushes the fibers, and by having a beveled wedge whose length is about twice the width of the spike rather than a short blunt point. The sizes of spikes recommended for different weights of rail are given in the table on the next page.

Railroad Spikes

Size measured under head in.	Average number per keg of 200 lb.	Ties 24 in. between centers, 4 spikes per tie, quantity per mile		Suitable rail, lb. per yd.
		Pounds	Kegs	
6 ×5/8	250	8448	42.24	120 to 150
5-1/2 ×5/8	275	7680	38.40	85 to 110
5-1/2 ×9/16	340	6220	31.10	45 to 90
5 ×9/16	360	5840	29.40	40 to 56
5 ×1/2	495	4260	21.30	40
4-1/2 ×1/2	540	3920	19.60	35
4 ×1/2	600	3520	17.60	30
4-1/2 ×7/16	655	3220	16.10	25 to 30

The specifications of the A.R.E.A. require the steel to made by the Bessemer or open-hearth process. The required properties are:

Ultimate strength, not less than 55 000 lb. per sq. in.
Elastic limit, not less than 50% of ultimate strength.
Elongation, not less than 25% in 2 in.

Bending, the finished spike when bent back upon itself 180° shall show no signs of fracture. When the head of the spike is bent backward cold it shall show no signs of fracture.

Other requirements cover workmanship, finish, inspection, etc.

For soft steel spikes, the requirements are:

Carbon not under 0.06% for Bessemer process or 0.12% for open-hearth.
Bending tests as above, also workmanship, finish, inspection, etc.

The Driving of Spikes should be by blows in line with the spike. Eccentric or glancing blows tend to enlarge the spike hole and reduce the holding power of the spike. The spikes should not be driven directly opposite each other, but should be staggered. The staggering should be reversed for the two rails on one tie; that is, the spikes on the inner side of the two rails should be near the same side of the tie.

Screw Spikes. The advantages of screw spikes are: (1) The increase in the life of the tie by the avoidance of spike killing; (2) The decreased cost of maintenance (although this is denied), and (3) their greater holding power. It is said, as an objection to their use, that when they are rusted or the head broken off by accident, it is difficult or impossible to extract the stump. They were adopted as standard by the D. L. & W. R. R. (Fig. 17), after extensive trials and are reported as continuing to give satisfactory service on that and other roads. The Pennsylvania Railroad, on the other hand, rejected them in 1918 after nine years of trial and careful observation in two sections of track carrying very heavy traffic. The figures in this case showed them to be more costly, both in first cost and maintenance, than drive spikes and the reported stated that they were unsatisfactory and less efficient than the ordinary spike. One strong objection was the entire loss of holding power when the wood fiber between the threads became worn as it did under the action of the extremely heavy traffic. The screws cost considerably more than common spikes, and require more work to put them in place. The screwing must be done with a track wrench. The auger hole should have the same diameter as that of the screw at the base of the thread, or perhaps 1/16 in. larger. The A. R. E. A. specifications require them to be made of

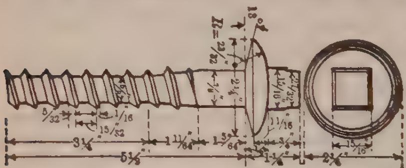


Fig. 17. Screw Spike

open-hearth steel, having not more than 0.05% of either phosphorus or sulfur. The minimum allowable figures for the finished spike are ultimate strength 60 000 lb., elastic limit 50% of ultimate strength, elongation 22% in 2 in., reduction of area 40%. The material must be capable of being bent 180 deg. and hammered down flat and the finished spike bent 90 deg. without sign of fracture.

Track Bolts. The length of track bolts necessarily depends on the distance between the outer faces of the angle plate, and to this must be added the thickness of the nut lock, if used, and the thickness of the nut. Sometimes there is a slight margin beyond this, but any unnecessary length is objectionable. The heads of track bolts are usually hemispherical. Directly under the head the bolt has an oval form which fits loosely in a corresponding oval hole in the angle plate. This prevents the bolt from turning. The table gives proper sizes of track bolts for various weights of rail and their corresponding angle bars. A typical design for a track bolt is shown in Fig. 18.

Track Bolts

Average number in a keg of 200 lb.

Size of bolt, in.	Square nut	Hexagonal nut	Suitable rail, lb. per yd.	Size of bolt, in.	Square nut	Hexagonal nut	Suitable rail, lb. per yd.
2-1/2 ×5/8	390	425	30	3-3/4 × 7/8	165	175	....
2-3/4 ×5/8	379	410	35	4 × 7/8	161	170	....
3 ×5/8	366	395	40	4-1/4 × 7/8	157	165	....
3 ×3/4	250	270	....	4-1/2 × 7/8	153	160	80–85
3-1/4 ×3/4	243	261	....	4-3/4 × 7/8	149	156	....
3-1/2 ×3/4	236	253	50	4-3/4 × 1	....	116	90–100
3-3/4 ×3/4	229	244	55–60	5-1/2 ×1	....	108	110
4 ×3/4	222	236	65–75	5-1/2 ×1-1/8	....	91	120
4-1/4 ×3/4	215	228	....	6 ×1-1/8	....	86	130–140
3-1/2 ×7/8	170	180	....	6-1/4 ×1-1/8	....	82	150

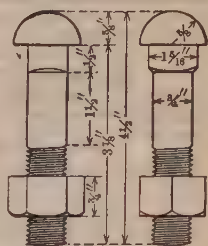


Fig. 18. Track Bolt

On the basis of 275 joints per mile, the number of bolts per mile of single track is 1100 for 4-bolt splices and 1650 per mile for 6-bolt splices. Divide 1100 (or 1650) by the number per keg to obtain the number of kegs per mile.

The specifications of the A.R.E.A. call for medium steel made by the Bessemer or open-hearth process and permit heat treatment if necessary to secure the desired properties which are:

Material	Ultimate tensile strength, lb. per sq. in.	Elastic limit, minimum, lb. per sq. in.	Elongation in 2 in. minimum, per cent	Reduction in area, per cent
Carbon steel, untreated *...	55 000	27 500	20	30
Carbon steel, treated.......	100 000	75 000	10	....
Alloy steel, treated.........	110 000	85 000	12	....

* Phosphorus not over 0.10% for Bessemer steel or 0.05% for open-hearth. For treated steels carbon must not be under 0.30% and phosphorus not over 0.04%.

The material for untreated bolts must bend cold through 180 deg. and flatten on itself without fracture; and treated bolts must bend cold through 45 deg. around a pin of the same diameter as the bolt, without cracking. Other requirements cover workmanship, finish, inspection, etc. Soft steel is required for the nuts.

Nut Locks to keep the nuts from working loose are not always effective and their use has been discontinued on some roads, notably the Lehigh Valley. It is claimed that the joints are easily kept tight by the trackwalkers without much additional vigilance or work, and of course costs of locks and their installation are eliminated. The most common kind of lock is essentially an open ring made of spring steel, the ends of the ring being bent outward. Screwing up the nut compresses the spring and makes the sharp edges bite into the nut and thereby prevents its turning backward. Another form combines a nut and nut lock, the nut being slightly open on one side. The hole is drilled through the nut and the thread is cut slightly smaller than the bolt. When the nut is screwed up, it is forced slightly open, which makes such a pressure on the screw threads that vibration cannot jar it loose. Other types use slightly different threads on the bolts from those of the nuts, but it is doubtful if the extra expense is justifiable.

Fig. 19. Nut Locks

The specifications of the A.R.E.A. cover " rail joint spring washers " of two types: elliptical springs exerting tensions on two bolts, and helical springs for individual bolts. There are two classes of the latter type, high-spring pressure and low-spring pressure. Carbon steel or an acceptable alloy steel manufactured by either basic or acid open-hearth, electric furnace or crucible process is required, with not more than 0.05% phosphorus or 0.04% sulfur. Elliptical springs are tested by subjecting them to ten applications of a load of 30 000 lb. and then to a load of 20 000 lb., under which the distance between platens must be at least 1/32 in. greater than under the last application of the 30 000-lb. load. Their hardness under the Brinnell scale must be between 321 and 418. Helical springs are tested in a similar manner in accordance with the accompanying table:

Spring washer number	For diameter of bolt in inches		Low-spring pressure		High-spring pressure	
	Body	Thread	Preliminary load, lb.	Test load, lb.	Preliminary load, lb.	Test load, lb.
1	3/4	13/16	down to solid	750	15 000	7 500
2	12/16	7/8	down to solid	850	17 500	8 500
3	7/8	15/16	down to solid	1 000	20 000	10 500
4	15/16	1	down to solid	1 250	25 000	11 500
5	1	1-1/16	down to solid	1 500	28 000	12 000
6	1-1/16	1-1/8	down to solid	2 000	36 000	15 000
7	1-1/4	1-1/4	down to solid	....	45 000	18 000

Distance between platens under tenth and eleventh applications of the preliminary load must not differ by more than 0.001 in. Washer is then removed from machine and maximum thickness of material measured by micrometer to 0.001 in. This thickness plus a reserve factor of 0.02 in. is called the test height and must be equaled or exceeded when the washer is subjected to the test load. The Brinnell hardness of helical washers must be from 402 to 477. All tests must be made with the specimen between 60° and 110° F.

Tie Plates reduce the unit pressure of the rail on the tie, and prevent it from cutting the tie. They relieve the lateral pressure of the rail against a spike, since the tie plate distributes the pressure between the two spikes. The pressure against the spikes is still further relieved by the lugs or corrugations which are found on the under side of some forms of plates, or if the plates are fastened to the tie by an independent fastening. The wear of spikes called "necking," which is caused by the vertical vibration of the rail against the spike, is very much reduced.

Design of Tie Plates. The Manual of the American Railway Engineering Association gives the following data for designing tie plates.

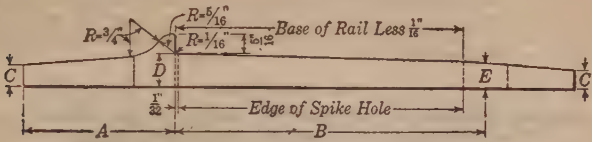


Fig. 20. Tie Plate Dimensions—A.R.E.A.

Dimensions for Tie Plates in Inches

American Railway Engineering Association

Length of plate	Range of rail bases	Same for flat and canted			Flat plates, D & E	Canted 1 in 40		Canted 1 in 20	
		A	B	C		D	E	D	E
9	4-3/4–5-1/2	2-3/8	5-3/8	5/16	9/16	9/16	7/16	3/4	15/32
9-1/2	4-3/4–5-1/8	2-5/8	5-9/16	5/16	5/8	5/8	1/2	3/4	15/32
10	4-7/8–5-1/2	2-7/8	5-3/4	5/16	5/8	5/8	15/32	3/4	15/32
10-1/2	5-1/8–5-3/4	3	5-15/16	3/8	11/16	11/16	17/32	13/16	17/32
11	5-3/8–6	3-1/4	6-1/8	3/8	11/16	11/16	17/32	13/16	1/2
11-1/2	5-3/4–6-1/4	3-1/2	6-5/16	3/8	3/4	3/4	19/32	13/16	1/2
12	6–6-1/2	3-5/8	6-1/2	3/8	3/4	3/4	19/32	13/16	1/2

Ribs not exceeding 1/4 in. deep on the bottom of plates are recommended, but the thicknesses given are exclusive of such ribs, if used. For extra heavy service D and E should be increased 1/8 in. Widths of 7 or 7-1/2 in. are recommended for hardwood ties. Spike holes should be made the size of the spike plus 1/8 in., and if splice bars are slotted, holes should be located accordingly. Plates may be rolled with a camber or crown of 1/16 in. if desired, the height of the shoulder then being measured from the center of the plate.

The specifications of the American Railway Engineering Association provide for steel, both Bessemer and open-hearth, of two grades, soft and medium, the soft to be used unless otherwise specified; and also for wrought iron and malleable iron.

The chemical requirements are that the steel shall not contain more than 0.10% phosphorus if made by the Bessemer process or more than 0.05% for the open-hearth. Carbon must be not less than 0.08% for soft steel or 0.12% for medium for the Bessemer process, not less than 0.15% and 0.20%, respectively, for open-hearth steel.

Bend test specimens taken from the finished plates or from rolled bars and longitudinally with the rolling must bend cold for 180 deg. around a pin the diameter of which is equal to the thickness of the specimen for the soft grade of steel and for wrought iron and to twice the thickness of the specimen for the medium grade of steel, without cracking on the outside of the bent portion.

An Order for Tie Plates should include the following items of information: (a) width of rail base; (b) size of spike under head; (c) number of spike holes, two, three or four; (d) spacing, lengthwise of rail, of slots in opposite angle bars on one tie; (e) net width, measured perpendicular to rail, between such slots. The proportion of joint plates to intermediates will depend on the number of ties per rail, on whether the joints are supported, suspended, or are three-tie joints. In the latter case there will be no slots in the angle bars over the middle joint tie, and the spacing of the holes must be a little wider than the extreme width, out to out, of the angle plates as assembled.

Setting Tie Plates. The efficiency depends largely on proper setting. Plates may be set on soft-wood ties by merely raising the rail, setting the plate squarely in place and driving the spikes. The first train passing over will force any corrugations or claws into the tie. The spikes should at once be driven home. Plates must be set in hardwood ties by direct driving. A follower plate of steel about 3/4 to 7/8 in. thick and about the size of the tie plate, will protect the tie plate from the direct blows of the swages used in driving. A still better method is to use a maul or beetle to strike a straddler which just straddles the rail and rests on the ends of the plate. A hand car carrying a hammer similar to a small-scale piledriver is far better and more effective than a hand maul. They are most economically and efficiently set in large numbers by means of a tie-plating machine.

14. Switches

Mechanical Features. At switches the flanges on the wheels make it necessary either to lift each wheel so that the flange may pass over the rail or else to make a break in the rail so that the flange may pass through the head of the rail. The latter method is virtually always used. The two types of switches are sketched in Figs. 21 and 22. Both use a frog at the intersection of the main rail and outer turnout rail, F, to provide **flangeways.** In the **stub switch** both of the main rails are cut at H and K (Fig. 21). The rails

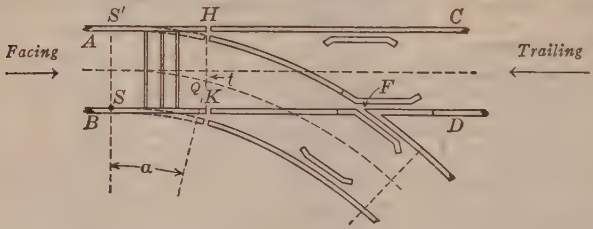


Fig. 21. Stub Switch

from S' to H and from S to K are not fastened to the ties, but are tied to each other by four or more tie-rods. The gaps at H and K are always sufficient to cause a considerable jar to the rolling stock. When operated as a trailing switch, a derailment is inevitable if the switch is incorrectly set. Their use is now confined to switches running from side tracks; they should never be usea in main tracks. In the **point switch** one main rail is unbroken. The point rail S'' H varies in length from 11 to 30 ft. and is planed and bent to give a straight gage line with a thickness of 1/4 in. at the point. The point is afterward ground to a thickness of 1/8 in. beginning at a point 2 ft. from the end. The angle of the switch point varies from about 0 deg. 45 min. to about 2 deg. 30 min. according to the dimensions and frog angle of the switch. There is an abrupt bend in the main rail at the point S', equal to the angle of the point rail. The point rails, $S''H$ and SK, are tied together with tie-rods. They are held in place by a very stiff spring (see Fig. 23) which will yield sufficiently to permit the wheels to remain on the rails if a train trails through with the switch incorrectly set. The **switch points** should fit the stock rail throughout the planing. The planing must be such that the point is 1/2 in. below the top of stock-rail at the end, rising to 1/4 in. above it in about 40% of length of point in order to care for hollow treads on car wheels. The rails of the frog are

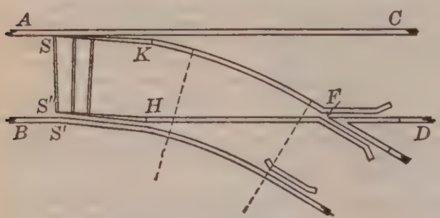


Fig. 22. Point Switch

always made straight; therefore the lead rails between the switch point and the frog must be curved to a circular arc which is tangent to both the switch rail and the wing rail.

Frogs. A diagrammatic design of a frog is given in Fig. 24. The frogs are always built up of four pieces of rail, which are usually of the cross-section and

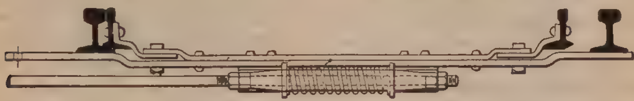


Fig. 23. Section of Main and Switch Rails about 2 ft. back of Point

weight used on the main track. The actual point of the frog is rounded off as shown. **Fixed frogs** are made either by riveting the four rails to a base plate,

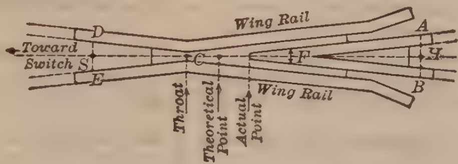


Fig. 24. Diagrammatic Design of Frogs

or else by placing cast-iron fillers between the rails and bolting or clamping the rails and fillers together or by a combination of riveting and bolting.

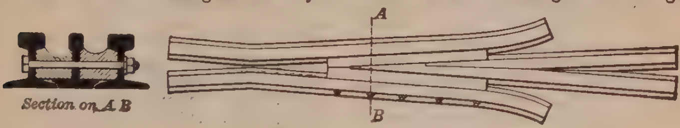


Fig. 25. Stiff Frog

Spring rail frogs usually have one wing rail (the one connecting with the main rail) movable (both may be made so) and yet normally pressing against

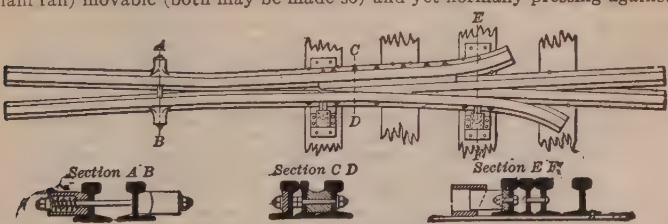


Fig. 26. Spring Rail Frog

the frog point so that there is no gap to be passed over by the wheels running on the main track. When wheels are running through the frog onto the switch, the guard rail opposite the frog forces the opposite wheel to run in its proper line, and this causes the inside of the flange of the wheel running through the frog to press against the wing rail and force it back so as to leave a sufficient opening between the wing rail and the frog point for the wheel flange to pass through. **Movable point frogs** operated with and in the same manner as switches controlling routes through them are used at slip switches in yards to avoid breaks in rail for high-speed traffic. The **frog number (n)** may be determined by dividing CH by AB (Fig. 24), or SH by $(AB + DE)$. When no tape or scale is available, use any unit of measure about 4 in. long, such as a stick or piece of paper, and find by trial two points on the frog where the distance between the gage lines (as at AB and DE) just equals the assumed unit of length. Then step off that unit of length between the points H and S and note the number of times the unit is contained in the distance HS. The result is twice the frog number.

Unbroken Main Rails. A switch which permits the use of unbroken main rails necessarily lifts the train a vertical height somewhat greater than the depth of the wheel flange, and as this must be accomplished in a distance of a very few feet, it is impossible to operate such switches at high speed. Although there are undoubted advantages in having an unbroken main rail, these devices have not come into common use.

The Dimensions of Stub Switches are usually computed on the basis that the lead rails are circular from the switch point to the frog point. For blunt frog angles such a theory may be practically applied, and the computations

Trigonometrical Functions of the Frog Angles

Frog No.	Frog angle F	Nat sin F	Nat cos F	Log sin F	Log cos F	Log cot F	Log vers F	Frog No.
4	14° 15′ 00″	.24615	.96923	9.39121	9.98643	10.59522	8.48812	4
4.5	12° 40′ 49″	.21951	.97561	.34145	.98927	.64782	.38721	4.5
5	11° 25′ 16″	.19802	.98020	.29671	.99131	.69461	.29670	5
5.5	10° 23′ 20″	.18033	.98360	.25606	.99282	.73676	.21467	5.5
6	9° 31′ 38″	.16552	.98621	.21884	.99397	.77513	.13966	6
6.5	8° 47′ 51″	.15294	.98823	.18453	.99486	.81033	.07059	6.5
7	8° 10′ 16″	.14213	.98985	.15269	.99557	.84288	8.00655	7
7.5	7° 37′ 41″	.13274	.99115	.12301	.99614	.87313	7.94691	7.5
8	7° 09′ 10″	.12452	.99222	.09522	.99661	.90138	.89111	8
8.5	6° 43′ 59″	.11724	.99310	.06909	.99699	.92791	.83865	8.5
9	6° 21′ 35″	.11077	.99385	.04442	.99732	.95290	.78915	9
9.5	6° 01′ 32″	.10497	.99448	9.02107	.99759	.97652	.74232	9.5
10	5° 43′ 29″	.09975	.99501	8.99891	.99783	10.99892	.69787	10.0
10.5	5° 27′ 09″	.09502	.99548	.97782	.99803	11.02021	.65559	10.5
11	5° 12′ 18″	.09072	.99588	.95770	.99821	.04051	.61527	11
11.5	4° 58′ 45″	.08679	.99623	.93849	.99836	.05987	.57676	11.5
12	4° 46′ 19″	.08319	.99653	.92007	.99849	.07842	.53986	12
14	4° 05′ 27″	.07134	.99747	.85332	.99889	.14735	.40616	14
15	3° 49′ 06″	.06659	.99778	.82343	.99904	.17561	.34631	15
16	3° 34′ 47″	.06244	.99804	.79544	.99915	.20371	.29028	16
18	3° 10′ 56″	.05551	.99846	.74438	.99933	.25494	.18807	18
20	2° 51′ 51″	.04997	.99875	.69869	.99946	.30076	7.09663	20
24	2° 23′ 13″	.04165	.99913	8.61959	9.99962	11.38003	6.93835	24

are very simple. Let l = the lead or the distance SF in Fig. 21, r = mean radius of the switch rails, n = number of the frog, g = gage of track. Then $\mathbf{l = 2\,gn}$ and $\mathbf{r = nl = 2\,gn^2}$. The length of switch rails ($S'H = SK$ in Fig. 21) must be such that the offset to the curve at the point Q shall equal the required switch-throw. If t = the switch-throw, then t/r = vers of the angle (α) subtended by the switch rails; then length of switch rails = $r \sin \alpha$. The above theory ignores the fact that the wing rail between the frog point and its junction with the switch rails is straight rather than curved.

A Point Switch has straight point rails also, Fig. 27. The effect of these two details, straight wing and switch rails, is to shorten the lead and therefore decrease the radius of the switch rails.

Let α = angle of point rails;
F = angle of frog = ϕ in Fig. 27;
g = gage of track;
n = frog number;
w = length of wing rail = FH;
c = chord of switch rail arc = HM;
t' = thickness of switch point;
s = length of switch rail = MD;
L = length of lead = BF;
r = radius of center line of lead curve = OT = OV;
h = heel distance = MQ;
f = distance FF',
t = thickness of frog point.

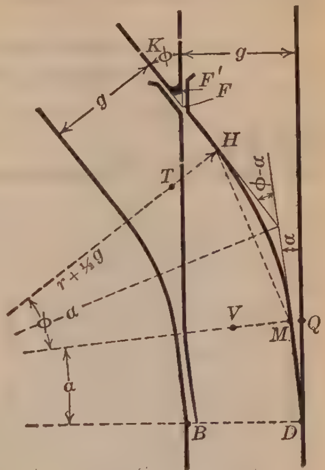


Fig. 27. Point Switch

Then

$$c = \frac{g - w \sin F - s \sin \alpha - t'}{\sin 1/2\,(F + \alpha)}$$

$$r = \frac{c}{2 \sin 1/2\,(F - \alpha)} - 1/2\,g$$

or $$r = \frac{g - w \sin F - h}{\cos \alpha - \cos F} - 1/2\,g$$

$$L = (g - w \sin F - h) \cot 1/2\,(F + \alpha) + w \cos F + s \cos \alpha + f$$

or $$L = (r + 1/2\,g)(\sin F - \sin \alpha) + w \cos F + s \cos \alpha + f$$

or $$L = (s - w)\frac{\sin 1/2\,(F - \alpha)}{\sin 1/2\,(F + \alpha)} + (g - t') \cot 1/2\,(F + \alpha) + f$$

$$\alpha = \sin^{-1}\left(\frac{h - t'}{s}\right)$$

The dimensions of a point switch for a given frog number, as computed from the above equations, depend on the values of w, s, t' and h, all of which may be chosen at pleasure with certain narrow limitations. The length of the wing rail ($w = FH$), as well as other frog dimensions, depends (within narrow limitations) on the switch manufacturer. The A.R.E.A. recommends as standard practice for standard gage (g = 4 ft. 8-1/2 in.), 6-1/4 in. for h and 1/4 in. for t' (for computation purposes, actual thickness is 1/8 in. as already given) and values of s and w for the various frog numbers as given in the table on page 2046.

The lead (L) is measured from the point of the switch to the actual blunt point of the frog (F'). This distance is greater than the distance to the theoretical point of the frog by the amount of the "frog bluntness (f)." On the basis that the actual width of the blunt point is 1/2 in. as recommended the frog bluntness equals the frog number times 1/2 in. or times 0.0417 ft. and is given in the second column in the table.

The closure is the distance from the heel of the switch rail, or "point," to the toe of the frog and equals, for the straight rail, $L-(s + w + f)$ and, for the curved rail, $2\pi\,(R + 1/2\,g) \times (F - \alpha)/360$, or, approximately $(F - \alpha)/D$.

The lengths and closures given in the table are the theoretical ones for the dimensions given, those for other values of s, t', h and w are easily computed by means of the formulas. The theoretical closures are usually modified in practice to avoid, if possible, cutting more than one rail per turnout, giving "practical" closures and "practical" leads (or lengths) as given in the table below.

Switch and Frog Dimensions

(Condensed from Manual of A.R.E.A., 1921 Ed.)

Frog number	Frog bluntness	Frog		Switch rail		Switch dimensions				
		Wing rail	Total length	Length	Angle	Radius	Degree of lead curve	Length	Closure	
								$L = BF'$	Straight rail	Curved rail
n	f	w	HK	s	α	r	D			
	ft.	ft. in.	ft. in.	ft.	° ′ ″	ft.	° ′ ″	ft.	ft.	ft.
5	0.21	3 4	9 0	11	2 36 19	185.59	31 15 28	43.15	28.61	28.96
6	0.25	3 6	10 0	11	2 36 19	280.48	20 32 14	48.66	33.91	34.18
7	0.29	4 5	12 0	16-1/2	1 44 11	364.88	15 47 19	62.23	41.02	41.24
8	0.33	4 9	13 0	16-1/2	1 44 11	488.71	11 44 40	67.80	46.22	46.42
9	0.37	6 0	16 0	16-1/2	1 44 11	616.27	9 18 27	72.61	49.64	49.92
10	0.42	6 0	16 6	16-1/2	1 44 11	790.25	7 15 18	77.93	55.01	55.17
11	0.46	6 0	17 0	22	1 18 08	940.21	6 05 48	92.52	64.06	64.20
12	0.50	6 5	18 6	22	1 18 08	1136.34	5 02 38	97.75	68.83	68.96
14	0.58	7 3	21 6	22	1 18 08	1600.73	3 34 48	107.74	77.91	78.03
15	0.62	7 8	22 6	30	0 57 18	1764.69	3 14 50	126.49	88.20	88.31
16	0.67	8 0	24 0	30	0 57 18	2032.74	2 49 08	131.82	93.15	93.25
18	0.75	8 10	26 6	30	0 57 18	2632.76	2 10 35	141.93	110.35	110.44
20	0.83	9 8	29 0	30	0 57 18	3334.16	1 43 06	151.60	120.10	120.18

Practical Leads and Closures and Ordinates for Curving Lead Rails

For Standard Turnouts of A.R.E.A.

Frog number	Lead	Closures		Ordinates			
		Straight rail	Curved rail	Center		Quarter points	
	ft.			ft.	in.	ft.	in.
5	42.54	1–28	1–28.31	0.55	6-5/8	0.41	4-7/8
6	47.50	1–32.75	1–33	0.51	6-1/8	0.38	4-1/2
7	62.08	1–26 1–14.87	1–26 1–15.12	0.58	7	0.43	5-1/8
8	68.00	1–30 1–16.42	1–30 1–16.58	0.55	6-5/8	0.41	4-7/8
9	72.29	1–33 1–16.42	1–33 1–16.58	0.50	6	0.38	4-1/2
10	78.75	1–28 1–27.83	2–28	0.48	5-3/4	0.36	4-3/8
11	94.31	1–33 1–32.85	2–33	0.55	6-5/8	0.41	4-7/8
12	100.80	2–24 1–23.88	3–24	0.52	6-1/4	0.39	4-3/4
14	106.27	2–30 1–16.44	2–30 1–16.56	0.47	5-5/8	0.35	4-1/4
15	126.19	2–30 1–27.90	2–30 1–28	0.55	6-5/8	0.41	4-7/8
16	131.56	1–30 1–33 1–29.90	2–30 1–33	0.53	6-3/8	0.40	4-3/4
18	138.50	2–33 1–32.92	3–33	0.50	6	0.38	4-1/2
20	151.46	2–33 1–30 1–14.96	2–33 1–30 1–15.04	0.47	5-5/8	0.35	4-1/4

Rails shorter than 12 or 13 ft. should not be used. The maximum modification from the theoretical leads is slightly over 3%, but modifications up to 6 to 8% in either direction may be made without seriously affecting the riding qualities of the turnout.

Turnouts from Curved Tracks. (a) *Stub Switch, lead rails considered to be curved with uniform curvature from switch point to frog point.*

1. Dimensions for turnout from the inner side of a curved track; see Fig. 28.

$$\tan 1/2\,\theta = gn/R \qquad (r + 1/2\,g) = (R - 1/2\,g)\frac{\sin\theta}{\sin(F+\theta)}$$

$$\text{Lead} = BF = 2\,(R - 1/2\,g)\sin 1/2\,\theta$$

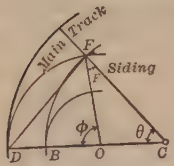


Fig. 28

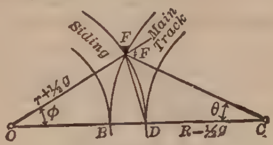


Fig. 29

2. Dimensions for turnout from the outer side of a curved track; see Fig. 29.

$$\tan 1/2\,\theta = gn/R \qquad (r + 1/2\,g) = (R + 1/2\,g)\frac{\sin\theta}{\sin(F-\theta)}$$

If the curvature of the main track is very sharp, θ may be greater than F and then the center of curvature C will be on the concave side of the curved main track. Figure 28 may then be used for this case by merely transposing O and C, ϕ and θ, and "main track" and "siding." The equation for r then becomes

$$(r - 1/2\,g) = (R + 1/2\,g)\frac{\sin\theta}{\sin(\theta - F)} \qquad \text{Lead} = BF = 2\,(R + 1/2\,g)\sin 1/2\,\theta.$$

The lead is practically the same as for straight track and is most easily found from the formula, **Lead = BF = 2 gn.** Then $\theta = 2\,gnD$ where D is degree of main track curve. The degree of the turnout curve is also approximately that of the main track curve plus or minus that of the turnout curve from straight track for the same frog number according to whether the turnout is from the inner or outer side of the main track curve.

(b) *Stub Switch, with straight frog.* Frogs are usually made straight, with the result on straight track of shortening each tangent to the turnout curve, and therefore the lead, by the length of the leg of the frog. The radius and degree of the new turnout curve are easily computed for the new tangent length. For curved track the effect on lead may be assumed the same as on straight track and the new degree of curve used as above.

(c) *Point Switch.* The gage lines of the switch rails have the curvature of the track so that the turnout at the heel of the switch rail makes the angle s with the main track the same as for straight track. The precise mathematical computations are quite complicated and it will be sufficient to assume the same difference in lead between stub and point switches as obtains on straight track, that is, use the table for leads, and to find the degree of the turnout curve in the manner described above for the stub switch.

Double Turnout from Straight Track. The computations are quite simple when it is assumed that all of the frog rails as well as the switch rails are circular throughout. As may be seen from Fig. 30, the dimensions would be considerably changed by assuming the frog rails to be straight, and much depends on the length of the straight rails. In any case, the sections of curved rails would be very short. Only the equations for curved lead rails will here be given.

$$\text{vers } 1/2\, F_m = \frac{g}{2\,(r + 1/2\, g)} \qquad MF_m = \frac{r}{2\, n_m}$$

in which n_m is the frog number corresponding to the middle frog F_m. The above equations also depend on the assumption that F_l and F_r are equal. Their values are obtained from the relation that vers $1/2\ F_m = 1/2$ vers $F_l = 1/2$ vers F_r. Although approximate, there is no sensible error in the relation $n_m = 0.707\ n$ between the frog numbers, in which n_m is the frog number of F_m, and n is the frog number of F_l and F_r.

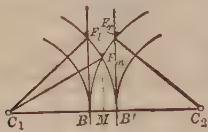


Fig. 30

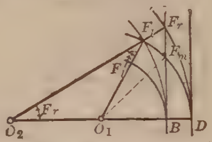


Fig. 31

If both turnouts run to the same side as indicated in Fig. 31, the relation of the frogs will be practically identical with those in the previous paragraph, since the inner switch may be regarded merely as a curved main track, and since, as previously shown, the dimensions of switches are but little altered by a moderate curvature of the main track.

Connecting Curves from Turnouts. The following solutions for connecting curves and crossovers between straight parallel tracks are based on the use of straight frog rails beyond the frog point. Let d = distance between track centers, g = gage, F = frog angle, w' = length of wing rail back of theoretical frog point = $(HK - w)$ of the Switch Table = DF in the figures,

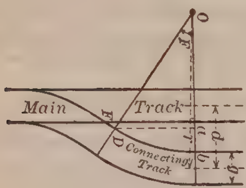


Fig. 32

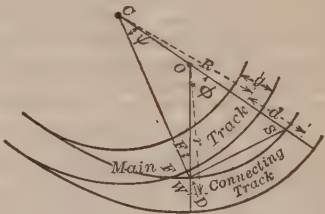


Fig. 33

R = radius of main curved track, r = radius of connecting curved track, n = f og number corresponding to frog angle F, ψ = central angle, measured at center C of main curved track, required for connecting curve, ϕ = central angle, required for connecting curve, measured at center O of connecting curve.

For a connecting curve from a straight track, see Fig. 32.

$$(r - 1/2\, g) = \frac{d - g - w' \sin F}{\text{vers } F}$$

Although the distance from F to the "fouling point" (which is on the line ab) may be shortened somewhat by using a sharper curve and using a short tangent at each end, the method indicated in Fig. 32 is preferable. Under

Connecting Curve from Turnout, Straight Main Track

Frog number	Distance, ft.	Radius, ft.	Degree of curve	Frog number	Distance, ft.	Radius, ft.	Degree of curve
			° ′ ″				° ′ ″
5	85.06	424.93	13 29 04	12	186.93	2102.35	2 43 32
6	93.41	525.63	10 54 16	14	217.91	2855.30	2 00 24
7	108.87	713.19	8 02 25	15	234.04	3292.44	1 44 25
8	124.68	935.56	6 07 37	16	248.45	3723.27	1 32 20
9	139.38	1168.59	4 54 16	18	281.30	4749.75	1 12 23
10	156.55	1464.51	3 54 47	20	312.23	5863.15	0 58 38
11	171.38	1768.46	3 14 25				

such conditions, using the standard dimensions given in the Switch Table, the distance from F to the " fouling point " measured along the main track, is a definite quantity for any frog number, and is as given in the accompanying table for $d = 13$, $g = 4{,}708$.

For a connecting curve on the outside of a main curved track, see Fig. 33.

$$\tan 1/2\, \psi = \frac{2\,n\,(d - g - w' \sin F)}{2\,R + d + w' \sin F}$$

$$(r - 1/2\, g) = (R + 1/2\, g + w' \sin F\, \frac{\sin \psi}{\sin (F + \psi)}$$

$$DS = 2\,(r - 1/2\, g) \sin 1/2\, (F + \psi)$$

For a connecting curve on the inside of a main curved track, see Fig. 34.

$$\tan 1/2\, \psi = \frac{2\,n(d - g - w' \sin F)}{2\,R - d - w' \sin F}$$

$$(r + 1/2\, g) = (R - 1/2\, g - w' \sin F)\, \frac{\sin \psi}{\sin (\psi - F)}$$

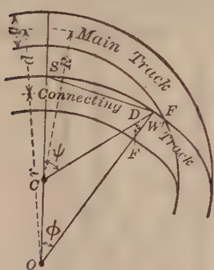


Fig. 34

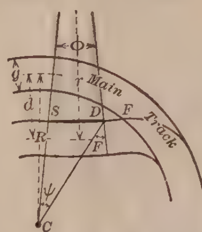


Fig. 35

In case the frog angle F is larger than ψ, the center of curvature of the connecting curve rails will be on the other side of the main track as shown in Fig. 35. Then

$$(r - 1/2\, g) = (R + 1/2\, g - w' \sin F)\, \frac{\sin \psi}{\sin (F - \psi)}$$

In case ψ should exactly equal F, the connecting rails would be straight and r = infinity. This is true when

$$2\,R - d - w' \sin F = 4\,n^2\,(d - g - w' \sin F)$$

The solution of this equation will show the value of R which makes this case possible for any given values of n, d, g and w'.

Crossover between Straight Parallel Tracks. With straight connecting track, the length between the frog wing rails

$$VZ = \frac{d - g}{\sin F} - 2\,w' - g \cot F.$$

The distance between frog points measured along either track

$$F_2Y = (d = g) \cot F - \frac{g}{\sin F}$$

The distance (measured along either track) from the switch point on one track to the switch point on the other track equals F_2Y plus twice the theoretical, or practical, lead minus twice the frog bluntness.

Distance between Switch Points, Crossover between Straight Parallel Tracks, with 13 Ft. between Track Centers

Frog number	F_2Y	Total distance, ft.		Frog number	F_2Y	Total distance, ft.	
		Theoretical	Practical			Theoretical	Practical
5	17.29	103.17	101.95	12	42.73	237.23	243.33
6	20.96	111.78	115.46	14	49.95	264.27	261.33
7	24.62	148.50	148.20	15	53.54	305.26	304.68
8	28.25	163.19	163.59	16	57.15	319.45	318.93
9	31.89	176.37	175.73	18	64.32	346.68	339.82
10	35.51	190.53	192.17	20	71.50	373.04	372.76
11	39.13	223.25	226.83				

With reversed curve connecting track, but with $F_1 = F_2 = F$, $w_1 = w_2 = w'$, and $r_1 = r_2 = r$, an indefinite number of combinations of r and θ may be selected by choosing some reasonable value of r in the following equation and solving the equation for θ, which is the only remaining unknown quantity,

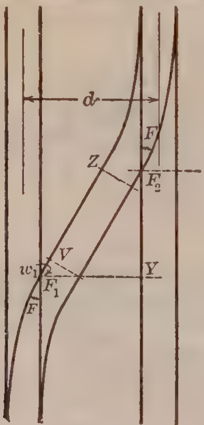


Fig. 36

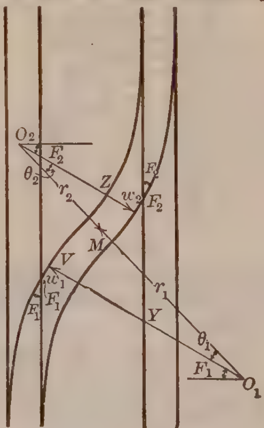


Fig. 37

provided the frog angles are equal. (Similar to Fig. 37.) In this case the point M is midway between the tracks and between the frogs. The use of reversed curves saves distance measured along the main track, as shown in the illustration.

$$1/2\,(d - g) = w' \sin F + 1/2\, g \cos F + r\,[\text{vers}\,(F + \theta) - \text{vers}\,F]$$

and $$F_2Y = 2\,[w' \cos F + 1/2\, g \sin F + r\,[\sin\,(F + \theta) - \sin F]]$$

In case it is desired to make the crossing with two unequal frogs it may be done within limitations as shown in Fig. 37. The following equation may be derived:

$$d - g = w_1 \sin F_1 + 1/2\, g\,(\cos\, F_1 + \cos F_2) + r_1\,[\text{vers}\,\,(F_2 + \theta_1) - \text{vers}\,F_1] + r_2\,[\text{vers}\,(F_2 + \theta_2) \to \text{vers}\,F_2] + w_2 \sin F_2.$$

But since $(F_1 + \theta_1)$ must always equal $(F_2 + \theta_2)$ above equation becomes

$$\text{vers}\,\,(F_1 + \theta_1) = \frac{d - g - w_1 \sin F_1 - w_2 \sin F_2 - 1/2\, g\,(\cos F_1 + \cos F_2) + r_1\,\text{vers}\,F_1 + r_2\,\text{vers}\,F_2}{r_1 + r_2}$$

All of the quantities in the right-hand side of the above equation are known except r_1 and r_2. Their values may be selected at pleasure (within limitations) and may be equal or unequal. The distance along the track

$$F_2Y = w_1 \cos F_1 - 1/2\, g \sin F_1 + r_1\,[\sin\,(F_1 + \theta_1) - \sin F_1] + w_2 \cos F_2 - 1/2\, g \sin F_2 + r_2\,[\sin\,(F_2 + \theta_2) - \sin F_2]$$

Ties for Turnouts and Crossovers. The following table gives the number of ties of various lengths for typical turnouts and crossovers given in the book of "Track Plans" of the Amer. Rwy. Eng. Assoc. These are believed sufficient to give the necessary flexibility but others may be easily worked up on the same basis. "Practical" leads with corresponding closures are used throughout. Standard ties are 7 × 9 in., 8 ft. 6 in. long and distance between track centers is 13 ft.

Number and Lengths of Ties in Turnouts and Crossovers

A.R.E.A. "Track Plans"

Length	Turnouts Frog number							Crossovers Frog number						
	6	7	8	10	11	16	20	6	7	8	10	11	16	20
ft. in.														
9 0	6	9	9	9	13	17	17	12	18	18	18	26	34	34
9 6	4	6	6	7	11	9	12	8	12	12	14	22	18	24
10 0	3	4	5	5	6	9	12	6	8	10	10	12	18	24
10 6	3	3	4	5	5	9	8	6	6	8	10	10	18	16
11 0	2	3	3	4	4	7	8	4	6	6	8	8	14	16
11 6	2	2	2	4	4	7	8	4	4	4	8	8	14	16
12 0	2	2	3	3	3	7	8	4	4	6	6	6	14	16
12 6	2	2	3	4	4	6	8	4	4	6	6	8	12	16
13 0	2	3	2	4	3	5	6	4	6	4	....	....	....	....
13 6	2	2	3	3	4	5	7	....	....	....	....	....	....	....
14 0	2	2	2	3	4	5	6	....	....	....	....	....	....	....
14 6	2	2	2	3	3	5	6	....	....	....	....	....	....	....
15 0	3	4	4	5	5	7	8	4	4	4	4	4	4	4
15 6	2	2	3	3	3	5	6	....	....	....	....	....	....	....
16 0	2	2	2	3	3	4	6	....	....	....	....	....	....	....
16 6	2	3	3	4	4	4	6	....	....	....	....	....	....	....
21 6	....	....	....	....	....	....	....	15	18	19	31	32	45	53
Total B.M.	2617	3203	3523	4407	4925	6967	8393	4880	6059	6507	8157	9282	13160	15191
Distance	19.54	22.04	26.13	35.71	37.50	54.57	73.42	5.96	6.97	7.97	9.97	10.98	15.98	19.99

These standards show a maximum spacing of 21 in. and a minimum of 17 in., the detailed spacing being arranged to give increased support to switch points and frogs and to give suspended joints with the arrangement of lead rails recommended. The two ties for the switch block are 15 ft. long and the maximum length for turnouts is 6 in. less than double the standard. For crossovers the long ties (21 ft. 6 in.) are used for the entire distance between the toes of the frogs.

Total B.M. is in feet, board measure, on the basis of 7 × 9-in. ties and the distance given in the last line for turnouts is from the actual frog point to the center of the last tie provided for in the table. Beyond that point standard ties are used. For crossovers the distance is the change in distance between frog points measured along either track due to change of 1 ft. in distance between track centers.

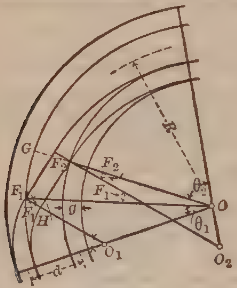


Fig. 38

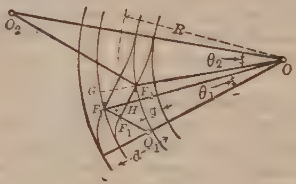


Fig. 39

Crossover between Curved Parallel Tracks. (a) *Using a straight connecting track*: There are two cases as illustrated in Figs. 38 and 39, depending on whether F_2 is greater or less than θ_2. The following equations apply to both figures. One frog (say F_1) may be chosen at pleasure within narrow limitations, then the relation between F_1 and F_2 is given by

$$\cos \mathbf{F}_2 = \cos \mathbf{F}_1 \frac{\mathbf{R} + 1/2\,\mathbf{d} - 1/2\,\mathbf{g} - \mathbf{g} \sec \mathbf{F}_1}{\mathbf{R} - 1/2\,\mathbf{d} + 1/2\,\mathbf{g}}$$

The distance between the frogs measured along the inner rail of the outer track is

$$\mathbf{GF}_1 = 2\,(\mathbf{R} + 1/2\,\mathbf{d} - 1/2\,\mathbf{g}) \sin 1/2\,(\mathbf{F}_1 - \mathbf{F}_2)$$

F_2 will not in general equal the angle of any standard frog, even though the frog F_1 is standard.

(b) *Using a reverse curve for the connecting curve*: In Fig. 40 the lead rails are assumed to be circular throughout, and the lead rail curves are assumed to have been continued until they meet at the point of reverse curve. Within narrow limitations F_1 and F_2 may be selected at pleasure, and will of course be made of standard numbers. The problem should be solved on this basis, which will determine the dimensions of the connecting curve, and then the switch rails may be altered if desired.

$$\text{vers}\,\psi = \frac{d\,(r_1 + r_2 - 1/2\,d)}{(R - 1/2\,d + r_2)\,(R + 1/2\,d - r_1)};\ \sin OO_2O_1 = \sin \psi \frac{R + 1/2\,d - r_1}{r_1 + r_2}$$

$$O_2O_1D = \psi + O_1O_2O;\ NF_2 = 2\,(R - 1/2\,d + 1/2\,g) \sin 1/2\,(\psi - \theta_1 - \theta_2).$$

Crossings. For *two straight tracks* the frog angles are the angle (and its supplement) made by the two alignments. For *one straight and one curved track*, Fig. 41 applies with these equations:

$$\cos F_1 = \frac{R\cos M + 1/2\,g}{R - 1/2\,g}, \qquad \cos F_2 = \frac{R\cos M + 1/2\,g}{R + 1/2\,g}$$

$$\cos F_3 = \frac{R\cos M - 1/2\,g}{R + 1/2\,g}, \qquad \cos F_4 = \frac{R\cos M - 1/2\,g}{R - 1/2\,g}$$

$$F_3F_4 = (R + 1/2\,g)\sin F_3 - (R - 1/2\,g)\sin F_4$$

$$HF_4 = (R - 1/2\,g)(\sin F_4 - \sin F_1)$$

$$F_1F_2 = (R + 1/2\,g)\sin F_2 - (R - 1/2\,g)\sin F_1$$

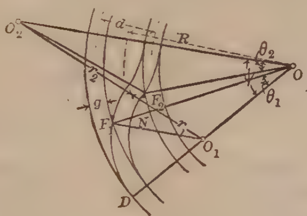


Fig. 40

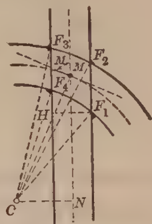


Fig. 41

For *two curved tracks* all four frogs are unequal, see Fig. 42. The radii of the two tracks are of course known and also the angle of intersection of their tangents (M). Then r_1, r_2, r_3, and r_4 become known by adding (or subtracting) 1/2 g to the known track center radii. In the triangle C_1MC_2,

$$1/2\,(C_1 + C_2) = 90° - 1/2\,M$$

$$\tan 1/2\,(C_1 - C_2) = \cot 1/2\,M\,\frac{R_2 - R_1}{R_2 + R_1}$$

From these equations C_1 and C_2 become known.

Then $$c = C_1C_2 = R_2\,\frac{\sin M}{\sin C_1}$$

For abbreviation $s_1 = 1/2\,(c + r_1 + r_4)$; $s_2 = 1/2\,(c + r_2 + r_4)$; $s_3 = 1/2\,(c + r_1 + r_3)$ and $s_4 = 1/2\,(c + r_2 + r_3)$.

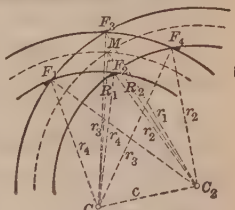


Fig. 42

Then $$\operatorname{vers} F_1 = \frac{2\,(s_1 - r_1)(s_1 - r_4)}{r_1r_4} \qquad \operatorname{vers} F_2 = \frac{2\,(s_2 - r_2)(s_2 - r_4)}{r_2r_4}$$

$$\operatorname{vers} F_3 = \frac{2\,(s_3 - r_1)(s_3 - r_3)}{r_1r_3} \qquad \operatorname{vers} F_4 = \frac{2\,(s_4 - r_2)(s_4 - r_3)}{r_2r_3}$$

$$\sin C_1C_2F_4 = \sin F_4\,\frac{r_3}{c};\quad \sin C_1C_2F_2 = \sin F_2\,\frac{r_4}{c};\quad F_2C_2F_4 = C_1C_2F_4 - C_1C_2F_2$$

$$\sin F_1C_1C_2 = \sin F_1\,\frac{r_1}{c};\quad \sin F_2C_1C_2 = \sin F_2\,\frac{r_2}{c};\quad F_1C_1F_3 = F_1C_1C_2 - F_2C_1C_2$$

From these equations the chords F_1F_2 and F_2F_4 are readily computed.

15. Stock Guards

Essential Features. A stock guard consists essentially of wing fences extending from the right-of-way fences to **aprons** which are short sections of fence, usually flaring, set parallel with the track at the track end of the wing fences; and a pit or a rough surface, on which stock will not walk, which extends the entire width of the track between the outer ends of the ties. A pit is about 3 ft. deep, 5 ft. long (parallel with track) and as wide as necessary; it has vertical walls of masonry or wood; the rails are supported on two wooden stringers spanning the pit. A surface guard covers the ties for a distance of about 8 ft. with some one of a variety of wooden or metal slats or tiles.

Recommended Practice. The American Railway Engineering Association recommends surface guards in preference to pit guards, chiefly on account of the disastrous consequences of derailment at the guard or a failure of the pit structure. A surface guard should be so designed that it will not catch dragging brake chains or other rigging; it should not endanger employees who must walk over it; it should not rattle during the passage of trains; it should be reasonable in first cost, durable and easily applied or removed for track repairs: finally, it should be effective to deter all kinds of livestock from attempting to cross and yet should not catch and hold such as should make the attempt.

Surface Guards. Home-made wooden guards consist of slats about 8 ft. long, 2-1/2 in. wide, 4 in. high, spaced 4-1/2 in. c.c. using 2-in. fillers and tied together with three 3/8-in. round rods running through slats and fillers. The slats are chamfered on the upper corners. Metal guards are usually patented and consist of rods (replacing the slats), or some one of a variety of designs in stamped sheet metal which, by bending out lugs, make a rough surface, or of rollers. A surface may also be made by a combination of interlocking tiles about 4 in. wide, 15 ft. long, and with their upper surface forming ridges. This type is free from rust or decay but is more liable to breakage.

16. Yards and Terminals

(Condensed from Definitions and Recommended Practice of the American Railway Engineering Association, as published in its " Manual.")

A Yard is a system of tracks within defined limits for receiving, separating and making up trains, storing cars and other purposes. A large yard will have some or all of the following features. An incoming train leaves the main track and enters a receiving yard where the train may wait temporarily until the separation of the cars begins or may be moved to a separate holding yard if not ready for the separating yard (or classification yard). The train runs from the receiving yard to the separating yard, in which the cars are separated according to district, commodity or other required order. Some cars will be run to a classification yard in which cars are classified or grouped in accordance with requirements preliminary to forwarding in trains although sometimes a sorting yard, in which cars are classified in greater detail, is provided between the classification and departure yards. Some cars may be sent to a storage yard, awaiting further disposition. Others will be sent to special tracks referred to later. The cars which are to be sent out of the yard are then sent to the departure or forwarding yard, where they are assembled into trains. The distribution of separate cars onto various tracks is often accomplished by pushing them over a summit, beyond which they run by gravity. Such a yard is called **a summit or hump yard.** In rare cases natural grades are such that cars will run down them onto classification tracks, giving a true **gravity yard.** A yard in which the classification is accomplished by the use of a pole operated by an engine on an adjacent parallel track is called a **poling yard.** A yard, usually small or cramped for room, in which movement of cars is accomplished entirely by engines, is called a **tail-switching** or **"push**

and pull" yard. Tracks in a group or a set of parallel tracks all used for some one purpose are called **body tracks.** They should be spaced from 13 to 14 ft. c.c. A group of such tracks all lead into a **ladder track** which should be not less than 15 ft. c.c. from any parallel track. No. 8 is the minimum frog number which should be used on such tracks. The track connecting either end of the yard with the main line is called a **lead track**, which should be interlocked with the main line. Running tracks and open tracks are provided so as to permit the free movement of cars and switching engines from any portion of the yard to any other. When riders are used to control cars on classification tracks a rider track may be used for the gasoline car which takes them back to the hump or they may be required to walk back. Many large yards are now equipped with car retarders which control the speed of cars by pressing against the sides of the wheels. These, together with switches, and "skates" for stopping cars quickly, are operated from towers by one man in each tower, thus eliminating riders and switch tenders. Caboose tracks should be located between the receiving and forwarding yards, and are preferably so constructed that a caboose may be readily pushed thereon from a receiving track and then dropped by gravity to the train departing in the direction from which the caboose has arrived. Scale tracks should be located between the receiving and separating yards. Coaling, ash-pit, sand and engine tracks should be located on the route to and from the engine house; they should be so arranged that water, coal and sand may be taken and ashes disposed of in convenient rotation and also provide that switching engines may clean fires, take coal, water and sand and pass around the waiting engines. Bad-order tracks should be convenient to the classification yard, so that bad-order cars may be set off and easily run to the repair tracks. The repair tracks should have a maximum capacity of about 15 cars each and should be spaced alternately 16 and 24 ft. apart c.c. Fifty linear feet of track should be estimated in rating the capacity of freight-car repair tracks, in order to provide working room about each car. Parts of the yards should be provided with air and water pipes with outlets 50 ft. apart for testing cars. A materials supply track should be placed in the space between each pair of repair tracks. Heavy freight car repairs should be made under cover in a shop provided with overhead traveling cranes to facilitate heavy lifting. Icing tracks should be located between the receiving and separating yards so that the cars to be iced may readily be moved from the receiving to the icing tracks and thence to the separating yard. A coach-cleaning yard should be located for ready and quick access to and from the station. The tracks should be long enough to accommodate trains without cutting, and should preferably be stub-ended, with a car cleaners' repair supply building located at right angles at their ends. Yard track capacity is estimated by allowing 42 linear ft. of track to each car. Team delivery yards should be located convenient to the freight house so that the receipt and shipment of freight may be easily under the control of the freight agent's force. The tracks should be stub tracks in parallel pairs, the tracks of each pair 12 ft. between centers and the pairs 52 ft. between centers. For convenience of shifting, the tracks should have a capacity of about 20 cars each. A crane for handling heavy freight should be provided. If possible, ingress and egress for teams should be provided for at each end of each teamway. Wagon scales should be provided near the team entrance of the yard, and track scales should be provided and located for convenient switching.

A Hump Yard should have receiving, classification and departure tracks. Trains may be handled through it faster and at less cost than through any other form of yard. The receiving tracks should be of sufficient length to hold a maximum train, and should be sufficient in number to hold as many trains, arriving in quick succession, as the character of the traffic renders

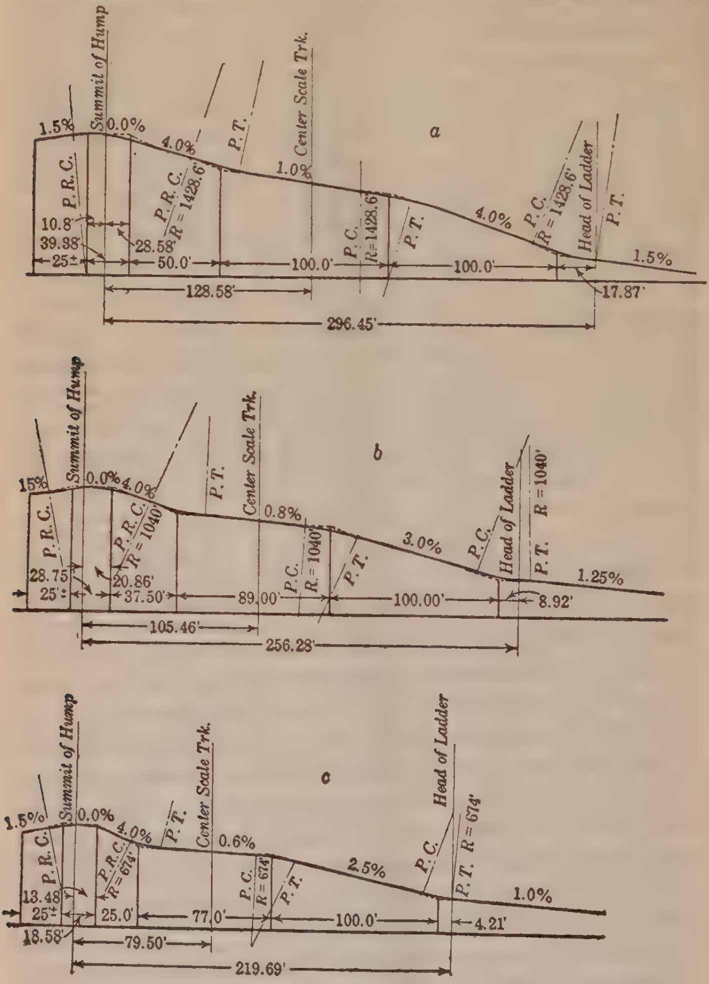


Fig. 43

probable. If possible the grades of the receiving tracks should be such that one engine can push the maximum train over the hump. The length and number of the classification tracks depend on the number of classifications required, number of cars in each classification, whether or not a sorting yard is used, etc. Classification of through traffic may often be shifted from one yard to another with advantage. The departure tracks should be of full train length and sufficient in number to provide ample standing room for trains while being tested for air and while waiting for engines. An air hose and an air brake testing plant shall be provided conveniently near the departure tracks. The grades recommended for hump yards by the American Railway Engineering Association are shown in Fig. 43, *a* being for cold, *b* for moderate and *c* for warm climate. These are for mixed traffic; empty cars require somewhat steeper grades and loaded cars lighter ones. Where the traffic and climatic conditions require it, the grades should be made steeper in winter and restored again in the spring. When required, scales should be located as shown so that when the car to be weighed reaches the scales it will be properly spaced from the following cars and running slowly enough to render correct weighing easy. For average conditions it is recommended that a No. 8 frog be the sharpest used for classification yards.

Freight Houses. For an inbound-freight house 50 ft. is recommended as good average width. Platforms should be provided beside the tracks and continuous doors so as to obviate the necessity for spotting cars. For an outbound freight house the recommended width is 30 ft. It is not advisable to load through more than four to six cars. For a very great number of cars there should be stub tracks in pairs with covered platforms between the pairs, the platforms leading to the freight house at the ends of the tracks. For a roadway between freight house on one side and wall on the other, minimum width 30 ft.; with team track or another freight house on the other side, minimum clear width 40 ft.

Stock Yards for receiving cattle vary from a small pen at a way station to a series of large pens at a terminal. They include arrangements for feeding and watering, as well as loading and unloading.

STEAM RAILROAD EQUIPMENT

17. Water Stations

Chemical Treatment is necessary when the water contains objectionable amounts of incrusting or corrosive matter or of alkali salts and is desirable for practically all natural waters, especially those containing even a fraction of a grain of calcium or magnesium sulfate. When water is boiled at a pressure of over 60 lb. the carbonates of lime and magnesia will precipitate and form mud or soft scale whose presence in the boiler is objectionable. It may be blown off but this wastes water and more or less heat. The sulfates of lime and magnesia when boiled form a hard scale which adheres to the tubes and is only removed with difficulty. Treatment with lime in a tank will precipitate the carbonates, which are thus easily removed. The cost of this treatment is very moderate. The use of sodium carbonate (or soda ash) in a water containing sulfate of lime will produce carbonate of lime, which precipitates forming soft scale and also sulfate of soda which is objectionable because of its effect on "foaming." **Foaming** is primarily due to suspended matter in the water but is aggravated by the alkali salts, sulfate, carbonate or chloride of sodium, and is objectionable because it is difficult to maintain the proper amount of water in the boiler and keep wet steam out of the cylinders. If the sulfate hardness is very great the water is practically worthless, since too much alkali salts would be developed by treatment. Even when the sulfate hardness is very low hard scale and corrosion will result unless a little soda-ash is used and boilers blown down frequently. Filters of various types have been tried in connection with water softening plants with varying results. They are more used for purifying water for drinking purposes.

The Quantity of pure reagents required to remove 1 lb. of various scaling and corroding substances is given in the following table. These should be increased to give equivalent quantities of pure reagents in case commercial products are used. The pounds of matter per 1000 gal. may be found by dividing parts per 100 000 by 12 or grains per gal. by 7. The total amount of lime must be sufficient to take care of the free carbonic acid as well as the solids.

Reagents Required for Water Softening. Per Pound of Substance

American Railway Engineering Association

Substance	Reagent and amount	Foaming Matter Increased
Sulfuric acid	0.57 lb. lime and 1.08 lb. soda ash	1.45 lb.
Free carbonic acid	1.27 lb. lime	None
Calcium carbonate	0.56 lb. lime	None
Calcium sulfate	0.78 lb. soda ash	1.04 lb.
Calcium chloride	0.96 lb. soda ash	1.05 lb.
Calcium nitrate	0.65 lb. soda ash	1.04 lb.
Magnesium carbonate	1.33 lb. lime	None
Magnesium sulfate	0.47 lb. lime and 0.88 lb. soda ash	1.18 lb.
Magnesium chloride	0.59 lb. lime and 1.11 lb. soda ash	1.22 lb.
Magnesium nitrate	0.38 lb. lime and 0.72 lb. soda ash	1.15 lb.
Calcium carbonate	3.15 lb. barium hydrate	None
Magnesium carbonate	3.76 lb. barium hydrate	None
Magnesium sulfate	2.62 lb. barium hydrate	None
*Calcium sulfate	2.32 lb. barium hydrate	None

* In precipitating the calcium sulfate, there would also be precipitated 0.74 lb. of calcium carbonate or 0.31 lb. of magnesium carbonate, the 2.32 lb. of barium hydrate performing the work of 0.41 lb. of lime and 0.78 lb. of soda ash, or for reacting on either magnesium or calcium sulfate, 1 lb. of barium hydrate performs the work of 0.18 lb. of lime and 0.34 lb. of soda ash, and the lime treatment can be correspondingly reduced.

These data will enable one to determine the quantities of reagents necessary to treat a given water and local quotations will supply data for a cost estimate. The cost of operation (aside from chemicals) will depend largely on the method used and local conditions and may be very little in addition to the cost of operating a water plant.

Concentration of foaming solids reaches the critical point between 100 and 200 grains per gal., depending on the character of the alkali salts and the amount of suspended matter in the water. Systematic and frequent blowing off and occasional complete blowing down and washing out at terminals are the best means for keeping below this limit.

The cost equals the cost of pumping, treating and heating to boiler temperature the amount of water blown off, plus cost of washing out boiler. With the limit 100 gr. per gallon the percentage of wastage equals the number of grains per gallon in the water supplied, for other limits the per cent varies inversely as the limit.

A committee of the American Railway Master Mechanics Association estimated in 1873 that the use of hard or muddy water in parts of the country cost $750 per locomotive per year. The Committee on Water Service and Sanitation of the A. R. E. A. estimated the cost at $4000 to $6000 per year per locomotive for present locomotives at present prices. This cost is made up of extra repairs, extra fuel, boiler washing, time out of service, etc.

As a further illustration of the importance of the subject, the El Paso and Southern

Railway found that even after chemical treatment of the hard-water supply on a division 128 miles long, the engine tonnage was reduced 25% and the cost of locomotive maintenance was increased $1000 per year per engine over the normal amount. To avoid this a waterworks system from a supply of pure mountain water 130 miles distant was constructed at a cost of $1 300 000. Even this expenditure was proven to be amply justified.

Tests have proven that for a scale thickness of, say, 1/8 in. the loss of heat transmission may amount to 10 or 12%. A porous scale increases the heat loss even more than a solid scale. The chemical composition has no practical effect except as it may produce a porous scale. The heat loss increases as the thickness of the scale.

In 1928 there were nearly 900 lime-soda plants treating about 120 million gallons daily, 650 plants using soda-ash, or sodium aluminate, only, treating 45 million gallons daily, 330 plants mixing anti-scale compound with 39 million gallons of water daily in wayside tanks and 35 zeolite plants softening 8 million gallons daily, while anti-scale compounds were used with 40 million gallons daily in boilers.

Supply. The American Railway Engineering Association recommends the purchase of water where it can be obtained in sufficient quantity and of suitable quality at a reasonable price. Otherwise the source may be springs, lakes, ponds, creeks, rivers or wells and it may be possible to supply the tanks by gravity, although usually pumping will be necessary. The quantity and quality of the water should be investigated for a sufficient time to give accurate results, future increased demands and possible necessity for treatment being taken into account. Ordinarily the quantity should be sufficient if economically possible, to give a 24-hour supply in 7 hours at terminal stations and 4 hours at intermediate stations, except for large stations when one may figure on 10-hour or even, 20-hour pumping service.

Pumping. The American Railway Engineering Association recommends the use of the following table for determining the size of pumping plants.

Quantity per 24 hours, gallons	Terminal stations		Intermediate stations	
	Time pump to run in 24 hours	Gallons per Minute	Time pump to run in 24 hours	Gallons per Minute
	Hours		Hours	
2 000 000	20	1 666	20	1 666
1 750 000	20	1 458	20	1 458
1 500 000	20	1 250	20	1 250
1 250 000	20	1 042	20	1 042
1 000 000	20	833	20	833
900 000	20	733	20	733
800 000	20	666	20	666
700 000	20	583	20	583
600 000	20	500	10	1 000
500 000	7	1 189	10	833
450 000	7	1 071	10	750
400 000	7	928	10	666
350 000	7	838	10	583
300 000	7	714	10	500
250 000	7	595	4	1 041
200 000	7	476	4	833
150 000	7	357	4	625
100 000	7	238	4	416
50 000	7	119	4	208
25 000	7	60	4	104

The effective horsepower

$$= \frac{\text{Gallons per minute} \times (\text{static head} + 1.5 \text{ friction head})}{3960}$$

The friction head should be determined from tables and the 50% added is to allow for ageing of the pipes. Heads are in feet.

With Q = the quantity pumped in 24 hours in 1000 gallon units and H = friction head in feet per 1 ft. of pipe for the given flow, the sizes recommended for discharge pipe are given herewith.

Cast-iron pipe, diameter, in.	Discharge when $\sqrt{Q}$ is most nearly
4	0.355
6	0.437
8	0.519
10	0.656
12	0.820
14	1.162

The Committee on Water Service of the A. R. E. A. stated in its report of October, 1927, that steam plants will usually prove most economical at points where it is necessary to maintain a steam plant for other purposes, as at engine terminals or where track pans are maintained in cold climates. Otherwise the medium pressure oil engine, or so-called semi-Diesel type, is generally the most economical and practicable power unit for all intermittent power service, except where the economies of automatic or remote control make the use of electricity advisable. The relative costs should be computed in detail for each case.

Tanks of ordinary size are commonly built of wood, redwood, cypress, white pine or other approved species. Steel is also used especially for the larger sizes and permanent installations, and a few reinforced-concrete tanks have been constructed with varying results. No doubt these will be more used in the future as they should be permanent and satisfactory if properly constructed. The construction of wooden and steel tanks is now a specialized business which may be left to the builders under general specifications. See Manual of the American Railway Engineering Association. Their capacity usually varies from 10 000 to 80 000 gal. Two or even three smaller tanks are preferable to one excessively large tank.

Capacity, in U. S. Gallons, of Tanks of Various Inside Dimensions

Height, feet	Diameter, feet	Gallons	Height, feet	Diameter, feet	Gallons	Height, feet	Diameter, feet	Gallons
10	12	8 460	14	16	21 057	18	22	51 185
	13	9 929		18	26 650		24	60 914
	14	11 515		20	32 901		26	71 489
	15	13 219		22	39 810		28	82 910
12	14	13 817	16	18	30 457	20	24	67 682
	15	15 863		20	37 601		26	79 432
	16	18 049		22	45 498		28	92 123
	18	22 843		24	54 146		30	105 752

The total cost of wooden tanks per 1000 gal. capacity may be roughly estimated at from \$30 to \$35 for 100 000-gal. tanks, \$45 to \$50 for 30 000-gal. tanks, \$70 to \$75 for 15 000-gal. tanks, to \$130 to \$140 for 5000-gal. tanks. Steel tanks will cost from 25% to 50% more.

Track Tanks, or Pans, are used to give a supply of water while the train is in motion. A scoop, lowered from under the tender, is attached to a pipe leading to the tender tank. The rapid motion forces the water up the pipe from the long shallow pan between the rails. The height of the top of the tender tank above the track pan is usually about 9 ft. and this means that water will not flow into the tank unless the speed of the engine is more than 16 miles per hour. Even at 20 miles an hour more water is wasted by slopping over the sides than reaches the tender. The minimum wastage is about 12-1/2% and occurs at a speed of 45 to 50 miles per hour, although with a properly designed pan the wastage is fairly constant at speeds from 25 to 60 miles per hour.

The track tank is commonly made of 3-16-in. plate, is 19 in. wide, 6 in. deep, bottom rounded to 1-1/2-in. radius, plates are 15 ft. long, riveted with 7/16-in. rivets, 20 rivets per joint; on each side is riveted an angle 1-1/8 by 2-1/4 in.; the edges are stiffened by a molding of 1-1/8 by 1/2-in. bar iron. The Pennsylvania R. R. uses 25-ft. flange sections with special lead joints, thus avoiding the use of rivets. Sides as high as practicable and lips along the sides reduce wastage. For two engines, "double-headers," it is better to increase length of pan rather than to provide storage by increased width. The ties are dapped 1-1/2 in. deep so as to lower the tank a little. At the center of its length it is rigidly attached to the ties. Ordinary track spikes are driven beside the angles, thus holding the tank in place laterally and vertically, but permitting longitudinal expansion with changes of temperature. There are inclined planes at each end, both inside and outside the tank, which will automatically raise the scoop in case the fireman neglects to raise it or lowers it too soon. About 1200 ft. is an ordinary length, although double this is sometimes used where trains are frequent, and the track must be absolutely level, which frequently requires reconstruction of the roadbed. Track tanks are usually placed on tangents but are successfully used on flat curves when necessary. To prevent freezing in winter, steam pipes having 1/8-in. nozzles are inserted every 40 to 50 ft. throughout the length of the tank, and jets of live steam are forced through them from the boiler in the pump house in the "direct" system. In the "circulatory" the water is kept in circulation and steam is fed in with it at the inlets. The size of the required boilers depends on the amount of pumping required and on the severity of the climate, as it affects the demand for steam to prevent freezing. Although 25 hp. would generally suffice for warm-weather pumping, 150 hp. might be required to prevent freezing. The cost of installation may amount to \$10 000 and the cost of maintenance may be \$125 to \$150 per month.

18. Miscellaneous Structures

Coaling Stations. These are located at division terminals and in general near engine houses. **Hand shoveling** of coal from a gondola car directly to engine tender involves delay of car until coal is required. A trestle costing \$250 to \$500, or a convenient natural embankment which lifts the car about 5 ft. above the locomotive track, facilitates the work of shoveling. The shoveling has been done for about 17 cents per ton. To save delaying the cars the coal is sometimes shoveled onto a platform, or bin with low sides, and again shoveled into the tender. This doubles the cost of shoveling and with bituminous coal produces more slack. As an improvement a **jib crane** is set on a platform, which lifts buckets of about one ton capacity and stores them on the platform from which they are again lifted and dumped in the tender when needed. Two quoted costs for this method are 23.2 and 32.0 cents per ton, but in these cases the plants handled only 12 and 13 tons per day. The use of small dump cars rather than buckets at a plant handling 235 tons a day reduced the cost to 17.8 cents per ton. The next step in reduction of operating costs by increase in cost of plant is to construct a **trestle** with coal-car track 30 to 40 ft. above the engine track, the grade of approach not exceeding 5%. Self-dumping cars, dumping the coal directly into bins, eliminate the cost of shoveling and reduce breakage of the coal. The cost of coal handling for 26 such plants varied from 9 to 12 cents per ton. When space for a 5% approach is not available, the grade may be increased to 20%, and the cars hauled up the grade by the use of cable and hoisting engine. Another type is a **locomotive crane,** which is essentially a jib crane mounted on a self-moving car. Such a crane, costing about $7500, can load 70 engines per day. It loads directly from the cars to the locomotive tender and can handle cinders and ashes as well as coal. It is also particularly useful when self-dumping cars are not regularly obtainable, especially since flat-bottom cars are almost useless for many types of coal-handling plants. The average cost of operating seven locomotive crane plants, each handling 106 to 230 tons of coal per day, was 7.3 cents per ton. Another crane handling only 45 tons per day cost 14.0 cents per ton. An added advantage in it is that such a crane may be transported and can be utilized for temporary emergencies, especially where permanent construction is for any reason inadvisable. The bucket conveyor type has the advantage of minimum ground space and indefinite flexibility to suit local conditions, provided the amount of coal to be handled is very large. The advantage and economy are in these particulars. The cost of handling coal in nine plants varied from 9.4 cents to 14.3 cents per ton. Storage bins should be constructed with hopper bottoms so as to facilitate the movement of the coal and also to prevent the accumulation of slack coal which sometimes ignites spontaneously. Coal is sometimes weighed by passing it through auxiliary bins holding 5 to 10 tons and mounted on scales. Sometimes the scales are omitted, and the measuring is done by volume. When sand and cinders are handled in the same plant, the handling machinery should be separate, since sand and cinders produce excessive wear on moving parts. The growing scarcity of lumber is constantly increasing the relative ultimate economy of plants of reinforced concrete and fireproof construction is desirable.

Ash-Pits. The simplest form of ash-pit is made by laying the rails on 12-in. by 14-in. wooden stringers, which rest on cross-ties leaving a net space 4 ft. wide. The stringers and ties are covered with old boiler plate to protect them from the hot ashes. Water service with hose to quench burning cinders and suitable drainage are essential. Although such a pit is justifiable for a road doing small business, mechanical means of handling ashes are recommended for pits serving 25 or more engines per 24 hours. The next step is to construct a pit with concrete walls and bottom and with a clear depth of about 3 ft. Light all-metal cars may be provided in the bottom of such a pit for the prompt removal of the ashes. Another method makes the pit wider and runs one rail on one side wall and the other rail on a series of cast-iron or concrete columns. This affords greater freedom in the removal of the ashes. A variety of designs agree in dumping the ashes from the locomotive into a hopper or car directly under the locomotive. The hopper or car is then drawn out sidewise, or, after the locomotive has passed on, the car is lifted vertically by some form of mechanical hoist and dumped into a gondola car on a storage track. Ashes are also handled by the belt type of conveyor, the same as coal. For large terminals the pits are often filled with water and the ashes removed by a grab bucket operated from a gantry, traveling, or locomotive crane.

Sand Houses. A sand house consists essentially of a wet sand storage bin and a sand drier and screen. The dried sand may be stored in buckets from which it may be dumped by hand into the sand boxes on the engine. A more elaborate and economical plan uses compressed air to pump the dry sand into an elevated storage tank from which it falls by gravity through a pipe to the engine.

Oil Houses. The essential features are a fireproof place of economical construction for the storage of oil, combined with conveniences for its distribution. The oil should be stored in tanks located in the basement, the basement having masonry walls and the floor above being made preferably of reinforced concrete. A trap door of fireproof materials should be the only opening into the vault beside the pipes. No illumination except electric lights should be permitted. The vault is surmounted by a house which may be of wood, and which is used for pumping the oil from the tanks into small cans for distribution. Since the oil is put in and removed entirely by pumping, the vault need not be opened except for occasional inspection and repairs.

Track Scales for freight-car service have a capacity of from 100 to 150 tons. The platform is generally from 36 to 46 ft. long, but the length is sometimes increased to 60 ft., which permits the weighing of cars without stopping them as they cross the scales on a gravity track. Scales 100 ft. long have been built, but their use is diminishing on account of greater unreliability. To economize yard room and yet not subject the scale mechanism to needless wear, a pair of through rails is run about 10 to 12 in. from the weighing rails. One of these through rails rests on the wall of the scale pit and the other rests on iron posts which extend through the platform and are separate from it. The weighing rails switch into the through rails by means of point rails at about one rail length each side of the scales. The cost varies from about $35 to $45 per ton capacity, according to capacity and style, the lower prices applying to the larger-capacity scales.

Turntables. Length for modern locomotives must be from 60 up to 100 ft., the most common length being about 80 ft. The pivot pier foundation must be designed for a load of perhaps 300 tons. The depth of the pit depends on whether a deck table or one made of two pony girders is used. The circular walls of the pit should be made of masonry, but are sometimes made of wood. They must support the circular rail on which the end of the table runs when unbalanced. The pit floor should be paved. A drain pipe with suitable outfall should be provided. The cost of an 80-ft. steel turntable will vary from $4500 to $8000 according to the details of depth of foundation, lining of pit, etc. Temporary structures which will serve the purpose for light-weight engines may be built for $700 to $800 where timber is cheap.

19. Signaling and Interlocking

(The following has been condensed by permission from the Manual of the American Railway Engineering Association. It is largely a statement of "recommended practice.")

Signals, if practicable, should be placed either over (on a signal bridge) or upon the right of and adjoining the track to which they refer. Semaphore arms which govern should be displayed to the right of the signal mast, as seen from an approaching train. A mast may have a crosspiece on which two uprights (no more) may be mounted on which to place signals. One upright may be a stub to indicate that the corresponding track has no governing signals. Not more than one track should intervene between a bracket signal mast and the track for which its left upright carries a signal arm. There should be a definite place for flags and hand lanterns when used for signals; they should be fixed by a flag socket and lantern hook on the side of the signal station toward the direction of an approaching train and convenient for the operator to reach from one of the windows. The common semaphore for either train order, interlocking or automatic block signal is as shown in Fig. 44 for indications in the lower quadrant, the upper quadrant is now recommended and many roads are discarding the semaphore signal entirely in favor of color-light or position-light indications. The greater simplicity and elimination of moving parts tend toward greater reliability. Light signals are usually mounted in a circular box much like the older "banjo" signals. The arm should have a sweep of 90 deg. The design may be used for either a two-position or a three-position signal. When the lamp is on the side (as in figure) spectacle *A* is always blank. Spectacle *B* has red glass. Spectacle *C* is red for a two-position signal and yellow or green (whichever color is used for "caution") in a three-position signal. Spectacle *D* is white or green, whichever color is used for "clear." The American Railway Association recommends use of red for stop, yellow for caution and green for clear. Sometimes for economy on a single-track road the post is cut off so that the lantern is placed on top of the post directly back of (*A*) in the figure and another arm and spectacle casting is swung on the other side of the pole. In such a case spectacle *A* is red; *B* is red for two-position and yellow or green for three-position signal (like spectacle *C* in other design); spectacle *C* is white or green and spectacle *D* is blank. It is recommended that the "electric slot," an appliance for automatically disengaging the signal arm connection from its actuating lever and returning signal arm to "stop," as a train passes, should be utilized. High-speed movements should be governed by high signals and low-speed movements by low signals. Not more than two high-speed signals should be displayed on one mast, the top arm to govern unrestricted speed and the lower arm to govern all other high speeds. All low-speed movements should be governed by one-arm low signals of dwarf construction. A distant signal should be provided for each high-speed route. "Red" should be the "color" stop indication, and the "horizontal" position of the arm should be the "position" stop indication for all home signals. A mark of distinction should be made between automatic block signals and all other home signals, whether interlocking, train-order or manually operated block signals. Home block signals should be provided at all interlocking plants used as block stations. All mechanically operated high-speed semaphore signals should be pipe-connected. Low-speed signals may be wire-connected. One distant signal only should be provided for a high-speed route, and when "clear" it should mean that all high-speed home signals along that route through the interlocking plant, including the home block signal, are "clear." Every movement within the limits of an interlocking plant should be governed by an interlocked signal.

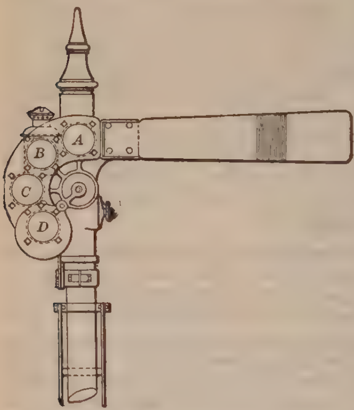


Fig. 44. Semaphore

The decrease in 1928 was due to the hesitancy of the roads to proceed with signaling programs while the question of the adequacy of existing train control and signaling installations was before the Interstate Commerce Commis-

Miles of Road Equipped with Automatic Signals, 1908-1928

Year	Miles	Year	Miles	Year	Miles
1908	1387.6	1915	1296.0	1922	1254.0
1909	2047.1	1916	1891.0	1923	2019.0
1910	3473.8	1917	2482.0	1924	2515.0
1911	2623.4	1918	1934.0	1925	2726.0
1912	1883.9	1919	1923.0	1926	4992.0
1913	4350.5	1920	546.0	1927	5127.0
1914	3294.2	1921	830.0	1928	3121.2

sion. The increasing preference for light signals is indicated by the installation of 3601 light signals out of a total of 5536 automatic signals installed in 1928; of these 3226 were color-light and 375 position-light.

The trend in interlocking practice as indicated by construction in 1928 is as follows: 88 new plants were constructed of which 24 were mechanical, 33 electrical, 8 electro-pneumatic and 23 electro-mechanical. The 382-lever electric plant installed at Buffalo, N. Y., by the New York Central is said to have the largest number of levers of any one plant in the world.

Automatic Train Control installations in 1928 were virtually limited to the completion of projects already under way and voluntary extensions by some roads of existing installations, as the Interstate Commerce Commission, believing that a far greater measure of safety will result from vigorous efforts to curtail the large number of accidents arising from other causes than disregard of signals, issued no new orders. The use of cab signal indications, however, increased rapidly and it is believed that future development will be in this direction without automatic brake control. The substitution of remote control of switches and signals for train orders and manual operation of switches is another important development which is being used to effect economies both in dispatching and operation of trains, since they do not have to stop to throw switches or for signals if the block is clear. Spring switches are also being used increasingly to eliminate stops at the ends of double track, at yard leads and at certain sidings where movements in one direction predominate.

20. Locomotives

Classification, according to wheel arrangement, is now usually indicated by three numbers, of which the first is the number of pilot-truck wheels, the second the number of drivers and the third the number of trailing wheels

Classification of Locomotives

Name	Type	Name	Type
Mikado	2–8–2	Santa Fe	2–10–2
Switching	0–8–0	Ten-Wheel	4–6–0
Switching	0–6–0	Mogul	2–6–0
Switching	0–4–0	Mastodon	4–8–0
Consolidation	2–8–0	Mountain	4–8–2
Mallett	A–B–B–C*	Atlantic	4–4–2
Pacific	4–6–2	American	4–4–0
Decapod	2–10–0	Mohawk	4–8–2

* A and C are truck wheels, usually 2 or 0, and B are drivers, usually 6 or 8. There may be three sets of drivers, as 2–8–8–8–2 for the large Mallett mentioned below, the third set being under the tender in this case.

although originally each type was given a name. All three numbers are given even though one, or two, of them is zero. For example, the common American type having four pilot wheels, four drivers and no trailing wheels is indicated by 4-4-0; the six-wheel switching engine, having neither pilot nor trailing wheels, by 0-6-0.

Steam Locomotives Built Each Year, 1896 to 1928

From "The Railway Age"

Year	Domestic	Foreign	Total	Year	Domestic	Foreign	Total
1896	866	309	1176	1913†	4561	771	5332
1897	865	386	1251	1914†	1962	273	2235
1898	1321	554	1875	1915†	1250	835	2035
1899	1951	514	2475	1916†	2708	1367	4075
1900	2648	505	3153	1917†	2585	2861	5446
1901	....	....	3384	1918†	3668	2807	6475
1902	....	....	4070	1919†	2162	1110	3272
1903	....	....	5152	1920†	2002	1650	3672
1904	....	....	3441	1921†	1185	638	1823
1905*	4896	595	5491	1922†	1303	231	1534
1906*	6232	720	6952	1923†	3505	280	3795
1907*	6564	798	7362	1924†	1810	226	2036
1908*	1886	456	2342	1925†	994	291	1285
1909*	2596	291	2887	1926†	1585	185	1770
1910*	4441	314	4755	1927†	1009	167	1176
1911*	3143	387	3530	1928†	636	111	747
1912†	4403	512	4915				

* Includes Canadian output.
† Includes Canadian output and those built in railroad shops.

Taken year by year the numbers are rather erratic but reflect in a general way economic conditions, modified of course by estimates of railroad officials of probable traffic requirements. Just prior to 1918 the rapid increase in traffic stimulated buying, but since that time passenger traffic has decreased approximately one-third and the rate of increase of freight traffic has been slower. These decreases together with the better utilization of equipment have made possible the storing of about 5000 serviceable locomotives for the past two years and naturally purchases have been low with this amount of power in reserve.

The trend as to types as indicated by the locomotives ordered in 1928 is definitely toward the heavier types with trailing trucks, probably influenced greatly by the increasing use of boosters to assist in starting and at low speeds though 58 tenders with auxiliary engine for booster service were ordered also. The Mountain type, 4-8-2, seems to have the preference with a total of 146, although 100 of these were ordered by one road, the New York Central. Excluding this large order, several other types had nearly the same number as follows: 2-10-4, 46; 4-8-4, 41; 2-8-4, 39; 4-6-4, 35; 2-8-2, 30 and 4-12-2, 23. Of the older types there were: 4-6-0, 11; 4-6-2, 9; 2-8-0, 7; 2-6-0, 2 and 4-4-0, 1. Eighty-six switching locomotives were ordered, a majority of them of the heavy 0-8-0 type, also 31 Malletts, 25 geared, 5 of the 4-6-6 type and 2 of the 2-6-2. Besides these steam locomotives 13 oil-electric and 50 electric locomotives were ordered for domestic use, all with all the weight on drivers except 25 electrics of the 2-6 + 6-2 type and one electric of the 2-8-2 type. On Dec. 31, 1927, there were 449 electric locomotives in use on Class 1 railways, about one-third in freight service and two-thirds in passenger.

Number and Average Tractive Force — Steam Locomotives Class 1 Railways 1916-1927

End of year	Number	Average tractive effort	End of year	Number	Average tractive effort
1916	60 990	33 188	1922	64 140	37 441
1917	61 533	33 932	1923	64 939	39 177
1918	63 531	34 995	1924	65 006	39 891
1919	64 618	35 789	1925	63 612	40 666
1920	64 368	36 365	1926	62 342	41 886
1921	64 585	36 935	1927	60 895	42 798

The steady and fairly uniform increase in the average tractive force of the locomotives operated on Class I railways is shown in the statistics of the Interstate Commerce Commission given in the above table. That this increase is likely to continue is indicated by the fact that the average tractive force for the 322 locomotives ordered in 1928 for which data are given was 64 372 pounds. The average tractive force for the 13 oil-electric locomotives was 41 923 lb. and for the 13 electric locomotives for which data are given was 24 426 lb.

Weights. These vary from about 25 tons for a 4–4–0 type of narrow-gage locomotive to that of a Mallett standard-gage locomotive weighing 853 050 lb. with 63 466 lb. on each of twelve driving axles. Between these limits there are locomotives of any total weight and any reasonable ratio of driving weight to total weight though the limits in any given type would of course be smaller.

Some general data for typical recent locomotives are given in the following table. All are simple engines except the Norfolk & Western Mallett (see cylinder dimensions). The largest locomotive built in 1928 was a single-expansion articulated, one of the 2–8–8–4 type weighing with tender 1 116 000 lb. It is exceeded in weight on drivers 554 000 lb. and tractive force (with booster) 153 000 lb. only by the Erie Triplex locomotive built in 1914 and the Virginian Triplex built in 1916. It was built by the American Locomotive Co. for the Northern Pacific Railway and was designed to burn lignite of relatively low heating value. Hence it has the largest combined heating surface, 10 892 sq. ft., and by far the largest grate area, 182 sq. ft., of any locomotive in existence. The stoker is designed to deliver a maximum of 20 tons of fuel per hour and the maximum evaporating capacity of the boiler is 14 400 gal. of water per hour.

The Tractive Force of a locomotive may be limited by any one of three factors:

(a) Frictional force or "adhesion" between drivers and rail, a function of the weight on the drivers.

(b) Capacity of the boiler to produce steam, depending on design of boiler, fuel, water and stoking.

(c) Capacity of the mechanism to convert the energy of steam into motion, depending on size of cylinders and drivers.

At low speeds, adhesion is usually the limiting factor as most locomotives are "over-cylindered" and the rate of steam consumption is low; hence they are able to "slip their drivers" and the tractive force using sand may be as much as one-third the weight on the drivers, although it is not usual to count

General Data for Some Recent Steam Locomotives

Year	1928	1928	1928
Railroad	B. & A.	N. & W.	B. & O.
Type	4-6-6*	2-8-8-2	4-6-2
Service	Suburban	Switching	Passenger
Weights, lb.:			
On drivers	180 000	478 000	203 500
Total	352 000	531 000	329 500
Tender	...	264 040	240 000
Cylinders, in.	23-1/2×26	25×39×32	27×28
Diameter of drivers, in.	63	57	80
Wheel bases, ft. and in.:			
Driving	15 0	42 4†	14 0
Engine	42 8	58	37 1
Engine and tender	...	...	78 2-7/16
Steam pressure, lb. per sq. in.	215	240	230
Grate area, sq. ft.	60.8	96.3	70
Heating surfaces, sq. ft.:			
Evaporative	2 761	...	4 595
Superheating	788	...	1 188
Total	3 549	7 639	5 783
Tender:			
Fuel capacity, tons	6	16	17-1/2
Water capacity, gal.	5 000	12 000	12 000
Tractive force, lb.:			
Drivers	41 600	107 373	50 000
With boosters	...	141 873‡	...

Year	1929	1929	1929
Railroad	Ga. Northern	Can. Pac.	Cent. Vt.
Type	2-8-2	4-8-4	2-10-4
Service	Freight	Passenger	Freight
Weights, lb.:			
On drivers	136 260	250 000	285 000
Total	189 470	423 000	419 000
Tender	140 930	286 000	269 600
Cylinders, in.	20×28	25-1/2×30	27×32
Diameter of drivers, in.	57	75	60
Wheel bases, ft. and in.:			
Driving	15 3	19 9	22 0
Engine	31 4	45 9-1/2	44 2
Engine and tender	65 5-1/2	87 0-3/4	82 2-1/4
Steam pressure, lb. per sq. in.	200	275	250
Grate area, sq. ft.	41.5	93.5	84.4
Heating surfaces, sq. ft.:			
Evaporative	2 482	4 931	4 703
Superheating	603	2 112	2 208
Total	3 085	7 043	6 911
Tender:			
Full capacity, tons	12	20	20
Water capacity, gal.	7 000	12 422	13 500
Tractive force, lb.:			
Drivers	33 400	60 800	75 800
With boosters	...	...	89 000§

* Double end locomotive, 6 trailing wheels are under tank.
† Rigid, 15 ft. 9 in.
‡ Two auxiliary engines on tender.
§ Franklin trailer booster.

on more than one-fourth as a maximum. As the speed increases a point is reached where the boiler is no longer able to furnish steam at full cut-off and maintain its normal pressure. So the cut-off is reduced and the "Mean Effective Pressure" becomes smaller and smaller as the speed increases. Hence boiler capacity limits tractive force at speeds beyond the point mentioned above as could be expected, since weight on drivers and size of cylinders can be increased easily while the size of the boiler is limited.

Determination of Tractive Force. The tractive force of a locomotive may be determined by actual measurement with a dynamometer car or at a locomotive testing plant or by computation. With a dynamometer car, speeds, alignment and grades must be accurately known in order to make the proper allowances. The American Railway Engineering Association recommends that speeds be determined to the nearest 0.1 mile per hour. Cylinder power may be determined by means of indicator cards and used to find tractive force. By computation: for low speeds (or maximum tractive force) one may simply take one-fifth to one-fourth the weight on the drivers, assuming reasonably good design. For higher speeds, recourse must be had to the theoretical formula with empirical coefficients or to empirical methods. Let P = the boiler pressure in pounds per square inch; S, the length of stroke in inches; d, the diameter of piston for simple engines; d_h and d_l, the diameters of high-pressure and low-pressure pistons respectively for compound engines; R, the ratio of the area of the low-pressure to the high-pressure cylinders; D, the diameter of drivers; and T.F. the tractive force at the circumference of the drivers. In a simple engine the work done by both cylinders during a complete revolution of the drivers = piston area × effective average cylinder pressure × stroke × 2 × 2. But the work also equals the tractive adhesion developed at the circumference of the drivers × the distance traveled by the drivers during one revolution, which of course equals the circumference of the drivers. Therefore, for simple engines,

$$\left.\begin{array}{l}\textbf{Theoretical}\\ \textbf{tractive force}\end{array}\right\} = \frac{\textbf{Piston area} \times \textbf{effective cylinder pressure} \times \textbf{stroke} \times 2 \times 2}{\textbf{circumference of drivers}}$$

The effective area of the piston is reduced about 1.5% on account of the area of the piston rod. The effective energy at the wheel rim is reduced on account of friction of the piston, piston rod, crosshead and the various bearings. The effective steam pressure in the cylinder is always considerably less than that in the boiler, even at low speed and full cut-off. These reductions may be allowed for by figuring the steam pressure (effective at the drivers) to be 80 to 85% of the boiler pressure. Therefore, dividing both numerator and denominator by π(3.1416) we have

$$\text{T.F.} = 0.8\ Pd^2S/D \quad \text{(for simple engines)}$$

To prevent racking the engine frame, two-cylinder compounds or four-cylinder compounds of the Baldwin type are designed with R at such a ratio that the work done by the high- and low-pressure cylinders is approximately equal. Although there is not such necessity for balancing with a tandem compound, the same ratio is used, which averages 2.81. The formula is

$$\text{T.F.} = \frac{PS}{D}(.67d_h^2 + .25d_l^2) \qquad \left(\begin{array}{c}\text{for Baldwin compounds and}\\ \text{tandem compounds}\end{array}\right)$$

Even greater power may be temporarily obtained when starting a train, or, in an emergency, on a grade (if not limited by poor adhesion) by exhausting from the high-pressure cylinder directly to the atmosphere and by admitting high-pressure steam directly into the low-pressure cylinders. Of course all economy due to compounding is lost while this is done.

The Tractive Force of a Two-Cylinder Compound, when the work is equal in both cylinders, and when working compound, is

$$\text{T.F.} = \frac{.8\ Pd_l^2S}{(R+1)D} \qquad \text{(for two-cylinder compounds)}$$

As before, the tractive force may be increased when starting by exhausting the high-pressure cylinder directly into the atmosphere and by admitting boiler steam to the low-pressure cylinder through a special valve which proportionately reduces its pressure. Here the T. F. is the same as that of a simple engine with both cylinders of the same diameter as the high-pressure cylinder.

The Equivalent Diameter of Simple Cylinders which, with the same boiler pressure, diameter of drivers, and stroke, will have the same tractive force, may be expressed as follows, when all cylinders perform the same work:

$$\mathbf{d} = \sqrt{\frac{\mathbf{d}_l^2\mathbf{d}_h^2}{\mathbf{d}_l^2 + \mathbf{d}_h^2}} \qquad \text{(for two-cylinder compounds)}$$

$$\mathbf{d} = 1.41\sqrt{\frac{\mathbf{d}_l^2\mathbf{d}_h^2}{\mathbf{d}_l^2 + \mathbf{d}_h^2}} \qquad \text{(for four-cylinder compounds)}$$

The Tractive Force as a Function of the Velocity is expressed by the following formula by Isaacs and Adams (Bulletin 112, Am. Ry. Eng. and M. W. Assoc.), the notation being somewhat revised to correspond with that given above, and in which V = train speed in miles per hour,

$$\text{T.F.} = d^2P\frac{S}{D}\left(0.95 - \frac{392\ S}{11\ 000\ D}V\right)$$

The reduction of tractive force with increase in velocity is shown in the table (computed on the basis of the above formula) in which is given the ratio of tractive force at speeds between 10 and 30 miles per hour to the tractive force at 10 miles per hour for engines with five combinations of ratio of stroke to diameter of drivers.

Ratio of Tractive Force at Various Speeds to Tractive Force at 10 Miles per Hour

Stroke = Drivers = Ratio S/D =	24 in. 56 in. 0.429	28 in. 62 in. 0.453	24 in. 50 in. 0.480	28 in. 56 in. 0.500	30 in. 56 in. 0.536
Velocity, miles per hour					
10	1.000	1.000	1.000	1.000	1.000
11	0.981	0.980	0.978	0.977	0.975
12	.962	.959	.956	.954	.950
13	.942	.939	.934	.931	.925
14	.923	.918	.912	.908	.899
15	.904	.898	.890	.885	.874
16	.885	.877	.868	.862	.849
17	.866	.857	.846	.838	.824
18	.847	.837	.824	.815	.799
19	.827	.816	.802	.792	.774
20	.808	.796	.780	.769	.748
21	.789	.775	.758	.746	.723
22	.770	.755	.736	.723	.698
23	.751	.734	.714	.700	.673
24	.731	.714	.692	.677	.648
25	.712	.694	.671	.654	.623
26	.693	.673	.649	.631	.597
27	.674	.653	.627	.608	.572
28	.655	.632	.605	.584	.547
29	.636	.612	.583	.561	.522
30	.616	.592	.561	.538	.497

Average Evaporation in Locomotive Boilers in Pounds of Steam per Pound of Coal

Thermal value of coal B.t.u.	Pounds of coal fired per hour per sq. ft. of heating surface											
	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
10 000	5.24	4.87	4.55	4.25	3.98	3.74	3.51	3.31	3.13	2.96	2.80	2.66
12 000	6.29	5.85	5.46	5.10	4.78	4.49	4.22	3.98	3.75	3.55	3.37	3.19
14 000	7.34	6.82	6.37	5.95	5.57	5.24	4.92	4.64	4.38	4.14	3.93	3.73

Feed water at 60° F. boiler pressure 200 lb. per sq. in. In bad water districts deduct 10% for each 1/16 in. accumulated scale and 1% for each grain per gal. of foaming salts. Heating surface does not include superheater if used but is water evaporating surface only. The quantities of steam evaporated for intermediate quantities or qualities of fuel may be found by interpolation.

Weight of Steam Used in One Foot of Stroke in Locomotive Cylinders

For Locomotives Using Saturated Steam

Diameter of Cylinder	Gage pressures						
	160 lb.	170 lb.	180 lb.	190 lb.	200 lb.	210 lb.	220 lb.
in.	lb.	lb.	lb.	lb.	lb.	lb.	lb.
12	0.304	0.321	0.337	0.354	0.370	0.389	0.405
13	0.357	0.376	0.396	0.415	0.435	0.456	0.475
14	0.414	0.436	0.459	0.482	0.504	0.529	0.551
15	0.476	0.501	0.527	0.553	0.579	0.607	0.633
15-1/2	0.508	0.535	0.562	0.590	0.618	0.649	0.675
16	0.541	0.570	0.599	0.629	0.658	0.691	0.720
17	0.611	0.643	0.676	0.710	0.744	0.780	0.812
18	0.685	0.722	0.759	0.796	0.834	0.875	0.911
18-1/2	0.724	0.762	0.801	0.841	0.881	0.924	0.962
19	0.763	0.804	0.845	0.887	0.928	0.975	1.015
19-1/2	0.804	0.847	0.890	0.934	0.978	1.027	1.069
20	0.846	0.891	0.936	0.983	1.029	1.080	1.125
20-1/2	0.888	0.936	0.984	1.032	1.081	1.134	1.181
21	0.932	0.982	1.032	1.083	1.134	1.191	1.240
22	1.023	1.078	1.133	1.189	1.245	1.307	1.361
23	1.118	1.178	1.238	1.300	1.361	1.428	1.487
28	1.657	1.745	1.835	1.926	2.017	2.117	2.204
For simple locomotives using superheated steam							
18	0.415	0.443	0.470	0.498	0.524	0.551	....
19	0.465	0.496	0.526	0.557	0.587	0.618	....
20	0.515	0.549	0.582	0.617	0.650	0.684	....
21	0.565	0.605	0.641	0.679	0.715	0.752	....
22	0.623	0.665	0.705	0.747	0.787	0.827	....
23	0.682	0.728	0.772	0.818	0.861	0.905	....
24	0.741	0.791	0.838	0.889	0.931	0.984	....
25	0.804	0.859	0.910	0.965	1.016	1.065	....
26	0.868	0.927	0.983	1.041	1.097	1.150	....
27	0.937	1.000	1.057	1.123	1.183	1.241	....
28	1.008	1.078	1.143	1.209	1.275	1.340	....
29	1.083	1.156	1.225	1.199	1.368	1.438	....
30	1.157	1.234	1.308	1.387	1.460	1.533	....

Cylinder diameter is for high-pressure cylinders in compounds. Superheat of 200° F. and drop of 5 lb. per sq. in. between boilers and cylinders are assumed.

The American Railway Engineering Association recommends the following method of determining tractive force. Assuming that the maximum amount of coal that can be fired is 4000 lb. per hour for hand-fired locomotives and 6000 lb. per hour for stoker-fired locomotives with grates of less than 70 sq. ft. and 8000 lb. per hour for stoker-fired locomotives with grates of more than 70 sq. ft., find the rate of steam production from the first table on p. 2071.

The steam used per revolution at full cut-off equals the quantity given in the second table on p. 2071 multiplied by four times the length of the stroke in feet for simple and four-cylinder compounds and by twice the length of stroke in feet for two-cylinder compounds. The weight of steam produced by the boiler per minute divided by the weight used per revolution will give the maximum number of revolutions per minute at which full cut-off can be maintained. The corresponding train speed is found by multiplying by the diameter of the driver in inches and dividing by 336.13 and this speed is called M.

The pounds of steam used per i. hp.-hr. at speed M are:

	For Locomotives Using Saturated Steam						
Gage pressure...	160	170	180	190	200	210	220
Simple.........	39.45	39.10	38.80	38.53	38.30	38.11	37.99
Compound.....	26.57	26.34	26.14	25.95	25.80	25.67	25.59
	For Locomotives Using Superheated Steam						
Simple.........	24.72	24.50	24.31	24.14	24.00	23.88	23.81

The steam produced per hour, divided by the amount used per i. hp.-hr. will give the i. hp. at speed M and the corresponding tractive force = 375 times i. hp. divided by M (in miles per hour). The tractive force at other speeds may then be found by use of the preceding tables.

The cylinder tractive force must be reduced by the engine resistance (see p. 2074) to give draw-bar pull and the tractive force between drivers and rail found by the theoretical method should be reduced by items " b " and " c " of those resistances, although the uncertainty in the value of the coefficients makes it useless to attempt refinements.

The Horsepower Developed by a Locomotive equals the actual tractive force developed, multiplied by the speed in miles per hour, and divided by 375; if the speed is given in feet per second, divide by 550 instead of 375. An actual test of the draw-bar pull of a locomotive measured with a dynamometer shows that at very low velocities the pull was about 30 000 lb., which corresponded to an adhesion ratio of about 22%. At a velocity of 10 miles per hour the pull had decreased to 26 500 lb. which indicated a development of 706 hp. At 20 miles per hour the tractive force had dropped to 17 000 lb., in spite of the fact that owing to the greater velocity the horsepower developed had increased to 906. At 30 miles per hour the tractive force had dropped to 10 500 lb., which indicated 840 hp. Since the above figures represented draw-bar pulls at the rear of the tender, the actual power developed at the drivers was greater, and the actual horsepower developed by the cylinders was still greater in each case. At low velocities the horsepower increases almost directly as the velocity. At velocities above 13 miles per hour, the horsepower increases very slowly, and at high speeds it decreases as the velocity increases.

Fuel Consumption. Wood is used only where it is comparatively cheap and coal is comparatively expensive. Green wood contains about 50% of

Per cent of Cylinder Tractive Force for Various Multiples of M

For Locomotives Using Saturated Steam

Speed	Compound per cent	Simple per cent	Speed	Compound per cent	Simple per cent	Speed	Compound per cent	Simple per cent
M			*M*			*M*		
Start	135.00 *	106.00	3.6	32.40	44.75	6.4	....	23.59
0.5	103.00	103.00	3.7	31.25	43.56	6.5	....	23.18
1.0	100.00	100.00	3.8	30.10	42.39	6.6	....	22.79
1.1	96.28	95.57	3.9	29.14	41.24	6.7	....	22.42
1.2	92.55	91.53	4.0	28.24	40.10	6.8	....	22.06
1.3	88.83	87.83	4.1	27.38	39.00	6.9	....	21.71
1.4	85.12	84.46	4.2	26.56	37.96	7.0	....	21.38
1.5	81.40	81.37	4.3	25.77	36.97	7.1	....	21.06
1.6	77.68	78.55	4.4	25.03	36.03	7.2	....	20.75
1.7	73.96	75.97	4.5	24.34	35.13	7.3	....	20.45
1.8	70.25	73.60	4.6	23.69	34.26	7.4	....	20.16
1.9	66.54	71.41	4.7	23.07	33.41	7.5	....	19.88
2.0	63.21	69.37	4.8	22.48	32.59	7.6	....	19.61
2.1	60.20	67.47	4.9	21.92	31.82	7.7	....	19.34
2.2	57.48	65.67	5.0	21.38	31.11	7.8	....	19.08
2.3	54.97	63.94	5.1	20.87	30.42	7.9	....	18.82
2.4	52.68	62.22	5.2	20.37	29.75	8.0	....	18.57
2.5	50.42	60.55	5.3	19.89	29.10	8.1	....	18.33
2.6	48.16	58.92	5.4	19.43	28.48	8.2	....	18.09
2.7	46.08	57.33	5.5	18.99	27.87	8.3	....	17.86
2.8	44.10	55.78	5.6	....	27.33	8.4	....	17.64
2.9	42.29	54.26	5.7	....	26.81	8.5	....	17.43
3.0	40.57	52.78	5.8	....	26.30	8.6	....	17.22
3.1	38.95	51.33	5.9	....	25.81	8.7	....	17.01
3.2	37.42	49.91	6.0	....	25.34	8.8	....	16.82
3.3	35.98	48.55	6.1	....	24.88	8.9	....	16.63
3.4	34.66	47.24	6.2	....	24.44	9.0	....	16.45
3.5	33.53	45.97	6.3	....	24.01			

* Operated as simple engine.

For Simple Locomotives Using Superheated Steam

Speed	Per cent	Speed	Per cent	Speed	Per cent	Speed	Per cent
M		*M*		*M*		*M*	
Start	106.00	2.7	47.12	4.5	31.19	6.3	22.90
0.5	103.00	2.8	45.82	4.6	30.61	6.4	22.56
1.0	100.00	2.9	44.61	4.7	30.05	6.5	22.21
1.1	92.42	3.0	43.49	4.8	29.52	6.6	21.89
1.2	86.55	3.1	42.30	4.9	29.00	6.7	21.57
1.3	81.20	3.2	41.21	5.0	28.48	6.8	21.24
1.4	76.95	3.3	40.17	5.1	27.96	6.9	20.92
1.5	73.00	3.4	39.22	5.2	27.47	7.0	20.62
1.6	69.55	3.5	38.30	5.3	27.00	7.1	20.32
1.7	66.60	3.6	37.42	5.4	26.53	7.2	20.07
1.8	63.66	3.7	36.61	5.5	26.10	7.3	19.78
1.9	61.27	3.8	35.89	5.6	25.69	7.4	19.52
2.0	58.96	3.9	35.11	5.7	25.26	7.5	19.26
2.1	56.94	4.0	34.39	5.8	24.86	7.6	19.01
2.2	55.12	4.1	33.72	5.9	24.46	7.7	18.76
2.3	53.26	4.2	33.06	6.0	24.04	7.8	18.52
2.4	51.53	4.3	32.40	6.1	23.66	7.9	18.28
2.5	48.50	4.4	31.79	6.2	23.28	8.0	18.06
2.6	48.50						

moisture and has far less calorific value than dry wood. About 2.5 lb. of dry wood are required to produce the same heat as 1 lb. of average soft coal. One cord of dry hickory will weigh about 4500 lb. and in calorific value equals about 1800 lb. of average soft coal. One cord of average pine will weigh only 2000 lb. and is the equivalent of 800 lb. of coal. **Oil** is used for locomotive fuel where crude petroleum is cheap. One pound has a heat value of about 21 000 B.t.u., or 1.5 times that of the best coal. Less weight to be carried in the tender, ease of handling and firing, more uniform heat, less repairs to firebox and no expenses for ash handling are additional advantages to be considered in comparing relative economy. On Dec. 31, 1927, 7415 locomotives out of a total of 60 895, or over 12%, burned oil for fuel. Its use is constantly increasing. **Anthracite coal** is used on a few railroads passing through the anthracite regions. Its calorific value is no higher than that of the best grades of bituminous and it costs considerably more, but it makes less smoke and soot and for passenger traffic this has its advantage. Wood, oil and anthracite may be considered the exceptions. **Bituminous coal** is the standard locomotive fuel. The average cost of fuel per train-mile in the U. S. increased from 16.8 cents in 1907 to 31.6 cents in 1927. This is chiefly due to the large increase in weight of locomotives and in the weight of trains hauled.

Steam Locomotive Resistances. The American Railway Engineering Association gives (Manual, 1929 Ed.) the following formulas for steam locomotive resistances: (a) Cylinder to rim of drivers: resistance = 18.7 (tons weight on drivers) + 80 (number of driving axles); (b) Engine and tender trucks: resistance = 2.6 (tons weight on trucks) + 20 (number of truck axles); (c) Head end or " air " resistance = $0.02\ V^2A$, in which V = velocity in miles per hour and A = end area (average for locomotives, 125 sq. ft., when resistance = $0.25\ V^2$). Total resistance equals the sum. For example, a consolidation engine weighs, engine and tender, 330 000 lb., with 170 000 lb. (= 85 short tons) on the four driving axles. The resistance between the cylinders and the rims of the drivers = $18.7 \times 85 + 80 \times 4 = 1910$ lb.; the truck resistance = $2.6 \times (165 - 85) + 20 \times 5 = 308$; air resistance at say 20 m. p. h. = $0.02 \times 400 \times 125 = 100$; the total resistance is therefore 2318. At the assumed velocity of 20 m. p. h. the horsepower consumed would be $2318 \times 29.33/550 = 123$, which must be subtracted from the total indicated horsepower to determine the effective power at the draw bar. The above does not include the effect of grade. On a 1% grade there is an additional tax on the engine, merely on account of the weight of the engine and tender of $3300 \times 29.33/550 = 176$ hp. which must also be deducted. It has been found that the resistance between cylinders and rims of drivers is practically constant, regardless of speed, after the bearings have become warmed up.

Rail Motor Cars and Trailers. The use of motor cars on branch and light traffic lines is steadily increasing, the number on Class I Railways powered by self-contained units other than storage batteries having increased from 314 to 502 in the two years, end of 1925 to end of 1927. Over 80% of these are passenger-carrying cars, the remainder being about equally distributed between " company service " and other passenger-train cars. During the same period (1925–27) the number of trailers increased from 447 to 803, nearly all being passenger-cars, 153 motor cars and 22 trailers were ordered in 1928. Most of the rail motor cars built in 1928 have power plant capacities of from 200 to 400 hp., although some have run as high as 800 hp., and weights of from 100 000 to 150 000 lb. Somewhat more than half of the rail motor cars in use have electric transmission, i.e., are gas-electric or oil-electric.

21. Cars

The Total Number of Cars in Service on the roads of the United States on Dec. 31, 1927, was 2 578 535, of which 2.2% were in passenger service; 4.4% were in company service, and the remaining 93.4% were in freight service. There were 9.6 freight cars per mile of line, but only 0.22 passenger cars per miles of line, or one passenger car for 4.6 miles of line. The above figures for the number of cars do not include 288 446 cars, privately owned, largely tank and refrigerator cars. A very large proportion of the sleeping cars in use are "leased." The cars in "the company's service" include chiefly cabooses and gravel cars, together with derrick cars, officers' and pay cars, and other road cars such as maintenance-of-way cars.

The average capacity of all freight cars in 1927 was 45.5 tons. In 1916 it was 40.5 tons and in 1904 it was only 30 tons; this illustrates the growth in average capacity from about 15 tons, which was once the standard.

Only 46 060 freight cars were built for domestic use in 1928, the number each year having steadily decreased from a peak of 175 748 in 1923, this peak having followed 5 years of fairly uniform production averaging about 66 000 per year. The decrease is partly due to better utilization of equipment and partly to the use of steel or steel under-frame construction, which is not only more durable than wood construction but also lends itself better to rebuilding.

General Data for Some Recent Freight Cars

Type	Light weight	Capacity	Length	Width		Height	
				Outside	Inside	Inside	Total
	lb.	lb.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
Furniture....	58 800	80 000	50 6	9 6	9 0	10 0	13 9
Box.........	59 700	100 000	50 6	....	9 0	8 6	....
Refrigerator..	53 700	80 000	33 2-3/4	....	8 2-3/4	7 0	....
Stock.......	47 700	80 000	40 0	9 7-1/2	....	....	13 6
Gondola.....	49 800	100 000	40 6-1/2	....	8 11	....	....
Flat........	42 000	100 000	52 8	9 4	....	....	....
Flat........	69 000	182 000	36 8	....	....	....	....
Hopper.....	47 000	140 000	29 6	....	....	....	....

Usual Dimensions of Passenger Cars are: length, 50 to 80 ft.; width 10 ft.; height above the rail, 14 ft. The width and height above the rail are approximately constant, the variations being chiefly in the length. The weight varies from 25 to 70 tons. The all-steel passenger car weighs about 65 tons. The nominal capacity of an ordinary passenger coach varies from 50 to 80 passengers. At 125 lb. per passenger, the total live load will vary from 6250 lb. to 10 000 lb., which is only 8 or 10% of the dead load. On the other hand, a freight car with 100 000 lb. capacity will weigh about 50 000 lb., and the live load is therefore twice the dead load. The proportion of paying load to dead load is therefore many times greater in freight service than in passenger service, which is one reason for the disparity in rates. Practically all passenger cars ordered are now all-steel, and the number averages about 1500 per year. The total number in service in 1928 was about 57 000. All-steel cars weigh from 2 to 4 tons more than wooden cars of the same dimensions but their life is greater and they are superior from the standpoints of safety and sanitation.

Car Maintenance. The relative economy of steel and wooden cars is indicated from a report made on the question of retiring from service 4600 wooden coal cars with a capacity of from 40 000 to 60 000 lb. and with ages varying from 9 to 23 years. It was shown that the annual cost of repairs per wooden car was $95.98 or 37.8% of their present value, whereas 3000 steel cars would have 20% greater capacity than 4600 wooden cars. The amount actually spent on the maintenance of the wooden cars would pay 6% on the cost of the steel cars and leave an annual surplus of $215 000. Experience on the Harriman lines has shown that the average cost of repairs on steel and wooden cars was in the ratio of 100 to 161, respectively.

22. Stations

Essential Requirements of a railroad station, given approximately in the order of their importance, are (1) platform; (2) shelter, developing from shed to waiting room; (3) station agent's office; (4) toilet facilities, developing from a mere privy to modern toilet room; (5) separate waiting room for ladies; (6) baggage and express room. In addition there is the legal requirement in many states that separate waiting rooms shall be provided for colored people. The policy of adding a freight room and agent's living quarters to the station building is debatable. For small stations there are advantages in the method.

The Cost of stations may, when necessary or advisable, be reduced to a very few hundred dollars, which will build a "shelter." As facilities and size are added the cost increases indefinitely. The reader is referred to Orrock's "Railroad Structures and Estimates," Berg's "Buildings and Structures of American Railroads," and similar works for detailed plans, accompanied by estimates of cost, of various types of structures. For use in preliminary estimates the following approximate costs of wooden frame structures, compiled from Orrock's "Railroad Structures," will be found useful, though costs should be modified to take into account differences in prices of materials and labor.

(1) Shelter: platform, 50 × 6 ft.; house, 22 × 12 ft.	$125 to $200
(2) Station: platform, 250 × 8 ft.; waiting room, 10 × 20 ft.; office, 10 × 10 ft.; baggage and express room, 10 ft. × 10 ft. 6 in.	$1000 to $1500
(3) Station: same as (2) but with four rooms in second story for agent's dwelling	$1500 to $2000
(4) Station: same as (2) but with freight room 16 × 20 ft.	$1400 to $1800
(5) Station: waiting room, 16 × 16 ft.; ladies' waiting room, 10 × 20 ft.; office, 12 × 10 ft.; baggage and express room, 16 × 16 ft.; corridor between ladies' room and waiting room and two lavatories; platform 8 × 300 ft. broadened and surrounding the station except at rear	$2000 to $2600
(6) Station: similar to (5) but with four rooms in second story for agent's dwelling	$2500 to $3500

The following unit costs per square foot of ground covered are taken from Gillette's "Hand Book of Cost Data" as being applicable to station buildings in the Mississippi Valley region. The unit costs are somewhat lower than Orrock's estimates. These figures do not include the cost of platform.

	Per square foot
Station, frame, with living rooms, on piles	$1.30
Station, frame, with living rooms, stone foundation	1.50
Station, passenger and freight, frame on piles	1.15
Station, passenger and freight, brick	1.80
Station, modern passenger, brick and stone, slate roof, hardwood finish	3.50

Another method is to allow from 7 to 10 cents per cubic foot for the cost of a frame structure with shingle roof, and from 10 to 12 cents per cubic foot for a brick building.

Add 1/2 cent per cubic foot for slate or metal roof rather than shingle. The higher figures apply to smaller buildings and the figures only apply to plain, unpretentious buildings.

23. Engine Houses and Shops

(Condensed from recommendations in Manual of American Railway Engineering Association)

The Circular Form is preferable although a rectangular house may be desirable when not more than three or four engines are to be housed, or when it is more economical to provide a Y than a turntable, or where engines need not be turned, or at shops where a transfer table is available which may serve engine house also.

A Round House implies a turntable located at its center. The turntable, preferably of the deck type, should be long enough to balance the engine when tender is empty and should be operated by power, preferably electric except where only a few light engines are turned. The turntable pit should be well drained and paved, with side walls of brick or concrete. Pivot masonry of concrete with stone cap. Ties under the circular rail should be supported on concrete walls. The distance from center of turntable to inner line of roundhouse is determined by number of stalls required in a full circle. The angle between consecutive stall tracks should be an even divisor of 180 deg. so that tracks at opposite ends of the turntable will simultaneously line up with it. The clear stall length should not be less than 15 ft. greater than the over-all length of the locomotive. The **interior arrangement** is designed on the basis that the locomotive is always run in with the tender toward the turntable. The entrance doorways should be 13 ft. by 16 ft. clear. The doors should be made of non-corrosive material, fit snugly, operate easily and permit the use of small doors. The walls and roof should be made of non-corrosive material unless protected against corrosion. Security against interruption to traffic from fire warrants the serious consideration of fire-proof construction. Engine pits should be not less than 60 ft. long, with convex floor and drainage toward the turntable. Walls and floors may be of concrete. Supports for jacking timbers should be provided. Smoke jacks with flues not less than 7 sq. ft. in area should be fixed, should have dampers and large hoods and be made of non-corrosive and non-combustible material. The bottom of the jack should be as low as the height of the locomotive stacks will permit, at least 42 in. wide and long enough to receive smoke from stack at its limiting positions, due to adjustments of the driving wheels to bring the side rods at proper position for repairs. The floor should be permanent and be crowned between pits. Drop pits should be provided for truck wheels, driving wheels and tender wheels. When hot-air heating is used, the air should be led through ducts under the floor to the pits under the engine portion of the locomotive. Ducts should be provided with dampers so that air can be turned off when workmen are in the pit. A general temperature of 50° F. to 60° F. is recommended. Air should be heated by exhaust steam in so far as possible. The supply should be taken from the external air and no recirculation allowed. Light should be obtained from the exterior as far as possible and from windows rather than skylights. General illumination, avoiding shadows, should be obtained by using a number of lights between stalls and a plug for incandescent lights should be placed in each alternate space between stalls. Provision should be made for a few necessary machine tools, preferably electrically driven. Piping for air, steam and water supply should be provided.

STEAM RAILROAD OPERATION

24. Statistics of U. S. Railroads

	1917	1927
Mileage		
Total line	253 626	249 131
Average increase for previous ten years	2 367	450*
Per 100 square miles of territory	8.38	8.23
Per 10 000 inhabitants	24.28	21.00
Total track	379 254	399 214
Average increase for previous ten years	5 128	1 996
Net Capitalization		
Total ... thousands	$16 401 786	$18 136 691
Per mile of line	$64 669	$72 800
Passenger Traffic		
Train-miles ... thousands	593 338	578 355
Passengers ... thousands	1 109 943	840 030
Passenger-miles ... millions	40 100	33 798
Average number of passengers per train	66	58
Average journey per passenger ... miles	36.13	40.23
Freight Traffic		
Train-miles ... thousands	646 402	598 435
Tons ... thousands	2 453 423	2 510 454
Ton-miles ... millions	398 263	432 014
Ton-miles per mile of road	1 538 211	1 668 800
Haul per ton ... miles	288.18	314.75
Revenue tons per train	588.29	689.68
Revenue tons per loaded car	24.75	24.60
Revenues		
Operating ... thousands	$4 115 413	$6 245 715
Non-operating ... thousands	$350 745	$466 770
Gross earnings ... thousands	$4 466 158	$6 712 485
Gross earnings per mile of line	$13 666	$26 943
Gross earnings per inhabitant	$42.76	$56.58
Gross earnings per train-mile	$3.60	$5.70
Passenger earnings per passenger mile ... cents	2.097	2.901
Freight earnings per ton-mile ... cents	0.728	1.095
Passenger earnings per passenger train-mile	$1.42	$1.69
Freight earnings per freight train-mile	$4.48	$7.72
Expenses		
Operating ... thousands	$2 906 283	$4 662 521
Per train-mile, all trains	$2.34	$3.96
Maintenance of way and structures ... thousands	$442 110	$868 581
Maintenance of equipment ... thousands	$685 429	$1 219 052
Employees, Class I Railroads		
Total	1 732 876	1 735 105
Per mile of line operated	7.46	7.27

* Decrease.

25. Cost of Operation

The Average Cost of a Train Mile for the whole United States in the year 1890 was 96.0 cents. It decreased to 91.8 cents in 1895, and steadily increased to nearly $5.00 in 1920, decreasing again to nearly $4.00 in 1922 since which time it has been fairly constant. Although the variations from these figures are large in many cases for individual roads, the average for individual trunk lines is seldom very different from the general average. Whereas the averages for the individual short lines with light traffic will vary considerably from the general average, the average for any group of short lines is usually but little different from the general average, which means that there is no general tendency for the average figure for a short light-traffic line to be either less or greater than the general average.

Classes of Railways. For the purpose of analyzing costs of operation, etc., the Interstate Commerce Commission has divided the railroads into three classes on the basis of the gross annual operating revenue: Class I, over

Average Cost, for United States, of Operating a Train One Mile

Year	Cents	Year	Cents	Year	Cents	Year	Cents
1890	96.006	1900	107.288	1910	148.865	1919	403.972
1891	95.707	1901	112.282	1911	154.338	1920	498.554
1892	96.580	1902	117.960	1912	159.077	1921	429.113
1893	97.272	1903	126.604	1913	170.375	1922	410.594
1894	93.478	1904	131.375	1914	176.917	1923	414.850
1895	91.829	1905	132.140	1915	177.641	1924	397.221
1896	93.838	1906	137.060	1916	183.279	1925	391.123
1897	92.918	1907	146.993	1917	234.427	1926	394.795
1898	95.635	1908	147.340	1918	347.175	1927	396.207
1899	98.390	1909	143.370				

A comparison of the various classes is given in the following table:

Comparison of Classes I, II, and III Railroads
Year Ending Dec. 31, 1927

	Class I	Class II	Class III	Total
Number of companies	167	265	334	766
Lessor	332	21	18	371
Total operated *	499	286	352	1 137
Miles owned	174 108	13 365	5 263	192 736
Lessor	40 267	532	166	40 965
Miles operated	238 634	15 302	5 703	259 639
Locomotives †	61 363	1 544	590	63 497
Passenger cars †	53 822	1 242	578	55 642
Freight cars †	2 354 884	34 133	4 792	239 809
Ton-miles ... thousands	428 736 962	3 019 091	257 926	432 013 979
Per cent	99.24	0.70	0.06	100
Passenger-miles † .. thousands	33 649 706	132 848	15 200	33 797 754
Per cent	99.57	0.39	0.04	100
Operating revenues,† thousands	$6 136 360	$92 923	$16 492	$6 245 715
Per cent	98.24	1.49	0.27	100
Operating expenses,† thousands	$4 574 178	$72 679	$15 664	$4 662 521
Per cent	98.11	1.56	0.33	100
Investment† ... thousands	$18 976 654	$543 655	$99 155	....
Lessor ... thousands	$3 896 406	$13 609	$5 297	....
Total ... thousands	$22 873 060	$557 254	$104 452	$23 534 766
Per cent	97.19	2.37	0.44	100

* Excludes switching and terminal, proprietary, circular and unofficial companies, not assigned to one of the classes.

† Excludes switching and terminal companies.

$1 000 000; Class II, $1 000 000 to $100 000; Class III, less than $100 000. About 90% of the mileage comes under Class I.

Classification of Revenues and Expenses. The classification of accounts required of Class I Railways by the Interstate Commerce Commission is quite detailed as will be seen from the following list giving principal groups and number of accounts in each, as well as total amount for the year 1927 and percentage for each group.

Railway Operating Revenues

No.		Totals	Per cent
	Transportation—Rail line, Accounts, 1–16, *inclusive*		
17	Total rail-line transportation revenue	$5 987 603 165	97.58
	Transportation—Water line, Accounts 18–25, *inclusive*		
26	Total water-line transportation revenue	$14 314 408	0.23
	Incidental, Accounts 27–39, *inclusive*		
40	Total incidental operating revenue	$124 792 868	2.03
	Joint Facility, Accounts 41 *and* 42		
43	Total joint facility operating revenue	$9 589 829	0.16
44	Total railway operating revenues	$6 136 300 270	100

Rail-line transportation of freight brought in $4 632 321 165 and of passengers, $974 950 863, the same items for water-line transportation were $11 195 846 and $1 226 773, respectively. Rail-line mail was $143 364 766 and water-line, $12 911. The largest incidental item was dining and buffet, $33 804 372, while revenue from demurrage was $18 472 560.

Railway Operating Expenses

No.		Totals	Per cent
	Maintenance of way and structures, Accounts 1–106, *inclusive*		
107	Total maintenance of way and structures expenses	$868 581 432	18.91
	Maintenance of equipment, Accounts 108–146, *inclusive*		
147	Total maintenance of equipment expenses	$1 219 051 528	26.51
	Traffic, Accounts 148–156, *inclusive*		
157	Total traffic expenses	$120 349 440	2.63
	Transportation—Rail line, Accounts 158–207, *inclusive*		
208	Total transportation—Rail-line expenses	$2 127 505 968	46.31
	Transportation—Water line, Accounts 209–211, *inclusive*		
212	Total transportation—Water-line expenses	$9 481 244	0.21
	Miscellaneous operations, Accounts 213–218, *inclusive*		
219	Total miscellaneous operations expenses	$55 766 966	1.22
	General, Accounts 220–231, *inclusive*		
232	Total general expenses	$191 081 367	4.18
233	Transportation for investment, Cr.	$17 640 124	.....
234	Total railway operating expenses	$4 574 177 821	.....
235	Ratio of operating expenses to operating revenues (per cent)		74.54

The accounts for Class II and Class III railways are successively simplified into a much smaller number, giving less detail.

Some selected major items in the various groups are as follows: Maintenance of way and structures: Superintendence, $57 694 596; roadway maintenance (other than yard), $80 622 921; bridges, trestles and culverts (other than yard), $39 931 216; ties (other than yard), $102 900 409; rails (other than yard), $46 098 032; track laying and surfacing (other than yard), $193 650 381; station and office buildings, $33 451 157 and signals and interlockers, $32 847 323. Maintenance of equipment: Superintendence, $45 858 746; steam locomotives — repairs, $435 564 729; steam locomotives — depreciation, $56 187 354; steam locomotives — retirements, $4 612 516; freight-train cars — repairs, $340 695 795–depreciation, $114 126 553, and–retirements, $17 393 635; and passenger-train cars — repairs, $81 219 354,–depreciation, $18 178 446 and–retirements, $1 603 544. Transportation — rail line: Superintendence, $69 020 593; station employees, $304 859 956; yard conductors and brakemen, $154 362 250; yard enginemen, $90 005 177; train enginemen, $224 609 355; fuel for train locomotives, $321 442 975; fuel for yard locomotives, $64 213 939; trainmen, $265 024 707; and loss and damage — freight, $33 976 470. Miscellaneous operations: Dining and buffet service, $41 365 319 and hotels and restaurants, $7 432 294. General: Salaries and expenses of general officers, $19 720 910; same for clerks and attendants, $93 539 620 and pensions, $24 637 019.

Compensation of Employees increased moderately in the last five years, as shown by the following table, compared with the large increases during the war period with which comparison is difficult due to changes in classification.

Average Number and Compensation of Employees

Group		1922		1927		Percentage increase
		Number	Compensation	Number	Compensation	
Executives, officials and staff assistants	D	15 250	$15.970	16 710	$17.242	7.96
Professional, clerical and general	D	48 451	6.739	52 860	7.290	8.18
	H	229 063	0.566	223 735	0.609	7.60
Maintenance of way and structures	D	4 591	8.875	5 019	9.201	3.67
	H	358 196	0.418	411 753	0.439	5.02
Maintenance of equipment and stores	D	15 336	8.929	16 241	9.028	1.11
	H	439 975	0.602	468 799	0.616	2.33
Transportation (other than train, engine and yard)	D	25 927	3.238	25 536	3.449	6.52
	H	178 782	0.533	176 731	0.564	5.82
Transportation (yardmasters, switch tenders and hostlers)	D	6 042	8.466	6 682	8.727	3.08
	H	17 548	0.614	15 896	0.680	10.75
Transportation (train and engine service)	H	302 082	0.791	315 143	0.840	6.19
All employees	D	116 376	7.541	123 048	8.133	7.85
	H	1 528 857	0.588	1 612 057	0.615	4.59

D indicates daily basis and H indicates hourly basis.

26. Taxes on Steam Railroads

From Interstate Commerce Commission Reports for 1907, 1917, and 1927

The table on p. 2082 gives the total taxes paid per mile of line in the various States during the fiscal years ending June 30, 1907, 1917, and 1927. The figures are given for the three years so as to indicate the increase and its rate. The table is useful to give some idea of the probable taxes on some existing road or a new project, for a few years in the near future. Although all of the increase in some States is undoubtedly due to a proportionate increase, through betterments, in the real value of the roads, in many cases the increase is undoubtedly due to a modification in the method of assessment.

No figures are given to show the relation between the taxes per mile in any State and the capitalization (or any other measure of value) of the roads in that State. The total taxes paid during 1927 amounted to $399 005 000, which is 1.79% of the total capitalization, $24 302 008 000.

Taxes on Class I Railroads in the United States

State	Taxes per mile			State	Taxes per mile		
	1907	1917	1927		1907	1917	1927
Alabama	218	419	714	New Hampshire	358	571	985
Arizona	142	621	1 370	New Jersey	2 047	3 945	8 259
Arkansas	224	431	642	New Mexico	139	591	866
California	390	780	1 911	New York	686	1 593	3 072
Colorado	287	446	880	North Carolina	177	445	1 444
Connecticut	1 339	1 311	2 101	North Dakota	265	563	833
Delaware	391	451	575	Ohio	569	1 347	2 356
Florida	176	407	1 105	Oklahoma	114	659	896
Georgia	166	383	668	Oregon	228	678	1 291
Idaho	233	727	1 185	Pennsylvania	510	924	1 319
Illinois	472	893	1 898	Rhode Island	1 100	2 262	3 750
Indiana	481	824	1 727	South Carolina	176	410	1 052
Iowa	243	410	712	South Dakota	101	385	629
Kansas	296	576	966	Tennessee	267	542	880
Kentucky	366	615	1 419	Texas	153	375	481
Louisana	218	622	1 219	Utah	320	606	1 047
Maine	292	558	1 054	Vermont	172	542	593
Maryland	620	991	1 823	Virginia	376	793	1 899
Massachusetts	1 525	1 199	2 096	Washington	415	906	1 462
Michigan	398	500	1 273	West Virginia	413	792	1 755
Minnesota	429	710	925	Wisconsin	414	750	937
Mississippi	214	591	1 283	Wyoming	141	741	1 005
Missouri	206	339	505	Dist. of Columbia	1 480	1 687	3 578
Montana	271	601	917				
Nebraska	429	499	837	Average	367	972	1 666
Nevada	265	706	1 006				

27. Territorial Subdivisions of the United States

In order to compare results in different sections of the country the Interstate Commerce Commission at one time divided it into ten districts, later reducing the number to three, the Eastern, Southern, and Western. Finding these rather large for some purposes, the commission divided them as shown in Fig. 45. Some data are now given by Region, others by District.

The following tables make possible interesting comparisons between the various territories. Note for instance the heavy freight traffic and consequent high earnings in the Pocahontas Region and the reverse in the Northwestern and Southwestern Regions with sparse traffic, both passenger and freight.

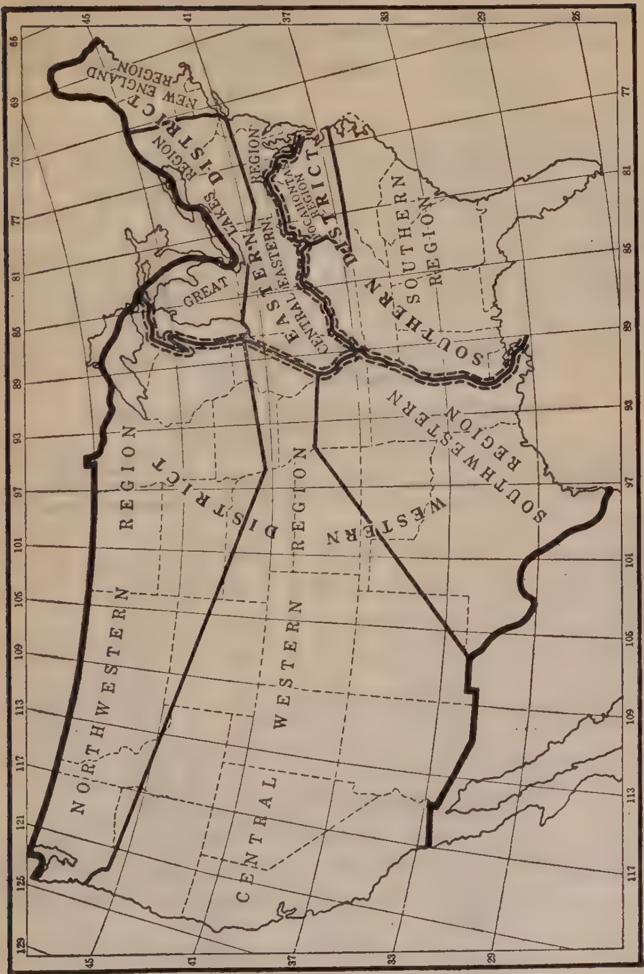


Fig. 45. Map showing subdivisions of the Interstate Commerce Commission

Mileage and Traffic by Regions, Class I Railways, 1927

District and region	Mileage	Trackage	Traffic in train-miles: Passenger, millions	Freight, millions	Total,* millions	Freight ton-miles, millions	Passenger miles, millions
Eastern:							
New England..	7 321	13 845	27 317	17 989	46 109	9 843	2 670
Great Lakes....	24 720	55 033	88 513	94 237	185 414	79 822	6 455
Central Eastern	26 927	59 389	120 377	114 478	237 040	108 808	9 041
Southern:							
Pocahontas....	5 582	10 714	12 823	26 717	39 850	39 685	582
Southern......	39 669	61 531	88 047	100 669	193 194	65 534	3 926
Western:							
Northwestern..	48 474	71 921	72 627	73 526	152 577	56 076	3 055
Central Western	52 201	8 593	108 008	105 551	216 228	73 353	5 862
Southwestern...	33 106	46 126	50 755	59 915	114 044	38 671	2 060
Total United States.....	238 000	399 214	568 538	588 081	1 184 455	471 793	33 650

* Including mixed and special.

Revenues, Expenses and Traffic per Mile of Road Class I Railways by Regions, 1927

District and region	Revenues	Expenses	Train-miles	Car-miles: Passenger	Car-miles: Freight
Eastern:					
New England......	$36 839	$27 897	6 298	23 924	96 583
Great Lakes.......	44 616	33 896	7 501	27 461	205 545
Central Eastern....	51 354	39 062	8 803	30 318	207 247
Southern:					
Pocahontas........	49 747	32 150	7 139	14 715	288 589
Southern..........	20 499	15 595	4 870	14 363	104 491
Western:					
Northwestern......	14 744	11 065	3 148	9 544	71 988
Central Western....	19 736	14 000	4 142	15 323	97 659
Southwestern......	15 988	12 101	3 445	9 512	81 058
Total United States.	$25 716	$19 159	4 977	16 355	119 309

Summary of Traffic in United States, 1927

District..........	Eastern		Southern		Western		Total United States	
Products	Tonnage	Per cent	Tonnage	Per cent	Tonnage	Per cent	Tonnage	Per cent
	thousands		thousands		thousands		thousands	
Agriculture.........	72 361	5.85	33 155	7.37	115 868	17.13	221 385	9.37
Animals............	18 020	1.45	4 414	0.98	24 261	3.59	46 696	1.97
Mines..............	729 555	58.94	267 928	59.58	274 821	40.64	1 272 304	53.83
Forests............	50 216	4.06	53 709	11.94	88 849	13.14	192 774	8.15
Manufactures *.....	334 034	26.99	76 281	16.96	154 318	22.82	564 643	23.89
L. C. L............	33 482	2.71	14 257	3.17	18 100	2.68	65 838	2.79
Total United States	1 237 668	100	449 753	100	676 217	100	2 363 639	100

* Includes miscellaneous.

Operating Data by Regions, Class I Railways, 1927

District and region	Revenues per train-mile	Expenses per train-mile	Revenues per ton-mile	Revenues per passenger-mile	Average journey per passenger	Average haul per ton	Operating ratio
Eastern:							
New England.....	$5.85	$4.43	$0.01780	$0.02825	26.99	111.88	75.73
Great Lakes......	5.95	4.52	0.01111	0.02807	39.82	150.96	75.97
Central Eastern...	5.83	4.44	0.01017	0.02716	27.36	148.58	76.06
Southern:							
Pocahontas.......	6.97	4.50	0.00646	0.03449	51.98	256.40	64.63
Southern.........	4.21	3.20	0.01073	0.03232	52.75	185.99	76.08
Western:							
Northwestern.....	4.68	3.51	0.01116	0.02940	63.53	195.01	75.05
Central Western...	4.76	3.38	0.01202	0.02871	69.63	244.81	70.93
Southwestern.....	4.64	3.51	0.01226	0.03250	100.36	193.92	75.69
Total United States	$5.17	$3.85	$0.01080	$0.02896	40.55	178.46	74.50

28. Accidents

During the year 1928, 6509 persons were killed, about 80% of whom were neither passengers nor employees but "others," mainly trespassers. Only 85 of the killed were passengers while 1243 were employees, of whom 824 were

Railroad Accidents in the United States

Year	Passengers		Employees		Others		Total	
	Killed	Injured	Killed	Injured	Killed	Injured	Killed	Injured
1906	359	10 764	3 929	76 701	6 330	10 241	10 618	97 706
1907	610	13 041	4 534	87 644	6 695	10 331	11 839	111 016
1908	381	11 556	3 405	82 487	6 402	10 187	10 188	104 230
1909	253	10 311	2 610	75 006	5 859	10 309	8 722	95 626
1910	324	12 451	3 382	95 671	5 976	11 385	9 682	119 507
1911	356	13 433	3 602	126 039	6 438	10 687	10 396	150 159
1912	318	16 386	3 635	142 442	6 632	10 710	10 585	169 538
1913	403	16 539	3 175	171 417	6 846	12 352	10 964	200 308
1914	265	15 121	3 259	165 212	6 778	12 329	10 302	192 662
1915	222	12 110	2 152	138 092	6 247	11 838	8 621	162 040
1916	283	8 379	2 687	160 663	6 394	11 333	9 364	180 375
1917	301	7 582	3 199	174 247	6 587	12 976	10 087	194 805
1918	471	7 316	3 419	156 013	5 396	11 286	9 286	174 575
1919	273	7 456	2 138	131 018	4 567	10 579	6 978	149 053
1920	229	7 591	2 578	149 414	4 151	11 304	6 958	168 309
1921	205	5 584	1 446	104 530	4 345	10 571	5 996	120 685
1922	200	6 153	1 648	116 757	4 477	11 961	6 325	134 871
1923*	138	5 847	1 563	39 476	5 221	11 141	7 385	171 712
1924*	149	5 354	1 192	32 174	4 874	10 843	6 617	143 739
1925	176	5 643	1 523	118 635	5 067	13 157	6 766	137 435
1926	155	5 149	1 590	111 241	5 348	13 845	7 090	130 235
1927	91	4 567	1 493	87 567	5 408	12 682	6 992	104 816
1928	85	4 027	1 243	69 692	5 181	11 482	6 509	85 561

* Non-train accidents included in totals only.

killed in train service accidents, 138 in train accidents, and 281 in non-train accidents. As about 800 000 000 passengers were carried during that year, only one in about nine and one-half million was killed; similarly, the total passenger mileage was 31 500 000 000, which means one passenger killed per 370 million passenger miles. Of courses the chances of injury are much greater, as indicated by the table at bottom of preceding page.

Accidents are more or less fortuitous and hence the data are somewhat erratic, but those for passengers and employees give some indication of results from the " safety first " movement, since accidents to both have decreased markedly, especially in the last decade, a period of fairly constant total traffic in train-miles. Nearly one-half of those killed in 1928 (2968) were killed in grade-crossing accidents, and 6667 were injured.

29. Cost of Railroads

The following items of cost are inserted as a catalog of the expenses which are usually met with and also to give an approximate idea of average costs per mile of track, which may be utilized in approximate preliminary calculations. Except where definite unit costs are given, the estimates are frequently subject to wide variations.

Right-of-way. Land for railroad purposes is valued at a higher rate than for farm purposes on account of consequential damages and injury to adjoining property. Even State Commissions for appraising railroads make an allowance. The Minnesota Commission generally allowed three times the ordinary farm value for farm lands. In cities from 1.25 to 1.75 times the ordinary market value was allowed. In Michigan from 2 to 2.25 times the ordinary market value is allowed.

Clearing. Usually $25 to $100 per acre.

Grubbing. Usually $100 to $200 per acre where required.

Earthwork. When the grading is light, fills are frequently made from borrow pits and the material in cuts is wasted, and then the price per cubic yard applies to the sum of the yardage of cut, and fill plus shrinkage. This price is usually 25 to 60 cents per cubic yard for earth; about 90 cents to $1.50 for loose rock and $1.50 to $3.00 for solid rock. When earthwork is heavy, care is taken to make the fills from the material in the cuts. The disposition of material is directed by the railroad engineer, and the contractor is paid according to the amount excavated. Borrowing is reduced to a minimum, and the contractor is often allowed extra for overhaul, usually 1 to 3 cents per cubic yard per 100 ft., for material hauled in excess of some limit, say 500 to 1000 ft. When only the yardage excavated is paid for and the contractor must make the embankments without extra pay, the price per yard for earth is nearly double the price given above unless machine methods are used when the cost may be about the same as above or even less. Much depends on haul and on local conditions. The prices for loose rock and solid rock will be but little more than the values given above, since the extra work is chiefly due to the loosening. Work in and near cities and on lines under traffic will usually cost more. The grading on all the railroads of Wisconsin was estimated to average $5098 per mile of main line, those in Michigan, $2778 per mile; those in Minnesota, $7372. The variations in this item are evidently very large.

Tunnels. This item is so exceedingly variable that general figures are worthless except those of unit cost. The average of a large number of cases is shown in the following tabular form, condensed from Drinker's Tunneling:

Material	Cost per cubic yard				Cost per lineal foot	
	Excavation		Masonry			
	Single	Double	Single	Double	Single	Double
Hard rock	$5.89	$5.45	$12.00	$8.25	$69.76	$142.82
Loose rock	3.12	3.48	9.07	10.41	80.61	119.26
Soft ground	3.62	4.64	15.00	10.50	135.31	174.42

Bridges, Trestles and Culverts. The price per mile of road is evidently very variable. The average for the railroads of Minnesota was computed at $2576 per mile; those of Michigan, $1027 per mile. Trestle timber in place is worth approximately $85 per M feet; trestle piling in place, 75 cents per lineal foot of pile; pile trestling from $12.00 to $18.00 per foot of trestle; vitrified culvert pipe in place, $2.00 per lineal foot for 18-in., $3.50 for 24-in., cast-iron pipe culverts in place about 8 cents per pound.

Rails. Tons 2240 lb. per mile of track equals 11/7 weight of rail per yard in pounds. Allow 2% for cutting and waste. The freight from mill to delivery point is frequently a considerable gross item. The present price for standard rails is $34 f. o. b. at the mills, while light and rerolled rails are quoted at $34 to $36. Prices of **rail fastenings** at Pittsburgh are: steel splice bars 2.75 cents per pound; spikes 2.8 cents per pound; track bolts 3.8 cents per pound. Prices at Chicago and Birmingham are 0.65 to 0.75 cent higher and at St. Louis and San Francisco about 1 cent higher than at Pittsburgh. Tieplates are 2.15 cents per pound at the Pittsburgh and Chicago mills.

Cross Ties. 75 cents to $1.10 per tie, untreated, depending on kind of wood and locality. Treated ties cost 40 to 90 cents more, depending on process used.

Frogs, Switches and Crossings. Switches in place will cost from $380 to $750 per switch. The cost of a crossing of two railroads will depend on the angle of intersection, and will cost from $200 up.

Ballast. Quantities and unit costs are given under the heading of **ballasting** under track. The cost may run from $1500 to $5000 per mile for the best of broken stone. An ordinary average is $2000 per mile of track.

Track Laying and Surfacing. This really includes the three items of track laying, surfacing and the train service of hauling track materials from the point of delivery to the places where used. The train service is frequently furnished by the railroad company, but its actual cost is from $125 to $200 per mile. The contract price for track laying alone is frequently $450 per mile at present prices for labor, but there are many records of track laying by means of special track-laying cars for as little as $150 per mile. Sometimes ballasting is deferred until the road is in operation, at least for construction trains. The surfacing may then be a separate contract at about $400 per mile. Allowing an average figure of $175 per mile for the train service, the total cost to the company for this item will average about $900 per mile.

Fencing will average about $250 per mile of fence, or about $500 per mile of road when both sides are completely fenced. Often only a small fraction of the total length is fenced.

Buildings and Miscellaneous Structures. Station buildings and fixtures in Wisconsin averaged $476 per mile; those in Minnesota, $771; those in Michigan, $526. Unit allowance in valuation of a Texas railroad for small frame passenger stations, $1 per square foot; for platforms, 16 cents per square foot. The analyzed cost of six section and tool houses averaged 30.7 cents per square foot of area. The cost of water stations of ordinary capacity will vary from 3 to 4 cents per gallon of capacity. Sign boards, $10 to $15 each. Whistle posts, mile posts and rail rests, about $2 each. Road crossings require about 260 ft. b. m., which at an average price in place of $70 per M will $18.20 each. A pair of stock guards is usually required for each crossing, which may cost about $75 per pair, including the short fences from the right-of-way lines to the ends of the ties. A Y can take the place of a turntable providing that land is available at no special cost. Using a radius of 300 ft., with 100 ft. of track for the engine at the tail of the Y, about 1050 ft. of track will be required together with three switches. The tail of the Y will require land for 400 ft. from the main track.

Turntables may cost anywhere from $2000 to $4000. The turntable itself is made of cast iron or of structural steel, or sometimes of a combination of structural steel trusses with a cast-iron center.

Coaling Stations may vary from a mere platform or bunker from which the coal is shoveled into the tender (although at a considerable cost per ton) to the very elaborate and costly coal pockets in which coal is deposited by coal conveyors or dumped from cars drawn up an incline, and from which the coal slides through chutes to the tender. The cost must evidently be computed for individual cases. **Ash pits** likewise vary from a mere pit between the rails to an elaborate adjunct of a coaling station by which the ashes are immediately transported on a " conveyor " to an ash car.

Terminal Grounds. Although the area of terminal grounds may be not more than 3 or 4% of the total property area, its gross cost or its cost per mile of road may be actually more than the cost of all the remaining right-of-way. In Minnesota, the valuation of terminal grounds was 71% of the total valuation of all the line right-of-way, gravel pits, station grounds and terminal grounds.

Miscellaneous: Shops, roundhouses and shop tools and machinery will average about 3% of the total cost of a road. Snow fences, bridge ticklers, track scales, mail cranes and bumping posts must be allowed for on all large roads, but their relative cost is insignificant.

Grain elevators, warehouses, docks and wharves are mentioned as possible items of expense, but do not apply to many roads. For signals the cost is so variable, depending on the degree of elaboration of the system, that average unit prices per mile of road would only be misleading.

Telegraph Lines. Safe estimate $350 per mile of road for a single-wire line. If poles are very cheap even this cost may be materially cut.

Freight on Construction Material. This applies chiefly to track material such as cross ties, rails, etc. While very variable, it may amount to nearly 1% of the cost of the road. Frequently the item is ignored, the freight being added to the cost of each item.

Contingencies. Usually estimated at 5 to 10% of the above items.

Engineering, Superintendence and Legal. Frequently figured at 5% of all the construction work. The legal work will cost about 1%.

Equipment. The approximate cost of locomotives and cars is about $7000 per mile of road. Marine equipment must occasionally be allowed for.

Organization Expenses will amount to 1 to 1.5% of the cost and frequently even more.

Interest on Cost during Construction. Theoretically this should mean interest on money from the time that bills are payable until the road begins operation. This

State Appraisals of Steam Railroads. Average Cost per Mile

No.	Items	Minnesota (1907) Cost	Minnesota (1907) Per cent	Michigan (1900) Cost	Michigan (1900) Per cent	Wisconsin (1903) Cost	Wisconsin (1903) Per cent
1	Land; right-of-way, yards and terminals	$9 637	17.78	$3 665	14.00	$3 719	12.03
2	Grading, clearing and grubbing	7 372	13.60				
3	Protection work, riprap, retaining walls	318	0.59	2 778	10.63	5 098	16.50
4	Tunnels	33	0.06	147	0.56	122	0.39
5	Cross ties and switch ties	2 303	4.25	1 426	5.46	1 529	4.95
6	Ballast	1 239	2.29	476	1.83	788	2.55
7	Rails	4 348	8.02	3 674	14.05	3 773	12.21
8	Track fastenings	782	1.44	492	1.88	617	2.00
9	Switches, frogs, and railroad crossings	183	0.34	188	0.72	151	0.48
10	Track laying and surfacing	703	1.30	839	3.21	447	1.45
11	Bridges, trestles and culverts	2 576	4.75	1 027	3.93	2 372	7.67
12	Track and bridge tools	27	0.05	......	......	19	0.06
13	Fences, cattle guards and signs	354	0.67	431	1.66	277	0.90
14	Stockyards and appurtenances	74	0.14	......	......	......	......
15	Water stations	211	0.39	93	0.36	161	0.52
16	Fuel stations	95	0.18	39	0.15	54	0.17
17	Station buildings and fixtures	771	1.42	526	2.01	476	1.54
18	Miscellaneous buildings	572	1.05	158	0.60	353	1.14
19	General repair shops	543	1.00				
20	Engine houses, turntables, cinder pits	373	0.69	276	1.05	428	1.38
21	Shop machinery and tools	241	0.44	142	0.54	182	0.59
22	Track scales	24	0.04	......	......	......	......
23	Docks and wharves	799	1.47	707	2.71	260	0.84
24	Interlocking plants	53	0.10	64	0.25	52	0.17
25	Signal apparatus	20	0.40				
26	Telegraph lines and appurtenances	173	0.32				
27	Telephone lines and appurtenances	12	0.02	33	0.13	19	0.06
28	Grain elevators and warehouses	......	......	204	0.78	163	0.52
29	Adaptation and solidification of roadbed	1 546	2.85	......	......	......	......
30	Engineering, superintendence and legal	1 598	2.95	776	2.97	940	3.04
31	Locomotives	2 249	4.15	1 155	4.42	1 342	4.35
32	Passenger equipment	871	1.61	408	1.57	627	2.03
33	Freight car equipment	6 176	11.40	2 527	9.67	3 630	11.75
34	Miscellaneous equipment	175	0.32	89	0.34	70	0.22
35	Marine equipment	6	0.01	220	0.84	0	0.00
36	Steam-, gas- and electric-power plants	105	0.19	13	0.05	9	0.03
37	Freight on track material	478	0.88	......	......	193	0.62
38	Organization expenses	......	......	340	1.30	275	0.89
39	Contingencies	2 353	4.34	2 360	9.02	1 512	4.90
40	Stores and supplies	686	1.27	188	0.72	427	1.38
41	Interest during construction	4 115	7.59	677	2.59	825	2.67
	Total average cost per mile	$54 184	100.00	$26 138	100.00	$30 910	100.00

Item 30 assumed to be 4.5% of items 1–28, inclusive, for Michigan and Wisconsin and of items 1–29, inclusive, and also item 36 for Minnesota.

Item 38 assumed to be 1.5% for Michigan and 1% for Wisconsin of items 1–37, inclusive.

Item 39 assumed to be 10% for Michigan, 5.5% for Wisconsin and 5% for Minnesota of items 1–37, inclusive. Item 41 assumed to be 3% for Michigan and Wisconsin of items 1–37, inclusive. Basis for Minnesota estimate not stated, but corresponding figure would be 8.75%.

should average about half the time required to build the road. Financiers sometimes require interest on the full amount from the time of signing the contract, and this will more than double this item.

Appraisal commissions appointed in Wisconsin, Michigan and Minnesota have appraised all of the railroads of each state, the appraisal value being made both from the standpoint of the cost of reproduction and also the present value as affected by depreciation. The systems of analyzing the cost are not identical, which makes comparison difficult and even slightly inaccurate as given on p. 2089, but the average prices are very instructive. The cost per mile of main line and branches and the percentage of the item to the total cost are given separately for each state.

Miles of U. S. Railroads in Operation on Dec. 31 of Each Year

Year	Miles	Year	Miles	Year	Miles	Year	Miles
1830	23	1855	18 374	1880	93 262	1905	217 341
1831	95	1856	22 016	1881	103 108	1906	222 766
1832	229	1857	24 503	1882	114 677	1907	228 128
1833	380	1858	26 968	1883	121 422	1908	232 046
1834	623	1859	28 789	1884	125 345	1909	238 356
1835	1 098	1860	30 626	1885	128 320	1910	242 107
1836	1 273	1861	31 286	1886	136 338	1911*	243 979
1837	1 497	1862	32 120	1887	149 214	1912	246 777
1838	1 913	1863	33 170	1888	156 114	1913	249 777
1839	2 302	1864	33 908	1889	161 276	1914	252 105
1840	2 818	1865	35 085	1890	166 703	1915	253 789
1841	3 535	1866	36 801	1891	170 729	1916	254 251
1842	4 026	1867	39 050	1892	175 170	1916	254 037
1843	4 185	1868	42 226	1893	177 516	1917	253 626
1844	4 377	1869	46 844	1894	179 415	1918	253 529
1845	4 633	1870	52 922	1895	181 115	1919	253 152
1846	4 930	1871	60 301	1896	182 769	1920	252 845
1847	5 598	1872	66 171	1897	184 591	1921	251 176
1848	5 996	1873	70 268	1898	186 810	1922	250 413
1849	7 365	1874	72 385	1899	190 818	1923	250 222
1850	9 021	1875	74 096	1900	194 262	1924	250 156
1851	10 982	1876	76 808	1901	198 743	1925	249 398
1852	12 908	1877	79 082	1902	202 938	1926	249 138
1853	15 360	1878	81 747	1903	207 335	1927	249 131
1854	16 720	1879	86 556	1904	212 394		

* 1911 and later years to 1916 are for June 30.

30. Train Resistances

Atmospheric Resistance depends on end area and length of train and is independent of tonnage. The end-area pressure is approximately one-half of the unit pressure times the area of the car end. According to elaborate experiments by Goss, the resistance of trains of locomotives and cars of ordinary dimensions may be expressed by the following equations, in which P = total atmospheric resistance in pounds, n = the number of cars, L = length of the whole train, l = combined length of the cars composing the train, and V = velocity in miles per hour.

Locomotive, tender, and freight cars........	$P = (0.13 + 0.01n)V^2$
Locomotive, tender, and passenger cars.....	$P = (0.13 + 0.02n)V^2$
Freight cars following locomotive...........	$*P = (0.016 + 0.01n)V^2$
Passenger cars following locomotive........	$*P = (0.016 + 0.02n)V^2$
Locomotive and any kind of train..........	$P = 0.0003(L + 347)V^2$
Any train of cars (freight or passenger) following locomotives......................	$*P = 0.0003(l + 53)V^2$

* Excludes atmospheric resistance of locomotive and tender.

Oscillation and Concussion. These resistances are usually considered to vary as the square of the velocity. Their determination, independent of other forms of resistance, is impracticable and even useless, since they vary with the particular condition of the track and roadbed at the time.

Rolling Friction. The resistance to the wheel rolling on the rail, independent of other forms of resistance, has not yet been definitely determined. It evidently depends on the stiffness of the rail and on the rigidity of the support given to the rail by the ties.

Journal Friction per ton of load is (a) less for higher pressures or for heavy wheel loads; (b) greater for very low velocities; (c) a minimum for speed corresponding to train velocity of about 10 miles per hour; (d) greater for higher velocities; (e) less for higher temperatures; and (f) very dependent on perfection of lubrication. The use of roller or ball bearings very materially lowers this item but is rather limited as yet and probably will be for a long time as far as freight equipment is concerned.

Total "Train Resistance." The aggregate effect of the four kinds of resistance given above is known as the **Train Resistance.** It is of course the total resistance only on straight, level track, as neither the resistance (or assistance) due to gradients nor the extra resistance on curves is taken into account. Even with these variable resistances eliminated, no one simple formula can be devised which is equally applicable to all conditions of weight and character of cars, condition of track, weight of rail, etc. Dennis' tests, corroborated by those of Shurtleff, indicated a practically uniform resistance for freight trains at all velocities between 7 and 35 miles per hour. Later experiments by Professor Schmidt at the University of Illinois indicate some increase in resistance with increase in velocity for both freight and passenger equipment and the resistance of high-speed passenger trains unquestionably increases with the velocity, and some say, with the square of the velocity. Formulas based on tests made many years ago, when the rails were light, track conditions relatively poor, wheel loads light, and running gear not as perfect as present standards, are certainly not applicable to present standard conditions; and results obtained by tests of passenger trains are not applicable to freight trains, or vice versa.

A very few of the multitudinous formulas which have been proposed are here quoted.

Engineering News, 1892..................................	$R = 2 + 1/4\ V$
Baldwin Locomotive Works, prior to 1892......................	$R = 3 + 1/6\ V$
Shurtleff, 1906..	$R = 0.5 + (90/w)$

Manual of A.R.E.A., 1929 Ed.

A Rating, temperature above 35° (Fahr.)......................	$T = 2.2\,t + 122\,n$
B Rating, temperature 20° to 35°..........................	$T = 3.0\,t + 137\,n$
C Rating, temperature 0° to 20°...........................	$T = 4.0\,t + 153\,n$
D Rating, temperature below 0°............................	$T = 5.4\,t + 171\,n$

in which R = total resistance in pounds per short ton, V = velocity in miles per hour, T = the resistance in pounds per train, n = number of cars in train, t = weight of train in short tons, w = average weight in short tons of cars in train.

The first two are based mainly on tests with the passenger equipment of 1890–92 and may be compared with the results obtained by Prof. Schmidt in 1916 as shown below.

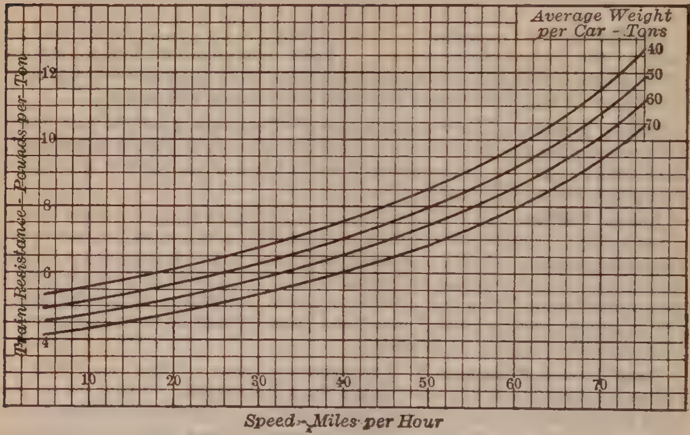


Fig. 46. Passenger Train Resistance

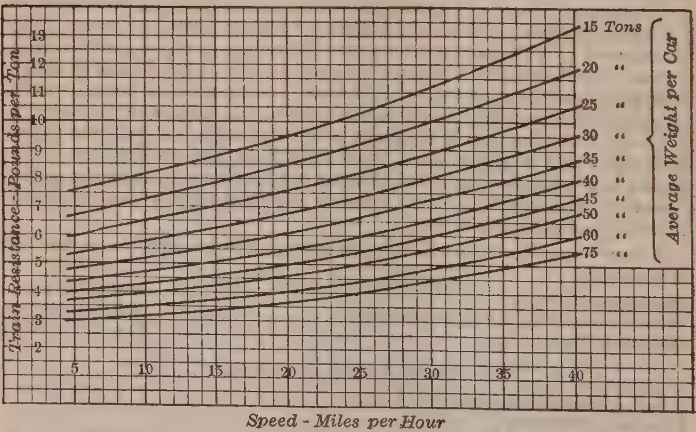


Fig. 47. Freight Train Resistance

The second two formulas are for heavy freight service and assume that the resistance per ton is constant at the ordinary freight speeds of 7 to 35 miles per hour. They may in turn be compared with the results obtained by Prof. Schmidt with freight equipment in 1908–09 given in Fig. 47.

Effect of Temperature. Decrease of temperature increases journal friction, but on the other hand, a frozen roadbed if smooth decreases the oscillatory resistances. At very low temperatures the increase in journal friction (combined with decreased power of the locomotive) requires a material reduction in the permissible train loading. Some railroads have a percentage scale for the reduction of the rating of locomotives according to the temperature. The increased tractive resistance for winter traffic over summer traffic is about two pounds per ton, with two pounds per ton additional for temperatures below zero. The American Railway Engineering Association formulas take the temperature into account by changing the constants as shown above.

Extra Resistance of Starting. The resistance of journals and axles is very much greater at the instant of starting and until they have become warmed up. Cars which have been stationary overnight, especially in cold weather, will become "frozen up" and will require a much greater force to start them, but the added resistance is only momentary and consumes but little energy, measured in foot-pounds. The frequent practice among locomotive engineers of backing the locomotive for a few feet and then immediately reversing and starting ahead has three effects, all of which help to start the train: (1) the journals are loosened from the somewhat rigid condition they will assume even during a short stop; (2) the springs in many of the couplers are more or less compressed during the backward movement, and their expansion materially assists in starting the cars; (3) if the train is very long, the total slack in the couplers is very considerable, and the locomotive will have moved forward several feet and will have a considerable velocity before the last car starts; the cars are therefore started one by one. Since so much depends on the method of handling the locomotive, there is a corresponding variation in the results of tests which have been made to determine the value of this resistance. Thirty-five tests on the Rock Island system, with trains of 34 to 45 cars, gave results varying from 10.6 to 18.2 lb. per ton. The weighted mean of the values was 14.1 lb. per ton. The same tests quoted 30 lb. per ton when a train had stood overnight and was "frozen up"; also a resistance of only 6 lb. when the stop was merely instantaneous. Other tests have shown an average of 14 lb. per ton.

Due to the above practice in starting trains and to the increased tractive force at starting this extra resistance is of little importance, i.e., it practically never limits the weight of the train.

Effect of Weight of Cars. The resistance per ton of horse cars, light trolley cars and contractors' dump cars is as high as 20 lb. per ton and even more when the rails are light or the track in poor condition. The heavier the wheel loads, the less the resistance per ton. The resistance of an empty standard gage car, weighing say 17 tons, on good track, is about 8 lb. per ton, and for a loaded car, weighing say 60 to 70 tons, about 4 lb. per ton or even less on a good track.

The American Railway Engineering Association formulas allow for the weight of cars by using two constants, one depending on the weight of the train and the other depending on the number of cars in the train. Schmidt gives curves for different weights of cars as shown in Figs. 46 and 47.

Grade Resistance is 20 lb. per short ton (2000 lb.) for each per cent of grade. This rule is not mathematically precise but the error is less than

0,5% for a 10% grade, 0.08% for a 4% grade, and is inappreciable for ordinary railroad grades. The **grade of repose** is the grade on which the effect of gravity just equals the tractive resistance. If a wagon or car is started down such a grade, it would continue to move indefinitely at a low velocity as long as the conditions are constant. The total resistance up such a grade is precisely twice that on a level.

Curve Resistance is considered the equivalent of a 0.04% grade per degree of curve, or 0.8 lb. per short ton per degree of curve. See also p. 2097.

Brake Resistance wastes the kinetic energy of the train by transforming it into heat and, unless such energy has been acquired by running down a grade, constitutes a source of double loss since the application of the brakes also requires power. For ordinary stops it may be considered as equivalent to a 7.5% grade, or 150 lb. per short ton; whereas for emergency stops it may reach double that amount. The corresponding distances may be obtained from the table on page 2096.

Rating of Locomotives. Since grade resistance varies with the total weight, while train resistance per ton depends on the weight of cars, the gross weight of cars which may be attached to a locomotive depends not only on the grade, but also on whether the cars are heavy or light. The greater the number of cars for a given weight of train, the greater the necessary drawbar pull, and therefore, for a given drawbar pull, the gross weight of a trainload of empties must be less than that of a train of loaded cars. For example, assume that an engine, weighing with the tender 336 500 lb., has a tractive effort at the rim of the drivers of 32 075 lb. On the basis of the resistance formula for "A" rating and constants given below the rating for a 0.6% grade is 2091 tons, with an allowance or "adjustment" of 8.6 tons for each car.* Then the permissible weight of the train which could be attached to that engine when the ruling grade is 0.6% may be expressed by the formula $W = A - 8.6\,n$, in which W = the weight of cars, A = the rating, and n = the number of cars.

For 30 loaded cars................2091 − 258 = 1833 tons; average 61 tons per car
For 50 partly loaded cars..........2091 − 430 = 1661 tons; average 33 tons per car
For 90 empties...................2091 − 774 = 1317 tons; average 15 tons per car

But a ruling grade of 0.6% is unusually low. As another example, with a ruling grade of 1.6%, the adjustment allowance is 3.6 tons per car, but the rating A is reduced to 770 tons. The possible train loads would be:

For 10 loaded cars..................770 − 36 = 734 tons; average 73 tons per car
For 25 partly loaded cars............770 − 90 = 680 tons; average 27 tons per car
For 40 empties.....................770 − 144 = 626 tons; average 16 tons per car

On the 1.6% grade the weight of the train of empties is 85% that of the loaded train; on the 0.6% grade it is only 72%; which shows the relatively greater importance of heavy loads with low ruling grades. Formulas for tractive force of locomotives are given in Art. 20.

The following derivation of the formula with a table for its use is condensed from a report of the Economics Committee to the American Railway Engineering Association in 1910. Let p = "pulling power of locomotive" or the tractive force as measured at the rim of the drivers; e the weight of the engine and tender; w the weight of the train, exclusive of locomotive; r the rate of grade; k a constant, depending on the weight of the train (w); n the

* This assumes that the engine and tender form part of the train which is approximately correct for A and B ratings on account of track and atmospheric resistances, not otherwise considered.

number of cars; c a constant, depending on the number of cars; and A = the "rating." Then

$$p = (e + w)(r + k) + nc$$

Transforming, $\frac{p}{r + k} - e = w + n\frac{c}{r + k}$; but $w + n\frac{c}{r + k} = A$

therefore $$A = \frac{p}{r + k} - e$$

The first formula for train resistance on a level given on p. —— is: $T = 2.2\,t + 122\,n$ in which t corresponds to $(e + w)$. But since the formula applies only to level track, $r = 0$. Therefore, the constant k, which depends on the weight of the train, = 2.2 lb. per ton or .0011 lb. per pound, and the constant c, which depends on the number (n) of cars, = 122 lb. The rating A for any grade is therefore some definite weight (depending on the grade), from which must be subtracted a constant for that grade times the number (n) of cars to obtain the actual weight (w) which may be hauled. The constant for each grade, $c/(r + k)$, is readily computed. For example, for a 0.6% grade, it equals 122/(.006 + .0011) = 17 183 lb. or 8.6 tons per car. Therefore on a 0.6% grade, $A = w + 8.6\,n$, or $w = A - 8.6\,n$. Similarly, the constant with which to multiply may be determined for any grade, and, as this is independent of the type of locomotive, it is given in the following table:

Values of c/(r + k) for Various Grades

Grade, per cent	Tons per car	Grade, per cent	Tons per car	Grade, per cent	Tons per car	Grade, per cent	Tons per car	Grade, per cent	Tons per car
Level	55	0.5	10.0	1.0	5.5	1.5	3.8	2.0	2.88
0.1	29	0.6	8.6	1.1	5.0	1.6	3.6	2.1	2.75
0.2	20	0.7	7.5	1.2	4.7	1.7	3.4	2.2	2.63
0.3	15	0.8	6.7	1.3	4.3	1.8	3.2	2.3	2.52
0.4	12	0.9	6.0	1.4	4.0	1.9	3.0	2.4	2.42

The rating of some particular engine for any and all grades may be determined by noting its hauling capacity on some one grade. For example, it hauls 50 cars, weighing 3033 tons, up a 0.3% grade at a constant speed. The adjustment for that grade (taken from the table) at the same speed is 15 tons per car. Therefore the rating is 3033 + (15 × 50) = 3783 tons or 7 566 000 lb. The engine weighs 336 500 lb. Then $A + e$ = 7 566 000 + 336 500 = 7 902 500 = $p/(r + k)$. But $(r + k)$ = .003 + .0011 = .0041. Therefore p = 32 400 lb., the pulling power of the locomotive. Then, knowing p, the rating A for any grade may be determined by substituting in the formula various values for r. For example, the ruling grade of the road is 1.2%; it is desired to know the power of that locomotive on the ruling grade.

$$A = p/(r + k) - e = 32\,400/(.012 + .0011) - 336\,500 = 2\,473\,000$$
$$- 336\,500 = 2\,136\,500 \text{ lb.} = 1068 \text{ tons.}$$

The actual tonnage of cars which may be hauled is less than this, since on this grade 4.7 tons must be subtracted for each car of the train. If there are 20 cars in the train, their aggregate weight must not exceed 1068 − (20 × 4.7) = 974 tons, or an average of 48.7 tons per car.

Using any one of the other formulas, or the curves of Professor Schmidt, the rating may be computed by finding the train load whose total resistance is slightly less than the tractive effort at the speed desired. Formerly this was the speed of maximum tractive force, but now many roads are rating for higher speeds, or the economical speed as noted below.

Locomotives may also be rated by actual trial, in fact, operating results of the use of computed ratings should be constantly studied to insure the best results.

Economical Speed. When the velocity of trains is very low (say 5 miles per hour) the coal burned per mile is greater and train wages are greater both on train under consideration and others due to greater interference with traffic, but any saving is small. For high velocities the cost per train mile is likewise comparatively high. The most economical speed is about 17 miles per hour for single track and nearly 20 miles per hour for double track according to results of computations by Isaacs and Adams, in Bul. 115 of the Amer. Rwy. Eng. and M. W. Assoc. These were, of course, based on certain assumptions but there is no doubt that it is often not most economical to haul maximum train loads or full " ratings " on account of low speeds.

Accelerative (or Retarding) Force in Pounds per Short Ton

(Original)

Distances s, feet	Velocity in miles per hour, V									
	15	20	25	30	35	40	45	50	55	60
100	158	281	439	632	860	......	......	......	......	......
200	79	140	219	316	430	562	711	878	......	......
300	53	94	146	211	287	375	474	585	708	843
400	40	70	110	158	215	281	355	439	531	632
500	32	56	88	126	172	225	284	351	425	506
600	26	47	73	105	143	187	237	293	354	421
700	23	40	62	90	123	161	203	251	303	361
800	20	35	55	79	108	141	178	218	266	316
900	18	31	48	70	96	124	158	195	236	281
1000	16	28	44	63	86	112	142	176	212	253
1100	14	25	40	57	78	102	129	160	193	230
1200	13	23	37	53	72	94	117	146	177	211
1300	12	22	34	49	66	86	109	135	163	194
1400	11	20	31	45	61	80	102	125	152	181
1500	10	19	29	42	57	75	95	117	142	168
1600	9	18	27	40	54	70	89	109	133	158
1700	9	17	25	37	51	66	84	103	125	149
1800	8	16	24	35	48	62	79	97	118	140
1900	8	15	23	33	45	59	75	92	112	133
2000	8	14	22	32	43	56	71	88	106	126
2100	7	13	21	30	41	54	68	84	101	119
2200	7	13	20	29	39	51	65	80	97	115
2300	7	12	19	27	37	49	62	76	92	110
2400	7	12	18	26	36	47	59	73	88	106
2500	6	11	18	25	34	45	57	70	85	102
2600	6	11	17	24	33	43	55	67	82	97
2640	6	11	17	24	33	43	54	66	80	96

Formula: $P = 70.22\ V^2/s$, in which P = force in pounds, s = distance in feet, V = velocity in miles per hour, 5% being allowed for rotative energy of wheels.

Distances, Speeds and Times. Let P represent the net force in pounds per short ton, s be the distance in feet between two points at which the train velocities are V_1 and V_2 respectively. Then $P = 70.22(V_2^2 - V_1^2)/s$, if V_1 and V_2 are in miles per hour. V_1 is the lower velocity; when the train is starting from rest, or coming to rest V_1 is zero.

The above table may be used to find the distance in which a train may be accelerated (or retarded) from any speed to any other, or to find the change in speed for a known distance. For instance, a train traveling 20 miles an hour enters a stretch of track 900 ft. long on which the accelerative force is 94 lb. per ton. The table shows that it takes 300 ft. to accelerate from 0 to 20 miles per hour and that in 900 ft. more or a total of 1200 ft. the velocity will be 40 miles per hour. Knowing the distances and speeds, the times are easily computed, and the sum of the time intervals is the schedule time of the run. Of course the net tractive force changes at each change of rate of grade and at each curve and it is faster to plot speed-time and distance-time curves if the motive power and load are known.

31. Compensation for Curvature

The Principle involved is that the grade should be reduced by such an amount that the saving in grade resistance will compensate for the additional resistance caused by the curvature. The proper rate of compensation evidently depends somewhat on the speed of the train; it also depends largely on the resistance to the rotation of the car trucks about their king-bolts, being very greatly reduced when ball bearings are used under the center and side plates; it is apparently far less per degree of curve on very sharp curvature than on easy curvature. The rate of compensation should be made greater on a curve occurring immediately above a stopping place for trains. Since the added resistance of curvature virtually increases the grade, it is unimportant whether there is any compensation on curves which are on minor grades, provided the total resistance does not become greater than that of the ruling grade. The rules recommended by the American Railway Engineering Association are as follows: (1) Compensate 0.03 ft. per degree when the length of curve is less than one-half the length of the longest train, when the curve occurs within the first 20 ft. of rise of a grade or when curvature is in no sense limiting. (2) Compensate 0.035 ft. per degree when the curve is between one-half and three-fourths as long as the longest train and when the curve occurs between 20 ft. and 40 ft. of rise from the bottom of the grade. (3) Compensate 0.04 ft. per degree when the curve is habitually operated at low speed, when the length of curve is more than three-fourths that of the longest train, when the elevation is excessive for freight trains or at all places where curvature is likely to be limiting. (4) Compensate 0.05 ft. per degree wherever the loss of elevation can be spared.

32. Economics of Location

Economic Location is the determination of the proper relation between construction and operating expenses and the discovery, in any individual case, of that location which will afford the most economical combination of cost and operating expenses for the given or estimated traffic. The **principles** are practically applied by comparing proposed plans for new construction or by investigating changes in existing lines including abandonment of existing facilities. For new work the method must include for each proposed route these features: (1) the estimated cost of the line; (2) the annual interest charge on capital at a rate which the circumstances of the project make proper; (3) the estimated annual traffic, not only the gross tonnage, but also the number of trains required to handle it; (4) the effect, if any, of change of location on the traffic obtainable; (5) the estimated annual cost of handling such traffic; (6) the effect, as far as it is possible or proper to predict it, of future changes in traffic or in operating conditions. Having determined the gross revenue, the interest on first cost and the operating expenses, the line giving the largest net revenue is evidently the most economical line. When revenue is not affected (generally true for small changes) the line having a minimum sum of operating expenses and interest on first cost is the most economical. When studying changes of an existing line, in addition to the features above mentioned, the possible injury to existing facilities must be considered. The proposed plan should, in such a case, so increase the net income that the excess will at least pay a proper return on the entire cost of the change.

The general method of analysis consists in separating the different items of cost of operation and determining the proportion of each item which is affected by each physical element in the proposed change of line. The physical elements are Distance, Curvature, Minor Grades and Ruling Grades. The table gives the aggregate results for each of the main groups of operating expenses and illustrates the method. Both the grouping and the percentages are now changed, see present groups operating expenses on p. 2080.

The lack of agreement of these figures is partly due to some differences in the basis of computation, partly to the different times at which they were computed and partly to the fact that such figures can never be constant for all times and conditions, but must be modified to suit local conditions. It is, however, remarkable that the mean values for the large and small changes give 38.1%, 41.4% and 38.5% respectively as the percentage of total operating expenses which are affected by changes of length of line.

J. B. Berry's estimate divides changes of distance into three classes: Class (A), distance so short as not to affect wages of engine or train men, 32.10%; Class (B), distance affecting train wages but not requiring additional side tracks, 45.94%; Class (C), distances so great as to require additional side tracks and stations, 59.5%.

W. G. Raymond's estimate is 35.3% + a flat amount of $350 per year per mile for expenses which are independent of the number of trains. This flat amount is a little less than $1 per day. With ten trains per day each way or twenty train-miles over each mile, it would add a little less than five cents and make the total approximately 40%.

Reduction of Distance may also reduce the receipts as well as operating expenses and thus reduce the advantage gained by shorter distance, provided the change of distance is so great that it is measured in miles. The **effect on receipts** is not computable except in the most approximate way; it depends on the following elements: (1) Since the cost of operating additional distance is approximately 40% of the average, whereas passenger receipts and some other kinds are based more or less strictly on actual mileage, there is a real advantage and profit in added distance for such kinds of traffic. (2) For competitive business, where receipts are absolutely independent of distance, any added distance is a pure loss, without any compensation. (3) For through business, where the division of receipts between roads is more or less strictly according to the relative mileage of hauls on the two or more roads, there is profit or loss according to the percentage of length of haul on the home road to the total haul, the larger the percentage the larger the possible profit, which also depends on whether the rate is competitive or non-competitive. (4) Since a road always has varying proportions of these several kinds of traffic, the net advantage or disadvantage depends on the combined effect.

Distance. Percentage of Operating Expenses Affected by Length of Line According to Three Estimates

Division of total cost of railroad operation	Wellington, 1887			Webb, 1906			Harriman Line,* 1909		
	Total per-cent-age	Am't af-fected	Per cent to total	Total per-cent-age	Am't‡ af-fected	Per cent to total	Total per-cent-age	Amount af-fected	Per cent to total
Maintenance of way.	†23.0	§11.6 19.0	50.4 82.7	21.0	§15.7 19.0	75.0 90.3	24.97	23.45	93.9
Maintenance of equipment.	15.6	5.7 8.2	36.6 52.6	18.0	6.7 7.6	37.2 42.1	20.53	0	0
Conducting transportation	31.4	7.5 24.2	23.9 77.1	56.6	8.9 25.0	15.8 44.0	50.22	15.02	29.8
General expenses	30.0	0 0	0 0	4.3	0 0	0 0	4.28	0	0
Totals..........	100.0	24.8 51.4	....	100.0	31.3 51.6	....	100.00	38.47	.....

* Auditor's estimate for one of the Harriman lines.

† Interpretation of first line of figures: the maintenance of way expenses are estimated at 23.0% of total cost of operation; 11.6% of the total are affected by changes of length of line; the 11.6% is 50.4% of 23.0%. Other groups interpreted similarly.

‡ Including several sub-items really belonging under maintenance of way or transportation which should decrease materially the percentages for "conducting transportation."

§ The upper figures show the percentage on the basis of changes so small that they are measured only in feet, which will not appreciably affect several items of expense; the lower figures are for changes measured in miles. The Harriman figures make no such distinction.

As a final conclusion, there is always some addition to receipts to partially offset the added cost of operating added distance; for a road whose business is very largely non-competitive the increase in receipts due to added distance might be greater than the increase in operating expenses; for a road whose business is chiefly competitive the net loss will be great; in any case there is some reduction on the advantage computed above by decreasing the distance.

Curvature. The basis of the analysis of cost is the determination, first, of the degree of curve on which the power requirements are precisely double the power used on a straight level track, and then the determination of the additional cost of the doubled resistance It is then assumed that train resistance and added cost of curvature are proportional to the number of degrees of central angle and independent of the radius of curvature. The degree of curve on which the resistance is precisely double that on a level tangent has been variously estimated from 10° to 12° 30′. A mile of such continuous curvature will have from 528° to 660° of curvature. Doubling the power requirements on the engine will increase certain items of cost of a train-mile by a computable percentage of such cost. A continuous 10° (or 12° 30′) curve will increase items of maintenance of way and equipment by percentages which may be closely estimated. Many of the items of cost of a train-mile are unaffected. In this way the total increase in the cost of one mile of such curved track is computed. Dividing the total extra cost by the number of degrees (528 to 660) gives the value of 1°. The chief items affected by curvature are rails, of which the extra wear may be from 200 to 300%; repairs to locomotives, 100 to 200%, and repairs of cars which are increased from 50 to 100%. Some other items of maintenance of way are increased 25 to 50%, while fuel and water would be increased about 50%. The extra cost per degree has been variously computed as follows:

Author	Date	Degree per mile	Percentage	Extra cost per degree per cent
Wellington..................	1887	600°	35.6	0.0593
Berry.....................	1904	660	35.35*	0.0536
			29.88†	0.0498
Webb......................	1908	528	34.6	0.0655

* Uncompensated curvature. † Compensated curvature.

The percentage of extra cost per degree chosen should be multiplied by the estimated cost per train-mile and this value multiplied by the number of trains per year, which gives the yearly cost of one degree of curvature. Capitalizing this at the proper rate of interest gives the amount which may justifiably be spent to avoid one degree. For calculations of this kind, estimates must be made on the following items which will vary according to the local conditions: (1) the local cost per train-miles; (2) the percentage by which the various items of cost per train-mile are affected by curvature; (3) the number of trains which will use the tracks. For new construction even the last item is an uncertain quantity.

Minor Grades. The basis of computing the cost is to determine, first, the rate of grade which will exactly double the resistance on a level tangent; second, to compute the effect on operating expenses of a mile of such a grade; and third, on the assumption that the cost of lifting a train through so many vertical feet is proportional to the height, the extra cost of one foot of elevation equals the added cost of operating a mile of such grade divided by the number of vertical feet in a mile of that grade. It is assumed that the grade which doubles tractive resistance on a level tangent does not limit the length or weight of trains it is designed to haul with one engine, or that any limiting effect is a separate matter, and that the only effect of the grade is to increase the operating expenses over those required for operation over a level track. Minor grades are necessarily divided into three classes on account of a distinction in their effects on the operation of trains. Class A includes those grades which on account of the rate of grade or the length, do not require any change in the handling of the train, no brakes and no change in the throttle or cut-off. Class B includes those grades which are steep enough or long enough to require shutting off steam (but no brakes) in order to avoid objectionable velocity while descending and to require more steam or longer cut-off when ascending. Class C includes the grades which require that steam shall be shut off and brakes applied on down grades and full steam power used on the up grades. Since a fast passenger train may safely attain very high speed in a sag and since the effect of a hump is a relatively small reduction of a previously high velocity, a grade which should properly be classified as an A grade for fast passenger trains should be classified as a B or even a C grade for slow freight trains. The average tractive resistance of a train at ordinary velocity is about 10 lb. per short ton. For slow freight trains the figure is too high; for fast passenger trains it is too low. Ten lbs. per ton is the equivalent of the grade resistance on a 0.5% grade which has a rise of 26.4 ft. per mile. **Sags and humps:** This section applies only to the possible elimination of sags and humps, since a long uniform grade connecting two predetermined points is practically unchangeable. The problem therefore becomes the determination of the value of removing a hump or filling a sag whose height is the vertical distance from the vertex of the sag or hump from the uniform grade by which it would be replaced. **Value of 1 ft. or rise-and-fall:** Class A grades have no effect on the power required of the locomotive, and no effect on operation except a harmless temporary increase or decrease in velocity; they have also no appreciable effect on operating expenses and can therefore be ignored. The effect of Class B and Class C grades has been variously computed as follows:

Author	Date	Percentage affected		Annual value *	
		Class B, per cent	Class C, per cent	Class B, per cent	Class C, per cent
Wellington..........	1887	3.03	9.67	84	276
Webb...............	1908	6.08	7.92	168	219
Berry...............	1904	3.38	7.13	93	197

* Annual value in per cent of average cost per train-mile for each round trip daily train per foot of rise-and-fall.

The above table is based on an average train resistance of 10 lb. per ton whereas 6 to 7 lb. is more nearly correct for present rolling stock.

Ruling Grades are those which limit the length or weight of the trains which may be handled by one locomotive. The classification does not include steep grades which will always be operated by two or more engines and are called pusher grades, nor does it include very short steep grades which may always be operated by momentum. The maximum train loads which may be hauled on a given ruling grade and on the proposed lighter grade by an engine of given weight, dimensions and boiler power, may be determined by dividing the tractive force by the total resistance per ton, both taken for the same speed.

The cost per train-mile of each additional train for hauling the same tonnage of cars up a somewhat steeper grade has been computed by Webb as about 53% of the average cost of a train-mile. J. B. Berry (Bul. 49, Am. Ry. Eng. & Main. Way Assoc.) computes the cost similarly as 43.29%. Isaacs and Adams (Bul. 112 of same association) computed that 30.64% of the total operating expenses are " affected by locomotive mileage." They use the figure by multiplying the additional number of train-miles required by the higher grade by 30.6% of the average cost per train-mile and consider this as the additional cost of the higher grade.

The number of daily trains required for each grade may be found by dividing the given, or estimated, total tonnage by the maximum net train load for each grade, the difference is then the number of daily trains saved.

Pusher Grades. A road may be designed for pusher grades by determining a pair of grades such that two engines of a certain type can haul up the steeper grade the same train that will just tax the capacity of one such engine on the lower grade. This pair of grades must be selected according to the character of the country and must be such that all the grades of the line

Per Cent of Trains Necessary to Handle a Given Traffic with Single Engine, if Gradients are Changed from Those in First Column to Any Other Given Gradient

Gradient to be reduced, per cent	Proposed gradient, per cent, with per cent of trains to handle same traffic as on original gradient															
	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
2.0	26	30	34	38	42	46	51	55	60	65	69	74	79	84	89	95
1.9	27	31	36	40	44	49	54	58	63	68	73	78	84	89	94	100
1.8	29	33	38	42	47	52	57	62	67	72	78	83	89	94	100	
1.7	31	35	40	45	50	55	60	66	71	77	82	88	94	100		
1.6	33	37	43	48	53	59	64	70	76	82	87	94	100			
1.5	35	40	46	51	57	63	68	75	81	87	93	100				
1.4	37	43	49	55	61	67	73	80	87	93	100					
1.3	40	46	53	59	66	72	79	86	93	100						
1.2	43	50	57	64	71	78	85	92	100							
1.1	47	54	61	69	76	84	92	100								
1.0	51	59	67	75	83	91	100									
0.9	55	64	73	82	91	100										
0.8	61	71	80	90	100											
0.7	68	79	89	100												
0.6	76	88	100													
0.5	87	100														
0.4	100															

This table is taken from Bulletin 49 of Am. Ry. Eng. & M. W. Assoc., by J. B. Berry; calculated for engine and tender weighing 150.3 tons in working order, tractive force 35 268 lb., weight of caboose 17 tons, velocity and frictional resistance 6 lb. per ton. As percentages, the figures apply approximately to almost any freight engine.

may be reduced, without unreasonable cost, to the limit of the grade for one engine, except such grades as it is intended to operate with pusher engines. Little or nothing is saved by reducing these pusher grades below the limit computed for two engines. In exceptional cases a combination of three grades may be computed, of which the two lower grades are to be operated as above, and the highest grade is such that three engines are required to haul the same train over it. The following table is computed for one type of engine. The precise balance between any through grade and the corresponding grades for one and for two pushers depends on the type of engine, the available ratio of adhesion and the tractive resistance; but it may be demonstrated that reasonable variations in the type of engine or tractive resistance will only alter the required grades by a few hundredths of one per cent. The corresponding pusher grades may therefore be accepted as approximately true for all cases. The accuracy of the table is theoretically impaired by the fact that the rating tonnage on the pusher-engine service is somewhat different from that on the through-engine service. The variation will depend on the ratio of rating-ton resistance to tare-ton resistance. As an illustration, a 1.9% one-pusher grade corresponds to a 0.92% through grade according to the table. Working out the corresponding grade, using the values proposed by Dennis of 2.6 lb. as the resistance of a rating ton and 9 lb. as the resistance of a tare ton, the corresponding through grade is computed to be 0.99%. Some increase should be expected since the pusher locomotive has a higher resistance per ton than average train resistance. This shows that, although no table can be compiled which will suit all conditions, the differences will not probably exceed a few hundredths of one per cent. The variations are of less importance with the higher pusher grades. The cost of a pusher engine mile as a percentage of the average cost of a train-mile was estimated by Wellington (8187) as 38.3%; by Berry (1904) 34.4%; by Webb (1908) 38.8%.

Balanced Grades for One, Two, and Three Engines

Through grade, per cent	Track resistance, 6 lb.			Track resistance, 8 lb.		
	Net load for one engine in tons (2000 lb.)	Corresponding pusher grade for same net load, per cent		Net load for one engine in tons (2000 lb.)	Corresponding pusher grade for same net load, per cent	
		One pusher	Two pushers		One pusher	Two pushers
Level	3868	0.28	0.55	2874	0.37	0.72
0.10	2874	0.47	0.82	2278	0.56	0.98
0.20	2278	0.66	1.08	1880	0.74	1.23
0.30	1880	0.84	1.33	1596	0.92	1.47
0.40	1596	1.02	1.57	1384	1.09	1.70
0.50	1384	1.19	1.80	1218	1.27	1.92
0.60	1218	1.37	2.02	1085	1.44	2.14
0.70	1085	1.54	2.24	977	1.60	2.36
0.80	977	1.70	2.46	887	1.77	2.56
0.90	887	1.87	2.66	810	1.93	2.76
1.00	810	2.03	2.86	745	2.09	2.96
1.10	745	2.19	3.06	688	2.24	3.15
1.20	688	2.34	3.25	638	2.40	3.33
1.30	638	2.50	3.43	594	2.55	3.51
1.40	594	2.65	3.61	555	2.70	3.68
1.50	555	2.80	3.78	521	2.85	3.85
1.60	521	2.95	3.95	489	2.99	4.02
1.70	489	3.09	4.12	461	3.13	4.17
1.80	461	3.23	4.27	435	3.27	4.33
1.90	435	3.37	4.43	411	3.42	4.49
2.00	411	3.52	4.59	390	3.55	4.63
2.10	390	3.65	4.73	370	3.68	4.78
2.20	370	3.78	4.88	352	3.81	4.92
2.30	352	3.91	5.02	335	3.94	5.05
2.40	335	4.04	5.15	319	4.07	5.19
2.50	319	4.17	5.29	304	4.20	5.32

Basis: Through and pusher engines alike; consolidation type; total weight, 107 tons; weight on drivers, 53 tons; adhesion, 9/40, giving a tractive force for each engine of 23 850 lb.; normal track resistance 6 (also 8) lb. per ton. It is assumed that the engine is so designed and that the velocity and steam pressure are such that the engine can always develop a tractive force of 23 850 lb.

The mileage must be computed as twice the total distance between the sidings at top and bottom of the grade, this distance being always somewhat in excess (and sometimes very much so) of the length of the pusher grade on the profile.

Momentum Grades are grades which, although somewhat steeper than the ruling grade, can invariably be approached at such a velocity that the kinetic energy of the train will assist the engine sufficiently to carry the train past the summit with a minimum velocity of say 10 miles per hour. If the resistance

Velocity Head of Trains Moving at Various Velocities

Velocity, m. p. h.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10	3.51	3.58	3.65	3.72	3.79	3.87	3.95	4.02	4.10	4.17
11	4.25	4.33	4.41	4.49	4.57	4.65	4.73	4.81	4.89	4.97
12	5.06	5.15	5.23	5.32	5.41	5.50	5.58	5.67	5.75	5.84
13	5.93	6.02	6.12	6.21	6.31	6.40	6.50	6.59	6.69	6.78
14	6.88	6.98	7.08	7.19	7.29	7.39	7.49	7.60	7.70	7.80
15	7.90	8.00	8.11	8.22	8.33	8.44	8.55	8.66	8.77	8.88
16	8.99	9.10	9.21	9.32	9.43	9.55	9.67	9.79	9.91	10.03
17	10.15	10.27	10.39	10.51	10.63	10.75	10.87	10.99	11.12	11.25
18	11.38	11.50	11.63	11.76	11.89	12.02	12.15	12.28	12.41	12.55
19	12.68	12.81	12.95	13.08	13.22	13.35	13.49	13.63	13.77	13.91
20	14.05	14.19	14.33	14.47	14.61	14.75	14.89	15.04	15.19	15.34
21	15.49	15.64	15.79	15.94	16.09	16.24	16.39	16.54	16.69	16.84
22	17.00	17.15	17.30	17.46	17.62	17.78	17.94	18.10	18.26	18.42
23	18.58	18.74	18.90	19.06	19.22	19.38	19.55	19.72	19.89	20.06
24	20.23	20.40	20.57	20.74	20.91	21.08	21.25	21.42	21.59	21.77
25	21.95	22.12	22.30	22.48	22.66	22.84	23.02	23.20	23.38	23.56
26	23.74	23.92	24.10	24.28	24.46	24.65	24.84	25.03	25.22	25.41
27	25.60	25.79	25.98	26.17	26.36	26.55	26.74	26.93	27.13	27.33
28	27.53	27.73	27.93	28.13	28.33	28.53	28.73	28.93	29.13	29.33
29	29.53	29.73	29.93	30.13	30.34	30.55	30.76	30.97	31.18	31.39
30	31.60	31.81	32.02	32.23	32.44	32.65	32.86	33.08	33.30	33.52
31	33.74	33.96	34.18	34.40	34.62	34.84	35.06	35.28	35.50	35.72
32	35.95	36.17	36.39	36.62	36.85	37 08	37.31	37.54	37.77	38.00
33	38.23	38.46	38.69	38.92	39.15	39.38	39.62	39.86	40.10	40.34
34	40.58	40.82	41.06	41.30	41.54	41.78	42.02	42.26	42.51	42.76
35	43.01	43.26	43.51	43.76	44.01	44.26	44.51	44.76	45.01	45.26
36	45.51	45.76	46.01	46.26	46.52	46.78	47.04	47.30	47.56	47.82
37	48.08	48.34	48.60	48.86	49.12	49.38	49.64	49.91	50.18	50.45
38	50.72	50.99	51.26	51.53	51.80	52.07	52.34	52.61	52.88	53.15
39	53.42	53.69	53.96	54.23	54.51	54.79	55.07	55.35	55.63	55.91
40	56.19	56.47	56.75	57.03	57.31	57.59	57.87	58.16	58.45	58.74
41	59.03	59.32	59.61	59.90	60.19	60.48	60.77	61.06	61.35	61.64
42	61.94	62.23	62.52	62.82	63.12	63.42	63.72	64.02	64.32	64.62
43	64.92	65.22	65.52	65.82	66.12	66.43	66.74	67.05	67.36	67.67
44	67.98	68.29	68.60	68.91	69.22	69.53	69.84	70.15	70.46	70.78
45	71.10	71.42	71.74	72.06	72.38	72.70	73.02	73.34	73.66	73.98
46	74.30	74.62	74.94	75.26	75.59	75.92	76.25	76.58	76.91	77.24
47	77.57	77.90	78.23	78.56	78.89	79.22	79.55	79.89	80.23	80.57
48	80.91	81.25	81.59	81.93	82.27	82.61	82.95	83.29	83.63	83.97
49	84.32	84.66	85.00	85.34	85.69	86.04	86.39	86.74	87.09	87.44
50	87.79	88.14	88.49	88.85	89.20	89.55	89.91	90.26	90.61	90.97

of trains were independent of velocity, the mechanics of the problem would be much simplified, but the extensive dynamometer tests of Dennis, confirmed by those of Shurtleff, have shown that the resistance per ton of freight trains at velocities between 10 and 35 miles per hour is so nearly uniform that the following method gives a close approximation to accuracy. It is also fortunately true that a very considerable error in velocity height at high velocities is comparatively unimportant. Assuming that a train could run without track resistance, its kinetic energy at a velocity v in feet per second would lift it up a grade to a total vertical height $h = v^2/2\,g$, in which g is the acceleration of gravity. If V = velocity in miles per hour, $h = 0.03344\ V^2$; adding 5% for the rotative kinetic energy of the wheels, we have $h = 0.0351\ V^2$. The values of h (or " velocity heights ") are given in the table.

An engine properly loaded will haul its train up the ruling grade and the test of the momentum grade or hump is the possibility of hauling the train up the excess grade with the help of momentum. For example, a train, with the engine loaded for a 0.7% grade, approaches a 1.2% grade 7850 ft. long with a velocity of 35 miles per hour. The excess grade is 0.5%, and in a distance of 7850 ft., the excess rise is 39.25 ft. The velocity head corresponding to 35 miles per hour is 43.01 ft.; subtracting 39.25, we have left a velocity head of 3.76, which corresponds to a velocity of 10.3 miles per hour, which is a safe speed for going over the summit. The secondary question, whether it is safe to assume that no fully loaded freight train will ever need to stop, or even slacken speed, along this grade of 7850 ft., is a matter of practical operation rather than of engineering.

The accuracy of this method depends on the assumptions that the resistance is uniform within the range of the velocities, and also that the work done by the engine is uniform. Although the resistance may be considered substantially uniform, the tractive force of the engine decreases as the velocity increases. When, as in the above illustration, the train approaches the grade at a velocity of 35 miles per hour, the engine is not producing as much tractive force as it can produce at a lower velocity. But if the rating of the engine (see p. 2094) is based on a fairly high velocity on the ruling grade, the excess tractive force at lower velocities may so balance the deficiency at the higher velocities that the method is applicable without material error. Experience has shown this to be true.

Sags in Grade Line. Economies are often possible where sags having grades exceeding the ruling grade on one or both sides may be harmlessly introduced. Their harmlessness may be tested by the velocity-head table. It is of course assumed that running through the sag without a stop may be depended on. Deciding first the maximum-permissible speed for freight trains in the bottom of the sag, the ability of the train to mount the grade in either direction and have sufficient speed at the top of the grade may be tested. It must also be determined what speed of approach is necessary at the other end of the sag in order to attain that maximum speed at the bottom, and whether it is practicable to approach the sag at that speed.

As an illustration: the ruling grade is 1.2%; economical construction suggests a down grade of 2.0% for a distance of 4500 ft., followed by an up grade of 2.0% for 7000 ft. Assume a maximum-permissible freight-train velocity of 45 miles per hour, the velocity head being 71.10 ft. The excess grade is 0.8%, which in 7000 ft. has a rise of 56 ft. At the top of the grade the remaining velocity head is 15.10 ft., which corresponds to 20.7 miles per hour, which shows that a speed of 45 miles per hour through the sag would be unnecessary. The drop on the other slope of the sag is 90 ft., which would give to the train a velocity of over 50 miles per hour, even without using steam. There is therefore no question in this case of the ability of the train to acquire that speed at the bottom of the sag. It will also be still easier to run the train through the sag in the opposite direction.

Classification of Grades. The velocity-head table may be utilized to predict the behavior of trains, and therefore to classify the grades into the A, B, and C classes discussed under Minor Grades. The above case should be classified as C grade, since steam must be shut off and brakes used to prevent objectionably high speed in the sag, for either direction of movement.

SECTION 22

ELECTRIC RAILROADS

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GENERAL

1. Classes of Traffic Suitable for Electric Traction

General. The essential characteristic of electric traction which distinguishes it from other forms, is the separation of the source of motive power from the rolling stock. This gives it a virtual monopoly for street and interurban systems because it permits the generation of motive power to be concentrated in a single economical plant instead of being distributed over a number of comparatively uneconomical ones, as in the case of steam traction.

Conditions for Electric Traction on Railroads. On electrified railroad trunk lines the generating and distributing systems must have large capacity in order to handle the heavy trains, but are not kept fully employed due to the comparative infrequency of traffic. This results in the investment of a great deal of capital which becomes idle, the fixed charges on which generally exceed the operating economies Exceptions to this generalization occur under the following conditions:

(a) When the cost of coal is very high, the operating economies may be sufficient to counterbalance the fixed charges, as electric traction requires only half the amount of coal needed with steam traction.

(b) When cheap water power is available, a similar condition may occur.

(c) When it is desired to increase the weight of trains to be hauled on steep grades, the elimination of the power generating system from the locomotive, in electric traction, permits an increased concentration of motors on the locomotive, thereby obtaining the desired increase of hauling power, at a lower cost than by the elimination of grades.

(d) When it is desired to increase the capacity of a mountain division and this can be accomplished by increasing the weight of trains as described above, rather than by increasing the number of tracks so as to permit more trains to be hauled.

(e) When it is necessary to eliminate smoke and gases in tunnels or city streets, the element of economy being necessarily subordinated to safety or convenience.

(f) When the traffic density is unusually great, as in certain suburban zones.

(g) When by the elimination of smoke and gases, trains through cities can be run underground and the ground surface utilized for lucrative building purposes.

2. Systems of Electric Traction

Economic Differences between Systems. The stationary equipment of an electric traction system consists of three parts — the generating, transmission, and distribution systems. These must have sufficient capacity to carry the peak loads and are therefore likely to be comparatively idle during a large part of the time. The activity of these parts depends upon the number of cars in operation.

The moving equipment of an electric railway system consists of the motors, controllers and other parts of the rolling stock necessary for electric operation. The activity of these parts does not depend upon the number of cars in operation, but only upon the activity of each car.

There are several systems of electric traction which differ in the distribution of the investment between stationary and moving equipment. Stationary equipment costs less than moving equipment of the same capacity, because it is concentrated in larger units and its design is not restricted by the limitations of car construction. It is, therefore, obvious that for railways operating a large number of cars, it pays to have a comparatively large investment in stationary equipment: whereas, railways with few cars must economize in stationary equipment and concentrate their investment as far as practicable on their limited rolling stock.

Technological Differences between Systems. The various systems of electric traction now in use may be classified as follows according to the above principle:

Low-voltage, direct current
High-voltage, direct current
Three-phase
Single-phase.

In every practical electric railway system current is generated for transmission at a high voltage and utilized in the motors at low voltage. In the direct-current systems, the transformation from high to low voltage is accomplished at stationary substations, which also change the alternating current from the transmission lines, into direct current, which is fed to the trolley or third rail, and thence to the motors. In the alternating-current systems, the high-voltage current is delivered to the trolley lines and transformed down to a voltage suitable for the motors by means of transformers on the cars or locomotives. Locomotives used with high voltage alternating-current transmission may have direct-motors, which are supplied with direct current from a motor generator set in the locomotive. As the direct-current systems carry a greater proportion of investment in their stationary equipment, they are especially adapted to urban and important interurban lines. The alternating-current systems, which carry a proportionately greater investment in moving equipment, are especially suitable for trunk lines having infrequent service.

The high-voltage, direct-current system is intermediate, in the above respect, between the low-voltage, direct-current and the alternating-current systems.

The choice of systems does not depend merely upon the investment, but also upon the operating costs. It has been found, however, that, with the present high rate of interest, these are seldom the governing factor.

3. Costs and Economics

General. Even more than in the case of steam railroads, electric urban and interurban railways labor under the difficulty that their rates are fixed by law, whereas their operating costs are constantly rising. This, added to the uncertainties of limited franchises and hostile regulation, makes it difficult to obtain capital for electric railways except at excessive rates of interest. The consequence is that the utmost economy must be exercised in construction and operation.

Revenues and Expenses of Street and Interurban Railways. The tables on page 2110 show the average annual revenues and expenses of 231 street and interurban railways for the year 1926.

Cost per Mile for Street and Interurban Railways. Later in this article is given the prescribed classification of capital charges used by the New York State Public Service Commission (2d District). It is useful in preparing estimates. An average total capital cost, including all items shown in the following classification, is $72 000 per mile of road. This figure covers all electric railways in New York State except those in the city of New York, and two lines of a very special character, namely, the New York, Westchester and Boston R.R. and the Niagara R.R. The former cost $1 269 219 and the latter $308 602 per mile of road owned.

Cost of Steam Railroad Electrification. The total cost of steam railroad electrification depends so much upon the amount of traffic and the character of the line that no general average figures are of use. The distribution of those features of annual expense which are purely electrical is shown for a typical electrified steam railroad, in a table near the end of this Article. Valuable data on this subject were published by J. B. Cox for the Butte, Anaconda & Pacific Railway, Trans., A.I.E.E., 1914, Vol. 33, p. 1369, and by W. S. Murray for the New York, New Haven & Hartford R. R., Trans. A.I.E.E., 1915, Vol. 34, p. 85, the data in both cases being too voluminous to abstract.

Annual Revenues and Expenses of Electric Railways

(Street and Interurban)

(From Reports to the American Electric Railway Association 1926)

Car Mile Basis

Income	**Cents per car mile**
Railway operating revenue	44.83
Railway operating expense	33.77
Net operating revenue	11.06
Net revenue — auxiliary operation	0.22
Taxes	2.80
Operating income	8.48
Non-operating income	0.84
Gross income	9.32
Deductions from gross income	7.50
Net income	1.82

Operating expense	**Cents per car mile**
Way and structures	4.86
Equipment	4.45
Power	4.57
Conducting transportation	14.74
Traffic	0.18
General and miscellaneous	4.87
Transportation for investment (Cr.)	0.04
Total operating expense	33.77

Note. The above figures are averaged from reports of 98 companies operating city lines exclusively; 50 operating interurban lines exclusively and 83 operating combined city and interurban lines. (From A.E.R.A., May, 1927.)

Unit Costs of Electrical Construction. Due to rapid engineering and economic changes in the electrical industry, a list of unit costs can never be relied upon. The following are typical average costs in 1927.

Third rails cost from $5000 to $8000 per mile, depending largely upon whether the railroad is operating during construction.

Ordinary trolley construction costs from $1500 to $2500 per mile of single-track road or from $3500 to $5000 per mile of double track. The cost of catenary construction is between $10 000 and $15 000 per track-mile.

Substations cost from $40 to $60 per kilowatt capacity, the former figure being for large substations, cost of real estate and building being omitted in each case.

Power stations cost from $100 to $125 per kilowatt capacity.

Underground conduits for railways cost from 75 cents to $1.25 per duct-foot, the latter figure being appropriate where the conduit lines are built along the narrow right-of-way of an operating railroad, or where paving must be replaced.

Transmission pole lines cost from $2000 per mile for wooden poles, to $4000 per mile for light steel poles.

Classification of Capital Expenditures

(New York State Public Service Commission, Second District)

Intangible Street Railway Capital

1. Organization.
2. Franchises.
3. Patent rights.
4. Other intangible street railway capital.

Direct Expenditures for Tangible Street Railway Capital

5. Right-of-way.
6. Other street railway land.
7. Grading.
8. Ballast.
9. Ties.
10. Rails, rail fastenings and joints.
11. Special work.
12. Underground construction.
13. Track laying and surfacing.
14. Paving.
15. Roadway tools.
16. Tunnels.
17. Elevated structures and foundations.
18. Bridges, trestles and culverts.
19. Crossings, fences and signs.
20. Interlocking and other signal apparatus.
21. Telephone and telegraph lines.
22. Poles and fixtures.
23. Underground conduits.
24. Transmission system.
25. Distribution system.
26. Dams, canals and pipe lines.
27. Power plant buildings.
28. Substation buildings.
29. General office buildings and equipment.
30. Shops and car houses.
31. Stations, waiting rooms and miscellaneous buildings.
32. Docks and wharves.
33. Park and resort properties.
34. Furnaces, boilers and accessories.
35. Steam engines.
36. Turbines and water wheels.
37. Gas power equipment.
38. Power plant electric equipment.
39. Miscellaneous power plant equipment.
40. Substation equipment.
41. Cable power equipment.
42. Shop equipment.
43. Locomotives.
44. Revenue cars.
45. Electric equipment of cars.
46. Other rail equipment.
47. Miscellaneous equipment.

General Expenditures for Street Railway Fixed Capital

48. Engineering and superintendence.
49. Law expenditures during construction.

50. Injuries during construction.
51. Taxes during construction.
52. Miscellaneous construction expenditures.
53. Interest during construction.
54. Total classified by prescribed accounts.
57. Total not classified by prescribed accounts.
58. Total fixed capital.

Distribution of Annual Expenses for Typical Electrified Eailroad

	Per cent
Fixed charges	59
Operating charges:	
Power station	12
Transmission and distribution	5
Substations	3
Signals and communications	5
Trains	16
	100

Notes. The above table excludes all items not peculiar to electric operation. It includes multiple-unit cars but not standard cars drawn by electric locomotives. The cost of train crews, except for electric locomotives, is omitted. Taxes on real estate are also omitted.

Life Expectancy Table of Electrical Equipment

(W. S. Gorsuch)

Equipment	Total life in years
Steam power plants:	
Buildings	75
Boilers	30
Stokers and grates	20
Conveyors, elevators and hoists	20
Turbines, complete	15
Engines and condensers	15
Piping	15
Heaters	15
Pumps	15
Alternators	15
Switchboard apparatus and instruments	15
Synchronous converters	30
Transformers	20
Exciters	25
Motors	25
Storage batteries	10
Tools and sundries	15
Substations:	
Synchronous converters	30
Transformers	20
Motor generator sets	25
Switchboard apparatus and instruments	20
Storage	10
Tools and sundries	15
Transmission and distribution:	
Conduit (vitrified clay)	50
Feeders (direct-current)	20
Feeders (alternating-current)	25

Note. In estimating the time in years during which equipment of a generating station may be reasonably expected to perform its functions, three elements must be clearly kept in mind; namely, obsolescence, inadequacy and inefficiency. How long apparatus or equipment will remain in service before it becomes obsolete, inadequate or ineffective, is purely a speculative matter, and can only be predicted in advance by past experience and a knowledge of the art, and with careful judgment.

4. Power Consumption in Relation to Schedule

Forces Acting on a Train. The forces tending to accelerate a train are the tractive effort developed by the motors and the component of the weight along the track on down-grades. The forces which retard the motion of the train are the various frictional forces and the component of the weight along the track on up-grades; also in braking, the frictional force due to the brakes. All the various frictional forces, except the braking resistance, such as track friction, journal friction, air friction, etc., which oppose the motion of a train on a straight track, are usually considered together and are referred to as the "train resistance." The extra friction due to track curvature is usually considered as an equivalent up-grade.

Train Resistance for Electric Train. Let

N = number of cars (including electric locomotive, if any);
w = average weight of car loaded, in tons (= total weight of train divided by N);
r = train resistance in pounds per ton;
v = speed in miles per hour;
a = cross-section of car in square feet;

A and B are constants in the formula; K taken as 0.0030 throughout.

$$r = A + Bv + \frac{Ka(0.9 + 0.1\,N)v^2}{Nw} \tag{1}$$

The constant A depends chiefly upon the average total weight of car and load. Thus

for w =	15	20	25–30	35	40–45	50	70
A =	6.0	5.5	5.0	4.5	4.0	3.5	3.0

The constant B depends primarily upon the nature of the track and roadbed and also to some extent upon the weight and type of the car. Burch gives the following values:

Passenger cars on excellent track	0.06–0.11
Passenger cars on ordinary track	0.10–0.15
Freight cars on ordinary track	0.05–0.06

The heavier the car the higher the value of this coefficient.

Grades and Curvatures. An actual up-grade of $G\%$ produces a retarding force of $20\,G$ lb. per ton and a down-grade of $G\%$ produces an accelerating force of $20\,G$ lb. per ton. A curve always gives rise to a retarding force, which ranges from 0.5 to 1 lb. per ton per degree of curvature. Using the higher figure, each degree of curvature may be taken equivalent to an up-grade of 0.05%. Note that for angles of curvature up to 12 deg. the angle in degrees may be taken equal to $5730 \div R$, where R is the radius of curvature in feet.

Average Acceleration Rates

Service	Miles per hour per second
Steam locomotive, freight service	0.1 to 0.2
Steam locomotive, passenger service	0.2 to 0.5
Electric locomotive, passenger service	0.3 to 0.6
Electric motor cars, interurban service	0.8 to 1.3
Electric motor cars, city service	1.5 to 2.0
Electric motor cars, rapi. transit service	1.5 to 2.0
Highest practical rate	2.0 to 2.5

Acceleration Constant. The tractive effort required to give to 1 ton (2000 lb.) a linear acceleration of 1 mphps is 91.2 lb. To accelerate a train of W tons requires a tractive effort of 91.2 aW lb. to produce a linear acceleration of a mphps; but on account of the accompanying angular acceleration of the rotating parts, an additional force is required.

The acceleration constant is raised by the flywheel effect by about 5% (i.e., $Wr/W = 0.05$) for heavy cars and locomotives, and between 5% and 10% for light, low-speed cars, 8% being an average figure. However, C is usually taken as 100, corresponding to an increase in effective weight of about 10%. A given linear acceleration of a mphps then requires an accelerating force of 100 a lb. per ton.

Tractive Effort and Adhesion Coefficient. Let

F = tractive effort, in pounds per ton, exerted by motors;
G = per cent actual grade (+ for up-grade);
g = degrees of curvature;
r = train resistance, in pounds per ton;
a = acceleration in mphps (− for retardation).

Then the tractive effort required per ton of total train weight is

$$F = 100\,a + r + 20\,G + g \qquad (2)$$

The adhesion or "tractive" coefficient is the quotient (expressed usually as per cent) of the tractive effort in pounds which will slip the drivers, divided by the weight in pounds on the drivers. Burch gives the values in the accompanying table. The maximum possible tractive effort is the product of the adhesion coefficient (as a decimal fraction) by the weight (in pounds) on the drivers.

Adhesion Coefficients

Condition of track	Without sand	With sand
Most favorable condition	35	40
Clean, dry rail	28	30
Thoroughly wet rail	18	24
Greasy moist rail	15	25
Sleet-covered rail	15	20
Dry snow-covered rail	11	15

The Weight of Locomotive required to accelerate a train weighing W tons at the rate of a miles per hour per second up a grade of G% on a g degree curve against a frictional resistance of r lb. per ton, when q% of the weight is on the drivers and the coefficient of adhesion, is p%, is given by the following formula:

$$\text{Weight of locomotive} = \frac{5\,W}{pq}\,(100\,a + r + 20\,G + g).$$

Example. What weight of locomotive is required to accelerate a 400-ton train at the rate of 0.5 mile per hour per second up a 0.1% grade against a frictional resistance of 8 lb. per ton, when 80% of the weight is on the drivers and the coefficient of adhesion is 20%?

$$\text{Weight of locomotive} = \frac{5 \times 400}{20 \times 80}\,(50 + 8 + 2) = 75 \text{ tons}.$$

Maximum Over-all Efficiency of Motors and Gears at Rated Voltage

Horsepower, 1-hour rating	Kind of motor	Maximum efficiency, per cent
30–100	D-C. geared	83–88
100–250	D-C. geared	88–89
250–500	D-C. gearless	91–83
50–200	A-C. series geared	70–80 *
200–500	3-phase induction geared	85–89

* Including step-down transformers.

Power Required at Given Speed. Let

r = train resistance in pounds per ton of total weight of train;
G = per cent grade;
g = degree of curvature;
a = degree of acceleration in mphps;
v = speed in mph;
W = total weight of train in tons.

Then the power required *at the rims of the drivers* is $1.99\,v\,(r + 20\,G + g + 100\,a)$ watts per ton, or $2.67 \times 10^{-3}\,vW\,(r + 20G + g + 100\,a)$ horsepower, total.

The power input, p_i, to the car or locomotive is equal to the power at the rims of the drivers divided by the over-all efficiency ϵ of the controller, motors and gears, that is,

$$p_i = \frac{1.99\,Wv(r + 20\,G + g + 100\,a)}{1000\,\epsilon} \text{ kilowatts} \qquad (5)$$

Approximate Method of Calculating Energy Consumption. The following method is based upon simple kinetic principles, and, if certain characteristics of the run are known, gives the actual energy output at the wheel rims. This fact makes the method useful, not only for rough calculations, but also to check calculations made by the more accurate step-by-step method.

When the method is applied to checking purposes, the column of the table below headed " Actual energy output " should be used and the input calculated from the known efficiencies. When applied to rough calculations, the column headed " Approximate electrical energy input " should be used. In the latter case the maximum speed and length of run with power on are not known, but it is possible to assume certain values, based upon experience, which will give a rough approximation to the energy required. The total energy in watt-hours per ton-mile will be the sum of the amounts required for acceleration and overcoming frictional train resistance grades and curves.

Let V = average running speed in miles per hour;
V_m = maximum speed in miles per hour;
L = length of run in miles;
L_p = distance traveled, with power on, in miles;
$n = l/L$ = number of stops per mile including one terminus;
r = average train resistance, in pounds per ton (say that corresponding to a speed from 10 to 20% greater than the average speed);
G = average equivalent grade, in per cent;
g = average curvature in degrees;
$K = \frac{V_m}{V}$ = ratio of maximum to average speed; see accompanying table of values of K;
$Q = \frac{L}{L_p}$ = ratio of length of run to distance traveled with power on.

See table of values of K and Q on next page.

Output at Wheel Rim and Input to Cars in Watt-hours per Ton-mile

Energy for	Actual energy output at wheel rims of cars	Approximate electric energy input to cars
Acceleration........................	$\dfrac{V^2m}{36.2\ L}$	$\dfrac{K^2nV^2}{25}$
Frictional train resistance................	$\dfrac{1.99\ r\ L_p}{L}$	$\dfrac{2.9\ r}{Q}$
Grades..............................	$\dfrac{39.8\ GL_p}{L}$	$\dfrac{57\ G}{Q}$
Curves..............................	$\dfrac{1.99\ gL_p}{L}$	$\dfrac{2.9\ g}{Q}$
Total..............................	Sum	Sum

Note. $25 = 36.2\ \epsilon$, $57.8 = 39.8 \div \epsilon$, and $2.9 = 1.99 \div \epsilon$, where ϵ is the efficiency, taken as 0.7. The formula for energy due to curves assumes each degree of curvature to be equivalent to a train resistance of 1 lb. per ton, which is probably high.

Values of K and Q

Stops per mile, n	K		Q
	Locomotive passenger trains	Single cars multiple unit trains and freight trains	All trains
0	1.0	1.0	1.0
0.1	1.2	1.1	1.1
0.2	1.35	1.2	1.25
0.3	1.5	1.25	1.4
0.4	1.6	1.3	1.5
0.5	1.7	1.35	1.7
0.6	1.75	1.4	1.8
0.7	1.8	1.45	1.9
0.8	1.85	1.45	2.0
0.9	1.9	1.5	2.1
1.0	1.95	1.5	2.15
1.2	1.95	1.6	2.2
1.4	1.95	1.6	2.3
1.6	1.95	1.6	2.4
1.8	1.95	1.65	2.5
2.0	1.95	1.7	2.6
2.5	1.95	1.75	2.7
3.0	2.0	1.8	2.8
3.5	2.0	1.85	2.9
4.0	2.0	1.9	2.9
4.5	2.0	1.95	2.95
5.0	2.0	2.0	3.0

More exact methods involve a knowledge of the motor characteristics, gears, brakes, and control.

Power Required for Car Heating and Lighting. In addition to the energy required for propelling the cars, a very appreciable amount is also required in the winter, for heating them, and a small amount at night for lighting. See table below. In making up a load diagram this energy should be included. See Art. 17 for power required for heating.

Lighting of Cars

Length of car, feet	Average * Kw for lighting
14–20	0.25
20–28	0.35
28–34	0.55
34–40	0.70

* During the hours lights are on, using tungsten lamps.

POWER STATIONS AND SUBSTATIONS

5. Power Stations

General. The power plant of an electric railroad should be located with reference to the load center, the water and coal supply, and the cost of real estate. The load center is usually the least important consideration as the cost of transmission lines and the losses therein do not rank in importance with considerations of plant economy.

Water is required for boilers and condensers. The coal supply should be brought by water where practicable, with auxiliary rail connection.

Buildings. Buildings are usually of brick with steel frames and concrete floors. Floors are of concrete with cement or tile finish for engine rooms and concrete with cement finish or bricks laid on edge in cement mortar for boiler rooms. Foundations for machinery are usually made of concrete and separate from the wall foundations in order to reduce vibration. Boiler-room equipment is carried on the steel structure.

Tidal limits and flood-water stages fix the levels of both condenser intake and outlet tunnels, and sometimes of engine- and boiler-room floors.

Standard Design. Present practice for large stations provides a boiler room and generating room side by side and separated by a thick wall. The generating room is divided into two parts, either with or without a separating wall, the part nearer the boiler room containing the generating equipment and provided with a crane; the other part, without a crane, having the electrical control apparatus.

Equipment. The table on page 2119 shows the principal items in a typical electric railway power station, and their relation to the general processes of the plant. At the head of the upper part of the table are listed the raw materials which enter the plant, followed by the machinery and apparatus through which they pass to the boiler. At the head of the lower part of the table are listed the products of the boilers, followed by the machinery and apparatus through which they are dissipated.

6. Substations

General. A substation is a group of apparatus or machinery which receives current from a transmission system, changes its kind or voltage, and delivers it to a distribution system. Most commonly, substations transform high-voltage alternating current to low-voltage direct current, the former affording the most economical transmission, and the latter the most economical conversion of electrical into mechanical energy. Substations are usually housed in substantial brick and concrete buildings, although where the current has to be changed only in voltage and not in kind, as on single-phase railway systems, the building is often unnecessary, as no rotating machinery is required, and the transformers and switching apparatus can be built for out-door operation. In some locations, the high tension bus, transformers, and switching equipment may be installed in an out-door substation adjacent to the building housing the rotating equipment, meters, etc.

Economical Location. Substations are located at intervals of 3 to 20 miles along the railroad, the spacing being limited by consideration of line voltage, regulation at the cars and economy, both of first cost and operation. Theoretically, substations should be located so as to give the least annual expense, taking into account both the fixed charges on the investment in substations and feeders, the operating charges and the cost of energy losses in the system. An investment in copper feeders, especially in the case of bare or weatherproof aerial cables, causes little annual expense beyond the interest and taxes on the investment, whereas, a substation involves, in addition, heavy depreciation charges and labor charges for operation. In recent years the use of automatic substations has been spreading rapidly, as it enables the operating force to be reduced, and in many cases eliminated with the exception of a small force for inspection and maintenance, one group of men being responsible for the care of several substations. Automatic substations have the further advantage that they act instantly in case of trouble, reducing the damage to equipment caused by abnormal conditions within or external to the station.

Automatic stations are usually controlled from a central point by a load dispatcher or supervisor by means of one of several forms of supervisory control which by the use of a very few wires, usually from one to four, enables all switches in a large group of substations to be operated, and all desired indications such as switch positions, machine temperatures, station output, to be relayed to the supervisor's board.

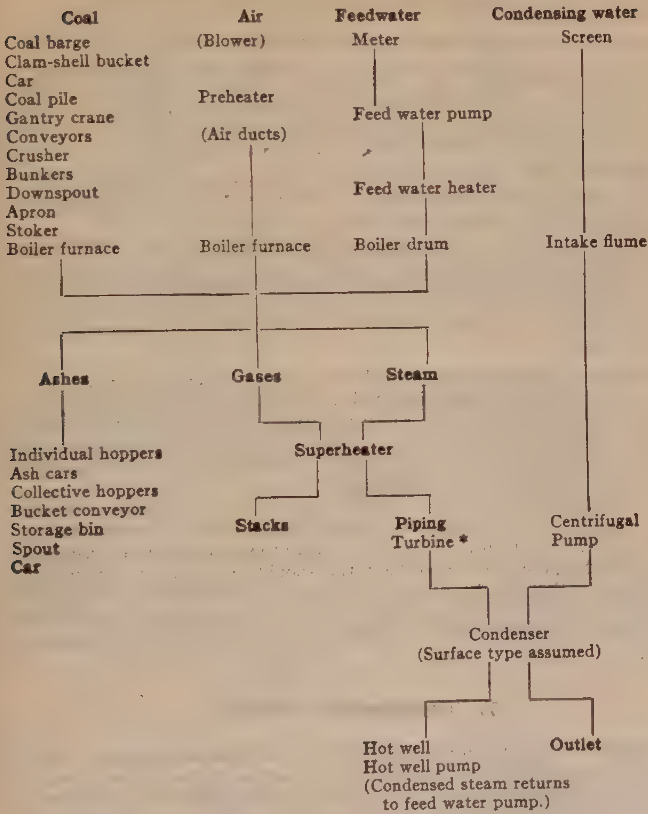
On interurban systems where the traffic density is low automatic stations may be equipped to start automatically as a car approaches the section fed by the substations, and shut down as soon as the load has dropped.

The permissible distance between substations depends on the voltage of distribution, being greater the higher the voltage. On an ideal line, doubling the operating voltage quarters the distance between substations. This ratio is not strictly adhered to, however, as the location of substations is also influenced by physical conditions such as availability of a suitable location for the substation, or the location of concentrated loads caused by yards, grades or sharp curves.

Where a length of track receives its power from only one direction, the distance that can be fed, with a given voltage drop, is one-fourth the distance allowable between substations.

Apparatus. In the usual type of substation for the conversion of high-pressure alternating current into low-pressure direct current, the high-pressure current first passes through oil-switches to the high-pressure bus, then through oil-switches to the primary windings of the transformers. From the secondary windings of the transformers, the low-pressure alternating current enters the slip-rings of the converters, and emerges from the commutators as direct current of a slightly higher pressure. From the commutators it passes through circuit-breakers to the direct-current bus bars and thence through circuit-

Relation of Machinery in Typical Power-house



* Turbine drives electric generators, which supply current to the switching and protective apparatus and thence to the outgoing feeders.

breakers to the outgoing feeders. In the design of substations, it is good practice to arrange all the apparatus so as to make the route of the electric current as direct as possible.

All substations are not equipped with synchronous converters, motor-generators having been used especially for pressures in excess of 750 volts. Such a motor-generator usually consists of a synchronous motor driving a direct-current generator.

For voltages below 750 volts, the rotary converter has a virtual monopoly, although in special cases both motor-generator sets and mercury-arc rectifiers are used.

For direct-current voltages of the order of 1500 volts, all three classes of equipment are used, the rotary converter and motor-generator set now meeting competition from the mercury-arc rectifier.

For voltages above 1500 volts, motor-generator sets have been used exclusively, but the development of the mercury-arc rectifier is making rapid strides and many installations are now operating at this voltage in Europe.

Portable Substations. Portable substations, that is, substations of small size installed in a box car, are sometimes used where unexpected or occasional loads are liable to occur. They are especially useful to carry holiday loads for which it would not pay to install a permanent substation.

Power Capacity. The power capacity required in a substation depends upon the size, number, acceleration and speed of cars; on the length of line fed and upon the number of stops per mile.

Railway substation machines and transformers are usually rated by the kilovolt-ampere output which, having produced a constant temperature, may be increased 50% for 2 hours without producing temperatures or temperature rises exceeding by more than 5° C. the standard limiting values established by the American Institute of Electrical Engineers. This is known as the nominal rating.

Buildings. Substations having all apparatus on one level require about 0.2 sq. ft. per kilowatt of nominal rating. Buildings are usually of brick with concrete foundations for walls and machines, except where the machines are small, in which case the machines are often carried by the floor beams. It is usual to provide a crane which, in important substations, may be equipped for both manual and electrical operation.

Provision must be made for ventilation and good illumination, but roofs must be constructed so that there will be no danger of leaks over the machines and other live apparatus. In locations where the air contains a considerable amount of dust, the air entering the station should be drawn through air filters, circulation of the air being obtained by means of motor-driven fans blowing the air into the station through louvers in the floor near the machine. Lavatory, toilet and telephone booth should be conveniently near the operators' desks.

7. Transformer Stations

General. A transformer station, as used on alternating-current railway systems, is an aggregation of apparatus for converting high-voltage alternating current from transmission lines into lower-voltage alternating current for use in the trains, the apparatus being usually out-of-doors and protected only by a fence.

Special designs of oil-cooled transformers, switching apparatus, and lightning arresters must be used for this purpose.

All apparatus is set on concrete foundations, and steel structures are used to support overhead wires and switching apparatus.

Auto-transformer System. The New York, New Haven & Hartford Railroad system transmits at 22 000 volts between trolley and feeder, but the locomotives receive current at 11 000 volts between trolley and track rail. This is accomplished by transformer stations about 2 miles apart, where the trolley and feeder are each connected to the track rails through a separate auto-transformer. This arrangement not only effects economy in transmission, but also greatly reduces the current in the track rails and earth, with consequent reduction of disturbances to telephone lines.

TRANSMISSION AND DISTRIBUTION

8. Duct Lines and Underground Conductors

General. Underground cables in the United States are almost invariably installed in conduits made either of glazed tile, concrete or fiber. The conduits are laid so as to form a series of continuous ducts in length not over 400 ft., which are terminated in brick or concrete chambers from which the cables are pulled into the ducts.

Conduit lines are used for the distribution of electrical energy wherever the unsightliness, danger or instability of pole lines prohibit the use of the latter. They are used for both the transmission and distribution cables of many railways, and for telephone and telegraph lines.

Types of Conduits. Figs. 1 and 2 show the tile conduits used by the New York Central Railroad Company, and are representative of the best modern practice except that 4-in. and 4-1/2 in. sizes are now preferred. The single-duct conduit weighs 16-1/2 lb. per length of 18 in., and the four-duct conduit weighs 100 lb. per length of 3 ft.

Fiber conduit consists of tubes made of wood pulp or paper, saturated with bituminous compound and formed around a mandrel. Lengths of fiber conduit are joined by screw and coupling, socket, sleeve or drive joints. Fiber conduit should be laid in concrete, with from two to three inches of concrete between each duct at the point of minimum concrete section, to allow for thorough tamping of the concrete. Typical dimensions of fiber conduits are given in the table.

Sizes and Weights of Fiber Conduits

Inside diameter, inches	Thickness of walls, inches	Approximate average weight, per foot, pounds	Length of section, inches	Inside diameter, inches	Thickness of walls, inches	Approximate average weight, per foot, pounds	Length of section, inches
Sleeve joint				Sleeve joint			
1	1/4	0.45	30	1	1/4	0.45	30
1-1/2	1/4	0.75	60	1-1/2	1/4	0.75	60
1	1/4	0.90	60	2	1/4	0.90	60
2-1/2	1/4	1.05	60	2-1/2	1/4	1.05	60
3	1/4	1.30	60	3	1/4	1.30	60
3-1/2	1/4	1.60	60	3-1/2	7/16	2.50	60
4	1/4	1.85	60	4	1/2	3.20	60
Screw joint				Drive joint			
1-1/2	5/16	0.85	60	2	1/4	0.90	60
2	3/8	1.35	60	2-1/2	1/4	1.05	60
2-1/2	3/8	1.70	60	3	1/4	1.30	60
3	7/16	2.20	60	3-1/2	1/4	1.60	60
3-1/2	7/16	2.50	60	4	1/4	1.85	60
4	1/2	3.20	60				

Wrought-iron pipe is used in city streets where the conduit line has to twist about sub-surface obstructions. It is more expensive than tile conduit, and does not last as long on account of rusting. The usual sizes are 3-in. and 3-1/2-in. pipe, 20 ft. long, provided with threaded ends and couplings.

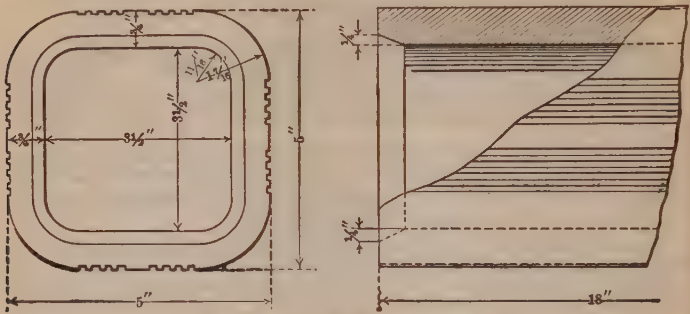


Fig. 1. Typical Single-Duct Conduit

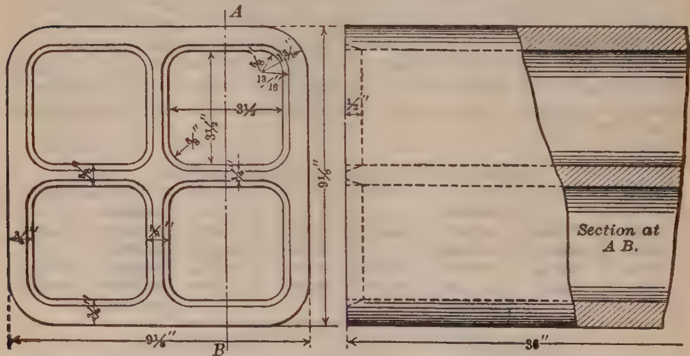


Fig. 2. Typical Four-Duct Conduit

Splicing Chambers. Splicing chambers for straight runs are usually built in the shapes shown in Figs. 3, 4, 5, and 6. The height of large splicing cham-

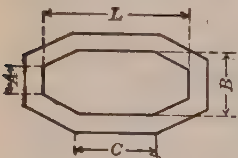


Fig. 3

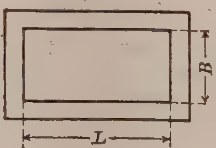


Fig. 4

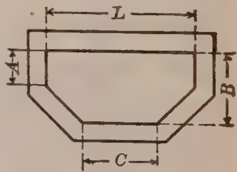


Fig. 5

Splicing Chambers

bers is usually determined by the height in which a man can stand upright and is seldom less than 6-1/2 ft. The width is similarly influenced by the space required for working, which is at least 4 ft. The length depends upon the length of splice and the space required to curve the cable from the ducts to the supporting shelves or racks, considerations which make a length of 8 ft. a practical minimum where there are large cables. The usual dimensions are tabulated herewith.

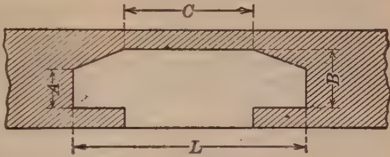


Fig. 6. Splicing Chamber

Cables in splicing chambers are usually supported on iron brackets attached to the chamber walls. This construction is very satisfactory, especially if the power cables are protected with fireproofing; but some engineers consider a shelf preferable, as it affords insulation and provides more protection between cable splices.

Type	A	B	C	L
Fig. 3	2 ft. 2 in.	5 ft. 0 in.	3 ft. 6 in.	9 ft. 0 in.
Fig. 4	....	4 6	....	8 0
Fig. 5	2 0	4 6	3 8	10 0
Fig. 6	1 6	3 0	4 0	10 0

Testing of Duct Lines. Ducts are usually tested by rodding with a mandrel in order to ascertain whether they are continuous and unobstructed. The rods used for this purpose are of hickory 1 in. in diameter and 3 or 4 ft. long, and are fitted at the ends with steel couplings such as that shown in Fig. 7. The first rod is attached to a mandrel and pushed into the duct. Another rod is coupled to the first, and the pair pushed farther into the duct. By successively coupling other rods and pushing them into the duct, the mandrel is made to travel from one chamber to another.

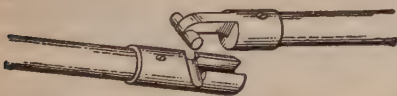


Fig. 7. Testing Mandrel

As soon as the mandrel emerges into the receiving chamber, the rods are pulled through and uncoupled. If an obstruction stops the mandrel, an attempt is made to force it through by repeated blows, failing which it becomes necessary to break into the conduit line from the side. It is usual to attach a No. 10 or No. 12 galvanized steel wire to the last rod, and leave the wire in the duct after the removal of the rods. This wire is subsequently used for drawing a heavy rope through the duct, by means of which the cable is pulled in place.

Types of Cables Used. Cables for underground conduit lines are almost invariably insulated with impregnated paper and covered with a lead sheath. Three-conductor cables are used for high-pressure, three-phase transmission, and either single-conductor or two-conductor concentric cables for low-pressure distribution. Single conductor cables are now being applied to high-voltage three-phase transmission in metropolitan areas.

9. Pole Lines and Aerial Cables

General. The most usual construction for street and interurban railways consists of wooden poles spaced about 100 ft. apart, carrying wooden cross-arms, porcelain insulators, and strung with bare or weatherproof copper wire.

Where the transmission line parallels the railway, the same poles are used for supporting both trolley system and transmission lines. Steel towers are used on electrified steam railways.

Tubular steel poles are largely used for urban railways and reinforced-concrete poles are gaining favor.

Dimensions of Wooden Poles. Poles are specified by their total length and their diameter or circumference at the top. Standard lengths are multiples of 5 ft. and vary from 30 to 60 ft. Standard diameters are even multiples of 1 in. and vary from 7 to 8 in., except on the Pacific Coast, where 8 to 10 in. prevail. The taper is expressed as the difference in inches between two circumferences 10 ft. apart and is usually as follows:

Michigan white cedar	5.2
Maryland chestnut	3.8–4.0
California yellow pine	4.0
Montana lodgepole pine	3.0
Texas loblolly pine	2.4
Washington cedar	3.5

The minimum dimensions specified by the American Electric Railway Association are given in a table on page 2126.

Tubular Steel Poles. Tubular steel poles are made of three pieces, each of a different size, of steel tubing. The three pieces are joined by shrinking the larger onto the smaller, until an overlap of 18 in. is obtained. Such poles are made in lengths of 28 to 35 ft. and vary in weight from 384 to 1688 lb. Comprehensive tables of data for such poles are given in A.E.R.A. Engineering Manual.

Reinforced-Concrete Poles. Reinforced-concrete poles are made in lengths of 28 to 35 ft. and with butt sections from 5 to 15 in. square, the corners, however, being eliminated. They are reinforced with four steel rods. Hexagonal poles are also used, containing six steel rods. Comprehensive tables of data for such poles are given in A.E.R.A. Engineering Manual.

Location of Poles. Side poles should have a minimum clear distance of 7 ft. 6 in. from the center line of track at level of top of rail; center poles should have a minimum clear distance of 7 ft. from the center of track, these clearances to be suitably increased at curves. Poles are usually spaced between 90 and 110 ft. apart.

Depth of Setting. The usual depth of pole setting for tangent construction is shown in the table on page 2127. On curves, and other special localities, an extra depth may be necessary.

Wires and Cables for Pole Lines. Three classes of wire are used for aerial conductors: Bare, weatherproof and insulated. Bare and weatherproof wires are usually made of hard-drawn copper, the stranding being as shown in the table below. Bare wires are used for operating pressures over 2500 volts, but where the pressure is not above this amount, weatherproof wire is preferred by many engineers, because it affords some protection from grounds due to the contact of foreign bodies, especially tree twigs.

Insulated cables, except telephone cables, are seldom used on pole lines on account of their expense, but where used are made of soft annealed copper either insulated with paper and sheathed in lead, or insulated with varnished cambric or rubber and covered with impregnated cotton braid. Such cables must be supported from a steel messenger cable by means of clips spaced from 12 to 18 in. apart. Where rubber insulation is used on aerial lines, a high-grade compound is essential if it is to withstand the effects of the weather.

Stranding of Concentric Lay-Cables for Aerial Use

Size (See note 1)	Number of wires (See note 2) Bare, insulated or weatherproof cables for aerial use	Size (See note 1)	Number of wires (See note 2) Bare, insulated or weatherproof cables for aerial use
Cir. in.			
2.0	91	0000 A.W.G.	19 or 7 (See note 3)
1.5	61	00 A.W.G.	7
1.0	61	2 A.W.G.	7
0.6	37	7 and smaller	1
0.5	37		
0.4	19		

1. For intermediate sizes, use stranding for next larger size.
2. Conductors of 0000 A.W.G. and smaller are often made solid and this table of stranding should not be interpreted as excluding this practice.
3. Cables of sizes 0000 and 000 A.W.G., are usually made of 7 strands when bare and 19 strands when insulated or weatherproof.

Guy Wires. Galvanized iron cables are used for guying poles. They are usually made of 7 strands of No. 12 or 14 B.W.G. wire, having an ultimate breaking strength of 2300 or 5000 lb. per cable, respectively.

Preservation of Poles. It is estimated that it takes 190 years to grow a 30-ft. cedar pole, which, when set in the ground, will not last over 15 years. When butt-treated, the life is increased to about 20 years. The preservatives most generally used are creosote, zinc chloride and bichloride of mercury. (See U. S. Forest Service Circulars 84 and 147.) Decay occurs at the butt of the pole, which must be particularly well protected.

Cross Arms. Cross arms are rated by the number of pins they are made to carry, the number being from 2 to 10. The usual cross-sections are 3-1/4 or 3-3/4 by 4-1/4 or 4-3/4, the length depending upon the number of wires to be carried and their voltage. Cross arms, 3-1/4 by 4-1/4 in. in cross-section are attached to wooden poles by a through-bolt driven from the back of pole toward and through the arm, having a washer at each end, with hole at back of pole counterbored to secure a good seat for the washer. Cross arms should be steadied by strap braces secured to the pole by a large screw and to side of arm away from pole by carriage bolts on the center line of arm, with nuts next to the braces. Arms up to and including 48 in. in length should have braces 24 in. long fastened to arm 16 in. from center; arms over 48 in. long should have braces 28 in. long fastened to arms 19 in. from center. Cross arms larger than 3-1/4 by 4-1/4 in. should be steadied by angle braces.

10. Trolley Systems

General. The trolley wire is usually of hard-drawn copper but under unusual conditions, bronze or steel wires are sometimes used. The wire is suspended from insulators some 16 to 30 ft. above the ground, and presents a continuous contact surface to a trolley wheel or bow attached to the rolling stock. There are two classes of trolley construction, the span wire and the side bracket; each may have either simple or catenary suspension. (See Art. 9 for details of poles.)

Minimum Circumference of Poles in Inches

(A.E.R.A. Engineering Manual)

A. For span construction where a 35-ft. span is required, or for heavy feeder lines carrying from one to six cross arms

B. For span or bracket construction where spans are not more than 35 ft. or bracket line or construction carrying two transmission circuits, one feeder arm and two telephone and signal arms.

C. For telephone, signal and other light auxiliary lines where no side strain is required.

Chestnut Poles

Total Length, ft.	Class A		Class B		Class C	
	Top	6 ft. from butt	Top	6 ft. from butt	Top	6 ft. from butt
25	24	36	21	31	20	30
30	24	40	22	36	20	33
35	24	43	22	40	20	36
40	24	45	22	43	20	40
45	24	48	22	47	20	43
50	24	51	22	50	20	46
55	22	54	22	53	20	49
60	22	57	22	56	....	....
65	22	60	22	59	....	....
70	22	63	22	62	....	....
75	22	66	22	65	....	....

Eastern White Cedar Poles

Total length, ft.	Class A	Class B	Class C
	Top 24 in.	Top 22 in.	Top 18-3/4 in.
	6 ft. from butt	6 ft. from butt	6 ft. from butt
30	40	36	33
35	43	38	36
40	47	43	40
45	50	47	43
50	53	50	46
55	56	53	49
60	59	56	....

Western Red Cedar Poles

Total length, ft.	Class A	Class B	Class C
	Top 28 in.	Top 25 in.	Top 22 in.
	6 ft. from butt	6 ft. from butt	6 ft. from butt
30	37	34	30
35	40	36	32
40	43	38	34
45	45	40	36
50	47	42	38
55	49	44	40
60	52	46	41
65	54	48	43

Depth of Setting Poles in Ground

Total length of pole, ft.	Trolley construction		Transmission line construction
	Rock or concrete	Earth	Earth
30	5 ft. 0 in.	6 ft. 0 in.	5 ft. 0 in.
35	5 6	6 0	5 6
40	5 6	6 6	6 0
45	6 0	6 6	6 5
50	6 6	7 0	6 5
55	6 6	7 6	7 0
60	7 0	8 0	7 0
65	7 0	8 6	7 6
70	7 0	9 0	7 6
75	....	....	8 0
80	....	....	8 0

Span-wire and Side-bracket Construction. In the simple span-wire construction the trolley wire is supported by wires stretched across the tracks between poles or building walls. The side-bracket construction resembles the span-wire except that instead of the supporting wire being stretched between two poles, it is stretched between two supports on the same pole. In both of these types of construction, the trolley wire is supported at intervals of 100 ft. or more and sags between supports, making it necessary for the trolley to be in vertical vibration as the cars move.

Catenary Construction. The speed attained upon modern electric roads makes it difficult to obtain satisfactory service with a trolley wire which dips between supports and sways with every impulse. The catenary construction was devised to meet this condition. In general, it consists of a grooved copper trolley wire suspended horizontally from a sagging messenger cable, which is suitably insulated and firmly held in place. The supporting structure employed for interurban single- or double-track roads usually is of the side-bracket type, but for some conditions cross-span construction becomes necessary. The latter method of support differs only in the substitution of a catenary cross-span for the bracket arm and doubling the number of poles required for single track.

Applications of Various Types of Construction. The overhead trolley system is used on urban railways, wherever the unsightliness or danger of its exposed construction is not considered objectionable. It is used on interurban, suburban and trunk lines, wherever, by reason of low load or high voltage, the current taken by the trains is not too great to be economically carried on copper wires.

Center-pole construction is approved practice for double-track railways, side-pole construction being generally used for single-track lines. Span-wire construction is used where, for any reason, it is impracticable to have the poles near the tracks or where, as is commonly the case in Europe, the span wires are supported from the walls of buildings. Prejudice against overhead lines is often due to the excessive loading of poles, which is both unsightly and dangerous.

Parts of Overhead Trolley Construction. The trolley wire is secured to an "ear" by soldering, clamps or other means and the ear is bolted to a "suspension" which may or may not be provided with an insulating portion. These suspensions are carried by span wires which in turn are fastened to the poles or brackets, or they may be fastened directly to the bracket. If the "suspension" is not insulated, "strain" insulators are inserted in the span wire between the ear and its point of attachment to the poles or brackets. A slightly different form of suspension, called a "pull-off," is used on curves. At turnouts "trolley frogs" must be used to guide the trolley wheel. These various parts are illustrated in the accompanying cuts.

Trolley Ears. Figs. 8 to 12 show ears for round wire. Fig. 8 shows an ear with flaps, which are bent around the wire to hold it in place; this type is

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Trolley Ears for Round Wire

seldom used on account of arcing at the flaps. The ear shown in Fig. 9, which has a deep groove into which the wire is soldered, is the type in common use. These ears may be provided with rings as in Fig. 10 to which guy wires are attached to relieve the strain at curves and for steadying the line at intervals. Fig. 11 shows the type of ear used at points where the trolley is spliced, which should always be at an ear. Fig. 12 shows an ear with a terminal for a feeder connection.

Figs. 13 to 15 are designed for grooved or "figure 8" trolley wire. Special grooved (Fig. 16) or "figure 8" wire affords a smoother running surface for the

Fig. 13

Fig. 14

Fig. 15

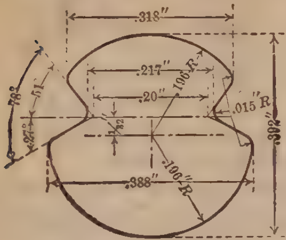
Trolley Ears for "Figure 8" Wires

trolley wheel or bow as the ear only grips the upper part, leaving the lower part absolutely even. This construction is practically essential for bow trolleys.

Suspensions for Straight Line Work. Fig. 17 shows an insulated suspension with ear attached at each end; this is also used on curves for double-track work. Figs. 19 and 20 show solid insulated suspension for span wire and side-bracket construction respectively. These types are frequently used, and are quite satisfactory if the insulating material used in their construction is properly made. Fig. 21 shows a section of an assembled cap and cone suspension. Fig. 22 shows a cap and cone suspension with ear attached. The type in which cap and cone are made separate is preferred by some engineers on account of the possibility of replacing injured bolts and insulation without removing the whole suspension. This advantage is partly offset by the greater liability to trouble due to multiplicity of parts.

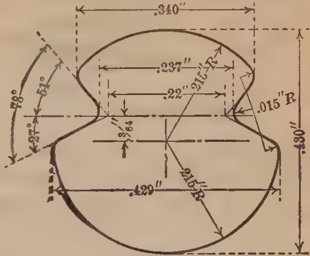
Strain Insulators. Fig. 23 shows a "globe" strain insulator, the type most commonly used. Fig. 24 shows a "Brooklyn" strain insulator. This type is used on wooden pole and light iron pole construction to draw span wires taut. It is required for heavy iron pole construction if spans are long and temperature variation great. Bolts may be provided at both ends if an extra amount of adjustment is required. A globe strain and a Brooklyn strain may be used in series where extra insulation is required.

Pull-offs. Fig. 25 shows a cap and cone pull-off for single-curve construction and Fig. 26, a cap and cone pull-off for double-track curve construction.



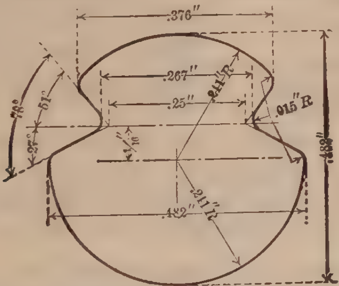
2/0 Wire

Area 0.1083 *sq. in.*
3/6 Round Wire 0.1045 *sq. in.*



3/0 Wire

Area 0.1314 *sq. in.*
3/6 Round Wire 0.1318 *sq. in.*



4/0 Wire

Area 0.1665 *sq. in.*
4/6 Round Wire 0.1662 *sq. in.*

Fig. 16. Standard Grooved Wire Sections

Pull-offs of the types corresponding to the uninsulated and solid insulated suspension shown in Fig. 17 and 18, are also used.

Fig. 17

Fig. 18

Fig. 19

Fig. 20

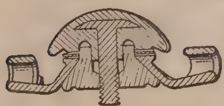


Fig. 21

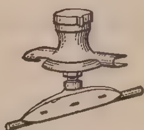


Fig. 22

Types of Suspensions

Trolley Frogs. A trolley frog is a malleable iron casting used at switches or crossovers where trolley wires from different tracks unite. Its function is to hold the diverging wires together and afford a smooth running path to the trolley wheel when a car passes or enters a switch. A common type is illus-

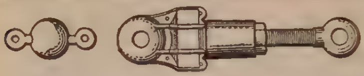


Fig. 23 Fig. 24

Strain Insulators

Fig. 25 Fig. 26

Pull-offs

trated in Fig. 27. Frogs are made for various angles of divergence and both right- and left-handed. The usual angles are 8, 15 and 20 degrees.

Sag and Tension in Overhead System. Particular attention must be paid to designing the overhead structure in such a manner that it will safely stand the extra tension due to the contraction of the wires at low temperatures and the extra loads due to wind and sleet, and a sufficient allowance should be made in the height of the trolley wire to take care of the extra sag which it experiences at high temperatures.

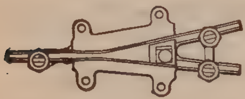


Fig. 27. Trolley Frog

Height of Trolley above Rail. The height of wire varies between a minimum of 16 ft. and a maximum of 22 ft., the usual height being about 18 ft. It is usual to raise the wire at railroad crossings to a height of 22 ft. or more.

Rake of Poles. Bracket-arm poles on tangent construction should have a rake backwards not exceeding 3 in., and span-wire poles in hard ground a rake of from 4 to 5 in. In soft ground a rake of 12 in. is not uncommon. Center poles should be set vertically except at curves, where they should bend away from the curve along the perpendicular to the tangent at that point of the curve.

Anchorage. At both ends of every grade and curve, there should be a permanent anchorage. If there are not many grades and curves, anchorages should be provided at intervals of from 1/4 to 3/4 mile. An anchorage is made by means of a steel cable running from the trolley wire to one or more anchor poles through an anchor ear and strain insulators.

Curves. At curves in the track the trolley wire should be made to follow the curve by means of pull-offs or wires pulling the trolley wire outward as shown in Fig. 28 and 29.

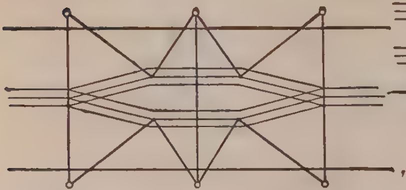


Fig. 28

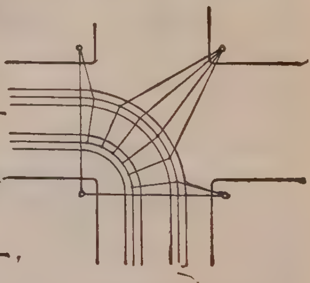


Fig. 29

Simple Pull-off Arrangement

Wherever possible the pull-off wires should be radial to the trolley wire. This, however, requires a large number of poles, and is therefore impracticable

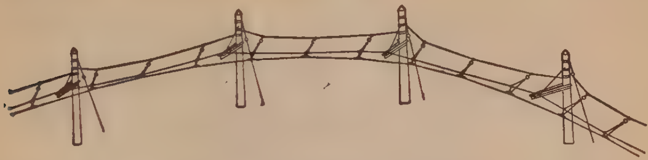


Fig. 30. Bridle Construction at Curve

in cities. In such cases the bridle, bowstring or backbone construction shown in Fig. 30, 31 and 32 is resorted to. Here the pull-off wires, instead of being

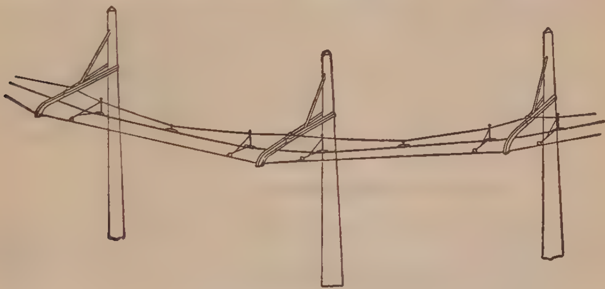


Fig. 31. Bow String Construction at Curve

anchored to individual poles, are fastened to a wire which is stretched between poles. While this construction is almost universally used in cities, it is more

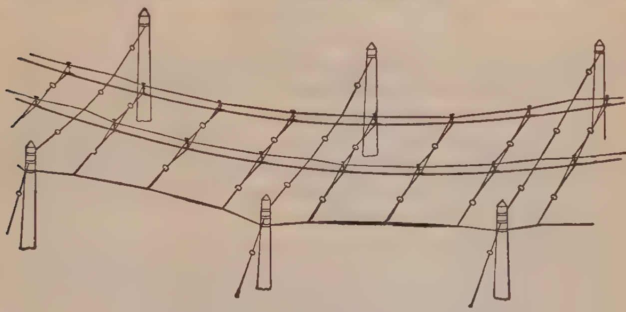


Fig. 32. Backbone Construction at Curve

expensive to maintain than single pull-offs, and is therefore less favored for interurban lines.

A combination of the two types of construction is shown in Fig. 33 (Fig. 28 to 33 are from General Electric Co. publications).

Spacing of Pull-offs at Curves. The number of pull-offs should be sufficient to keep the wire within about 2-1/2 in. from the theoretical curve. This may

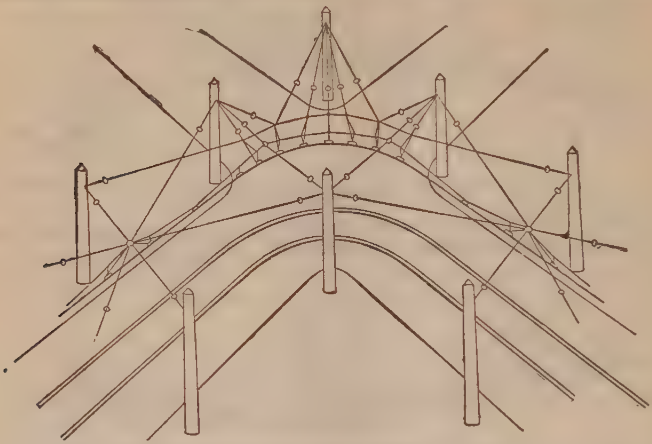


Fig. 33. Combination Construction at Curve

be accomplished by spacing the successive ears in accordance with the following relations:

Let L = distance between pull-offs in feet;
R = radius of curve in feet;
a = offset of wire in inches from theoretical curve, midway between pull-offs, the ears being assumed to lie on the theoretical curve.

Then
$$L = \sqrt{\frac{2\,aR}{3} - \left(\frac{a}{6}\right)^2},$$

Or
$$L = 0\,815\,\sqrt{aR},$$

with an error less than 1-4% for all radii greater than 40 ft.

If a is to be 2-1/2 in., then,approximately

$$L = 1.29\,\sqrt{R}$$

If the ears are set exactly on the theoretical curve, the wire will depart 2-1/2 in. from the correct position half way between the two ears; if the ears are set 1-1/4 in. from the curve, the mid-point of the wire will be only 1-1/4 in. when L is taken equal to 1.29 $\sqrt{R}$.

Offset of Trolley Wire at Curves. It is usual, at curves, to offset the trolley wire from the center of the track because the trolley wheel is tilted inward due to the elevation of the outer rail, and because in order to keep the wheel on the wire, the latter must be so placed that the projection of the pole on the plane of the track is always tangential to the wire.

(a) The former offset, measured horizontally toward the inside of the curve, equals $h \tan \theta$ where h is the normal height of the wire above the top of rail, and θ is the angle of elevation between the plan of the track and the horizontal. For standard gage and wire 18 ft. above the top of rail this offset is about 4 in. for every inch of elevation.

(b) The latter offset, also measured horizontally toward the center of the curve, is calculated as follows, assuming the curve to be longer than the car itself; for shorter curves the offset will be less.

Let R = radius of curve;
L = horizontal distance of center of trolley wheel to center of trolley base usually about 11 ft.;
G = distance from center of car to center of truck.

In the case where the trolley base is located over the truck center, as on cars with two trolleys, the offset is

$$R = \sqrt{R^2 - L^2}.$$

If the trolley base is located over the car center as on cars with one trolley, the offset equals

$$R - \sqrt{R^2 - G^2 - L^2}.$$

The total offset is the sum of the offsets (a) and (b).

Curves of Small Curvature. If the curvature * is less than 10 deg., the poles are frequently spaced closer together and no pull-offs used. The table may be considered typical, the divergence of the wire from the theoretical curve being kept within 3-1/2 in.

Spacing of Poles on Curves, Interurban Road

Degree of curvature of track	Pole spacing, feet	Divergence of trolley wire from track center, inches	Degree of curvature of track	Pole spacing, feet	Divergence of trolley wire from track center, inches
Tangent	120	0.	6	60	2.87
1	120	1.87	7	50	2.34
2	110	3.37	8	50	2.64
3	90	3.06	9	50	2.94
4	80	3.37	10	50	3.24
5	70	3.06			

Turnouts. The location of the trolley frog at turnouts may be determined as follows: Referring to Fig. 34, from switch point A draw a line to center point D of track frog distance BC and from switch point B draw a line to center point E of arc AEC. The intersection of these two lines at F will be the proper location of the frog. While certain variables, such as superelevation of the outer rail on the curve, length of wheel base and projection of trolley pole rearward from center of car, may necessitate slight variation of setting, this location will be found so nearly correct that a very small alteration, which must be determined by experiment, will compensate for the variable conditions.

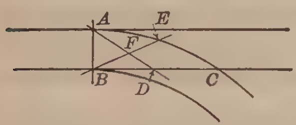


Fig. 34. Location of Frog

* The degree of curvature of a track is the angle which subtends a chord 100 ft long, between points on the center line of the track.

The following table gives the range of distance from track switch point to track frog with which each set of trolley frogs may be most satisfactorily used:

Track-frog distance	Divergence angle of trolley frog
Up to 22 ft.	20°
From 20 to 30 ft.	15°
Above 28 ft.	8°

The minimum frog distance given in the table with which the 15° frogs may be used to best advantage corresponds to a turnout radius of 40 ft., but when suburban cars, using high-speed trolley wheels, run over city tracks it is advisable to use 15-deg. frogs rather than 20-deg. frogs throughout the city construction even where the minimum frog distance is less than 20 ft.

Simple and Compound Catenary Construction. Catenary construction may be divided into simple and compound. In the former the trolley wire or wires are carried by one or two messenger cables which are supported only at the poles, bents or span wires; in the latter, the horizontal wire or wires are carried by a messenger cable, which is itself suspended from another messenger cable which is supported at the poles, bents or span wires. The advantages of the compound catenary are greater flexibility, reduced stresses in the supporting wires, shorter hangers, better lightning protection and superior curve construction.

To obtain a line which will not require frequent readjustment, the messenger cable must be installed with practically uniform tension throughout its length, making it necessary to have less sag in the shorter spans. For this reason certain definite pole spacings and corresponding hanger lengths have been standardized.

Number of Suspensions. The number of suspensions depends upon the speed at which the cars are to be run, and upon whether a bow or wheel trolley is used. The three-point suspension, in which, with 150-ft. spacing, the hangers are 50 ft. apart, has been found ample for wheel collectors. With the sliding pantograph or bow trolley an eleven-point suspension has been found sufficient, with 150-ft. pole spacing. (See following Table.)

Hangers. Where only one horizontal wire is suspended from the messenger cable, the hangers should hang loosely from the cable and should be screwed fast to the trolley wire. This permits the trolley wire to rise slightly as the trolley passes under it, thereby making the wire equally flexible along its entire length. Where a steel contact wire is clipped to the horizontal conductor, the hangers are usually rigidly attached at both ends, as the duplication of horizontal wires assures uniform flexibility, regardless of the hangers. (See following Table.)

Wear of Trolley Wheel Bow and Trolley Wire. The wear of the trolley wires is not serious with either wheel or bow collectors. On the lines of the Indianapolis & Cincinnati Traction Company, a copper trolley wire lost less than 1% in weight after it has experienced 39,000 car movements, each car taking an average of about 40 amperes by an aluminum slider.

The vertical wear of the steel contact wire on the N. Y., N. H. & H. R.R. was 0.028 in. in 30 months, which is practically 4.5% per year of the half diameter of the wire (one-half taken to permit wire to be held in clips) which, even on this vertical diameter basis, indicates a life of over 20 years; but as a

Tangent Construction

Number of Hangers per Span. Pantograph or Bow Trolleys

Length pole spacing, feet	Number of points of suspension	Length of hangers, inches										
		6	6-3/4	8-1/2	11	12	13-1/2	14-3/4	16	17-1/2	19-1/4	20-1/2
150	11	1	2	2	2	...	......	2	...	......	2	......
125	9	...	......	......	1	2	2	......	2	......	2	......
110	8	...	......	......	...	...	2	2	...	2	......	2
95	7	...	......	......	...	...	......	......	3	2	......	2
80	6	...	......	......	...	...	......	......	...	2	2	2
70	5	...	......	......	...	...	......	......	...	......	3	2
55	4	...	......	......	...	...	......	......	...	......	......	4

Number of Hangers per Span. Wheel Trolleys

Length pole spacing, feet	Number of points of suspension	Length of hangers, inches							
		6	11	13-1/2	14-3/4	16	17-1/2	19-1/4	20-1/2
150	3	1	......	......	2	......	......	......	......
125	3	......	1	......	......	......	2	......	......
110	3	......	......	1	......	......	1	......	......
95	3	......	......	......	......	1	......	2	......
80	3	......	......	......	......	......	1	2	......
70	2	......	......	......	......	......	......	......	2
55	2	......	......	......	......	......	......	......	2

matter of fact it will be much more than this, for the reason that as the vertical diameter lessens the breadth of contact increases throughout, thus diminishing the rate of vertical wear. Of further interest is the fact that there is practically no corrosion of the wire as the wire is covered with a film of grease deposited by the pantograph shoe (W. S. Murray).

Change in Length of Trolley Wire. The change in length of copper due to changes in temperature is one of the greatest difficulties in the maintenance of overhead work. A drop of 100° F. in temperature will cause a copper bar to contract approximately 1 in. for every 100 ft. of length. If it is restrained at the ends this will cause an additional stress of 2500 lb. in the case of a No. 0000 trolley.

European catenary lines are usually maintained at constant tension by means of weights pulling on the free ends of the trolley wire at the end of every section.

Prevention of Formation of Sleet. Sleet may be prevented from forming on the wires by greasing the latter with petroleum jelly and if it does form it may be easily removed by any of the numerous commercial forms of sleet cutters.

11. Third Rail Systems

General. Third rails are used for low voltage railways where the power requirements are in excess of the current carrying capacity of a trolley wire, and where there is private right-of-way.

Collector Shoes. Current is collected from the third rail by projecting collector shoes on the cars. These shoes are placed on each side of each truck, and are all alive, when any one is alive.

Rails are usually made of special high conductivity steel, it being not unusual to have a conductivity 1/6 or 1/7 that of copper. (See Article 12 for resistance of rails of various sections.)

Types of Construction. Third-rail construction may be classified into the top-contact and under-contact types, each of which is susceptible of important variations in design, especially with reference to the type of protection.

(a) **Interborough Top-contact Type.** One of the most commonly used type is illustrated in Fig. 35. It is often called the "Interborough Type" on account of its use in the subways of the Interborough Rapid Transit Co., of New York. The rail is a standard T-section and rests on reconstructed granite insulators. A board protection is attached to the rail itself by means of clamps and uprights and is thereby kept in perfect alignment.

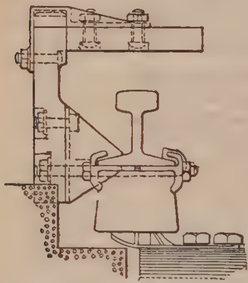


Fig. 35. Interborough Top-Contact Type

(b) **Pennsylvania Top-contact Type.** (Fig. 36.) Another type has the protection supported on separate brackets independent of the third rail itself. It is claimed that this reduces the amount of labor which has to be done on the live rail, when repairs are being made, but it cannot be relied upon as well as the Interborough type to keep the rail and protection in perfect alignment.

(c) **Under-contact Types.** (Fig. 37.) While the top-contact types have given first-class service, they are considered to have certain disadvantages for exposed locations as they cannot be wholly protected from snow, ice and sleet. The lower part is only a few inches from

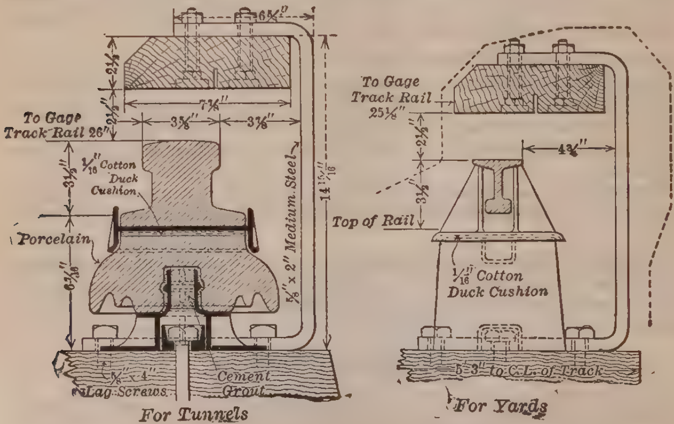


Fig. 36. Pennsylvania Top-Contact Type

the ties, while holding clips generally reduce this clearance, increasing the danger of grounding from accumulation of wet snow, and from flooding. The occasional suspension of traffic during sleet and snowstorms and floods, on railroads using the top-contact type of third rail, led to the idea of an under-contact third rail loosely clasped in insulators by hook-bolts hung from brackets, with the top and sides of the rail completely sheathed in a flexible insulating material for protecting the rail from accidental contact with man and beast, and from sleet, snow and spray. With this type of rail the protection is of such character that there is no packing of snow between the sheathing and the contact rail, as in some other forms, and in sleet storms no ice forms on the contact surface; some icicles may form at the edge of the petticoats, but, hanging down clear of the edge of the rail, are easily broken off by the passing shoe.

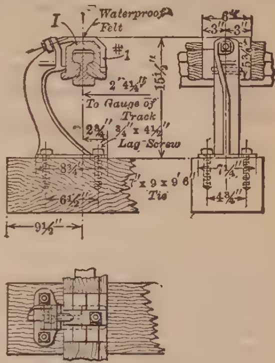


Fig. 37. N. Y. C. R. R. Under-Contact Type

Where the rail is buried in snow, the passage of the contact shoe breaks the snow away, leaving the rail surface clear, instead of ironing the snow down on the rail, as may happen with the top-contact type.

Location and Weight of Third Rails. There is no standard gage for contact rails, corresponding to the standard track gage. This unfortunate condition arises from lack of uniformity in the clearance lines of the right-of-way and in the maximum equipment lines of various railroads. The following standard has been recommended (1911 and 1912) by the American Electric Railway Engineering Association:

The gage line of the third rail to be located not less than 26 in. and not more than 27 in. from the gage line of the track and the contact surface of the third rail to be not less than 2-3/4 in. or more than 3-1/2 in. above the plane of the top of the track rail.

This standard has since been abandoned and three standard clearance lines adopted instead. The maximum equipment line for rolling equipment is given in Art. 13; those for third rail structures and permanent way, respectively, are given below:

Third-rail Structures Must be Contained within the Line Indicated by the Following Ordinates and Abscissas

Height above top of rail, inches	Distance out from gage line, inches	Height above top of rail, inches	Distance out from gage line, inches
Top of tie	19-3/4	9-7/8	33-3/4
5/8	19-3/4	6-1/8	36-3/4
3-11/16	25	1/4	36-1/4
8	25	−2-1/4	35-1/4
9-7/8	26-7/8	Top of tie	35-1/4

(On curves of less radii than 800 ft. and at switches, this line must be moved out 2-1/2 in.)

Permanent Way Structures Must not Encroach upon the Line Indicated by the Following Ordinates and Abscissas

Height above top of rail, inches	Distance out from gage line, inches	Height above top of rail, inches	Distance out from gage line, inches
Top of tie	36-1,4	7-5/16	37-3/4
=2-1/2	36-1/4	10-7/8	34-1/4
0	37-3/4		

(On curves of less radii than 800 ft. and at switches, this line must be moved out 2-1/2-in.)

Location and Weight of Third Rail

Name of railroad	Center of third rail to near gage line, inches	Contact face above top of running rail, inches	Weight, pounds, per yard
Albany & Hudson	27	6	....
Aurora, Elgin & Chicago	20-1/8	6-5/16	100
Baker St. & Waterloo Ry	....	....	90
Baltimore & Ohio	30	3-1/2	....
Berlin Elevated and Subway	13.25	7	....
Boston Elevated and Subway	20-3/8	6	85
Brooklyn & Manhattan Rapid Transit	21-3/4	6	70–140
Camden & Atlantic City R.R.	26	3-1/2	100
Central London R.R.	Center	1-1/2	....
Columbus, London & Springfield	27	6	....
Columbus & Newark	27	6	....
Fayet-Chamounix	23	9	....
Grand Rapids, Gd. Haven & Muskegon	20-3/8	5-3/4	....
Great Northern Ry., England	19-1/4	....	80
Hudson Tunnels, New York	26	4	75
Interborough Rapid Transit	26	4	75–140
Kings County El., Brooklyn	19-1/2	5-1/4	....
Lackawanna & Wyoming	20-1/2	3	75
Lake St. El., Chicago	20-1/8	6-1/2	....
Lancashire & Yorkshire Ry	19-1/4	3	70
Liverpool Elevated	Center	1-1/2	....
Long Island R.R.	27	3-1/2	100
Manhattan Ry., New York	20-3/4	7-1/2	100
Mersey Ry	22	4-1/2	....
Metropolitan & District, London	16	3	100
Metropolitan Elevated, Chicago	20-1/8	6-1/4	48
*New York Central R.R.	28-1/4	2-3/4	70
Northeastern Ry., England	19-1/4	3-1/4	80
Northwestern Elevated, Chicago	20-1/8	6-1/2	48
North Shore R.R. Cal	27	6	50–60
Paris Orleans Ry	25-5/8	7-7/8	....
Paris Versailles Ry	25-5/8	7-7/8	....
Pennsylvania R.R.	27	3-1/2	150
*Philadelphia & Western	27	3-5/8	....
*Philadelphia Rapid Transit	27	6	70
Seattle & Tacoma R.R.	20	7-1/2	100
South Side Elevated, Chicago	20-1/8	6-3/4	....
Wamseebahn (Berlin)	33-1/2	12-5/8	....
Waterloo & City Ry	28-1/4	0	....
West Jersey & Seashore	26	3-1/2	100
*West Shore R.R.	32	2-3/4	70
Wilkesbarre & Hazelton	28	5	80

* Bottom-contact surface. All others have top-contact surface.

Inclines. Where it is necessary to break the third rail at cross-overs, grade crossings or electrical section breaks, **end declines** are required to catch the contact shoes and bring them to the normal level of the third rail.

The exact location of the inclines on each side of a track-rail intersection depends upon a number of conditions, an important one of which is the extent to which the car equalizer bar and journal box project outward. If there is a train-bus line connecting all the cars, the end inclines may be situated many feet back from the switch point of frog, but if there is no such bus line, the contact rail will probably have to be terminated in a cross incline extending as near as possible to the switch point or frog.

12. Rail Bonds

General. Rail bonds are electrical conductors for bridging the joints of rails. They consist either of a series of thin strips of annealed copper, or of one or more cables of copper wire, the ends of which are usually pressed or cast into solid terminals. Ribbon is more compact, but stranded wire is more flexible; the latter should always be used if space permits.

Types of Bonds. Bonds may be classified, according to the method of fastening them to the rail, as soldered bonds, brazed bonds and bonds applied

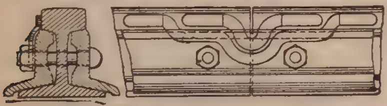


Fig. 38

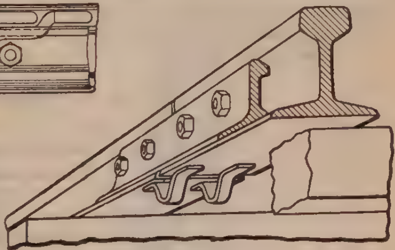


Fig. 39

by mechanical pressure. Soldered bonds and compressed terminal head bonds find their best application in third-rail work, where good electrical contact is of greater importance than mechanical strength. Expanded terminal web bonds, especially of the concealed type with two stranded conductors, are regarded as the best for heavy track work, where mechanical strength and cost of installation are of the utmost importance.

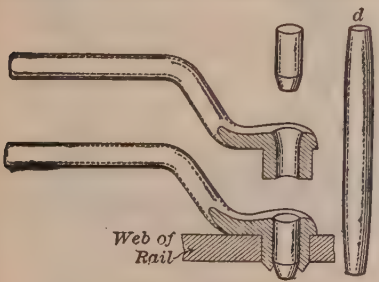


Fig. 40

Soldered Bonds usually consist of a series of thin strips of annealed copper with tinned terminals as shown in Figs. 38 and 39. They are soldered direct to the head, foot or web of the rail. One or more bonds per joint may be used.

Brazed Bonds resemble soldered bonds except that the terminals are enveloped in brass. They are brazed or welded to the rail by heat generated electrically in a carbon electrode which constitutes one jaw of a clamp holding the bond against the rail.

Expanded and Compressed Terminal Bonds. Bonds fastened to the rail by mechanical pressure may be divided into two general classes, expanded terminal and compressed terminal bonds.

Pin-expanded Terminal Bonds (Fig. 40) have their heads drilled with an axial hole, through which a tapered steel pin *d* is driven, forcing the copper outward and against the steel. This type of bond is fastened to the web of the rail.

Compressed Terminal Bonds. (Figs. 41 and 42.) There are two kinds of compressed terminal bonds, in one of which direct pressure is applied at both ends of the head, and in the other, at one end only. The first type of bond is usually applied to the web of the rail by means of a heavy screw or hydraulic press (Fig. 43) which engages the bond head and causes it to compress longitudinally and expand laterally as the pressure is applied, bringing

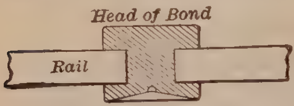


Fig. 41

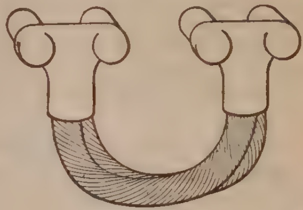


Fig. 42

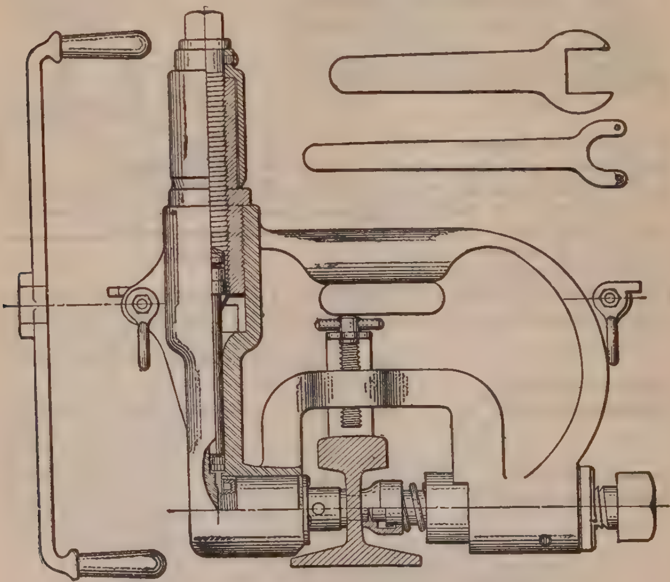


Fig. 43

the copper into firm contact with the steel and spreading the projecting end of the terminal into a button-shaped rivet-head, as shown in Fig. 41. The second type of bond (Fig. 42) is applied only to the head of the rail, the terminal lugs being set in holes therein and expanded into contact by means of hammer blows.

Exposed Versus Concealed Bonds. Whether soldered, brazed, expanded or compressed, bonds may be either exposed or concealed (Fig. 44) under the fish-plates. The former condition is preferable, if there is no likelihood of theft, as it permits inspection to be easily made. Where the bonds are exposed to theft, as, for example, on track rails unprotected by paving, concealed bonds are almost a necessity.

While concealed bonds are necessarily applied to the web of the rail, exposed bonds may be applied to the foot or head. Head bonds have the advantage of greater contact surface at the terminal studs, while foot bonds are less exposed to mechanical violence. Web bonds, unless concealed, have to be excessively long in order to span the fish-plates.

Substitutes for Bonding. Several efficient substitutes for bonding are now in use, such as electrical welding and thermit welding. (See Article 21.)

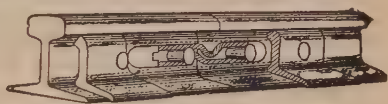


Fig. 44

Installation of Bonds. The foremost consideration in the installation of bonds is the cleanliness of the bonds and bond holes, or other adhesion surface. Unless this is secured the bonds will be electrically defective whatever their mechanical strength may be.

Soldered Bonds. The rail surface is brightened by means of a carborundum or emery wheel, and tinned, using an acid flux. The bond is then clamped in place and the rail and bond heated by means of a blow-torch, to a temperature at which the solder will melt and cause the bond to adhere firmly to the rail.

Brazed Bonds. The preliminary processes are the same as for soldered bonds except that a special clamp is used, the terminals of which are the electrodes of an electrical circuit, one being of copper and the other of carbon. The surface of the rail being previously ground bright at the point where the braze is to be made, the brass-enveloped bond terminal is pressed against the rail by the carbon electrode, the copper electrode being in contact with the opposite side of the rail. The current on passing from one electrode to the other, traverses the bond terminal and rail, the carbon becoming incandescent. The incandescent carbon (pressing the copper against the rail) quickly transmits sufficient heat at exactly the point where it is required to produce the weld.

Welding Outfit. It is claimed by the manufacturers that an average of over 100 bonds per day are readily installed by a car operating with four men, a bonder and three helpers. Such a car carries a rotary converter and transformer, with accessory apparatus. To weld an average-sized rail bond to the rail, an alternating current of about 2000 amperes at 5 volts is employed. On direct-current railways this is obtained by converting and transforming current taken from the trolley wire.

Pin-expanded Bonds. The rail is drilled, usually through the web, with or without lubricant, using some form of drill especially adapted to the service. Drilling without lubricant has the advantage of giving a perfectly clean hole, but is believed by some to cause excessive wear of the drills. It is doubtful, however, whether the small amount of oil which would be used could be kept constantly at the cutting edges, the only places where it would be useful. Dry drilling has been found successful on many railroads. If oil is used, it should be wiped out with a clean cloth saturated with gasoline, in which case the joint resistance will be increased less than 3%. Lubricants containing water are likely to cause rust, especially if the drilling gang precedes the bond installers by any considerable time. The hole having been drilled, the bond head is inserted into it and a long taper punch lubricated with grease is driven entirely through the terminal. Then a short drift pin is driven home, as shown in Fig. 40.

This type of bond requires a smaller equipment in tools and materials than most other types and does not necessitate the use of any apparatus which obstructs the track and would thereby endanger traffic.

Compressed-Terminal Web and Foot Bonds. The drilling having been performed as for a pin-expanded bond, and the bond heads inserted into their respective holes, a screw or hydraulic compressor, as shown in Fig. 43, is applied at both ends of the bond head, the conical point of the press fitting into the conical depression of the bond. Pressure is applied, either until a collar on the ram touches the rail, or until the head of the bond acquires the proper shape. Where no collar is used the point of the press (if of the screw type) sometimes cuts into the bond head; this may be avoided by placing a small amount of flake graphite mixed with oil in the depression of the bond head.

Compressed-Terminal Head Bonds. A four-spindle drill is used to drill four holes simultaneously in the rail heads. It is important to avoid drilling the holes too deep lest the copper should not touch bottom and therefore be unable to expand laterally. If, on the other hand, the hole is too shallow, expansion will occur too soon.

Tests of Bonds after Insulation. Every rail in service should be periodically tested and a complete record of the tests kept. The frequency of the tests will depend upon local conditions, once in 9 to 12 months being an average.

Resistance Test. The usual method of testing is to measure the drop of potential across the bonded joint and find simultaneously the length of con-

Resistance of T-rails, A.S.C.E. Standard Section (20° C.)

(Full Cross-section)

Weight, pounds per yard	Cross-section, square inch	Area, millions of circular mills	Specific resistance 12.5 times that of copper		Specific resistance 8 times that of copper *	
			Ohms per 1000 ft.	Ohms per mile	Ohms per 1000 ft.	Ohms per mile
70	6.9	8.77	0.0148	0.0779	0.00944	0.0499
75	7.5	9.45	0.0138	0.0729	0.00884	0.0467
80	7.8	9.9	0.0131	0.0689	0.00835	0.0441
85	8.3	10.5	0.0122	0.0645	0.00781	0.0413
90	8.8	11.2	0.0115	0.0609	0.00738	0.0390
95	9.3	11.8	0.0109	0.0570	0.00701	0.0370
100	9.8	12.5	0.0104	0.0547	0.00664	0.0350

* To find the resistance of rails of any specific resistance x referred to copper as unity multiply these resistances by x and divide by 8.

tinuous rail in which the same drop occurs, that is, the "equivalent" length of the bonded joint. Several ingenious instruments have been devised for making this comparison with ease and accuracy.

Mechanical Strength. The mechanical adhesion of soldered bonds may be tested by means of a lever as shown in Fig. 45. It may be used as the terminals are cool. The operation of testing consists simply in submitting each bond terminal to a predetermined pull. A properly soldered bond should stand a shearing force of 1200 lb. per square inch of contact. Calling S this shearing force per square inch, A the square inch of contact, and P the pull, as registered on the balance, then

$$P = \frac{ASl}{L}.$$

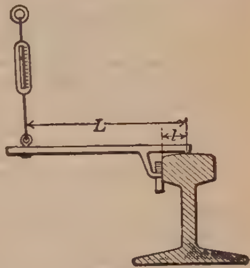


Fig. 45

Rebonding. The resistance at which a joint should be rebonded depends upon how much potential drop is permissible in the tracks, and upon the relative cost of the energy loss and the cost of rebonding. The equivalent length, calculated on this basis, ranges from 2 to 12 ft., with an average of 6 to 7 ft.

CARS AND LOCOMOTIVES

13. Cars

City cars vary in length from 42 to 50 ft., the average of twenty typical city cars being 45 ft. 6 in. over the bumpers. The width varies between 8 ft. and 8 ft. 10 in., the average of twenty typical cars being 8 ft. 4 in. Truck centers vary from 19 to 24 ft., averaging 21 ft. 10 in. The number of seats varies from 32 to 56, the average of twenty typical cars being 47.

The standard height of couplers for city cars, measured from top of rail to center of coupler, is 1 ft. 8 in. The standard height of bumpers is 2 ft. 7 in. from top of rail to top of bumper. The standard height of platform is 2 ft. 7 in. from top of rails.

Recent trend has been toward the drop platform type with rear entrance, and front exit, as it is one of the safest types and seems ideal for quiet loading and unloading of passengers. The center door type has also found favor. These types lend themselves readily to the pay-as-you-enter system of fare collection.

Another design of car which facilitates rapid loading and unloading employs front entrance, with center exit. Passengers having exact fare stay in the front of the car while passengers desiring change pay their fare to the conductor who is stationed in the center of the car and pass to the rear of the car from which they leave at will.

Interurban Cars vary in dimensions from those of city cars to those of standard steam passenger cars, depending on the class of service, speed and length of run. Interurban cars vary in length from 40 to 75 feet, with corresponding weights of from 25 000 to 75 000 pounds.

Suspension of Motors. Motors, on double-truck cars, are hung between an axle and the transom. A lug cast on the side of the motor frame away from the axle rests on the transom either with or without intervening springs. About 60% of the weight is carried directly on the axles.

Maximum Equipment Line. The maximum equipment line for cars on third rail systems has been standardized as follows by the American Electric Railway Engineering Association and other interested bodies.

Points Determining Maximum Equipment Line

Height above top of rail, inches	Distance out from gage line, inches	Height above top of rail, inches	Distance out from gage line, inches
6	0	24	9
6	2-1/2	25-7/8	10-7/8
20	2-1/2	31	10-7/8
24	4-11/16	34-1/4	24

Trucks. Cars not more than 20 ft. long can be satisfactorily carried on single trucks having a wheelbase of about 7 ft. Longer cars should have two trucks of the swivel type, having a wheelbase of from 4 ft. to 6 ft. 6 in., the shorter wheelbase being used where there are sharp curves. Maximum traction trucks have unequal-sized pairs of wheels, the bolster being near the larger pair, to the axle of which the motor is geared. This has the effect of throwing from 70% to 85% of the total weight on the driving wheels, although only two motors are used on the car, an especial advantage where high acceleration is desired.

Wheels. Electric railways use the standard gage of 4 ft. 8-1/2 in. A typical standard form of wheel flange is shown in Fig. 46.

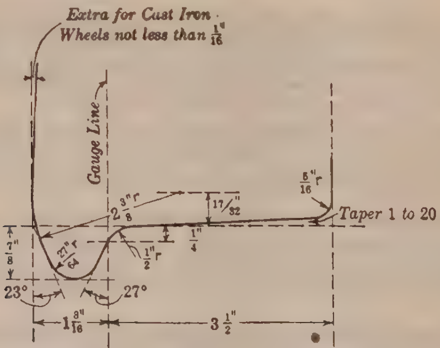


Fig. 46. Standard Sections of Wheel Flanges

14. Locomotives

General. Electric locomotives, unlike steam locomotives, can be made of practically any desired power capacity without excessive complication and without increase in the number of operators. This arises from the absence of boilers and the possibility of controlling a number of motors or even a number of complete locomotive units as if they were a single unit.

Rating. The rating of electric locomotives is usually expressed in terms of the weight on drivers, the nominal 1-hour tractive effort, the continuous tractive effort and corresponding speeds.

(a) Weight on Drivers. The weight on drivers, expressed in pounds, shall be the sum of the weights carried by the drivers and of the drivers themselves.

(b) Nominal Tractive Effort. The nominal effort, expressed in pounds, shall be that exerted at the rims of the drivers, when the motors are operating at their nominal (1-hour) rating. (See Art. 15.)

(c) Continuous Tractive Effort. The continuous tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers when the motors are operating at their full voltage continuous rating. (See Art. 15.)

In the case of locomotives operating on intermittent service, the continuous tractive effort may be given for one-half or three-fourths voltage, but in such cases the voltage shall be clearly specified.

(d) Speed. The rated speed, expressed in miles per hour, shall be that at which the continuous tractive effort is exerted.

Classification. Locomotives may be classified as follows, with reference to trucks:

1. Rigid wheelbase.
   (a) Without leading and trailing trucks.
   (b) With leading and trailing trucks.
2. Separated bogie truck (the trucks being connected through the upper frames as in a car):
   (a) Symmetrical.
   (b) Unsymmetrical.
3. Articulated (wherein the two trucks are hinged back to back).

All of these types of locomotives are practically steady at speeds under 40 mph., but above this speed marked differences appear: the steadiest riding machines, according to G. Gibbs, having high centers of gravity and long unsymmetrical wheelbases. Considerable difference of opinion prevails in regard to proper mechanical design for high speeds.

Transmissions. Motors are connected to locomotive drivers by the following forms of transmission:

(a) Gearless, the motor armatures being mounted direct on the axles.

(b) Gearless quill, each motor armature being mounted on a hollow shaft or quill which surrounds the axle, but is free with respect to it. At each end of the quill is a disk having projecting arms which fit into spaces between the spokes of the driving wheels. The arms are connected to the driving wheels by springs which tend to keep the arms centered between spokes but allow sufficient play to relieve the excessive starting stresses in the armature.

(c) Direct gears, the motors being mounted either between or over the axles, a motor having a pinion engaging a gear wheel on the locomotive axle.

(d) Quilled gears, the gear wheel being on a hollow shaft or quill which engages the drivers through springs, thereby relieving both the armature and gears of excessive starting stresses.

(e) Cranks and Scotch yoke, in which two motors are connected together and have crank shafts which operate three sets of drivers.

(f) Crank and countershaft, in which the motors are located above the tracks and are connected through a countershaft to side rods joining the drivers.

Control. Practically all modern electric locomotives have multiple-unit control, so that two or more locomotives may be coupled together under the control of one operator. Indeed, some locomotives consist of two sections —

each of which is virtually a separate locomotive — coupled together permanently.

Weight. The weight per driver axle for high speed locomotives should not exceed 40 000 lb. with ordinary track, and 50 000 lb. with very good track, and in slow-speed service more usual weights are 15 000 to 20 000 lb. per driver axle. European practice indicates a maximum of 35 000 to 40 000 lb. per axle.

The weight in pounds per foot of total wheel base usually varies from 4500 to 7500 lb., but is occasionally as high as 11 000 lb.

The following weight analysis by E. P. Burch (Electric Traction for Railway Trains) is based upon data for about twenty representative American and European locomotives.

Weight Distribution, Electric Locomotives

Per cent

Distribution of weight	Direct-current		Three-phase		Single-phase		Motor-generator
		Ave.		Ave.		Ave.	Ave.
Mechanical parts.......	50–72	68	48–76	51	46–59	58	43
Motor................	20–27	24	26–40	30	26–36	27	30
Transformer...........	0	0	0–10	10	8	7	6
Other electrical parts....	5–10	8	7–10	9	7–11	8	21
Hp. 1 hr. per ton, about.	....	16	....	18	....	14	8

The mechanical parts of direct-current locomotives are high in percentage because the total weights are low. Three-phase motor weights appear high because European designers use light frames.

15. Motors and Controllers

Selection of Motor. The following information, relative to the service to be performed, is required, in order that an appropriate motor may be selected.

(a) Weight of total number of cars in train (in tons of 2000 lb.) exclusive of electrical equipment and load.

(b) Average weight of load and durations of same, and maximum weight of load and durations of same.

(c) Number of motor cars or locomotives in train, and number of trailer cars in train.

(d) Diameter of driving wheels.

(e) Weight on driving wheels, exclusive of electrical equipment.

(f) Number of motors per motor car.

(g) Voltage at train with power on the motors — average, maximum and minimum.

(h) Rate of acceleration in miles hour per second.

(i) Rate of braking (retardation in miles per hour per second).

(j) Speed limitations, if any (including slowdowns).

(k) Distances between stations.

(l) Duration of station stops.

(m) Schedule speed including station stops in mph.

(n) Train resistance in pounds per ton of 2000 lb. at stated speeds.

(o) Moment of inertia of revolving parts, exclusive of electrical equipment.

(p) Profile and alignment of track.

(q) Distance coasted as a per cent of the distance between station stops.

(r) Time of layover at end of run, if any.

Rating of Motors. The *one-hour rating* of a railway motor shall be the output at the motor shaft measured in horsepower (or kilowatts) which the motor can carry for one hour on stand test, starting cold, at its rated voltage and frequency (in case of alternating-current motor) with the ventilation system as in service without exceeding the temperature limits given in the table of limiting temperature rises.

The *continuous rating* of a ventilated railway motor shall be the output at the motor shaft measured in horsepower (or kilowatts) which the motor can carry for an unlimited period on stand test, at its rated voltage and frequency (in the case of alternating-current motor), with the ventilation system as in service without exceeding the temperature limits given in the table. Direct-current ventilated railway motors may also be given a continuous rating in amperes at full, three-fourths and half-rated voltage.

The *continuous rating of totally enclosed* direct-current railway motors shall be given in amperes at three-fourths, and half rated voltage.

In the absence of any specification as to the kind of rating the *one-hour rating* shall be understood.

The ratings of a *field-control direct-current motor* shall relate to its performance with the field connection which gives the maximum motor rating. (This is usually the minimum field.) Each section of the field winding shall

Limiting Temperature Rises

Item		Type of enclosure	Method of temperature determination to be employed	Limiting temperature rise, deg. C.			
				One-hour rating		Continuous rating	
				Class A insulation	Class B insulation	Class A insulation	Class B insulation
1	Armature and field winding	Ventilated	Resistance	100	120	85	105
			Thermometer	80	95	65	80
		Totally * enclosed	Resistance	110	130	95	115
			Thermometer	90	105	75	90
2	Cores and mechanical parts in contact with or adjacent to insulation	Ventilated	Thermometer	80	95	65	80
		Totally * Enclosed	Thermometer	90	105	75	90
3	Commutators	Ventilated	Thermometer	95	110	80	95
		Totally * enclosed	Thermometer	105	120	90	105
4	Miscellaneous parts (such as brush holders, brushes, pole tips, etc.) other than those whose location is such that they may injuriously affect the adjacent insulation may attain such temperatures as will not be injurious in any other respect.						

* The temperature rises of totally enclosed motors are taken as 10° C. higher than the ventilated motors since the cooling on stand test will be inferior to that obtained in service.

be adequate to perform the service for which it is designed without exceeding specified temperature rises.

Voltages. Direct-current motors are usually made for line voltages of 500, 550 or 600 volts and sometimes for 1200 volts or somewhat more. On 2400-volt lines, it is usual to have the motors in pairs connected permanently in series.

Alternating-current single-phase motors, are usually made for 400 to 500 volts, the required voltage being obtained by transforming down the line voltage by means of a transformer or auto-transformer on the train.

Weight. Motors vary in weight from 30 lb. per horsepower (40 lb. per kilowatt) for the largest sizes (200 hp.) to 70 lb. per horsepower (93 lb. per kilowatt) for the smaller sizes (35 hp.), the ratings being the nominal.

Controllers. The speed of direct-current motors is controlled by connecting resistance in series with the motors, by connecting the motors at first in series and then in parallel, and sometimes by varying the strength of the fields. Alternating-current motors are similarly controlled, except that auto-transformers are used instead of resistances.

The direction of rotation, in both kinds of motors, is changed by reversing the current in *either* the fields *or* armature, this being usually accomplished by rotating an auxiliary handle of the controller.

The type *K* controller is largely used for light weight cars, and consists of an operating handle which moves a cylindrical drum with projecting contact pieces which come in contact with stationary fingers. The first three points correspond to accelerating steps, by means of which the resistance in series with the motors is gradually cut out. The fourth step, full series, gives about half speed and is a continuous running point. The following steps restore the resistance and put the motors in parallel. The last step leaves the motors in parallel without any resistance.

Multiple-unit control is used on large cars, especially where the combined motor capacity exceeds 300 hp. The equipment consists of a small master controller which enables a comparatively weak current to operate contactors large enough to make the necessary changes in the circuit, as in the case of type *K* controllers. The master controller on any car will operate the contactors on all cars, if a jumper cable be run from car to car connecting together all the control circuits.

The weight of control equipment of type *K* varies from 1200 to 2250 lb., and of multiple-unit equipment from 2800 to 3200 lb.

16. Brakes

General Principles. Practically all existing brakes make use of the frictional adhesion between the wheels and track, and between the brake shoes and wheels. The frictional adhesion between wheels and track varies from 15% to 30% of the weight on the wheels. An adhesion of 15% applies to normal track and the higher values to sanded track. This refers to rolling friction; if the wheel begins to slide, the coefficient of friction drops. For this reason braking must be such as to to allow the wheels to slip. The frictional adhesion between brake shoes and wheels is given by the formula,

$$h = \frac{1 + 0.000\,472\,l}{1 + 0.00\,2390\,l} F,$$

where h = coefficient of friction after elapsed distance l;
l = distance wheel has traveled in frictional contact with the brake shoe;
F = coefficient of brake shoe friction at speed and pressure at which brake shoe was applied at the beginning of the distance l

$$= \frac{0.382}{1 + 0.02933\,S}, \text{ where } S = \text{speed, mph.}$$

The braking retardation depends upon the various factors involved, approximately as indicated by the following formula:

$$a = 0.01098\,k\,(Ph + fW);$$

where a = retardation, mph. per second;
k = ratio of linear inertia to total inertia of train;
P = total braking pressure applied normal to wheel treads by brake shoes, pounds;
h = coefficient of brake friction;
f = train resistance, pounds per ton weight of train;
W = weight of train, tons.

If we neglect rotational energy and train resistance

$$a = 0.01098\,\frac{R}{W},$$

where R = total retarding force, in pounds.

The usual rate of retardation for multiple-unit electric trains is from 1.5 to 2.0 mphps., and on street cars from 2 to 2-1/2 mphps.

Construction. Any brake which depends upon the friction between the wheels and brake shoes is composed of four parts; the shoes, the truck rigging, the foundation rigging and the source of braking force.

Brake Shoes. In order to avoid excessive costs for the renewal of brake shoes, the weight should be limited so that no individual shoe should weigh more than 24 lb. To avoid an excessive loss of weight in scrap, the minimum weight should be 20 lb. The cast iron should be closely granular, of uniform texture and with the combined and graphitic carbon evenly balanced. The graphitic carbon should be in the form of nodules rather than flakes.

Wear averages 3.75 to 6.5 lb. per 1000 wheel-miles and the minimum scrap weight should be 6-1/4 lb. per shoe; at 5-1/2 lb. there is danger of cutting the head.

Ordinarily, brake shoes are placed to bear on the inside of the truck frame; that is, between wheels. With this arrangement, it is possible, by varying the angularity of the hanger link, to introduce a force which will equalize, to any desired degree, the transfer of weight from the rear to the forward axle.

Truck Rigging. The braking force must be distributed equally between the brake shoes, or there will be danger of sliding the wheels. Separate levers are used for each shoe. On double-truck cars, which must travel around sharp curves, the bar connecting the brake levers is made of circular form, and is then known as a "radius bar"; and the brake pull rod is connected to it through, a roller working in a clevis, which allows the brakes to act in spite of the swiveling of the truck.

Foundation Brake Rigging. The pull rods which pull on the radius bar rods are each attached to one end of the cylinder levers, the centers of these levers (which are normally about parallel) being joined by a rod. The other ends of the cylinder levers are connected to the piston and the slack adjuster, respectively.

The purpose of the slack adjuster is to pull back the piston without loosening the brakes in case it travels so far as to unduly reduce the cylinder pressure. This is usually accomplished by causing the excess piston motion to operate a device which shifts the fulcrum of one of the cylinder levers. The normal piston travel is 8 in. on standard equipment.

Hand Brakes. It is usual to provide all cars with hand brakes, even if also equipped with air brakes. Hand brakes communicate their motion to the foundation rigging by means of a chain which winds on the brake staff at one end and is attached, through a multiplying lever, to the pull rods. In high-ratio hand brakes, the bottom of the brake staff carries a gear, the chain connection being made through the meshing gear.

Air Brakes. Air brakes have the advantage of being capable of quicker application than hand brakes, and they save power both by permitting more coasting and by making sure that the brake shoes do not rub when the car is running, a condition which frequently occurs with hand brakes due to the desire of the motorman to be able to apply the brakes as quickly as possible.

The maximum pressure of the brake shoes on the wheel may be definitely limited by proper design and wheel skidding prevented. Air brakes also relieve the motorman of severe manual strain.

There are two kinds of air brakes: the " straight " and the " automatic." In the former, the pressure of a main reservoir acts directly on the foundation rigging; in the latter, the pressure comes from an auxiliary reservoir, the main air pressure being used to control the admission of air to and from the auxiliary reservoir and to release the air from the brake cylinder to the outside atmosphere. In " straight " air brakes, the control consists essentially of a valve which can connect the cylinder to the main reservoir, can disconnect the cylinder yet allow it to retain the air, or can release the air from it. This system is used for single car operation.

In long trains, the air has so far to flow from the reservoir on the forward car or locomotive that the brakes on the rear cars are applied later than on the front cars. This may impose severe stresses in the draw bars and may cause the train to break in two. This trouble does not exist with the " automatic " air brake. In this system, under normal running conditions, the auxiliary reservoirs on each car are fully charged to the pressure of the train pipe and the brake cylinders are open to the atmosphere. To apply the brakes, the train pipe pressure is reduced, an air operated " triple valve " automatically disconnects the auxiliary reservoir from the train pipe and connects it with the brake cylinder, meanwhile closing the cylinder exhaust. Where rapid brake applications are desired on long trains, the air-operated triple valve is not fast enough, and electrical train lines are now being used for this purpose.

Inspection of Air Brakes. The American Electric Railway Association gives the following instructions for inspection: Start the air pump to its maximum capacity; see that brake-valve handle is in release position and where automatic air is used see that gages indicate 20 lb. difference between train line and auxiliary. If they do not, the governors need to be reset. Apply brake to show reduction of 40 lb. Place brake-valve handle to lap position; see that air gage operates properly and that no leaks are in or around the brake-valves or pipes leading thereto; examine all pipes, reservoirs, triple valves, cylinders, etc., while brake is set and see that none are leaking and that brake does not release while the brake-valve handle is in lap position. If the cylinder piston has a travel of more than 5 in. an adjustment of brakes is necessary. Where slack adjuster is used, see that it is placed to its minimum of travel before any adjustment of brakes.

Inspect all shoes and see that they are in alignment with the wheel so that none are broken and renew those that will not give sufficient wear until the next inspection.

In renewing brake shoes put shoes of same thickness on opposite wheels, be they either old or new.

Examine all keys, bolts, pins, beams, turn-buckles, rods, etc., and lubricate where necessary.

Compressors. Independent motor-driven air compressors are most commonly used for air brakes, and are usually directly connected. Compressors should be of a size such as will not be required to operate more than one-third the time.

17. Car Heating

General. Street cars should be heated to a temperature of 55° to 60° F. which is found to be comfortable for passengers in street attire. Suburban and interurban cars, which have longer runs, should be heated to between 65° and 70° F. in order that the passengers may be comfortable without their wraps.

Systems. Three systems of heating are in use on modern railways. The steam boiler, hot water furnace, and the electric heater. Steam is used on locomotive trains, hot water for some interurban cars making long runs and electric heaters for urban cars and most interurban cars. In locomotive trains, the steam boiler is oil-fired and built into the locomotive or carried on a tender behind the locomotive.

Electric Heating. The electric system of heating, although more expensive to operate than the others, finds the greatest favor for urban and interurban service and has the advantage of good distribution of heat, cleanliness, ease of regulation, low fire hazard and no attendance.

Electric heaters are all equally efficient in regard to the amount of heat developed but may differ in respect to durability and cost of maintenance. They are usually made of resistance wire wound on porcelain forms, but sometimes the wire is embedded in insulating material. The latter type has greater heat storage capacity than the former.

The power required for car heating is as follows:

Power for Car Heating

Length of car, feet	Average kilowatts for heating	
	Average conditions	Severe conditions
14 to 20	3.5	4.
20 to 28	4.5	5.5
28 to 34	5.5	7.5
34 to 40	7.5	10.5

CAR BARNS AND OTHER BUILDINGS

18. Car Houses and Inspection Sheds

General. Car houses are used for the storage and inspection of street and interurban cars and on small railways are combined with the repair shop. On electrified railroads, car houses are for inspection only, as out-of-door storage is the usual practice.

When about to design a car house, the engineer should first acquaint himself with municipal and underwriters' regulations and, conforming with these, should provide room for car storage and inspection, administration offices, line department, road department, car employees' lobby, car sign storage, sand drying and storage, salt storage, oil storage, wash-room and toilets.

Concrete foundations are used for walls and columns, and the walls are usually made of brick; but where cheapness is essential, a mill frame with 2-in. cement curtain wall may often be used. Heavy wooden columns and roof trusses are generally used; although cast-iron columns are favored by some designers. Wooden roofs covered with felt, pitch and gravel are usual, and they are provided with copper flashings and counter-flashings of either copper or lead.

Floors are made of concrete with cement finish, and are sloped to provide ample drainage. It is especially important to keep the inspection pits clear of water.

Inspection Pits. Inspection pits are placed between the rails and are usually 4 ft. 6 in. deep below the top of rails. The track rails are carried on wooden stringers (usually 10 by 12) on the masonry side walls of the pit. More modern practice in large car houses is to have a basement as deep as an ordinary pit but extending under all the tracks. In this case the track rails are supported on stringers or T-beams resting on posts.

Tracks. Car tracks should be spaced not less than 11 ft. between centers where there are no posts between tracks, and not less than 13 ft. where there are posts. Ordinary T-rails are used. The dead ends of the car-house tracks should be provided with bumpers. The trolley wire should be at least 16 ft. 6 in. above the top of rail in order to avoid injuring the trolley springs by keeping them in undue compression. It is usual to support the trolley wire on a flat board to prevent the trolley rising and striking the building structure if it should leave the wire.

Tracks at Entrance. The tracks at the entrance to the car house should be be designed to facilitate the rapid entrance and departure of cars with the least interruption to traffic on the main tracks. This is sometimes accomplished by having an extra track in front of the car house and running all storage tracks into it; the extra track being connected at either or both ends to the main tracks.

Heating. Car houses are best heated by means of a blower system where the air is blown over steam coils and through the building. The heating plant should be either in a separate building or enclosed in fireproof walls.

Doors. Where there is sufficient clearance, swinging and sliding wooden doors are preferred; but where space is restricted, rolling steel doors are used for large openings.

Lighting may be either by incandescent lamps or mercury vapor lamps, the former being most usual. Lamps should be fed, wherever practicable, from a regular lighting system, as the voltage fluctuations of the trolley or third rail circuit are usually excessive. Where, however, the lights must be fed from the traction circuits, the requisite number of lamps must be grouped together in series. The pits or basements should be well lighted.

Painting. Car-house walls are usually covered with cold water paint except for distances of several feet from the ground, where oil paint is to be preferred.

Fire Risks. Due to the great value of cars and the loss of revenue which would result from their destruction, it is important to make the fire risk as low as possible by the adoption of the following precautions:

(1) Use of automatic sprinklers.
(2) Provision of two sources of water supply such as mains and tank.
(3) Division of building by fire-proof walls. The underwriters stipulate that no section shall contain cars to the value of over $200 000.
(4) Avoiding proximity to inflammable buildings.
(5) Having ample provision of water pails, sand pails, chemical extinguishers and short hoses, the last being usually 2-1/2 in. in diameter and provided with 1-1/8-in. nozzles.
(6) Locating car house near a fire station.
(7) Building tracks on a grade so that cars may be easily pushed out by hand.
(8) Having numerous auxiliary fire alarms.
(9) Consulting underwriters when planning.

The Operating Force should be located at the front of the house and starters should be in a position to see all incoming and outgoing cars.

19. Repair Shops

General. Car repair shops are used for the repair and renovation of cars and are generally similar in construction to car houses, being often combined therewith. The details of building construction, heating and lighting given under Car Houses apply equally well to car shops, except that only about half

Space Distribution in Car Repair Shops

No.	Department	Work of department	Per cent of floor area occupied: Departments with car tracks	Per cent of floor area occupied: Departments without car tracks
1	Repair shop.....	General repair work on cars:		
		(a) Main repair shop..........	15	....
		(b) Truck repair shop.........	....	5
2	Machine shop...	Metal work on parts, including slotting commutators..........	....	10
3	Blacksmith shop.	Forgings, welds, etc., on parts....	....	4
4	Carpenter or erecting shop.	Heavy repairs on car bodies......	17	....
5	Store-room and offices.	Stock storage exclusive of heavy material....................	....	15
6	Electrical shop..	Electrical repairs to armatures, etc., except as cited under No. 2.	....	5
7	Wood mill......	Machine work on wood.........	....	8
8	Paint shop and wash-rooms.	Painting of cars and storage of painting materials, etc.:		
		(a) General paint shop..........	18	....
		(b) Cabinet room, varnish room..	....	3
	Total.......	....	50	50

the floor area need be equipped with tracks, the remainder being devoted to departments where unassembled parts are handled. Where tracks are installed, the usual spacing is from 15 to 16 ft. between centers.

Heating. While large repair shops are usually heated by the blower system (see " Car House "), an exception must be made in the case of the paint shop where direct radiation is preferred. Small shops are usually heated directly by steam or hot-water radiators.

Space Distribution. The table on 2153 gives the principal subdivisions of a typical repair shop with the average space distribution, the actual space required being from 140 to 240, averaging about 200 sq. ft. of shop floor per car owned, exclusive of yard space or transfer tables. Other departments, not included in the table either because they are affected by the amount the shop depends upon outside aid or because of their smallness, are: Dry kiln, lumber store, boiler room, brass foundry, pattern store, indoor transfer table, locker room and toilets.

Means for Moving Cars. Cars are lifted by any of the following means: Traveling cranes, jacks, hydraulic lifts, screw hoists and chain hoists. Traveling cranes are generally used except in the smallest shops.

Transfer tables are preferred to ladder tracks for moving cars between departments.

Proportion of Cars in Shops. The percentage of cars in the shops varies from 8 to 12 depending upon the quality of the equipment, and its usage.

Storage Space around Shops. Shop buildings should be surrounded with sufficient ground to serve for storage of trucks, lumber, scrap material, etc. The total area of land required for shops and storage varies from 1 to 2 acres per 100 cars owned, depending upon whether all departments are concentrated in one building or scattered among several.

SIGNALS AND TRACKS

20. Signals

General. The purposes of signals are to promote safety and to increase line capacity.

In addition to audible signals such as bells, whistles, and torpedoes, movable visible signals, such as lanterns, flags and fuses, single-aspect fixed signals such as slow, stop, whistle and drawbridge signs, electric railways use two-aspect or three-aspect fixed signals, such as switch targets, train order signals, interlocking signals and block signals.

Signal Indications. The two- or three-aspect fixed signals are either semaphores or lamps, or more commonly, a combination of both. Where semaphore signals are used, they are usually arranged to indicate three positions in the upper right-hand quadrant. These positions are, horizontal for " stop," 45 deg. for " proceed with caution " and vertical for " proceed."

Where semaphores have pointed ends, the horizontal position indicates " stop and proceed " and 45 deg. indicates " proceed, next signal at stop."

Where colored lights are used, a large variety of indications may be obtained, a simple set of which would be as follows:

$$\left.\begin{matrix}\text{Red}\\\text{Red}\\\text{Red}\end{matrix}\right\}\text{Stop and stay}$$

Red Red Red Yellow	Stop and proceed
Yellow Yellow Yellow	Proceed, next signal at stop
Green Green Green	Proceed

A " stop " signal combined with a yellow flag indicates " proceed under control, prepared to stop short of any obstructions."

Location of Poles. The standard location for a signal pole is 7 ft. 9 in. from the center of the track to the center of the pole, with semaphore 15 ft. from top of rail to bottom of arm. The wooden blade of the semaphore is 3 ft. 6 in. by 7 in. and from top to vertical projection of center of rotation when horizontal is 3 ft. 11-1/2 in., the center of rotation being 16 ft. 7 in. from top of rail.

Lenses. (a) For high-speed interurban service, a lens not less than 8-3/8 in. in diameter should be used on all light signals operated by continuous track circuits.

(b) For moderate speed roads, a lens not less than 5-3/8 in. in diameter should be used where light-signals are operated by trolley contact or other end-set devices.

Manually Operated Signals. Signals employing a wire circuit are used on the shorter interurban roads and on some city roads. A typical system is shown in Fig. 47 in which a double-throw electric switch is installed on a pole

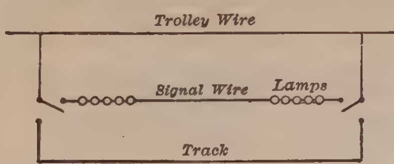


Fig. 47. Manually Operated Signal System

at each end of the block, so as to be accessible to the motormen. When a car enters the block at either end, the conductor or motorman throws the switch to the opposite position, thereby lighting the lamps at both ends. (If there is already a car in the block, the lamps will not light, as the motorman coming the other way will have thrown up his switch.) When he leaves the block he throws up the switch, thereby extinguishing lamps at both ends. The man on the next car puts down the switch and if it lights he proceeds. A light always means safety, whereas no light means unsafe.

A development of this is a semaphore in place of lamps and automatic throwing of the switches by mechanical trip or magnet on the car.

Manual Block System. This is similar to the system used on steam roads, wherein block signals are operated manually upon information received by telephone or telegraph.

Controlled Manual Block System. This is similar to the simple manual block system except that the signals at the end of each block are interconnected electrically, so that a clear signal cannot be obtained without the cooperation of the signalmen at each end of the block.

Token System. This is a manual block system applied to single-track lines. In this system no train is allowed to occupy a block section without obtaining possession of a tablet, staff or other characteristic token. These tokens are taken from signals at the ends of blocks and form part of an interconnected electric system, such that but one token can be removed at a time from a pair of signals, and until the token has been replaced in one signal, no token can be taken from the other. The possession of a token is authority to proceed.

Where it is desired to admit two trains to a block, divided tokens are used, each train taking one, and no further tokens can be withdrawn until all of the pieces of the first are replaced.

Automatic Block System. This is a system in which signals controlled by trains govern the entrance to each block, thereby affording both head and rear protection.

Protection is obtained either by a sectionalized trolley wire, sectionalized track rails or short insulated sections of track rail at the ends of blocks.

Due to the use of the track rails as part of the traction current circuit, some means must be adopted to keep the signal and traction currents separate. On direct-current railways, this may be accomplished either by bridging the insulated rail joints by inductance bonds, which permit the traction current to flow and choke back the alternating signal current, or by confining the traction current to one rail, and the signal current to the other, the latter system being favored where the track work is complicated, as in yards, junctions, etc. On alternating-current railroads, it is usual to reserve one track rail for signal purposes.

Insulating Rail Joints. Insulating joints at the ends of blocks are made by inserting a piece of insulating fiber between the rail ends, insulating bushings around the bolts and a plate of insulating fiber between the joint plate and rails, the former being prepared especially for the purpose.

Effect of Preservatives in Ties. Zinc-treated ties when new tend to short-circuit the rails, due to the conductivity of the zinc salts. Circuits 2000 ft. long may be operated successfully with 50% of ties so treated, but 5000-ft. circuits with all new zinc-treated ties will not operate.

Crossing Protection. Automatic highway crossing protection is usually accomplished by means of a gong (200 cycles per min.) operated by a " setting switch " placed in the trolley wire at the approach to the crossing and a " restoring switch " at the crossing.

Dispatcher's Signal Systems. These are for the operation of special signals to control train movement from a dispatcher's office.

One system has a pendulum of a different length at each signal, and a duplicate of each of these in the dispatcher's office. When the dispatcher starts one of his pendulums, it makes and breaks an electric circuit, thereby periodically energizing electromagnets near all the line pendulums. Only that pendulum which is synchronized with it starts into motion and it trips a release which both sets the signal and closes a sounder circuit in the dispatcher's office, thus indicating that the signal has been set.

Standard Practice. The reports of the Joint Committee on Block Signals for Electric Railways, American Electric Railway (Engineering) Association, give full details of actual and recommended practice. See proceedings American Electric Railway Engineering Association and A.E.R.E.A. Engineering Manual. The standards cited in this article are those of the above association.

21. Track

(By Walter Loring Webb)

General. Track for interurban electric roads is identical with that for steam roads of equal carrying capacity except that the rails must be so effectively bonded as to facilitate the return of the current to the power house with minimum power loss and electrolytic effect. Urban tracks have the additional difference of requiring high rails to comply with paving and street traffic requirements.

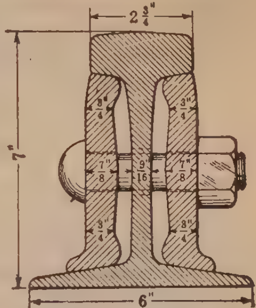


Fig. 48. Shanghai Rail

Rails. Some of the forms of high rails together with suitable rail joints are shown in Figs. 48, 49 and 50. All such rails, being embedded in pavement and prevented from lateral displacement, are laid with close joints, no allowance being made for temperature changes. Being buried in the pavement the range of temperature is less than if fully exposed to the atmosphere, and temperature changes merely produce a harmless tension or compression in the metal. This permits the use of **welded joints**, which have the advantage of greater permanency, less cost for maintenance and better conductivity.

Thermit Process: A mixture of powdered aluminum and oxide of iron is placed in a crucible and ignited. The chemical reaction develops a heat of about 5000° F. The oxygen combines with the aluminum, and the iron becomes a molten low-carbon steel. The molten iron sinks to the bottom of the crucible. An iron pin, covered and protected by an asbestos washer, forms a plug for a

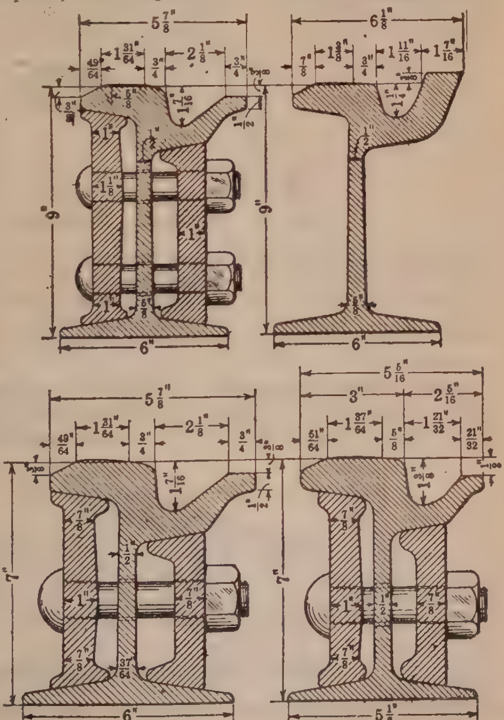


Fig. 49. Girder Rails

hole in the bottom of the crucible. When the iron is thoroughly melted, the pin is struck up so that the molten metal escapes from the bottom and pours into a mold which has been made surrounding the joint. The intense heat welds the ends of the rails together and to the molten iron surrounding them, producing a solid welded joint.

Jacobs process, as used by the Lorain Steel Co., is applicable only to new work. A flux is interposed between the ends of the rails; a comparatively small

Fig. 50*a*.

current of electricity starts chemical action in the flux which develops intense heat and heats the ends of the rails above the melting point of steel, the rail ends being forced together by hydraulic pressure. While the end of the rails are being heated, they are surrounded by the flux, which prevents air from oxidizing the rail ends and thus permits a clean strong weld.

The **bar weld process**, see Fig. 50*b*, as used by the Lorain Steel Co., welds two plates 18 in. long by 3 in. wide and 1 in. thick to the webs of the rails.

Fig. 50*b*. Standard Bar for Welding

Each plate has three raised bosses on the surface, the bosses being the only portions of the plate actually welded to the rail web. The plates are pressed against the web with a pressure as high as 35 tons, while a current which may amount to 25 000 amperes runs through the joint. The mechanism for making such joints is carried on a car and the current taken directly from the trolley wire.

Repair of Broken-Down Joints. A reinforcement of the ends of the rails at a joint may be made by placing a welded chock under the head for support, as shown in the figure. See Fig. 50*c*. In case the ends of one or both rails

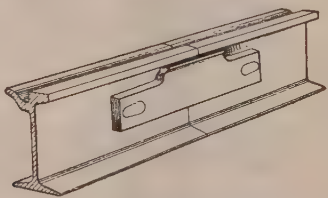


Fig. 50*c*. Welded Joint with Head Support

are broken down, it may be necessary to cut out a foot or more of old rail, replace it with new rail and make an extended joint as shown. By welding a chock under the head at each end of the short piece, the rail ends are secured against breaking down. By such means the life of an otherwise worn-out track may be extended many years. See Fig. 50*d*.

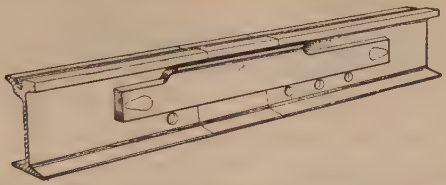


Fig. 50*d*. Welded Joint with Head Support of Special Length

Track Gage. The convenience of operating standard-gage steam railroad equipment on electric roads has resulted in the almost universal adoption of standard gage (4 ft. 8-1/2 in.) for electric roads. Philadelphia (5 ft. 2-1/4 in.) and St. Louis (4 ft. 10 in.) are the most prominent exceptions to this rule. The flanges of wheels have conoidal surfaces and there are therefore no definite points on their surfaces between which the gage of the wheels may be measured. The gage is therefore arbitrarily measured between points on the flanges 1/4 in. from (or below) the treads, and this gage-width for the wheels is made 1/4 in. less than the gage-width between rails.

Track Clearances. The spacing between track centers for double track or turnouts depends on the width overall of the cars in use, and also indirectly on the speed, which would affect the clearance. The extreme width of cars out to out varies from 8 ft. to 9 ft. 1 in. This width plus the clearance, which usually varies from 5 to 13 in., equals the distance between track centers, which therefore varies from 8 ft. 6-1/2 in. in Philadelphia to 9 ft. 5 in. in Chicago. When streets are very wide, the clear distance between cars may be increased to 2 ft. For center-pole construction even greater space must be

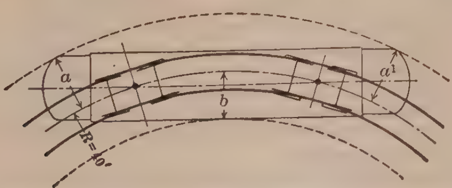


Fig. 51. Clearance for Car on Curve

used and the distance between track centers may be increased to 15 ft. The large cars now commonly used overhang the track so much when going around sharp curves that the standard clearance between the tracks on the tangents is too narrow to permit the passing of two cars on sharp curves. It is frequently impossible to increase the track clearance so that the largest cars may pass on the curves, and it is then necessary to require one car to wait for another and not permit them to turn on the curve simultaneously. The amount of added clearance depends on whether the car is mounted on single or double trucks, its length (including the possible fender at either end) and the spacing between the center pins of the trucks. The simplest rule for determining the extra width between track centers is to draw at a proper scale the curves at the desired radius, and draw a plan view of a car together with its wheel base, somewhat as shown in Fig. 51. The excess over the width of the car of the sum of the distances (*a* and *b* in Fig. 51) from the track center to the extreme projecting points of the car on both sides is the required excess in track clearance. When no excess clearance is provided, both tracks have a common center of curvature. By using curves with different centers of curvature and with suitable radii, the desired extra clearance may be obtained. Since the projection of the car over the rails varies with each change of curvature, the necessary combination of curves to provide sufficient clearance for a given type of rolling stock must usually be determined by trial.

Transition Curves or Spirals should be used at the beginning and ending of all curves. The spirals for easy curves should be those already developed for steam railway work. The curves around street corners are usually so sharp that spirals having more rapid changes of curvature must be used. The manufacturers of " special work " now use spirals not only in constructing approaches to simple curves but also for branch-offs, Y's, etc. The design of such spiral work may be left to them by furnishing them with the following items: (a) total central angle of curve, (b) total required clearance at center of curve between curb and nearest rail or distance from vertex to inner rail, (c) distance from rails to curb in each street.

The American Electric Railway Engineering Association has recommended a system of spirals especially designed for the sharp curves of street railway work, which has been generally adopted. In principle, it is the Searles Railroad Spiral, of multiform compound curves, but using much shorter chord lengths. Characteristics of these curves are quoted as follows:

" All spirals are similar to Searles Spirals and consist of compound curves composed of a series of circular arcs of equal length.

" Rate of change of central angles of arcs is uniformly one degree.

" Corresponding angles and angular functions of all spirals are the same.

" Corresponding lineal dimensions of all spirals are proportional in their respective arc lengths.

" For convenience the base spiral has arc lengths of 10.0 ft.

" The number of any spiral is the length in feet and tenths of its individual circular arcs measured on the center line of standard gage track.

" C = central angle for any arc. S = total central angle of spiral.

" A = arc lengths. R = radius for any arc = $360\ A/2\ \pi C = 57.2958\ A/C$.

" C and S are the same for corresponding arcs of all spirals.

" A is constant for each spiral and its center line length in feet and tenths is the same as the spiral number.

" The standard spirals are numbers 4.0, 4.5, 5.3, 6.5, 8.5 and 10.0.

The coordinates for the commonly used " 5.3 plain end spiral " are tabulated herewith.

Radius	Angle A	X	Y	S°	Total
R_1 = 301.313	1° 00′	0.046	5.259	1° 00′	5.259
R_2 = 149.480	2° 00′	0.228	10.473	3° 00′	10.477
R_3 = 98.868	3° 00′	0.634	15.633	6° 00′	15.654
R_4 = 73.563	4° 00′	1.349	20.718	10° 00′	20.789
R_5 = 58.379	5° 00′	2.451	25.690	15° 00′	25.884
R_6 = 48.257	6° 00′	4.012	30.494	21° 00′	30.939
R_7 = 41.027	7° 00′	6.089	35.052	28° 00′	35.950
R_8 = 35.604	8° 00′	8.721	39.265	36° 00′	40.912

Fig. 51, which is drawn to scale, graphically illustrates the form of one of the spirals using six chords. The spiral commences at B' (or B) with a radius of 301.313 ft. At the end of such a curve 5.259 ft. long, the curve compounds into a curve with 149.480-ft. radius, and so on to the end of the spiral at the point C'. At the other end of the uniform circular curve (C), the spiral is repeated in inverse order.

Each system of chord lengths (5.3, 6.5, etc.) has a corresponding " switch easement " system " which will permit the conversion of a plain curve into a

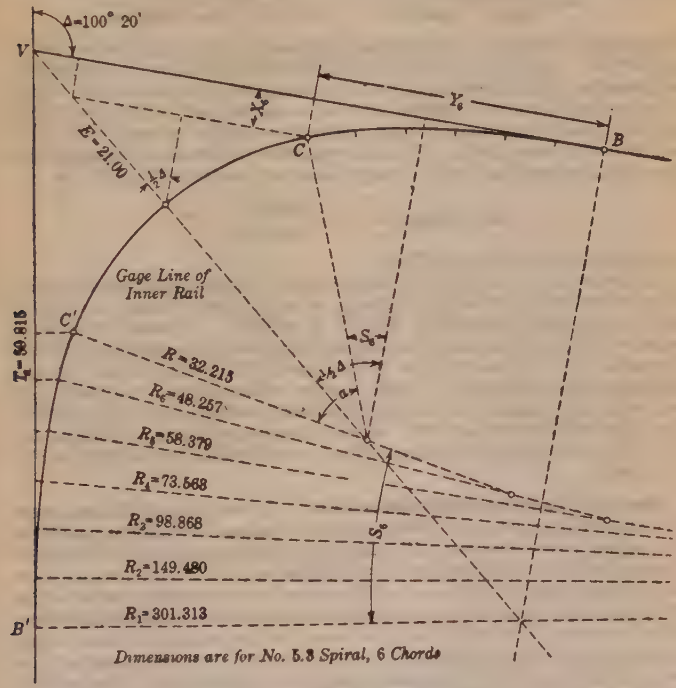


Fig. 52

branch-off with a minimum disturbance of existing work." The total angle S of the spiral is the same and the sharper-curved part is identical, but the flatter part of the spiral is modified so that a switch may be subsequently introduced with a minimum change in trackage.

A 25-page book of tables gives the ordinates and other measurements for the various spirals and their corresponding switch easements.

To Select a Curve with Spirals. Given the central angle Δ and the desired external distance E from the intersection of the tangents to the desired middle point of the inner rail curve. As a trial $R' = E/\text{exsec } 1/2\,\Delta$, which gives the radius of a simple curve through that point and connecting the tangents. The radius R for the central part of the curve must be somewhat less than R'

and also somewhat less than the radius of the last chord of the spiral which it is proposed to use. For example, with $\Delta = 100° 20'$ and $E = 21'$, $R' = 37.424$. (Note that R, E and R' here refer to the gage line of the inner rail and not to the track center line.) The proper spiral must be selected by trial, perhaps after several computations. The six-chord spiral, No. 5.3, shown in the tabular form and Fig. 52, has for ordinates at the end of the spiral, $X = 4.012$; $Y = 30.494$; and $S = 21° 0'$.

$$\mathbf{R} = \frac{\mathbf{E \cos 1/2\,\Delta - X}}{\mathbf{\cos S - \cos 1/2\,\Delta}} = \frac{\mathbf{E \cos 1/2\,\Delta - X}}{\mathbf{2 \sin (S + 1/4\,\alpha) \sin 1/4\,\alpha}}$$

The central part of the curve, with radius R, has a central angle of $\alpha = \Delta - 2S = 100° 20' - 42° 0' = 58° 20'$. Solving the above equation, $R = 32.215$ ft., which is less than the radius of the sixth chord, which is 48.257 ft. Similar computations for a seven-chord, and for an eight-chord, spiral would show them as possible solutions, but the radius R would be reduced to 30.375 ft. and 28.084 ft. respectively. The easier radius would evidently be preferable. The distance from the vertex V to the point of tangency B (and B') is

$$\mathbf{T_s = X \tan 1/2\,\Delta + R \frac{\sin 1/2\,\alpha}{\cos 1/2\,\Delta} + Y}$$

which on the above basis, = 59.815 ft.

As another example, where the central angle is comparatively small, assume a curve in a track following a highway, where the central angle $\Delta = 30° 20'$ and the maximum permissible distance $E = 6.5$ ft. A simple curve (inner rail curve) through the desired middle point and joining the tangents will have the radius $R' = E/\text{exsec } 1/2\,\Delta = 6.5$ ft./exsec $15° 10' = 180.11$ ft. R must be somewhat less than this. Using the first three chords of a 10.0-ft. plain end spiral, the ordinates of the end of the third chord point are $X = 1.210$ ft. and $Y = 29.715$ ft. $S° = 6° 0'$. Solving the above equation for R, we find $R = 171.73$ ft., which is less than 188.632 ft., the radius of the third chord of the spiral.

Spacing of Turnouts; Single-Track Road. Economical and efficient operation absolutely requires that turnouts shall be located as nearly as possible at the points where the cars will naturally meet (according to the system adopted) without requiring one car to wait for another at any meeting point. Before these meeting points can be determined, the maximum number of cars to be operated on one section and their effective schedule time must be known. When there are unusually steep grades on the line which will cause certain sections to be run more slowly than others, allowance must be made by a corresponding shortening of the sections having the steep grades. The necessity for slow speed through village streets and the possibility of comparatively high speed through the open country or on private right-of-way, and even excessive curvature, will have substantially the same effect in modifying the distance between adjacents turnouts. The principles of spacing turnouts may be best stated by an example. Assume a road 6 miles long, operating six cars; assume maximum velocity of 20 miles per hour with stops averaging 800 ft. apart. Assume that the grades are equivalent to an average of 2%, but that there is a half-mile stretch of 6%. The energy which will move the car at 20 miles per hour on a 2% grade will move it only 9 miles per hour on the 6% grade. The half mile will require 200 sec., which at 20 miles per hour would run the car 1-1/9 miles or 5867 ft. Therefore that half-mile stretch must be increased to 5867 ft. in Fig. 53, and the virtual length of the line must be considered as increased by 5867 − 2640 = 3227 ft. = 0.61 mile, or that the line is virtually

6.61 miles long. On the basis of 10-sec. stops every 800 ft., 20 mph. maximum velocity, and an acceleration (and deceleration) of 1.25 mphps., the mean velocity will be 10.2 miles per hour and the entire 6.61 miles will be run in 38.9 min. With an allowance of say 6 min. for terminal wait, the run must be practically on 45-min. schedule. Lay off on profile or cross-section paper, at suitable scales, times as vertical ordinates and distances as horizontal spaces. The 2640 ft. of 6% grade is expanded to the 5867 ft. of virtual distance requiring the same time interval. Two inclined lines represent the course of

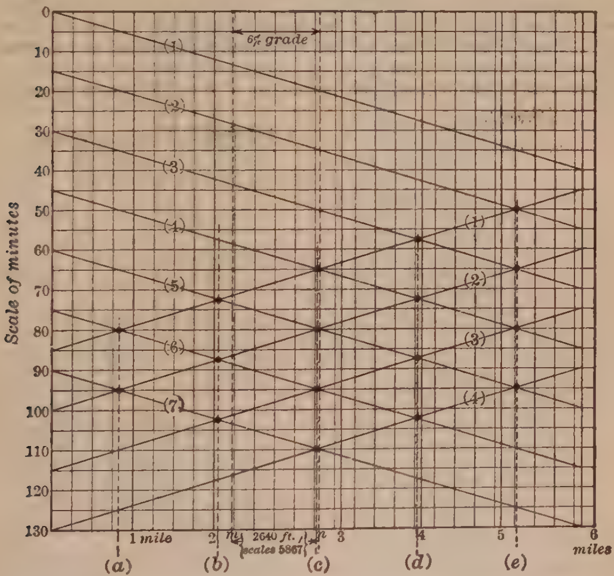


Fig. 53. Chart to Determine Location of Turnouts or Passing Points

the car during the round trip. The six cars run at 15-min. intervals. The location of the meeting places is graphically determined. The location of turnout (*c*) must be determined by scaling 2640/5867 of the space from *m* (or *n*) from the line (*c*). From the above example the following general principles may be determined:

(a) The number of cars is one greater than the number of turnouts.

(b) The running time between consecutive turnouts is one-half the running-time interval between cars.

(c) The time interval between consecutive turnouts is necessarily uniform throughout.

(d) When, as illustrated above, the velocity for any section is necessarily reduced, the spacing must be reduced accordingly.

(e) If variation in traffic density seem to require a temporary or periodical reduction in number of cars, it must be done by cutting out one or more cars from the regular schedule, or perhaps by cutting out every alternate car, and operating the others precisely as before. This merely means that some turnouts are not always utilized.

(f) Since in the example given the lay-over is 6 min. and the time interval between successive car passings at (*a*) is 15 min., the time interval from the terminus to (*a*) is 9 min. and the distance is 0.6 times the normal distance between turnouts. The distance between (*e*) and the other terminal is the same. There are therefore 4 + 0.6 + 0.6 = 5.2 intervals in the virtual distance of 6.61 miles. The interval is therefore 6.61/5.2 = 1.271 miles, = 6711 ft., which is the distance (*a*) to (*b*) and (*d*) to (*e*). If (*b*) to *m* is 920 ft., then *m* to (*c*) is virtually 6711 − 920 = 5791 ft., which multiplied by 2640/5867 = 2606 ft., the actual distance. The spacing from (*b*) to (*c*) is therefore 920 + 2606 = 3526 ft. 5867 − 5791 = 76, which multiplied by 2640/5867 = 34 ft., the true distance from (*c*) to *n*. As a check, 2606 + 34 = 2640. (*a*) and (*e*) are each distant from their termini by 6/10 of 1.271, or 0.76 mile.

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